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Self-Selection and Wage-Tenure Profiles  
for Heterogeneous Labor

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## Self-Selection and Wage-Tenure Profiles for Heterogeneous Labor

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### Zusammenfassung:

In this paper I develop a theoretical model explaining optimal wage-tenure profiles for heterogeneous labor. My findings entail that high productive people have steeper profiles than low productive individuals. I find strong empirical evidence for these findings. At the end of my paper, I utilize the basic model to describe the labor market entry of college graduates.

JEL Klassifikation : J31, J44, I21

Schlüsselwörter : Wages, Seniority, Educational decisions

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# Self-Selection and Wage-Tenure Profiles for Heterogeneous Labor

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August 31, 2004

## **Abstract**

In this paper I develop a theoretical model explaining wage-tenure profiles for heterogeneous labor. My findings entail that high productive people have steeper profiles than low productive individuals. In the literature, I find strong empirical evidence for these findings. At the end of the paper, I utilize the basic model to describe the labor market entry of college graduates.

Keywords: wages; seniority; educational decisions.

JEL classifications: J31; J44; I21.

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\*Thanks to Friedrich Breyer and Mathias Kifmann for helpful comments. All errors are mine.

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# 1 Introduction

A rising wage-tenure profile is a stylized fact in labor economics and empirical evidence is provided by several papers.<sup>1</sup> But what are the reasons for this phenomenon? Lazear (1981) argues that firms prefer upward-sloping wage curves in order to avoid shirking among workers, since the opportunity costs of shirking are larger for rising wage-tenure profiles than for flat ones. Freeman (1977) utilizes risk-aversion of the workers as the explanatory factor for seniority wages. People accept lower initial wages which function as an insurance against decreasing future productivity. Seniority wages also occur within tournament models.<sup>2</sup> Here, workers have to pay an admission fee to join the tournament. At the end the winner receives a promotion in form of money from the defeated colleague. Frank and Hutchens (1993) argue that workers seek to obtain a positively sloped earnings profile because their consumption-age curve corresponds to such a profile.<sup>3</sup> Finally, Hartog (1981) and Jovanovic (1979) describe a screening process which causes an upward-sloping wage curve. In these papers, firms and employees learn about each other with tenure and thus, productivity increases and firms are able to pay higher wages for the workers with an increasing tenure.

Next to the preceding approaches explaining increasing wage-tenure profiles, the self-selection approach is the most commonly cited. This approach is based on a paper by Salop and Salop (1976). The authors consider a labor market incorporating two types of workers. The employees differ in their

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<sup>1</sup>See among others Abraham and Farber (1987), Altonji and Shakotko (1987), Altonji and Williams (1997), Dustmann and Meghir (2003), Luchsinger, Wild, and Lalive (2001) and Topel (1991).

<sup>2</sup>See Lazear and Rosen (1981).

<sup>3</sup>The authors assume that people prefer larger future incomes to savings today.

propensity to leave the firm. Given job replacement costs, a worker with a low propensity is preferred by firms. Selection among workers occurs if firms offer increasing wage profiles<sup>4</sup>, since slow quitters prefer the rising profile to the flat one and fast quitters do not.

A paper by Guasch and Weiss (1981) discusses that the self-selection mechanism maximizes the profit of a firm. The authors model an application process with two types of applicants, a high-productive type 1 and a low-productive type 2. In this paper the selection framework is based on an application fee, an informative assessment test and a wage above the market wage if the applicant passes the test. The authors show that such a selection mechanism implying an increasing wage induces applications just from the high-productive type.

The papers by Salop and Salop (1976) and Guasch and Weiss (1981) explain fundamentally the selection process which implies a rising earnings-tenure profile. However, these approaches do not discuss the wage profiles for heterogeneous labor, since in these papers only the favorable workers are employed. Empirically, there is evidence that the earnings curve over tenure varies among heterogeneous workers. In this paper, I utilize an adverse selection mechanism to explain theoretically the observed variation in wage-tenure profiles.

The remainder of this paper is organized as follows: Section 2 describes the basic model which derives optimal wage profiles for heterogeneous labor. The results and the empirical evidence are discussed in Section 3. Using the basic model, Section 4 analyzes the labor market entry of college graduates. Section 5 concludes.

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<sup>4</sup>Thereby, the authors impose an admission fee to receive a job and a bonus upon the market wage in the next period.

## 2 A Model

This model incorporates heterogeneous labor supply with a competitive labor market. I assume that there are two types of applicants for the labor market, type 1 and type 2 which differ in productivity  $V$ , with  $V_1 > V_2$ . Since firms have no information about the productivity of the applicants ex ante,<sup>5</sup> they introduce an informative probationary at the beginning of the cooperation.<sup>6</sup> The informative test phase yields that type 1 has a higher probability  $p$  to pass this monitoring time than his counterpart.<sup>7</sup> Furthermore, I assume that  $V_i$  is constant while being in the labor force and the overall productivity exceeds the aggregate reservation benefit for each type

$$p_i \int_{t_1}^T V_i dt + \int_0^{t_1} V_i dt > \int_0^T w_i dt \quad \text{for } i = 1, 2, \quad (2.1)$$

where  $w_i$  denotes the reservation wage per unit of time for type  $i$ , with  $w_1 > w_2$ . The duration of the probationary is depicted by  $t_1$ .<sup>8</sup> Condition (2.1) assures that the employment of both types is worthwhile for the welfare. For simplicity, I assume that all agents are risk-neutral and the discount rate is zero. Additionally, the overall utility for type  $i$  is

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<sup>5</sup>Firms may have information about social background, education indicators and labor market experiences. However, the information do not detect the true productivity of the applicants, for instance a graduate from a top college could be less productive than a graduate from an unknown college.

<sup>6</sup>At the end of this section, we see that merely the announcement of a probationary reveals the true productivity.

<sup>7</sup>Since all applicants have full information about themselves, there is already an automatic preselection of the applicants. See also Guasch and Weiss (1981).

<sup>8</sup>The reservation wage is defined by the value of worker's time used outside the firm including for instance unemployment aids and the valuation of leisure. See also Lazear (1981).

$$U_i = p_i \int_{t_1}^T (a_i + b_i t) dt + \int_0^{t_1} (a_i + b_i t) dt + (1 - p_i) \int_{t_1}^T w_i dt \quad \text{for } i = 1, 2,$$

where  $a$  denotes the initial wage level and  $b$  the wage-tenure-effect.<sup>9</sup> Until the end of the qualifying period  $t_1$ , the worker obtains a guaranteed income. Afterwards, he either stays in or out of the firm.

## The Monopoly

At first I consider the case in which we have one firm. This firm maximizes its profit

$$p_i \int_{t_1}^T V_i dt + \int_0^{t_1} V_i dt - (p_i \int_{t_1}^T (a_i + b_i t) dt + \int_0^{t_1} (a_i + b_i t) dt) \quad \text{for } i = 1, 2,$$

subject to the participation constraints

$$\int_0^T w_i dt \leq U_i \quad \text{for } i = 1, 2$$

and the incentive compatibility constraints (ICCs):

$$p_1 \int_{t_1}^T (a_1 + b_1 t) dt + \int_0^{t_1} (a_1 + b_1 t) dt > p_1 \int_{t_1}^T (a_2 + b_2 t) dt + \int_0^{t_1} (a_2 + b_2 t) dt$$

and

$$p_2 \int_{t_1}^T (a_2 + b_2 t) dt + \int_0^{t_1} (a_2 + b_2 t) dt > p_2 \int_{t_1}^T (a_1 + b_1 t) dt + \int_0^{t_1} (a_1 + b_1 t) dt.$$

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<sup>9</sup>The assumption of linear wage profiles is empirically confirmed by Frank and Hutchens (1993).

Therefore, the maximization problem for the firm is

$$\begin{aligned}
L = & \max_{a_1, b_1, a_2, b_2, \mu, \nu, \omega, \rho} p_1 \int_{t_1}^T V_1 dt + \int_0^{t_1} V_1 dt - (p_1 \int_{t_1}^T (a_1 + b_1 t) dt + \int_0^{t_1} (a_1 + b_1 t) dt) \\
& + p_2 \int_{t_1}^T V_2 dt + \int_0^{t_1} V_2 dt - (p_2 \int_{t_1}^T (a_2 + b_2 t) dt + \int_0^{t_1} (a_2 + b_2 t) dt) \\
& + \mu \{ p_1 \int_{t_1}^T (a_1 + b_1 t) dt + \int_0^{t_1} (a_1 + b_1 t) dt + (1 - p_1) \int_{t_1}^T w_1 dt - \int_0^T w_1 dt \} \\
& + \nu \{ p_1 \int_{t_1}^T (a_1 + b_1 t) dt + \int_0^{t_1} (a_1 + b_1 t) dt - (p_1 \int_{t_1}^T (a_2 + b_2 t) dt + \int_0^{t_1} (a_2 + b_2 t) dt) \} \\
& + \omega \{ p_2 \int_{t_1}^T (a_2 + b_2 t) dt + \int_0^{t_1} (a_2 + b_2 t) dt + (1 - p_2) \int_{t_1}^T w_2 dt - \int_0^T w_2 dt \} \\
& + \rho \{ p_2 \int_{t_1}^T (a_2 + b_2 t) dt + \int_0^{t_1} (a_2 + b_2 t) dt - (p_2 \int_{t_1}^T (a_1 + b_1 t) dt + \int_0^{t_1} (a_1 + b_1 t) dt) \}.
\end{aligned}$$

Using the Kuhn-Tucker-Theorem, I receive the following solutions:<sup>10</sup>

$$\omega = 1, \quad (2.2)$$

$$\mu = 1. \quad (2.3)$$

This result emphasizes that the monopolist pays only the reservation wage in order to maximize the profit. From (2.2) and (2.3) we know that  $\frac{\partial L}{\partial \omega} = 0$  and  $\frac{\partial L}{\partial \mu} = 0$ . Hence, it exists the following relation between the optimal values of  $a_1, b_1$  and  $a_2, b_2$ :

$$a_1 = \frac{-b_1(t_1^2 + p_1(T^2 - t_1^2)) + 2(p_1(T - t_1) + t_1)w_1}{2(p_1(T - t_1) + t_1)} \quad (2.4)$$

and

$$a_2 = \frac{-b_2(t_1^2 + p_2(T^2 - t_1^2)) + 2(p_2(T - t_1) + t_1)w_2}{2(p_2(T - t_1) + t_1)}. \quad (2.5)$$

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<sup>10</sup>See appendix for the mathematical proof.

Accounting for the ICCs, the set of optimal wage profiles is characterized by

$$\begin{aligned} a_1 &< \frac{-(p_1(T-t_1)+t_1)(t_1^2+p_2(T^2-t_1^2))w_1+(p_2(T-t_1)+t_1)(t_1^2+p_1(T^2-t_1^2))w_2}{T(p_1-p_2)(T-t_1)t_1} < a_2, \\ b_1 &> \frac{2(p_1(T-t_1)+t_1)(p_2(T-t_1)+t_1)(w_1-w_2)}{T(p_1-p_2)(T-t_1)t_1} > b_2. \end{aligned} \quad (2.6)$$

According to the seniority parameter  $b_i$ , the model predicts that high productive people have a steeper profile than less productive individuals and the wage-tenure track slopes strictly upwards for the former. From (2.6) we recognize that the closer  $w_1, w_2$ , the flatter the profiles to distinguish between both types, since low productive workers have less incentive to choose the profile for the high productive worker. Whereas closeness between  $p_1$  and  $p_2$  leads to steeper profiles.<sup>11</sup> The latter is caused by the ICCs. Furthermore, an increasing length of the probationary also causes steeper wage-tenure profiles, because the post-assessment tenure decreases and hence the wages within the probationary has to be fairly low in order to select both types of worker. Due to the fact that the rotation of the wage profiles results in many optimal paths without changing the objective function, the optimal value of  $\rho$  and  $\nu$  is zero. Figure 2.1 illustrates the results.<sup>12</sup>

Before I describe the wage profiles for a competitive market, I show that the above results guarantee a maximal profit.

**Lemma 1** *Given asymmetric information, the ICCs have to be fulfilled in order to maximize the profit of a firm.*

**Proof:** Suppose the firm offer a single wage profile  $z$  such that:

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<sup>11</sup>Since the probationary cannot guarantee a full informative assessment, the similarity of  $V_i$  and  $p_i$  can differ between both types.

<sup>12</sup>It does not matter for the analysis, whether  $w_1 \stackrel{\leq}{\geq} w_2$ .

1.  $z < \int_0^T w_2 dt$ . In this case, no worker applies. The profit  $\pi_1 = 0$ .
2.  $z = \int_0^T w_2 dt$ . In this case, type 1 applies. The profit  $\pi_2 > 0$ .
3.  $\int_0^T w_2 dt < z < \int_0^T w_1 dt$ . In this case, type 1 applies. The profit  $\pi_3 < \pi_2$ .
4.  $z = \int_0^T w_1 dt$ . In this case, type 1 and type 2 apply. The profit  $\pi_4 \begin{matrix} \geq \\ \leq \end{matrix} \pi_2$ .
5.  $z > \int_0^T w_1 dt$ . In this case, type 1 and type 2 apply. The profit  $\pi_5 < \pi_4$ .

We see that only strategy 2 and 4 are not strictly dominated. However, neither strategy 2 or 4 delineates an optimal strategy, because there is a separation equilibrium enhancing the firm's profit.

Due to asymmetric information, a low productive applicant could assert to be high productive. If the ICCs hold, the firm is capable to offer two different profiles separating both groups of applicants. Regarding strategy 4, the firm increases its profit, if it offers  $z_1 = \int_0^T w_1 dt$  for the high productive applicants and  $z_2 = \int_0^T w_1 dt - \epsilon$ , with  $\epsilon > 0$ , for the low productive applicants. Thus, in order to maximize the firm's profit, the unique equilibrium is  $z_1 = \int_0^T w_1 dt$  and  $z_2 = \int_0^T w_2 dt$ .

**Proposition 2.1** *Under asymmetric information, the monopolist maximizes his profit subject to that the ICCs and the participation constraints hold. The wage profiles fulfill  $z_i = \int_0^T w_i dt$ , for  $i = 1, 2$ .*

## The competitive market

Suppose we have now  $n$  identical firms.

**Proposition 2.2** *Under asymmetric information, the competitive market equilibria include zero profits and the equilibria is characterized by:*

1. The wage profiles  $z_i$  satisfy  $p_i \int_{t_1}^T V_i dt + \int_0^{t_1} V_i dt =$  for  $i = 1, 2$ .

2. The ICCs must be fulfilled in order to employ both types of worker or only the high productive worker.

**Proof:** Suppose the firms offer two wage profiles such that  $p_i \int_{t_1}^T V_i dt + \int_0^{t_1} V_i dt = z_i$  for  $i = 1, 2$  and the ICCs are fulfilled. Now, there is no profitable deviation for any firm.

The increment of a profile such that  $z_i + \epsilon > p_i \int_{t_1}^T V_i dt + \int_0^{t_1} V_i dt$  yields a profit  $\pi < 0$ .

If a firm offers the profiles  $z_i - \epsilon < p_i \int_{t_1}^T V_i dt + \int_0^{t_1} V_i dt$ , no worker will apply. In addition,  $z_i - \epsilon < p_i \int_{t_1}^T V_i dt + \int_0^{t_1} V_i dt$  cannot be an equilibrium, because there are profitable deviations like  $z_i - \frac{\epsilon}{2}$ .

We see that in the separating equilibrium, the high productive workers obtain  $z_1 = p_1 \int_{t_1}^T V_1 dt + \int_0^{t_1} V_1 dt$ . If the ICCs do not hold, the wage profile  $z_1$  will attract both types of worker and yield  $\pi < 0$ . Additionally, every wage profile  $\bar{z}$ , with  $z_1 > \bar{z} > z_2$  only attracts low productive workers and yields also  $\pi < 0$ .<sup>13</sup>

Once we know that the zero profit constraints and the ICCs have to be fulfilled<sup>14</sup> in order to maximize the firm's profit, we can easily derive the wage profiles for the competitive market case. The only change to the monopoly case is that the aggregate profile corresponds now to the aggregate productivity instead of the aggregate reservation wage. Thus, we obtain for

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<sup>13</sup>See also Mas-Colell, Whinston, and Green (1995).

<sup>14</sup>I look at interesting case meaning to attract both types of workers or at least the high productive workers.

the competitive market case the following wage-tenure profiles

$$\begin{aligned}
 a_1 &< \frac{-(p_1(T-t_1)+t_1)(t_1^2+p_2(T^2-t_1^2))V_1+(p_2(T-t_1)+t_1)(t_1^2+p_1(T^2-t_1^2))V_2}{T(p_1-p_2)(T-t_1)t_1} < a_2, \\
 b_1 &> \frac{2(p_1(T-t_1)+t_1)(p_2(T-t_1)+t_1)(V_1-V_2)}{T(p_1-p_2)(T-t_1)t_1} > b_2.
 \end{aligned} \tag{2.7}$$

We see that the profiles are almost identical to the monopoly case. In the competitive market case, the wages hinge on the productivities  $V_i$ , whereas in the monopoly case, the reservation wages determine the slope of the profiles. But in both cases, the high productive people have a steeper profile than less productive individuals and the wage-tenure track slopes strictly upwards for the former.

In general, we have seen that high productive workers have positive sloped profiles. Given a constant productivity, the wages must be initially below and at the end above their productivity. Hence, firms could fire the employees after the probationary to obtain a profit  $\pi > 0$ . But is this an optimal strategy in the long run? If firms fire their employees after the probationary, the high productive workers would only accept flat profiles which correspond to their productivity. This causes the impossibility to select the worker, because all applicants would prefer the profile  $z_1$ . Hence, no firm has an incentive to dismiss its employees after the probationary.<sup>15</sup>

The values of  $a_i$  and  $b_i$  indicate that there must be a single crossing between the two profiles to separate the two group of workers. This finding corresponds to the fact that many "high flyers" accept low wages during an initial trainee time in order to earn high wages afterwards.

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<sup>15</sup>See also Lazear (1981).

### 3 Results and Empirical Evidence

In the previous section I have derived wage-tenure profiles for heterogeneous labor. Individuals with high productivity possess over strictly upward-sloping profiles, whereas people with a lower productivity have not necessarily a seniority effect on wages.

An empirical study by Dustmann and Meghir (2003) investigates wage profiles for the German labor force. The authors focus on young workers with and without an apprenticeship. Using a search model, the study identifies only a modest seniority effect for unskilled workers.<sup>16</sup> Moreover, the paper detects a strict positive seniority effect on wages for skilled workers.

Another empirical study by Abraham and Farber (1987) discloses the seniority effect for managerial, technical and professional employees and for blue-collar employees in the US. After controlling for the expected job duration, the authors find a stronger effect for the first subsample.

A paper by Topel (1991) confirms the preceding results. The author estimates the tenure effect for white-collar and blue-collar workers and obtains a slightly higher effect of tenure on wages for the white-collar workers.

Consequently, the empirical evidence totally coincides with the results of my model.

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<sup>16</sup>Dustmann and Meghir (2003) finds out that there is only an upward-sloping profile for the first two years of work experience.

## 4 Implications for the Labor Market Entry of College Graduates

In many European countries the late labor market entry of college graduates is a major issue. In these countries, there is an ongoing discussion about reforms reducing the completion time at universities.

Regarding completion time, clearly, time-to-degree varies among individuals. Thus, late graduation occurs only to a certain type of students. To be sure, students with higher ability and less working time while studying at university usually are faster than their counterparts. Moreover, the consumption of leisure postpones the labor market entry. Crucially, the decision to skip a term at college hinges on the opportunity costs, namely the post-college wage.

Referring to my model,<sup>17</sup> we have heterogeneity with respect to the major field of study. In order to model the labor market entry decision, I assume that the reservation wage while being a student  $w_c$  is higher than as a worker.<sup>18</sup> Using a dynamic approach, it is easy to show that the decision in each unit of time hinges on the following inequality:<sup>19</sup>

$$p_i(V_i - w_i) + w_i \geq w_c \quad \text{for } i = 1, 2. \quad (4.8)$$

The right-hand side of the inequality shows the benefit to postpone the study and the left-hand side consists in the expected foregone earnings. The deci-

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<sup>17</sup>In this section, I discuss the competitive market case.

<sup>18</sup>The assumption is based on the fact that students have more degrees of freedom than employees.

<sup>19</sup>Due to the zero profit condition, the decision does not depend on the seniority factor  $b$ . This occurs, since in my model the wage curve becomes steeper if the student postpones his labor market entry.

sion to enter the labor market depends on the individual characters  $V_i$ ,  $p_i$ ,  $w_i$  and the reservation wage  $w_c$ . In addition,  $p_i$  includes also the general situation at the labor market. Assuming that the difference between  $w_1$  and  $w_2$  are negligible small, we see that the higher the productivity and the higher the expected probability to stay in a job the higher the likelihood that students enter the labor market. Moreover, if there is no difference in productivity, variations in  $p_i$  are sufficient to explain differences in time-to-degree.

Comparing the return to schooling among the major fields, there are sizeable distinctions. Empirical papers by Naylor, Smith, and McKnight (2002), Blackaby, Murphy, and O'Leary (1999) and Blundell, Dearden, Goodman, and Reed (2000) have shown that there is a remarkable difference in returns between a degree in Law and Humanities. The papers analyze the post-college wages for British graduates. However, a study by Ederer and Schuller (1999) confirms the finding also for German students.

Given these empirical facts, my model predicts that Humanity students have a longer completion time than Law students because they have lower opportunity costs.

Using the German data from Wissenschaftsrat (2001) and controlling for the standard time of study, the completion time for the median student in Humanities lies 48,8 percent above the standard time, whereas the median student in Law needs 1,1 percent less than the standard time to attain his degree. Evidently, the time-to-degree varies enormously between both fields and corresponds completely to the results of my model.<sup>20</sup>

Moreover, due to my basic model, Law students obtain a steeper wage-tenure profile than students in Humanities. Unfortunately, there is no empirical paper which could confirm my prediction about seniority wages for college

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<sup>20</sup>Due to the lack of information about ability and wealth status within each field, the difference may be less after controlling for these parameters.

graduates.

## 5 Conclusion

In this paper, I developed a model which explains wage-tenure profiles for heterogeneous labor. My findings suggest that highly productive individuals receive steeper profiles than their counterparts. In addition, only the wage tenure curve of highly productive individuals is strictly positive sloped. I also find strong empirical evidence for my findings.

In section 4 the paper explains why the time to degree varies among major field of studies. Undoubtedly, the expected probability to receive a job and the individual valuation of the firm influence the labor market entry decision. Since Law students possess higher values in both parameters than students in Humanities, the completion time for Law students is dramatically shorter. Finally, the model also predicts that Law students face steeper wage-tenure profiles.

## Appendix

The Kuhn-Tucker conditions are for the monopoly case:

$$\begin{aligned} \frac{\partial L}{\partial a_1} &= -p_1(T - t_1) + \mu(p_1(T - t_1) + t_1) + t_1 + \nu(p_1(T - t_1) + t_1) \\ &+ \rho(-p_2(T - t_1) - t_1) \leq 0 \quad a_1 \geq 0 \quad \frac{\partial L}{\partial a_1} a_1 = 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial b_1} &= \frac{t_1^2}{2} + p_1\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right) + \mu\left(\frac{t_1^2}{2} + p_1\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right)\right) + \nu\left(\frac{t_1^2}{2} + p_1\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right)\right) \\ &+ \rho\left(-\frac{t_1^2}{2} - p_2\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right)\right) \leq 0 \quad b_1 \geq 0 \quad \frac{\partial L}{\partial b_1} b_1 = 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial a_2} &= p_2(T - t_1) + \omega(p_2(T - t_1) + t_1) + t_1 + \nu(-p_1(T - t_1) - t_1) \\ &+ \rho(p_2(T - t_1) + t_1) \leq 0 \quad a_2 \geq 0 \quad \frac{\partial L}{\partial a_2} a_2 = 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial b_2} &= \frac{t_1^2}{2} + p_2\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right) + \omega\left(\frac{t_1^2}{2} + p_2\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right)\right) \\ &+ \nu\left(-\frac{t_1^2}{2} - p_1\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right)\right) + \rho\left(\frac{t_1^2}{2} + p_2\left(\frac{T^2}{2} - \frac{t_1^2}{2}\right)\right) \leq 0 \quad b_2 \geq 0 \quad \frac{\partial L}{\partial b_2} b_2 = 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial \mu} &= a_1 t_1 + \frac{1}{2} b_1 t_1^2 + p_1\left(T a_1 + \frac{T^2 b_1}{2} - a_1 t_1 - \frac{1}{2} b_1 t_1^2\right) - T w_1 \\ &+ (1 - p_1)(T w_1 - t_1 w_1) \geq 0 \quad \mu \geq 0 \quad \frac{\partial L}{\partial \mu} \mu = 0, \end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \omega} &= a_2 t_1 + \frac{1}{2} b_2 t_1^2 + p_2 \left( T a_2 + \frac{T^2 b_2}{2} - a_2 t_1 - \frac{1}{2} b_2 t_1^2 \right) - T w_2 \\ &+ (1 - p_2)(T w_2 - t_1 w_2) \geq 0 \quad \omega \geq 0 \quad \frac{\partial L}{\partial \omega} \omega = 0.\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \rho} &= -a_1 t_1 + a_2 t_1 - \frac{1}{2} b_1 t_1^2 + \frac{1}{2} b_2 t_1^2 - p_2 \left( T a_1 + \frac{T^2 b_1}{2} - a_1 t_1 - \frac{1}{2} b_1 t_1^2 \right) \\ &+ p_2 \left( T a_2 + \frac{T^2 b_2}{2} - a_2 t_1 - \frac{1}{2} b_2 t_1^2 \right) \geq 0 \quad \rho \geq 0 \quad \frac{\partial L}{\partial \rho} \rho = 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \nu} &= a_1 t_1 - a_2 t_1 + \frac{1}{2} b_1 t_1^2 - \frac{1}{2} b_2 t_1^2 + p_1 \left( T a_1 + \frac{T^2 b_1}{2} - a_1 t_1 - \frac{1}{2} b_1 t_1^2 \right) \\ &- p_1 \left( T a_2 + \frac{T^2 b_2}{2} - a_2 t_1 - \frac{1}{2} b_2 t_1^2 \right) \geq 0 \quad \nu \geq 0 \quad \frac{\partial L}{\partial \nu} \nu = 0.\end{aligned}$$

Solving the above system of equations yields

$$\omega = 1,$$

$$\mu = 1,$$

$$\rho = 0,$$

$$\nu = 0,$$

and

$$a_1 < \frac{-(p_1(T-t_1)+t_1)(t_1^2+p_2(T^2-t_1^2))w_1+(p_2(T-t_1)+t_1)(t_1^2+p_1(T^2-t_1^2))w_2}{T(p_1-p_2)(T-t_1)t_1} < a_2,$$

$$b_1 > \frac{2(p_1(T-t_1)+t_1)(p_2(T-t_1)+t_1)(w_1-w_2)}{T(p_1-p_2)(T-t_1)t_1} > b_2.$$

q.e.d.

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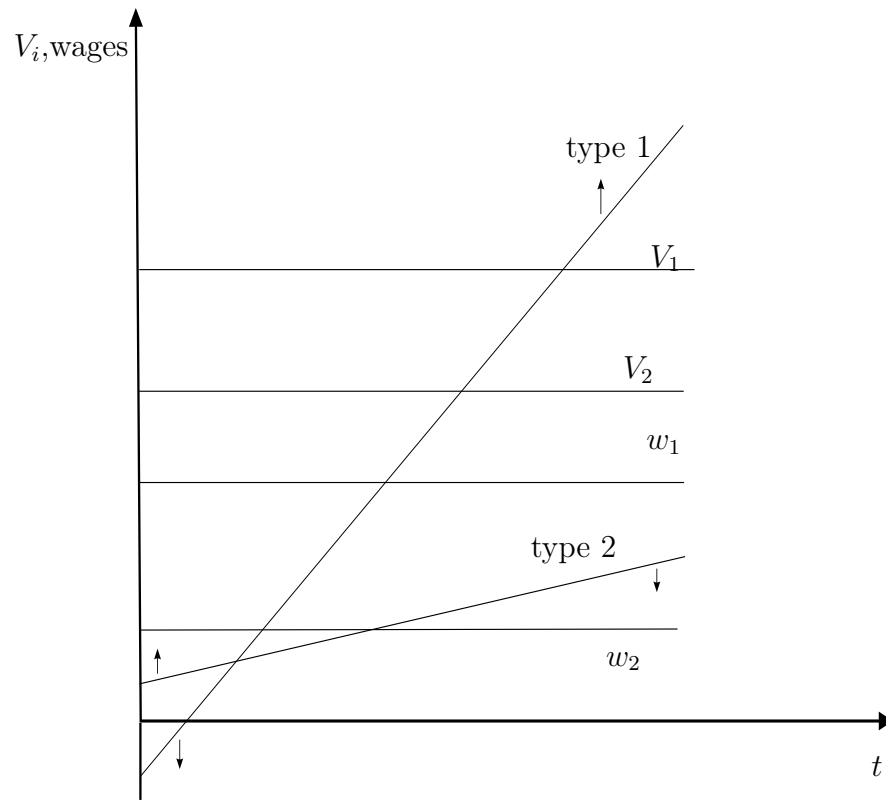


Figure 2.1: Wage profiles for heterogeneous labor: The monopoly case.