

# Restricted Export Flexibility and Risk Management with Options and Futures\*

Axel F. A. Adam-Müller\*\*

Center of Finance and Econometrics  
Department of Economics  
University of Konstanz, D - 78457 Konstanz, Germany  
Tel.: +49-7531-882346, Fax: +49-7531-883559  
email: axel.adam-mueller@uni-konstanz.de

Kit Pong Wong

School of Economics and Finance  
University of Hong Kong, Pokfulam Road, Hong Kong  
Tel.: +852-2859-1044, Fax: +852-2548-1152  
email: kpwong@econ.hku.hk

March 19, 2002

JEL classification: F31; D21; D81

Keywords: restricted export flexibility; risk management; currency futures; currency options

---

\*We would like to thank Jan Beran, Günter Franke, Christian Gollier, Roland Jeske, Henri Loubergé, and participants at the 2001 European Financial Management Association Annual Meeting in Lugano and the 8th Annual Meeting of the German Finance Association in Vienna for helpful comments and suggestions. Any remaining errors are our responsibility. We gratefully acknowledge financial support from the Germany/Hong Kong Joint Research Scheme provided by the German Academic Exchange Service (DAAD) and the Hong Kong Research Grants Council (RGC).

\*\*Corresponding author.

# Restricted Export Flexibility and Risk Management with Options and Futures

This paper examines the production, export and risk management decisions of a risk-averse competitive firm under exchange rate risk. The firm is export flexible in allocating its output to either the domestic market or a foreign market after observing the exchange rate. Export flexibility is restricted by certain minimum sales requirements that are due to long-term considerations. Currency options are sufficient to derive a separation result under restricted export flexibility. Under fairly priced currency futures and options, full hedging with both instruments is optimal. Introducing fairly-priced currency options stimulates production provided that the currency futures market is unbiased.

JEL classification: F31; D21; D81

Keywords: restricted export flexibility; risk management; currency futures; currency options

## 1 Introduction

Foreign exchange rate fluctuations became a major source of risk for international firms since the Bretton Woods Agreement collapsed in 1973. Consequently, these firms have been using various hedging strategies to cope with the adverse effects of exchange rate risk on their profits.<sup>1</sup> On the one hand, international firms can adopt a real hedge by following a flexible sales or input/output policy which allows them to alter their operations according to realized exchange rates. On the other hand, these firms can rely on a financial hedging strategy which is typically based on currency derivatives such as currency futures and options. In any case, there is a close link between the hedging activities in the markets for goods and services and the financial hedging measures. This paper analyzes the interaction between the firm's real and financial risk management decisions in the

---

<sup>1</sup>In a survey conducted by Rawls and Smithson (1990), foreign exchange risk management is indicated by financial managers to be among their primary objectives.

context of a competitive exporting firm. In addition, the paper provides a particularly simple framework in which the joint use of currency futures and options is optimal.

In the literature on the competitive exporting firm under exchange rate risk, it is typically assumed that the risk-averse firm makes its production and export decision prior to the resolution of exchange rate uncertainty (see, e.g., Benninga *et al.* (1985), Kawai and Zilcha (1986) and Adam-Müller (1997, 2000). In this case, the firm is *inflexible* since it cannot react on the realized exchange rate. Its profits are linear in the exchange rate. Consequently, the existence of currency futures is sufficient to derive a separation theorem which states that the firm's production decision is independent of its attitude towards risk and the exchange rate distribution. In an unbiased currency futures market, the firm completely eliminates exchange rate risk by holding a full hedge position. As shown by Lapan *et al.* (1991) and Battermann *et al.* (2000), fairly priced currency options play no role for an inflexible firm.

In an alternative approach, the firm is allowed to decide whether to export or not after observing the exchange rate. This approach, originally proposed by Ware and Winter (1988), has been further developed by Broll and Wahl (1997) in a rigorous formal model. While production takes place prior to the resolution of uncertainty, the firm makes its export decision (i.e. sales allocation between the domestic market and a foreign market) after the resolution of uncertainty.<sup>2</sup> Hence, the firm is *fully flexible* in exporting or refraining from exports. Profits of a fully export flexible firm are piecewise linear in the exchange rate with zero slope for low exchange rate realizations. The existence of currency call options is sufficient to derive the separation result. Fairly priced call options are the only hedging instrument used. The existence of unbiased currency futures is irrelevant.

The first class of models can explain the use of currency futures, the second can explain the use of currency options. But exporting firms typically employ various types of derivatives for managing their exchange rate risk (see, e.g., Bodnar and Gebhardt 1999). Thus, models in which an exporting firm relies exclusively on currency futures or

---

<sup>2</sup>Ben-Zvi and Helpman (1992) argue that international transactions are better described by such a sequence of moves. This is supported by the empirical evidence in Magee (1974).

exclusively on currency options might seem unsatisfactory. Our model extends the work of Broll and Wahl (1997) in a way that provides a rationale for the joint use of currency futures and options. This is done by restricting the firm's export flexibility.

Under *restricted* export flexibility, the firm has to maintain certain minimum levels of domestic sales and exports such that the degree of flexibility enjoyed by the firm varies inversely with the tightness of these minimum levels. The reason for assuming the existence of such minimum levels of domestic sales and exports is the observation that firms typically have explicit or implicit obligations to remain present in a market even under (temporarily) unfavorable conditions. These obligations may either be due to already signed contracts with customers or be simply due to the necessity to maintain a minimum level of activity in a market in order to remain visible to future customers.<sup>3</sup> This minimum level of activity is the result of a longer-term consideration in which market exit and entry costs determine whether a firm is currently in the market with at least the minimum level of activity or whether the firm is not in the market at all. However, it is not the purpose of this paper to analyze this longer-term market entry decision. Instead, it is taken as given that market entry and exit costs are such that it is currently optimal for the firm to be present in the domestic and the export markets for longer-term reasons even if this market presence is not necessarily favorable in the period of time considered in this paper.

Given restricted export flexibility, the firm's optimal sales allocation rule is state contingent: It exports more than the minimum level to the foreign market when the realized exchange rate is sufficiently favorable such that the foreign price (measured in units of the domestic currency) exceeds the domestic price; otherwise, it maintains the minimum level of exports and sells the rest in the domestic market. Alternatively put, exports are like a real option with a strike price equal to the domestic price. The sales allocation between the domestic and foreign markets provides the firm an implicit real hedge against adverse

---

<sup>3</sup>Bagwell and Staiger (1989) and Bagwell (1991) show that export subsidies facilitate the entry of high-quality firms under asymmetric information. Shy (2000) goes one step further and argues that the decision to export is chosen to signal product quality, despite the fact that exporting is dominated by non-exporting under symmetric information.

exchange rate changes.

It will be shown that the separation theorem can be derived even in the absence of currency futures. If currency futures and options are fairly priced, it is optimal to fully hedge with a portfolio that consists of both currency futures and options. The joint use of these derivatives is due to the fact that the restricted export flexible firm's exposure is piecewise linear in the exchange rate with strictly positive slope everywhere. In addition, the paper analyzes optimal production and risk management decisions when there are no currency options available. It is also shown that making fairly-priced currency options available to the firm enhances production provided the currency futures market is unbiased.

The argument for the *joint* use of currency futures and options proposed in this paper has the advantage of being particularly simple since it relies on a one-period model with one single source of risk. Other models explaining the joint use of futures and options are much more complex since they either require the existence of several sources of risk as in Lapan *et al.* (1991), Lapan and Moschini (1994), Moschini and Lapan (1995), Broll *et al.* (2001), Frechette (2001) and Mahul (2002) or a multi-period framework as in Lence *et al.* (1994).

The model which comes closest to the spirit of ours is the model of Moschini and Lapan (1992) who analyze a competitive firm with production flexibility under output price uncertainty. In their model, there are two types of inputs. The decision on the use of quasi-fixed inputs has to be made before price uncertainty is resolved whereas the decision on other inputs can be made under certainty. Hence, this firm's flexibility is restricted by the obligation to decide on the level of quasi-fixed inputs before price uncertainty is resolved. Moschini and Lapan (1992) show that the optimal hedging portfolio consists of both futures and options if the profit function is quadratic and the price distribution is symmetric. In contrast to their model of restricted production flexibility, we analyze restricted export flexibility without imposing a similar symmetry requirement on the distribution. However, the profit function analyzed by Moschini and Lapan (1992) is

more general than ours. In both models, the joint use of futures and options is optimal.

The paper is organized as follows. Section 2 delineates the model. Section 3 characterizes the firm's optimal production and risk management decisions when both currency futures and options are available. Section 4 derives the firm's optimal production and risk management decisions when there are currency futures only. Section 5 concludes.

## 2 The model

Consider a risk-averse competitive firm which produces a single commodity  $Q$ . The cost function is  $c(Q)$ , where  $c(0) \geq 0$ ,  $c'(Q) > 0$  and  $c''(Q) > 0$ . The firm supplies its entire output to two markets: the domestic and a foreign market. The per-unit price in the domestic market,  $P_d$ , is denominated in domestic currency. The per-unit price in the foreign market,  $P_f$ , is denominated in foreign currency.  $P_d$  and  $P_f$  are fixed and known to the firm. Due to the segmentation of the domestic and the foreign market, commodity arbitrage is unprofitable so that the law of one price does not necessarily hold.

At date 0, when the firm makes its production decision, it does not know the exchange rate (in units of domestic currency per unit of foreign currency). The exchange rate, denoted by  $\tilde{S}^4$ , is distributed according to a cumulative distribution function,  $G(S)$ , over support  $[\underline{S}, \overline{S}]$ , where  $0 \leq \underline{S} < \overline{S} < \infty$ . Prior to making its export decision at date 1, i.e. before the sales allocation between the domestic and foreign market, the firm observes the realization of the exchange rate. For high exchange rate realizations, it is attractive to export since the domestic currency value of the firm's foreign exchange revenue is also high. In contrast, for low exchange rate realizations, the firm will sell on the domestic market. Hence, the sales allocation decision at date 1 depends on the realization of the exchange rate. In this sense, the firm is export flexible. The possibility to export can thus be regarded as a real option held by the firm. This option is exercised if the exchange rate is sufficiently high. The time structure is summarized in Figure 1.

---

<sup>4</sup>Throughout the paper, random variables have a tilde ( $\sim$ ) while their realizations do not.

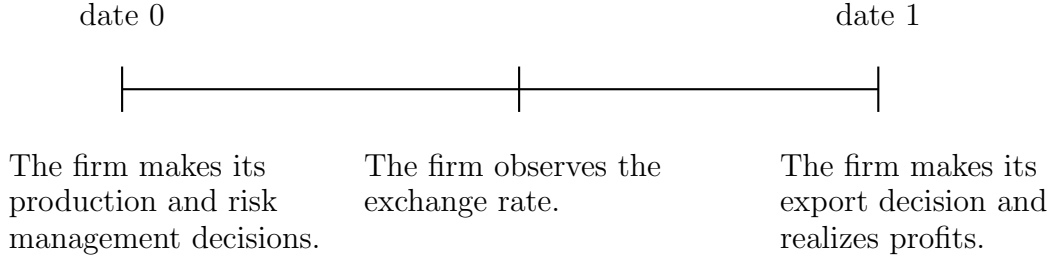


Figure 1: *Time line*

However, it seems realistic that export flexibility is restricted to some extent. Due to various explicit and implicit obligations, the firm has to maintain certain minimum levels of domestic sales and exports. These quantities are exogenously given and are denoted by  $Q_d$  for the domestic market and by  $Q_f$  for the foreign market. Thus, the firm's flexibility only applies to the amount of output which exceeds the sum of these minimum levels of domestic sales and exports, i.e. restricted flexibility only applies to  $Q - Q_d - Q_f > 0$ .

Given an exchange rate realization  $S$  and the firm's restricted export flexibility, its optimal decision on the allocation of output between the domestic and foreign markets is as follows: If  $SP_f > P_d$ , the domestic currency revenue from exporting is higher than that from selling in the domestic market. Hence, the firm exercises its real option and exports as much as possible to the foreign market,  $Q - Q_d$ , while still meeting the minimum level of domestic sales,  $Q_d$ . For  $SP_f \leq P_d$ , the firm maintains only the minimum level of exports,  $Q_f$ , and sells the rest,  $Q - Q_f$ , in the domestic market. It is assumed that there is at least some probability mass for realizations of  $\tilde{S}$  below  $P_d/P_f$  and at least some mass for realizations above this value,  $\underline{S} < P_d/P_f < \bar{S}$ .

Given the optimal sales allocation rule, the firm's domestic currency revenue,  $\tilde{R}$ , is given by

$$R = \begin{cases} SP_f Q_f + P_d(Q - Q_f) & \text{if } SP_f \leq P_d, \\ SP_f(Q - Q_d) + P_d Q_d & \text{if } SP_f > P_d. \end{cases} \quad (1)$$

Writing the above equation in a compact way yields

$$\tilde{R} = \tilde{S} P_f Q_f + P_d(Q - Q_f) + P_f \max(\tilde{S} - P_d/P_f, 0)(Q - Q_d - Q_f). \quad (2)$$

It is evident from the last summand in equation (2) that there are options embedded in the

firm's domestic currency revenue. Figure 2 illustrates this graphically. The steeper thick line represents the firm's domestic currency revenue if it exports  $Q - Q_d$  to the foreign market. The slope is  $P_f(Q - Q_d)$ . The flatter thick line represents the firm's domestic currency revenue if the firm exports only the minimum level,  $Q_f$ , to the foreign market. Here, the slope is only  $P_f Q_f$ . Since the firm is export flexible, it will always choose a sales allocation between the domestic and the foreign market that maximizes its domestic currency revenue. In Figure 2, the domestic currency revenue is represented by the solid part of the two thick lines, which is convex in  $S$  and piecewise linear with positive slope everywhere. This convexity is created by the possibility to export. Inspection of equation (2) reveals that this real option is exercised if  $S$  exceeds  $P_d/P_f$ .

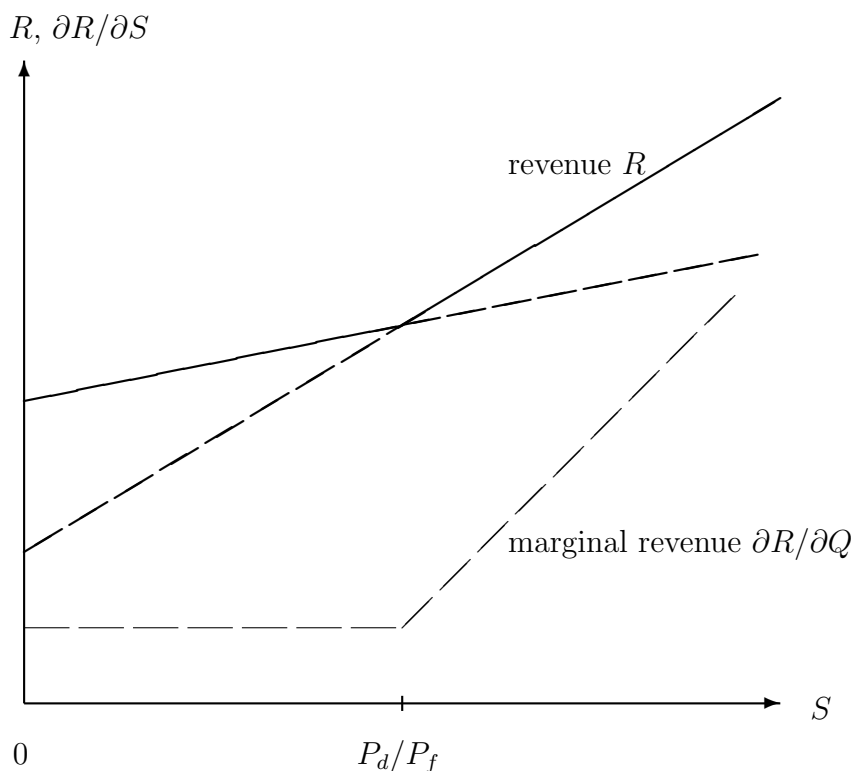


Figure 2: *Total and marginal revenue in domestic currency*

The thin dashed line in Figure 2 represents the dependence of the firm's marginal revenue  $\partial R/\partial Q$  on the exchange rate  $S$ . For exchange rates below  $P_d/P_f$ , any additional unit of output that exceeds  $(Q_d + Q_f)$  is sold in the domestic market. Hence, marginal



revenue is unaffected by the exchange rate. For exchange rates above this level, additional output is exported such that the domestic currency value of marginal revenue linearly increases in the exchange rate with slope  $P_f$ .

To hedge its exchange rate risk exposure, the firm can trade currency futures as well as currency call and put options on the delivery of the domestic currency per unit of the foreign currency. Of course, the firm makes its risk management decision at date 0, i.e. before the realized exchange rate is known. Since the payoffs of any combination of futures, call options and put options can be replicated by any two of these three financial instruments using put-call parity (see, e.g., Sercu and Uppal, 1995), one of them is redundant. Without loss of generality, we restrict the firm to use currency futures and currency call options. Let  $F$  be the futures price and  $H$  be the number of currency futures sold by the firm. In addition, let  $C$  denote the premium of a call option with strike price  $K$  and  $Z$  denote the number of currency call options written by the firm. For simplicity,  $K$  is chosen to be equal to  $P_d/P_f$ .<sup>5</sup> The currency derivatives markets are competitive such that  $F$  and  $C$  are not affected by the firm's positions in these markets.

Taking the optimal sales allocation rule described above as given, the firm's domestic currency profits at date 1, denoted by  $\tilde{\Pi}$ , can be written as<sup>6</sup>

$$\tilde{\Pi} = \tilde{R} + (F - \tilde{S})H + [C - \max(\tilde{S} - P_d/P_f, 0)]Z - c(Q), \quad (3)$$

where  $\tilde{R}$  is defined in equation (2). The firm's decision problem at date 0 is to choose an output level,  $Q$ , and a hedge portfolio,  $(H, Z)$ , so as to maximize the expected utility of its domestic currency profits:

$$\max_{Q, H, Z} E[U(\tilde{\Pi})], \quad (4)$$

where  $E[\cdot]$  is the expectation operator,  $\tilde{\Pi}$  is defined in equation (3) and  $U(\Pi)$  is a von Neumann-Morgenstern utility function defined over the firm's domestic currency profits. The firm is risk averse,  $U'(\Pi) > 0$  and  $U''(\Pi) < 0$ .

---

<sup>5</sup>In practice, it is relatively easy to trade in currency options with any strike price since the majority of currency options is traded in the over-the-counter markets where products are not standardized.

<sup>6</sup>For simplicity, we assume an interest rate of zero such that costs  $c(Q)$  can simply be subtracted at date 1.

The first-order conditions for program (4) are given by

$$\mathbb{E}\left[U'(\tilde{\Pi}^*)\left(P_d + P_f \max(\tilde{S} - P_d/P_f, 0) - c'(Q^*)\right)\right] = 0, \quad (5)$$

$$\mathbb{E}\left[U'(\tilde{\Pi}^*)(F - \tilde{S})\right] = 0, \quad (6)$$

$$\mathbb{E}\left[U'(\tilde{\Pi}^*)\left(C - \max(\tilde{S} - P_d/P_f, 0)\right)\right] = 0, \quad (7)$$

where an asterisk (\*) indicates an optimal level. The second-order conditions for the unique maximum,  $(Q^*, H^*, Z^*)$ , are satisfied given risk aversion and the convexity of the cost function. For ease of exposition,  $Q_d$ ,  $Q_f$ , the distribution of  $\tilde{S}$  and the cost function are assumed to be such that the firm possesses some degree of export flexibility at the optimum,  $Q^* > Q_d + Q_f$ .<sup>7</sup>

### 3 Optimal production and risk management with futures and options

This section characterizes the firm's optimal production and risk management decisions on the premise that the firm can trade both currency futures and currency call options.

First, examine the firm's optimal production decision. Rewriting condition (7) as  $\mathbb{E}[U'(\tilde{\Pi}^*)]C = \mathbb{E}[U'(\tilde{\Pi}^*) \max(\tilde{S} - P_d/P_f, 0)]$  and substituting this into condition (5) yields

$$\mathbb{E}\left[U'(\tilde{\Pi}^*)\right]\left[P_d + P_f C - c'(Q^*)\right] = 0. \quad (8)$$

Since  $U'(\Pi) > 0$  for all  $\Pi$ , equation (8) implies  $c'(Q^*) = P_d + P_f C$ . Thus, the following separation result is established.

---

<sup>7</sup>The case where  $Q^* < Q_d + Q_f$  is a completely different problem from an economic point of view. In order to exclude this case, the condition that  $Q^* \geq Q_d + Q_f$  had to be considered in the optimization problem. Hence, assuming the existence of some flexibility in the optimum only avoids lengthy discussions of Kuhn-Tucker conditions and the corner solution ( $Q^* = Q_d + Q_f$ ) which is of no interest since there is no flexibility at all.

**Proposition 1 (Separation)** *When currency call options<sup>8</sup> with strike price  $P_d/P_f$  are available, the restricted export flexible firm's optimal output,  $Q^*$ , is implicitly given by  $c'(Q^*) = P_d + P_f C$ . Thus,  $Q^*$  depends neither on the preferences of the firm nor on the distribution of the exchange rate.*

Since the derivation of  $c'(Q^*) = P_d + P_f C$  does not involve equation (6), i.e. the use of currency futures, the above separation result holds even when  $H \equiv 0$ . The intuition behind Proposition 1 can be explained using the thin dashed line in Figure 2 which exhibits the firm's marginal revenue with respect to the exchange rate as given by  $P_d + P_f \max(\tilde{S} - P_d/P_f, 0)$ . Since the shape of this function exactly mirrors the shape of the call option's payoff (plus a constant), call options with strike price  $P_d/P_f$  span the firm's exchange rate exposure. Thus, the production decision is based on the market price for this exposure as given by the call premium  $C$ . The optimal production decision is to equate marginal costs with deterministic marginal revenue. If the condition of Proposition 1 is violated, the firm can make a riskless profit. If, for example, output is less than  $Q^*$ , then increasing output and selling the associated exchange rate exposure by writing a call option on  $P_f$  units of foreign currency results in a deterministic profit of  $P_d + P_f C - c'(Q) > 0$ . Hence, the degree of risk aversion and the distribution of the exchange rate cannot affect the optimal production decision.

A natural question to ask in the context of restricted export flexibility is whether the tightness of the restrictions affects the firm's optimal output. The following statement is a direct consequence of Proposition 1.

**Corollary 1** *When currency call options with strike price  $P_d/P_f$  are available, the restricted export flexible firm's optimal output,  $Q^*$ , is not affected by the tightness of the restrictions arising from  $Q_d$  and  $Q_f$ .*

This result is a direct implication of the optimality condition,  $c'(Q^*) = P_d + P_f C$ , which is unaffected by  $Q_d$  and  $Q_f$ . Since marginal revenue is independent of  $Q_d$  and  $Q_f$ ,

---

<sup>8</sup>Due to put-call parity, the availability of currency futures and put options yields the same result.

the optimal production is unaffected by these restrictions as well. Hence, the optimal output is the same irrespective of whether the firm's flexibility is restricted or not. This is due to the fact that marginal revenue with respect to the exchange rate is independent of the restrictions. That is why Broll and Wahl (1997) derive an equivalent result for a fully flexible firm. In contrast to the production decision, the optimal hedge portfolio,  $(H^*, Z^*)$ , depends on the restrictions as will become clear later.

Proposition 1 states that the distribution of the exchange rate does not affect optimal production. This statement, however, has to be interpreted with care since it only holds for a given call option premium  $C$ . As is well-known from the option pricing literature, an increase in the volatility of the exchange rate makes currency options more valuable (see, e.g., Sercu and Uppal, 1995). Thus, it will result in an increase in the call option premium  $C$ . Then,  $c'(Q^*) = P_d + P_f C$  and the convexity of the cost function imply that the firm's optimal output increases in  $C$  and, hence, in the volatility of the exchange rate. This is summarized in the following statement.

**Corollary 2** *When currency call options with strike price  $P_d/P_f$  are available, the restricted export flexible firm's optimal output,  $Q^*$ , increases in the call option premium  $C$  which in turn increases in the volatility of the exchange rate. It follows that the firm produces more as the exchange rate becomes more volatile.*

An immediate implication of Corollary 2 is that export volume and exchange rate volatility should be positively related in countries where export flexibility prevails.<sup>9</sup>

We now turn to the question of how the firm's optimal production decision is affected by the existence of export flexibility. As shown by Benninga *et al.* (1985), Kawai and Zilcha (1986) and others, the optimal output of an export-inflexible firm,  $Q_{\text{inflex}}^*$ , which is obliged to export its entire output is implicitly given by  $c'(Q_{\text{inflex}}^*) = FP_f$ . Comparing this optimality condition and the one given in Proposition 1 yields  $c'(Q_{\text{inflex}}^*) = FP_f <$

---

<sup>9</sup>Together with the theoretical results of Franke (1991), Dellas and Zilberfarb (1993) and Broll and Eckwert (1999), Corollary 2 might therefore explain the positive empirical relation between exchange rate volatility and the volume of international trade found in a number of studies that are surveyed by McKenzie (1999).

$P_d + P_f C = c'(Q^*)$ . The inequality follows directly from put-call parity.<sup>10</sup> Then, the convexity of the cost function implies  $Q_{\text{inflex}}^* < Q^*$ . This proves the next corollary.

**Corollary 3** *When currency call options with strike price  $P_d/P_f$  are available, the (restricted) export flexible firm's optimal output is higher than that of an otherwise identical exporting firm which possesses no export flexibility.*

The opportunity to refrain from exporting at low realizations of the exchange rate stimulates the export flexible firm to produce more. Export flexibility creates additional value for the firm which could be sold in the currency call options market at a positive price per unit of potential exports. This creates a wedge between the marginal cost of a flexible firm and that of an inflexible firm as shown above.

Now, turn to the firm's optimal risk management decision. Suppose that the currency futures and options markets are jointly unbiased:  $F = E[\tilde{S}]$  and  $C = E[\max(\tilde{S} - P_d/P_f, 0)]$ . Joint unbiasedness implies that the firm's expected profits are unaffected by its positions in the currency futures and options markets. Using the covariance operator,  $\text{Cov}[\cdot]$ , conditions (6) and (7) can be written as

$$\text{Cov}[U'(\tilde{\Pi}^*), \tilde{S}] = 0, \quad (9)$$

$$\text{Cov}[U'(\tilde{\Pi}^*), \max(\tilde{S} - P_d/P_f, 0)] = 0. \quad (10)$$

Rewrite the firm's profits as

$$\Pi = \tilde{S}[P_f Q_f - H] + \max(\tilde{S} - P_d/P_f, 0)[P_f(Q - Q_d - Q_f) - Z] + J \quad (11)$$

where  $J = CZ + FH + P_d(Q - Q_f) - c(Q)$ . Substituting  $H = P_f Q_f$  and  $Z = P_f(Q - Q_d - Q_f)$  into equation (11) yields  $\Pi = J$ , which is non-stochastic. Inspection of conditions (9) and (10) reveals that these two equations hold simultaneously at these values of  $H$  and  $Z$  since  $U'(\cdot)$  is constant if  $\Pi$  is deterministic, which in turn implies zero covariances.

---

<sup>10</sup>By put-call parity, the premium of a put option with strike price  $P_d/P_f$  must equal  $C$  plus  $P_d/P_f$  minus  $F$ , where the option premiums are compounded to date 1. Since the put option premium is positive,  $C + P_d/P_f > F$ .

Due to the uniqueness of the optimum, the firm's optimal hedge portfolio is indeed given by  $H^* = P_f Q_f$  and  $Z^* = P_f(Q^* - Q_d - Q_f)$ . This hedge portfolio makes the firm's profits riskless but does not change its expected value, given the joint unbiasedness of the currency futures and options markets. Hence, this portfolio is optimal. This establishes the following proposition.

**Proposition 2 (Full hedging)** *Suppose that the currency futures and options markets are jointly unbiased. The restricted export flexible firm's optimal hedge position,  $(H^*, Z^*)$ , satisfies  $H^* = P_f Q_f$  and  $Z^* = P_f(Q^* - Q_d - Q_f)$ .*

The optimal futures position is aimed at hedging the exchange rate exposure created by selling the minimum level  $Q_f$  in the export market against foreign currency. As is obvious, the minimum sales requirement for the export market directly affects the optimal futures position.

The optimal call option position, on the other hand, is used to hedge the conditional exchange rate exposure created by export flexibility. The existence of additional foreign exchange revenue of  $P_f(Q^* - Q_d - Q_f)$  is conditional on the exchange rate exceeding  $P_d/P_f$ . By writing call options on this amount with strike price  $P_d/P_f$ , the firm creates a conditional obligation to deliver foreign exchange. Since the firm becomes less export flexible the higher the minimum levels of domestic sales and exports,  $Q_d$  and  $Q_f$ , the optimal call option position declines in these parameters.

The firm's net foreign currency position sums up to zero. For exchange rate realizations below  $P_d/P_f$ , the call options are not exercised and the optimal futures position provides a full hedge for the export revenue of  $P_f Q_f$ . For exchange rate realizations above  $P_d/P_f$ , the call options are exercised. In both cases, the firm has to deliver its entire foreign exchange revenue in order to satisfy the obligations from the hedge portfolio  $(H^*, Z^*)$ . Therefore, the optimal portfolio of futures and call option positions makes the firm's profits invariant to different realizations of the exchange rate (full hedging). These hedging mechanics are illustrated in Figure 3.

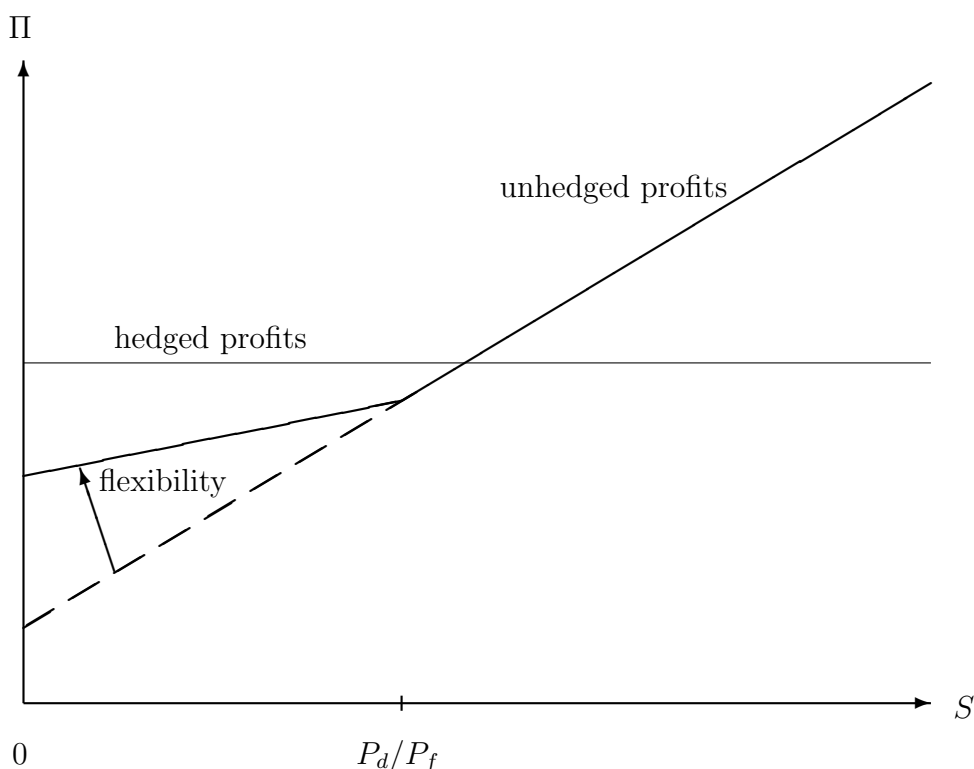


Figure 3: *Hedged and unhedged profits with futures and options*

If there is no export flexibility, the firm's revenue is linear in the exchange rate. This is represented by the dashed line and its solid continuation in Figure 3. In this case, unbiased currency futures are the preferred hedging instrument since they are also linear in the exchange rate. As shown by Battermann *et al.* (2000), fairly-priced options will not be used by an inflexible firm.

On the other hand, a fully flexible firm will entirely rely on fairly priced currency call options. Broll and Wahl (1997) have shown that full hedging with call options eliminates exchange rate risk. This is due to the fact that revenue is piecewise linear in the exchange rate with a zero slope for low exchange rate realizations. It follows that a risk averse and fully flexible firm will never use unbiased currency futures since this would increase risk while leaving expected profits unchanged.

Restricted export flexibility allows the firm to implicitly hedge against its exchange rate risk exposure by the sales allocation between the domestic and foreign markets.

Specifically, for realizations of  $\tilde{S}$  below  $P_d/P_f$ , the firm optimally allocates less, but still some, output to the foreign market and more output to the domestic market. This is illustrated in Figure 3 by bending up the dashed line at  $P_d/P_f$ . This implicit real hedge has two consequences on the firm's unhedged domestic currency profits. First, unhedged profits are less volatile. Second, unhedged profits become convex in the exchange rate with strictly positive slope everywhere. The convexity requires the use of currency options, similar to the case of a fully flexible firm. Due to the minimum export level, the slope is positive even at low realizations of  $\tilde{S}$ . This requires the use of currency futures in addition to currency options. This is not the case for a fully flexible firm.

Loosely speaking, adding some export flexibility to the inflexible firm results in adding currency options to the hedging position which consisted of currency futures only. Alternatively, restricting a fully flexible firm to some extent results in adding currency futures to the hedging position which consisted of currency options. Hence, the restricted export flexible firm has two appealing characteristics: First, it seems to be more realistic than totally inflexible or fully flexible firms. Second, it optimally uses a portfolio of currency futures *and* currency options which coincides with observable risk management behavior.

## 4 Optimal production and risk management with futures

This section analyzes the firm's optimal production and risk management decisions under the assumption that currency futures are the only hedging instrument available to the firm. Since currency options are absent, this section applies to export markets in countries where currency derivatives markets just begin to develop. Currency futures, because of their relatively simple structure, are readily available but currency options are not.

The absence of currency call options implies  $Z = 0$  in equation (3). Furthermore, condition (7) is irrelevant. Let  $\Pi_\diamond$ ,  $Q_\diamond$  and  $H_\diamond$  denote the firm's profits and the decisions



variables in this case. Then, the first-order conditions for an optimum become

$$\mathbb{E}\left[U'(\tilde{\Pi}_\diamond^*)\left(P_d + P_f \max(\tilde{S} - P_d/P_f, 0) - c'(Q_\diamond^*)\right)\right] = 0, \quad (12)$$

$$\mathbb{E}\left[U'(\tilde{\Pi}_\diamond^*)(F - \tilde{S})\right] = 0, \quad (13)$$

where an asterisk (\*) again indicates an optimal level. The second-order conditions for the unique maximum,  $(Q_\diamond^*, H_\diamond^*)$ , are satisfied given risk aversion and the convexity of the cost function. It is still assumed that  $Q_\diamond^* > Q_f + Q_d$  so that the firm has some degree of export flexibility at the optimum.

The firm's optimal production decision is analyzed first. Rewriting condition (12) yields

$$c'(Q_\diamond^*) = P_d + P_f \frac{\mathbb{E}\left[U'(\tilde{\Pi}_\diamond^*) \max(\tilde{S} - P_d/P_f, 0)\right]}{\mathbb{E}\left[U'(\tilde{\Pi}_\diamond^*)\right]}. \quad (14)$$

Inspection of condition (14) reveals that, in general, the firm's optimal output,  $Q_\diamond^*$ , depends on the firm's attitude toward risk and on the nature of the underlying exchange rate uncertainty. This implies the following result.

**Proposition 3** *If currency futures are the only hedging instrument available to the restricted export flexible firm, its optimal output,  $Q_\diamond^*$ , is neither separable from the firm's attitude toward risk nor from the distribution of the exchange rate.*

Since the available currency derivatives do not allow for complete elimination of exchange rate risk from marginal revenue, the firm's willingness to assume risk and the characteristics of the exchange rate distribution have an adverse impact on the firm's optimal production decision.

Now, the firm's optimal futures position is characterized. If the currency futures market is unbiased,  $F = \mathbb{E}[\tilde{S}]$ , condition (13) can be written as

$$\text{Cov}\left[U'(\tilde{\Pi}_\diamond^*), \tilde{S}\right] = 0. \quad (15)$$

Based on condition (15), the following proposition can be established where a proof is given in Appendix A.

**Proposition 4** *Suppose that the restricted export flexible firm can trade unbiased currency futures only. Then, its optimal futures position,  $H_\diamond^*$ , satisfies  $P_f Q_f < H_\diamond^* < P_f(Q_\diamond^* - Q_d)$ .*

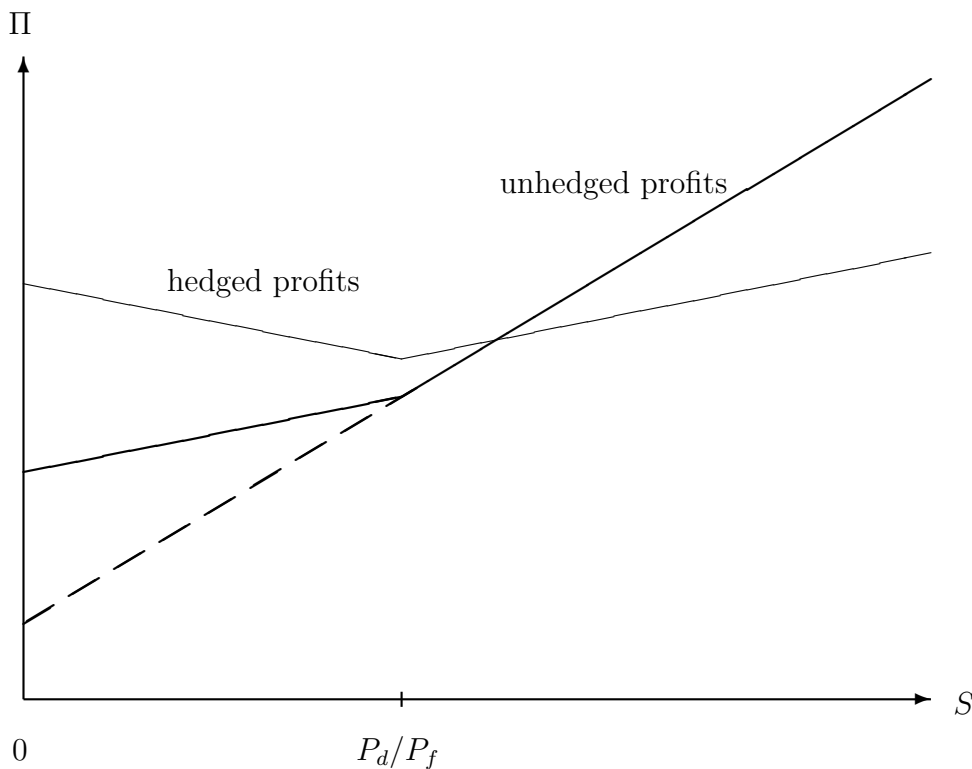


Figure 4: *Hedged and unhedged profits with futures only*

Proposition 4 can be illustrated using Figure 4. Without hedging, the firm's profits are piecewise linear and convex in the exchange rate. However, currency futures only allow for hedging against a linear exposure. For  $S > P_d/P_f$ , the optimal export policy is to export as much as possible which generates foreign currency revenue of  $P_f(Q - Q_d)$ . This is the steeper part of the unhedged profits line in the north-east of Figure 4. Setting  $H = P_f(Q - Q_d)$ , the firm could eliminate exchange rate risk for high exchange rate realizations,  $S > P_d/P_f$ . For lower realizations,  $S < P_d/P_f$ , export revenue only amounts to  $P_f Q_f$ . For these realizations, exchange rate risk can be eliminated by setting  $H = P_f Q_f$ . This shows that there is a conflict between hedging exchange rate risk for high and for low realizations of the exchange rate. Proposition 4 states that the firm prefers a

compromise between these two futures positions. The dependence of the firm's profits on the exchange rate, given the optimal futures position, is depicted by the V-shaped line in Figure 4.

Proposition 4 shows that the interval containing  $H_\diamond^*$  narrows if either  $Q_d$  or  $Q_f$  increases. Tightening the restrictions on the firm's export flexibility means that the convexity of unhedged profits in Figure 4 becomes smaller. This reduces the conflict between hedging for high and hedging for low exchange rate realizations. In the limit, for a firm without any export flexibility, the interval degenerates and the convexity of unhedged profits disappears. Then, the optimal futures position is unequivocally determined and the present model reduces to the classical model of an inflexible exporting firm as analyzed by Benninga *et al.* (1985) and others.

Finally, it is of interest to compare  $Q^*$  and  $Q_\diamond^*$  in order to find out whether introducing fairly-priced currency options to the firm stimulates production, thereby expected exports and expected domestic sales. Using the above notation for an unbiased currency option,  $C = E[\max(\tilde{S} - P_d/P_f, 0)]$ , condition (14) can be written as

$$c'(Q_\diamond^*) = P_d + P_f C + P_f \frac{\text{Cov}[U'(\tilde{\Pi}_\diamond^*), \max(\tilde{S} - P_d/P_f, 0)]}{E[U'(\tilde{\Pi}_\diamond^*)]}. \quad (16)$$

Comparing condition (16) with the optimality condition in Proposition 1,  $c'(Q^*) = P_d + P_f C$ , yields  $c'(Q^*) > c'(Q_\diamond^*)$  if and only if the covariance term in equation (16) is negative. It then follows from the convexity of the cost function that  $Q^* > Q_\diamond^*$ . Signing the covariance in equation (16) requires some tedious algebra. As the following proposition indicates, this covariance term is indeed negative if the currency futures and options markets are jointly unbiased. A proof is relegated to Appendix B.

**Proposition 5** *Suppose that the currency futures and options markets are jointly unbiased. Then, making currency options with strike price  $P_d/P_f$  available to the firm enhances production.*

The intuition behind this result is as follows. Introducing fairly-priced currency options allows the firm to sell the risk associated with the option to export without altering its

expected profits. As shown in Proposition 2, it is optimal for the firm to eliminate all exchange rate risk if the two currency derivative markets are jointly unbiased, i.e. when full hedging is costless in terms of expected profits. Proposition 5 compares two situations with identical expected marginal revenue. In the first situation, characterized by the absence of an unbiased currency options market, marginal revenue is risky. In the second, with jointly unbiased currency futures and options markets, marginal revenue is riskless at the optimum. It follows that a risk-averse firm produces more under riskless marginal revenue than under risky marginal revenue.

## 5 Conclusions

Foreign exchange risk management and its interaction with real operations play a significant role for an international firm's success. This paper has examined the optimal production and risk management decisions of an export flexible firm under exchange rate uncertainty. The paper focuses on restrictions of export flexibility in that the firm is assumed to serve both the domestic market and a foreign market with certain minimum levels of domestic sales and exports. The separation theorem requires the existence of currency call options only. Optimal production is unaffected by the tightness of the restrictions of the firm's flexibility. The optimal hedge portfolio eliminates all exchange rate risk if the currency derivatives markets are jointly unbiased. The hedge portfolio consists of both currency futures and currency options. Hence, our simple model of a restricted export flexible firm is sufficiently rich to provide a rationale for the joint use of currency futures and options in exchange rate risk management. In contrast to the production decision, the structure of the optimal hedge portfolio directly depends on how severe the restrictions are.

In the absence of currency options, neither separation nor full hedging can be derived. Since currency futures do not allow for complete elimination of the firm's piecewise linear exchange rate risk, the firm has to bear some exchange rate risk whatever its futures position will be. In this case, it is clear that preferences and the assessment of the

exchange rate distribution affect the firm's optimal production and risk management decisions. Making fairly priced currency options available to the firm has a positive effect on production and, consequently, on expected exports and expected domestic sales if the currency futures market is unbiased.

## Appendix

### A. Proof of Proposition 4

Partially differentiating the firm's profits,  $\Pi_\diamond$ , as given in equation (3) with  $Z = 0$ , with respect to  $S$  yields<sup>11</sup>

$$\frac{\partial \Pi_\diamond}{\partial S} = P_f Q_f - H_\diamond + \frac{\partial}{\partial S} \max(S - P_d/P_f, 0) P_f(Q_\diamond - Q_d - Q_f). \quad (17)$$

The remainder of the proof is by contradiction. Inspection of equation (17) reveals that  $\partial \Pi_\diamond / \partial S \leq 0$  if  $P_f(Q_\diamond - Q_d) \leq H_\diamond$ . Given risk aversion and  $P_f(Q_\diamond - Q_d) \leq H_\diamond$ ,  $U'(\Pi_\diamond)$  is non-decreasing in  $S$  for  $S > P_d/P_f$  and strictly increasing for  $S \leq P_d/P_f$ . Hence,  $\text{Cov}(U'(\tilde{\Pi}_\diamond), \tilde{S}) > 0$  for  $P_f(Q_\diamond - Q_d) \leq H_\diamond$ . Thus, equation (15) requires  $H_\diamond^* < P_f(Q_\diamond^* - Q_d)$ .

Likewise, equation (17) implies  $\partial \Pi_\diamond / \partial S \geq 0$  if  $P_f Q_f \geq H$ . Thus, given  $P_f Q_f \geq H$ ,  $U'(\Pi_\diamond)$  is non-increasing in  $S$  for  $S \leq P_d/P_f$  and decreasing in  $S$  for  $S > P_d/P_f$  such that the covariance in equation (15) is negative. Hence, (15) implies  $H_\diamond^* > P_f Q_f$ .  $\square$

### B. Proof of Proposition 5

We have to show that  $\text{Cov}[U'(\tilde{\Pi}_\diamond^*), \max(\tilde{S} - P_d/P_f, 0)]$  is negative. Partially differentiating  $\Pi_\diamond^*$  with respect to  $S$ , given the optimal futures position  $H_\diamond^*$  as stated in Proposition 4, results in<sup>12</sup>

$$\frac{\partial \Pi_\diamond^*}{\partial S} = \begin{cases} P_f Q_f - H_\diamond^* < 0 & \text{for } S < P_d/P_f, \\ P_f(Q_\diamond^* - Q_d) - H_\diamond^* > 0 & \text{for } S > P_d/P_f. \end{cases}$$

<sup>11</sup> $\Pi_\diamond$  is continuous at  $S = P_d/P_f$  but not differentiable.

<sup>12</sup>Like  $\Pi_\diamond$ ,  $\Pi_\diamond^*$  is continuous at  $S = P_d/P_f$  but not differentiable.

Therefore,  $\Pi_\diamond^*$  attains a unique minimum at  $S = P_d/P_f$ . (See Figure 4.) Since  $U''(\cdot) < 0$ ,  $U'(\Pi_\diamond^*)$  reaches a unique maximum at  $S = P_d/P_f$ . Thus, there exists a unique  $\hat{S} > P_d/P_f$  defined by  $U'(\Pi_\diamond^*(\hat{S})) = E[U'(\Pi_\diamond^*(\tilde{S}))]$ . Similarly, there exists a unique  $S^\# < P_d/P_f$  defined by  $U'(\Pi_\diamond^*(S^\#)) = E[U'(\Pi_\diamond^*(\tilde{S}))]$ .

First, consider case (a), defined by  $E[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \geq P_d/P_f]$  not exceeding  $E[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \leq P_d/P_f]$ , where  $E[\cdot | \cdot]$  is the conditional expectation operator. By the definition of the covariance operator, one can write

$$\begin{aligned}
& \text{Cov}\left[U'(\tilde{\Pi}_\diamond^*), \max(\tilde{S} - P_d/P_f, 0)\right] \\
&= \int_{\underline{S}}^{P_d/P_f} \left\{U'(\Pi_\diamond^*(S)) - E[U'(\tilde{\Pi}_\diamond^*)]\right\} \left\{-E[\max(\tilde{S} - P_d/P_f, 0)]\right\} dG(S) \\
&\quad + \int_{P_d/P_f}^{\bar{S}} \left\{U'(\Pi_\diamond^*(S)) - E[U'(\tilde{\Pi}_\diamond^*)]\right\} \\
&\quad \quad \quad \times \left\{(S - P_d/P_f) - E[\max(\tilde{S} - P_d/P_f, 0)]\right\} dG(S) \\
&= \int_{P_d/P_f}^{\bar{S}} \left\{U'(\Pi_\diamond^*(S)) - E[U'(\tilde{\Pi}_\diamond^*)]\right\} (S - P_d/P_f) dG(S) \\
&= \int_{P_d/P_f}^{\bar{S}} \left\{U'(\Pi_\diamond^*(S)) - E[U'(\tilde{\Pi}_\diamond^*)]\right\} \left\{(S - P_d/P_f) - (\hat{S} - P_d/P_f)\right\} dG(S) \\
&\quad + \int_{P_d/P_f}^{\bar{S}} \left\{U'(\Pi_\diamond^*(S)) - E[U'(\tilde{\Pi}_\diamond^*)]\right\} (\hat{S} - P_d/P_f) dG(S) \\
&< (\hat{S} - P_d/P_f) \int_{P_d/P_f}^{\bar{S}} \left\{U'(\Pi_\diamond^*(S)) - E[U'(\tilde{\Pi}_\diamond^*)]\right\} dG(S) \\
&= (\hat{S} - P_d/P_f) \left\{ \int_{P_d/P_f}^{\bar{S}} U'(\Pi_\diamond^*(S)) dG(S) G(P_d/P_f) \right. \\
&\quad \quad \quad \left. - \int_{\underline{S}}^{P_d/P_f} U'(\Pi_\diamond^*(S)) dG(S) [1 - G(P_d/P_f)] \right\} \\
&= (\hat{S} - P_d/P_f) G(P_d/P_f) [1 - G(P_d/P_f)] \\
&\quad \times \left\{ E[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \geq P_d/P_f] - E[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \leq P_d/P_f] \right\},
\end{aligned}$$

where the inequality follows from the fact that  $\{U'(\Pi_\diamond^*(S)) - E[U'(\tilde{\Pi}_\diamond^*)]\}$  and  $(S - P_d/P_f) - (\hat{S} - P_d/P_f) = (S - \hat{S})$  have opposite signs for all  $S > P_d/P_f$ . The curly bracketed term in the last line is non-positive by assumption. Since  $(\hat{S} - P_d/P_f)$ ,  $G(P_d/P_f)$  and  $[1 - G(P_d/P_f)]$  are all positive,  $\text{Cov}[U'(\tilde{\Pi}_\diamond^*), \max(\tilde{S} - P_d/P_f, 0)]$  is negative for case (a).

Now, consider case (b), in which  $E[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \geq P_d/P_f]$  is greater than  $E[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \leq$

$P_d/P_f$ ]. Since the covariance operator is linear, equation (15) implies  $\text{Cov}[U'(\tilde{\Pi}_\diamond^*), \tilde{S}] = \text{Cov}[U'(\tilde{\Pi}_\diamond^*), \tilde{S} - P_d/P_f] = 0$ . Using the fact that  $(S - P_d/P_f) = \max(S - P_d/P_f, 0) - \max(P_d/P_f - S, 0)$  and the linearity again results in  $\text{Cov}[U'(\tilde{\Pi}_\diamond^*), \max(\tilde{S} - P_d/P_f, 0)] = \text{Cov}[U'(\tilde{\Pi}_\diamond^*), \max(P_d/P_f - \tilde{S}, 0)]$ . Using the definition of the covariance operator again yields

$$\begin{aligned}
& \text{Cov}[U'(\tilde{\Pi}_\diamond^*), \max(P_d/P_f - \tilde{S}, 0)] \\
&= \int_{\underline{S}}^{P_d/P_f} \{U'(\Pi_\diamond^*(S)) - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*)]\} \\
&\quad \times \{(P_d/P_f - S) - \mathbb{E}[\max(P_d/P_f - \tilde{S}, 0)]\} dG(S) \\
&\quad + \int_{P_d/P_f}^{\bar{S}} \{U'(\Pi_\diamond^*(S)) - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*)]\} \{-\mathbb{E}[\max(P_d/P_f - \tilde{S}, 0)]\} dG(S) \\
&= \int_{\underline{S}}^{P_d/P_f} \{U'(\Pi_\diamond^*(S)) - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*)]\} (P_d/P_f - S) dG(S) \\
&= \int_{\underline{S}}^{P_d/P_f} \{U'(\Pi_\diamond^*(S)) - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*)]\} \{(P_d/P_f - S) - (P_d/P_f - S^\#)\} dG(S) \\
&\quad + \int_{\underline{S}}^{P_d/P_f} \{U'(\Pi_\diamond^*(S)) - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*)]\} (P_d/P_f - S^\#) dG(S) \\
&< (P_d/P_f - S^\#) \int_{\underline{S}}^{P_d/P_f} \{U'(\Pi_\diamond^*(S)) - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*)]\} dG(S) \\
&= (P_d/P_f - S^\#) \left\{ \int_{\underline{S}}^{P_d/P_f} U'(\Pi_\diamond^*(S)) dG(S) [1 - G(P_d/P_f)] \right. \\
&\quad \left. - \int_{P_d/P_f}^{\bar{S}} U'(\Pi_\diamond^*(S)) dG(S) G(P_d/P_f) \right\} \\
&= (P_d/P_f - S^\#) G(P_d/P_f) [1 - G(P_d/P_f)] \\
&\quad \times \left\{ \mathbb{E}[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \leq P_d/P_f] - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*) | \tilde{S} \geq P_d/P_f] \right\},
\end{aligned}$$

where the inequality follows from the fact that  $\{U'(\Pi_\diamond^*(S)) - \mathbb{E}[U'(\tilde{\Pi}_\diamond^*)]\}$  and  $(P_d/P_f - S) - (P_d/P_f - S^\#) = (S^\# - S)$  have opposite signs for all  $S < P_d/P_f$ . Since the curly bracketed term in the last line is negative by assumption and  $(P_d/P_f - S^\#)$  is positive,  $\text{Cov}[U'(\tilde{\Pi}_\diamond^*), \max(\tilde{S} - P_d/P_f, 0)]$  is negative for case (b) as well.  $\square$

## References

- Adam-Müller, A.F.A.** (1997). ‘Export and hedging decisions under revenue and exchange rate risk: a note’, *European Economic Review*, **41**, 1421–6.
- Adam-Müller, A.F.A.** (2000). ‘Exports and hedging exchange rate risk: the multi-country case’, *Journal of Futures Markets*, **20**, 843–64.
- Bagwell, K.** (1991). ‘Optimal export policy for a new-product monopoly’, *American Economic Review*, **81**, 1156–69.
- Bagwell, K. and Staiger, R.W.** (1989). ‘The role of export subsidies when product quality is unknown’, *Journal of International Economics*, **27**, 69–89.
- Battermann, H.L., Braulke, M., Broll, U., and Schimmelpfennig, J.** (2000). ‘The preferred hedge instrument’, *Economics Letters*, **66**, 85–91.
- Benninga, S., Eldor, R., and Zilcha, I.** (1985). ‘Optimal international hedging and output policies in commodity and currency forward markets’, *Journal of International Money and Finance*, **4**, 537–52.
- Ben-Zvi, S. and Helpman, E.** (1992). ‘Oligopoly in segmented markets’, in G.M. Grossman (ed.), *Imperfect Competition and International Trade*, MIT-Press, Cambridge (Mass.).
- Bodnar, G.M. and Gebhardt, G.** (1999). ‘Derivatives usage in risk management by U.S. and German non-financial firms: A comparative survey’, *Journal of International Financial Management and Accounting*, **10**, 153–87.
- Broll, U., Chow, K.W., and Wong, K.P.** (2001). ‘Hedging and nonlinear risk exposure’, *Oxford Economic Papers*, **53**, 281–96.
- Broll, U. and Eckwert, B.** (1999). ‘Exchange rate volatility and international trade’, *Southern Economic Journal*, **66**, 178–85.
- Broll, U. and Wahl, J.E.** (1997). ‘Export flexibility and hedging’, *Bulletin of Economic Research*, **49**, 205–11.
- Dellas, H. and Zilberfarb, B.-Z.** (1993). ‘Real exchange rate volatility and international trade: a reexamination of the theory’, *Southern Economic Journal*, **59**, 641–7.
- Frechette, D.L.** (2001). ‘The demand for hedging with futures and options’, *Journal of Futures Markets*, **21**, 693–712.
- Franke, G.** (1991). ‘Exchange rate volatility and international trading strategy’, *Journal of International Money and Finance* **10**, 292–307.
- Kawai, M. and Zilcha, I.** (1986). ‘International trade with forward-futures markets under exchange rate and price uncertainty’, *Journal of International Economics*, **20**, 83–98.
- Lapan, H. and Moschini, G.** (1994). ‘Futures hedging under price, basis, and production risk’, *American Journal of Agricultural Economics*, **76**, 465–77.
- Lapan, H., Moschini, G., and Hanson, S.D.** (1991). ‘Production, hedging, and speculative decisions with options and futures markets’, *American Journal of Agricultural Economics*, **73**, 66–74.



- Lence, S.H., Sakong, Y., and Hayes, D.J.** (1994). ‘Multiperiod production with forward and option markets’, *American Journal of Agricultural Economics*, **76**, 286–95.
- McKenzie, M.D.** (1999). ‘The impact of exchange rate volatility on international trade flows’, *Journal of Economic Perspectives*, **13**, 71–106.
- Magee, S.P.** (1974). ‘U.S. import prices in the currency contract period’, *Brookings Papers on Economic Activity*, **1**, 117–64.
- Mahul, O.** (2002). ‘Hedging in futures and options markets with basis risk’, *Journal of Futures Markets*, **22**, 59–72.
- Moschini, G. and Lapan, H.** (1992). ‘Hedging price risk with options and futures for the competitive firm with production flexibility’, *International Economic Review*, **33**, 607–18.
- Moschini, G. and Lapan, H.** (1995). ‘The hedging role of options and futures under joint price, basis, and production risk’, *International Economic Review*, **36**, 1025–49.
- Rawls, S.W. and Smithson, C.W.** (1990). ‘Strategic risk management’, *Journal of Applied Corporate Finance*, **1**, 6–18.
- Sercu, P. and Uppal, R.** (1995). *International Financial Markets and the Firm*. South-Western College Publishing, Cincinnati.
- Shy, O.** (2000). ‘Exporting as a signal for product quality’, *Economica*, **67**, 79–90.
- Ware, R. and Winter, R.** (1988). ‘Forward markets, currency options and the hedging for foreign exchange risk’, *Journal of International Economics*, **25**, 291–302.