

Exact ground-state properties of disordered Ising systems

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Exact ground states are calculated with an integer optimization algorithm for two- and three-dimensional site-diluted Ising antiferromagnets in a field (DAFF) and random field Ising ferromagnets (RFIM), the latter with Gaussian- and bimodal-distributed random fields. We investigate the structure and the size distribution of the domains of the ground state and compare it to earlier results from Monte Carlo (MC) simulations for finite temperature. Although DAFF and RFIM are thought to be in the same universality class we found differences between these systems as far as the distribution of domain sizes is concerned. In the limit of strong disorder for the DAFF in two and three dimensions the ground states consist of domains with a broad size distribution that can be described by a power law with exponential cutoff. For the RFIM this is only true in two dimensions while in three dimensions above the critical field where long-range order breaks down the system consists of two infinite interpenetrating domains of up and down spins—the system is in a two-domain state. For DAFF and RFIM the structure of the domains of finite size is fractal and the fractal dimensions for the DAFF and the RFIM agree within our numerical accuracy supporting that DAFF and RFIM are in the same universality class. Also, the DAFF ground-state properties agree with earlier results from MC simulations in the whole whereas there are essential differences between our exact ground-state calculations and earlier MC simulations for the RFIM which suggested that there are differences between the fractality of domains in RFIM and DAFF. Additionally, we show that for the case of higher disorder there are strong deviations from Imry-Ma-type arguments for RFIM and DAFF in two and three dimensions.

I. INTRODUCTION

Many aspects of the influence of random field disorder on a system of interacting spins are still not well understood (for a review, see Ref. 1). Since it has been argued that the diluted Ising antiferromagnet in a homogeneous field (DAFF) and the random field Ising ferromagnet (RFIM) are in the same universality class^{2,3} experimental investigations focus on the DAFF (Ref. 4) as experimental realizations of a system with random field disorder while theorists usually focus on the RFIM.⁵

However, there have been investigations on the domain structures of the RFIM⁶ and the DAFF,^{7,8} both based on Monte Carlo (MC) simulations which suggest that there might be essential differences between DAFF and RFIM, at least as far as the domain structure is concerned. In the limit of strong disorder for the RFIM the domains are fractal in three dimensions and more compact in two dimensions⁶ while the domains of the DAFF have a fractal structure both in two and three dimensions.^{7,8} Additionally, essential deviations from Imry-Ma-type⁹ behavior which can be thought to be valid for small disorder have been found for both the RFIM and the DAFF but the corresponding exponents are different for these two systems. In order to clarify the situation and to prove how far the results from MC simulations which are out of equilibrium for these systems with frozen dynamics¹⁰ can be transferred to the equilibrium $T=0$ case we performed exact ground-state calculations on DAFF and RFIM with an integer optimization algorithm. With this method we get exact information on equilibrium properties at zero temperature.

We investigate the distribution of domain sizes and the structure of the domains and we compare our findings for the

DAFF and the RFIM. Hopefully, the results may help to improve our understanding of the critical behavior of these systems. Apart from this also for an analytic description of the dynamics of a disordered system the knowledge of domain structures and distributions is often used as a starting point.^{10,11}

The Hamiltonian of the DAFF in units of the nearest-neighbor coupling constant J is

$$H = \sum_{\langle ij \rangle} \epsilon_i \epsilon_j \sigma_i \sigma_j - B \sum_i \epsilon_i \sigma_i \quad (1)$$

with the uniform field $B > 0$ on all sites of the quadratic $L \times L$, respectively, cubic $L \times L \times L$ lattice. Here $\sigma_i = \pm 1$ denotes Ising spins and a fraction p of the sites is occupied with a spin (quenched disorder: $\epsilon_i = 0, 1$). The phase diagram of the two-dimensional (2D) DAFF is thought to consist of an antiferromagnetic low-temperature phase for magnetic field $B=0$ and a disordered phase for all finite values of B . In three dimensions there exists a long-range ordered phase^{12,13} for magnetic fields B smaller than the critical fields B_c which is $B_c \approx 1.4$ for a dilution of $p=0.5$.⁷ For higher fields and low temperatures the DAFF develops a frozen disordered domain state in two and three dimensions. This domain state has many of the characteristics of a spin glass, as for instance a remanent magnetization and an irreversibility line scaling such as the de Almeida-Thouless line.⁷

The Hamiltonian of the RFIM in units of the nearest-neighbor coupling constant J is

$$H = - \sum_{\langle ij \rangle} \sigma_i \sigma_j - \sum_i B_i \sigma_i. \quad (2)$$

Here, all sites are occupied and the random fields B_i are taken from a Gaussian (Gaussian RFIM), respectively, bimodal ($\pm \Delta$ RFIM) probability distribution corresponding to

$$P(B_i) = \frac{1}{\sqrt{2\pi}\Delta} e^{-(B_i/\sqrt{2}\Delta)^2} \quad (3)$$

and

$$P(B_i) = \frac{1}{2} [\delta(B_i - \Delta) + \delta(B_i + \Delta)], \quad (4)$$

respectively.

As in the case of the DAFF the phase diagram of the 2D system is thought to consist of a long-range ordered low-temperature phase for zero random fields and a disordered phase for all finite values of the random field. In three dimensions for weak random fields and low temperatures there exists a long-range ordered phase.^{12,13} The critical value for the random field at which long-range order disappears was found to be $\Delta_c = 2.35$ at zero temperature.¹⁴ Even for the RFIM a possible glassy transition has been discussed.¹⁵

We calculate exact ground states using an optimization algorithm well known in graph theory. The Ising system is mapped on an equivalent transport network, and the maximum flow is calculated using the Ford-Fulkerson algorithm.¹⁶⁻¹⁸ In order to investigate the distribution and the structure of domains we perform a cluster analysis with a suitable adjusted Hoshen-Kopelman-type algorithm.¹⁹ This algorithm pieces the system into domains, where a domain is defined as a group of spins which are connected and antiferromagnetically (DAFF), respectively, ferromagnetically (RFIM) ordered.

The investigations for the DAFF are carried out for systems of size 400×400 with an average over 50 different dilution configurations with $p = 0.7$ for each value of B . For some structural aspects even ground states of size up to 700×700 were calculated. Systems of size $50 \times 50 \times 50$ averaged over 40 dilution configurations with $p = 0.5$ are used for $D = 3$. The calculation of ground states for the RFIM is more time consuming. Here we considered 300×300 systems with 50 different random field configurations in 2D and 40 systems of size $30 \times 30 \times 30$ in 3D.

II. DISTRIBUTION OF DOMAIN SIZES

A. DAFF

In two dimensions for $B > 0$ no long-range order exists and the ground state of the system consists of antiferromagnetic domains of finite size. Within a ground state for each domain the domain-wall energy E_w , i.e., the number of broken bonds, must be smaller than the field-energy Bm_D , where m_D is the absolute value of the magnetization of the domain which is not zero in these antiferromagnetic domains due to the dilution. Therefore, only domains with $m_D \geq E_w/|B|$ exist in the system. The minimum wall energy of a domain is one broken bond, $E_w = 1$. Additionally, it is always $V_D \geq m_D$, where V_D is the size of the domain (number of spins), so that with $V_D \geq 1/|B|$ we get a rigorous lower cutoff for possible domain sizes. This relation indicates that except from the isolated spin clusters only large domains can

TABLE I. Field dependence for the exponent δ of the domain-size distribution for the 2D DAFF.

B	δ
0.6	1.76 ± 0.06
0.9	1.76 ± 0.06
1.2	1.68 ± 0.04
1.4	1.61 ± 0.06
1.5	1.58 ± 0.05
2.2	1.44 ± 0.03
2.5	1.42 ± 0.06
2.8	1.37 ± 0.04
3.5	1.32 ± 0.05

be realized for very weak fields. Due to this finite-size effect we consider here fields $B \geq 0.5$ only and hence, all domain sizes $V_D \geq 3$ exist.

As has been shown earlier¹¹ for the two-dimensional case the distribution of domains in a DAFF is well described by a power law with an exponential cutoff,

$$N_D(V_D) \sim V_D^{-\delta} \exp(-V_D/V_0) \quad (5)$$

where V_0 depends strongly on the field. Because of the inaccuracy in the determination of V_0 no significant p dependence can be observed for this quantity. We increased the numerical effort in order to investigate a possible concentration and field dependence of δ . We find δ weakly decreasing for increasing field but no p dependence. The results are summarized in Table I.

Figure 1 shows the size distribution of the domain state of the three-dimensional DAFF for different fields. For the largest value of the field a reasonable fit to Eq. (5) can be carried out yielding $\delta = 1.8 \pm 0.3$ and $V_0 = 50 \pm 20$. However, a detailed investigation of the field or concentration dependence of V_0 and δ is not possible with satisfactory precision for the following reasons: As has been shown earlier⁸ for $B \approx B_c$ the domain state consists mainly of two large interpenetrating domains (for $B = 4$ these domains contain approximately 80% of the spins). In Fig. 1 the two isolated peaks in the size

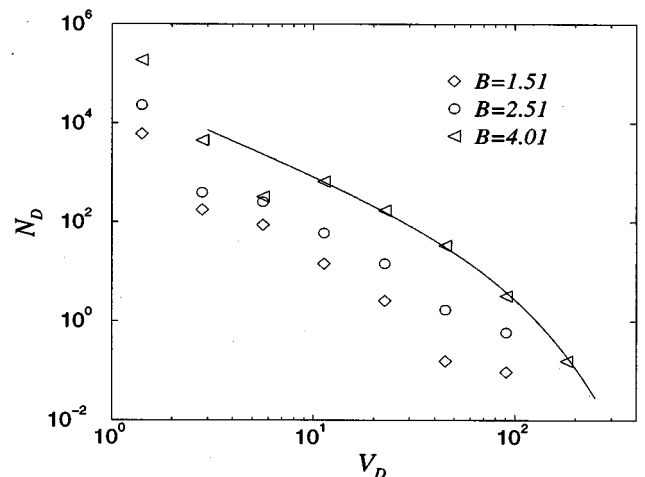


FIG. 1. N_D vs V_D for the DAFF in three dimensions. For $B = 4.01$ a fit to Eq. (5) is also shown as solid line.

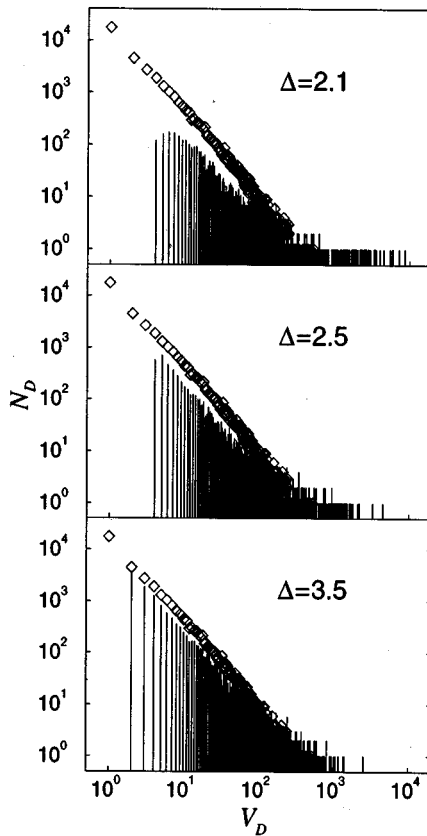


FIG. 2. Domain-size distributions for the two-dimensional $\pm\Delta$ RFIM (impulses) and size distribution of the random field clusters (open symbols).

distribution corresponding to the two percolating domains are not shown but due to the existence of these two infinite domains there are only few data for domains of finite size so that the statistics for these domains is rather bad. In the limit of very high fields the domains become very small. For $B > 6$ all spins are polarized by the field, i.e., all domains have size 1. Hence, there is only a small region of values of the magnetic field where a broad distribution of domain sizes exist. As we will show in the next subsection the existence of two infinite domains has even more dramatic consequences for the RFIM. Here, there are no values of random fields where relevant domains of finite size can exist.

B. RFIM

In the limit of strong random fields all spins follow the direction of the random field. Hence, the ferromagnetic domains are the clusters of sites with uniform random field signs. In this limit the domains of the RFIM correspond to clusters of the percolation problem²⁰ with a concentration of 50%.

The distribution of domain sizes for different Δ is shown in Fig. 2 for the two-dimensional $\pm\Delta$ RFIM. For comparison the size distribution for the percolation problem with a concentration of 50% is also shown. With increasing Δ , the domain-size distribution approaches the cluster-size distribution of the percolation problem which follows Eq. (5) with $\delta = 1.55 \pm 0.05$ and $V_0 = 120 \pm 20$.

For lower random fields there is a striking difference between the Gaussian RFIM on the one hand and the $\pm\Delta$

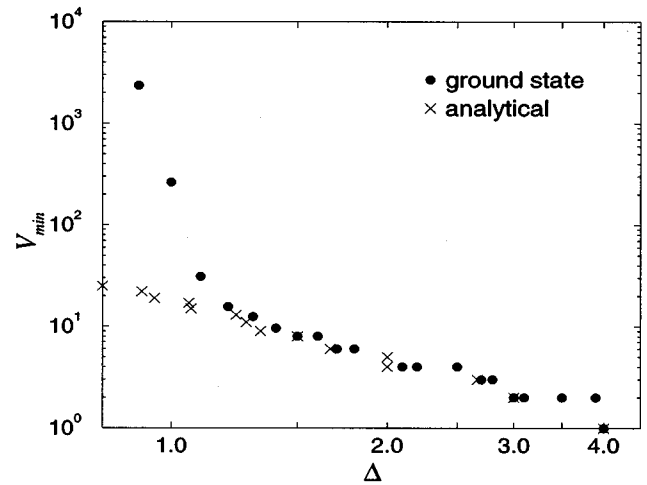


FIG. 3. Minimum domain size for given values Δ of the field of a $\pm\Delta$ RFIM in two dimensions.

RFIM on the other hand. For the latter system there is a lower cutoff for the possible domain sizes depending on the strength of the random field Δ . This lower cutoff is the only qualitative difference we found between $\pm\Delta$ and Gaussian RFIM. It can be seen in Fig. 2 and it can be determined analytically: Similar to the arguments given for the DAFF for each domain the relation $E_w < E_B = \sum_{i=1}^V \sigma_i B_i$ must hold. The positive domain-wall energy has to be compensated by the negative field energy. In the most favorable case all random fields acting on the spins of a domain have the same sign, $E_B = V_D \Delta$ so that for the smallest possible domains we get $V_D \geq E_w / \Delta$. With this relation as starting point a lower cutoff can be determined as the sizes of those domains which have a minimum surface. For large domain sizes the minimum surface of a domain is the surface of a circle (2D), $E_w \sim V_D^{1/2}$, respectively, sphere (3D), $E_w \sim V_D^{2/3}$. For smaller sizes, the domain with the minimum surface V_{\min} can be determined numerically. Figure 3 shows this dependence of V_{\min} from Δ for the $\pm\Delta$ RFIM in two dimensions. In addition, the minimum domain volumes of the ground states of the RFIM for different Δ are depicted. For $\Delta \leq 1$ there is a strong deviation of the observed domains of minimum size from the theoretical curve. Obviously, this is due to the fact that for larger domains it is less probable that all random fields within a domain with minimized surface have the same orientation. Hence, either the field energy E_B is overestimated or the shapes of the domains are more complicated than those with minimum surface. The first possibility would correspond to a crossover to a behavior following the Imry-Ma argument,⁹ where E_B is thought to scale with the root of the domain size due to simple statistical fluctuations of random fields within a domain. However, in the range of fields where this crossover may occur the minimum domain size is larger than the system sizes that we can investigate with numerical methods.

Figure 4 shows the volumes of the two largest domains in the ground state of the $\pm\Delta$ and the Gaussian RFIM. For lower fields $\Delta < 0.8$ the order parameter is finite. Obviously, this is the region where finite-size effects lead to wrong results for the system sizes we investigated since in this regime

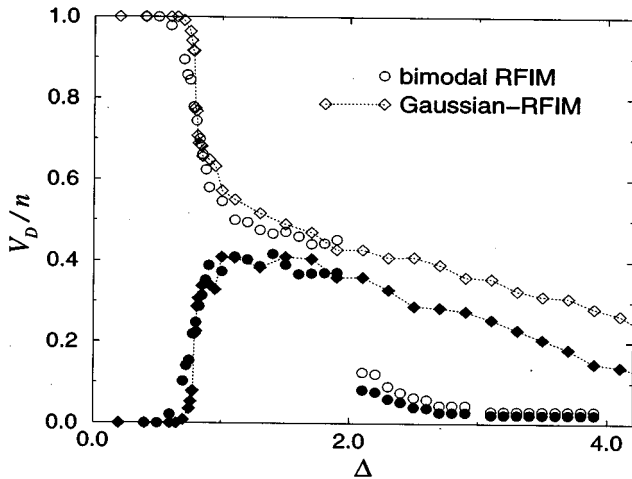


FIG. 4. Largest (open symbols) and second largest (filled symbols) domain size divided by the system size $n=L \times L$ for the $\pm \Delta$ RFIM in two dimensions.

minimum domain sizes are larger than the system. Therefore, in the following we consider only results for $\Delta > 0.8$.

An additional, striking feature of Fig. 4 is the jump in the data for the $\pm \Delta$ RFIM at $\Delta = 2$. Considering Fig. 5 (left) one realizes $\Delta = 2$ as a special value: The random fields are marked with $+$ and $-$ signs and the arrows represent the spin orientations (\uparrow spins are oriented parallel to a positive Δ). The energy of the center spin with $\sigma = \pm 1$ is

$$E_\sigma = 2\sigma - \sigma\Delta. \quad (6)$$

In the ground state it is $\sigma = +1$ for $\Delta > 2$ and the bold drawn domain wall is realized. $\Delta < 2$ favors $\sigma = -1$ and the center spin belongs to the down domain. Generally, for $\Delta > 2$ the domain wall arranges in such a way that the sign of the random fields changes at each position of a domain wall, i.e., the domain walls run along the surfaces of the random field clusters (clusters of sites with uniform random field sign). Figure 5 (right) demonstrates that nevertheless even for $\Delta > 2$ domains larger than these random field clusters can exist. A cluster with a magnetization that is oriented antiparallel to the random fields is enclosed such that its surface lies within the domain. Hence, for $\Delta > 4$ each domain is a random field cluster while for $2 < \Delta < 4$ each domain wall is a part of the surfaces of the random field clusters. Consequently, the domains in this range of Δ are larger. In three dimensions each spin has six next neighbors. Here, the corresponding effect occurs at $\Delta = 4$.

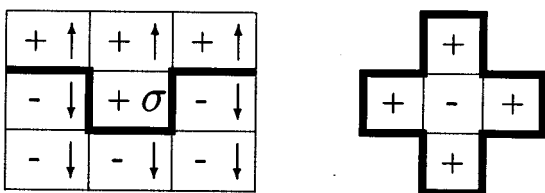


FIG. 5. $\pm \Delta$ RFIM; left: cut of a domain wall; right: a domain for $\Delta > 2$ consisting of multiple random field clusters.

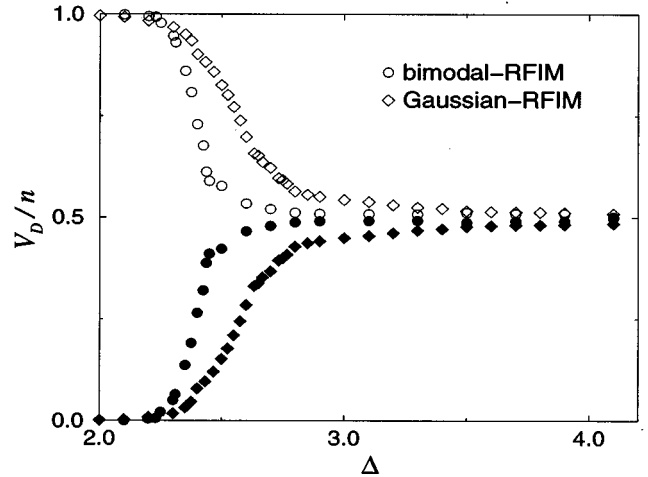


FIG. 6. Largest and second-largest domain size corresponding to Fig. 4 for the RFIM in three dimensions.

Both, for $D=2$ and $D=3$ the RFIM approaches the percolation problem with a concentration of 50% for large Δ , but while for $D=2$ in this case there exists no percolating cluster of random fields in three dimensions 50% is above the percolation threshold. This means that as mentioned above for the case of the DAFF in three dimensions one will find two infinite interpenetrating domains.

The normalized volumes of the largest and second-largest domain for the $D=3$ RFIM are shown in Fig. 6. Adding the sizes of these two largest domains we found that more than 97% of the spins of the system belong to these two domains in the whole range of random fields. Consequently, the phase transition from the ordered to the disordered state is not due to a breakup of the system into finite domains. Instead, two infinite interpenetrating domains with antiparallel oriented magnetizations cause the order parameter to vanish.

In contrast to $D=2$ there is no broad range of domain sizes for $D=3$ and in contrast to the DAFF there is no range of fields where a relevant number of domains of finite size exist. For this reason it is senseless to analyze the distribution of the domains in more detail.

III. DOMAIN STRUCTURES

A. DAFF

In a DAFF, domains are composed in such a way that as many spins as possible are oriented parallel to the applied field with a minimum number of broken bonds. Hence, domain walls favorably run along vacancies to minimize the wall energy. These two competing requirements for minimizing the total energy lead to highly nontrivial structures. Figure 7 shows the antiferromagnetic domains in the ground state. Here, the configuration of occupied sites and vacancies is kept fixed, while the external field B is tuned. In order to investigate the structural properties of the domains we consider the following quantities and corresponding scaling relations: (i) volume V_D : number of spins, (ii) surface S_D : sum of all unsatisfied bonds (broken or “open” due to vacancies) with $S_D \sim V_D^{1/D_{vf}}$, (iii) radius of gyration R_D , i.e., root of mean squared distance of all spins of a domain with

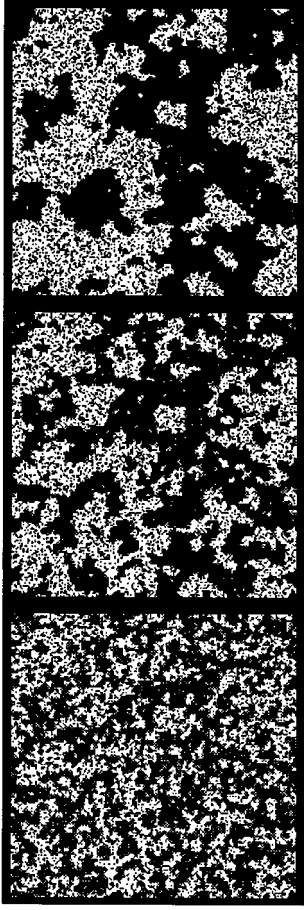


FIG. 7. Ground states of a 200×200 DAFF with a fixed dilution configuration for increasing fields $B=0.8, 1.4, 2.2$ (from above). The two antiferromagnetic phases are represented in black and white. The vacancies are also black.

$V_D \sim R_D^{D_v}$, (iv) absolute value of the magnetization of a domain m_D with $m_D \sim V_D^\theta$, and (v) wall energy E_W , sum of all broken bonds with $E_W \sim V_D^\sigma$.

For large domains the scaling relations above hold and the exponents are determined by analyzing all domains in the ground states and taking an average of the quantities above for all domains of equal size V_D . In Ref. 11 we showed an example for the two-dimensional case. In the meantime we investigated the three-dimensional system also. Our resulting exponents are summarized in Table II and compared to the values resulting from MC simulations.^{7,8}

The domains are fractal with fractal dimension $D_v = 1.65$ for $D=2$ and $D_v = 2.14$ for $D=3$. This fractality is also reflected in the proportionality of surface and volume

TABLE II. Structural exponents for the DAFF.

D	MC simulation		Exact ground states	
	2	3	2	3
θ	1.00 ± 0.01	0.996 ± 0.001	0.97 ± 0.01	0.99 ± 0.01
σ	0.98 ± 0.01	0.995 ± 0.001	0.96 ± 0.02	0.98 ± 0.03
D_{vf}	1.001 ± 0.002	1.000 ± 0.001	1.00 ± 0.01	1.00 ± 0.01
D_v	1.56 ± 0.03	2.0 ± 0.1	1.64 ± 0.06	2.14 ± 0.08

($D_{vf} \approx 1$). A great part of the surface is inside the domains. Slight deviations of exponents from exact ground states from those resulting from MC simulation are due to a systematic inaccuracy which is not reflected by the error bars representing the fluctuations of the data only. Probably, a small, systematic error is due to the fact that the data are slightly curved in a log-log plot so that the linear range for large domains can hardly be determined (see also the corresponding Fig. 10 for the RFIM). From comparison with the corresponding figures^{7,8} it is found that in spite of similar exponents the values $m_d(V_D)$ are larger for exact ground states—the domains of the ground states are obviously better optimized.

In good agreement with MC simulations is $\theta \approx 1$ both in two and three dimensions. Altogether the fractal structures for strong disorder lead to deviations from Imry-Ma-type arguments⁹ based on compact domains. Here, a statistical distribution of vacancies in a domain would yield $\theta \approx 1/2$ which can clearly be ruled out by our data for the range of fields that we investigated. The implication of this finding will be discussed in connection with the RFIM where we have found similar results. A field dependence of any of the exponents could not be found.

B. RFIM

As we showed in the previous section there is no broad distribution of domains for the three-dimensional RFIM—domains of finite size are irrelevant for the ground state. Therefore, in this subsection we restrict ourselves to the case $D=2$.

Figure 8 shows the ferromagnetic domain configuration of a $\pm \Delta$ RFIM in the ground state. The configuration of signs of the random fields is kept fixed, while the value Δ of the field is tuned. As for the DAFF we study the structural quantities volume, surface and radius of the domains. Since there is no dilution the surface and the domain wall energy are identical. Also, the magnetization and the volume of the domains are identical. We study the following quantities: (i) local random field fluctuations b_D , which is the absolute value of the sum of the random fields of a domain, (ii) random field fluctuations of all sites of a domain along the domain wall inside the domain b_{in} , (iii) random field fluctuations of all sites of a domain along the domain wall outside the domain b_{out} .

Figure 9 shows b_{in}/S_D and b_{out}/S_D each multiplied with the sign of the magnetization of the considered domain for the $\pm \Delta$ RFIM. As we demonstrated with Fig. 5 for $\Delta > 2$ the domain walls run exactly along the boundaries of random field clusters. Hence, in this range of fields it is $|b_{in}/S_D| = |b_{out}/S_D| = 1$. Even for smaller fields the domain walls are presumably located along these random field surfaces and for large domains these fluctuations are proportional to the surface of the domains.

Corresponding to the DAFF the exponents D_v , D_{vf} , and θ with $b_D \sim V_D^\theta$ are determined from appropriate fits (Fig. 10). In Table III the exponents are summarized. The exponents resulting from MC are from Ref. 6.

The investigations are carried out for relative strong random fields, $\Delta = 3.01$ (Gaussian RFIM) and $\Delta = 4.01$ ($\pm \Delta$ RFIM), because then—as explained in the previous

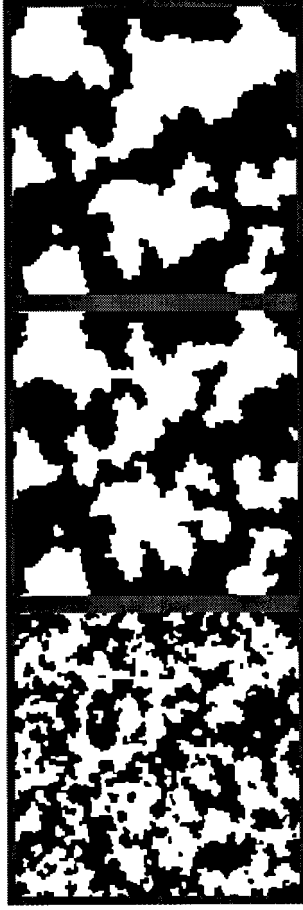


FIG. 8. Ground states of a $200 \times 200 \pm \Delta$ RFIM for a fixed configuration of random fields and $\Delta = 1.0, 1.1, 1.5$ (from above). The two ferromagnetic phases are represented in black and white.

chapter—a broad range of domain sizes exists. It is found that the exponents do not depend on the kind of random field distributions within the given precision.

In agreement with the findings for the DAFF the domains are fractal with nearly the same exponents $D_v \approx 1.6$ and

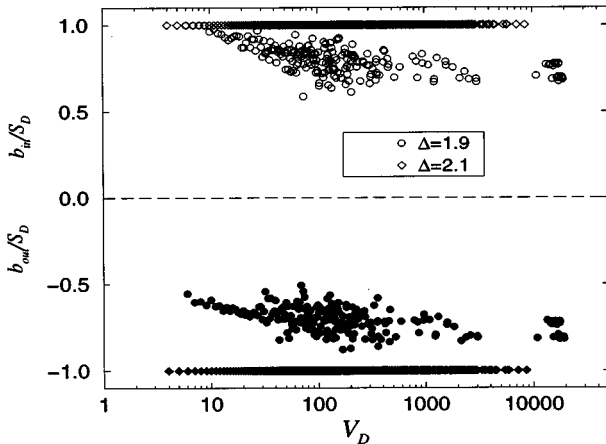


FIG. 9. Random field fluctuations b_{out} and b_{in} of the domain surfaces each multiplied with the sign of the domain-magnetization and divided by the surface S_D for the two-dimensional $\pm \Delta$ RFIM.

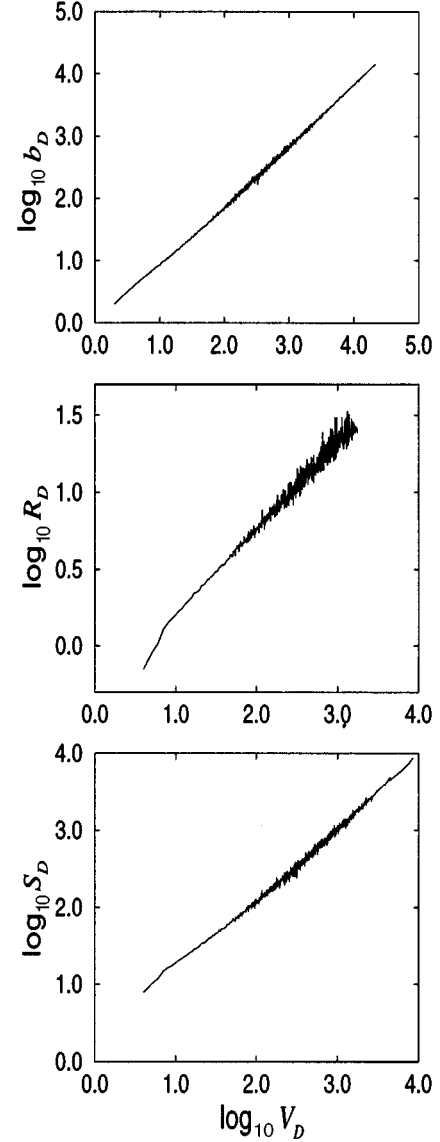


FIG. 10. Figures for the determination of the structural exponents θ , D_v , and D_{vf} for the two-dimensional $\pm \Delta$ RFIM.

$D_{vf} \approx 1$. Also, it is $\theta \approx 1$ indicating that the random fields are distributed nonstatistically within a domain. These results are in contradiction to the earlier investigations using MC methods⁶ where results were found corresponding to more compact domains. We interpret this discrepancy as being due to the fact that the MC simulations were carried out with $\Delta = 1$ and $T/k_B = 1$. As we showed before for this value of Δ only very large domains can exist in the ground state. Therefore, the broad range of analyzed domains in Ref. 6

TABLE III. Structural exponents for the 2D RFIM; $\Delta = 4.01$ for $\pm \Delta$ and $\Delta = 3.01$ for Gaussian-distributed random fields.

	MC simulation		Exact ground states	
	$\pm \Delta$	Gaussian	$\pm \Delta$	Gaussian
D_v			1.65 ± 0.08	1.67 ± 0.08
D_{vf}	0.59 ± 0.04		0.98 ± 0.05	0.97 ± 0.05
θ	0.66 ± 0.04		1.00 ± 0.01	1.00 ± 0.01

results obviously from thermal fluctuations. Apart from this for small domains one cannot observe a hierarchy of domains inside domains which is an important feature of the domains we observed. For larger fields and lower temperature we would expect agreement of these two methods above.

Our results deviate once more from the assumption of Imry-Ma-type random field fluctuations⁹ for compact domain structures with $\theta=1/2$. Also, no Δ dependence for our exponents can be determined as indication for a crossover to Imry-Ma behavior within the range of fields that we investigated.

We interpret this as being due to the fact that for the strength of disorder we consider domains are not compact and they do not have a shape for which a statistical argument holds. Instead very complicated structures arise. Domains are interpenetrating and there is a hierarchy of domains inside domains so that even large domains for which the Imry-Ma argument was thought to hold can draw advantage from random field fluctuations on small length scales.

Note that all exponents found for the fractal behavior of the domains of the RFIM as well as the DAFF are very close to those for the lattice animals of the percolation problem.^{20,21} This is plausible since as mentioned before the domains of the RFIM in the limit of high random fields have the same distribution and structure like the clusters from the percolation problem. As we discussed in the previous subsection systematic inaccuracies are not reflected by the error bars and systematic error can be due to a slight curvature of the data.

IV. CONCLUSIONS

We investigated the distribution and structure of the domains of the ground state of DAFF and RFIM in two and three dimensions. Since except for the Gaussian RFIM there is always a lower cutoff for the minimum domain size one is restricted to rather large disorder due to finite size effects, i.e., high dilution and fields of the order of the spin-spin coupling constant.

The domain-size distribution for the DAFF can be well described by a power law with a field-dependent exponential cutoff for a broad range of the applied fields in 2D and for strong fields ($B>3$) in 3D. For the RFIM with increasing random field strength there is a continuous transition to the cluster-size distribution of the percolation problem which also follows a power law with an exponential cutoff. For the $\pm\Delta$ RFIM a nontrivial minimum domain size is found, that can be estimated based on energetic considerations.

Investigating structural properties we have found that the finite domains of the domain state are fractal for the DAFF ($D=2,3$) as well as for the RFIM ($D=2$) with the same fractal exponents. Surprisingly, the domain state of the RFIM in 3D is found to consist mainly (>97%) of two infinite interpenetrating domains of opposite phase in the whole range of random fields for which the long-range order is broken. In this sense, the phase diagram of the RFIM at $T=0$ consists only of two regions, a one-domain state (long-range order below the critical field) and this two-domain state.

While for the RFIM the results for the structural scaling exponents differ essentially from earlier findings at finite temperature carried out using MC simulations, our results are in good agreement with Monte Carlo simulations for the DAFF both in two and three dimensions. Also, the results for DAFF and RFIM agree.

Due to the complex shape of the domains the magnetization of the domains (DAFF) and the distribution of random fields within the domains (RFIM), respectively, strongly deviates from the assumptions of the Imry-Ma argument which, however, can still be thought of as correct in the limit of small disorder. A more detailed investigation of the structure of domains for weak random fields is desirable. Unfortunately, for this the numerics exceeds current capacities.

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