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How Turnout Depends on the Number of Parties: A Logical Model

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ABSTRACT *We illustrate the power of "logical models" (Taagepera, 2007) by offering a three parameter model of the relationship between the effective number of parties and electoral turnout that makes use of the constraints on what parameter values are internally coherent given boundary conditions to specify functional form, and seeks not optimal curve fitting but rather a direct model testing. In our model, one parameter reflects an effect that generally acts to increase turnout as the effective number of parties increases, another an effect that generally acts to decrease turnout as the effective number of parties increases, while a third parameter allows for baseline variation in turnout across countries (or within countries across elections). We fit this model to district level data from 237 elections held in 17 countries, representing a wide range of electoral system types generating multi party contests, with over 20,000 district election observations. The basic intuition, that turnout rises to a peak as the effective number of parties increases and then falls slowly, fits our data pretty well.*

1. Introduction

This article has three main goals. First and foremost, we wish to illustrate the power of "logical" models (Taagepera, 2007) in that we start off with a problem (how should turnout vary with the number of parties?) and consider from a theoretical perspective what kinds of factors might affect this relationship, seeking to identify effects that are central. We then show how features of model building emphasized in "logical models," such as the constraints imposed by boundary conditions, a focus on only a handful of key variables, considerations of marginal effects, and clearly testable hypotheses which are theoretically linked to potential mechanisms, allow us to specify a functional form for this relationship. We get what looks like a relatively complex function, but actually it has only two key parameters (with a third parameter allowing for country-specific or election-specific variations).

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Second, we wish to illustrate an important statistical estimation procedure that allows us to simultaneously deal with variation across elections within a given country and variation across countries within a maximum likelihood framework by making use of electoral data at the district level. Moreover, because our underlying model has a clear theoretical prediction (along with some general expectations about signs and orders of magnitude) we can present our results in a way that allows readers to directly visually (and viscerally) see how well the data fit the single humped expectations of the model.

Third, our work seeks to make a substantive contribution to the understanding of variations in turnout (see Blais, 2006; Blais & Aarts, 2006; Blais & Carty, 1990; Blais & Dobrzynska, 1998). Blais and his colleagues are concerned that comparisons of types of electoral systems show turnout higher in PR systems than in single-seat plurality systems and the standard reason for why we get this result is that more political parties give greater choice and so higher levels of turnout; yet, one does not find turnout rising linearly with the (effective) number of parties (or the number of seat-winning parties). By developing a model that identifies factors which lead to rises in turnout with increasing N, and others that lead to declines in turnout with rising N, and which then incorporates combined effects into a two-parameter model generating curvilinearity, we implicitly show how misleading cross-national results that fit a linear model to data that is curvilinear at the national level can be. However, we would also note that the three-parameter model we present and estimate has implications well beyond the context of voter turnout. In particular, it can be adapted to virtually any setting where there is choice among a multiplicity of objects but also the possibility of abstention (i.e., in effect, rejecting all the available choices). In addition, the types of processes we emphasize, e.g., involving cognitive complexity of choice (Brockington, 2004; Geys & Heyndels, 2006; Huber et al., 2005), or ability to find parties close to one's preferences (Cox, 1999; Crepaz, 1990), are important for our understanding of fundamental processes of representation.[1]

It is important for us to emphasize that our aim is not to account for variation in turnout T as fully as possible, but to assess the relationship between the (effective) number of parties N and T.[2] What we seek to do is to find ways to simplify models to the point where specific outcomes can be predicted and predictions can be tested, rather than adding variables whose effects we cannot clearly identify in advance.

Multi-seat PR elections tend to enable many parties to run and tend to have more voters turning out than single-seat plurality elections (Blais & Carty, 1990; Blais & Dobrzynska, 1998; Brockington, 2004; Crewe, 1981; Jackman, 1987; Jackman & Miller, 1995; Norris, 2004; Selb, 2009; Siaroff & Merer, 2002). However, within the PR systems an increase in the effective number of electoral parties actually is observed to lower turnout (Blais, 2006; Blais & Aarts, 2006; Blais & Carty, 1990; Blais & Dobrzynska, 1998; Brockington, 2004; Jackman, 1987; Jackman & Miller, 1995). This article proposes a single, logically anchored equation (Taagepera, 2008) to explain this reversal, which Grofman and Selb (2011) refer to as "Blais'

Paradox." This model can account for empirical findings not only directionally (first goes up as the number of parties increases, then down), but also in quantitative detail how much up or down, at a given number of parties. Since the number of parties and district magnitude, i.e., the number of seats to be filled in a constituency, are intimately linked theoretically as well as empirically, most of our assertions about the effects of the number of parties on turnout apply equally to effects of district magnitude on turnout.

The two best known qualitative explanations predicting an upward trend of turnout with increasing number of parties *N* are that (1) more parties make it more likely that a voter locates a party making it worthwhile to go and vote because there is a greater likelihood that there will be a party close to the voter's own preferred position(s), and (2) as *M* (district magnitude) increases in a PR system, any given party has greater chance to get some representation because of the decline in the threshold of representation (Cox, 1999; Grofman & Selb, 2009; Selb, 2009). One well-known explanation for the subsequent decline of turnout with increasing number of parties argues that it does not suffice for a small party to win seats in the legislative assembly. It also must (1) be included in a coalition government, and (2) once in, have some impact on cabinet policy. An increasing number of parties makes such impact increasingly unlikely (Jackman, 1987; see also Jackman & Miller, 1995), especially since many of the new parties are apt to be small and the number of possible coalitions goes up exponentially. Hence a very large number of parties would depress incentives to go and vote (Powell, 2000). An alternative explanation for the lack of a rise in turnout with an increasing number of parties is that, contrary to intuition, having more parties is not necessarily correlated with a greater likelihood that a vote for a small party will be efficacious, since, as *M* increases, the number of parties motivated to contest the election may increase faster than the decrease in the *Threshold of Exclusion* an account closely related to Taagepera and Shugart's *Law of Conservation of Disproportionality* (Taagepera & Shugart, 1989). If so, incentives to turnout may actually fall with increasing numbers of parties. A third explanation is that a negative relationship between the number of parties and electoral turnout might indicate that increasing party system complexity confronts ordinary voters with choices that overwhelm their choice capacities and thus reduce turnout (Brockington, 2004; Huber et al., 2005).

Drawing on the notion of "logical models" involving constraints on what parameter values are internally coherent given boundary conditions, and an attempt to use the simplest models that make sense, and to seek to understand behavior "on average" in terms of factors that are theoretically key (Taagepera, 2008), we offer a three-parameter model of the relationship between effective number of parties and turnout that seeks to make sense of the observed empirical patterns. We later apply this model to current and historical district-level election data from 17 countries.

The next section of the article lays out this model in detail (with full derivation provided in an online appendix available at http://dx.doi.org/10.1080/17457289.2013.858345). In this model, one parameter allows for baseline variation in turnout across countries (or within countries across elections), another parameter reflects one or more effects that generally act to increase turnout as the effective number of parties increases (greater range of voter choice, greater likelihood of finding a party close to one's preferred position), and another parameter reflects one or more effects that generally act to decrease turnout as the effective number of parties increases (increased complexity of choice).

If we wish to truly understand the linkage between turnout and number of parties we need to make use of within-country data. When we study the relationship between (effective) number of parties and turnout at the national level, making cross-national comparisons, we risk committing ecological fallacies (Grofman & Selb, 2011). In the third and fourth sections of the article we use district-level data in a country-specific fashion, and we show empirically that linear models of the relationship between effective number of (electoral) parties and turnout (or district magnitude and turnout) are misleading since there are fundamental nonlinearities.

In the empirical section of the article we fit the three-parameter model to district-level election data from 17 countries, allowing for country- and election-specific effects. We show that the basic intuition that turnout rises to a peak as the effective number of parties increases and then falls slowly seems to fit our data well.

In the concluding discussion we briefly consider the implications of our work for theories of turnout and for broader questions about modeling strategies in the study of electoral and party systems.

2. A "Logical" Model of Turnout

Turnout can be expected to depend on a huge variety of factors in addition to the number of parties. Focusing on a single country eliminates at least the inter-country factors, though we still expect variation across elections. The empirical work of Grofman and Selb (2011) on turnout in electoral districts in Switzerland and Spain, where the local effective number of parties (N) covers a wide range, from less than two to more than six, inspired the qualitative features of the model we propose. The first figure in Grofman and Selb (2011) shows that even intra-country variation in turnout is huge, but, in the two countries, the mean pattern of T as N increases is an increase followed by a gradual decrease when N surpasses about 3.0 (see Figure 1). Thus, linear correlations between turnout and effective number of parties can miss important nonlinearities.[3] After identifying this pattern *within* individual countries, Grofman and Selb use it to account for the seeming anomaly observed in *inter-country* studies that, while turnout is higher in PR systems than in SMD systems, with the latter having many fewer parties, turnout does not monotonically rise with the effective number of parties.

The basic insight with which we begin our own modeling is that there are some factors that would lead to an increase in turnout with the effective number of parties, and other factors that would lead to a decrease in turnout with the effective number of parties. On the face of it this insight might seem a recipe for confusion. But we can translate this fundamental insight regarding direction of impact of effective number of parties into equations that can generate testable models, if we keep in mind the advice ascribed to Albert Einstein: "Our models should be as simple as possible but no simpler," and if we recognize that there are logical constraints on the structure of the relationships that affect turnout, such as the fact that turnout as a share of eligible electors is bounded from above and below, by 0 and 1.

Furthermore we wish to generate a model that can account for what is already known about the link between turnout and N. Thus, we wish a model that, for the data that has previously been studied, (1) would place the peak turnout at its empirically observed number of parties and (2) generally fit the mean values of turnout (T) at any effective number of electoral parties, N. In particular, it has to reproduce the initial increase in T and the shallower slope of the subsequent decrease. This is where the second part of the dictum enters: "But no simpler."

2.1. *Formulating a Logical Model of the Relationship Between the Effective Number of Parties and Turnout*

The competing hypotheses reviewed in the introduction suggest interaction of two (or more) processes that work against each other: e.g., as the effective number of parties (N) increases, choice options (C) increase, but potential for policy input (P) decreases. Here we will develop a model that uses those labels, C and P, as shorthand labels for the set of upward and the set of downward forces that affect turnout as N increases. Given the aggregate data we work with, teasing out which particular mechanisms are generating the effects of C and P is not possible.

In addition, total turnout (T) probably should include a baseline level (T_0), which is independent of N, and varies from country to country (and perhaps within a country from election to election). T_0 includes the impact of sense of civic duty and all other unmeasured country-specific (and perhaps also election-specific) factors apart from N.

Thus, turnout consists of T_0 plus a function $T_1(C,P)$ of choice and policy input:

$$T = T_0 + T_1(C, P) \tag{1}$$

We seek to specify the functional form of such a model.

The model that we arrive at (see http://dx.doi.org/10.1080/17457289.2013.858345 for derivation), with parameters T_0, h and k to be estimated, is general enough to be applicable to most observed patterns involving turnout and effective number of parties; it also can be modified to include other covariates.

$$T = T_0 + (1 - T_0)\big(1 - \exp[-k(N-1)]\big)\exp[-h(N-1)] \tag{2}$$

2.2. *The Expected Range of Parameters and the Location of the Peak*

From our previous general knowledge about turnout in two-party and multi-party systems, we might expect the number of parties to matter, but along with other factors, which are represented by T_0. If T_0 were close to the actual turnout in the country there would be no range of values for N to act in; thus we expect T_0 to be below the mean turnout in the country.

As for the parameters k and h, we would expect the turnout-boosting k to be larger than the turnout-depressing h, because otherwise the boosting would be squashed before having time to act at all. Hence $k > h$ is expected. These parameters represent the initial slopes of the respective exponential curves, at $N = 1$. If the initial slope k continued, this tangent to the exponential curve would take us to full turnout at $N = 1 + 1/k$. In particular, for $k = 1$, full turnout would be reached at $N = 2$, as if two parties could offer sufficient choice. Compared to this initial tangent, the exponential model bends down, of course, but $k = 1$ still offers some sort of a baseline. We would be surprised if k were much larger than that.

On the other hand, a value of h smaller than 0.1 would imply that the initial tangent to the slowdown exponential would take the turnout to the floor (T_0) when $N = 1 + 1/0.1 = 11$. Compared to this initial tangent, the exponential model curves up, of course, but $h = 0.1$ still offers some sort of a baseline. A single party out of 11 is not likely to affect policy detectably. Indeed, we might expect the turnout-depressing effect of excessive parties to set in forcefully much earlier, i.e., at h larger than 0.1.

In sum, we might expect the relationship $1 > k > h > 0.1$. Contrary outcomes are not impossible, but they would be surprising, making us check the data and look into the special characteristics of such a country.

How much turnout would result from extreme combinations of k and h? For given T_0, turnout would be maximized for high k and low h, such as for $k = 1$ and $h = 0.1$. Turnout would be minimized, conversely, for h matching k, such as for $k = h = 0.5$. But at which number of parties would the respective peak turnouts be reached? It can be calculated that the function

$$f = \left(1 - \exp[-k(N-1)]\right)\exp[-h(N-1)] \tag{3}$$

reaches its peak at

$$N_{\text{opt}} = \frac{\ln(1+r)}{k}, \tag{4}$$

where $r = k/h$. At this value of N,

$$f_{\max} = \frac{r}{(1+r)^{1+\frac{1}{r}}} \tag{5}$$

For $k = 1$ and $h = 0.1$, $r = 10$, $N_{\text{opt}} = 3.40$ and $f_{\max} = 0.715$. Thus, even for the most optimistic combinations for parties to offer choice yet have policy impact, it is hard to raise the party impact on turnout above 70% of its conceivable maximum. For $T_0 = 0.25$, $T_{\max} = 0.786$. For $T_0 = 0.50$, $T_{\max} = 0.858$. For $k = h = 0.5$, $r = 1$, $N_{\text{opt}} = 2.38$ and $f_{\max} = 0.25$. For $T_0 = 0.25$, $T_{\max} = 0.438$. For $T_0 = 0.50$, $T_{\max} = 0.625$.

Thus, turnout can be expected to be maximized when the effective number of parties is around two to three, regardless of the values of the parameters k and h, in their expected range. The peak turnout, however, depends heavily on the ratio of these parameters, as well as on the basic level of turnout, which does not depend on the number of parties.

Note, too, that in calculating the expected maximum, we tacitly presume that $N \quad 3$ represents something like 34-33-33, not 50-17-17-16 or 54-10-10-9-9-7-1. In the presence of some very large and many tiny parties, the effective number of parties becomes a less reliable predictor for turnout.

It becomes now a matter of determining the values of parameters T_0, k and h that best fit the observed mean pattern and finding out how good this fit is, and whether the theoretically motivated expectations about parameter values described above are satisfied. There certainly can be other factors that affect the link between turnout and number of parties, e.g., we might expect more parties in districts with high district magnitude and these high magnitude districts might be disproportionately urban and thus have somewhat different demographic characteristics that affect turnout, but we can still treat such differences within the broad framework of there being two kinds of forces, one operating to increase turnout with increased number of parties, one operating to reduce turnout with increased number of parties. (See the concluding discussion about issues of causal inference.)

3. Data

We will fit the model to district-level election data from 17 countries: Australia, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, India, Ireland, Japan, Netherlands, Norway, Portugal, Spain, Sweden and Switzerland. All these countries currently employ "simple" electoral systems, or did so in the past. By simple we mean single-tiered electoral systems in which all the seats are distributed at the local level of the primary electoral districts (see Taagepera, 2007).[4] What these systems have in common is that substantive parts of the incentives for mobilization and turnout emanate from local-level competitions (of which the number of parties is one essential feature). Such systems represent a large segment of the types of institutional arrangements governing elections today. Thus, though only involving 17 countries, the empirical analysis to follow can be considered a reasonably global test of our theoretical model.

Most of the data we use were taken from the Constituency-level Elections Archive (CLEA), a huge repository of detailed election results at the constituency level for lower house legislative elections from around the world (Kollman et al., 2011).[5] We have updated the data where necessary. Considering district-level contests from simple electoral systems just multiplies the number of data points to test the model's implications. Table 1 gives an overview over the data used. Altogether, we consider 237 national elections with a total of almost 23,000 district-election observations. Countries and districts in which voting is compulsory have been excluded from the analysis for obvious reasons. Because we are most interested in studying the effects on turnout of increasing numbers of parties, we focus on multi-party PR systems.[6]

4. Empirical Model

In terms of statistical modeling, there are three options to fit the model in Equation (2) to such district-level data covering multiple elections:

1. A *pooled model* assuming that T_0, k and h are constant across elections (and countries). Results of preliminary analyses suggested that this assumption is utterly misguided.
2. *Separate models* that produce election-specific estimates of T_0, k and h. Often however, we will just have a finite number of district observations per election possibly too little to fit a nonlinear three-parameter model. This will be particularly true if the variation in N is limited. As is obvious from Table 1, the number of parties per country varies quite a bit across districts and elections, but not excessively in most countries, and the number of districts which make up the estimation sample for separate models are clearly below 100 in most countries considered, with a minimum of just 15 districts in Finland.
3. A *multilevel model* which constitutes a compromise between 1) and 2) by giving distributions to T_0, k and h, so that all the district observations contribute information about mean estimates of T_0, k and h, while election-specific estimates can be retrieved as weighted averages of the pooled estimates from 1) and the separate estimates from 2). In one such model, the multilevel or *Empirical Bayes (EB)* approach, our estimate of k for election j will be

$$k_j^{\mathrm{EB}} = \lambda_j k_j^{\mathrm{Separate}} + (1 - \lambda_j) k^{\mathrm{Pooled}}, \tag{6}$$

where λ_j is the precision of the estimate from the separate modeling strategy in 2), which is largely a function of the scatter around the model's curve and the number of district observations per election j. In effect, this multilevel estimator shrinks the separate estimate from 2) to the pooled estimate from 1) relative to the (im)precision of the separate estimates from 2).

Following the multilevel modeling strategy, the empirical version of Equation (2) is a mixed effects model that allows the parameters T_0, k and h to vary across elections j, while borrowing strength for parameter estimation across elections in order to compensate for the paucity of data points in any single election:

$$\begin{aligned} T_{ij} = T_{0j} + (1 - T_{0j})\left(1 - \exp[-k_j(N_{ij} - 1)]\right)\exp[-h_j(N_{ij} - 1)] + e_{ij} \\ e_{ij} \sim \mathrm{Normal}(0, \sigma_e^2) \\ T_{0j} \sim \mathrm{Normal}\left(\tau_0, \sigma_{T_0}^2\right) \\ k_j \sim \mathrm{Normal}(\kappa, \sigma_\kappa^2) \\ h_j \sim \mathrm{Normal}\left(\eta, \sigma_\eta^2\right). \end{aligned} \tag{7}$$

Table 1. Descriptive statistics for 17 countries included in the empirical analysis

Country	Elections			Turnout		Effective number of parties		Number of districts	
	First	Last	Number	Mean	S.D.	Mean	S.D.	Mean	S.D.
Australia	1901	1917	7	0.644	0.043	2.055	0.076	64.6	2.2
Belgium	1847	1898	28	0.771	0.015	1.548	0.048	24.3	1.6
Denmark	1901	1915	7	0.654	0.043	1.942	0.091	95.1	14.5
Finland	1907	2011	36	0.668	0.015	3.902	0.127	15.0	0.4
France	1973	2002	7	0.720	0.028	3.730	0.235	519.7	16.6
Germany	1871	1912	13	0.682	0.029	2.171	0.071	395.2	1.8
Greece	1952	1952	1	0.727	0.000	2.184	0.000	99.0	0.0
Iceland	1919	1931	4	0.720	0.047	1.758	0.093	23.0	2.5
India	1977	2004	11	0.580	0.036	2.664	0.085	437.8	62.4
Ireland	1969	2011	12	0.713	0.014	3.174	0.145	52.3	5.8
Japan	1947	1993	18	0.727	0.007	3.037	0.110	110.1	3.3
Netherlands	1888	1917	9	0.720	0.046	2.232	0.122	77.0	6.9
Norway	1882	1985	20	0.735	0.026	2.768	0.190	56.4	9.5
Portugal	1975	2011	13	0.717	0.032	3.131	0.143	20.0	0.0
Spain	1977	2011	11	0.719	0.022	3.062	0.104	52.0	0.0
Sweden	1911	1960	16	0.691	0.026	3.217	0.080	34.9	3.1
Switzerland	1919	2011	24	0.564	0.015	2.847	0.126	23.1	0.3

Thus far, the empirical model sketched is a two-level model with electoral districts nested within elections. The model can easily be extended to also include random effects at the country level, to allow unobserved country factors to have an impact on T_0, k and h. Each model parameter is then composed of:

1. a fixed component that is common to all the 237 elections considered, which gives an average characterization of the relationship between N and T;
2. a random component that depicts the deviations specific for country l from that overall pattern;
3. and an election-specific random component that picks up idiosyncratic deviations from the general country pattern:

$$T_{ijl} = T_{0jl} + \left(1 - T_{0jl}\right)\left(1 - \exp\left[-k_{jl}\left(N_{ijl} - 1\right)\right]\right)\exp\left[-h_{jl}\left(N_{ijl} - 1\right)\right] + e_{ijl} \quad (8)$$

The model has been fitted by maximizing the restricted log likelihood (REML) using nmle, an R-package for linear and nonlinear mixed effects models (Pinheiro et al., 2013). Computer code and detailed results are available from the authors on request.

5. Empirical Results

Table 2 gives REML estimates of this multilevel model fitted to the district-level election data. The mean estimates are 0.540 for parameter T_0, 0.668 for k and 0.272 for h, indicating that the average pattern of T versus N is indeed characterized by a steep initial increase followed by a subsequent gradual decrease. For this pooled data, our expectation that $1 > k > h > 0.1$ holds true. However, as the country- and election-level σ-estimates of the standard deviations around these mean parameter estimates suggest, it is important to look at the data in a more disaggregated fashion to see if there are national and election-specific distinctions. Overall, the model seems to fit the data reasonably well. The average difference between predicted and observed turnout in the sample as expressed by the residual standard deviation is 0.09. By contrast, the residual standard deviation of a pooled model that constrains all the model parameters to be constant across countries and elections is considerably higher (0.134). An intermediate two-level model that lets the parameters vary by country produces a residual standard deviation of 0.116.[7] Once again, these results demonstrate that it is important to take differences between countries and elections into account.

5.1. *Country-Specific Variation in T_0, k and h*

What about inter-country comparisons? Well, we would certainly expect that random scatter would be extensive with cross-national pooled data. But it seems worth a try. The broad comparative question is whether the values of T_0, k and h are all country-specific, or whether some universal values prevail for at least some of the cases. And if so, might we be able to develop a meta-theory that would allow us to predict which countries might have similar values?

There is, indeed, substantive variation in the country-specific parameter estimates. We show in Table 3 the empirical Bayes predictions of the country-level parameters. In eight of the 17 country cases (Australia, Germany, Greece, Iceland, India, Ireland, Spain and Switzerland), our expectation that $1 > k > h > 0.1$ holds on (cross-election) average. As Figure 2 illustrates, the *T* versus *N* patterns in those countries are all characterized to a greater or lesser extent by an initial increase and a subsequent decrease in turnout, as the numbers of parties increase. The amount of scatter around most country-specific curves is still substantial. However, when looking at Figure 2 one has to keep in mind that not all the scatter around the curves in Figure 2 reflects variance in turnout our model is unable to predict. The curves in that figure only include the country-specific deviations from the overall parameter estimates (i.e., the level-3 random effects), not the election-specific deviations (i.e., the level-2 random effects) which account for a fair amount of the scatter. This can be seen in Figure 3, whose curves include both the country- and election-specific deviations for one specific country case.

For another four countries (Belgium, Denmark, Japan and Norway), it seems still true that $k > h$. However, the estimates of *k* for Belgium and Japan exceed a value of 1 and, in the case of Belgium, the resulting steep increase of turnout with rising numbers of parties is not counteracted by estimated values of *h* greater than 0.1. In these cases, turnout is predicted (and observed) to monotonically rise with increasing

Table 2. REML estimates of the nonlinear multilevel model of turnout fitted to district level election data from 237 national elections in 17 countries.

Fixed	Estimate	S.E.	t value
τ_0	0.540	0.030	18.147
κ	0.668	0.110	6.079
η	0.272	0.060	4.497
Random: Countries	S.D.	Correlation	
σ_{t0}	0.113	t_0	
σ_k	0.427	0.287	k
σ_h	0.230	0.248	0.146
Random: Elections	S.D.	Correlation	
σ_{t0}	0.099	t_0	
σ_k	0.284	0.201	k
σ_h	0.098	0.104	0.003
σ_e	0.090		
Countries	17		
Elections	237		
District observations	22,981		

Table 3. Empirical Bayes predictions of the country specific parameters T_0, k and h. Parameter estimates that do not correspond to theoretical expectations are printed in bold. Optimal numbers of effective parties and maximum expected turnout is given in the final columns.

Country	T_0	k	h	N_{opt}	T_{max}
Australia	0.426	0.841	0.296	2.60	.690
Belgium	0.608	**1.573**	**0.094**	2.82	.920
Denmark	0.572	0.200	**−0.152**	*	*
Finland	0.423	0.325	**0.094**	5.60	.713
France	0.674	0.471	0.659	2.15	.738
Germany	0.574	0.566	0.413	2.52	.705
Greece	0.495	0.750	0.177	3.21	.771
Iceland	0.647	0.414	0.272	3.23	.763
India	0.476	0.715	0.631	2.06	.618
Ireland	0.575	0.947	0.447	2.20	.744
Japan	0.584	**1.305**	0.495	1.99	.767
Netherlands	0.418	**1.102**	0.157	2.89	.797
Norway	0.392	0.862	**0.090**	3.74	.822
Portugal	0.586	0.269	**0.056**	7.54	.824
Spain	0.626	0.549	0.475	2.40	.729
Sweden	**0.732**	**−0.078**	0.214	*	*
Switzerland	0.369	0.539	0.201	3.42	.652

Note: * Values undefined, see Equations (2) to (4).

numbers of parties. This is also true for Denmark, for which we even get a slightly negative estimate of h. Finally, for France and Sweden, the statistical results indicate h values larger than k, which contradicts our expectations. Other minor violations of

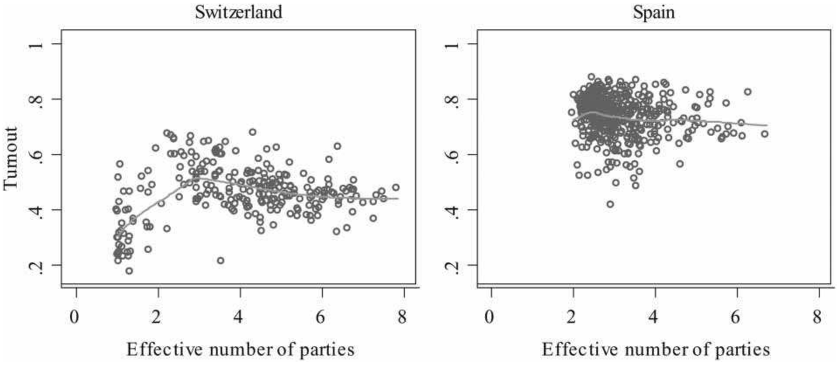

Figure 1. Turnout versus the effective number of electoral parties at the district level in Swiss National Council elections, 1971 2007, and in Spanish congressional elections, 1977 2004. Line represents locally weighted regression fit (LOWESS). Figure taken from Grofman and Selb (2011: 97).

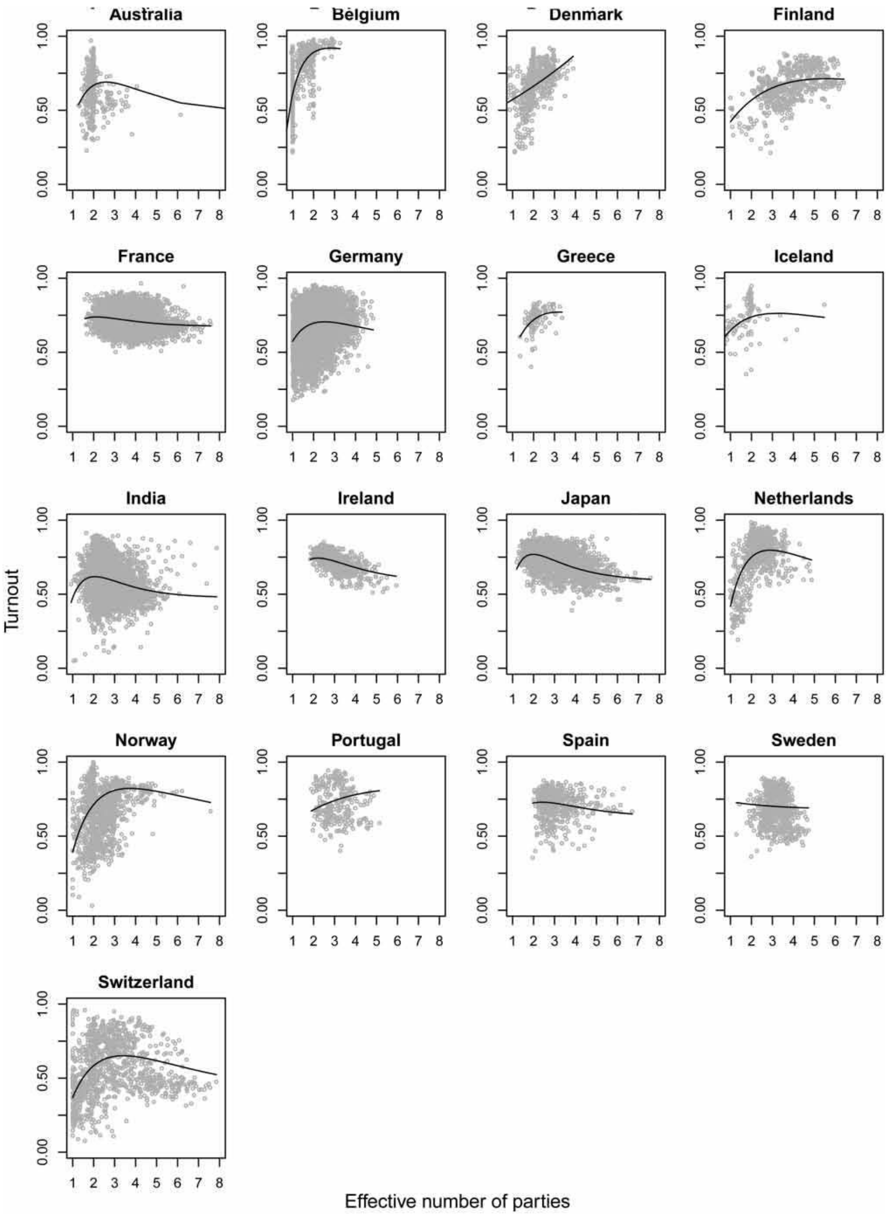

Figure 2. Turnout versus effective number of parties: observed turnout and predicted from the nonlinear multilevel model of turnout (country level).

expectations occur in the Netherlands, where we get a k value greater than 1; and in Portugal, where we get an h value less than 0.1.

We also developed a theoretical expectation for the T_0 parameter, namely that, in each country, it would be below the mean turnout shown in Table 1. This expectation is confirmed for all but Sweden.

The parameters that contradict one or more of our theoretical expectations are highlighted in Table 3. However, as we see when we compare the highlighted items in Table 3 with the graphs shown in Figure 2, generally speaking, the fact that some parameter value is outside the predicted range, has little effect on the curvilinearity of the relationship between effective number of parties and turnout, with Belgium, Denmark and Finland the only clear outliers.

5.2. *Within-Country Evidence: The Swiss Case*

Thus far, we have looked at country-specific distinctions from the general pattern observed, and the estimates for the model's variance components (the σ's) indicate that countries are indeed the major source of variation in T_0, k and h. However, there still is considerable variation in the parameters within countries at the level of elections, too, which is driven by the tremendous scatter of the empirical data around the country-level curves observed in Figure 2. Part of this dispersion is probably due to shifts in average turnout across elections, which tend to blur otherwise quite clear election-specific patterns of T versus N. By including both national- and election-specific parameters, our empirical model allows for such shifts.

To illustrate such cross-election variability within a country, we will use Switzerland 1919 2011 as an example. We chose Switzerland since, of all the countries included in our empirical analysis, Switzerland exhibits the widest variation in both T and N, probably owing to the fact that the Swiss electoral system features district magnitudes ranging from 1 to 35 parliamentary seats. A closer look at the Swiss panel in Figure 2 suggests that there is, in fact, substantial variation in T that our model's fixed and country-level random effects alone (represented by the solid curve) are unable to account for. Last but not least, it was the Swiss case that motivated this piece of research (see Figure 1). Observed and predicted turnout as a function of N for Swiss national parliamentary elections 1919 2011 is plotted in Figure 3. The respective parameter estimates are given in Table 4.

While the general nonlinear pattern seems to hold in each and every Swiss national election considered, some cross-election patterns are evident, too. First, there is tremendous over time variation in $T_{0.}$ As T_0 should reflect very permanent attitudes stable from decade to decade plus the salience of issues at stake in the given election, one may argue that the latter fluctuates appreciably. A more plausible ad hoc interpretation, the stark variation in T_0 may also be a matter of whether uncontested district elections are actually fought or not. There are plenty of historical instances where party elites in single-member districts perfectly coordinate, for example, by dividing the lower and upper house seats among them before the election, so that the election is suspended in the district (euphemistically labeled "tacit elections").

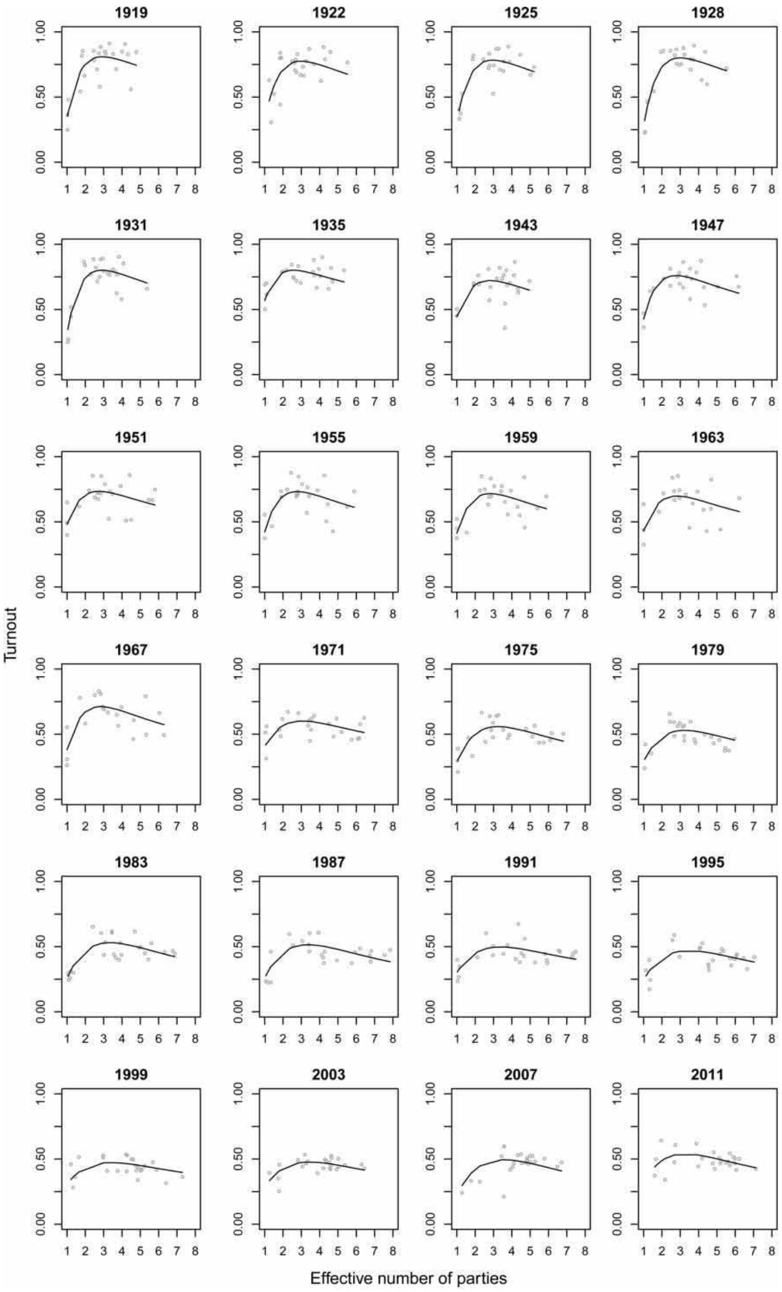

Figure 3. Turnout versus effective number of parties: observed turnout and predicted turnout from the nonlinear multilevel model of turnout versus district magnitude, Swiss National Council Elections, 1919 2011.

Table 4. Empirical Bayes predictions of the election specific parameters T0, k and h for Swiss National Council elections, 1919 2011. Parameter estimates that do not correspond to theoretical expectations are printed in bold.

Year	T_0	k	h	N_{opt}	$T_{\max}$
1919	0.363	0.954	0.104	3.432	0.809
1922	0.409	0.710	0.115	3.775	0.779
1925	0.344	0.917	0.125	3.313	0.776
1928	0.274	**1.163**	0.115	3.071	0.795
1931	0.307	**1.198**	0.129	2.946	0.794
1935	0.578	0.616	0.184	3.386	0.787
1943	0.459	0.624	0.211	3.204	0.713
1947	0.440	0.755	0.178	3.194	0.747
1951	0.486	0.631	0.219	3.149	0.724
1955	0.428	0.835	0.224	2.860	0.725
1959	0.419	0.735	0.217	3.012	0.709
1963	0.436	0.736	0.265	2.806	0.693
1967	0.382	0.902	0.233	2.755	0.708
1971	0.404	0.449	0.293	3.069	0.601
1975	0.292	0.470	0.245	3.279	0.558
1979	0.271	0.648	0.327	2.686	0.550
1983	0.260	0.449	0.244	3.325	0.532
1987	0.262	0.434	0.259	3.268	0.519
1991	0.304	0.307	0.267	3.493	0.495
1995	0.241	0.413	0.283	3.179	0.484
1999	0.284	0.439	0.330	2.927	0.500
2003	0.311	0.214	0.238	3.997	0.471
2007	0.262	0.229	0.171	4.711	0.486
2011	0.318	0.395	0.268	3.293	0.538

In other instances, some challenger blows the deal so that the election has to be fought. In such instances, only the most undaunted (20% or so) of the voters show up at the polls. We think it is the latter instances that drive T_0 down so much in some elections.

Second, it seems that initial turnout increases, as indicated by large *k*, have weakened over the observation period, while subsequent decreases have gotten more pronounced a development that might be attributed to the fact that Swiss national elections got more competitive over the years, particularly in the districts hosting small numbers of parties.

Moreover, it seems that there are two quite distinct periods in Swiss electoral history, one up through and including 1967, and one afterward. One possible explanation is that the electorate more or less doubled from 1967 to 1971, due to the late enfranchisement of women, which might have structurally changed the nature of electoral competition. Thus, a closer inspection of the Swiss case clearly indicates how important it is to look at the data in a more disaggregated fashion to see if there are time trends or break points that suggest the need for caution. Nevertheless, the general nonlinear pattern in the relationship between *N* and *T*, which has been the focus of this article, holds true.

6. Conclusions

The evidence in this article is that simple models can work quite well, if they are "not too simple." A linear model of the relationship between the effective number of parties and turnout is too simple. A three-parameter model based on logical boundary constraints that recognizes that there are two forces affecting turnout that push in opposite directions as we increase the number of parties (or district magnitude) does, however, seem just about simple enough.

Looking at the data at the country level, as shown in Figure 2, we find the rising then falling pattern of turnout we have posited in all but Belgium, Denmark and Finland. Recall, too, that we hypothesized that the relationships among our parameter estimates should, in general come close to satisfying $1 > k > h > 0.1$, though we also indicated that contrary outcomes are not impossible. For the pooled data, we indeed find perfect agreement with this prediction. For the country-specific parameters in Table 2, however, we see that this expectation holds true in only a bare majority of cases, but several of the exceptions are minor, and in only three countries is there a pattern that contradicts our expectation that turnout will be falling for values of the effective number of parties above three indeed, in a number of countries it appears to begin dropping for even lower values of the Laakso-Taagepera index. Moreover, our expectation that T_0 will be less than mean turnout is met for 16 of our 17 countries (with Sweden the only exception). When we look below the national level at the Swiss case, our fit is equally good. For the within-nation Swiss National Council elections, 1919 2011, we find with two minor exceptions (see Table 4) perfect fit for the expectation that $1 > k > h > 0.1$.

We believe that the type of modeling we offer here has implications beyond the specific empirical relationships we examine. There are five key elements.

First, we make use of a model whose functional forms take into account logically necessary boundary constraints. Second, we build in nonlinearities in a theoretically motivated fashion in a way that allows us to take into account previous empirical work. We do not model merely for the sake of elegant modeling. Third, the model we offer is as simple as possible, but no simpler than necessary. In particular, it is complex enough to account for the puzzling empirical finding previously reported for Switzerland and Spain of a curvilinear pattern of turnout rising then falling with effective number of parties. Fourth, we have been deliberate in our choice of cases, looking only at countries that use what Taagepera (2007) calls "simple" electoral rules, but which nonetheless include a wide range of cases. This allows us to offer a very general model that avoids the need for a plethora of dummy variables to seek to account for idiosyncratic features of particular aggregation processes. Fifth, we illustrate MLE methods that can combine pooled data with case-specific data so as to deal with problems caused by estimates with high standard errors derived from small samples.

We would emphasize, however, that we are not interested in optimizing model fit by finding the best fitting curve.[8] We are seeking to fit a model chosen for its theoretical features, including simplicity. In model fitting, if we hypothesize a particular parameter value (as in the work of Taagepera on square root laws and related results involving other exponents: see, e.g., Taagepera, 2007, 2008; Taagepera & Shugart, 1989) then, if data confirm this slope, the high degree of scatter (as we observe here) and R-square are irrelevant; indeed, the logically predicted slope or pattern emerging *despite* scatter makes it even more impressive.[9] Our aim in this article has not been to account for variation in turnout T as fully as possible, but to assess a specifically predicted and theoretically derived form of the relationship between the number of parties N and T.[10]

We would acknowledge that there are limits to what we can say about causality from the kinds of data that we examine here.[11] In particular, we are limited in what we can say about the specific mechanisms that generate our upward sloping and downward sloping effects. As we see it, the next step in the line of research we have added to here, is to move to more micro-level analyses, including survey data, to examine the processes that generate the curvilinear results we observe at the district level. We proposed several mechanisms that would operate to either reduce turnout as the number of parties increases (e.g., increased cognitive complexity due to the existence of very similar choices) or increase turnout as the number of parties increases (e.g., greater likelihood of a choice close to the voter's own ideal point), and identified other factors whose effects are harder to access *a priori* (e.g., decisiveness of elections, disproportionality) but with the kind of data we have we cannot empirically assess their relative importance.

As we noted at the beginning of this article, our aim has never been to account for variation in turnout T as fully as possible, but to assess the relationship between the number of parties N and T. What we sought to do is to find ways to simplify models to the point where specific outcomes can be predicted and predictions can be tested, rather than adding variables whose effects we cannot clearly identify in advance. We address one specific effect, in a *ceteris paribus* environment. We hypothesize that for very low N (however caused), the number of distinct choices are limited and hence, *ceteris paribus*, the incentive to vote might be slightly depressed; and that for very high N (however caused), the distinction among choices is reduced and the number of choices is high leading to limits on cognitive processing and hence, *ceteris paribus*, the incentive to vote might again be slightly depressed. This simple model predicts a central hump. The remarkable thing is that for most countries our data analysis (involving 237 national elections in 17 countries, with a total of 23,000 district observations) does find this hump, often in the face of scatter that should make detection impossible.

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Notes

1. For example, one of the factors involved in complexity in multi party settings is attempting to assess what governing coalition will form (Kedar, 2009).
2. The general reasons why the effective number of parties (Laakso & Taagepera, 1979) is used so widely in comparative politics rather than the raw number of parties is that the total number of parties that run in elections depends heavily on the ease of registering parties and independent candidates that net only a few votes, and there is considerable variation in party size. The effective number of parties provides a single meaningful number. Nonetheless, when we have replicated our analyses with the raw number of parties, we get similar results (data omitted for space reasons).
3. Since most modeling in political science still uses linear models, whether they be OLS or otherwise, it is often easy to miss curvilinearity in the data in the absence of theoretical expectations that lead the analyst to look for such nonlinearities. There are, however, a number of recent papers that offer theoretical reasons to expect curvilinearity. For example, in the election context, Ashworth et al. (2006) argue that, rather than high competition levels leading to greater turnout, the relationship between competition and turnout should be curvilinear. The reason they offer is that, while the relationship should be positive for "Downsian" voters, it should, they claim, be negative for "expressive" voters who vote as an expression of identity politics, and the fact that there is a mix of the two types in the population will lead to a curvilinear pattern. Stoll (n.d., cited in Moser and Scheiner, 2012: 186 195; see also Madrid, 2005) argues that, rather than the number of parties increasing with social heterogeneity, as is commonly supposed to be the case, the pattern should instead be curvilinear. As summarized by Moser and Scheiner (2012: 186 187) her argument is that, past some point, "further increases in diversity when there are larger numbers of (especially small groups actually reduce the likelihood that any of the groups will play a meaningful governing role. Under these circumstances, political entrepreneurs have no incentive to create parties (or promote) candidates" for small groups, leading to "a consolidation of the party system." However, while both these papers propose a curvilinear model, neither seeks to model the functional form that governs that curvilinearity.
4. Simple systems include first past the post/single member plurality (SMP) electoral systems such as the United States or India, single transferable vote (STV) systems like Australia and Ireland, single non transferable vote (SNTV) systems like (pre 1996) Japan, or systems of so called "districted proportional representation (DPR)" such as Spain or Switzerland (Monroe & Rose, 2002) in which there is no cross district compensation for disproportionalities resulting from the vote seat translation at the district level.
5. The data are freely available from the CLEA website at http://www.electiondataarchive.org/index.html. We use download release 4 (September 14, 2011).
6. For SMD countries, there are also some difficulties in obtaining a sufficiently long time series that reports votes for minor parties at the district level. Countries such as the United States and Great Britain are not in the database we used.
7. Detailed results are available from the authors on request.
8. *Curve fitting* is a statistical endeavor entirely or mostly oblivious to the nature of the data and the substantive implications of the findings, and (adjusted) R square is the supreme criterion. In the present case, the curve fitter might well be satisfied with a linear fit, because, given the huge scatter, no other curve would raise the dismally low R square to any appreciable extent. A curve fitter is not disturbed by this line projecting to turnouts larger than 100% (if, as expected, the linear fit is upward sloping) since there are no data points where the unrealism of this prediction might be noted.

9. *Model fitting* involves two stages: (1) developing a theoretically grounded set of expectations and developing a functional form that is appropriate, and (2) evaluating the goodness of result. The first stage is often art as much as standard procedure. For the second, model fitting is in stark contrast to what is usually referred to as curve fitting that aims at maximizing R^2 or related empirical goodness of fit measures. For example, if we hypothesize that a given parameter has a certain value or range of values then we would reject the model fit, regardless of R^2, if that parameter was clearly outside the predicted range rather than testing a null hypothesis of no relationship. In the approach to model testing we use here, for each of the countries, we simply report which parameters violate our theoretical expectations discovering that our model does quite well. (An alternative approach might constrain a parameter at a specific value and then use a likelihood ratio test to compare the log likelihood of this model to that of the unconstrained model where the parameter is freely estimated.)
10. The chief concern with omitted variables is that their omission will lead to a misspecification of the relationship between the included variables and the dependent variable of interest. Factors that affect *T* but are unrelated to *N* are irrelevant. To the degree *T* and *N* interact through other processes (and through third factors) not included in our simple model, they would be likely to produce different patterns. Thus they would tend to blur not reinforce the relatively simple pattern our simple model expects. Two potential confounders that might affect both *N* and *T* are electoral system strength and social heterogeneity. Our implicit assumption is that those factors indirectly affect *T* via *N*. Problems would only occur if those factors had an impact on *T* other than through *N*, for instance, electoral system → competition → *T* (Cox, 1999), or social heterogeneity → conflict → *T* (Huckfeldt, 1979). But as far as we are aware, none of those theories would suggest the particular pattern of nonlinearities between *N* and *T* we observe for almost all the countries in our data set.
11. Determination of causality requires attention to issues of model specification. At the suggestion of a reviewer, we have now replicated our analyses with raw numbers of party instead of effective number of parties, *N*, and the results look quite similar. This reinforces our view that one primary causal process affecting turnout involves number of parties, e.g., some aspect of complexity of choice lowering turnout while more options for choice raising turnout (e.g. Brockington, 2004; Geys & Heyndels, 2006). We deliberately have not made direction of causality central in our article because, in the sciences, many processes involve simultaneous determination, e.g., in Ohm's Law about the relationships between measures of resistance and measures of flow. While we do agree that it is quite likely that *T* and *N* interact reciprocally by various other processes, and third factors act on both, thus connecting them indirectly, our various theoretical lines of argument all suggest that *T* is primarily a function of *N* rather than *N* a function of *T*.

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