

## Analytical description for current-induced vortex core displacement

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In this article, an analytical model for current-induced vortex core displacement is developed. By using this model, one can solve the equations of motion analytically to determine the effects of the adiabatic and nonadiabatic spin-torque terms. The final displacement direction of the vortex core due to the two torque terms mirrors their relative strengths. The resulting vortex core displacement direction combined with the amplitude of the displacement is thus a measure for both torque terms.

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For the development of novel applications such as magnetic solid-state storage devices,<sup>1</sup> a controlled manipulation of the magnetization is required. To achieve this, the injection of spin-polarized electrons is believed to be a viable alternative approach to conventional magnetic fields, as field-induced magnetization dynamics suffer from unfavorable scaling. As the design rule of devices is decreased, the current density necessary in the striplines to generate the switching fields goes up and eventually reaches levels at which structural degradation sets in. Spin-polarized currents, on the other hand, show a more favorable scaling as a means of manipulating the magnetization because, for a constant current density, the total current, and thus the power consumption, goes down with decreasing feature sizes.

Spin currents interact with the magnetization via the adiabatic and nonadiabatic spin torque as theoretically predicted.<sup>2,3</sup> Experimentally, for instance, switching of spin valves as well as in domain wall and vortex core (VC) motion was observed.<sup>4,5</sup> To utilize this effect, an in-depth understanding of the torque terms needs to be developed; this has previously been hampered by the difficulty of separating the spin-torque terms.

In particular, determining the nonadiabatic torque<sup>6,7</sup> has proven difficult, with a wide range of values having been determined from measurements of domain wall motion in wires.<sup>8–11</sup> The large variation and the strong assumptions and simplifications used in the analysis to extract the torque terms show that a reliable determination of an absolute value is not straightforward in the wire geometry.

A different geometry that has received much attention is the disk where the vortex state is present. The displacement of the VC under injected current in disks entails a number of advantages compared to the study of domain wall displacement in wires, where edge roughness and other edge defects can play a major role. The VC is always far away from the disk edge (for reasonable excitations) and thus is less influenced by defects located at the edge.

In this article, we use an analytical model to determine the displacement direction of a VC under current injection. We study the VC motion in this model and find that the final equilibrium displacement direction depends on the adiabatic and nonadiabatic spin-torque terms. In particular, we show that this direction can be used to determine the nonadiabaticity parameter, which is key to understanding the underlying spin current transport.<sup>6,7</sup>

To analytically derive the final current-induced position of the VC, we start with the extended Landau–Lifshitz–Gilbert equation, where the third and fourth terms are the adiabatic and nonadiabatic spin-torque terms, respectively.<sup>6,7</sup>

$$\dot{\vec{m}} = \gamma_0 \vec{H} \times \vec{m} + \alpha \vec{m} \times \dot{\vec{m}} - (\vec{u} \cdot \vec{\nabla}) \vec{m} + \beta \vec{m} \times [(\vec{u} \cdot \vec{\nabla}) \vec{m}]. \quad (1)$$

Thiele derived an equation of motion for a spin texture, which has been employed to describe domain wall motion and is commonly called a one-dimensional model.<sup>12</sup> It can also be used to describe the displacement of a single VC with a fixed magnetization profile, and it has been extended by Thiaville to include the spin-torque terms from Eq. (1):<sup>6</sup>

$$\vec{F}_s(\vec{X}) + \vec{G} \times \left( \vec{u} + \frac{d\vec{X}}{dt} \right) + \bar{D} \left( \beta \vec{u} + \alpha \frac{d\vec{X}}{dt} \right) = 0, \quad (2)$$

with the VC position  $\vec{X}$ . Assuming an electron flow the in  $x$ -direction,  $\vec{u}$  becomes  $\vec{u} = jPg\mu_B/(2eM_s)\vec{e}_x$ , with the spin polarization of the current  $P$  and the saturation magnetization  $M_s$ . The gyrovector  $\vec{G}$  points out-of-plane in the direction of the VC and equals  $\vec{G} = pG\vec{e}_z = p2\pi M_s\mu_0 t/\gamma\vec{e}_z$ , with  $p$  as the direction of the out of plane component of the VC (polarity  $\pm 1$ ) and the disk thickness  $t$ . The dissipation tensor  $\bar{D}$  is defined as:

$$\bar{D} = -\frac{M_s\mu_0}{\gamma} \int dV \left( \vec{\nabla} \vartheta \vec{\nabla} \vartheta + \sin^2(\theta) \vec{\nabla} \varphi \vec{\nabla} \varphi \right), \quad (3)$$

with  $\vartheta$  being the out-of-plane angle and  $\varphi$  the in-plane angle of the local magnetization. For a rigid vortex centered in a disk with radius  $r$ ,  $\bar{D}$  can be numerically evaluated (as shown in Ref. 13) and turns out to be a diagonal tensor with:

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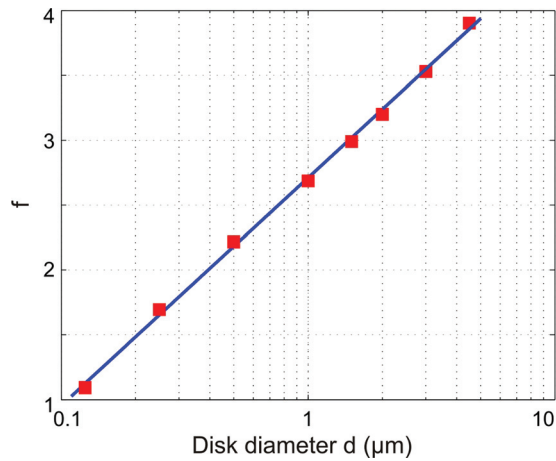


FIG. 1. (Color online) The factor  $f$  is plotted as a function of the disk diameter for disks 30 nm thick. The red squares correspond to the numerical results, and the blue line is a logarithmic fit.

$$D_{zz} = 0, \quad D_{xx} = D_{yy} = D \approx -\frac{\pi M_s \mu_0 t}{\gamma} \ln(2.0r/\delta) = -fG. \quad (4)$$

The radius of the VC  $\delta$  (about 10 nm in the permalloy structures<sup>14</sup>) depends on the exchange length and, slightly, the thickness of the disk  $t$ .<sup>15</sup> Due to the logarithm, and because  $d \gg \delta$ , the factor  $f$  is not very sensitive to variations in the core profile, and therefore  $f$  can be reliably calculated. In disks, the confining potential is radially symmetric, resulting in a force  $\vec{F}_s = -\kappa\vec{X}$  that tries to push the VC back to the disk center. The stiffness  $\kappa$  is given by the disk dimension and material parameters.<sup>16</sup>

When current is injected, the VC will be displaced according to Eq. (2) until the restoring force equals the spin torque. To determine the final displacement  $\vec{X}$  under current injection, we look for solutions where  $d\vec{X}/dt = 0$  and thus a steady state is obtained. Equation (2) then simplifies to:

$$x_{VC}\vec{e}_x + y_{VC}\vec{e}_y = Gu/\kappa(f\beta\vec{e}_x + p\vec{e}_y). \quad (5)$$

A similar calculation was done by Shibata *et al.*, but without including nonadiabatic contributions.<sup>17</sup> It now becomes obvious that the adiabatic spin-torque term is responsible for a displacement perpendicular to the electron direction ( $y_{VC}$ ), whereas the nonadiabatic term leads to a displacement in the direction of the electron flow ( $x_{VC}$ ). By measuring the angle of displacement  $\theta$  with respect to the electron direction, the nonadiabaticity parameter  $\beta$  can be directly evaluated:

$$\tan(\theta) = x_{VC}/y_{VC} = pf\beta. \quad (6)$$

For the sake of simplicity, the calculation was done for harmonic potentials, but it holds for any potential, as long as it is radially symmetric. It is also important to note that uncertainties with regard to the current density, sample thickness, material parameters, etc. do not affect the displacement direction, making this relationship very robust.

To directly evaluate this expression, one needs to know the values of  $p$  and  $f$ . To set  $p$ , one can initialize the vortex spin structure by applying a strong out-of-plane field to set the polarity of the VC. What remains is to calculate  $f$ . To check whether the analytical calculation described above holds, we have carried out corresponding full micromagnetic simulations to determine  $f$  numerically. We use the usual parameters for permalloy<sup>11</sup> to determine the factor  $f$  in disks 30 nm thick and of varying diameters. The result is shown in the semilogarithmic plot in Fig. 1. The predicted logarithmic dependence of the factor  $f$  on the disk diameter is very well confirmed (Fig. 1), so the relation above can be used to determine  $\beta$  if the two spin torques are the governing torques and other torques, such as the Oersted field,<sup>13,18</sup> play a minor role.

In conclusion, we have used an analytical model for the VC dynamics to determine the current-induced VC displacement direction. The direction is found to be directly dependent on the ratio of the adiabatic and nonadiabatic torques. We have determined the proportionality factor so that potentially measuring the displacement direction allows for the determination of the nonadiabaticity parameter.

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