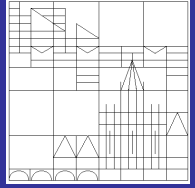




University of Konstanz  
Department of Economics



# Endogenous Credit Constraints, Human Capital Investment and Optimal Tax Policy

*Hongyan Yang*

Konstanz Working Paper  
2010-04

*Konstanzer Online-Publikations-System (KOPS)*  
URL: <http://nbn-resolving.de/urn:nbn:de:bsz:352-134367>

<http://www.wiwi.uni-konstanz.de/workingpaperseries>

# Endogenous Credit Constraints, Human Capital Investment and Optimal Tax Policy\*

Hongyan Yang  
University of Konstanz

September 30, 2010

## Abstract

This paper employs a two-period life-cycle model to derive the optimal tax policy when educational investments are subject to credit constraints. Credit constraints arise from the limited commitment of debtors to repay loans and are endogenously determined by private banks under the non-default condition that individuals cannot be better off by defaulting. We show that the optimal redistributive taxation trades the welfare gain of reducing borrowing demand and of changing the credit constraints against the efficiency costs of distorting education and labor supply. In addition, we compare the optimal taxation with that when credit constraints are taken as given. If income taxation decreases (increases) the borrowing limit, taking credit constraints as given leads to a too high (low) labor tax rate. Thus, ignoring the effects of tax policy on credit constraints overestimates (underestimates) the welfare effects of income taxation. Numerical examples show that income taxation tightens the credit constraints and the optimal tax rates are lower when credit constraints are endogenized. The intuition is that redistributive taxation reduces the incentive to invest in education and to work, thus exaggerating the moral hazard problems associated with credit constraints.

*JEL classification:* H21, I2, J2

*Keywords:* labor taxation, human capital investment, credit constraints

---

\*I am grateful to Dan Anderberg, Søren Bo Nielsen, Dominik Sachs as well as to participants in the research seminar on public finance at CESifo institute for very valuable comments. Financial support by the Fritz-Thyssen-Stiftung is gratefully acknowledged.

# 1 Introduction

Redistributive policy and education subsidies are often justified by the existence of credit constraints faced by poor individuals when they invest in education. Credit constraints arise from the limited commitment of debtors to repay loans. In addition, moral hazard problems due to the non-observability of effort in education as well as in labor supply make human capital bad collateral and thus make the credit for educational investment more difficult to access. In the presence of credit constraints, family income would play an important role in determining educational attainment. Using the changes in income distribution in the US as instrument, Acemoglu and Pischke (2001) find that family income is the main explanation for the different enrollment rates of children from different income groups. Belley and Lochner (2007) conclude that the importance of income has substantially increased from the early 1980s to the early 2000s, after comparing the causal effects of income on educational attainment found in the NLSY97 data with those found in the NLSY79 data. Lochner and Monge-Naranjo (2008) show further that the increase in the role of family income can only be explained by more severe credit constraints in response to increasing income inequality and rising tuition costs. Evidence for credit constraints is found as well in other empirical studies. Kane (1995) and Van der Klaauw (2002) identify large impacts of financial aids and tuition costs on college enrollment, which indicates the presence of credit constraints. Stinebricker and Stinebricker (2008) find an important causal role of credit constraints in explaining the drop-out decision of students from low-income families. Credit constraints are also shown by Kean and Wolpin (2001) to significantly affect students' consumption and working choices. Kane (1996) argues that borrowing constraints can explain the higher tendency of delayed entrance in college observed in high tuition states.

To mitigate the negative effects of credit constraints, optimal policy should feature public provision of education and redistribution from rich to poor individuals (see e.g. Glomm and Ravikumar, 1992; Fernandez and Rogerson, 1996, 1998 and Benabou, 1996). However, these studies take credit constraints as given and ignore the fact that governmental policy can change credit constraints by affecting the incentive to repay. Therefore, optimal policy derived under the assumption of exogenously given credit constraints could be misleading. Krueger and Perri (1999) show that a more redistributive taxation can exaggerate credit constraints and leads to a lower welfare. Andolfatto and Gervais (2006) argue that education subsidies and a pension program financed by taxes on working population may actually lead to a lower level of human capital investment when the effects of taxation on credit constraints are considered. By simulating an educational investment model with endogenously determined credit constraints, Lochner and Monge-Naranjo (2002) find that education subsidies have substantially greater effects than implicated in a similar model with exogenous credit constraints.

This paper aims to analyze optimal tax policy when individuals face credit constraints in educational investment, while taking the effects of taxation on credit constraints into account. To that end, we employ a two-period life-cycle model with identical agents, who invest in education in the first period and work in the second period. There are private banks providing loans for the individuals. We assume that in case of default banks can force the debtor to pay back the loans if the repayment is covered by the collateral, which is a given fraction of the debtor's earnings. Otherwise, banks can only seize that part of the earnings. Moreover, the debtor has to pay a fixed cost associated with the enforcement of repaying. We make the further assumption that educational investment is not verifiable. Educational investment in this paper is mainly secondary and higher education. The costs of education include tuition fees, expenditure for computer and books etc, additional costs for accommodation and forgone earnings. These costs are hardly verifiable except for tuition fees, which are only a small share of the total costs.<sup>1</sup> Labor supply in the second period is not observable either. The non-observability of education and labor supply leads to the moral hazard problems that the collateral, i.e. the debtor's seizable income, is unknown when credit is extended. To avoid defaults, banks would limit the amount of credit the individuals can take. Under the assumption of a perfectly competitive loan market the equilibrium borrowing limit is determined by the condition that the individuals are indifferent between repaying and defaulting.

We show that, when credit constraints are binding, both educational investment and the first period consumption are too low compared to those without credit constraints. When government has access to age-specific transfers, the optimal policy is to transfer income from the second to the first period via lump-sum transfers, which is in fact governmental loans. When age-specific transfers are not available, government has to use distortionary labor tax to alleviate credit constraints. We derive the optimal linear labor tax rate, which trades the welfare effects of redistributing income across the life-cycle periods and the welfare effects of changing the borrowing limit against the efficiency costs of distorting education and labor supply. Compared to the case where credit constraints are taken as given, the optimal labor tax rate is lower (higher) if redistributive taxation tightens (relaxes) the borrowing limit. Numerical examples show that for reasonable model parameters the borrowing limit decreases with labor tax rate; and ignoring the effects of tax policy on credit constraints overestimates the welfare effects of taxation. The intuition is that individuals planning to default invest less in education and work less in order to reduce default costs. Redistributive taxation reduces the incentive to invest in education and to work, thus exaggerating the moral hazard problems associated with credit constraints.

---

<sup>1</sup>Becker (1964) and Boskin (1975) show that the costs of goods investment and time investment (which is the forgone earnings) in the total costs of education is one-quarter and three-quarters. The share of tuition costs in total costs are thus less than one-quarter.

This paper is closely related to Jacobs and Yang (2010), who analyze the optimal taxation under exogenous credit constraints. They show that a redistributive taxation mitigates credit constraints by redistributing from high-income to low-income (and constrained) life-cycle periods. In this paper we endogenize credit constraints and show how this affects the optimal tax policy. Close to our analysis is also Andolfatto and Gervais (2006), who also analyze the optimal tax policy under endogenous credit constraints. However, they only analyze the optimal age-specific lump-sum transfers and argue that it is optimal to redistribute from young and old to working individuals. In our model it would imply redistributing from working to young individuals. The completely opposite results of our paper arise from the different assumptions about default punishment. They assume that defaulting debtors are punished by being excluded from capital market. Consequently, redistributing income from old to working individuals relaxes credit constraints by increasing default costs, namely the costs of no-saving for old age. Differently, we assume that creditors can seize part of the debtors' earnings. Redistributing from working to young individuals via lump-sum transfers increases the costs of losing part of the earnings and relaxes credit constraints.

Lochner and Monge-Naranjo (2002) show that education subsidies relax credit constraints and have greater impact on human capital formation than in a similar model with exogenous credit constraints. Intuitively, education subsidies increase educational investment and thus future earnings. Default becomes less attractive since default costs, i.e., losing part of the earnings, increase. Their analysis assumes exogenous labor supply and the verifiability of educational investment. As a result, the amount that the creditors can seize in case of default is known when credit is extended. In this paper, however, the non-observability of educational investment and labor supply leads to moral hazard problems, which are the main reason why human capital is badly collateralizable. Krueger and Perri (1999) analyze quantitatively the effects of tax system on welfare with credit constraints. They show that, under plausible conditions, the increase of tax progressivity leads to a lower welfare by exaggerating credit constraints. Thus, our results seem to be consistent with their findings. However, they did not consider educational investment and labor supply decision.

The rest of the paper is organized as follows. In section 2 we lay out the model environment and derive the borrowing constraints under non-default condition. Section 3 analyzes the optimal tax policy for both the case of age-specific lump-sum transfers and the case of uniform transfers. Numerical examples are given in section 4. The last section concludes.

## 2 The Model

The economy is populated by a continuum of identical individuals whose mass is normalized to one. The individuals live for two periods. In the first period the representative individual does not work, but invests in education and consumes. Since we are interested in credit constraints, we assume that the individual does not have sufficient initial wealth to finance its optimal educational investment and consumption in the first period. Without loss of generality, we set the initial wealth to zero. After acquiring human capital, the agent supplies labor in the second period and consumes all its wealth. There are private banks providing loans for individuals on a perfectly competitive capital market. The economy is a small open economy, which implies that banks can raise funds at a constant interest rate  $r$ .

### 2.1 Preferences and human capital technology

The utility of the individuals is separable in consumption and labor and between periods

$$U = u(c^1) + \beta u(c^2) - \beta v(l), \quad (1)$$

with  $c^1$  and  $c^2$  denoting consumption in the first and the second period respectively and  $l$  the labor supply in the second period. The separability is assumed for reason of simplicity and does not affect the main results of the paper.  $\beta$  is the time preference. The subutility function  $u(\cdot)$  is assumed to be increasing and concave, whilst the disutility function  $v(\cdot)$  is increasing and convex. Furthermore, the Inada-conditions are fulfilled to avoid corner solutions.

When the agent is young, it decides on consumption  $c^1$ , investment in human capital  $e$  and the required borrowing  $a$ . The interest rate the agent pays is also  $r$ . The costs of human capital investment  $e$  are assumed to be only monetary and may include tuition fees, forgone earnings and additional costs for computer, accommodation and books etc. than what would otherwise occur. We assume that these costs are private information, since additional costs for computer, accommodation and books etc. and forgone earnings are badly verifiable. Although tuition fees are observable, the share of tuition fees in the total costs of education is almost zero for public secondary school and small for higher education. In the second period the agent becomes adult and supplies labor according to the wage rate  $w(e)$  with  $w'(e) > 0$  and  $w''(e) < 0$ . Inada-conditions for  $w(e)$  are fulfilled as well.

The tax system consists of a labor tax with the flat rate  $t$  and lump-sum transfers in both periods,  $g^1$  and  $g^2$ , where the superscript denotes the periods. The agent's budget

constraints for both periods are consequently

$$c^1 = -e + a + g^1, \quad (2)$$

and

$$c^2 = (1 - t) w(e) l - (1 + r)a + g^2. \quad (3)$$

Following standard literature in optimal taxation, only gross income is observable. Since educational investment is private information, neither wage rate  $w(e)$  nor labor supply  $l$  can be verified.

## 2.2 First-best allocation

For comparison we first describe the optimal allocation with a perfect capital market. The representative agent maximizes utility (1) subject to (2) and (3). As a result, consumption is smoothed according to the Euler's equation  $\frac{u_1}{\beta u_2} = 1 + r$ , where  $u_1$  and  $u_2$  denote the derivative of the first and the second period utility of consumption respectively. Moreover, the agent equals the marginal costs of education to its marginal return,  $(1 - t) w'(e) l = 1 + r$ , and the optimal labor supply is given by  $(1 - t) w(e) = v'(l)$ .

Labor taxation distorts educational investment as well as labor supply. Since we have identical agents and thus no distributional concern, there is no need for governmental intervention. First-best allocation could be achieved by setting labor tax rate to zero and using lump-sum taxes to raise exogenous governmental expenditure.

## 2.3 Endogenous credit constraint

Credit constraints arise from the agent's limited ability to commit itself to the repayment. We assume that a given fraction  $\gamma$  of the agent's future earnings is taken as collateral. If the agent defaults, banks can get the repayment  $(1 + r)a$  back if it is covered by the collateral. Otherwise, banks can only get that part of the earnings. Furthermore, the defaulting agent has to pay a fixed cost of  $F$ , which covers the banks' costs in processing defaults, e.g. costs for courts and garnishees. The default costs  $C$  can be summarized as follows:

$$C = \begin{cases} (1 + r)a + F & \text{if } (1 + r)a \leq \gamma(1 - t)w(e)l, \\ \gamma(1 - t)w(e)l + F & \text{if } (1 + r)a > \gamma(1 - t)w(e)l. \end{cases}$$

We assume that banks cannot seize lump-sum transfer the agent receives in the second period, which can be thought of as public goods and social insurance that cannot be seized. Punishment of exclusion from credit markets as in Kehoe and Levine (1993, 2000) and Andolfatto and Gervais (2006) is not considered in our two-period model.

An agent would default if the utility of defaulting is higher than that of repaying. Unlike in Lochner and Monge-Naranjo (2002, 2008), educational investment and labor supply in our model are private information that cannot be observed by banks. As a result, the amount of collateral, i.e. the fraction  $\gamma$  of the agent's future earnings, is unknown when credit is extended to the agent in the first period. The agent who plans to default can reduce default costs by reducing educational investment and labor supply. The non-default condition is thus that the agent cannot be better off by defaulting and by adjusting its choices correspondingly. Because the credit market is per assumption perfectly competitive, the equilibrium borrowing limit would be such that the agent is indifferent between repaying and defaulting.

Now we first derive the indirect utility of the agent when it plans to repay. After inserting budget constraints (2) and (3) in the utility function (1), the lagrangian function for the agent's maximization problem when credit limit is  $\bar{a}$  and when the agent repays is

$$\max_{e,a,l} \mathcal{L} = u(-e + g^1 + a) + \beta u((1-t)w(e)l - (1+r)a + g^2) - \beta v(l) + \mu(\bar{a} - a), \quad (4)$$

where  $\mu$  is the Kuhn-Tucker multiplier for the credit constraint  $a \leq \bar{a}$ .  $\mu$  gives the shadow price of relaxing credit limit by one unit. We assume that the agent is credit constrained, since the case of slack credit constraint is not interesting. This assumption implies that

$$a = \bar{a}; \mu = u_1 - \beta(1+r)u_2 > 0, \quad (5)$$

$$\frac{u_1}{\beta u_2} = (1-t)w'(e)l > 1+r, \quad (6)$$

$$\frac{v'(l)}{u_2} = (1-t)w(e). \quad (7)$$

A credit constrained agent cannot borrow the unconstrained optimal amount of credit to finance its consumption and educational investment. As a result, both first period consumption and educational investment are distorted downwards compared to the first-best allocation:  $\frac{u_1}{\beta u_2} > 1+r$  and  $(1-t)w'(e)l > 1+r$ . The agent would like to consume more and invest more in education if it could borrow more than  $\bar{a}$ .

Binding credit constraints act like an implicit tax on borrowing and educational investment. We define this implicit tax as

$$\pi = 1 - (1+r) \frac{\beta u_2}{u_1}. \quad (8)$$

Accordingly, we can rewrite the first-order-condition for educational investment as

$$(1-\pi)(1-t)w'(e)l = 1+r. \quad (9)$$



Therefore,  $\pi$  measures the extent to which the inter-temporal consumption and educational investment are distorted by credit constraints. The lower the credit limit  $\bar{a}$ , the higher is the implicit tax  $\pi$  and the more severe are the credit constraints. Substituting the optimal decisions given by the first-order-conditions into the utility function (1), we can denote the indirect utility of the repaying agent as a function of tax policy, interest rate and credit limit,  $V(t, g^1, g^2, r, \bar{a})$ .

The lagrangian function for the agent's maximization problem if it plans to default is

$$\begin{aligned} \max_{e, a, l} \mathcal{L} = & u(-e + g^1 + a) + \beta u((1-t)w(e)l - \min[(1+r)a, \gamma(1-t)w(e)l] - F + g^2) \\ & - \beta v(l) + \mu(\bar{a} - a). \end{aligned} \quad (10)$$

It follows immediately that defaulting always leads to a lower utility if  $(1+r)a \leq \gamma(1-t)w(e)l$ , since in this case defaulting only causes the additional cost of  $F$  and brings no benefit. Consequently, credit constraints can only arise where  $(1+r)a > \gamma(1-t)w(e)l$ . Therefore, we derive the first-order-conditions only for the case  $(1+r)a > \gamma(1-t)w(e)l$ .

Since the agent does not repay the loans in the second period, it would borrow as much as possible, i.e.  $a_d = \bar{a}$ . We use the subscript  $d$  to denote the variables in case of default. The first-order-conditions for the defaulting agent are

$$a_d = \bar{a}; \mu_d = u_{1d} > 0, \quad (11)$$

$$\frac{u_{1d}}{\beta u_{2d}} = (1-t)(1-\gamma)w'(e_d)l_d, \quad (12)$$

$$\frac{v'(l_d)}{u_{2d}} = (1-t)(1-\gamma)w(e_d). \quad (13)$$

We can see that the defaulting agent would choose education and labor supply differently than the agent who repays. Again, the indirect utility of the defaulting agent can be denoted as a function of tax policy, interest rate, credit limit and the punishment parameters,  $V_d(t, \gamma, F, g^1, g^2, r, \bar{a})$ .

For both maximization problems (4) and (10) the second-order-conditions require that the marginal utility of consumption should decrease sufficiently fast, the productivity of education in wage rate is not too high and the marginal disutility of labor should increase fast enough (see Appendix A.1). These conditions ensure that the positive feedback between education and labor supply is not too strong such that interior solutions are obtained. We assume that the second-order-conditions are always fulfilled.

We denote the optimal borrowing of the repaying agent with a perfect capital market as  $a^*$ , for given tax policy and interest rate. We make the assumption that

$$V(t, g^1, g^2, r, a^*) < V_d(t, \gamma, F, g^1, g^2, r, a^*), \quad (14)$$

which implies that the agent would default if it can borrow  $a^*$ . Consequently, no banks would lend the amount  $a^*$ , since they know for sure that the agent would default. This assumption ensures the existence of credit constraints.

The indirect utility  $V_d$  is increasing and concave in borrowing limit  $\bar{a}$ , since  $\frac{\partial V_d}{\partial \bar{a}} = \mu_d = u_{1d}$  applies and the first period consumption always increases with borrowing limit. The indirect utility  $V$  is increasing and concave in  $\bar{a}$  as well, as long as  $\bar{a} < a^*$ . This is because  $\frac{\partial V}{\partial \bar{a}} = \mu = u_1 - (1+r)\beta u_2$  is positive and decreasing in  $\bar{a}$  for  $\bar{a} < a^*$  and equal to zero for  $\bar{a} \geq a^*$ . From the concavity of both  $V$  and  $V_d$  in  $\bar{a}$  for  $\bar{a} < a^*$ , the fact that  $V(t, g^1, g^2, r, \bar{a}) > V_d(t, \gamma, F, g^1, g^2, r, \bar{a})$  for very small  $\bar{a}$  and the assumption (14), we can conclude that in a  $(\bar{a}, V)$  diagram  $V_d$  would cut  $V$  only once from below in the interval  $[0, a^*]$ . Note that for small loans where  $(1+r)a \leq \gamma(1-t)w(e)l$ , the first-order-conditions for the optimal choices of the defaulting agent are the same as for the repaying agent and  $V_d$  is always lower than  $V$ .

Denote the equilibrium borrowing limit as  $A$ , then  $A$  is determined by the equation

$$V(t, g^1, g^2, r, A) = V_d(t, \gamma, F, g^1, g^2, r, A). \quad (15)$$

Solving the equation (15), we can denote the equilibrium credit limit as the function  $A(t, g^1, g^2, r, \gamma, F)$ . By construction we have  $A < a^*$ . In equilibrium, banks would lend up to the amount of  $A$ . The borrowers would take the highest possible loan  $A$  and pay it back in the second period. If the agent borrows more than  $A$ , it would default.

Since both the marginal return to education and the net wage rate are lower for the defaulting agent, it is optimal to invest less in education and to work less if the agent plans to default. The following lemma compares the optimal choices in equilibrium by the agent if it plans to default and if it plans to repay.

**Lemma 1** *In equilibrium the agent who plans to default invests less in education and works less than the agent who plans to repay ( $e > e_d$  and  $l > l_d$ ). It follows straightforward from the first period budget constraint (2) and the condition for equilibrium  $V = V_d$  that  $c^1 < c_d^1$  and  $c^2 > c_d^2$ .*

Proof see Appendix A.2.

The comparative statics of the credit limit  $A$  depend on how the change in one parameter affects the indirect utility of repaying agents compared to that of defaulting agents.

By totally differentiating the equation (15) and by using the Roy's lemma we can derive:

$$\frac{\partial A}{\partial t} = \beta \frac{u_2 w(e) l - u_{2d} (1 - \gamma) w(e_d) l_d}{u_1 - (1 + r) \beta u_2 - u_{1d}} \quad (16)$$

$$\frac{\partial A}{\partial g^1} = \frac{u_{1d} - u_1}{u_1 - (1 + r) \beta u_2 - u_{1d}} > 0 \quad (17)$$

$$\frac{\partial A}{\partial g^2} = \beta \frac{u_{2d} - u_2}{u_1 - (1 + r) \beta u_2 - u_{1d}} < 0 \quad (18)$$

$$\frac{\partial A}{\partial \gamma} = - \frac{\beta u_{2d} (1 - t) w(e_d) l_d}{u_1 - (1 + r) \beta u_2 - u_{1d}} > 0 \quad (19)$$

$$\frac{\partial A}{\partial F} = - \frac{\beta u_{2d}}{u_1 - (1 + r) \beta u_2 - u_{1d}} > 0 \quad (20)$$

$$\frac{\partial A}{\partial r} = \frac{\beta u_2 A}{u_1 - (1 + r) \beta u_2 - u_{1d}} < 0 \quad (21)$$

Using Lemma 1 all of the comparative statics can be signed except for the labor tax rate. From  $c^1 < c_d^1$  we have  $u_1 > u_{1d}$ . Moreover, we have  $u_1 - (1 + r) \beta u_2 - u_{1d} = \mu - \mu_d < 0$ , since the shadow price for a marginal increase of borrowing limit in equilibrium is higher for the defaulting agent than for the repaying one.<sup>2</sup> Therefore,  $\frac{\partial A}{\partial g^1} > 0$  and the borrowing limit increases with  $g^1$ , ceteris paribus. Intuitively, since the repaying agent consumes less in the first period, increasing first period consumption benefits the repaying agent more than the defaulting one. On the other hand, increasing the second period transfer  $g^2$  tightens the incentive constraint of repaying and lowers the borrowing limit. This is because lump-sum transfer is not seizable and it makes the punishment of losing part of the earnings less severe. A higher interest rate tightens the incentive constraint as well. The higher the interest rate, the higher the costs of repaying and the more attractive is defaulting. Making the default punishment more severe, either by increasing the fixed cost  $F$  or the fraction of income that can be seized  $\gamma$ , increases the default costs and therefore also the borrowing limit.

However, the effect of a higher  $t$  is ambiguous. On the one hand, a higher tax rate harms the defaulting agent more by reducing the second period income, since the defaulting agent consumes less in the second period and has a higher marginal utility of consumption. On the other hand, a higher tax rate reduces the after tax seizable income and makes the punishment less severe. The total impact of a higher tax rate depends therefore on which effect dominates.

---

<sup>2</sup>In equilibrium the agent is indifferent between repaying and defaulting,  $V = V_d$ . If the agent can borrow one unit more than the equilibrium borrowing limit, it would default,  $V < V_d$ . Consequently, we have  $\mu < \mu_d$ .

### 3 Optimal Tax Policy

In this section we first formulate the governmental problem and then derive the optimal tax policy. The government is benevolent and can fully commit to announced tax policy. The tax system consists of a flat labor tax and lump-sum transfers in both periods. The time structure of the model is as follows: the government first announces labor tax and lump-sum transfers. Then private banks determine the borrowing limit under the non-default condition (15). After observing tax policy and borrowing constraints, individuals decide on educational investment, borrowing and labor supply.

We assume without loss of generality that there is no exogenous governmental expenditure<sup>3</sup>. The governmental budget constraint is thus given by

$$tw(e)l = (1+r)g^1 + g^2. \quad (22)$$

The government chooses  $g^1$ ,  $g^2$  and  $t$  to maximize the indirect utility of a representative agent, whereby taking the responses of banks in determining credit constraints into account. The lagrangian function for the governmental problem is

$$\mathcal{W} = V + \eta (tw(e)l - (1+r)g^1 - g^2), \quad (23)$$

where the lagrangian multiplier  $\eta$  measures the shadow price of governmental revenue.

#### 3.1 Optimal age-specific transfers

We first derive the optimal tax policy when the government can use age-specific transfers. With exogenous credit constraints, Jacobs and Yang (2010) argue that the availability of age-specific transfers enables the government to remove credit constraints perfectly. The government only has to provide the amount of credit that individuals could not borrow on the private loan market and requires them to pay it back plus interest in forms of lump-sum taxes. In fact, government acts through age-specific lump-sum transfers as a lender to replace the missing (or imperfect) private credit market. As a result, agents are not credit constrained anymore and the optimal labor tax rate is zero.

In our model with endogenous credit constraints, however, such age-specific transfers would change the individuals' incentive to repay and thus affect the borrowing limit. As shown before, a higher first period transfer increases the borrowing limit whilst a higher second period transfer reduces the borrowing limit. Consequently, age-specific transfers that redistribute income from the second to the first period ( $g^1 = -(1+r)g^2 > 0$ ) would increase borrowing limit. The lowest level of lump-sum transfers needed to remove

---

<sup>3</sup>Any exogenous governmental expenditure can be financed by lump-sum taxes in such a way that credit constraint is not affected.

binding credit constraints perfectly is characterized by

$$A(g^{1*}) + g^{1*} = a^{LP}, \quad (24)$$

where  $a^{LP}$  is the optimal borrowing in a laissez-faire economy with a perfect credit market. The government should provide the amount of credit to young agents such that the credit constraints are not binding any more. If government transfers more resources than  $g^{1*}$ , banks would like to lend more than agents want to borrow. The public lending crowds out private lending. For  $g^1 = a^{LP}$  there would be no private lending any more. Compared to the case of exogenous credit constraints, a lower level of lump-sum transfers is needed to remove credit constraints perfectly.

Andolfatto and Gervais (2006) also analyze the optimal age-specific lump-sum transfers with endogenous credit constraints. However, they argue that the optimal policy should be transferring income from young and old to working agents. The difference of our results to theirs arises from the different assumptions about the default punishment. In their model default is punished by being excluded from the capital market, which means that agents cannot save any more for old age if they default. Transferring income from old and young to middle-aged agents makes the costs of no-saving higher and relaxes the incentive constraint of repaying thereafter. In our two-period model, however, the punishment of exclusion from capital market is not encompassed. The defaulting agent is punished by losing part of its labor income. Transferring income from adults to young agents not only reduces the credit demand but also relaxes the incentive constraint, since the defaulting agent benefits less from the (mandatory) public lending on which it cannot default.

The first-best result with age-specific transfers arises from the assumption that the government has higher enforcement power than private banks and the agent cannot default on governmental loans. This assumption is not very harmful since the government does have higher enforcement power through the tax system. In addition, government faces lower costs of collecting repayment and a less severe tracking problem than private banks.

### 3.2 Optimal tax policy with uniform lump-sum transfers

Age-specific lump-sum transfers (or taxes) are normally difficult to implement in an overlapping-generation world due to age-discrimination, which is legally forbidden in some countries. Therefore, in this section, we analyze the optimal tax policy when age-specific lump-sum transfers are not available, i.e.  $g^1 = g^2 \equiv g$ . The lagrangian function for governmental optimization becomes

$$\mathcal{W} = V + \eta (tw(e)l - (2 + r)g). \quad (25)$$

If credit constraints are exogenous, a redistributive taxation, i.e. a positive tax rate on labor income and positive lump-sum transfers, alleviates credit constraints by redistributing income from high-income to low-income (and constrained) period. In fact, government still acts like a lender through redistributive taxation to supplement the imperfect capital market (see Jacobs and Yang, 2010). With endogenous credit constraints, there is an additional effect of tax policy, namely its effect on the borrowing limit, that has to be taken into account when designing the optimal policy.

Analogously to the definition of the net social marginal valuation of income by Diamond (1975) we define the net social marginal valuation of a higher borrowing limit as

$$\psi \equiv \frac{u_1 - (1+r)\beta u_2}{\eta} + tw'(e)l \frac{\partial e}{\partial A} + tw(e) \frac{\partial l}{\partial A}. \quad (26)$$

$\psi$  gives the welfare effects, measured in terms of governmental revenue, of a marginal increase in  $A$ . The increase of borrowing limit by one unit increases the utility of the agents by  $u_1 - (1+r)\beta u_2$ , which is positive for constrained agents. The last two terms on the right-hand-side of (26) are the effects on tax revenue of a higher borrowing limit due to the induced changes in the agents' choices. A higher  $A$  enables the agents to invest more in education and leads to a higher labor supply due to the positive feedback between education and labor<sup>4</sup>. The income effects are thus positive as well.

Using definition (26) we can characterize the optimal lump-sum transfers as (see Appendix A.4)

$$\frac{u_1 + \beta u_2}{\eta} + tw'(e)l \frac{\partial e}{\partial g} + tw(e) \frac{\partial l}{\partial g} + \psi \frac{\partial A}{\partial g} = 2 + r. \quad (27)$$

The left-hand-side of equation (27) gives the net social marginal valuation of one unit of income given in both periods, including the income effects on tax revenue. The welfare effects of income by affecting borrowing limit  $A$  is given by  $\psi \frac{\partial A}{\partial g}$ . If a higher income increases the borrowing limit, one unit of income is more valuable than that in case of exogenous credit constraints; and vice versa. In optimum, lump-sum transfers equal the net social marginal valuation of income to its resource costs,  $2 + r$ , both measured in terms of the second period income.

The first-order-condition for the optimal tax rate  $t$  can be reformulated as (see Appendix A.4)

$$(1 - \rho) \pi + \left( \rho \frac{\partial A}{\partial g} + \frac{\partial A}{\partial t} \frac{1}{z} \right) \psi = \frac{t}{1-t} (\theta \varepsilon_e + \varepsilon_l), \quad (28)$$

where  $\varepsilon_e \equiv -\frac{\partial e^c}{\partial t} \frac{1-t}{e}$  and  $\varepsilon_l \equiv -\frac{\partial l^c}{\partial t} \frac{1-t}{l}$  are the compensated tax elasticities of education and labor supply,  $z \equiv w(e)l$  denotes the gross labor income, and  $\rho \equiv \frac{1-\pi}{2+r-\pi}$ . The optimal income taxation balances the welfare gain of alleviating credit constraints against the

---

<sup>4</sup>Labor supply increases with education as long as the substitution effect dominates the income effect.

efficiency costs of doing so. The latter, as given by the right-hand-side of equation (28), arises from tax distortions in education and labor supply, measured by the compensated tax elasticities.  $\theta \equiv \frac{w'(e)e}{w(e)}$  is the elasticity of gross wage rate w.r.t education. The more important education is, the higher is  $\theta$  and the higher are the efficiency costs of tax distortion.

The welfare gain of taxing labor income is given by the left-hand-side of equation (28). The first term is the welfare effects for fixed borrowing constraints. Taxing labor income and reimbursing tax revenue in forms of lump-sum transfers redistributes income from the second to the first period and thus reduces the credit demand. Since credit constrained agents value the first period income more than the second period income ( $u_1 - \beta(1+r)u_2 > 0$ ), such redistribution increases welfare. The more agents are credit constrained, as shown by a higher value of  $\pi$ , the higher is the welfare gain of transferring one unit of income from the second to the first period.

However, since the same amount of transfer has to be given in the second period as well, only part of the tax revenue can be transferred to the first period. The parameter  $\rho \equiv \frac{1-\pi}{2+r-\pi} = \frac{1}{\frac{1+r}{1-\pi}+1} < 1$  gives the increase in the uniform transfers if tax revenue is increased by one unit, while taking into account that the relative shadow price of the first period income compared to the second period is  $\frac{1+r}{1-\pi}$ . Note that for one unit of tax revenue we have  $\frac{1+r}{1-\pi}\rho + \rho = 1$ , i.e., the values of the first and the second period transfer should sum up to 1. Consequently,  $1-\rho = \frac{1+r}{1-\pi}\rho$  gives the value of the first period transfer. The higher the shadow price  $\frac{1+r}{1-\pi}$ , the higher the value of the first period transfer and the higher is the welfare gain of taxation. A higher interest rate and more severe credit constraints increase the value of the first period transfer and thus the welfare gain of taxation.

The second term on the left-hand-side of (28) is the welfare effects of taxation by affecting the borrowing constraints. As defined by (26),  $\psi$  gives the welfare effects of relaxing borrowing constraints by one unit. The term  $\left(\rho \frac{\partial A}{\partial g} + \frac{\partial A}{\partial t} \frac{1}{z}\right)$  gives the total change in borrowing limit for one unit increase in tax revenue.  $\rho$  is by definition the increase in  $g$ , while taking the relative price of the first period income into account. Therefore,  $\rho \frac{\partial A}{\partial g}$  gives the change in  $A$  due to higher lump-sum transfers when tax revenue is increased by one unit. Similarly,  $\frac{\partial A}{\partial t} \frac{1}{z}$  is the change in  $A$  due to a higher labor tax rate, whilst  $\frac{1}{z}$  is the required increase in tax rate to increase the tax revenue by one unit, ceteris paribus.

To sum up, the welfare effects of taxing labor income are the sum of the welfare gain of reducing borrowing demand while keeping the borrowing limit as given and the welfare effect of changing the borrowing limit. The aforementioned comparative statics (equation (16) to (18)) show that both labor tax  $t$  and uniform lump-sum transfer  $g$  have ambiguous effect on borrowing limit<sup>5</sup>. Therefore, the second welfare effect can be either positive or

<sup>5</sup>The comparative static for the uniform transfer is  $\frac{\partial A}{\partial g} = \frac{u_{1d}-u_1+\beta u_{2d}-\beta u_2}{u_1-(1+r)\beta u_2-u_{1d}}$ .

negative.

Rewriting condition (28), we can characterize the optimal tax rate as

$$\frac{t}{1-t} = \frac{(1-\rho)\pi + \left(\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z}\right)\psi}{\theta\varepsilon_e + \varepsilon_l} \quad (29)$$

The optimal tax rate depends on the total welfare gain of alleviating credit constraints and its efficiency costs. The higher the total welfare gain and the lower the tax distortions, the higher is the optimal tax rate.

If credit constraints are exogenous, equation (29) reduces to

$$\frac{t}{1-t} = \frac{(1-\rho)\pi}{\theta\varepsilon_e + \varepsilon_l}, \quad (30)$$

since  $\frac{\partial A}{\partial g} = \frac{\partial A}{\partial t} = 0$  for exogenous credit constraints. This result is the same as that in Jacobs and Yang (2010) for identical agents, where exogenous credit constraints are assumed. Compared to equation (30), the optimal tax rate with endogenous credit constraints is additionally determined by the term  $\left(\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z}\right)\psi$ , which is the welfare effect of taxation by changing the borrowing limit  $A$ .

If  $\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z} < 0$ , a more redistributive taxation tightens the credit constraints. We know from Lemma 1 that the agent who plans to default invests less in education and works less in order to reduce default costs. Since redistributive taxation reduces the incentive to invest in education and to work, it exaggerates the moral hazard problems associated with credit constraints. As a result, the borrowing limit decreases when taxation becomes more redistributive. Therefore, redistributive taxation has two opposite effects on welfare. It increases welfare by mimicking governmental loans, but reduces welfare by exacerbating the credit constraints. Compared to the case of exogenous credit constraints, the optimal tax rate is lower. Moreover, it can even turn negative, if the negative effect of redistributive taxation by reducing borrowing limit dominates its positive effect. In this case, the optimal taxation is reverse redistributive, i.e. the tax system consists of lump-sum taxes and labor income subsidies, which redistributes from the first to the second period. Such a taxation would increase welfare, because the tax-induced increase in borrowing limit overcompensates the tax-induced increase in borrowing demand.

If  $\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z} > 0$ , a more redistributive taxation not only reduces credit demand but also relaxes the borrowing limit. Redistributive taxation is therefore more efficient in mitigating credit constraints compared to the case of exogenous credit constraints. Consequently, a higher labor tax rate is optimal.

Lochner and Monge-Naranjo (2002) show that subsidizing education reduces the incentive to default and thus has a larger welfare effect than in a similar model with exogenous credit constraints. In this paper, we rule out education subsidies by the assumption



of unobservable educational investment. Since education subsidies mimic age-specific transfers, the availability of education subsidies would reduce the desirability of labor taxation. However, as long as the share of observable educational costs in total costs is not very high, the effects of education subsidies are limited. Moreover, subsidizing verifiable costs would distort the efficient composition of the verifiable and the non-verifiable investment (see Bovenberg and Jacobs, 2005).

We summarize our results in the following proposition:

**Proposition 1** *The optimal labor tax rate balances the welfare gain of redistributing income across periods and of changing the borrowing limit against the efficiency costs of distorting educational investment and labor supply. If a higher tax rate and higher lump-sum transfers tighten (relax) borrowing constraints, the optimal income taxation is less (more) redistributive compared to the case of exogenous credit constraints.*

## 4 Numerical Examples

This section uses numerical examples to illustrate how borrowing limit responds to changing tax policy and changing parameters. Moreover, we compare the optimal tax rates with those under exogenous credit constraints. We should note that these numerical examples are only for the purpose of illustration. Political interpretation of the results should be taken with caution.

Following Saez (2001) we assume the utility function to be logarithmic:

$$U = \ln c^1 + \beta \ln c^2 - \beta \ln \left( 1 + \frac{l^{1+\epsilon^{-1}}}{1 + \epsilon^{-1}} \right), \quad (31)$$

where  $\epsilon$  is a parameter governing the (un)compensated elasticity of labor. Furthermore, we assume the wage rate function to be Cobb-Douglas:

$$w(e) = ne^\theta. \quad (32)$$

$\theta$  is the elasticity of wage rate w.r.t education and  $n$  reflects the individual innate ability in generating income.

For the parameterization of the benchmark case we set the elasticity of wage rate  $\theta$  to 0.5. Trostel (1993) uses the value of 0.45 for the elasticity of human capital in time investment and the value of 0.15 for the elasticity in goods investment. Jacobs (2005) uses the values of 0.3 and 0.1 respectively. Therefore the total share of education in the wage rate of 0.5 lies between the value of 0.4 by Jacobs (2005) and 0.6 by Trostel (1993). For the parameter  $\epsilon$  we take the value of 0.2 as benchmark case. Saez (2001) uses the value of 0.25 and 0.5 to match the empirical estimates of compensated tax elasticity of

earnings. With endogenous education in our model, the tax elasticity of earnings is higher than the value of  $\epsilon$ . Moreover, we set the interest rate  $r$  to 0.63, which equals an annual interest rate of 5% for a period of 10 years. The time preference  $\beta$  is assumed to be 0.62 such that  $\beta(1+r) \approx 1$ . For the punishment parameter  $\gamma$  we use the value of 0.2, which is a bit higher than the calibrated value of 13% by Lochner and Monge-Naranjo (2002) for the US economy. The fixed cost  $F$ , ability  $n$  and initial wealth  $\omega$  are calibrated to ensure the existence of credit constraints and to avoid corner solutions. We use for the benchmark case  $F = 1$ ,  $n = 6$  and  $\omega = 5$ .

The procedure to find the borrowing limit is as follows: we first calculate the optimal individual borrowing when capital market is perfect. Then we check if the agent can be better off by defaulting when the credit limit is at the level of the optimal borrowing. If this is the case, we reduce the credit limit by one unit and check again if the agent can be better off by defaulting. We keep doing this till the agent cannot be better off by defaulting. The corresponding credit limit is then the endogenous credit limit. To find the optimal tax rate we follow the method used by Jacobs (2005). For each given tax rate we find the uniform lump-sum transfer which maximizes the utility of the representative agent under the governmental budget constraint, whereby the credit constraints are endogenously determined by the respective tax policy. We then search for the tax rate that leads to the highest utility of the agent. For comparison we also calculate the optimal tax rate when credit limit is held constant. The exogenous credit limit is set to equal the credit limit in laissez-faire. To reduce computational time, we searched for the optimal tax rate in the range between 0 and 0.5 for the case of exogenous credit constraints and searched in the range between -0.2 and 0.4 for the case of endogenous credit constraints<sup>6</sup>.

We first depict in Figure 1 the borrowing limit for each given tax rate in benchmark case, where lump-sum transfer  $g$  is chosen optimally. The credit limit falls almost monotonically with increasing tax rate, implying that for our benchmark case labor taxation tightens the credit constraints.

---

<sup>6</sup>For the case of exogenous credit constraints the governmental maximization problem is well-behaved. We found that for our simulations the optimal tax rates always lie between 0 and 0.5. With endogenous credit constraints, the indirect utility is almost concave in tax rate and there are only very small local fluctuations. We also found that the global maximum always lies between -0.2 and 0.4. The Gauss-programmes for simulation are available upon request.

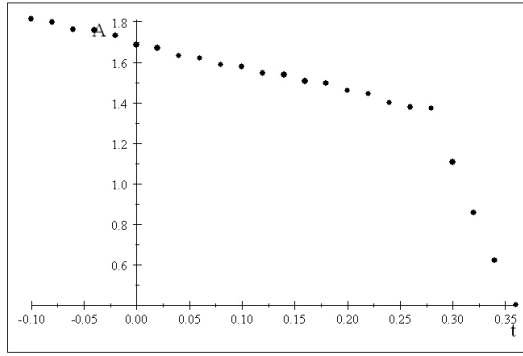


Figure 1: Endogenous borrowing limit

For the benchmark case, the optimal tax rates and the corresponding allocations under endogenous and exogenous credit constraints and in laissez-faire are shown in Table 1. The credit limit is decreased by optimal taxation compared to laissez-faire. However, the distortion caused by credit constraints, which is measured by  $\pi$ , is reduced. This is because labor taxation redistributes income to constrained life-cycle period and thus mitigates the distorting effects of credit constraints. Compared to the case of exogenous credit constraints, the optimal tax rate is lower due to the negative effect of labor taxation on the credit constraints. Moreover, the equilibrium education, labor supply and welfare are lower when credit constraints are endogenized<sup>7</sup>. This implies that assuming exogenous credit constraints overestimates the welfare effects of taxation.

	$t$	$g$	$e$	$l$	$A$	$\pi$	$v$
Endogenous	0.22	0.778	2.047	1.084	1.442	0.08	2.5727
Exogenous	0.25	0.930	2.223	1.093	1.686	0.01	2.5886
Laissez-faire	0	0	2.032	1.107	1.686	0.30	2.5530

Table 1: Benchmark case simulation,  $n=6$ ,  $\varepsilon=0.2$ ,  $\theta=0.5$ ,  $\gamma=0.2$ ,  $F=1$

Table 2 shows the comparative statics of the borrowing limit. Starting from laissez-faire and from the benchmark case values, the row  $dA/dx$  gives the relative change in credit limit for given changes in tax policy or in parameters. Ceteris paribus, increasing the labor tax rate by 1 unit lowers the credit limit by 1.78 unit. However, increasing the lump-sum transfer by 1 unit relaxes the credit limit by 3.78 unit. Consistent to the comparative statics derived in (19) to (21), a higher  $\gamma$  and a higher  $F$  increase the credit limit, whereby increasing  $\gamma$  has a much larger effect than increasing  $F$ . A higher interest rate  $r$  tightens the credit constraints.

	$dt = 0.01$	$dg = 0.01$	$d\gamma = 0.01$	$dF = 0.1$	$dr = 0.01$
$dA/dx$	-1.78	3.74	10	1	-1.38

Table 2: Comparative statics

We further simulated the optimal tax rates under exogenous and endogenous credit

<sup>7</sup>As Saez (2001) pointed out,  $l$  does not necessarily represent the working time.

constraints for different values of the parameters  $\epsilon$ ,  $\gamma$  and  $\theta$ . The results are reported in Table 3, 4 and 5 respectively, where the second row gives the optimal tax rates and the third row gives the corresponding utility. The figures in the bracket are values for the case of exogenous credit constraints. For all parameter values the endogeneity of credit constraints reduces the optimal tax rate and the maximized welfare. This suggests that redistributive taxation has a negative effect on the borrowing limit. However, for lower parameter values the optimal tax rates are reduced only to a small degree. When the parameter values are higher, the effect of the endogeneity of credit constraints tends to increase.

We first look at Table 3. Since  $\epsilon$  measures the wage rate elasticity of labor supply, it reflects the magnitude of the positive feedback effect between labor and education. A higher  $\epsilon$  implies that the agent would like to invest more in education and to work more. Consequently, credit constraints are more severe. This can be seen by the fact that the optimal tax rate under exogenous credit constraints increases with  $\epsilon$ , since a higher tax rate is then needed to alleviate the more severe credit constraints. However, a higher  $\epsilon$  implies that the agent responds more elastically to the changing credit limit. The negative welfare effects of taxation by reducing credit limit are consequently larger. As a result, the endogeneity of credit constraints lowers the optimal tax rate to a larger degree.

	$\epsilon = 0.1$	$\epsilon = 0.15$	$\epsilon = 0.2$	$\epsilon = 0.25$
$t$	0.2 (0.22)	0.23 (0.24)	0.22 (0.25)	0.22 (0.27)
$v$	2.5963 (2.6061)	2.5829 (2.5962)	2.5727 (2.5886)	2.5671 (2.5871)

Table 3: optimal tax rates for different values of  $\epsilon$

Increasing the value of  $\gamma$  reduces the optimal tax rates both in case of endogenous and exogenous credit constraints. A higher value of  $\gamma$  implies higher borrowing limit and less severe credit constraints, since defaulting is less attractive due to higher default costs. Consequently, a lower tax rate is required to mitigate credit constraints. At the same time, the borrowing limit is more responsive to labor taxation, because the after tax income is the more important the higher is the value of  $\gamma$ . As a result, the optimal tax rate decreases much faster than those under exogenous credit constraints.

	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
$t$	0.29 (0.30)	0.22 (0.25)	0.18 (0.21)	0.06 (0.16)
$v$	2.5437 (2.5527)	2.5727(2.5886)	2.6036 (2.6233)	2.6360 (2.6518)

Table 3: optimal tax rates for different values of  $\gamma$

Increasing the value of  $\theta$  has similar effects on the optimal tax rates as increasing the value of  $\epsilon$ . The higher is  $\theta$ , the more important is education and the more the agent would

like to invest in education. Therefore, when  $\theta$  is higher, the credit constraints are more severe and the optimal tax rate is correspondingly higher. As for  $\varepsilon$ , the difference between the optimal tax rates under exogenous and endogenous credit constraints increases with  $\theta$ . A higher value of  $\theta$  implies more distortionary effects of credit constraints and thus a larger negative effect of taxation by reducing credit limit.

	$\theta = 0.4$	$\theta = 0.45$	$\theta = 0.5$	$\theta = 0.55$
$t$	0.11 (0.12)	0.18 (0.19)	0.22 (0.25)	0.26 (0.31)
$v$	2.4959 (2.4988)	2.5309 (2.5381)	2.5727 (2.5886)	2.6234 (2.6531)

Table 3: optimal tax rates for different values of  $\theta$

Our simulation results suggest that the optimal tax rate should be set lower when the effect of tax policy on credit constraints is considered. When labor is elastic and education is important and also when banks can seize a large share of the earnings, we find a significant effect on the optimal tax policy, when credit constraints are endogenized. Moreover, our simulation shows that, notwithstanding the fact that redistributive taxation reduces the borrowing limit, taxation with reverse redistribution is not very probably to be optimal.

## 5 Conclusion

This paper derives the optimal tax policy when credit constraints arise from the limited commitment of individuals to repay the loans for educational investment and consumption smoothing. The optimal redistributive taxation balances the total welfare gain of reducing borrowing demand and of changing the credit limit against the efficiency costs of distorting labor supply and education. If a more redistributive taxation tightens (relaxes) credit constraints, the optimal tax rate is lower (higher) compared to the case of exogenous credit constraints. Thus, assuming exogenous credit constraints leads to a too high (low) tax rate and over- (under) estimates the welfare effects of taxation. Numerical examples show that for reasonable model parameters the optimal tax rate is lower than that under exogenous credit constraints.

In future research it would be interesting to analyze optimal tax policy with endogenous credit constraints when individuals differ in ability and in initial wealth. In this case, the level of borrowing limit at which the individual is indifferent between defaulting and repaying would differ individually and is private information due to the unobservable ability. Consequently, conditions for equilibrium credit constraints would change, which would also have different political implications.

# A Appendix

## A.1 Second-order-conditions for the households' problem

We first derive the second-order-conditions for the agent's maximization problem when it repays the loans. With binding credit constraints, savings are equal to the borrowing limit  $\bar{a}$ . Hence, we can obtain an unconstrained maximization problem upon substitution of the two budget constraints (2) and (3) in the utility function (1):

$$\max_{\{e,l\}} u(-e + g^1 + \bar{a}) + \beta u((1-t)w(e)l - (1+r)\bar{a} + g^2) - \beta v(l) \quad (33)$$

The first-order-conditions are given by

$$-u_1 + \beta u_2(1-t)w'(e)l = 0, \quad (34)$$

$$\beta u_2(1-t)w(e) - \beta v'(l) = 0. \quad (35)$$

The Hessian matrix  $H$  with the second-order partial derivatives is given by

$$H \equiv \begin{bmatrix} \begin{pmatrix} u_{11} + u_{22}\beta(1-t)^2(w'(e))^2 l^2 \\ +\beta u_2(1-t)w''(e)l \end{pmatrix} & \begin{pmatrix} \beta u_{22}(1-t)^2 w(e) w'(e)l \\ +\beta u_2(1-t)w'(e) \end{pmatrix} \\ \begin{pmatrix} \beta u_{22}(1-t)^2 w(e)w'(e)l \\ +\beta u_2(1-t)w'(e) \end{pmatrix} & \begin{pmatrix} \beta u_{22}(1-t)^2 (w(e))^2 \\ -\beta v''(l) \end{pmatrix} \end{bmatrix} \quad (36)$$

where  $u_{11}$  and  $u_{22}$  are the second derivative of the first and the second period utility of consumption respectively. For the Hessian matrix to be negative semi-definite, the principal minors should switch signs. The first principal minor

$$u_{11} + u_{22}\beta(1-t)^2(w'(e))^2 l^2 + \beta u_2(1-t)w''(e)l \quad (37)$$

is negative if the utility function of consumption and the wage rate function are concave, i.e.  $u_{11} < 0$ ,  $u_{22} < 0$  and  $w''(e) < 0$ , which are assumed to be fulfilled. The second principal minor should be positive:

$$\begin{aligned} & \left( u_{11} + u_{22}\beta(1-t)^2(w'(e))^2 l^2 + \beta u_2(1-t)w''(e)l \right) \times \left( \beta u_{22}(1-t)^2 (w(e))^2 - \beta v''(l) \right) \\ & - \left( \beta u_{22}(1-t)^2 w(e)w'(e)l + \beta u_2(1-t)w'(e) \right)^2 > 0. \end{aligned} \quad (38)$$

Next, define  $\delta \equiv \left( \frac{lw''(l)}{v'(l)} \right)^{-1}$  as the elasticity of the marginal disutility  $v'(l)$ ,  $\theta = \frac{w'(e)e}{w(e)}$  as the elasticity of wage rate and  $\alpha = \frac{w''(e)e}{w'(e)}$  as the elasticity of the marginal return to education. Furthermore, define  $(1-t)w(e)l = Z$  as the net labor income. Using the

first-order-conditions we can reformulate the above inequation as

$$\left( \frac{u_{11}}{\beta z} \frac{e^2}{\theta^2} + u_{22}Z + u_2 \frac{\alpha}{\theta} \right) \times \left( u_{22}Z - \frac{u_2}{\delta} \right) > (u_{22}Z + u_2)^2 \quad (39)$$

Define  $-\frac{u_{22}}{u_2}c_2 = \sigma$  as the elasticity of the marginal utility of consumption  $u_2$ .  $\sigma$  measures how fast the marginal utility of the second period consumption declines. Further reformulations of equation (39) lead to

$$\delta^{-1} > \frac{1 + \sigma \frac{Z}{c_2} \left( \frac{u_{11}}{u_2 \beta Z} \frac{e^2}{\theta^2} + \frac{\alpha}{\theta} - 2 \right)}{\sigma \frac{Z}{c_2} - \frac{u_{11}}{u_2 \beta Z} \frac{e^2}{\theta^2} - \frac{\alpha}{\theta}} \quad (40)$$

From the concavity of the consumption utility function and of the wage rate function we have  $u_{11} < 0$ ,  $\alpha < 0$  and  $\sigma > 0$ . Therefore, the right-hand-side of (40) is the smaller, the larger  $\sigma \frac{Z}{c_2}$ ,  $\frac{u_{11}}{u_2 \beta Z} \frac{e^2}{\theta^2}$  and  $\frac{\alpha}{\theta}$  are in absolute value. Consequently, the second-order-condition (40) requires that the elasticity  $\delta$  is not too high, the marginal utility of consumption decreases sufficiently fast ( $\sigma$  and  $u_{11}$  are not too small in absolute value) and the productivity of education is not too high (the elasticity  $\theta$  is sufficiently small and the marginal return to education decreases fast enough). These conditions ensure that the positive feedback between education and labor supply dampens and that interior solutions are obtained. The unconstrained maximization problem for the defaulting agent is

$$\max_{\{e, l\}} u(-e + g^1 + \bar{a}) + \beta u((1 - \gamma)(1 - t)w(e)l - F + g^2) - \beta v(l) \quad (41)$$

for the case that  $(1 + r)a > \gamma(1 - t)w(e)l$ . Thus, the second-order-conditions are the same as for the repaying agent except that the after-tax income becomes  $Z = (1 - \gamma)(1 - t)w(e)l$ , which does not change the results qualitatively.

## A.2 Proof of Lemma 1

We prove in this appendix that in equilibrium the agent who plans to default invests less in education and work less than the agent who plans to repay. First we establish that the equilibrium credit limit  $A$  is characterized by the following inequations:

$$\gamma(1 - t)w(e^*)l^* + F > (1 + r)A, \quad (42)$$

$$\gamma(1 - t)w(e_d^*)l_d^* + F < (1 + r)A, \quad (43)$$

where  $(e^*, l^*)$  are the optimal choices of the agent who plans to repay and  $(e_d^*, l_d^*)$  are the optimal choices of the agent who plans to default. We denote the indirect utilities for the repaying agent and for the defaulting one as  $V$  and  $V_d$  respectively. In equilibrium we have  $V = V_d$ . The first inequation (42) can be shown as follows: Suppose that

$\gamma(1-t)w(e^*)l^* + F < (1+r)A$ , the repaying agent could then achieve a higher utility than  $V = V_d$  by defaulting, since its second period consumption would increase while the first period consumption and labor supply remain unchanged. This is not possible since  $V_d$  is the highest utility the agent can achieve by defaulting. Let's now look at the case of  $\gamma(1-t)w(e^*)l^* + F = (1+r)A$ . If the agent changes its plan and decides to default, its utility would remain the same as long as it does not change its education and labor supply choices. However, the choices  $(e^*, l^*)$  are not optimal any more for defaulting, since  $\frac{u_{1d}}{\beta v'_{2d}} > (1-t)(1-\gamma)w'(e^*)l^*$  and  $\frac{v'(l^*)}{u_{2d}} > (1-t)(1-\gamma)w(e^*)$ . Consequently, the agent can achieve a higher utility than  $V = V_d$  by defaulting and optimizing its choices accordingly. This contradicts again the fact that  $V = V_d$  is the highest utility the defaulting agent can obtain. Therefore we can conclude that the inequation (42) must be fulfilled. The second inequation (43) can be shown analogously: if  $\gamma(1-t)w(e_d^*)l_d^* + F \geq (1+r)A$ , the defaulting agent can achieve a higher utility than  $V = V_d$  by repaying its loan and optimizing its choices accordingly. This is a contradiction to the fact that  $V$  is the highest utility the repaying agent can obtain. From the two inequations (42) and (43) it follows that in equilibrium

$$w(e^*)l^* > w(e_d^*)l_d^*.$$

Thus the defaulting agent must have a lower gross labor income than the repaying agent. There are however 3 cases for  $w(e^*)l^* > w(e_d^*)l_d^*$ : 1)  $e^* > e_d^*$  and  $l^* > l_d^*$  2)  $e^* \leq e_d^*$  and  $l^* > l_d^*$  3)  $e^* > e_d^*$  and  $l^* \leq l_d^*$ . Now we show that only the first case is possible. Dividing the first-order-condition for education by the first-order-condition for labor supply we find for the repaying agent that

$$\frac{u_1}{\beta v'(l^*)} = \frac{w'(e^*)l}{w(e^*)}, \quad (44)$$

and for the defaulting agent that

$$\frac{u_{1d}}{\beta v'(l_d^*)} = \frac{w'(e_d^*)l_d^*}{w(e_d^*)}. \quad (45)$$

When  $e^* \leq e_d^*$  and  $l^* > l_d^*$ , it follows that  $\frac{u_1}{\beta v'(l^*)} < \frac{u_{1d}}{\beta v'(l_d^*)}$  and  $\frac{w'(e^*)l}{w(e^*)} > \frac{w'(e_d^*)l_d^*}{w(e_d^*)}$ . Therefore, (44) and (45) cannot be fulfilled simultaneously. Analogously, if  $e^* > e_d^*$  and  $l^* \leq l_d^*$ , we have  $\frac{u_1}{\beta v'(l^*)} > \frac{u_{1d}}{\beta v'(l_d^*)}$  and  $\frac{w'(e^*)l}{w(e^*)} < \frac{w'(e_d^*)l_d^*}{w(e_d^*)}$ . Again, for this case (44) and (45) cannot be fulfilled simultaneously. Therefore, we can conclude that in equilibrium  $e^* > e_d^*$  and  $l^* > l_d^*$ . It follows straightforward that the defaulting agent consumes more in the first period  $c^1 < c_d^1$ . Since  $V = V_d$  in equilibrium, the repaying agent consumes more in the second period,  $c^2 > c_d^2$ .



### A.3 Slutsky equations

To derive the Slutsky equations we calculate how much lump-sum income  $g$  given in both periods an individual should receive (pay) in order to keep its utility constant when the tax rate  $t$  changes. This is equivalent to deriving the expenditure function and applying Shephard's lemma. Totally differentiating the utility function (1) and the budget constraints of the households (2) and (3) gives:

$$dU = u_1 dc^1 + \beta u_2 dc^2 - \beta v'(l) dl, \quad (46)$$

$$dc^1 = -de + dg + da, \quad (47)$$

$$dc^2 = (1-t)w'(e)lde + (1-t)w(e)dl - w(e)ldt - (1+r)da + dg. \quad (48)$$

Substitute  $dc^1$  and  $dc^2$  in  $dU$  to get

$$\begin{aligned} dU &= (\beta u_2(1-t)w'(e)l - u_1)de + u_1dg + (u_1 - \beta u_2(1+r))da \\ &\quad + \beta(u_2(1-t)w(e) - v'(l))dl - \beta u_2w(e)ldt + \beta u_2dg \\ &= 0. \end{aligned} \quad (49)$$

$(\beta u_2(1-t)w'(e)l - u_1)de$  and  $\beta(u_2(1-t)w(e) - v'(l))dl$  are both equal to zero from the first-order-conditions (6) and (7). The term  $(u_1 - \beta u_2(1+r))da$  is equal to zero as well since with binding credit constraints  $da = 0$ . Thus, we have

$$dU = -\beta u_2w(e)ldt + (u_1 + \beta u_2)dg = 0. \quad (50)$$

The following compensation in  $g$  for changes in  $t$  is obtained

$$\frac{dg}{dt} = \frac{\beta u_2w(e)l}{u_1 + \beta u_2}.$$

The Slutsky equations are therefore given by

$$\frac{\partial e}{\partial t} = \frac{\partial e^c}{\partial t} - \frac{\beta u_2w(e)l}{u_1 + \beta u_2} \frac{\partial e}{\partial g}, \quad (51)$$

$$\frac{\partial l}{\partial t} = \frac{\partial l^c}{\partial t} - \frac{\beta u_2w(e)l}{u_1 + \beta u_2} \frac{\partial l}{\partial g}. \quad (52)$$

where  $e^c$  and  $l^c$  denote the compensated demand function for education and the compensated labor supply function respectively.

### A.4 Optimal tax policy with uniform lump-sum transfers

In this appendix we derive the optimal tax policy when government has no access to age-specific lump-sum transfers, which implies  $g^1 = g^2 \equiv g$ . The lagrangian function for

governmental maximization problem becomes

$$\mathcal{L} = V + \eta (tw(e)l - (2+r)g), \quad (53)$$

where  $\eta$  is the Lagrangian multiplier for governmental budget constraint. The first-order-conditions are respectively

$$\frac{\partial \mathcal{L}}{\partial t} = \frac{\partial V}{\partial t} + \eta \left( w(e)l + tw'(e)l \left( \frac{\partial e}{\partial t} + \frac{\partial e}{\partial A} \frac{\partial A}{\partial t} \right) + tw(e) \left( \frac{\partial l}{\partial t} + \frac{\partial l}{\partial A} \frac{\partial A}{\partial t} \right) \right) = 0, \quad (54)$$

$$\frac{\partial \mathcal{L}}{\partial g} = \frac{\partial V}{\partial g} + \eta \left( -(2+r) + tw'(e)l \left( \frac{\partial e}{\partial g} + \frac{\partial e}{\partial A} \frac{\partial A}{\partial g} \right) + tw(e) \left( \frac{\partial l}{\partial g} + \frac{\partial l}{\partial A} \frac{\partial A}{\partial g} \right) \right) = 0. \quad (55)$$

Note that for the constrained agent  $a = A$  and  $e = e(t, g, A)$ . Assuming that credit constraints remain binding in the neighborhood of policy parameters, we use general envelope theorem to get

$$\frac{\partial V}{\partial g} = u_1 + \beta u_2 + \mu \frac{\partial A}{\partial g} = u_1 + \beta u_2 + (u_1 - (1+r)\beta u_2) \frac{\partial A}{\partial g}, \quad (56)$$

$$\frac{\partial V}{\partial t} = -\beta u_2 w(e)l + \mu \frac{\partial A}{\partial t} = -\beta u_2 w(e)l + (u_1 - (1+r)\beta u_2) \frac{\partial A}{\partial t}, \quad (57)$$

where  $\mu$  is the marginal utility of an increase in borrowing limit by one unit and is equal to  $u_1 - (1+r)\beta u_2$ . We define

$$\psi \equiv \frac{u_1 - (1+r)\beta u_2}{\eta} + tw'(e)l \frac{\partial e}{\partial A} + tw(e) \frac{\partial l}{\partial A} \quad (58)$$

as the net social marginal valuation of one unit increase in credit limit measured in terms of tax revenue, including the income effect. Using (56) and (58), we can rewrite the first-order-condition for  $g$  as

$$\frac{u_1 + \beta u_2}{\eta} + \psi \frac{\partial A}{\partial g} + tw'(e)l \frac{\partial e}{\partial g} + tw(e) \frac{\partial l}{\partial g} = 2 + r. \quad (59)$$

The optimal lump-sum transfer requires that the net social marginal valuation of income should be equal to its resource costs  $2+r$ , whereby the effect of income on the borrowing limit is taken into account. Using (57), the Slutsky equations (51) and (52), the definition of  $\psi$  (58) and the equation (59), the first-order-condition for tax rate  $t$  can be reformulated as

$$w(e)l - \frac{\beta u_2 w(e)l}{u_1 + \beta u_2} (2+r) + \frac{\beta u_2 w(e)l}{u_1 + \beta u_2} \psi \frac{\partial A}{\partial g} + \psi \frac{\partial A}{\partial t} = \frac{t}{1-t} \theta \varepsilon_e w(e)l + \frac{t}{1-t} \varepsilon_l w(e)l \quad (60)$$

where we define the elasticity of wage rate in education as  $\theta \equiv \frac{w'(e)e}{w(e)}$  and the tax elasticities of education and labor supply as  $\varepsilon_e \equiv -\frac{\partial e^c}{\partial t} \frac{1-t}{e}$  and  $\varepsilon_l \equiv -\frac{\partial l^c}{\partial t} \frac{1-t}{l}$  respectively. Using the definitions  $\pi \equiv 1 - (1+r) \frac{\beta u_2}{u_1}$  and  $\rho \equiv \frac{1-\pi}{2+r-\pi}$  we derive the following equation for the optimal tax rate

$$(1 - \rho) \pi + \left( \rho \frac{\partial A}{\partial g} + \frac{\partial A}{\partial t} \frac{1}{z} \right) \psi = \frac{t}{1-t} (\theta \varepsilon_e + \varepsilon_l), \quad (61)$$

where  $z \equiv w(e)l$  is the gross labor income. The optimal tax rate balances the welfare gain of alleviating credit constraints, including the welfare effect of induced changes in the borrowing limit, against the efficiency costs of distorting educational investment and labor supply.

## References

- Acemoglu, D. and J.-S. Pischke (2001) Changes in the Wage Structure, Family Income, and Children's Education, *European Economic Review*, Vol. 45, No. 4-6, pp. 890-904.
- Andolfatto, D. and M. Gervais (2006) Human Capital Investment and Debt Constraints, *Review of Economic Dynamics*, Vol. 9, No. 1, pp. 52-67.
- Becker, G.-S. (1964) *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*, NBER, New York.
- Belley, P. and L. Lochner (2007) The Changing Role of Family Income and Ability in Determining Educational Achievement, *Journal of Human Capital*, Vol. 1, No.1, pp. 37- 89.
- Benabou, R. (1996) Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance, *The American Economic Review*, Vol. 86, No. 3, pp. 584-609.
- Bovenberg, A.-L. and B. Jacobs (2005) Redistribution and Education Subsidies are Siamese Twins, *Journal of Public Economics*, Vol. 89, No. 11-12, pp. 2005-2035.
- Boskin, M.-J. (1975) Notes on the Tax Treatment of Human Capital, NBER Working Paper No. 116.
- Diamond, P.-A. (1975) A Many-Person Ramsey Tax Rule, *Journal of Public Economics*, Vol. 4, No. 4, pp. 227-244.

- Fernandez, R. and R. Rogerson (1996) Income Distribution, Communities, and the Quality of Public Education, *The Quarterly Journal of Economics*, Vol. 111, No. 1, pp. 135-164.
- Fernandez, R. and R. Rogerson (1998) Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform, *The American Economic Review*, Vol. 88, No. 4, pp. 813-833.
- Glomm, G. and B. Ravikumar (1992) Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality, *Journal of Political Economy*, Vol. 100, No. 4, pp. 818-834.
- Jacobs, B. (2005) Optimal Income Taxation with Endogenous Human Capital, *Journal of Public Economic Theory*, Vol. 7, No. 2, pp. 295-315.
- Jacobs, B. and H. Yang (2010) Second-Best Income Taxation with Borrowing Constraints, Erasmus University Rotterdam/University of Konstanz, Mimeo.
- Kane, T.-J. (1995) Rising Public College Tuition and College Entry: How Well Do Public Subsidies Promote Access to College? NBER Working Paper No. 5164.
- Kane, T.-J. (1996) College Cost, Borrowing Constraints and the Timing of College Entry, *Eastern Economic Journal*, Vol. 22, No. 2, pp. 181-194.
- Keane, M.-P. and K.-I. Wolpin (2001) The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment, *International Economic Review*, Vol. 42, No. 4, pp. 1051-1103.
- Kehoe, T.-J. and D.-K. Levine (1993) Debt Constrained Asset Market, *Review of Economic Studies*, Vol. 60, No. 4, pp. 865-888.
- Kehoe, T.-J. and D.-K. Levine (2000) Liquidity Constrained Markets versus Debt Constrained Markets, *Econometrica*, Vol. 69, No. 3, pp. 575-598.
- Krueger, D. and F. Perri (1999) Risk Sharing: Private Insurance Markets or Redistributive Taxes? Federal Reserve Bank of Minneapolis, Research Department Staff Report 262.
- Lochner, L. and A. Monge-Naranjo (2002) Human Capital Formation with Endogenous Credit Constraints, NBER Working paper No. 8815.
- Lochner, L. and A. Monge-Naranjo (2008) The Nature of Credit Constraints and Human Capital, NBER Working Paper No. 13912.

- Saez, E. (2001) Using Elasticities to Derive Optimal Income Tax Rates, *Review of Economic Studies*, Vol. 68, No. 1, pp. 205-229.
- Stinebrickner, R. and T. Stinebrickner (2008) The Effect of Credit Constraints on the College Drop-Out Decision: A Direct Approach Using a New Panel Study, *The American Economic Review*, Vol. 98, No. 5, pp. 2163-2184.
- Trostel, P. A. (1993) The Effect of Taxation on Human Capital, *Journal of Political Economy*, Vol. 101, No. 2, pp. 327-350.
- Van der Klaauw, W. (2002) Estimating the Effect of Financial Aid Offers on College Enrollment: A Regression-Discontinuity Approach, *International Economic Review*, Vol. 43, No. 4, pp. 1249-1287.