

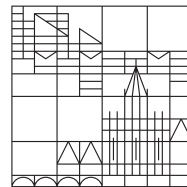
# Three Essays on Market Frictions and Wage Inequality

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# Summary

This dissertation consists of **three stand-alone research papers** on wage development in frictional markets. All three papers (henceforth: chapters) have in common that they consider at least one sort of market friction. Labor market frictions in the form of search costs or imperfect job mobility make it difficult for workers to obtain the job which offers them the best wage and can thereby lead to wage dispersion. Credit market frictions slow down the reallocation of capital to its most productive use and can thereby cause a spread in equity returns.

Only few models of labor market search take into account that workers face a finite planning horizon. However, the time until retirement plays a role for optimal job search behavior, human capital investment decisions, and the wage that a worker is willing to accept. Understanding these optimal behavior is important for the design of labor market policies and the focus of the first two chapters of this thesis.

**Chapter 1** explores whether job search is a potential driving force behind wage dispersion over the life cycle. In line with earlier literature, I find that the variance of residual log wages across workers in the U.S. follows a U-shape with age. The variance is high for young workers who have just entered the labor market. As workers grow older, it falls at first and starts to increase again in the second half of the working life. There are also substantial differences between age groups concerning search behavior. Young workers switch employers three times more often than older workers do.

In order to explore wage dispersion over the life cycle in conjunction with on-the-job search, I develop a quantitative life-cycle model of labor market search with strategic wage bargaining, heterogeneous firm-worker matches, and endogenous search effort. Firms can counter the outside offers of their workers. The option of on-the-job search lowers the wage that workers are willing to accept because a high match quality offers workers more opportunities to obtain wage rises because of possible outside job offers. This option value of on-the-job search diminishes when workers approach the retirement age. Searching for a job is costly. Older workers choose

to search less than young workers as the value of a job is lower when the time horizon until retirement shortens. Young workers switch employers often and are gradually matched to better jobs. Therefore, the variance of log wages initially falls with age. Middle-aged and older workers switch employers less frequently and have a longer search history. As workers are differently successful in the labor market, the variance of match productivities rises in the second half of the working life. I calibrate the model to U.S. panel data and show that the model captures the U-shape of the age-inequality profile of wages in conjunction with the hump-shaped age profile of average wages, as well as employment-to-employment transitions that decrease with age.

**Chapter 2** presents a life-cycle model with endogenous job search and investments in general human capital. Search efforts and training are efficient in the sense that they maximize the joint value of a firm-worker match. Given these optimal decisions, the wage is negotiated in a strategic bargaining game in which the employer can counter the worker's outside offers.

Job search of unemployed and employed workers is costly and the value of a job offer diminishes when the expected duration of employment declines. Search efforts are therefore reduced when workers approach the retirement age. Investments in human capital decrease over most of the life cycle because the time span in which human capital is productive shortens. Therefore and because worker skills depreciate during spells of unemployment, the life-cycle profile of the average stock of human capital is hump-shaped. The average match quality increases over most of the life cycle because workers are gradually matched to better jobs. Young workers accept low starting wages because a job gives them a better position in future wage negotiations and because they are offered training on the job. The life-cycle profiles of human capital, match productivity, and reservation wages translate into a concave life-cycle profile of average wages. Skill depreciation during unemployment induces workers with more human capital to search more and to leave unemployment at a higher rate.

**Chapter 3** is devoted to a different area of research: Since the early 1980's the share of credit in gross domestic product (GDP) has increased in the U.S.. This increase has been accompanied by a decline in the volatility of GDP growth as well as an increase in wage inequality. The chapter explores the impact of financial development on macroeconomic volatility and wage inequality. Related theoretical work establishes a link between financial development and macroeconomic stability. The main contribution of the chapter is the introduction of labor market frictions into a real business cycle model with collateral-based credit constraints and to make visible interactions between the credit and the labor market.

I develop a model with two sectors. Workers earn the marginal productivity of labor that prevails in their sector. Always one sector produces at the technology frontier while the other sector has low productivity. Sectoral productivity shocks occur at a stochastic rate. In an economy with perfect mobility of capital and labor, all workers work in the currently most productive sector and the less productive sector lends all its capital. In such a frictionless market, volatility and wage inequality do not occur and output is maximized. When firms are credit-constrained, aggregate output depends on the distribution of wealth between sectors. Improvements in the access to credit for firms reduce the volatility of aggregate output because the wealth distribution becomes less important. When workers are perfectly mobile between sectors, there exists one economy-wide wage. An increase in financial development implies that more capital is employed in the sector with higher total factor productivity and aggregate output, wage income as well as wage stability increase. In contrast, when workers are not perfectly mobile between sectors, wage inequality may arise. In this case, financial development increases the volatility of sector-specific wages and the wage dispersion between sectors increases. The mechanism behind this result is that workers cannot always react to each productivity shock at once by switching to the currently most productive sector. In each period only a fraction of workers employed in the low-wage sector can move into the high-wage sector. Financial development causes capital to move more quickly to the sector with higher returns and this increases the correlation of wages with sector-specific total factor productivity.

# Zusammenfassung

Die vorliegende Dissertation besteht aus **drei eigenständigen Forschungspapieren** über Lohnentwicklung in Märkten mit Friktionen. Jedes dieser Papiere (im Folgenden: Kapitel) betrachtet mindestens eine Art von Marktfriktion. Arbeitsmarktfriktionen in Form von Suchkosten oder unvollkommener Jobmobilität erschweren es Arbeitnehmern, in den Jobs zu arbeiten, welche ihnen den besten Lohn bieten; sie sind deshalb ein Grund für Lohndispersion. Kreditmarktfriktionen verlangsamen die Umverteilung von Kapital zu seiner produktivsten Verwendung und können somit eine Streuung von Kapitalrenditen verursachen.

Es gibt bisher nur wenige Arbeiten, die ein Suchmodell mit einem endlichen Planungshorizont der Arbeitnehmer betrachten. Die verbleibende Zeit bis zum Renteneintritt spielt jedoch eine Rolle beim optimalen Arbeitssuchverhalten, bei Entscheidungen über Investitionen in Humankapital und bei der Höhe des Lohns, welchen der Arbeitnehmer bereit ist zu akzeptieren. Das Verständnis dieser optimalen Verhaltensweisen ist wichtig für die Gestaltung von Arbeitsmarktprogrammen und bildet den Forschungsgegenstand der ersten beiden Kapitel der Dissertation.

**Kapitel 1** geht der Frage nach, ob Arbeitssuche eine treibende Kraft hinter Lohndispersion über den Lebenslauf darstellt. In Einklang mit früherer Literatur stelle ich einen u-förmigen Verlauf des Altersprofils von Lohnungleichheit in den USA fest. Die Varianz der residualen logarithmierten Löhne ist für junge Arbeitnehmer hoch und sinkt zunächst mit dem Alter. Für Arbeitnehmer mittleren Alters steigt sie wieder an. Auch gibt es große Unterschiede im Arbeitssuchverhalten zwischen den Altersgruppen. Junge Arbeitnehmer wechseln dreimal häufiger als ältere Arbeitnehmer den Arbeitgeber. Um Lohndispersion über den Lebenslauf zusammen mit on-the-job Arbeitssuche zu untersuchen, entwickle ich ein quantitatives Lebenszyklusmodell der Arbeitssuche mit strategischen Lohnverhandlungen, heterogenen Matchproduktivitäten und endogener Suchintensität. Unternehmen können auf externe Angebote an ihre Arbeitnehmer mit Gegenangeboten reagieren. Die Option, auch on-the-job nach Stellen zu suchen, senkt den Lohn, der für Arbeitnehmer akzeptabel ist, da eine hohe Matchproduktivität dem Arbeitnehmer

mehr Möglichkeiten für Lohnerhöhungen durch Außenangebote eröffnet. Der Optionswert der on-the-job Arbeitssuche ist umso geringer, je kürzer die Zeit bis zum Renteneintritt ist. Die Arbeitssuche verursacht Kosten. Ältere Arbeitnehmer suchen weniger als junge Arbeitnehmer, da der Wert einer neuen Stelle umso geringer ist, je weniger Zeit bis zum Renteneintritt verbleibt. Junge Arbeitnehmer wechseln häufig den Arbeitgeber und kommen so sukzessive zu besseren Jobs. Dadurch verringert sich zunächst die Varianz der logarithmierten Löhne mit dem Alter. Arbeitnehmer mittleren Alters wechseln weniger häufig den Arbeitgeber und haben zudem eine längere Suchhistorie. Da Arbeitnehmer im Arbeitsmarkt unterschiedlich erfolgreich sind, steigt die Varianz der Matchproduktivität ab der zweiten Hälfte des Berufslebens. Ich kalibriere das Modell mit Paneldaten aus den USA und zeige, dass das Modell das u-förmige Altersprofil der Lohn dispersion zusammen mit dem buckelförmigen Profil der Durchschnittslöhne und den fallenden Job-to-Job Übergangsraten abbilden kann.

**Kapitel 2** entwickelt ein Lebenszyklusmodell mit endogener Arbeitssuche und Investitionen in allgemeines Humankapital. Arbeitssuche und Training sind in dem Sinne effizient, dass sie den gemeinsamen Wert des Matches maximieren. Unter Berücksichtigung dieser Aspekte wird der Lohn in strategischen Lohnverhandlungen festgelegt. Arbeitgeber können versuchen, externe Jobangebote zu überbieten. Für Arbeitslose wie für Arbeitnehmer ist die Jobsuche kostenintensiv, und der Wert eines Jobangebots ist umso geringer je kürzer die erwartete Dauer einer Beschäftigung ist. Deshalb nimmt die Suchintensität wenige Jahre vor der Rente ab. Investitionen in Humankapital sinken über den Großteil des Lebenslaufs, da sich die Zeitspanne, in der Humankapital produktiv ist, verkürzt. Aus diesem Grund und wegen der Reduktion von Humankapital während Phasen der Arbeitslosigkeit ist das Altersprofil des durchschnittlichen Humankapitalstocks buckelförmig. Die durchschnittliche Matchproduktivität nimmt über einen Großteil des Lebenslaufs zu, da Arbeitnehmer nach und nach auf bessere Stellen wechseln. Junge Arbeitnehmer akzeptieren geringe Einstiegsgehälter, da ihnen eine Tätigkeit mit hoher Matchproduktivität eine bessere Position bei zukünftigen Lohnverhandlungen gibt und ihnen Training on-the-job geboten wird. Die Altersprofile von Humankapital, Matchproduktivität und Reservationsgehältern führen zu einem konkaven Altersprofil der Durchschnittslöhne. Da Humankapital während der Arbeitslosigkeit an Wert verliert, suchen Arbeitslose mit mehr Humankapital intensiver und verlassen die Arbeitslosigkeit deshalb schneller.

**Kapitel 3** widmet sich einem anderen Forschungsbereich: Seit den frühen 1980er Jahren ist in den USA der Anteil von Krediten am Bruttoinlandsprodukt (BIP) gestiegen. Begleitet wurde dieser Anstieg sowohl von einer Abnahme in der Volatilität des BIP-Wachstums als auch

von einer Zunahme der Lohnungleichheit. Das Kapitel untersucht den Einfluss fortschreitender Finanzmarktentwicklung auf makroökonomische Volatilität und Lohnungleichheit. Ähnliche theoretische Arbeiten stellen einen Zusammenhang zwischen Finanzmarktentwicklung und makroökonomischer Stabilität her. Der Hauptbeitrag dieses Kapitels liegt in der Einführung von Arbeitsmarktfriktionen in ein Modell realer Konjunkturzyklen, in welchem die Kreditaufnahme von Unternehmen auf den Umfang verfügbarer Sicherheiten begrenzt ist. Dadurch werden Wechselwirkungen zwischen Kredit- und Arbeitsmarkt sichtbar. Ich entwickle ein Modell mit zwei Sektoren. Arbeitnehmer werden nach der in ihrem Sektor herrschenden Grenzproduktivität entlohnt. Sektorale Produktivitätsschocks treten mit stochastischer Rate auf. Wären Kapital und Arbeit uneingeschränkt mobil zwischen den Sektoren, so würden alle Arbeitnehmer in dem aktuell produktiveren Sektor arbeiten, und der weniger produktive Sektor würde sein gesamtes Kapital verleihen. In solch einem Markt ohne Friktionen würden weder Volatilität noch Lohnungleichheit auftreten; der Output wäre maximiert. Wenn Unternehmen jedoch in ihrer Kreditaufnahme eingeschränkt sind, hängt die Gesamtproduktion von der Vermögensverteilung zwischen den Sektoren ab. Verbesserter Zugang zu Krediten reduziert die Volatilität des Output, da die Vermögensverteilung an Bedeutung verliert. Wenn Arbeitnehmer vollkommen mobil zwischen den Sektoren sind, existiert in der gesamten Volkswirtschaft nur ein Lohn. Eine bessere Finanzmarktentwicklung bedeutet, dass mehr Kapital in dem Sektor mit der höheren totalen Faktorproduktivität eingesetzt wird. Dadurch steigen sowohl der gesamtwirtschaftliche Output als auch die Löhne und die Lohnstabilität. Wenn Arbeitnehmer jedoch nicht vollkommen mobil sind, kann Lohnungleichheit entstehen. In diesem Fall erhöht Finanzmarktentwicklung die Volatilität sektorspezifischer Löhne, und die Lohndispersion zwischen den Sektoren steigt. Der Mechanismus hinter diesem Ergebnis ist der, dass Arbeitnehmer nicht bei jedem Produktivitätsschock sofort in den aktuell produktivsten Sektor wechseln können. In jeder Periode wechselt nur ein Teil der Arbeitnehmer aus dem Niedriglohnsektor in den Hochlohnsektor. Als Folge fortschreitender Finanzmarktentwicklung fließt Kapital schneller in den Sektor mit höheren Erträgen; dies verstärkt die Korrelation der Löhne mit der sektorspezifischen totalen Faktorproduktivität.

# Chapter 1

## Job Search and the Age-Inequality Profile

### 1.1 Introduction

The objective of this paper is to explore whether job search is a driving force behind wage dispersion over the life cycle. Understanding the sources of lifetime wage inequality is necessary for the design of welfare policies and insurance programs. Furthermore, the age structure of the population might be an important factor behind differences in income inequality between countries or changes in the wage structure across time. Search frictions and on-the-job search are potentially important determinants of residual wage dispersion (Burdett and Judd, 1983; Burdett and Mortensen, 1998; Postel-Vinay and Robin, 2002; Burdett and Coles, 2003; Hornstein *et al.*, 2011). Topel and Ward (1992) find that about one third of wage growth among young workers can be attributed to employment-to-employment transitions.

In line with several studies (Mincer, 1974; Dooley and Gottschalk, 1984; Heckman *et al.*, 2003), I find that the variance of residual log wages across workers in the U.S. follows a U-shape with age. The variance is high for young workers who have just entered the labor market. As workers grow older, it falls at first and starts to increase again in the second half of the working life. Polachek (2003) explores the variance of log wages for nine other countries<sup>1</sup> and finds the U-shaped relationship between the variance and age for most of them.<sup>2</sup>

In order to explore the age-inequality profile in conjunction with on-the-job search, I develop a life-cycle model of labor market search. Wages are determined by bargaining and workers' search intensity is endogenous. Firm-worker matches have different productivities, workers

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<sup>1</sup>The nine countries considered are Australia, Belgium, Canada, Czech Republic, France, Mexico, Taiwan, Spain, and Sweden.

<sup>2</sup>Deaton and Paxson (1994), Storesletten *et al.* (2004), and Huggett *et al.* (2011) find that wage dispersion rises with age.

search on and off the job, and incumbent employers can counter outside wage offers. Searching for a job is costly. Older workers choose to search less than young workers as the value of a job is lower when the time horizon until retirement shortens. I calibrate the model to U.S. panel data and show that the calibrated model captures the U-shaped age-inequality profile of wages in conjunction with the observed hump-shaped age profile of average wages and the employment-to-employment transition rate which decreases with age.

A large fraction of the overall wage inequality in the model is driven by match heterogeneity. Indeed, the endogenous age-variance profile of match qualities is U-shaped. Search on and off the job leads to two opposing effects on wage dispersion. Because of employment-to-employment transitions, workers are gradually matched to better jobs and this decreases the variance of match productivities. Because of unemployment-to-employment transitions, there is a permanent flow of workers into the lower tail of the productivity distribution. The first effect is dominant for young workers who switch employers often. The second effect dominates in the second half of the working life. Older workers have a longer search history. Workers who have obtained many good job offers are employed in high productivity matches. At the same time, some workers who became unemployed again have to accept low productivity matches. Older workers sort themselves more slowly into better matches as the optimal search effort decreases with age.

The wage formation mechanism is based on the strategic wage bargaining model of Cahuc *et al.* (2006). The model provides rich wage dynamics and allows for wage rises within an employment while remaining solvable. In contrast to the wage-posting model of Burdett and Mortensen (1998), firms make wage offers that depend on worker characteristics. Furthermore, firms can counter the outside offers of their workers. In Cahuc *et al.* (2006), a high match quality offers workers more opportunities to obtain wage rises because of possible outside job offers. This option value effect lowers the wage that workers are willing to accept. There are additional implications in the present model with a finite time horizon. The shorter the remaining time horizon before retirement, the lower is the option value of on-the-job search. Hence, workers who accepted a low starting wage when young might have a credible threat to quit into unemployment when growing older as their option value of on-the-job search is lower. In that case, they negotiate wage rises from the current employer without any outside job offer.

The U-shaped age-variance profile of match qualities only translates into a U-shaped age-variance profile of wages if workers' bargaining power is sufficiently high. If the workers' bargaining power is too low, the option value effect is very high for young workers and older workers' reservation wages increase strongly. The standard deviation of wages for workers close to retirement then falls sharply. If the bargaining power of workers is sufficiently high,



there is a modest increase in the reservation wage only for low quality matches prior to retirement. Apart from that, the reservation wage decreases for older workers since the probability of obtaining a better job offer by waiting decreases. For the same reason, also the observed hump-shaped age profile of the average wage is better matched if the bargaining power of workers is sufficiently high.

Related models with a finite working life and on-the-job search are Jung and Kuhn (2012) and Menzio *et al.* (2012). Jung and Kuhn (2012) explore earnings losses after displacement for workers with high tenure in conjunction with worker flows. Menzio *et al.* (2012) develop a life-cycle model with directed search and human capital accumulation. Their objective is to explain the age profile of worker transitions across employment states, while I focus on the wage distribution for different age groups. The random search model of the present paper features similar life-cycle profiles of transitions from unemployment to employment and between employers as Menzio *et al.* (2012). While their channel is directed search, the channel in the present model is endogenous search intensity. The main advantage of a directed search model is its solvability not only in steady state but also when the economy is not in steady state (Menzio and Shi, 2011). Here, I only consider the steady state, which is tractable since one can use the value of retirement as a terminal condition.

Other authors have explored the effects of a finite working life on labor market outcomes within search-theoretic models in which workers can only search when unemployed (Hairault *et al.*, 2010; Hahn, 2009; De la Croix *et al.*, 2009; Chéron *et al.*, 2008). These models can explain the hump-shaped age profile of employment, but without additional assumptions, they imply a decreasing age-wage profile. In order to obtain the empirically observed increasing and concave age-wage profile, Hairault *et al.* (2010) calibrate age-specific wage offer distributions. De la Croix *et al.* (2009) assume that workers' productivities increase with age and then decrease as workers approach retirement. Chéron *et al.* (2008) introduce human capital accumulation into their model.

This paper also relates to Bagger *et al.* (2011) and Yamaguchi (2010), who also explore the driving forces of wage dynamics over the life cycle in a bargaining model with counteroffers. They focus on the importance of job search and human capital accumulation for individual wage growth in a model with an infinite time horizon, while I focus on the importance of job search and a finite working life for shaping the age-inequality profile of wages.

There are different alternative approaches to the U-shape of the age-inequality profile. Heterogeneous age-tenure profiles are one potential source. Another approach attributes the high residual wage dispersion of young and older workers to investment in human capital accumulation (Mincer, 1974). Rubinstein and Weiss (2006) explore the implications of the human capital

investment model and a search model of the labor market for life-cycle wages. They find empirical support for both theories. While they argue that a search model cannot give rise to a U-shaped age-inequality profile, the present paper shows that search theory is sufficient to explain the U-shape. Rubinstein and Weiss (2006) argue that in a search model workers become increasingly heterogeneous at first as they can search on-the-job and are differently successful in finding good job offers. Since workers move up the wage ladder and since the probability of obtaining a higher wage decreases in the current wage, wage dispersion finally falls. In the model developed in the present paper some unemployed older workers accept low wages and then upgrade their wages only slowly. It is shown that this channel is one potentially important reason for the rise in wage dispersion among middle-aged and older workers.

The paper is organized as follows: Section 1.2 describes the data and derives the empirical age profile of wage inequality to be explained using the model framework set out in section 1.3. In section 1.4, I calibrate the model economy and quantitatively investigate the performance of the model in capturing the age-inequality profile of wages as well as age profiles of transition rates and average wages. Section 1.5 discusses the mechanisms that shape the age-inequality profile of wages. Section 1.6 concludes.

## **1.2 The empirical age profile of wage inequality**

This section discusses the empirical age profile of wage inequality. The finding that the variance of the residuals of a wage regression follows a U-shape with age has its origin in the work of Mincer (1974). Mincer's log earnings function is estimated by a regression of log earnings on years of experience, years of experience squared, and years of schooling. It has been estimated in several studies interested in the returns to schooling or post-school human capital investment. The theory states that human capital investments mostly take place when workers are young. Workers who invest in human capital on-the-job early in their career earn initially a low wage but have higher wage growth than non-investors. The standard deviation of residual log wages is then the lowest for middle-aged workers when the wage profiles of investors and non-investors cross. This implies a U-shaped age profile of wage inequality but also a negative correlation of the current wage with wage growth for young workers and a positive correlation of the current wage with wage growth for older workers. Rubinstein and Weiss (2006) find a negative correlation of the current wage with wage growth for all age groups, which is a feature of many search models in which better wage offers become less likely when the current wage is already high.

In the next section I develop a model of labor market search to explore the role of search

frictions for age-specific wage dispersion more closely. In order to calibrate the model, I use data from the 1996 panel of the U.S. Census' Survey of Income and Program Participation (SIPP), which spans the time period from December 1995 to February 2000.<sup>3</sup> The SIPP contains monthly data on the worker's employment status, earnings, weekly hours, primary job, and information on whether the worker has changed the employer. I restrict the analysis to a subsample of non-unionized men between the ages of 18 and 66, whose highest educational attainment is a high school degree, and who do not have any income from self-employment. Furthermore, I do not consider any workers in the armed forces and workers who stop working for school or training reasons. The data set comprises 10,340 individuals and 242,159 observations.

Residual log wages are derived from a fixed-effects regression of monthly log-wages on occupational dummies, a dummy for disabled workers, regional dummies, a dummy for marital status, and weekly hours. Time fixed effects are included. The estimated model is

$$\ln w_{it} = \alpha_i + \beta X_{it} + \varepsilon_{it},$$

where  $w_{it}$  is monthly earnings of worker  $i$  in period  $t$ ,  $\alpha_i$  is the unknown intercept for worker  $i$ ,  $\beta$  is a vector of coefficients,  $X_{it}$  is a vector of regressors, and  $\varepsilon_{it}$  is the error term. A description of the regressors and estimation results are presented in Appendix 1.7.1.

The age-inequality profile is determined by the standard deviations of the residual,  $\hat{\varepsilon}_{it}$  given age (in years). The residual is given by

$$\hat{\varepsilon}_{it} = \ln w_{it} - \widehat{\ln w_{it}},$$

where  $\widehat{\ln w_{it}}$  denotes the prediction of  $\alpha_i + \beta X_{it}$ . Figure 1.1 shows that the age-inequality profile is U-shaped. This result is robust to several alternative model specifications.<sup>4</sup> Table 1.1 contains the standard deviation of residual wages for larger age groups. It is 22 percent higher for young workers aged 18 to 27 than for middle-aged workers aged 38 to 47. The standard deviation for older workers aged 58 to 66 is 27 percent higher than for middle-aged workers.

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<sup>3</sup>Data source: Center for Economic and Policy Research. 2012. SIPP Uniform Extracts, Version 2.1.7. Washington, DC.

<sup>4</sup>A very similar age-inequality profile is obtained if number of kids, age, age squared, and/or interaction between occupation and age are included, and if weekly hours is excluded. Also if only full-time workers are considered, the age-inequality profile is U-shaped. The corresponding figures are shown in Appendix 1.7.1.

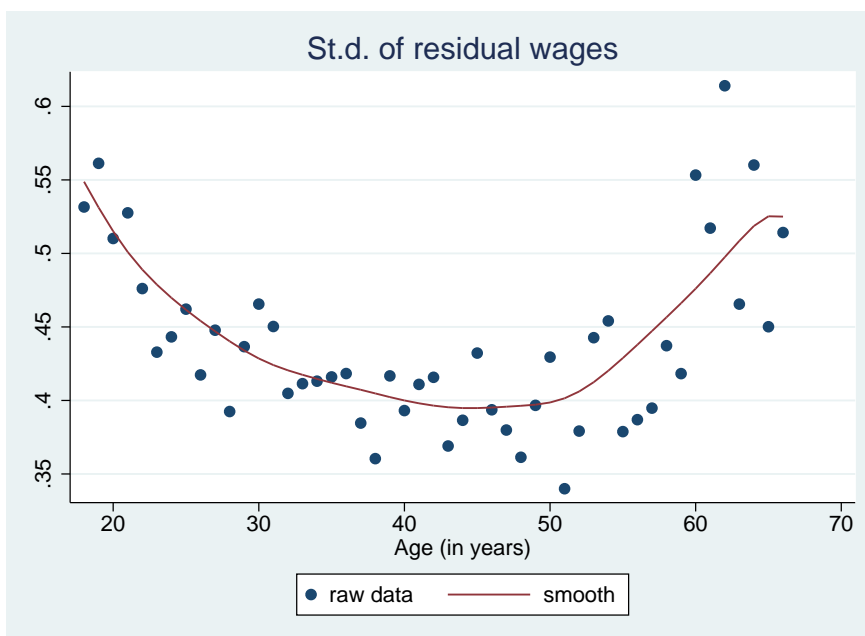


Figure 1.1: Age-inequality profile of residual log wages

Table 1.1: Standard deviation of residual wages

age group	st.d. of wages	number of residuals
18 – 27	0.4835	50,143
28 – 37	0.4198	53,509
38 – 47	0.3964	42,987
48 – 57	0.3971	24,141
58 – 66	0.5033	10,068

### 1.3 A life-cycle model with on-the-job search

In this section, I develop a basic life-cycle model of labor market search. The labor market is populated by a continuum of competitive firms and a unit mass of risk-neutral workers of different ages  $k = 1, 2, \dots, K$ . Time is discrete and the economy is in steady state. Firms produce a unique multipurpose good, maximize profits, and live forever. Each worker lives a finite life of  $K$  periods. In steady state, all workers that leave the labor market at age  $K + 1$  are replaced by unemployed workers of age 1. Hence, the fraction of the population aged  $k$  is given by  $l$  for all  $k < K$ .

Firm-worker matches differ in their productivities denoted as  $a_i$  with  $i = 1, \dots, n$  and  $a_{j-1} < a_j$ ,  $j = 2, \dots, n$ . The probability that a potential match has productivity  $a_i$  is given by  $p_i$ . The cumulative distribution of potential match qualities is denoted by  $P_i$ . When a firm and a worker meet, the quality of the potential match is revealed. For convenience, I describe a firm that offers a worker a match of quality  $a_i$  as a type  $i$  firm. Output per period in a firm-worker match does not depend on the worker's age and equals the marginal productivity of labor  $a_i$ . Unemployed workers receive an income flow of  $b_U$ . Workers derive utility from consumption and discount future utility at the factor  $\beta \in (0, 1)$ . I am interested in the importance of search frictions and a finite planning horizon for life-cycle wage inequality. The model therefore abstracts from experience effects and does not contain accumulation of human capital.

Workers search on and off the job. Searching for a job is costly for the worker. The cost of spending an effort  $e$  on searching is given by a cost function  $c(e)$ , with  $c(0) = 0$ . The cost function is increasing and strictly convex. The offer arrival rate per search effort is  $\lambda > 0$ . The search effort is derived endogenously by the worker's optimizing behavior. The timing of events is as follows. In the beginning of a period,  $g(k, a_i)$  workers aged  $k$  are employed at a match  $a_i$ . Each of these firm-worker matches is hit by an exogenous separation shock with probability  $\delta \in [0, 1]$ . Workers who become unemployed can immediately search for a new job that starts in the next period. The mass of unemployed workers of age  $k$  is then

$$u(k) = l - (1 - \delta) \sum_{j=1}^n g(k, a_j). \quad (1.1)$$

All workers that enter the labor market are unemployed, hence  $u(1) = l$ .

For the quantitative analysis of section 1.4, I apply a richer model taking into account that the rate at which workers become unemployed is age-dependent and that not all workers enter the labor market at the same age. I further account for age-dependent flows in and out of the labor force. For reasons of clarity I initially abstract from these details.

### 1.3.1 Wage bargaining

The wage formation rules are based on the bargaining model of Cahuc *et al.* (2006). If an employed worker obtains an outside wage offer, the incumbent employer can counter the outside offer. Workers and employers have complete information over each other's type and over the worker's wage and job offers. Wage contracts specify a wage that can only be renegotiated by mutual agreement. A renegotiation can occur if the worker has a credible threat to quit. Wage cuts within an employment do not take place since the productivity remains constant throughout

the duration of the match. Consider a worker of age  $k$  employed at a type  $i$  firm earning wage  $w$ . When the worker contacts a type  $h$  firm, the incumbent and the poaching employer compete for the worker. The maximum wage a firm is able to offer equals the match productivity. The worker chooses the firm that offers the highest lifetime utility. The outcome of the bargaining process depends on the productivity of both firms and on the current wage. Three cases can occur. If  $h > i$ , the worker switches to the poaching employer since the type  $h$  firm will offer the worker a wage that has a higher value than the highest wage the type  $i$  firm can offer. Note that the wage from the new employer can be smaller than  $w$  as the worker takes into account possible future wage rises. Such a wage cut is possible because of the option value of on-the-job search. An employment within a high productivity match gives the worker a better position for future wage negotiations. If  $h < i$ , the worker stays with the incumbent employer. The worker obtains a wage rise from the incumbent employer if and only if  $i \geq h \geq q(k, w, a_i)$ . If  $h$  is smaller than the threshold marginal productivity index  $q(k, w, a_i)$ , nothing changes for the worker. Table 1.2 gives an overview of the bargaining game.  $\phi(k, a_i, a_h)$  denotes the wage that is the outcome of a bargaining game between a type  $i$  firm and a type  $h$  firm, with  $h > i$ , and a worker of age  $k$ .

Table 1.2: Outcome of the wage bargaining game between a worker earning wage  $w$ , the incumbent employer of type  $i$ , and a poaching employer of type  $h$

	<b>negotiation outcome</b>
$h > i$	new employer $h$ and a wage $\phi(k, a_i, a_h)$
$i \geq h \geq q(k, w, a_i)$	wage rise $\phi(k, a_h, a_i) - w$ from current employer
$h < q(k, w, a_i)$	no change

The mechanisms of wage bargaining discussed so far are the same as in Cahuc *et al.* (2006). However, while they assume that workers have an infinite life, workers leave the labor market at a given age in the present model. A young worker's wage bargain outcome is different than that of a worker close to retirement. The option value of on-the-job search makes workers accept a low starting wage. The shorter the time horizon before retirement, the lower is the option value of on-the-job search. Hence, it can occur that workers negotiate wage rises from the current employer without any outside job offer when they have a credible threat to quit into unemployment.

Let  $\mathcal{W}(k, w, a_i)$  denote the value of a job to a worker of age  $k$  earning wage  $w$  in a match with productivity  $a_i$ . When the two competing firms have productivities  $i$  and  $h$  with  $i < h$ , type

$h$  firm wins the bargain by offering a wage  $\phi(k, a_i, a_h)$  that is determined by

$$\mathcal{W}(k, \phi(k, a_i, a_h), a_h) = \mathcal{W}(k, a_i, a_i) + \gamma[\mathcal{W}(k, a_h, a_h) - \mathcal{W}(k, a_i, a_i)], \quad (1.2)$$

where the parameter  $\gamma \in [0, 1]$  is the worker's bargaining power. The worker obtains a value  $\mathcal{W}(k, \phi(k, a_i, a_h), a_h)$  that equals his outside option  $\mathcal{W}(k, a_i, a_i)$  - the highest value the lower productivity firm can offer - plus a share  $\gamma$  of the match surplus.<sup>5</sup>

Consider a worker of age  $k$  earning wage  $w$  in a type  $i$  firm. The productivity index of the poaching firm must be at least equal to  $q(k, w, a_i)$  such that the worker obtains a higher lifetime utility in the bargaining game. Hence, the threshold productivity index  $q(k, w, a_i)$  is the lowest index for which

$$\mathcal{W}(k, w, a_i) < \mathcal{W}(k, a_{q(k, w, a_i)}, a_{q(k, w, a_i)}) + \gamma[\mathcal{W}(k, a_i, a_i) - \mathcal{W}(k, a_{q(k, w, a_i)}, a_{q(k, w, a_i)})] \quad (1.3)$$

is fulfilled. It follows that  $q(k, a_i, a_i) = i + 1$ . If the poaching employer has productivity  $h$  and  $i \geq h \geq q(k, w, a_i)$ , the negotiation outcome is a wage  $\phi(k, a_h, a_i)$  at the incumbent firm that is determined by

$$\mathcal{W}(k, \phi(k, a_h, a_i), a_i) = \mathcal{W}(k, a_h, a_h) + \gamma[\mathcal{W}(k, a_i, a_i) - \mathcal{W}(k, a_h, a_h)].$$

The outside option of an unemployed worker aged  $k$  is the value of unemployment denoted by  $\mathcal{U}(k)$ . A match between an unemployed worker and a type  $i$  firm is formed if and only if  $\mathcal{W}(k, a_i, a_i) \geq \mathcal{U}(k)$ . Provided this condition is satisfied, the firm offers a wage  $\phi_0(k, a_i)$  that solves

$$\mathcal{W}(k, \phi_0(k, a_i), a_i) = \mathcal{U}(k) + \gamma[\mathcal{W}(k, a_i, a_i) - \mathcal{U}(k)]. \quad (1.4)$$

A higher match quality offers the worker more opportunities to obtain wage rises because of possible outside job offers. This option value effect makes wages decrease in match quality. However, the higher the productivity of the firm that wins the bargain, the higher is the match surplus. The higher the worker's bargaining power, the more the worker captures of the match surplus. The bargaining power effect makes wages increase in match quality. In Cahuc *et al.* (2006), wages decrease in the productivity of the firm that wins the bargain if  $\gamma$  is sufficiently small such that the option value effect dominates. If  $\gamma$  is large enough, the bargaining power effect dominates and wages increase in productivity. There are additional implications in a model with a finite time horizon. The shorter the remaining time horizon before retirement, the

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<sup>5</sup>Here, I assume that wages are determined by a linear sharing rule. For a foundation of this wage equation by a bargaining game of alternating offers see Cahuc *et al.* (2006).

lower is the option value of on-the-job search.

### 1.3.2 Value functions

Each period, a worker decides how much effort  $e$  to spend on job search. The problem of an unemployed worker of age  $k < K - 1$  is summarized by

$$\mathcal{U}(k) = \max_{e \geq 0} \left\{ b_U - c(e) + \beta \left[ \mathcal{U}(k') + (1 - \delta)e\lambda \sum_{j=r(k')}^n [\mathcal{W}(k', \phi_0(k', a_j), a_j) - \mathcal{U}(k')] p_j \right] \right\},$$

where  $k' = k + 1$  and  $r(k')$  is the minimum productivity index of a match that a worker of age  $k'$  accepts. The unemployed worker's value is the flow income of unemployment  $b_U$  minus search costs plus the discounted continuation value. In the next period the worker obtains at least the value of unemployment. With probability  $e\lambda$  he receives a job offer. The expected gain in value of an offer to the worker is  $\sum_{j=r(k')}^n [\mathcal{W}(k', \phi_0(k', a_j), a_j) - \mathcal{U}(k')] p_j$ . With probability  $\delta$  the newly formed match is hit by a separation shock.

The reservation productivity  $a_{r(k')}$  is the lowest productivity level for which

$$\mathcal{W}(k', a_{r(k')}, a_{r(k')}) \geq \mathcal{U}(k')$$

holds. Since unemployed and employed workers face the same search cost function and the same offer arrival rate per search effort, the lowest acceptable match productivity for a worker equals the flow income when unemployed,  $b_U$ . In the remainder of the paper, I set

$$a_1 = a_{r(k)} = b_U,$$

such that all matches have a positive surplus. Using equation (1.4), the value of unemployment becomes

$$\mathcal{U}(k) = \max_{e \geq 0} \left\{ b_U - c(e) + \beta \left[ \mathcal{U}(k') + (1 - \delta)e\lambda \gamma \sum_{j=1}^n [\mathcal{W}(k', a_j, a_j) - \mathcal{U}(k')] p_j \right] \right\}. \quad (1.5)$$

The optimal search effort of an unemployed worker aged  $k$ ,  $e_U(k)$ , is the solution to the first order condition (FOC) of the maximization problem

$$c'[e_U(k)] = \beta(1 - \delta)\lambda \gamma \sum_{j=1}^n [\mathcal{W}(k', a_j, a_j) - \mathcal{U}(k')] p_j. \quad (1.6)$$

The value of a job to a worker of age  $k < K - 1$  earning wage  $w$  in a match with productivity  $a_i$



is derived as follows:

$$\begin{aligned}
\mathcal{W}(k, w, a_i) = \max_{e \geq 0} & \left\{ w - c(e) + \beta \left[ \delta \mathcal{U}(k') + (1 - \delta) \left[ \right. \right. \right. \\
& \left. \left. \left. \left( 1 - e\lambda [1 - P_{q(k', w, a_i) - 1}] \right) \max \left\{ \mathcal{W}(k', w, a_i), \mathcal{U}(k') + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{U}(k')] \right\} \right. \right. \right. \\
& + e\lambda \sum_{j=q(k', w, a_i)}^i \left( \mathcal{W}(k', a_j, a_j) + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{W}(k', a_j, a_j)] \right) p_j \\
& \left. \left. \left. + e\lambda \sum_{j=i+1}^n \left( \mathcal{W}(k', a_i, a_i) + \gamma [\mathcal{W}(k', a_j, a_j) - \mathcal{W}(k', a_i, a_i)] \right) p_j \right] \right] \right\}.
\end{aligned} \tag{1.7}$$

The worker's value is the current wage minus search costs plus the discounted continuation value. The worker becomes unemployed and earns a value  $\mathcal{U}(k')$  with probability  $\delta$ . The employed worker does not meet an outside firm that has a productivity larger than  $a_{q(k', w, a_i) - 1}$  with probability  $1 - e\lambda(1 - P_{q(k', w, a_i) - 1})$ . In this case the worker stays in his current match. As the option value of on-the-job search decreases with age, the worker renegotiates the wage if  $\mathcal{W}(k', w, a_i)$  becomes smaller than  $\mathcal{U}(k') + \gamma[\mathcal{W}(k', a_i, a_i) - \mathcal{U}(k')]$ . If the worker meets an outside firm with lower productivity than  $a_i$  but above  $a_{q(k', w, a_i) - 1}$ , he expects a wage rise from the incumbent employer and a bargain outcome with value  $\sum_{j=q(k', w, a_i)}^i (\mathcal{W}(k', a_j, a_j) + \gamma[\mathcal{W}(k', a_i, a_i) - \mathcal{W}(k', a_j, a_j)]) p_j$ . If the worker meets an outside firm with match productivity larger than  $a_i$ , he switches to the poaching firm and expects a value  $\sum_{j=i+1}^n (\mathcal{W}(k', a_i, a_i) + \gamma[\mathcal{W}(k', a_j, a_j) - \mathcal{W}(k', a_i, a_i)]) p_j$ . Let  $e_W(k, w, a_i)$  denote the optimal search effort of a worker earning wage  $w$  at a type  $i$  firm. The optimal search effort is the solution to the FOC

$$\begin{aligned}
c' [e_W(k, w, a_i)] = & \beta(1 - \delta)\lambda \left( \sum_{j=q(k', w, a_i)}^i \left[ (1 - \gamma)\mathcal{W}(k', a_j, a_j) + \gamma\mathcal{W}(k', a_i, a_i) \right] p_j \right. \\
& + \sum_{j=i+1}^n \left[ (1 - \gamma)\mathcal{W}(k', a_i, a_i) + \gamma\mathcal{W}(k', a_j, a_j) \right] p_j \\
& \left. - (1 - P_{q(k', w, a_i) - 1}) \max \left\{ \mathcal{W}(k', w, a_i), \mathcal{U}(k') + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{U}(k')] \right\} \right).
\end{aligned} \tag{1.8}$$

Alternatively, an employed worker's search effort could be chosen such that it is jointly efficient as in Lentz (2010). In this case, the corresponding employment contract would specify not only a wage but also the search effort that maximizes the joint surplus of the firm-worker match. However, since the search effort is private choice of the worker, the implementability

of jointly setting the search effort via a wage contract is unclear. Furthermore, the search effort that maximizes the joint surplus of a match of highest quality ( $a_n$ ) is zero even if the worker earns a low wage. As a worker in a type  $n$  match cannot upgrade his wage under such a contract, an unemployed worker who obtains a job offer at a type  $n$  match earns always a lower wage than a worker who switches from a type  $i$  match with  $1 < i < n$  to a type  $n$  match. It is hard to justify the enforceability of zero search effort especially in this case. I therefore assume that a worker chooses the search effort to maximize his own value of the match.

Let  $\mathcal{R}(K)$  be the value of retirement.<sup>6</sup> A worker aged  $K - 1$  faces the following values:

$$\mathcal{U}(K - 1) = b_U + \beta \mathcal{R}(K), \quad (1.9)$$

$$\mathcal{W}(K - 1, w, a_i) = w + \beta \mathcal{R}(K). \quad (1.10)$$

### 1.3.3 Steady-state labor market flows

The labor market dynamics lead to the following stationary distribution of workers across employment states. Let  $g(k, w, a_i)$  be the fraction of the population aged  $k$ , earning wage  $w$ , and being employed at a type  $i$  firm. The fraction of the population aged  $k$  being employed at a type  $i$  firm is given by  $g(k, a_i) = \int g(k, w, a_i) dw$ . The fraction of the population aged  $k'$  being employed at a type  $i$  firm is made up of the pool of unemployed workers that form a match with a type  $i$  firm, the workers that are recruited out of lower productivity jobs, and the workers that have stayed in a type  $i$  match:

$$\begin{aligned} g(k', a_i) = & e_U(k) \lambda u(k) p_i + (1 - \delta) \lambda p_i \sum_{j=1}^{i-1} \int e_W(k, w, a_j) g(k, w, a_j) dw \\ & + (1 - \delta) \int [1 - e_W(k, w, a_i) \lambda (1 - P_i)] g(k, w, a_i) dw. \end{aligned} \quad (1.11)$$

### 1.3.4 Wage distribution

Let  $G(w|k, a_i)$  be the cumulative distribution of wages conditional on age and productivity. The maximum wage a type  $i$  firm can offer is  $a_i$ . Hence

$$G(a_i|k, a_i) = 1.$$

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<sup>6</sup>The value of  $\mathcal{R}(K)$  has an effect only on the scale of the value functions but not on equilibrium wages or search efforts.

All newborns are unemployed. Employed workers aged  $k = 2$  earn a wage  $\phi_0(2, a_i)$  since they were hired out of unemployment and have not yet searched on the job. The cumulative distribution of wages conditional on age and productivity for workers of age  $k' \geq 3$  is determined by

$$G(w|k', a_i) = I_{w \geq \phi_0(k', a_i)} \left\{ e_U(k) \lambda u(k) p_i + (1 - \delta) \lambda p_i \sum_{j=1}^{q(k', w, a_i) - 2} \int e_W(k, \tilde{w}, a_j) g(k, \tilde{w}, a_j) d\tilde{w} \right. \\ \left. + (1 - \delta) \int^w [1 - e_W(k, \tilde{w}, a_i) \lambda (1 - P_{q(k', w, a_i) - 1})] g(k, \tilde{w}, a_i) d\tilde{w} \right\} / g(k', a_i), \quad (1.12)$$

where  $I_{w \geq \phi_0(k', a_i)}$  is a dummy variable equal to 1 if  $w \geq \phi_0(k', a_i)$  and 0 otherwise. The conditional cumulative distribution of wages  $G(w|k', a_i)$  is the sum of unemployed workers with reservation wage  $\phi_0(k', a_i) \leq w$  who meet a type  $i$  firm, workers that switch from a lower productivity firm to a type  $i$  firm for a wage  $\leq w$ , and workers that stay in their current match of type  $i$  who do not earn a wage larger than  $w$ . Workers are only willing to switch to a type  $i$  firm for a wage  $\leq w$  if the match productivity of the current employment is smaller than  $q(k', w, a_i)$ .<sup>7</sup> The cumulative distribution of wages conditional on age is determined by

$$G(w|k) = \frac{\sum_{j=1}^n G(w|k, a_j) g(k, a_j)}{g(k)}. \quad (1.13)$$

### 1.3.5 Equilibrium

A *stationary equilibrium* consists of

- the optimal search efforts  $e_U(k)$  and  $e_W(k, w, a_i)$  given by the first-order conditions (1.6) and (1.8),
- the reservation wages  $\phi_0(k, a_i)$  and  $\phi(k, a_i, a_h)$  derived from equations (1.4) and (1.2),
- the threshold productivities  $q(k, w, a_i)$  of employed workers derived from condition (1.3),
- a stationary employment distribution of unemployed workers  $u(k)$ , of employed workers  $g(k, a_i)$ , and the cumulative distribution of wages  $G(w|k, a_i)$ , given by equations (1.1), (1.11), and (1.12)

for all combinations of age  $k < K$ , wages  $w$ , and match productivities  $a_i$ , given an exogenous productivity distribution, a constant mass of new workers of age  $k = 1$ , and the value of retire-

<sup>7</sup>The index above the summation sign in equation (1.12) is set equal to  $q(k', w, a_i) - 2$  such that the equation fulfills  $G(a_i|k', a_i) = 1$  and equation (1.11).

ment  $\mathcal{R}(K)$ .

**Proposition 1.1.** *The stationary equilibrium exists and is unique.*

*Proof.* Given a value of retirement  $\mathcal{R}(K)$ , the optimal search efforts ( $e_U(k)$  and  $e_W(k, w, a_i)$ ), reservation wages ( $\phi_0(k, a_i)$  and  $\phi(k, a_i, a_h)$ ), and threshold productivities ( $q(k, w, a_i)$ ) can be computed starting with age  $k = K - 1$  and continuing backwards in age. Using this and the condition that all newborns are unemployed ( $u(1) = l$ ), the stationary employment distribution ( $u(k)$  and  $g(k, a_i)$ ) and wage distribution ( $G(w|k, a_i)$ ) can be computed for all combinations of age  $k < K$ , wages  $w$ , and match productivities  $a_i$  starting with age  $k = 2$  and continuing upwards in age.  $\square$

## 1.4 A quantitative analysis

In this section, I calibrate the model and derive the equilibrium life-cycle profiles of the unemployment-to-employment transition rate, the employment-to-employment transition rate and the wage distribution.

The definition of employment states and transition rates derived from the SIPP data follows Menzio *et al.* (2012). A worker is assigned an employer based on his primary job where he worked the most hours. A worker is not in the labor force (N) if he reports having no job, not looking for work, and not being on layoff. A worker is unemployed (U) if he reports having no job and looking for work or being on layoff. A worker is employed (E) if he reports having a job and being either on layoff or not and absent without pay or not. A worker is in the labor force (L) if he is either employed or unemployed. The unemployment-to-employment transition (UE) rate is defined as the number of workers that experience a transition from unemployment to employment in a given month divided by the number of unemployed workers at the beginning of the month. The other transition rates are defined analogously.

The data shows that there are workers who flow in and out of the labor force across all age groups (see Figure 1.2). Furthermore, the empirical rate at which employed workers become unemployed is decreasing with age (see Figure 1.3). These transitions influence the wage distribution and are therefore taken into account when calibrating the model. They are directly calibrated. Figure 1.4 shows how the labor market dynamics enter the model calibration. The complete model is described in Appendix 1.7.2.

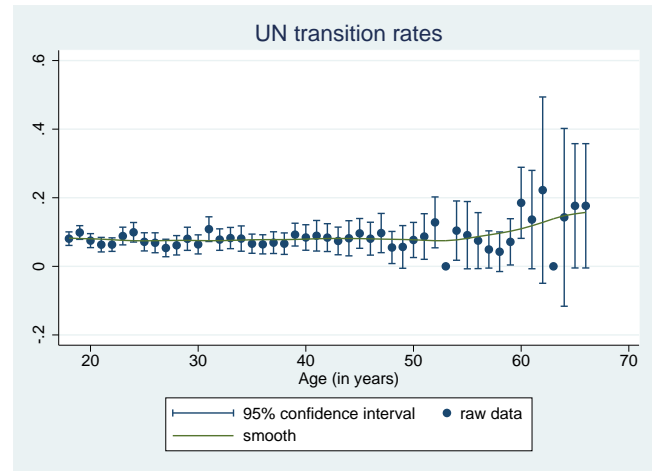
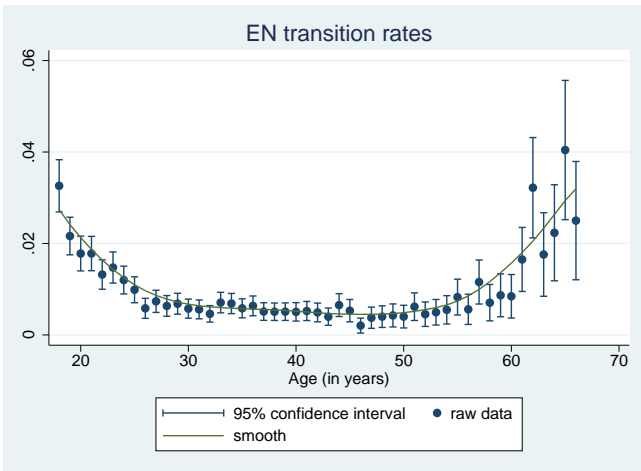
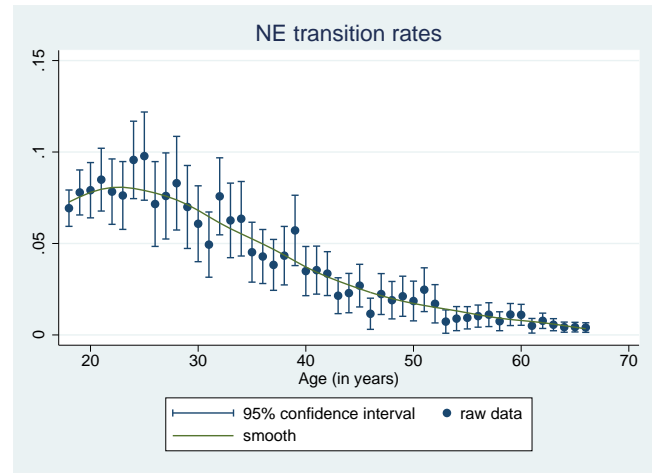
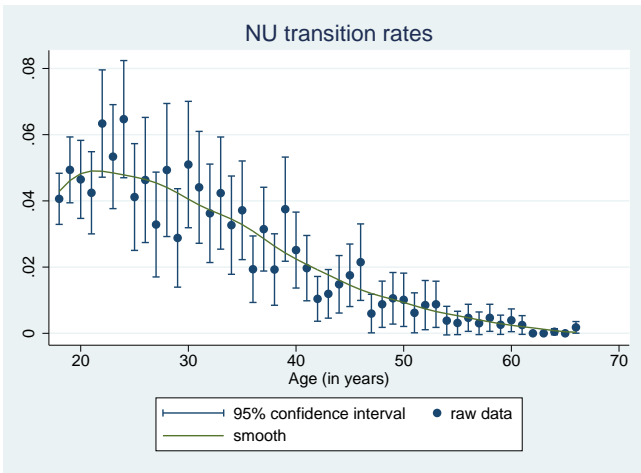


Figure 1.2: Flows in and out of the labor force

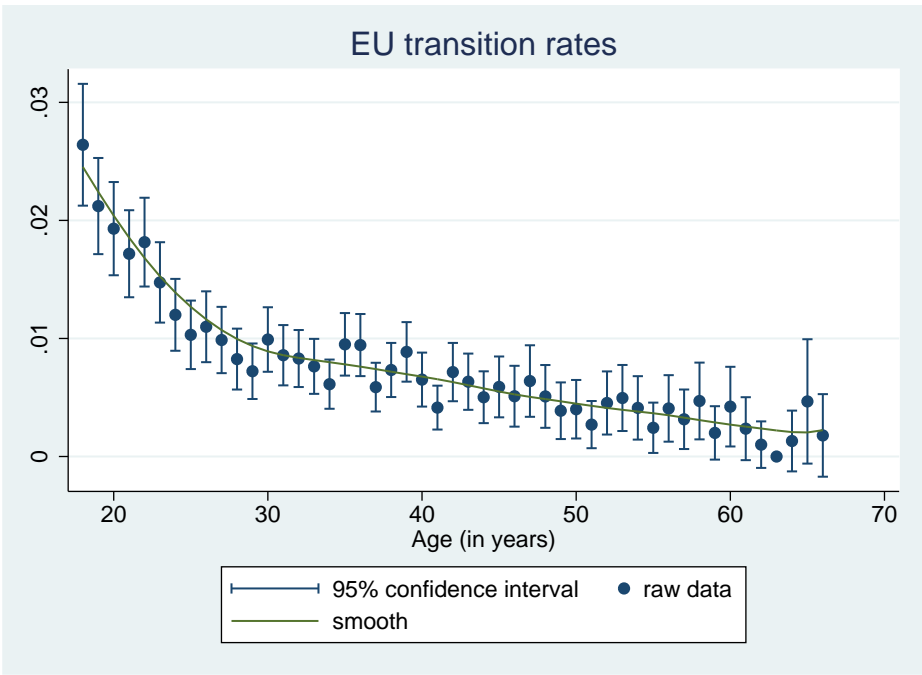


Figure 1.3: Life-cycle profile of the EU rate

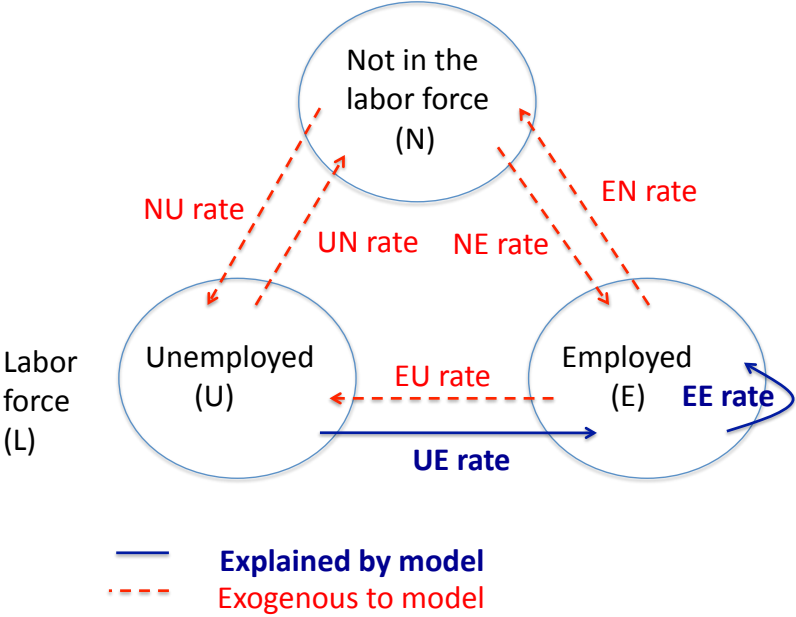


Figure 1.4: Labor market transitions

## 1.4.1 Calibration

The model is calibrated to monthly data. I therefore set the discount factor to  $\beta = 0.9967$ , which implies an annual real interest rate of 4 percent. Workers retire after 49 years in the labor market, i.e.  $K = 588$ . The distribution of match qualities is Weibull with scale parameter  $\phi$ , shape parameter  $\tau$ , and a location parameter that equals  $b_U = 0.05 (= a_1)$ .<sup>8</sup> The number of grid points is  $n = 40$ . The cost of spending an effort  $e$  on searching is given by the quadratic cost function<sup>9</sup>

$$c(e) = ce^2.$$

I set  $c = 0.5$ .<sup>10</sup> The estimate of the vector of structural parameters  $\theta = (\gamma, \lambda, \tau, \phi)$  minimizes the distance between simulated moments and corresponding moments obtained from the SIPP 1996 panel. I use 53 estimation targets: Average wages within each age group (49 targets), the standard deviation of residual wages, the EE rate, the UE rate, and the skewness of the wage distribution.

The simulated moments depend on the four structural parameters to be estimated. The transition rates are mainly influenced by the offer arrival rate per search effort  $\lambda$ . In addition, a higher bargaining power of workers  $\gamma$  has a positive effect on the UE rate as it rises the expected value of a job. As workers obtain higher wages,  $\gamma$  has a negative effect on the EE rate. The EE rate contains also information on the parameters of the distribution of match qualities. A more dispersed productivity distribution induces more EE transitions. The standard deviation of residual wages and the skewness contain mainly information on the productivity distribution. The age-wage profile contains information on the life-cycle profile of reservation wages and thereby on the bargaining power parameter  $\gamma$ . The influence of  $\gamma$  on the life-cycle profile of wages is discussed in more detail in section 1.5.

Table 1.3 contains the calibration targets and Table 1.4 the estimated parameters. The model captures the hump shape of the age-wage profile (Figure 1.5). It furthermore matches well the standard deviation of wages, the average UE rate, and the average EE rate. It captures the negative sign of the skewness of the distribution of log wages, though underestimates it.

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<sup>8</sup>The choice of the location parameter mainly influences the scale of wages.

<sup>9</sup>Christensen *et al.* (2005) estimate a model with on-the-job search in which the search effort is endogenous and the offer arrival rate per search effort is the same for employed and unemployed workers. Their results support a quadratic cost of search function.

<sup>10</sup>The FOCs (1.6) and (1.8) show that  $\lambda$  and  $c$  cannot be identified separately but only the ratio  $\lambda/c$ . One can therefore normalize arbitrarily  $c$  and then calibrate the parameter  $\lambda$ .

Table 1.3: Calibration Targets

Target	Data	Model
Age-wage profile		see Figure 1.5
St.d. of residual wages	0.4379	0.4340
Average UE rate	0.1685	0.1782
Average EE rate	0.0154	0.0150
Skewness	-2.5406	-1.9599

Table 1.4: Point Estimates

Description	Parameter	Estimate
Workers' bargaining power	$\gamma$	0.7295
Offer arrival rate per search effort	$\lambda$	0.0615
Shape parameter	$\tau$	1.9661
Scale parameter	$\phi$	4.9306

## 1.4.2 Life-cycle profiles

Let us turn to the standard deviation of wages illustrated in Figure 1.6. The age-inequality profile of wages is U-shaped in the data and in the model. The age-inequality profile falls for young workers because the EE rate is high for this age group and workers are gradually matched to better jobs. However, better job offers become less frequent for workers in a high quality match. For middle-aged and older workers the longer search history plays a dominant role. The standard deviation of match qualities increases. This occurs because workers are differently successful in finding good job offers and career paths diverge. Some workers have obtained many good job offers and are employed in a high productivity match at a high wage. At the same time, there are workers who flow from unemployment to employment for a low productivity match and a low wage. Workers reduce their search effort when they approach the retirement age and move therefore more slowly to higher productivity matches. Hence, the effect that led to the reduction in inequality among young workers is too weak for middle-aged and older workers. This is reflected in the increase in the standard deviation of this age group.



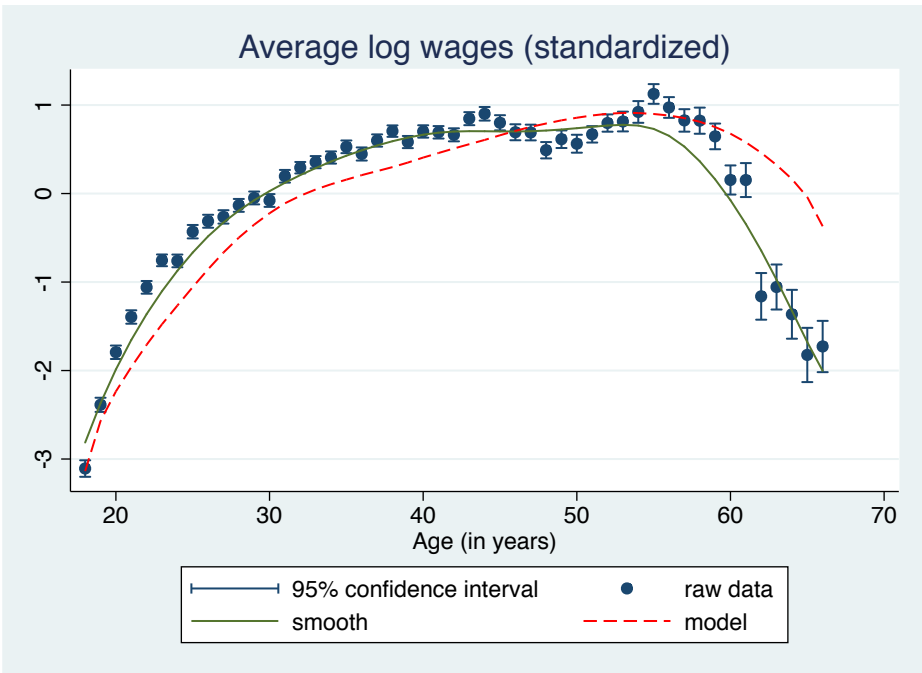


Figure 1.5: The age profiles of average log wages are standardized for comparability between model and data. The standardized age-specific average wage is derived by dividing the difference between the age-specific average wage and its mean by its standard deviation.

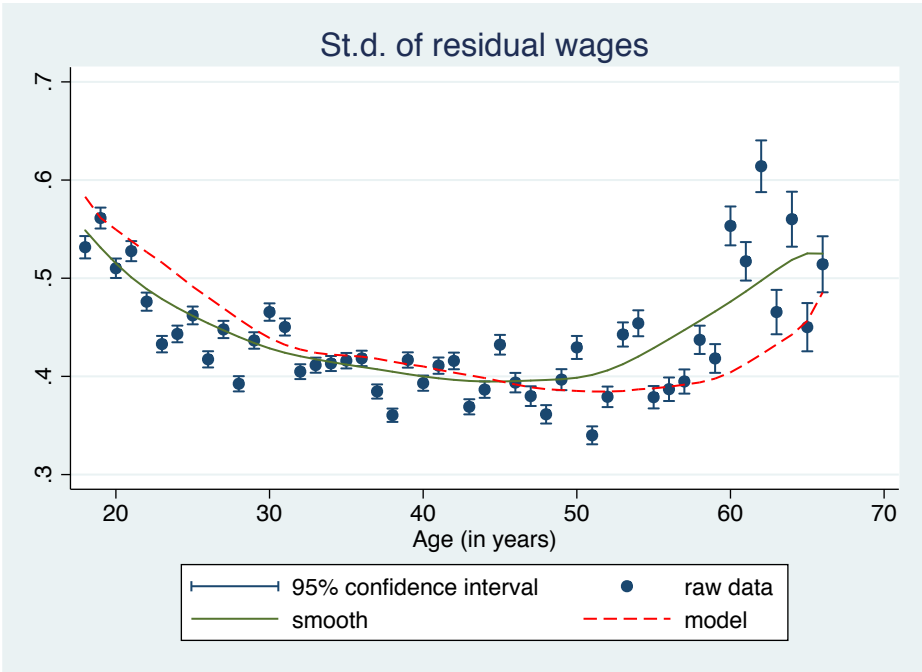


Figure 1.6: Wage dispersion

Figure 1.7 displays the life-cycle profiles of the empirical and of the model EE rate. The life-cycle profile of the EE rate decreases with age. The decreasing EE age profile is the result of two effects. A worker's search history increases with age and so does the average match quality. The probability of obtaining a better match decreases in the quality of the current match. Furthermore, older workers reduce their search effort since the remaining time horizon in the labor market shortens. The EE rate approaches zero for workers close to retirement because these workers reduce the search effort substantially. The simulated EE rate matches the empirical one well. Compared with the data, the EE rate obtained from the simulation declines sharply for older workers as all workers retire at the same age in the model economy.

The life-cycle profile of the UE rate remains relatively constant until a few years before retirement, then declines dramatically as workers reduce their search effort substantially when they approach the retirement age (see Figure 1.8). Searching for a job is costly and the expected value of a job offer is small for workers close to retirement. The model UE rate slightly increases until age 48 because the rate at which workers quit employment decreases until this age group. This reduction in the quit rate has a positive effect on the value of a job and thereby also on the search effort of unemployed workers.

Figure 1.9 shows the age profile of the unemployment rate. It is decreasing strongly for young workers, since workers are initially unemployed and gradually matched to their first jobs.

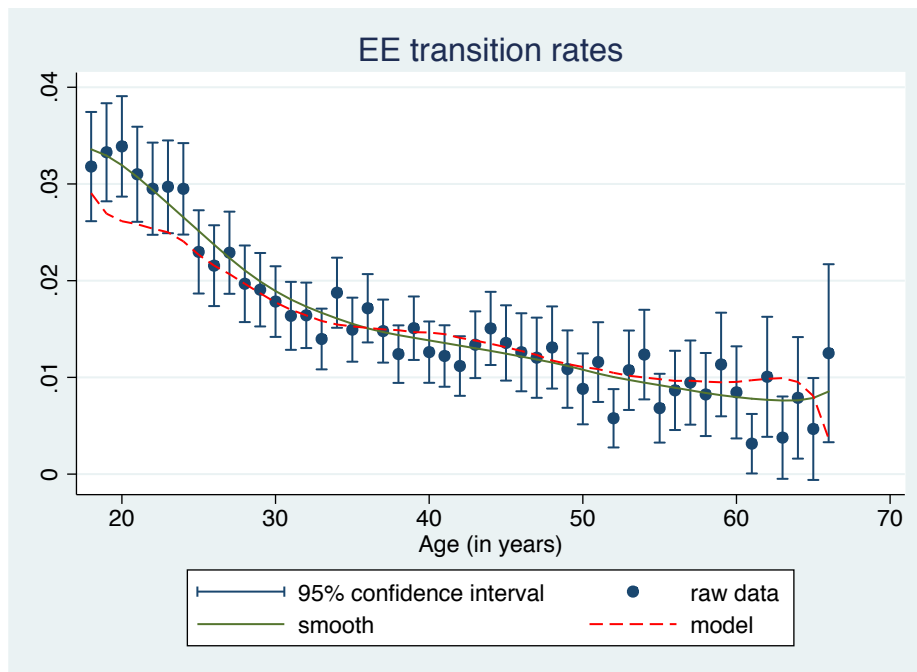


Figure 1.7: Average employment-to-employment transition rates

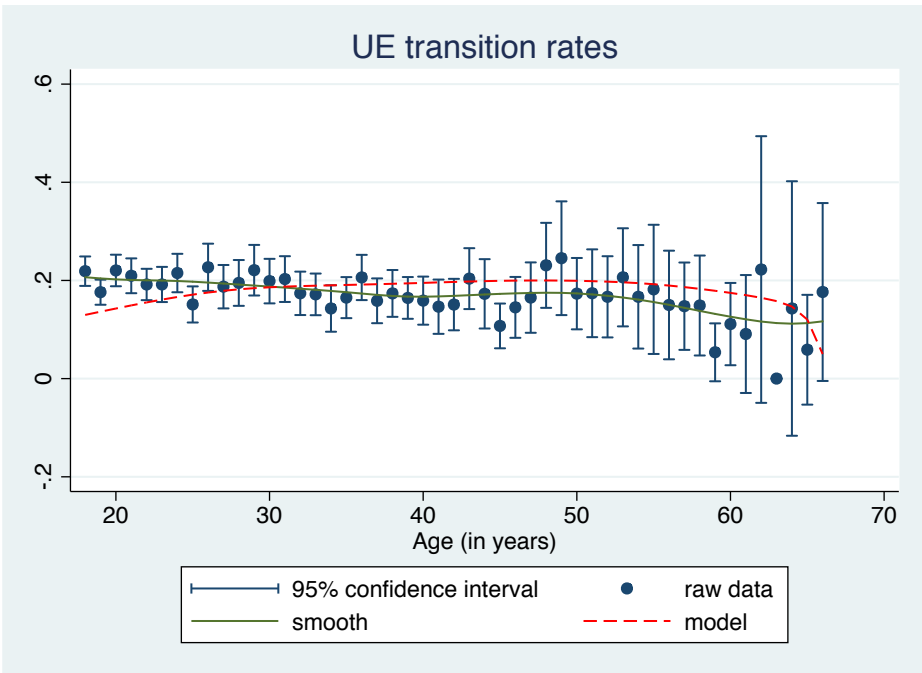


Figure 1.8: Average unemployment-to-employment transition rates

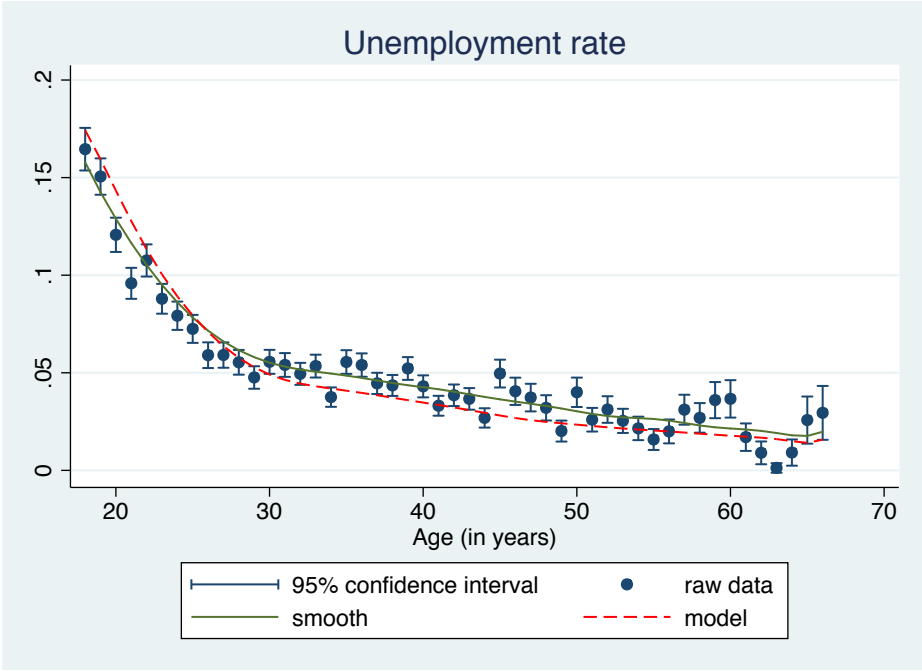


Figure 1.9: Unemployment rates

The endogenous life-cycle profiles of EE and UE transitions in the present model of random search are similar to those in the directed search model of Menzio *et al.* (2012). Endogenous search effort is the main channel at work in the present model. Workers search less when the time until retirement is short. Workers also search less when they are well matched or earn a high wage as the expected additional value of a job offer is lower.<sup>11</sup> The second channel that is responsible for the strong decrease in the EE rate for young workers is sorting into better matches. Workers are gradually matched to better jobs and this lowers their probability of obtaining an even better job offer. Figure 1.10 illustrates that the reduction in search effort and the decreasing probability that the poaching employer has a higher productivity are responsible for the decrease in the EE rate until the age of 53. Afterwards, the EE rate only falls because of the reduction in search effort while the probability that a job offer leads to an EE transition rises again. Menzio *et al.*'s model does not contain search effort. The mechanism in their model is directed search. There is a continuum of submarkets. Each submarket is targeted at workers of a specific age and productivity and offers workers an employment contract with a specified value. Workers face a tradeoff between a high offer arrival rate and a high value of a job when choosing in which submarket to search. Workers choose to search in a submarket that offers a high value but has a low vacancy to applicant ratio and therefore low offer arrival rate if the value of their current position is high. Firms choose in which submarkets to create how many vacancies. In the calibrated model, workers search in submarkets with a lower vacancy to applicant ratio when they grow older, are employed in a good match or are less experienced and therefore less productive.<sup>12</sup>

## 1.5 Discussion

The endogenous age-inequality profile of match qualities is U-shaped. This translates into a U-shaped age-inequality profile of wages if the bargaining power of workers is sufficiently high (see Figure 1.11). If the bargaining power of workers is much lower than the calibrated value, the standard deviation of wages sharply decreases for older workers. Figure 1.12 illustrates this by showing the age-inequality profile of wages derived from a model calibration in which  $\gamma$  is set equal to 0.5. The vector of the remaining structural parameters  $\theta_{\gamma=0.5} = (\lambda, \tau, \phi)$  was

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<sup>11</sup>Better matched workers search less in most cases. However, when two workers earn the same wage but have different match qualities, the worker with higher match quality searches more because he has a better position in wage negotiations and therefore a higher probability of obtaining a wage rise. The same applies if the worker in the higher quality match earns a lower wage.

<sup>12</sup>In Menzio *et al.* (2012) more experienced workers are more productive as learning-by-doing increases their stock of human capital.

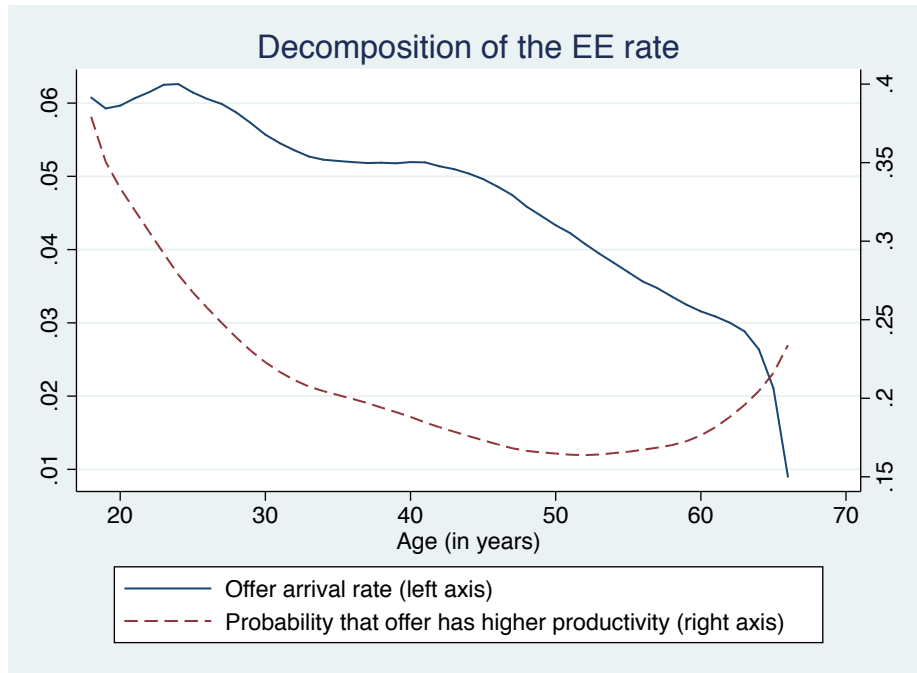


Figure 1.10: Decomposition of the EE rate into the average offer arrival rate ( $e\lambda$ ) and the average probability ( $1 - P_i$ ) that the poaching employer has higher productivity than the incumbent employer (of type  $i$ ).

estimated using the same targets as before.



Figure 1.11: Dispersion of wages and match qualities; estimated  $\gamma = 0.7295$

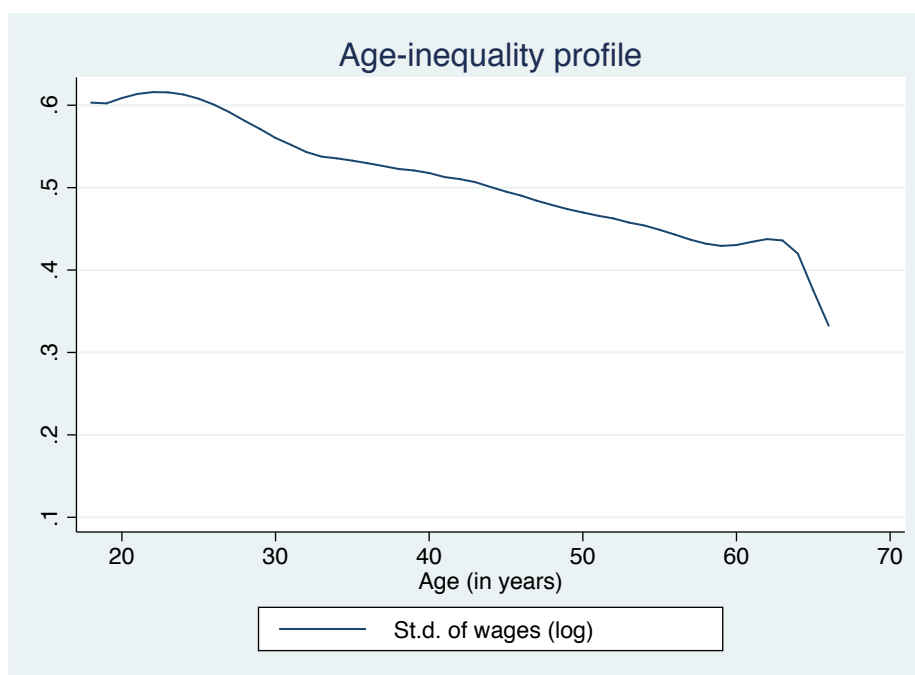


Figure 1.12: Calibrated model if  $\gamma = 0.5$

The bargaining power parameter has an effect on the age-inequality profile of wages through the worker's reservation wage. When  $\gamma$  is too low, young workers accept wages far below the match productivity as their option value of on-the-job search is high. The shorter the time until retirement, the lower is the option value of on-the-job search. Older workers therefore demand higher wages. This increases the lowest bound of the conditional wage distribution given the match productivity. Hence, the standard deviation of wages decreases for workers close to retirement. When the bargaining power parameter is sufficiently large and the match quality is relatively high, the decreasing time horizon has the opposite effect on wages. Older workers accept lower wages since the probability of obtaining a better job offer by waiting decreases.

Figures 1.5 and 1.13 compare the age-wage profiles of the model economy with the empirical one. The age-wage profile in the U.S. economy is hump-shaped.<sup>13</sup> Average wages increase with age for young and middle-aged workers. They decrease with age a few years before retirement. The worker's bargaining power must be sufficiently high such that the present model reproduces a hump-shaped age-wage profile. Because workers are gradually matched to better jobs, the average match quality and the average wage in the model economy increase with age. It increases at a decreasing rate because job offers from higher quality matches become less probable the higher the productivity of a match. Although the UE rate decreases when workers

<sup>13</sup>A concave age-wage profile in the U.S. can be found in several empirical studies including Kambourov and Manovskii (2009) and Mincer (1974).

approach the retirement age, there are permanent flows from unemployment to employment until one period before retirement. All workers recruited out of unemployment who have not obtained any outside offer, have the same distribution of match productivities with a low average match quality independent of age. Because also the search effort of employed workers decreases with age, an increasing fraction of the workers in low quality matches does not move to higher quality matches. As a result, the average match quality decreases some years prior to retirement. When the worker's bargaining power  $\gamma$  is high, the age-wage profile is similar to the age-match productivity profile and depicts the empirically observed hump-shaped age-wage profile. When  $\gamma$  is low, the above explained increase of the reservation wage has a positive effect on the average wage prior to retirement.

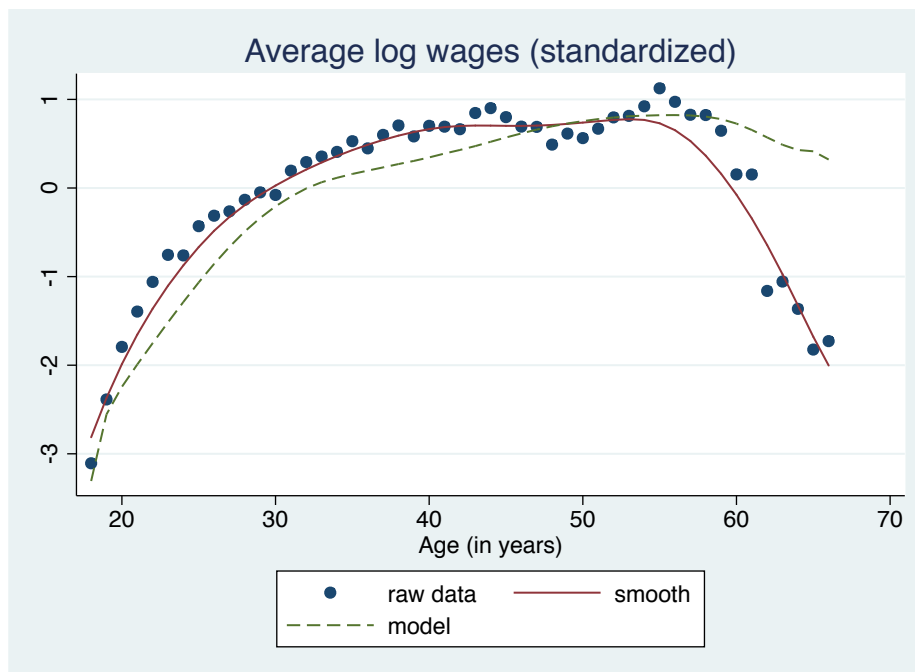


Figure 1.13: Calibrated model if  $\gamma = 0.5$

When the bargaining power parameter  $\gamma$  is chosen to match the empirically supported hump-shaped age-wage profile and the U-shape of the age-inequality profile of wages, it must be rather high (roughly 0.7) in this model. This is in contrast to Cahuc *et al.* (2006) who find for French data that  $\gamma$  lies between 0 and 0.35. An exception is the high value of  $\gamma = 0.98$  for high skilled workers in the construction sector. Bagger *et al.* (2011) explore the importance of human capital accumulation and labor market competition for life-cycle wage dynamics in a bargaining framework similar to Cahuc *et al.* (2006). They find in their analysis of Danish data that the bargaining power  $\gamma$  lies between 0.2475 and 0.4141 and declines with education. In both papers, workers have an infinite working life. The present paper provides a different interpretation of

the bargaining power parameter. It contains information on the relative importance of the option value of on-the-job search over the life cycle.

## 1.6 Conclusions

I consider a life-cycle model of labor market search with strategic wage bargaining, counteroffers, match heterogeneity and endogenous search effort. I show that the model can reproduce the U-shape of the age-inequality profile of wages if the bargaining power of workers is sufficiently high. Furthermore, the present model captures the shapes of the empirically observed age profiles of average wages, the unemployment-to-employment transition rate, and the employment-to-employment transition rate. The shape of the age-inequality profile of wages is mainly driven by the age profile of reservation wages, by transitions into employment, and transitions between employers. The optimal search effort of employed workers depends on the worker's time horizon before retirement, the current wage, and the quality of the firm-worker match. Furthermore, the probability of meeting an outside firm with a higher match quality decreases in the quality of the current match. This leads to frequent employment-to-employment transitions of young workers, a moderate employment-to-employment transition rate of middle aged workers, and a sharp decrease in the employment-to-employment transition rate of workers close to retirement. The bargaining power parameter plays an important role in the model because the option value of on-the-job search decreases when the time horizon before retirement shortens. A low bargaining power makes young workers accept a wage far below the productivity of the firm-worker match. Since the option value of on-the-job search is low for workers close to retirement, the reservation wage increases for older workers. This leads to a decline in the standard deviation of wages for older workers when the workers' bargaining power is too low.

This paper focuses on job search as an important factor for the shape of the age-inequality profile of wages. There is evidence that residual wage dispersion is well explained by both human capital and search on-the-job (Burdett *et al.*, 2011; Tjaden and Wellschmied, 2012). An obvious extension would therefore be the introduction of human capital accumulation through learning-by-doing. Furthermore, it would be interesting to introduce optimal human capital investments into the model in order to assess the relative contribution of job search and post-school human capital investments to the life-cycle wage inequality. The literature that combines search theory and the theory of on-the-job training (Acemoglu and Pischke, 1998; Moen and Rosén, 2004; Wasmer, 2006; Stevens, 2012) shows that there are important interactions between labor turnover and endogenous human capital investments. Exploring these interactions in a life-cycle model is the focus of chapter 2.



## 1.7 Appendix

### 1.7.1 Estimation

Table 1.5: Regressors

<b>Variable</b>	<b>Description</b>
Major occupations (occ14)	1 Executive, Administrative, and Managerial 2 Professional Speciality 3 Technicians and Related Support 4 Sales 5 Administrative Support , Including Clerical 6 Private Household Services 7 Protective Services 8 Services, except Household and Protective 9 Farming, Forestry, and Fishing 10 Precision Production, Craft, and Repair 11 Machine Operators, Assemblers, and Inspectors 12 Transportation and Material Moving 13 Handlers, Equipment Cleaners, Helpers, and Laborers
Disability that limits work (disabled)	0 not disabled 1 disabled
Census Region (region)	1 New England 2 Middle Atlantic 3 E. North Central 4 W. North Central 5 South Atlantic 6 E. South Central 7 W. South Central 8 Mountain 9 Pacific
Marital Status (ms)	0 Never Married 1 Married, Widowed, Divorced, or Separated
Weekly hours (hours)	Weekly hours in dominant job

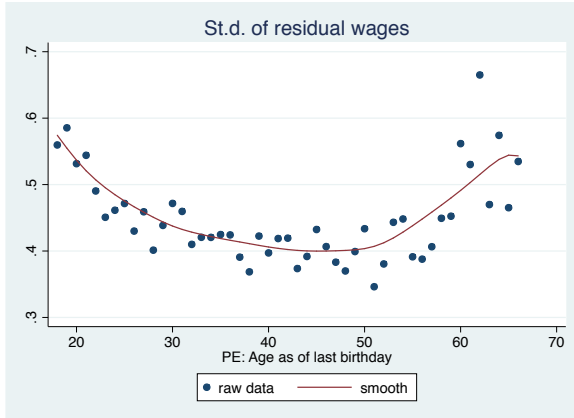
Table 1.6: Estimation results

Variable	Coefficient	(Robust Std. Err.)
1b.occ14	0.000	(0.000)
2.occ14	-0.006	(0.048)
3.occ14	-0.050	(0.046)
4.occ14	-0.087**	(0.031)
5.occ14	-0.082*	(0.032)
6.occ14	-0.666**	(0.236)
7.occ14	-0.127*	(0.057)
8.occ14	-0.184**	(0.033)
9.occ14	-0.280**	(0.048)
10.occ14	-0.051 <sup>†</sup>	(0.028)
11.occ14	-0.068*	(0.030)
12.occ14	-0.074*	(0.036)
13.occ14	-0.110**	(0.030)
0b.disabled	0.000	(0.000)
1.disabled	-0.081**	(0.022)
1b.region	0.000	(0.000)
2.region	0.090	(0.099)
3.region	0.193	(0.131)
4.region	0.013	(0.137)
5.region	0.083	(0.108)
6.region	0.378**	(0.135)
7.region	0.052	(0.107)
8.region	0.165	(0.130)
9.region	0.213	(0.133)
0b.ms2	0.000	(0.000)
1.ms2	0.094**	(0.024)
hours	0.017**	(0.001)
Intercept	6.617**	(0.102)
Observations		180,848
Groups		8,422
R <sup>2</sup> within		0.077
R <sup>2</sup> between		0.356
R <sup>2</sup> overall		0.244
F <sub>(72,8421)</sub>		46.028

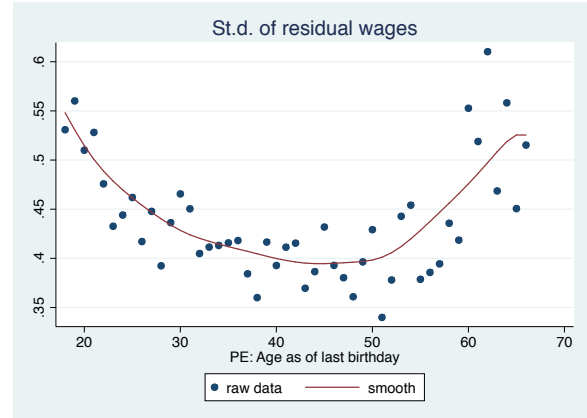
A <sup>†</sup>/<sub>\*</sub>/<sub>\*\*</sub> next to the coefficient indicates significance at the 10/5/1% level.

## Robustness

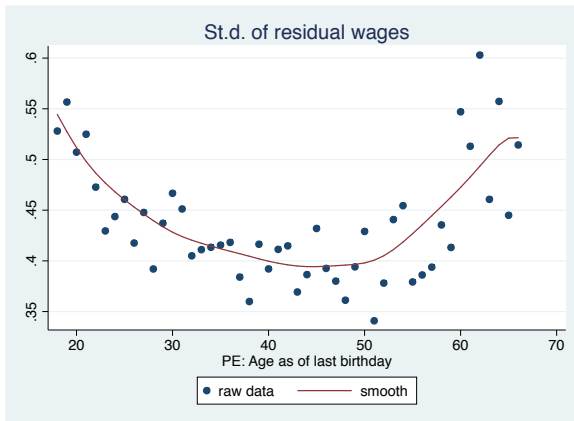
The U-shaped relationship between the wage dispersion and age remains if number of kids, age, age squared, and/or interaction between occupation and age are included, and if weekly hours is excluded in the regression (see Figure 1.14). The age-inequality profile is also U-shaped if only full-time workers (working at least 30 hours per week) are considered (see Figure 1.15).



(a)



(b)



(c)

Difference to baseline regression:

- (a) Weekly hours is excluded.
- (b) Interaction between occupation and age is included.
- (c) Number of kids, age, age squared, and interaction between occupation and age are included.

Figure 1.14: Age-inequality profile of residual log wages

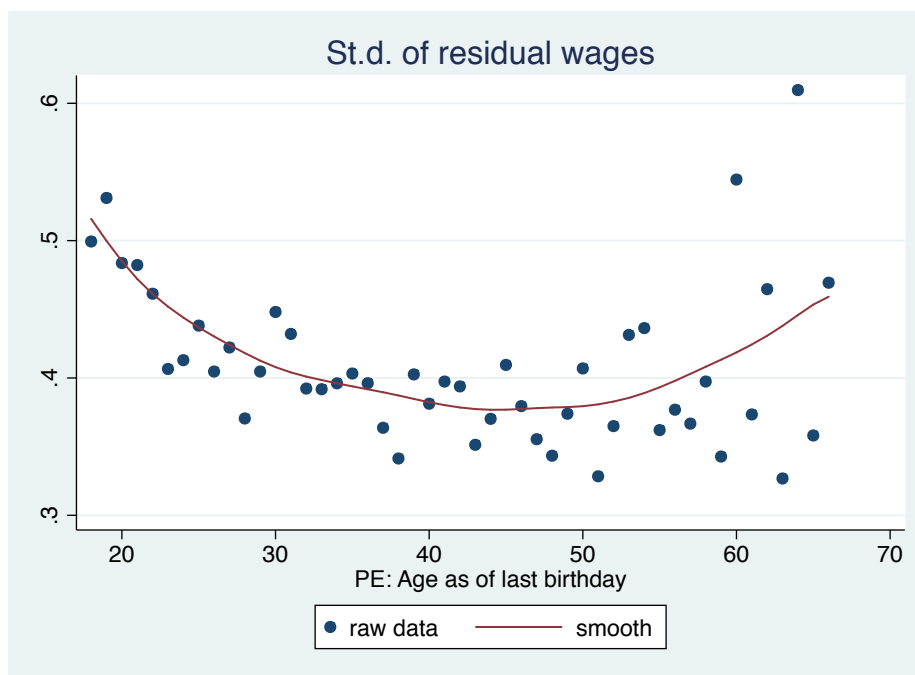


Figure 1.15: Age-inequality profile of residual log wages for full-time workers

## 1.7.2 Complete model

This section introduces the complete model which is used for the calibration. Let  $\delta_k$  denote the age-dependent job destruction rate which is given by the empirical smoothed EU rate. Unemployed workers in the model experience a transition out of the labor market at rate  $\eta_k$  given by the empirical smoothed UN rate. Employed workers leave the labor market at rate  $\zeta_k$  given by the empirical smoothed EN rate. Transitions from non-participation to unemployment (employment) occur at rate  $\mu_k$  ( $\nu_k$ ) given by the empirical smoothed NU (NE) rate. Let  $l(k)$  be the mass of workers aged  $k$  who participate and  $n(k)$  be the mass of workers who do not participate in the labor market.

### Value functions

A worker who does not participate in the labor market obtains at least his value of unemployment. Otherwise he would search for a job. I assume that the value of being not in the labor market makes the worker indifferent between searching for a job and not searching. I therefore set the value of non-participation equal to the value of unemployment.<sup>14</sup> The value of

<sup>14</sup>Setting the value of non-participation equal to the value of unemployment plus a constant does not impact the results.

unemployment becomes

$$\mathcal{U}(k) = \max_{e \geq 0} \left\{ b_U - c(e) + \beta \left[ \mathcal{U}(k') + (1 - \delta_k)(1 - \zeta_k)e\lambda \gamma \sum_{j=1}^n [\mathcal{W}(k', a_j, a_j) - \mathcal{U}(k')] p_j \right] \right\}. \quad (1.14)$$

The optimal search effort of an unemployed worker aged  $k$ ,  $e_U(k)$ , is the solution to the first order condition (FOC) of the maximization problem

$$c'[e_U(k)] = \beta(1 - \delta_k)(1 - \zeta_k)\lambda \gamma \sum_{j=1}^n [\mathcal{W}(k', a_j, a_j) - \mathcal{U}(k')] p_j. \quad (1.15)$$

The value of a job to a worker of age  $k < K - 1$  earning wage  $w$  in a match with productivity  $a_i$  is derived as:

$$\begin{aligned} \mathcal{W}(k, w, a_i) = \max_{e \geq 0} \left\{ w - c(e) + \beta(1 - \zeta_k) \left[ \delta_k \mathcal{U}(k') + (1 - \delta_k) \left[ \right. \right. \right. \\ \left. \left. \left. \left( 1 - e\lambda [1 - P_{q(k', w, a_i) - 1}] \right) \max \left\{ \mathcal{W}(k', w, a_i), \mathcal{U}(k') + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{U}(k')] \right\} \right. \right. \right. \\ \left. \left. \left. + e\lambda \sum_{j=q(k', w, a_i)}^i \left( \mathcal{W}(k', a_j, a_j) + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{W}(k', a_j, a_j)] \right) p_j \right. \right. \right. \\ \left. \left. \left. + e\lambda \sum_{j=i+1}^n \left( \mathcal{W}(k', a_i, a_i) + \gamma [\mathcal{W}(k', a_j, a_j) - \mathcal{W}(k', a_i, a_i)] \right) p_j \right. \right. \right. \\ \left. \left. \left. + \beta \zeta_k \mathcal{U}(k') \right] \right\}. \quad (1.16) \end{aligned}$$

The FOC that determines an employed worker's search effort is

$$\begin{aligned} c'[e_W(k, w, a_i)] = \beta(1 - \delta_k)(1 - \zeta_k)\lambda \left( \sum_{j=q(k', w, a_i)}^i \left[ (1 - \gamma)\mathcal{W}(k', a_j, a_j) + \gamma\mathcal{W}(k', a_i, a_i) \right] p_j \right. \\ \left. + \sum_{j=i+1}^n \left[ (1 - \gamma)\mathcal{W}(k', a_i, a_i) + \gamma\mathcal{W}(k', a_j, a_j) \right] p_j \right. \\ \left. - (1 - P_{q(k', w, a_i) - 1}) \max \left\{ \mathcal{W}(k', w, a_i), \mathcal{U}(k') + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{U}(k')] \right\} \right). \quad (1.17) \end{aligned}$$

## Transition rates and wage distribution

The wage distribution is discretized.  $u(k)$ ,  $n(k)$ ,  $g(k, w_s, a_i)$ ,  $g(k, a_i)$ , and  $G(w_s|k, a_i)$  with  $s = 1, \dots, S$  and  $w_{s-1} < w_s$  are determined as follows:

- The initial mass of unemployed workers  $u(1)$  and of workers who do not participate in the labor force  $n(1)$  is given.
- The mass of workers aged  $k = 2$  who are employed at a type  $i$  firm is determined as the mass of workers aged  $k = 1$  who obtain a job offer from a type  $i$  firm:

$$g(2, a_i) = [e_U(1)\lambda u(1) + v_1 n(1)] p_i.$$

- Workers aged  $k = 2$  earn their reservation wages  $\phi_0(2, a_i)$ . The cumulative distribution of wages for workers aged  $k = 2$  in a type  $i$  match is therefore given by

$$G(w_s|2, a_i) = \begin{cases} 1 & \text{if } w \geq \phi_0(2, a_i) \\ 0 & \text{if } w < \phi_0(2, a_i) \end{cases}.$$

- The mass of workers aged  $k = 2$  earning wage  $w$  in a type  $i$  match is given by

$$g(2, w_s, a_i) = \begin{cases} g(2, a_i) G(w_1|2, a_i) & \text{if } s = 1 \\ g(2, a_i) [G(w_s|2, a_i) - G(w_{s-1}|2, a_i)] & \text{if } s > 1 \end{cases}.$$

- The mass of unemployed workers aged  $k = 2$  consists of the unemployed workers who did not find a job and did not leave the labor market and the workers who enter the labor market and do not immediately obtain a job:

$$u(2) = (1 - \lambda e_U(1) - \eta_1) u(1) + \mu_1 n(1).$$

- The mass of workers who do not participate in the labor market is given by

$$n(2) = (1 - \mu_1 - v_1) n(1) + \eta_1 u(1).$$

For  $k \geq 2$  the following steps are repeated:

1. The steady-state mass of workers aged  $k + 1$  in a type  $i$  match is given by

$$\begin{aligned} g(k+1, a_i) &= [e_U(k)\lambda u(k) + v_k n(k)] p_i \\ &+ (1 - \delta_k - \zeta_k) \lambda p_i \sum_{j=1}^{i-1} \sum_{s=1}^S e_W(k, w_s, a_j) g(k, w_s, a_j) \\ &+ (1 - \delta_k - \zeta_k) \sum_{s=1}^S [1 - e_W(k, w_s, a_i) \lambda (1 - P_i)] g(k, w_s, a_i). \end{aligned}$$

2. The cumulative distribution of wages in steady state for workers aged  $k + 1$  who are employed in a type  $i$  match is derived as

$$\begin{aligned} G(w_s | k+1, a_i) &= I_{w_s \geq \phi_0(k+1, a_i)} \left\{ [e_U(k)\lambda u(k) + v_k n(k)] p_i \right. \\ &+ (1 - \delta_k - \zeta_k) \lambda p_i \sum_{j=1}^{q(k+1, w_s, a_i) - 2} \sum_{r=1}^S e_W(k, w_r, a_j) g(k, w_r, a_j) \\ &+ (1 - \delta_k - \zeta_k) \sum_{r=1}^S [1 - e_W(k, w_r, a_i) \lambda (1 - P_{q(k+1, w_s, a_i) - 1})] g(k, w_r, a_i) \left. \right\} \\ &/ g(k+1, a_i). \end{aligned}$$

3. The mass of workers earning wage  $w$  in a type  $i$  match is determined by

$$g(k+1, w_s, a_i) = \begin{cases} g(k+1, a_i) G(w_1 | k+1, a_i) & \text{if } s = 1 \\ g(k+1, a_i) [G(w_s | k+1, a_i) - G(w_{s-1} | k+1, a_i)] & \text{if } s > 1 \end{cases}.$$

4. The mass of unemployed workers is given by

$$u(k+1) = (1 - \lambda e_U(k) - \eta_k) u(k) + \mu_k n(k) + \delta_k \sum_{j=1}^n g(k, a_j).$$

5. The mass of workers who do not participate in the labor market is derived as

$$n(k+1) = (1 - \mu_k - v_k) n(k) + \eta_k u(k) + \zeta_k \sum_{j=1}^n g(k, a_j).$$

In the SIPP data 53 percent of all workers aged 18 are employed. I obtain the wage distribution of workers aged 18 ( $k = 1$ ) in the model economy by initially computing the stationary employment distribution for the first 13 months assuming that all workers enter the labor market

at the same age.



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## Chapter 2

# Human Capital Investments and Worker Mobility over the Life Cycle

### 2.1 Introduction

In the literature, job search as well as human capital have been identified as potential sources of individual wage development (Rubinstein and Weiss, 2006; Tjaden and Wellschmied, 2012). Topel and Ward (1992) find that job changing contributes substantially to wage growth of young workers. Brown (1989) finds a link between training and wage growth. The objective of this paper is to develop a framework that combines the theory of training-on-the-job and labor market search in a life-cycle setting.

I present a life-cycle model of labor market search with general human capital and match heterogeneity. Human capital is accumulated via on-the-job training. Workers search on and off the job. Firms and their employees decide jointly about training and search intensities. Given these efficient decisions which maximize the joint value of the match, wages are negotiated in a strategic bargaining game in which the value of the worker's outside option serves as the threat point.

Optimal human capital investments decrease with age because the time span in which human capital can be used productively shortens. Also search efforts are reduced when workers approach the retirement age as search is costly and the additional value of a job offer is lower. The model predicts that workers accumulate different amounts of human capital over the life cycle if they experience spells of unemployment or non-participation of different duration and during different periods of their lifetime. During spells of non-employment, workers do not invest in human capital but their stock of human capital depreciates. Because young workers receive more training than older workers, these spells of non-employment are more severe when

they occur in the early period of a career. Interestingly, the simulated model predicts that not the youngest workers (aged 18) receive the highest level of training but workers aged 26. The model mechanism behind the result is that employment of the youngest workers is very unstable and that skills acquired on the job depreciate during periods of non-employment.

The depreciation of human capital is more severe for workers with a high stock of human capital. Therefore, unemployed workers with more general human capital search more and have a shorter unemployment duration. This is conform to the finding by Kriechel and Pfann (2005) that general human capital lowers unemployment duration.<sup>1</sup> The model further predicts that unemployed workers demand higher wages as they get older. Young workers accept low starting wages because a job gives them a better position in future wage negotiations and because they are offered training on the job which raises their productivity and therefore raises their expected earnings.

Because investments in human capital decrease over most of the life cycle and human capital depreciates during spells of unemployment, the life-cycle profile of the average stock of human capital is hump-shaped. The average match specific productivity increases over most of the life-cycle at a decreasing rate because workers search on the job and are gradually matched to better jobs. Search off the job has in contrast a negative effect on the average match quality because the workers who were just recruited out of unemployment have the lowest average match quality. Since older workers switch jobs at a low rate, the second effect eventually dominates and the average match quality falls when workers approach the retirement age. The life-cycle profiles of human capital, match productivity, and reservation wages translate into a concave life-cycle profile of average wages. A concave age-wage profile is also a typical empirical finding (Mincer, 1974; Heckman *et al.*, 2003; Menzio *et al.*, 2012). Hellerstein and Neumark (2007) estimate relative productivities in the US for workers under age 35, workers aged 35 to 54 and workers over age 54 using the 1990 Decennial Employer-Employee Dataset. They estimate that productivity and wage profiles over the life-cycle are hump-shaped but the fall in productivity among older workers is stronger than the wage reduction.<sup>2</sup> Also Haltiwanger *et al.* (1999) exploit a US matched employer-employee dataset. They find that productivity is the lowest among older workers.

This paper contributes to the theoretical literature on market imperfections and employer-provided general training by adding the life-cycle setting. Theoretical work has shown that market imperfections which allow the employer to share the rent from its worker's general

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<sup>1</sup>In contrast, Kriechel and Pfann (2005) estimate that specific human capital measured by tenure increases unemployment duration.

<sup>2</sup>In an earlier study using the Worker Establishment Characteristics Database, Hellerstein *et al.* (1999) find that the life-cycle profile of productivities is increasing. The estimates are however less precise.

human capital give firms an incentive to invest in general training of their workers (Stevens, 1994; Acemoglu and Pischke, 1998b,a; Fu, 2011).<sup>3</sup> Empirical evidence in favour of the view that employers obtain a fraction of the returns to general training is provided by Loewenstein and Spletzer (1998). They estimate that general training at a previous employer has higher returns than general training at the current employer. Wasmer (2006) and Stevens (2012) show that in a labor market with search frictions investments in training and labor turnover interact. Moen and Rosén (2004) argue that investment in human capital and worker reallocation is efficient if firms and their employees maximize their joint expected income. However, if wages are set such that they maximize the ex-post profit of training firms, turnover is too high and human capital investment too low.

Theoretical work that combines search frictions and learning-by-doing includes Yamaguchi (2010), Bagger *et al.* (2011), Burdett *et al.* (2011), and Menzio *et al.* (2012).<sup>4</sup> This paper has the strategic wage bargaining mechanism in common with Yamaguchi (2010) and Bagger *et al.* (2011). It is related to Menzio *et al.* (2012) as they also explore worker transitions and human capital development in a life-cycle setting. An important difference is that in the present paper, human capital accumulates as a result of optimal training decisions.

The paper is organized as follows: In section 2.2, I develop the life-cycle model of labor market search and training on the job. In section 2.3, I simulate the model and explore job search, training and wage development over the life cycle. Section 2.4 concludes.

## 2.2 Life-cycle model

In this section I develop a baseline life-cycle model of labor market search with training on the job. Consider a labor market that is populated by a continuum of competitive firms and a unit mass of workers. A worker is either inactive, unemployed or employed. Transitions in and out of non-participation and the rate at which employed workers become unemployed are exogeneous while unemployment-to-employment transitions and job-to-job transitions are endogenous outcomes of the model. Time is discrete and the economy is in steady state. Workers have different ages  $k = 1, 2, \dots$  and retire at age  $R$ . Each period, all workers who retire are replaced by workers of age 1. Each worker starts with the same human capital endowment  $h_1$ . During spells of unemployment or non-participation, the stock of human capital depreciates at rate  $\zeta$  but it does not fall below  $h_1$ . When employed, the worker and the firm decide jointly on

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<sup>3</sup>Firm specific human capital and endogenous on-the-job search is explored by Jovanovic (1979).

<sup>4</sup>Yamaguchi (2010), Bagger *et al.* (2011), and Menzio *et al.* (2012) find that most wage growth takes place when workers are young and that human capital accumulation is the most important contributor to young workers' wage growth.

the amount of training the worker receives in the current and all subsequent periods. Agents derive utility from consumption. They are risk-neutral and discount future utility at the factor  $\beta \in (0, 1)$ .

Each period unemployed and employed workers choose how much effort  $e$  to spend on job search. The cost of spending an effort  $e$  on searching is given by an increasing and strictly convex cost function  $c(e)$ , with  $c(0) = 0$ . A worker meets a firm at endogenous rate  $e\lambda$ , where  $\lambda > 0$ . Firm-worker matches have different productivities. The match specific productivity  $a_i$  is revealed once a worker meets a firm and it remains constant throughout the duration of the match. Labor productivity is given by a function

$$y(i, h),$$

where  $i$  is the index of the match-specific productivity, with  $i = 1, \dots, n$  and  $a_{j-1} < a_j$ ,  $j = 2, \dots, n$ , and  $h$  denotes the worker's stock of human capital. Labor productivity is increasing in both arguments  $i$  and  $h$ . When unemployed, the worker obtains a flow income  $b(h)$  that is a positive function of the worker's human capital endowment. In the beginning of a period, each firm-worker match faces the risk of being hit by an exogenous separation shock. Let  $\zeta_k \in [0, 1]$  denote the rate at which employed workers leave the labor market and  $\delta_k \in [0, 1]$  denote the rate at which employed workers become unemployed. Workers who become unemployed can immediately search for new jobs that start in the next period.

### 2.2.1 Bargaining

Wages are set in a strategic bargaining game in which firms are allowed to counter the outside offers of their workers. Wages are determined according to a linear sharing rule. The rules of the bargaining game are based on Cahuc *et al.* (2006).

An employment contract specifies a wage that can only be renegotiated by mutual agreement. Workers and employers have complete information over productivities and job offers. When a worker meets a poaching employer, the two employers compete for the worker and the outcome of the bargaining process depends on the match productivity of both firms, on the current contract, the worker's age and human capital endowment. The worker chooses the employer that offers the higher lifetime utility, which is the employer with higher productivity. When a worker of age  $k$  and human capital  $h$  employed at a type  $i$  firm contacts a type  $j$  firm with  $j > i$ , the worker switches employers. The contract with firm  $j$  specifies a wage  $\phi$  that equalizes the worker's value at firm  $j$  denoted by  $\mathcal{W}(k, h, j, \phi)$  with the worker's outside option plus a fraction  $\gamma$  of the match surplus. This fraction can be interpreted as the worker's

bargaining power. The worker's outside option is the highest value the lower-productivity firm can offer. That is the joint value of the match at firm  $i$  denoted by  $\mathcal{J}(k, h, i)$ . Hence, the worker obtains the following value at firm  $j$ :

$$\mathcal{W}(k, h, j, \phi) = \mathcal{J}(k, h, i) + \gamma[\mathcal{J}(k, h, j) - \mathcal{J}(k, h, i)]. \quad (2.1)$$

If the incumbent employer has higher productivity than the poaching employer ( $i \geq j$ ), the worker can renegotiate the wage under the condition that the poaching employer's productivity is sufficiently high. When the worker's current wage is  $w$ , the corresponding threshold productivity index of the poaching firm is the lowest index  $q(k, h, i, w)$  for which

$$\mathcal{W}(k, h, i, w) < \mathcal{J}(k, h, q) + \gamma[\mathcal{J}(k, h, i) - \mathcal{J}(k, h, q)] \quad (2.2)$$

is fulfilled. When  $i \geq j \geq q(k, h, i, w)$  the worker negotiates a wage rise of  $\phi(k, h, j, i) - w$  from  $i$ , where the new wage  $\phi(k, h, j, i)$  is derived from the condition

$$\mathcal{W}(k, h, i, \phi) = \mathcal{J}(k, h, j) + \gamma[\mathcal{J}(k, h, i) - \mathcal{J}(k, h, j)].$$

Table 2.1 gives an overview of the bargaining game.

Table 2.1: Outcome of the bargaining game between a worker earning wage  $w$ , the incumbent employer of type  $i$ , and a poaching employer of type  $j$

	<b>negotiation outcome</b>
$j > i$	new employer $j$ and a wage $\phi(k, h, i, j)$
$i \geq j \geq q(k, h, i, w)$	wage rise $\phi(k, h, j, i) - w$ from the current employer $i$
$j < q(k, h, i, w)$	no change

A match between an unemployed worker and a type  $i$  firm is formed if and only if the joint value of the match is at least as high as the value of unemployment denoted by  $\mathcal{U}(k, h)$ , that is if  $\mathcal{J}(k, h, i) \geq \mathcal{U}(k, h)$ . Provided this condition is satisfied, the firm offers a wage that is the outcome of a Nash bargaining game in which the value of unemployment is the worker's outside option. The negotiated wage  $\phi_0(k, h, i)$  solves

$$\mathcal{W}(k, h, i, \phi_0) = \mathcal{U}(k, h) + \gamma[\mathcal{J}(k, h, i) - \mathcal{U}(k, h)]. \quad (2.3)$$



## 2.2.2 Search

Training only takes place within firms. Under the assumption that the value of non-participation equals the value of unemployment, the problem of an unemployed worker of age  $k < R - 1$  is summarized by

$$\mathcal{U}(k, h) = \max_{e \geq 0} \left\{ b(h) - c(e) + \beta \left[ \mathcal{U}(k+1, h') + (1 - \delta_k)(1 - \zeta_k)e\lambda \gamma \sum_{j=r(k+1, h')}^n [\mathcal{J}(k+1, h', j) - \mathcal{U}(k+1, h')] p_j \right] \right\}, \quad (2.4)$$

where  $r(k+1, h')$  denotes the index of the reservation productivity and  $h'$  is the stock of human capital in the next period. The unemployed worker's value is the flow income of unemployment  $b(h)$  minus search costs plus the discounted continuation value. In the next period, the worker obtains at least the value of unemployment  $\mathcal{U}(k+1, h')$ . With probability  $(1 - \delta_k)(1 - \zeta_k)e\lambda$ , the worker obtains a job offer and the newly formed match is not immediately hit by a separation shock. The expected gain in value of an offer to the worker equals  $\gamma \sum_{j=r(k+1, h')}^n [\mathcal{J}(k+1, h', j) - \mathcal{U}(k+1, h')] p_j$ , where  $p_j$  is the probability that a potential match has productivity  $a_j$ .

When unemployed or inactive, the worker's stock of human capital evolves according to

$$h' = \begin{cases} (1 - \varsigma)h & \text{if } (1 - \varsigma)h \geq h_1 \\ h_1 & \text{else} \end{cases}, \quad (2.5)$$

where  $\varsigma$  denotes the rate of depreciation. The reservation productivity  $a_{r(k+1, h')}$  is the lowest productivity level for which

$$\mathcal{J}(k+1, h', r(k+1, h')) \geq \mathcal{U}(k+1, h')$$

holds. Optimal search intensity fulfils the first-order condition

$$c'[e_U(k, h)] = \beta(1 - \delta_k)(1 - \zeta_k)\lambda \gamma \sum_{j=r(k+1, h)}^n [\mathcal{J}(k+1, h', j) - \mathcal{U}(k+1, h')] p_j. \quad (2.6)$$

Search is jointly efficient in the sense that the search effort of employed workers maximizes the joint value of the firm-worker match as in Lentz (2010). Also the amount of training on the job is chosen in the present model such that it maximizes the joint value of the match. These joint investment and search decisions are taken as given in wage negotiations.

### 2.2.3 Investment in general human capital

I consider general human capital only.<sup>5</sup> Hence, the worker can use the human capital acquired in one firm also in any other firm. The stock of human capital increases only when investments in human capital take place. The firm and the worker jointly choose to invest  $g$  units of the consumption good in the worker's human capital development. The amount  $g$  may represent direct costs as well as indirect costs in the form of foregone earnings that arise because the worker spends some of his time on training activities.<sup>6</sup> Spending  $g$  units of consumption on training increases the worker's stock of human capital by an amount  $Ag^{\rho}$ . The Cobb-Douglas-type human capital production function goes back to Ben-Porath (1967). The stock of human capital of an employed worker then evolves according to the following equation of motion

$$h' = h + Ag^{\rho}. \quad (2.7)$$

The amount  $g$  devoted to training activities and the search effort  $e$  are chosen to maximize the joint value of the firm-worker match:

$$\begin{aligned} \mathcal{J}(k, h, i) = \max_{g \geq 0, e \geq 0} & \left\{ y(i, h) - g - c(e) + \beta(1 - \zeta_k) \left[ \delta_k \mathcal{U}(k + 1, h') + (1 - \delta_k) \right. \right. \\ & \left. \left[ (1 - e\lambda[1 - P_i]) \mathcal{J}(k + 1, h', i) \right. \right. \\ & \left. \left. + e\lambda \sum_{j=i+1}^n (\mathcal{J}(k + 1, h', i) + \gamma[\mathcal{J}(k + 1, h', j) - \mathcal{J}(k + 1, h', i)] p_j) \right] \right] \\ & \left. + \beta \zeta_k \mathcal{U}(k + 1, h') \right\}, \\ \text{s.t. } & h' = h + Ag^{\rho}. \end{aligned}$$

The joint value equals current output minus training and search costs plus the discounted continuation value. If the worker stays in the labor market, he becomes unemployed with probability  $\delta_k$  and earns a value  $\mathcal{U}(k + 1, h')$ . The firm's value is zero in this case. If the worker remains employed at the current match, the joint value equals  $\mathcal{J}(k + 1, h', i)$ . This happens with probability  $(1 - \delta_k)(1 - e\lambda[1 - P_i])$ , where  $P_i$  denotes the cumulative distribution of potential match qualities. If the worker remains employed and meets a poaching employer with productivity  $j > i$ , he switches employers. In this case, the worker obtains a value  $\mathcal{J}(k + 1, h', i) + \gamma[\mathcal{J}(k + 1, h', j) - \mathcal{J}(k + 1, h', i)]$  at the new match and the previous em-

<sup>5</sup>According to a study by Loewenstein and Spletzer (1999) most of human capital acquired in firms is general.

<sup>6</sup>Almeida and Carneiro (2009) find that training costs are mainly direct costs and only 25 percent of training costs are foregone earnings.

ployer's value is zero. With probability  $\zeta_k$  the worker becomes inactive. The above maximization problem is equivalent to

$$\begin{aligned} \mathcal{J}(k, h, i) = \max_{h' \geq h, e \geq 0} & \left\{ y(i, h) - ([h' - h]/A)^{1/\rho} - c(e) + \beta(1 - \zeta_k) \right. \\ & \left[ \delta_k \mathcal{U}(k+1, h') + (1 - \delta_k) [(1 - e\lambda\gamma[1 - P_i]) \mathcal{J}(k+1, h', i) \right. \\ & \left. \left. + e\lambda\gamma \sum_{j=i+1}^n \mathcal{J}(k+1, h', j) p_j \right] + \beta \zeta_k \mathcal{U}(k+1, h') \right\}. \end{aligned} \quad (2.8)$$

Optimal search satisfies

$$c'[e_W(k, h, i)] = \beta(1 - \delta_k)(1 - \zeta_k)\lambda\gamma \sum_{j=i+1}^n [\mathcal{J}(k+1, h', j) - \mathcal{J}(k+1, h', i)] p_j. \quad (2.9)$$

The optimal search effort does not directly depend on the current stock of human capital  $h$ , but depends on the optimal value of  $h'$  which itself depends on  $k$ ,  $h$ , and  $i$ . Using the condition for optimal search simplifies maximization problem (2.8):

$$\mathcal{J}(k, h, i) = \max_{h'} \mathcal{J}_0(k, h', i). \quad (2.10)$$

I derive the optimal value of  $h'$  by applying numerical optimization methods.

Given optimal search and training, the value of a job to a worker of age  $k$  earning wage  $w$  in a match with productivity  $a_i$  is given by

$$\begin{aligned} \mathcal{W}(k, h, i, w) = & w - c(e_W) + \beta(1 - \zeta_k) \left[ \delta_k \mathcal{U}(k+1, h') + (1 - \delta_k) \left[ (1 - e_W\lambda[1 - P_{q(k+1, h', i, w)-1}]) \right. \right. \\ & \left. \left. \max \left\{ \mathcal{W}(k+1, h', i, w), \mathcal{U}(k+1, h') + \gamma [\mathcal{J}(k+1, h', i) - \mathcal{U}(k+1, h')] \right\} \right] \right] \\ & + e_W\lambda \sum_{j=q(k+1, h', i, w)}^i \left( \mathcal{J}(k+1, h', j) + \gamma [\mathcal{J}(k+1, h', i) - \mathcal{J}(k+1, h', j)] \right) p_j \\ & + e_W\lambda \sum_{j=i+1}^n \left( \mathcal{J}(k+1, h', i) + \gamma [\mathcal{J}(k+1, h', j) - \mathcal{J}(k+1, h', i)] \right) p_j \quad \left. \right] \\ & + \beta \zeta_k \mathcal{U}(k+1, h'). \end{aligned}$$

The worker's value is the current wage minus search costs plus the discounted continuation value. Given that the worker stays in the labor market, he becomes unemployed and earns a value  $\mathcal{U}(k+1, h')$  with probability  $\delta_k$ . The employed worker stays in his current match with

probability  $1 - e_W \lambda [1 - P_{q(k+1, h', i, w) - 1}]$ . As the option value of on-the-job search decreases with age, it is possible that  $\mathcal{W}(k+1, h', i, w)$  falls below  $\mathcal{U}(k+1, h') + \gamma[\mathcal{J}(k+1, h', i) - \mathcal{U}(k+1, h')]$ . The worker has then a credible threat to quit and renegotiates the wage. If the worker can renegotiate the wage because of an outside job offer, the bargain outcome has an expected value of  $\sum_{j=q(k+1, h', i, w)}^i \left( \mathcal{J}(k+1, h', j) + \gamma[\mathcal{J}(k+1, h', i) - \mathcal{J}(k+1, h', j)] \right) p_j$ . If the worker meets an outside firm with higher match productivity than the current one, he switches employers and has an expected value  $\sum_{j=i+1}^n \left( \mathcal{J}(k+1, h', i) + \gamma[\mathcal{J}(k+1, h', j) - \mathcal{J}(k+1, h', i)] \right) p_j$ . The worker becomes inactive with probability  $\zeta_k$ .

## 2.2.4 Equilibrium

A *stationary equilibrium* consists of

- optimal human capital investments  $g(k, h, i)$  derived from maximization problem (2.10),
- the optimal search efforts  $e_U(k, h)$  and  $e_W(k, h, i)$  given by the first-order conditions (2.6) and (2.9),
- the wages  $\phi_0(k, h, i)$  and  $\phi(k, h, i, j)$  derived from equations (2.3) and (2.1),
- and a stationary employment and wage distribution

for all combinations of age  $k < R$ , human capital, and match productivities  $a_i$ , given an exogenous productivity distribution, a constant mass of new workers of age  $k = 1$ , and a terminal condition which is the value of retirement  $\mathcal{R}$ .<sup>7</sup>

Given a value of retirement  $\mathcal{R}$ , the optimal search efforts  $e_U(k, h)$  and  $e_W(k, h, i)$ , wages  $\phi_0(k, h, i)$  and  $\phi(k, h, i, j)$ , and optimal human capital investments  $g(k, h, i)$  can be computed starting with age  $k = R - 1$  and continuing backwards in age. Given these policy functions and the condition that all newborns are unemployed, there exists a unique and stationary employment distribution and wage distribution.

## 2.3 Simulation

In this section the stationary equilibrium is simulated for reasonable parameter values. To approximate the stationary employment and wage distribution, I simulate a large number of employment histories.

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<sup>7</sup>The value of retirement does not need to depend on human capital if one assumes a fair pension scheme. For example, in such a scheme the worker's pension is financed by a fraction of his wage. When the worker retires, he receives all payments plus interest rate. His utility is the same as if he consumed this pension at once.

### 2.3.1 Parametrization

Several empirical studies find that the longer a worker stays out of the labor market, the larger is the wage reduction at re-entry compared to the wage earned at labor market exit (Mincer and Ofek, 1982; Addison and Portugal, 1989). I set the monthly depreciation rate for unemployed workers to  $\zeta = 0.0135$  such that the annual depreciation rate lies in the range estimated by Keane and Wolpin (1997).<sup>8</sup> The literature that uses the Ben-Porath (1967) model usually assumes a curvature parameter  $\rho$  between 0.8 and 0.95 (Heckman *et al.*, 1998; Guvenen and Kuruscu, 2010). I therefore set  $\rho = 0.85$ . Bagger *et al.* (2011) consider a labor market search model with strategic bargaining and learning-by-doing. They find that human capital accumulation contributes to a large extent to wage growth among young workers but becomes negative for older workers. They calibrate age-specific rates of human capital accumulation that decrease with age and become negative for older workers. In the present model, the decreasing rate of human capital accumulation is the result of optimal training decisions. The average stock of human capital decreases among older workers because skills depreciate during unemployment and because older workers obtain little training.

In order to set reasonable values to the remaining parameters, I make use of the quantitative analysis in chapter 1 in which I calibrate a life-cycle model of labor market search similar to the one developed in this paper but without human capital. The reader is referred to chapter 1 for a more detailed description of the dataset used, the construction of the calibration targets, and the calibration strategy. The dataset is the 1996 panel of the US Census' Survey of Income and Program Participation (SIPP), which contains monthly data from December 1995 to February 2000. The subgroup considered comprises non-unionized men with a high-school degree and between 18 and 66 years old. Furthermore, I only consider workers who do not have any income from self-employment, are not in the armed forces and do not stop working for school or training reasons.

The model takes into account that there are flows in and out of labor force participation across all age groups. These age-specific transition rates and the rate at which employed workers become unemployed are directly calibrated. Unemployed workers receive a flow income that equals output at the least productive match

$$b(h) = y(1, h).$$

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<sup>8</sup>Keane and Wolpin (1997) find an annual depreciation rate of 0.096 for unemployed blue collar workers and of 0.365 for unemployed white collar workers.

The production technology is given by

$$y(i, h) = a_i + h. \quad (2.11)$$

The additive production technology implies that the productivity of human capital is independent of match quality. This ensures that the derived relationships between investment and search do not simply arise because of complementarities in production.<sup>9</sup>

I set the bargaining power parameter  $\gamma = 0.7$  which is sufficiently high to ensure that young workers do not accept negative wages and which corresponds to the value estimated in chapter 1. The cost of search function is quadratic with

$$c(e) = 0.5e^2.$$

A period in the model corresponds to a month. I set the monthly discount factor to  $\beta = 0.9967$  to match an annual real interest rate of 4 percent. Workers retire at age 66, i.e.  $R = 588$ . The distribution of match qualities is Weibull with a location parameter equal to  $a_1 = 1$ . The number of productivity grid points is  $n = 10$ . The scale and shape parameter of the productivity distribution are chosen to match the standard deviation of residual wages obtained from a fixed-effects regression<sup>10</sup> and the employment-to-employment transition (EE) rate. The offer arrival rate per search effort  $\lambda$  is chosen to match the unemployment-to-employment transition (UE) rate. The parameter of the human capital production function is fixed to  $A = 0.005$  which results in average training expenditures of 1 % of output for workers aged between 18 and 39. The parameters are presented in table 2.2 and the resulting simulated moments are shown together with their empirical counterparts in table 2.3.

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<sup>9</sup>What are the consequences of the mentioned complementarities? Let us assume the following alternative production function:

$$y(i, h) = a_i h.$$

This functional form of output implies that it is more efficient to have workers with high human capital employed in high-productivity matches. Indeed, workers - employed and unemployed - search more, the higher their level of human capital. Employed workers with high human capital therefore sort themselves into good matches. Optimal investment in human capital increases in the current match quality because of the assumed multiplicative production technology: human capital is employed more efficiently in high-productivity matches.

<sup>10</sup>See chapter 1 for details.

Table 2.2: Parameter values

Description	Parameter	Value
Match quality distribution	shape	1.6000
	scale	12
Human capital production	$A$	0.005
	$\rho$	0.85
Workers' bargaining power	$\gamma$	0.7
Offer arrival rate per search effort	$\lambda$	0.0340

Table 2.3: Comparison of model and data moments

Target	Data	Model
St.d. of residual wages	0.4379	0.4262
Average UE rate	0.1685	0.1631
Average EE rate	0.0154	0.0116

### 2.3.2 Job search, training, and wages over the life cycle

In the following, I discuss the equilibrium life-cycle profiles of job search intensity, training, and wages.

**Search.** Searching for a job is costly and the additional value of a job offer is smaller the shorter the time until retirement. The optimal search effort of employed and unemployed workers therefore decreases when workers approach the retirement age (Figures 2.1 and 2.2). The initial rise in search effort arises because young workers leave employment at a relatively high rate (Figures 2.3 and 2.4).

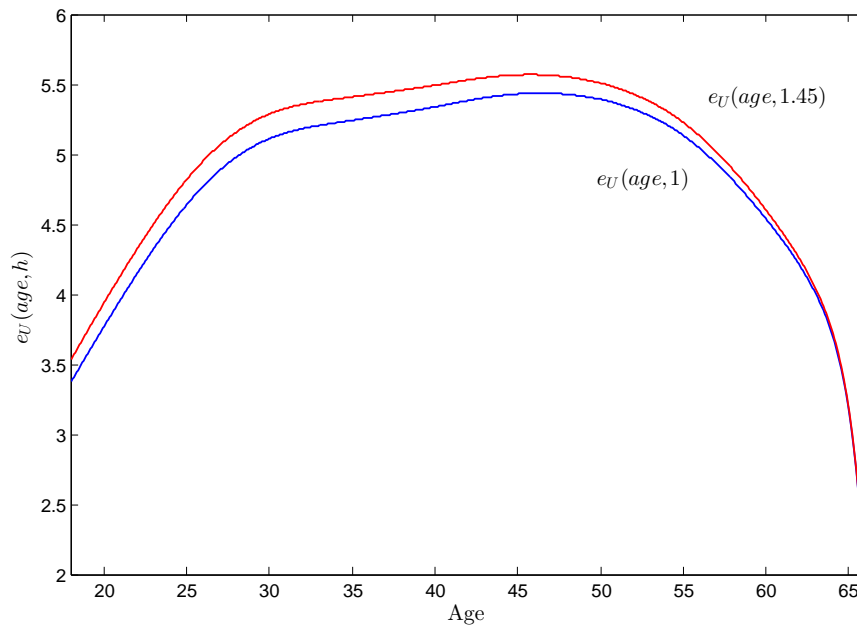


Figure 2.1: Search effort of unemployed workers (rises in human capital)

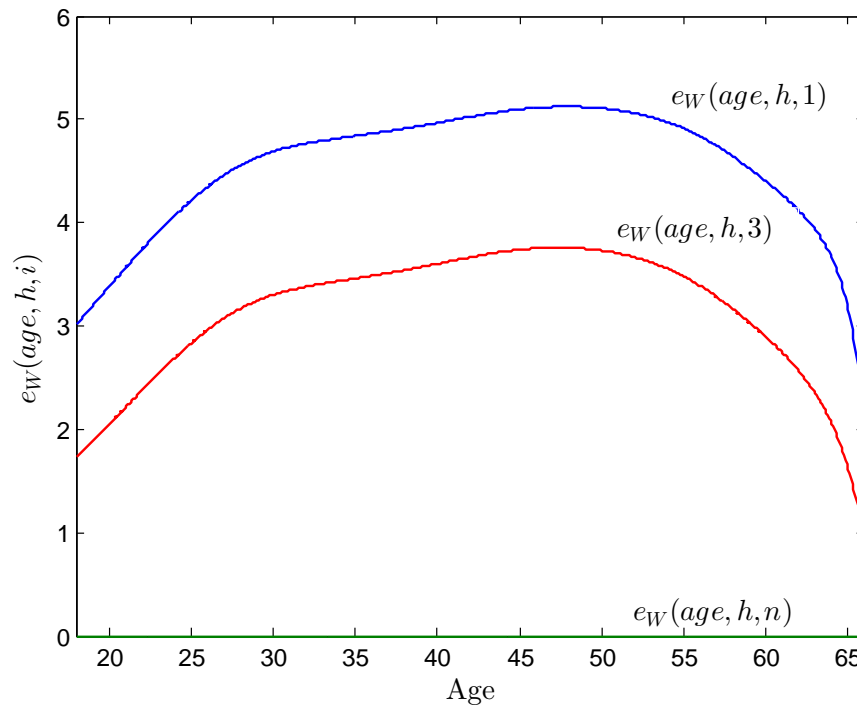


Figure 2.2: Search effort of employed workers (falls with match quality)



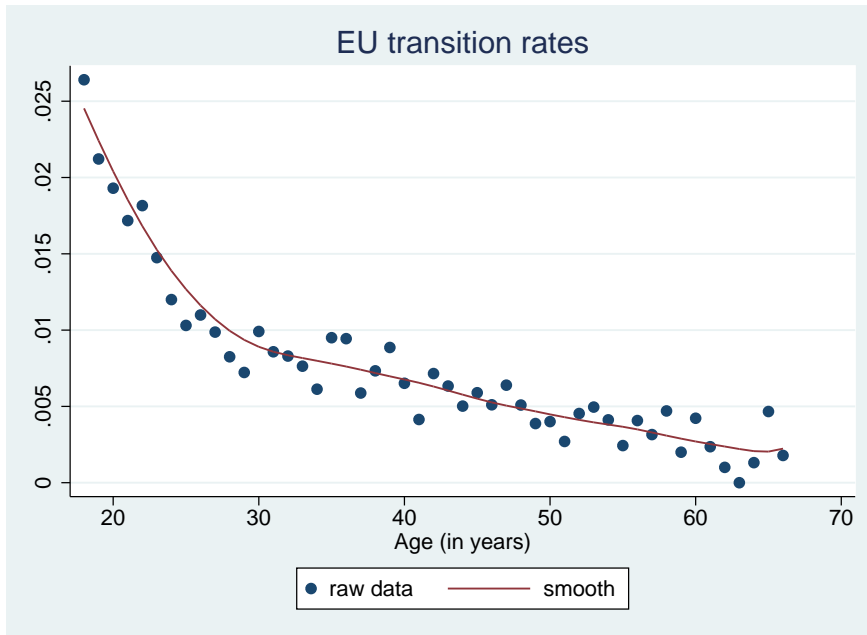


Figure 2.3: Job destruction rate ( $\delta_k$ ), SIPP data

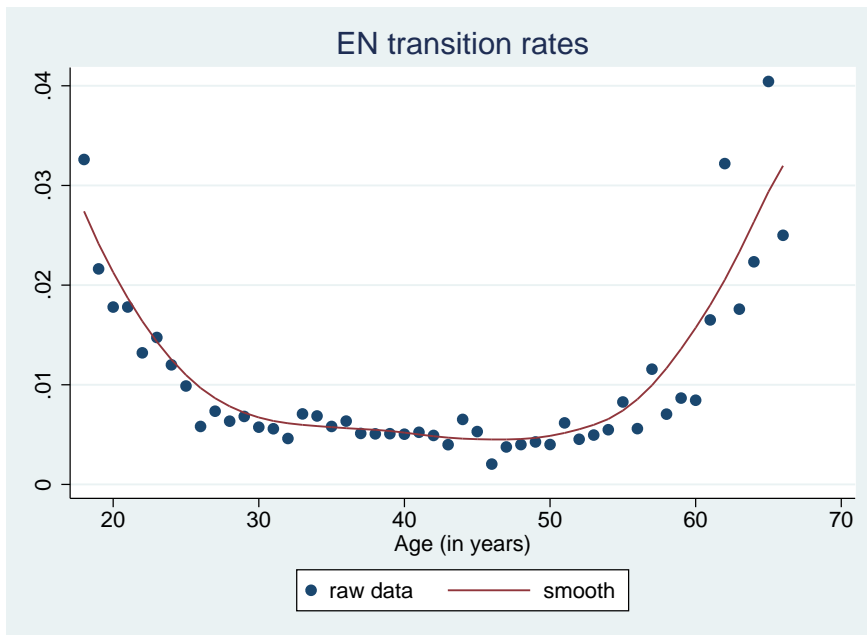


Figure 2.4: Flows out of the labor force ( $\zeta_k$ ), SIPP data

Low-productivity matches only exist in this labor market because of search frictions. They are however not efficient. Efficient search on-the-job ensures that search intensity decreases in match quality. Hence, high-productivity matches last longer.

The loss in human capital due to depreciation when unemployed is more severe for high skilled workers than it is for low skilled. Unemployed workers with more human capital therefore search more and have a shorter spell of unemployment. Search of employed workers does not depend on human capital as the assumed additive production technology implies that the productivity of human capital is independent of match quality.

**Investment.** Optimal investment in human capital decreases over most of the life cycle because the time span in which the human capital can be used productively decreases with age. The model predicts that workers aged 18 obtain less training than workers aged 41 or younger (Figure 2.5). The initial increase of investments up to age 26 arises here because young workers leave employment at a relatively high rate.<sup>11</sup>

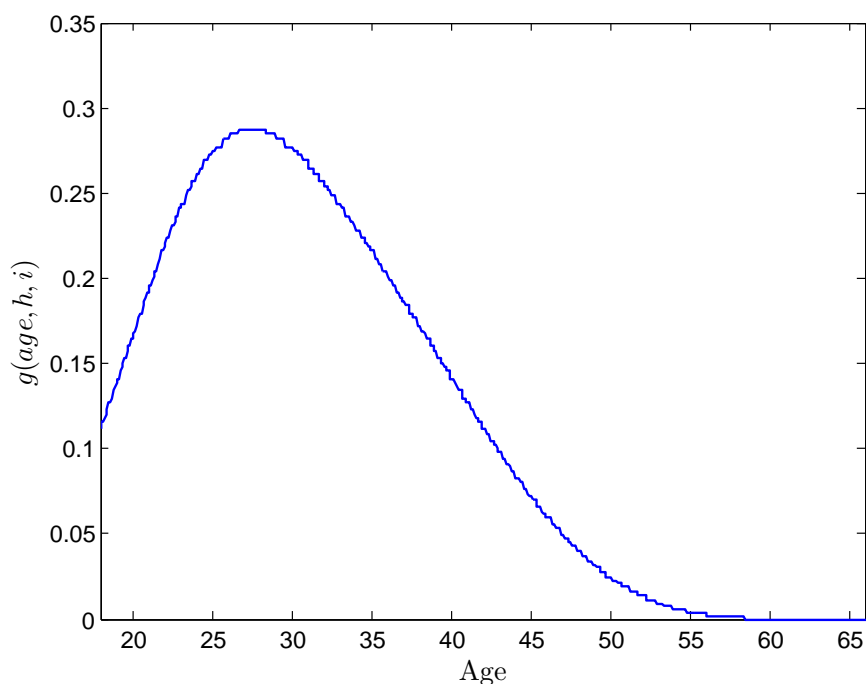


Figure 2.5: Human capital investments (independent of the match quality and of human capital)

The assumed production technology implies that the productivity of human capital does not increase in match quality. As I consider general human capital only and because training is chosen such that it maximizes the joint value of the firm-worker match, the longer expected duration of a high-quality match does not influence human capital investment decisions. It does nevertheless have an impact on how much the worker has to bear of the training costs in the

<sup>11</sup>  $\delta_k$  and  $\zeta_k$  are relatively high for young workers.

form of lower wages.

**Wages.** Simulation results show that wages negotiated by unemployed workers,  $\phi_0(k, h, i)$ , increase with age over most of the life cycle (Figure 2.6). The workers' bargaining power  $\gamma$  influences the life-cycle profile of reservation wages (see chapter 1). While the chosen value of  $\gamma$  leads to increasing starting wages in an economy with human capital development, it leads to a decreasing life-cycle profile of starting wages in the search-only-case with constant human capital. The worker's age has two opposing effects on  $\phi_0$ . Older workers have a shorter time horizon in the labor market and therefore a lower probability to obtain better alternative offers in the future by waiting. In the search-only case, older workers therefore accept lower wages than young workers when  $\gamma$  is sufficiently high. Otherwise, young workers accept very low starting wages because a job gives a worker a better position in future wage negotiations. As this option value of on-the-job search is much smaller for older workers, older workers demand higher wages.

The prospect of human capital development on the job raises the option value of on-the-job search and lowers the reservation wages of young unemployed workers. The simulated life-cycle profile of  $\phi_0$  as shown in Figure 2.6 displays a relatively high initial value of  $\phi_0$  which decreases at first. This occurs because of the high rates at which young workers leave employment and because employment becomes more stable. Figure 2.6 also displays a fall in reservation wages for older workers when the match quality is relatively high. Obtaining a job with such a high match quality has low probability especially if the time horizon until retirement is short. Therefore, older workers accept lower wages if the match quality is particularly high.

The average match quality rises at first with age because workers search on-the-job and are gradually matched to better jobs (Figure 2.7). The group of workers who were just recruited out of unemployment is the one with the lowest average match quality. Search off the job has therefore a negative effect on match quality. Because older workers reduce their search effort substantially, the second effect eventually dominates and the average match quality falls again. The average stock of human capital increases at first with age because of training on the job. However, during spells of unemployment, human capital depreciates. As older workers obtain only little training, the average stock of human capital falls again (Figure 2.8). The life-cycle profiles of match quality, human capital, and reservation wages translate into a hump-shaped life-cycle profile of average wages which is consistent with empirical evidence (Figures 2.9 and 2.10). Average wages of older workers however decrease less strongly than average output because starting wages increase with age.

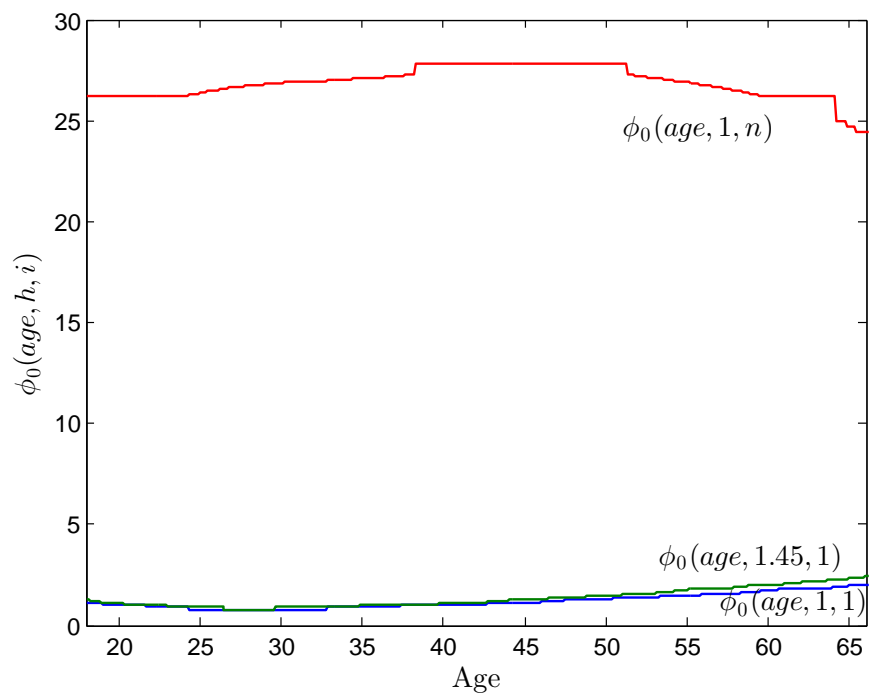


Figure 2.6: Wages bargained by unemployed workers (rise in match quality and in human capital)

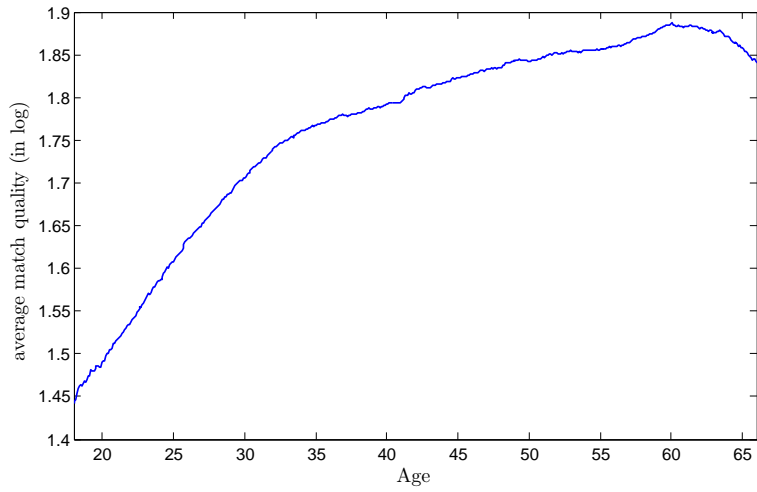


Figure 2.7: Average match productivity ( $\log(a)$ )

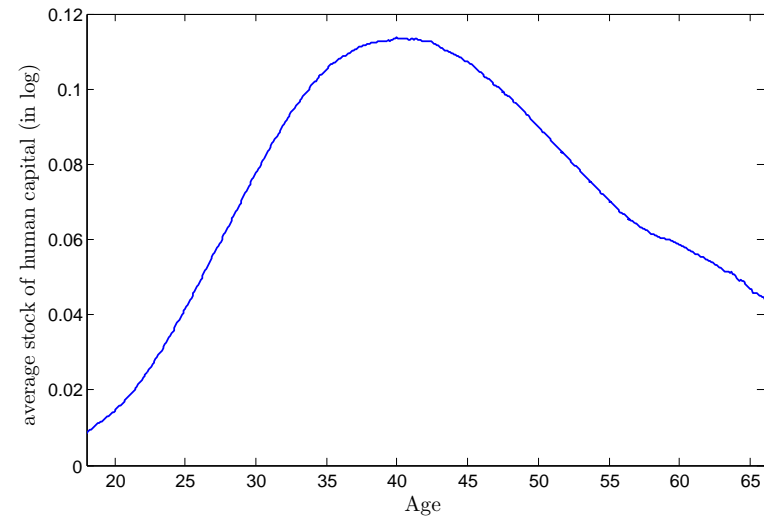


Figure 2.8: Average stock of human capital ( $\log(h)$ )

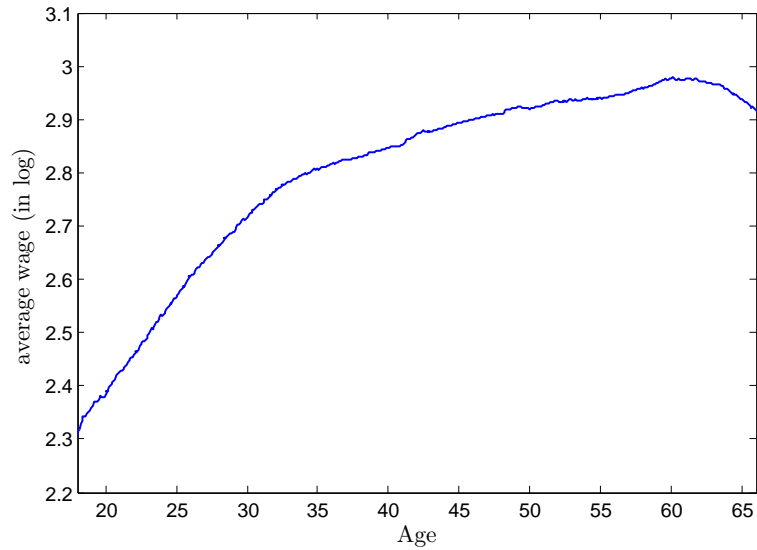


Figure 2.9: Average log wage ( $\log(w)$ )

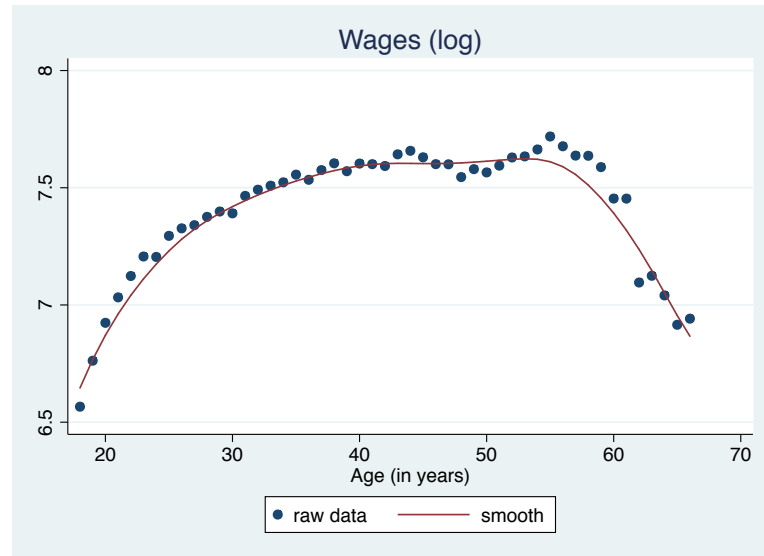


Figure 2.10: Life-cycle profile of average (log) wages, SIPP data

## 2.4 Conclusions

In this paper, I develop a life-cycle model of endogenous job search and training. It is shown that optimal human capital investments decrease over most of the life-cycle and search efforts are reduced when workers approach the retirement age. Because the depreciation of human capital is more severe for workers with a high stock of human capital, unemployed workers with more general human capital have a shorter unemployment duration. The model further predicts that wages accepted by unemployed workers increase with age. Although workers' learning ability ( $A$ ) and the rate of human capital depreciation do not depend on age, the average stock of human capital increases at first at a decreasing rate and then decreases with age after the second half of the working life. This is the result of decreasing investments in human capital and depreciation during spells of unemployment. As workers search on the job, they are gradually matched to better jobs and the average match quality increases over most of the life cycle. When workers approach the retirement age, the effect of off-the-job search dominates and the average match quality slightly falls again. The life-cycle profiles of human capital, match productivity, and reservation wages translate into a concave life-cycle profile of average wages.

Search and training are chosen in the present model such that they maximize the joint value of the firm-worker match. For future work, I consider it interesting to explore how the results are altered if employer-provided training is chosen to maximize the firm's value of the match while workers choose the search intensity that maximizes their own value of the match. Interactions between search and training over the life cycle might arise in such a setting. When high-productivity matches are expected to last longer, it seems plausible that workers obtain more training, the higher the match-specific productivity even if human capital and match quality are no complements in production.

Several labor market policies could be explored in the model framework. For instance, training of unemployed workers could prevent the reduction of the average stock of human capital for middle-aged and older workers. It might however also lower the optimal search intensities especially of high-skilled unemployed workers and raise the unemployment rate. Early retirement schemes have the potential to lower human capital investments because they shorten the time span in which human capital is productive.

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## Chapter 3

# Macroeconomic Stability and Wage Inequality: A Model with Credit and Labor Market Frictions

### 3.1 Introduction

Since the early 1980's, the US experienced a decline in the volatility of real GDP growth (Stock and Watson, 2002; Davis and Kahn, 2008). One possible explanation is the increase in financial development in the meaning of easier access to credit for firms (Dynan *et al.*, 2006). Support for the positive link between macroeconomic stability and financial development is given in the cross-country study of Denizer *et al.* (2002). There is however no evidence for higher stability at the household level. Kambourov and Manovskii (2009) find that volatility of individual wages and wage inequality increased since the 1970's. The increase occurred especially within narrowly defined age-education subgroups. Figures 3.1 and 3.2 show a decline in the volatility of GDP growth as well as an increase in the ratio of gross earnings at the top decile to those at the bottom decile, that have been accompanied by an increase in the share of credit in GDP. The present paper argues that an increase in financial development can contribute to both changes: increasing macroeconomic stability and increasing wage inequality within groups of similar workers.

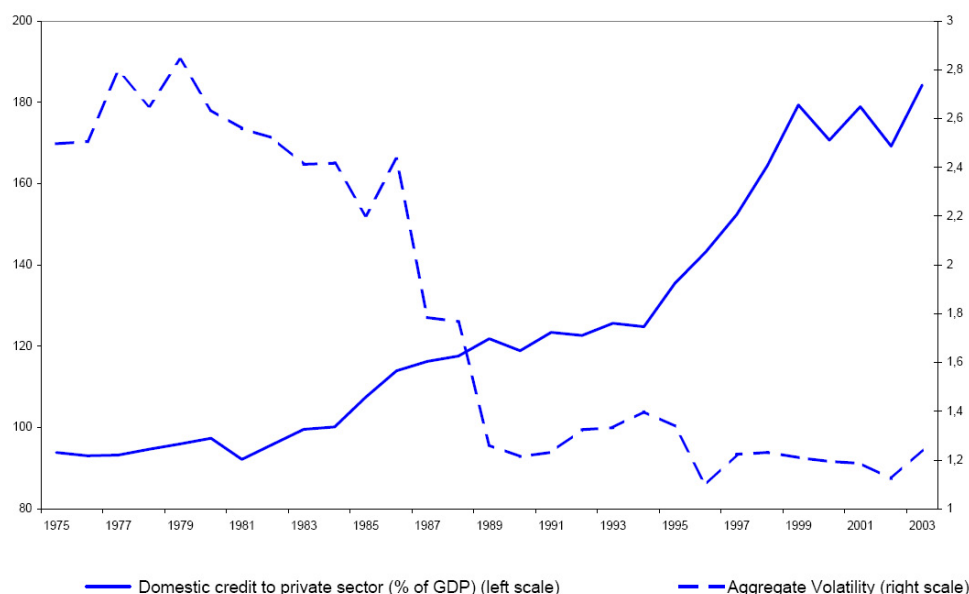


Figure 3.1: Volatility and Credit:

Domestic credit to private sector (% of GDP). Data source: World Development Indicators (World Bank).

Aggregate Volatility: Rolling standard deviation of GDP growth (annual %) over 10-year time windows. Data source for GDP growth: World Development Indicators.

An increasing amount of theoretical work demonstrates the link between financial development, macroeconomic fluctuations and growth. The models often dispense with labor as a production input (Kiyotaki and Moore, 1997; Kiyotaki, 1998; Azariadis and Kaas, 2009) or they assume perfect mobility of labor (Aghion *et al.*, 1999; Kiyotaki and Moore, 2012; Kocherlakota, 2009; Kaas, 2009). However, labor reallocation has been identified as an important factor in explaining macroeconomic variables (Lilien, 1982; Burgess and Mawson, 2003). Lagos (2006), for example, shows in a model of frictional labor market and sectoral shocks how labor market policies affect total factor productivity (TFP).

There are few papers that study the effect of financial development on within-group wage inequality. Jerzmanowski and Nabar (2011) find empirical evidence for a positive link between financial development and within-group wage inequality in the US. Their theoretical approach attributes financial development to organizational change, and thereby to higher wage inequality for skilled workers.

The objective of this paper is to explore the link between macroeconomic stability, volatility of individual wages, and wage inequality when debt constraints slow down capital reallocation and when workers are not perfectly mobile. It is shown that looking at frictions in financial

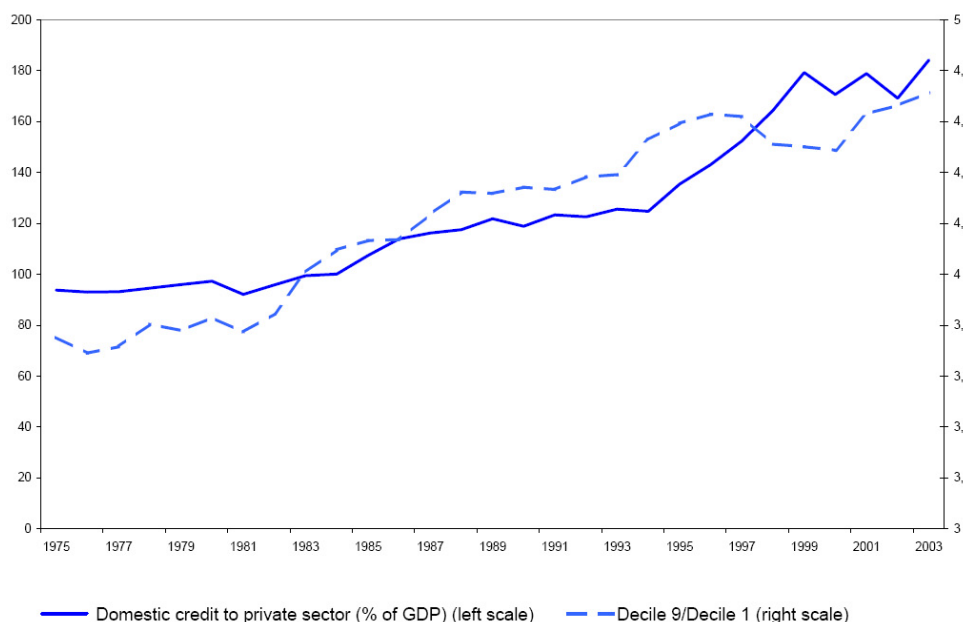


Figure 3.2: Wage Inequality and Credit:

Domestic credit to private sector (% of GDP). Data source: World Development Indicators (World Bank).

Decile 9/Decile 1. Data source: Decile ratios of gross earnings (OECD.Stat)

markets and labor markets separately may be misleading. There are important interactions between both markets. As a result, the effectiveness of improvements in the credit market depend on the degree of labor market frictions.

I develop a model with two sectors. Credit market frictions arise in the form of collateral-based credit constraints as for example in Kiyotaki and Moore (1997), Kiyotaki (1998), and Azariadis and Kaas (2009). I introduce a simple form of labor market frictions: In each period only a given fraction of workers employed in the low-wage sector can move into the high-wage sector. One can think of various factors that make it difficult for a worker to switch sectors. Barriers may arise from sector-specific skills or workers may have to move to another town if they want to switch sectors. Wages within one sector are determined competitively. In the model, volatility is the result of sectoral productivity shocks. I do not consider aggregate productivity shocks that would affect both sectors in the same way. One sector always produces at the technology frontier. If capital and labor were perfectly mobile, all production factors would flow to the productive sector, and there would be no volatility and no wage inequality. In addition, output and wage income would be maximized. When capital market frictions are introduced, aggregate output depends on the distribution of wealth between sectors. When financial development increases, the wealth distribution becomes less important and the volatility of aggregate

output decreases. When labor is mobile, each worker earns the same wage, which behaves similar to aggregate output. Wage income increases in financial development as more capital is employed in the high-TFP sector, and wage volatility decreases in financial development. If, in addition, labor market frictions are introduced, wage inequality can arise. When financial development increases now, more capital flows to the high-TFP sector. As a result, wages in the high-TFP sector increase even more while wages in the low-TFP sector decrease even more. Hence, financial development increases the correlation of wages with sector-specific TFP, and thereby wage inequality and volatility of individual wages.

A related strand of literature introduces capital market frictions into a search and matching model of equilibrium unemployment (Wasmer and Weil, 2004; Petrosky-Nadeau, 2009; Petrosky-Nadeau and Wasmer, 2010; Dromel *et al.*, 2010). While these papers explore the impact of credit market frictions on unemployment in the presence of macroeconomic shocks, the present paper examines the impact of credit market frictions on wage inequality in the presence of sector-specific productivity shocks.

The rest of the paper is organized as follows: Section 3.2 describes the model environment. Equity returns, wages, and the sectoral distribution of wealth in equilibrium are determined in section 3.3. Section 3.4 explores the effect of financial development on volatility and inequality. Section 3.5 concludes.

## 3.2 The model

Consider a discrete-time economy with two labor markets or sectors indexed by  $j = 1, 2$  and infinitely-lived workers and entrepreneurs. There is one representative worker household in the economy consisting of a continuum of workers. The worker household does not save or borrow, but simply consumes its labor income each period.<sup>1</sup> The worker household assigns its members to the sectors in order to maximize labor income. The labor market within one sector is competitive, but wages may differ between sectors when labor is not perfectly mobile. Workers wish to move to the sector with the highest wage. However, each period only up to a fraction  $\gamma$  of workers in the low-wage sector can move into the high-wage sector. The parameter  $\gamma$  captures stochastic costs of switching sectors. Each sector consists of a continuum of entrepreneurs that can be represented by one firm in each sector. Entrepreneurs may lend or borrow capital, hire workers, and produce. They derive logarithmic utility from consumption.

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<sup>1</sup>Workers may not borrow since they have no collateral. There is one big representative worker household that insures its members against sectoral income shocks and that does not save. The assumption that workers do not save is a common simplification in models with credit market frictions (e.g. Kocherlakota, 2009).

The expected utility of firm  $j$  at date  $t$  is  $E_t \sum_{\tau \geq t} \beta^{\tau-t} \ln(C_\tau^j)$ , where  $E_t$  denotes expectations formed at date  $t$ ,  $\beta \in (0, 1)$  is the discount factor of firms, and  $C_\tau^j$  is firm  $j$ 's consumption at date  $\tau$ . All firms produce the same homogeneous good with the Cobb-Douglas production technology  $Y_t^j = A_t^j (K_t^j)^\alpha (L_t^j)^{1-\alpha}$ , where  $L_t^j$  and  $K_t^j$  denote labor and capital input in sector  $j$ . Let  $Y_t^j$  include current output as well as undepreciated capital.<sup>2</sup> The good produced in period  $t$  is used for next period's consumption and investment. There are two states of productivity ( $s_t = 1, 2$ ) given by

$$A_t^j = A^j(s_t) = \begin{cases} A & \text{if } j = s_t \\ zA & \text{if } j \neq s_t \end{cases}$$

with  $0 < z < 1$ ,

and with transition probabilities

$$\pi(s_{t+1}|s_t) = \begin{cases} \pi \in (0.5, 1) & \text{if } s_{t+1} = s_t \\ 1 - \pi & \text{if } s_{t+1} \neq s_t \end{cases}.$$

Firms in one sector produce at the productivity frontier  $A$ , and firms in the other sector have low TFP  $zA$ . Productivity states are positively autocorrelated. In the following, the sector with high (low) TFP  $A$  ( $zA$ ) is indexed by  $i = H$  ( $L$ ).

Firms may borrow and lend at gross interest rate  $R_t$ . Let  $D_t^i$  be the debt position of firm  $i$  in period  $t$ . If firm  $i$  is a borrower,  $D_t^i > 0$ . If firm  $i$  is a lender,  $D_t^i < 0$ . Only a fraction  $\lambda \in [0, \alpha z^{\frac{1}{\alpha}})$  of output is pledgeable collateral and firm  $i$  may only borrow up to the value of its collateral  $D_t^i \leq \frac{\lambda Y_t^i}{R_t}$ . Firms observe the state of productivity and decide based on that information whether to borrow or lend, and whether they want to produce and hire workers. Firms collect profits out of production, redeem debt or collect returns from saving.

Firms choose consumption  $C_t^i$ , capital input  $K_t^i$ , labor input  $L_t^i$ , and debt position  $D_t^i$  in order to maximize the entrepreneurs' expected utility subject to budget and debt constraints:

$$\max E_t \sum_{\tau \geq t} \beta^{\tau-t} \ln(C_\tau^i) \tag{3.1}$$

$$\text{s.t. } C_\tau^i + K_\tau^i - D_\tau^i = A_{\tau-1}^i (K_{\tau-1}^i)^\alpha (L_{\tau-1}^i)^{1-\alpha} - w_{\tau-1}^i L_{\tau-1}^i - R_{\tau-1} D_{\tau-1}^i, \tau \geq t,$$

$$R_\tau D_\tau^i \leq \lambda A_\tau^i (K_\tau^i)^\alpha (L_\tau^i)^{1-\alpha}, \tau \geq t,$$

where  $w_\tau^i$  denotes the (real) wage paid in period  $\tau$  in sector  $i$ . Equity of firm  $i$  is given by

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<sup>2</sup>Other papers using this simplification are for example Azariadis and Kaas (2009) and Kaas (2009).

$E_t^i = K_t^i - D_t^i$ . It is shown below that the budget constraint in period  $t$  simplifies to

$$C_{t+1}^i + E_{t+1}^i = R_t^i E_t^i,$$

where the equity return  $R_t^i$  equals the interest rate  $R_t$  if firm  $i$  is a lender. The equity return is given by  $\tilde{R}_t \geq R_t$  if firm  $i$  is a borrower. Solving the maximization problem of firm  $i$  yields the Euler equation

$$\frac{1}{C_t^i} = \beta R_t^i E_t^i \frac{1}{C_{t+1}^i}$$

with the solutions

$$C_{t+1}^i = (1 - \beta) R_t^i E_t^i \text{ and } E_{t+1}^i = \beta R_t^i E_t^i.$$

### 3.3 Equilibrium

A *market equilibrium* is defined as a sequence of consumption plans, allocations of capital and labor, as well as debt positions  $\{C_t^i, K_t^i, L_t^i, D_t^i\}$  for each sector, consumption for the workers  $\{C_t^w\}$ , and wages and interest rate  $\{w_t^H, w_t^L, R_t\}$  such that for a given initial capital stock, and initial wealth and labor distribution

- the entrepreneurs' maximization problem (3.1) is solved
- as many workers as possible are allocated in the high-wage sector and workers consume all their labor income  $C_t^w = w_t^H L_t^H + w_t^L L_t^L$
- the markets for output, labor, and capital clear.

Let  $D_t^i = D_t > 0$  if firm  $i$  is a borrower. Equilibrium in the credit market ensures that  $D_t^i = -D_t$  if firm  $i$  is a lender. Next period's aggregate capital stock equals then

$$K_{t+1} = \alpha \beta Y_t, \tag{3.2}$$

where  $Y_t$  denotes aggregate output. In the following, the total amount of labor is normalized to one and  $k_t^i \equiv \frac{K_t^i}{L_t^i}$ . The market equilibrium depends on financial development ( $\lambda$ ) as well as on the degree of labor mobility ( $\gamma$ ). Equilibrium wages, equity returns, and the development of wealth are separately determined for the case of perfect labor mobility and the case when labor is only mobile to some degree.

### 3.3.1 Credit market frictions only: $\gamma = 1$

Competitive wages are equal to the marginal productivity of labor. As workers can move freely between sectors, wages are the same in both sectors:

$$w_t = (1 - \alpha)A (k_t^H)^\alpha = (1 - \alpha)zA (k_t^L)^\alpha. \quad (3.3)$$

It follows that  $\frac{k_t^H}{k_t^L} = z^{\frac{1}{\alpha}}$  must hold. Suppose high-TFP firms are debt constrained. Credit is then given by  $D_t = \frac{\lambda Y_t^H}{R_t}$ . Wealth of the productive (and credit constrained) firm at the end of period  $t$  is derived as

$$\begin{aligned} & Y_t^H - w_t L_t^H - R_t D_t^H \\ &= \frac{R_t(\alpha - \lambda)}{\underbrace{R_t A^{-1/\alpha} [w_t / (1 - \alpha)]^{(1-\alpha)/\alpha} - \lambda}_{\tilde{R}_t}} E_t^H. \end{aligned} \quad (3.4)$$

Wealth of the low-TFP firm at the end of period  $t$  is given by

$$R_t E_t^L.$$

Let  $x_t \equiv \frac{E_t^H}{K_t}$  denote the wealth share of the high-TFP firm. The interest rate is determined as a function of  $x_t$  and of the total capital stock  $K_t$  (see Appendix 3.6 for the derivation):

$$R(x_t, K_t) = \begin{cases} \alpha z A \left( \frac{\alpha z^{\frac{1}{\alpha}} - \lambda + \alpha x_t - \alpha z^{\frac{1}{\alpha}} x_t}{\alpha z^{\frac{1}{\alpha}} - \lambda} \right)^{\alpha-1} K_t^{\alpha-1} = MPK_t^L & \text{if } x_t \leq 1 - \frac{\lambda}{\alpha z^{1/\alpha}} \\ \frac{\lambda A}{1-x_t} K_t^{\alpha-1} \in (MPK_t^L, MPK_t^H) & \text{if } x_t \in \left[ 1 - \frac{\lambda}{\alpha z^{1/\alpha}}, 1 - \frac{\lambda}{\alpha} \right], \\ \alpha A K_t^{\alpha-1} = MPK_t^H & \text{if } x_t \geq 1 - \frac{\lambda}{\alpha} \end{cases}, \quad (3.5)$$

where  $MPK_t^i = \alpha A^i (k_t^i)^{\alpha-1}$  denotes the marginal product of capital in sector  $i$ . Equation (3.5) describes the three cases that can occur: If the wealth share of high-TFP firms is smaller than  $1 - \lambda / (\alpha z^{1/\alpha})$ , all firms produce. Otherwise, low-TFP firms lend all their equity to high-TFP firms and only high-TFP firms produce. Borrowers are not debt constrained if and only if  $x_t \geq 1 - \lambda / \alpha$ . In this case, the equity return of the high-TFP sector  $\tilde{R}_t$  equals the interest rate  $R_t$ .



Next period's wealth share of the high-TFP firm is

$$x_{t+1} = \begin{cases} X_0(x_t) & \text{if the productivity state does not change} \\ X_1(x_t) = 1 - X_0(x_t) & \text{if the productivity state changes} \end{cases}$$

with

$$X_0(x_t) = \frac{\tilde{R}(x_t, K_t)x_t}{\tilde{R}(x_t, K_t)x_t + R(x_t, K_t)(1-x_t)} = \begin{cases} \frac{(\alpha-\lambda)x_t}{(\alpha-\lambda)x_t + (\alpha z^{\frac{1}{\alpha}} - \lambda)(1-x_t)} & \text{if } x_t \leq 1 - \frac{\lambda}{\alpha z^{1/\alpha}} \\ 1 - \frac{\lambda}{\alpha} & \text{if } x_t \in [1 - \frac{\lambda}{\alpha z^{1/\alpha}}, 1 - \frac{\lambda}{\alpha}] \\ x_t & \text{if } x_t \geq 1 - \frac{\lambda}{\alpha} \end{cases} \quad (3.6)$$

The stochastic dynamics of borrower wealth depends on the collateral share  $\lambda$ , and is similar to the dynamics of a model in which capital is the only production input (see Azariadis and Kaas, 2009). Economies with high collateral  $\lambda \geq \frac{\alpha}{2} \equiv \lambda_2^{comp}$  converge to an equilibrium with efficient production, non-binding credit constraints and no volatility. Economies with medium collateral  $\frac{\alpha z^{\frac{1}{\alpha}}}{1+z^{\frac{1}{\alpha}}} \leq \lambda < \frac{\alpha}{2}$  converge to a cycle with efficient production. However credit constraints bind in a fraction  $1 - \pi$  of periods. Economies with small collateral  $\lambda < \frac{\alpha z^{\frac{1}{\alpha}}}{1+z^{\frac{1}{\alpha}}} \equiv \lambda_1^{comp}$  converge to a cycle with a finite number of states. Production is efficient only in three states. For a more detailed description of the dynamics and for a proof, see Azariadis and Kaas (2009).

### 3.3.2 Credit and labor market frictions: $0 < \gamma < 1$

Let  $b_t$  be the fraction of workers in the high-TFP sector at the beginning of period  $t$  before the new productivity state is drawn and before labor reallocation takes place. After labor is reallocated, labor input in period  $t$  is  $L_t^H = b_{t+1}$  in the high-TFP sector and  $L_t^L = 1 - b_{t+1}$  in the low-TFP sector. In each period, only a fraction  $\gamma$  of workers in the low-wage sector can move to the high-wage sector.

If the labor market constraint does not bind, all workers earn the same wage after labor reallocation. The distribution of workers between sectors for this case is derived in Appendix

3.6. The resulting labor input in the high-TFP sector in period  $t$  is

$$b_{t+1} = B^{nc}(x_t) = \begin{cases} \frac{\alpha x_t}{\alpha z^{\frac{1}{\alpha}} (1-x_t) - \lambda + \alpha x_t} & \text{if } x_t < 1 - \frac{\lambda}{\alpha z^{\frac{1}{\alpha}}} \\ 1 & \text{if } x_t \geq 1 - \frac{\lambda}{\alpha z^{\frac{1}{\alpha}}} \end{cases}. \quad (3.7)$$

A fraction  $1 - \gamma$  of workers in the low-wage sector cannot move to the high-wage sector. Hence, both sectors produce. Wages do not only depend on sectoral TFP, but also on the amount of capital and labor employed. Workers in the high-TFP sector usually earn the highest wage. However, when labor is sufficiently scarce in the low-TFP sector, workers in the low-TFP sector are paid the highest wage. Given  $x_t$  and  $b_t$ , labor input in the high-TFP sector in period  $t$  is given by

$$b_{t+1} = B_0(x_t, b_t) = \begin{cases} \min(b_t + \gamma(1 - b_t), B^{nc}(x_t)) & \text{if } B^{nc}(x_t) \geq b_t \\ \max((1 - \gamma)b_t, B^{nc}(x_t)) & \text{if } B^{nc}(x_t) < b_t \end{cases} \quad (3.8)$$

if the productivity state remains the same, and by

$b_{t+1} =$

$$B_1(x_t, b_t) = B_0(x_t, 1 - b_t) = \begin{cases} \min(1 - b_t + \gamma b_t, B^{nc}(x_t)) & \text{if } B^{nc}(x_t) \geq 1 - b_t \\ \max((1 - \gamma)(1 - b_t), B^{nc}(x_t)) & \text{if } B^{nc}(x_t) < 1 - b_t \end{cases}$$

if the productivity state changes.

When workers are not perfectly mobile, the high-TFP firm is not necessarily the borrower. The productivity of capital within one sector increases in the amount of labor employed. When labor is sufficiently immobile, it may happen that there is so few labor in the high-TFP sector that the productivity of capital is higher in the low-TFP sector, and the low-TFP firm borrows. A firm borrows capital as long as its marginal productivity of capital exceeds the interest rate. It is shown in Appendix 3.6 that there exists a threshold of  $x_t$  below which high-TFP firms borrow and above which low-TFP firms borrow. The threshold increases in the fraction of labor employed in the high-TFP sector.

**Case 1: High-TFP firms borrow, i.e.**  $x_t \leq \frac{b_{t+1}}{b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}}$

When the high-TFP firm is borrowing constrained, debt is given by  $D_t = \frac{\lambda A (K_t^H)^\alpha (L_t^H)^{1-\alpha}}{R_t}$ . Wealth of the productive firm at the end of the period is then

$$\underbrace{\frac{R_t(\alpha - \lambda)}{R_t A^{-1} (k_t^H)^{1-\alpha} - \lambda}}_{\tilde{R}_t} E_t^H. \quad (3.9)$$

Since some workers stay in the low-TFP sector, both sectors produce and the interest rate equals the marginal product of capital of low-TFP firms. When borrower wealth is large enough, credit constraints are not binding, and the interest rate equals also the marginal product of capital of high-TFP firms.<sup>3</sup> The interest rate is therefore given by

$$R_t = MPK_t^L =$$

$$\begin{cases} \alpha z A (f(x_t, b_{t+1}) b_{t+1} + 1 - b_{t+1})^{1-\alpha} K_t^{\alpha-1} & \text{if } x_t < \frac{(\alpha-\lambda)b_{t+1}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \\ MPK_t^H = \alpha A \left( b_{t+1} + z^{\frac{1}{1-\alpha}} (1 - b_{t+1}) \right)^{1-\alpha} K_t^{\alpha-1} & \text{if } x_t \geq \frac{(\alpha-\lambda)b_{t+1}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \end{cases}, \quad (3.10)$$

where  $f(x_t, b_{t+1})$  gives the equilibrium value of  $\frac{k_t^H}{k_t^L}$  and is derived in Appendix 3.6. The transitional dynamics of the wealth share of high-TFP firms is described by

$$X_0(x_t, b_{t+1}) = \begin{cases} \frac{(\alpha-\lambda)x_t}{(\alpha-\lambda)x_t + (\alpha z [f(x_t, b_{t+1})]^{1-\alpha} - \lambda)(1-x_t)} & \text{if } x_t < \frac{(\alpha-\lambda)b_{t+1}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \\ x_t & \text{if } x_t \geq \frac{(\alpha-\lambda)b_{t+1}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \end{cases} \quad (3.11)$$

if the productivity state does not change. If the productivity state changes,  $x_{t+1} = 1 - X_0(x_t, b_{t+1})$ .

<sup>3</sup>The corresponding value of  $x_t$  is derived in Appendix 3.6.

**Case 2: Low-TFP firms borrow, i.e.**  $x_t > \frac{b_{t+1}}{b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}}$

Consider the case when the low-TFP firm is borrowing constrained. Using  $D_t = \frac{\lambda Y_t^L}{R_t}$  one obtains wealth of the low-TFP firm at the end of the period as

$$\underbrace{\frac{R_t(\alpha - \lambda)}{R_t(zA)^{-1} (k_t^L)^{1-\alpha} - \lambda}}_{\tilde{R}_t} E_t^L. \quad (3.12)$$

The interest rate equals the marginal product of capital of high-TFP firms. When wealth of the low-TFP firm is sufficiently high, credit constraints are not binding and the interest rate equals also the marginal product of capital of low-TFP firms.<sup>4</sup> It follows that the interest rate is given by

$$R_t = MPK_t^H =$$

$$\begin{cases} \alpha A \left( \frac{g(x_t, b_{t+1})}{g(x_t, b_{t+1})^{b_{t+1} + 1 - b_{t+1}}} \right)^{\alpha-1} K_t^{\alpha-1} & \text{if } x_t > \frac{\alpha b_{t+1} + \lambda(1-b_{t+1})z^{\frac{1}{1-\alpha}}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \\ MPK_t^L = \alpha A \left( b_{t+1} + z^{\frac{1}{1-\alpha}}(1-b_{t+1}) \right)^{1-\alpha} K_t^{\alpha-1} & \text{if } x_t \leq \frac{\alpha b_{t+1} + \lambda(1-b_{t+1})z^{\frac{1}{1-\alpha}}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \end{cases}, \quad (3.13)$$

where  $g(x_t, b_{t+1})$  gives the equilibrium value of  $\frac{k_t^H}{k_t^L}$  and is derived in Appendix 3.6. The transitional dynamics of the wealth share of high-TFP firms is described by

$$X_0(x_t, b_{t+1}) = \frac{R_t x_t}{R_t x_t + \tilde{R}_t (1-x_t)} = \begin{cases} \frac{\left( \frac{\alpha}{z} [g(x_t, b_{t+1})]^{\alpha-1} - \lambda \right) x_t}{\left( \frac{\alpha}{z} [g(x_t, b_{t+1})]^{\alpha-1} - \alpha \right) x_t + \alpha - \lambda} & \text{if } x_t > \frac{\alpha b_{t+1} + \lambda(1-b_{t+1})z^{\frac{1}{1-\alpha}}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \\ x_t & \text{if } x_t \leq \frac{\alpha b_{t+1} + \lambda(1-b_{t+1})z^{\frac{1}{1-\alpha}}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]} \end{cases}. \quad (3.14)$$

The values of  $b_t$  and  $x_t$  determine whether the high-TFP firm is a lender or a borrower in period  $t$ , and which sector can attract workers. Figure 3.3 illustrates the thresholds of  $x_t$  as functions of  $b_t$  indicating which case occurs when the productivity state does not change ( $s_t = s_{t-1}$ ):

<sup>4</sup>The corresponding value of  $x_t$  is derived in Appendix 3.6.

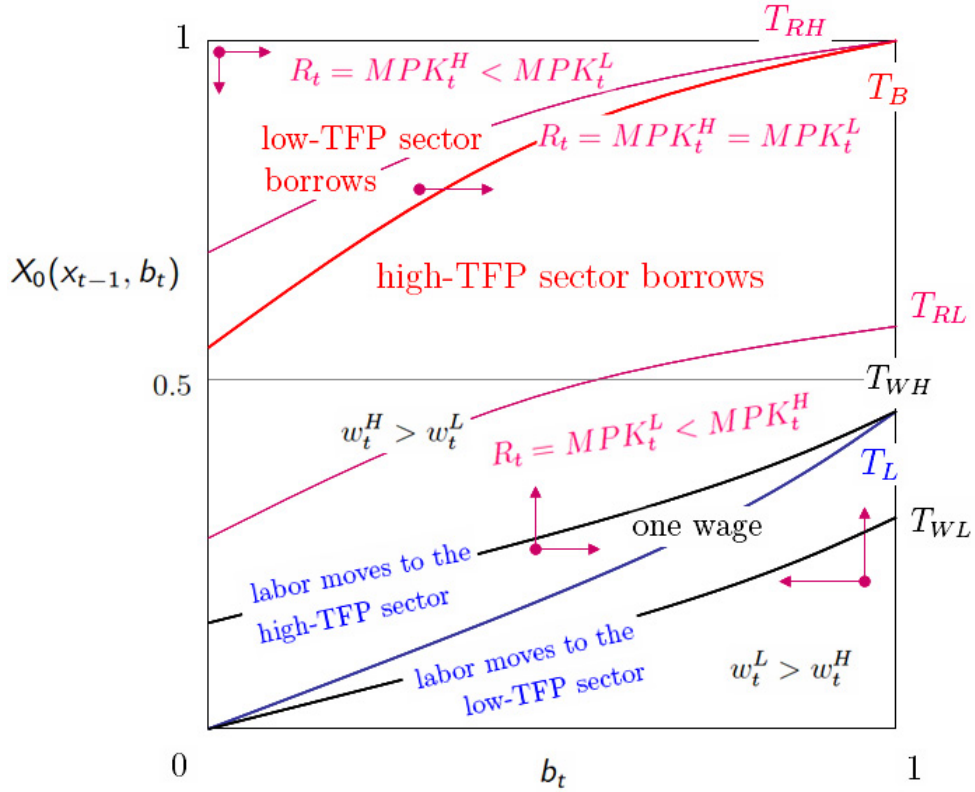


Figure 3.3: Dynamics when the productivity state remains the same ( $s_t = s_{t-1}$ ). When  $s_t \neq s_{t-1}$ ,  $b_t$  and  $X_0(x_{t-1}, b_t)$  are replaced by  $1 - b_t$  and  $1 - X_0(x_{t-1}, b_t)$ .

## Labor market

- Workers move to the high-TFP sector in period  $t$  as long as  $b_t \leq B^{nc}(x_t)$  which is equivalent to

$$x_t \geq \frac{(\alpha z^{\frac{1}{\alpha}} - \lambda) b_t}{\alpha [1 - (1 - z^{\frac{1}{\alpha}}) b_t]} \equiv T_L(b_t).$$

If  $x_t < T_L(b_t)$ , workers move to the low-TFP sector.

- Wages in the high-TFP sector exceed wages in the low-TFP sector if  $b_t + \gamma(1 - b_t) < B^{nc}(x_t)$  which is equivalent to

$$x_t > \frac{(\alpha z^{\frac{1}{\alpha}} - \lambda) (\gamma + (1 - \gamma) b_t)}{\alpha [1 - (1 - z^{\frac{1}{\alpha}}) (\gamma + (1 - \gamma) b_t)]} \equiv T_{WH}(b_t).$$

- Wages in the low-TFP sector exceed wages in the high-TFP sector if  $(1 - \gamma)b_t > B^{nc}(x_t)$ , i.e.

$$x_t < \frac{(\alpha z^{\frac{1}{\alpha}} - \lambda)(1 - \gamma)b_t}{\alpha \left[ 1 - \left( 1 - z^{\frac{1}{\alpha}} \right) (1 - \gamma)b_t \right]} \equiv T_{WL}(b_t).$$

- All workers earn the same wage if

$$T_{WL}(b_t) \leq x_t \leq T_{WH}(b_t).$$

## Credit market

The following threshold functions are derived in Appendix 3.6.

- The high-TFP sector borrows if

$$x_t \leq \frac{\gamma + (1 - \gamma)b_t}{\gamma + (1 - \gamma) \left( b_t + (1 - b_t)z^{\frac{1}{1-\alpha}} \right)} \equiv T_B(b_t).$$

If  $x_t > T_B(b_t)$ , the low-TFP sector borrows.

- When the high-TFP sector borrows, credit constraints bind if

$$x_t < \frac{\alpha - \lambda}{\alpha} \frac{\gamma + (1 - \gamma)b_t}{\gamma + (1 - \gamma) \left( b_t + (1 - b_t)z^{\frac{1}{1-\alpha}} \right)} \equiv T_{RL}(b_t).$$

- When the low-TFP sector borrows, credit constraints bind if

$$x_t > \frac{\alpha [\gamma + (1 - \gamma)b_t] + \lambda(1 - \gamma)(1 - b_t)z^{\frac{1}{1-\alpha}}}{\alpha \left[ \gamma + (1 - \gamma) \left( b_t + (1 - b_t)z^{\frac{1}{1-\alpha}} \right) \right]} \equiv T_{RH}(b_t).$$

- Credit constraints do not bind if

$$T_{RL}(b_t) \leq x_t \leq T_{RH}(b_t).$$

It is shown in Appendix 3.6 that  $T_{RH}(b_t) > T_B(b_t) > T_{RL}(b_t) > T_{WH}(b_t) > T_L(b_t) > T_{WL}(b_t)$  for all  $b_t \in (0, 1)$ . In the beginning of period  $t$ , before labor is reallocated between sectors, a fraction  $b_t$  of workers is in the high-TFP sector, and the wealth share of high-TFP firms is given

by  $x_t = X_0(x_{t-1}, b_t)$ . When  $(b_t, x_t)$  is located above the  $T_B$  curve, the high-TFP firm has a high wealth share and employs relatively few workers. It will therefore lend capital to the low-TFP firm. Below the  $T_B$  threshold, in contrast, the high-TFP firm borrows capital. It is profitable for a firm to borrow capital as long as its marginal productivity exceeds the interest rate. When  $(b_t, x_t)$  is located in the area between the  $T_{RH}$  and the  $T_{RL}$  curve, the borrower's wealth share is sufficiently large that credit constraints are not binding. Marginal productivities of capital are equalized across sectors. Equity returns are equalized as well. The wealth distribution does not change. When  $(b_t, x_t)$  is located above the  $T_{RH}$  curve, the interest rate equals  $MPK_t^H$  and the wealth share of the high-TFP sector decreases. Below the  $T_{RL}$  curve, the interest rate equals  $MPK_t^L$  and the wealth share of the high-TFP sector increases.

Workers earn a wage equal to the marginal productivity of labor within their sector. They wish to move to the sector with the highest wage. That is usually the sector with higher TFP. However, when the wealth share in the high-TFP sector is very low, workers in the low-TFP sector earn the highest wage. This occurs in the area below the  $T_{WL}$  curve. Workers move to the high-TFP (low-TFP) sector when  $(b_t, x_t)$  is located above (below) the  $T_L$  curve. Wages in both sectors are equalized by labor reallocation, when  $(b_t, x_t)$  is located in the area between the  $T_{WH}$  and the  $T_{WL}$  curve. Above the  $T_{WH}$  curve wages in the high-TFP sector exceed wages in the low-TFP sector.

The arrows in Figure 3.3 indicate whether  $x$  and/or  $b$  decrease or increase. When the productivity state changes, the fraction of workers in the high-TFP sector before labor reallocation is given by  $1 - b_t$ , and the wealth share in the new high-TFP sector is  $x_t = 1 - X_0(x_{t-1}, b_t)$ .

Assume the productivity state does not change for several periods. When a lot of workers and only few capital is allocated in the high-TFP sector, workers will leave and capital flows to the high-TFP sector. The wealth share increases and eventually the sector can again attract workers. When there are only few workers and a lot of capital in the high-TFP sector, workers will move to the high-TFP sector while capital leaves. When the productivity state changes, the wealth share and labor input in the new high-TFP sector is again located below the  $T_{RL}$  threshold. When financial development is sufficiently high,  $(b_t, x_t)$  eventually stays in the area between the  $T_{RH}$  and the  $T_{RL}$  curve where equity returns are equalized across sectors.

**Proposition 3.1.** *If  $\lambda \geq \frac{\alpha}{2}$ , equity returns are equalized across sectors in the long run for a given  $\gamma \in (0, 1]$ .*

Appendix 3.6 contains the proof of *Proposition 3.1*. The critical value of  $\lambda$  equals the threshold  $\lambda_2^{comp}$  that holds in the case of perfect labor mobility.

### 3.4 Simulation

This section examines the dynamics of the model for  $0 < \gamma \leq 1$  by varying the value of  $\lambda$ . The simulation is not meant to replicate real data, but to highlight the effects of different degrees of capital and labor market imperfections on the development of output and wages. The model period is one year.<sup>5</sup> The discount factor is set to  $\beta = 0.95$ . Let  $Y$  include output as well as undepreciated capital. Using this interpretation, it is reasonable to choose a capital share  $\alpha = 0.8$  (Kaas, 2009). The remaining parameters are set to  $A = 1$ ,  $z = 0.9$ , and  $\pi = 0.6$ . One obtains the thresholds  $\lambda_1^{comp} = 0.37$  and  $\lambda_2^{comp} = \frac{\alpha}{2} = 0.40$ . The variables of interest are determined by the sample means of

- aggregate output:  $Y_t = Y_t^H + Y_t^L$
- the share of credit in aggregate output:  $D_t/Y_t$
- the average wage:  $b_{t+1}w_t^H + (1 - b_{t+1})w_t^L$
- wage inequality:  $w_t^H/w_t^L$

The volatility of a variable is measured as its standard deviation over all periods. The volatility of individual wages is calculated as the standard deviation of wages within one sector.

The simulation results are illustrated in Figures 3.4 to 3.9. Since labor and capital are complementary input factors, the effect of a policy improving the mobility of capital depends on the mobility of labor. I simulated each series for three different degrees of labor market frictions: low labor mobility ( $\gamma = 0.1$ ), high labor mobility ( $\gamma = 0.9$ ), and perfect mobility of labor ( $\gamma = 1$ ). The simulation results show that increasing financial development has, in general, a higher effect when workers are more mobile. Higher financial development increases the share of credit in aggregate output (Figure 3.4). Recall that the credit share in GDP has been taken as a measure of financial development in the introductory section of this paper. Financial development has a similar effect on aggregate output as it has on the average wage. Aggregate output increases in financial development (Figure 3.5). The volatility of aggregate output decreases in financial development (Figure 3.6). Financial development has a higher potential effect on volatility when the labor market is more flexible. Aggregate volatility is zero only if neither the capital nor the labor market constraint binds. Note, however, that for economies with poor financial development, volatility is higher when workers are more mobile. When capital mobility is low, in some periods, a lot of capital is allocated in the low-TFP sector. Hence, the

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<sup>5</sup>I simulated time series of 50,000 periods.



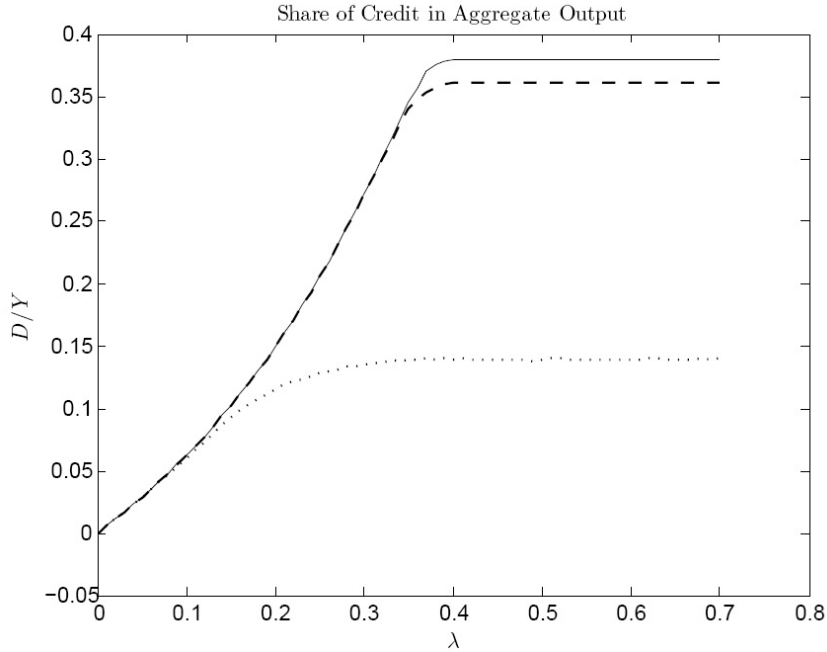


Figure 3.4: The effect of financial development on the share of credit in aggregate output for  $\gamma = 0.1$  (dotted line),  $\gamma = 0.9$  (dashed line),  $\gamma = 1$  (solid line)

low-TFP sector withdraws workers from the high-TFP sector and aggregate output is low. In other periods, a lot of capital is allocated in the high-TFP sector and workers want to work in the high-TFP sector. As a result, aggregate output is high. These fluctuations are amplified when worker mobility is increased. Wage inequality as well as the volatility of individual wages increase in financial development when labor is not perfectly mobile between sectors (Figures 3.7 and 3.8). When labor is assumed to be perfectly mobile, each worker earns the same wage and the simulation shows that volatility of wages decreases in financial development. It is by the introduction of labor market frictions that a positive relationship between wage inequality, volatility of individual wages, and financial development emerges.

What is the intuition behind the results? Volatility in the model framework is the result of sectoral productivity shocks. When there are no credit and no labor market frictions, capital and labor always flow to the sector with high TFP. There is no volatility, and no inequality. Labor income and output are maximized. When capital market frictions are introduced, the sector with lower TFP also produces and the distribution of wealth between the high and the low TFP sector matters. The wealth distribution becomes less important when financial development increases. Increasing financial development decreases the volatility of aggregate output and of the single wage. When labor market frictions are introduced as well, it may occur that not

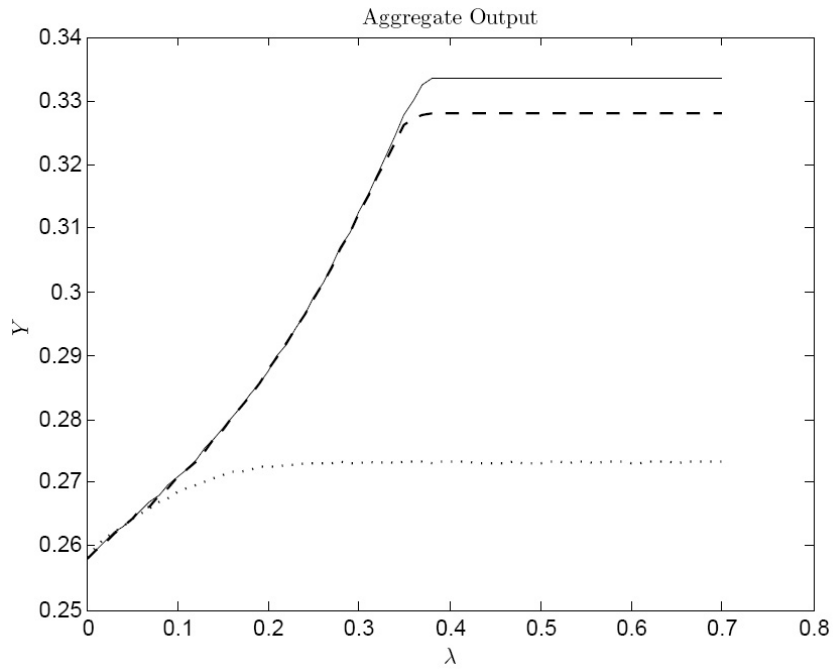


Figure 3.5: The effect of financial development on aggregate output for  $\gamma = 0.1$  (dotted line),  $\gamma = 0.9$  (dashed line),  $\gamma = 1$  (solid line)

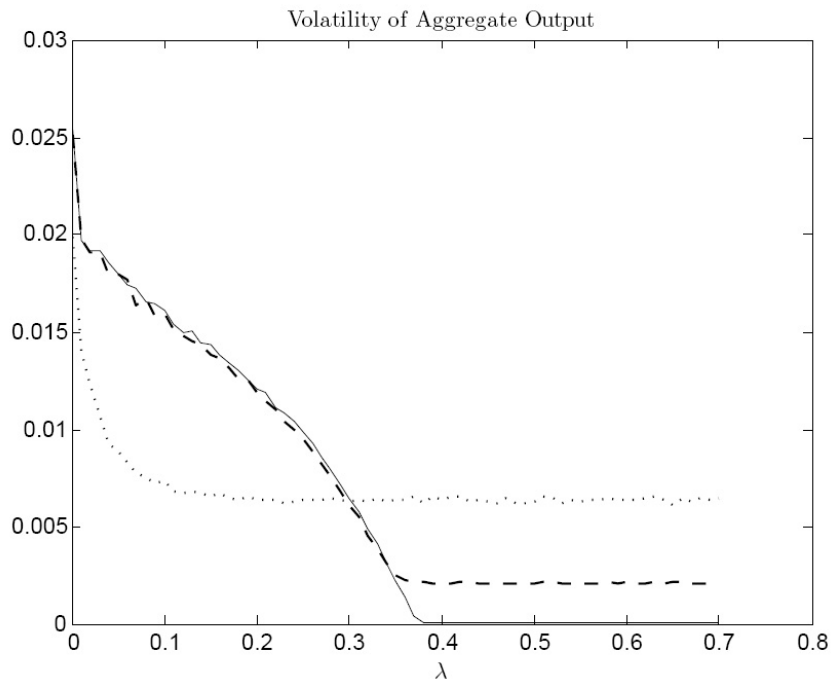


Figure 3.6: The effect of financial development on the volatility of aggregate output for  $\gamma = 0.1$  (dotted line),  $\gamma = 0.9$  (dashed line),  $\gamma = 1$  (solid line)

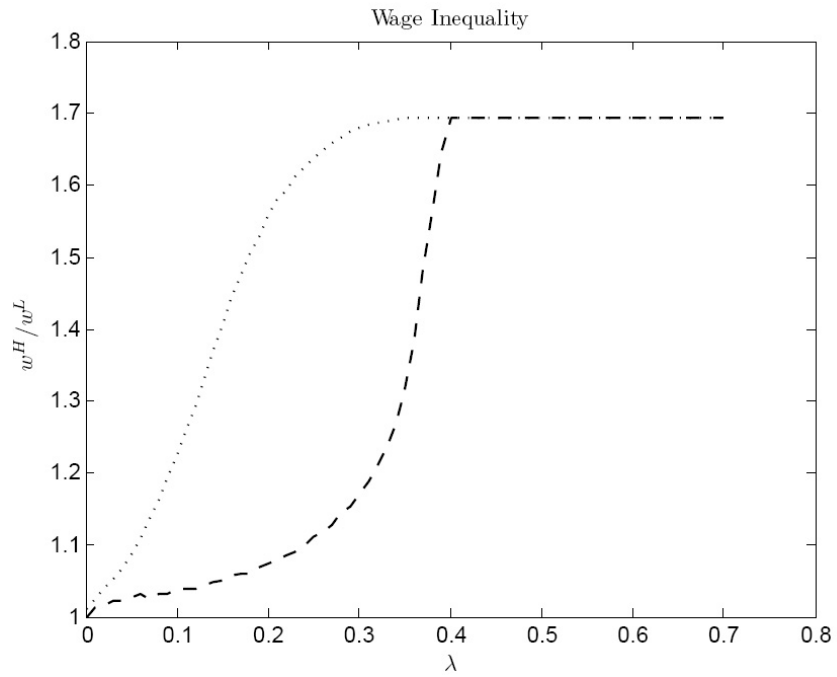


Figure 3.7: The effect of financial development on wage inequality for  $\gamma = 0.1$  (dotted line),  $\gamma = 0.9$  (dashed line),  $\gamma = 1$  (solid line)

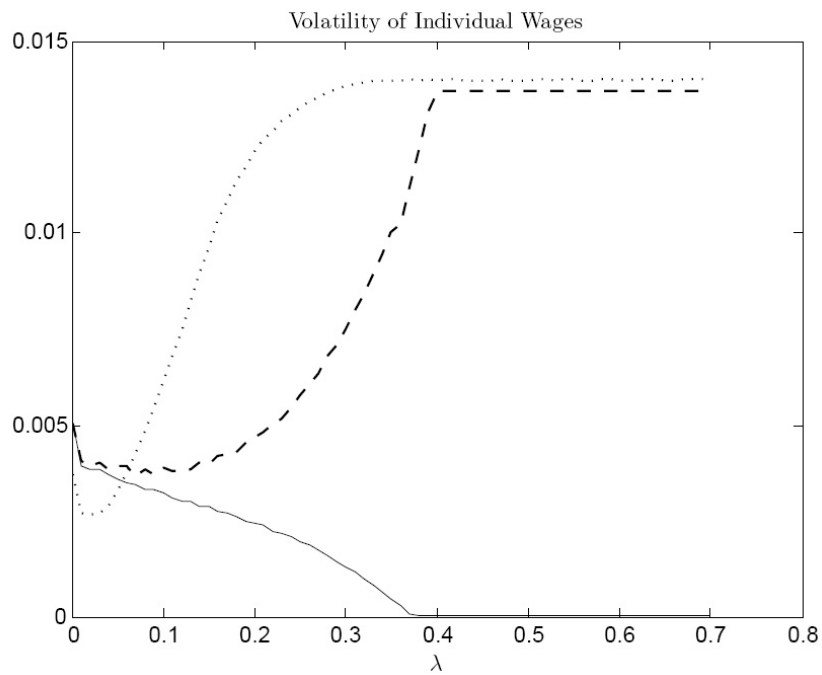


Figure 3.8: The effect of financial development on the volatility of individual wages for  $\gamma = 0.1$  (dotted line),  $\gamma = 0.9$  (dashed line),  $\gamma = 1$  (solid line)

enough workers manage to move to the high-wage sector to equalize marginal productivities of labor across sectors. Workers in one sector earn then lower wages than workers in the other sector. If financial development increases now, more capital flows to the high-TFP sector. As a result, wages in the high-TFP sector increase even more while wages in the low-TFP sector decrease even more. Wages are more correlated with sectoral TFP (Figure 3.9). Wage inequality and volatility of individual wages increase.

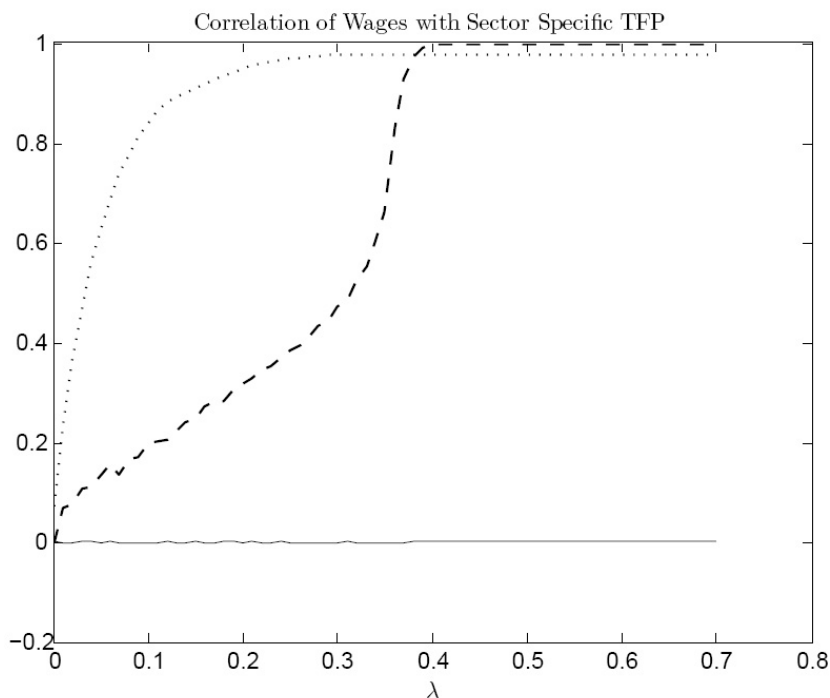


Figure 3.9: The effect of financial development on the correlation of wages with sector-specific TFP for  $\gamma = 0.1$  (dotted line),  $\gamma = 0.9$  (dashed line),  $\gamma = 1$  (solid line)

### 3.5 Conclusions

A real business cycle model with sectoral productivity shocks and labor as well as credit market frictions can explain a simultaneous increase in macroeconomic stability and in wage inequality. In line with other theoretical work on financial frictions, it was shown that financial development has a positive effect on output and macroeconomic stability. The main contribution of the present paper is to make visible the interaction between the labor and the credit market. In the presence of labor market frictions, an increase in financial development increases the correlation of wages with sector-specific TFP and thereby wage inequality, and volatility of individual wages.

## 3.6 Appendix

### Derivation of equation (3.5)

When  $\gamma = 1$ , one of three cases can occur:

- **Only high-TFP firms produce and debt constraints are binding**

When low-TFP firms do not produce, and debt constraints are binding, the debt/equity ratio of high-TFP firms is

$$\theta_t^H = \frac{D_t}{K_t^H - D_t} = \frac{\lambda A(k_t^H)^{\alpha-1}}{R_t - \lambda A(k_t^H)^{\alpha-1}}.$$

As low-TFP firms lend all their equity to high-TFP firms in this case, the debt/equity ratio also equals

$$\theta_t^H = \frac{(1-x_t)K_t}{x_t K_t}.$$

The interest rate is then

$$R(x_t, K_t) = \frac{\lambda A K_t^{\alpha-1}}{1-x_t} \in (MPK_t^L, MPK_t^H),$$

where  $MPK_t^i = \alpha A_t^i (k_t^i)^{\alpha-1}$  denotes the marginal product of capital in sector  $i$ .

- **All firms produce and debt constraints are binding**

The interest rate will not fall below the marginal product of capital of low-TFP firms. If the wealth share of productive firms is small, the interest rate is

$$R(x_t, K_t) = MPK_t^L.$$

The critical value of  $x_t$  below which this happens, is derived as follows:

$$\alpha_z A(k_t^L)^{\alpha-1} > \frac{\lambda A(k_t^H)^{\alpha-1}}{1-x_t}$$

$$\Leftrightarrow x_t < 1 - \frac{\lambda}{\alpha_z^{1/\alpha}}.$$

In this case, both types of firms produce, and high-TFP firms are debt constrained. Using

$D_t = \frac{\lambda Y_t^H}{R_t}$ ,  $K_t^H = x_t K_t + D_t$  and  $\frac{k_t^H}{k_t^L} = z^{\frac{1}{\alpha}}$  yields

$$D_t = \frac{\lambda x_t}{\alpha z^{\frac{1}{\alpha}} - \lambda} K_t. \quad (3.15)$$

- **Only high-TFP firms produce and borrowers are not debt constrained**

The interest rate will not exceed the marginal product of capital of high-TFP firms. If the wealth share of high-TFP firms is large, borrowers are not debt constrained and the interest rate is

$$R(x_t, K_t) = MPK_t^H.$$

The equity return of the productive sector  $\tilde{R}_t$  is then equal to the interest rate. The critical value of  $x_t$  is derived as follows:

$$\begin{aligned} \alpha A (k_t^H)^{\alpha-1} &\leq \frac{\lambda A (k_t^H)^{\alpha-1}}{1 - x_t} \\ \Leftrightarrow x_t &\geq 1 - \frac{\lambda}{\alpha}. \end{aligned}$$

## Derivation of equation (3.7)

The following derivation makes use of results obtained in section 3.3.1. In a competitive labor market, only high-TFP firms produce in period  $t$  if  $x_t \geq 1 - \frac{\lambda}{\alpha z^{\frac{1}{\alpha}}}$ . All workers are then employed in the high-TFP sector, i.e.  $b_{t+1} = 1$ . When  $x_t < 1 - \frac{\lambda}{\alpha z^{\frac{1}{\alpha}}}$  both firms produce, the high-TFP firm borrows, and debt constraints are binding. The marginal products of labor are equalized between both sectors. Hence,  $k_t^H = k_t^L z^{\frac{1}{\alpha}}$  holds. This is equivalent to

$$\frac{x_t K_t + D_t}{b_{t+1}} = \frac{(1 - x_t) K_t - D_t}{1 - b_{t+1}} z^{\frac{1}{\alpha}}.$$

Using equation (3.15), one obtains

$$\Leftrightarrow b_{t+1} = \frac{\alpha x_t}{\alpha z^{\frac{1}{\alpha}} (1 - x_t) - \lambda + \alpha x_t}.$$

## Threshold of $x_t$ below which high-TFP firms borrow

The high-TFP firm borrows as long as  $MPK_t^H \geq MPK_t^L$ . This is equivalent to

$$\begin{aligned} \frac{K_t^H}{K_t^L} &\leq \frac{b_{t+1}}{1-b_{t+1}} z^{\frac{1}{\alpha-1}} \\ \Leftrightarrow \frac{x_t}{1-x_t} &\leq \frac{b_{t+1}}{1-b_{t+1}} z^{\frac{1}{\alpha-1}} \\ \Leftrightarrow x_t &\leq \frac{b_{t+1}}{b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}}. \end{aligned} \quad (3.16)$$

## Derivation of $f(x_t, b_{t+1})$

Using  $\frac{K_t^H}{K_t^L} = \frac{K_t x_t + D_t}{K_t(1-x_t) - D_t}$  and  $\frac{K_t^H}{K_t^L} = \frac{b_{t+1}}{1-b_{t+1}} \frac{k_t^H}{k_t^L}$  yields

$$\frac{K_t x_t + D_t}{K_t(1-x_t) - D_t} = \frac{b_{t+1}}{1-b_{t+1}} \frac{k_t^H}{k_t^L}. \quad (3.17)$$

When  $x_t < \frac{(\alpha-\lambda)b_{t+1}}{\alpha[b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]}$ , the debt constraint is binding and

$$D_t = \frac{\lambda x_t}{\alpha z \left(\frac{k_t^H}{k_t^L}\right)^{1-\alpha} - \lambda} K_t.$$

Substituting this into equation (3.17) yields

$$\alpha z (1-x_t) \frac{k_t^H}{k_t^L} - \alpha z x_t \frac{1-b_{t+1}}{b_{t+1}} = \lambda \left(\frac{k_t^H}{k_t^L}\right)^\alpha. \quad (3.18)$$

The value of  $f(x_t, b_{t+1})$  is determined as the value of  $\frac{k_t^H}{k_t^L}$  that solves equation (3.18). The left-hand side of this equation is linear and increasing in  $\frac{k_t^H}{k_t^L}$ . The right-hand side is increasing at a decreasing rate. Hence,  $f(x_t, b_{t+1})$  is determined as the unique solution of equation (3.17). The solution is illustrated in Figure 3.10.

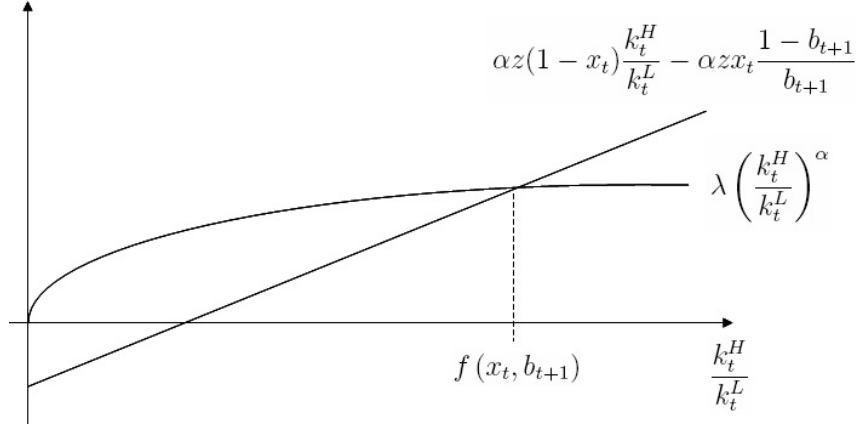


Figure 3.10: Determination of  $f(x_t, b_{t+1})$

## Derivation of $g(x_t, b_{t+1})$

The derivation of  $g(x_t, b_{t+1})$  is similar to the derivation of  $f(x_t, b_{t+1})$ . The value of  $g(x_t, b_{t+1})$ , with  $x_t > \frac{\alpha b_{t+1} + \lambda(1-b_{t+1})z^{\frac{1}{1-\alpha}}}{\alpha [b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}}]}$  is uniquely determined as the value of  $\frac{k_t^H}{k_t^L}$  that solves

$$\alpha x_t - \alpha(1-x_t) \frac{b_{t+1}}{1-b_{t+1}} \frac{k_t^H}{k_t^L} = \lambda z \left( \frac{k_t^H}{k_t^L} \right)^{1-\alpha}.$$

## Critical value of borrower wealth above which credit constraints do not bind for $0 < \gamma < 1$

There are two cases to distinguish:

### Case 1: The high-TFP firm borrows

When the high-TFP firm is not borrowing constrained, the following conditions are fulfilled:

$$D_t \leq \frac{\lambda Y_t^H}{R_t} \quad (3.19)$$

$$R_t = MPK_t^L = MPK_t^H \quad (3.20)$$

$$\frac{k_t^H}{k_t^L} = z^{-\frac{1}{1-\alpha}} \quad (3.21)$$

$$K_t^H = x_t K_t + D_t \quad (3.22)$$



Equation (3.21) follows from (3.20). Substituting (3.20) and (3.22) into (3.19) yields

$$D_t \leq \frac{\lambda x_t}{\alpha - \lambda} K_t. \quad (3.23)$$

Since  $\frac{x_t K_t + D_t}{(1-x_t)K_t - D_t}$  is increasing in  $D_t$ ,

$$\frac{x_t K_t + D_t}{(1-x_t)K_t - D_t} \frac{1-b_{t+1}}{b_{t+1}} \leq \frac{x_t K_t + \frac{\lambda x_t}{\alpha - \lambda} K_t}{(1-x_t)K_t - \frac{\lambda x_t}{\alpha - \lambda} K_t} \frac{1-b_{t+1}}{b_{t+1}}.$$

Using  $\frac{x_t K_t + D_t}{(1-x_t)K_t - D_t} \frac{1-b_{t+1}}{b_{t+1}} = \frac{k_t^H}{k_t^L}$  and (3.21), one obtains

$$x_t \geq \frac{(\alpha - \lambda)b_{t+1}}{\alpha \left[ b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}} \right]}. \quad (3.24)$$

## Case 2: The low-TFP firm borrows

When the low-TFP firm is not borrowing constrained, the following conditions are fulfilled:

$$D_t \leq \frac{\lambda Y_t^L}{R_t} \quad (3.25)$$

$$R_t = MPK_t^H = MPK_t^L \quad (3.26)$$

$$\frac{k_t^H}{k_t^L} = z^{-\frac{1}{1-\alpha}} \quad (3.27)$$

$$K_t^L = (1-x_t)K_t + D_t \quad (3.28)$$

Substituting (3.26) and (3.28) into (3.25) yields

$$D_t \leq \frac{\lambda(1-x_t)}{\alpha - \lambda} K_t. \quad (3.29)$$

Since  $\frac{x_t K_t - D_t}{(1-x_t)K_t + D_t}$  is decreasing in  $D_t$ ,

$$\frac{x_t K_t - D_t}{(1-x_t)K_t + D_t} \frac{1-b_{t+1}}{b_{t+1}} \geq \frac{x_t K_t - \frac{\lambda(1-x_t)}{\alpha - \lambda} K_t}{(1-x_t)K_t + \frac{\lambda(1-x_t)}{\alpha - \lambda} K_t} \frac{1-b_{t+1}}{b_{t+1}}.$$

Using  $\frac{x_t K_t - D_t}{(1-x_t)K_t + D_t} \frac{1-b_{t+1}}{b_{t+1}} = \frac{k_t^H}{k_t^L}$  and (3.27), one obtains

$$x_t \leq \frac{\alpha b_{t+1} + \lambda(1-b_{t+1})z^{\frac{1}{1-\alpha}}}{\alpha \left[ b_{t+1} + (1-b_{t+1})z^{\frac{1}{1-\alpha}} \right]}. \quad (3.30)$$

## Threshold functions when $0 < \gamma < 1$

The following calculations hold when the productivity state remains the same ( $s_t = s_{t-1}$ ). Using equations (3.8) and (3.16), one obtains the condition under which high-TFP firms borrow. The conditions under which credit constraints are binding is derived by substitution of (3.8) into (3.24) and (3.30).

- When  $w_t^H > w_t^L$ , the high-TFP sector borrows if

$$x_t \leq \frac{\gamma + (1-\gamma)b_t}{\gamma + (1-\gamma) \left( b_t + (1-b_t)z^{\frac{1}{1-\alpha}} \right)} \equiv T_B(b_t).$$

The high-TFP firm is credit constrained if

$$x_t < \frac{\alpha - \lambda}{\alpha} \frac{\gamma + (1-\gamma)b_t}{\gamma + (1-\gamma) \left( b_t + (1-b_t)z^{\frac{1}{1-\alpha}} \right)} \equiv T_{RL}(b_t).$$

When the low-TFP sector borrows, credit constraints bind if

$$x_t > \frac{\alpha [\gamma + (1-\gamma)b_t] + \lambda(1-\gamma)(1-b_t)z^{\frac{1}{1-\alpha}}}{\alpha \left[ \gamma + (1-\gamma) \left( b_t + (1-b_t)z^{\frac{1}{1-\alpha}} \right) \right]} \equiv T_{RH}(b_t).$$

- When  $w_t^L > w_t^H$ , the high-TFP sector borrows if

$$x_t \leq \frac{(1-\gamma)b_t}{z^{\frac{1}{1-\alpha}} + (1-\gamma)b_t \left( 1 - z^{\frac{1}{1-\alpha}} \right)} \equiv T_{HB}(b_t).$$

The high-TFP sector is borrowing constrained if

$$x_t < \frac{\alpha - \lambda}{\alpha} \frac{(1-\gamma)b_t}{z^{\frac{1}{1-\alpha}} + (1-\gamma)b_t \left( 1 - z^{\frac{1}{1-\alpha}} \right)} \equiv T_{HC}(b_t).$$

When the low-TFP firm borrows, the credit constraint binds if

$$x_t > \frac{\alpha(1-\gamma)b_t + \lambda(1-(1-\gamma)b_t)z^{\frac{1}{1-\alpha}}}{\alpha \left[ z^{\frac{1}{1-\alpha}} + (1-\gamma)b_t \left( 1 - z^{\frac{1}{1-\alpha}} \right) \right]} \equiv T_{LC}(b_t).$$

- If  $w_t^L = w_t^H$ , always high-TFP firms borrow. To see this, substitute equation (3.7) into (3.16). This yields

$$x_t \leq \frac{\alpha - \left( \alpha z^{\frac{1}{\alpha}} - \lambda \right) z^{\frac{1}{1-\alpha}}}{\alpha - \alpha z^{\frac{1}{\alpha}} z^{\frac{1}{1-\alpha}}}.$$

Since the right-hand side is larger than 1, the condition that high-TFP firms borrow is always fulfilled if  $w_t^L = w_t^H$ . Further, we know from section 3.3.1 that credit constraints bind in this case.<sup>6</sup>

## Location of the threshold curves in $(b_t, x_t)$ space

**Proposition 3.2.** *When  $w_t^L > w_t^H$ , high-TFP firms borrow and credit constraints bind.*

*Proof.* Simple algebra proves that the  $T_{LC}$  curve and the  $T_{HB}$  curve are located above the  $T_{HC}$  curve. It remains to show that the  $T_{HC}$  curve lies above the  $T_{WL}$  curve for all  $b_t \in (0, 1)$ . Since the  $T_{WL}$  curve is convex and the  $T_{HC}$  curve is concave, and since both curves are increasing and start at  $b_t = 0$  and  $x_t = 0$ , it suffices to show that the  $T_{HC}$  threshold exceeds the  $T_{WL}$  threshold for  $b_t = 1$ :

$$\begin{aligned} \frac{\alpha - \lambda}{\alpha} \frac{1 - \gamma}{z^{\frac{1}{1-\alpha}} + (1-\gamma) \left( 1 - z^{\frac{1}{1-\alpha}} \right)} &\geq \frac{\left( \alpha z^{\frac{1}{\alpha}} - \lambda \right) (1 - \gamma)}{\alpha \left[ 1 - \left( 1 - z^{\frac{1}{\alpha}} \right) (1 - \gamma) \right]} \\ \Leftrightarrow \gamma \left[ \alpha - \lambda - z^{\frac{1}{1-\alpha}} \left( \alpha z^{\frac{1}{\alpha}} - \lambda \right) \right] + \lambda (1 - \gamma) \left( 1 - z^{\frac{1}{\alpha}} \right) &\geq 0 \end{aligned}$$

The condition is satisfied for  $0 < \gamma < 1$  and  $\lambda \in [0, \alpha z^{\frac{1}{\alpha}})$ . □

It remains to show that  $T_{RH}(b_t) > T_B(b_t) > T_{RL}(b_t) > T_{WH}(b_t) > T_L(b_t) > T_{WL}(b_t)$  for all  $b_t \in (0, 1)$ . Figure 3.11 illustrates the threshold functions and includes their corresponding values at  $b_t = 0$  and  $b_t = 1$ . Since the  $T_{RL}(b_t)$  curve is increasing and concave, and the  $T_{WH}(b_t)$  curve is increasing and convex, it suffices to compare both threshold functions at  $b_t = 0$  and

<sup>6</sup>It was shown in section 3.3.1 that credit constraints bind if both firms produce,  $R_t = MPK_t^L$ , and  $w_t^L = w_t^H$ .

$b_t = 1$ . It can be shown that  $T_{RL}(0) > T_{WH}(0)$  if

$$\gamma < \frac{\alpha - \lambda - (\alpha z^{\frac{1}{\alpha}} - \lambda) z^{\frac{1}{1-\alpha}}}{\alpha - \lambda - (\alpha z^{\frac{1}{\alpha}} - \lambda) z^{\frac{1}{1-\alpha}} - \lambda(1 - z^{\frac{1}{\alpha}})}.$$

Since the right-hand side is larger than 1,  $T_{RL}(0) > T_{WH}(0)$  is always satisfied for  $0 < \gamma < 1$ . Further, it can be shown that  $T_{RL}(1) > T_{WH}(1)$ . The relative location of the other threshold functions is obtained by simple algebra.

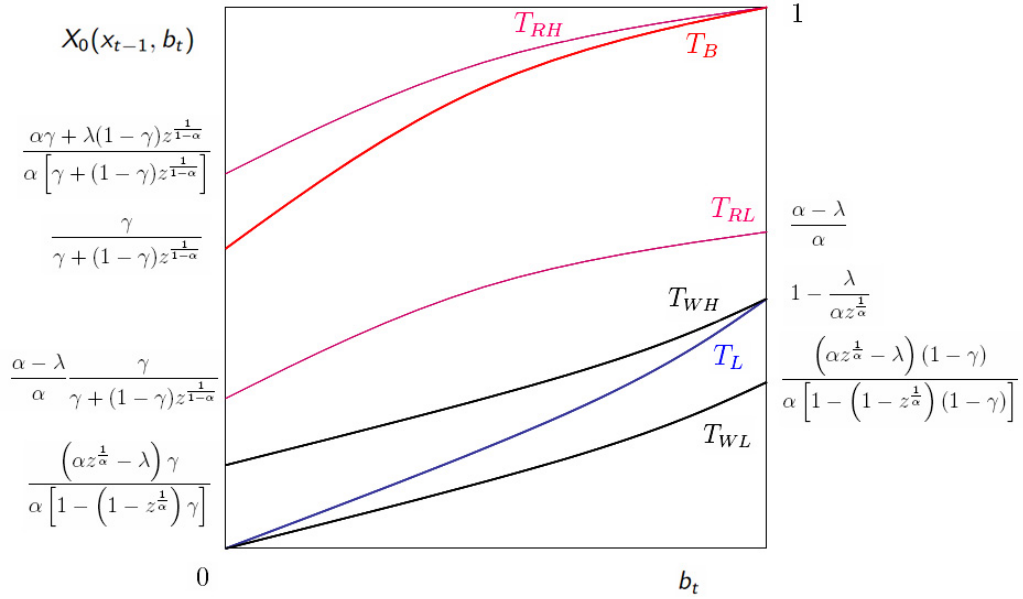


Figure 3.11: Threshold functions for  $0 < \gamma < 1$

## Proof of Proposition 3.1

Step 1. Development of  $x$

$$s_t = s_{t-1}:$$

1. If the high-TFP firm borrows, the transitional dynamics of the wealth share of high-TFP firms is described by

$$x_t = X_0(x_{t-1}, b_t) = \frac{\tilde{R}_{t-1}x_{t-1}}{\tilde{R}_{t-1}x_{t-1} + R_{t-1}(1 - x_{t-1})}.$$

The wealth share of the high-TFP firm increases if

$$X_0(x_{t-1}, b_t) > x_{t-1},$$

which is equivalent to  $\tilde{R}_{t-1} > R_{t-1}$ . This condition holds if the high-TFP firm is credit constrained, i.e. if  $(b_t, x_t)$  is located below the  $T_{RL}$  curve.

2. If the low-TFP firm borrows, the transitional dynamics of the wealth share of high-TFP firms is described by

$$x_t = X_0(x_{t-1}, b_t) = \frac{R_{t-1}x_{t-1}}{R_{t-1}x_{t-1} + \tilde{R}_{t-1}(1 - x_{t-1})}.$$

The wealth share of the high-TFP firm decreases if

$$X_0(x_{t-1}, b_t) < x_{t-1},$$

which is equivalent to  $\tilde{R}_{t-1} > R_{t-1}$ . This condition holds if the low-TFP firm is credit constrained, i.e. if  $(b_t, x_t)$  is located above the  $T_{RH}$  curve.

3. It follows that the wealth distribution does not change if  $(b_t, x_t)$  is located in the area between the  $T_{RL}$  and the  $T_{RH}$  curve.

If  $s_t \neq s_{t-1}$ , the wealth share of high-TFP firms at the end of period  $t - 1$  is given by  $1 - X_0(x_{t-1}, b_t)$ .

Step 2. The functions  $T_{RL}(b_t)$  and  $T_{RH}(b_t)$  are monotonously increasing in  $b_t$  and have the same derivative:

$$T'_{RL}(b_t) = T'_{RH}(b_t) = \frac{\alpha - \lambda}{\alpha} \frac{(1 - \gamma)z^{\frac{1}{1-\alpha}}}{\left(\gamma + (1 - \gamma)\left(b_t + (1 - b_t)z^{\frac{1}{1-\alpha}}\right)\right)^2} > 0.$$

It follows that  $(b_t, x_t)$  stays in the area between the  $T_{RL}$  and the  $T_{RH}$  curve if the following conditions are true:

Condition 1:  $T_{RL}(1) \leq 0.5$

Condition 2:  $T_{RH}(0) \geq 0.5$

Using  $T_{RL}(1) = \frac{\alpha - \lambda}{\alpha}$ , Condition 1 is satisfied if

$$\lambda \geq \frac{\alpha}{2}.$$

Using  $T_{RH}(0) = \frac{\alpha\gamma + \lambda(1-\gamma)z^{\frac{1}{1-\alpha}}}{\alpha[\gamma + (1-\gamma)z^{\frac{1}{1-\alpha}}]}$ , *Condition 2* is satisfied if

$$\alpha\gamma + (2\lambda - \alpha)(1 - \gamma)z^{\frac{1}{1-\alpha}} \geq 0.$$

It follows that *Condition 1* also ensures that *Condition 2* is satisfied.

Hence,  $\lambda \geq \frac{\alpha}{2}$  is a sufficient condition that  $(b_t, x_t)$  stays in the area between the  $T_{RL}$  and the  $T_{RH}$  curve in the long run.  $\square$

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