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Diskussionspapiere der DFG-Forschergruppe (Nr.: 3468269275):

Heterogene Arbeit: Positive und Normative Aspekte der Qualifikationsstruktur der Arbeit

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Education, Wage Uncertainty and the ‘Flat Tax Debate’

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Abstract:

Educational risk and wage uncertainty are important features in human capital investment. Therefore, we apply an OLG-model, wherein human capital formation is exposed to idiosyncratic risk. Extending the instruments available for social insurance, a (Norwegian-type) two-bracket progressive labor tax system and education subsidies are shown to enhance the efficiency-insurance trade-off and to increase social welfare compared to a standard Eaton-Rosen-world. Progression is a superior instrument for insurance, and education subsidies are desirable to alleviate efficiency losses. Finally, we link our results to the flat tax debate. Direct tax progression becomes the more likely the more welfare costs are caused by income risk, the less elastic is labor supply and the more education is boosted by education subsidies.

JEL Klassifikation : H21, I2, J2
Schlüsselwörter : progressive labor taxation, human capital investment, education subsidies, idiosyncratic risk
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Education, Wage Uncertainty and the 'Flat Tax Debate' *

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September 23, 2009

Abstract

Educational risk and wage uncertainty are important features in human capital investment. Therefore, we apply an OLG-model, wherein human capital formation is exposed to idiosyncratic risk. Extending the instruments available for social insurance, a (Norwegian-type) two-bracket progressive labor tax system and education subsidies are shown to enhance the efficiency-insurance trade-off and to increase social welfare compared to a standard Eaton-Rosen-world. Progression is a superior instrument for insurance, and education subsidies are desirable to alleviate efficiency losses. Finally, we link our results to the flat tax debate. Direct tax progression becomes the more likely the more welfare costs are caused by income risk, the less elastic is labor supply and the more education is boosted by education subsidies.

JEL Classification: H21, I2, J2

Keywords: progressive labor taxation, human capital investment, education subsidies, idiosyncratic risk

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1 Introduction

As savings in real capital, education and accumulation of human capital is another way to transfer resources into the future. However, as real capital investment, investment into human capital is a decision under uncertainty. The risk effects of human capital investment are manifold: on the one hand, investment in human capital reduces the probability of being unemployed (see, e.g. Chapman, 1993) and can act as a kind of unemployment insurance. On the other hand, education increases the risky components in wage income (see, i.e., Mincer, 1974).[1]

Following the Mincer tradition, educational risk then has a negative feedback effect on investment in education, if risk is increasing with educational investment: unpredictable income risks make risk-averse households demand a risk-premium, thereby causing underinvestment in education and an inefficient resource allocation (see Levhari and Weiss, 1974).

What can be done in order to reduce the effects of educational and wage risk and in mitigating "precautionary underinvestment" in education? In case of educational and wage risks, it is often argued that private insurance contracts cannot be signed due to, e.g., moral hazard or contract incompleteness. However, the government can provide some insurance via labor taxation and has to trade-off this welfare gain against induced distortions.[2]

This paper shows that, even in case of elastic supply of skilled labor, this trade-off can be optimally implemented, if the government can use a progressive labor tax, targeting the source of risk directly. In particular, human capital risk is mirrored in the skill premium, which is then taxed at a surtax rate. In order to alleviate created distortions in educational investment and in labor choice, it is necessary to grant education subsidies. Concerning the latest reforms towards a flat tax system (see section 8), our results raise additional and new doubts against the desirability of such a flat tax. Insuring human capital risk by directly progressive income taxation contradicts the flat tax system, where the labor tax rate is constant.

In short, optimal tax and education policy in case of risky human capital formation requires a combination of progressive taxation and education subsi-

[1] The results by Mincer (1974) have gained renewed support by, e.g., Carneiro et al (2003).

[2] See the following section for a brief overview of the literature on socially insuring educational risk and our paper's relationship to it.

dies. This 'Siamese-Twins-principle' is well-known from Bovenberg and Jacobs (2005), who argue that optimal redistribution via progressive income taxation requires education subsidies in order to mitigate distortions. As it is shown in the present paper, this mechanism can also be applied in case of risky skilled labor income in order to insure against ex-post income inequality *and* in order to mitigate ex-ante underinvestment in education, caused by uncertainty and incomplete insurance markets.[3]

Thus, insurance motives are another argument for progressive (labor) taxation besides redistributional concerns or efficient human capital investment à la Nielsen and Sørensen (1997). Progression allows to target the risky component in labor income directly, whereas tuition fees (or education subsidies) provide an instrument in order to influence education decisions directly. Although each instrument ceteris paribus enforces distortions in educational and labor choice, the combination of both increases the degrees of freedom for the government.

Do educational and wage risk, however, really matter? One can observe (still) increasing wage inequality among workers of different skill grades. This is driven by increasing educational wage differentials, but also by wage differentials between industries and it is – predominantly – driven by skill-biased technological change (Katz and Autor, 1999). Jacobs (2004) supposes that the wage differential will increase even more in the future, due to the growth rate of skilled labor supply falling short of the increase in the demand for skilled workers. In reverse, the increase of skilled wages and the increase of its variance turn into educational risk, because at the time of deciding on educational investments future wages are hardly predictable and graduating failure produces the risk of low incomes. Chen (2008, p. 285) claims that "at least 80% of potential wage inequality is due to uncertainty". Carneiro et al (2003) show for high-school and college graduates that the overwhelming part of variance in returns to education cannot be predicted by students at the time of investment. Moreover, a significant share of about 10% of college graduates suffers from even negative returns on educational investment. Cunha et al. (2005) confirm that a lot of graduates of both high-schools and col-

[3]This is also the major difference to a setting with ex-ante heterogenous households in a deterministic world, as in, e.g., Bovenberg and Jacobs (2005). Both approaches are very similar from an ex-post point of view, but uncertainty causes inefficient investment incentives and differs therefore ex ante.

leges would change their prior decision, based on ex-post knowledge, and that schooling choice is strongly affected by outcome uncertainty.

In a nutshell, education creates several types of risk: students may fail at university, their acquired skills, to handle the latest production technology, may deteriorate due to idiosyncratic (technological) shocks and prospective incomes of skilled workers face a higher variance, because after graduation salaries differ across sectors and professions.[4]

Our approach provides conditions for the 'Siamese-Twins-mechanism' to be optimal as social insurance in such a risky economy. We apply an OLG-model of the Nielsen and Sørensen (1997) type with a (Norwegian-type) two-bracket labor tax and augment the model by risky human capital formation in an educational sector. Hereby, ex-ante homogenous households differ ex-post due to private risk realization and uncertainty in their educational process.

The remainder of the paper is organized as follows: in section 2 we provide a brief survey of related literature, we then present the model in section 3 and we examine household choices in section 4. In section 5 the optimization problem is stated and welfare-maximizing conditions for optimal tax and education policies are derived. In section 6, we analyze the special case of exogenous leisure demand first. In a next step, we generalize the major results to the case of endogenous skilled labor supply in section 7. Section 8 finally links our results to the recent flat tax debate. The paper closes with some conclusions.

2 Relationship to the Literature

The analysis of effects of various kinds of risk on the human capital decision dates back to Levhari and Weiss (1974). However, they neglect the effects of tax instruments on this decision margin. Eaton und Rosen (1980a,b) are the first to show in a seminal paper series that there is a trade-off between distortions in human capital investment and labor supply on the one hand and insurance, provided by proportional income taxation, on the other hand. The main intuition of their results is that the government can diversify idiosyncratic risk at no costs and grant

[4]See Anderberg and Andersson (2003) for another discussion of the various risk aspects.

a deterministic lump-sum transfer. This reduces the income risk. Moreover, they show that in case of risk a proportional income tax can increase human capital investment under some assumptions. Their model is extended for capital income taxation as indirect subsidy on education by Hamilton (1987).[5] Educational policy as direct instrument to mitigate distortions from income taxation is analyzed in Anderberg and Andersson (2003) for centrally decided (compulsory) educational investment and in Jacobs et al (2009b) for education subsidies in a decentralized economy.

Wigger and von Weizsäcker (2001) analyze optimal public insurance against educational risk in a model with two types of individuals and two different states of nature (success and failure in educational investment). In case of unobservable learning effort and of heterogeneity of individuals, they show that full insurance is never optimal, because both of ex-ante moral hazard, i.e., putting too less (learning) effort into education, and of ex-post moral hazard, i.e., being a successful graduate, but mimicking a failed individual and over-consuming leisure. Consequently, any program of financing public education has to leave some risk on behalf of the individuals.

The papers, being closest to the set of governmental instruments in our approach, are, however, García-Peñalosa and Wälde (2000) and Jacobs and van Wijnbergen (2007). The first paper uses an education subsidy and contingent lump-sum (graduate) taxes, which have to be paid, if human capital investment is successful. The latter paper focuses on capital market failure and adverse selection problems in credit financing. Both papers apply a binary model and exclude endogenous labor supply and its distortions, created by graduate taxes. Therefore, the result of optimal income insurance with governmental full equity stakes in human capital returns in Jacobs and van Wijnbergen can be seen as analogon to our result in the (special) case of exogenous leisure demand.

[5] Varian (1980) moreover includes a section on non-linear taxation, but focuses on risky returns in real capital and uses a model, where savings are the only choice variable. Whilst this model is not able to analyze our questions, the basic intuition of progressive taxation as superior social insurance is similar to our approach.

3 The Model

We apply a two-period OLG-model based on Nielsen and Sørensen (1997). Their standard model is augmented by stochastic shocks in the individual human capital formation technology and by an explicitly modeled educational sector, including tuition fees.

We assume a continuum of ex-ante homogenous individuals, whose mass is normalized to unity. The representative individual lives for two periods and is provided with one unit of time in each period. In period 1, the individual invests time e in education and goes to work for the remaining time span. Moreover, the household chooses its first period consumption and, thereby, determines its savings in real capital. In period 2, the individual supplies skilled labor L and consumes leisure $l = 1 - L$.[6] Progressive labor taxation and tuition fees should mainly affect the labor supply of the old-age, skilled households. In order to simplify the analysis, we assume that the young generation either supplies labor $(1 - e)$ or goes to university (e), hence, there is no leisure in the first period. Note that this setting is the most disadvantageous one for arguing in favor of progressive taxation.

In period 1, the individual has an initial human capital stock of 1, only supplying unskilled labor. After investing in education, it acquires a human capital stock $\tilde{H}(e, \theta)$ in period 2,[7] which is a function of time investment e and a random variable θ.

The human capital production technology is supposed to be concave in time effort. Thus, we have a positive, but decreasing marginal productivity, $\tilde{H}_e = \frac{\partial \tilde{H}(e,\theta)}{\partial e} > 0$, $\tilde{H}_{ee} < 0 \quad \forall\theta$. We assume that the marginal productivity of human capital production is large enough at the point of $e = 0$ and is small enough around $e = 1$ to insure an inner solution in the education decision. Moreover, we assume that, in the second period, the household can supply unskilled labor without incurring any risk, if it does not invest in education, $e = 0$. This implies $H(0, \theta) = 1$, independently of the realization of θ.

The random variable θ captures the risk property of human capital formation.

[6] Leisure in the second period can also be interpreted as retirement. Accordingly, labor time L would then represent utilization of human capital.

[7] Variables indicated with a tilde depend on the realization of θ and are stochastic.

It can be, for instance, interpreted as individual ability to learn, which is assumed to be unknown ex-ante; it can mirror sector-specific or technological shocks affecting the utilization of specific human capital; or it can be seen as individual fortune in final exams.[8] Following the Mincer-tradition, we focus throughout the paper on the case that θ has a positive effect on human capital formation, $\frac{\partial H(e,\theta)}{\partial \theta} > 0$. Thus a higher θ can be interpreted, e.g., as a higher realized learning ability or as a better grade in the exam at university, which is relevant for the effective wage as skilled worker. Furthermore, we assume that θ positively effects the marginal productivity of education, $H_{e\theta} > 0$, as well – implying that individuals with higher ability are learning more effectively than less gifted individuals. In other words, $H_{e\theta} > 0$ accords to risk, increasing in schooling. This results in underinvestment in education due to risk, accompanied by a positive risk premium for education, as shown later on.[9] The density function of θ is $f(\theta)$, which is known to the individuals and the government.

The idiosyncratic educational risk realizes at the beginning of period 2, and, depending on its realization of θ, an individual then supplies $\tilde{H}(e,\theta) \cdot L$ units of effective (skilled) labor. Thus, the households are ex-post heterogenous and differ in their human capital stock, due to their different outcomes of educational investment. However, as we assume θ to be an idiosyncratic risk factor, there is no aggregate risk, and in aggregate all stochastic variables will take their expected values. This implies that total human capital stock is deterministic.

The distribution of θ and the human capital formation function are assumed to guarantee that $\tilde{H}(e,\theta) > 1$ for all values of θ, given $e > 0$. Any educational investment increases the human capital stock of a household due to $\tilde{H}_e > 0$, but this increase is in part stochastic, which implies that second period income and consumption will be risky.

Following the major line of the literature, we assume that private insurance against education risk is not available, i.e., insurance markets to insure idiosyn-

[8]The latter argument is based on the idea that better exam grades imply higher wages. However, the success in final exams can be negatively affected, if the student has a bad hair day due to, e.g., illness.

[9]See Levhari and Weiss (1974, pp. 953) and Anderberg and Andersson (2003, pp. 1527) for a related discussion on $H_{e\theta} = \frac{\partial^2 H}{\partial e \partial \theta} \gtrless 0$. Following Levhari and Weiss, the case of $H_{e\theta} > 0$ and, therefore, underinvestment in education strikes us to be more relevant.

cratic labor income risks are incomplete. This can be due to moral hazard, adverse selection and contract incompleteness (e.g., individuals are too young to write binding insurance contracts, when they decide on their human capital investment).[10]

The utility of the individual depends on consumption in the first period C_1 and in the second period C_2, respectively, as well as on second period leisure l. It takes the form

$$U = U(C_1, C_2, l). \tag{1}$$

The utility function is assumed to be twice differentiable in all arguments and marginal utilities are assumed to be positive, but decreasing. Consequently, $U_{C_1},\ U_{C_2},\ U_l\ >\ 0$ and $U_{C_1C_1}, U_{C_2C_2}, U_{ll} <\ 0$, respectively. Thus, marginal utilities of consumption and leisure are decreasing, guaranteeing risk-averse behavior in both periods. Furthermore, we assume the Inada conditions to hold.

Following Nielsen and Sørensen (1997), the economy is a small open economy. The aggregate production function of a homogenous good has constant returns to scale, and the production function can be formulated in intensive form as $y = f(k)$. Hereby y is the output and k represents the physical (or real) capital stock per unit of effective labor. The price of the good is normalized to unity. The world capital market is perfectly integrated, and the real interest rate $r = f'(k)$ is exogenously given from the perspective of the home economy. This also implies that the wage rate per unit of effective labor is determined by $W = f(k) - rk$ and is exogenous as well.

We assume that higher education is acquired at public universities, which are financed by the government. For the sake of simplicity, we assume that universities are a "club good:" Consuming higher education is non-rival, but students can be excluded. The educational sector causes fix costs of $\overline{G}$ and these can either be financed by taxes or by tuition fees.

In our model, we adopt the Norwegian two-bracket labor tax system, which is also used in Nielsen and Sørensen (1997). There is a basic tax rate t_1 for labor income below a threshold value $X \cdot W$. If the household earns more labor income

[10] See, e.g., Eaton and Rosen (1980b), pp. 707, and Sinn (1996) for an overview. Some discussion of this assumption and an opposing view is to be found in Andersson and Konrad (2003).

than the threshold, the entire part of income above this threshold will be liable to a marginal tax rate t_2. Accordingly, $t_2 > t_1$ implies that the labor tax structure is progressive.

The labor income of the individual is equal to $W \cdot (1-e)$ in period 1, and equal to $W \cdot \tilde{H}(e,\theta) \cdot L$ in period 2. It is assumed that $W \cdot (1-e) < WX < W \cdot \tilde{H}(e,\theta) \cdot L \, \forall\theta$ such as t_1 is the marginal tax rate for unskilled workers. t_2 is the marginal tax rate for the skilled.

The assumption also has two other implications. First, even in the worst state of nature, a silled worker is, after realization of risk, more productive than an unskilled worker, who never visited university, and (marginal) return to human capital will be liable to the high-bracket rate t_2. Second, we implicitly assume that the marginal productivity of the first units of time investment in human capital production is high enough to ensure an inner solution of e, avoiding any problems around the kink in the household's budget constraint.

In order to focus on the risk effects of human capital and the insurance property of the labor tax, we want to keep the model as simple as possible and assume that there is no taxation of real capital. An interest tax acts as a subsidy on human capital investment and also calls for either progressive taxation (see Nielsen and Sørensen, 1997) or for reducing pre-loaded education subsidies. As it will turn out later on, a tax on real capital would not change the main results of our model, whereas the analysis became much more complicated.

However, the government can collect tuition fees P_e per semester, spent at university. Each household has to pay $P_e \cdot e$ fees, which turn into a scholarship, if $P_e < 0$, thus if the government decides to implement direct subsidies.

Taken together, we apply the model of Nielsen and Sørensen (1997) and augment it by a risky human capital production and an educational sector, where the government can also use tuition fees. Compared to the standard Eaton-Rosen-world (Eaton and Rosen, 1980a,b), we extend the instruments of the government by a two-bracket tax system and tuition fees. The motivation behind this is that a progressive labor tax is a superior instrument (compared to a proportional tax system) in order to tax and to insure risky returns to human capital, and tuition fees (or subsidies) provide another, direct, instrument in order to control the education decision of households.

4 Individual Optimization

At the beginning of period 1, the individual chooses its educational investment e, its first period consumption C_1 and leisure demand in the second period l, without knowing the outcome of its educational investment. The individual pays tuition fees $P_e \cdot e$, and it forgoes $(1 - t_1)W \cdot e$ labor income. The budget constraint of the individual for the first period is:

$$S = w_1(1 - e) - P_e \cdot e - C_1,$$

whereby $w_1 = (1 - t_1)W$ is the after tax wage rate for unskilled labor income. S stands for savings if positive and private debt if negative.

In period 2, the individual consumes its entire wealth, which is given by the sum of savings in period 1 plus interest and its after tax labor income in period 2, $(1 - t_2)W \cdot \tilde{H}(e, \theta) \cdot L$. Consequently, the consumption in period 2, $\tilde{C}_2$, is equal to

$$\tilde{C}_2 = (1 + r)S + w_2\tilde{H}(e, \theta) \cdot L + (w_1 - w_2)X,$$

with $w_2 = (1 - t_2)W$. Remind that labor income of period 2, $W \cdot \tilde{H}(e, \theta) \cdot L$, is liable to two different tax rates: t_1 is the basic tax rate on labor income $W \cdot X$, and t_2 is levied on the part above the threshold, $W \cdot (\tilde{H}(e, \theta) \cdot L - X)$.

The intertemporal budget constraint can then be written as

$$\tilde{C}_2 = (1 + r)[w_1(1 - e) - P_e \cdot e - C_1] + w_2 \cdot \tilde{H}(e, \theta) \cdot L + (w_1 - w_2)X \quad (2)$$

Due to the risk in the human capital formation process, second period labor income and, therefore, consumption in period 2 are uncertain. The individual only knows the distribution of possible outcomes for any given investment e, when deciding over its consumption, educational investment and leisure demand. Thereby, it maximizes a von Neumann–Morgenstern expected utility function, $\mathrm{E}[U(C_1, \tilde{C}_2, l)]$, where leisure l is given by $l = 1 - L$.

The optimization problem of the individual is:

$$\max_{C_1, e, L} \quad \mathrm{E}[U(C_1, \tilde{C}_2, 1 - L)] \quad \text{s.t.} \quad (2). \quad (3)$$

The first order conditions for the solution of this problem are given by:

$$\mathrm{E}[U_{C_1}] - (1+r)\mathrm{E}[U_{C_2}] = 0 \quad \Rightarrow \quad \frac{\mathrm{E}[U_{C_1}]}{\mathrm{E}[U_{C_2}]} - 1 = r, \tag{4}$$

$$\mathrm{E}[U_{C_2}\{w_2\, \tilde{H}_e\, L - (1+r)(w_1 + P_e)\}] = 0, \tag{5}$$

$$\mathrm{E}[U_l] - w_2 E[U_{C_2}\, \tilde{H}] = 0. \tag{6}$$

Equation (4) states the standard condition that the rate of time preferences must be equal to the real interest rate r. Equation (5) implies that the effort level e is chosen optimally, if risk-adjusted expected marginal productivity and marginal costs of educational investment are equalized. Thereby, costs of educational investment are given by the indirect costs of forgone earnings plus direct costs of tuition fees. Hence, average and marginal costs of investment into education are, in terms of second period consumption, given by $(1 + r)(w_1 + P_e)$.

For optimal choice of leisure in equation (6), the expected marginal utility of leisure should be as large as the risk-adjusted expected marginal costs, represented by reduced consumption. Higher human capital in the second period implies higher opportunity costs of leisure and, accordingly, reduces leisure demand and boosts labor supply.

According to Jacobs (2005), decreasing marginal utilities do not guarantee the second order conditions to be fulfilled, due to a positive feedback effect of educational investment on labor supply. Assuming, however, the cross-effects of changes in marginal utilities to be sufficiently small, the second order conditions are fulfilled, and we can still focus on the first order conditions.[11]

In doing so, we can apply Steiner's rule, $\mathrm{E}[\tilde{X} \cdot \tilde{Y}] = \mathrm{E}[\tilde{X}]\mathrm{E}[\tilde{Y}] + \mathrm{Cov}(\tilde{X}, \tilde{Y})$, and define the negatively normalized covariance between marginal utility of second period consumption and marginal return to education

$$\pi_e = -\frac{\mathrm{Cov}(U_{C_2}, H_e)}{\mathrm{E}[U_{C_2}]\mathrm{E}[H_e]}$$

as the risk premium in educational investment. π_e measures the monetarized utility costs of having a risky instead of a deterministic return to educational invest-

[11] See appendix 10.1 for a formal derivation.

ment. Thus, we receive

$$w_2 L\overline{H}_e - (1+r)(w_1 + P_e) = \frac{\pi_e}{1-\pi_e}(1+r)(w_1 + P_e) > 0 \tag{7}$$

from equation (5), and incomplete insurance markets drive a risk wedge $\frac{\pi_e}{1-\pi_e}(1+r)(w_1+P_e)$ between marginal return to education and its marginal costs.

In a deterministic world, there is no risk premium, implying $\pi_e = 0$, and marginal return in human capital must be equal to marginal return in real capital in order to guarantee a household optimum. Increased labor supply L enhances the utilization of human capital, therefore raising the marginal return in education. Consequently, labor supply has a positive effect on education. In a world where human capital formation is stochastic and $H_{e\theta} > 0$ holds, risk averse individuals, additionally, demand a positive risk premium $\pi_e > 0$ and will end up with socially undesirable underinvestment in human capital: expected marginal return to human capital is larger than marginal costs (and as marginal productivity of physical capital). Underinvestment into education serves as self-insurance device for decreasing the exposure to income risk. Note that the risk wedge is present even in absence of tuition fees and of progressive taxation.

From a closer examination of equation (7) it can be taken that, if tuition fees are absent ($P_e = 0$), a proportional labor tax ($w_1 = w_2$) affects investment in education only indirectly via labor supply and by risk aversion, the latter embedded in the risk premium. For the case of exogenous skilled labor supply, returns and costs of human capital investment are reduced proportionally and there is no distortion in labor supply, as already shown by Eaton and Rosen (1980b).

Optimal consumption and education demand functions can be characterized by $C_1^* = C_1^*(w_1, w_2, P_e, r)$, $e^* = e^*(w_1, w_2, P_e, r)$, $l^* = 1 - L^*(w_1, w_2, P_e, r)$ and indirect utility of an individual results as

$$V = U(C_1^*(w_1, w_2, P_e, r), e^*(w_1, w_2, P_e), L^*(w_1, w_2, P_e, r), \theta) = V(w_1, w_2, P_e, r, \theta).$$

Using the envelope theorem, the derivatives of the indirect utility function w.

r. t. the two after tax wage rates and tuition fees are given by

$$\frac{\partial V}{\partial w_1} = [(1+r)(1-e) + X] \cdot \mathrm{E}[U_{C_2}], \tag{8}$$
$$\frac{\partial V}{\partial w_2} = [(1-\xi)\overline{H}(e) \cdot L - X] \cdot \mathrm{E}[U_{C_2}], \tag{9}$$
$$\frac{\partial V}{\partial P_e} = -(1+r)e \cdot \mathrm{E}[U_{C_2}]. \tag{10}$$

and will be used in the following sections. Thereby, $\overline{H}(e)$ is the expected human capital stock, and $\xi = -\frac{\mathrm{Cov}(U_{C_2},H)}{\mathrm{E}[U_{C_2}]\overline{H}(e)} > 0$ is the insurance characteristic. The latter gives the marginal welfare loss of income risk from stochastic human capital formation, expressed in monetary units. Simultaneously, it represents the government's concern for providing social insurance.

5 Optimal Taxation and Educational Policy

We assume that governmental expenditures for universities are fixed and do not depend on education demand of households. This exogenous public spending is financed by labor tax revenue and tuition fees.

Following Nielsen and Sørensen (1997), the government implements a Pareto-improving tax reform: in order to avoid windfall gains and / or losses, the old generation still faces the old tax rules, but the young and all following generations are liable to the new, post-reform tax parameters. Thus, the government chooses tax rates on labor income and tuition fees in order to maximize the welfare of a representative consumer, subject to the government's budget constraint, but it keeps the utility of the current old generation constant.

This allows to implement the new steady-state tax parameters within one period, but requires a transition scheme in order to fulfill both constraints simultaneously. Such a Pareto-improving mechanism, which does not affect the welfare of the current old, can be achieved by using a one-time debt policy in the transition period.[12]

[12] See Nielsen and Sørensen (1997), pp. 318. The advantage of this approach is that one does not focus on steady-state utility only.

Defining the budget constraint for the transition period and keeping the tax parameters and the stock of debt constant for all following periods, the consolidated intertemporal budget constraint of the government results after some rearrangements as

$$(W - w_1)[(1+r)(1-e)+X] + (W - w_2)(\overline{H} \cdot L - X) + (1+r)P_e \cdot e = \overline{R}, \quad (11)$$

where $\overline{R}$ now is the exogenous public spending minus the yield on tax revenue collected from the old generation during the transition period. As we have assumed only idiosyncratic risk in human capital formation, all risk vanishes in aggregate and total human capital stock in period 2 equals its expected value:

$$\overline{H} = \overline{H}(e) = \mathrm{E}[\tilde{H}(e, \theta)]. \quad (12)$$

Thus, government's revenue is deterministic, because it can diversify educational risk perfectly by pooling (and at no costs) due to the law of large numbers.

The government chooses the optimal tax rates t_1, t_2 (and accordingly after tax wages w_1, w_2) and tuition fees P_e in order to maximize social welfare. The optimization problem of the government is given by:

$$\max_{w_1, w_2, P_e} V(w_1, w_2, P_e, r, \theta) \quad \text{s. t.} \quad (11) \quad (13)$$

The first order conditions for this problem are:

$$\frac{\partial V}{\partial w_1} + \lambda \Bigg\{ -[(1+r)(1-e)+X] + (W-w_2)\overline{H} \cdot \frac{\partial L}{\partial w_1} + [(W-w_2)\overline{H}_e L - (1+r)(W-w_1-P_e)] \frac{\partial e}{\partial w_1} \Bigg\} = 0, \tag{14}$$

$$\frac{\partial V}{\partial w_2} + \lambda \Bigg\{ -(\overline{H}L - X) + (W-w_2)\overline{H} \cdot \frac{\partial L}{\partial w_2} + [(W-w_2)\overline{H}_e L - (1+r)(W-w_1-P_e)] \frac{\partial e}{\partial w_2} \Bigg\} = 0, \tag{15}$$

$$\frac{\partial V}{\partial P_e} + \lambda \Bigg\{ (1+r)e + (W-w_2)\overline{H} \cdot \frac{\partial L}{\partial P_e} + [(W-w_2)\overline{H}_e L - (1+r)(W-w_1-P_e)] \frac{\partial e}{\partial P_e} \Bigg\} = 0 \tag{16}$$

with λ as marginal costs of governmental revenue, and $\overline{H}_e$ as deterministic marginal productivity of educational investment. The latter is equal to the expected value of marginal productivity across all households, $\overline{H}_e = \overline{H}_e(e) = \mathrm{E}[\tilde{H}_e]$.

6 A Special Case: Exogenous Leisure Demand

Before starting the general analysis of optimal education and taxation policies, we first analyze the special case of exogenous skilled labor supply, i.e., $l = 0$ and $L = 1$. This allows to sharpen the intuition for the insurance effect of tax progression.

If leisure is exogenously given, utility only depends on consumption in both periods. Moreover, the threshold value for the upper tax bracket X can be set at $X = 1$ for convenience. First order conditions for household behavior and optimal governmental policy can be inferred from equations (4) – (6), respectively (14) – (16) in the previous sections by substituting $L = 1$ and $X = 1$.

Applying the envelope results (8) to (10) and the covariance rule, combining and rearranging conditions (14) and (15), results in

$$-\xi \overline{H} \cdot N + \Delta_e \cdot \Bigg\{ (\overline{H} - 1)\frac{\partial e}{\partial w_1} - [(1+r)(1-e)+1]\frac{\partial e}{\partial w_2} \Bigg\} = 0. \tag{17}$$

Combining conditions (14) and (16) in the same manner leads to

$$\Delta_e \cdot \left\{(1+r)e\frac{\partial e}{\partial w_1} + [(1+r)(1-e)+1]\frac{\partial e}{\partial P_e}\right\} = 0, \tag{18}$$

whereby $\Delta_e = (W-w_2)\overline{H}_e - (1+r)(W-w_1-P_e)$ is the tax wedge of education, measuring the change in tax revenue, if educational investment increases by one unit. $N = -[(1+r)(1-e)+1] + \Delta_e \cdot \frac{\partial e}{\partial w_1}$ is the tax revenue effect of increasing net unskilled wage w_1.

Substituting some comparative-static results, equations (40) and (41) in appendix 10.2, in (18) and simplifying, we obtain

$$\Delta_e \cdot \frac{(1+r)\alpha \mathrm{E}[U_{C_2}](2+r)}{SOC} = 0, \tag{19}$$

whereby SOC stands for the determinant of the Hessian matrix.

Assuming an inner solution, SOC must be positive. Moreover, the expected marginal utility of second period consumption is positive, and it is that $\alpha = \frac{\partial\{\mathrm{E}[U_{C_1}]-(1+r)\mathrm{E}[U_{C_2}]\}}{\partial C_1} \neq 0$ by assuming the Hessian matrix to be negative definite. Hence, condition (19) can only be fulfilled, if

$$\Delta_e = (W-w_2)\overline{H}_e - (1+r)(W-w_1-P_e) = 0. \tag{20}$$

This implies that educational investment should optimally not be distorted, i.e., it should neither be taxed nor subsidized on a net basis.

From inserting (20) in (17) follows

$$-\xi\overline{H} \cdot N = 0, \tag{21}$$

where N reduces to $-[(1+r)(1-e)+1] < 0$, because the tax base of the basic labor tax rate must be positive as $e \in [0,1]$.

Therefore, optimal tax policy is described by

$$\xi = 0 \quad \Leftrightarrow \quad \mathrm{Cov}(U_{C_2}(\theta), \tilde{H}(\theta)) = 0, \tag{22}$$

which states that the marginal utility of second period consumption should be uncorrelated with risk in human capital formation. This is only the case, if second period consumption does not depend on the human capital stock H, implying $w_2 = 0$ and, consequently, $t_2 = 1$ from equation (2).

The intuition for this result is as follows: given our model and the fact that educational risk is assumed to be idiosyncratic, the government can provide full insurance against income risk by taxing away all (risky) returns to human capital. By the law of large numbers, the risk is entirely diversified in the aggregate budget constraint. Thus, the government can diversify at no costs. However, taxing away the entire skill premium will sweep out any incentive for investing in human capital and, therefore, lead to inefficiency.

This can be avoided by using additionally education subsidies. In our case, the government can control the education decision fully, because it simultaneously provides incentives for educational investment via scholarships, paid out per semester spent at university. This can be seen from substituting $t_2 = 1$ in the first order condition (5) of household optimization, which leads to

$$P_e = -w_1. \tag{23}$$

From (23) and enforcing Inada-conditions on the utility function, $t_1 \geq 1$ would imply $w_1 \leq 0$ and $P_e \geq 0$, which cannot be a social optimum, as household income would be zero or even turn negative and there would be no private consumption. It follows that $t_1 < 1$, and thus direct tax progression $t_2 > t_1$.

Next, by applying $t_2 = 1$ and $P_e = -w_1$, equation (20) simplifies to

$$\overline{H}_e = (1 + r). \tag{24}$$

Hence, optimal tax and education policy guarantee that the socially optimal level of educational investment is reached, and they equate marginal productivities in educational investment and real capital investment. With full insurance and no distortions in educational investment individuals choose the efficient level of education. Moreover, there is no distortion in labor supply, because we have assumed that leisure demand is exogenous. Therefore, the basic tax rate t_1 is (in combination with education subsidies P_e) a lump-sum tax and can be used for balancing the budget. Accordingly, we can simultaneously reach an efficient allocation of resources and full insurance.[13]

Taken together, we can state our first result:

Proposition 1 *If educational risk is idiosyncratic and leisure demand is exogenous, optimal tax and education policy is characterized by full insurance and education subsidies. The government taxes skilled labor income with* $t_2 = 1$ *and subsidizes education directly with negative tuition fees* $P_e = -w_1 < 0$.

Of course, this result is a kind of artefact due to the fact that we have assumed exogenous leisure demand, and educational investment is the only decision margin, which is distorted by labor tax. However, the results show that private (educational) risk itself is not the problem, even if it is uninsurable in the private sector. The result can be seen as an analogon to the results in Jacobs and van Wijnbergen (2007, Propositions 6 and 8), who argue that optimal risk diversification should imply that all human capital investment is financed by an 'equity stake' of the government, which also takes all risky returns.

In the standard risk models, based on the seminal work by Eaton and Rosen (1980a,b), the government can only use a proportional labor tax and lump-sum transfers. Compared to this, extending the governmental instruments by a progressive labor tax and by education subsidies increases the degree of freedom.[14]

The risk is not embedded in labor income, but in the skill premium. This premium can be taxed directly using the two-bracket tax system. This avoids any effect on returns to unskilled (raw) labor supply. Furthermore, education subsidies ($P_e < 0$) allow to control the education decision and guarantee efficiency, although full insurance is provided. If, instead, a proportional labor tax at a tax rate $t = 1$ would be implemented in such a model and all revenue would be returned in a pure lump-sum manner, i.e., there were no education subsidies, there would be neither any incentive for working in the second period, nor for investing in human capital at all. Therefore, it is worthwhile to extend governmental

[13]In fact, individuals are indifferent between any level of educational investment. We assume that they still choose the efficient one.

[14]In addition, the major difference to the non-linear taxation result in Varian (1980) is that tuition fees increase the degree of freedom even more and enable a better alleviation of distortions. Translated into the Varian-setting, this would imply to use investment subsidies additionally, therein.

instruments compared to standard analysis, because this enables the government to handle risk better, without sacrificing (more) efficiency.

Last but not least, Proposition 1 shows that there is another linkage between income taxes and educational systems. Bovenberg and Jacobs (2005) state that progressive taxation and educational subsidies are 'Siamese Twins'. The intuition is that endogenous education decisions increase the elasticity of labor supply and, thus, increase the costs of redistribution by income taxation. This effect can be countered by introducing education subsidies. A similar effect can emerge in Schindler (2009), who introduces tuition fees and an educational sector in the model by Nielsen and Sørensen (1997), where there is a distortionary real capital taxation, which is to be countered by progressive labor taxation.

Proposition 1 now shows that progressive taxation and education subsidies are also 'Siamese Twins' due to risky educational investment. In order to improve the insurance function of income taxation, the government must grant scholarships to students. The argument behind this is again analogous to Bovenberg and Jacobs (2005): education subsidies are needed in order to avoid efficiency losses and to control the education decision.

At first sight and from an ex post perspective, our approach seems to double Bovenberg and Jacobs (2005). However, ex ante, there are major differences between a setting with heterogenous households versus educational risk in incomplete insurance markets: in the latter case the lack of sufficient insurance leads to inefficient underinvestment in education, and taxation is less distortive due to an insurance effect, which mitigates ceteris paribus underinvestment. Nevertheless, it turns out in our analysis that full subsidization of educational investment is necessary in the special case of exogenous labor supply.

Two criticism force on: First, full insurance is supposed to create moral hazard,[15] second, exogenous leisure demand is – as mentioned – unrealistic. In our model, investing less time than socially efficient, is not optimal given the per-semester subsidies. Hence, an individual will not receive transfers, if it does not invest in education and does not pay its taxes on the skill premium. Thus, there is no moral hazard in sense of 'shirking.' However, if the success in human capital

[15] See, e.g., Wigger and von Weizsäcker (2001), who examine public versus private financing of higher education and focus on moral hazard problems in section II.5.

formation does not only depend on the time spent at university, but also on the learning intensity – the way *how* time is spent at university, – the moral hazard problem is re-introduced, and full insurance will not to be optimal.

If leisure choice is not exogenous, the (skilled) wage tax will cause major distortions in labor supply. These cannot be (entirely) avoided by education subsidies. Thus, there emerges a trade-off between the insurance effect of taxation on the one hand, and efficiency losses on the other hand. Although full insurance appears to be unlikely then, the intuition for some progressive taxation (at $t_1 < t_2 < 1$) and education subsidies, $P_e < 0$ should, however, survive. This will be examined in the next section.

7 Educational Risk and Siamese Twins

Turning to the more realistic case of endogenous second period leisure, skilled labor supply will reduce to $L < 1$, and labor taxation will be distortive. We are going to show conditions under which the intuition for progressive taxation and education subsidization can still be generalized.

For being able to reproduce a standard flat tax, let us assume that the government also grants a lump-sum transfer T to individuals. In case of $t_2 = t_1 = t > 0$ and $T > 0$, the tax structure would be fully equivalent to a flat tax regime. If the transfer turns negative, the government instead implements a (state-independent) poll tax $T < 0$.

Accordingly, the governmental budget constraint slightly changes to

$$t_1 \cdot W \cdot [(1+r)(1-e)+X] + t_2 \cdot W \cdot [\overline{H} \cdot L - X] + (1+r) \cdot P_e \cdot e = T + \overline{R}. \quad (25)$$

In order to show the optimality of progressive taxation and education subsidies, we focus on a balanced budget policy reform concerning the instruments t_2 and P_e. Totally differentiating (25) and rearranging gives

$$\left.\frac{dP_e}{dt_2}\right|_{d\overline{R}=0} = \quad (26)$$

$$-\frac{\left[t_2 \cdot W \cdot \overline{H}_e \cdot L - (1+r)(t_1 \cdot W - P_e)\right] \cdot \frac{\partial e}{\partial t_2} + t_2 \cdot W \cdot \overline{H} \cdot \frac{\partial L}{\partial t_2} + W \cdot [\overline{H} \cdot L - X]}{\left[t_2 \cdot W \cdot \overline{H}_e \cdot L - (1+r)(t_1 \cdot W - P_e)\right] \cdot \frac{\partial e}{\partial P_e} + t_2 \cdot W \cdot \overline{H} \cdot \frac{\partial L}{\partial P_e} + (1+r) \cdot e}.$$

If we first look at the case, where the entire public spending is financed by the poll tax, $-T = \overline{R}$ and $t_1 = t_2 = P_e = 0$, the balanced budget condition (26) simplifies to

$$\frac{dP_e}{dt_2}\bigg|_{d\overline{R}=0} = -\frac{W \cdot [\overline{H}(e,\theta) \cdot L - X]}{(1+r) \cdot e}. \tag{27}$$

In this case, introducing a positive surtax rate will implement a progressive tax system, which equals a pure graduate tax. Returning tax revenue as education subsidies, the effect of such a compensated tax reform on social welfare can be calculated from

$$\frac{dV}{dt_2} = \frac{\partial V}{\partial t_2} + \frac{\partial V}{\partial P_e} \cdot \frac{dP_e}{dt_2}\bigg|_{d\overline{R}=0}. \tag{28}$$

Using the Envelope effects in (9) and (10), as well as the simplified balanced-budget effect (27), we infer from (28) at $t_2 = t_1 = P_e = 0$

$$\begin{aligned} \frac{dV}{dt_2}\bigg|_{t_2=t_1=P_e=0} &= W \cdot \left\{ \mathrm{E}[U_{C_2}] \cdot [\overline{H} \cdot L - X] - [(1-\xi)\overline{H} \cdot L - X] \cdot \mathrm{E}[U_{C_2}] \right\} \\ &= \xi \cdot W \cdot \overline{H} \cdot L \cdot \mathrm{E}[U_{C_2}] > 0, \end{aligned} \tag{29}$$

because $\xi > 0$.

Hence, we can conclude:

Proposition 2 *In case of risky human capital formation, it is not optimal to finance the education system by a pure lump-sum tax. Introducing a graduate tax, accompanied by education subsidies, increases social welfare.*

Beginning in an undistorted allocation, progressive labor taxation with $t_2 > t_1 = 0$, which is in fact a graduate tax, insures against income risk and distorts both investment in education and skilled labor supply. These distortions can be countered in part by granting education subsidies. The combination of both distorting instruments increases welfare, because around $t_2 = t_1 = P_e = 0$, the welfare increasing insurance effect is more valuable than the net efficiency losses created by distortions.

Whilst Eaton and Rosen (1980b) show that distortionary labor taxation and lump-sum transfers increase welfare, we show that the combination of two distorting instruments can deliver a welfare increasing insurance effect. The intuition of the inelastic leisure demand case still applies: progressive labor taxation can tackle the risky income base in a better way, and education subsidies are a superior instrument in order to avoid distortions in education.

Analyzing these effects, instead, in an economy, where a flat tax with positive tax rate t is in place, is more realistic, but also much more complicated. The economy is already distorted and increasing the surtax rate t_2 will then amplify these distortions in a non-negligible way. Nevertheless, we are able to derive conditions, for which tax progression is optimal, and we can draw some conclusions on the favorability of a flat tax regime in the following section.

For the analysis to come, we will meet the following assumptions:

Assumption 1 *(i) The Laffer curve concerning tuition fees has a positive derivative around $P_e = 0$, thus $D = \frac{\partial \bar{R}}{\partial P_e}\Big|_{P_e=0} > 0$.*

(ii) The tax base for the surtax encompasses at least the skill premium, accordingly $W \cdot L \geq W \cdot X$.

(iii) Tuition fees have negative effects on uncompensated labor supply and educational investment, $\frac{\partial e}{\partial P_e} < 0$, $\frac{\partial L}{\partial P_e} < 0$.

(iv) Uncompensated labor supply and educational investment depend positively on net skilled wages, hence $\frac{\partial L}{\partial t_2} < 0$, $\frac{\partial e}{\partial t_2} < 0$.

The negative effects of tuition fees and the positive effects of skilled wages on the demand for education are empirically well tested. In Leslie and Brinkman's review (1987) they conclude that the modal result of about 30 empirical studies is a 1.8 enrollment decline per 100 dollar increase in tuition fees. An overview on the literature, analyzing the effects of higher skilled wages on enrollment has been provided by Freeman (1986), showing an elasticity of higher education demand to salaries in a range of 0.5 to 2.0.

The estimates for the uncompensated wage elasticity of labor supply are, instead, mostly very low and sometimes even negative, ranging from 0.14 to -0.29 for US men, with a median of -0.10 (see Pencavel, 1986). Nevertheless, we assume the uncompensated labor supply not to be backward bending, implying $\frac{\partial L}{\partial t_2} < 0$. $\frac{\partial L}{\partial P_e} < 0$ is then implied by the fact that an increase in tuition fees will reduce human capital and consequently reduce earnings, leading to a decrease in labor supply.

The welfare effect of altering the surtax rate t_2 in a flat tax environment $t_2 = t_1 = t$ and in the absence of tuition fees $P_e = 0$ can be derived as[16]

$$\left.\frac{dV}{dt_2}\right|_{t_2=t_1=t,P_e=0} = -\frac{W\,\overline{H}\,L\mathrm{E}\left[U_{C_2}\right]\,(1+r)\,e}{D} \cdot \left\{\frac{t}{1-t}\cdot\Big(\varphi\left[\gamma\,\pi_e\,\eta_{eP_e}+\eta_{LP_e}\right]-\left[\gamma\,\pi_e\,\eta_{et_2}+\eta_{Lt_2}\right]\Big)-\xi\right\} \tag{30}$$

where $\gamma = \frac{\overline{H}_e e}{\overline{H}}$ is the expected production elasticity of human capital. ξ stands for the insurance characteristics, and η_{ij} represents uncompensated elasticities in labor supply and educational investment.

$\varphi = \frac{W\cdot[\overline{H}L(1-\xi)-X]}{(1+r)\cdot W\cdot e}$ can be interpreted as the risk-adjusted average return on educational investment before taxes. $(1+r)We$ gives the accumulated (present value) costs of educational investment (remind that tuition fees are zero, $P_e = 0$). The risk-adjusted skill premium in income before taxes is $W\cdot(\overline{H}L(1-\xi)-L)$. Due to $X < L$ from Assumption 1, the numerator of φ, $W\cdot[\overline{H}L(1-\xi)-X] > 0$, exceeds the real skill premium, and we have $\varphi > 1$, because otherwise there would not be any educational investment. Simultaneously, in case of a linear wage tax $t_2 = t_1 = t$, φ also indicates the risk-adjusted relative return in tax revenue from increasing educational investment by one unit. The numerator represents the (risk-adjusted) tax base of returns to education and the denominator gives tax revenue forgone by educational investment. Hence, φ roughly mirrors the self-financing effect of fostering education as well.

According to Assumption 1, the denominator of equation (30) D is positive. Progressive taxation to be welfare-enhancing, implying $\left.\frac{dV}{dt_2}\right|_{t_2=t_1=t,P_e=0} > 0$, requires thereafter

$$\frac{t}{1-t}\cdot\Big(\varphi\cdot\left[\gamma\pi_e\eta_{eP_e}+\eta_{LP_e}\right]-\left[\gamma\,\pi_e\,\eta_{et_2}+\eta_{Lt_2}\right]\Big) < \xi. \tag{31}$$

[16]See Appendix 10.3.

Accordingly, a tax reform, introducing both progressive wage tax and subsidization of education, has three welfare-relevant effects: first, a progressive tax provides better insurance against income risks and increases the utility of risk-averse individuals. This unambiguously welfare-increasing effect is measured by the magnitude of the insurance characteristics $\xi > 0$. Second, progressive wage taxation has negative incentive effects on both labor supply and educational investment ($\eta_{zt_2} < 0, z = L, e$), causing excess burden. Third, though being per-se distorting as well, education subsidies ($\eta_{zP_e} < 0, z = L, e$) have a positive welfare effect for two reasons: (i) by fostering educational investment and stabilizing labor supply, they alleviate the distortions from increased labor taxation t_2. (ii) Educational subsidies mitigate the underinvestment problem into education. The latter is the stronger, the larger is the risk premium in education π_e. Underinvestment turns into a fiscal externality, because there is a 'risk wedge' between marginal return and marginal costs, and increasing education will increase both total income and tax revenue (see Jacobs et al, 2009b, for a detailed analysis).

In a nutshell, the allocative net effect is ambigous in sign and is represented by the first term on the LHS of equation (31). Taken together, for tax progression to be desirable, the (potentially) harmful net effect of induced distortions in labor supply and educational investment has then to be compensated by the welfare-enhancing insurance effect ξ.

If the disincentive effects of increasing the surtax rate t_2 are strong and dominate the alleviating effects of education subsidies (e.g., in underinvestment in education), the allocative net effect of the tax reform ceteris paribus decreases welfare. Then, there is the classical trade-off between insurance and efficiency and whether such a tax reform can improve welfare, depends on which effect dominates. A welfare-enhancing tax reform requires that the initial flat tax rate t is not too high:

$$\frac{t}{1-t} < \frac{\xi}{\varphi \cdot [\gamma \pi_e \eta_{eP_e} + \eta_{LP_e}] - [\gamma \, \pi_e \, \eta_{et_2} + \eta_{Lt_2}]}. \tag{32}$$

The higher the initial tax rate t, the larger are the induced distortions by an increase of t_2 and the less likely is a welfare-improvement. The more risk matters, however, i.e., the higher is the insurance characteristic ξ, the more importance is attached to insurance and the more likely is a welfare-improvement by the tax reform.

If the allocative net effect is, instead, welfare-enhancing itself, there is no trade-off at all and we can state from examining condition (31):

Proposition 3 *Starting from* $t_2 = t_1 = t > 0$ *and* $P_e = 0$*, a sufficient condition for a welfare-enhancing introduction of tax progression* $t_2 > t_1 > 0$ *and simultaneous redemption of additional tax revenue as education subsidies per semester* $P_e < 0$ *is*

$$|\gamma\pi_e\eta_{et_2} + \eta_{Lt_2}| \leq |\varphi\left(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}\right)|.$$

Proof: According to Assumption 1 both $\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}$ and $\gamma\pi_e\eta_{et_2} + \eta_{Lt_2}$ are negative. If $|\gamma\pi_e\eta_{et_2}+\eta_{Lt_2}| \leq |\varphi\cdot(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e})|$, the inequality (31) is fulfilled irrespectively of the magnitude of the insurance effect $\xi > 0$. □

Proposition 3 characterizes a situation, where the distortive effects of increased wage taxation are more than compensated by the introduction of educational subsidies. In case Proposition 3 holds, the tax reform provides efficiency gains instead of an excess burden and should be implemented even on pure efficiency grounds and irrespectively of any insurance effect.

This case is the more likely the more inelastic labor supply and the less educational investment reacts on changes in the net wages, consequently, the lower are the elasticities η_{et_2} and η_{Lt_2}. In this case, the distortionary effects are very small. Moreover, the effect on education is weighted by the product of the risk premium in educational investment π_e and the expected production elasticity of human capital γ, indicating whether labor taxation amplifies underinvestment in education substantially.

The likelihood for fulfilling the condition in Proposition 3 increases in the sensitivity of labor supply and educational investment with respect to education subsidies, η_{eP_e} and η_{LP_e} respectively, measuring the mitigating allocative effects. Once more, the effect on education is weighted by $\pi_e \cdot \gamma$, measuring the relevance of underinvestment again. Additionally, the effect of education subsidies is weighted by $\varphi > 1$, which serves as a proxy for the self-financing effect of subsidizing education (around $P_e = 0$).

How realistic is such a situation and can this proposition be backed by some empirical evidence? In the discussion of Assumption 1, we have already seen that uncompensated labor supply is rather very inelastic. This implies that the sensitivity of uncompensated labor supply to education subsidies should be around zero, as well, because the complementarity of labor supply and education mainly works via increasing wages. If we assume $\eta_{Lt_2} = \eta_{LP_e} = 0$, the condition in Proposition 3 boils down to

$$|\eta_{et_2}| \leq \varphi \cdot |\eta_{eP_e}|, \tag{33}$$

where $\varphi \geq 1$.

Leslie and Brinkman (1987) conclude that the modal result of about 30 empirical studies is a 1.8 enrollment decline per 100 dollar increase in tuition fees. More recently, Chang and Hsing (1996) report that for the U.S. the elasticity of enrollment in private institutions of higher education (IHE), relative to that in public institutions, is -13.561 for the years $1990 - 1991$ w. r. t. average tuition fees and costs per student at private IHEs, relative to those at public IHEs. In our case of pure public schools the elasticity of enrollment w. r. t. tuition fees might be expected to be smaller, but should still not be too small and remain negative. Dynarski (1999) points out that each $1,000$ dollar increase in student benefits by Social Security Student Benefit Programm increases the share of high school graduates who attended college before 1996 by 3.6 percentage points. Therefore an increase in tuition fees and an increase in student aid (education subsidies) respectively seem to have a significant effect on education demand.

The effect of a higher wage rate on student enrollment has also been estimated by some studies. Freeman (1986) provides a partial survey of this literature, showing an elasticity of higher education demand to salaries in a range of 0.5 to 2.0. Kodde (1985) reports a smaller elasticity of enrollment to the future monthly income of 0.14, using data from Dutch high school graduates in 1982.[17] In a more recent study by Fredriksson (1997) the elasticity of the enrollment rate of higher school leavers w. r. t. the university graduate wage rate is estimated to equal 2.8 for Sweden.

Psacharopoulos (1973), instead, estimates the elasticity of freshman enroll-

[17] However, he also finds only small responses of enrollment to tuition fees.

ment at public institutions for higher education in Hawaii for the years $1956-1968$ to be 0.45 w. r. t. relative earnings of college graduates to high school graduates, but to be -1.12 w. r. t. tuition fees.

Taken together, the requirements for Proposition 3 to be applicable may or may not be fulfilled. Though it appears likely that the requirements are met, it is in any case worthwhile to have a closer look at the determinants of the insurance characteristics ξ. In order to be able to derive some clear-cut results here, we make some additional assumptions:

Assumption 2 *(i) The subutility function in the second period is separable in consumption and labor supply, i.e.,* $U_{c_2 l} = 0$*.*

(ii) There is multiplicative wage risk, i.e., $\tilde{H}(e, \theta) = \theta \cdot h(e)$*.*

(iii) The shock is normally distributed with mean $E[\theta] = 1$ *and variance* σ_θ^2*.*

Modeling wage risk in a multiplicative way is in line with Eaton and Rosen (1980a,b) and Hamilton (1987), whereas assuming that it is normally distributed might be a little cumbersome at first glance. However, this allows to apply a Rubinstein-theorem, which should be a reasonable approximation for other distributions, as well. We conclude:

Proposition 4 *Given Assumption 2, the insurance effect and therefore the preferability of progression in the wage tax and of introducing direct education subsidies* $P_e < 0$ *are increasing in*

(i) expected net labor earnings $(1 - t) \cdot W \cdot h(e) \cdot L$*,*

(ii) global risk aversion of consumption $ARA(C)$*,*

(iii) and the variance of the shock σ_θ^2*.*

Proof: See Appendix 10.4. □

The more risk is in the economy and the more this risk affects well-being, the more valuable social insurance ceteris paribus gets – calling then for tax progression. This intuition is mirrored in Proposition 4, where the expected net wage income measures a household's exposure to risk and where risk aversion determines, how the household is affected by this exposure. The variance of the shock is finally a measure for the magnitude of the risk in the economy.

Altogether, if the insurance effect of progressive taxation is more important than its net distortionary effects, progressive income taxation in combination with education subsidies is superior to a proportional (flat) income tax. If so, progression in the labor tax guarantees superior insurance effects, compared to proportional taxation, and education subsidies avoid that the efficiency losses become too strong. Thus, the intuition of the result in case of entirely exogenous leisure demand can also be applied in case of elastic skilled labor supply. The result also fits to insights in capital taxation, where directly taxing the risk premium with a special tax rate allows improved insurance at very low efficiency costs (see Schindler, 2008).

However, full insurance will not be optimal, because the induced efficiency losses would be too high, if $t_2 = 1$. Moreover, we have to note that non-progressive taxation can nevertheless be optimal under certain conditions. A necessary condition for this to be true can be easily derived from Proposition 3.

Corollary 1 *A necessary condition for having non-progressive taxation in a Second-best optimum is that*

$$|\gamma \pi_e \eta_{et_2} + \eta_{Lt_2}| > |\varphi \left(\gamma \pi_e \eta_{eP_e} + \eta_{LP_e} \right)|.$$

This condition implies that the negative welfare effects of increasing t_2 have to dominate the effects of fostering educational investment by subsidization $P_e < 0$.

In fact, a regressive tax structure might be optimal, in case the initial flat tax rate t is too high. If so, it matters more to directly decrease distortions in labor supply by reducing the marginal tax rate on skilled labor than to provide enhanced insurance via a progressive tax system and to foster eduction via subsidies. In a nutshell, tax regression is optimal, in case net distortions dominate the insurance effect. This can be the case, if, and only if, Corollary 1 is fulfilled, but following our discussion of Proposition 3, we think that, for real-world values of elasticities, it is unlikely having the net distortions dominating the insurance effect.

8 Implications to the Flat Tax Debate

In which way are our results related to the debate, whether a flat tax should be implemented and which new insights can be derived? The main idea of a flat tax dates back to a very influential proposal by Hall and Rabushka (1983). In fact, they proposed a tax system with a tax base, containing all labor income, but tax-exempting capital income on the personal level, and only one, constant tax rate, accompanied by a tax-credit.[18]

This tax system contradicts standard results in optimal taxation models, where non-linear taxation turns out to be beneficial in case society values redistribution.[19] However, flat tax systems are easier to run and some simulations show that the optimal non-linear structure can be well-approximated by a flat tax (Myles, 1995, section 5.4). More important, the standard models on non-linear taxation neglect (labor market) participation distortions, endogenous human capital formation and labor market imperfections. They all tend to decrease marginal tax rates.[20]

These might be the reasons, why flat tax systems became more and more popular and why they had been recently introduced in a couple of Eastern European countries (e.g., Slovakia and Estonia), and why they are discussed in Western Europe, too.[21] Nevertheless, a flat tax remains doubtful even if standard optimal taxation results are neglected.

Nielsen and Sørensen (1997) show that a flat tax in a broader sense, incorporating a constant tax rate on capital income, cannot be optimal even in a model without redistribution, if there is endogenous human capital formation. The capital tax acts as subsidy on educational investment and *calls* for progressive taxation in order to mitigate distortions.

Our results now raise doubts whether a flat tax can be any optimal in a world with human capital formation, if there are risky returns to education and wage risk in general. These doubts are derived in a model with ex-ante homogenous households and without capital taxation, which then should be the most favorable

[18] See Atkinson (1995) for a detailed description and analysis of the flat tax.

[19] See, e.g., Myles (1995, chapter 5) for a survey and Saez (2001) for a more recent contribution.

[20] See, e.g., Jacobs et al. (2009a, section 2) or Keen et al. (2008, section 3.1) for a brief overview and discussion.

[21] See Keen et al. (2008) and Paulus and Peichl (2009). In Germany, e.g., the conservative party CDU supported a flat tax like proposal by Paul Kirchhof in its election campaign 2005.

setting for a flat tax. However, in our model a constant flat tax rate $t > 0$ is only optimal, if simultaneously

$$\frac{t}{1-t} = \frac{\xi}{\varphi\left(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}\right) - \left(\gamma\pi_e\eta_{et_2} + \eta_{Lt_2}\right)} \tag{34}$$

and Corollary 1, $|\gamma\pi_e\eta_{et_2} + \eta_{Lt_2}| > |\varphi\left(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}\right)|$, are fulfilled.

Thus, there have to be net distortions from introducing tax progression and these distortions have to cancel exactly against the insurance effect at the proportional tax rate t.

Even without insurance effect, it is doubtful, whether Corollary 1 can be sustained by empirical observations. Taking into account the insurance effect and Proposition 4, it appears more than unlikely that there is a flat tax rate $t > 0$ guaranteeing sufficient insurance in a Second-best optimum. Insurance and distortionary effects should cancel each other only by chance.

9 Conclusions

We have shown that in a two-period model with endogenous, but risky human capital formation, the optimal labor tax structure is most likely to be progressive in order to insure against income risk and that (direct) education subsidies are used in order to alleviate induced distortions, if the educational risk is idiosyncratic and the government can diversify the risk at no costs.

If leisure demand is entirely inelastic, the government will provide full insurance. In the more realistic case of elastic skilled labor supply, progression survives under some assumptions even then, when the starting point is a positive proportional labor tax. In a nutshell, extending the instruments of the government in a standard Eaton-Rosen-world leads to the Bovenberg-Jacobs effect of 'Siamese Twins', where education subsidies are needed in order to alleviate efficiency losses. Thus, the mechanism for redistribution (or, in our case, social insurance), identified in Bovenberg and Jacobs (2005), carries over to the case of income risk and helps to avoid inefficient underinvestment in education in a laissez-faire-economy.

Our results also apply to the flat tax debate. Here, we raise severe doubts, whether a flat tax can be optimal, even in a model with ex-ante homogenous households and without capital taxation. The reason is that, in a trade-off against distortions, it is unlikely to guarantee sufficient insurance against risky returns to educational investment and wage risk without relying on directly progressive taxation.

A critical point in the model is the assumption of idiosyncratic risk. The government can diversify the risk at no costs, whereas it is in general assumed that private insurance is not possible. Thus, the critical question, which appears in all such models is: Why can the government do better than private insurers? This problem can easily be solved, if the risk is assumed to be aggregate risk. In this case the government can provide diversification of risk on private and public consumption (see e.g., Kaplow, 1994). However, tax revenue then turns risky, which will have some impact on the results. This aspect is left for further research.

10 Appendix

10.1 Second order conditions of household optimization

The Hessian Matrix in the households' optimization problem is

$$H = \begin{pmatrix} \frac{\partial \mathrm{E}[U]}{\partial C_1 C_1} & \frac{\partial \mathrm{E}[U]}{\partial C_1 e} & \frac{\partial \mathrm{E}[U]}{\partial C_1 l} \\ \frac{\partial \mathrm{E}[U]}{\partial e C_1} & \frac{\partial \mathrm{E}[U]}{\partial ee} & \frac{\partial \mathrm{E}[U]}{\partial el} \\ \frac{\partial \mathrm{E}[U]}{\partial l C_1} & \frac{\partial \mathrm{E}[U]}{\partial le} & \frac{\partial \mathrm{E}[U]}{\partial ll} \end{pmatrix}$$

The second order conditions for maximizing utility require $\mathrm{E}[U_{C_1C_1}] < 0$ and the second leading minor $\mathrm{E}[U_{C_1C_1}] \cdot \mathrm{E}[U_{ee}] - \mathrm{E}[U_{C_1e}]^2$ to be positive. The third leading minor must be negative, implying

$$\begin{aligned} \mathrm{E}[U_{C_1C_1}] \cdot \mathrm{E}[U_{ll}] \cdot \mathrm{E}[U_{ee}] \quad &+ \quad 2\mathrm{E}[U_{el}] \cdot \mathrm{E}[U_{C_1e}] \cdot \mathrm{E}[U_{C_1l}] < \mathrm{E}[U_{C_1C_1}] \cdot \mathrm{E}[U_{el}]^2 \\ &+ \quad \mathrm{E}[U_{ll}] \cdot \mathrm{E}[U_{C_1e}]^2 + \mathrm{E}[U_{ee}] \cdot \mathrm{E}[U_{C_1l}]^2. \end{aligned}$$

Rearranging leads to

$$\begin{aligned}2(\mathrm{E}[U_{C_1e}]\cdot\mathrm{E}[U_{el}]\cdot\mathrm{E}[U_{C_1l}]-\mathrm{E}[U_{C_1C_1}]\cdot\mathrm{E}[U_{ee}]\cdot\mathrm{E}[U_{ll}])<\\ \mathrm{E}[U_{C_1C_1}](\mathrm{E}[U_{el}]^2-\mathrm{E}[U_{ee}]\cdot\mathrm{E}[U_{ll}])+\mathrm{E}[U_{ll}](\mathrm{E}[U_{eC_1}]^2-\mathrm{E}[U_{ee}]\cdot\mathrm{E}[U_{C_1C_1}])\\ +\mathrm{E}[U_{ee}](\mathrm{E}[U_{C_1l}]^2-\mathrm{E}[U_{C_1C_1}]\cdot\mathrm{E}[U_{ll}])\end{aligned} \tag{35}$$

If we change the order of Hessian Matrix, it follows that the second leading minors of the respective Hessian Matrix $\mathrm{E}[U_{ee}]\cdot\mathrm{E}[U_{ll}]-\mathrm{E}[U_{el}]^2$ and $\mathrm{E}[U_{C_1C_1}]\cdot\mathrm{E}[U_{ll}]-\mathrm{E}[U_{C_1l}]^2$ are both positive. Therefore we have:

$$\mathrm{E}[U_{el}]^2 < \mathrm{E}[U_{ee}]\cdot\mathrm{E}[U_{ll}] \tag{36}$$

$$\mathrm{E}[U_{C_1l}]^2 < \mathrm{E}[U_{C_1C_1}]\cdot\mathrm{E}[U_{ll}] \tag{37}$$

$$\mathrm{E}[U_{C_1e}]^2 < \mathrm{E}[U_{ee}]\cdot\mathrm{E}[U_{C_1C_1}] \tag{38}$$

Inequations (36), (37) and (38) imply that

$$(\mathrm{E}[U_{el}]\cdot\mathrm{E}[U_{eC_1}]\cdot\mathrm{E}[U_{C_1l})]^2<(\mathrm{E}[U_{ee}]\cdot\mathrm{E}[U_{ll}]\cdot\mathrm{E}[U_{C_1C_1})]^2 \tag{39}$$

(39) states that the absolute value of the three cross-effects in household utility multiplied together should be smaller than the second-derivatives of e, l and C_1.

Using (39) in equation (35), its LHS is positive and the RHS of (35) turns out to be positive, as well – due to (36) – (38).

Therefore, all restrictions for guaranteeing the Hessian matrix to be negative definite, (35) – (39), are fulfilled if the cross-effects are sufficiently small in absolute values (or even tend to zero).

This is the standard second-order condition in case of positive or negative feedback effects between different household decisions, which we will assume to hold throughout the paper: the cross effects are small enough in comparison to diminishing marginal utilities, in order to avoid corner solutions.

10.2 Comparative Statics of Household Choice

Assuming exogenous labor supply in the second period and totally differentiating the two first order conditions (4) and (5) of the individual maximization problem results in

$$\begin{aligned}
&\alpha dC_1 + \beta de = \\
&-\{\mathrm{E}[U_{C_1C_2}][(1+r)(1-e)+1] - (1+r)\mathrm{E}[U_{C_2C_2}][(1+r)(1-e)+1]\}dw_1 \\
&-\{\mathrm{E}[U_{C_1C_2}(\tilde{H}-1)] - (1+r)\mathrm{E}[U_{C_2C_2}(\tilde{H}-1)]\}dw_2 \\
&-\{-(1+r)e\mathrm{E}[U_{C_1C_2}] + (1+r)^2 e\mathrm{E}[U_{C_2C_2}]\}dP_e,
\end{aligned}$$

$$\begin{aligned}
&\gamma dC_1 + \delta de = \\
&-\{\mathrm{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}[(1+r)(1-e)+1]] - (1+r)\mathrm{E}[U_{C_2}]\}dw_1 \\
&-\{\mathrm{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}(\tilde{H}-1)] + E[U_{C_2}\tilde{H}_e]\}dw_2 \\
&-\{\mathrm{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}(1+r)(-e)] - (1+r)\mathrm{E}[U_{C_2}]\}dP_e,
\end{aligned}$$

whereby

$$\begin{aligned}
\alpha &= \frac{\partial \mathrm{E}[U_{C_1}] - (1+r)\mathrm{E}[U_{C_2}]}{\partial C_1}, \\
\beta &= \frac{\partial \mathrm{E}[U_{C_1}] - (1+r)\mathrm{E}[U_{C_2}]}{\partial e}, \\
\gamma &= \frac{\partial \mathrm{E}[U_{C_2}\{w_2 H_e - (1+r)(w_1+P_e)\}]}{\partial C_1}, \\
\delta &= \frac{\partial \mathrm{E}[U_{C_2}\{w_2 H_e - (1+r)(w_1+P_e)\}]}{\partial e}.
\end{aligned}$$

Using Cramer's rule the derivatives of optimal education decision e w.r.t. w_1 and P_e can be derived as:

$$\frac{\partial e}{\partial w_1} = \frac{-\alpha\{\eta[(1+r)(1-e)+1] - (1+r)\mathrm{E}[U_{C_2}]\} + \gamma\{\epsilon[(1+r)(1-e)+1]\}}{SOC}, \tag{40}$$

$$\frac{\partial e}{\partial P_e} = \frac{\alpha\{\eta(1+r)e + (1+r)\mathrm{E}[U_{C_2}]\} - \gamma\epsilon(1+r)e}{SOC}, \tag{41}$$

where

$$\eta = \mathrm{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}], \ \epsilon = \mathrm{E}[U_{C_1C_2}] - (1+r)\mathrm{E}[U_{C_2C_2}],$$

and SOC stands for the determinant of the Hessian matrix.

10.3 Proof of Equation (30)

Applying $t_2 = t_1 = t$ and $P_e = 0$ in the balanced-budget condition (26), leads to

$$\frac{dP_e}{dt_2} = -\frac{tW[\overline{H}_e L - (1+r)]\frac{de}{dt_2} + tW\overline{H}\frac{dL}{dt_2} + W[\overline{H}L - X]}{tW[\overline{H}_e L - (1+r)]\frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e} \tag{42}$$

Moreover, from household choice and equation (7) follows

$$\overline{H}_e \cdot L - (1+r) = \overline{H}_e \cdot L \cdot \pi_e, \tag{43}$$

as $P_e = 0$ and $w_1 = w_2$ due to $t_1 = t_2$.

This allows to rewrite equation (42) as

$$\frac{dP_e}{dt_2} = -\frac{tW\ \overline{H}_e\ L\ \pi_e \cdot \frac{de}{dt_2} + tW\ \overline{H}\frac{dL}{dt_2} + W[\overline{H}L - X]}{tW\ \overline{H}_e \cdot L \cdot \pi_e \cdot \frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e} \tag{44}$$

Assuming, according to Assumption 1, that the Laffer curve is increasing around $P_e = 0$, $\frac{\partial \bar{R}}{\partial P_e}\Big|_{dP_e=0} > 0$, implies that the denominator of (44) is positive:

$$D = \frac{\partial \bar{R}}{\partial P_e} = tW\ \overline{H}_e \cdot L \cdot \pi_e \cdot \frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e > 0. \tag{45}$$

Substituting equation (44) as well as the Envelope effects in (9) and (10) into equation (28) results in

$$\begin{aligned}\frac{dV}{dt_2}\Big|_{t_2=t_1=t, P_e=0} &= -W\mathrm{E}[U_{C_2}(HL - X)] + (1+r)\,e \cdot \mathrm{E}[U_{C_2}] \\ &\quad \cdot \frac{t\overline{H}_e LW\pi_e\frac{de}{dt_2} + tW\overline{H}\frac{dL}{dt_2} + W[\overline{H}L - X]}{t\overline{H}_e LW\pi_e\frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e}\end{aligned} \tag{46}$$

and by factoring out the denominator of the second summand on the RHS

$$
\begin{aligned}
&\frac{dV}{dt_2}\Big|_{t_2=t_1=t,P_e=0} = \\
&-\frac{W}{D}\cdot\Big\{\mathrm{E}[U_{C_2}\cdot(\tilde{H}\cdot L-X)]\cdot\left[tW\overline{H}_e\,L\,\pi_e\cdot\frac{\partial e}{\partial P_e}+tW\overline{H}\cdot\frac{\partial L}{\partial P_e}\right] \\
&-\mathrm{E}[U_{C_2}](1+r)\cdot e\cdot\left[t\overline{H}_e\,L\,\pi_e\cdot\frac{\partial e}{\partial t_2}+t\overline{H}\cdot\frac{\partial L}{\partial t_2}\right] \\
&+\Big(\mathrm{E}[U_{C_2}\cdot(\tilde{H}\cdot L-X)]-\mathrm{E}[U_{C_2}][\overline{H}\cdot L-X]\Big)\cdot(1+r)\cdot e\Big\}
\end{aligned} \tag{47}
$$

Applying Steiner's Rule, the first bracket in the last line of (47) reduces to

$$
\mathrm{E}[U_{C_2}\cdot(\tilde{H}\cdot L-X)]-\mathrm{E}[U_{C_2}][\overline{H}\cdot L-X]=-\mathrm{E}[U_{C_2}]\,\overline{H}\,L\,\xi<0, \tag{48}
$$

where we have defined the insurance characteristics (according to Feldstein's distributional characteristic) as

$$
\xi=-\frac{\mathrm{Cov}(U_{C_2},H)}{\mathrm{E}[U_{C_2}]\cdot\overline{H}}>0. \tag{49}
$$

Substituting equation (48) into (47), factoring out $\overline{H}\;L\;\mathrm{E}[U_{C_2}]\;(1+r)\;e$ now, and relying in the second line on Steiner's Rule again, leaves us with

$$
\begin{aligned}
&\frac{dV}{dt_2}\Big|_{t_2=t_1=t,P_e=0} = -\frac{W\overline{H}\,L\,\mathrm{E}[U_{C_2}](1+r)e}{D}\cdot\Big\{\frac{\overline{H}\,L\cdot(1-\xi)-X}{(1+r)\cdot e}\cdot \\
&\left[\frac{t}{1-t}\frac{\overline{H}_e\cdot e}{\overline{H}}\cdot\pi_e\cdot\frac{(1-t)W}{e}\frac{\partial e}{\partial P_e}+\frac{t}{1-t}\frac{(1-t)W}{L}\cdot\frac{\partial L}{\partial P_e}\right] \\
&-\left[\frac{t}{1-t}\frac{\overline{H}_e\cdot e}{\overline{H}}\cdot\pi_e\cdot\frac{(1-t)}{e}\frac{\partial e}{\partial t_2}+\frac{t}{1-t}\frac{(1-t)}{L}\cdot\frac{\partial L}{\partial t_2}\right]-\xi\Big\}
\end{aligned} \tag{50}
$$

Evaluating at $t_2=t_1=t$ and $P_e=0$, we define the uncompensated elasticities of labor supply L and educational investment e with respect to the tax rate t_2 and overall education costs $w_1 + P_e = (1-t)W$ as

$$\begin{aligned}
\eta_{Lt_2} &= \frac{1-t_2}{L} \cdot \frac{\partial L}{\partial w_2} = \frac{1-t}{L} \cdot \frac{\partial L}{\partial w_2} \\
\eta_{et_2} &= \frac{1-t_2}{e} \cdot \frac{\partial e}{\partial w_2} = \frac{1-t}{e} \cdot \frac{\partial e}{\partial w_2} \\
\eta_{LP_e} &= \frac{w_1 + P_e}{L} \cdot \frac{\partial L}{\partial P_e} = \frac{(1-t)W}{L} \cdot \frac{\partial L}{\partial P_e} \\
\eta_{eP_e} &= \frac{w_1 + P_e}{e} \cdot \frac{\partial e}{\partial P_e} = \frac{(1-t)W}{e} \cdot \frac{\partial e}{\partial P_e}
\end{aligned}$$

Moreover, we define $\gamma = \frac{e}{\overline{H}} \cdot \overline{H}_e$ as the (expected) production elasticity of educational investment. Applying these definitions in equation (50), we finally end up with equation (30) in the text:

$$\begin{aligned}
\frac{dV}{dt_2} &= -\frac{W\ \overline{H}\ L\mathrm{E}[U_{C_2}]\ (1+r)\ e}{D} \cdot \\
&\left(\frac{t}{1-t} \left[\frac{\left(\overline{H}L\,(1-\pi_H) - X\right)}{(1+r)e} \left(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}\right) - \left(\gamma\pi_e\eta_{et_2} + \eta_{Lt_2}\right) \right] - \xi \right).
\end{aligned}$$

10.4 Proof of Proposition 4

For proving Proposition 4, we have to show the decomposition of the insurance effect ξ into the factors mentioned in the proposition. If the shock θ is normally distributed, second period consumption $\tilde{C}_2(\theta)$ is normal as well, and we can apply a Rubinstein-theorem (see Rubinstein, 1976, pp. 421) in order to receive

$$\mathrm{Cov}(U_{c_2}, \theta) = \mathrm{E}[U_{c_2c_2}] \cdot \mathrm{Cov}(C_2, \theta). \tag{51}$$

For $t_2 = t_1 = t$ and $P_e = 0$, the household's budget constraint reads

$$\tilde{C}_2 = (1+r)\ S_1 + (1-t)\ W\ \theta\ h(e)\ L, \tag{52}$$

where $S_1 = (1-t)\, W\, L - C_1$. Inserting this in the RHS of (51) results in

$$\begin{aligned} \mathrm{E}[U_{c_2c_2}] \cdot \mathrm{Cov}(C_2, \theta) &= \mathrm{E}[U_{c_2c_2}] \cdot \mathrm{Cov}((1+r)\, S_1 + (1-t)\, W\, \theta\, h(e)\, L, \theta) \\ &= \mathrm{E}[U_{c_2c_2}] \cdot \mathrm{Cov}(\theta, \theta) \cdot (1-t)\, W\, h(e)\, L \end{aligned} \tag{53}$$

by applying some covariance rules.

Collecting terms and recognizing that $\mathrm{Cov}(\theta, \theta) = \sigma_\theta^2$ is the variance of the shock parameter θ, ξ can be rearranged to

$$\begin{aligned} \xi &= -\frac{\mathrm{Cov}(U_{C_2}, H)}{\mathrm{E}[U_{C_2}] \cdot \overline{H}} = -\frac{\mathrm{E}[U_{C_2C_2}]}{\mathrm{E}[U_{C_2}] \cdot \overline{H}} \cdot \mathrm{Cov}(C_2, \theta) \cdot h(e) \qquad (54) \\ &= -\frac{\mathrm{E}[U_{C_2C_2}]}{\mathrm{E}[U_{C_2}]} \cdot (1-t) W\, h(e)\, L \cdot \sigma_\theta^2 = (1-t) W\, h(e)\, L \cdot ARA(C) \cdot \sigma_\theta^2 \end{aligned}$$

whereby $\overline{H} = h(e)$, and $(1-t)\, W\, h(e)\, L$ is second-period expected wage income of a skilled worker as $\mathrm{E}[\theta] = 1$. Moreover, we have defined global absolute risk aversion in consumption as $ARA(C) = -\frac{\mathrm{E}[U_{c_2c_2}]}{\mathrm{E}[U_{c_2}]} > 0$ according to Varian (1992, p. 380). Obviously, the insurance effect ξ is increasing in the three economic variables, mentioned in Proposition 4.

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