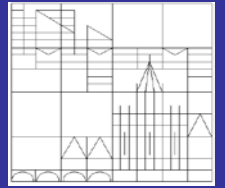




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Evidence from Asymmetric Impulse Responses***

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The Stock Return - Trading Volume Relationship in European Countries: Evidence from Asymmetric Impulse Responses^{*}

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Abstract

We investigate non-linearities in the stock return - trading volume relationship by using daily data for 16 European countries in an asymmetric vector autoregressive model. In this framework, we test for asymmetries and analyze the dynamic relationship using a simulation based procedure for computing asymmetric impulse response functions. We find that stock returns have a significant influence on trading volume, but there is no evidence for the influence of trading volume on returns. Our analysis indicates that responses of trading volume to return shocks are non-linear and the sign of the response depends on the absolute size of the shock. Thus, using linear VAR models may lead to wrong conclusions concerning the return - volume relationship. We also find that after stock markets go up (down), investors trade significantly more (less) in small and mid cap stocks, supporting evidence for the theories of overconfidence, market participation, differences of opinion, and disposition effect.

JEL Classification: G12, G14, G17, C32

Keywords: asymmetric vector autoregression, asymmetric impulse response functions, stock return, trading volume

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1 Introduction

The relationship between stock returns and trading volume reveals important insights into the workings of financial markets and makes it possible to test and discriminate between a number of competing theories for market participants' behaviour (see e.g. Moosa and Silvapulle (2000) or Griffin et al. (2007)). Yet, the nature of the return-volume relationship is still heavily debated, as the research results have not been unequivocal regarding issues as whether the relationship is uni- or bidirectional, contemporaneous or dynamic, whether there are some asymmetric effects, and whether the responses to shocks depend on the size of the shock. The last two issues are in the focus of this paper, as their presence implies non-linearity and requires non-standard modelling methods.

Many studies dealing with the dynamic interactions between returns and volume apply linear vector autoregressive (VAR) models (see e.g. Lee and Rui (2002) or Statman et al. (2006)). This is quite surprising, as non-linearities in the form of asymmetries are omnipresent in financial markets. Stock returns in general, and the returns of some investment strategies in particular display skewness (see e.g. Daniel and Moskowitz (2013)), correlations are stronger in downward moving markets (see e.g. Daigler and You (2010)), and investors react differently to positive and negative returns in bull and bear markets (see e.g. Kim and Nofsinger (2007), Chen (2012)).

Many researchers also point to asymmetric effects in the return-volume relationship (see e.g. Karpoff (1987) or Chordia et al. (2007)). It is important to note here that there is no single definition of asymmetry in this context in the literature. Some researchers understand asymmetry as the situation when the reaction of one variable is different in absolute terms, depending on whether the shock to another variable is negative or positive. Thus, they concentrate on the strength of the reaction to the shock, depending on the shock direction (see e.g. Moosa et al. (2003)). Other researchers pay more attention to the nature of the relation between the variables. They interpret a situation when trading volume is correlated positively with positive price changes and negatively with negative price changes as asymmetry (see e.g. Karpoff (1987) or Chen (2012)), whereas the strength of the reaction to a positive or a negative shock is not in the centre of their analysis.

We see asymmetry in the return-volume relationship in opposition to the conclusions that can be drawn from a standard VAR model. A linear VAR per construction cannot capture any non-linearities, so if there are asymmetric effects in the data, such a model will lead to biased results. To avoid this potential bias in linear VAR models, we use an asymmetric VAR that allows negative and positive shocks to have fundamentally different impact on the analyzed variables. For this purpose we adapt the approach of Kilian and Vigfusson (2011) who show in a macroeconomic application that ignoring non-linearities, resulting from asymmetries, causes inconsistency of the model coefficient estimates and subsequently of estimated impulse response functions (IRFs). The asymmetric VAR provides consistent estimates both in the case when the asymmetric effects are present and when the data generating process is symmetric. Thus, when we obtain different results with the asymmetric VAR than one could get with a linear VAR model, we conclude that the asymmetric effects in our model are necessary and the relationship between trading volume and stock returns can

be described as asymmetric. In contrast to the models commonly used to tackle the problem of non-linearity, the asymmetric VAR does not require sophisticated estimation techniques and can be estimated by ordinary least squares (OLS) applied equation by equation.

In this paper, we focus on impulse response functions from asymmetric VARs to analyze the return-trading volume relationship. Impulse responses for this model are computed using a simulation-based method in the spirit of Koop et al. (1996), as the standard impulse responses based on the moving average representation provide biased results in the presence of non-linearities. In order to assess the statistical significance of the reaction to a shock, we use a wild bootstrap approach to obtain confidence intervals for the response functions. To the best of our knowledge, we are the first to fully examine the non-linearities in the relationship of stock returns and trading volume with the help of an asymmetric impulse response analysis.¹ By analyzing the impulse responses, we are able to shed light on the question whether non-linearities in the return-volume relationship exist, and whether more sophisticated models than a linear VAR are needed to accurately model this relationship.

In order to test for asymmetric effects in the return-volume relationship, we run two types of Wald tests. First, we conduct Wald tests for the joint significance of model coefficients (slope-based test), which is a standard instrument in the literature. Second, we also use a novel Wald test, based on impulse-response functions, to check for asymmetries, following the approach of Kilian and Vigfusson (2011). These authors argue that slope-based tests are useful for single equation models, but they become uninformative for a dynamic, multi-equation system, as the coefficients themselves contain little information about the dynamics of the impulse response functions and their potential asymmetric behaviour. Thus, compared to the standard approach in the literature, the impulse-response based asymmetry test may provide additional insights into our analysis.

Our empirical analysis provides a number of interesting results. For stock markets of 16 selected European countries the descriptive analysis and statistical inference based on daily data yield strong evidence for non-linearities in the return-volume relationship. We find that the size of the shock in returns has a crucial effect on the direction and magnitude of the response in trading volume. We find that trading volume increases for medium and large (± 1 and ± 2 standard deviations) absolute return shocks, whereas it decreases for small (± 0.25 standard deviation) absolute shocks. For most countries we also find that shocks are transitory and their effects vanish after a few trading days. Finally, we provide some evidence for the overconfidence, market participation, differences of opinion, and disposition effect theories, as volume in small and mid cap stocks significantly increases (decreases) after stock markets go up (down), and this effect persists for at least 20 trading days. In contrast, the effect for large cap stocks is not significant at a number of medium response horizons and in the long run. All results are robust with respect to a number of variations in the empirical model specification.

Our paper is related to a number of other studies dealing with non-linearities in the return-trading volume context. According to Epps (1975), traders react more strongly to

¹Griffin et al. (2007) used non-linear impulse responses for a threshold VAR in their robustness checks. However, they only present a tiny portion of the results for this model, as they find the return-volume relationship quite symmetric, and the focus of their paper is on a linear VAR.

positive than to negative returns. Wang (1994) finds a positive correlation between volume and absolute price changes. Hiemstra and Jones (1994) address the problem of non-linearities by applying non-linear Granger causality tests. They find strong evidence of bidirectional non-linear causality relationship between daily stock returns and NYSE trading volume. Moosa et al. (2003) consider oil futures markets and show that linear models can only detect unidirectional causality (from returns to trading volume), whereas non-linear models detect bidirectional causality. They also find evidence for asymmetry using a threshold vector autoregressive model - negative price and volume changes have more influence on each other than positive changes. Gerlach et al. (2006) find strong evidence that returns and volatility are non-linear functions of trading volume.

Gebka and Wohar (2013) use quantile regression to show that both low and high returns imply more trading volume. In a recent study, Chen (2012) applies a regime-switching model and finds strong evidence for contemporaneous asymmetric effects in the return - trading volume relationship, which depend on the state of the world (bull vs. bear market regime). The author also shows that when linear models are used, results depend heavily on the sample period and unequivocal conclusions cannot be drawn based on such analysis.

Overall, there is some evidence in the literature that the relationship between stock returns and trading volume is non-linear and asymmetric and thus, linear models might provide misleading results. Compared to the existing literature, our analysis is based on a flexible econometric framework, tailored to give more detailed insights into the nature of the return-trading volume relationship.

The remainder of the paper is organized as follows. Section 2 introduces the asymmetric VAR model. Section 3 contains the description of the data, while Section 4 provides empirical results on asymmetry and the impulse response analysis. Section 5 presents a number of robustness checks before Section 6 concludes.

2 The Asymmetric Vector Autoregressive Model

We introduce an asymmetric vector autoregressive (asymmetric VAR) model that allows for asymmetric effects of both trading volume on returns and of returns on trading volume. For this purpose, we define a vector y_t , which includes the stock returns r_t and the growth rate of the trading volume tv_t , i.e. we set $y_t = (r_t, tv_t)'$. In what follows, a generalization of a model used by Kilian and Vigfusson (2011) is employed. To be more precise, we use structural models of the form

$$A_0 y_t = c + \sum_{i=1}^p A_i y_{t-i} + \sum_{i=0}^p B_i y_{t-i}^+ + \varepsilon_t, \quad (2.1)$$

where y_t is a K -dimensional vector of endogenous variables as defined above, c is a fixed $K \times 1$ vector of intercepts, A_i , B_i are fixed $K \times K$ coefficient matrices and $y_t^+ = (r_t^+, tv_t^+)'$

is defined to capture possible asymmetries. In particular, we define

$$r_t^+ = \begin{cases} r_t & \text{if } r_t > 0 \\ 0 & \text{else} \end{cases} \quad \text{and} \quad \text{tv}_t^+ = \begin{cases} \text{tv}_t & \text{if } \text{tv}_t > 0 \\ 0 & \text{else} \end{cases}.$$

To identify the structural shocks and to avoid problems related to endogeneity in the model, we use a recursive system, i.e. we use a lower triangular matrix A_0 :

$$A_0 = \begin{bmatrix} 1 & 0 \\ -a_{21,0} & 1 \end{bmatrix}.$$

Furthermore, we impose

$$B_0 = \begin{bmatrix} 0 & 0 \\ b_{21,0} & 0 \end{bmatrix} \quad \text{and} \quad B_i = \begin{bmatrix} 0 & b_{12,i} \\ b_{21,i} & 0 \end{bmatrix}, \quad i = 1, \dots, p.$$

This recursive structure ensures that $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ is a vector of contemporaneously uncorrelated structural shocks with zero mean and non-singular diagonal covariance matrix Σ_ε . In addition, we assume that ε_t is serially uncorrelated. This structure implies that shocks to stock returns $\varepsilon_{1,t}$ may have an immediate - within one day - effect on the trading volume, whereas the converse is not true. This assumption is justified by the efficient market rationale - the revelation of information on financial market variables should not contain any information on future stock returns. Moreover, given that the contemporaneous effects of returns are included in the second equation of (2.1), the error terms $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are uncorrelated.² Thus, given the structure on A_0 and B_0 , the parameters of the equations in (2.1) can be estimated consistently and efficiently by ordinary least squares (OLS) applied equation by equation (see Kilian and Vigfusson (2011) for a detailed discussion on asymmetric VARs). Moreover, note that the structure on the $B_i, i = 1, \dots, p$ implies that there may be asymmetric spillovers from returns to trading volume and vice versa. In addition, we assume an immediate asymmetric effect from returns to trading volume.

In order to test whether the asymmetric effects are significant and thus necessary in the model, we implement a suitable testing procedure. A standard choice in the existing literature that examines the interactions between returns and trading volume is the Wald test based on slope coefficients. Following this approach, we test in the system (2.1) the null hypothesis $b_{12,i} = 0$ for $i = 1, \dots, p$ and $b_{21,i} = 0$ for $i = 0, \dots, p$, respectively. In other words, we test whether the relevant slope coefficients are jointly significant (slope-based Wald test). Under the null hypothesis the asymmetric effects are jointly insignificant and the model reduces to a linear bivariate VAR for returns and trading volume.

In addition to the commonly used slope-based test, we apply an alternative test for asymmetry, based on impulse response functions obtained from the VAR system in (2.1). Kilian and Vigfusson (2011) argue that while slope-based tests are useful for single equation models, they become uninformative for a dynamic, multi-equation system, as the coefficients

²This can be easily seen by noting that for $B_i = 0, i = 1, \dots, p$, the system reduces to a standard recursive VAR.

themselves contain little information about the dynamics of the impulse response functions and their potential asymmetric behaviour. Thus, it is possible that the responses to a shock are symmetric, even if the slope-based test hints to asymmetries. Moreover, although slope-based tests can distinguish between effects from positive and negative shocks, they do not discriminate between shocks of different size. This is a major drawback as there is some evidence that the response of trading volume depends on the size of the shock in returns (see Griffin et al. (2007)). The impulse-response based Wald test is thoroughly described later at the end of this section after the computation of asymmetric IRFs has been explained.

Due to the presence of the asymmetric terms r_{t-i}^+ and tv_{t-i}^+ , the standard approach of estimating impulse responses based on the moving average representation³ leads to inconsistent parameter estimates. This point is convincingly illustrated by Kilian and Vigfusson (2011) and is especially relevant for the case of small and moderate shocks, which are most common on the stock market, as the bias of the impulse response estimates obtained by standard methods is then especially high. In contrast, a simulation based approach for the equations in (2.1) provides consistent estimates for impulse responses, even if asymmetries are present. Thus, we follow the approach in Kilian and Vigfusson (2011) to obtain asymmetric impulse response function (IRF) estimates:

1. First we obtain parameter estimates for (2.1) using OLS equation by equation on the whole sample of observations. These estimates will be used for the next steps of the algorithm.
2. We take non-overlapping blocks of p consecutive values of r_t and tv_t , starting from period 1 until $T - p$. Each of these blocks is called a history. Each history s , with $s = 1, \dots, [T/p]$, is the starting point for calculating conditional impulse response functions.
3. Given the history s , we simulate two time paths for r_{t+h} and tv_{t+h} for $h = 0, 1, \dots, H$, where h is the time horizon for the impulse response function. We set $H = 20$ in our analysis. When generating the first time path, for $h = 0$ we set $\varepsilon_{1,0}$ to a prespecified value δ , where δ is the size of the shock for which the impulse response function is calculated. The realizations of $\varepsilon_{1,t+h}$ for $h = 1, \dots, H$ are drawn as random blocks of length H from the empirical distribution of the estimated residual $\hat{\varepsilon}_1$. The realizations of $\varepsilon_{2,t+h}$ for $h = 0, \dots, H$ are drawn as random blocks from the empirical distribution of the estimated residual $\hat{\varepsilon}_2$. When generating the second time path, all $\varepsilon_{1,t+h}$ and $\varepsilon_{2,t+h}$ for $h = 0, \dots, H$ are drawn as random blocks from their respective empirical distributions.
4. We calculate the difference $tv_{t+h}(\delta, s) - tv_{t+h}(s)$ between the two time paths for $h = 0, \dots, H$, obtained for the shock scenario in step 3.
5. We make $m = 500$ repetitions of steps 3 and 4 and we average the difference obtained in step 4 across the m repetitions to obtain the impulse response function of tv_{t+h} at horizon $h = 0, 1, \dots, H$ to a shock of size δ , conditional on history s .

³See e.g. Breitung et al. (2004).

6. The unconditional response function $I_{tv}(h, \delta)$ is the mean of the conditional responses from step 5 across all histories s .

To account for estimation uncertainty around the estimated IRFs, we report bootstrap confidence intervals. Given the difficulties in the analytical derivation of the intervals and the strong GARCH effects in the residuals of our estimated models, we propose a wild bootstrap procedure in the spirit of Goncalves and Kilian (2004) for calculating the confidence intervals around the IRFs $I_{tv}(h, \delta)$:

1. Based on the estimated residuals $\hat{\varepsilon}_{1,t}$ and $\hat{\varepsilon}_{2,t}$ of (2.1), we simulate new residuals $\tilde{\varepsilon}_{1,t} = \eta_{1,t} \cdot \hat{\varepsilon}_{1,t}$ and $\tilde{\varepsilon}_{2,t} = \eta_{2,t} \cdot \hat{\varepsilon}_{2,t}$ with $\eta_{i,t} \stackrel{i.i.d.}{\sim} N(0, 1)$, $i = 1, 2$. This step is done for $j = 500$ simulations.
2. With the simulated residuals we generate $j = 500$ new paths of \tilde{r}_t^j and \tilde{tv}_t^j .
3. For each pair of simulated time series \tilde{r}_t^j and \tilde{tv}_t^j we follow steps 1 to 6 from the IRF calculation algorithm described above.

We end up with $j = 500$ paths of the unconditional $I_{tv}^j(h, \delta)$, of which we take the 2.5%- and 97.5%-percentile as a lower and upper value of the confidence interval.

Having obtained the responses of trading volume to return shocks we apply the impulse response based Wald test for asymmetry.⁴ Under the null hypothesis of symmetry positive and negative shocks of the same size are the exact opposite to each other. In other words we test $H_0 : I_{tv}(h, \delta) = -I_{tv}(h, -\delta)$ for all $h = 0, 1, \dots, H$. If the null hypothesis can be rejected, the impulse responses to a negative and positive shock of the same absolute size do not create a mirror-image effect, and thus are very different from the pattern that is imposed by the linear VAR model. Such a result would therefore be in favour of the asymmetric VAR.

In order to conduct the test, we first calculate the unconditional IRFs of trading volume to stock returns for both positive and negative shocks of size δ , as described above. Then we jointly test for the symmetry of impulse responses for all h , where $h = 0, 1, \dots, H$. The $H \times H$ variance-covariance matrix of $I_{tv}(h, \delta) + I_{tv}(h, -\delta)$, necessary for the calculation of the Wald test statistic, is estimated by the same bootstrap procedure that is used for the calculation of the confidence intervals. Given the asymptotic normality of the parameter estimators in (2.1) the test statistic has an asymptotic χ^2 distribution with $H + 1$ degrees of freedom.

3 Data

We use country-specific value-weighted indices, as well as indices constructed by aggregating data for all analyzed countries (aggregated data). We use all available stocks for 16 selected European countries (Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United

⁴This test is taken from Kilian and Vigfusson (2011), p. 437.

Kingdom) and obtain their respective daily stock prices and trading volume.⁵ The data covers the period between January 1990 and July 2012 (5891 observations⁶) and is obtained from Thomson Reuters Datastream. We choose the countries according to data availability and their importance in terms of market capitalization. We focus only on daily data from European financial markets, as this helps us to avoid the problem of nonsynchronous trading, which would complicate the analysis if data from Asian or U.S. markets were also taken into account (see e.g. Glaser and Schaarschmidt (2012)). Summary statistics for all considered countries are shown in Table A.1 in the Appendix.

We include delisted stocks until they disappear, in order to prevent a possible survivorship bias as indicated by Brown et al. (1992). In addition, we apply a data screening procedure similar to Ince and Porter (2006). We sort out firms with market capitalization smaller than 0.5 million EUR, absolute daily returns higher than 50% and stock prices smaller than 1 EUR.

We calculate stock returns by taking first differences of the log prices: $r_t = \ln(\text{price}_t) - \ln(\text{price}_{t-1})$, which is a standard approach in the literature. When it comes to trading volume, different ways are proposed in the literature to handle its non-stationarity. Some studies take first differences of the log trading volume (e.g. Chen (2012)), whereas others use moving averages to detrend the time series (e.g. Griffin et al. (2007)). There are also some authors who apply alternative detrending methods (see Statman et al. (2006)), such as Hodrick and Prescott (1997) filtering. As there is no consensus about the appropriate transformation, we decide to use first differences of the log trading volume, i.e. we consider $\text{tv}_t = \ln(\text{volume}_t) - \ln(\text{volume}_{t-1})$ in our analysis, which is in line with the results of the unit root tests.⁷ We check the sensitivity of our results with respect to different detrending methods in Section 5.

Figure 1 provides typical plots of log returns, log trading volume, and first differences of log trading volume from January 1990 to July 2012 for aggregated data.

4 Empirical Results

In this section we report results of the empirical analysis of the return-volume relationship. Using impulse-response analysis and different types of Wald tests, we investigate possible asymmetries in the return-volume relationship, and whether those asymmetries depend on the absolute size of the shock in returns. We also find a link between our empirical results and behavioural finance explanations such as the theories of overconfidence, market participation,

⁵“All available stocks” means all major securities, which are equities. We set the option “Primary Quote” to “Yes”. All other features are set to “Default”. For prices we use the adjusted price (datatype “P”). For all countries except for Germany we download daily trading volume (padded, datatype “VP”), which is common trading volume (datatype “VO”), replaced by the value from the previous day, if the stock is not traded for some reason at this particular day. In order to prevent a possible illiquidity bias, we conduct a thorough stock price and stock volume screening (as described in the text).

⁶For Ireland and Spain the sample is shorter and has 3173 and 5848 observations respectively.

⁷We conduct an Augmented Dickey-Fuller and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests (see Dickey and Fuller (1979) and Kwiatkowski et al. (1992)). In unreported results both tests show for all countries and for the aggregated data that the first differences of log stock prices and log trading volume are stationary, whereas raw series or log trading volume are not.

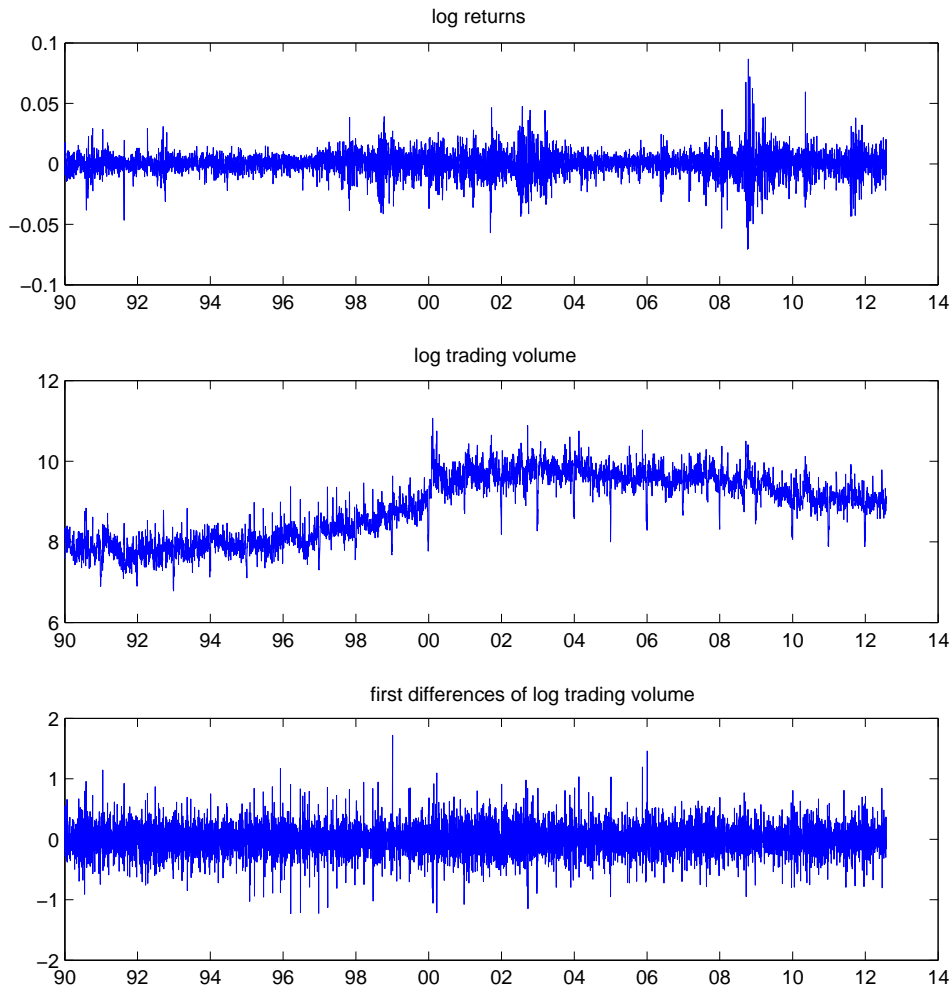


Figure 1: Plots of stock returns and trading volume (log level and growth rate) for aggregated data

disposition effect, and differences of opinion. We start with some descriptive evidence for asymmetries in Section 4.1 and provide inference on impulse responses in Section 4.2. Then we analyze to what extent volume reaction depends on the size of a return shock in Section 4.3, followed by some explanations for our results in Section 4.4.

4.1 Slope-based Tests and Descriptive Evidence for Asymmetry

We first investigate the presence of asymmetric effects in the return equation via a Wald slope-based test. For each country and for the aggregated data we estimate the asymmetric VAR from equation (2.1), choosing the lag length p by the Schwarz information criterion (see Schwarz (1978)). Then we test the null hypothesis $H_0 : b_{12,i} = 0$ for all $i = 1, \dots, p$, using autocorrelation and heteroscedasticity consistent covariance estimator and robust standard errors of Newey and West (1987). Table 1 summarizes the results. Instead of reporting all estimated coefficients of the models, we provide a concise summary of the sign and significance of the estimated asymmetry coefficients. The first line of each country entry refers to the coefficients in B_i of the first VAR equation, i.e. $b_{12,i}$. Clearly, for all countries and for the aggregated data we observe that almost none of the $b_{12,i}$ estimates is significantly

Table 1: Slope-based asymmetry test

Country	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	p -values
Austria	0 0 + 0	0 · + 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · · 0	0 · - 0		0.9766 < 0.001
Belgium	0 0 + 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · · 0	0 · · 0		0.8831 < 0.001
Denmark	0 0 + 0	0 · · 0	0 · - 0	0 · · 0	0 · - 0	0 · - 0	0 · · 0	0 · - 0	0 · - 0	0 · · 0		0.4147 < 0.001
Finland	0 0 + 0	0 · · 0	0 · - 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0		0.3431 < 0.001
France	0 0 + 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0						0.2813 < 0.001
Germany	0 0 + 0	0 · + 0	0 · · 0	0 - - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · · 0		0.4929 < 0.001
Greece	0 0 + 0	0 - + 0	0 · - 0	0 - - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0				0.0599 < 0.001
Ireland	0 0 + 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0							0.7408 < 0.001
Italy	0 0 + 0	0 · + 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · · 0	0 · · 0	0 · - 0	0.9344 < 0.001
Netherlands	0 0 + 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · · 0	0 · · 0	0 · · 0		0.2018 < 0.001
Norway	0 0 + 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0							0.4446 < 0.001
Portugal	0 0 + 0	0 · + 0	0 · - 0	0 · - 0	0 · · 0	0 · - 0	0 · - 0					0.8146 < 0.001
Spain	0 0 + 0	0 · + 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0	0 · - 0		0.2138 < 0.001
Sweden	0 0 + 0	0 · + 0	0 · · 0	0 · - 0	0 · - 0	0 · - 0	0 · · 0	0 · · 0	0 · - 0	0 · - 0	0 - - 0	0.0975 < 0.001
Switzerland	0 0 + 0	0 · + 0	0 · - 0	0 · - 0	0 · · 0	0 · - 0	0 · - 0	0 · · 0	0 · · 0	0 · · 0	0 + · 0	0.1808 < 0.001
UK	0 0 + 0	0 · · 0	0 · · 0	0 · - 0	0 · - 0	0 - - 0	0 · - 0	0 · · 0	0 · · 0	0 · - 0		0.1675 < 0.001
Aggregated data	0 0 + 0	0 · · 0	0 · · 0	0 · - 0	0 · · 0	0 · - 0	0 · · 0	0 · · 0	0 · + 0	0 + · 0	0 · · 0	0.5240 < 0.001

Note: The table reports sign and significance of estimated parameters in asymmetric VAR of the form (2.1). We report results for asymmetry parameters in B_i only. 0 denotes a restricted coefficient, + positive significant, - negative significant, · not significant (at 5% significance level). For each country, the first row denotes the coefficients of the return equation, the second row the coefficients of the trading volume equation. The last column reports p -values of the slope-based Wald tests for asymmetry in equation (2.1). For each country, the first row entry gives the p -value for the test of $H_0 : b_{12,i} = 0$ for $i = 1, \dots, p$, while the second row gives the p -value for the test of $H_0 : b_{21,i} = 0$ for $i = 0, \dots, p$. All statistics are based on HAC covariance estimators of Newey and West (1987). Sample range from January 1990 to July 2012.

different from zero (as indicated by the ‘.’). According to the p -values for the Wald statistic of joint significance reported in the last column, we cannot reject the null hypothesis on the 5% significance level for any country and the aggregated data. Greece and Sweden are the only countries for which we reject the null hypothesis at the 10% significance level (p -values of 0.0599 and 0.0975 respectively). We also report estimates for a typical country (France) in Table A.2 in the Appendix.⁸ As most coefficients relating trading volume to returns in the first equation are insignificant, we do not find evidence that trading volume has any impact on returns. This confirms the results of many previous papers and is consistent with the efficient market hypothesis, according to which financial market variables - such as trading volume - cannot help to forecast stock returns. Basing on the results of the Wald test and on previous research, we decide not to include asymmetric effects in the first (return) equation for further analysis.

Looking at the estimation results for the second equation in (2.1) (see results for France in Table A.2) we infer that both past trading volume and past, as well as contemporaneous returns, have an effect on trading volume, as the respective coefficients are highly significant. Positive stock returns imply a positive instantaneous reaction of trading volume for all countries and for the aggregated data (as the sums of coefficients $a_{21,0}$ and $b_{21,0}$ are positive). On the other hand, negative return shocks also imply a positive instantaneous reaction of trading volume for all countries and for the aggregated data (as coefficients $a_{21,0}$ are negative). We can also observe a similar pattern for the first lags of returns.

For the trading volume equation in (2.1) we investigate the presence of asymmetric effects, i.e. we test whether positive return shocks affect trading volume in a different way than negative return shocks. The asymmetry is indicated by the significance of $b_{21,i}$ ($i = 0, 1, \dots, p$) coefficients. We conduct a Wald test of joint significance of all $b_{21,i}$ and find that the $H_0 : b_{21,i} = 0$ for $i = 0, 1, \dots, p$ is rejected at the 1% significance level for all countries and for the aggregated data. Hence, the slope-based test indicates strong asymmetric effects in the direction from stock returns to trading volume.

To sum up, slope-based tests provide evidence for trading volume being affected in a different way by stock returns, conditional on the sign of the shock. Thus, allowing for asymmetric effects in the second equation of (2.1) (the trading volume equation) is necessary. In compliance with the efficient market rationale we find no evidence for trading volume affecting stock returns or for any asymmetries in this relationship. Based on these results, we modify the model from Section 2 by imposing that $b_{12,i} = 0$ for all $i = 1, 2, \dots, p$. In the following, we therefore proceed with a VAR as in (2.1) but using

$$A_0 = \begin{bmatrix} 1 & 0 \\ -a_{21,0} & 1 \end{bmatrix} \quad \text{and} \quad B_i = \begin{bmatrix} 0 & 0 \\ b_{21,i} & 0 \end{bmatrix}, \quad i = 1, \dots, p \quad (4.1)$$

While slope-based tests are useful to assess asymmetries in single equation models, they are not as informative as impulse-response based tests in systems of equations. Even when the relevant coefficients are significant and indicate asymmetries, the impulse response functions may still be symmetric, so the extension of the standard VAR model into a non-linear

⁸Detailed estimation results for other countries are available on request.

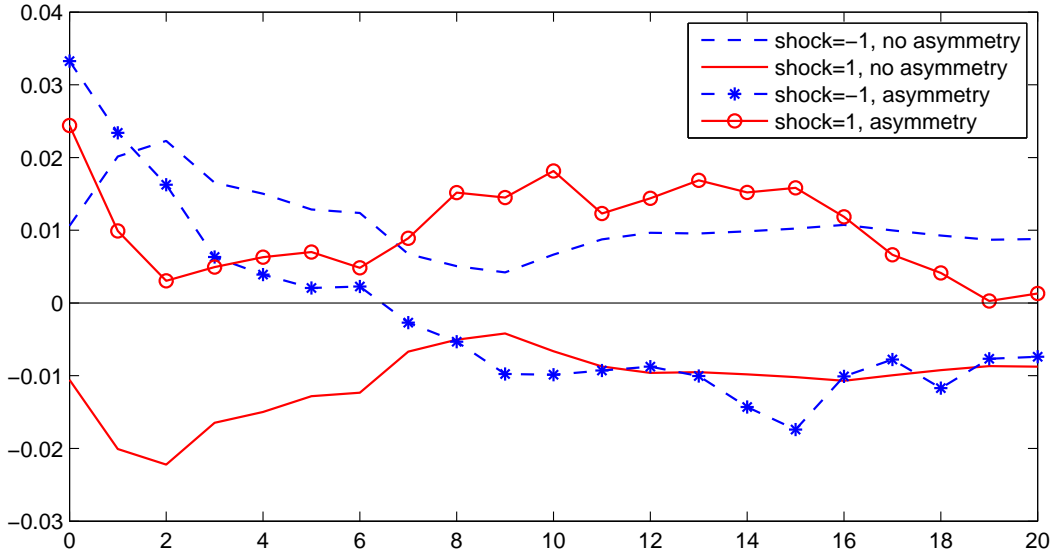


Figure 2: Response of trading volume to ± 1 standard deviation shock to stock returns over the period of 20 trading days, obtained from linear and asymmetric VAR models. Results for aggregated data and sample range January 1990 - July 2012.

one may not be justified. In order to check whether there are differences between the results obtained from the symmetric and asymmetric models, we run a linear VAR on returns and volume and compare it to the asymmetric VAR. To ensure comparability, we allow for contemporaneous effects of stock returns on trading volume in the linear VAR and compute simulation-based impulse responses for both models. Thus, the linear VAR we estimate corresponds to the model defined by equation (2.1) without the asymmetric components, i.e. by imposing $B_i = 0$ for all i . We compare impulse responses from the asymmetric VAR with those obtained for the linear VAR in order to evaluate the relevance of asymmetries. Figure 2 shows accumulated responses of trading volume growth to shocks in returns of size ± 1 standard deviation.⁹ The responses for the individual countries look similar, so we refrain from showing them here.

For both models trading volume responds immediately to the shock in returns (at horizon $h = 0$). This is due to the fact that we implicitly allow for instantaneous reactions of volume to return shocks. However, the shape of responses differs substantially for both models. In the linear VAR setting, a $+1$ standard deviation return shock implies a response of about -0.01% of trading volume. Due to symmetry, a -1 standard deviation shock implies an exact opposite reaction - trading volume increases by 0.01% . Looking at the asymmetric VAR model, we find completely different results. A $+1$ standard deviation return shock is followed by an immediate increase of trading volume of more than 0.02% , while a negative shock of the same size causes an even higher immediate reaction of volume. For larger h , the response function of trading volume for both negative and positive shock moves slowly towards zero. However, after about 5 trading days volume increases again for a positive shock and decreases for a negative one. After about 20 trading days, volume reverts back to its

⁹A return shock of 1 standard deviation corresponds to an increase in returns of about 1% for the aggregated data, and from 0.81% to 1.86% for individual countries.

initial pre-shock level, indicated by the response functions approaching zero. These results point to asymmetry in the return-volume relationship. Thus, our findings contradict the results of Griffin et al. (2007), who use a linear VAR in their main analysis after concluding that negative and positive return shocks have a fairly symmetric impact on turnover.

4.2 Inference on Impulse Responses

To account for estimation uncertainty we have computed approximate 95% wild bootstrap confidence intervals (as described in Section 2) for the asymmetric IRFs. Figure 3 shows the results for different shock sizes for the aggregated data. Figures A.1 and A.2 in the Appendix present the results for ± 1 standard deviation shock for all 16 analyzed countries. A brief look at the results confirms the findings from the previous section, i.e. the IRFs for positive and negative shocks of the same absolute size do not resemble a mirror-image, imposed per construction by a linear VAR. Thus, linear VARs should not be used to model the return - volume relationship, as that would lead to wrong conclusions about the nature of this relation.

Looking specifically at the results for a ± 1 standard deviation shock for aggregated data (lower left hand panels of Figure 3), we observe that trading volume significantly increases instantaneously after a positive/negative return shock by about 0.03%, and approaches zero after about twenty trading days. When individual countries are considered, the pattern is very similar - trading volume rises instantaneously in reaction to a ± 1 standard deviation shock in returns. However, in most cases the IRFs revert to zero after just 2-3 trading days. The size of the instantaneous reaction of trading volume for individual countries ranges from about 0.01% to 0.1%. Although statistically significant, such a reaction may seem small at first glance, but we think that it is not economically unimportant. Firstly, bearing in mind that the average trading volume for our sample exceeds 5 million EUR, a response of 0.03% as a reaction to a very moderate shock is in absolute terms not negligible. Secondly, this kind of moderate shocks occur quite frequently in financial markets and thus should not be underestimated.¹⁰ And thirdly, looking at the IRFs for bigger shocks (± 2 standard deviations) from the lower right hand panels of Figure 3, one can see that the response of trading volume does not change linearly with the shock size, but grows overproportionally when shock increases from 1 to 2 standard deviations. Thus, for even bigger shocks the response of trading volume might be large.

Our results are in line with Chen (2012), who finds a positive correlation of returns and trading volume in bull markets and a negative correlation in bear markets. Since in bull markets positive returns dominate negative returns, results are comparable. However, Chen (2012) applies Markov-switching models in his analysis, and thereby assumes two different data generating processes for both regimes. We, on the other hand, rely on the asymmetric VAR model that treats positive and negative returns differently, and is much easier to estimate.

¹⁰On average about 78% of stock returns in our data are within the range of ± 1 standard deviation.

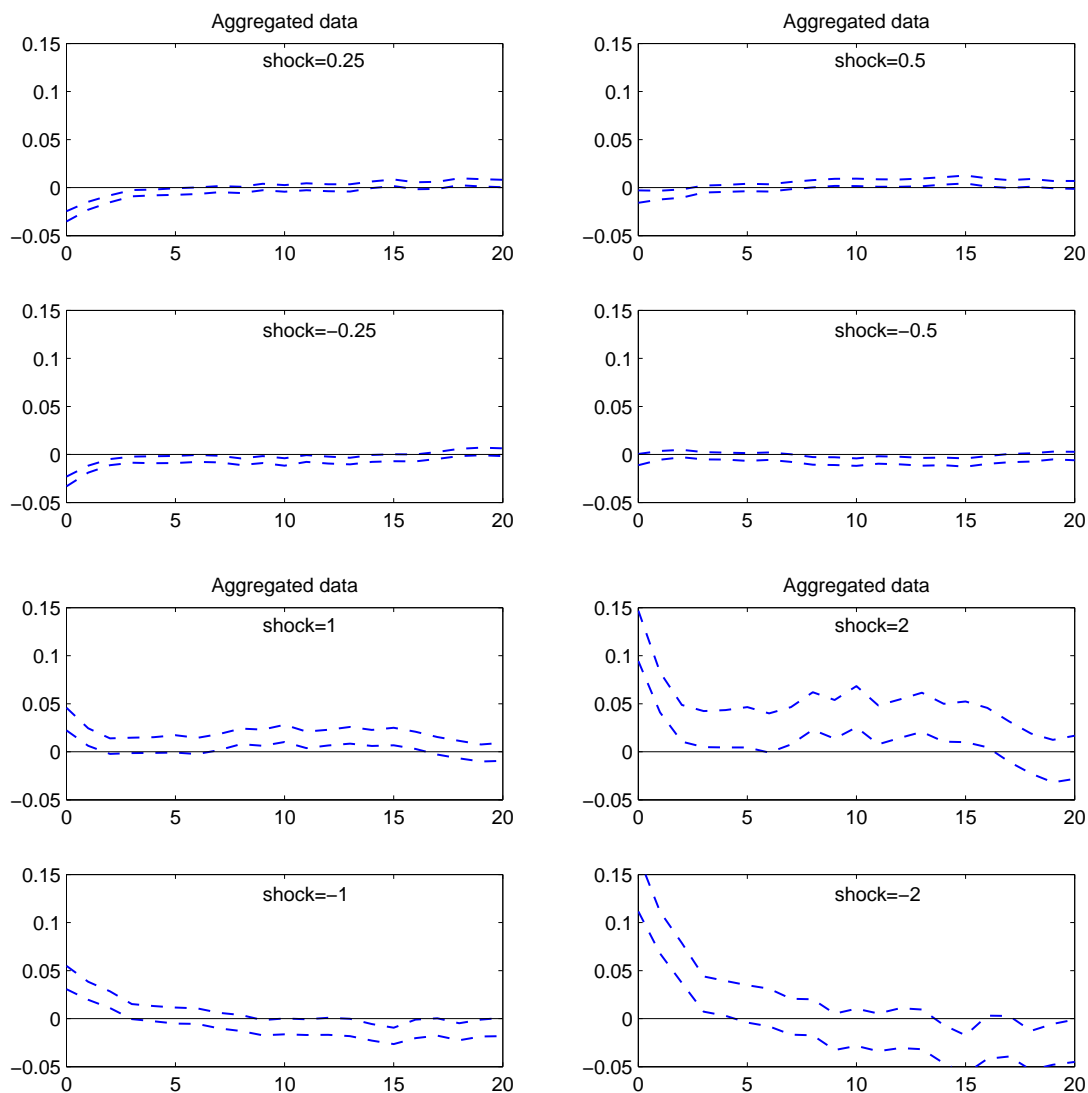


Figure 3: 95% bootstrap confidence intervals for responses of trading volume to shocks in stock returns over the period of 20 trading days. Shock size is given in standard deviations. Results are shown for aggregated data and sample range January 1990 - July 2012.

4.3 Asymmetry and Shock Size

Linear VARs per construction cannot capture any asymmetric effects present in the return-volume relationship. In addition, a standard VAR imposes a linear relationship between returns and volume with respect to the shock size. That means that e.g. a +2 standard deviation shock causes a reaction in the response variable that is twice the size of a reaction to a +1 standard deviation shock. This is a very limiting assumption, especially in the light of the evidence that small (absolute) shocks have proportionally different effects than large (absolute) shocks. For instance, Griffin et al. (2007) find that small positive return shocks lead to a positive reaction of turnover, while small negative shocks lead to a turnover decrease. However, they also find that large and medium absolute shocks tend to increase the trading volume, irrespective of their sign. That points to non-linearities in the return-turnover relation. Therefore, in this section we focus on the magnitude and direction of

Table 2: Impulse-response based Wald tests for responses of trading volume

Shock Size	$H_0 : I_{tv}(h, \delta) + I_{tv}(h, -\delta) = 0$				$H_0 : I_{tv}(h, \delta) - I_{tv}(h, -\delta) = 0$			
	0.25 s.d.	0.5 s.d.	1 s.d.	2 s.d.	0.25 s.d.	0.5 s.d.	1 s.d.	2 s.d.
Austria	< 0.001	< 0.001	< 0.001	< 0.001	0.9971	0.9908	0.9508	0.9699
Belgium	< 0.001	< 0.001	< 0.001	< 0.001	>0.9999	>0.9999	0.9999	0.9999
Denmark	< 0.001	< 0.001	< 0.001	< 0.001	0.8677	0.8678	0.9100	0.8558
Finland	< 0.001	< 0.001	< 0.001	< 0.001	0.9956	0.9955	0.9866	0.9880
France	< 0.001	< 0.001	< 0.001	< 0.001	0.9664	0.9470	0.9715	0.9757
Germany	< 0.001	< 0.001	< 0.001	< 0.001	0.9994	0.9975	0.9966	0.9962
Greece	< 0.001	< 0.001	< 0.001	< 0.001	0.1893	0.0375	0.0048	0.0014
Ireland	< 0.001	< 0.001	0.0014	< 0.001	0.9995	0.9996	0.9994	0.9988
Italy	< 0.001	< 0.001	< 0.001	< 0.001	0.4318	0.3071	0.3083	0.3118
Netherlands	< 0.001	< 0.001	< 0.001	< 0.001	0.9355	0.8937	0.8713	0.8546
Norway	< 0.001	< 0.001	< 0.001	< 0.001	0.9967	0.9991	0.9987	0.9984
Portugal	< 0.001	< 0.001	< 0.001	< 0.001	0.8335	0.6769	0.5712	0.5173
Spain	< 0.001	< 0.001	< 0.001	< 0.001	>0.9999	0.9999	>0.9999	>0.9999
Sweden	< 0.001	< 0.001	< 0.001	< 0.001	0.9750	0.9606	0.8857	0.8588
Switzerland	< 0.001	< 0.001	< 0.001	< 0.001	0.9999	0.9999	0.9999	0.9999
UK	< 0.001	< 0.001	< 0.001	< 0.001	0.8325	0.6313	0.7327	0.7276
Aggregated	< 0.001	< 0.001	< 0.001	< 0.001	0.0064	0.0070	0.0061	0.0052

Note: We report p -values for Wald-type tests of the null hypotheses $H_0 : I_{tv}(h, \delta) + I_{tv}(h, -\delta) = 0$ and $H_0 : I_{tv}(h, \delta) - I_{tv}(h, -\delta) = 0$ for all $h = 0, 1, 2, \dots, H$ (with $H = 20$). The variance-covariance matrix of the vector sum of responses is estimated by bootstrap simulation with $m = 500$ replications. The test statistic is asymptotically χ^2_{H+1} -distributed. We report results for shock sizes of 0.25, 0.5, 1, and 2 standard deviations. p -values below 0.1 are in boldface. Sample range from January 1990 to July 2012.

volume responses depending on the return shock size, as motivated by previous research.

Figure 3 presents trading volume responses to return shocks of different sizes and magnitudes ($\pm 0.25, \pm 0.5, \pm 1, \pm 2$ standard deviations¹¹) for the aggregated data. One can notice that medium and large absolute shocks (± 1 and ± 2 standard deviations) result in an instantaneous, significant increase in trading volume, whereas very small absolute shocks (± 0.25 standard deviations) result in a significant decrease in trading volume. For ± 0.5 standard deviation shocks, we only find a borderline significant reaction of trading volume. Overall, the direction of the response of trading volume depends on the magnitude, but not on the sign of the shock.

Furthermore, while a linear VAR model implies that volume reaction to stock return shocks is proportional, e.g. a return increase of 2 standard deviations is twice the reaction to a return increase of 1 standard deviation, the asymmetric VAR yields different results. We find that e.g. for the aggregated data a return decrease of 2 standard deviations leads to a four times higher reaction of trading volume (+0.12%) than a return decrease of 1 standard deviation (+0.03%), which confirms the non-linearity of the return-volume relationship. The results for individual countries are similar (in many cases the differences between the reactions to a ± 1 and ± 2 standard deviation shocks are even higher than for the aggregated data), and also point to strong non-linearities in the return-volume relationship. These results can be seen in Figures A.3 and A.4 in the Appendix.

To formally investigate the symmetry of the response functions obtained from the asymmetric VAR for different shock sizes, we conduct an impulse-response based Wald test,

¹¹On average 95% of stock returns in our data are within the range of ± 2 standard deviations, so in our analysis we cover the most common shock scenarios.

following the approach of Kilian and Vigfusson (2011). We test the null hypothesis that $I_{tv}(h, \delta) + I_{tv}(h, -\delta) = 0$ for all $h = 0, 1, 2, \dots, H$ jointly, which implies that the responses create a mirror-image effect, just as in the case of a linear VAR model (see our discussion in Section 2 for details).

Analyzing the effects of a shock in returns on trading volume, we find for the aggregated data, as well as for all individual countries that, regardless of the shock size, symmetry is rejected according to the impulse response based Wald test at the 1% significance level (see left panel of Table 2). Thus, apart from graphical evidence we obtain evidence for asymmetries from a formal statistical test, and confirm that the return-volume relation should not be modelled via a linear VAR.

Inspecting Figure 3 for the aggregated data, as well as Figures A.1 and A.2 for individual countries, one can notice that the impulse response functions of volume to positive and negative return shocks of the same absolute size are very similar. This supports the result of Ying (1966), who claims a positive relationship between trading volume and absolute stock price changes. Consequently, we test whether $I_{tv}(h, \delta)$ and $I_{tv}(h, -\delta)$ can be seen as equal. If the differences between $I_{tv}(h, \delta)$ and $I_{tv}(h, -\delta)$ are statistically insignificant, we could simply use absolute returns in further analysis. We modify the impulse response based Wald test from Section 2 and test the null hypothesis $H_0 : I_{tv}(h, \delta) - I_{tv}(h, -\delta) = 0$ for all $h = 0, 1, 2, \dots, H$ jointly. The results of the test are presented in the right panel of Table 2. For all countries (except for Greece) and for all considered shock sizes the H_0 cannot be rejected, pointing to the equality of IRFs for negative and positive shocks of the same size. Thus, it seems that in most considered cases one could use absolute returns for the analysis. However, for the aggregated data the null hypothesis can be rejected at the 1% significance level, which means that in spite of the apparent similarity of $I_{tv}(h, \delta)$ and $I_{tv}(h, -\delta)$, they cannot always be treated as identical. Thus, one should be careful when using absolute returns in their research.

Our results are partly in line with the findings of Griffin et al. (2007) who also find some non-linearities in the return-volume relationship with respect to the sign and magnitude of shocks. However, Griffin et al. (2007) conclude that the return-volume relationship is quite symmetric and thereby use a linear VAR model in their main analysis. We come to a different conclusion. Both graphical evidence and the results of tests for asymmetry clearly reject the hypothesis of symmetric responses of trading volume to a shock in returns for all considered countries and shock magnitudes, and point to strong non-linearities in the return-volume relationship. Thus, we find conclusive and significant evidence against the use of a linear VAR for modelling of this relationship.

4.4 Possible Explanations for Trading Volume Responses

The question of how our results relate to the theories on the behaviour of financial market participants is of utmost importance for the understanding of the mechanisms that drive the return-volume dynamics and for the market participants themselves. Many theories concentrate on the explanation of the behaviour of private investors and are especially relevant for assets that are mainly traded by these investors. A widely used criterion to identify such

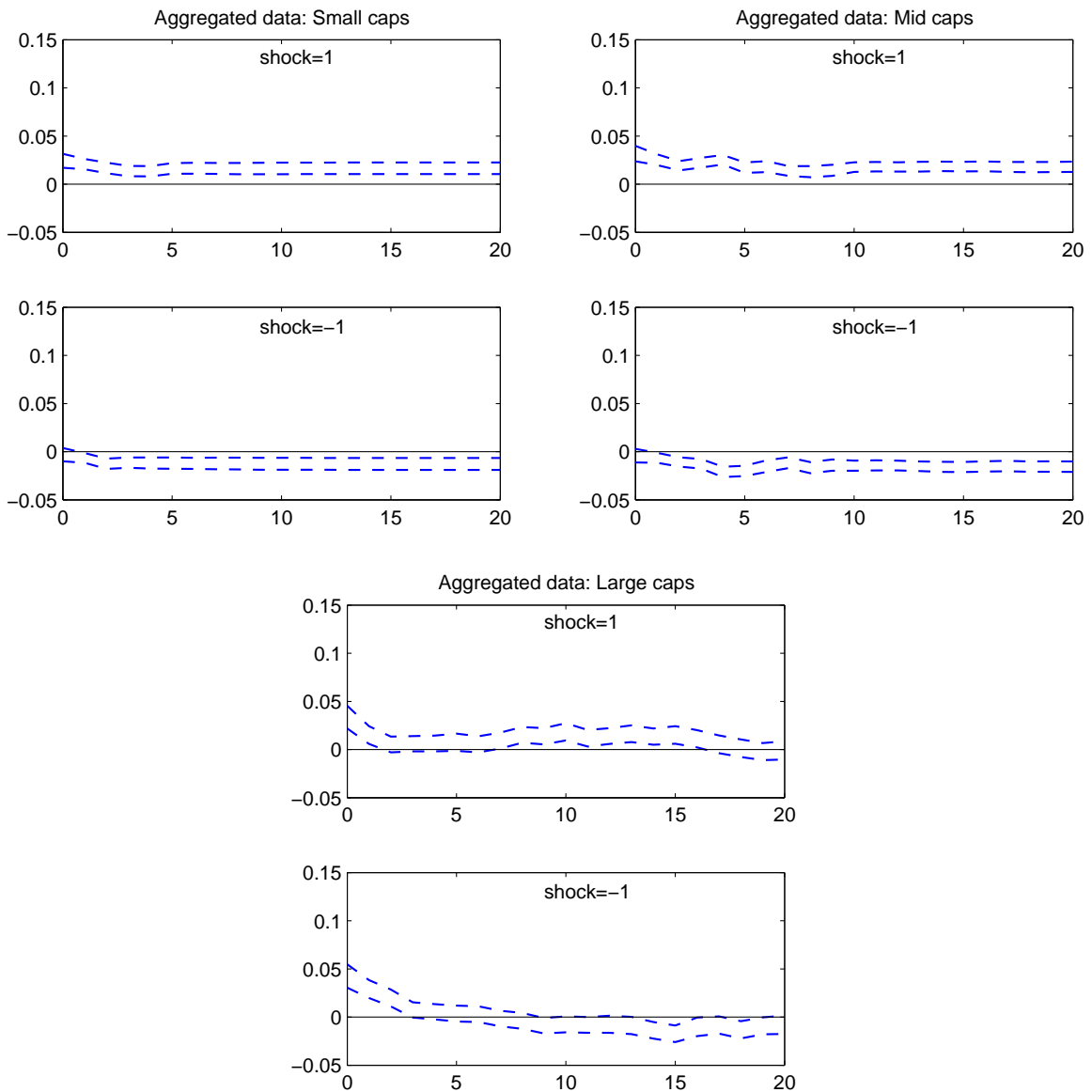


Figure 4: 95% bootstrap confidence intervals for response of trading volume to ± 1 standard deviation shocks in stock returns over the period of 20 trading days. Results for small, mid and large cap firms using aggregated data. Sample range January 1990 - July 2012.

assets is firm size. Small and middle cap stocks are mainly traded by private investors, contrary to large stocks, which are mainly traded by institutional investors (see e.g. Statman et al. (2006)). Sorting firms by size on a monthly basis, we obtain three groups of firms according to their market capitalization - in the small cap group we have the 30% smallest firms, in the large cap group we have the 30% largest firms, and we call the remaining group the mid cap group. We calculate the asymmetric IRFs for each of these groups separately.

The results for the trading volume responses to a ± 1 standard deviation shock in returns are shown in Figure 4 for the aggregated data. We find significant effects (for at least 20 trading days) of the return shocks on trading volume for mid and small cap firms, whereas for large cap firms these effects are not significant at a number of medium response horizons and in the long run. This is an interesting result, as it shows that there are some fundamental differences between the behaviour of private and institutional investors. To explain these

differences, we resort to some theories concerning the behaviour of private investors.

The first theory that can explain the IRFs we obtain from the asymmetric model for mid and small caps is the theory of overconfidence (see e.g. Daniel et al. (1998), Odean (1998*b*), Gervais and Odean (2001), Statman et al. (2006) or Glaser and Weber (2009)). The theory states that high market returns result in some investors being overconfident about the accuracy of their information about the market. The investors falsely interpret their gains from trading as the result of their ability to pick up the right portfolio. The consequence is that they underestimate the trading risk and thus tend to trade more in the following periods and engage in more risky investments. This implies, as Gervais and Odean (2001) state in their paper, that trading volume is higher after positive market returns, and lower after negative market returns. We can confirm this finding in our analysis for small and mid cap firms, where private investors are strongly involved.

The second theory that can explain our results is the participation theory. Orosel (1998) finds that market participation rises after an increase, and falls after a decrease in stock market returns. After an increase in returns investors perceive market participation as more profitable than it used to be (relative to participation costs), and new traders enter the market. After a decrease the trend is opposite, which is reflected in our results for small and mid cap firms.

The third theory that may drive our results is the differences of opinion hypothesis (see e.g. Harris and Raviv (1993) or Kandel and Pearson (1995)). According to this theory investors are heterogeneous with respect to their prior beliefs about the market and/or their interpretation of the public information about the market. High absolute returns may increase the differences of opinion among the investors, and the higher these differences are, the higher the trading volume (see e.g. Bamber et al. (1999) and Antweiler and Frank (2004)). This statement can, however, only explain the results for large cap firms, as in their case trading volume rises immediately after a big shock, regardless of its sign. For small and mid cap firms the volume increases after a positive, but decreases after a negative shock. Glaser and Weber (2009) argue that high differences of opinion resulting from a negative return shock do not necessarily lead to the trading volume increase of the same extent as a positive shock of the same size. This is related to the fact that negative returns go together with paper losses, which investors are unwilling to realize (see e.g. Odean (1998*a*)). Another explanation for different results for mid and small cap firms may be the fact that when differences of opinion grow, some private investors may behave differently than institutional investors. Thus, when overall the trading volume grows for the whole market after a negative shock (which is reflected by the results for large caps), some groups of investors may trade less (the case of mid and small caps).

Finally, the disposition effect hypothesis may also explain part of our results (see Odean (1998*a*)). According to this theory, private investors are more willing to hold stocks that are in a loss position and sell stocks that perform well. As a result, trading volume decreases in bear periods with many negative returns, and increases in bull periods, when private investors reach their individual break-even stock prices and gain on their investment position. This behaviour is reflected by our results for mid and small cap firms.

It is very probable that the results we obtained are caused by a mixture of the above-mentioned theories. Stock market investors are highly heterogeneous and one cannot plausibly assume that only one behavioural pattern drives them all.

5 Robustness

5.1 Volatility

In our main analysis, we consider a bivariate asymmetric VAR model of stock returns and trading volume. However, there is a branch of theoretical and empirical research that relates volume to return volatility (see, e.g. Harris and Raviv (1993) and Shalen (1993)). Thus, ignoring volatility in the analysis might lead to biased results on the relationship between returns and volume. In order to control for volatility in our framework, we include contemporaneous and lagged return volatility obtained from a GARCH(1,1) model for the respective returns. Thus, the enlarged VAR system may be written in the form of the equation (2.1) by using $y_t = (r_t, \text{vola}_t, \text{tv}_t)'$ and $y_t^+ = r_t^+$, and imposing a recursive structure with

$$A_0 = \begin{bmatrix} 1 & 0 & 0 \\ a_{21,0} & 1 & 0 \\ a_{31,0} & a_{32,0} & 1 \end{bmatrix} \quad \text{and} \quad B_i = \begin{bmatrix} 0 \\ b_{21,i} \\ b_{31,i} \end{bmatrix}, \quad i = 0, \dots, p \quad (5.1)$$

This particular ordering of the variables ensures that returns can have an immediate effect on both trading volume and volatility, and also volatility may affect volume instantaneously.

Our results do not change qualitatively after controlling for volatility. However, the return effects on trading volume are more persistent, as for most analyzed countries the accumulated volume responses converge to zero more slowly than in the case of the bivariate VAR. We provide graphs of the IRFs from the trivariate asymmetric VAR model for all countries in Figures A.5 and A.6 in the Appendix.

5.2 Detrending Method

Different ways of dealing with the non-stationarity of trading volume are proposed in the literature. In order to test the robustness of our results to these different methods, we investigate the asymmetric impulse responses obtained for trading volume detrended in various ways. Our first alternative detrending method is the moving average. We subtract the average trading volume over the last q trading days¹² from the actual trading volume, as proposed and applied by Griffin et al. (2007). Our second detrending method is the Hodrick-Prescott (HP) filter of Hodrick and Prescott (1997), which has been used in the related literature for instance by Statman et al. (2006).¹³ For both alternative detrending methods the impulse responses are similar to the ones obtained using first differences of log trading volume. Thus we refrain from showing these results in the paper.

¹²We use window length parameters $q = \{20, 50, 100, 200\}$.

¹³We use $\lambda = 1.000.000$ and $\lambda = 1.600 \cdot 90^4$ as the smoothing parameters for daily data.

5.3 Weighting Method

Apart from weighting portfolios according to their market capitalization in order to construct country indices for returns and trading volume, we use equally-weighted stock portfolios. While the results for portfolios weighted according to market capitalization are driven by large firms, the impulse responses for equally-weighted portfolios resemble the graphs for mid and small cap stocks (see Figure 4). Due to this similarity we refrain from showing the graphs here.

5.4 Data Frequency

As the next robustness check, we analyze weekly data for stock returns and trading volume.¹⁴ By doing so, we can not only check the robustness of our results to data frequency, but also compare our findings directly to a study by Griffin et al. (2007), who use weekly data in their main analysis.

For the weekly data analysis, for most countries, we can observe a very similar pattern as for daily data: large shocks imply a positive and instantaneous trading volume reaction, whereas small shocks imply an instantaneous trading volume decrease. For weekly data we also observe asymmetric effects, which cannot be captured by standard VAR models. We report graphs for the weekly data analysis in Figures A.7 and A.8 in the Appendix.

On average Griffin et al. (2007) find a positive relation of stock returns and trading volume in their analysis. This implies that investors trade more when past returns are positive, and less when past returns are negative. We cannot confirm these finding in our weekly data analysis. We find that for most countries moderate or big absolute return shocks (± 1 or ± 2 standard deviations) result in an immediate increase in trading volume, whereas for small absolute shocks (± 0.25 standard deviation) in an immediate decrease of trading volume. Thus, in contrast to the finding of Griffin et al. (2007), we find that there are strong non-linearities in the return-volume relationship.

5.5 Winsorizing

It is a well known fact that the empirical distribution of daily stock returns exhibits fat tails, which means that extreme negative or positive returns are more frequent than a normal distribution would imply. One could thus argue that the relationship between trading volume and stock returns is in fact linear for the majority of observations in the sample, and that the non-linearity is driven only by extreme negative and positive returns. To limit the influence of outliers, we winsorize the data (log differences in trading volume and log returns) at the 10% level. This means that we set all values that are larger than the 95% quantile or smaller than the 5% quantile of the data distributions to the respective quantile. By doing so, we make sure that our results are not driven by some extreme values. Excluding extreme returns from the sample does not change our results. We refrain from showing the relevant graphs here due to their similarity to the figures for the non-winsorized data.

¹⁴For weekly frequencies we use the volume datatype “VO” and the adjusted stock price datatype “P” for all countries.

6 Conclusion

In this paper we investigate the dynamic relationship between daily stock returns and trading volume in 16 selected European countries. For this purpose we use an asymmetric vector autoregressive (VAR) model. For this model we compute non-linear impulse responses, using a simulation based procedure. We test for asymmetric effects via slope-based and impulse-response based Wald tests. Contrary to the commonly used linear VARs our framework allows the IRFs to change non-linearly with the sign and magnitude of a shock. Thus, our analysis is based on a more flexible econometric framework, tailored to give more detailed insights into the nature of the return-trading volume relationship.

Our analysis indicates that stock returns have a significant influence on trading volume, but there is no evidence for the influence of trading volume on returns. We also find strong evidence that the responses of trading volume to stock returns are asymmetric. From the impulse-response based Wald tests, we find that asymmetry is present regardless of the size of the shock.

Furthermore, we conclude that the sign of the responses depends on the absolute size of the shock. Trading volume increases for medium (± 1 standard deviation) and large (± 2 standard deviations) return shocks, whereas it decreases in reaction to small (± 0.25 standard deviation) shocks.

Looking at the results of the analysis for small, mid and large cap firms separately, we find that a positive (negative) shock in returns results in a significant, positive (negative) and long-lasting effect on trading volume for small and middle cap firms with a high share of private investors. For large cap firms, however, this effect is less pronounced. This result provides supportive evidence for the theories of overconfidence, market participation, differences of opinion, and disposition effect.

Overall, we find that the relationship between stock returns and trading volume is strongly non-linear and asymmetric. Consequently, using linear VAR models to analyze this relationship may be misleading. Thus, non-linear methods, such as the asymmetric VAR proposed in this paper, should be used to deal with the problem.

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A Appendix

Table A.1: Summary Statistics

	Start Date	Mean	Std.Dev.	Firm MCAP	Volume	# Firms	MCAP
Austria	01:1990	-0.0041	1.15	653	285	76	49,965
Belgium	01:1990	0.0031	1.09	1,439	438	92	131,932
Denmark	01:1990	0.0127	0.81	452	478	210	95,023
Finland	01:1990	0.0068	1.86	1,374	8,056	79	108,951
France	01:1990	0.0053	1.19	1,891	2,132	491	928,030
Germany	01:1990	-0.0024	1.20	1,457	7,155	490	714,379
Greece	01:1990	-0.0185	1.72	355	454	178	63,227
Ireland	07:2000	-0.0181	1.48	3,189	1,793	20	64,148
Italy	01:1990	-0.0077	1.36	2,117	14,126	173	365,654
Netherlands	01:1990	0.0077	1.21	2,284	6,414	162	370,300
Norway	01:1990	0.0091	1.25	540	3,404	204	110,123
Portugal	01:1990	-0.0065	1.06	808	2,579	51	41,474
Spain	03:1990	0.0055	1.26	2,574	11,090	128	330,440
Sweden	01:1990	0.0086	1.37	576	4,065	385	221,457
Switzerland	01:1990	0.0217	0.97	2,419	3,473	221	535,245
UK	01:1990	0.0047	1.01	1,722	18,412	1,104	1,900,591

Note: Summary statistics for the dataset used in the analysis. Sample range: January 1990 - July 2012. Data source: Thomson Reuters Datastream. Average daily returns (Mean) and their respective standard deviation (Std.Dev.) are denoted in percentage points, average firm size (Firm MCAP) and average total market capitalization (MCAP) in millions, whereas average trading volume (Volume) in thousands of EUR.

Table A.2: Estimated asymmetric VAR for returns r_t and growth rate of trading volume (tv_t) for France

$$\begin{aligned}
 r_t = & -0.000119 + 0.0334 r_{t-1} - 0.0280 r_{t-2} - 0.0529 r_{t-3} + 0.0333 r_{t-4} - 0.0381 r_{t-5} \\
 & \quad (0.00033) \quad (0.016) \quad (0.0245) \quad (0.0192) \quad (0.0218) \quad (0.0212) \\
 & - 0.000731 tv_{t-1} + 0.000337 tv_{t-2} + 0.0012 tv_{t-3} - 0.000105 tv_{t-4} + 0.000733 tv_{t-5} \\
 & \quad (0.00068) \quad (0.0007) \quad (0.00083) \quad (0.00073) \quad (0.0007) \\
 & + 0.0019 tv_{t-1}^+ + 0.000615 tv_{t-2}^+ - 0.0013 tv_{t-3}^+ + 0.0014 tv_{t-4}^+ \\
 & \quad (0.0012) \quad (0.0011) \quad (0.0012) \quad (0.0013) \\
 & - 0.0012 tv_{t-5}^+ + \hat{\varepsilon}_{1t} \\
 & \quad (0.0012) \\
 tv_t = & -0.0023 - 10.174 r_t - 0.451 r_{t-1} + 0.725 r_{t-2} + 2.5 r_{t-3} + 1.25 r_{t-4} \\
 & \quad (0.0074) \quad (0.58) \quad (0.54) \quad (0.54) \quad (0.57) \quad (0.54) \\
 & + 4.4 r_{t-5} - 0.526 tv_{t-1} - 0.4 tv_{t-2} - 0.341 tv_{t-3} - 0.24 tv_{t-4} \\
 & \quad (0.63) \quad (0.018) \quad (0.019) \quad (0.017) \quad (0.016) \\
 & - 0.0166 tv_{t-5} + 19.42 r_t^+ - 0.106 r_{t-1}^+ - 2.71 r_{t-2}^+ \\
 & \quad (0.014) \quad (1.1) \quad (0.93) \quad (0.95) \\
 & - 4.21 r_{t-3}^+ - 2.9 r_{t-4}^+ - 8.59 r_{t-5}^+ + \hat{\varepsilon}_{2t} \\
 & \quad (0.99) \quad (0.93) \quad (1.00)
 \end{aligned}$$

Note: Table reports OLS estimates of asymmetric VAR in (2.1). HAC standard errors are in parentheses. Sample range: January 1990 - July 2012.

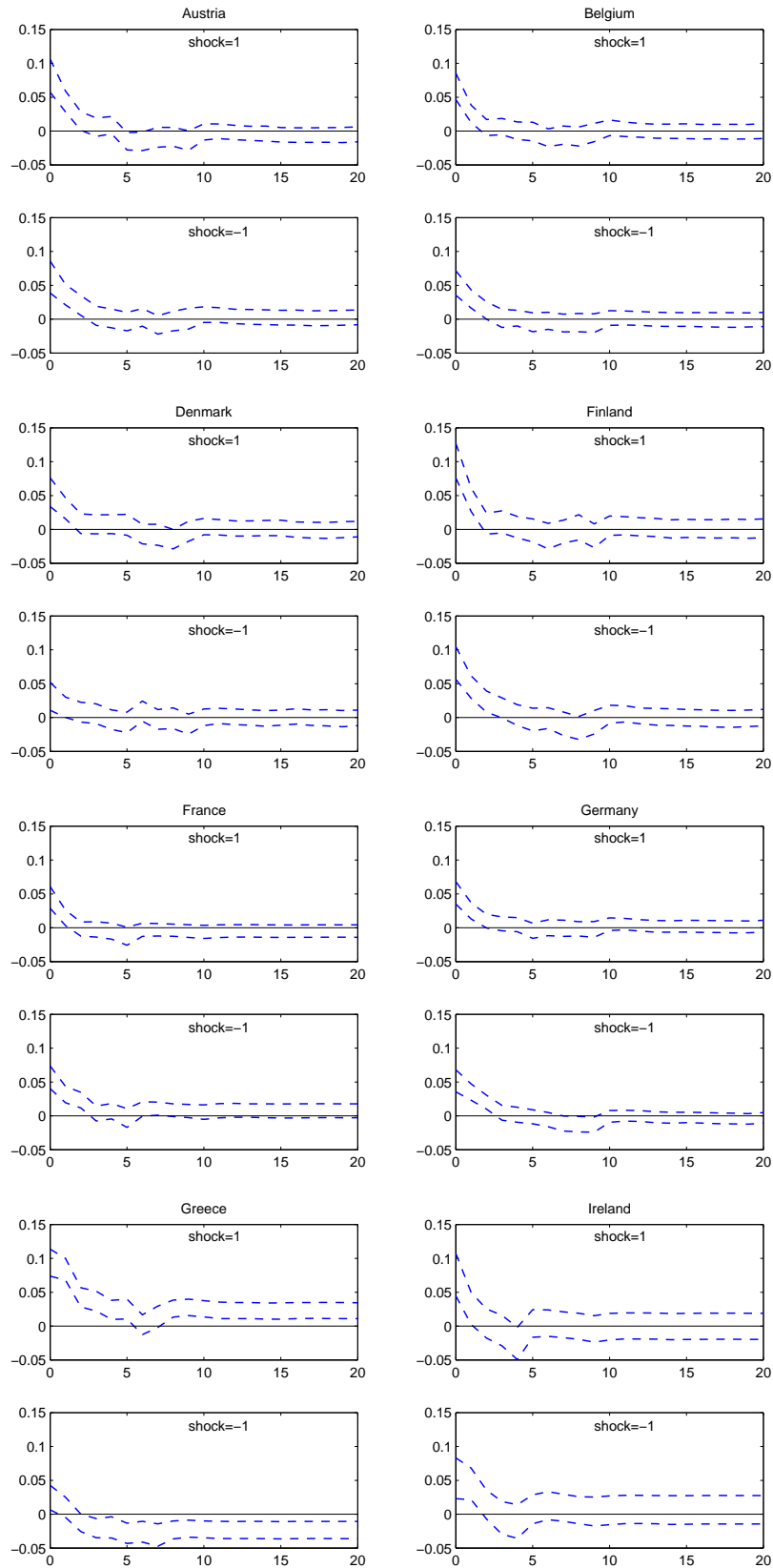


Figure A.1: 95% bootstrap confidence intervals for responses of trading volume to ± 1 standard deviation shocks in stock returns over the period of 20 trading days. Results for different European countries. Sample range: January 1990 - July 2012.

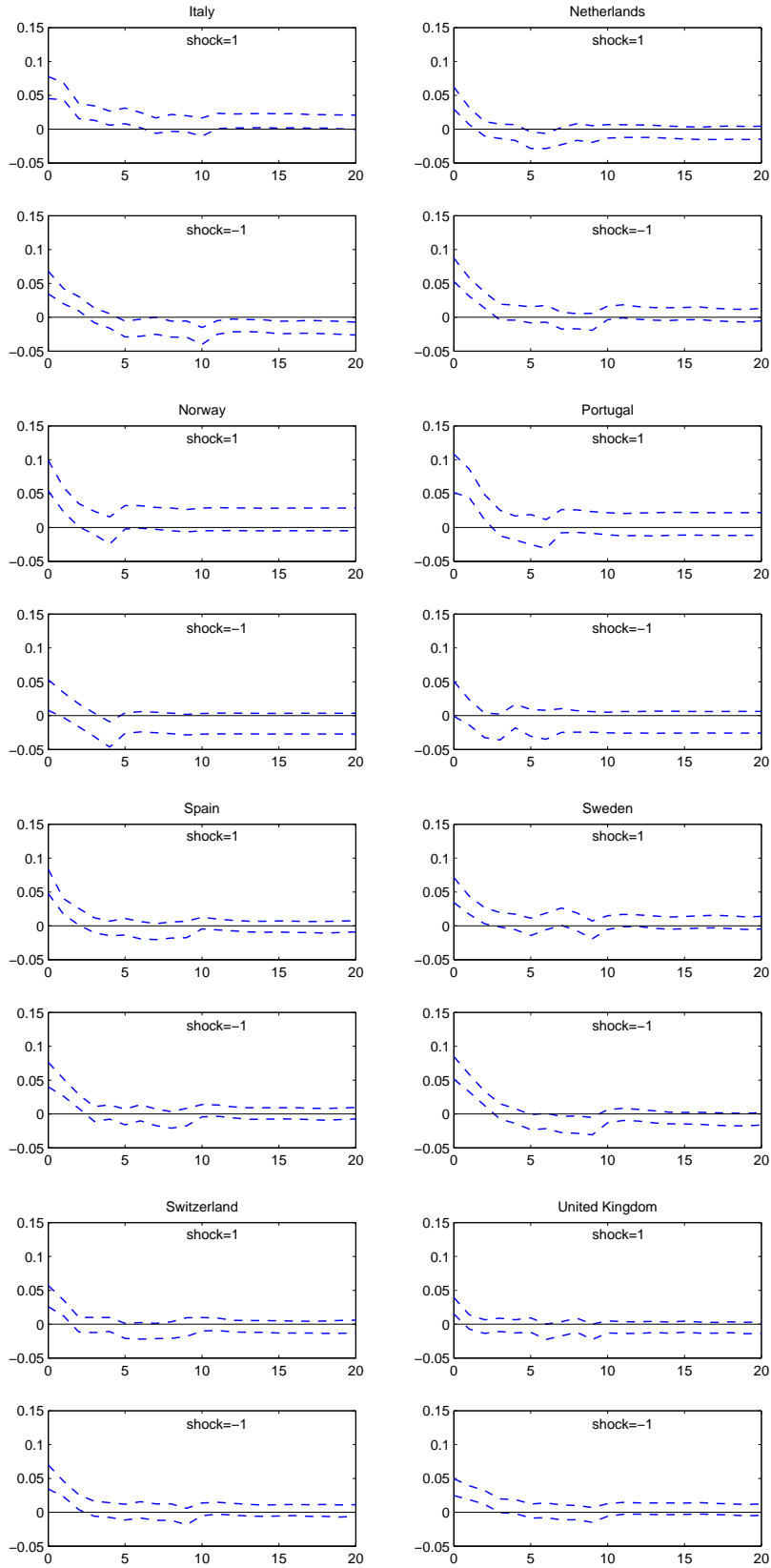


Figure A.2: 95% bootstrap confidence intervals for responses of trading volume to ± 1 standard deviation shocks in stock returns over the period of 20 trading days. Results for different European countries. Sample range: January 1990 - July 2012.

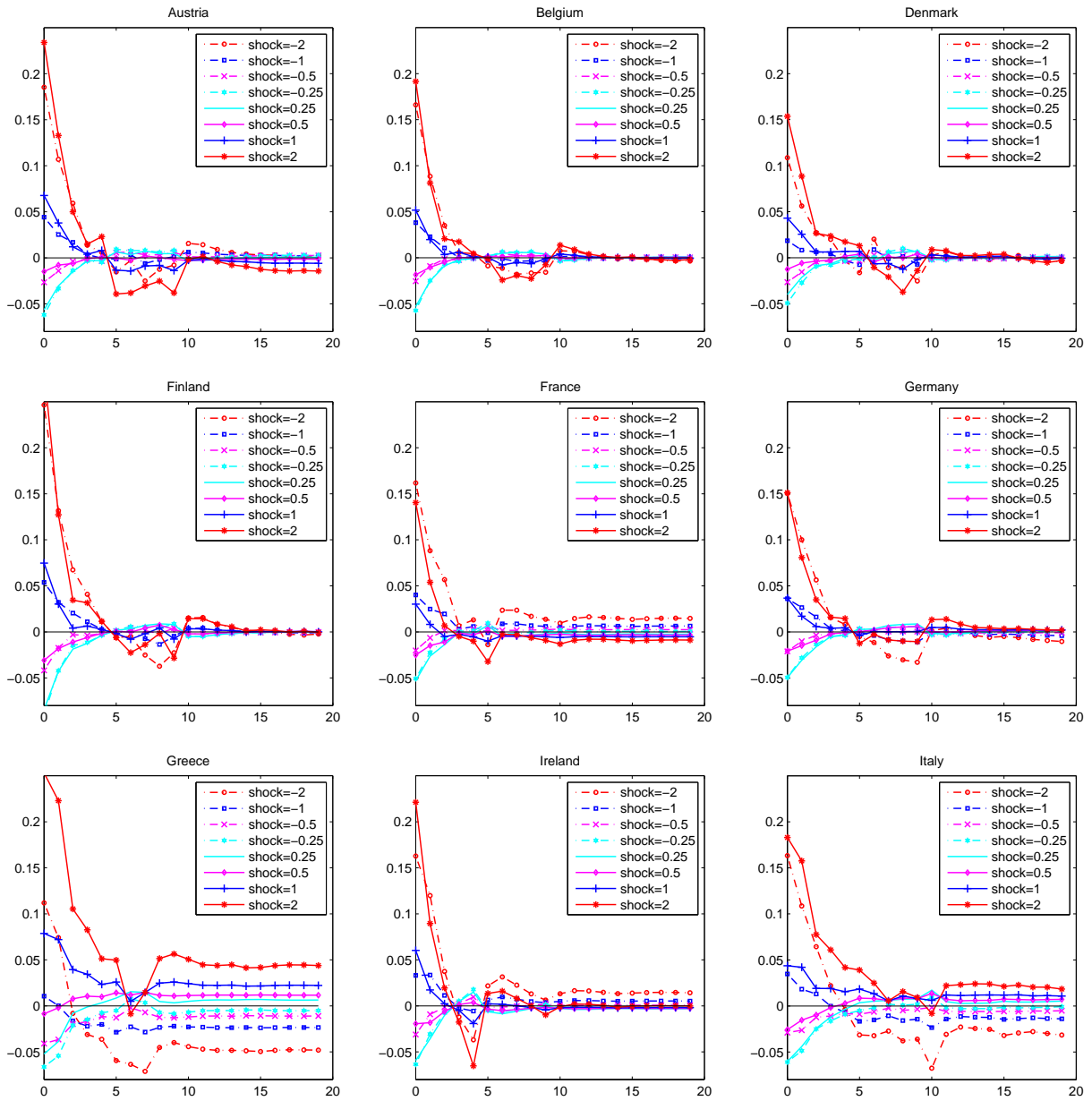


Figure A.3: Response of trading volume to shocks in stock returns of different size over the period of 20 trading days. Results for different European countries. Sample range: January 1990 - July 2012.

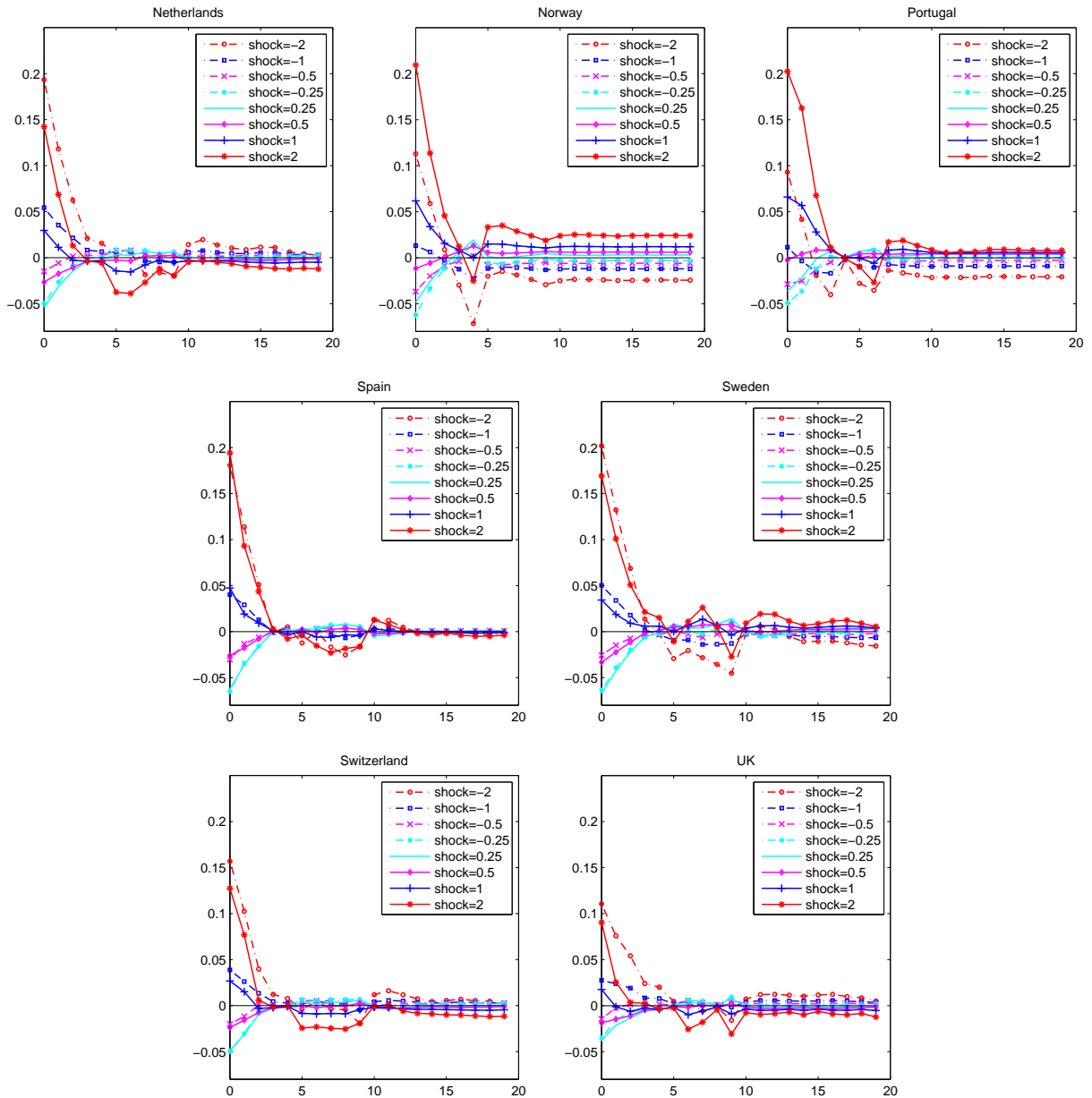


Figure A.4: Response of trading volume to shocks in stock returns of different size over the period of 20 trading days. Results for different European countries. Sample range: January 1990 - July 2012.

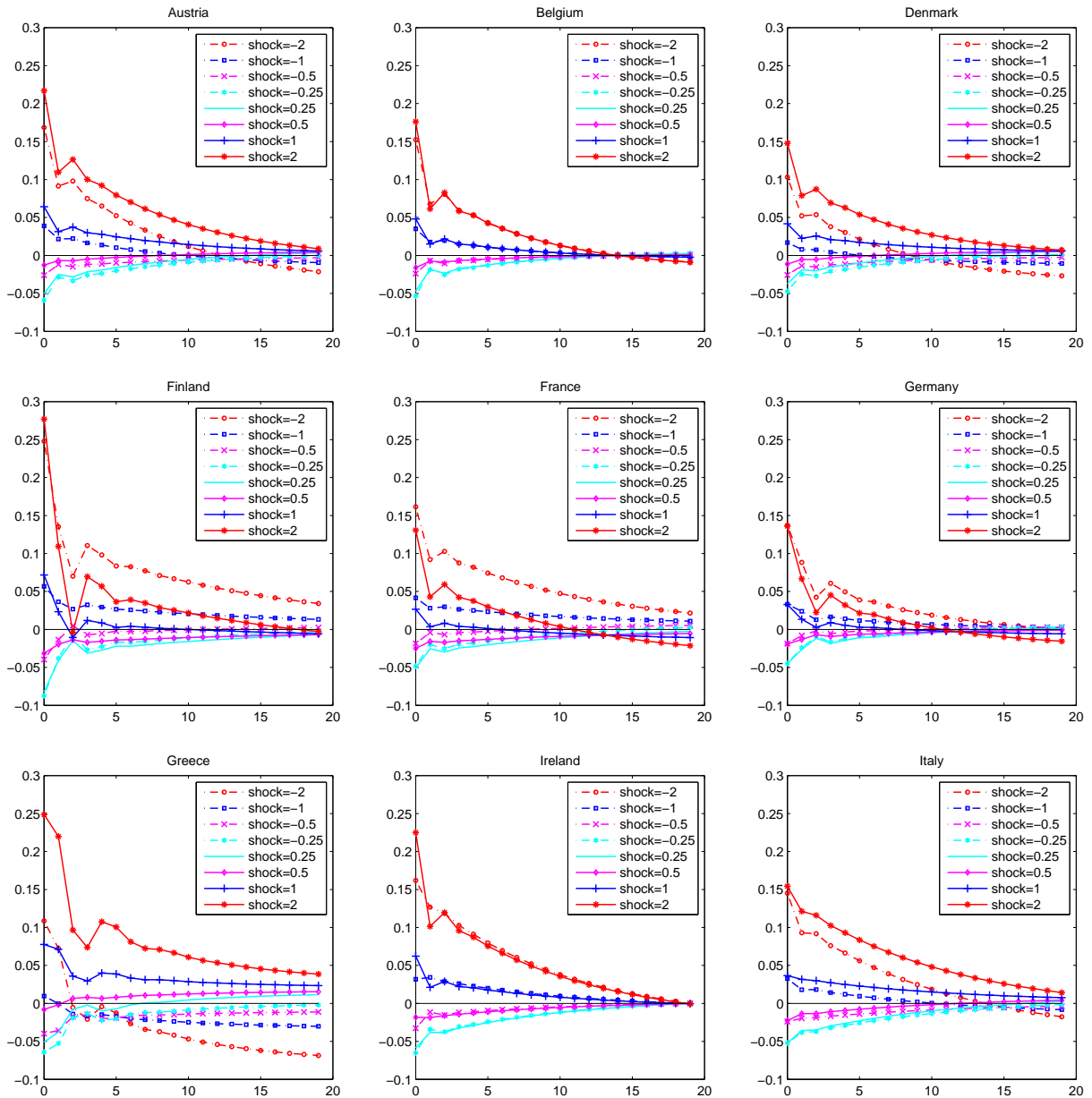


Figure A.5: Response of trading volume to shocks in stock returns of different size over the period of 20 trading days. Results from asymmetric VAR including volatility (see eq. (5.1)) for different European countries. Sample range: January 1990 - July 2012.

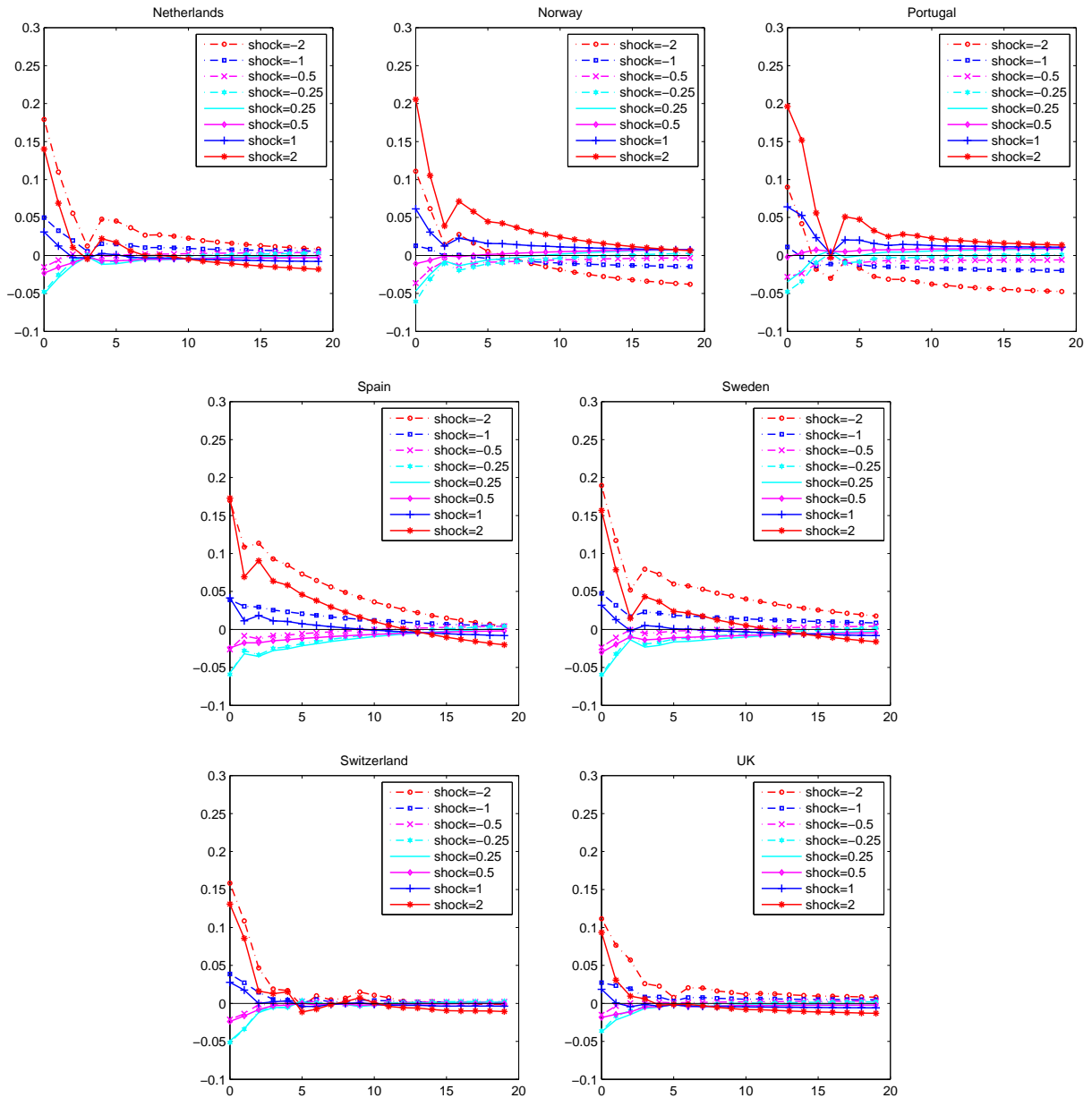


Figure A.6: Response of trading volume to shocks in stock returns of different size over the period of 20 trading days. Results from asymmetric VAR including volatility (see eq. (5.1)) for different European countries. Sample range: January 1990 - July 2012.

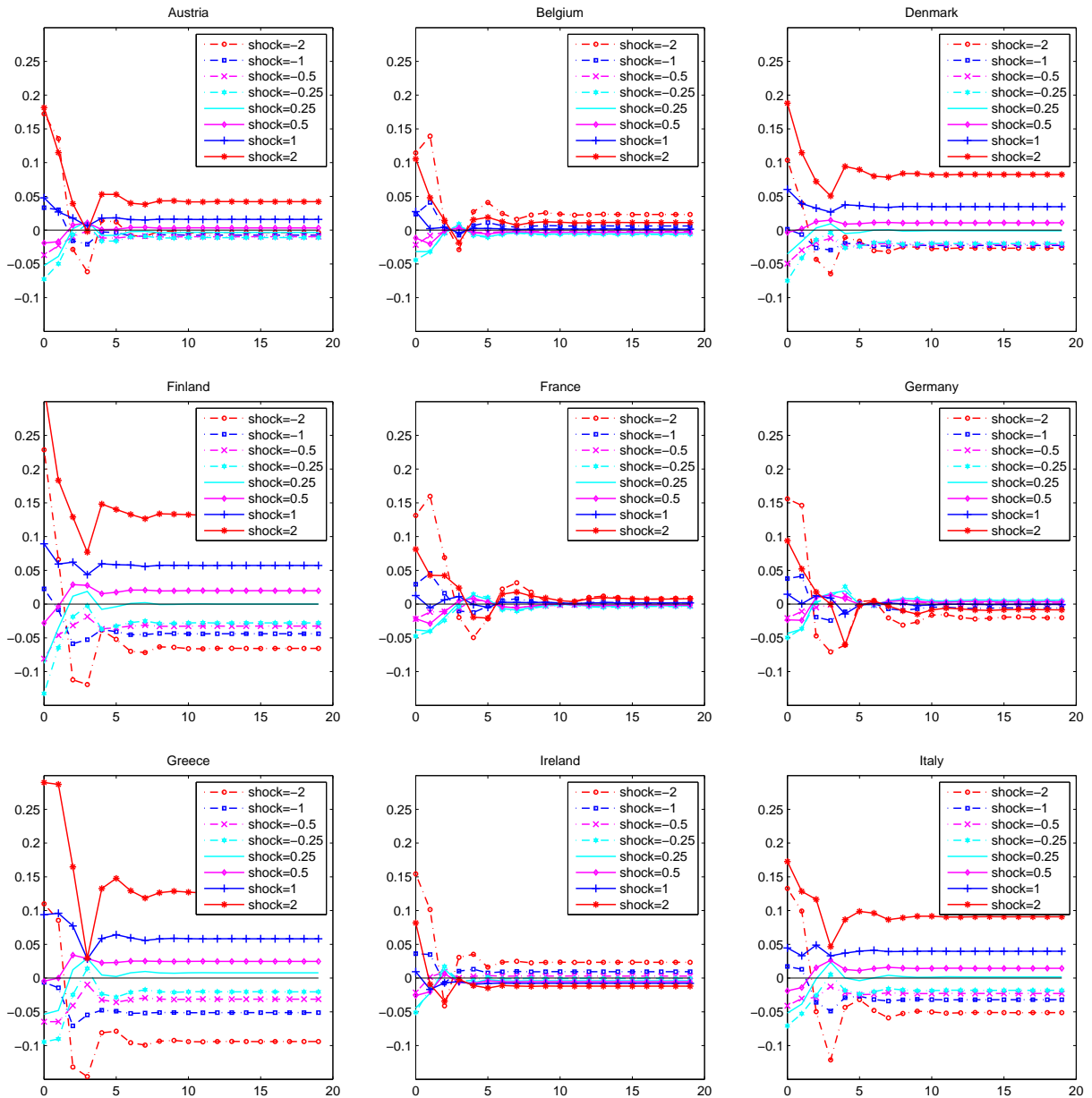


Figure A.7: Response of trading volume to shocks in stock returns of different size over the period of 20 trading days. Results from asymmetric VAR using weekly data for different European countries. Sample range: January 1990 - July 2012.

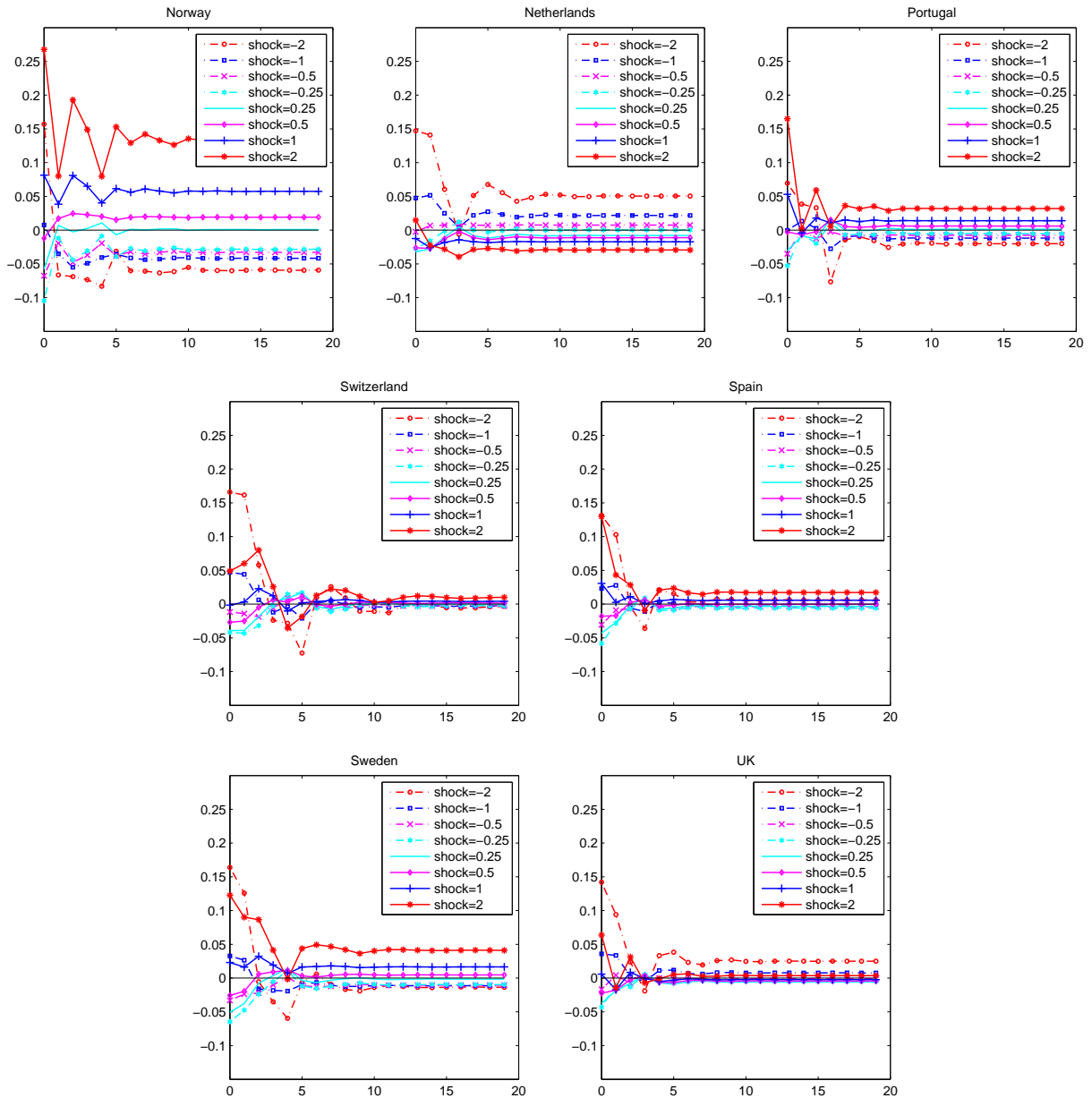


Figure A.8: Response of trading volume to shocks in stock returns of different size over the period of 20 trading days. Results from asymmetric VAR using weekly data for different European countries. Sample range: January 1990 - July 2012.

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