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Panel Intensity Models with Latent Factors:

An Application to the Trading Dynamics on the Foreign Exchange Market*

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Abstract

We develop a panel intensity model, with a time varying latent factor, which captures the influence of unobserved time effects and allows for correlation across individuals. The model is designed to analyze individual trading behavior on the basis of trading activity datasets, which are characterized by four dimensions: an irregularly-spaced time scale, trading activity types, trading instruments and investors. Our approach extends the stochastic conditional intensity model of Bauwens & Hautsch (2006) to panel duration data.

We show how to estimate the model parameters by a simulated maximum likelihood technique adopting the efficient importance sampling approach of Richard & Zhang (2005).

We provide an application to a trading activity dataset from an internet trading platform in the foreign exchange market and we find support for the presence of behavioral biases and discuss implications for portfolio theory.

JEL classification: $G10, F31, C32$

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1 Introduction

High-frequency data in finance spurred research in financial econometrics in many directions such as modelling of irregularly spaced and discrete data, market microstructure analysis as well as volatility measurement with intradaily data. Recently, even more detailed, so called trading activity datasets, have become available, which contain information on the whole trading history of individual investors in particular markets. These datasets can be characterized through a panel structure with four dimensions: an irregularly-spaced time scale, trading activity types, trading instruments and investors. The richness of information allows us to examine behavioral aspects of trading and investment decisions, as well as to study in detail the trading strategies of investors.

Trading behavior of investors is influenced by a broad set of decision variables. If we were able to observe this complete information set, we could fully characterize the time varying correlation structure across individuals based on this observable information. Individual investment opportunity sets as well as unobservable macroeconomic factors are just two examples of information which is not observed by the econometrician. Such unobservable factors induce a certain correlation structure across individuals which cannot be accounted for by considering only the observable variables. Time varying latent factors can be used to approximate this unobservable information and improve the characterization of the correlation structure in the model.

The aim of this paper is to develop an econometric model which can cope with those characteristics in order to investigate the factors influencing the trading decisions of investors in multiple assets over time and the dynamics of the trading process within a panel intensity model framework which we extend by introducing a dynamic latent factor. This framework allows for a rigorous exploration of financial decision making theories such as rational expectations and behavioral finance theories.

The proposed model can be viewed on the one hand as an extension of the stochastic conditional intensity (SCI) model of Bauwens & Hautsch (2006) to panel data and on the other hand as an augmentation of the class of panel survival models by a latent factor. The intensity based specification is chosen, since it allows us to account for the impact of time-varying covariates on the trading process. The latent factor is assumed to evolve on a pooled arrival process resulting from the aggregation of individual point processes for each investor and trading instrument. We use a simulated maximum likelihood (SML) technique to estimate the proposed model by adjusting the efficient importance sampling method of Richard & Zhang (2005).

The model is used to analyze the trading behavior of retail investors in the foreign exchange market based on a trading activity dataset of OANDA FXTrade, which allows us to trace every action of around 2500 investors in up to 30 currency pairs over the period from 1^{st} October 2003 to 31^{st} October 2003.

The paper is structured as follows: in Section 2 we provide a theoretical description of the model, in Section 3 the SML estimation procedure is presented in detail. Section 4 contains the empirical analysis, and Section 5 concludes.

2 Panel Intensity Model

Let $t \in [0, T]$ denote the physical calendar time, $n = 1, \ldots, N$ denote the n^{th} investor and $k = 1, \ldots, K$ denote the k^{th} currency pair in which an investor can trade. The i^{th} action[1] of the n^{th} investor in the k^{th} currency pair is denoted by $i = 1, \ldots, I^{k,n}$ and the corresponding arrival time is denoted by $t_i^{k,n}$. For all n and all k the sequences $\{t_i^{k,n} | 0 \leq t_{i-1}^{k,n} \leq t_i^{k,n} \leq T; i = 1, \ldots, I^{k,n}\}$ represent point processes with corresponding right-continuous counting processes $N^{k,n}(t) = N^{k,n}([0,t]) = \sum_{i=1}^{I^{k,n}} \mathbb{1}_{\{t_i^{k,n} \leq t\}}$ which count the number of actions in the time interval $[0, t]$. The corresponding left-continuous counting process is denoted by $\breve{N}^{k,n}(t) = N^{k,n}([0,t)) = \sum_{i=1}^{I^{k,n}} \mathbb{1}_{\{t_i^{k,n} < t\}}$. Let $\{\Omega, \mathfrak{F}, \mathfrak{F}_t, \mathcal{P}\}$ denote the associated joint probability space, where the filtrations of the individual processes are denoted by $\mathfrak{F}_t^{k,n} \subset \mathfrak{F}_t$. We assume that each individual point process is orderly (simple), i.e.

$$\mathrm{P}(N^{k,n}(t+\delta) - N^{k,n}(t) > 1 | \mathfrak{F}_t^{k,n}) = o(\delta), \tag{1}$$

where $o(\cdot)$ denotes the little Landau symbol, which ensures that there are no simultaneous arrivals and it implies that $t_{i-1}^{k,n} < t_i^{k,n}$ (almost surely), for $i = 1, \ldots, I^{k,n}$. The inter-event duration between two consecutive actions is denoted by $\tau_i^{k,n} = t_i^{k,n} - t_{i-1}^{k,n}$. By $u^{k,n}(t) = t - t_{\breve{N}^{k,n}(t)}^{k,n}$ we denote the corresponding backward recurrence time at t. For each investor and for each currency pair the arrival times $\{t_i^{k,n} | \, i = 1, \ldots, I^{k,n}\}$ constitute a pooled process, induced by S sub-processes. The corresponding arrival times of the s^{th} sub-process is denoted by $t_i^{s,k,n}$ with $i = 1, \ldots, I^{s,k,n}$. Since the pooled process is orderly the sub-processes are orderly as well. With $N^{s,k,n}(t) = \sum_{i=1}^{I^{s,k,n}} \mathbb{1}_{\{t_i^{s,k,n} \leq t\}}$ being the corresponding counting functions we get that $N^{k,n}(t) = \sum_{s=1}^{S} N^{s,k,n}(t)$. In our application we observe $S = 2$ sub-processes which are:

[1] By action we understand any event that changes the investor's portfolio value. Thus it can be initiated by the investor at that particular time or be a consequence of an earlier activity of the investor, e.g. an executed limit order.

- $s = 1$: The process which is related to an increase in a given currency pair exposure, i.e. the process which characterizes whether a position is (further) opened;

- $s = 2$: The process which is related to a decrease in a given currency pair exposure, i.e. the process which characterizes whether a position is (partly) closed.

The likelihood function of the complete model without a latent factor is given by

$$\mathcal{L}(W;\theta) = \prod_{n=1}^{N}\prod_{k=1}^{K}\left(\prod_{i=1}^{I^{k,n}} f^{k,n}(\tau_i^{k,n} \mid \mathfrak{F}^-_{t_i^{k,n}})\right)^{d_n^k}, \tag{2}$$

where $f^{k,n}(\tau_i^{k,n} \mid \mathfrak{F}^-_{t_i^{k,n}})$ is the conditional density function of the durations. With $\mathfrak{F}^-_{t_i^{k,n}}$ we denote the filtration, which consists of all information up to but excluding time $t_i^{k,n}$. W denotes the generic symbol for all relevant data and θ is the generic symbol for all relevant parameters used in the estimation. By d_n^k we denote the dummy which takes on the value of one if the n^{th} investor is active in currency pair k at least once, and zero otherwise.

We can write the conditional probability of the duration $\tau_i^{k,n}$ between two arbitrary consecutive actions as the conditional probability that all processes have survived during the period $[t_{i-1}^{k,n}, t_i^{k,n})$ times the instantaneous probability for arrival in the next instant $t_i^{k,n}$, which is formally given by

$$\mathrm{P}\left(\tau_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right) = \prod_{s=1}^{S} \bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right)\left(\theta^{s,k,n}\left(t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right)\right)^{d_i^{s,k,n}}, \tag{3}$$

where $d_i^{s,k,n}$ is a dummy, which takes on the value of one whenever the corresponding duration ends with an arrival of type s, and zero otherwise. $\bar{F}^{s,k,n}$ denotes the "survivor" function of the s-type process given by

$$\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right) = \mathrm{P}\left(t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} \notin [t_{i-1}^{k,n}, t_i^{k,n}), t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} = t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right) \tag{4}$$

and

$$\theta^{s,k,n}\left(t_i^{k,n}\middle|\mathfrak{F}^-_{t_i^{k,n}}\right) = \lim_{h\to 0}\frac{\mathrm{P}\left(t_i^{k,n} \le t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} < t_i^{k,n} + h\middle| t^{s,k,n}_{N^{s,k,n}(t_{i-1}^{k,n})+1} \notin [t_{i-1}^{k,n}, t_i^{k,n}), \mathfrak{F}^-_{t_i^{k,n}}\right)}{h} \tag{5}$$

denotes the corresponding intensity of type s. It follows that

$$\begin{aligned} \bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}_{t_i^{k,n}}^-\right) &= \exp\left(-\int\limits_{t_{i-1}^{k,n}}^{t_i^{k,n}} \theta^{s,k,n}(u \mid \mathfrak{F}_u^-)du\right) \\ &= \exp\left(-\Theta^{s,k,n}(t_{i-1}^{k,n}, t_i^{k,n} \mid \mathfrak{F}_{t_i^{k,n}}^-)\right), \end{aligned}$$

where $\Theta^{s,k,n}(t_{i-1}^{k,n}, t_i^{k,n} \mid \mathfrak{F}_{t_i^{s,k,n}}^-)$ denotes the s-type integrated intensity between $t_{i-1}^{k,n}$ and $t_i^{k,n}$. Therefore, the likelihood function of the model without a latent factor in equation (2) can be rewritten as

$$\mathcal{L}(W;\theta) = \prod_{n=1}^{N}\prod_{k=1}^{K}\prod_{i=1}^{I^{k,n}}\prod_{s=1}^{S} \bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}_{t_i^{k,n}}^-\right)\left(\theta^{s,k,n}\left(t_i^{k,n} \middle| \mathfrak{F}_{t_i^{k,n}}^-\right)\right)^{d_i^{s,k,n}}. \tag{6}$$

Since we believe that investors' behavior is influenced by unobservable factors, like an unobservable time effect, we introduce a latent factor denoted by λ_i. To model the dynamic behavior of the latent factor, we need to introduce a time scale over which the latent factor evolves. Therefore, we define the ordered pooled point process as the sequence of arrival times $t_i, i = 1, \dots, I$ for all actions of all investors in all currency pairs, where simultaneous arrivals at the same time are treated as one arrival only, i.e. formally,

$$\begin{aligned} \{t_i | t_{i-1} < t_i\} = &\bigcup_n \left\{\bigcup_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\} \setminus \bigcap_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\}\right\} \setminus \\ &\bigcap_n \left\{\bigcup_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\} \setminus \bigcap_k \{t_i^{k,n} | \, t_{i-1}^{k,n} < t_i^{k,n}\}\right\}. \end{aligned}$$

The corresponding counting processes are denoted by $N(t) = \sum_{i=1}^{I} \mathbb{1}_{\{t_i \leq t\}}$ and $\breve{N}(t) = \sum_{i=1}^{I} \mathbb{1}_{\{t_i < t\}}$. Thus, at $t \in \{t_i\}$ we have $N(t) = \breve{N}(t) + 1$, whereas for $t \notin \{t_i\}$ it holds that $N(t) = \breve{N}(t)$. This pooled process serves as the time scale on which the latent factor evolves. In particular, we assume that the duration $\tau_{N^{k,n}(t)}^{k,n}$ depends on the latent factor, i.e. we assume that $\tau_{N^{k,n}(t)}^{k,n} = \tau_{N^{k,n}(t)}^{k,n}(\lambda_{\breve{N}(t)+1})$ at $t \in \bigcup_n \bigcup_k \{t_i^{k,n}\}$ is a function of the latent factor. Note, that this definition ensures that at every time t where an action occurs there is a corresponding value of the latent factor. Since the latent factor is unobservable and stochastic we need to integrate it out, which results in the following likelihood function

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^I} \prod_{n=1}^{N}\prod_{k=1}^{K}\prod_{i=1}^{I^{k,n}} f^{k,n}(\tau_i^{k,n}, \lambda_{\breve{N}(t_i^{k,n})+1} \mid \mathfrak{F}^-_{t_i^{k,n}}) d\Lambda, \tag{7}$$

where $\Lambda = (\lambda_1, \dots, \lambda_I)'$ and the integral is taken over $\mathbb{R}^I$, and where $f^{k,n}(\tau_i^{k,n}, \lambda_{\breve{N}(t_i^{k,n})+1} \mid \mathfrak{F}^-_{t_i^{k,n}})$ is the joint conditional density of the duration $\tau_i^{k,n}$ and its corresponding latent factor $\lambda_{\breve{N}(t_i^{k,n})+1}$. The likelihood can then be factorized as the product of the density conditional on the latent factor times the conditional density of the latent factor as

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^I} \prod_{n=1}^{N}\prod_{k=1}^{K}\prod_{i=1}^{I^{k,n}}\prod_{s=1}^{S} \bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)$$
$$\left(\theta^{s,k,n}\left(t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)\right)^{d_i^{s,k,n}} \rho(\lambda_{\breve{N}(t_i^{k,n})+1} | \mathfrak{F}^-_{t_i^{k,n}}) d\Lambda, \tag{8}$$

where $\rho(\lambda_{\breve{N}(t_i^{k,n})+1} | \mathfrak{F}^-_{t_i^{k,n}})$ is the conditional density of the latent factor and the exact specification of the intensities and the corresponding integrated intensities is presented below.

The model described by the likelihood function in equation (8) is formulated in terms of $t_i^{k,n}$, which is the pooled (orderly) point process over the S subprocesses of the n^{th} investor in the k^{th} currency pair. As the latent factor which has to be integrated out is defined on t_i, we also provide a reformulation of the model in equation (8) in terms of the pooled times t_i. Since the pooled process may not be orderly there may be several pairs (k, n) associated with the arrival time t_i. We denote the set of such pairs by $\mathcal{C}_i = \{(k,n) | t_i = t^{k,n}_{N^{k,n}(t_i)}\}$. The likelihood in (8) can then be rewritten as

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^I} \prod_{i=1}^{I}\prod_{\mathcal{C}_i}\prod_{s=1}^{S} \bar{F}^{s,k,n}\left(t^{k,n}_{N^{k,n}(t_i)-1}, t^{k,n}_{N^{k,n}(t_i)} \middle| \mathfrak{F}^-_{t_i}, \lambda_i\right)$$
$$\left(\theta^{s,k,n}\left(t^{k,n}_{N^{k,n}(t_i)} \middle| \mathfrak{F}^-_{t_i}, \lambda_i\right)\right)^{d^{s,k,n}_{N^{k,n}(t_i)}} \rho(\lambda_i | \mathfrak{F}^-_{t_i}) d\Lambda. \tag{9}$$

As suggested by the model presentation above there are several ways to model the likelihood function. One can either specify the likelihood function (7) for the durations of the pooled process $t_i^{k,n}$ directly or one can specify the likelihood function (8) based on the intensities of the s sub-processes $t_i^{s,k,n}$ which generate the pooled process $t_i^{k,n}$. Although in different ways, both approaches ultimately allow to make inference about the durations $\tau_i^{k,n}$ of the pooled process.

An attractive feature of the intensity based modelling is that it accounts for changes in the values of time varying covariates during a duration spell in a very intuitive way since it is set up in continuous time. The duration based approach, which is a discrete time model can also account for time varying covariates (e.g. Lunde & Timmermann (2005)), but then the likelihood function has to be additionally adjusted (effectively this again amounts to adjusting the intensity to reflect the changes in the values of the covariates). Furthermore, the intensity based approach allows to characterize the dynamic behavior among the s sub-processes, which is a source of additional information, whereas the duration approach considers the pooled process only. One possibility to model the duration based likelihood (7) is to adopt the stochastic conditional duration (SCD) approach of Bauwens & Veredas (2004), whereas likelihood (8) can be modelled by augmenting the stochastic conditional intensity (SCI) model of Bauwens & Hautsch (2006). We rely on the latter strategy and parameterize $\theta^{s,k,n}(t|\mathfrak{F}_t^-, \lambda_{\breve{N}(t)+1})$ generally in the following way:

$$\theta^{s,k,n}(t|\mathfrak{F}_t^-, \lambda_{\breve{N}(t)+1}) = \left(b^{s,k,n}(t) S^{s,k,n}(t) \Psi^{s,k,n}(t|\mathfrak{F}_t^-)(\lambda_{\breve{N}(t)+1})^{\delta^{s,k,n}}\right) D^{s,k,n}(t). \qquad (10)$$

Thereby $b^{s,k,n}(t)$ denotes a (possibly investor, currency pair or state dependent) baseline intensity, $S^{s,k,n}(t)$ a deterministic seasonality function, $\Psi^{s,k,n}(t|\mathfrak{F}_t^-)$ the intensity component capturing the dynamic information processing and $\delta^{s,k,n}$ is a parameter which controls for the impact of the latent component on the corresponding intensities. In our application we need to take into account that after an action which sets the exposure in a given currency pair to zero, i.e. closes the position completely, there is no possibility for a subsequent close. Hence, the intensity $\theta^{2,k,n}(t \mid \mathfrak{F}_t^-, \lambda_{\breve{N}(t)+1})$ is zero in this case. We model this through the variable

$$D^{s,k,n}(t) = \begin{cases} 1, & \text{if} \quad s = 1 \\ 1 - d_{cc}^{k,n}(t), & \text{if} \quad s = 2, \end{cases} \qquad (11)$$

where $d_{cc}^{k,n}(t)$ denotes the dummy which takes on the value one, if the previous arrival time is associated with a complete close of the position in the given currency pair k for investor n, and zero otherwise. In the following we will parameterize the separate intensity components in a parsimonious way:

Baseline Intensity

We assume that there are different baseline intensities for the different states, but that they are identical across currency pairs and investors, i.e. we assume that

$$b^{s,k,n}(t) = b^s(t) \quad \text{for } k = 1, \ldots, K \text{ and } n = 1, \ldots, N.$$

In our application we use a multivariate Weibull specification of the following form:

$$b^s(t) = \exp(\omega^s) \prod_{r=1}^{S} u^{r,k,n}(t)^{\alpha_r^s - 1} \quad \text{for } s = 1, \ldots, S.$$

Diurnal Seasonality and Weekend Effects

The seasonality function $S^{s,k,n}(t)$ incorporates a diurnal seasonality component $\tilde{S}^{s,k,n}(t)$ and a weekend component $\tilde{W}^{s,k,n}(t)$ multiplicatively as

$$S^{s,k,n}(t) = \tilde{S}^{s,k,n}(t)\tilde{W}^{s,k,n}(t).$$

In order to capture the deterministic intraday seasonality pattern of the intensity processes we assume that

$$\tilde{S}^{s,k,n}(t) = \tilde{S}(t) \quad \text{for } k = 1, \ldots, K \text{ and } n = 1, \ldots, N.$$

where

$$\tilde{S}(t) \equiv \tilde{S}(\nu, \tau, K) \equiv \exp\left(\nu_0 \tau + \sum_{k=1}^{K} \nu_{2k-1} \sin(2\pi(2k-1)\tau) + \nu_{2k} \cos(2\pi(2k)\tau)\right)$$

which is an exponentially transformed Fourier flexible form, where τ denotes the intraday trading time standardized to $[0, 1]$ and ν is a $2K + 1$ dimensional parameter vector.
To model the lower trading activity on weekends in a parsimonious way we specify $\tilde{W}(t)$ as

$$\tilde{W}(t) = \exp(\varpi D_W(t)),$$

where ϖ denotes a scalar and $D_W(t)$ a weekend dummy, which is one during weekends and zero otherwise. According to this specification the intensity process is dampened for $\varpi < 0$, which is the effect that we expect, and amplified for $\varpi > 0$.

Dynamics and Explanatory Variables

The dynamic structure and the influence of the explanatory variables is modelled through $\Psi^{s,k,n}(t|\mathfrak{F}_t^-)$ in the same fashion as suggested by Russell (1999). Let $z_j^{s,k,n}$ denote the vector of all (time-varying) possibly investor, currency pair and state dependent covariates, where at least one covariate is updated at time $\tilde{t}_j^{s,k,n}$ with $j = 1, \ldots J^{s,k,n}$. $\breve{M}^{s,k,n}(t) = \sum_{j=1}^{J^{s,k,n}} \mathbb{1}_{\{\tilde{t}_j^{s,k,n} < t\}}$ is the corresponding left continuous counting function of the update times $\tilde{t}_j^{s,k,n}$. Furthermore, let $\{\check{t}_h^{s,k,n}\}$ denote the process resulting from the pooling of the process $\{t_i\}$ and the covariate process $\{\tilde{t}_j^{s,k,n}\}$, with $H^{s,k,n}(t) = \sum_{h=1}^{H^{s,k,n}} \mathbb{1}_{\{\check{t}_h^{s,k,n} \leq t\}}$ denoting the corresponding right continuous counting function. We assume that

$$\Psi^{s,k,n}(t|\mathfrak{F}_t^-) = \exp\left(\tilde{\Psi}^{s,k,n}_{\check{N}^{k,n}(t)+1} + \left(z^{s,k,n}_{\check{M}^{s,k,n}(t)}\right)' \gamma^{s,k,n}\right).$$

Note, that $\tilde{\Psi}^{s,k,n}_{\cdot}$ is indexed by $\check{N}^{k,n}(t)+1$, which ensures that $\tilde{\Psi}^{s,k,n}_{\cdot}$ is updated with the value of $\tilde{\Psi}_i^{s,k,n}$ directly after but excluding $t_{i-1}^{k,n}$ and stays constant until and including $t_i^{k,n}$. The coefficient vector is denoted by $\gamma^{s,k,n}$. The vector $\tilde{\Psi}_i^{k,n} = (\tilde{\Psi}_i^{1,k,n}, \ldots, \tilde{\Psi}_i^{S,k,n})'$ is parametrized multivariately as

$$\tilde{\Psi}_i^{k,n} = \sum_{s=1}^{S} \left(A^{s,k,n}\varepsilon_{i-1}^{k,n} + B^{k,n}\tilde{\Psi}_{i-1}^{k,n}\right) d_{i-1}^{s,k,n},$$

where $A^{s,k,n}$ is an $S \times 1$ parameter vector and $B^{k,n}$ is an $S \times S$ parameter matrix. The innovation term $\varepsilon_i^{k,n}$ is given by

$$\varepsilon_i^{k,n} = \sum_{s=1}^{S} d_i^{s,k,n}\varepsilon_i^{s,k,n},$$

where

$$\varepsilon_i^{s,k,n} = 1 - \Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\check{N}(t_i^{s,k,n})+1}\right) \tag{12}$$

or

$$\varepsilon_i^{s,k,n} = -0.5772 - \ln\Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\check{N}(t_i^{s,k,n})+1}\right), \tag{13}$$

where the integrated intensity is computed as

$$\Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n}, \mid \mathfrak{F}^-_{t_i^{s,k,n}}\lambda_{\check{N}(t_i^{s,k,n})+1}\right) = \sum_{h=H^{s,k,n}(t_{i-1}^{s,k,n})}^{H^{s,k,n}(t_i^{s,k,n})-1} \int_{\check{t}_h^{s,k,n}}^{\check{t}_{h+1}^{s,k,n}} \theta^{s,k,n}\left(u\middle|\mathfrak{F}_u^-, \lambda_{\check{N}(u)+1}\right) du. \tag{14}$$

Note, that the intensity is integrated between $t_{i-1}^{s,k,n}$ and $t_i^{s,k,n}$ piecewise, where the pieces are determined either by an arrival time t_i, which includes the arrival times $t_i^{k,n}$, or by an arrival time $\tilde{t}_j^{s,k,n}$. The innovation term in equation (12) is defined in that way, since $\Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\breve{N}(t_i^{s,k,n})+1}\right) \sim$ i.i.d. Exp(1) and hence its mean value is 1. Equation (13) uses that $\ln \Theta^{s,k,n}\left(t_{i-1}^{s,k,n}, t_i^{s,k,n} \mid \mathfrak{F}^-_{t_i^{s,k,n}}, \lambda_{\breve{N}(t_i^{s,k,n})+1}\right)$ follows an i.i.d. standard extreme value type I distribution with mean -0.5772.

The survivor function $\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)$ in equation (8) is given by

$$\bar{F}^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right) = \exp\left(-\Theta^{s,k,n}\left(t_{i-1}^{k,n}, t_i^{k,n} \mid \mathfrak{F}^-_{t_i^{k,n}}, \lambda_{\breve{N}(t_i^{k,n})+1}\right)\right),$$

where the integrated intensity is obtained piecewise according to equation (14).

Latent Factor

We assume that the dynamics of the latent factor are defined on the time scale t_i. This means the latent factor changes whenever there is an action of some investor in some currency pair. Since each intensity $\theta^{s,k,n}$ and each integrated intensity $\Theta^{s,k,n}$ depend at every time t on the current value of the latent factor we induce at every time t a contemporaneous correlation between all intensities $\theta^{s,k,n}$ through the latent factor. The magnitude of this possibly investor, currency pair or state dependent correlation is determined by the parameters $\delta^{s,k,n}$. The latent factor therefore can be interpreted as an unobservable time effect which affects the decisions (open, close) of all investors at every time t by influencing the intensities of the corresponding processes. We can justify the existence of such an unobservable time effect in our model in several ways: i) (News) effects of news announcements, not modelled due to data limitations, ii) (Order Flow) buy or sell pressure from the interbank market, which we do not observe directly since we consider an internet trading platform or iii) (Herding) similar behavior of traders, due to similar interpretations of any kind of technical chart patterns.
In our model we assume that the latent factor follows, conditional on $\mathfrak{F}^-_{t_i}$, a lognormal distribution, i.e.

$$\ln \lambda_i | \mathfrak{F}^-_{t_i} \overset{i.i.d.}{\sim} \mathrm{N}(\mu_i, 1)$$

where the dynamics is modelled through an AR(1) process

$$\ln \lambda_i = a \ln \lambda_{i-1} + \epsilon_i \quad \text{for} \quad i = 1, \ldots, I,$$

with $\epsilon_i \overset{i.i.d.}{\sim} \mathrm{N}(0,1)$. Let l_i denote the log of latent factor at t_i, i.e

$$l_i \equiv \ln \lambda_i,$$

and let L_i denote the history of the log latent factor up to and including t_i, i.e.

$$L_i = \{l_j\}_{j=1}^{i}.$$

With this specification, the (log) latent factor depends only on its own past, so we denote its conditional distribution by $p(l_i|L_{i-1})$. From equation (10) it follows that the influence of the log latent factor on the s type intensity is given by $\delta^{s,k,n} \ln \lambda_i$, which we can denote by $\lambda_i^{s,k,n}$. Then we have that

$$\lambda_i^{s,k,n} = a\lambda_{i-1}^{s,k,n} + \delta^{s,k,n}\epsilon_i \quad \text{for} \quad i = 1, \ldots, I.$$

Therefore the variance of ϵ_i is set to unity, so that the conditional variance of $\lambda_i^{s,k,n}$ is equal to $(\delta^{s,k,n})^2$, which eases the interpretation of the parameter.[2]

In order to summarize and visualize the model specification, data characteristics, and the different time scales we depict the stylized panel structure in Figure 1.

[2]Note that this does not preclude that $\delta^{s,k,n}$ could be negative.

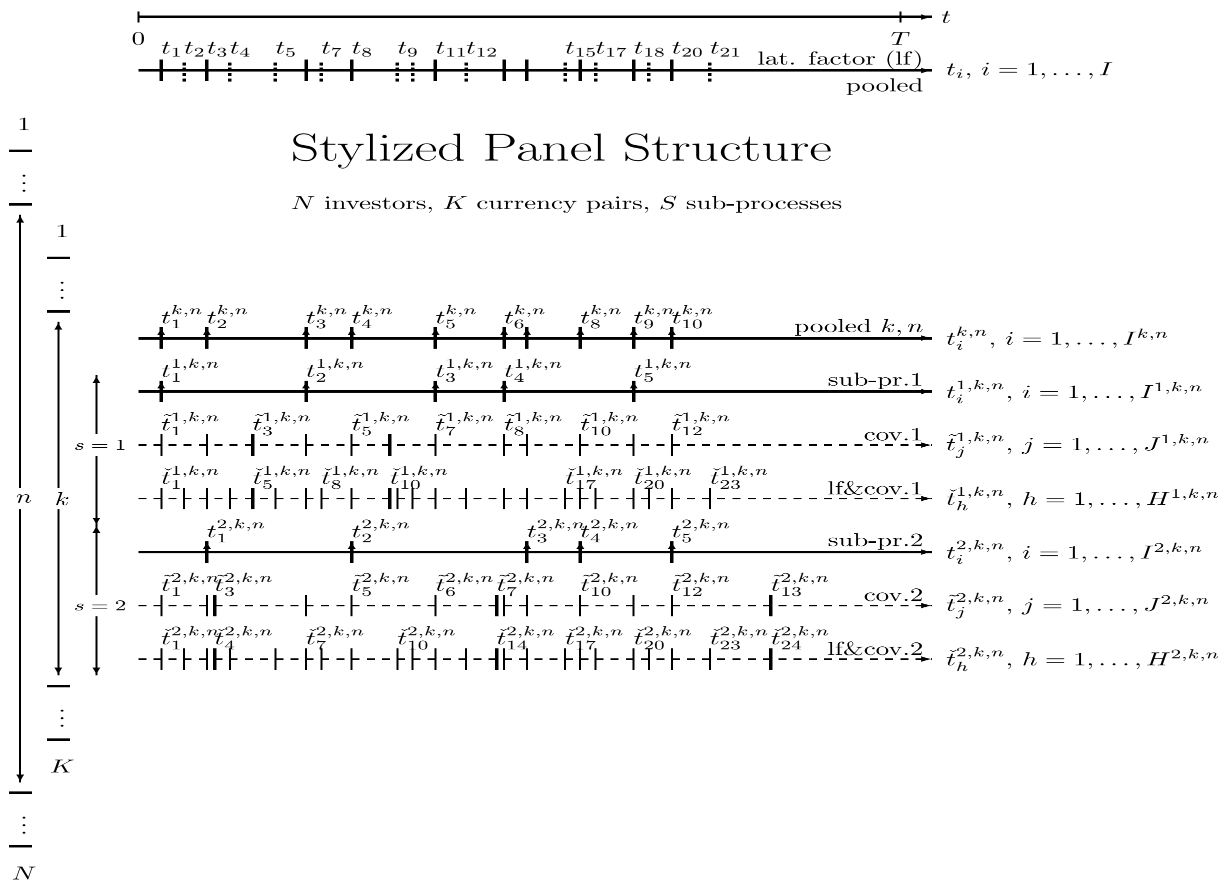

Figure 1: Stylized Model Structure. The figure represents for s=2 the time scales associated with the arrival times of the processes (sub-pr.1 and sub-pr.2), the times of the covariate processes (cov.1 an cov.2) as well as the pooled arrival processes $\check{t}_h^{s,k,n}$ and t_i.

3 Estimation of the Panel Intensity Model

We now consider the explicit form and the estimation of the parameters in the likelihood function. Let W denote the set of data matrices $W^{k,n}$ for each currency pair $k = 1, \ldots, K$ and investor $n = 1, \ldots, N$ where the i^{th} row of $W^{k,n}$, $w_i^{k,n}$, consists of the following data:

$$w_i^{k,n} = (t_i^{k,n}, d_i^{1,k,n}, \ldots, d_i^{S,k,n}), \quad \text{with } i = 1, \ldots, I^{k,n}.$$

With $W_i^{k,n}$ we denote the history of $w_i^{k,n}$ up to and including $t_i^{k,n}$, i.e.

$$W_i^{k,n} = \{w_j^{k,n}\}_{j=1}^{i}.$$

Furthermore, let $\breve{Z}_i^{k,n}$ for $k = 1, \ldots, K$ and $n = 1, \ldots, N$ denote the set which consists of the following time-varying covariate data:

$$\breve{Z}_i^{k,n} = \left\{ \{z_j^{1,k,n} | j = 1, \ldots, \breve{M}^{1,k,n}(t_i^{k,n})\}, \ldots, \{z_j^{S,k,n} | j = 1, \ldots, \breve{M}^{S,k,n}(t_i^{k,n})\} \right\}.$$

Recall that the likelihood function of our model is given by

$$\begin{aligned}
\mathcal{L}(W;\theta) &= \int\limits_{\mathbb{R}^I} \prod_{n=1}^{N} \prod_{k=1}^{K} \prod_{i=1}^{I^{k,n}} \prod_{s=1}^{S} \bar{F}^{s,k,n} \left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}_{t_i^{k,n}}^{-}, \lambda_{\breve{N}(t_i^{k,n})+1} \right) \\
&\quad \left(\theta^{s,k,n} \left(t_i^{k,n} \middle| \mathfrak{F}_{t_i^{k,n}}^{-}, \lambda_{\breve{N}(t_i^{k,n})+1} \right) \right)^{d_i^{s,k,n}} \rho(\lambda_{\breve{N}(t_i^{k,n})+1} | \mathfrak{F}_{t_i^{k,n}}^{-}) d\Lambda \\
&= \int\limits_{\mathbb{R}^{+I}} \prod_{n=1}^{N} \prod_{k=1}^{K} \prod_{i=1}^{I^{k,n}} \prod_{s=1}^{S} \bar{F}^{s,k,n} \left(t_{i-1}^{k,n}, t_i^{k,n} \middle| \mathfrak{F}_{t_i^{k,n}}^{-}, \exp(l_{N(t_i^{k,n})}) \right) \\
&\quad \left(\theta^{s,k,n} \left(t_i^{k,n} \middle| \mathfrak{F}_{t_i^{k,n}}^{-}, \exp(l_{N(t_i^{k,n})}) \right) \right)^{d_i^{s,k,n}} p(l_{N(t_i^{k,n})} | L_{N(t_i^{k,n})-1}) dL \\
&= \int\limits_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} \prod_{s=1}^{S} \bar{F}^{s,k,n} \left(t_{N^{k,n}(t_i)-1}^{k,n}, t_{N^{k,n}(t_i)}^{k,n} \middle| \mathfrak{F}_{t_i}^{-}, l_i \right) \\
&\quad \left(\theta^{s,k,n} \left(t_{N^{k,n}(t_i)}^{k,n} \middle| \mathfrak{F}_{t_i}^{-}, l_i \right) \right)^{d_{N^{k,n}(t_i)}^{s,k,n}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(l_i - \mu_i)^2}{2} \right) dL.
\end{aligned}$$

where $L = \ln \Lambda$ and the second equality follows from a change of the variable λ to l. Using the datasets defined above the likelihood function can be rewritten as

$$\begin{aligned}
\mathcal{L}(W;\theta) &= \int\limits_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} g^{k,n} \left(w_{N^{k,n}(t_i)}^{k,n} | W_{N^{k,n}(t_i)-1}^{k,n}, L_i, \breve{Z}_{N^{k,n}(t_i)}^{k,n} \right) p(l_i | L_{i-1}) dL \\
&= \int\limits_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w_{N^{k,n}(t_i)}^{k,n}, l_i | W_{N^{k,n}(t_i)-1}^{k,n}, L_{i-1}, \breve{Z}_{N^{k,n}(t_i)}^{k,n} \right) dL, \qquad (15)
\end{aligned}$$

where $g^{k,n}$ denotes the product of the survivor and the intensity functions, p the density of the conditional normal distribution and $\varphi^{k,n}$ denotes the resulting corresponding joint conditional density. Since this likelihood involves the computation of an I-dimensional integral, we employ the Efficient Importance Sampling (EIS) technique of Liesenfeld & Richard (2003), which has been used for estimating stochastic conditional intensity models by Bauwens & Hautsch (2006). The EIS technique is based on simulation of the likelihood function (15) which can be rewritten as

$$\mathcal{L}(W;\theta) = \int_{\mathbb{R}^{+I}} \prod_{i=1}^{I} \prod_{\mathcal{C}_i} \frac{\varphi^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}, l_i | W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)}{m(l_i|L_{i-1},\phi_i)}$$

$$\prod_{i=1}^{I} \prod_{\mathcal{C}_i} m(l_i|L_{i-1},\phi_i) dL,$$

where $m(l_i|L_{i-1},\phi_i)$ is a sequence of auxiliary importance samplers which are used to draw a trajectory of the latent factor, given some additional parameters ϕ_i of the sampler. The estimation then proceeds by generating R trajectories of the latent factor and averaging over the draws

$$\mathcal{L}_R(W;\theta) = \frac{1}{R}\sum_{r=1}^{R} \frac{\prod_{i=1}^{I}\prod_{\mathcal{C}_i} \varphi^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L^{(r)}_{i-1}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)}{\prod_{i=1}^{I}\prod_{\mathcal{C}_i} m(l_i^{(r)}|L^{(r)}_{i-1},\phi_i)}, \qquad (16)$$

where the bracketed superscript r indicates the values of the corresponding variable or set for the r-th repetition. The idea of the EIS approach is to find the values of the parameters ϕ_i for $i = 1, \ldots, I$ such that the sampling variance of $\mathcal{L}_R(W;\theta)$ is minimized. For ease of illustration denote the numerator in equation (16) by $\varphi(W, L^{(r)}|\theta) = g(W|L^{(r)},\theta)p(L^{(r)})$, where the generic parameter vector θ appears now, and the denominator by $m(L^{(r)}|\phi)$. A more elaborate presentation can be found in Richard & Zhang (2005). The sampling variance of $\mathcal{L}_R(W;\theta)$ is given by

$$\begin{aligned} \mathrm{V}\left(\mathcal{L}_R(W;\theta)\right) &= \frac{\mathcal{L}(W;\theta)}{R}\frac{1}{\mathcal{L}(W;\theta)}\mathrm{V}\left(\frac{\varphi(W,L^{(r)}|\theta)}{m(L^{(r)}|\phi)}\right) \\ &= \frac{\mathcal{L}(W;\theta)}{R}\frac{1}{\mathcal{L}(W;\theta)}\int_{\mathbb{R}^{+I}}\left(\frac{\varphi(W,L|\theta)}{m(L|\phi)} - \mathcal{L}(W;\theta)\right)^2 m(L|\phi)dL \quad (17) \end{aligned}$$

If we are able to choose ϕ such that $m(L|\phi) = \frac{\varphi(W,L|\theta)}{\mathcal{L}(W;\theta)}$ the sampling variance would be zero. Since this case is very unrealistic the aim is to find ϕ such that $m(L|\phi)$ is very close to $\varphi(W,L|\theta)$ under the restriction that $m(L|\phi)$ is analytically integrable.

Furthermore $m(L|\phi)$ can be decomposed into

$$m(L|\phi) = \frac{k(L,\phi)}{\chi(\phi)} \tag{18}$$

where $k(L,\phi)$ and $\chi(\phi) = \int_{\mathbb{R}^{+I}} k(L,\phi)dL$ can either be interpreted as joint and marginal density or as kernel and integration constant. Defining $d(L;\varphi,\theta)$ as

$$\begin{aligned} d(L;\phi,\theta) &= \ln\left(\frac{\varphi(W,L|\theta)}{\mathcal{L}(W;\theta)m(L|\phi)}\right) && (19)\\ &= \ln(\varphi(W,L|\theta)) - \ln(\mathcal{L}(W;\theta)) - \ln(m(L,\phi)) && (20)\\ &= \ln(\varphi(W,L|\theta)) - \ln(\mathcal{L}(W;\theta)) + \ln(\chi(\phi)) - \ln(k(L,\phi)) && (21) \end{aligned}$$

and defining $h(x)$ as

$$h(x) = \exp(\sqrt{x}) + \exp(-\sqrt{x}) - 2 \tag{22}$$

allows to rewrite equation (17) as

$$\mathrm{V}\left(\mathcal{L}_R(W;\theta)\right) = \frac{\mathcal{L}(W;\theta)}{R}\int\limits_{\mathbb{R}^{+I}} h\left(d(L;\phi,\theta)^2\right)\varphi(W,L|\theta)dL. \tag{23}$$

This equation defines a nonlinear Generalized Least Squares problem in ϕ, since h is monotone and convex on $\mathbb{R}^+$. The power series representation of h is given by

$$h(x) = \sum_{i=1}^{\infty}\frac{x^i}{(2i)!}. \tag{24}$$

Using the series expansion of order one for h, which is $h(x) = x$ equation (23) simplifies to

$$\mathrm{V}\left(\mathcal{L}_R(W;\theta)\right) = \frac{\mathcal{L}(W;\theta)}{R}\int\limits_{\mathbb{R}^{+I}} d(L;\phi,\theta)^2\varphi(W,L|\theta)dL, \tag{25}$$

and the minimization problem becomes

$$\begin{aligned} \hat{\phi}(\theta) &= \underset{\phi}{\operatorname{argmin}}\int\limits_{\mathbb{R}^{+I}} d(L;\phi,\theta)^2\varphi(W,L|\theta)dL \\ &= \underset{\phi}{\operatorname{argmin}}\int\limits_{\mathbb{R}^{+I}} d(L;\phi,\theta)^2 g(W|L,\theta)p(L)dL && (26) \end{aligned}$$

The integral in equation (26) is computed by its Monte Carlo proxy given by

$$\frac{1}{R}\sum_{r=1}^{R} d(L^{(r)};\phi,\theta)^2 g(W|L^{(r)},\theta)$$

where $L^{(r)}$ denote trajectories of length I sampled from the initial sampler p and $\hat{\phi}(\theta)$ is determined based on this approximation. Since the $L^{(r)}$ generate a high variance of g Richard & Zhang (2005) propose to drop the weight function g from the equation and compute $\hat{\phi}(\theta)$ on the basis of the unweighted problem. Therefore the minimization problem is given by

$$\hat{\phi}(\theta) \quad = \quad \underset{\phi}{\operatorname{argmin}} \sum_{r=1}^{R} d(L^{(r)}; \phi, \theta)^2. \tag{27}$$

Writing $d(L^{(r)}; \phi, \theta)$ explicitly yields

$$d(L^{(r)}; \phi, \theta) \\ = \ln \left(\frac{\prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right)}{\prod_{i=1}^{I} \prod_{\mathcal{C}_i} m(l_i^{(r)} | L_{i-1}^{(r)}, \phi_i)} \right) - \ln \left(\mathcal{L}(W; \theta) \right) \tag{28}$$

Substituting

$$m(l_i^{(r)} | L_{i-1}^{(r)}, \phi_i) = \frac{k(L_i^{(r)}, \phi_i)}{\chi(\phi_i, L_{i-1}^{(r)})} \tag{29}$$

yields

$$\begin{aligned} d(L^{(r)}; \phi, \theta) \quad &= \quad \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right) \chi \left(\phi_i, L_{i-1}^{(r)} \right) \right) \\ &\quad - \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} k(L_i^{(r)}, \phi_i) \right) - \ln \left(\mathcal{L}(W; \theta) \right) \\ &= \quad \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right) \chi \left(\phi_{i+1}, L_i^{(r)} \right) \right) \\ &\quad - \ln \left(\prod_{i=1}^{I} \prod_{\mathcal{C}_i} k(L_i^{(r)}, \phi_i) \right) - \ln \left(\mathcal{L}(W; \theta) \right) + \ln \left(\chi \left(\phi_1, L_0^{(r)} \right) \right) \end{aligned}$$

where $\chi \left(\phi_{I+1}, L_I^{(r)} \right) \equiv 1$. The thereto related minimization problem (27) can now be solved sequentially using a backward recursion from $I \to 1$ which yields $\phi = \{\phi_i | i = I, \ldots, 1\}$. The sequential problem consists then at each $i = 1, \ldots, I$ of approximating

$$\ln \left(\prod_{\mathcal{C}_i} \varphi^{k,n} \left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)} | W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)} \right) \chi \left(\phi_{i+1}, L_i^{(r)} \right) \right)$$

by

$$\ln\left(k\left(L_i^{(r)},\phi_i\right)\right).$$

Thus $\hat{\phi}_i(\theta)$ is obtained through

$$\hat{\phi}_i(\theta) = \underset{\phi_i}{\operatorname{argmin}} \sum_{r=1}^{R}\left(\ln\left(\prod_{\mathcal{C}_i}\varphi^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}, l_i^{(r)}|W^{k,n}_{N^{k,n}(t_i)-1}, L_{i-1}^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)\chi\left(\phi_{i+1}, L_i^{(r)}\right)\right)\right.$$
$$\left. - \phi_{0,i} - \ln\left(k\left(L_i^{(r)},\phi_i\right)\right)\right)^2 \tag{30}$$

The additional coefficients $\phi_{0,i}$ are scalars which capture corresponding components of $\ln(\mathcal{L}(W;\theta))$, which are still unobservable. As Liesenfeld & Richard (2003) note, a sensible choice for the class of kernels for the auxiliary samplers m is a parametric extension to the direct samplers p given by

$$k(L_i,\phi_i) = p(l_i|L_{i-1})\zeta(l_i,\phi_i),$$

where ζ is itself a Gaussian density kernel given by

$$\zeta(l_i,\phi_i) = \exp\left(\phi_{1,i}l_i + \phi_{2,i}l_i^2\right).$$

Since a product of normal kernels is a normal kernel as well, we obtain for $k(L_i,\phi_i)$

$$\begin{aligned} k(L_i,\phi_i) &\propto \exp\left((\phi_{1,i}+\mu_i)l_i + \left(\phi_{2,i}-\frac{1}{2}\right)l_i^2 - \frac{1}{2}\mu_i^2\right) \\ &= \exp\left(-\frac{1}{2\pi_i^2}(l_i-\kappa_i)^2\right)\exp\left(\frac{\kappa_i^2}{2\pi_i^2}-\frac{1}{2}\mu_i^2\right), \end{aligned}$$

where

$$\pi_i^2 = (1-2\phi_{2,i})^{-1}, \qquad \text{and} \tag{31}$$
$$\kappa_i = (\phi_{1,i}+\mu_i)\pi_i^2. \tag{32}$$

It follows that

$$\chi(\phi_i, L_{i-1},) = \exp\left(\frac{\kappa_i^2}{2\pi_i^2}-\frac{\mu_i^2}{2}\right). \tag{33}$$

Under this choice of kernels class, $p(l_i|L_{i-1})$ cancels out in the minimization problem (30), which can then be rewritten as

$$\hat{\phi}_i(\theta) = \underset{\phi_i}{\operatorname{argmin}} \sum_{r=1}^{R}\left(\ln\left(\prod_{\mathcal{C}_i}g^{k,n}\left(w^{k,n}_{N^{k,n}(t_i)}|W^{k,n}_{N^{k,n}(t_i)-1}, L_i^{(r)}, \breve{Z}^{k,n}_{N^{k,n}(t_i)}\right)\chi\left(\phi_{i+1}, L_i^{(r)}\right)\right)\right.$$
$$\left. - \phi_{0,i} - \ln\left(\zeta\left(l_i^{(r)},\phi_i\right)\right)\right)^2. \tag{34}$$

The implementation of the sequential ML-EIS approach can be summarized in the following steps:

STEP 1. Draw R trajectories $\{l_i^{(r)}\}_{i=1}^I$ from $\{\mathrm{N}(\mu_i, 1)\}_{i=1}^I$.
STEP 2. For each i with $i : I \to 1$ solve the R-dimensional OLS problem in (34).
STEP 3. Calculate the sequences $\{\pi_i^2\}_{i=1}^I$ and $\{\kappa_i\}_{i=1}^I$ from equations (31) and (32) and draw R trajectories of $\{l_i^{(r)}\}_{i=1}^I$ from $\{\mathrm{N}(\kappa_i, \pi_i^2)\}_{i=1}^I$ to compute the likelihood function given in (16).

4 Empirical Analysis

4.1 Data Description

We analyze an activity dataset of 2120 investors trading on the Internet trading platform OANDA FXTrade for the period from 00:00:00 on the 1^{st} October 2003 until 23:59:59 on the 31^{st} October 2003, which is a total of 31 days[3]. The investors can trade in up to 30 currency pairs, including the most active ones such as EUR/USD, GBP/USD, USD/CHF, EUR/JPY, USD/JPY, etc. Trades can be initiated by market orders, limit orders, stop-loss or take-profit orders. Additionally, a trader can cancel an order, modify an existing limit order or change the stop-loss or take-profit limits. In our analysis we will only consider those actions, which either lead to opening a new position, changing an existing position, or closing a position. Those are market orders, executed limit orders, or executed stop-loss and take-profit orders.
Since the traders on OANDA FXTrade are rather heterogeneous with respect to their trading activity and volume, we classify them into "big", "moderate" and "small" with respect to their total trading volume in USD over the whole period, corresponding to the largest 3%-, middle 3%- and smallest 3%-quantile of the distribution of total trading volume. Additionally, we require that each trader should have at least 30 transactions during the month, resulting in 36 investors for each category. From each group we choose 5 investors randomly for which we estimate the model. Table 1 contains descriptive statistics for the traders in each group.

[3] A detailed description of the dataset is contained in Lechner & Nolte (2005) and Nolte (2006).

	Big	Moderate	Small
Realized Profit (USD)	38.9T	-69.3	0.93
Median Transaction Volume (USD)	325.3T	2800.1	4.1
Maximum Transaction Volume (USD)	1.3M	7768.3	46.4
Total Transaction Volume (USD)	114.5M	405.6T	962.6
Number of Transactions	299.4	240.6	169.8
Number of open	156.0	134.4	103.8
Number of close	143.4	106.2	66.0
Number of full close (thereof)	87.4	63.4	45.4

Table 1: Descriptive statistics for small, moderate and big investors. All figures are averages over the 5 investors within each group. All currency values have been converted to USD. M $\triangleq$ Million and T $\triangleq$ Thousand.

Although we observe large differences with respect to trading volume (total as well as per trade), the trading activity does not differ so much. The average number of transactions corresponds to 9.6, 7.7 and 5.5 trades per day on average for the big, moderate and small investors, respectively.

In addition to the activity data set from OANDA FXTrade we include in our analysis the bid-ask spreads for each of the 30 currency pairs from the interbank market, which are provided by Olsen Financial Technologies. As a further descriptive tool to analyze the deterministic intradaily trading patterns we estimate a Nadaraya-Watson kernel regression separately for opening and closing trades. To check if there are differences across traders located in different areas we separate the traders into three groups – America, Europe, and Asia as follows: traders with accounting currency USD or CAD (America), traders with accounting currency EUR, CHF or GBP (Europe), and traders with accounting currency JPY or AUD (Asia).

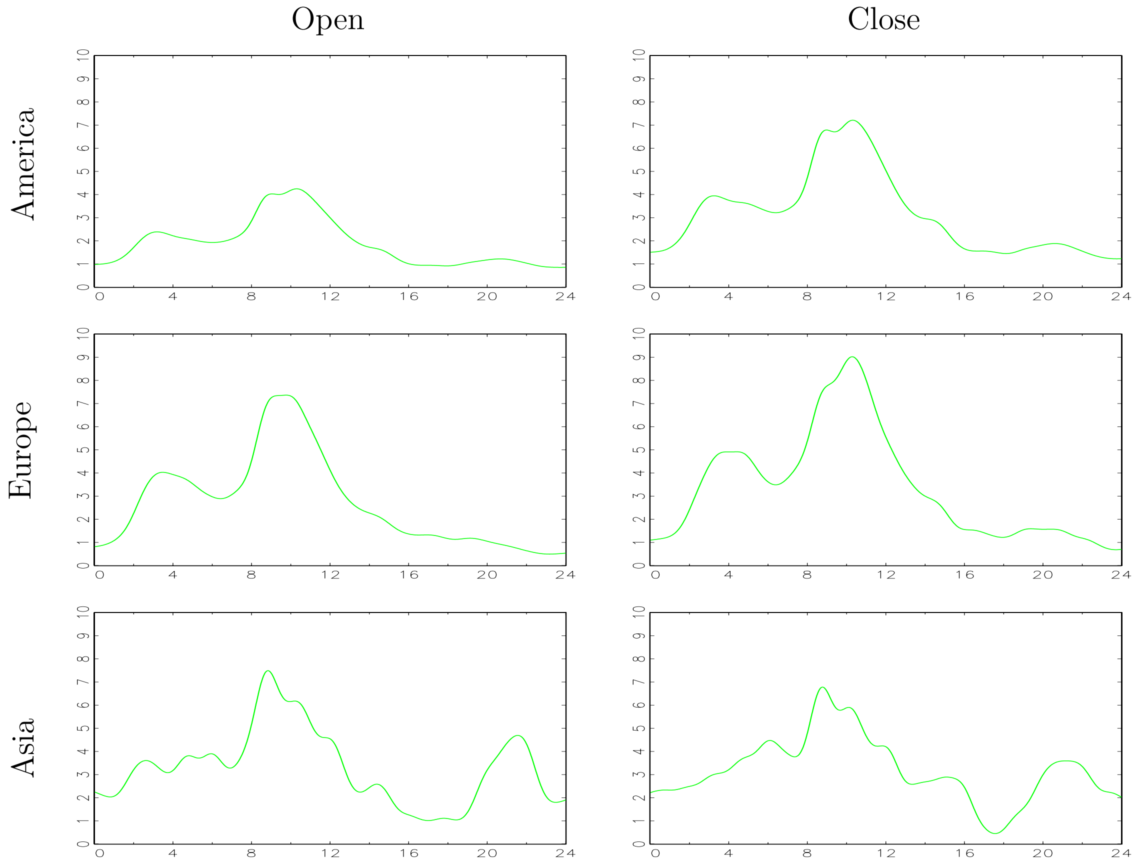

Figure 2: Seasonality patterns of trader activity. Traders are assigned to each group according to their accounting currency: USD and CAD – America, EUR, CHF and GBP – Europe, and JPY and AUD – Asia. The x-axis denotes time of day in Eastern Standard Time. Each function is estimated by Nadaraya-Watson kernel regression with 1440 nodes (24 hours × 60 minutes).

It is evident from the figure that the diurnal seasonality pattern is similar across traders and transaction type (open or close). One general pattern emerges among all traders: a pronounced peak in activity from 8:30 to 10:00 EST and a minor peak at around 3:00 – 4:00 EST, which corresponds to 8:00 – 9:00 GMT. The peak at 23:00 EST for the traders with JPY or AUD as an accounting currency coincides with the after-lunch time in Tokyo (13:00). This pronounced similarity in the seasonality among all traders led us to use a common seasonality component in the intensity specification.

4.2 Estimation Results

This section presents the estimation results of our intensity-based model and reports some model diagnostics. First, we report some properties of the "raw" interevent duration data, which we will compare to the properties of the residuals from the model. In an intensity-based framework, the integrated intensities (see equation (14)) can be considered as generalized residuals which under the correct model specification should be i.i.d exponentially distributed with mean 1.

In Table 2 we report on the parameter estimates and their standard errors for the three groups of investors. The number of observations for each model is the number of pooled events over all currency pairs and investors, i.e., the dimension I of the latent factor vector. For each investor category (small to big) we have 813, 1181, and 1473 observations, respectively.

We have grouped the estimates into several categories: baseline intensity, latent factor, seasonality, dynamics and covariates. The covariates correspond to observable variables in the traders' information set, which can vary during the interevent durations. In our specification we include a news dummy, the bid-ask spread on the interbank market, the current paper profit/loss in the currency pair, and the paper profit/loss in the portfolio of open positions into both the opening and closing intensity processes. The news indicator is based on data from Reuters and Money Market Services which collect survey data on expectations for the development of leading macroeconomic indicators. The news dummy is constructed as an indicator of surprise, which takes on the value of 1 when the median survey value was lower than the actual announced value, and -1 when the median survey value was higher than the announced value. The bid-ask spread can be regarded as a proxy for market liquidity or uncertainty, so that a larger spread should invoke less activity in both opening and closing positions. The variables which measure the paper profit/loss in the given position and the total portfolio can be used to investigate the disposition effect, as well as to study whether investment decisions are made based on portfolio considerations or only on the profit/loss in the single currency position. The disposition effect (Shefrin & Statman (1985)) describes the tendency to hold positions with a paper loss longer than positions with the symmetric paper profit. The disposition effect is considered as a behavioral bias and Shapira & Venezia (2001), Dhar & Zhu (2002), and Chen, Kim, Nofsinger & Rui (2004) among others, show that professional and more sophisticated investors are less prone to the disposition effect and to behavioral biases in general. Although most of the empirical evidence is based on single position considerations, we also investigate it with respect to the paper profit/loss of the total portfolio and we expect the disposition effect to play a smaller role for big investors in comparison to small investors.

Parameter	Small Investors Estimate	Std.	Moderate Investors Estimate	Std.	Big Investors Estimate	Std.
			Baseline Intensity			
ω^o	-3.9185***	0.3481	-2.5985***	0.2736	-2.1907***	0.1261
α_o^o	0.5146***	0.0200	0.8436***	0.0266	0.8595***	0.0256
α_c^o	0.9862***	0.0324	0.5417***	0.0234	0.4848***	0.0208
ω^c	-3.1793***	0.4430	-3.1023***	0.3647	-2.4077***	0.1345
α_o^c	0.7653***	0.0390	0.9438***	0.0326	0.6862***	0.0238
α_c^c	0.7683***	0.0455	0.6635***	0.0340	0.8132***	0.0268
			Latent Factor			
a	-0.0660	0.3540	-0.9711***	0.0110	-0.9277***	0.0052
δ^o	-0.3282***	0.0467	0.3940***	0.0247	0.4602***	0.0244
δ^c	0.4802***	0.1278	0.0596	0.1853	-0.0938	0.0645
			Dynamics			
A_o^o	0.1216***	0.0361	0.1042**	0.0424	0.0292	0.0231
A_c^o	-0.0620*	0.0354	-0.0034	0.0319	-0.0375***	0.0098
A_o^c	0.0507	0.0403	0.0556	0.0381	0.0837*	0.0486
A_c^c	0.1601***	0.0492	0.1119***	0.0203	-0.1151***	0.0309
$B_{o,o}$	0.9781***	0.0162	0.8050***	0.0451	-0.2921	0.4183
$B_{c,c}$	0.9946***	0.0208	0.9986***	0.0051	-0.9034***	0.0291
			Seasonality			
ν_0	-0.3538	0.2848	0.1945	0.4363	-0.3552***	0.1282
ν_1	-0.2373***	0.0584	-0.2223***	0.0711	-0.1799***	0.0424
ν_2	-0.1006*	0.0575	0.2700***	0.0673	0.0509	0.0383
ν_3	0.2415**	0.1130	0.5578***	0.1665	0.0156	0.0262
ν_4	-0.1443**	0.0686	-0.1293	0.0885	-0.1170***	0.0406
ϖ	-1.8941***	0.2694	-2.9254***	0.4821	-2.0883***	0.2916
			Covariates			
γ^o_{news}	2.2484***	0.6160	-1.2592	2.7157	-0.3757	1.3417
γ^o_{spread}	-2.9344	1.9705	-8.4875***	3.1980	-6.8857***	1.9379
$\gamma^o_{\text{P/L 1}}$	-0.0279	0.0252	-0.0849***	0.0288	-0.0584***	0.0109
$\gamma^o_{\text{P/L pf}}$	0.1069	0.0701	0.0094	0.1341	-0.0216	0.0390
γ^c_{news}	2.5533***	0.8074	0.5776	0.6199	-0.6108	0.6746
γ^c_{spread}	-7.7845*	4.4347	-9.4089**	3.8779	-8.5294***	2.1863
$\gamma^c_{\text{P/L 1}}$	0.0894***	0.0167	0.0799***	0.0083	-0.0405***	0.0072
$\gamma^c_{\text{P/L pf}}$	0.0968	0.1089	-0.1098***	0.0233	0.1192**	0.0585

Table 2: Estimation results. The $\gamma^{\cdot}_{\cdot}$ coefficients on the covariates should be interpreted as follows: superscript "o" for opening intensity, superscript "c" for closing intensity. The subscripts stand for the corresponding variable, where "news" is the news dummy, "spread" is the bid-ask spread in the interbank market, "P/L 1" is the paper profit/loss in the corresponding currency pair, and "P/L pf" is the paper profit/loss in the total portfolio. All other coefficients are detailed in the main text. Quasi-maximum likelihood standard errors reported.
***,**, and * denote significance at the 1%, 5% and 10% level, respectively.

The coefficients for the baseline intensity for all three groups result in a monotonically decreasing intensity, which implies that, ceteris paribus, the longer the periods of no activity, the lower is the instantaneous probability for an open or close trade.
The autoregressive parameter of the latent factor "a" is significant only for the big and moderate investors. A possible explanation for this fact, is that the set of observable variables and the dynamics specification is sufficient to capture the heterogeneity and the true trading dynamics among the small investors, whereas for big and moderate investors the observable variables alone are inadequate to explain the more complex correlation structure, so that we additionally require at least an AR(1) process in the latent factor.This explanation is further supported by the model evaluation analysis presented below. The autoregressive parameter is negative which corresponds to an alternating open-close trading pattern.
For the big and moderate investors we observe that the latent factor influences the opening intensity through the coefficient δ^o, while the coefficient δ^c is insignificant. This can be interpreted in connection with the significance of the covariates coefficients, since we observe that for the closing intensity process most of the observable covariates play a significant role, whereas for the opening intensity process only a few of them have a significant influence. Thus, we can conclude that the observable covariates capture the dynamics of the closing intensity process sufficiently, whereas they are inadequate to characterize the dynamics of the opening intensity completely, which necessitates the existence of the latent factor.
The autoregressive parameters in the matrices A and B vary considerably across investor groups which also underly the different trading dynamics. The model diagnostics which we report later reveals that the dynamics have been captured reasonably well within each group.

The shape of the seasonality pattern corresponds closely to the one resulting from the Nadaraya-Watson kernel regression and we refrain from plotting it again. The weekend dummy is significantly negative for all three groups which is in line with the lower trading activity during the weekends.

The news indicator is only significant for small investors, which might be caused by the fact that these investors are less sophisticated than moderate and big ones, so that they rely instead of on own private experience, possibly generated through their own trading strategies and their current portfolio state, on common public information when opening or closing positions.

This explanation can be underpinned by the observation, that contrary to small investors the moderate and big investors, have a pronounced aversion to trade when the spread is large, which is again a sign for more sophisticated and careful trading strategies.
Additionally, when moderate and big investors have an open position generating profits, the probability of increasing the exposure (further open) declines, since $\gamma^o_{\mathrm{P/L\ 1}}$ is significantly negative. The explanation for this effect can be that the investors follow contrarian strategies, rather than momentum strategies and hence do not buy/sell when the price rises/falls.

The decision to close a position is influenced much stronger by the observed information set. For all investors most explanatory variables (except the news) are highly significant. The effect of a large spread is much stronger here compared to the same effect on the opening intensity. Although one could attribute the trading cost in terms of half the spread to both the opening and the closing trade, it is evident that the traders are much more sensitive when this cost is actually paid by a closing trade.
The parameter $\gamma^c_{\mathrm{P/L\ 1}}$ can be interpreted in light of the disposition effect. A positive sign corresponds to an increasing closing intensity as the profit of the single currency position grows, and decreasing closing intensity as the loss grows, which exactly describes the disposition effect. We observe positive signs and similar magnitudes for this coefficient for the small and moderate investors, while for the big investors we have an inverse disposition effect of a much smaller magnitude based on the single position profit/loss. Thus, as expected, one could conclude that larger (and possibly more sophisticated) investors are less prone to behavioral biases.
Additionally, we have the impact of the total portfolio profit/loss on the opening and closing intensities, captured in the parameter $\gamma^c_{\mathrm{P/L\ pf}}$, which is highly significant for moderate and big investors, but insignificant for small ones. This observation can again be attributed to the level of sophistication between the three groups of investors. Whereas big and moderate investors rely on complex trading decisions, meaning considering both the single position and the portfolio profit/loss, the small investors base their closing decision only on the single position profit/loss, which is again a sign for being less sophisticated or narrow framed.
The sign of $\gamma^c_{\mathrm{P/L\ pf}}$, however, is negative for moderate investors but positive for big ones, which in combination with $\gamma^c_{\mathrm{P/L\ 1}}$ for the single position profit/loss paves the way for an interesting interpretation. $\gamma^c_{\mathrm{P/L\ pf}}$ implies an inverse disposition effect for moderate investors but a disposition effect for big investors based on the portfolio profit/loss.

Confessing and assuming that moderate and even big investors are subject to an overall disposition effect, which may be smaller in magnitude for big ones, we can conclude that big investors, although taking the single position profit/loss into account, decide on closing their positions primarily based on portfolio considerations and are being affected here by the disposition effect. On the contrary, moderate investors take the portfolio profit/loss into account, but focus primarily on the single position profit/loss when closing positions, since they are affected there by the disposition effect. To conclude the argument, small investors ignore the portfolio profit/loss completely and consider only the single position profit/loss and are of course prone to the disposition effect as well.

In general, it is important to note that the portfolio profit/loss matters when a decision about closing a single position is made. This finding has implications for the investigations of behavioral aspects of trading. In particular, focusing only on the impact of single positions on the trading decisions could be insufficient. Conversely, in classical portfolio theory (Markowitz (1952), Sharpe (1964), Lintner (1965), and Elton, Gruber & Brown (2006)), trading decisions are only based on the portfolio of assets. In a simple mean-variance framework, portfolio weights are adjusted to meet some mean-variance trade-off for the whole portfolio. While in our analysis, the portfolio of currency positions plays a role, our findings show that investors tend to view their single positions in isolation as well when deciding to close a position.

We evaluate the model statistically by means of goodness-of-fit diagnostics, which are given in Table 3. Although we specified the model ad hoc, without an initial model selection analysis, the proposed specification fits the underlying data generating process quite well. The mean and the standard deviation of the generalized residuals are close to 1 for all three specifications and the QQ-plots in Figure 3 show that they are nearly exponentially distributed, except for extreme values. To test the i.i.d. assumption we apply the Ljung-Box test and the Brock, Dechert & Scheinkman (1987) (BDS) test. We observe, that the Ljung-Box statistics of the generalized residual series decreased considerably in comparison to those of the raw data series. The same observation also holds for the BDS test, which is not only a test for uncorrelatedness but rather a test for i.i.d.ness.

	Small Investors		Moderate Investors		Big Investors	
	Raw Series	Resid.	Raw Series	Resid.	Raw Series	Resid.
Mean	2448.5	0.9740	979.07	1.0510	944.63	1.0567
Std	3877.0	0.9417	3286.7	1.2103	3386.2	1.0610
LB(20)	277.61	25.938	358.51	32.842	356.74	50.597
LB(50)	456.87	52.781	491.12	71.962	380.99	99.538
BDS(m=2)	9.9766	1.1283	12.202	0.5545	16.965	1.5331
BDS(m=3)	10.105	0.0671	11.073	0.0863	18.046	1.6848
BDS(m=4)	10.816	-0.2172	11.359	1.0605	19.333	2.3731

Table 3: Diagnostics for the raw and the residual series. Both series are pooled series over sub-processes, currency pairs and investors. LB ≜ Ljung-Box test statistic, BDS(m=embedding dimension) ≜ Brock-Dechert-Scheinkman test statistic.

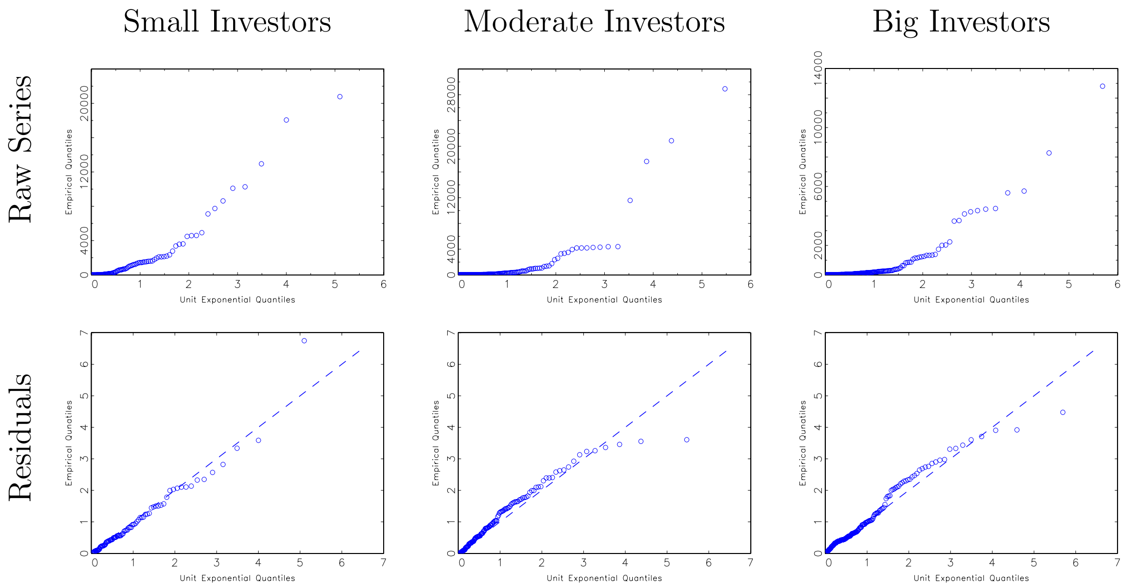

Figure 3: Quantile-Quantile plots of raw and residual series against unit exponential distribution.

5 Conclusion

In this paper we propose an econometric model for the analysis of trading activity datasets. Such datasets contain very detailed information about the trading history of single traders, and provide even more insights into the market microstructure and investors' trading behavior which goes beyond the information contained in typical high-frequency datasets. From an econometric point of view, analyzing activity datasets is rather challenging, as they can be considered as a panel data with irregularly spaced observations with four dimensions: time, type of trading activity, trading instruments, and investors. The model developed in the paper, is suited to cope with this data structure.

A particularity of our approach is the presence of a latent time-varying factor which is responsible to capture hidden correlation structures, not accounted for by observable variables. In this aspect, our specification can be seen as an extension to the class of stochastic conditional intensity models to panel data. Alternatively, our model can be regarded as an augmentation of the panel duration models by a latent factor. The intensity-based framework is suitable to capture the impact of time-varying covariates on the underlying processes.

We show how to adjust the efficient importance sampling algorithm of Richard & Zhang (2005) in order to estimate the model by a simulated maximum likelihood technique. As an application the model is estimated for a trading activity dataset from OANDA FXTrade. Due to the investor heterogeneity, we classify the traders into three groups according to their trading volume and study them separately. Considering the behavioral finance aspects, we find that larger, and therefore probably more sophisticated investors, are less affected by behavioral biases as the disposition effect. Furthermore, our results have implications for classical portfolio theory, as we obtain that traders pay close attention to their single positions within the portfolio when they make an investment decision.

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