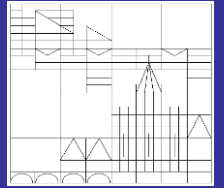




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Optimal Contracting with Reciprocal Agents in a Competitive Search Model

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Abstract

The presented paper offers a simple search model of the labor market to explain the empirical findings on the role of reciprocity for labor market outcomes as reported by Dohmen et al. (2009). In an agency setting where profit-maximizing firms compete for heterogeneous reciprocal workers, with full information about workers' types, reciprocal workers who are willing to engage in gift exchange are approached by more firms, get higher wages and exert higher efforts than selfish workers.

Keywords: reciprocity, gift exchange, competitive search equilibrium, optimal contracts, wage differentials, unemployment

JEL classification: D03, D21, E24, J31, J64

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1 Motivation

Dohmen et al. (2009) analyze survey data and relate measures of reciprocity to actual and future labor market outcomes. They show that positively reciprocal workers are associated with higher wages, higher efforts, and a higher probability to be employed compared to selfish workers. To explain these empirical findings I investigate identical profit-maximizing firms that engage in competitive search for workers with heterogeneous reciprocal preferences. I assume that there is perfect information on worker types. The corresponding labor market is based on Moen (1997) and characterized by search frictions. However, while Moen (1997) assumes homogeneous workers and exogenous productivity differentials among firms which results in workers building queues for best paid jobs, in my model the firms line up for the desired workers and thus determine the probability to fill the vacancy.

To the best of my knowledge, the presented model is the first attempt to introduce social preferences as well as optimal linear incentive contracts into a competitive search model. My approach based on *ex ante* homogeneous firms that adjust the provision of incentives according to the workers' *ex ante* heterogeneous reciprocity concerns is different to Moen and Rosén (2011) who first investigated linear incentive contracts within a competitive search model. In their setting, output is match-specific due to an exogenous stochastic matching term and firms use wage contracts to attract and motivate *ex ante* identical workers. Moreover, workers' heterogeneous social preferences as modeled here allow for output differentials driven by the workers' responsiveness to incentives. This represents a different explanation for productivity differentials compared to traditional labor search models (see, e.g., Inderst (2005), Albrecht and Axell (1984) or Rogerson et al. (2005) for an overview).

2 Model Setup

2.1 Production and Labor Market

Consider a labor market with $n \geq 2$ different types of workers. Workers have heterogeneous preferences for gift exchange captured by a publicly observable reciprocity concern $\rho_i \in [0, 1]^1$ with $i = \{0, R_1, \dots, R_{n-1}\}$. Workers are either selfish, i.e., $\rho_S = 0$, or reciprocal, i.e., $\rho_{R_1}, \dots, \rho_{R_{n-1}} \in (0, 1)^2$, and can be ordered according to their reciprocity concerns such that $\rho_{R_{n-1}} > \dots > \rho_{R_1} > \rho_S$.

By participating in labor market search, firms consisting of one job want to be matched to a worker to produce a verifiable output flow denoted by Y which is either high or low, i.e., $Y \in \{0, 1\}$. The probability for $Y = 1$ is given by $Pr[Y = 1 | e_i] = e_i$ with $e_i \in [0, 1]$ as the worker's unobservable effort which comes at a cost

¹The reciprocity parameter ρ_i has an upper bound to avoid very high reciprocal utilities and to ensure the concavity of the problem.

²In Section 3 I integrate the subscripts $i = R_1, \dots, R_{n-1}$ for reciprocal workers to one subscript R .

$$c(e_i) = \frac{1}{2}e_i^2.$$

The firm offers a contract (α_i, β_i) that consists of a fixed payment α_i and a bonus $\beta_i \in [0, 1]$ whenever the worker realizes a high output flow $Y = 1$. The corresponding expected wage is given by $w_i(e_i) = \alpha_i + \beta_i e_i$ and generates an expected utility flow $\Omega_i(w_i)$ when employed.³ Workers have a reservation utility of zero and are protected by limited liability, i.e., the actual wage must be non-negative, which prescribes $\alpha_i \geq 0$ (LL₁) and $\alpha_i + \beta_i e_i \geq 0$ (LL₂). Moreover, both, firms and workers, are risk neutral.

Public information concerning worker types allows firms to approach workers directly. The labor market can thus be divided into n different sub-markets indexed by $i = \{S, R_1, \dots, R_{n-1}\}$ where a worker of type ρ_i can never enter a sub-market $j \neq i$.

Firms have to take on a cost $k \geq 0$ to open a vacancy in a sub-market i , i.e., to approach workers of type ρ_i . To capture frictions in the labor market, let the matching function $x(u_i, v_i)$ describe the flow of new worker-firm matches in each sub-market i . It depends on u_i , the measure of unemployed workers and on v_i , the measure of vacancies in a sub-market i . Further, $x(u_i, v_i)$ is concave and homogeneous of degree one in (u_i, v_i) and matches $x(u_i, v_i)$ are separated at an exogenous rate s .

Let θ_i denote the corresponding market tightness $\frac{v_i}{u_i}$ in a sub-market i . Then $p(\theta_i) = \frac{x(u_i, v_i)}{u_i} = x(1, \theta_i)$ denotes the job-finding-rate of a worker in a sub-market i and is characterized by $\lim_{\theta_i \rightarrow 0} p(\theta_i) = 0$ and $\lim_{\theta_i \rightarrow \infty} p(\theta_i) = \infty$ with $\frac{\partial p(\theta_i)}{\partial \theta_i} > 0$. As the market tightness of sub-market i decrease, indicating that the number of vacancies v_i decreases compared to the number of unemployed workers u_i , it takes infinitely long for a worker of type ρ_i to find a job. Similarly, the job-filling-rate in a sub-market i is given by $q(\theta_i) = \frac{x(u_i, v_i)}{v_i} = x(\frac{1}{\theta_i}, 1)$ with $\lim_{\theta_i \rightarrow 0} q(\theta_i) = \infty$ and $\lim_{\theta_i \rightarrow \infty} q(\theta_i) = 0$ with $\frac{\partial q(\theta_i)}{\partial \theta_i} < 0$.

Both, firms and workers, are assumed to be price takers, i.e., they choose their actions given the expected wages w_i . The exact timing of the model is as follows: First, given α_i and β_i , firms decide whether to participate in the labor market and which sub-market to enter. Then, workers who are matched to firms according to $x(u_i, v_i)$ within a sub-market i start production by choosing effort e_i at cost $c(e_i)$. Finally, outputs are realized, wages are paid, and existing matches separate at the rate s .

2.2 Asset Value Equations

Workers' asset values $r\mathcal{W}_i = \Omega_i(w_i) - s(\mathcal{W}_i - \mathcal{U}_i)$ and $r\mathcal{U}_i = p(\theta_i)(\mathcal{W}_i - \mathcal{U}_i)$ capture the workers' discounted values of employment and unemployment, respectively, and are set in continuous time with the discount factor r . The discounted value of employment $r\mathcal{W}_i$ consists of $\Omega_i(w_i)$, the utility flow of a worker of type i who is employed at a wage w_i net of $s(\mathcal{W}_i - \mathcal{U}_i)$ as the expected loss if his match is separated. In the absence of unemployment benefits the discounted value of unemployment $r\mathcal{U}_i$ is the expected gain from participating in

³The corresponding utility functions are defined in Section 3.1.

job search, i.e., $p(\theta_i)(\mathcal{W}_i - \mathcal{U}_i)$. The information of both asset values can be captured by a single equation by substituting out \mathcal{W}_i .

Lemma 1. *The worker's asset value equation is given by $r\mathcal{U}_i = \frac{p(\theta_i)\Omega_i(w_i)}{p(\theta_i)+r+s}$.*

Similarly, firms' asset values $r\mathcal{J}_i = e_i - w_i - s(\mathcal{J}_i - \mathcal{V}_i)$ and $r\mathcal{V}_i = -k + q(\theta_i)(\mathcal{J}_i - \mathcal{V}_i)$ describe firms' discounted values of a filled job and a vacancy, respectively. More specifically, the asset value of a filled job $r\mathcal{J}_i$ is a flow of expected payoff net of wage costs $e_i - w_i$ less an expected loss of $s(\mathcal{J}_i - \mathcal{V}_i)$ if the match is separated. In contrast, a vacancy in sub-market i generates an asset value of $r\mathcal{V}_i$ that consists of the cost k and the gain from a filled job if the firm is matched with a worker.⁴ Again, the two asset value equations can be summarized to obtain a single equation by substituting out \mathcal{J}_i .

Lemma 2. *The firm's asset value equation is given by $r\mathcal{V}_i = \frac{q(\theta_i)[e_i - w_i] - (r+s)k}{q(\theta_i)+r+s}$.*

3 Optimal Contracts and Worker Types

The worker chooses his effort to maximize the expected gain from labor market search, i.e., $\max_{e_i} r\mathcal{U}_i$, which boils down to $\max_{e_i} \Omega_i(w_i)$.

3.1 Selfish Workers

As a benchmark, first consider selfish workers who are characterized by $\rho_S = 0$. They choose their efforts to maximize their monetary payoff $\Omega_S(w_S) = \alpha_S + \beta_S e_S - c(e_S)$. The incentive compatibility constraint is given by $e_S = \beta_S$ (IC_S). Limited liability implies $\alpha_S \geq 0$ (LL_1) and $\alpha_S + \beta_S e_S \geq 0$ (LL_2). In addition, the participation constraint prescribes $\Omega_S(w_S) \geq 0$ (PC_S) but since LL_1 , LL_2 , and IC_S ensure PC_S , PC_S has not to be considered explicitly.

The firm chooses (α_S, β_S) to maximize its expected income from labor market search. The corresponding problem is

$$\max_{\alpha_S, \beta_S} \Pi_S = e_S(1 - \beta_S) - \alpha_S$$

$$\text{s.t. } IC_S, LL_1, LL_2.$$

Lemma 3. *The firm offers $\alpha_S^* = 0$ and $\beta_S^* = \frac{1}{2}$ to a selfish worker.*

⁴Note that a firm will enter a sub-market i only if $\frac{e_i - w_i}{r+s} \geq k$.

3.2 Reciprocal Workers

When employed, a reciprocal worker characterized by $\rho_R \in (0, 1)$ has an expected utility flow

$$\begin{aligned} \Omega_R(w_R) = & \alpha_R + \beta_R e_R - c(e_R) \\ & + \rho_R [\alpha_R + \beta_R e_S^* - \alpha_S^* - \beta_S^* e_S^*] \left[c'(e_R) - \frac{\partial w_R(e_R)}{\partial e_R} \right]. \end{aligned}$$

The first part of $\Omega_R(w_R)$ is the worker's monetary payoff whereas the second part represents his reciprocal utility with the firm's friendliness multiplied by the worker's friendliness and weighted by ρ_R which measures the importance of reciprocal utility to the worker.⁵

The firm's friendliness $[\alpha_R + \beta_R e_S^* - \alpha_S^* - \beta_S^* e_S^*]$ is positive whenever the the expected wage w_R exceeds w_S^* for a given effort level e_S^* . The worker's friendliness $\left[c'(e_R) - \frac{\partial w_R(e_R)}{\partial e_R} \right]$ is positive whenever the worker increases effort beyond the selfishly optimal amount, i.e., $c'(e_R) > \frac{\partial w_R(e_R)}{\partial e_R}$, implying a monetary loss in form of a suboptimal monetary payoff for the worker. According to this specification, friendly firm behavior results in additional reciprocal utility for the worker if he reciprocates by choosing his effort e_R such that $c'(e_R) > \frac{\partial w_R(e_R)}{\partial e_R}$.⁶

The reciprocal preferences as modeled here are based on Englmaier and Leider (2012) and are related to intention-based reciprocity as modeled by Rabin (1993). The difference is that instead of comparing total payoffs to a reference payoff, in my model, reciprocal workers evaluate offered wages directly. This helps to overcome the charging feature of the former models which originates from the comparison of the actual outcome to a reference outcome where costly gifts decrease the total payoff of the giving party and thus reduce the friendliness of the receiving party although the gift was provided voluntarily.

Given Lemma 3 and $\Omega_R(w_R)$ the corresponding IC_R can be derived as

$$e_R = \begin{cases} \beta_R + \frac{1}{4}\rho_R [4\alpha_R + 2\beta_R - 1] & \text{for } 0 < \beta_R + \frac{1}{4}\rho_R [4\alpha_R + 2\beta_R - 1] < 1 \\ 1 & \text{for } \beta_R + \frac{1}{4}\rho_R [4\alpha_R + 2\beta_R - 1] \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Limited liability prescribes $\alpha_R \geq 0$ (LL₁) and $\alpha_R + \beta_R e_R \geq 0$ (LL₂) and together IC_R , LL₁, and LL₂ ensure PC_R,

⁵ $\rho_R \in (0, 1)$ can be interpreted as the relative weight that the worker puts on reciprocity as compared to his monetary net-payoff.

⁶Note that an offer $w_R = w_S^*$ exactly meets the reciprocal worker's reference point and is thus perceived as neutral implying payoff maximizing behavior.

i.e., $\Omega_R(w_R) \geq 0$. The firm's problem is then

$$\begin{aligned} \max_{\alpha_R, \beta_R} \Pi_R &= e_R(1 - \beta_R) - \alpha_R \\ \text{s.t.} \quad & \text{IC}_R, \text{LL}_1, \text{LL}_2. \end{aligned}$$

Lemma 4. *The firm offers $\alpha_R^* = 0$ and $\beta_R^* = \frac{4+3\rho_R}{8+4\rho_R}$ to a reciprocal worker.*

3.3 Optimal Contracts in the Labor Market

Proposition 1. *Optimal contracts offer $\alpha_i^* = 0$ and $\beta_i^* = \frac{4+3\rho_i}{8+4\rho_i}$, resulting in expected wages $w_i^* = \frac{(4+\rho_i)(4+3\rho_i)}{32(2+\rho_i)}$. They implement efforts $e_i^* = \frac{4+\rho_i}{8}$ and generate expected net profits $\Pi_i^* = \frac{(4+\rho_i)^2}{32(2+\rho_i)}$.*

In equilibrium, the expected wage w_i^* increases in the worker's reciprocity concern ρ_i .⁷ As a result, workers with stronger reciprocal preferences earn higher wages because they are willing to provide higher efforts in return. This is in line with Dohmen et al. (2009) who find that positively reciprocal workers are associated with higher wages and efforts compared to selfish workers.

The benefits from increased gift exchange outweigh the increased wage costs, such that the firm's expected net profits increase in ρ_i . Consequently, firms are especially interested in employing workers with high reciprocity concerns ρ_R which affects the market tightness and the job-filling-rate in the corresponding sub-market.

4 Reciprocity and Unemployment

The number of firms is determined endogenously through free entry implying $r\mathcal{V}_i(\theta_i^*, w_i^*) = 0$. Rearrangement leads to Lemma 5:

Lemma 5. *The equilibrium job filling rate in a sub-market i is given by $q(\theta_i^*) = \frac{32k(2+\rho_i)(r+s)}{(4+\rho_i)^2}$.*

Due to $\frac{\partial q(\theta_i)}{\partial \theta_i} < 0$ the following must hold:

Proposition 2. *The equilibrium market tightness θ_i^* increases in ρ_i .*

The market tightness balances the job filling rate according to the realizable net payoff. As a result, more vacancies are posted in sub-markets with higher worker types.

Corollary 1. *Workers with stronger reciprocal preferences find a job more quickly than workers with weaker reciprocal preferences.*

⁷Accordingly, selfish workers earn the lowest wage implying that the reference point of reciprocal workers corresponds to the lowest wage in the labor market.

This is also supported by Dohmen et al. (2009) where, compared to selfish workers, positively reciprocal workers are associated with a higher probability to be employed at any moment in time. Moreover, Proposition 2 predicts an additional testable connection between reciprocity and unemployment:

Corollary 2. *Long-term unemployment should arise more often among less reciprocal workers.*

5 Conclusion

In a labor market with search frictions where homogeneous profit-maximizing firms can approach heterogeneous reciprocal workers directly due to perfect information (e.g. from screening), reciprocal workers with stronger preferences for gift exchange find a job more quickly, are less often hit by long-term unemployment, get higher wages and exert higher efforts. The reason is that the benefits from increased gift exchange outweigh the increased wage costs, such that the firm's expected net profits rise as workers become more reciprocal.

The more favored the worker type a firm wants to attract, the longer it takes to be successful in hiring this particular worker type due to higher competition. In contrast, less popular worker types can be hired more quickly. As a result, firms face a trade-off between the resources spent on hiring and the "quality" of the hired worker. This implies that whenever a vacancy must be filled quickly and at low costs, hired workers will tend to be of lower "quality".

These findings stress the importance of non-cognitive skills for labor market success. Increased soft skills in form of the willingness to return favors can thus decrease the length of unemployment. Accordingly, soft skills development can represent an important part in the reeducation of unemployed workers.

References

- Albrecht, J. W. and Axell, B. (1984). An Equilibrium Model of Search Unemployment. *Journal of Political Economy*, 92(5):824 – 840.
- Dohmen, T., Falk, A., Huffman, D., and Sunde, U. (2009). Homo Reciprocans: Survey Evidence on Behavioural Outcomes. *Economic Journal*, 119(536):592 – 612.
- Englmaier, F. and Leider, S. (2012). Contractual and Organizational Structure with Reciprocal Agents. *American Economic Journal: Microeconomics*, 4(2):146 – 183.
- Inderst, R. (2005). Competitive Search Markets with Heterogeneous Workers. *European Economic Review*, 49(6):1525 – 1542.
- Moen, E. R. (1997). Competitive Search Equilibrium. *Journal of Political Economy*, 105(2):385 – 411.
- Moen, E. R. and Rosén, A. (2011). Incentives in Competitive Search Equilibrium. *The Review of Economic Studies*, 78(2):733 – 761.
- Rabin, M. (1993). Incorporating Fairness into Game Theory and Economics. *The American Economic Review*, 83(5):1281 – 1302.
- Rogerson, R., Shimer, R., and Wright, R. (2005). Search-Theoretic Models of the Labor Market: A Survey. *Journal of Economic Literature*, 43(4):959 – 988.

Proofs

Proof of Lemma 3 By inserting the corresponding IC into the firm's maximization problem, the problem can be rewritten to

$$\begin{aligned} \max_{\alpha_S, \beta_S} \Pi_S &= \beta_S(1 - \beta_S) - \alpha_S \\ \text{s.t. } \alpha_S &\geq 0, \alpha_S + \beta_S^2 \geq 0. \end{aligned} \quad (1)$$

Since the firm can only reduce its profits by paying a positive fixed wage α_S , it will choose α_S as small as possible, i.e., $\alpha_S^* = 0$. Positive net profits are only ensured under an interior solution for β_S which is characterized by

$$1 - 2\beta_S = 0 \quad (2)$$

and results in $\beta_S^* = \frac{1}{2}$.

Proof of Lemma 4 Assume that the firm wants to implement an interior solution of e_R , i.e., $0 < \beta_R + \frac{1}{4}\rho_R[4\alpha_R + 2\beta_R - 1] < 1$. Then the firm's maximization problem can then be rewritten to

$$\begin{aligned} \max_{\alpha_R, \beta_R} \Pi_R &= (1 - \beta_R) \left(\beta_R + \frac{1}{4}\rho_R[4\alpha_R + 2\beta_R - 1] \right) - \alpha_R \\ \text{s.t. } \alpha_R &\geq 0, \alpha_R + \beta_R \left(\beta_R + \frac{1}{4}\rho_R[4\alpha_R + 2\beta_R - 1] \right) \geq 0 \end{aligned} \quad (3)$$

which boils down to

$$\begin{aligned} \max_{\alpha_R, \beta_R} \Pi_R &= (1 - \beta_R) \left(\beta_R + \frac{1}{4}\rho_R[4\alpha_R + 2\beta_R - 1] \right) - \alpha_R \\ \text{s.t. } \alpha_R &\geq 0 \end{aligned} \quad (4)$$

since $\alpha_R + \beta_R \left(\beta_R + \frac{1}{4}\rho_R[4\alpha_R + 2\beta_R - 1] \right) \geq 0$ is ensured by $0 < \beta_R + \frac{1}{4}\rho_R[4\alpha_R + 2\beta_R - 1] < 1$, $\alpha_R \geq 0$ and $\beta_R \in [0, 1]$.

The derivative regarding α_R is

$$\rho_R(1 - \beta_R) - 1 < 0 \quad (5)$$

for all $\beta_R \in [0, 1]$ and $\rho_R \in (0, 1)$. Consequently, the optimal contract is characterized by $\alpha_R^* = 0$. Moreover, positive net profits are only ensured under an interior solution for β_R which is characterized by

$$1 + \frac{3}{4}\rho_R - \beta_R(2 + \rho_R) \quad (6)$$

and results in $\beta_R^* = \frac{4+3\rho_R}{8+4\rho_R}$. Together α_R^* and β_R^* ensure $0 \leq \beta_R + \frac{1}{4}\rho_R[4\alpha_R + 2\beta_R - 1] \leq 1$ for $\rho_R \in (0, 1)$ and

the highest net profits for the firm, i.e., $\Pi_R^* = \frac{(4+\rho_i)^2}{32(2+\rho_i)}$, compared to the implementation of the corner solutions $e_R = 0$ and $e_R = 1$ which result in net profits $\Pi_R(e_R = 0) = 0$ and $\Pi_R(e_R = 1) = \frac{\rho}{(4+2\rho)}$, respectively.

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