Three Essays on Using High Frequency Data in Estimating Financial Risks

Dissertation
zur Erlangung des akademischen Grades
des Doktors der Wirtschaftswissenschaften (Dr. rer. pol.)
am Fachbereich Wirtschaftswissenschaften
der Universität Konstanz

vorgelegt von
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Tag der mündlichen Prüfung: 10. April 2013

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To Till

Harry Potter:	Sirius, what are you doing here? If somebody sees you. . .
Sirius Black:	I had to see you off, didn't I? What's life without a little risk?
Harry Potter:	I just don't want to see you get shut back in Azkaban.

~J.K. Rowling, *Harry Potter and the Order of the Phoenix*, 2003.

Acknowledgements

The completion of this thesis would not have been possible without the help and support of many people.

First and foremost, I would like to thank my supervisor, Prof. Winfried Pohlmeier for his kindness and support during my PhD. It was he who had led me into high frequency econometrics research. His enthusiasm for research and teaching of econometrics, as well as his personability to students, are highly motivating. The passion for econometrics in his Chair is highly contagious. I would also like to thank my second supervisor, Prof. Ralf Brüggemann, for his help in the first year in helping me search for topics that might be interesting to work on.

I would also like to thank my friends and colleagues at the University of Konstanz, namely Hao Liu, Jing Zeng, Dr. Roxana Halbleib, Zlatina Balabanova, Ruben Sieberlich, Derya Usyal, Petra Marotzke, Fabian Krüger, Frieder Mokinski, Laura Wichert, and many others, for making research life in Konstanz so pleasant. They have always shown concern for me and provided numerous interesting discussions during lunches and breaks. Special thanks go to my friends Katarina Zigova and Peter Schanbacher whom I've also shared offices with. Their faith, humour and warmth of spirit made my daily life in the university enjoyable.

During my PhD, I had the opportunity to go for two research visits abroad in Manchester Business School where I did most of my work in Chapter 2. I would like to thank Prof. Ser-Huang Poon, my co-author in Chapter 2, and the researchers at Manchester Business School, Yong Woong Lee, Anton Golub, Dr. Jan Novety and many others, for their hospitality and the numerous fruitful discussions about my work. My special thanks to Dr. Heikki Seppala for his help with checking through the mathematical proofs in Chapter 2.

The support of my parents, who have always encouraged my love for learning, was invaluable. Last but not least, I would like to thank my husband Till for standing by me during difficult times.

A large part of my PhD study was financed by the Marie Curie Early Research Fellowship from the European Community's. Seventh Framework Programme FP7-PEOPLE-ITN-2008 under grant agreement number PITN-GA-2009-237984. This fellowship also financed my research stays at Manchester Business School. Another part of my PhD was funded by a scholarship from the Ministry of Science, Research and the Arts of the Federal State of Baden-Wüttemberg. I am extremely grateful for these fundings.

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Summary

Increasingly powerful and affordable data computing and storage has led to the availability of high frequency financial data, which gave rise to breakthrough research in the last decade in financial econometrics. The tick-by-tick data of trading and limit order books in exchanges and trading platforms are valuable in allowing for detailed characterizations of the trading dynamics, liquidity supply and demand, assets' dependence structures and contagion effects, traders' and investors' behaviour in the markets, as well as for constructing econometric estimators that characterise the current market conditions and the risks that markets players face. There are endless possibilities to what that can be learnt about financial markets from these data, such that today's research using high frequency data is still at its infancy, despite having begun more than a decade ago.

This dissertation comprises of three stand-alone research papers, all considering the use of high frequency financial data for financial market risk measurement. The thesis is organised as follows: The first chapter considers the extraction of liquidity information from the intraday limit order book to enhance the daily market risk measures for the dimension of liquidity risk. While liquidity risk in trading is understood to be important for practitioners, it is not well understood how it should be quantified and included in estimating market risks. The second and third chapter contribute to the ongoing research on realized measures in the multivariate context. Realized measures are nonparametric ex-post measures of the volatility of financial assets using intraday data. Since their introduction by Barndorff-Nielsen and Shephard (2002), they have been shown to be more accurate measures of volatility as compared to using squared daily returns. In the last two chapters, we provide some new ideas about how to characterise the ex-post dependence, which we term as "realized dependence", between two or more assets using intraday data, as well as describe a new approach to deal with the bias-correction problem that currently plague multivariate realized measures due

to the inherent properties of high frequency data.

The first chapter of the thesis is joint work with Hao Liu which introduces an intuitive method of enhancing low-frequency volatility measures used to compute Value-at-Risk (VaR) by incorporating intradaily liquidity information from the limit order book. While the dimension of liquidity risk has long been recognised to be important when considering market risk, research in this area is limited and is often not compatible with the Basel framework. Our model is not only compatible with the Basel framework, it also allows for dynamics in liquidity. We use the quote slope of Hasbrouck and Seppi (2001), a compound liquidity measure comprising the dimensions of bid-ask spread and log depths, which are extracted from the intraday limit order book, as a proxy for latent liquidity. As with Bangia et al. (2001), we find evidence that liquidity proxies tend to have multimodal distributions, and thus assign states of liquidity that the asset instantaneously resides in to allow only extremal shocks to liquidity to influence volatility. Instantaneous liquidity is then incorporated into a simple EGARCHX(1,1) volatility forecast model. As such, our specification allows for instantaneous causality between liquidity and volatility. To forecast the liquidity states, we use the dynamic autoregressive conditional multinomial model of Liesenfeld et al. (2006).

We test the model on 25 individual stocks and equally-weighted portfolios of these stocks. We find that the influence of liquidity on volatility is nonlinear, with both high liquidity and high illiquidity causing volatility to increase. Furthermore, the influence of liquidity is greater as the VaR quantile under consideration becomes more extreme. When the stocks are grouped by industry sectors, we find that for stocks in financial and technological sectors, only the extremal shocks to liquidity affects volatility and such a liquidity-state adjusted volatility is likely to improve VaR forecast. Stocks in other sectors are less affected by extremal shocks to liquidity but are sensitive to the mean continuous liquidity proxy. This phenomenon could be explained by investors regarding liquidity information differently for different sectors as well as by the increasing presence of financial analysts who often specialise only within certain industries.

The second chapter is joint work with Ser-huang Poon that introduces a new idea of estimating the daily ex-post copula dependence between different assets by using high-frequency data. Copula models can help characterize linear or nonlinear dependences, but capture only static dependences. Time-varying copula models have been developed to remedy this problem, but they often assume a certain parametric form for the dynamics of the dependence measure. We hence consider using intraday data to estimate dependence measures in the framework of Semiparametric COpula-based Multivariate Dynamic (SCOMDY) models introduced by Chen and Fan (2006). We consider one class of SCOMDY, the GARCH-semiparametric copula model, where the univariate variances are captured by the univariate GARCH, and the residuals are fitted to the parametric copula form with empirical marginals. Using intraday data, we obtain the ex-post copula dependence measure for each day and term this the ”realized dependence”. The advantage of such a method is that it is allows for the nonparametric conditional dependence between assets to be estimated. As such it belongs in the class of time-varying copulas.

We derive some of the properties of realized dependence and discuss some estimation issues. We then provide two simulation experiments that confirm our theoretical arguments. We show empirically that the "realized dependence" performs better in-sample in estimating the daily’s true dependence as compared to some often-used time-varying copula approaches. We also find that, like other realized measures, realized dependence exhibits long memory and hence forecast the dependence using the long memory HAR model of Corsi (2003). The Value-at-Risk estimated using the out-of-sample forecast of realized dependence was found to consistently perform better than when using other copula dependence estimators.

In the third chapter of the thesis, we deal with the problem of estimation of realized covariance and correlation, which is biased due to the high frequency data being plagued by market microstructure noise. Furthermore, in the multivariate setting, trades are often asynchronous, which leads to another source of biasness. Various methods have been proposed to obtain unbiased estimators but they often require certain unverifiable assumptions, for example that market microstructure noise are white noise processes. This chapter takes another approach altogether and argues that the inherent data problems make precise point

identification of realized covariance and correlation difficult. However identification bounds in the spirit of Manski (1995) can be derived and these identification bounds allow for a more robust approach to inference because it does not require making strong assumptions about the inherent data problems. We hence treat the problem of asynchronous observations as a missing data problem and use the approach of Horowitz and Manski (2006) for incomplete data to obtain the identification bounds. For microstructure noise, we use the approach by Horowitz and Manski (1995) for treatment of contamination and corrupted data to estimate the identification region. The contaminated sampling scheme assumes that the noise process is independent of the latent price process while the corrupted sampling scheme makes no such assumption. To obtain overall bounds, we add the two identification regions.

Our simulation study suggests that such an approach provides tight bounds to the estimators in the presence of such data problems. We also look at the sensitivity of the bounds to some tuning parameters used in the estimation methodology. We finally provide an empirical study using data of two stocks during the year of onset of the credit crisis and estimate the bounds of realized covariance and correlation. Like previous research, we find evidence that the noise process is not independent of the latent price process because bounds estimated using the contaminated sampling scheme provide inadequate coverage. We then forecast the identification bounds of realized correlation using the HAR model of Corsi (2003) and find that the bounds provide good predictive coverage of the realized correlation for both 1- and 10-step forecasts even in highly volatile periods.

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Zusammenfassung

Steigende Rechenleistung und Speicherkapazität hat in den letzten Jahren zu vermehrter Verfügbarkeit von Hochfrequenzdaten im Finanzsektor geführt. Darauf aufbauend sind im Bereich der Finanzmarktökonometrie wegweisende Forschungsarbeiten entstanden. Insbesondere Tick- by-Tick-Daten aus Handels- und Limit-Order-Buch ermöglichen die präzise Charakterisierung von Handelsdynamiken, von Liquiditätsangebot und Ðnachfrage, von Abhängigkeiten und den sogenannten Ansteckungseffekten zwischen verschiedenen Assets, und - im weiteren Sinne - der Beschreibung und Erklärung des Verhaltens von Wertpapierhändlern und Investoren. Aus ökonometrischer Sicht können diese Daten benutzt werden, um Schätzer der aktuellen Marktbedingungen und -risiken zu konstruieren. Dies sind nur einige der Möglichkeiten, die sich aus der Omnipräsenz von Hochfrequenzdaten ergeben. Die Forschung steht auch nach mehr als einem Jahrzehnt noch am Anfang, so dass mit einer weiteren Ausdifferenzierung und Vertiefung dieses Forschungszweigs zu rechnen ist.

Diese Dissertation enthält drei eigenständige Forschungsarbeiten, die sich mit unterschiedlichen Aspekten der Schätzung von Finanzmarktrisiken durch Hochfrequenzdaten beschäftigen. Die vorliegende Arbeit ist wie folgt gegliedert: Das erste Kapitel beschreibt, wie die Schätzung des täglichen Marktrisikos durch das Einbeziehen des Liquiditätsrisikos verbessert werden kann. Die Schätzung erfolgt dabei auf Basis der Daten des Limit-Order-Buches. Durch diese Arbeit wird das in der Praxis bedeutende Liquiditätsrisiko in die Forschung übersetzt, und das Verständnis wird gesteigert, wie Liquiditätsrisiko gemessen und mit in die Schätzung des Marktrisikos einbezogen werden sollte. Das zweite und dritte Kapitel tragen zur Forschung zu "realized measures" im multivariaten Kontext bei. Unter "realized measures" werden hier nicht-parametrische, ex-post Maße der Volatilität von Assets, basierend auf Intraday-Daten, verstanden. Dieses Forschungsthema wurde durch Barndorff-Nielsen and Shephard (2002) begründet und es konnte seitdem gezeigt werden, dass "realized measures" oft genauere Maße der Volatilität sind als zum Beispiel die Quadrate der Tagesreturns. Der Forschungsbeitrag der Kapitel 2 und 3 besteht in einer neuen Charakterisierung der ex-post Abhängigkeit zwischen zwei oder mehr Assets. Das hier neu entwickelte Abhängigkeitsmaßbezeichnen wir als "realized dependence". Außerdem beschreiben wir eine neue Methode, wie die - insbesondere für multivariate "realized measures" relevante - Problematik der Verzerrungskorrektur gemildert werden kann.

Das erste Kapitel dieser Dissertation ist in Zusammenarbeit mit Hao Liu entstanden. Hier wird eine intuitive Methode entwickelt, wie die - unter anderem zur Berechnung des Value at Risk (VaR) benutzten - Volatilitätsmaße durch die Einbeziehung von Intraday-Liquidität verbessert werden können. Obwohl der Einfluss des Liquiditäts- auf das Marktrisiko schon vor längerer Zeit erkannt wurde, konnte die Forschung dies bisher nur unvollständig abbilden. So wurden beispielsweise die Basel-Richtlinien in der bisherigen Forschung nur teilweise miteinbezogen. Das hier vorgestellte Modell ist nicht nur in Einklang mit den Basel-Richtlinien, sondern bindet auch Dynamiken in der Liquidität mit ab. Wir nutzen dafür das Liquiditätsmaß-"quote slope" -von Hasbrouck and Seppi (2001) als Schätzer für latente Liquidität. "Quote slope" umfasst sowohl die Dimension des Bid- Ask-Spreads, als auch die sogenannte "Log-depth" (die Summe der Logarithmen der Bid- und Ask- Volumina) und wird aus dem Limit-Order-Buch berechnet. Wie Bangia et al. (2001) finden wir Hinweise, dass die Liquiditätsschätzer einer multimodalen Verteilung folgen. Deshalb ordnen wir den Assets diskrete Liquiditätszustände zu, so dass nur extreme Liquiditätsschocks die Volatilität beeinflussen. Die augenblickliche Liquidität ("instantaneous liquidity") wird durch ein EGARCHX(1,1) modelliert. Dadurch ergibt sich in unserem Modell die Möglichkeit, den augenblicklichen Zusammenhang ("instantaneous causality") zwischen Liquidität und Volatilität zu untersuchen. Die Liquiditätszustände werden dabei durch das dynamische, autoregressive, konditional multinomiale Modell von Liesenfeld et al. (2006) vorhergesagt.

Wir testen das Modell mit 25 Einzelaktien und gleichgewichteten Portfolien dieser Aktien. Dabei ergeben sich folgende Ergebnisse. Der Einfluss von Liquidität auf die Volatilität ist nichtlinear: Sowohl hohe Liquidität, als auch hohe Illiquidität verursachen einen Anstieg in Volatilität. Der Einfluss von Liquidität ist am Größten, je extremer das VaR-Quantil ist. Die Volatilität von Aktien im Finanz- und Technologiesektor wird nur von extremen Liquiditätsschocks beeinflusst. Die VaR-Vorhersage in diesen Sektoren wird meist durch die Liquiditätszustand-adjustierte Volatilität verbessert. Aktien in anderen Sektoren sind weniger durch extreme Volatilitätsschocks, dafür aber stärker durch den stetigen Schätzer der Liquidität beeinflusst. Dieses Phänomen kann einerseits dadurch erklärt werden, dass Investoren Liquiditätsinformationen unterschiedlicher Sektoren unterschiedlich verarbeiten, andererseits durch eine Spezialisierung der Finanzanalysten in verschiedene Sektoren.

Das zweite Kapitel ist als gemeinsame Arbeit mit Ser-Huang Poon entstanden und führt eine neue Art der Schätzung von täglichen ex-post Copula-Abhängigkeiten zwischen unterschiedlichen Assets ein, wiederum basierend auf Hochfrequenzdaten. Copula-Modelle helfen in diesem Zusammenhang die linearen und nichtlinearen Abhängigkeiten zu erklären, beschränken sich allerdings oft auf statische Abhängigkeiten. Um dieses Problem zu beseitigen wurden zeitveränderliche Copula-Modelle entwickelt, die allerdings oft spezielle, parametrische Formen für die Dynamik der Abhängigkeit annehmen. Wir nutzen Intraday-Daten, um Abhängigkeitsstrukturen mit dem semi-parametrischen Copula-basierten, mulitivariaten, dynamischen Modell (SCOMDY) zu schätzen. Diese Modellklasse wurde durch Chen and Fan (2006) eingeführt. Wir betrachten eine spezifische Art von SCOMDY- Modell, das sogenannte GARCH-semiparametrische Copula-Modell, bei dem die univariaten Varianzen durch ein univariates GARCH und die Residuen durch eine parametrische Copula mit empirischen Grenzverteilungen modelliert werden. Basierend auf Intraday-Daten ermitteln wir ein ex-post Copula-Abhängigkeitsmaßauf Tagesbasis und nennen dieses "realized dependence". Der Vorteil dieser Methode ist, dass sie eine nicht-parametrische, bedingte Abhängigkeit zwischen den Assets erlaubt. Damit gehört dieses Modell zur Klasse der zeitveränderlichen Copula-Modelle.

Wir leiten einige Eigenschaften der "realized dependence" her und diskutieren wichtige Fragen der Schätzung. Danach führen wir zwei Simulationsexperimente durch, die unsere theoretischen Argumente bestätigen. Wir zeigen empirisch, dass die "realized dependence" besser die täglichen Abhängigkeiten schätzt als viele häufig verwendete zeitveränderliche Copula-Ansätze. Unser Ergebnis zeigt, dass die "realized dependence" ein "Long-Memory" besitzt, weshalb wir zur Vorhersage von Abhängigkeiten das Long-Memory HAR-Modell von Corsi (2003) benutzen. Die Schätzung des Value- at-Risk auf Basis der out-of-sample Vorhersage der "realized dependence" ist durchweg besser als die Vorhersagen mit anderen Copula-Schätzern.

Das dritte Kapitel dieser Dissertation handelt von der Schätzung der "realized covariance" und der "realized correlation", welche durch das sogenannte "Market Microstructure Noise" verzerrt wird. Eine zusätzliche Quelle der Verzerrung ist, dass im multivariaten Fall der Handel oft nicht synchron abläuft. Viele Methoden zur Herleitung unverzerrter Schätzer wurden in der Literatur vorgeschlagen. Diese unterliegen allerdings starken, nicht-verifizierbaren, Annahmen. Ein Beispiel dafür ist die Annahme, dass das "Market Microstructure Noise" ein White-Noise-Prozess ist. In diesem Kapitel wird ein vollständig anderer Weg eingeschlagen und argumentiert, dass eine genaue Punktidentifikation von "realized covariance" und "realized correlation" schwierig ist. Es können jedoch Identifikationsregionen im Sinne von Manski (1995) hergeleitet werden, ein Ansatz, der im Vergleich deutlich robuster ist, da er auf weniger starken Annahmen über die zugrundeliegenden Daten beruht. Wir behandeln also das Problem von nicht synchronen Beobachtungen als Missing-Data-Problem und nutzen dafür den Ansatz von Horowitz and Manski (2006), um die Identifikationsregionen herzuleiten. Zur Behandlung der Datenprobleme ("contamination", "corruption") im Rahmen des "Microstructure Noise" nutzen wir den Ansatz von Horowitz and Manski (1995). Dabei wird angenommen, dass beim "contaminated sampling" der Noise-Prozess unabhängig vom latenten Preisprozess ist, während diese Annahme beim "corrupted sampling" nicht getroffen wird. Zur Berechnung der Gesamtregionen müssen außerdem noch zwei Indentifikationsregionen hinzugenommen werden.

Unsere Simulationsstudie legt nahe, dass der beschriebene Ansatz - in Anbetracht der Datenproblematik - relativ kleine Identifikationsregionen liefert. Wir analysieren ebenfalls die Sensitivität dieser Regionen auf Veränderungen einiger der verwendeten Parameter. Abschließend führen wir eine empirische Studie mit Daten zweier Aktien zu Beginn der Finanzkrise durch. Wir schätzen die Regionen der "realized covariance" und der "realized correlation". Wie in früheren

Forschungsarbeiten finden wir Hinweise darauf, dass der Noise-Prozess nicht unabhängig vom latenten Preisprozess ist, da die auf Basis der "kontaminierten" Daten geschätzten Regionen keine ausreichende Abdeckung erzeugen. Zur Vorhersage der Identifikationsregionen der "realized correlation" nutzen wir das HAR-Modell von Corsi (2003) und finden heraus, dass die Regionen eine gute Abdeckung der "realized correlation" haben. Dies gilt sowohl für die 1- als auch für die 10- Schritt-Vorhersage und das sogar in hoch volatilen Perioden.

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Chapter 1

Liquidity and the Value at Risk

1.1 Introduction

In this paper, like Bangia et al. (2001), we consider the incorporation of exogenous liquidity risk in VaR models. We wish to obtain a practical model for use in financial institutions and for regulators that takes into account the characteristics of market liquidity, its close relationship with volatility, and hence produce a VaR that is enhanced by liquidity information. Our approach is in the spirit of the recent literature in financial econometrics of using measures derived from high frequency data to enhance low frequency estimations, such as the HEAVY model of Shephard and Sheppard (2010), and Maheu and McCurdy (2011) who use realized volatility to forecast the returns distribution.

The credit crisis that begun in mid-2007 was a sobering event for risk management, whereby confidence in risk management techniques has collapsed to a large degree. Government financial regulators, while trying to save commercial banks from sinking, had to revise banking regulations to bolster confidence in the financial system. The US government passed the Dodd-Frank Act in mid-2010, with a sweeping set of bank reforms aimed at preventing future crises. The Basel Committee on Banking Supervision revised the Basel II market risk framework in July 2009 to require banks to calculate stressed value-at-risk (VaR) which is VaR that is augmented with data that includes significant losses. This is in recognition of the inadequacy of VaR when data used is too recent (see for example Halbleib and Pohlmeier (2012) who discuss the effects of window length used on the VaR estimate and proposed optimal forecast combination of VaR methodologies for the reduction of model risk). In addition, VaR for trading books are required to include incremental risk capital charge for default and migration specific risk. The amendement does not recommend any specific approach for capturing the incremental risk, but states that "The bank must demonstrate that the approach used to capture incremental risks meets a soundness standard comparable to that of the internal-ratings based approach for credit risk...adjusted where appropriate to reflect the impact of liquidity, concentrations, hedging, and optionality" (Basel Committee on Banking Supervision (2009)). Such an amendment reflects the inadequancy of VaR in capturing dimensions of risk that are often hidden in trading books. This paper deals with the issue of incorporating market liquidity risk into trading book VaR.

Liquidity risk has been found to be priced (Pastor and Stambaugh (2003), Archarya and Pedersen (2005), Sadka (2006)), tends to fluctuate with time and its effects should thus be included in risk models. However incorporating liquidity risk into market risk is not straightforward, mainly due to the fact that it is not directly observable and its transmission into market risk is poorly understood. Bangia et al. (2001) classified market liquidity risk into two categories: (1) exogeneous liquidity which is common to all market players for an individual asset and is unaffected by individual traders' actions, and (2) endogenous liquidity that is specific to a trader's position in the market and thus varies across different traders. Our paper focuses on exogenous liquidity due to the lack of a unified approach in dealing with endogenous liquidity. Furthermore, we believe that exogenous intraday liquidity measures are able to capture market frictions and adverse selection due to information asymmetry, which traditional market microstructure theory cites as causes of market illiquidity. For a thorough discussion on market liquidity, we refer the reader to Amihud et al. (2005) who provide an excellent overview of the subject.

In another line of research in financial econometrics, the recent availability of high frequency financial data has provided more powerful information for inference (see e.g. Maheu and McCurdy (2011), Shephard and Sheppard (2010), Engle and Gallo (2006), and Brownlees and Gallo (2010)). In this paper, we adopt a similar idea that the inclusion of high frequency information into daily volatility forecast models is beneficial as it increases the information set available for forecasting. We focus on including liquidity risk in market risk measurement.

Hence we wish to exploit the availability of high frequency data in the limit order book as a rich source of information about the intraday market liquidity and use it to enhance volatility measures estimated using lower frequency data from the trading order book.

Following arguments by Renault and Werker (2011) who showed that not accounting for instantaneous causality between volatility and durations underestimates spot volatility by up to 50%, our model allows for instantaneous causality between liquidity and volatility, albeit at the daily level. This is reasonable as we would expect current intraday liquidity and conditional volatility of the same day to be instantaneously causal. We thus consider a GARCHX-type model but with the additional variable specified at the current period. This would then serve as a quantitatively reasonable method for incorporating incremental liquidity specific risk into market risk of the trading book.

In this paper we focus on obtaining VaR that is enhanced by high frequency liquidity information. We consider the quote slope liquidity proxy of Hasbrouck and Seppi (2001), a compound liquidity proxy that incorporates the depth and tightness dimensions of liquidity. The quote slope describes the state of the best buy and sell limit orders which would be executed against incoming market orders and is a parsimonous way of modelling information about the supply of liquidity from the intraday limit order book.

In addition, we wish to characterize the dynamics of liquidity since the assumptions of a constant mean and variance of liquidity shocks in liquidity shock models used in Karpoff (1986), Michaely and Vila (1996), Michaely et al. (1996), Fernando (2003) etc. are not realistic, especially when we consider the recent credit crisis. The benefit of such an approach is immense, first as such a dynamic model can help improve asset pricing models where liquidity risk is included, such as the liquidity adjusted CAPM of Archarya and Pedersen (2005) and Pastor and Stambaugh (2003), and second, we can make inference of the dynamics of liquidity supply beyond the mean (see e.g. Engle and Lange (2001) and Gomber et al. (2005)) to other risk measures like the second moment or a prespecified quantile of the liquidity density. Such information would be very useful for traders in optimizing their trading and liquidation strategies (see e.g. Bertsimas and Lo (1998), Almgren and Chriss (2000) and Subramanian and Jarrow (2001)).

The paper is organized as follows: Section 2 does a brief review of some existing liquidity-incorporated VaR models, while Section 3 introduces our liquidity information incorporated VaR model. Section 3 also elaborates on the modelling subtleties, discussing the issue of market liquidity, our choice of a liquidity proxy, and its modelling dynamics using states and market liquidity's relationship with volatility. It then describes the derivation of VaR in our model as well as some VaR evaluation methodologies. Section 4 describes our dataset, while Section 5 gives estimation results using individual stocks. Section 6 considers commonality in liquidity and extends the model for use in portfolios. Section 7 concludes.

1.2 VaR and the liquidity-adjusted VaR

VaR has been a standard risk metric to assess financial market risks since the 1996 Market Risk Amendment to the Basel Accord. It is defined as the most likely loss a financial institution or portfolio is likely to face for a certain percentage over a certain time horizon, usually 1 day or 10 days as required by Basel. Statistically speaking, it is the forecasted lower tail quantile estimate of the returns distribution. Research on VaR modelling issues have since proliferated, but papers on the inclusion of specific risks such as liquidity risk in VaR measures remain relatively few.

One of the earliest such papers by Bangia et al. (2001) proposes the inclusion of exogenous market liquidity risk into VaR based on arguments that exogenous liquidity affects market players of all sizes and display significant fluctuations. Furthermore, the data required to quantify exogenous liquidity effects is much more easily available than that required to quantify endogenous liquidity. They thus define a parametric liquidity-adjusted VaR (LVaR) that includes an add-on cost of liquidity, COL_t at time t to the traditional VaR measure, VaR_t[1]:

$$LVaR_t = VaR_t - COL_t = (\mu_r - z\sigma_r) - \frac{1}{2}(\mu_s + z_s\sigma_s),$$

where μ_r and σ_r are mean and volatility of mid-price returns, μ_s and σ_s are mean and volatility of the bid-ask spread, z is the percentile of the (normal) distribution while z_s is a multiple of the spread volatility to cover most of the spread situations at the interested percentile (e.g. 99%). Bangia et al. (2001) found the spread distributions to be far from normal and often exhibit multimodal distributions.

[1] In this paper, we define VaR in terms of returns and leave it as a negative number.

z_s is hence a scaling factor for spread volatility and they find it empirically to be between 2.0 to 4.0. Such an inclusion of liquidity risk into market risk assumes that market returns and liquidity risks are uncorrelated.

As an extension to Bangia et al. (2001) to account for the non-normality of spreads, Ernst et al. (2009) model z_s according to the Cornish-Fisher expansion that adjusts the normal distribution quantile z for skewness γ and excess kurtosis κ by:

$$z_s = z + \frac{1}{6}(z^2 - 1)\gamma + \frac{1}{24}(z^3 - 3z)\kappa - \frac{1}{36}(2z^3 - 5z)\gamma^2$$

Such an approach to include liquidity risk in VaR is convenient and easy to implement, especially when only the best bid and ask prices are available (such as in NYSE TAQ data). However as highlighted by Bangia et al. (2001), the non-normality of spreads is an issue, and the empirical determination of z_s is rather ad-hoc. The Cornish-Fisher expansion used by Ernst et al. (2009) is also unable to account for the multimodality observed in spreads distribution. Bangia et al. (2001) suggests the multimodality to be indicative of the presence of liquidity states, which we will discuss in Section 1.3.1.

Berkowitz (2000) argues that the endogenous liquidity risk a trader faces should be included in VaR and proposes a model that considers the possible depression in prices during liquidation. Using x_t to represent basic economy-wide risk factors (for instance interest rates or exchange rates) to asset prices, then the next period returns r_{t+1} can be given by

$$r_{t+1} = r_t + x_{t+1} - \theta q_t^*$$

where q_t^* is the number of shares to be sold over a given time period, and θ is a liquidity coefficient such that the term $-\theta q_t^*$ accounts for possible price drops due to market reaction to the sales. θ is estimated via OLS, and the forecast distribution of returns (after obtaining a forecast distribution of x_t) can be obtained and hence its VaR. The approach of Berkowitz (2000) is only of interest for the minority of institutions or traders with extremely large positions such that their actions will affect the market price. It is also doubtful that the assumption of independence of x_{t+1} and q_t^* is reasonable, which leads to issues with the estimation of θ.

Almgren and Chriss (2000) and a subsequent thread of literature discuss the optimal trading strategy to liquidate a portfolio with the aim of minimizing volatility risk and transaction costs. The LVaR is obtained when the objective function to be minimized is the VaR. This approach is rather complex and theoretical, with many areas of development in modelling (especially in the area of stochastic portfolio modelling). We do not follow this line of literature for this paper as our focus is on empirical applications rather than theoretical modelling, but leave the reader to delve further into this if interested.

Giot and Grammig (2006) rely on complete order book data to obtain time series of intraday bid and ask prices at a given volume. They define the actual returns as the log ratio of mid-quote and consecutive bid price at v volume:

$$r_{mb,t}(v) = \ln \frac{2b_t(v)}{a_{t-1}(1) + b_{t-1}(1)}$$

where $a(v)$ and $b(v)$ are the ask and bid prices at volume v. The VaR is then computed using intraday returns based only on mid-quotes to give a "frictionless VaR" and intraday returns using $r_{mb,t}$ to give an actual VaR. A liquidity premium based on the difference of these two measures can then be obtained. This is not within the Basel VaR framework required by financial institutions and regulators, but rather an assessment of the intraday liquidation risk that an impatient trader faces. This is also the case for Angelidis and Benos (2006) who obtain the intraday LVaR based on components of the bid-ask spread. They argue that such an approach allows the bias of classical VaR analysis, when liquidity risk is not considered, to be observed. They also find however that there is a U-shape pattern in the spread components, which means that this bias has a diurnal pattern. It is then an issue as to when during the intraday trading hours that the bias be measured for use to adjust the daily VaR. The adjustment of daily VaR for liquidity effects was thus not discussed in their paper.

1.3 Incorporating liquidity into VaR

We now introduce our model for enhancing VaR with liquidity information. Let the joint distribution of returns and liquidity be represented by $f_{r,l^*}(r_t, l_t^*|\mathcal{F}_{t-1})$, where r_t is the log return at time t, l_t^* is the continuous latent liquidity, f_{r,l^*} is the joint distribution of returns and latent liquidity and $\mathcal{F}_t$ is the information set at time t. Then, the conditional VaR at probability level α given past filtration and the one period return of an asset or portfolio, r_t, is the quantile of the joint distribution:

$$\int_{-\infty}^{VaR(\alpha)} \int_{-\infty}^{\infty} f_{r,l^*}(r_t, l_t^*|\mathcal{F}_{t-1}) dl_t^* dr_t = \int_{-\infty}^{VaR(\alpha)} \int_{-\infty}^{\infty} f_{r|l^*}(r_t|l_t^*, \mathcal{F}_{t-1}) \cdot g_{l^*}(l_t^*|\mathcal{F}_{t-1}) dl_t^* dr_t = \alpha \tag{1.1}$$

where $f_{r|l^*}(\cdot|\cdot)$ denotes the conditional distribution of returns conditional on liquidity and $g_{l^*}(\cdot|\cdot)$ is the marginal distribution of liquidity. As we have noted before, such a specification allows for the instantaneous causality effect of liquidity on current day returns. The drawback is that it becomes necessary to provide a predictive model for $g_{l^*}(\cdot|\cdot)$, which we will define using the autoregressive conditional multinomial (ACM) model in Section 1.3.1. The alternative specification $f_r(r_t|l_{t-1}^*, \mathcal{F}_{t-1})$ would mean that liquidity is a subset of past filtration and contemporaneous liquidity of the market is ignored.

When we define liquidity states (i.e. l^* is segregated by thresholds, see Section 1.3.1), we can consider a liquidity indicator, l_t, that is multinomially distributed. We denote high illiquidity ($l_t = 1$), moderate illiquidity ($l_t = 0$) and low illiquidity ($l_t = -1$). The joint process of returns and illiquidity state j can then be given as

$$f_{r,l}(r_t, l_t = j|\mathcal{F}_{t-1}) = f_{r|l}(r_t|l_t = j, \mathcal{F}_{t-1}) \cdot g_l(l_t = j|\mathcal{F}_{t-1}), \text{ for } j = -1, 0, 1 \tag{1.2}$$

where $g_l(l_t = j|\mathcal{F}_{t-1}) = Pr[l_t = j|\mathcal{F}_{t-1}]$ is the probability mass function of the multinomial random variable l_t and $f_{r|l}(r_t|l_t = j, \mathcal{F}_{t-1})$ is the return distribution for a given illiquidity state j.

The marginal distribution of returns after integrating over all three illiquidity states would then be given as

$$f_r(r_t|\mathcal{F}_{t-1}) = \sum_j f_{r,l}(r_t, l_t = j|\mathcal{F}_{t-1}) = \sum_j Pr[l_t = j|\mathcal{F}_{t-1}] \cdot f_{r|l}(r_t|l_t = j, \mathcal{F}_{t-1}). \tag{1.3}$$

In the next few subsections, we will discuss the modelling of liquidity and its states, $g_l(l_t = j|\mathcal{F}_{t-1})$, as well as the joint modelling of returns and liquidity, $f_{r,l}(r_t, l_t = j|\mathcal{F}_{t-1})$.

1.3.1 Modeling market liquidity

The term "liquidity" has no general consensus in terms of definition but is generally regarded as a desirable latent quality of the trading process. It can be thought of as the ease of trading, and much of the literature in market microstructure theory discusses reasons for the impairment of this ease, for example the existence of information asymmetry or inventory costs of the market maker (see also O'Hara (1995)) for an overview). Our purpose in this paper however is not to give a theoretical discussion on market liquidity, but rather attempt to include the risk posed by illiquidity into market risk in an easy-to-use framework.

The latent quality of liquidity however makes quantitative measurement challenging. Most empirical work on liquidity refer to Kyle (1985), who divides liquidity into three dimensions – tightness (usually measured by the bid-ask spread), depth (amount of one-sided volume that can be absorbed by the market without causing a revision in bid-ask prices), and resiliency (speed of return to equilibrium). In general, liquid markets are those that are able to accommodate large volumes (depth dimension) at low transaction costs (tightness dimension) and that asset mispricings should quickly return to their fundamentals (resilience dimension).

In modern automated auction markets, liquidity supply generally depends on the electronic order book. Liquidity providers submit limit orders that do not face immediate execution but provides standing orders that a trader faces when he requires immediacy. The state of the order book determines the price-volume relationship, and determines the degree of increased volatility and trading cost that impatient traders have to face (see for example Cushing and Madhavan (2000) and Keim and Madhavan (1997)).

Limit and market orders are closely linked, and empirical studies such as Biais et al. (1995), Handa and Schwartz (1996), and Beltran et al. (2005) conclude that equilibrium levels of limit order trading and returns volatility should exist as they are instantaneously causal –short run volatility discourages limit order trading and a decrease in limit order trading increases short run volatility. This leads us to postulate that (1) the intraday information in the limit order book is important when considering daily market risk; (2) market liquidity information derived from the state of the limit order book could enhance the accuracy of forecasting returns' volatility. We will discuss the nature of this interaction in greater detail in Section 1.3.2.

Market Liquidity Proxy

To quantify liquidity, we require a proxy for measurement. One of the most commonly used measures of ex-ante liquidity is the quoted inside spreads, which is available on an intraday basis from the limit order book. Other liquidity proxies that are used include trading volume, quote depths, price impact coefficients of Kyle (1985) and Admanti and Pfleiderer (1988), as well as Amihud (2002) liquidity ratios. In our empirical study we use the quote slope of Hasbrouck and Seppi (2001) as it is an aggregation of two important liquidity dimensions that is observable in the limit order book.[2] It also does not incur model and estimation risks as in the case when estimating price impact coefficients.

Hasbrouck and Seppi (2001) find that the quote slope displays the greatest degree of commonality with some commonly used liquidity proxies. The intraday quote slope for day t at time period i is a compound measure of the inside spread, but incorporates the additional dimension of depth in the measure.

The intraday quote slope for day t at time period i is defined as

$$l^*_{i,t} = \frac{a_{i,t} - b_{i,t}}{\ln(D^a_{i,t}) + \ln(D^b_{i,t})} \tag{1.4}$$

where a and b represent the best ask or best bid quote and D^a and D^b represent the corresponding depths[3]. An increase in $l^*_{i,t}$ denotes a decrease of liquidity (an increase of illiquidity).

This proxy thus captures the first level of the bid and ask side of the limit order book. Some remarks are in order here: First, this proxy does not capture the entire depth of bid and ask sides of the limit order book, and a very small depth at the inside bid ask spreads can easily skew l^*_t upwards. Second, such a measure is an ex ante measure of liquidity since it does not involve any information from an executed transaction. Only incoming market orders would affect the quote slope.

Modeling the dynamics of liquidity

Since our liquidity measure is an empirical proxy of liquidity, which is latent and has more dimensions than the measure can capture, it would be naive to as-

[2]We also tried other liquidity proxies such as the inside bid-ask spread, and similar results were obtained due to their high correlation with the quote slope, as was found by Hasbrouck and Seppi (2001).

[3]Note that depth is measured by the volume of bid or ask quotes in the limit order book.

sume that we have obtained an exact measure of liquidity. By assigning states, we assume that the empirical measure is merely an approximation of latent market liquidity. Furthermore, we posit that it is only the extremal liquidity conditions that should affect how volatility behaves.

The assignment of states is also reasonable from what is observed in the empirical data. Like Bangia et al. (2001), we find evidence that the distribution of the liquidity proxy is multimodal. Figure 1.1 graphs the probability density function of the log 90% quote slope of Johnson & Johnson (jnj) over a period between 2001-2008 (see Section 1.4 for the time series plot of jnj's liquidity proxy, and for explanation of why the 90% quote slope is used). It appears to have three modes. Similar graphs can be plotted for the liquidity proxy of other stocks.

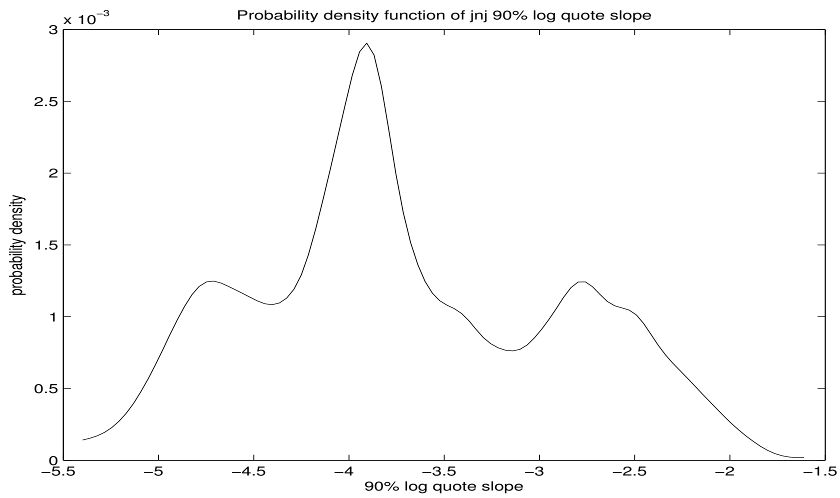


Figure 1.1: Probability density function of Johnson & Johnson's log 90% intradaily quote slope (2001-2008). It is smoothed using a Nadaraya-Watson kernel with a bandwidth of 0.1.

Thus we assume three liquidity states (low, medium, and high illiquidity) by using empirical quantiles of the proxy (in rolling window samples) as thresholds. Methods to set the thresholds is a potential opportunity for further research. Here we use a simple empirical approach to solve this problem. Defining states implies an approximate description of the latent liquidity process. We then forecast such states using a multinomial process which we will describe in the next subsection.

State Dynamics

The dynamics of liquidity states are modelled given past information by the autoregressive conditional multinomial model (ACM) of Liesenfeld et al. (2006), which we deem advantageous over Markov-switching models (see Hamilton (1989)) as it does not assume any latent driving process for state realizations. The ACM model belongs to the class of observationally driven models where time dependence is accounted for by a recursion on lagged endogenous variables.

We use a logistic link function to relate the probabilities of the occurrences $\pi_{j,t} \equiv Pr[l_t = j|\mathcal{F}_{t-1}]$ of the j states:

$$\pi_{jt} = \frac{\exp\{\Lambda_{jt}\}}{\sum_{j=-1}^{1} \exp\{\Lambda_{j,t}\}},\ j = -1, 0, 1, \tag{1.5}$$

where Λ_{jt}, which is termed the log-odds ratio, represents a function of a subset of $\mathcal{F}_{t-1}$. As a normalizing constraint, we assume $\Lambda_{0t} = 0,\ \forall t$. Hence the log-odds ratio reduces to a 2-by-1 matrix:

$$\Lambda_t \equiv (\Lambda_{j=-1,t}, \Lambda_{j=1,t})' \equiv (\ln[\pi_{-1,t}/\pi_{0,t}], \ln[\pi_{1,t}/\pi_{0,t}])'. \tag{1.6}$$

The serial dependence of l_t can be obtained by first defining the following state vector x_t in (1.7), where $x_{j,t} = \mathbb{I}_{l_t=j}$, and $\mathbb{I}$ is the indicator function

$$x_t \equiv (x_{-1,t}, x_{1,t})' \quad = \begin{cases} (1,0)', & j = -1 \\ (0,0)', & j = 0 \\ (0,1)', & j = 1. \end{cases} \tag{1.7}$$

Using state vectors, we can express the sample autocorrelation matrix for lag length p and sample size n as

$$\Upsilon(p) \equiv D^{-1}\Gamma(p)D^{-1},\ l = 1, 2, \dots \tag{1.8}$$

where

$$\Gamma(p) \equiv \frac{1}{n-p-1} \sum_{t=p+1}^{n} (x_t - \overline{x})(x_{t-p} - \overline{x})' \tag{1.9}$$

and D is the diagonal matrix containing the standard deviations of $x_{-1,t}$ and $x_{1,t}$.

The dynamics of the log-odds ratios are specified as a multivariate ARMA(P,Q) process with Z_t possibly containing additional explanatory variables capturing other marks of the trading day or macroeconomic information. The model is thus given by

$$\Lambda_t = \sum_{m=0}^{M} G_l Z_{t-m} + \alpha_t \quad (1.10)$$

$$\alpha_t = \mu + \sum_{p=1}^{P} B_p \alpha_{t-p} + \sum_{q=1}^{Q} A_q \xi_{t-q} \quad (1.11)$$

with B_p being the 2×2 autoregressive parameter term and A_q being the 2×2 matrix capturing parameters of the innovation terms, and $\mu = (\mu^{(1)}, \mu^{(2)})'$ the conditional mean vector. For our empirical study, we use $P = 1$, $Q = 1$, and do not specify any additional explanatory variable Z.

A noticable problem with the ACM model is the proliferation of parameters to be estimated as the number of states increases. We hence consider a restricted symmetric A matrix (i.e. $A^{(12)} = A^{(21)}$ and $A^{(11)} = A^{(22)}$) and further symmetry restrictions of $B^{(12)} = B^{(21)} = 0$ which implies that the shocks in the log-odds ratio are determined by the diagonal elements $B^{(11)}$ and $B^{(22)}$. We test the validity of the restricted case using a likelihood ratio test.

The vector of log-odds ratios is driven by the martingale difference sequence:

$$\xi_t = (\xi_{j=-1,t}, \xi_{j=1,t}), \text{ with } \xi_{jt} = \frac{x_{j,t} - \pi_{j,t}}{\sqrt{\pi_{j,t}(1 - \pi_{j,t})}}, \; j = -1, 1, \quad (1.12)$$

which is the state vector x_t that is standardized. Hence the model specification lets the conditional distribution of liquidity depend on the lagged conditional distribution of the liquidity process and the lagged values of the standardized state vector. The process is stationary if all values of z that satisfy $|I - B_1 z - B_2 z^2 - \ldots - B_p z^p| = 0$ lie outside the unit circle.

Finally, the ACM model is estimated using quasi-maximum likelihood, where the log-likelihood takes the form

$$\ln \mathcal{L} = \sum_{t=1}^{n} \sum_{j=-1}^{1} [\mathbb{1}_{(l_t = j)} \ln \pi_{j,t}]. \quad (1.13)$$

1.3.2 Instantaneous causality of volatility and liquidity

Andersen and Bollerslev (1997), Giot (2000), Giot and Schwienbacher (2005), Sokalska et al. (2005) and a large area of research thereafter characterize volatility on an intraday basis and find that its estimation is strongly affected by microstructure noise, sometimes termed as the hidden martingale. This noise is caused by a variety of microstructure frictions such as discreteness in price changes, gradual response of prices to block trade, etc. By decomposing total observed variance into a component attributable to the fundamental price signal and one attributable to market microstructure noise, Ait-Sahalia and Yu (2009) showed that microstructure noise exhibits strong positive correlation with various illiquidity proxies and is priced. These studies suggest that the modelling of volatility can be enhanced by the inclusion of illiquidity effects that may arise in the form of certain microstructure frictions.

Furthermore, Renault and Werker (2011) showed that spot volatility forecasts that disregard the instantaneous causality effect of duration on returns would severely underestimate spot volatility. In this light, we wish to allow for the instantaneous causality effect of illiquidity on volatility, which means that a contemporaneous specification of liquidity (i.e. non-lagged liquidity) should be used.

For forecasting daily financial time series, GARCH models (Bollerslev (1986)) are effective in capturing the clustering of volatility in asset returns and a plethora of GARCH-type models has since been developed (see review on GARCH models in Poon and Granger (2003)). Despite the numerous and increasingly sophisticated GARCH-type models, Hansen and Lunde (2005) showed that the simple GARCH(1,1) is in fact difficult to outperform for forecasting exchange rates. For equity data, however, models that accomodate asymmetric volatility (leverage effects), where negative returns are associated with larger increases in volatility as compared to positive returns, do outperform the simple GARCH(1,1). Other studies like Shephard and Sheppard (2010) have also found equity data to exhibit greater leverage effects.

The exponential GARCH (EGARCH) model of Nelson (1991) allows for leverage effects and has the additional benefit that it does not require positivity constraints for the coefficients, unlike linear GARCH models. To model the joint density of returns and liquidity, we assume a zero conditional mean[4] and use an EGARCHX(1,1) to model the conditional variance, where liquidity is incorporated as an exogeneous explanatory variable in the conditional variance equation.

[4]Conditional means of daily data are dominated by conditional variances, hence a zero-mean assumption is reasonable and often made, see for example Kim et al. (1999) and Christoffersen and Diebold (2006).

The conditional variance is thus given by:

$$\ln \sigma_t^2 = \omega_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|) + \beta_1 \ln(\sigma_{t-1}^2) + \delta l_t^*, \tag{1.14}$$

where ω_0 captures the conditional mean of volatility and α_1 captures the leverage effect, where if say $\alpha_1 < 1$, then negative returns cause the innovation in $\ln \sigma_t^2$ to be positive, and vice versa. γ_1 captures the magnitude effect of returns on volatility as in symmetric GARCH models, β_1 captures the autoregressive effects and δ captures the effect of liquidity on $\ln \sigma_t^2$.

From the conditional variance equations, we see that if we specify the exogenenous variable in the EGARCHX as $L_t^* = \ln l_t^*$, L_t^* has a multiplicative effect to the current period volatility, unlike the linear additive assumption of GARCHX. This is advantageous for interpretation, where the coefficient δ_0 captures the elasticity effect. Rewriting (1.14), we get:

$$\sigma_t^2 = \exp(\omega_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|))\sigma_{t-1}^{2\beta_1} L_t^{*\delta_0}. \tag{1.15}$$

When liquidity states are assumed, we get:

$$\ln \sigma_t^2 = \omega_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|) + \beta_1 \ln(\sigma_{t-1}^2) + \delta_1 \mathbb{I}_{(L_t^* \leq c^-)} + \delta_2 \mathbb{I}_{(L_t^* \geq c^+)} \tag{1.16}$$

where c^- and c^+ are the lower and upper thresholds for the liquidity proxy that separate the proxy into j states of $-1, 0, 1$. δ_1 is then the coefficient for the high liquidity state and δ_2 the coefficient for the high illiquidity state.

The combined estimation of the volatility and liquidity models is done via quasi-maximum likelihood. We assume the standardised errors to have a student-t distribution with v degrees of freedom. At high v, the standardised distribution of errors approaches normality, and at low v, the distribution is leptokurtotic. Hence the student-t distribution allows more flexibility for distributions with fat tails, and the degrees of freedom $\hat{v}$ is re-estimated at each time period. This implies maximising the joint log-likelihood function of EGARCHX and ACM given in (1.13):

$$\ln \mathcal{L} = \sum_{t=1}^{n} \ln f(r_t|l_t, F_{t-1}) + \sum_{t=1}^{n} \ln g(l_t|\mathcal{F}_{t-1})$$

$$= \sum_{t=1}^{n} \left[\ln \left(\Gamma \left(\frac{v+1}{2} \right) \right) - \ln \left(\Gamma \left(\frac{v}{2} \right) \right) - \frac{1}{2} \ln \left((v-2)\, \sigma_t^2 \right) + \sum_{j=-1}^{1} [\mathbb{1}_{(l_t=j)} \ln \pi_{j,t}] \right]. \tag{1.17}$$

1.3.3 Estimating and evaluating VaR

To integrate the mixture t-distribution in Eq. (1.3), we use Monte Carlo simulation by drawing 1,000,000 state realisations from a random uniform distribution that are compartmentalised by the forecasted state probabilities of the ACM model. This gives us the forecast density of r_t, and the empirical quantile of the forecast density is taken as $\widehat{VaR}$. In this paper, we consider the 1%, 5% and 10% quantiles, which corresponds to what is termed in practice as the 99%, 95% and 90% VaR respectively.

For comparison purposes, we evaluate the EGARCH-liq model (EGl) against the EGARCH with student-t distributed standard errors (EG), as well as the methods of RiskMetrics (RM), unconditional variance-covariance (VC) and historical simulation (HS) VaR methods. The RM, VC and HS methods are commonly used by many large financial institutions for computing VaR of their trading portfolios. HS is a nonparametric method, where VaR is computed from the empirical cdf of returns of the last N days that today's portfolio would earn. VC assumes normality and serial independence of returns and computes VaR as a multiple of the unconditional empirical standard deviation of the portfolio over a certain period. Both HS and VC are unconditional methods that assume returns to be i.i.d.. This makes them slower to respond to changes in return volatility. Despite the assumption of normally distributed returns, RM has been often found to perform well and is widely used in practice. It uses the exponentially weighted moving average (EWMA) method where the estimator for volatility is

$$\hat{\sigma}^2_{t+1} = (1 - \hat{\lambda}) \sum_{\tau=0}^{\infty} \hat{\lambda}^{\tau} r^2_{t-\tau}. \tag{1.18}$$

RM sets $\hat{\lambda} = 0.94$ which was found to give the best backtesting results using different portfolios.

We assess the VaR models for both adequacy and accuracy. Tests on the binary indicator of VaR failure also known as hits, $H_{t+1} \equiv \mathbb{I}_{\{r_{t+1} < \widehat{VaR}^{\alpha}_{t+1|t}\}}$, are aimed at assessing the adequacy of the VaR measure. An adequate VaR model has to fulfil two properties: unconditional coverage and independence. The unconditional coverage property requires that the hit rate (or percentage of exceedances) should be close to the α percentage that VaR is computed at. The Kupiec test (Kupiec (1995)) is a popular test for the property of unconditional coverage. The test of unconditional coverage is

$$H_{0,unc} : \hat{\alpha} = \alpha$$

and its test statistic takes the form

$$Ku = 2\log\left(\left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{N-\sum H_t}\left(\frac{\hat{\alpha}}{\alpha}\right)^{\sum H_t}\right)$$

where $\hat{\alpha} = \frac{1}{N}\sum H_t$ and N is the sample size.

The independence property requires that the hits are not clustered, i.e. VaR should be responsive to changing market risks. An early and influential test for independence is Christoffersen (1998) Markov test. The hit sequence is defined to follow a first-order Markov sequence with switching probability matrix

$$\Pi = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix}$$

where π_{01} is the probability of a non-hit ('0') at $t-1$ followed by a hit ('1') at t and π_{11} is the probability of consecutive hits at $t-1$ and t. The test of independence is then

$$H_{0,ind} : \pi_{01} = \pi_{11}.$$

Besides VaR adequacy, we wish to check for VaR accuracy by using the tick loss function of Komunjer (2005). The loss function is given by

$$TL_{\alpha,t} = (\alpha - \mathbb{I}_{r_t < \widehat{VaR}^{\alpha}_{t|t-1}})(r_t - \widehat{VaR}^{\alpha}_{t|t-1})$$

which measures the distance of the predicted VaR from the α quantile of the returns. Hence it penalises the VaR estimate when it is far from the true quantile, even in the case of non-exceedance. The relative performance of the forecasts from different models (M) are evaluated against the base model (M_0), which is our EGl model, with a Diebold-Mariano predictive ability test:

$$DM_{TL} = \frac{\overline{d}}{(\widehat{LRV}/N)^{1/2}}$$

where $\overline{d} = \frac{1}{N}\sum d_t$, $d_t = TL_{t,\alpha,M} - TL_{t,\alpha,M_0}$ and $\widehat{LRV} = \gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j$, $\gamma_j = cov(d_t, d_{t-j})$, is the long run variance that takes serial correlation into account. A significant value for the statistic (DM_{TL}) implies the rejection of the null of equal predictive accuracy, and a significant positive value indicates that the EGl model M_0 has better predictive ability than a comparative model M.

1.4 Data

We use 25 Dow Jones stocks[5] from the TAQ database from the period 1 Jan 2001 to 31 Dec 2008, a total of 2011 daily observations. Both the trade and quote data are cleaned according to the algorithm given in Barndorff-Nielsen et al. (2008). Daily log return series are constructed from the trade data after adjusting prices for dividends and stock splits. We group the stocks into 5 major sectors: "Financial" (axp, c, jpm), "Technology and Conglomerates", (hpq, ibm, mmm, ge, utx, ba) "Services" (hd, mcd, dis, wmt), "Energy, Basic Materials and Capital Goods" (aa, dd, cat, hon, xom) and "Consumer and Healthcare" (mo, ko, pg, gm, jnj, mrk, pfe). Table 1.6 in the Appendix gives the names of the stocks represented here by their ticker names as well as the descriptive statistics of the daily log returns.

Figure 1.2 shows Johnson & Johnson's daily log returns (top panel), log mean intraday quote-slope (middle panel), and log 90% intraday quote-slope (bottom panel) over the period 2001-2008. The daily returns show a share tumble on 19 July 2002 when the firm confirmed that it was under criminal investigation by the FDA for alleged record-keeping irregularities. There is also an increase in volatility of returns towards the end of 2008 from effects of the overall volatile stock market that is affected by the credit crisis.

The log mean quote slope series shows a steep fall after January 2001. This is due to the introduction of decimalization (from fractions) in the New York Stock Exchange, bringing about a strong improvement in market liquidity (as many research papers have found). It can also be observed that using mean quote slope averages away most intraday effects and it thus displays little variability. The log 90% quote slope, on the other hand, moves very much in tandem with the log returns. This was observed consistently for all stocks. Hence in our study, we use the 90% empirical quantile of the intraday quote slopes by making the following arguments: (1) our stocks used are highly traded liquid stocks. Using intraday averages of market illiquidity would average away any intraday temporal illiquidity effects; (2) our interest is on illiquidity (i.e. lack of liquidity) that would affect volatility. A 90% quantile measure would be a more effective intraday illiq-

[5]We use these 25 stocks instead of all 30 as the remaining 5 stocks included in Dow Jones were frequently changed during the 2001-2008 period. These 25 stocks were in the Dow Jones throughout or at least for most part of the period.

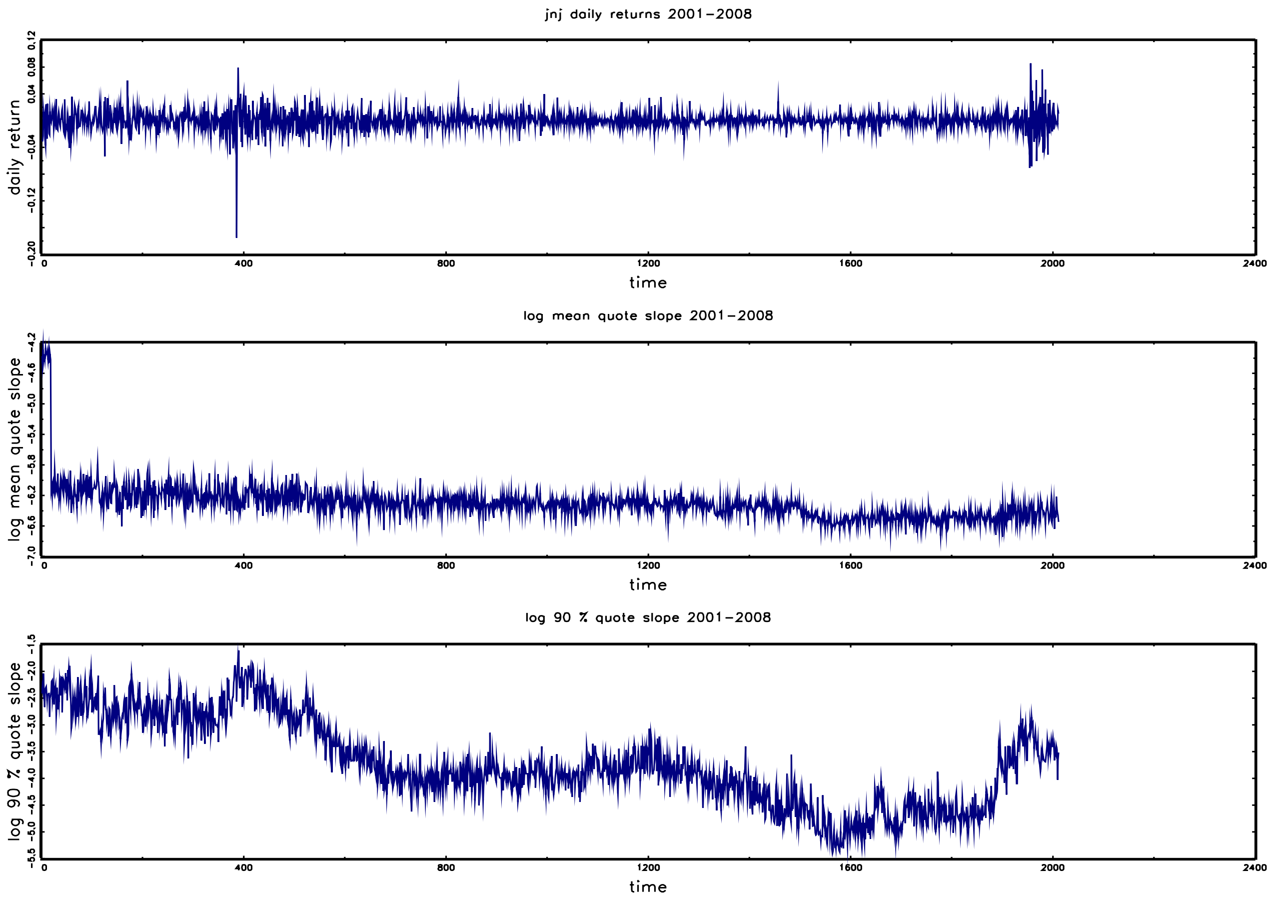


Figure 1.2: Johnson & Johnson's log returns (top), log mean intradaily quote slope (middle) and log 90% intradaily quote slope (bottom) for the period between Jan 2001 to Dec 2008.

uidity measure than using the mean; (3) we do not use the maximum intraday log quote slope as it is subjected to large jumps and outliers.

Table 1.1 gives the descriptive statistics of the log 90% quote slope series. The means of log quote slope are all negative due to the use of logs. From the statistics alone, we are not able to identify any obvious patterns about liquidity of stocks in different industry sectors. Ljung-Box Q statistics indicates presence of significant autocorrelation of the liquidity proxy for all stocks. Correlations of the liquidity proxy with returns are very low (not reported here), but correlations with squared returns are higher – between 0.063 to 0.294 (see column ρ_{sqret} in Table 1.1). This further motivates the addition of the contemporaneous liquidity effect into volatility rather than in the mean of returns.

	mean	std	max	min	skew	kurt	Q(10)	Q(25)	ρ_{sqret}
	Financial sector								
axp	-3.56	0.575	-1.46	-5.45	0.128	3.19	1323	1145	0.294
c	-4.18	0.887	-0.968	-6.00	0.313	2.24	1053	861	0.191
jpm	-3.95	0.931	-0.478	-5.49	0.719	2.59	1189	903	0.261
	Technology and Conglomerates								
hpq	-4.17	0.789	0.077	-5.52	0.828	3.36	451	286	0.255
ibm	-2.84	0.695	-0.492	-4.62	0.258	2.43	1244	1048	0.263
mmm	-2.65	0.744	-0.511	-4.51	0.355	2.33	1604	1508	0.175
ge	-4.62	0.845	-1.02	-6.08	0.670	2.67	845	592	0.294
utx	-2.85	0.633	-0.916	-5.11	-0.419	3.00	1246	1150	0.124
ba	-3.09	0.472	-1.79	-4.19	0.369	2.33	1280	1162	0.183
	Services								
hd	-3.96	0.823	-1.51	-5.66	0.459	2.62	1497	1395	0.190
mcd	-3.93	0.631	-1.83	-5.43	0.239	2.34	1345	1249	0.187
dis	-4.24	0.738	-0.329	-5.62	0.499	2.49	410	381	0.222
wmt	-3.77	0.912	-1.14	-5.48	0.255	2.06	1425	1281	0.226
	Energy, Basic Materials and Capital Goods								
aa	-3.75	0.745	-1.53	-5.28	0.404	2.44	1439	1354	0.093
dd	-3.50	0.614	-1.90	-5.01	0.211	2.30	1498	1393	0.232
cat	-2.88	0.542	-0.821	-5.41	-0.851	4.30	880	702	0.129
hon	-3.34	0.626	-0.632	-4.97	0.252	3.00	964	804	0.216
xom	-4.06	0.616	-1.35	-5.42	0.346	2.60	1210	1040	0.216
	Consumer and Healthcare								
mo	-3.78	0.565	-2.21	-5.76	-0.861	4.03	999	852	0.206
ko	-3.73	0.807	-1.91	-5.39	0.260	2.22	1541	1474	0.194
pg	-3.42	0.926	-1.17	-5.29	0.193	1.94	1665	1568	0.172
gm	-3.80	0.772	-1.71	-5.45	0.292	2.28	1465	1364	0.133
jnj	-3.71	0.800	-1.61	-5.40	0.268	2.23	1525	1456	0.146
mrk	-3.73	0.953	-1.01	-5.44	0.727	2.64	1486	1349	0.063
pfe	-4.63	0.922	-2.16	-6.12	0.459	2.01	1562	1517	0.223

Table 1.1: Descriptive statistics of log intraday 90% quote slope (2001-2008). The Ljung-Box Q statistics are given at 10 and 25 lags, while ρ_{sqret} is the correlation of the liquidity measure with squared returns of the stock.

1.5 Results

Table 1.2 gives the liquidity coefficient estimates of the EGARCHX (with continuous liquidity measure) δ_0 and EGARCH-liq (using liquidity states) δ_1 and δ_2 with their robust standard errors. The bold figures indicate that the estimates are statistically significant at 5% level or less. For 18 out of 25 stocks, δ_0 is significant and positive, which means that greater illiquidity is causal for greater volatility. The estimated δ_0 values show that the effect of liquidity on volatility is about 1.4 - 9.0% of volatility for these stocks.

δ_1 and δ_2 in Table 1.2 are estimated using the 0.8/0.2 quantiles as thresholds for the liquidity states. Table 1.8 in the Appendix gives the full estimation results using thresholds of 0.8/0.2 as well as 0.75/0.25 quantiles to examine the sensitivity of our results to the thresholds chosen. The differences are minor and we use the 0.8/0.2 threshold for the rest of our analysis. A rather surprising observation is that the signs on the δ parameters are mainly positive for both δ_1 and δ_2. This means that the high liquidity and high illiquidity both increases volatility and suggests that the relationship of liquidity with volatility is highly nonlinear. However the magnitudes of the effects do differ, with $\delta_2 > \delta_1$ consistently, which means that the effect of high illiquidity is greater on $\ln \sigma^2$ than that of high liquidity.

For stocks in the financial sector, both δ_1 and δ_2 are significant, but δ_0 displays no significance. This is indicative that volatility of stocks in the financial sector is not sensitive to liquidity except when large shocks occur, and that the effect is highly nonlinear. For stocks in the sectors "Capital Goods" and "Healthcare", δ_1 and δ_2 are almost always insignificant, but δ_0 is significant. It appears then that for stocks in these sectors, volatility is sensitive to the effect of illiquidity but not to large shocks. For stocks in "Technology" and "Services", δ_2 is significant, and δ_0 is often significant. The volatility of these stocks appears to be affected by the mean as well as by high illiquidity shocks (significant δ_2) but is insensitive to high liquidity shocks.

We now look at the effect of including liquidity on other parameters. The full set of parameter estimates for EGARCH, EGARCHX and EGARCH-liq models is given in Tables 1.7 and 1.8 in the Appendix. From these tables, we see that the β_1 terms are large, almost all above 0.9, which indicates strong persistency of volatility. For stocks that have significant δ_0, β_1 tends to be reduced, which means that the illiquidity proxy captures some degree of persistency of volatility. β_1 is increased when liquidity states are used because the liquidity shocks tend to be clustered and hence increase the degree of persistency.

The magnitude and leverage effects of volatility are similar to previous studies, with $\alpha_1 < 0$ and $\gamma_1 > 0$. The negative α_1 indicates that volatility tends to rise more when negative shocks are experienced and vice versa, while the positive γ_1 implies that volatility rises when the magnitude of returns movement is larger than expected. When δ_0 is significant, its effect on α_1 is small but γ_1 is increased significantly. This is due to the negative log liquidity proxy that causes γ_1 to compensate by being more positive. When δ_1 and δ_2 are significant, the magnitude of α_1 is decreased, indicating that liquidity state changes are able to capture some of the magnitude effects when there are large changes in volatility. v, the estimated degrees of freedom of the student-t distribution, indicate that most stocks are leptokurtotic or fat-tailed. The inclusion of the liquidity proxy, whether continuous or states, tends to increase v slightly (less fat-tailed) which means illiquidity explains some of the leptokurtosis of volatility.

The ACM estimation results for individual stocks are given in Tables 1.9 and 1.10 in the Appendix, Table 1.9 for the unrestricted model and Table 1.10 for the restricted case. For the unrestricted model, the $A^{(12)}$, $A^{(21)}$, $B^{(12)}$ and $B^{(21)}$ terms tend to be insignificant. The restricted ACM model assumes that the current period's high liquidity state has an equal effect as the current low liquidity state on the next period's liquidity state. Likelihood ratio tests indicate that the restricted model cannot be rejected and we thus use the restricted model for forecasting. For both the unrestricted and the restricted models, large $B^{(11)}$ and $B^{(22)}$ values indicate the high persistency of states. Overall, the persistency of high illiquidity is greater than that of high liquidity (i.e. $B^{(22)} > B^{(11)}$).

Our main findings show that as in previous research, an increase in illiquidity is associated with an increase in market volatility. We find however that this effect is highly nonlinear, as a sharp increase in liquidity is also accompanied by an increase in market volatility. To understand this, imagine a news release event that triggers interests of traders and investors. This leads to a large upswing in trading activity, which is associated with an increase in market liquidity (e.g. volumes of trade increases). This "buzz" of trading activity also results in increased market volatility. However, as our results suggest, high illiquidity is associated with larger market volatility as compared to a large increase in market liquidity. For risk management purposes, the greater concern is on the risk posed by market illiquidity. This non-linear effect of liquidity on volatility observed here suggests that in the construction of illiquidity proxies, information from ask side quotes might be more important than that of buy side quotes. However our interest in this paper is not on the construction of liquidity proxies and we leave this to future research.

Our observation that the volatility of stocks differ in sensitivity to liquidity depending on their industry sector suggests that investors regard liquidity differently for stocks in different industry sectors. For example, for stocks in the sectors "Capital Goods" and "Healthcare", investors tend to react to small changes in liquidity of the stocks, perhaps due to news releases, and are less responsive when there large shocks in liquidity. In contrast, investors in financial sector stocks tend to largely respond only to large shocks in liquidity and ignore small changes to liquidity. Because the manner in which liquidity affects volatility seems to be similar intra-industry and different inter-industry, we interpret this as a form of commonality, which we discuss in the next section.

	con't	high liq state	high illiq state
	δ_0	δ_1	δ_2
Financial			
axp	0.015	**0.015**	**0.026**
	(0.010)	(0.006)	(0.006)
c	-0.001	**0.012**	**0.019**
	(0.001)	(0.005)	(0.007)
jpm	0.005	**0.015**	**0.031**
	(0.006)	(0.005)	(0.006)
Technology & Conglomerates			
hpq	**0.094**	0.004	**0.023**
	(0.034)	(0.007)	(0.008)
ibm	**0.044**	**0.036**	**0.037**
	(0.017)	(0.011)	(0.010)
mmm	**0.029**	0.010	**0.022**
	(0.009)	(0.010)	(0.011)
ge	0.020	0.004	**0.018**
	(0.014)	(0.004)	(0.006)
utx	**0.022**	-0.001	**0.017**
	(0.008)	(0.006)	(0.008)
ba	**0.036**	0.023	**0.034**
	(0.013)	(0.012)	(0.013)
Services			
hd	0.000	0.000	**0.019**
	(0.003)	(0.007)	(0.009)
mcd	**0.019**	**0.022**	**0.031**
	(0.008)	(0.009)	(0.009)
dis	**0.031**	0.010	**0.019**
	(0.014)	(0.007)	(0.008)
wmt	**0.015**	0.008	**0.031**
	(0.007)	(0.008)	(0.010)

	con't	high liq state	high illiq state
	δ_0	δ_1	δ_2
Energy, Basic Materials & Capital Goods			
aa	**0.014**	-0.008	0.021
	(0.006)	(0.012)	(0.011)
dd	**0.035**	0.018	**0.033**
	(0.012)	(0.011)	(0.011)
cat	**0.026**	-0.019	0.000
	(0.009)	(0.011)	(0.012)
hon	**0.064**	0.011	0.014
	(0.018)	(0.013)	(0.013)
xom	**0.043**	-0.017	**0.064**
	(0.016)	(0.016)	(0.022)
Consumer & Healthcare			
mo	**0.213**	-0.009	0.023
	(0.071)	(0.016)	(0.022)
ko	**0.037**	-0.005	0.016
	(0.011)	(0.008)	(0.009)
pg	**0.023**	-0.014	0.018
	(0.007)	(0.012)	(0.014)
gm	-0.002	-0.015	-0.012
	(0.003)	(0.011)	(0.014)
jnj	**0.038**	0.009	0.019
	(0.009)	(0.010)	(0.013)
mrk	0.018	0.006	0.016
	(0.011)	(0.009)	(0.009)
pfe	**0.020**	0.014	**0.031**
	(0.010)	(0.008)	(0.012)

Table 1.2: Estimation results of the δ parameters in the EGARCHX (continuous liquidity- δ_0) and the EGARCH-liq (high liquidity state- δ_1 and high illiquidity state- δ_2) models for individual stocks. Bold figures indicate significance at 5% level.

1.6 Commonality in Liquidity

Earlier studies have found that liquidities of individual assets tend to be driven by common influences (e.g. Chordia et al. (2000), Sokalska et al. (2002), Korajczyk and Sadka (2008)). Individual asset liquidities tend to co-move with each other even after accounting for idiosyncratic effects such as trading volume and volatility. Our results in the previous section show that the dynamics of liquidity does indeed co-move in certain patterns intra-industry.

Individual asset proxies of liquidity tend to exhibit idiosyncratic noise that may reduce their explanatory power in volatility estimation. On the other hand, using a diversified liquidity measure would have higher correlation with overall market liquidity, and the reduction in noise could give it better explanatory power in volatility. In practice, a portfolio manager would be interested in diversifying the idiosyncratic liquidity risk of individual assets and forecasting the dynamics of the diversified measure that has a higher explanatory power in volatility. This enables him to manage larger portfolio turnovers without sacrificing on profitability, at the same time controlling for his risk appetite.

We hence consider an equally weighted portfolio of all 25 stocks (A) as well as equally weighted portfolios constructed by stocks in the same industry sectors, namely financial (F), technology (T), services (S), capital goods (C) and healthcare (H). To obtain the liquidity proxy of the portfolio, we use an equally weighted liquidity measure, l_p, to extract information about the liquidity of a portfolio. To check for the presence of commonality of liquidity, we run regressions of each individual stock's liquidity proxy on the constructed liquidity proxy of the portfolio:

$$l_{i,t} = \alpha + \beta l_{p,t} + \varepsilon_t. \tag{1.19}$$

Thereafter, we use l_p in the volatility estimation of the individual stocks and for the equally weighted portfolios.

Table 1.11 in the Appendix gives the results of individual assets' liquidity proxy regressions to an equally-weighted liquidity proxy consisting of all 25 stocks, as well as the EGARCHX estimation results for volatilities of individual stocks with the diversified liquidity proxy as the additional explanatory variable. To avoid regressing the individual proxy on itself, we exclude in turn the specific asset's liquidity in the construction of the diversified proxy. The p-values for the commonality regression are given in brackets. In general, beta parameters are found to be large and significant and the R^2s are high, suggesting presence of commonality. There is however no clear observable pattern of commonality inter- or intra-industry.

Using the common liquidity proxy in an EGARCHX regression for each individual stock shows the δ_0 parameter to be significant for almost all stocks. Except for stocks in the financial industry, where volatilities seems to be less sensitive to the diversified market liquidity proxy, other stocks do not show any significant pattern by industry, and are mostly positive and significant.

Table 1.3 gives the portfolio parameter estimates of EGARCH (EG), EGARCHX (EX) and EGARCH-liq (EGl) models, with robust standard errors given in brackets. β_1 parameters are decreased in the EX model, as was the case with individual stocks. In the EGl model, magnitude of β_1 tends to be increased due to the clustering of liquidity states. δ_0 parameters are all significant, but it is small and negative for the financial portfolio, which again suggests that the volatilities of stocks in the financial sector tend to be less sensitive to illiquidity except when there are large shocks. δ_1 and δ_2 parameters of the EGl model are significant for the portfolios "financials", "technology" and "services" but not for "capital goods" and "healthcare", indicative that stocks in financial, technology and services sectors are sensitive to large illiquidity movements while stocks in capital goods and healthcare sectors are sensitive to illiquidity in the continuous mean and less responsive to large shocks. For the overall portfolio, the parameters are significant, but δ_2 is not significant at the 1% level.

v estimates indicate the volatility of the financial portfolio to be leptokurtotic while the volatilities of the rest of the portfolios have near Gaussian tails. In general, the addition of the liquidity proxy whether continuous or states increases v slightly, which suggests that illiquidity explains some of the leptokurtosis of volatility.

Table 1.4 gives the unrestricted and restricted ACM estimation results for the portfolios. Like the ACM estimation for individual stocks, the likelihood ratio test given in column LR does not reject the restricted model. The Ljung-Box Q statistics of the liquidity states, given by $Q_s(10)$ and $Q_s(25)$ for 10 and 25 lags respectively, are very large since the states are indicator variables. The Ljung-Box Q statistics of the ACM residuals, $Q_r(10)$ and $Q_r(25)$, are substantially smaller,

		ω_0	α_1	β_1	γ_1	δ_0/δ_1	δ_2	v
Portfolio of all stocks								
All	EG	0.396	-0.109	0.962	5.953	-	-	10.283
		(0.057)	(0.014)	(0.005)	(0.814)	-	-	(2.336)
	EX	0.218	-0.127	0.936	7.694	**0.025**	-	10.346
		(0.077)	(0.016)	(0.010)	(1.061)	(0.008)	-	(2.338)
	EGl	0.457	-0.112	0.969	4.634	**0.017**	0.016	10.651
		(0.046)	(0.014)	(0.004)	(0.688)	(0.006)	(0.008)	(2.588)
Portfolio by industry								
Financial	EG	0.614	-0.072	0.982	2.533	-	-	6.486
		(0.024)	(0.008)	(0.002)	(0.332)	-	-	(0.834)
	EX	0.665	-0.069	0.989	1.840	**-0.003**	-	6.492
		(0.009)	(0.007)	(0.001)	(0.155)	(0.001)	-	(0.830)
	EGl	0.603	-0.0903	0.9820	2.3069	**0.023**	**0.021**	6.642
		(0.022)	(0.014)	(0.002)	(0.310)	(0.007)	(0.007)	(0.857)
Technology	EG	0.378	-0.092	0.959	5.727	-	-	9.409
		(0.058)	(0.013)	(0.006)	(0.791)	-	-	(1.913)
	EX	0.087	-0.101	0.907	7.577	**0.066**	-	10.837
		(0.010)	(0.016)	(0.015)	(1.163)	(0.018)	-	(2.38)
	EGl	0.432	-0.081	0.965	4.873	0.009	**0.022**	7.970
		(0.054)	(0.015)	(0.005)	(0.735)	(0.006)	(0.007)	(1.330)
Services	EG	0.436	-0.072	0.965	5.272	-	-	12.166
		(0.060)	(0.010)	(0.006)	(0.830)	-	-	(3.136)
	EX	0.209	-0.079	0.935	7.610	**0.018**	-	12.42
		(0.090)	(0.013)	(0.011)	(1.176)	(0.007)	-	(3.290)
	EGl	0.757	-0.063	0.996	-0.018	**0.012**	**0.024**	13.28
		(0.001)	(0.007)	(0.000)	(0.083)	(0.003)	(0.003)	(3.907)
Capital Goods	EG	0.273	-0.085	0.947	5.670	-	-	11.633
		(0.088)	(0.013)	(0.009)	(0.947)	-	-	(2.965)
	EX	0.052	-0.088	0.910	7.471	**0.035**	-	11.89
		(0.110)	(0.015)	(0.015)	(1.178)	(0.012)	-	(3.031)
	EGl	0.269	-0.079	0.947	5.521	-0.002	0.013	11.82
		(0.098)	(0.014)	(0.010)	(0.992)	(0.009)	(0.010)	(3.261)
Healthcare	EG	0.306	-0.080	0.955	8.251	-	-	10.644
		(0.084)	(0.013)	(0.008)	(1.462)	-	-	(2.168)
	EX	0.187	-0.089	0.939	9.383	**0.012**	-	11.14
		(0.102)	(0.014)	(0.011)	(1.759)	(0.005)	-	(2.415)
	EGl	0.308	-0.081	0.955	7.901	0.006	0.012	10.67
		(0.086)	(0.013)	(0.008)	(1.410)	(0.009)	(0.011)	(2.146)

Table 1.3: Parameter estimates of EGARCH (EG), EGARCHX (EX) and EGARCH-liq (EGl) models for equally weighted portfolios consisting of all stocks and by industry sectors. Bold font indicates significance of the delta parameters at the 5% level.

indicating that the ACM model does explain some of the autoregressive structure of the states.

1.6.1 Parameter stability

To observe the effect of the additional liquidity state variables on other estimated parameters as well as to check for the stability of the parameters, we plot the estimated coefficients using rolling windows of 1,000 observations for 250 time steps for both the EGARCH (dashed lines) and EGARCH-liq (solid lines) models. For the EGARCH-liq model, the high liquidity state parameter δ_1 is given by the solid line while the high illiquidity state parameter δ_2 is given by the dashed line.

We plot the parameter estimates for the financial portfolio in Figure 1.3 (a-f). The parameters for both models are in general rather stable, but some patterns can be observed. ω_0 and β_1 parameters (Figure 1.3a and Figure 1.3c) both show a decrease after inclusion of the liquidity states, with the decrease being particularly large between the 30-100th rolling window samples. This coincides with the δ_1 and δ_2 (Figure 1.3e) being large in those rolling window samples. This suggests that the liquidity states are able to explain some of the level and autoregressive effects of $\ln \sigma^2$. Figure 1.3e also shows that δ_2 is consistently greater than δ_1, confirming that the effect of high illiquidity is greater than the effect of high liquidity on volatility.

From Figure 1.3b, the effect of the liquidity states on the leverage parameter α_1 is less clear. For most of the period, α_1 appears to be increased (less negative) in the EGARCH-liq model, and the magnitude of the leverage effect is decreased, which indicates that the liquidity states are able to capture some of the leverage effects of volatility. There is a small period where α_1 is decreased (more negative) and the magnitude of the leverage effect is increased. This occurs during the period when δ_1 and δ_2 are at their peaks, hence this increase in leverage effect is likely a result from the large δ_1 and δ_2. Figure 1.3d shows that the effect of the additional liquidity state variables on γ_1 appears to be small. From Figure 1.3f, the degrees of freedom v are consistently greater (hence less fat-tailed) in the EGARCH-liq model, which strongly suggests that the liquidity states do explain some of the leptokurtosis of volatility.

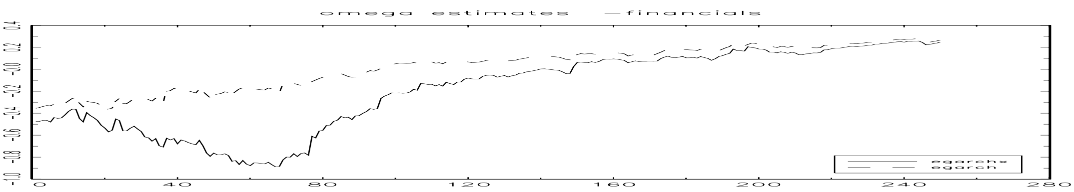
Unrestricted ACM estimations

	μ_1	μ_2	$A^{(11)}$	$A^{(12)}$	$A^{(21)}$	$A^{(22)}$	$B^{(11)}$	$B^{(12)}$	$B^{(21)}$	$B^{(22)}$	loglik	$Q_s(10)$	$Q_s(25)$
A	-0.200	-0.098	0.508	-0.087	-0.127	0.502	0.927	-0.058	-0.021	0.956	-0.576	19131	39640
	(0.107)	(0.075)	(0.060)	(0.134)	(0.095)	(0.051)	(0.033)	(0.036)	(0.025)	(0.025)		(0.000)	(0.000)
F	-0.182	-0.135	0.419	-0.124	0.007	0.439	0.919	-0.057	-0.034	0.939	-0.699	12840	26682
	(0.074)	(0.078)	(0.046)	(0.057)	(0.075)	(0.050)	(0.027)	(0.026)	(0.028)	(0.029)		(0.000)	(0.000)
T	-0.134	-0.066	0.443	-0.109	-0.012	0.461	0.951	-0.035	-0.011	0.965	-0.597	17923	37442
	(0.045)	(0.039)	(0.049)	(0.013)	(0.056)	(0.049)	(0.015)	(0.015)	(0.013)	(0.000)		(0.000)	(0.000)
S	-0.177	-0.026	0.396	-0.011	-0.121	0.318	0.928	-0.057	-0.001	0.985	-0.671	15252	32762
	(0.085)	(0.040)	(0.060)	(0.048)	(0.048)	(0.032)	(0.034)	(0.028)	(0.015)	(0.014)		(0.000)	(0.000)
C	-0.435	-0.390	0.473	0.047	-0.020	0.440	0.831	-0.126	-0.132	0.877	-0.639	16436	35589
	(0.173)	(0.166)	(0.056)	(0.056)	(0.063)	(0.054)	(0.060)	(0.049)	(0.058)	(0.048)		(0.000)	(0.000)
H	-0.401	-0.299	0.482	-0.084	-0.039	0.414	0.847	-0.127	-0.093	0.888	-0.652	16362	33953
	(0.317)	(0.193)	(0.078)	(0.069)	(0.055)	(0.048)	(0.108)	(0.103)	(0.065)	(0.064)		(0.000)	(0.000)

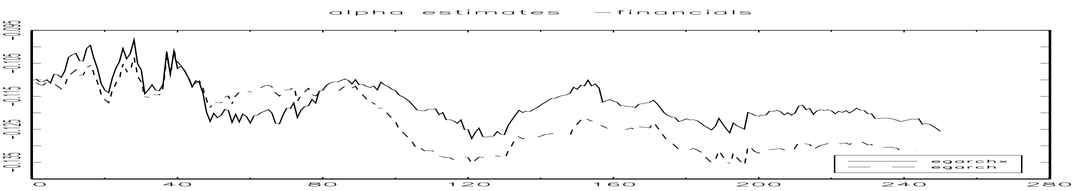
Restricted ACM estimations

	μ_1	μ_2	$A^{(11)}$	$A^{(12)}$	$B^{(11)}$	$B^{(22)}$	loglik	LR	μ_r	cov_r		$Q_r(10)$	$Q_r(25)$
A	-0.037	-0.048	0.468	-0.199	0.971	0.970	-0.579	-0.005	0.001	0.113	-0.007	11084	18460
	(0.02)	(0.02)	(0.03)	(0.05)	(0.01)	(0.01)		(1.00)	-0.218	-0.007	0.297	(0.00)	(0.00)
F	-0.041	-0.053	0.373	-0.186	0.964	0.965	-0.716	-0.003	-0.003	0.138	-0.015	9186	17316
	(0.01)	(0.02)	(0.01)	(0.01)	(0.03)	(0.04)		(1.00)	-0.268	-0.015	0.248	(0.00)	(0.00)
T	-0.047	-0.044	0.422	-0.068	0.965	0.965	-0.608	-0.005	0.009	0.116	-0.010	11359	20134
	(0.01)	(0.02)	(0.01)	(0.01)	(0.04)	(0.04)		(1.00)	-0.248	-0.010	0.296	(0.00)	(0.00)
S	-0.026	-0.033	0.346	-0.088	0.978	0.979	-0.676	-0.009	0.001	0.133	-0.014	10743	20058
	(0.01)	(0.01)	(0.03)	(0.04)	(0.01)	(0.01)		(1.00)	-0.258	-0.014	0.264	(0.00)	(0.00)
C	-0.029	-0.043	0.352	-0.136	0.973	0.975	-0.645	-0.011	-0.004	0.113	-0.009	10290	18814
	(0.01)	(0.01)	(0.04)	(0.06)	(0.01)	(0.01)		(0.01)	-0.228	-0.009	0.290	(0.00)	(0.00)
H	-0.041	-0.051	0.376	-0.145	0.968	0.966	-0.656	-0.007	0.010	0.133	-0.011	9817	15735
	(0.01)	(0.02)	(0.04)	(0.04)	(0.01)	(0.01)		(1.00)	-0.254	-0.011	0.232	(0.00)	(0.00)

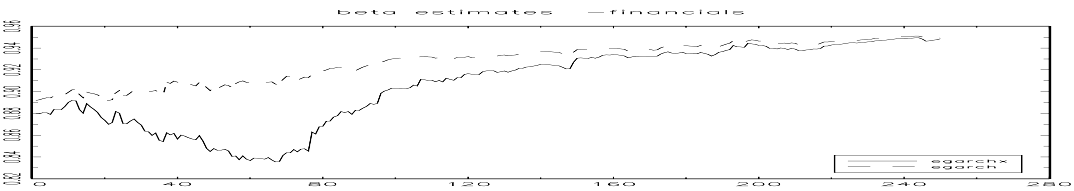
Table 1.4: Unrestricted (top) and restricted (bottom) ACM parameter estimation of portfolios consisting of all stocks (A), and stocks in the Financial (F), Technology (T), Services (S), Capital Goods (C) and Healthcare (H) sectors. LR gives the likelihood ratio test statistics. $Q_s(10)$ and $Q_s(25)$ give the Ljung-Box Q statistics of the liquidity states for 10 and 25 lags while $Q_r(10)$ and $Q_r(25)$ give the corresponding Q statistics of the residuals of the ACM model.



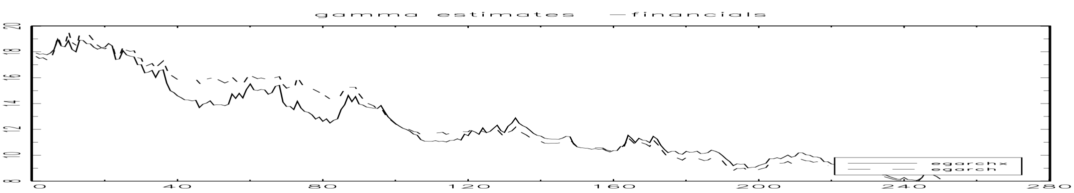
(a) ω_0 estimated parameters



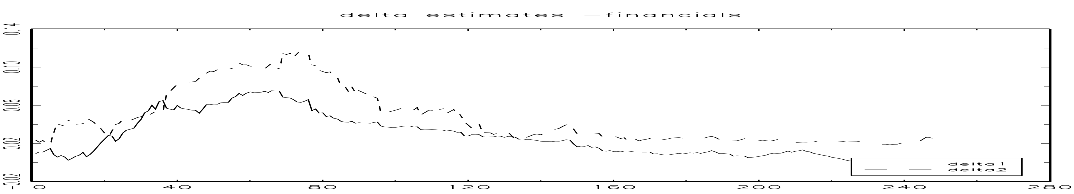
(b) α_1 estimated parameters



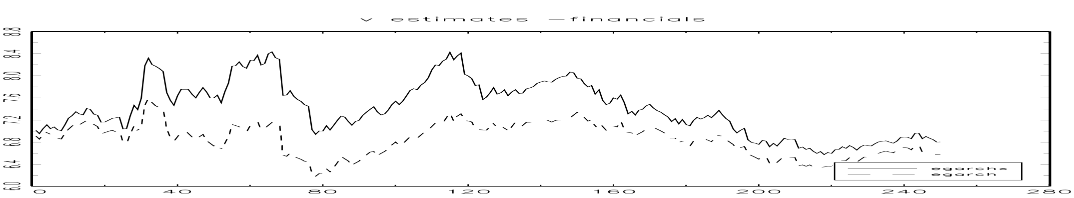
(c) β_1 estimated parameters



(d) γ_1 estimated parameters



(e) δ_1 and δ_2 estimated parameters



(f) v estimated parameters

Figure 1.3: Estimated parameters of the EGARCH (dashed lines) and EGARCH-liq (solid lines) models over 250 rolling window periods. Each window size consists of 1000 observations. For (e), the δ_1 (solid line) and δ_2 (dashed line) parameters of the EGARCH-liq model is given.

1.6.2 Forecast Evaluation

We estimate 250 1-step VaR forecasts using rolling windows of 1,000 observations (approximately 4 years) for the constructed portfolios. The forecast period spans the period 26 July 2007 to 24 July 2008, which coincides with the onset of the credit crisis. The model parameters are re-estimated with each rolling window. Figure 1.4 shows the 1-step forecast of VaR for the all-stocks portfolio using EGARCH-liq (EGl, solid lines), EGARCH (EG, broken lines), RiskMetrics (RM, dashed dotted lines), variance-covariance (VC, dotted lines), as well as historical simulation (HS, dashed lines) methods. Plots of the VaR forecasts for the sector portfolios are given in the Appendix.

The graphs show that the conditional models (EGl, EG, RM) are much more responsive to the fluctuations in returns as compared to the unconditional models (HS and VC), and are hence better forecast models. The EGl VaR tends to move in tandem with EG VaR, with its adjustments sometimes being upward or downward. When liquidity risk falls, the EGl VaR is of lower magnitude than the EG VaR, and when liquidity risk rises, the EGl VaR is of greater magnitude than the EG VaR. As we have seen in the earlier sections, for some sector stocks, both periods of high market illiquidity and high market liquidity are associated with an increase in returns volatility, hence the effect of liquidity on volatility is highly non-linear for such stocks. A direct increase of VaR by a multiple of the spread's volatility as done in Bangia et al. (2001) may not be the optimal method of adjusting VaR for liquidity risks.

It is also interesting to note that the differences between EGl and EG are smaller for the 90% VaR, and increase as the quantile of interest becomes more extreme. This suggests that liquidity risk becomes more important for VaR as we move into more extreme quantiles.

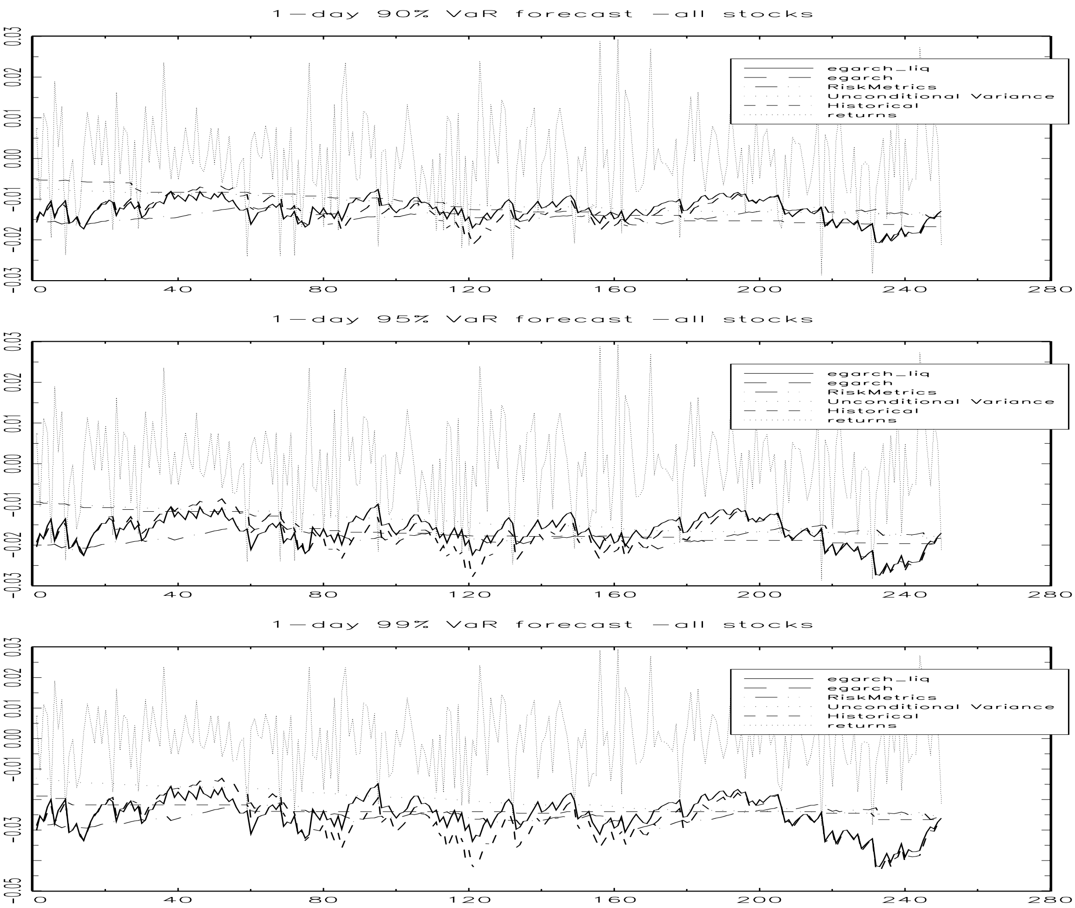


Figure 1.4: 1-step VaR forecast for an equally weighted portfolio consisting of all 25 stocks. Plots of the 90% (top), 95% (middle) and 90% (bottom) VaR forecasts are given for the EGl (solid lines), EG (broken lines), RM (dash-dot lines), VC (dotted lines) and HS (dashed lines) models

Table 1.5 gives the portfolio VaR evaluations for the EGl, EG, RM, VC, and HS VaR models. The p-values for the Kupiec test are given, and they show that the conditional models (EGl, EG, RM) perform better than the unconditional models (VC and HS). From the Kupiec statistics, the RM performs best in terms of VaR adequacy, followed by the EG and EGl models. None of the models fails the Christoffersen Independence test, but this may be due to the weak power of the test when the sample size is small (see e.g. Christoffersen and Pelletier (2004)).

The DM test using the tick loss function compares the EGl model against the EG, RM, VC and HS models for VaR accuracy, with bold figures indicating that the EGl model is superior at 5% significance level. Our main comparative model is the EG model, since RM is a different GARCH model and VC and HS are unconditional models. The statistics show that, as noted earlier, the advantage of including liquidity in the GARCH model increases at the more extreme quantiles (99% or 95%). EGl performs poorly in some cases, especially for the capital goods sector portfolio. From Table 1.3, we see that both δ_1 and δ_2 are insignificant for the capital goods sector portfolio, which explains the poor performance when using EGl. While the performance for the portfolio of healthcare stocks using EGl at the 99% VaR outperforms EG, it is less stellar than expected due to both δ_1 and δ_2 being insignificant. EGl for the portfolio consisting of all stocks performs well only at the 99% VaR, and poorly for 95% and 90%. This suggests that aggregating liquidity effects over different industry sectors is not an ideal approach if we wish to capture liquidity dynamics and their effects on volatility and VaR. It is also noteworthy that RM VaR performs well in terms of VaR accuracy, which accounts for its popularity among practitioners. Possible future extensions would be to enhance the RM model for liquidity effects.

1.7 Conclusion

We have introduced an intuitive way of enhancing VaR forecasts for liquidity effects by incorporating intraday liquidity information from the limit order book into a low frequency volatility forecasting model. Our model specification allows for instantaneous causality between volatility and illiquidity, as well as for liquidity to be modelled using a dynamic state process.

By using liquidity states assignment, we allow only extreme shocks in liquidity to affect volatility. We find that for stocks in the financial, technology and services sectors, the returns volatility is affected by large shocks in liquidity and is less sensitive to continuous changes in liquidity. Stocks in the capital goods and healthcare sectors have volatility that is less sensitive to large shocks in liquidity, but continuous changes in the mean of the proxy are able to explain some of the dynamics of their volatilities. Hence the incorporation of liquidity effects into an asset's volatility differs according to industry sectors. We construct portfolios by sectors and observe a similar phenomenon.

This suggests that investors regard liquidity differently for stocks in different industry sectors, and this is further aggravated by the increasing presence of financial analysts in the markets (see Piotroski and Roulstone (2004) who find that

	α	exceedance rate					Uncond Kupiec					Chistoffersen Indep					DM (Tick Loss)			
		EGl	EG	RM	VC	HS	EGl	EG	RM	VC	HS	EGl	EG	RM	VC	HS	EG	RM	VC	HS
Portfolio of all stocks																				
A	0.01	0.03	0.02	0.02	0.06	0.02	0.12	0.21	0.35	0.00**	0.21	0.55	0.63	0.77	0.39	0.72	**10**	-0.6	**29**	-15
	0.05	0.10	0.07	0.08	0.12	0.10	0.03*	0.32	0.18	0.00**	0.03*	0.33	0.62	0.62	0.26	0.46	-2	-27	**21**	-4
	0.10	0.15	0.14	0.12	0.18	0.15	0.11	0.23	0.49	0.02*	0.11	0.14	0.14	0.17	0.15	0.22	-18	-18	**10**	**13**
Portfolio by industry																				
F	0.01	0.01	0.01	0.01	0.05	0.03	0.82	0.82	0.84	0.00**	0.12	0.73	0.53	0.82	0.49	0.70	**48**	-82	**17**	-21
	0.05	0.06	0.05	0.07	0.14	0.11	0.64	0.92	0.41	0.00**	0.01**	0.93	1.00	0.26	0.46	0.33	**24**	-19	**21**	**11**
	0.10	0.14	0.10	0.14	0.20	0.20	0.23	1.00	0.18	0.00**	0.00**	0.18	0.25	0.57	0.34	0.50	-1	-20	**16**	**20**
T	0.01	0.02	0.02	0.02	0.06	0.02	0.21	0.21	0.21	0.00**	0.21	0.67	0.59	0.63	0.33	0.72	**8**	**3**	**30**	-3
	0.05	0.08	0.08	0.06	0.12	0.11	0.24	0.18	0.78	0.01**	0.02*	0.19	0.21	0.33	0.88	0.98	**7.2**	-7.6	**30**	**27**
	0.10	0.15	0.10	0.10	0.16	0.17	0.11	0.23	0.89	0.05*	0.03*	0.40	0.85	0.92	0.82	0.78	-2.2	-17	**18**	**26**
S	0.01	0.00	0.01	0.01	0.03	0.01	0.48	0.82	0.82	0.07	0.84	0.91	0.72	0.72	0.63	0.86	**31**	**7**	**22**	**27**
	0.05	0.07	0.06	0.06	0.10	0.08	0.41	0.78	0.64	0.02**	0.25	0.65	0.62	0.27	0.39	0.21	**33**	**33**	**28**	**24**
	0.10	0.12	0.10	0.09	0.13	0.14	0.41	0.89	0.79	0.34	0.18	0.37	0.33	0.46	0.19	0.29	**29**	**15**	**15**	**21**
C	0.01	0.04	0.04	0.01	0.04	0.00	0.04*	0.02*	0.84	0.01**	0.48	0.51	0.47	0.77	0.51	0.95	-4	-18	**15**	-17
	0.05	0.09	0.08	0.07	0.10	0.08	0.07	0.18	0.41	0.03*	0.18	0.88	0.13	0.22	0.46	0.22	-11	-26	**5**	-20
	0.10	0.14	0.14	0.12	0.14	0.16	0.18	0.18	0.58	0.23	0.05*	0.16	0.14	0.47	0.85	0.93	-4	-11	0.6	**18**
H	0.01	0.01	0.01	0.02	0.02	0.02	0.84	0.84	0.56	0.21	0.56	0.81	0.81	0.77	0.72	0.81	**2**	**3**	-1.6	-22
	0.05	0.06	0.05	0.05	0.08	0.08	0.78	0.92	0.92	0.13	0.24	0.30	0.33	0.43	0.90	0.24	**19**	**16**	**8**	**5**
	0.10	0.11	0.10	0.10	0.15	0.16	0.67	0.89	0.89	0.09	0.05*	0.82	0.54	0.55	0.55	0.65	-14	**3.3**	0.9	**11**

Table 1.5: VaR quantile evaluations for portfolios consisting of all stocks (A), financial stocks (F), technological stocks (T), service sector stocks (S), capital goods sector stocks (C) and healthcare sector stocks (H): the exceedance rates, p-values of the Kupiec and Christoffersen Independence test statistics, as well as the Diebold-Mariano (DM) test statistics are given here. * and ** indicates rejection of the test at 5% and 1% significance levels respectively. Bold font in the DM statistics indicates that the EGl model performs better than the respective models.

the participation of financial analysts increases stock return synchronicity). Our findings suggests that the market liquidity proxy should hence be aggregated by industry sectors and not across different sectors and this could explain why Clement (1999) finds that analysts who have portfolios across different industries tend to perform weaker than analysts who specialise in an industry.

From the results of VaR forecast for portfolios, we find that the improvement in VaR forecast when the effects of liquidity are accounted for is greater as the quantile of interest becomes more extreme. Improvements at the 1% quantile tend to be the greatest. A portfolio formed from grouping stocks from different industries does not perform well for the less extreme 5% and 10% quantiles. By segregating the stocks into portfolios of different industry sectors, our model produces more accurate VaR forecasts at 1% and 5% quantiles in industry sectors where liquidity states have explanatory power on conditional volatility. Overall, we find that the more extreme the quantile in consideration, the more important it is to include the effects of liquidity for forecasting.

The model we have presented here is a simple prototype of adjusting the conditional VaR for liquidity effects by using the EGARCH volatility model as a vehicle to incorporate liquidity. The flexibility of our model which allows for time-varying liquidity risks makes it potentially attractive for financial institutions in their risk management, as inclusion of the liquidity dimension in market risk may not only enhance their risk measure, but also possibly decrease their risk capital charges in periods when liquidity risk is low. The overall simplicity of our model has also much potential for extensions and adaptations to industry practice.

Further possible research areas would be to consider enhancing other volatility models like RiskMetrics for liquidity effects, finding better methods of determining the threshold for the liquidity states, and to study how to incorporate the effects of liquidity on the extreme quantiles directly without involving a volatility model, for instance, incorporating liquidity effects into extreme-value theory (EVT) type models for VaR estimation.

We should also make clear that while our liquidity proxy harnesses intradaily data from the limit order book, it is not a realized measure, and hence does not fully exploit the path variation of intraday liquidity information. Such a method is clearly more desirable but we leave this to future research.

Appendix

1.7.1 Tables

	company name	mean	std	max	min	skew	kurt	LB(10)	LB(25)
	Financial								
axp	American Express	-0.000	0.020	0.115	-0.145	-0.281	8.224	1.420	1.048
c	Citigroup	-0.001	0.019	0.116	-0.167	-0.229	11.646	4.925	2.542
jpm	JP Morgan Chase	-0.000	0.022	0.147	-0.183	0.062	11.640	2.413	1.714
	Technology and Conglomerates								
hpq	Hewlett-Packard	0.000	0.025	0.160	-0.207	0.072	11.078	1.901	1.693
ibm	IBM	0.000	0.017	0.117	-0.110	0.307	9.375	1.567	1.939
mmm	3M	0.000	0.014	0.076	-0.090	-0.099	8.074	2.075	1.902
ge	General Electric	-0.000	0.018	0.111	-0.117	-0.059	9.275	1.676	2.224
utx	United Technologies	0.000	0.019	0.119	-0.330	-2.651	54.049	5.813	3.567
ba	Boeing	0.000	0.019	0.124	-0.198	-0.632	11.863	2.425	1.827
	Services								
hd	Home Depot	-0.000	0.021	0.123	-0.149	0.062	8.385	1.850	1.628
mcd	Mc Donald's	0.000	0.016	0.087	-0.130	-0.247	8.536	1.299	0.890
dis	Walt Disney	-0.000	0.021	0.134	-0.200	-0.074	11.552	1.506	1.447
wmt	Wal-Mart	0.000	0.015	0.096	-0.080	0.322	6.772	2.503	2.172
	Energy, Basic Materials and Capital Goods								
aa	Alcoa	-0.000	0.023	0.140	-0.117	-0.155	6.926	2.362	1.611
dd	E.I. du Pont	-0.000	0.015	0.094	-0.106	-0.083	8.197	2.530	2.008
cat	Caterpiller	0.000	0.019	0.111	-0.147	-0.293	7.805	1.79	1.946
hon	Honeywell	-0.000	0.020	0.107	-0.181	-0.735	12.392	2.188	1.810
xom	Exxon Mobil	0.000	0.017	0.162	-0.150	0.082	15.912	12.833	6.642
	Consumer and Healthcare								
mo	Altria Group	0.000	0.014	0.087	-0.134	-0.941	15.844	2.257	2.119
ko	Coco Cola	0.000	0.013	0.085	-0.103	-0.247	10.342	1.347	1.613
pg	Procter & Gamble	0.000	0.012	0.092	-0.0746	-0.057	9.666	4.610	4.002
gm	General Motors	-0.001	0.021	0.130	-0.141	-0.009	7.214	1.373	1.526
jnj	Johnson & Johnson	0.000	0.013	0.086	-0.175	-1.034	24.230	2.994	2.780
mrk	Merck	-0.000	0.017	0.092	-0.256	-2.309	35.077	1.378	1.506
pfe	Pfeizer	-0.000	0.015	0.071	-0.106	-0.347	7.994	3.170	2.426

Table 1.6: Descriptive statistics of daily log returns of 25 individual stocks for the period between Jan 2001- Dec 2008. $Q(10)$ and $Q(25)$ gives the Ljung-Box Q statistics at 10 and 25 lags respectively.

	EGARCH(1,1)					EGARCHX(1,1)					
	ω	α	β	γ	v	ω	α	β	γ	δ	v
	Financial										
axp	0.559	-0.081	0.976	2.929	6.412	0.444	-0.089	0.958	4.253	0.015	6.361
	(0.041)	(0.012)	(0.004)	(0.497)	(0.886)	(0.073)	(0.015)	(0.010)	(0.860)	(0.010)	(0.872)
c	0.650	-0.066	0.986	1.949	5.712	0.664	-0.064	0.987	1.762	-0.001	5.722
	(0.017)	(0.007)	(0.002)	(0.243)	(0.640)	(0.013)	(0.007)	(0.001)	(0.187)	(0.001)	(0.646)
jpm	0.534	-0.053	0.974	3.419	5.572	0.436	-0.059	0.962	4.501	0.005	5.527
	(0.044)	(0.009)	(0.005)	(0.538)	(0.643)	(0.106)	(0.014)	(0.013)	(1.160)	(0.006)	(0.639)
	Technology and Conglomerates										
hpq	0.342	-0.049	0.951	4.657	4.451	-0.004	-0.048	0.857	5.489	**0.094**	4.717
	(0.085)	(0.014)	(0.009)	(0.845)	(0.468)	(0.107)	(0.020)	(0.030)	(0.836)	(0.034)	(0.527)
ibm	0.207	-0.062	0.941	7.649	6.072	-0.081	-0.066	0.896	9.605	**0.044**	6.134
	(0.102)	(0.013)	(0.010)	(1.249)	(0.784)	(0.144)	(0.016)	(0.020)	(1.441)	(0.017)	(0.806)
mmm	0.259	-0.081	0.946	6.849	4.937	0.068	-0.083	0.916	7.537	**0.029**	5.009
	(0.089)	(0.015)	(0.009)	(1.068)	(0.572)	(0.111)	(0.016)	(0.013)	(1.297)	(0.009)	(0.591)
ge	0.512	-0.051	0.973	4.035	6.758	0.229	-0.069	0.933	7.173	0.020	6.469
	(0.043)	(0.010)	(0.004)	(0.589)	(1.050)	(0.146)	(0.017)	(0.022)	(1.638)	(0.014)	(0.984)
utx	0.431	-0.070	0.962	4.027	6.774	0.202	-0.081	0.931	6.002	**0.022**	7.164
	(0.073)	(0.015)	(0.008)	(0.770)	(1.325)	(0.113)	(0.018)	(0.014)	(1.173)	(0.008)	(1.473)
ba	0.236	-0.062	0.941	5.665	8.367	0.193	-0.056	0.922	5.774	**0.036**	8.725
	(0.121)	(0.013)	(0.013)	(1.178)	(1.425)	(0.126)	(0.015)	(0.016)	(1.226)	(0.013)	(1.548)
	Services										
hd	0.453	-0.066	0.964	3.840	7.187	0.452	-0.066	0.964	3.848	0.000	7.186
	(0.068)	(0.010)	(0.007)	(0.736)	(1.053)	(0.075)	(0.010)	(0.009)	(0.788)	(0.003)	(1.052)
mcd	0.294	-0.051	0.949	6.208	6.391	0.234	-0.040	0.933	6.537	**0.019**	6.497
	(0.085)	(0.014)	(0.009)	(1.015)	(0.825)	(0.104)	(0.016)	(0.013)	(1.140)	(0.008)	(0.851)
dis	0.505	-0.063	0.970	3.342	6.687	0.189	-0.070	0.920	6.089	**0.031**	7.095
	(0.047)	(0.011)	(0.005)	(0.516)	(1.062)	(0.142)	(0.016)	(0.022)	(1.344)	(0.014)	(1.157)
wmt	0.339	-0.027	0.955	6.571	7.199	0.111	-0.026	0.924	8.219	**0.015**	7.108
	(0.086)	(0.014)	(0.009)	(1.184)	(0.983)	(0.132)	(0.017)	(0.016)	(1.495)	(0.007)	(0.966)
	Energy, Basic Materials and Capital Goods										
aa	0.296	-0.040	0.945	4.508	7.672	0.205	-0.040	0.928	5.136	**0.014**	7.493
	(0.094)	(0.015)	(0.011)	(0.716)	(1.220)	(0.092)	(0.015)	(0.012)	(0.743)	(0.006)	(1.158)
dd	0.286	-0.061	0.949	6.750	7.486	0.121	-0.067	0.917	7.544	**0.035**	7.681
	(0.076)	(0.013)	(0.008)	(0.991)	(1.175)	(0.107)	(0.016)	(0.015)	(1.267)	(0.012)	(1.248)
cat	0.293	-0.064	0.945	4.349	6.365	0.202	-0.061	0.925	4.852	**0.026**	6.375
	(0.087)	(0.012)	(0.010)	(0.812)	(0.925)	(0.101)	(0.014)	(0.013)	(0.965)	(0.009)	(0.950)
hon	0.208	-0.075	0.938	5.851	6.434	-0.050	-0.064	0.883	7.283	**0.064**	6.811
	(0.116)	(0.014)	(0.012)	(1.125)	(0.959)	(0.140)	(0.015)	(0.021)	(1.268)	(0.018)	(1.046)
xom	-0.204	-0.092	0.895	9.291	10.098	-0.329	-0.084	0.861	9.996	**0.043**	10.760
	(0.161)	(0.018)	(0.017)	(1.599)	(1.944)	(0.171)	(0.020)	(0.023)	(1.609)	(0.016)	(2.244)
	Consumer and Healthcare										
mo	0.038	-0.051	0.928	12.729	3.721	-0.317	-0.077	0.805	17.896	**0.213**	3.970
	(0.176)	(0.018)	(0.017)	(2.549)	(0.326)	(0.244)	(0.021)	(0.051)	(3.265)	(0.071)	(0.365)
ko	0.272	-0.053	0.950	8.713	5.546	-0.028	-0.065	0.905	11.224	**0.037**	5.842
	(0.107)	(0.015)	(0.010)	(1.566)	(0.684)	(0.138)	(0.018)	(0.017)	(1.947)	(0.011)	(0.764)
pg	0.144	-0.084	0.937	9.333	5.514	-0.066	-0.107	0.906	10.462	**0.023**	5.530
	(0.149)	(0.017)	(0.015)	(1.822)	(0.637)	(0.160)	(0.019)	(0.018)	(1.862)	(0.007)	(0.627)
gm	0.324	-0.035	0.950	5.440	5.006	0.332	-0.035	0.952	5.342	-0.002	5.026
	(0.093)	(0.011)	(0.010)	(1.064)	(0.562)	(0.090)	(0.010)	(0.010)	(1.024)	(0.003)	(0.568)
jnj	0.189	-0.090	0.942	8.858	5.607	0.016	-0.102	0.908	9.257	**0.038**	6.005
	(0.145)	(0.017)	(0.014)	(1.979)	(0.637)	(0.127)	(0.018)	(0.015)	(1.802)	(0.009)	(0.734)
mrk	0.385	-0.043	0.959	5.622	4.043	0.083	-0.047	0.919	8.195	0.018	4.001
	(0.071)	(0.011)	(0.007)	(0.886)	(0.420)	(0.249)	(0.014)	(0.031)	(2.411)	(0.011)	(0.417)
pfe	0.274	-0.063	0.948	6.984	5.613	0.019	-0.061	0.911	9.160	**0.020**	5.715
	(0.119)	(0.013)	(0.012)	(1.549)	(0.754)	(0.214)	(0.017)	(0.026)	(2.375)	(0.010)	(0.791)

Table 1.7: Estimated results for EGARCH(1,1) and EGARCHX(1,1) (with continuous liquidity proxy) for individual stocks. Note that β_1 decreases with the inclusion of the liquidity proxy.

	0.8/0.2 thresholds							0.75/0.25 thresholds						
	ω	α	β	γ	δ_1	δ_2	v	ω	α	β	γ	δ_1	δ_2	v
	Financial													
axp	0.664	-0.068	0.987	1.369	**0.015**	**0.026**	6.725	0.642	-0.073	0.985	1.614	**0.018**	**0.025**	6.730
	(0.01)	(0.01)	(0.00)	(0.14)	(0.006)	(0.006)	(0.98)	(0.02)	(0.01)	(0.00)	(0.19)	(0.006)	(0.006)	(0.98)
c	0.678	-0.067	0.989	1.373	**0.012**	**0.019**	5.873	0.671	-0.067	0.988	1.501	**0.012**	**0.016**	5.780
	(0.01)	(0.01)	(0.00)	(0.13)	(0.005)	(0.007)	(0.68)	(0.01)	(0.01)	(0.00)	(0.15)	(0.006)	(0.008)	(0.66)
jpm	0.652	-0.049	0.986	1.615	**0.015**	**0.031**	5.725	0.656	-0.046	0.987	1.469	**0.020**	**0.037**	5.850
	(0.01)	(0.01)	(0.00)	(0.21)	(0.005)	(0.006)	(0.66)	(0.01)	(0.01)	(0.00)	(0.18)	(0.005)	(0.006)	(0.69)
	Technology and Conglomerates													
hpq	0.536	-0.037	0.973	2.743	0.004	**0.023**	4.556	0.559	-0.035	0.975	2.480	0.008	**0.024**	4.575
	(0.06)	(0.01)	(0.01)	(0.60)	(0.007)	(0.008)	(0.48)	(0.04)	(0.01)	(0.01)	(0.48)	(0.006)	((0.007)	(0.49)
ibm	0.306	-0.060	0.953	6.172	**0.036**	**0.037**	6.208	0.329	-0.058	0.956	5.825	**0.036**	**0.037**	6.188
	(0.09)	(0.01)	(0.01)	(1.13)	(0.011)	(0.010)	(0.82)	(0.09)	(0.01)	(0.01)	(1.08)	(0.011)	(0.009)	(0.82)
mmm	0.346	-0.075	0.955	5.555	0.010	**0.022**	4.989	0.355	-0.074	0.957	5.384	0.014	**0.025**	5.001
	(0.09)	(0.02)	(0.01)	(1.04)	(0.010)	(0.011)	(0.57)	(0.09)	(0.02)	(0.01)	(0.99)	(0.010)	(0.012)	(0.57)
ge	0.598	-0.038	0.981	2.748	0.004	**0.018**	6.984	0.593	-0.038	0.981	2.800	0.005	**0.018**	6.985
	(0.02)	(0.01)	(0.00)	(0.34)	(0.004)	(0.006)	(1.12)	(0.02)	(0.01)	(0.00)	(0.35)	(0.005)	(0.006)	(1.12)
utx	0.534	-0.054	0.973	2.841	-0.001	**0.017**	6.862	0.506	-0.057	0.970	3.139	0.000	0.014	6.839
	(0.05)	(0.01)	(0.01)	(0.52)	(0.006)	(0.008)	(1.33)	(0.06)	(0.02)	(0.01)	(0.60)	(0.007)	(0.008)	(1.34)
ba	0.281	-0.060	0.947	5.038	0.023	**0.034**	8.577	0.271	-0.062	0.946	5.040	**0.029**	**0.037**	8.569
	(0.13)	(0.01)	(0.01)	(1.20)	(0.012)	(0.013)	(1.53)	(0.13)	(0.01)	(0.01)	(1.24)	(0.012)	(0.013)	(1.53)
	Services													
hd	0.473	-0.062	0.967	3.407	0.000	**0.019**	7.170	0.460	-0.063	0.965	3.543	0.003	0.017	7.157
	(0.07)	(0.01)	(0.01)	(0.72)	(0.007)	(0.009)	(1.05)	(0.07)	(0.01)	(0.01)	(0.73)	(0.007)	(0.009)	(1.05)
mcd	0.392	-0.048	0.960	4.686	0.022	**0.031**	6.488	0.390	-0.048	0.960	4.666	**0.023**	**0.031**	6.472
	(0.07)	(0.01)	(0.01)	(0.88)	(0.009)	(0.009)	(0.84)	(0.07)	(0.01)	(0.01)	(0.89)	(0.010)	(0.009)	(0.84)
dis	0.553	-0.056	0.975	2.697	0.010	**0.019**	6.882	0.548	-0.056	0.975	2.765	0.009	**0.018**	6.849
	(0.04)	(0.01)	(0.00)	(0.37)	(0.007)	(0.008)	(1.13)	(0.04)	(0.01)	(0.00)	(0.39)	(0.007)	(0.008)	(1.13)
wmt	0.443	-0.029	0.966	4.513	0.008	**0.031**	7.760	0.446	-0.030	0.966	4.465	0.013	**0.034**	7.738
	(0.06)	(0.01)	(0.01)	(0.79)	(0.008)	(0.010)	(1.13)	(0.06)	(0.01)	(0.01)	(0.77)	(0.009)	(0.011)	(1.12)
	Energy, Basic Materials and Capital Goods													
aa	0.354	-0.029	0.952	3.869	-0.008	0.021	7.827	0.348	-0.030	0.951	3.901	-0.006	0.021	7.862
	(0.09)	(0.01)	(0.01)	(0.65)	(0.012)	(0.011)	(1.28)	(0.09)	(0.01)	(0.01)	(0.66)	(0.012)	(0.012)	(1.28)
dd	0.339	-0.058	0.956	5.639	0.018	**0.033**	7.538	0.334	-0.055	0.955	5.717	0.011	**0.027**	6.22
	(0.07)	(0.01)	(0.01)	(0.91)	(0.011)	(0.011)	(1.20)	(0.07)	(0.01)	(0.01)	(0.93)	(0.012)	(0.012)	(1.22)
cat	0.331	-0.058	0.948	3.943	-0.019	0.000	6.324	0.332	-0.058	0.948	3.954	-0.021	-0.004	6.332
	(0.09)	(0.01)	(0.01)	(0.81)	(0.011)	(0.012)	(0.93)	(0.09)	(0.01)	(0.01)	(0.79)	(0.011)	(0.011)	(0.93)
hon	0.216	-0.075	0.939	5.658	0.011	0.014	6.438	0.216	-0.075	0.939	5.658	0.011	0.014	6.438
	(0.12)	(0.01)	(0.01)	(1.13)	(0.013)	(0.013)	(0.97)	(0.12)	(0.01)	(0.01)	(1.14)	(0.013)	(0.013)	(0.96)
xom	-0.263	-0.089	0.888	7.929	-0.017	**0.064**	12.214	-0.283	-0.091	0.886	8.059	-0.017	**0.062**	12.072
	(0.19)	(0.02)	(0.02)	(1.52)	(0.016)	(0.022)	(2.90)	(0.19)	(0.02)	(0.02)	(1.50)	(0.016)	(0.023)	(2.81)
	Consumer and Healthcare													
mo	0.045	-0.049	0.929	12.498	-0.009	0.023	3.729	0.016	-0.049	0.926	12.847	0.002	0.028	3.729
	(0.20)	(0.02)	(0.02)	(2.78)	(0.016)	(0.022)	(0.33)	(0.23)	(0.02)	(0.02)	(3.18)	(0.017)	(0.023)	(0.33)
ko	0.406	-0.049	0.963	6.450	-0.005	0.016	5.618	0.383	-0.050	0.961	6.825	-0.007	0.012	5.600
	(0.08)	(0.01)	(0.01)	(1.17)	(0.008)	(0.009)	(0.69)	(0.09)	(0.01)	(0.01)	(1.28)	(0.010)	(0.010)	(0.69)
pg	0.183	-0.085	0.940	7.950	-0.014	0.018	5.703	0.132	-0.088	0.934	8.658	-0.019	0.009	5.687
	(0.15)	(0.02)	(0.02)	(1.73)	(0.012)	(0.014)	(0.67)	(0.16)	(0.02)	(0.02)	(1.84)	(0.015)	(0.015)	(0.67)
gm	0.364	-0.035	0.954	5.076	-0.015	-0.012	5.042	0.336	-0.034	0.951	5.295	-0.012	-0.007	5.039
	(0.09)	(0.01)	(0.01)	(1.02)	(0.011)	(0.014)	(0.57)	(0.10)	(0.01)	(0.01)	(1.07)	(0.012)	(0.015)	(0.58)
jnj	0.229	-0.085	0.946	8.131	0.009	0.019	5.655	0.203	-0.087	0.944	8.424	0.013	0.020	5.677
	(0.15)	(0.02)	(0.01)	(1.97)	(0.010)	(0.013)	(0.65)	(0.15)	(0.02)	(0.01)	(2.02)	(0.012)	(0.014)	(0.65)
mrk	0.424	-0.047	0.963	4.908	0.006	0.016	4.056	0.419	-0.046	0.963	4.997	0.006	0.015	4.052
	(0.07)	(0.01)	(0.01)	(0.81)	(0.009)	(0.009)	(0.43)	(0.07)	(0.01)	(0.01)	(0.84)	(0.009)	(0.010)	(0.42)
pfe	0.343	-0.058	0.956	5.638	0.014	**0.031**	5.775	0.315	-0.059	0.953	6.008	0.016	**0.028**	5.747
	(0.10)	(0.01)	(0.01)	(1.27)	(0.008)	(0.012)	(0.78)	(0.11)	(0.01)	(0.01)	(1.392)	(0.010)	(0.01)	(0.782)

Table 1.8: Estimation results for EGARCH-liq model (with liquidity states) using individual stocks. Both sets of results when the liquidity states are determined using 0.8/0.2 and 0.75/0.25 thresholds are given.

	$\mu^{(1)}$	$\mu^{(2)}$	$A^{(11)}$	$A^{(12)}$	$A^{(21)}$	$A^{(22)}$	$B^{(11)}$	$B^{(12)}$	$B^{(21)}$	$B^{(22)}$	log lik	SIC
	Financial											
axp	-0.109	-0.046	0.337	-0.080	-0.048	0.299	0.947	-0.040	-0.010	0.968	-0.768	77.60
	(0.042)	(0.050)	(0.040)	(0.055)	(0.052)	(0.063)	(0.017)	(0.020)	(0.020)	(0.025)		
c	-0.231	-0.130	0.406	-0.011	-0.123	0.425	0.854	-0.101	-0.054	0.932	-0.722	77.51
	(0.141)	(0.104)	(0.066)	(0.066)	(0.066)	(0.047)	(0.075)	(0.064)	(0.056)	(0.048)		
jpm	-0.062	-0.057	0.261	-0.043	-0.179	0.379	0.963	-0.024	-0.013	0.955	-0.719	77.50
	(0.034)	(0.038)	(0.052)	(0.046)	(0.052)	(0.045)	(0.019)	(0.019)	(0.015)	(0.024)		
	Technology and Conglomerates											
ibm	-0.436	-0.354	0.477	-0.017	-0.022	0.508	0.783	-0.168	-0.152	0.836	-0.709	77.48
	(0.486)	(0.574)	(0.084)	(0.101)	(0.090)	(0.132)	(0.216)	(0.197)	(0.257)	(0.237)		
hpq	-0.044	-0.033	0.326	-0.174	-0.051	0.322	0.964	-0.014	-0.001	0.965	-0.746	77.56
	(0.020)	(0.030)	(0.045)	(0.047)	(0.041)	(0.051)	(0.016)	(0.011)	(0.016)	(0.020)		
mmm	-0.093	-0.126	0.332	-0.159	0.041	0.345	0.947	-0.034	-0.042	0.938	-0.741	77.55
	(0.058)	(0.107)	(0.042)	(0.062)	(0.058)	(0.079)	(0.027)	(0.025)	(0.046)	(0.047)		
ge	-0.176	-0.146	0.335	-0.058	-0.003	0.400	0.893	-0.095	-0.064	0.910	-0.709	77.48
	(0.074)	(0.070)	(0.039)	(0.045)	(0.056)	(0.068)	(0.038)	(0.040)	(0.036)	(0.041)		
utx	-0.043	-0.094	0.312	-0.099	-0.097	0.374	0.973	-0.012	-0.024	0.946	-0.742	77.55
	(0.025)	(0.043)	(0.030)	(0.042)	(0.045)	(0.048)	(0.009)	(0.012)	(0.016)	(0.023)		
ba	-0.134	-0.155	0.230	-0.075	-0.046	0.360	0.938	-0.066	-0.049	0.902	-0.799	77.66
	(0.106)	(0.109)	(0.047)	(0.047)	(0.066)	(0.056)	(0.045)	(0.055)	(0.047)	(0.056)		
	Services											
hd	0.030	-0.126	0.250	-0.113	-0.163	0.335	1.003	0.022	-0.046	0.933	-0.739	77.54
	(0.099)	(0.096)	(0.063)	(0.069)	(0.068)	(0.040)	(0.047)	(0.047)	(0.036)	(0.047)		
mcd	-0.058	-0.063	0.292	0.016	-0.034	0.263	0.960	-0.023	-0.031	0.962	-0.794	77.65
	(0.047)	(0.038)	(0.060)	(0.048)	(0.040)	(0.030)	(0.030)	(0.022)	(0.023)	(0.018)		
dis	-0.051	-0.026	0.242	-0.113	0.002	0.262	0.963	-0.023	0.001	0.977	-0.790	77.64
	(0.023)	(0.020)	(0.045)	(0.039)	(0.034)	(0.029)	(0.016)	(0.011)	(0.010)	(0.011)		
wmt	-0.028	-0.073	0.266	-0.111	-0.078	0.332	0.975	-0.005	-0.026	0.948	-0.758	77.58
	(0.025)	(0.040)	(0.035)	(0.065)	(0.059)	(0.052)	(0.016)	(0.013)	(0.018)	(0.020)		
	Energy, Basic Materials and Capital Goods											
aa	-0.701	-0.180	0.378	-0.082	-0.103	0.291	0.613	-0.314	-0.082	0.911	-0.754	77.57
	(0.984)	(0.166)	(0.079)	(0.090)	(0.081)	(0.052)	(0.514)	(0.441)	(0.084)	(0.075)		
dd	-0.145	-0.030	0.435	-0.086	0.018	0.318	0.872	-0.058	-0.002	0.974	-0.797	77.66
	(0.053)	(0.042)	(0.048)	(0.063)	(0.054)	(0.050)	(0.040)	(0.024)	(0.031)	(0.022)		
cat	-0.048	-0.157	0.308	-0.062	-0.073	0.378	0.965	-0.012	-0.064	0.942	-0.729	77.52
	(0.052)	(0.150)	(0.056)	(0.068)	(0.084)	(0.048)	(0.025)	(0.017)	(0.070)	(0.034)		
hon	-0.103	-0.081	0.315	-0.014	-0.027	0.293	0.925	-0.039	-0.039	0.955	-0.789	77.64
	(0.049)	(0.042)	(0.048)	(0.047)	(0.048)	(0.049)	(0.030)	(0.023)	(0.024)	(0.020)		
xom	-0.136	-0.047	0.342	-0.133	-0.073	0.394	0.948	-0.045	-0.012	0.969	-0.616	77.30
	(0.054)	(0.048)	(0.041)	(0.051)	(0.070)	(0.058)	(0.018)	(0.021)	(0.017)	(0.020)		
	Consumer and Healthcare											
mo	-0.039	-0.063	0.286	-0.082	-0.041	0.298	0.965	-0.010	-0.013	0.961	-0.799	77.66
	(0.029)	(0.024)	(0.031)	(0.041)	(0.044)	(0.030)	(0.013)	(0.014)	(0.011)	(0.013)		
ko	-0.197	-0.173	0.403	0.017	0.013	0.337	0.879	-0.085	-0.087	0.912	-0.757	77.58
	(0.076)	(0.058)	(0.052)	(0.049)	(0.047)	(0.052)	(0.042)	(0.034)	(0.029)	(0.028)		
pg	-0.103	-0.133	0.314	0.004	-0.113	0.324	0.933	-0.047	-0.063	0.924	-0.709	77.48
	(0.053)	(0.057)	(0.040)	(0.056)	(0.053)	(0.046)	(0.028)	(0.027)	(0.030)	(0.029)		
gm	-0.377	-0.369	0.354	-0.005	-0.056	0.386	0.801	-0.171	-0.165	0.816	-0.742	77.55
	(0.149)	(0.166)	(0.054)	(0.058)	(0.068)	(0.064)	(0.072)	(0.069)	(0.079)	(0.078)		
jnj	-0.111	-0.065	0.232	-0.022	-0.073	0.307	0.953	-0.052	-0.017	0.959	-0.735	77.53
	(0.100)	(0.065)	(0.068)	(0.049)	(0.073)	(0.043)	(0.040)	(0.049)	(0.026)	(0.031)		
mrk	-0.126	-0.083	0.411	-0.071	-0.091	0.300	0.920	-0.049	-0.031	0.954	-0.698	77.46
	(0.105)	(0.070)	(0.060)	(0.062)	(0.070)	(0.039)	(0.051)	(0.049)	(0.035)	(0.032)		
pfe	-0.055	-0.020	0.384	-0.088	-0.104	0.259	0.949	-0.021	0.000	0.980	-0.738	77.54
	(0.032)	(0.016)	(0.051)	(0.045)	(0.046)	(0.037)	(0.020)	(0.018)	(0.010)	(0.009)		

Table 1.9: Unrestricted ACM estimation results for individual stocks. SIC gives the Schwarz information criteria.

	$\mu^{(1)}$	$\mu^{(2)}$	$A^{(11)}$	$A^{(12)}$	$B^{(11)}$	$B^{(22)}$	log lik	SIC	LR test	$mean_{resid}$	var_{resid}	
	Financial											
axp	-0.034	-0.032	0.311	-0.102	0.972	0.970	-0.771	47.18	0.004	-0.001	0.132	-0.024
	(0.012)	(0.011)	(0.029)	(0.043)	(0.007)	(0.008)			(1.000)	-0.197	-0.024	0.287
c	-0.015	-0.054	0.358	-0.163	0.958	0.968	-0.726	47.09	0.009	0.003	0.153	-0.022
	(0.011)	(0.015)	(0.035)	(0.052)	(0.009)	(0.007)			(1.000)	-0.185	-0.022	0.210
jpm	-0.023	-0.043	0.320	-0.139	0.971	0.964	-0.722	47.08	0.007	0.004	0.134	-0.023
	(0.010)	(0.012)	(0.031)	(0.033)	(0.007)	(0.007)			(1.000)	-0.148	-0.023	0.244
	Technology and Conglomerates											
hpq	-0.028	-0.028	0.326	-0.119	0.971	0.964	-0.748	47.13	0.003	0.003	0.147	-0.024
	(0.011)	(0.012)	(0.027)	(0.035)	(0.009)	(0.007)			(1.000)	-0.156	-0.024	0.260
ibm	-0.049	-0.044	0.393	-0.176	0.952	0.962	-0.713	47.06	0.008	0.002	0.002	-0.018
	(0.016)	(0.015)	(0.040)	(0.053)	(0.014)	(0.008)			(1.000)	-0.152	-0.018	0.291
mmm	-0.025	-0.037	0.301	-0.116	0.975	0.971	-0.746	47.13	0.010	-0.003	0.136	-0.019
	(0.010)	(0.013)	(0.030)	(0.037)	(0.007)	(0.008)			(1.000)	-0.193	-0.019	0.243
ge	-0.013	-0.041	0.315	-0.107	0.970	0.969	-0.714	47.07	0.011	0.008	0.151	-0.023
	(0.010)	(0.012)	(0.035)	(0.041)	(0.009)	(0.007)			(1.000)	-0.159	-0.023	0.218
utx	-0.024	-0.041	0.335	-0.098	0.975	0.967	-0.743	47.12	0.003	-0.004	0.126	-0.020
	(0.012)	(0.018)	(0.035)	(0.074)	(0.006)	(0.009)			(1.000)	-0.189	-0.020	0.249
ba	-0.023	-0.042	0.248	-0.139	0.978	0.961	-0.803	47.24	0.007	-0.005	0.132	-0.024
	(0.009)	(0.013)	(0.033)	(0.037)	(0.006)	(0.010)			(1.000)	-0.183	-0.024	0.232
	Services											
hd	-0.020	-0.030	0.298	-0.149	0.974	0.976	-0.741	47.12	0.004	-0.003	0.130	-0.019
	(0.010)	(0.011)	(0.024)	(0.036)	(0.006)	(0.006)			(1.000)	-0.168	-0.019	0.239
mcd	-0.017	-0.017	0.256	-0.042	0.984	0.982	-0.796	47.23	0.005	0.001	0.145	-0.031
	(0.008)	(0.008)	(0.025)	(0.033)	(0.006)	(0.005)			(1.000)	-0.146	-0.031	0.273
dis	-0.016	-0.030	0.258	-0.066	0.978	0.973	-0.793	47.22	0.006	0.005	0.169	-0.028
	(0.009)	(0.010)	(0.026)	(0.033)	(0.008)	(0.007)			(1.000)	-0.220	-0.028	0.215
wmt	-0.024	-0.030	0.297	-0.100	0.971	0.970	-0.759	47.15	0.003	0.002	0.147	-0.023
	(0.010)	(0.011)	(0.027)	(0.039)	(0.008)	(0.006)			(1.000)	-0.173	-0.023	0.240
	Energy, Basic Materials and Capital Goods											
aa	-0.018	-0.040	0.273	-0.156	0.968	0.972	-0.760	47.16	0.012	-0.001	0.155	-0.024
	(0.010)	(0.011)	(0.033)	(0.041)	(0.007)	(0.006)			(1.000)	-0.165	-0.024	0.234
dd	-0.040	-0.035	0.370	-0.070	0.941	0.965	-0.802	47.24	0.008	0.002	0.154	-0.031
	(0.016)	(0.013)	(0.034)	(0.036)	(0.018)	(0.009)			(1.000)	-0.163	-0.031	0.263
cat	-0.025	-0.028	0.318	-0.071	0.974	0.978	-0.731	47.10	0.004	-0.003	0.122	-0.020
	(0.011)	(0.011)	(0.026)	(0.039)	(0.007)	(0.006)			(1.000)	-0.152	-0.020	0.292
hon	-0.029	-0.026	0.280	-0.075	0.963	0.976	-0.792	47.22	0.006	0.004	0.150	-0.030
	(0.011)	(0.010)	(0.031)	(0.032)	(0.010)	(0.006)			(1.000)	-0.168	-0.030	0.259
xom	-0.032	-0.019	0.355	-0.142	0.981	0.979	-0.619	46.87	0.006	0.005	0.113	-0.012
	(0.011)	(0.011)	(0.031)	(0.056)	(0.005)	(0.005)			(1.000)	-0.123	-0.012	0.325
	Consumer and Healthcare											
mo	-0.023	-0.041	0.284	-0.081	0.970	0.970	-0.800	47.24	0.001	-0.003	0.139	-0.026
	(0.010)	(0.013)	(0.022)	(0.026)	(0.007)	(0.008)			(1.000)	-0.210	-0.026	0.223
ko	-0.027	-0.034	0.323	-0.065	0.962	0.971	-0.764	47.17	0.015	0.006	0.151	-0.025
	(0.011)	(0.012)	(0.040)	(0.047)	(0.012)	(0.007)			(1.000)	-0.201	-0.025	0.228
pg	-0.017	-0.029	0.285	-0.124	0.975	0.975	-0.712	47.06	0.007	0.002	0.134	-0.022
	(0.008)	(0.011)	(0.030)	(0.047)	(0.006)	(0.005)			(1.000)	-0.106	-0.022	0.249
gm	-0.022	-0.042	0.280	-0.156	0.970	0.967	-0.746	47.13	0.007	0.004	0.146	-0.022
	(0.010)	(0.012)	(0.028)	(0.033)	(0.007)	(0.007)			(1.000)	-0.166	-0.022	0.222
jnj	-0.016	-0.033	0.253	-0.118	0.984	0.974	-0.739	47.12	0.008	0.001	0.133	-0.022
	(0.008)	(0.010)	(0.028)	(0.038)	(0.004)	(0.006)			(1.000)	-0.189	-0.022	0.234
mrk	-0.024	-0.033	0.329	-0.142	0.968	0.974	-0.701	47.04	0.005	0.005	0.140	-0.018
	(0.010)	(0.012)	(0.029)	(0.038)	(0.008)	(0.006)			(1.000)	-0.180	-0.018	0.233
pfe	-0.021	-0.030	0.317	-0.105	0.968	0.969	-0.740	47.12	0.004	0.008	0.151	-0.025
	(0.010)	(0.012)	(0.027)	(0.036)	(0.008)	(0.007)			(1.000)	-0.183	-0.025	0.212

Table 1.10: Restricted ACM estimation results for individual stocks. SIC gives the Schwarz information criteria, $LRtest$ gives the likelihood ratio test statistics, $mean_{resid}$ and cov_{resid} give the mean and covariance of the residuals respectively.

	commonality regression				EGARCHX -common liq					
	α_{meanqs}	β_{meanqs}	R^2	$\overline{R}^2$	ω	α	β	γ	δ	v
	Financial									
axp	0.008	0.673	0.664	0.663	0.689	-0.072	0.991	1.351	**-0.005**	6.492
	(0.001)	(0.011)			(0.008)	(0.009)	(0.005)	(0.142)	(0.001)	(0.915)
c	-0.008	0.812	0.727	0.727	0.216	-0.102	0.932	7.101	0.025	5.894
	(0.001)	(0.011)			(0.147)	(0.018)	(0.019)	(1.735)	(0.013)	(0.674)
jpm	-0.020	1.367	0.725	0.725	0.309	-0.069	0.943	5.975	0.019	5.470
	(0.001)	(0.019)			(0.136)	(0.017)	(0.019)	(1.518)	(0.014)	(0.626)
	Technology and Conglomerates									
hpq	-0.003	0.688	0.250	0.250	0.003	-0.056	0.859	5.418	**0.111**	4.736
	(0.001)	(0.027)			(0.142)	(0.021)	(0.031)	(1.036)	(0.036)	(0.525)
ibm	0.005	1.820	0.666	0.665	0.035	-0.067	0.911	9.036	**0.029**	6.132
	(0.001)	(0.029)			(0.113)	(0.015)	(0.015)	(1.291)	(0.012)	(0.799)
mmm	-0.011	2.783	0.867	0.867	0.063	-0.082	0.907	7.351	**0.044**	5.009
	(0.001)	(0.024)			(0.110)	(0.016)	(0.016)	(1.308)	(0.013)	(0.596)
ge	-0.005	0.509	0.520	0.520	0.019	-0.072	0.891	8.786	**0.076**	6.62
	(0.001)	(0.011)			(0.102)	(0.019)	(0.018)	(1.148)	(0.023)	(1.045)
utx	0.018	1.344	0.713	0.713	0.099	-0.090	0.903	5.973	**0.056**	7.869
	(0.001)	(0.019)			(0.124)	(0.019)	(0.019)	(1.198)	(0.016)	(1.768)
ba	0.022	0.775	0.601	0.601	0.032	-0.081	0.894	5.914	**0.054**	8.608
	(0.001)	(0.014)			(0.156)	(0.017)	(0.023)	(1.361)	(0.017)	(1.433)
	Services									
hd	-0.008	0.930	0.779	0.779	0.314	-0.069	0.943	5.022	0.014	7.014
	(0.001)	(0.011)			(0.106)	(0.013)	(0.014)	(1.045)	(0.009)	(1.029)
mcd	0.005	0.512	0.663	0.663	0.170	-0.045	0.927	6.907	**0.021**	6.583
	(0.000)	(0.008)			(0.114)	(0.017)	(0.015)	(1.186)	(0.009)	(0.879)
dis	-0.002	0.560	0.441	0.441	0.065	-0.071	0.890	6.428	**0.075**	7.485
	(0.001)	(0.014)			(0.138)	(0.017)	(0.023)	(1.228)	(0.023)	(1.210)
wmt	-0.010	1.186	0.807	0.807	-0.133	-0.021	0.878	9.049	**0.063**	6.995
	(0.001)	(0.013)			(0.205)	(0.021)	(0.033)	(1.764)	(0.030)	(0.940)
	Energy, Basic Materials and Capital Goods									
aa	-0.004	0.920	0.802	0.802	0.161	-0.042	0.922	5.290	0.017	7.426
	(0.001)	(0.010)			(0.110)	(0.016)	(0.015)	(0.804)	(0.009)	(1.142)
dd	0.005	0.813	0.809	0.809	0.065	-0.069	0.912	8.095	**0.034**	7.688
	(0.000)	(0.009)			(0.125)	(0.017)	(0.017)	(1.350)	(0.013)	(1.253)
cat	0.033	0.825	0.452	0.452	0.150	-0.063	0.916	4.854	**0.028**	6.441
	(0.001)	(0.020)			(0.108)	(0.014)	(0.015)	(0.997)	(0.009)	(0.983)
hon	0.006	0.988	0.607	0.607	-0.121	-0.071	0.871	7.522	**0.069**	6.864
	(0.001)	(0.018)			(0.148)	(0.016)	(0.024)	(1.227)	(0.023)	(1.049)
xom	0.005	0.420	0.515	0.515	-0.240	-0.090	0.888	9.579	0.009	10.09
	(0.000)	(0.009)			(0.163)	(0.018)	(0.019)	(1.612)	(0.008)	(1.933)
	Consumer and Healthcare									
mo	0.014	0.326	0.434	0.434	-0.179	-0.065	0.892	14.91	0.036	3.732
	(0.000)	(0.008)			(0.254)	(0.023)	(0.032)	(3.103)	(0.020)	(0.333)
ko	-0.005	1.014	0.847	0.847	-0.150	-0.063	0.881	12.22	**0.071**	5.970
	(0.000)	(0.010)			(0.176)	(0.020)	(0.025)	(2.256)	(0.022)	(0.807)
pg	-0.012	1.619	0.797	0.797	-0.129	-0.122	0.882	9.741	**0.067**	5.876
	(0.001)	(0.018)			(0.185)	(0.020)	(0.025)	(2.090)	(0.019)	(0.685)
gm	-0.002	0.860	0.758	0.758	0.318	-0.035	0.949	5.492	0.001	5.001
	(0.001)	(0.011)			(0.097)	(0.011)	(0.011)	(1.090)	(0.004)	(0.560)
jnj	-0.004	1.007	0.799	0.799	0.095	-0.106	0.913	8.274	**0.048**	6.020
	(0.001)	(0.011)			(0.204)	(0.018)	(0.025)	(2.681)	(0.016)	(0.735)
mrk	-0.021	1.618	0.682	0.682	-0.133	-0.047	0.881	9.819	0.054	4.013
	(0.001)	(0.025)			(0.345)	(0.017)	(0.049)	(3.070)	(0.031)	(0.424)
pfe	-0.005	0.527	0.804	0.804	-0.198	-0.051	0.872	10.31	**0.064**	5.754
	(0.000)	(0.006)			(0.259)	(0.021)	(0.037)	(2.440)	(0.028)	(0.826)

Table 1.11: Regression results of individual liquidity proxies with the equally weighted liquidity proxy and EGARCHX estimation results using the diversified liquidity proxy.

1.7.2 VaR forecasts by industry sector

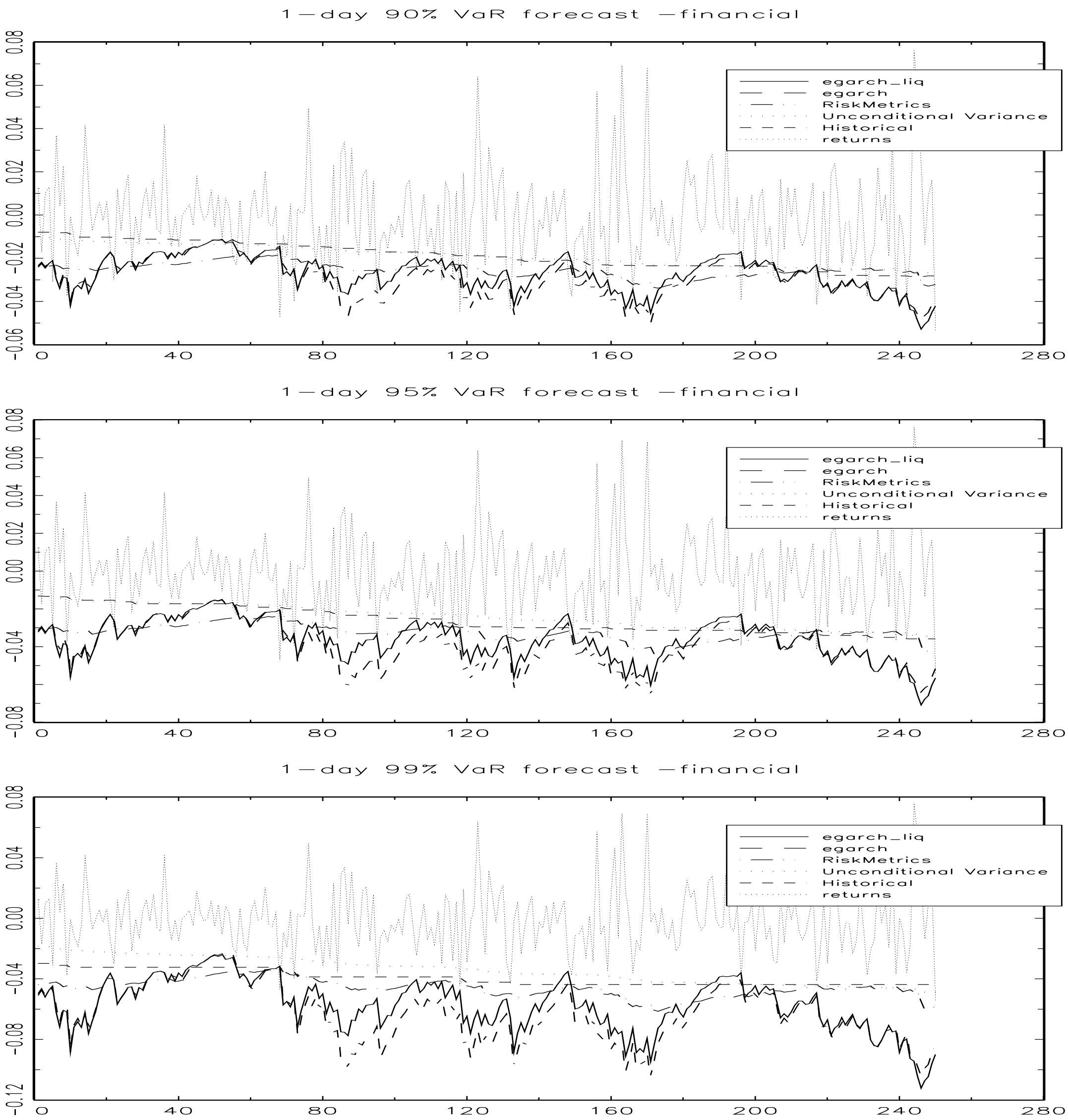


Figure 1.5: 1-step VaR forecast for an equally weighted portfolio consisting of financial sector stocks. Plots of the 90% (top), 95% (middle) and 90% (bottom) VaR forecasts are given for the EGl (solid lines), EG (broken lines), RM (dash-dot lines), VC (dotted lines) and HS (dashed lines) models

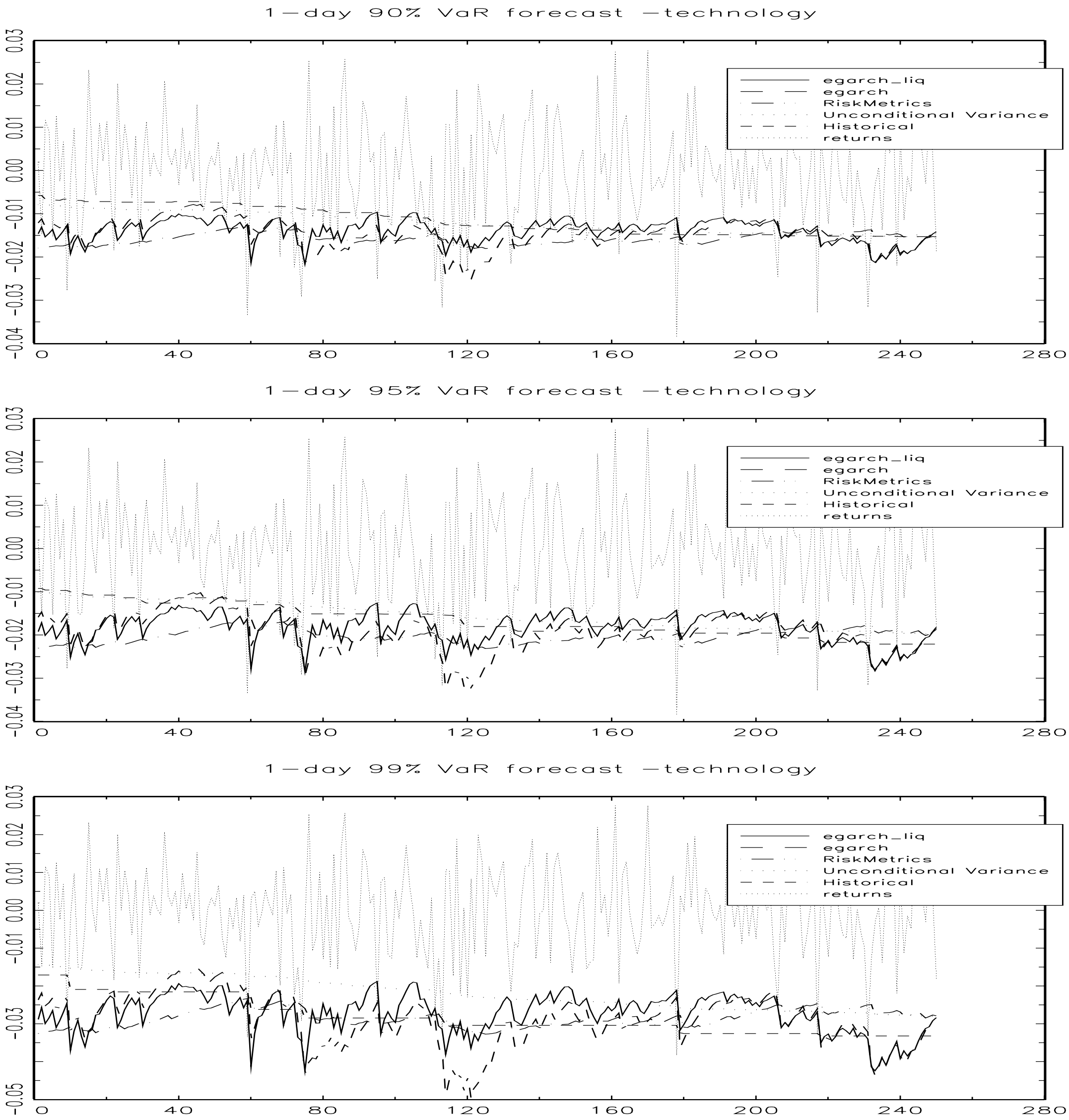


Figure 1.6: 1-step VaR forecast for an equally weighted portfolio consisting of technology sector stocks. Plots of the 90% (top), 95% (middle) and 90% (bottom) VaR forecasts are given for the EGl (solid lines), EG (broken lines), RM (dash-dot lines), VC (dotted lines) and HS (dashed lines) models

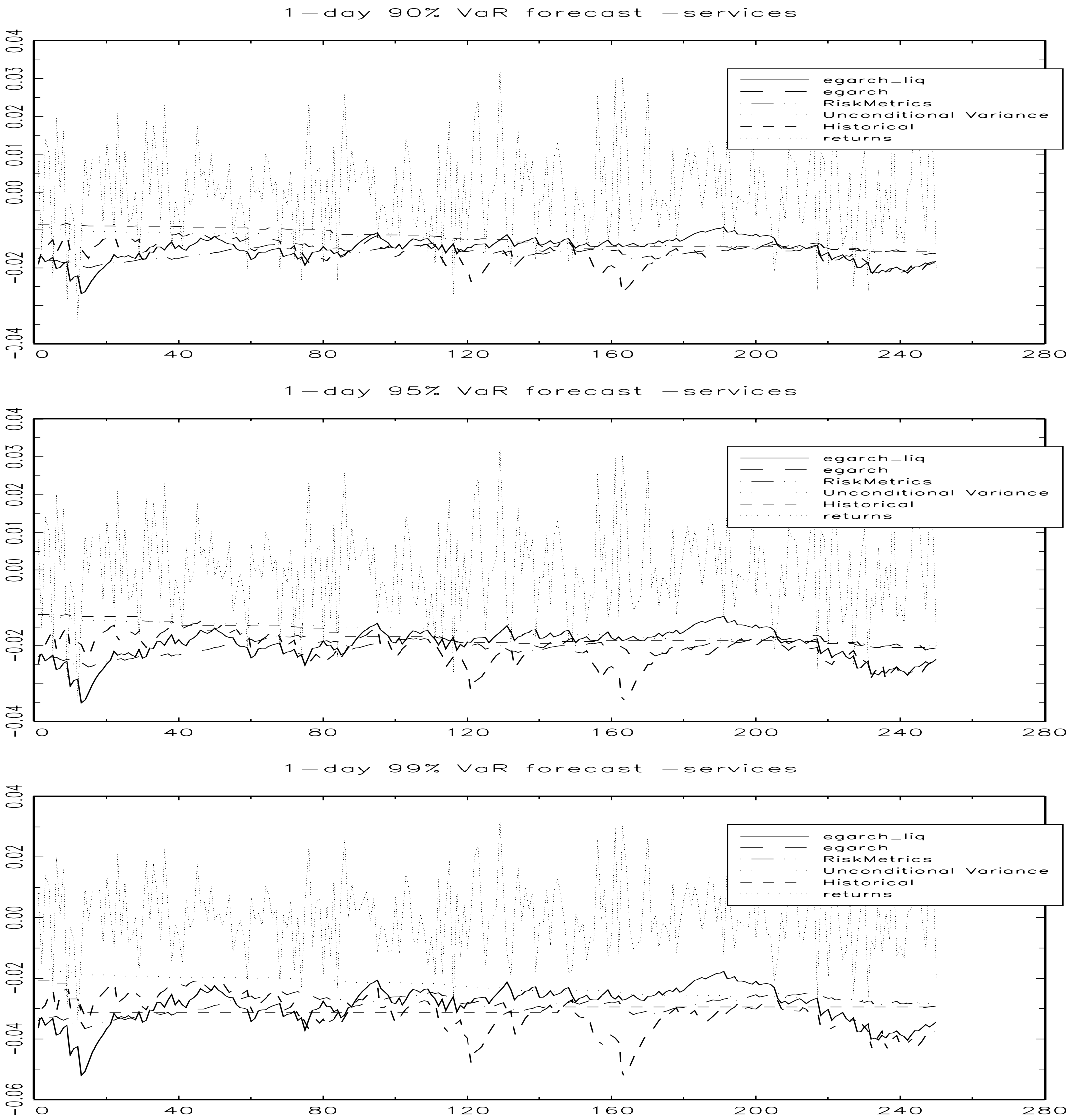


Figure 1.7: 1-step VaR forecast for an equally weighted portfolio consisting of service sector stocks. Plots of the 90% (top), 95% (middle) and 90% (bottom) VaR forecasts are given for the EGl (solid lines), EG (broken lines), RM (dash-dot lines), VC (dotted lines) and HS (dashed lines) models

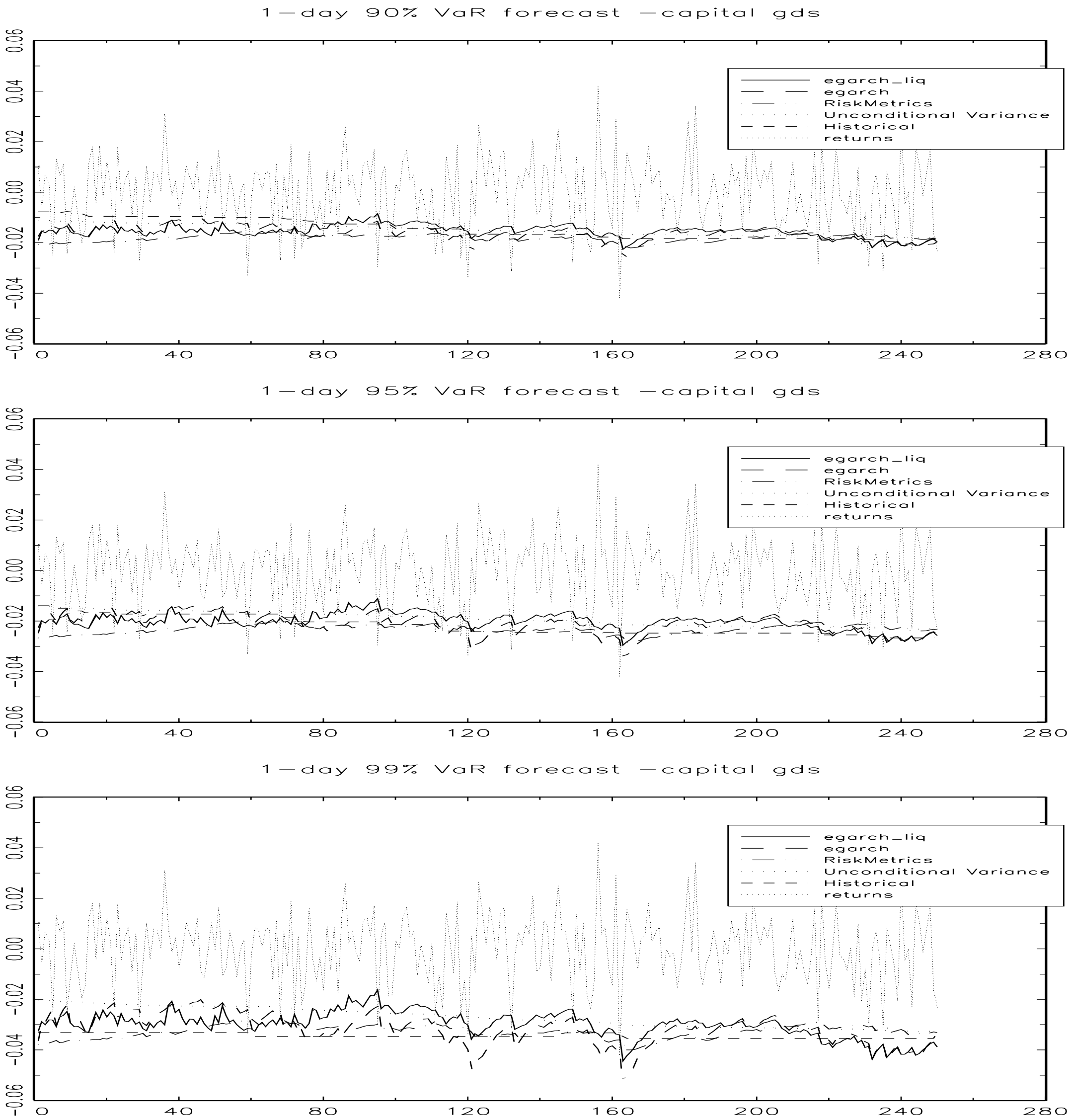


Figure 1.8: 1-step VaR forecast for an equally weighted portfolio consisting of capital goods sector stocks. Plots of the 90% (top), 95% (middle) and 90% (bottom) VaR forecasts are given for the EGl (solid lines), EG (broken lines), RM (dash-dot lines), VC (dotted lines) and HS (dashed lines) models

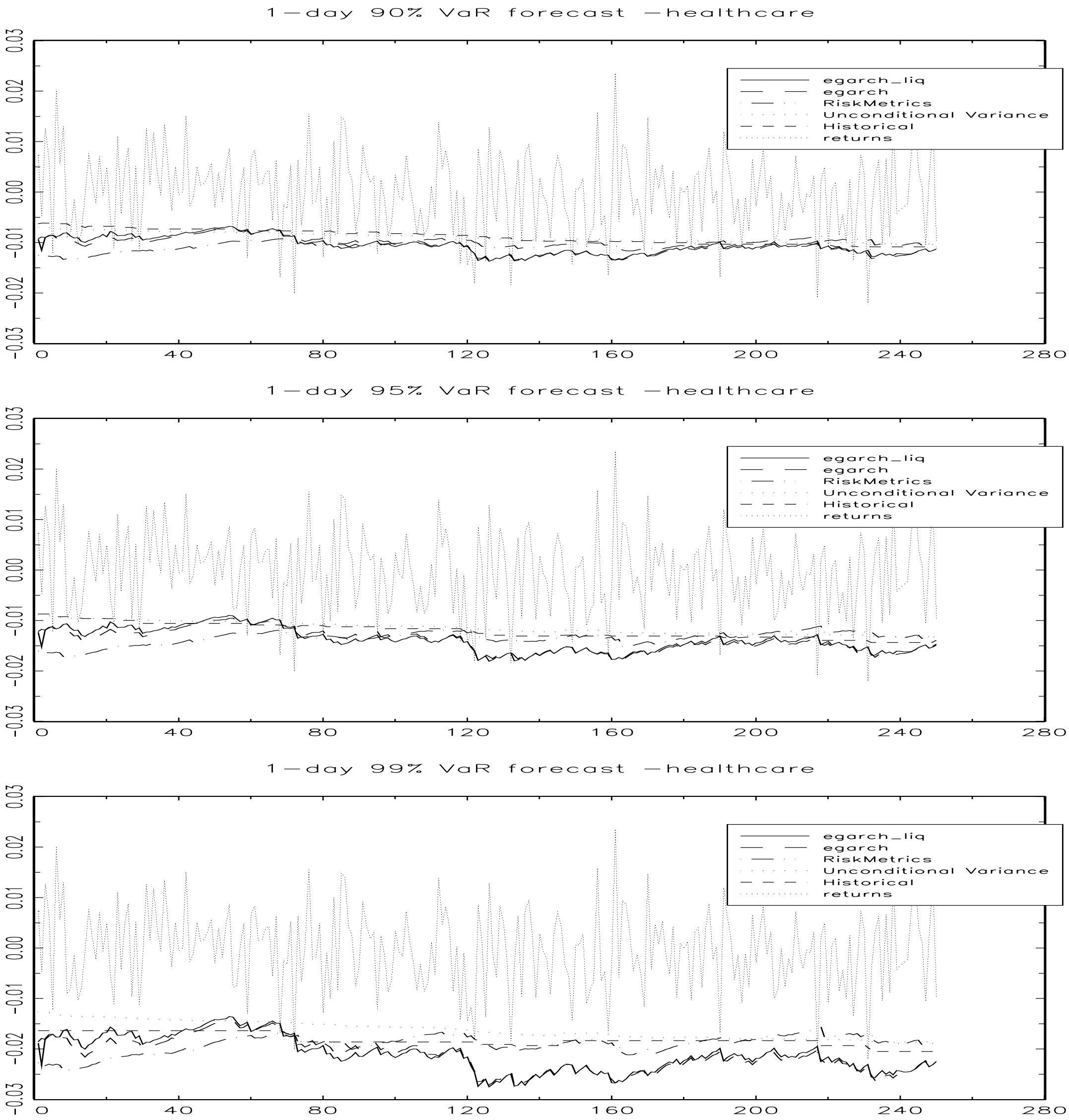


Figure 1.9: 1-step VaR forecast for an equally weighted portfolio consisting of healthcare sector stocks. Plots of the 90% (top), 95% (middle) and 90% (bottom) VaR forecasts are given for the EGl (solid lines), EG (broken lines), RM (dash-dot lines), VC (dotted lines) and HS (dashed lines) models

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Chapter 2

Estimating Dynamic Copula Dependence Non-parametrically Using Intraday Data

2.1 Introduction

In this paper, the realized dependence measure is estimated using intraday data applied to the class of Semiparametric Copula-Based Multivariate Dynamic (SCOMDY) models introduced by Chen and Fan (2006). In SCOMDY models, the individual series' conditional mean and conditional variance is parametrically specified while their joint distribution is specified by a semiparametric copula (a copula with empirical marginals). Chen and Fan (2006) noted that in such models, the limiting distribution of the copula dependence parameters are unaffected by the first-step estimation of the dynamic parameters specifying the conditional mean and variance. It is however affected by the marginal distribution of the standardized residuals. Here, we use the univariate GARCH model to filter the volatility of each series and fit the residuals to a semiparametric copula. We consider some of the residual properties and show empirically that the derived dependence from intraday data is a superior estimator of daily dependence as compared to commonly used time-varying copulas.

From a broader perspective, multivariate volatility models have been a long-standing challenge in financial econometrics, therein which an important issue is the estimation and modeling of correlations between time series. Recent financial crises have taught us that models based on normal distribution are inadequate, and when distributions depart from elliptical forms, dependence between assets cannot be captured by linear correlations alone (Embrechts et al. (1999)). This means that assets that have zero correlations cannot be interpreted as being independent of each other. This research brings together two main areas of econometric literature- that of time-varying copulas and that of econometric estimators that harnesses high frequency data.

First, the notion of copulas introduced by Sklar (1959) allows for a joint distribution function to be flexibly decomposed into its individual marginals and a dependence function, termed the copula function, linking the marginals. By specifying different copula functions, different dependence structures beyond linear correlations can be formed and estimated. This makes copulas popular tools in finance when dealing with multivariate distributions. A main shortcoming is that they are static, i.e. they assume i.i.d data, and hence only consider the spatial dependence. In reality, financial time series often exhibit time-varying dependences and structural breaks (see for example Patton (2001)).

It is thus important to use time-varying copula models. One of the most traditional methods of obtaining time-varying dependence is by using rolling windows of observations for estimation. More recent developments include the regime-switching models of Rodriguez (2007) and Chollete et al. (2009), and the autoregressive dependence parameter of Patton (2006). To avoid specifying the parameter changing scheme, Giacomini et al. (2009) use local change point analysis to detect intervals of local homogeneity for estimation of the copula parameter. These models face varying degrees of the same problem: when too much data from the distant past is used, the estimated parameters could be biased. However using only recent data would result in too few observations for inference. While the local change point analysis method attempts to circumvent this problem, it is only effective if sufficient post-break data is available.

Second, the availability of high-quality tick-by-tick data in financial time series has led to much innovation and research in the last decade of financial econometrics, with the development of realized volatility of Barndorff-Nielsen and Shephard (2002), realized covariances and other realized measures based on the theory of quadratic variation. The realized correlation (Barndorff-Nielsen and Shephard (2004), henceforth BNS) is derived from dividing realized covariances by the realized volatilities of the assets. We also wish to harness intraday data in estimating correlations and other non-linear dependence measures for use in copulas for two simple reasons: (1) they provide information on the most current status about the relationship between different assets of interest; (2) they provide a large quantity of observations for econometric inference. Since what we wish to obtain from intraday data is a daily nonparametric conditional dependence that is observed ex-post, we term this the "realized dependence"[1].

This paper harnesses intraday data to obtain a more powerful estimator for the daily time-varying copula dependence. The idea of using intraday data for copula dependence estimation has been considered before, but was focused on sampling frequency that is shorter than 1 day for use in estimating dependence across a large number of days (i.e. not within the same day). Dias and Embrechts (2004) use univariate ARMA-GARCH models and the residuals obtained were found to retain their contemporaneous dependence, and show evidence that the i.i.d property cannot be rejected. Since they do not model the time-varying dependence parameter explicitly, they apply the method of change-point analysis on the copula dependence parameter. Breymann et al. (2003) consider FX data and find that the empirical marginals of the residuals can be rejected for ellipticity for frequencies of 8 hours and shorter.

The rest of this paper is organized as follows: Section 2 discusses the class of semiparamatric copula-based multivariate dynamic models, and introduces the realized dependence and some time-varying copula models. Section 3 discusses the properties of the realized dependence estimator, while Section 4 reports the results of two simulation experiments that confirm the theoretical discussions in Section 3. Section 5 begins the empirical study, where we estimate the realized dependence measures, discuss some of its estimation issues, and use them to estimate distribution quantiles for three stock pairs. The dependence measures are then forecasted for the 1-step ahead Value-at-Risk (VaR) estimation. Section 6 concludes.

[1]While we term this "realized dependence", it is however not obtained within the framework of quadratic variation from which realized variances are derived.

2.2 Using High Frequency Data in SCOMDY models

We first introduce the general notion of copula and Sklar's theorem, and then apply this to SCOMDY. Assume a bivariate returns process of assets a and b. Let F_{ab} be the joint density function between random variables $\eta_{a,t}$ and $\eta_{b,t}$. The vector of random variables are then represented by $\boldsymbol{\eta_a}$ and $\boldsymbol{\eta_b}$ respectively. The joint distribution F_{ab} can be decomposed using Sklar's theorem into the individual marginal densities and a copula, i.e. $F_{ab}(\boldsymbol{\eta_a}, \boldsymbol{\eta_b}) = C(F_a(\boldsymbol{\eta_a}), F_b(\boldsymbol{\eta_b}); \theta)$, where F_a and F_b are marginals and C is a copula with dependence parameter θ. This illustrates the benefit of copulas in that it allows for flexible modeling of joint distributions by separating the dependence structure from the marginal distributions. Different joint distributions are derived by combining the same marginal distributions with different dependence structures, or by combining different marginal distributions with the same dependence structures[2].

One of the simplest examples of a dependence structure is the Gaussian copula, which is symmetric and has no tail dependence. The dependence parameter in the Gaussian copula is the linear correlation. Non-Gaussian copulas like the Clayton and Gumbel copulas are asymmetric and exhibit a stronger dependence among the tails of the distribution. A brief description of the form of some commonly used copulas is given in Appendix 2.9.

For the class of SCOMDY models, the term "semiparametric" refers to the use of a semiparametric copula– the copula form is specified parametrically but the marginals have no pre-specified parametric form. Instead the empirical marginals are used. This reduces the risk of misspecifying the marginals in the estimation of dependence but comes at a cost of decreasing efficiency of the ML method (as compared to when the parametric forms for the marginals are known)[3]. The joint distribution of the standardised residuals in SCOMDY is then given as

$$F_{ab}(\boldsymbol{\eta_a}, \boldsymbol{\eta_b}) = C(F_{n,a}(\boldsymbol{\eta_a}), F_{n,b}(\boldsymbol{\eta_b}); \theta),$$

where $\boldsymbol{\eta_a}$ and $\boldsymbol{\eta_b}$ are vectors of residuals from a parametric model, $F_{n,a}$ and $F_{n,b}$ are the empirical cdf of $\boldsymbol{\eta_a}$ and $\boldsymbol{\eta_b}$ respectively and θ is the dependence parameter

[2]For detailed discussion on copulas, refer to Nelsen (1999) and Joe (1997).

[3]See Genest (1995) which derives the asymptotic properties of the canonical maximum likelihood (CML) estimator. The term 'canonical' is used here as it is no longer a standard maximum likelihood estimation due to the use of empirical marginals.

of the copula.

In this paper, we consider one class of SCOMDY– the GARCH-semiparametric copula models, in which the specification of the univariate variances is captured by univariate GARCH. The GARCH residuals are then fitted to the semiparametric copula for the estimation of contemporaneous dependence θ. This two-step procedure is feasible as established by Chen and Fan (2006), who showed via heuristic arguments with stringent conditions that the asymptotic distribution of $\hat{\theta}$ is unaffected by the initial step of GARCH estimation. Chan et al. (2009) derive, under mild conditions, the weighted approximation of the empirical distribution of univariate GARCH models which is then used to derive the limiting distribution for the canonical ML (CML) estimator of θ.

The simple GARCH model for estimating the conditional variance is commonly used to model daily returns, see e.g. Kim et al. (1999), Christoffersen and Diebold (2006) and Giacomini et al. (2009). These papers pointed out that for daily equity data, conditional means are dominated by conditional variances, hence a zero-mean assumption is reasonable[4]. Conditional means matter more for intraday data than for daily data, but are still dominated by conditional variances, as noted by Andersen and Bollerslev (1997). We hence use a ARMA(1,1) to model conditional means.

An important characteristic of high frequency data is the presence of intraday seasonality. Wood et al. (1985) and Harris (1986) were the first to note this periodic U-shape pattern in return volatility during the trading day while Andersen and Bollerslev (1997) show that it is important to take into account intraday seasonality before applying standard volatility models to intraday data. They propose the Flexible Fourier Form (FFF) for estimating the seasonality pattern. Martens et al. (2002) show that FFF adjusted GARCH(1,1) forecasts intraday volatility better than other seasonality correction methods. We hence adopt FFF for seasonality adjustment.

To illustrate the resulting realized dependence estimate for both with and without seasonality adjustment, Figure 2.1 compares the BNS realized correlation[5] and the Gaussian copula realized dependence of Citigroup and JP Morgan

[4]We also checked our results by including a conditional mean ARMA(1,1) model. The conditional means are small and overall results remain unchanged.

[5]The realized covariance matrix that was used to compute realized correlation has been subsampled to reduce the effects of market microstructure noise.

returns in the period 2006-2008 using 1-minute sampling frequency. The dotted line is when the FFF seasonality adjustment is applied while the dash-dot line is when no seasonality adjustment is made. The graph shows the realized Gaussian dependence and the realized correlation to be closely matched. This means that dependence is preserved after the univariate ARMA-GARCH filtration, as was observed by Dias and Embrechts (2004).

Furthermore, we find that the inclusion of seasonal adjustment gives a dependence parameter that is very close to that obtained without seasonal adjustment. We also find that assuming a zero conditional mean gives a dependence parameter that is close to that when the conditional mean is modelled by an ARMA process (which is used in Figure 2.1).

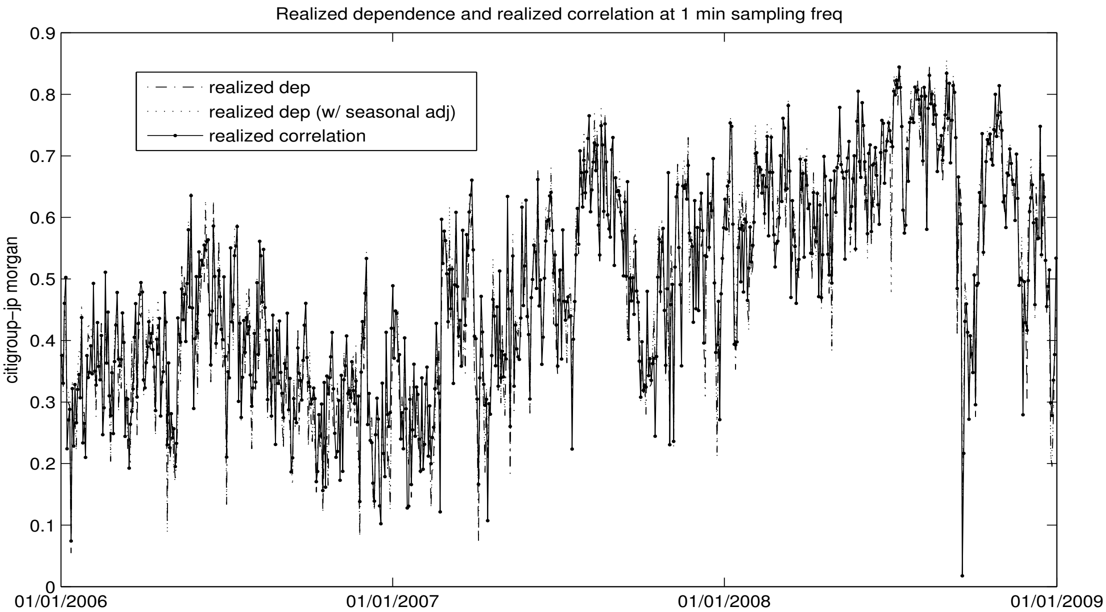

Figure 2.1: Plot of the Citigroup-JP Morgan realized correlation (solid line) and the realized dependence of the Gaussian copula using 1-minute sampling frequency without pre-applying seasonality adjustment (dash-dot line) and when FFF seasonality adjustment is applied (dotted line)

In summary, what we obtain is a dynamic nonparametric daily conditional dependence ("nonparametric" in the sense that the dynamics do not follow a prespecified parametric form). Exploiting intraday data allows us to observe the latent conditional dependence. This approach is much less sensitive to structural breaks in long time series as only information within the day is used without any assumption on the number of states of dependence, nor on the parametric form of the dependence dynamics.

Other recent methods for modelling time variation in dependence include estimation using rolling windows, regime switching dependence (Rodriguez, 2003), and autoregressive dependence (Patton (2006)). There are drawbacks to these methods. Rolling window is arbitrary on the choice of window size, is slow to react to market changes, and still suffers from effects of structural breaks. The regime switching model allows for the dependence parameter to react swiftly to changes in the market conditions, but it is subject to the number of states assumed. The autoregressive dependence of Patton (2006) assumes that the dependence parameter follows an ARMA(1,10) process. For the Gaussian correlation, the proposed dynamic is given as

$$\rho_t = \tilde{\Lambda}\left(\omega_\rho + \beta_\rho \rho_{t-1} + \alpha \frac{1}{10}\sum_{j=1}^{10} \Phi^{-1}(\eta_{a,t-j})\Phi^{-1}(\eta_{b,t-j})\right),$$

where $\tilde{\Lambda}(x) \equiv \frac{1-e^{-x}}{1+e^{-x}}$ is the modified log transformation used to restrict ρ between -1 and 1. The determination of the forcing (or updating) variable (see Patton (2006)) and the choice of number of lags are key issues to deal with. Estimation of the parameters are also biased in event of a structural break.

2.3 The realized dependence estimator

In this section, we derive the nonparametric estimator of the time-varying copula dependence. We then derive conditions under which this estimator is asymptotically unbiased, and discuss the reasonableness of these conditions.

Assume a univariate ARMA(1,1)-GARCH(p,q) process is used to model the conditional mean and variance of intraday returns $y_{k,i,j}$, $j = 1, \ldots, 1/m$, for m regular time-spaced sampling intervals within a day, $0 < m \leq 1$. Let us denote $\boldsymbol{\eta}^m_{1,i}, \ldots, \boldsymbol{\eta}^m_{k,i}$, $0 \leq i \leq t$ as the vector of GARCH residuals of k assets in day i, where t is the number of days in the sample period. For each day i and sampling frequency m, the intraday return at time j is given by

$$y_{k,i,j} = c^m_{k,i} + a^m_{k,i} y_{k,i,j-1} + \epsilon_{k,i,j} + b^m_{k,i}\epsilon_{k,i,j-1}, \tag{2.1}$$

$$\epsilon_{k,i,j} = \sigma_{k,i,j}\eta_{k,i,j}, \tag{2.2}$$

$$\sigma^2_{k,i,j} = \omega^m_{k,i} + \sum_{q_1=1}^{q} \beta^m_{k,i,q_1}\sigma^2_{k,i,j-q_1} + \sum_{p_1=1}^{p} \alpha^m_{k,i,p_1} y^2_{k,i,j-p_1}, \tag{2.3}$$

where $i = 1, \ldots, t$, $j = 1, \ldots, 1/m$ and $k = 1, \ldots, K$. The parameters $c^m_{k,i}$, $a^m_{k,i}$, $b^m_{k,i}$, $\omega^m_{k,i}$, β^m_{k,i,q_1} and α^m_{k,i,p_1} for $q_1 = 1, \ldots, q$, $p_1 = 1, \ldots, p$ are different for each asset k, each day i and are a function of m, hence the corresponding subscripts and superscript. Furthermore, $\omega^m_{k,i} > 0$, $\beta^m_{k,i,q_1} > 0$ and $\alpha^m_{k,i,p_1} > 0$.

The copula function using intradaily data is then given by

$$F_K(\boldsymbol{\eta^m_{1,i}}, \ldots, \boldsymbol{\eta^m_{K,i}}) = C_i(F^m_{1,i}(\boldsymbol{\eta^m_{1,i}}), \ldots, F^m_{K,i}(\boldsymbol{\eta^m_{K,i}}); \theta^m_i), \tag{2.4}$$

where $\boldsymbol{\eta_{k,i}} = (\eta_{k,i,1}, \ldots, \eta_{k,i,m})^T$ and θ^m_i is the realized dependence on day i for the K assets. When (2.4) is written as

$$F_{n,K}(\boldsymbol{\eta^m_{1,i}}, \ldots, \boldsymbol{\eta^m_{K,i}}) = C_i(F^m_{n,1,i}(\boldsymbol{\eta^m_{1,i}}), \ldots, F^m_{n,K,i}(\boldsymbol{\eta^m_{K,i}}); \theta^m_i), \tag{2.5}$$

where F_n represents the empirical cdf, the model setup belongs to the Semiparametric Copula-Based Multivariate Dynamic (SCOMDY) class of models.

We now investigate how θ^m_i is related to θ_0 which is the true dependence of the day. θ_0 is a latent variable that cannot be estimated (assuming each day's dependence is different) given that we have only one observation for each day's return per asset. Hence let us assume a hypothetical scenario where we can observe an adequate number of days in which the univariate returns process and the multivariate dependence are non time-varying throughout the period. For each day, the true dependence between k assets is given by θ_0 under a SCOMDY model,

$$F_{K,i}(\boldsymbol{\eta_1}, \ldots, \boldsymbol{\eta_K}) = C_i(F_{n,1}(\boldsymbol{\eta_1}), \ldots, F_{n,K}(\boldsymbol{\eta_K}); \theta_0),$$

where $\boldsymbol{\eta_1}, \ldots, \boldsymbol{\eta_K}$ are the vector of residuals from an ARMA(1,1)-GARCH(p,q) process at a daily sampling frequency:

$$y_{k,i} = c_k + a_k y_{k,i-1} + \epsilon_{k,i} + b_k \epsilon_{k,i-1}, \tag{2.6}$$

$$\epsilon_{k,i} = \sigma_{k,i}\eta_{k,i}, \tag{2.7}$$

$$\sigma^2_{k,i} = \omega_k + \sum_{q_1=1}^{q} \beta_{k,q_1}\sigma^2_{k,i-q_1} + \sum_{p_1=1}^{p} \alpha_{k,p_1} y^2_{k,i-p_1}, \tag{2.8}$$

where $i = 1, \ldots, t$ and $k = 1, \ldots, K$. Furthermore, (2.1) and (2.6) are related by $y_{k,i} = \sum_{j=1}^{1/m+1} y_{k,i,j}$.

The marginal distributions of $F^m_{k,i,\eta}(\boldsymbol{\eta^m_{k,i}})$ can be estimated by the empirical marginal distributions of the residuals for each day i[6]

$$F^m_{k,i,\eta}(x) = \frac{m}{1+m} \sum_{2\le j\le 1/m+1} I(\eta_{k,i,j} \le x), \ \ -\infty < x < \infty, \, 0 < m \le 1.$$

When the estimated standardized residuals $\hat{\boldsymbol{\eta}}$ are used, we denote this by $\hat{F}^m_{k,i,\eta}(\hat{\boldsymbol{\eta}}^{\boldsymbol{m}}_{\boldsymbol{k,i}})$

$$\hat{F}^m_{k,i,\eta}(x) = \frac{m}{1+m} \sum_{2\le j\le 1/m+1} I(\hat{\eta}_{k,i,j} \le x), \ \ -\infty < x < \infty, \, 0 < m \le 1.$$

The corresponding distribution functions for low frequency residuals are similarly denoted by $F_{k,\eta}(x)$ and $\hat{F}_{k,\eta}(x)$ respectively.

Hence, the copula canonical ML estimators $\hat{\theta}_0$ and $\hat{\theta}^m_i$ are given by

$$\hat{\theta}_0 = \arg\max_{\theta_0} \frac{1}{t} \sum_{i=1}^{t} \log c(\hat{F}_{1,\eta}(\hat{\boldsymbol{\eta}}_{\boldsymbol{1,i}}), \ldots, \hat{F}_{k,\eta}(\hat{\boldsymbol{\eta}}_{\boldsymbol{k,i}}); \theta_0),$$

and

$$\hat{\theta}^m_i = \arg\max_{\theta^m_i} \frac{m}{1+m} \sum_{j=1}^{1/m+1} \log c(\hat{F}^m_{1,i,\eta}(\hat{\boldsymbol{\eta}}_{\boldsymbol{1,i,j}}), \ldots, \hat{F}^m_{k,i,\eta}(\hat{\boldsymbol{\eta}}_{\boldsymbol{k,i,j}}); \theta^m_i)$$

respectively, where $c(x_1, \ldots, x_k; \theta)$ is the copula density function.

We have now defined our estimator $\hat{\theta}^m_i$. What we are interested in is how good $\hat{\theta}^m_i$ is as an estimator of θ_0. We consider two cases: (i) Under independence of the processes at a daily level and intradaily level and (ii) Under non-independence of the processes by considering temporal aggregation effects of GARCH. We find that under case (ii), $\hat{\theta}^m_i$ has a positive bias, but argue that the bias is small due to limited effects of non-independence.

For simplicity, let us assume a GARCH(1,1) process for returns. GARCH(1,1) models are compatible with continuous time processes and can be viewed as a discretised form of continuous stochastic models. Nelson (1990) showed that a sequence of discrete time GARCH(1,1) processes with i.i.d. normal innovations converges in distribution to Itô processes as the length of the discrete time intervals goes to zero. Denote the parameter vector of the true GARCH process by $\gamma_{k,0} = (\omega_k, \beta_k, \alpha_k)^T$, and the parameter vector of the intradaily GARCH process by $\gamma^m_k = (\omega^m_k, \beta^m_k, \alpha^m_k)^T$. Let u be the estimated parameter vector for the daily

[6]Note that the counter for j begins from 2 here as in Berkes and Horvath (2003) as the first estimated residual tends to be biased due to the initial values assumed for the conditional mean and variance. Other papers however include the first estimated residual, i.e. j begins from 1.

GARCH(1,1) process, $u = (\hat{\omega}, \hat{\alpha}, \hat{\beta})$ and u^* be the estimated parameter for the intraday GARCH(1,1) process $u^* = (\hat{\omega}^*, \hat{\alpha}^*, \hat{\beta}^*)$.

First consider independence of daily and intradaily processes. Berkes and Horvath (2003) proved that the asymptotic behaviour of the empirical process of squared GARCH residuals weakly converges to a Gaussian process. This result was used in Chan et al. (2009) to establish the CML consistency using weighted approximation of the empirical residuals and to show that the limiting distribution of $\hat{\theta}_0$ is independent of the GARCH filtering under sufficiently mild conditions. In this context, if we regard the intradaily process and the daily process as independent processes, their squared residuals then both converge weakly to the Gaussian process and it is not difficult to show as in Chan et al. (2009) that $\hat{\theta}_i^m \xrightarrow{p} \theta_0$ as $1/m \to \infty$ (see Appendix 2.7).

However, in reality, the intraday and daily processes are not independent. Another line of literature discusses the temporal aggregation of GARCH models. In a seminal paper, Drost and Nijman (1993) showed not only that $\gamma_k^m \neq \gamma_{k,0}$, but also defined the strong, semi-strong and weak GARCH, where strong GARCH assumes the residuals to be i.i.d. and have a distribution with zero mean and unit variance (e.g $\sim N(0,1)$). Semi-strong GARCH assumes that the residuals are uncorrelated while for weak GARCH, the estimated σ_t^2 is not the conditional variance but the best linear predictor of the squared residuals, i.e. $E(y_{t+1}^2 - \sigma_t^2)y_{t-i}^r = 0,\ i \geq 0,\ r = 0, 1, 2$. With these definitions, Drost and Nijman (1993) showed that GARCH models in the strong or semi-strong sense are not closed under aggregation. Only weak GARCH models are time-aggregating. This means that if the data generating process is weak GARCH(1,1) at a certain sampling frequency, then the weak GARCH(1,1) will be the data generating process for any other sampling frequency (Alexander and Lazar (2005)).

Alexander and Lazar (2005) derive the continuous limit of the weak GARCH(1,1) as a stochastic volatility model with price-volatility correlation that is related to the skewness and kurtosis of the returns density. The limit reduces to the results of Nelson (1990) if the returns density is normal. Drost and Werker (1996) show that both diffusion and jump-diffusion processes can be approximated by weak GARCH models sampled at any discrete time frequency, with the kurtosis of GARCH jump-diffusion processes being larger than that of GARCH diffusion processes.

The problem for us then lies in that for weak GARCH processes, the distribution of residuals at different frequencies cannot be assumed to be similar, hence the intraday realized dependence estimator cannot be shown to be an unbiased estimator of the true dependence of the day. In fact, Nelson (1990) showed that as sampling frequency increases, the residuals approach a limiting Student-t distribution with $2 + 4\tau/\alpha^{*2}$ degrees of freedom, where $\tau = \lim_{\Delta t \to 0} \Delta t^{-1}(1 - \beta^* - \alpha^*)$. Similarly, Drost and Werker (1996) show that the implicit assumption of an underlying continuous model implies the presence of heavy tails, and consistently find that leptokurtosis is less pronounced in aggregated series. This means that for any copula model which allows for tail dependences, $\hat{\theta}_i^m$ will be positively biased from the true θ_0. The simulation experiments in Section 4 confirm this theory.

Despite the possible biasness in the dependence estimate, we claim that the realized dependence estimator remains a good estimator for time-varying dependence for several reasons. First, simulations in Drost and Nijman (1992a) suggest that the QML estimates are close to the true parameters even if the model is weak GARCH. Second, Nelson (1990) and Alexander and Lazar (2005) showed that as sampling frequency increases, the estimated GARCH variance will offer a good approximation for the variance of the true process, even if the GARCH model is misspecified. Both these arguments reduces the importance of time aggregation effects of GARCH, which means that the magnitude of the bias is not very large.

2.4 Simulation Results

To check the verity of our theoretical discussion in Section 2.3, we study two cases in this section to observe the effect of time aggregation on the realized dependence estimator. In the first case, returns are simulated under a SCOMDY model. In the second case we attempt to observe the effect under a more neutral and realistic setting by using a bivariate Normal Inverse Gaussian Levy process, where the true nonlinear dependence is unknown.

For our first simulation case, we generate 1,000 random samples of size 10,000 from the Gaussian, Clayton and survival Gumbel (s. gumbel) copulas with true copula parameters set at $\theta_{gaussian} = 0.7$, $\theta_{clayton} = 2.5$ and $\theta_{s.gumbel} = 3.0$. To obtain the univariate returns series, we assume a zero conditional mean and

GARCH(1,1) parameters $c_1 = c_2 = 1$, $\alpha_{1,1} = \alpha_{2,1} = 0.2$ and $\beta_{1,1} = \beta_{2,1} = 0.6$, where the error terms are simulated from the respective copulas above. These values are used in the simulations in Chan et al. (2009). The simulated returns are aggregated by $k = 1, 2, 5, 10, 15, 20, 30, 60$ and 390 times (think of this as aggregating 1-minute returns to 2, 5, 10, etc. minutes, where there are 390 minutes in a trading day) to form lower frequency returns samples. The GARCH and copula dependence parameters are then re-estimated at each aggregation level.

Table 2.1 shows estimates of the mean and standard deviation of the pseudo maximum likelihood estimator for the Gaussian, Clayton and survival Gumbel. The full results including the estimated GARCH parameters are given in Appendix 2.10. The GARCH parameters are not stable under aggregation (as found in Drost and Nijman (1993)). There are different effects of sampling frequency on parameters: ω parameters increase, α parameters decrease, β parameters increase and the variance of all parameters increase.

More importantly, as argued before, the dependence parameter θ is relatively stable for the Gaussian copula (which has no tail dependence). For fat-tailed copulas, dependence declines as the number of times of aggregation increases. This is due to the decreasing kurtosis of the residuals as aggregation times increases.

		aggregation by k times								
copula	true	1	2	5	10	15	20	30	60	390
Gaussian	0.7	0.689	0.690	0.693	0.692	0.691	0.690	0.689	0.687	0.686
		(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)	(0.010)
Clayton	2.5	2.314	2.165	1.892	1.715	1.635	1.583	1.535	1.475	1.385
		(0.004)	(0.006)	(0.009)	(0.010)	(0.013)	(0.015)	(0.018)	(0.023)	(0.053)
s. Gumbel	3.0	2.935	2.935	2.856	2.776	2.736	2.710	2.683	2.643	2.597
		(0.003)	(0.004)	(0.007)	(0.008)	(0.010)	(0.014)	(0.014)	(0.020)	(0.048)

Table 2.1: Estimation results for copula dependence parameters under reestimation at 1, 2, 5, 10, 15, 20, 30, 60 and 390 times of aggregation. Numbers in brackets are standard deviations.

To observe how the realized dependence estimator behaves under time aggregation in a more realistic setting, we next simulate a fat-tailed bivariate Normal Inverse Gaussian (NIG) Levy process, a sub-class of the more general class of hyperbolic Levy processes. This class of processes are closed under convolution unlike GARCH processes. The NIG Levy process was introduced by Barndorff-Nielsen (1995) as a realistic model for log stock price returns. It exhibits non-

Gaussian semi-heavy tails and allows for skewness. The bivariate process is given by

$$X = \mu + Z\Gamma\beta + \sqrt{Z}\Gamma^{1/2}Y,$$

where $\beta \in \mathbb{R}^2$, $Z \sim IG[\delta^2, \alpha^2 - \beta^T\Gamma\beta]$, $IG[\chi, \psi]$, $\alpha, \delta, \chi, \psi > 0$. IG denotes the inverse Gaussian distribution which has the pdf

$$f(z) = \left(\frac{\chi}{2\pi z^3}\right)^{1/2} e^{\sqrt{\chi\psi}} \exp\left[-\frac{1}{2}(\chi z^{-1} + \psi z)\right],\ z > 0,$$

and $Y \sim N_d[0, I]$. The characteristic function of the NIG process is given by

$$\phi(u; \alpha, \beta, \delta, \mu) = \exp\left\{\delta(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + iu)^2}) + i\mu u\right\},$$

where $u \in \Re$, $\mu \in \Re$, $\delta > 0$, $0 \leq |\beta| \leq \alpha$, and the $\alpha, \beta, \delta, \mu$ parameters can be interpreted as the tail index, symmetry, scale and location parameters respectively. Table 2.2 gives the results of the estimated dependence under aggregation of the simulated NIG process[7]. The Gaussian copula dependence is accurate at approximately 0.5 throughout the aggregations. While we do not have a true theoretical value of θ for the survival Gumbel and Clayton copulas, we observe that there is a greater degree of stability in the NIG simulations as compared to the GARCH simulations. There is still a slight downward bias for survival Gumbel and a relatively greater downward bias for Clayton copula dependence as the number of times of aggregation increases.

The two simulation cases show that for copulas with tail dependence, there is an upward bias in the estimation of θ_0 as sampling frequency increases (sampling intervals get smaller). This is in line with the theoretical findings in Section 2.3. Overall, the results are promising in that despite the bias, the realized dependence estimator is relatively stable, and that when realistic returns processes are used, the effect of this bias is small.

[7]The following parameters are used: $\alpha = \begin{pmatrix} 1.892 \\ 1.892 \end{pmatrix}$, $\beta = \begin{pmatrix} 0.0092 \\ -0.0263 \end{pmatrix}$, $\delta = \begin{pmatrix} 0.9906 & 0.5 \\ 0.5 & 1.0098 \end{pmatrix}$ and $\mu = \begin{pmatrix} -0.0045 \\ -0.0043 \end{pmatrix}$. The resulting simulated process has an empirical mean= $\begin{pmatrix} -0.0059 \\ -0.0120 \end{pmatrix}$, covariance= $\begin{pmatrix} 0.3473 & 0.1753 \\ 0.1753 & 0.3541 \end{pmatrix}$, skewness= $\begin{pmatrix} -0.0071 \\ -0.0386 \end{pmatrix}$, and excess kurtosis = $\begin{pmatrix} 2.7598 \\ 2.7617 \end{pmatrix}$. These values are close to those obtained empirically for our dataset.

		aggregation by k times								
copula	true	1	2	5	10	15	20	30	60	390
Gaussian	0.5	0.494	0.496	0.498	0.499	0.500	0.500	0.499	0.499	0.495
		(0.003)	(0.023)	(0.028)	(0.008)	(0.009)	(0.011)	(0.014)	(0.019)	(0.047)
Clayton	-	0.728	0.704	0.682	0.675	0.672	0.671	0.670	0.672	0.705
		(0.007)	(0.033)	(0.040)	(0.019)	(0.022)	(0.027)	(0.033)	(0.044)	(0.117)
s. Gumbel	-	1.473	1.457	1.442	1.436	1.434	1.433	1.432	1.433	1.446
		(0.004)	(0.017)	(0.021)	(0.012)	(0.014)	(0.017)	(0.021)	(0.028)	(0.073)

Table 2.2: Estimation results for Gaussian, Clayton and s. Gumbel copula dependence when a heavy-tailed bivariate NIG process is simulated. Numbers in brackets are the standard deviations.

2.5 Empirical Study

In this section, we provide empirical evidence for both in-sample and out-of-sample forecasts of the realized dependence. The realized dependence proves to be more accurate in estimating the quantiles of a multivariate distribution than the constant, rolling window and autoregressive dependence estimators. For the out-of-sample forecasts, we estimate Value-at-Risk, an often-used risk measure in financial institutions. We should emphasize that our focus is not on copula choice, but on comparing the performance of our time-varying dependence estimator with other commonly-used time-varying dependence estimators.

2.5.1 Data

For our empirical study, we use the NYSE TAQ data of three stocks - Citigroup (c), JP Morgan (jpm) and IBM (ibm) between 2 Jan 2006 and 31 Dec 2008. The data is cleaned using the algorithm described in Barndorff-Nielsen et al. (2008), and we include only observations for each trading day from 9.30 to 16.00. We use quote data for our study as it contains a larger number of observations as compared to trade data, thus reducing the effect of asynchronous trading on the estimations. Finally, we use the calendar time sampling and the last-tick interpolation method to obtain regularly spaced return observations. Figure 2.2 plots the daily and 1-minute intraday log returns time series of the three stocks. We can see that for both daily and intradaily data, the volatility of returns increases sharply from mid-2007 onwards due to the 2007 subprime crisis.

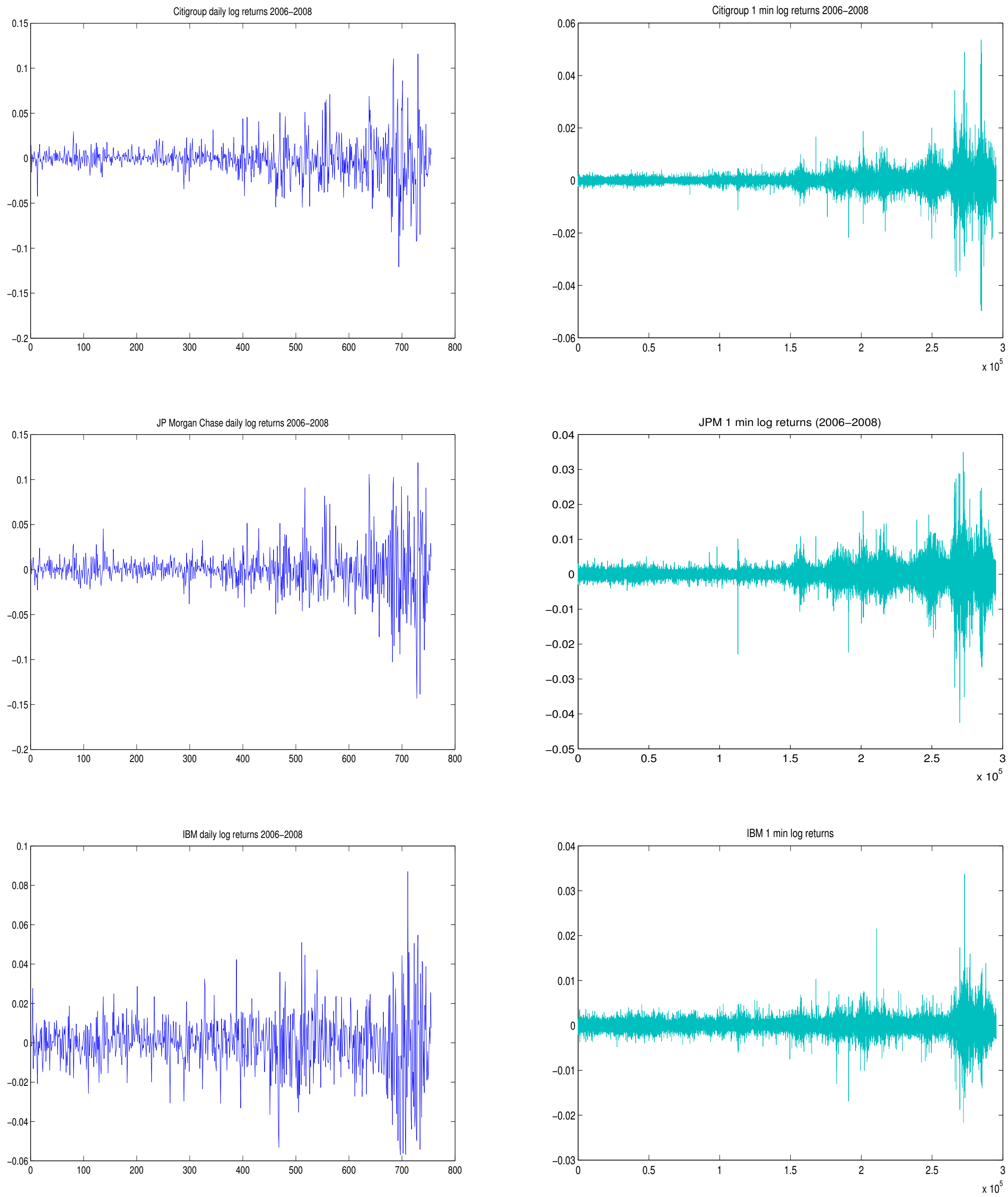

Figure 2.2: Return series of c, jpm and ibm (from top to bottom) at daily (left) and 1-minute (right) frequencies.

Intraday seasonality adjustment and other issues

We use FFF to adjust for intraday seasonality of the returns before passing them through a conditional mean and volatility model. Figure 2.3 shows that the estimated average intraday periodic patterns over the sample period using FFF (in dotted lines) provide a close approximation to the intraday absolute returns. [8]

We find however that the seasonality correction removes the extreme observations of each day (usually at the start and end of day), resulting in an intraday residual empirical distribution that has thinner tails. Figure 2.4 shows the scatter plots of the intraday residuals for the c-jpm stock pair for three different days: 3 January 2006, 1 August 2007 and 13 October 2008. The residual plots on the left side do not have deseasonalization adjustment while those on the right side are when the returns are first deseasonalized.

A first observation is that the dependence of the stock pair is definitely time-varying. On 3rd Jan 2006, the scatter plots are relatively elliptical with little tail dependence. On 1 Aug 2007, which is close to the start of the credit crisis, the dependence of the stock-pair has increased sharply. 13 Oct 2008, about the middle of the credit crisis, shows a reduction in linear dependence as compared to 1 Aug 2007, but still exhibits lower tail dependence. These dependence variations over time would have not been captured if we had used daily data though the period (see Figure 2.5).

The second observation is that the shapes of the scatter plots are similar when deseasonalization is applied (plots on right) and when it is not (plots on left), hence resulting in the dependence parameter estimates for deseasonalized and non-deseasonalized returns being rather close, as noted in Figure 2.1. However the density of tail observation is slightly thinner after deseasonalization as it removes the effect of the beginning and end of day returns, which tend to exhibit the largest intraday volatilities. Since these extreme tail observations are important for copula tail estimation, we decide to not apply deseasonalization to the returns before passing through the ARMA-GARCH filter.

Andersen and Bollerslev (1997), Martens et al. (2002) and other papers apply the ARMA-GARCH filtration to high frequency returns across multiple days and

[8]For FFF as described in Andersen and Bollerslev (1997), we use P=2 and J=0 for c and jpm and P=4 and J=0 for ibm, and find that this provides an adequate fit.

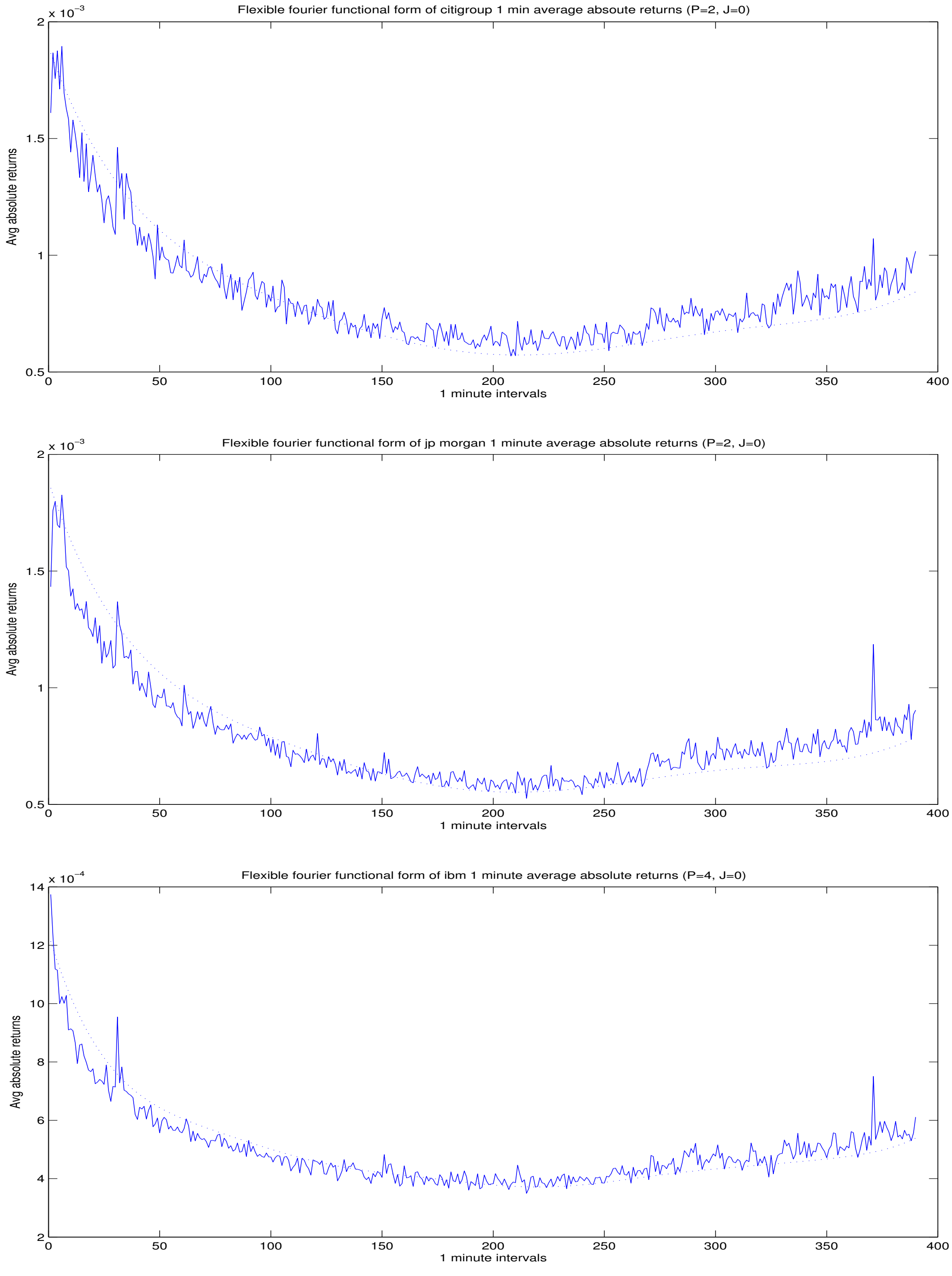

Figure 2.3: Flexible Fourier functional form of intraday average absolute returns over the sample period for the stocks c (top), jpm (middle) and ibm (bottom). x-axis is the average absolute returns, while the y-axis is the intraday time period.The dotted lines are the fitted FFF while the solid lines are the intraday average absolute returns.

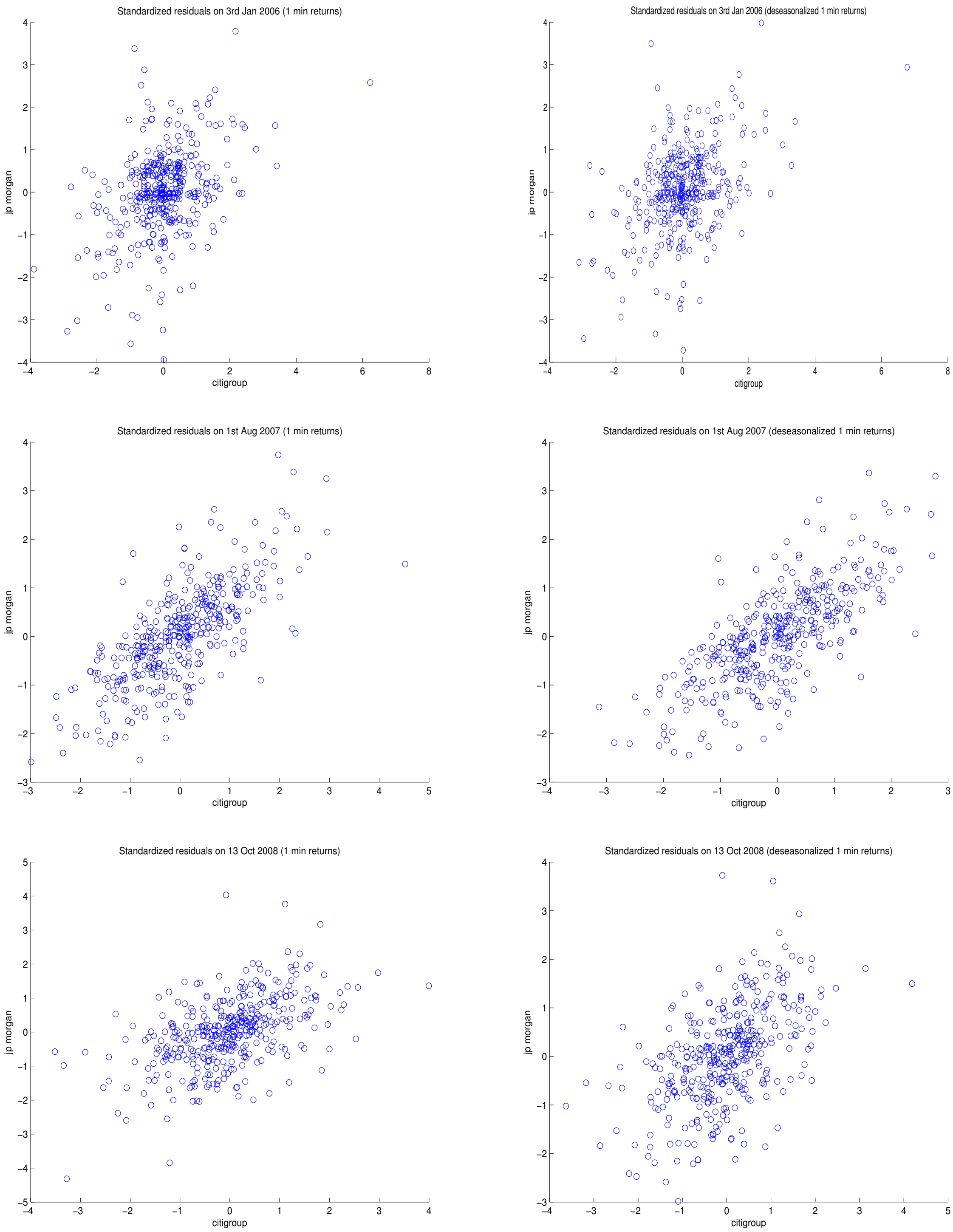

Figure 2.4: Scatter plots of 1-minute residuals for c-jpm returns pair for days 3 Jan 2006 (top), 1 Aug 2007 (middle) and 13 Oct 2008 (bottom) respectively. Plots of left side use seasonality-unadjusted returns while plots on right side use deseasonalized returns.

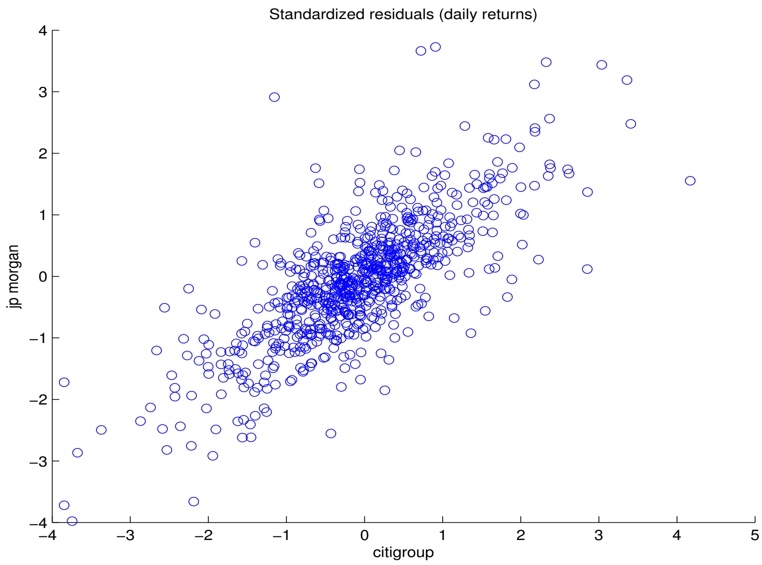

Figure 2.5: Daily residual scatter plot of stock pair c-jpm for the period between 3 Jan 2006 and 31 Dec 2008.

find a large bias in the estimated model parameters induced by the periodicity effect when seasonality is not first adjusted for. In this paper, however, the conditional mean and variance model is applied only to a single day's intraday return and reestimated everyday. We find that this provides some form of adjustment for the intraday seasonality pattern. Figure 2.6 shows the average intraday conditional variance of the stocks c, jpm and ibm captured by GARCH(1,1). They exhibit the signature lopsided U-shape where volatility is the highest at the beginning of the trading day, lowest at midday, and picks up towards trading day close. Hence standardising returns using a GARCH(1,1) is capable of removing the intradaily seasonality effect. An issue that arises with such an approach is that the GARCH variance looks non-stationary. However, we checked the estimated GARCH parameters and find them to fulfil the stationarity conditions.

We next turn to the issue of using ARMA to model the conditional mean. As with many earlier research papers, we find that the inclusion of the conditional mean has a negligible effect as compared to the conditional variance. The magnitude of the conditional mean of intraday data is very small but matter more than in the case of daily data. Andersen and Bollerslev (1997) find that allowing for an MA term in the conditional mean can account for the economically minor first order autocorrelation in returns, and the MA term is negative at sampling fre-

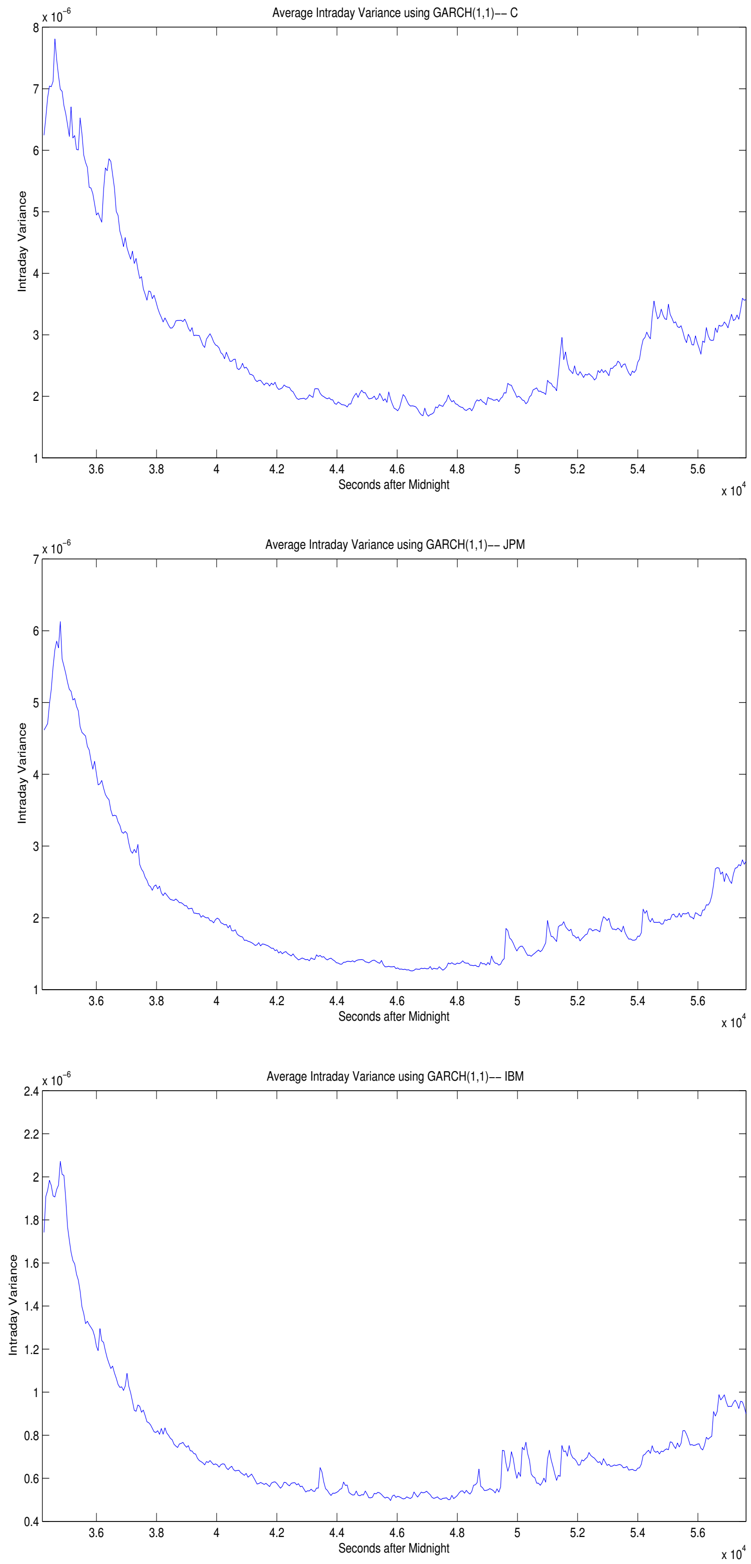

Figure 2.6: Average intraday variance of c (left), jpm(middle) and ibm (right) using GARCH (1,1) effectively captures the intradaily seasonality effects.

quencies of less than 5 minutes. The negative MA term is sometimes explained by the effect of bid-ask bounce in high frequency data due to dealers trading around the spread producing a slight mean reversion in returns[9].

We model the conditional mean for intraday returns using an ARMA process and find the estimated dependence measures to be very close to the case where the conditional mean is assumed to be zero. The residual distributions are similar but slightly more centred when the conditional mean is filtered out. However, we find the results of quantile estimation to be more accurate when the conditional mean is not filtered out of the intraday residual distribution (since this gives information about that day's stocks' performance). Since our aim is not to forecast intraday returns, and the Ljung-Box statistics of the residuals do not detect significant autocorrelations after a GARCH filtration, we do not filter out the conditional mean for intraday returns.

Finally we recognise that EGARCH or similar models that account for leverage effects may be more appropriate for daily equity data. We find however, by plots of the squared residuals against the lagged residuals, that the leverage effect is not prominent in intraday data.

Descriptive statistics

Descriptive statistics of the return series and GARCH residuals at daily and 1-minute sampling frequencies are given in Tables 2.3 and 2.4. The descriptive statistics indicate that intraday returns tend to have thinner tails than daily returns. However, the GARCH residuals display the reverse characteristic (as found by Nelson (1990) and Drost and Werker (1996)). Finally, the Ljung-Box statistics indicate that there is no significant autocorrelation in the residuals at both daily and intradaily frequencies.

Effect of sampling frequency

To investigate the effects of sampling frequency on the realized dependence estimator when empirical data is used, we estimate realized dependence of the stock pairs c-jpm, c-ibm and jpm-ibm at sampling frequencies ranging from 1 to 30 minutes. Note that all the dependence measures are estimated using GARCH

[9]However, for the highly liquid stocks that we are using, the effect of the bid-ask bounce is small, since we are not considering tick-by-tick returns.

	c daily	residuals	jpm daily	residuals	ibm daily	residuals
mean	-0.001	-0.059	0.000	-0.002	0.000	0.026
median	0.000	-0.067	0.000	-0.003	0.001	0.050
std dev	0.021	1.009	0.024	1.007	0.015	0.999
skew	0.035	0.004	-0.028	0.147	-0.141	-0.156
kurtosis	**9.853**	**4.674**	**9.866**	**4.662**	**6.657**	**4.220**
min	-0.121	-3.914	-0.143	-3.825	-0.057	-3.904
max	0.116	4.315	0.119	4.198	0.087	4.614
LB(10)	35.44	10.72	16.10	14.77	28.65	16.06
	(0.00)	(0.38)	(0.10)	(0.14)	(0.00)	(0.10)
LB(25)	55.70	21.11	64.14	26.92	61.79	33.00
	(0.00)	(0.69)	(0.00)	(0.36)	(0.00)	(0.13)

Table 2.3: Descriptive statistics of daily returns and univariate GARCH residuals. Numbers in brackets indicate the p-value of the Ljung-Box statistic with the null hypothesis of no autocorrelation. The kurtosis of the returns and residuals series are highlighted in bold for comparison with Table 2.4. ′c′, ′jpm′ and ′ibm′ are short for Citigroup, JP Morgan and IBM respectively.

residuals. Figure 2.7 shows the average realized dependence at the different sampling frequencies for year 2006 (left) and year 2008 (right) for the stock-pairs using the Gaussian, survival Gumbel and Clayton copulas. We use these three copulas since they are commonly used in empirical applications.

The graphs show that the parameters are fairly stable at different sampling frequencies but show a consistent slight upward trend as the sampling frequency decreases. (Note that the Gaussian dependence parameter is also increasing slightly but this increase is less obvious in the graphs due to scale effects.) This upward bias is due to the effect of finite sampling[10]. At 1-minute sampling frequency, there are 390 observations for estimation while at 30 minutes sampling frequency, there are only 12 observations for the estimation of the dependence parameter. Our simulation results (see Appendix Table 2.15) confirmed this upward bias to be due to finite sampling. We now see that there are two effects that overlap when we use actual empirical data: at too low sampling frequencies we have fewer data points and face a positive bias due to finite sampling, while at too high sampling frequencies (such as 10-30 seconds), we face a positive bias due to

[10]Epps effects may also be present at high sampling frequencies.

	c 1min	residuals	jpm 1min	residuals	ibm 1min	residuals
mean	0.000	-0.007	0.000	0.002	0.000	0.006
median	0.000	-0.004	0.000	-0.002	0.000	0.001
std dev	0.001	1.054	0.001	1.035	0.001	1.047
skew	0.056	0.042	0.031	0.005	0.077	-0.013
kurtosis	**6.729**	**6.596**	**6.969**	**6.007**	**7.986**	**6.639**
min	-0.005	-4.367	-0.005	-4.086	-0.003	-4.318
max	0.005	4.420	0.005	4.098	0.003	4.152
LB(10)	16.08	10.74	16.41	10.69	18.06	11.51
	(0.27)	(0.47)	(0.27)	(0.47)	(0.22)	(0.42)
LB(25)	33.90	24.82	35.21	25.72	36.25	26.38
	(0.29)	(0.52)	(0.26)	(0.48)	(0.24)	(0.46)

Table 2.4: Average descriptive statistics of 1 min returns and univariate GARCH residuals. Numbers in brackets are the average p-values of the Ljung-Box statistics. The kurtosis of the returns and residuals series are highlighted in bold for comparison with Table 2.3. 'c', 'jpm' and 'ibm' are short for Citigroup, JP Morgan and IBM respectively.

the distribution of the residuals tending to fatter tails. As a compromise of these two present effects in the data, we use the 1-minute sampling frequency. Naturally an interesting research question would be the determination of an optimal sampling frequency. We do not delve further into this issue here.

Copula choice

Given that there are hundreds of copula specifications, it is impossible to consider all of them. To reduce the estimation load, we estimate seven different copulas (Gaussian, Clayton, s. Gumbel, Plackett, Frank, Student-t and Symmetrized Joe-Clayton (SJC)) using the daily GARCH residuals of the three stock pairs (c-jpm, c-ibm, jpm-ibm) and pick the three copulas that give the three largest log likelihoods. The Gaussian, Plackett and Frank copulas do not exhibit tail dependence, while the Clayton and survival Gumbel copulas have lower tail dependence. The Student-t and SJC copulas have both upper and lower tail dependence. Refer to Appendix 2.9 for more description about these copulas. Table 2.5 gives the average log likelihood estimates for the three stock pairs. Finally, we always include the Gaussian copula as a basis of comparison, which gives four copulas for each stock pair.

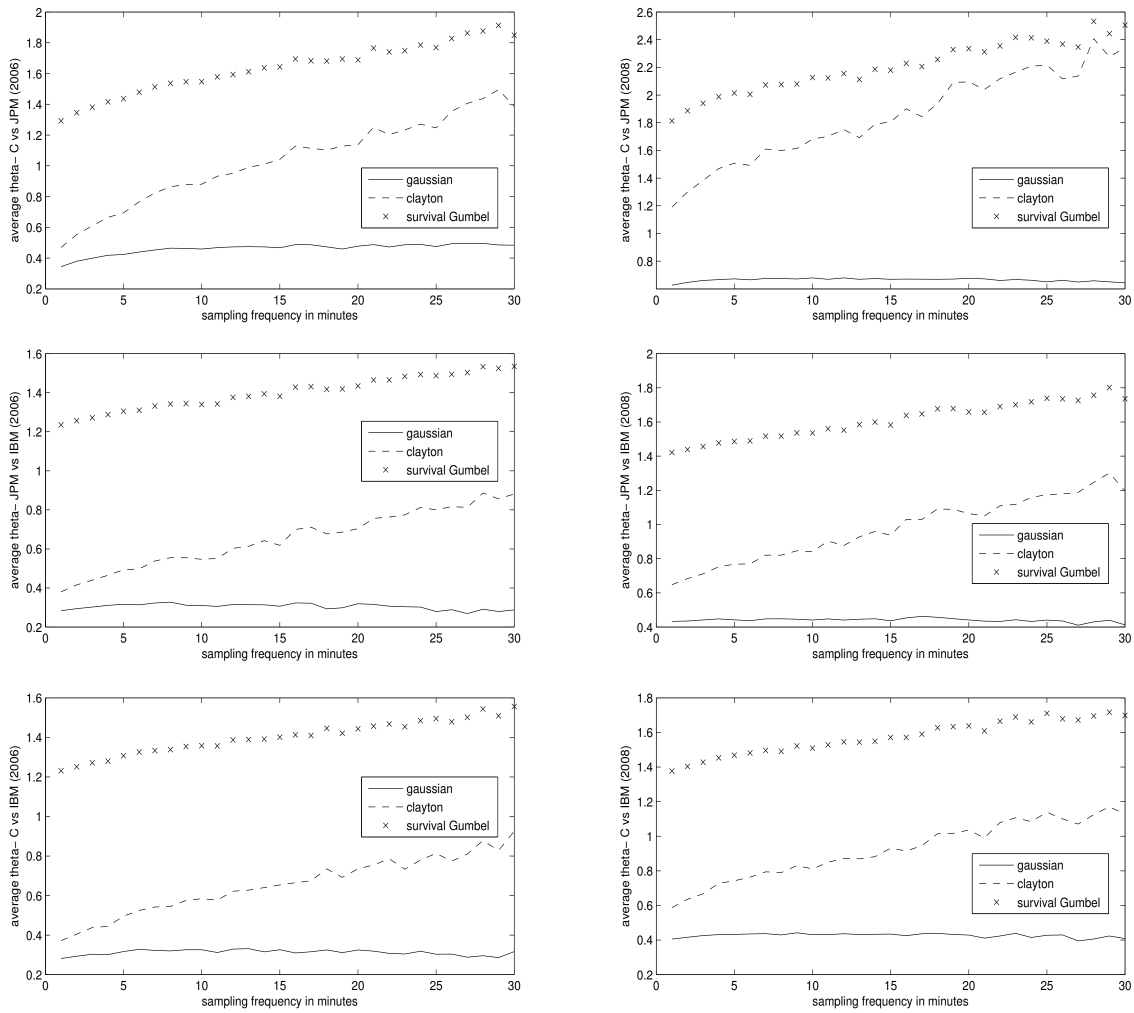

Figure 2.7: Realized dependence signature plots of c-jpm (top), c-ibm (middle) and jpm-ibm (bottom) for year 2006 and 2008 using the Gaussian (solid lines), Clayton (dashed lines) and s. Gumbel (x-lines) copulas

	Gaussian	Clayton	s. Gumbel	Plackett	Frank	Student-t	SJC
c-jpm	0.4335	0.3762	***0.4446***	0.4414	0.4064	***0.4723***	***0.4599***
c-ibm	0.1336	0.1111	0.1335	***0.1396***	0.1357	***0.1430***	***0.1359***
jpm-ibm	0.1226	0.1166	***0.1392***	***0.1463***	0.1355	0.1548	***0.1385***

Table 2.5: Average log likelihood of different copulas over intraday dependence estimated for the period 2006-2008. Bold font indicates it is one of the best three log likelihood estimates and is selected for further estimations

2.5.2 Comparing the intraday dependence estimator with time-varying dependence estimators

We compare the nonparametric realized dependence with the constant unconditional dependence estimator and two other time-varying dependence estimators: the rolling window estimator and the autoregressive estimator.

The constant unconditional dependence estimator estimates the copula parameter over the whole time series and one constant estimate over the whole period is thus obtained. The rolling window estimator estimates the copula parameter over a rolling window of 250 observations (approximately 1 year). After the first 250 observations, the window incorporates one new observation and drops the oldest observation at each incremental time point.

The autoregressive estimator is adapted from that of Patton (2006) and estimated using maximum likelihood. For the specification of the Gaussian and SJC time varying copulas, we use the specification of ARMA(1,10) as in Patton (2006). As noted in the paper, a key difficulty is in identifying the forcing (or updating) variable. For the other time varying copula models, we use the mean absolute difference between the GARCH residuals u_t and v_t as forcing variable with evolution equation for θ given by

$$\theta_t = \Lambda_2 \left(\beta_0 + \beta_1 \cdot \theta_{t-1} + \beta_2 \cdot \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right), \tag{2.9}$$

where $\Lambda_2(x) = x^2 + 1$ which keeps the parameter above 1 at all times. This is suitable for the Clayton, Gumbel, Student t's v parameter where the parameter must lie above 1. (Clayton and survival Gumbel copulas do not allow for negative dependence and have a parameter boundary value for θ of 1 and 2 respectively, while the degrees of freedom for a student-t copula should lie above 2 if we assume finite variance). For the time-varying Plackett copula, the same forcing variable was used and $\Lambda_2(x) = x$. Since for the Plackett copula, $\theta > 0$ and $\theta \neq 0$, penalties to the likelihood were imposed to deal with these restrictions.

Figures 2.8, 2.9 and 2.10 show the estimated dependence measures θ_t of c-jpm, c-ibm and jpm-ibm stock-pairs respectively using different copulas with the constant estimator (solid horizontal lines), realized dependence estimator (dotted lines), rolling window estimator (dashed lines) and autoregressive estimator (solid lines).

In Figure 2.8 which is for c-jpm, the realized dependence estimator shows steep increases from the second quarter of 2007 to about last quarter of 2008 (coinciding with the credit crisis of 2007). A large drop in dependence is observed in November 2008 when Citigroup experienced a 1 week crash in its stock prices. The rolling window estimator does effectively capture the increasing dependence but has a slight lag. The autoregressive estimator moves much in tandem with the realized dependence measure in capturing short term trends but on a long term basis moves largely about the constant unconditional estimator and and does not capture the increasing dependence over the period. Finally, the realized dependence estimator tends to lie below the other estimators except during the crisis period.

Figures 2.9 and 2.10 show the same observations for c-ibm and jpm-ibm stock pairs, except that the increase in dependence during the crisis period is less dramatic compared to that of c-jpm stock pair. For the Student-t copula in c-ibm stock pair, convergence for the autoregressive estimator could not be obtained for the ν (degrees of freedom) parameter. This illustrates a shortcoming of the autoregressive estimator in that such a parametric form of the dynamics is suitable for only certain copula dependences.

Overall, the graphs show that dependence between stocks is dynamic, and that in general, the time-varying copula parameters tend to trend together. This emphasizes the phenomenon of time-varying dependence. In general, the autoregressive estimator hovers around the unconditional constant estimator while the rolling window estimator displays a smooth trend. The realized dependence estimates tends to be lower than other estimators during the calm periods, but becomes greater than other estimators during the subprime crisis period.

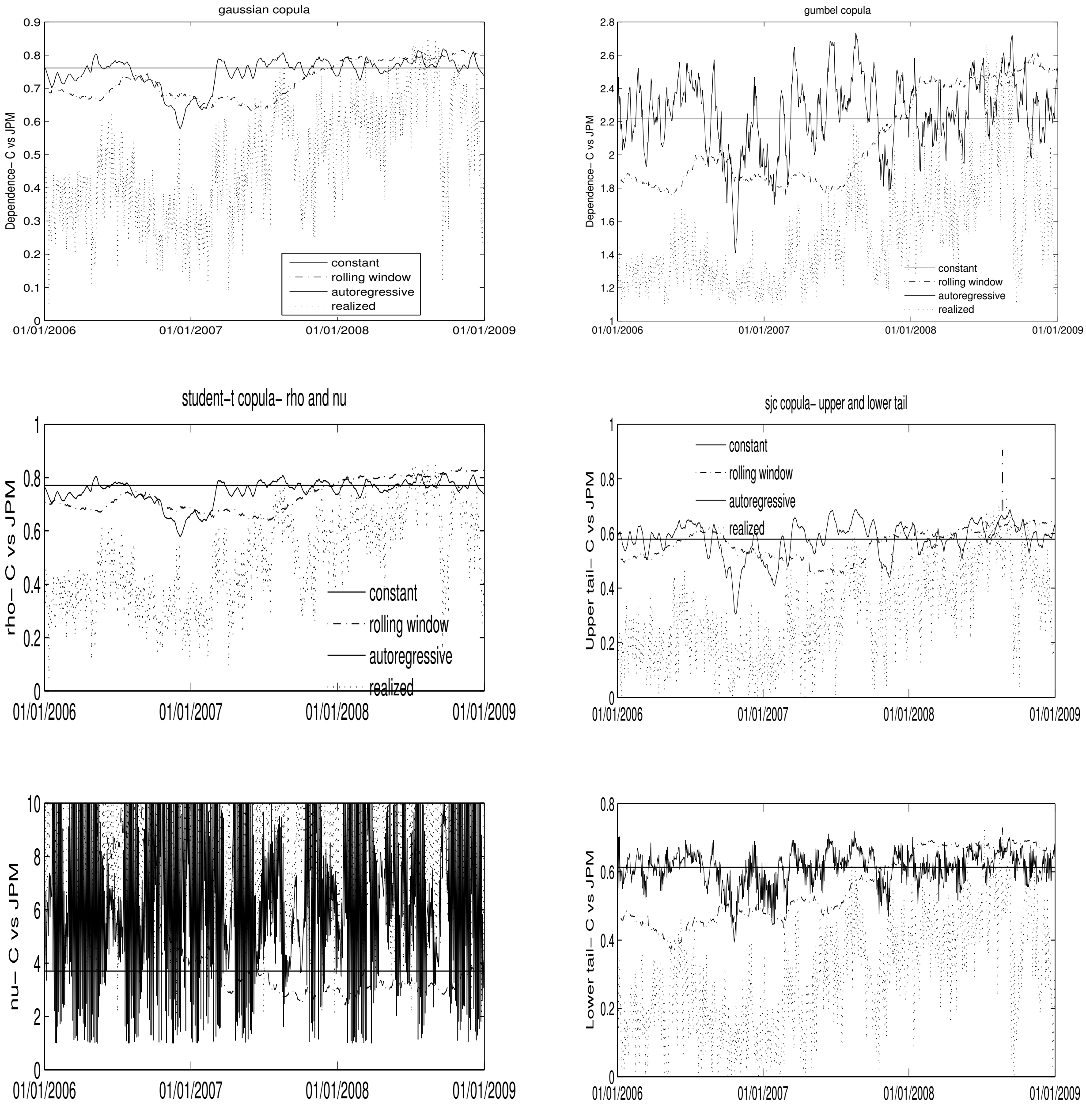

Figure 2.8: Constant and time varying dependence parameters for c-jpm between 2006-2008. Top row: Gaussian and s. Gumbel copulas. Left middle and bottom rows: Student-t copula (rho and v), right middle and bottom rows:- SJC copula (upper and lower tail parameters).

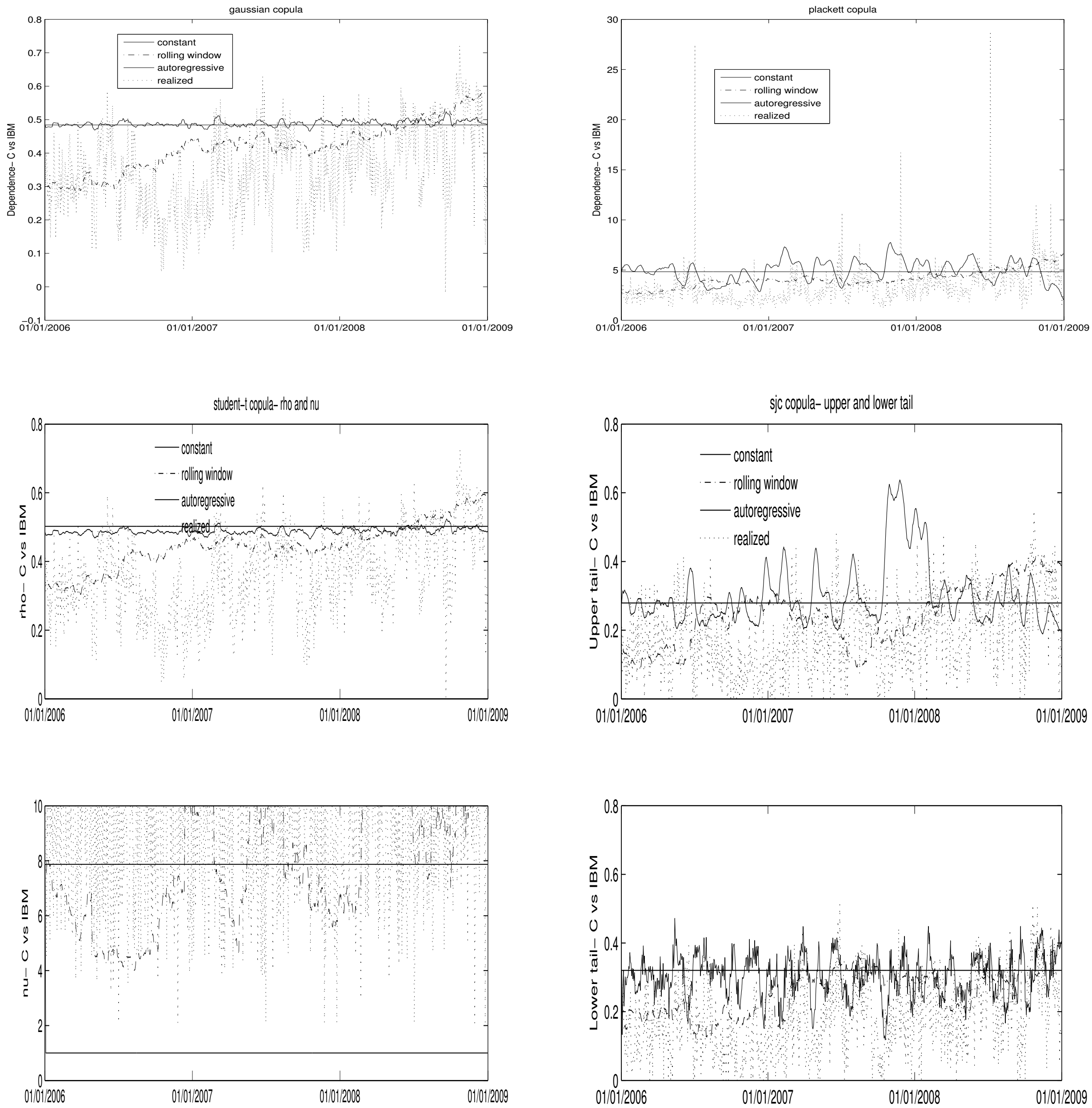

Figure 2.9: Constant and time varying dependence parameters for c-ibm between 2006-2008. Top row: Gaussian and Plackett copulas. Left middle and bottom rows: Student-t copula (rho and v), right middle and bottom rows: SJC copula (upper and lower tail parameters).

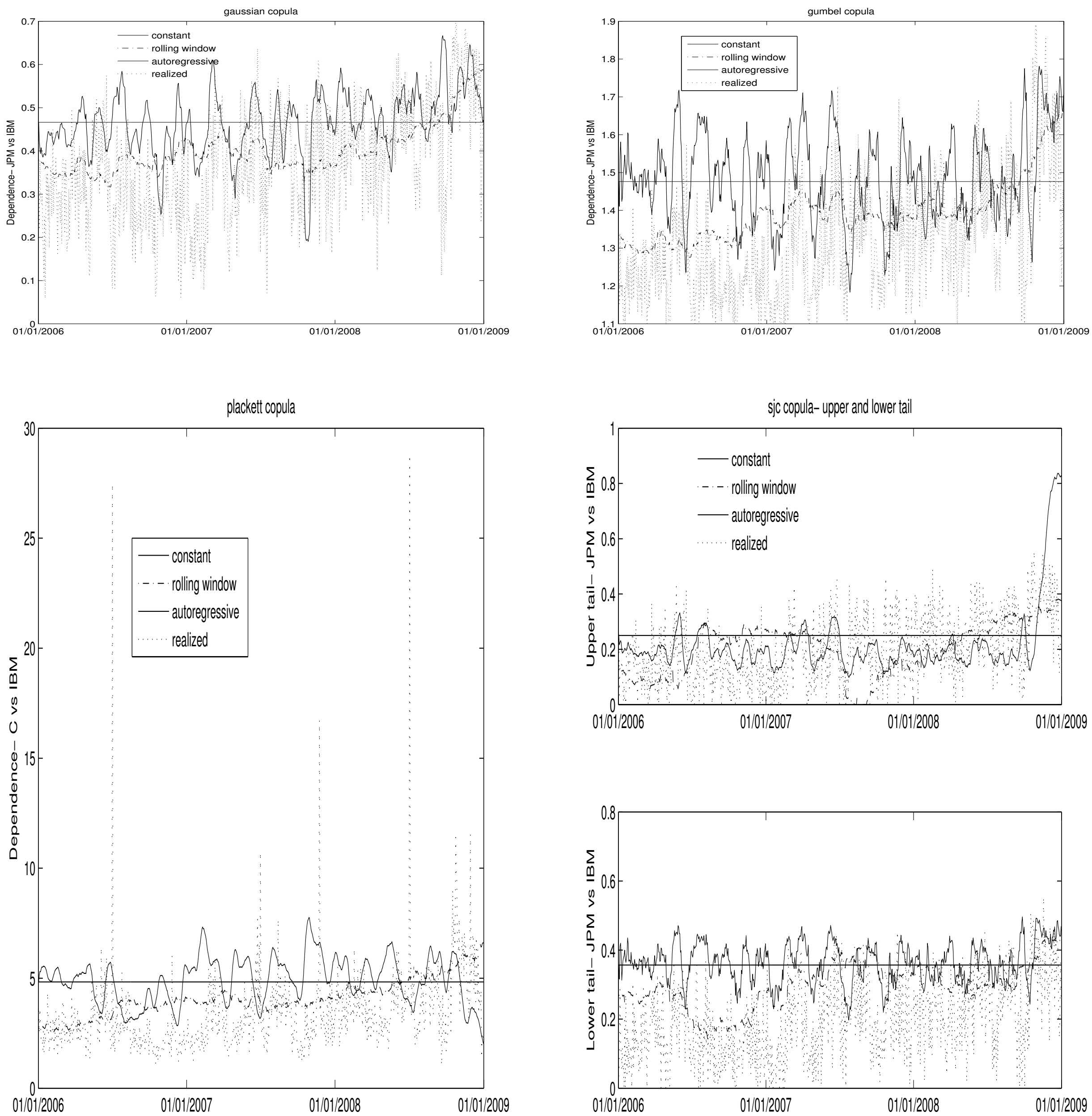

Figure 2.10: Constant and time varying dependence parameters for jpm-ibm between 2006-2008. Top row: Gaussian and s. Gumbel copulas. Left middle: Plackett copula; right middle and bottom rows: SJC copula (upper and lower tail parameters).

2.5.3 Estimating quantiles of the distribution

To illustrate the increased accuracy of the realized dependence estimator as compared to other time-varying copula dependence estimators that use daily data, we estimate different quantiles of a portfolio using the (i) constant parameter estimator (Co), (ii) the rolling-window estimator (RW), (iii) autoregressive estimator (AR) and (iv) realized dependence (RD) estimators. We would like to emphasize again that our aim is not in copula selection, but rather in the comparison of realized dependence against different dependence estimators. The joint cumulative distribution of uniform marginals $\{u, v\}$ is simulated using the parametric copula model with the estimated $\hat{\theta}$ and the residuals $\eta_{X,i} = F_{N,X}^{-1}(u_i)$ are then obtained by using the inverse of the empirical distribution. Individual returns of each asset, $X_{k,t}$ are obtained by $X_{k,t} = \sigma_{k,t}\eta_{k,t}$ and the returns of portfolio, P_t are computed by

$$P_t = \sum_{k=1}^{K} w_k X_{k,t},$$

where w_k are the fixed portfolio weights and $\sum_{k=1}^{K} w_k = 1$. For our purpose, we use $K = 2$, $w_1 = w_2 = 0.5$. The estimated quantile of interest $\widehat{Q}(\alpha)$ is then the empirical quantile of the distribution.

We use the tick loss function $TL_{t,p}$ of Komunjer (2005) to check for accuracy of the quantile estimation. The subscripts t refer to the day and p refers to the quantile of interest. Formally,

$$TL_{t,p} = (p - \mathbb{I}_{r_t < \widehat{Q}_t^p})(r_t - \widehat{Q}_t^p), \tag{2.10}$$

where r_t is the daily return, $\widehat{Q}_t^P$ the estimated quantile and $\mathbb{I}$ is the indicator function. This loss function considers the distance of the estimated quantile from the true quantile. To compare the different estimators which we denote by M, we use the Diebold-Mariano test with tick-loss function in (2.10). The constant estimator (Co) is used as the base model (M_0) for each copula. The DM test statistic is given by

$$DM_{TL} = \frac{\overline{d}}{(L\hat{R}V/N)^{1/2}},$$

where $\overline{d} = \frac{1}{N}\sum d_t$, $d_t = TL_{t,p,M} - TL_{t,p,M_0}$ and $L\hat{R}V = \gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j$, $\gamma_j = cov(d_t, d_{t-j})$ is the long run variance that takes into account of serial correlation. A significant value for the statistic (DM_{TL}) implies the rejection of the null of equal predictive accuracy, and a negative value indicates that model M has better predictive ability than base model M_0.

Tables 2.6 - 2.8 documents the DM_{TL} of the RW, AR and RD estimators against the Co estimator for the three stock-pairs using different copulas. The results consistently show that for central quantiles between 0.1-0.9, the RD estimator is significantly more accurate than other time-varying copula dependence estimators for all copulas tested.

p	0.01	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
	Gaussian copula								
RW	-0.440	-0.494	-0.384	-0.589	-0.365	-0.633	-0.242	-0.215	0.167
AR	0.956	0.427	0.334	-0.140	0.400	-1.10	-0.293	-0.422	0.163
RD	0.748	-1.47	-4.31***	-6.39***	-5.33***	-2.69***	-1.63*	-0.416	0.231
	s. Gumbel copula								
RW	-0.439	-0.538	-0.403	-0.679	-0.339	-0.282	-0.353	-0.263	0.021
AR	0.909	0.101	0.349	0.551	0.783	-0.979	0.765	1.01	0.656
RD	0.637	-1.47	-4.14***	-6.25***	-5.95***	-2.54***	-1.73*	-0.295	0.478
	Student-t copula								
RW	-0.459	-0.420	-0.331	-0.522	-0.479	-0.327	-0.524	-0.202	0.026
AR	0.017	-0.662	-0.357	-0.273	-0.357	-0.836	-0.188	0.259	-0.483
RD	0.735	-1.83*	-3.92***	-5.95***	-7.38***	-2.51***	-1.68*	-0.333	-0.003
	SJC copula								
RW	-0.360	-0.452	-0.324	-0.551	-0.302	-0.249	-0.144	-0.198	0.030
AR	2.72	-0.035	-0.423	-0.103	0.148	-0.132	-0.733	0.033	0.242
RD	0.670	-1.37	-4.32***	-6.11***	-5.88***	-2.93***	-1.70*	-0.574	-0.403

Table 2.6: Quantile evaluation at the p^{th} quantile for stock-pair c-jpm using the the rolling window estimator (RW), the autoregressive estimator (AR) and the realized dependence estimator (RD) for the Gaussian, s. Gumbel, Student-t and SJC copulas. Numbers indicate the DM statistics of the test model against the base model, which uses the constant estimator. * indicates 10% significance, ** indicates 5% significance and *** indicates 1% significance.

At the extreme tails, the performance of the RD estimator is not significantly better than the Co estimator. This is difficult to achieve statistically due to the scarcity of observations of the intraday return distribution of each day at the ex-

p	0.01	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
	Gaussian copula								
RW	-0.454	-0.805	-0.790	-0.543	-0.526	-0.597	-0.205	-0.492	-0.493
AR	-0.327	-0.856	0.741	0.308	0.514	0.313	0.221	0.337	0.407
RD	0.288	-1.71*	-4.05***	-6.99***	-8.65***	-3.45***	-1.11	0.112	0.659
	Plackett copula								
RW	-0.541	-0.856	-0.846	-0.524	-0.610	-0.619	-0.268	-0.771	-0.284
AR	-0.165	-0.404	-0.547	-0.355	0.102	0.795	-0.710	-0.094	1.246
RD	0.485	-1.85*	-3.68***	-6.02***	-8.59***	-2.82***	-0.950	0.219	0.959
	Student-t copula								
RW	-0.438	-0.665	-0.751	-0.451	-0.480	-1.06	-0.373	-0.727	-0.609
AR	0.585	-0.733	0.572	1.26	-0.063	0.110	0.055	-0.709	1.26
RD	0.389	-1.87*	-3.45***	-5.54***	-8.71***	-3.05***	-1.35	0.024	0.481
	SJC copula								
RW	-0.244	-0.789	-0.803	-0.476	-0.592	-0.448	-0.230	-0.788	-0.910
AR	1.514	0.696	-0.083	-0.441	-0.501	0.728	0.049	-0.348	-0.607
RD	0.375	-1.96*	-3.77***	-5.81***	-7.54***	-3.68***	-1.47	-0.295	0.364

Table 2.7: Quantile evaluation for stock-pair c-ibm using the constant estimator, the rolling window estimator, the autoregressive estimator and the realized dependence estimator for the Gaussian, Plackett, Student-t and SJC copulas. * indicates 10% significance, ** indicates 5% significance and *** indicates 1% significance.

treme tails, causing the quantile estimator to be highly unstable with large variances. We attempted to enhance the lower tail estimation (for financial losses) by fitting a generalised Pareto distribution (GPD) at the lower tail of the marginal distribution below a threshold μ_X. The improvement in the extreme quantile estimation after fitting a GPD was however only slight and the DM_{TL} test statistic remains insignificant.

2.5.4 Using realized dependence in VaR estimation

We now consider using the realized dependence estimator for Value-at-Risk (VaR) estimation. For in-sample period, we use data from 2 Jan 2006 to 31 Dec 2006 and forecast the period between 2 Jan 2007 to 31 Dec 2008. The forecast period is especially challenging given that it consists mainly of the credit crisis, whereas the in-sample period consists of only a calm market period. However

p	0.01	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
	Gaussian copula								
RW	-0.237	-0.694	-0.692	-0.853	-0.636	-0.544	-0.147	-0.596	-0.243
AR	-0.197	-0.124	-0.550	0.118	0.587	0.263	-0.002	-0.668	-0.103
RD	-0.140	-1.65	-2.71***	-4.89***	-7.36***	-3.59***	-0.909	-0.277	1.01
	s. Gumbel copula								
RW	-1.85	-1.90	-1.12	-1.08	-0.446	-0.913	-1.05	-0.659	-1.46
AR	-2.82	-0.823	-0.142	0.139	0.829	-0.218	-0.582	-0.238	-0.594
RD	-2.408***	-3.60***	-4.32***	-7.12***	-8.31***	-3.56***	-1.80*	-0.547	-0.044
	Plackett copula								
RW	-1.15	-1.51	-1.41	-0.880	-0.587	-0.890	-1.26	-1.22	-0.846
AR	-1.18	-1.56	-1.50	-0.809	0.322	-0.843	-1.29	-1.23	-0.848
RD	-1.17	-1.85*	-2.12**	-2.55***	-8.71***	-2.00**	-1.57	-1.27	-0.816
	SJC copula								
RW	0.581	-0.657	-0.709	-0.710	-0.613	-0.791	-0.287	-0.435	-0.146
AR	1.46	0.027	-0.179	0.105	0.078	-0.018	0.669	0.736	0.945
RD	0.092	-1.86*	-3.15***	-4.52***	-5.49***	-4.22***	-1.46	-0.338	0.891

Table 2.8: Quantile evaluation for stock-pair jpm-ibm using the constant estimator, the rolling window estimator, the autoregressive estimator and the realized dependence estimator for the Gaussian, s. Gumbel, Plackett and SJC copulas. * indicates 10% significance, ** indicates 5% significance and *** indicates 1% significance.

we wish to observe how the realized dependence performs in relation to other time varying dependence measures in times of extreme market fluctuations.

Since VaR is the next period's forecasted loss with a certain (extreme) probability, its estimation using copulas requires a forecast of the next day's dependence.

Forecasting Realized Dependence

Figure 2.11 shows the sample autocorrelations (SACFs) of the realized dependences in a Gaussian, survival Gumbel and SJC copula for the c-jpm stock pair.[11] It shows the slow decay of the dependence measures, a property also exhibited by realized volatility. The strong persistence is suggestive of long memory behaviour of the realized dependence. Long memory processes can be modelled

[11]The Student-t copula is not included here as the Student-t correlation is similar to that of the Gaussian copula dependence.

parametrically by fractionally integrated processes such as the ARFIMA model or by the heteroscedastic autoregressive (HAR) model introduced by Corsi (2009) and Corsi et al. (2008).

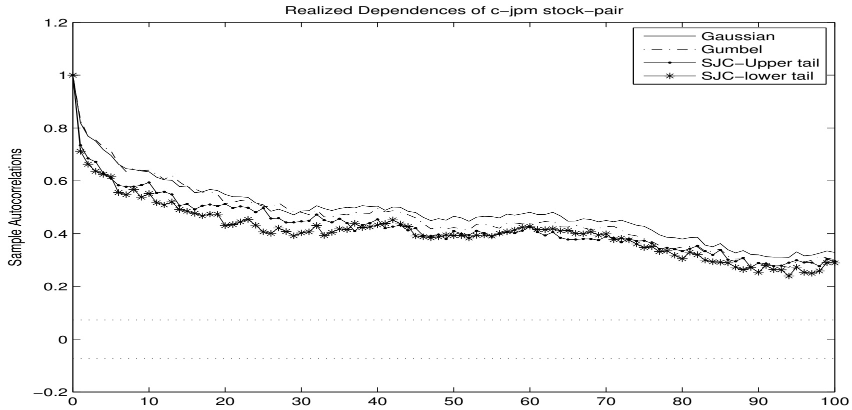

Figure 2.11: Sample autocorrelations of the realized dependences for the c-jpm stock pair using the Gaussian (solid line), s. Gumbel (dash-dot line) and SJC (line with dot) copulas.

We use a HAR model for forecasting realized dependence due to its simplicity in estimation and good performance in modelling realized volatility and correlations (see for example Corsi (2009), Audrino and Corsi (2010), Vortelinos (2010)). The form is a stochastic additive cascade of three different components corresponding to three different time horizons:

$$\hat{\theta}_{t+1}^{(d)} = a + b^{(d)}\theta_t^{(d)} + b^{(w)}\theta_t^{(w)} + b^{(m)}\theta_t^{(m)}, \tag{2.11}$$

where $\theta_t^{(d)}$, $\theta_t^{(w)}$ and $\theta_t^{(m)}$ are the daily, weekly and monthly dependences. For our purpose, the weekly and monthly dependences are computed by taking averages of realized dependences of the past 5 and 20 days respectively. Estimation of (2.11) is simply made via OLS.

VaR Estimation and Evaluation

To estimate VaR, we again assume an equally-weighted portfolio and forecast the next period's variance using univariate GARCH with residuals fitted to a copula. For the constant dependence estimator (Co), we keep the dependence constant over the whole period. For the rolling-window dependence estimator (RW), we use the last 250 days to estimate the dependence and assume this is the next period's best forecast. For the autoregressive estimator (AR), we use the last 250 days to update the parameters of the model and forecast the next period's dependence. The forecast of realized dependence estimator (RD) is as described in subsection 2.5.4.

To calculate the VaR, we simulate the joint cumulative distribution of the residuals from the copula model for each of the forecasted dependences from the four models (Co, RW, AR, RD). Linear interpolation is used to invert the simulated cdf using the empirical probability distribution of the last 250 days (in the case of realized dependence, we use the last 4 days of intraday data). The forecasted portfolio distribution is constructed as in Section 2.5.3 and VaR is obtained by taking the empirical quantiles. We estimate the 1, 5 and 10% VaR, and also include the 0.1 and 0.5% since our interest is in the extreme quantiles when considering VaR. Furthermore, differences between the different time-varying dependence measures when they are used in copulas with larger tail dependence may become more evident when we go into the extreme quantiles.

For VaR evaluation, we compute what is popularly known as "Hit Rate" which is simply the percentage of exceedances (% ex). This number should preferably be as close to the quantile of interest as possible. The Kupiec Test (Ku) (Kupiec, 1995) is a popular VaR backtest used to check for VaR adequacy and it considers the unconditional coverage, i.e. the number of violations (Campbell, 2005). The test statistic takes the form

$$\text{Ku} = 2\log\left(\left(\frac{1-\hat{p}}{1-p}\right)^{N-\sum \mathbb{I}_{r_t<\widehat{VaR}_t^p}} \left(\frac{\hat{p}}{p}\right)^{I(p)}\right),$$

where $\hat{p} = \frac{1}{N}\sum \mathbb{I}_{r_t<\widehat{VaR}_t^p}$ and N is the sample size. The statistic is asymptotically χ^2-distributed with 1 degrees of freedom with the null hypothesis that the VaR model is adequate. A rejection of the statistic hence indicates inadequate coverage.

We also use the Dynamic Quantile (DQ) test introduced by Engle and Manganelli (2004) to check if the VaR model is prone to "violation clustering", an undesirable property for VaR models. An ideal VaR model should not only fulfill the regulatory Hit Rate, but also have Hits (H_t) that are uncorrelated with the variables in the information set $\mathcal{F}_{t-1}$. To conduct the test, we regress the variable, $\tilde{H}_t = I(y_t < VaR_t(p)) - p$ against past lagged $\tilde{H}_t$ and the contemporaneous VaR forecast,

$$\tilde{H}_t = \lambda_0 + \sum_{i=1}^{h} \lambda_i \tilde{H}_{t-i} + \lambda_{h+1}\widehat{VaR}_t + \mu_t = \mathbf{X}\lambda + \mu_t. \tag{2.12}$$

The DQ test statistic is then given by $\frac{\hat{\lambda}' X' X \hat{\lambda}}{p(1-p)} \sim \chi^2(h+2)$, where $\hat{\lambda}$ is estimated using OLS. We use $h = 4$ lags as in Engle and Manganelli (2004).

Tables 2.9, 2.10 and 2.11 tabulate the VaR evaluation for the three stock pairs respectively. As noted in Section 2.5.3, the use of the autoregressive estimator for the degrees of freedom, v, in the Student-t copula is problematic and faces convergence issues. We hence leave out the autoregressive estimator for the Student-t copulas in our results. Furthermore, when no hits are observed, a singular matrix in the explanatory variables in (2.12) results and the DQ statistic cannot be computed. This sometimes occurs at the 0.1% quantile. The p-values for the Kupiec Test (Ku) and Dynamic Quantile test (DQ) are reported in the tables. Bold numbers indicate a rejection of the test at 1% significance level. A rejection of the Ku means that the VaR model produces a hit rate that is too far from acceptable, while a rejection of the DQ means that the VaR model either produces a hit rate that is unacceptable or that the hits cannot be rejected for no autocorrelation.

The VaR results are poor in general, given that the out-of-sample forecast period is an extremely turbulent period. The constant estimator performs especially poorly since it is estimated only using the in-sample calm period data. The use of rolling window improves the VaR performance but still has too many exceedances. The autoregressive dependence estimator gives much better performance than the rolling window, but the best performing estimator is the realized dependence, which often gives the exceedances that are closest to the desired p. In terms of the DQ statistics, the performance is poorer for the unconditional copula models (constant and rolling window estimators), and improve slightly with the use of autoregressive or realized dependence estimators. The overall poor DQ results are unsurprising, given the challenging time period.[12]

In terms of copula choice, our results show that distributions that allow for lower tail dependence such as the survival Gumbel, Student-t and SJC perform better than distributions which do not allow for tail dependence such as the Gaus-

[12]Note that even when other time periods are used, most models in Kuester et al. (2006) do not pass the DQ test, and for the few that do pass the test (GARCH with skewed student-t, Mixed GED and EVT), they do so only at a certain quantile (mostly at 1%) and none of the models pass the test at all three quantiles (1%, 2.5% and 5%).

sian and Plackett. Furthermore, the survival Gumbel and SJC are non-symmetric distributions and hence often perform better than the Student-t. The results are in line with prior research. Kuester et al. (2006) and Halbleib and Pohlmeier (2012) find that GARCH volatility with a skewed-t distribution or skewed EVT distribution gives adequate VaR performance while a GARCH with symmetrized residuals (normal or Student-t) performs poorly. Giot and Laurent (2004) also find that the APARCH, an extension of GARCH which accounts for asymmetry with Student-t innovations performs as adequately as when realized volatility is used.

It is well-known that the simple GARCH(1,1) gives less conservative VaR forecast which makes it attractive for financial institutions to implement (Berkowitz and O'Brien (2002)). However, we find that the GARCH(1,1) is inadequate for modelling volatility in turbulent times, as it gives too many exceedances. On the other hand, the use of realized dependence does improve the performance of this simple volatility model and even more so when the copula of choice allows for skewness and tail dependence in the distribution of the innovations. Overall performance can most certainly be improved by use of a more sophisticated GARCH model or realized volatility for volatility forecasting, coupled with the realized dependence and a suitable copula.

		Gaussian					s. Gumbel				
		0.001	0.005	0.010	0.050	0.100	0.001	0.005	0.010	0.050	0.100
Co	ex	0.010	0.026	0.046	0.123	0.185	0.008	0.018	0.044	0.123	0.179
	Ku	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.002**	**0.002**	**0.000**	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
RW	ex	0.008	0.014	0.028	0.113	0.167	0.008	0.012	0.024	0.117	0.163
	Ku	**0.002**	0.020	**0.001**	**0.000**	**0.000**	**0.002**	0.062	**0.008**	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**
AR	% ex	0.006	0.012	0.020	0.101	0.151	0.032	0.050	0.083	0.181	0.232
	Ku	0.017	0.062	0.050	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
	DQ	**0.000**	**0.004**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
RD	% ex	0.006	0.018	0.026	0.095	0.133	0.004	0.012	0.018	0.087	0.133
	Ku	0.017	**0.002**	**0.003**	**0.000**	0.018	0.112	0.062	0.110	**0.001**	0.018
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.027	**0.003**	**0.000**	**0.000**	**0.000**
		Student-t					SJC				
		0.001	0.005	0.010	0.050	0.100	0.001	0.005	0.010	0.050	0.100
Co	ex	0.030	0.048	0.077	0.181	0.232	0.006	0.024	0.044	0.129	0.193
	Ku	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.017	**0.000**	**0.000**	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
RW	ex	0.008	0.012	0.026	0.115	0.169	0.006	0.012	0.026	0.113	0.163
	Ku	**0.002**	0.062	**0.003**	**0.000**	**0.000**	0.017	0.062	**0.003**	**0.000**	**0.001**
	DQ	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**
AR	ex	-	-	-	-	-	0.006	0.008	0.018	0.105	0.153
	Ku	-	-	-	-	-	0.017	0.390	0.110	**0.000**	**0.002**
	DQ	-	-	-	-	-	**0.000**	0.043	**0.002**	**0.000**	**0.000**
RD	ex	0.008	0.016	0.020	0.091	0.131	0.004	0.014	0.020	0.099	0.143
	Ku	**0.002**	**0.006**	0.050	**0.000**	0.026	0.112	0.020	0.050	**0.000**	**0.002**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.027	**0.001**	**0.000**	**0.000**	**0.000**

Table 2.9: VaR evaluation for stock-pair c-jpm using the constant (Co), rolling window (RW), autoregressive estimator (AR) and the realized dependence (RD) estimators for the Gaussian, s. Gumbel, Student-t and SJC copulas. The exceedance rate and the p-values for the Ku and DQ tests are given. Bold font indicates rejection of the Ku or DQ test at the 1% significance level.

		Gaussian					Plackett				
		0.001	0.005	0.010	0.050	0.100	0.001	0.005	0.010	0.050	0.100
Co	ex	0.004	0.014	0.040	0.121	0.183	0.010	0.028	0.058	0.129	0.187
	Ku	0.112	0.020	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
	DQ	0.036	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
RW	ex	0.002	0.012	0.032	0.107	0.161	0.002	0.016	0.036	0.107	0.163
	Ku	0.538	0.062	**0.000**	**0.000**	**0.000**	0.538	**0.006**	**0.000**	**0.000**	**0.000**
	DQ	0.832	**0.003**	**0.000**	**0.000**	**0.000**	0.812	**0.000**	**0.000**	**0.000**	**0.000**
AR	ex	0.004	0.010	0.022	0.107	0.145	0.006	0.012	0.028	0.103	0.147
	Ku	0.112	0.168	0.021	**0.000**	**0.002**	0.017	0.062	**0.001**	**0.000**	**0.000**
	DQ	0.025	0.016	**0.000**	**0.000**	**0.000**	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**
RD	ex	0.004	0.008	0.024	0.095	0.131	0.006	0.010	0.028	0.089	0.129
	Ku	0.112	0.390	**0.008**	**0.000**	0.026	0.017	0.168	**0.001**	**0.000**	0.037
	DQ	0.079	0.150	**0.000**	**0.000**	**0.000**	**0.000**	0.017	**0.000**	**0.000**	**0.000**
		Student-t					SJC				
		0.001	0.005	0.010	0.050	0.100	0.001	0.005	0.010	0.050	0.100
Co	ex	0.018	0.050	0.064	0.145	0.218	0.002	0.010	0.038	0.131	0.194
	Ku	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.538	0.168	**0.000**	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	0.658	0.018	**0.000**	**0.000**	**0.000**	**0.000**
RW	ex	0.002	0.008	0.028	0.107	0.169	0.000	0.079	0.026	0.111	0.169
	Ku	0.538	0.390	**0.001**	**0.000**	**0.000**	0.315	0.390	**0.003**	**0.000**	**0.000**
	DQ	0.803	0.193	**0.000**	**0.000**	**0.000**	-	0.198	**0.000**	**0.000**	**0.000**
AR	ex	-	-	-	-	-	0.002	0.004	0.016	0.111	0.153
	Ku	-	-	-	-	-	0.538	0.733	0.222	**0.000**	**0.000**
	DQ	-	-	-	-	-	0.634	0.838	**0.002**	**0.000**	**0.000**
RD	ex	0.002	0.008	0.022	0.089	0.131	0.002	0.008	0.014	0.091	0.137
	Ku	0.538	0.390	0.021	**0.000**	0.026	0.538	0.390	0.408	**0.000**	**0.009**
	DQ	0.793	0.153	**0.000**	**0.000**	**0.000**	0.813	0.155	**0.000**	**0.000**	**0.000**

Table 2.10: VaR evaluation for stock-pair c-ibm using the constant (Co), rolling window (RW), autoregressive (AR) and realized dependence (RD) estimators for the Gaussian, Plackett, Student-t and SJC copulas. The exceedance rate and the p-values for the Ku and DQ tests are given. Bold font indicates rejection of the Ku or DQ test at the 1% significance level.

		Gaussian					s. Gumbel				
		0.001	0.005	0.010	0.050	0.100	0.001	0.005	0.010	0.050	0.100
Co	ex	0.016	0.028	0.044	0.115	0.177	0.014	0.020	0.034	0.113	0.183
	Ku	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
RW	ex	0.006	0.020	0.034	0.099	0.167	0.004	0.010	0.022	0.093	0.163
	Ku	0.017	**0.000**	**0.000**	**0.000**	**0.000**	0.112	0.168	0.021	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.017	0.010	**0.000**	**0.000**	**0.000**
AR	ex	0.004	0.014	0.018	0.093	0.157	0.026	0.036	0.042	0.075	0.119
	Ku	0.112	0.020	0.110	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**	0.015	0.165
	DQ	0.024	**0.000**	**0.003**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
RD	ex	0.002	0.014	0.020	0.073	0.133	0.000	0.008	0.014	0.071	0.135
	Ku	0.538	0.020	0.050	0.024	0.018	0.315	0.389	0.407	0.038	0.013
	DQ	0.730	**0.001**	**0.000**	**0.000**	**0.000**	-	0.182	0.068	**0.000**	**0.000**
		Plackett					SJC				
		0.001	0.005	0.010	0.050	0.100	0.001	0.005	0.010	0.050	0.100
Co	ex	0.020	0.036	0.052	0.119	0.177	0.014	0.020	0.034	0.123	0.185
	Ku	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
RW	ex	0.008	0.022	0.034	0.097	0.161	0.004	0.012	0.022	0.097	0.171
	Ku	**0.002**	**0.000**	**0.000**	**0.000**	**0.000**	0.112	0.062	0.021	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.020	**0.001**	**0.000**	**0.000**	**0.000**
AR	ex	0.010	0.018	0.020	0.095	0.159	0.002	0.008	0.018	0.091	0.157
	Ku	**0.000**	**0.002**	0.050	**0.000**	**0.000**	0.538	0.390	0.110	**0.000**	**0.000**
	DQ	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.575	0.052	**0.003**	**0.000**	**0.000**
RD	ex	0.002	0.014	0.020	0.071	0.129	0.000	0.010	0.016	0.075	0.133
	Ku	0.538	0.020	0.050	0.038	0.037	0.315	0.168	0.222	0.015	0.018
	DQ	0.776	**0.001**	**0.000**	**0.000**	**0.000**	-	0.096	0.017	**0.000**	**0.000**

Table 2.11: VaR evaluation for stock-pair jpm-ibm using the constant (Co), rolling window (RW), autoregressive (AR) and realized dependence (RD) estimators for the Gaussian, s. Gumbel, Plackett and SJC copulas. The exceedance rate and the p-values for the Ku and DQ tests are given. Bold font indicates rejection of the Ku or DQ test at the 1% significance level.

2.6 Conclusion

In this paper, we propose a new method for estimating the time-varying copula dependence parameter by using high frequency intraday data. Since the estimator is an ex-post estimate of the day's latent dependence, we term this estimator "realized dependence". As with findings of Goorbergh et al. (2005), Rodriguez (2007) and Patton (2006), we find evidence that the copula dependence parameter varies over time and a time-varying model is necessary to capture these changes. We showed that the realized dependence estimator is a superior time-varying copula dependence estimator compared to other commonly used methods for estimating time varying dependence.

By an in-sample application for three stocks pairs, we find that the realized dependence measure consistently and significantly improve the accuracy of quantile estimation of a portfolio in the central part of the distribution as compared to the use of constant, rolling window and autoregressive estimators. At extreme quantiles however, the improvement is insignificant due to data scarcity at the extreme tails of the intraday distribution. Application of methods of extreme value theory for estimation of the extreme tails, while improving the quantile estimation for days with outliers in the intraday distribution, was unable to give overall significant improvement.

We find that realized dependence exhibits the long-memory property akin to realized volatility and correlation, and forecast realized dependence using the HAR model of Corsi (2009). This forecasted dependence was then used to estimate the 1-day ahead VaR during the crisis period. While overall performance of the VaR is not good due to the high volatile out-of-sample period, the use of realized dependence is found to outperform the constant, rolling window or autoregressive estimators for estimating VaR. In agreement with previous research, we find that copulas that allow for skewness and tail dependence have better VaR estimation. Our findings suggest that more sophisticated GARCH models combined with the realized dependence in copulas that allow for skewness and tail dependence in the innovations will give the best out-of-sample VaR results, especially in turbulent times.

Appendix

2.7 Proofs

Notation: Definitions for $\gamma_{k,0}$, γ_k^m, u and u^m can be found in Section 3 page 9-10. Definitions for the rest of the parameters can be found in the beginning of Section 3. Definitions for $F_{k,i,\eta}^m(x)$, $F_{k,\eta}(x)$, $\hat{F}_{k,i,\eta}^m(x)$ and $\hat{F}_{k,\eta}(x)$ can be found in Section 3 page 9. Let $\tilde{F}_{k,i,\eta}^m(x)$ and $\tilde{F}_{k,\eta}(x)$ be the equivalent continuous distribution functions of $F_{k,i,\eta}^m(x)$ and $F_{k,\eta}(x)$ respectively:

$$\tilde{F}_{k,i,\eta}^m(x) = P(\eta_{k,i}^m \leq x),\ -\infty < x < \infty,$$

$$\tilde{F}_{k,\eta}(x) = P(\eta_k \leq x),\ -\infty < x < \infty.$$

Berkes et al. (2003) showed that the GARCH variance can be expressed as a recursive function:

$$w_{k,i}(u) = c_{k,0}(u) + \sum_{1 \leq p < \infty} c_{k,p}(u) y_{k,i-p}^2, \tag{2.13}$$

where $w_{k,i}(u)$ is shown to exist with probability one for $u \in U$. In the case of GARCH(1,1), U is defined as

$$U = \{\mathbf{u} : \hat{\beta} \leq \rho_{\mathbf{0}} \text{ and } \underline{\mathbf{u}} \leq \min(\hat{\omega}, \hat{\alpha}, \hat{\beta}) \leq \max(\hat{\omega}, \hat{\alpha}, \hat{\beta}) \leq \overline{\mathbf{u}}\},$$

where $\underline{u}$, $\overline{u}$ and ρ are such that $0 < \underline{u} < \overline{u}$, $0 < \rho_0 < 1$, $\underline{u} < \rho_0$. Furthermore, note that $w_{k,i}(\gamma_{k,0}) = \sigma_{k,i}^2$.

For the intradaily GARCH variance, the recursive expression is then given by

$$w_{k,i,j}(u^m) = c_{k,0}^m(u^m) + \sum_{1 \leq p < \infty} c_{k,p}^m(u^m) y_{k,i,j-p}^2. \tag{2.14}$$

The parameters $c_{k,0}$, $c_{k,p}$, $c_{k,0}^m$ and $c_{k,p}^m$ for GARCH(1,1) are given by

$$c_{k,0}(u) = \frac{\hat{\omega}_k}{1 - \hat{\beta}_k},\ \ c_{k,1}(u) = \hat{\alpha}_k, \tag{2.15}$$

and

$$c_{k,0}^m(u^m) = \frac{\hat{\omega}_{k,i}^m}{1 - \hat{\beta}_{k,i}^m},\ \ c_{k,1}^m(u^m) = \hat{\alpha}_{k,i}^m. \tag{2.16}$$

For $p > 1$,

$$c_{k,p}(u) = \hat{\beta}_k c_{k,p-1}(u),\ \ c_{k,p}^m(u^m) = \hat{\beta}_{k,i}^m c_{k,p-1}^m(u^m). \tag{2.17}$$

From the data, what can be realistically computed for daily GARCH variance is

$$\hat{w}_{k,i}(u) = c_{k,0}(u) + \sum_{1 \le p \le i-1} c_{k,p}(u) y_{k,i-p}^2, \;\; 2 \le i \le t, \tag{2.18}$$

and the intradaily GARCH variance can be given recursively by

$$\hat{w}_{k,i,j}(u^m) = c_0^m(u^m) + \sum_{1 \le p \le j-1} c_{k,p}^m(u^m) y_{k,i,j-p}^2, \;\; 1 < j \le 1/m, \, 1 \le i \le t. \tag{2.19}$$

Assume that the stationary conditions for GARCH processes are satisfied as in Bougerol and Picard (1992a, b). Furthermore, we make the following set of assumptions:

Assumption 1. *All parameters are strictly positive, i.e* $\omega > 0, \alpha > 0, \beta > 0, \omega^m > 0, \alpha^m > 0, \beta^m > 0$.

Assumption 2. $\{\eta_i, \, -\infty < i < \infty\}$ *and* $\{\eta_{i,j}, \, -\infty < i, j < \infty\}$ *are independent, identically distributed random variables with* $E[\eta_i] = E[\eta_{i,j}] = 0$ *and* $E[(\eta_i)^2] = E[(\eta_{i,j})^2] = 1$.

Assumption 3. γ *is in the interior of* $U = \{\mathbf{u}\}$ *and* γ^m *is in the interior of* $U^m = \{\mathbf{u^m}\}$.

Assumption 4. $E|\eta_0|^\rho < \infty$ *for some* $\rho > 0$.

Assumption 5. u^m *is a strongly consistent estimator for* γ^m *(i.e.* $u^m \to \gamma^m$ *a.s.) and* u *is a strongly consistent estimator for* γ *(i.e.* $u \to \gamma$ *a.s.).*

Assumption 6. $\log c(x_1, \ldots, x_k; \theta^m)$ *is a continuous function of* θ^m *for each* $(x_1, \ldots, x_k)^T \in [0,1]^k$.

Assumption 7. Θ^m *is a compact subset of* $\mathbb{R}^k$, *where* $\theta^m \in \Theta^m$.

Assumption 8. $E \sup_{\theta \in \Theta} |\log c(F_{1,\eta}(x), \ldots, F_{k,\eta}(x); \theta)| < \infty$.

Assumption 9. *For any* $\Delta_0 \in (0, 1/2)$ *and* $(\Delta_1 \in (1/2, 1))$, *there exist* $v_0 \in (0,1)$, $M_0 > 0$, $v_1 > 0$ *and* $M_1 > 0$ *such that*

$$\sup_{\theta \in \Theta} |\log c(x_1, \ldots, x_k; \theta)| \le M_0 \{\wedge_{i=1}^k x_i\}^{-v_o}$$

for $\wedge_{i=1}^k x_i \le \Delta_0$,

$$\sup_{\theta \in \Theta} |\log c(x_1, \ldots, x_k; \theta)| \le M_0 \{1 - \vee_{i=1}^k x_i\}^{-v_o}$$

for $\vee_{i=1}^{k} x_i \geq \Delta_1$, *and*

$$\sup_{\theta \in \Theta} |\log c(x_1, \ldots, x_k; \theta) - \log c(y_1, \ldots, y_k; \theta)| < M_1 \sum_{i=1}^{k} |x_i - y_i|^{v_1}$$

for $\Delta_0/2 < \wedge_{i=1}^{k} \leq \vee_{i=1}^{k} x_i < \Delta_1 + (1 - \Delta_1)/2$ *and* $\Delta_0/2 < \wedge_{i=1}^{k} y_i \leq \vee_{i=1}^{k} y_i < \Delta_1 + (1 - \Delta_1)/2$.

Assumptions 1-5 are conditions for the GARCH(1,1) process and assumptions 6-9 are conditions C1-C4 in Chan et al. (2009). Assumptions 6-8 are standard conditions for MLE. Assumption 9 is a boundary condition to control the speed of divergence of the log density (used in Genest (1995), Chen and Fan (2005) and Chan et al. (2009)) and is satisfied by all commonly used copula densities.

Theorem 2.7.1. *Following Lemma 2.7.3,*

$$\lim_{1/m \to \infty} \sup_{0 \leq x < \infty} |\hat{F}_{k,i,\eta}^{m}(x) - \tilde{F}_{k,\eta}(x)| = 0, \text{ a.s.}$$

Proof. The proof is similar to Theorem 2.1 in Berkes and Horvath (2003). By Lemma 2.7.3,

$$\frac{m}{1+m} \sum_{j=1}^{1/m} |\hat{F}_{k,i,\eta}^{m}(x) - \tilde{F}_{k,\eta}(x)| \xrightarrow{p} 0.$$

By continuity of $\tilde{F}_{k,\eta}(x)$ and Polya's lemma, the uniform convergence of theorem 2.7.1 is obtained. □

Theorem 2.7.2. *Suppose assumptions 4-8 hold, then by Lemma 2.7.3 and Theorem 2.7.1*

$$\hat{\theta}^{m} \xrightarrow{p} \theta_0, \; 1/m \to \infty.$$

Proof. From Lemma 2.7.3

$$\frac{m}{1+m} \sum_{j=1}^{1/m} |\hat{F}_{k,i,\eta}^{m}(x) - \tilde{F}_{k,\eta}(x)| \xrightarrow{p} 0,$$

and Theorem 2.7.1

$$\lim_{1/m \to \infty} \sup_{-\infty < x < \infty} |\hat{F}_{k,i,\eta}^{m}(x) - \tilde{F}_{k,\eta}(x)| = 0 \text{ a.s..}$$

Using assumptions 4-8, the rest of the proof is similar to that of theorem 2.2 in Chan et al. (2009).

□

Theorem 2.7.2 establishes the asymptotics of the realized dependence estimator at a certain value of m, the time interval between sampling. Some remarks are in order here: unlike the asymptotics established for realized volatility for the infill case, i.e. as $m \to 0$ for a fixed time domain T, the asymptotics of Theorem 2.7.2 belongs in the infill-increasing case, where one considers an increasing domain T with more and more dense observations (see e.g. Fazekas and Kukush (2000), Fazekas and Chuprunov (2001) and Fazekas (2003) which used infill-increasing approach to establish asymptotics of the least squares estimator, kernel density estimator and the empirical distribution for spatial case respectively).

Lemma 2.7.3. *By assumptions 1-4,*

$$\frac{m}{1+m}\sum_{j=1}^{1/m+1}|\hat{F}^m_{k,i,\eta}(\hat{\eta}_{k,i,j})-\tilde{F}_{k,\eta}(\eta_{k,i})| \xrightarrow{p} 0.$$

Proof. For convenience, we drop the k subscript here as it is not of relevance for the proof. First consider the case when $I(\eta_{i,j}\geq 0)$. By assumptions 1 and 4, we can use Lemma 4.3 in Berkes and Horvath (2003) such that

$$\sup_{u^*\in U^*}|w_{i,j}(u^m)-\hat{w}_{i,j}(u^m)|\leq C\rho_0^j\xi,\quad 0\leq j<\infty \tag{2.20}$$

for some $C>0$, and where $\xi=\sum_{0\leq j<\infty}\rho_0^j y_{-j}^2$. Then

$$\sup_{u^*\in U^*}\left|\frac{w_{i,j}(u^m)-\hat{w}_{i,j}(u^m)}{\sigma^2_{i,j}}\right|\leq\frac{C\rho_0^j\xi}{\sigma^2_{i,j}},\quad 0\leq j<\infty. \tag{2.21}$$

Then

$$\left[\frac{m}{1+m}\sum_{2\leq j\leq 1/m+1}I\{\eta_{i,j}\leq x\left(\frac{w_{i,j}(u^m)}{\sigma^2_{i,j}}-\frac{C\rho_0^j\xi}{\sigma^2_{i,j}}\right)^{1/2}\}|I(\eta_{i,j}\geq 0)\right]$$

$$\leq\frac{m}{1+m}\sum_{2\leq j\leq 1/m+1}I\{\eta_{i,j}\leq x(\hat{w}_{i,j}(u^m)/\sigma^2_{i,j})^{1/2}\}|I(\eta_{i,j}\geq 0)$$

$$\leq\left[\frac{m}{1+m}\sum_{2\leq j\leq 1/m+1}I\{\eta_{i,j}\leq x\left(\frac{w_{i,j}(u^m)}{\sigma^2_{i,j}}+\frac{C\rho_0^j\xi}{\sigma^2_{i,j}}\right)^{1/2}\}|I(\eta_{i,j}\geq 0)\right]. \tag{2.22}$$

The rest of the proof is similar to that of Lemma 5.1 in Berkes and Horvath (2003). Let there be a random variable t_0 such that if $j>t_0$,

$$\frac{C\rho_0^j\xi}{\sigma^2_{i,j}}\leq\frac{C\rho_0^j\xi}{\omega_i^m}\leq\epsilon. \tag{2.23}$$

Then

$$\frac{m}{1+m}\sum_{2\le j\le 1/m+1} I\{\eta_{i,j} \le x\left(\frac{w_{i,j}(u^m)}{\sigma_{i,j}^2}+\frac{C\rho_0^j\xi}{\sigma_{i,j}^2}\right)^{1/2}\}|I(\eta_{i,j}\ge 0)$$

$$\le \frac{t_0 m}{1+m}+\frac{m}{1+m}\sum_{2\le j\le 1/m+1} I\{\eta_{i,j} \le x\left(\frac{w_{i,j}(u^m)}{\sigma_{i,j}^2}+\epsilon\right)^{1/2}\}|I(\eta_{i,j}\ge 0). \quad (2.24)$$

From Lemma 4.1 in Berkes and Horvath (2003), for any u satisfying $|\gamma-u|\le\tau$ and u^m satisfying $|\gamma^m-u^m|\le\tau^m$,

$$\left[\frac{m}{1+m}\sum_{2\le j\le 1/m+1} I\{\eta_{i,j} \le x\left(\frac{w_{i,j}(u^m)}{\sigma_{i,j}^2}+\epsilon\right)^{1/2}\}|I(\eta_{i,j}\ge 0)\right]$$

$$\le\left[\frac{m}{1+m}\sum_{2\le j\le 1/m+1} I\{\eta_{i,j} \le x\left(\frac{w_{i,j}(u^{m(2)})}{\sigma_{i,j}^2}+\epsilon\right)^{1/2}\}|I(\eta_{i,j}\ge 0)\right] \quad (2.25)$$

for small τ such that $u^{(2)}=(\hat\omega+\tau,\hat\alpha+\tau,\hat\beta+\tau)$ and small τ^m such that $u^{m(2)}=(\hat\omega_i^m+\tau^m,\hat\alpha_i^m+\tau^m,\hat\beta_i^m+\tau^m)$.

By ergodicity and assumption 2,

$$\left[\frac{m}{1+m}\sum_{2\le j\le 1/m+1} I\{\eta_{i,j} \le x\left(\frac{w_{i,j}(u^{m(2)})}{\sigma_{i,j}^2}+\epsilon\right)^{1/2}\}|I(\eta_{i,j}\ge 0)\right]$$

$$\to EI\{\eta_{i,0}\le x\left(\frac{w_{i,0}(u^{m(2)})}{\sigma_{i,0}^2}+\epsilon\right)^{1/2}\}|I(\eta_{i,0}\ge 0)$$

$$= EI\{\eta_0\le x\left(\frac{w_0(u^{(2)})}{\sigma_0^2}+\epsilon\right)^{1/2}\}|I(\eta_0\ge 0). \quad (2.26)$$

By independence of η_0 and $w_0(u^{(2)})/\sigma_0^2$

$$EI\{\eta_0\le x\left(\frac{w_0(u^{(2)})}{\sigma_0^2}+\epsilon\right)^{1/2}\}|I(\eta_0\ge 0)$$

$$= E\tilde F_\eta\left(x\left(\frac{w_0(u^{(2)})}{\sigma_0^2}+\epsilon\right)^{1/2}\right). \quad (2.27)$$

By continuity of $w_0(u)$

$$\lim_{\tau\to 0}\frac{w_0(u^{(2)})}{\sigma_0^2}=1,\ \text{a.s.}. \quad (2.28)$$

Using dominated convergence theorem and (2.28)

$$\lim_{\tau\to 0} E\tilde{F}_\eta\left(x\left(\frac{w_0(u^{(2)})}{\sigma_0^2}+\epsilon\right)^{1/2}\right)=\tilde{F}_\eta(x(1+\epsilon)^{1/2}). \tag{2.29}$$

Using equations (2.22)-(2.29), there is a $\tau=\tau(\epsilon)$ such that

$$\lim\inf_{1/m\to\infty}\sup_{|u-\gamma|\le\tau}\frac{m}{1+m}\sum_{2\le j\le 1/m} I\{\eta_{i,j}\le x(\hat{w}_{i,j}(u^m)/\sigma_{i,j}^2)^{1/2}\}|I(\eta_{i,j}\ge 0)\le$$

$$(\tilde{F}_\eta(x)+2(\tilde{F}_\eta(x(1+\epsilon)^{1/2})-\tilde{F}_\eta(x))). \tag{2.30}$$

By similar steps, we can get

$$\lim\inf_{1/m\to\infty}\sup_{|u-\gamma|\le\tau}\frac{m}{1+m}\sum_{2\le j\le 1/m} I\{\eta_{i,j}\le x(\hat{w}_{i,j}(u^m)/\sigma_{i,j}^2)^{1/2}\}|I(\eta_{i,j}\ge 0)\ge$$

$$(\tilde{F}_\eta(x)-2(\tilde{F}_\eta(x)-\tilde{F}_\eta(x(1-\epsilon)^{1/2}))), \tag{2.31}$$

which gives

$$\lim\inf_{1/m\to\infty}\sup_{|u-\gamma|\le\tau}\left|\frac{m}{1+m}\sum_{2\le j\le 1/m} I\{\eta_{i,j}\le x(\hat{w}_{i,j}(u^m)/\sigma_{i,j}^2)^{1/2}\}\right||I(\eta_{i,j}\ge 0)$$

$$\le \tilde{F}_\eta(x)+2(\tilde{F}_\eta(x(1+\epsilon)^{1/2})-(\tilde{F}_\eta(x(1-\epsilon)^{1/2}))). \tag{2.32}$$

This gives

$$\frac{m}{1+m}\sum_{j=1}^{1/m}|\hat{F}_{i,\eta}^m(x)-\tilde{F}_\eta(x)|I(\eta_{i,j}\ge 0)\xrightarrow{p}0. \tag{2.33}$$

Applying the steps again for $I(\eta_{i,j}<0)$ would give

$$\frac{m}{1+m}\sum_{j=1}^{1/m}|\hat{F}_{i,\eta}^m(x)-\tilde{F}_\eta(x)|I(\eta_{i,j}<0)\xrightarrow{p}0. \tag{2.34}$$

By (2.33) and (2.34), we obtain the lemma. □

2.8 Realized Dependence versus Realized Correlation

In the case of the Gaussian copula, the dependence measure is simply linear correlation. We would like to relate the realized dependence in the Gaussian copula to the realized correlation (RC) estimator (Barndorff-Nielsen and Shephard (2004)) that is obtained by dividing the realized covariance by realized volatilities.

Let the observed intraday log asset price for time in unit interval $[0, 1]$ representing one day be $p_{t_j,m} = p^*_{t_j,m} + u_{t_j}$ where p^* is the unobserved equilibrium logarithmic price and u denotes market microstructure noise. The time index t_j represents the j^{th} observation in the M intraday observations with sampling interval equal to m and $M = \frac{1}{m} + 1$. Intraday returns are then $y_{t_j,m} = p_{t_j,1} - p_{t_j-1,m}$. Assume p^* evolves as a function of a stochastic volatility process σ_t: $p^*_{t_j} = \int_0^{t_j} \sigma_s dW_s$ where W_t is standard Brownian motion. The integrated variance over 1 day is $V_t = \int_0^1 \sigma_s^2 ds$ and for two assets a and b, the covariance is given by $C_t = \int_0^1 \Sigma_{a,b} ds$. The realized covariance estimator $RCov_t^{(m)} = \sum_{j=1}^{M} y_{a,j,m} y_{b,j,m}$ is found to be a consistent estimator of latent C_t (Barndorff-Nielsen and Shephard (2004)) and realized variance $RV_t^{(m)} = \sum_{j=1}^{M} y_{j,m}^2$ is a consistent estimator of latent V_t. Realized correlation is then given by

$$RC_t^{(m)} = \frac{RCov_t^{(m)}}{\sqrt{RV_{t,a}^{(m)}}\sqrt{RV_{t,b}^{(m)}}}.$$

The realized covariance estimator differs from the empirical intraday covariance given by

$$\frac{1}{M}\sum_{j=1}^{M} y_{a,j,i} y'_{b,j,i} - \left(\frac{1}{M}\sum_{j=1}^{M} y_{a,j,i}\right)\left(\frac{1}{M}\sum_{j=1}^{M} y_{b,j,i}\right) = \frac{1}{M}\sum_{j=1}^{M} y_{a,j,i} y_{b,j,i} - \frac{1}{M^2} y_{a,i} y'_{b,i} \quad (2.35)$$

which is approximately M times smaller than the realized covariance and converges in probability to a matrix of zeros as $M \to \infty$ (Barndorff-Nielsen & Shephard, 2006). The empirical correlation $EC_t^{(m)}$ estimated using intraday returns of day i is given by

$$EC_t^{(m)} = \frac{\frac{1}{M}\sum y_{aj} y_{bj} - \frac{1}{M^2} y_a \cdot y_b}{\sqrt{\frac{1}{M}\sum y_{aj}^2 - \frac{1}{M^2} y_a^2}\sqrt{\frac{1}{M}\sum y_{bj}^2 - \frac{1}{M^2} y_b^2}}$$

$$= \frac{\frac{1}{M}\sum y_{aj}y_{bj} - \frac{1}{M^2}y_a \cdot y_b}{\frac{1}{M}\sqrt{\sum y_{aj}^2 - \frac{1}{M}y_a^2}\sqrt{\sum y_{bj}^2 - \frac{1}{M}y_b^2}}$$

$$= \frac{\sum y_{aj}y_{bj} - \frac{1}{M}y_a \cdot y_b}{\sqrt{\sum y_{aj}^2 - \frac{1}{M}y_a^2}\sqrt{\sum y_{bj}^2 - \frac{1}{M}y_b^2}}$$

$$p\lim_{M\to\infty} EC_t^{(m)} = \frac{\sum y_{aj}y_{bj}}{\sqrt{\sum y_{aj}^2}\sqrt{\sum y_{bj}^2}} = RC^{(m)}.$$

Hence the empirical correlation converges in probability to the realized correlation. Figure 2.12 compares the realized correlation and the Gaussian realized dependence of Citigroup and JP Morgan in the period 2006-2008, using sampling frequency of 1, 5 and 10 minutes. The realized covariance matrix that was used to compute realized correlation has been subsampled to reduce the effects of market microstructure noise.

The top graph at 1-minute sampling frequency shows that both the realized dependence and realized correlation are extremely close. It means that correlation is not destroyed by the univariate degarch process. The middle and bottom graphs show that the two estimators diverge as sampling frequency increases to 5 and 10 mins.

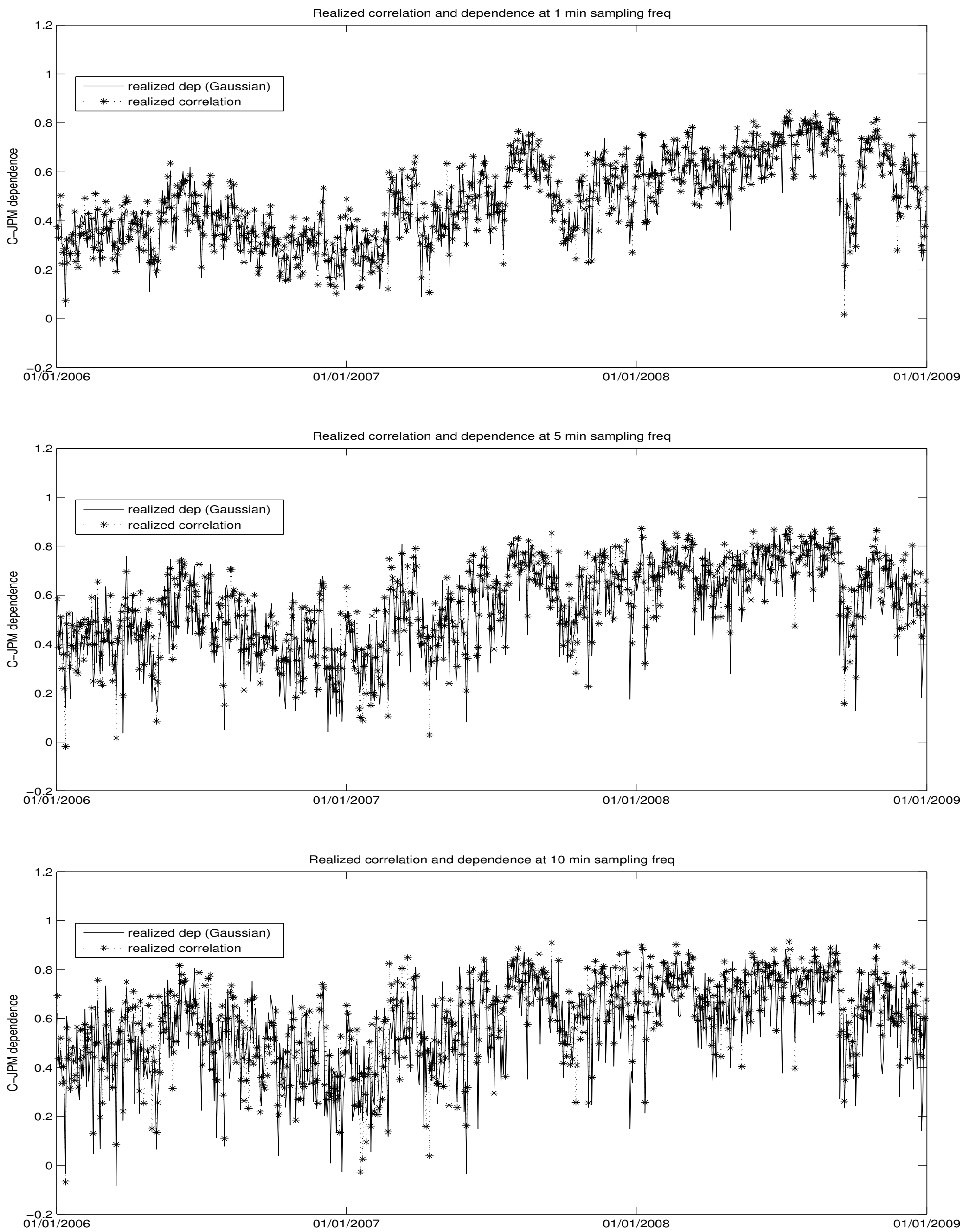

Figure 2.12: Plots of the realized correlation against the empirical correlation (which is the same as the realized dependence in the Gaussian copula) using 1 min (top), 5 mins (middle) and 10 mins (bottom) sampling frequencies.

2.9 Brief description of copula models used in this paper

This section briefly details the copulas that were used in this paper. They are presented in the order they were estimated rather than by copula family. The first five copulas (Gaussian, Clayton, Gumbel, Plackett and Frank) are single parameter copulas and the last 2 have two parameters for estimation (Student-t and SJC). The Gaussian and Student-t copulas belong to the elliptical family of copulas, while the Clayton, Gumbel, Frank and SJC copulas belong to the Archimedean family. The Gumbel copula is also an extreme-value copula due to its fat-tail distribution. The survival Gumbel copula was used in this paper (sometimes termed as rotated gumbel), and is simply the mirror image of the Gumbel copula. This terminology can be used on other copulas as well.

2.9.1 Gaussian copula

The cdf of a bivariate Gaussian copula is given by

$$C(u_1, u_2; \theta) = \Phi_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2)), \tag{2.36}$$

where $\Phi_\rho(\cdot, \cdot)$ is the standard bivariate normal distribution with correlation ρ and Φ^{-1} is the inverse of the univariate normal cdf.

The copula density function is given by

$$c(u_1, u_2; R) = |R|^{-1/2} \exp(-\frac{1}{2}\eta'(R^{-1} - \mathbf{I})\eta), \tag{2.37}$$

where R is the correlation matrix with off-diagonal element ρ and $\eta = (\Phi^{-1}(u)\Phi^{-1}(v))'$. The Gaussian copula belongs to the elliptical family of copulas and has no upper or lower tail dependence.

2.9.2 Clayton copula

Clayton copulas belong to the family of Archimedean copulas which have the form

$$C(u_1, u_2) = \phi^{-1}(\phi(u_1) + \phi(u_2)), \tag{2.38}$$

where ϕ is termed generator and is a function that is convex and decreasing. For Clayton copulas, $\phi(x) = \frac{x^{-\theta}-1}{\theta}$. The Clayton copula cdf is given by

$$C(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}. \quad (2.39)$$

The Clayton copula has positive lower tail dependence and no upper tail dependence.

2.9.3 Gumbel copula

The Gumbel copula belongs to the Archimedean family (see eq 2.38) and has generator $\phi(x) = (-\ln x)^{\theta}$. The Gumbel copula is also an extreme value copula, because the gumbel distribution is an extreme-value theory distribution. The copula cdf is given by

$$C(u_1, u_2; \theta) = \exp\left\{-[(-\ln u_1)^{\theta} + (-\ln u_2)^{\theta}]^{1/\theta}\right\}, \quad (2.40)$$

and the survival Gumbel is the mirror image with cdf

$$C(u_1, u_2; \theta) = u_1 + u_2 - 1 + \exp\left\{-[(-\ln(1-u_1))^{\theta} + (-\ln(1-u_2))^{\theta}]^{1/\theta}\right\} \quad (2.41)$$

where for both equations, $\theta \geq 1$. The survival Gumbel has lower tail dependence.

2.9.4 Plackett copula

Plackett copulas form a family of absolutely continuous copulas.
For $\theta > 0$, $\theta \neq 1$

$$C(u, v; \theta) = \frac{[1 + (\theta-1)(u+v)] - \sqrt{[1 + (\theta-1)(u+v)]^2 - 4uv\theta(\theta-1)}}{2(\theta-1)}. \quad (2.42)$$

$\theta = 1$ implies independence. The Plackett copula does not possess tail dependence.

2.9.5 Frank copula

Frank copula also belong to the Archimedean family (see eq 2.38), with generator given by $\phi(x) = -\ln\left(\frac{e^{-\theta x}-1}{e^{-\theta}-1}\right)$. The Frank copula is symmetric and has a cdf is given by

$$C(u_1, u_2; \theta) = -\frac{1}{\theta}\ln\left[1 + \frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1}\right], \quad (2.43)$$

where $\theta \in \mathbb{R}/\{0\}$. $\theta \to 0$ indicates independence of the variables. The frank copula does not possess tail dependence.

2.9.6 Student-t copula

The cdf of a Student-t copula is give by

$$C(u_1, u_2; \theta) = T(T_{v1}^{-1}(u_1), T_{v2}^{-1}(u_2)), \tag{2.44}$$

where T_{v1} and T_{v2} are univariate Student-t cdf with degrees of freedom v_1 and v_2 respectively. The copula parameter θ is a vector of 2 elements– R, the correlation matrix with off-diagonal element ρ, and v_c, the degrees of freedom of the copula. The copula density function is given by

$$c(u_1, u_2, R, v_c) = |R|^{-1/2} \frac{\Gamma(\frac{v_c+2}{2})\Gamma(\frac{v_c}{2})}{\Gamma(\frac{v_c+1}{2})^2} \frac{(1 + \frac{\eta' R^{-1} \eta}{v_c})^{-\frac{v_c+2}{2}}}{\prod_{i=1}^{2}(1 + \frac{\eta_i^2}{v_c})^{-\frac{v_c+1}{2}}}. \tag{2.45}$$

The Student-t copula also belongs to the elliptical family of copulas and possesses upper and lower tail dependence which are equal due to the symmetry of the copula density. In particular,

$$\lambda_U = \lambda_L = 2 - 2t_{v_c+1}(\sqrt{v+1}\sqrt{1-\rho}/\sqrt{1+\rho}). \tag{2.46}$$

2.9.7 SJC

The symmetrized Joe-Clayton (SJC) copula suggested by Patton (2006) is a modified Joe-Clayton copula (also known as the BB7 copula) The original Joe-Clayton copula (Joe (1997)) has two tail dependence parameters for the upper and lower tail (θ^U and θ^L) respectively and the copula is given by

$$C_{JC}(u, v|\theta^U, \theta^L) = 1 - (1 - \{[1-(1-u)^\kappa]^{-\gamma} + [1-(1-v)^\kappa]^{-\gamma} - 1\}^{-1/\gamma})^{1/\kappa}, \tag{2.47}$$

where $\kappa = 1/\log_2(2 - \theta^U)$ and $\gamma = -1/\log_2(\theta^L)$.
The SJC copula below corrects for the slight asymmetry in the functional form of the Joe-Clayton copula, allowing for symmetry when upper and lower tail dependence parameters are equal,

$$C_{SJC}(u, v|\theta^U, \theta^L) = 0.5\cdot(C_{JC}(u, v|\theta^U, \theta^L) + C_{JC}(1-u, 1-v|\theta^L, \theta^U) + u + v - 1). \tag{2.48}$$

2.10 Simulation Results including GARCH parameters

		aggregation by k times								
	true	1	2	5	10	15	20	30	60	390
$\hat{\omega}_{1,1}$	1	3.451	8.867	30.29	62.05	88.21	112	168	274	360
		(0.057)	(0.181)	(1.051)	(4.906)	(17.12)	(30.79)	(50.71)	(110.5)	(456.2)
$\hat{\omega}_{2,1}$	1	3.459	8.869	30.41	61.66	89.26	113.1	170.6	281.1	283.7
		(0.071)	(0.185)	(0.904)	(5.539)	(16.47)	(31.69)	(46.31)	(119.0)	(184.7)
$\hat{\alpha}_{1,1}$	0.2	0.096	0.100	0.060	0.024	0.012	0.007	0.004	0.005	0.006
		(0.002)	(0.002)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.006)	(0.012)
$\hat{\alpha}_{2,1}$	0.2	0.096	0.099	0.060	0.023	0.012	0.007	0.004	0.005	0.004
		(0.002)	(0.002)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.008)
$\hat{\beta}_{1,1}$	0.6	0.386	0.234	0.031	0.045	0.106	0.153	0.157	0.309	0.845
		(0.009)	(0.015)	(0.032)	(0.073)	(0.172)	(0.231)	(0.253)	(0.278)	(0.196)
$\hat{\beta}_{2,1}$	0.6	0.385	0.234	0.027	0.051	0.095	0.144	0.141	0.292	0.877
		(0.011)	(0.014)	(0.027)	(0.084)	(0.165)	(0.238)	(0.231)	(0.300)	(0.084)
$\hat{\theta}$	0.7	0.689	0.690	0.693	0.692	0.691	0.690	0.689	0.687	0.686
		(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)	(0.010)

Table 2.12: Estimation results for Gaussian copula dependence under reestimation at 1 minute and aggregation of 2, 5, 10, 15, 20, 30, 60 and 390 minutes. Numbers in brackets are standard deviations

		aggregation by k times								
	true	1	2	5	10	15	20	30	60	360
$\hat{\omega}_{1,1}$	1	3.449	8.833	30.14	62.33	88.08	104.98	168.1	300.7	261.9
		(0.061)	(0.191)	(1.144)	(5.061)	(17.94)	(34.83)	(47.06)	(99.34)	(223.9)
$\hat{\omega}_{2,1}$	1	3.450	8.881	30.22	62.22	86.71	102.6	167.5	281.3	365.0
		(0.070)	(0.180)	(1.017)	(5.055)	(19.41)	(37.60)	(50.28)	(112.2)	(441.6)
$\hat{\alpha}_{1,1}$	0.2	0.096	0.100	0.060	0.024	0.013	0.008	0.005	0.005	0.003
		(0.002)	(0.002)	(0.006)	(0.004)	(0.005)	(0.005)	(0.004)	(0.005)	(0.006)
$\hat{\alpha}_{2,1}$	0.2	0.096	0.100	0.061	0.024	0.012	0.006	0.004	0.004	0.005
		(0.001)	(0.002)	(0.003)	(0.004)	(0.005)	(0.004)	(0.004)	(0.006)	(0.009)
$\hat{\beta}_{1,1}$	0.6	0.387	0.237	0.036	0.041	0.107	0.205	0.155	0.244	0.888
		(0.010)	(0.015)	(0.033)	(0.075)	(0.180)	(0.261)	(0.235)	(0.250)	(0.097)
$\hat{\beta}_{2,1}$	0.6	0.387	0.233	0.032	0.043	0.122	0.225	0.158	0.293	0.843
		(0.012)	(0.014)	(0.031)	(0.076)	(0.195)	(0.282)	(0.250)	(0.280)	(0.193)
$\hat{\theta}$	2.5	2.314	2.165	1.892	1.715	1.635	1.583	1.535	1.475	1.385
		(0.004)	(0.006)	(0.009)	(0.010)	(0.013)	(0.015)	(0.018)	(0.023)	(0.053)

Table 2.13: Estimation results for clayton copula dependence under reestimation at 1 minute and aggregation of 2, 5, 10, 15, 20, 30, 60 and 390 minutes. Numbers in brackets are standard deviations

		aggregation by k times								
	true	1	2	5	10	15	20	30	60	360
$\hat{\omega}_{1,1}$	1	3.435	8.849	30.30	60.80	87.81	107.0	175.4	279.2	260.0
		(0.060)	(0.191)	(1.079)	(6.329)	(16.97)	(27.37)	(42.03)	(104.3)	(143.3)
$\hat{\omega}_{2,1}$	1	3.433	8.845	30.34	61.16	87.78	111.3	173.6	297.3	266.0
		(0.072)	(0.188)	(0.960)	(6.785)	(18.66)	(24.83)	(44.11)	(111.9)	(250.5)
$\hat{\alpha}_{1,1}$	0.2	0.096	0.100	0.059	0.024	0.014	0.013	0.009	0.004	0.004
		(0.017)	(0.002)	(0.003)	(0.004)	(0.005)	(0.005)	(0.005)	(0.004)	(0.006)
$\hat{\alpha}_{2,1}$	0.2	0.096	0.099	0.060	0.024	0.015	0.017	0.013	0.006	0.004
		(0.002)	(0.002)	(0.003)	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.010)
$\hat{\beta}_{1,1}$	0.6	0.389	0.235	0.031	0.063	0.108	0.185	0.115	0.298	0.888
		(0.010)	(0.015)	(0.032)	(0.095)	(0.170)	(0.205)	(0.210)	(0.261)	(0.063)
$\hat{\beta}_{2,1}$	0.6	0.389	0.236	0.029	0.059	0.108	0.148	0.120	0.250	0.885
		(0.012)	(0.015)	(0.029)	(0.102)	(0.188)	(0.187)	(0.221)	(0.280)	(0.110)
$\hat{\theta}$	3.0	2.935	2.935	2.856	2.776	2.736	2.710	2.683	2.643	2.597
		(0.003)	(0.004)	(0.007)	(0.008)	(0.010)	(0.014)	(0.014)	(0.020)	(0.048)

Table 2.14: Estimation results for s. Gumbel copula dependence under reestimation at 1 minute and aggregation of 2, 5, 10, 15, 20, 30, 60 and 390 minutes. Numbers in brackets are standard deviations

	aggregation by k times						
	1	2	5	10	15	20	30
$\hat{\theta}_{gaussian}$	0.685	0.684	0.677	0.667	0.659	0.647	0.635
	(0.030)	(0.042)	(0.064)	(0.093)	(0.114)	(0.144)	(0.178)
$\hat{\theta}_{clayton}$	2.330	2.216	2.018	1.991	2.056	2.188	2.362
	(0.232)	(0.301)	(0.442)	(0.630)	(0.785)	(1.064)	(1.484)
$\hat{\theta}_{s.gumbel}$	2.939	2.962	2.926	2.942	2.970	3.092	3.267
	(0.166)	(0.235)	(0.363)	(0.517)	(0.641)	(0.884)	(1.189)

Table 2.15: Estimation results for the Gaussian, Clayton and s. Gumbel copula dependence under finite sampling. Here, sample sizes of 390 (rather than 10,000) are simulated 1000 times and the mean and variance of the dependence at different aggregation levels are recorded.

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Chapter 3

Obtaining and Predicting the Bounds of Realized Correlations

3.1 Introduction

Given the plethora of bias correction methods for the estimation of realized covariance and correlation that only work well under certain conditions, this paper proposes a different approach to the problem. We argue that the inherent data problems render point identification of realized covariance and correlation unreliable, especially when the level of asynchronicity and microstructure noise is high. Under such circumstances, the data only allows for partial identification (Manski (1995)) of the realized covariance and correlation, whereas point identification of these measures requires prior assumptions about the data. Given the data limitations, partial identification analysis identifies the bounds that the mean of the distribution of interest lies in. Although conservative, the estimation of bounds is a more robust approach when estimating realized correlations, because both econometricians and practitioners should be aware of the worst and best case scenarios when assumptions about the data conditions that are needed to yield point estimations cannot or should not be made.

The availability of high frequency data in recent years has allowed financial econometrics to shift away from parametric conditional variance and covariance estimation based on daily or weekly data towards nonparametric ex-post measures termed as realized measures. Barndorff-Nielsen and Shephard (2002) and Mykland and Zhang (2006) have shown that as sampling intervals get smaller, the realized variance or covariance is a consistent estimator of integrated variation or covariation and has an asymptotic variance that is mixture normally distributed. In this paper we consider the realized correlation, which is analogous to the realized covariance. It is almost trivial to state that the estimation of the correlation structure of asset returns is important for many areas in finance, such as in risk management and portfolio optimization.

The estimation of realized covariance (RC) encounters several problems. First of all, trading of different assets rarely occur simultaneously, i.e. trading is asynchronous. This causes the realized covariance to tend to zero as sampling frequency increases, a phenomenon termed as "Epps effect" (Epps (1979)). The most common approach to estimating realized covariance is to construct approximately synchronised pairs using either previous-tick interpolation or linear interpolation. However asynchronicity renders these interpolated estimators to be biased (see e.g. Dacorogna et al. (2001)) and Zhang (2011) derives the analytical bias of the previous-tick RC.

To correct for the bias due to asynchronicity, Barndorff-Nielsen et al. (2011) propose the incorporation of lead and lag autocovariance terms based on the idea that returns sampled at regular calendar time will correlate with preceding and succeeding returns on other assets even if the underlying correlation is purely contemporaneous. Another approach to dealing with asynchronicity is the Hayashi and Yoshida estimator (Hayashi and Yoshida (2005)) which accumulates cross-products of all fully and partially overlapping event-time returns to obtain unbiased covariance estimators.

Unfortunately asynchronicity is not the sole problem that realized covariance and correlation encounter. A further problem is that high frequency data is characteristically plagued by what is termed as "market microstructure noise", a distortion of the true latent efficient (or "frictionless") price which according to classical market microstructure theory (O'Hara (1995)) should evolve as a martingale. The existence of market microstructure noise is explained by market frictions that distort efficient prices (Roll (1984)). These frictions could be induced by price discreteness, the existence of bid-ask spreads or the lack of liquidity. Market microstructure noise renders the true price process unobservable. Aït-Sahalia et al. (2005) and Zhang et al. (2005) showed that the realized variance estimator (the realized covariance in the univariate case) is biased in the presence of market microstructure noise, and the bias becomes larger as the sampling frequency increases. This led to the literature about obtaining the optimal sampling frequency to reduce the effects of noise in a bias-variance tradeoff (see e.g. Zhou (1996), Bandi and Russell (2008)).

To deal with the combination of both problems, methods such as subsampling (Zhang et al. (2005)), pre-averaging (Jacod et al. (2009)) and the two- and multi-scales estimators (Zhang (2011)) have been proposed to restore consistency of the estimators. Voev and Lunde (2007) proposed a bias correction method for the Hayashi and Yoshida estimator in the presence of dependent microstructure noise while Nolte and Voev (2009) propose a least squares approach to obtain the unbiased integrated volatility or co-volatility. Griffin and Oomen (2011) however showed that under a high enough noise level and low degree of correlation, the previous-tick RC that is not bias-corrected may be more efficient in terms of log mean-squared error than the Hayashi and Yoshida estimator and the lead-lag estimator of Barndorff-Nielsen et al. (2011).

Whichever the bias-correction method of the realized covariance and correlation, they require making some assumptions about the data problems and these assumptions may or may not be fulfilled in practice. We on the other hand take a conservative approach by seeking identification bounds in the spirit of Manski (1995) on the previous-tick covariance estimator and the corresponding realized correlation. The issue of asynchronicity is regarded as a missing data problem because in the case of the previous-tick estimator, the problem is that the true latent price of one or more assets is not observed at the sampling time. We use the partial identification approach of Horowitz and Manski (2006) for incomplete data to obtain bounds of the identification region. The presence of microstructure noise can be regarded as a data corruption or contamination problem because the issue is that the sampling process is a distortion of the true latent price process. We use the approach by Horowitz and Manski (1995) for the treatment of contaminated and corrupted data to estimate the identification region.

The estimated bounds provide the worst and best case scenarios that can be found using information that the data provides without having to make assumptions about the inherent data problems. They require no structure to be imposed on the sample space and are attempts to guard against the worst outcomes that the data problems could possibly produce by using ex-post knowledge of the data. Altogether this serves as a more robust approach to inference, which is especially valuable when the realized covariance and correlation are used for estimating other useful measures such as sharpe ratios or the Value-at-Risk.

The paper is organised as follows: Section 2 gives the mathematical description of the previous tick realized covariance and correlation as well as the subsampled estimator of Zhang et al. (2005); Section 3 describes the idea of partial identification, how we apply such identification analysis to estimate bounds of the realized covariance and correlation when the problems of asynchronicity and microstructure noise are present in the data, some practical issues involved in the bounds estimation, and the forecasting of the bounds; Section 4 gives the results of a simulation exercise to study the efficacy of these bounds and their sensitivity to the tuning parameters; Section 5 gives an empirical application using two stocks, first describing the dataset and then the results of the bounds estimation and forecasting efficacy. Finally Section 6 concludes.

3.2 Realized Covariance and Correlation

Consider the price processes of two assets $\{X_t\}$ and $\{Y_t\}$. To estimate their integrated covariation $\langle X, Y\rangle_T$, the standard assumption is that both processes follow an Itô stochastic process with standard Brownian motion B^X and B^Y. Further assume that the processes have a drift coefficient μ_t^X and μ_t^Y with instantaneous variance $\sigma_t^{2,X}$ and $\sigma_t^{2,Y}$. Under such assumptions, the integrated covariation is given by

$$\langle X, Y\rangle_T = \int_0^T \sigma_t^X \sigma_t^Y d\langle B^X, B^Y\rangle_t. \tag{3.1}$$

Barndorff-Nielsen and Shephard (2002) and Mykland and Zhang (2006) show that using the limit theorem for stochastic processes, an estimator for the integrated covariation is the realized covariance,

$$RC_T = \sum_{i:\tau_i \in [0,T]} (X_{\tau_i} - X_{\tau_{i-1}})(Y_{\tau_i} - Y_{\tau_{i-1}}), \tag{3.2}$$

which is a consistent estimator as sampling intervals get smaller and has an asymptotic mixture normal distribution. The realized correlation $RCorr$ is obtained by dividing realized covariance by the realized volatilities (square root of realized variances) of the individual assets:

$$RCorr_T = \frac{RC_T}{\sqrt{\sum_{i:\tau_i\in[0,T]}(X_{\tau_i} - X_{\tau_{i-1}})^2}\sqrt{\sum_{i:\tau_i\in[0,T]}(Y_{\tau_i} - Y_{\tau_{i-1}})^2}}. \tag{3.3}$$

RC is most commonly estimated using previous-tick interpolation (or last-tick interpolation) which was found to be less biased than linear interpolation (Dacorogna et al 2001). As the name implies, the previous-tick RC is simply RC estimated using prices on or immediately preceding a regularly spaced sampling time grid.

Consider a fixed time period $[0, T]$, usually a single trading day, to have a regularly sampled time grid denoted by $\mathcal{T}_N \in [0, T]$, $\mathcal{T}_N = \{\tau_0, \tau_1, \ldots, \tau_{M_N}\}$, where M_N is the sampling frequency, N is the total number of observations or transactions of both X and Y, and $\Delta t = \tau_i - \tau_{i-1}, \forall i$, is the regular sampling interval that is constant, for example 1, 5, or 10 minutes. Hence a 5 minutes RC refers to RC with $\Delta t = 300s$. We can view M_N as a filter on N and it is a function of Δt.

Let the irregular transaction times of X and Y be denoted by grids $\mathcal{G}_A$ and $\mathcal{H}_B$ respectively, with $\mathcal{G}_A = \{g_0, g_1, \ldots, g_A\}$ and $\mathcal{H}_B = \{h_0, h_1, \ldots, h_B\}$. Hence $N = A + B$. Also, $0 \le g_0 \le g_1 \le \ldots \le g_A \le T$ and $0 \le h_0 \le h_1 \le \ldots \le h_B \le T$. The previous ticks are then defined to be $a_i = \max\{g \in \mathcal{G}_A : g \le \tau_i\}$ and $b_i = \max\{h \in \mathcal{H}_B : h \le \tau_i\}$. The previous tick RC is thus given by

$$[X, Y]_T = \sum_{i=1}^{M_N}(X_{a_i} - X_{a_{i-1}})(Y_{b_i} - Y_{b_{i-1}}). \tag{3.4}$$

Zhang (2011) gives the analytical characterisation of the previous tick RC estimator in the presence of asynchronous trading and microstructure noise.

The bias of the previous tick RC due to asynchronicity alone (assuming no microstructure noise) is $-\int_0^T \langle X, Y\rangle'_u dF_N(u) + O_p\left(\frac{1}{N}\right)$ where $F_N(t) = \sum_{i:max(a_i,b_i)\le t} |a_i - b_i|$ (Zhang (2011)), under the assumption that there is at least one pair of observations (g, h) within each $[\tau_i, \tau_{i+1}]$. While this condition is usually satisfied for highly traded assets, it does not hold for less liquid assets or in times of sudden liquidity shortages ('liquidity black holes') and inconsistency of the bias-corrected previous tick RC estimator results. Analytical solution of the bias can be obtained by assuming that the transaction time arrival rates follow independent intensity processes.

The correction for the effect of microstructure noise involves empirical and theoretical modelling subtleties, because both microstructure noise and the efficient price are latent variables and a direct measurement of microstructure noise is not possible. Zhang (2011) derives the bias of the previous-tick RC due to microstructure noise by assuming additive noise processes, with the analytical solution being available when the noise processes of X and Y (ϵ^X and ϵ^Y processes) are assumed to be independent white noise. However Phillips and Yu (2006) argue that the complexity of microstructure noise cannot be adequately captured by a simple white noise specification. They show that the properties of microstructure noise evolve over time, and may exhibit local non-stationarity and perfect correlation with the efficient price.

Zhang et al. (2005) proposed subsampling the estimator which reduces the bias caused by asynchronicity and microstructure noise. The subsampled estimator $[X, Y]_T^{(avg)}$ is constructed by averaging the estimators $[X, Y]_T^{(k)}$ across K grids, where the original regular spaced grid $\mathcal{T}$ is partitioned into K non-overlapping subgrids $\mathcal{T}^{(k)}$, $k = 1, \ldots, K$. Consider the original grid $\mathcal{T} = \{\tau_0, \tau_1, \ldots, \tau_{M_N}\}$, then the k^{th} subgrid is given by $\mathcal{T}^{(k)} = \{\tau_{k-1}, \tau_{k-1+K}, \ldots, \tau_{k-1+M_N K}\}$. This creates K realized covariance estimates $[X, Y]_T^{(k)}, k = 1, \ldots, K$ and the subsampled RC ($ssRC$) estimator is obtained by averaging:

$$ssRC_T = [X, Y]_T^{(avg)} = \frac{1}{K} \sum_{k=1}^{K} [X, Y]_T^{(k)} \tag{3.5}$$

and the corresponding subsampled $RCorr$ ($ssRCorr$) estimator is

$$ssRCorr_T = [X, Y]_T^{(avg)} = \frac{1}{K} \sum_{k=1}^{K} \frac{[X, Y]_T^{(k)}}{\sqrt{[X, X]_T^{(k)}} \sqrt{[Y, Y]_T^{(k)}}}. \tag{3.6}$$

While there are many proposed methods (e.g. pre-averaging, realized kernels, etc.) for correcting the bias of the RC and $RCorr$ estimators, we retain here only the subsample estimator as an indicative bias-reduced estimator. Our purpose is not to obtain bias correction but to obtain identification bounds on the RC and $RCorr$. Proper identification bounds should then include not only the RC and $RCorr$ estimators but also the subsampled and other bias-corrected RC and $RCorr$ estimators.

3.3 Identification bounds of realized covariance/correlations

Given that data problems can potentially complicate point identification of realized correlations and the use of bias-correction methods involves making assumptions about latent variables, we adopt another perspective to the problem by studying the identification of bounds of realized correlations under the prevalent data conditions of asynchronicity and microstructure noise. Partial identification using bounds does not require assumptions to be made about the way asynchronicity is present and about the latent time-series structure of microstructure noise.

Partial identification analysis departs from traditional robust statistics, which also deals with inference in the presence of data errors or problems. Robust estimation considers the stability and sensitivity of the estimators when the underlying data distribution deviates from the distribution used in the assumed model, and the objective of robust estimation is to guard against worst outcomes *ex ante*. Partial identification analysis considers these data problems in an *ex-post* setting by giving the range of the possible values of the parameter of interest given what is known about the empirical distribution. The narrower the bounds, the more information they provide.

We deem identification analysis to be well-suited for application to RC and $RCorr$ given that they are *ex-post* or *realized* measures. Use of traditional robust estimation techniques would be unsuitable and overly conservative. However, the bounds derived in Horowitz and Manski (2006) and Horowitz and Manski (1995) are made for the case of a static framework, where time-series effects are not considered. Time dependencies would complicate the analysis of the data problems since the errors in observations would also affect the next period's observations (see for example Chen and Liu (1993) and Tsay et al. (2000)). Fortunately for our case, RC and $RCorr$ are derived under the framework of assuming that the returns follow Itô stochastic processes, with i.i.d. increments (Brownian motion increments are assumed). It then becomes reasonable to use identification analysis in a such a framework.

Hence we treat asynchronicity as a missing data problem and microstructure noise as a corrupted or contaminated data problem to obtain bounds on RC and $RCorr$ via partial identification analysis. To investigate the effect of different data problems on the bounds, we derive the bounds under three cases: (i) asynchronous data without microstructure noise, (ii) synchronous data with microstructure noise, and (iii) asynchronous data with microstructure noise. While only the last case provides realistic bounds to RC, the first two cases allow us to observe the marginal effect of each data problem alone.

3.3.1 Bounds due to asynchronicity

The standard approach to characterise the bias due to asynchronicity is to assume a Poisson arrival rate for an observation (i.e. a trade or quote) that is independent of the price process (see for example Assumption 2 in Griffin and Oomen (2011) and Section 6 of Zhang (2011)). Renault and Werker (2011) however showed that durations and price processes are not independent but exhibit instantaneous causality. This suggests that the estimated bias due to asynchronicity would be inaccurate.

We hence adopt a more robust approach to the problem by seeking to identify the bounds of the estimator using the methodology of Horowitz and Manski (2006), which provides sharp informative bounds on parameters when the sampling process produces incomplete data due to missing observations. The advantage of this approach is that these bounds do not rely on assumptions about the process of arrival rates of the quotes/trades, i.e. the degree of asynchronicity. To identify the bounds under asynchronicity, if there are no observations within the immediate vicinity of the sampling grid, we consider the observation as latent or "missing".

Let the log returns be given as $r^X_{\tau_i} = (X_{\tau_i} - X_{\tau_{i-1}})$ and $r^Y_{\tau_i} = (Y_{\tau_i} - Y_{\tau_{i-1}})$, where $\tau_i,\ i = 1, \ldots, M_N$ are points on a regular time grid within a day. When there are no observations within the vicinity of τ_i, we consider the data to be "missing" at τ_i. We term this vicinity as "tolerance" and it is a tuning parameter that determines the width of the bounds (See Section 3.3.3 for further discussion). We define four states of "missingness" and let integer valued random variable Z_i represent the state of "missingness" of the data. $Z_i = 1$ implies all components (both $r^X_{\tau_i}$ and $r^Y_{\tau_i}$) are observed. $Z_i = 2$ if $r^X_{\tau_i}$ is missing (i.e. X_{τ_i} or $X_{\tau_{i-1}}$ or both are missing) while $r^Y_{\tau_i}$ is observed. $Z_i = 3$ if $r^Y_{\tau_i}$ is missing (i.e. Y_{τ_i} or $Y_{\tau_{i-1}}$ or both are missing) but $r^X_{\tau_i}$ is observed. $Z_i = 4$ is when both $r^X_{\tau_i}$ and $r^Y_{\tau_i}$ are missing.

Let the price pair be given by random vector $V_i = (r^X_{\tau_i}, r^Y_{\tau_i})$. Assume V_i is a discrete random variable with support $\{v_j : j = 1, \ldots, J\}$. Define $\pi_z = P(Z_i = z)$ and $p_{zj} = P(V_i = v_j | Z_i = z)$. The sampling process identifies π_z and p_{1j} for all $j = 1, \ldots, J$. For $z = 2, 3, 4$, p_{zj} is unidentified due to missingness of data, but it obeys certain restrictions. First of all, $p_{zj} \geq 0,\ z = 2, 3, 4$ and $\sum_{j=1}^{J} p_{zj} = 1,\ z = 2, 3, 4.$

For $z = 2$, r^Y is observed. Let the support of r_Y be given by Λ_2, and suppose there are K_2 distinct points in Λ_2. Let q_{2k_2} denote the probability of point $k_2 \in \Lambda_2$ conditional on $Z = 2$. Then $\sum_{j:v_j \in k_2} p_{2j} = q_{2k_2}$, $k_2 = 1, \ldots, K_2$, where q_{2k_2} is the marginal probability of $r^Y_{\tau_i}$ in state 2.

The same applies to $z = 3$, where this time r^X is observed and r^Y is missing. Let Λ_3 refer to the support of r^X. q_{3k_3} denotes the probability of point $k_3 \in \Lambda_3$ conditional on $Z = 3$. Then $\sum_{j:v_j \in k_3} p_{3j} = q_{3k_3}$, $k_3 = 1, \ldots, K_3$, where q_{3k_3} is the marginal probability of $r^X_{\tau_i}$ in state 3. In reality, when π_2 and π_3 are small, consistent estimation of q_{2k_2} and q_{3k_3} is difficult. However as argued by Horowitz and Manski (2006), the effects of such imprecision are limited by the multiplication with the small π_2 and π_3.

The probability mass function of the returns vector is then given by $P(V_i = v_j) = \sum_{z=1}^{4} \pi_z p_{zj}$. The upper and lower bounds can be obtained by maximising and minimising the realized covariance or correlation with respect to the unknown probabilities (p_{2j}, p_{3j} and p_{4j}) and computed using V_i over its support v_j, $j = 1, \ldots, J$ subjected to the above given constraints.

Estimation of bounds

The relevant optimisation problem for estimating bounds of realized covariance is given as[1]

$$\text{maximize (minimize)}_{p_{zj}:z\geq 2; j=1,\ldots,J} : \sum_{\tau_i, i\in[0,M_N]} \sum_{z=1}^{4} \sum_{j=1}^{J} \pi_z p_{zj} r^X_{\tau_i} r^Y_{\tau_i} \tag{3.7}$$

subject to:

$$p_{zj} \geq 0,\ j = 1, \ldots, J,$$

$$\sum_{j\in[0,J]} p_{zj} = 1,\ z = 2, 3, 4,$$

$$\sum_{j:v_j \in k_z} p_{zj} = q_{zk_z},\ z = 2, 3;\ k_z = 1, \ldots, K_z.$$

For realized correlation, the optimisation problem is:

$$\text{maximize (minimize)}_{p_{zj}:z\geq 2; j=1,\ldots,J} :$$

[1]Here maximising the objective function gives the upper bound and minimising the objective function gives the lower bound. This is not to be confused with a max-min optimisation problem.

$$\frac{\sum_{\tau_i, i\in[0,M_N]}\sum_{z=1}^{4}\sum_{j=1}^{J}\pi_z p_{zj} r_{\tau_i}^X r_{\tau_i}^Y}{\sqrt{\sum_{\tau_i, i\in[0,M_N]}\sum_{z=1}^{4}\sum_{j=1}^{J}\hat{\pi}_z p_{zj} r_{\tau_i}^X r_{\tau_i}^X} \times \sqrt{\sum_{\tau_i, i\in[0,M_N]}\sum_{z=1}^{4}\sum_{j=1}^{J}\hat{\pi}_z p_{zj} r_{\tau_i}^Y r_{\tau_i}^Y}} \tag{3.8}$$

subject to:

$$p_{zj} \geq 0,\ j = 1, \ldots, J,$$

$$\sum_{j\in[1,J]} p_{zj} = 1,\ z = 2, 3, 4,$$

$$\sum_{j:v_j\in k_z} p_{zj} = q_{zk_z},\ z = 2, 3;\ k_z = 1, \ldots, K_z.$$

The data sample allows identification of π_z (for $z = 1, \ldots, 4$), p_{1j}, q_{2k_2} and q_{3k_3}. The empirical estimators are

$$\hat{\pi}_z = \frac{1}{M_N}\sum_{i=1}^{M_N} I(Z_i = z),\ z = 1, \ldots, 4, \tag{3.9}$$

$$\hat{p}_{1j} = \frac{1}{\hat{\pi}_1 M_N}\sum_{i=1}^{M_N} I(V_i = v_j, Z_i = 1),\ j = 1, \ldots, J, \tag{3.10}$$

and for $z = 2, 3$,

$$\hat{q}_{zk_z} = \frac{1}{\hat{\pi}_z M_N}\sum_{i=1}^{M_N} I(V_i \in k_z, Z_i = z),\ z = 2, 3. \tag{3.11}$$

When $\hat{\pi}_1 = 0$, the right hand side of (3.10) is defined to be zero, and when $\hat{\pi}_z = 0$ for $z = 2, 3$, the right hand side of (3.11) is defined to be zero. The upper and lower bounds of RC and $RCorr$ are then obtained by replacing π_z, p_{1j}, q_{2k_2}, and q_{3k_3} with $\hat{\pi}_z, \hat{p}_{1j}, \hat{q}_{2k_2}$, and $\hat{q}_{3k_3}$ and solving (3.7) and (3.8) respectively.

Analytical solutions of (3.7) and (3.8) are not possible[2] hence we use numerical techniques to obtain the bounds. To achieve global optimization, we use the genetic algorithm in MATLAB to solve the nonlinear programming problem, and then refine the optimisation locally using constraint optimisation. Horowitz and Manski (2006) found that the genetic algorithm performs the optimization faster, but alternative global optimisation techniques such as simulated annealing may be used. Optimization at each time point (per day) is approximately 10 minutes for realized covariance using a 2.66 GHz Intel computer, and 15 minutes per time point for realized correlation. Asymptotically valid confidence intervals for the bounds may be obtained using bootstrap (see Horowitz and Manski (2006) for details), but due to the huge computational time required, we do not do this here.

[2]Horowitz and Manski (2000) derived the analytical solution for the optimization problem in the case of binary outcomes. Our problem here is however more complex.

3.3.2 Bounds due to microstructure noise

To examine the effects of microstructure noise alone, let us assume that the degree of asynchronicity is negligible. Let the product of the price pair be given by random variable $W_i = r^X_{\tau_i} \times r^Y_{\tau_i}$. Let ϵ_{W_i} be a random variable representing microstructure noise that has an unknown distribution. The inferential problem is then that the true latent martingale price returns W^*_i is not observed but W_i which is contaminated by ϵ_{W_i} with probability p. Let F_W indicate the cdf of W_i. Then the sampling process only allows for the identification of F_W

$$F_W = (1-p)F_{W^*} + pF_{\epsilon_W}, \tag{3.12}$$

where F_{W^*} is the distribution of the observations that belong in the efficient price distribution and F_{ϵ_W} is the distribution of noise.

If the occurrence of microstructure noise ϵ_W can be assumed to be random and independent of the sampling process, then inference on F_W can be assumed to be equivalent to inference on F_{W^*}. This is termed "contaminated sampling". When this independence assumption is not made, it is termed the "corrupted sampling" model and is the more general model (i.e. bounds under a corrupted sampling model would be wider than bounds under a contaminated sampling model, see Horowitz and Manski (1995)). Both of these error models are commonly assumed in robust statistics. For example, the contaminated sampling scheme is assumed in obtaining the influence function in robust statistics, while the corrupted sampling scheme is assumed in high-breakdown estimation (see a brief review by Čížek and Härdle (2006)). We consider here both cases of contaminated and corrupted sampling schemes.

Horowitz and Manski (1995) showed that for parameters which respect stochastic dominance, sharp bounds can be obtained under corrupted and contamination sampling. RC respects stochastic dominance with respect to W_i, for $W_i \in Y$ where Y is the extended real line. This means that $RC(F^1_W) \geq RC(F^2_W)$ when $F^1_W \leq F^2_W$. Hence we estimate these sharp bounds to obtain the bounds of RC and $RCorr$ under the presence of microstructure noise.

Estimation of upper bound of p

While we need not assume a value for p, the analysis requires that an upper bound of p can be estimated or known a priori. Denote this upper bound as $\lambda < 1$

such that $\lambda \geq \max(\lambda^X, \lambda^Y)$, for $p_X \leq \lambda^X < 1$ and $p_Y \leq \lambda^Y < 1$, where p_X and p_Y are contamination probabilities of X and Y respectively, and λ^X and λ^Y are their corresponding upper bounds. To estimate the level of microstructure noise, we use Eq (14) of Bandi and Russell (2008) who derive the bias in realized variances (RV) due to microstructure noise:

$$E(\widehat{RV} - RV) = M_N E(\epsilon_X^2) \tag{3.13}$$

where $E(\epsilon_X^2)$ can be estimated consistently using Theorem 2 (Bandi and Russell (2008)):

$$\frac{1}{M_N}\sum_{j=1}^{M_N} r_j^{2,X} \xrightarrow{p} E(\epsilon_X^2) \text{ as } M_N \to \infty. \tag{3.14}$$

This result is also obtained in Zhang et al. (2005) and the intuition is that at very high sampling frequencies, the realized variance is a consistent estimator of the variance of microstructure noise. While this bias term is obtained under the assumption that microstructure noise follows a white noise process that is independent of the price process, we use this bias term to obtain a rough estimate of the percentage of microstructure noise in the data. Bandi and Russell (2008) propose using an average of bias estimation over n samples (rolling windows can be used to allow time-variation in bias), but we will use the maximum estimated bias over the period to estimate the upper bound of microstructure noise λ^X and λ^Y separately. λ is then a value such that $\lambda \geq \max(\lambda^X, \lambda^Y)$.

Estimation of bounds

Under Proposition 4 in Horowitz and Manski (1995), the bounds of RC under contamination, $RC(W_{cont})$, is given by

$$RC(W_{cont}) \in \left[\sum_{i=1}^{M_N}\{W_{i,L}\}, \sum_{i=1}^{M_N}\{W_{i,U}\}\right] \tag{3.15}$$

where $W_{i,L}$ and $W_{i,U}$ are random variables drawn from distributions

$$L[-\infty, t] \equiv \begin{cases} F_W[-\infty, t]/(1-\lambda), & \text{if } t < F_W^{-1}(1-\lambda) \\ 1, & \text{if } t \geq F_W^{-1}(1-\lambda) \end{cases} \tag{3.16}$$

and

$$U[-\infty, t] \equiv \begin{cases} 0, & \text{if } t < F_W^{-1}(\lambda) \\ (F_W[-\infty, t]-\lambda)/(1-\lambda), & \text{if } t \geq F_W^{-1}(\lambda) \end{cases} \tag{3.17}$$

respectively.

For corrupted sampling, let $\delta_{-\infty}$ and δ_{∞} be the limiting probability measures on W at $-\infty$ and ∞ respectively, then the bounds are

$$RC(W_{corr}) \in \left[\sum_{i=1}^{M_N}\{W_{i,L_2}\}, \sum_{i=1}^{M_N}\{W_{i,U_2}\}\right] \tag{3.18}$$

where $L_2 = (1-\lambda)L + \lambda\delta_{-\infty}$ and $U_2 = (1-\lambda)U + \lambda\delta_{\infty}$.

Consistent estimation of the bounds under contaminated sampling can be obtained by $[0,1] \cap [\{\hat{F}_W - \hat{\lambda}\}/(1-\hat{\lambda}), \hat{F}_W/(1-\hat{\lambda})]$ and the bounds under corrupted sampling by $[0,1] \cap [\hat{F}_W - \hat{\lambda}, \hat{F}_W + \hat{\lambda}]$, where $\hat{F}_W$ is estimated using the empirical cdf of W. The corresponding bounds for $RCorr$ under the corrupted ($RCorr(W_{corr})$) and contaminated ($RCorr(W_{cont})$) schemes are then obtained by dividing the bounds obtained in RC by the realized volatilities of X and Y.

3.3.3 Overall bounds and estimation issues

We assume that the effects of asynchronicity and microstructure noise are independent, separate data problems, and obtain overall bounds on RC and $RCorr$ by summing the individual bounds. Since the bounds obtained due to microstructure noise alone are symmetric, we add half of the bound due to noise to the upper bound due to asynchronicity, and subtract half of the bound due to noise from the lower bound due to asynchronicity to obtain the overall bounds on RC and $RCorr$, i.e. $U_{\text{overall}} = U_{\text{asyncronicity}} + 1/2(U_{\text{noise}} - L_{\text{noise}})$ and $L_{\text{overall}} = L_{\text{asyncronicity}} - 1/2(U_{\text{noise}} - L_{\text{noise}})$, where U_p and L_p refer to upper and lower bounds due to p effect.

In reality, these two effects may not be independent and may be instantaneously causal for each other. Then the actual overall bounds may be narrower than what we estimate here, given that we are measuring these effects ex-post – some of the effects of asynchronicity would already be included when estimating the bounds due to microstructure noise and vice versa. By adding the bounds of the two effects, we would likely obtain a more conservative estimate of the overall bounds. From our empirical application (see Section 3.5), we find that this method gives rather tight bounds, hence we do not delve further into the issue of instantaneous causality of the two effects.

Estimation of the bounds encounters several issues: First, for computational tractability in estimating bounds due to asynchronicity, the support of the returns is assumed to be finite, but in theory the support is $[-\infty, \infty]$. What this entails is that if there is a large proportion of missingness that in reality would lie outside the support of the observed returns, the width of the bounds would be underestimated.

Second, the estimation of bounds is subjected to error due to discretization of the support. In our applications, we used 10-by-10 bins for approximating the two-dimensional support to obtain bounds due to asynchronicity. Finer discretization would reduce the degree of error but would require greater computational power.

Third, the overall width of the bounds is subject to two tuning parameters. For bounds due to asynchronicity, the "tolerance" or vicinity of the grid for assigning "missingness" (i.e. if no observations are observed within x seconds before the sampling grid, the observations are considered missing) is the tuning parameter. The smaller the tolerance chosen, the wider the bounds. At high tolerance levels and low asynchronicity, the bounds narrow towards the previous-tick RC. We consider a base case of 15 seconds for the 5 minutes RC and $RCorr$, or 5% of the sampling interval. In Section 3.4.1, we use simulations to observe the sensitivity of the bounds to the tolerance level. For bounds due to microstructure noise, the tuning parameter is the upper bound of microstructure noise λ. The higher the estimated or assumed λ, the wider the bounds. Simulations in Section 3.4.2 provide a brief evaluation on the effect of this tuning parameter on the bounds.

3.3.4 Forecasting correlation bounds using HAR

Besides providing an identification region for an estimator, bounds could be useful in providing prediction of the region where the parameter could potentially lie in. The forecasting of RC is a challenging issue because it is usually made in the context of forecasting the entire variance-covariance matrix, and in this multivariate dimension, the forecast model engages in problems with parameter proliferation and ensuring the positive definiteness of the variance-covariance matrix. Hence we consider the forecasting of the bounds of realized correlations rather than realized covariance. Inherent in conducting such a forecast is the assumption that the degree of asynchronicity and microstructure noise remains constant.

Corsi and Audrino (2007) and Vortelinos (2010) forecast realized correlations using the Heterogeneous Autoregressive Model (HAR) (Corsi (2003) and Corsi (2009)). HAR models are AR-type stochastic additive cascades of three different time horizon components- daily, weekly and monthly. The explanation given as to why HAR models are effective in modelling long memory processes is via the "Heterogeneous Market Hypothesis" of Müller et al. (1993), which posits that long memory processes are superimpositions of several processes that operate on different time scales. Correlations tend to empirically exhibit a high degree of persistence, hence HAR is a simple and suitable forecasting model. The HAR model for forecasting the 1-step ahead realized correlations is given by

$$RCorr_{t+1}^{(d)} = c + \beta^{(d)} RCorr_t^{(d)} + \beta^{(w)} RCorr_t^{(w)} + \beta^{(m)} RCorr_t^{(m)} + \varepsilon_{t+1} \qquad (3.19)$$

where $RCorr_t^{(d)}$, $RCorr_t^{(w)}$ and $RCorr_t^{(m)}$ are the daily, weekly and monthly realized correlations respectively.

We extend the application of the HAR model to forecast the bounds of realized correlations[3]. We obtain the weekly $RCorr^{(w)}$ and monthly $RCorr_t^{(m)}$ by taking averages of the last 5 and 20 days $RCorr$ respectively. Estimation of (3.19) is made via ordinary least squares. To achieve multi-period forecasting, we use iterative forecasts to obtain the h-step ahead forecast.

3.4 Simulations

Before we attempt to estimate the bounds using empirical data, it is worth doing a simulation study to check the efficacy of these bounds and their sensitivity to the tuning parameters (described in Section 3.3.3).

We simulate a bivariate Brownian process with zero drift and constant covariance using the Euler discretization scheme to obtain prices at 1 second intervals. For simplicity, we do not consider leverage effects. The price process is then

$$dP = \Sigma_t^{1/2} dB$$

where $\Sigma_t^{1/2}$ is the Cholesky factorisation of the covariance matrix Σ_t and B is now a (2×1) vector of independent standard Brownian motions. The integrated covariance is then

$$IC = \int_0^1 \Sigma_t dt.$$

[3]Note that we forecast the bounds directly, and not obtain the bounds of the forecast $RCorr$.

To obtain non-synchronous price pairs, we simulate durations using independent Poisson processes with constant intensities ϕ_X and ϕ_Y. An additive noise process in the form of Gaussian white noise $u \sim N\left(0, \begin{pmatrix} \omega_{XX} & \omega_{XY} \\ \omega_{XY} & \omega_{YY} \end{pmatrix}\right)$ is added to the price processes.

We use simulation values $IC = \begin{pmatrix} 1.0e^{-4} & 2.5e^{-4} \\ 2.5e^{-4} & 5.0e^{-4} \end{pmatrix}$ which are close to what is observed empirically. Multivariate normal contemporaneously correlated noise is added with $\omega_{XX} = 1.0e^{-5}$, $\omega_{YY} = 2.5e^{-5}$ and $\rho = -0.1$. This gives a noise-signal ratio of approximately 10%. We use this simple simulation setup to evaluate the estimated bounds with respect to the RC estimators and the true RC.

3.4.1 Bounds' sensitivity to asynchronicity

There does not exist at present a standard measure for the degree of asynchronicity, hence we use simple simulated trials to observe the effect of the tuning parameter on the estimated bounds due to asynchronicity. Figure 3.1 plots the width of 5 minutes RC bounds due to asynchronicity against tolerance using simulated data with asynchronicity alone for 3 cases: (i) $\phi_X = 1,\ \phi_Y = 1/5$ (durations of X and Y are 1 and 5 seconds respectively), (ii) $\phi_X = 1,\ \phi_Y = 1/10$ (durations of X and Y are 1 and 10 seconds respectively) and (iii) $\phi_X = 1/5,\ \phi_Y = 1/10$ (durations of X and Y are 5 and 10 seconds respectively). Cases (i) and (iii) have the same approximate degree of asynchronicity (differences in durations are 4 and 5 seconds respectively), while case (ii) has the greatest degree of asynchronicity (with difference in durations of 9 seconds).

Figure 3.1 shows that as expected, for all three cases, the width of bounds decreases as tolerance increases (i.e. as the vicinity of what we consider 'missing' becomes narrower, the estimated bounds become wider). Second, bounds of case (ii) are wider than bounds of cases (i) and (iii), and width of case (iii) is larger than case (i), which confirms that the greater the degree of asynchronicity, the larger is the width of the bounds. Third, although case (i) and case (iii) have approximately the same degree of asynchronicity, the width of case (iii) is much larger than case (i), which suggests that the lower the intensities of the arrival rates (i.e. larger durations), the larger the width of the bounds. This is intuitive since the lower the intensity, the greater the probability that no observations will be observed within the vicinity or "tolerance" of the grid.

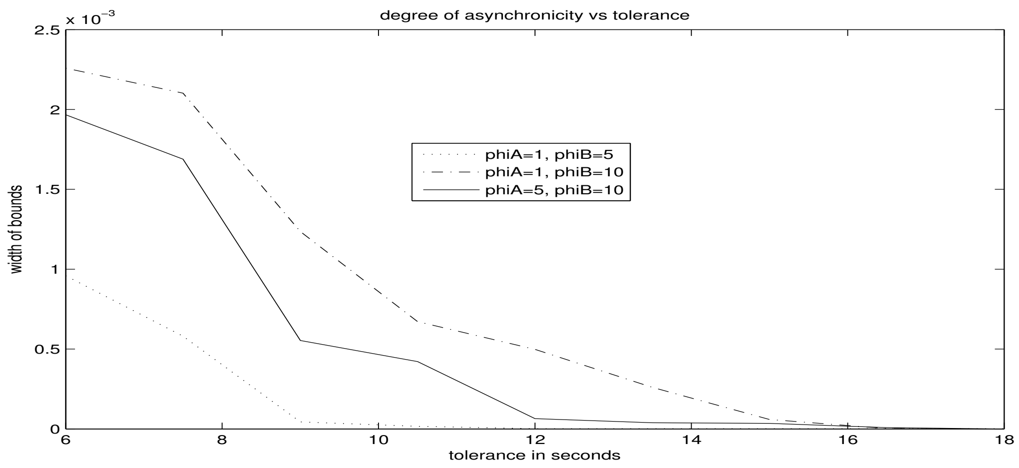


Figure 3.1: Plots of the width of 5 min RC bounds due to asynchronicity against tolerance (vicinity of grid) using simulated data without noise. Three cases are plotted here: (i) $\phi_X = 1$, $\phi_Y = 1/5$ (1 and 5 sec durations) (dotted line), (ii) $\phi_X = 1$, $\phi_Y = 1/10$ (1 and 10 sec durations) (dash-dot line) and (iii) $\phi_X = 1/5$, $\phi_Y = 1/10$ (5 and 10 sec durations) (solid line).

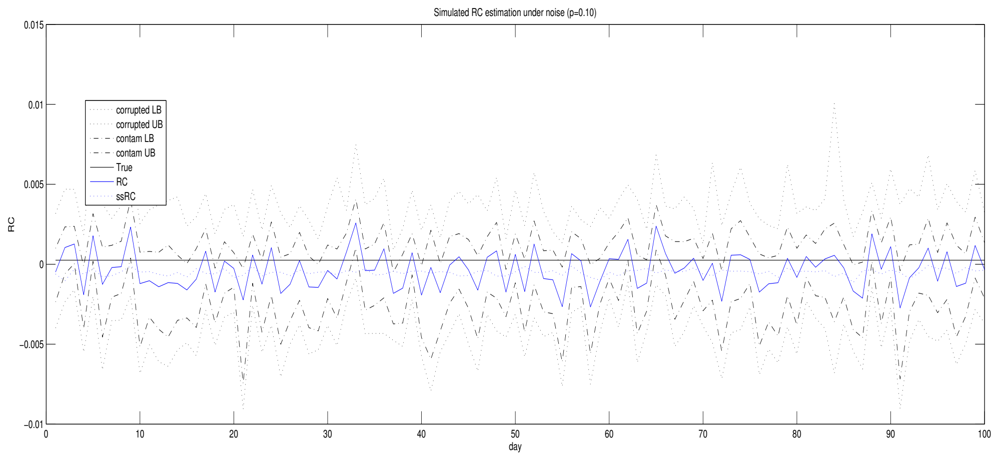
3.4.2 Bounds' sensitivity to level of microstructure noise

We now turn to the sensitivity of bounds due to the effect of microstructure noise alone. Figure 3.2 shows the 5 minutes RC and its bounds with synchronous observations for 100 simulated days using constant noise-signal ratios of 10% ($p = 0.1$, Fig 3.2a) and 20% ($p = 0.2$, Fig 3.2b). The horizontal solid line close to the x-axis is the true RC, the solid fluctuating line is the previous-tick RC and the dotted line that exhibits lesser fluctuations than previous-tick RC is the $ssRC$. The $ssRC$ tends to lie consistently under the true RC, hence the $ssRC$ still incurs a bias but it is much less biased than the previous-tick RC.

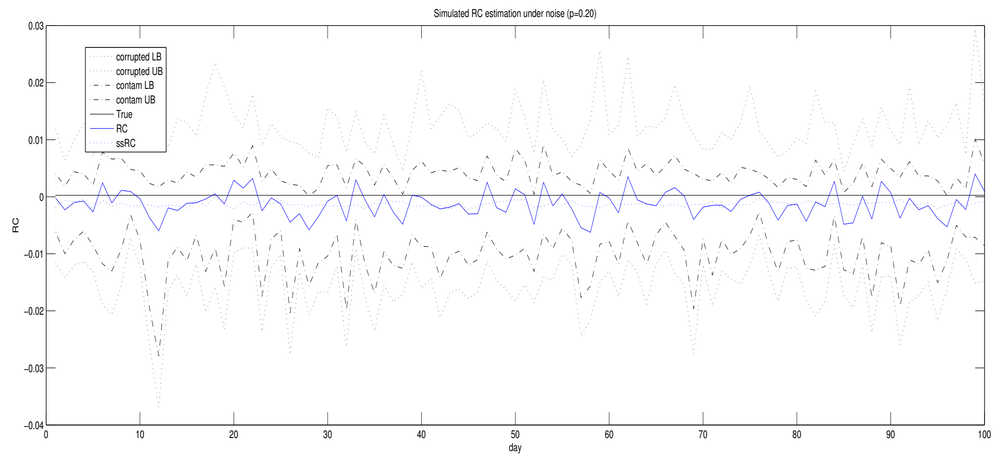
The inner and outer bounds are the bounds under contaminated and corrupted sampling respectively (bounds under the corrupted sampling scheme are wider than those under the contaminated sampling scheme, see Section 3.3.2). The bounds provide total coverage of the previous-tick RC and $ssRC$. For the true RC (solid constant line), the bounds under the corrupted sampling scheme provide 100% coverage, but under contaminated sampling, it is less that 100%.

This implies that the bounds under contaminated sampling are not as reliable in the inclusion of the true RC. This also applies when adding a 20% microstructure noise (Figure 3.2b).

Finally, as expected, the bounds widen as the level of additive microstructure noise increases. This is observed in Figure 3.2 where the bounds in (b) are wider than those in (a).



(a) 10% noise-signal ratio (p=0.1)



(b) 20% noise-signal ratio (p=0.2)

Figure 3.2: True RC (solid constant line), previous tick RC (fluctuating solid line), $ssRC$ (fluctuating dotted line) and estimated bounds under microstructure noise alone at 5 minutes sampling intervals. Additive microstructure noise is at (a) 10% and (b) 20% noise-signal levels. Estimated bounds assume the corrupted sampling scheme (dotted lines) and contaminated sampling scheme (dash-dot lines).

3.4.3 Overall bounds and sampling frequency

Figure 3.3 shows simulations under 10% microstructure noise ($p = 0.1$) and asynchronicity of $\phi_X = 1/5$ and $\phi_Y = 1/10$. (X and Y have 1 and 10 minute durations respectively). The estimations of the 5 minutes RC is given in Figure 3.3a while the 1 minute RC is given in Figure 3.3b. The tolerance is set at 15 seconds (i.e. if no observations are within 15 seconds of the grid, the observation is considered "missing").

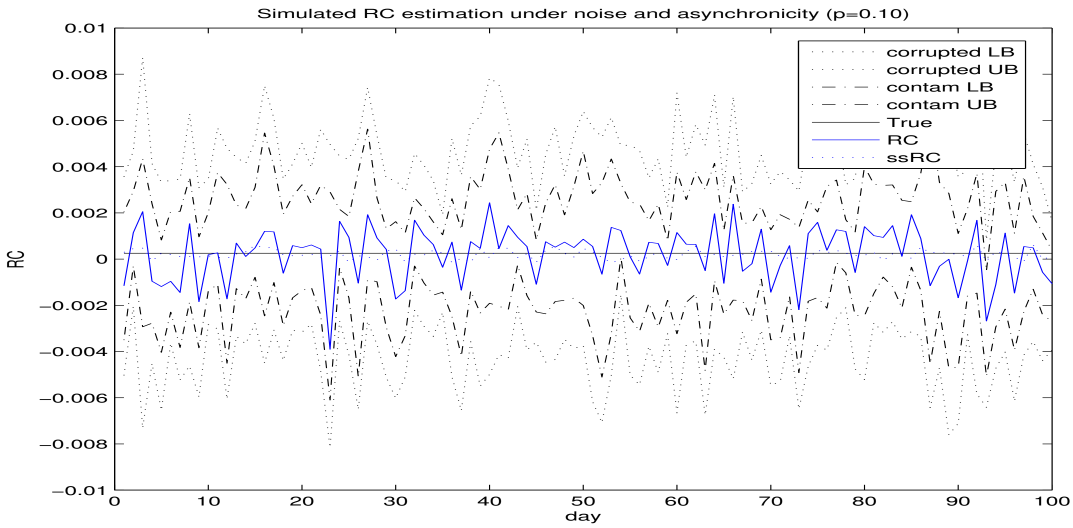
For both cases, the bounds provide complete coverage under both types of sampling (except for one day in Figure 3.3a under contaminated sampling). The bounds under 1 minute sampling (Figure 3.3b) are wider than those under 5 minutes sampling (Figure 3.3a). This is expected as under 1 minute sampling, there are more points on the sampling grid and hence a greater probability for "missingness" to occur.

3.5 Data

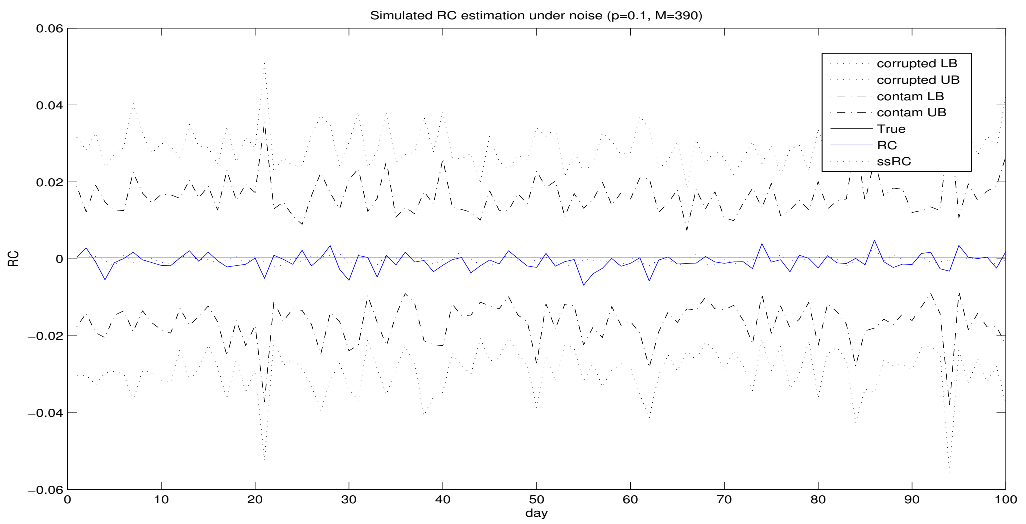
For our empirical study, we use the NYSE TAQ quote data of Citigroup (c) and JP Morgan Chase (jpm) for the year 2007 (2 Jan 2007 to 31 Dec 2007), a total of 251 days. The mid-quotes during regular trading hours between 9.30 and 16.00 are extracted. For multiple quotes within the same second, the average mid-quote is used.

Figure 3.4 shows the signature plot of realized covariance, which is a plot of the average realized covariance using different sampling frequencies. The x-axis gives the sampling frequency in minutes (i.e. Δt, the interval between each sampling). The smallest sampling interval used is one second. The y-axis gives the average estimated previous-tick RC over the sample period.

As is characteristic of realized volatility signature plots, there is a large upward slope at very high sampling frequencies due to the effect of microstructure noise (at very high sampling frequencies, only microstructure noise is measured, see for example Figure 1 in Bandi and Russell (2008)). This is in line with the asymptotic theory for microstructure noise by Bandi and Russell (2008). However for realized covariance, an additional Epps effect is observed between 2-5 minutes sampling interval, where there is a bias towards zero as the sampling frequency increases. From Figure 3.4, the RC stabilises at the 5 minutes sampling



(a) 5 minute RC and corresponding bounds



(b) 1 minute RC and corresponding bounds

Figure 3.3: Estimated RC and overall bounds for 100 simulated days using additive microstructure noise of 10% ($p = 0.1$), $\phi_X = 1/5$, $\phi_Y = 1/10$ at (a) 5 minutes sampling intervals and (b) 1 minute sampling intervals.

interval, hence we will use the 5 minutes RC and $RCorr$.

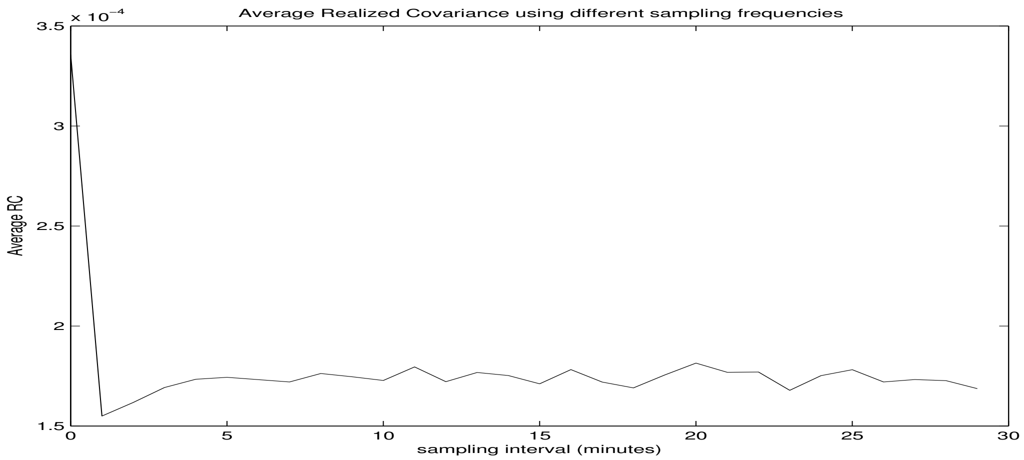


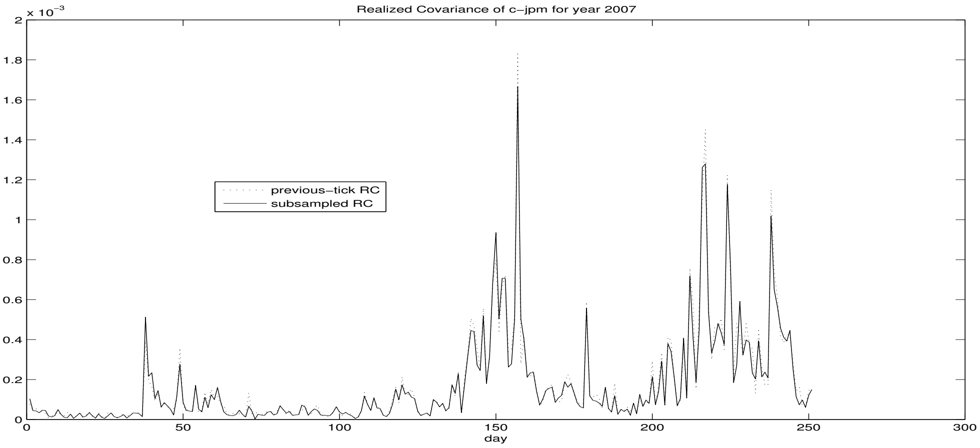
Figure 3.4: Previous-tick realized covariance signature plot: the realized covariance tends to zero as sampling frequency increases due to the Epps effect (observed here between 1-5 minutes sampling frequencies). In the case of the previous-tick realized covariance, the effect of microstructure noise at the highest sampling frequency (here 1 second sampling frequency) is also observed.

Figure 3.5 shows the realized covariance and correlation of c and jpm at 5 minutes sampling frequency using the previous-tick RC and subsampled estimator $ssRC$. The graphs show sharp increases in both covariance and correlations in the second half of 2007 due to the effects of the credit crisis. The previous-tick RC and $ssRC$ also produce substantially different values for covariance and correlations. This difference is more noticeable for realized correlations due to scaling effects.

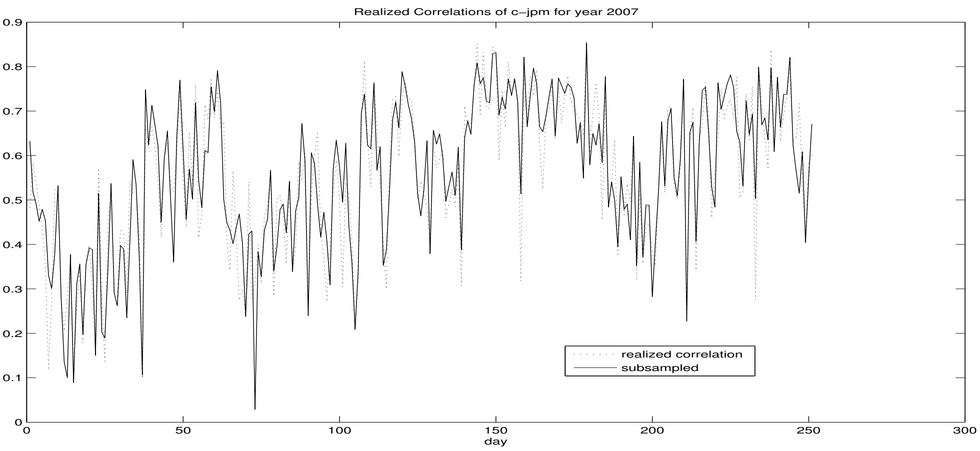
3.5.1 Results

Estimated bounds due to asynchronicity

We first present the results for bounds due to asynchronicity of data alone. If there are no observations within a "tolerance" vicinity of the sampling grid, we consider the observation as latent or "missing". For our case, we set the tolerance level to 5% of the sampling interval Δt. Since we use 5 minutes sampling frequency, the state of "missingness" is conferred if there is no observed data within the last 15 secs of the sampling grid.



(a) Realized Covariance



(b) Realized Correlations

Figure 3.5: Plots of the previous-tick (dotted line) and the subsampled (solid line) realized covariance (top) and realized correlations (bottom) of the sample period using 5 minutes sampling frequency.

Figure 3.6 shows the average estimated probabilities obtained that are used to estimate the maximum bounds of realized covariance. The x- and y-axis give the jpm and c returns respectively, while the z-axis gives the average estimated probabilities over the sample period. Figure 3.6a graphs the average observable probabilities p_{1j} (i.e. $z = 1$). It has a cone shape that peaks at the centre where returns are approximately 0 for both c and jpm. This is expected as 5 minutes intraday returns are relatively small with approximately zero mean. Figure 3.6b shows the average estimated probabilities at $z = 2$, where returns of c are observed but returns of jpm are missing. Peaks tend to be along the x-y axis, which gives the maximum covariance. Figure 3.6c gives the estimated probabilities at $z = 3$, where returns for jpm are observed but returns of c are missing. Since c is more frequently traded than jpm, π_3 tends to be small. Under $\pi_3 = 0$, the probabilities do not enter the optimisation problem and are relegated to the first bin (smallest c and jpm returns), hence giving the sharp peak at that corner. The rest of the probability surface is rather flat with some increase at the largest value for c and positive values of jpm. Figure 3.6d shows the average estimated probabilities when both c and jpm returns are not observed. Again there is a peak in the first bin (smallest c and jpm returns) for days when $\pi_4 = 0$ and another peak at the opposite end where both jpm and c returns are largest, which gives the largest covariance.

Figure 3.7 shows the average estimated probabilities used to estimate the minimum bounds of realized covariance. Figure 3.7a gives the average observable probabilities p_{1j}, which is the same as in Figure 3.6a. Figure 3.7b gives the probabilities for $z = 2$ when returns for c are observable and returns for jpm are missing. The peak occurs at the maximum value for c and the minimum value for jpm, which gives the smallest covariance. A smaller peak is observed at the largest value for jpm and smallest value for c which also decreases the value of realized covariance. The peak at the first bin (smallest c and jpm returns) is again caused by days when $\pi_2 = 0$. Figure 3.7c gives the probabilities for $z = 3$. The large peak in the first bin is caused by days when $\pi_3 = 0$, and there is some small increments along the negative c values and positive jpm values which gives smaller realized covariances. Figure 3.7d gives the probabilities for $z = 4$ where both c and jpm returns are unobserved. Besides the peak at the first bin for days when $\pi_4 = 0$, there are two peaks opposite each other at maximum jpm returns and minimum c and vice versa, which both give the smallest realized covariance.

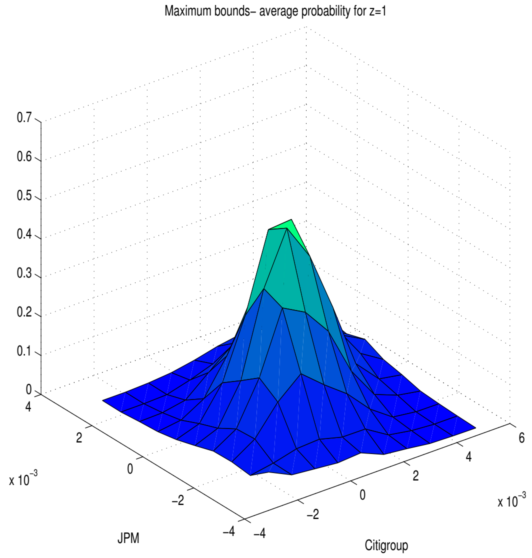
The corresponding estimated probabilities used for deriving the bounds due to asynchronicity of realized correlations are given in Figures 3.13 and 3.14 in the Appendix. The shapes of the probability surfaces are more complicated due to the division by realized volatilities (Eq. 3.8), but the analysis remains similar.

The maximum and minimum bounds of the realized covariance and realized correlations due to asynchronous trading alone are plotted in Figure 3.8. As c and jpm are both highly traded stocks, the effect of asynchronicity is often small (maximum and minimum bounds are close to each other). For realized covariance, the bounds tend to widen at points of sharp increases or decreases of RC. This implies that for sharp increase or decrease in return of one stock, trading of the other stock tends to temporarily slow down, hence resulting in unobservability of its returns. This illustrates that asynchronicity is linked to an increase in investors' uncertainty. Similarly for realized correlations, the bounds are observed to widen around the time of the onset of the credit crisis, implying greater uncertainty during that period.

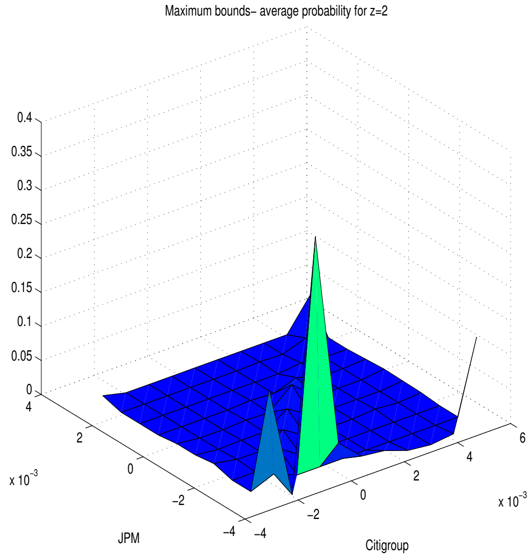
Estimated bounds due to microstructure noise

We now turn to bounds due to the effects of microstructure noise alone. Figure 3.9 shows the percentage of microstructure noise p_X and p_Y for c and jpm respectively over the time period. The percentage of noise seems to be relatively low at about 1-2 % with a sharp spike for c to over 11.5% in the last quarter of 2007 (right-hand side of Figure 3.9). Noise levels tend to be higher in the second half of the year as compared to the first half. The sharp spike observed for c occurs on 5 Nov, when Citigroup's rating was downgraded by Moodys. We interpret this spike to be the degree of deviation from the true latent efficient process and take the upper bound of noise levels λ to be 12%, in recognition that the percentage noise computed here may not be precise due to assumptions involved in obtaining the bias term. We will check the reasonableness of this upper bound later. In any case, as argued in Horowitz and Manski (1995), even if there are no obvious ways to set a firm bound, it will still be useful to analyse the sensitivity of the bounds under different error probabilities.

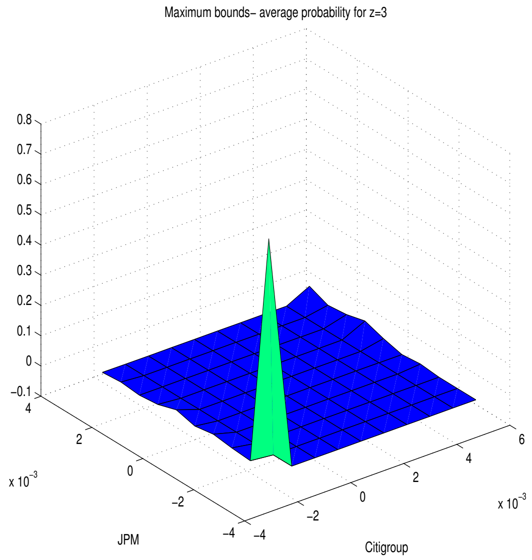
The bounds obtained for realized covariance and correlations due to effects of microstructure noise alone are plotted in Figure 3.10. The figures give both



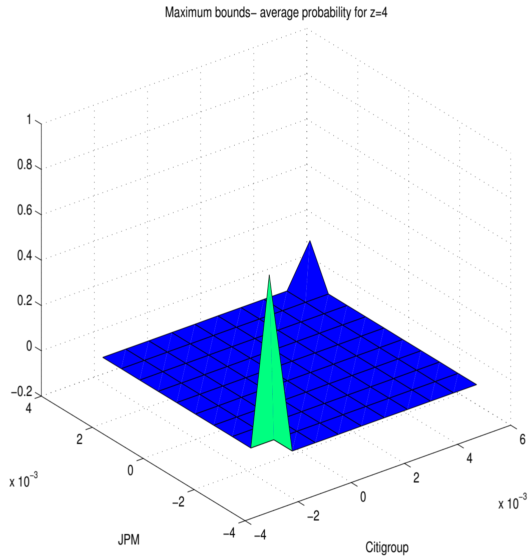
(a) $\hat{p}_{1j}$ (both jpm and c returns are observed)



(b) $\hat{p}_{2j}$ (c returns observed, jpm returns missing)

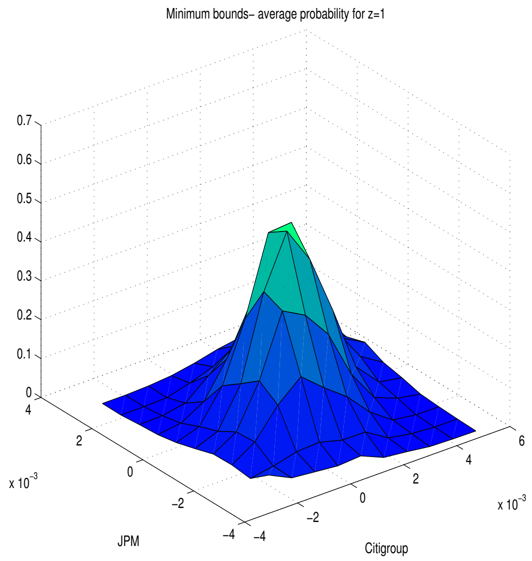


(c) $\hat{p}_{3j}$ (jpm returns observed, c missing)

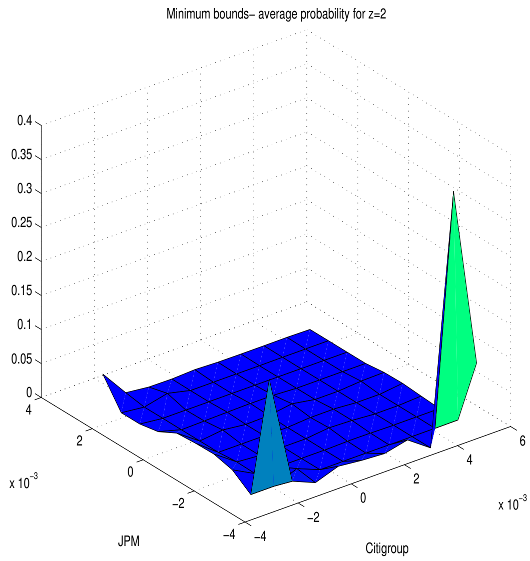


(d) $\hat{p}_{4j}$ (both c and jpm returns missing)

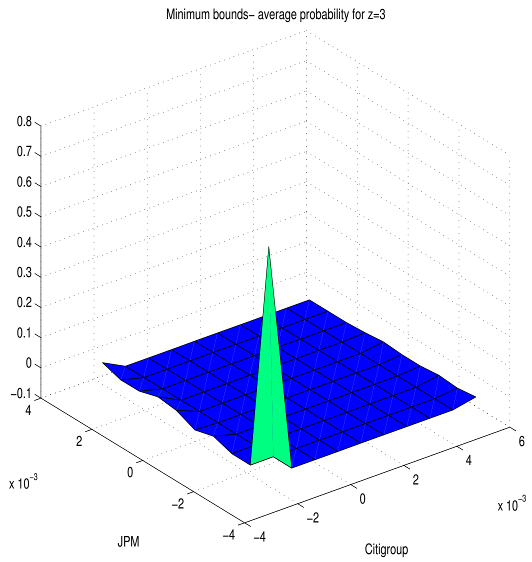
Figure 3.6: Average estimated probabilities $\hat{p}_{zj}$ for estimating maximum bounds due to asynchronicity for realized covariance.



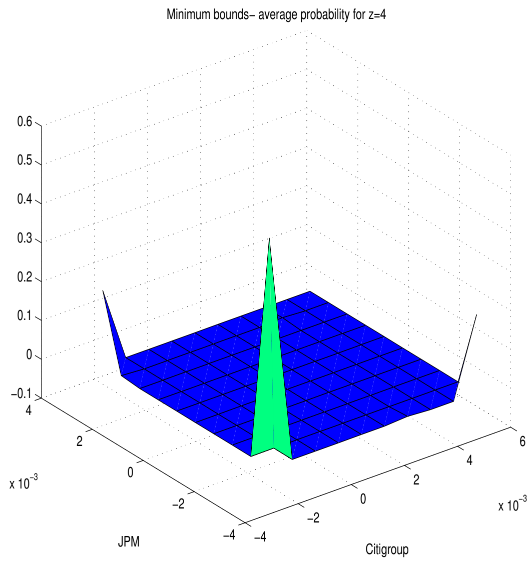
(a) $\hat{p}_{1j}$ (both jpm and c returns are observed)



(b) $\hat{p}_{2j}$ (c returns observed, jpm returns missing)



(c) $\hat{p}_{3j}$ (jpm returns observed, c missing)



(d) $\hat{p}_{4j}$ (both c and jpm returns missing)

Figure 3.7: Average estimated probabilities $\hat{p}_{zj}$ for estimating minimum bounds due to asynchronicity for realized covariance.

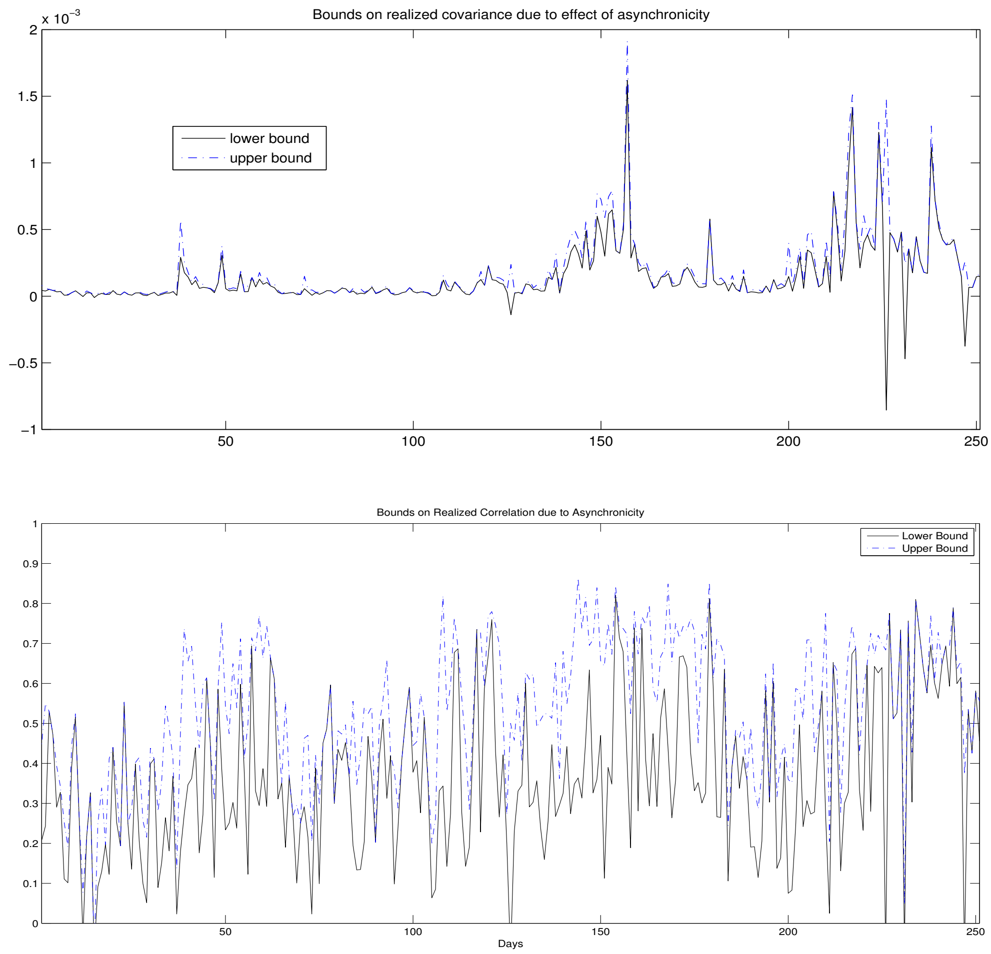


Figure 3.8: Bounds on realized covariance RC (top) and correlations $RCorr$ (bottom) due to effect of asynchronicity. Upper bounds are represented by dash-dot lines while lower bounds are represented by solid lines

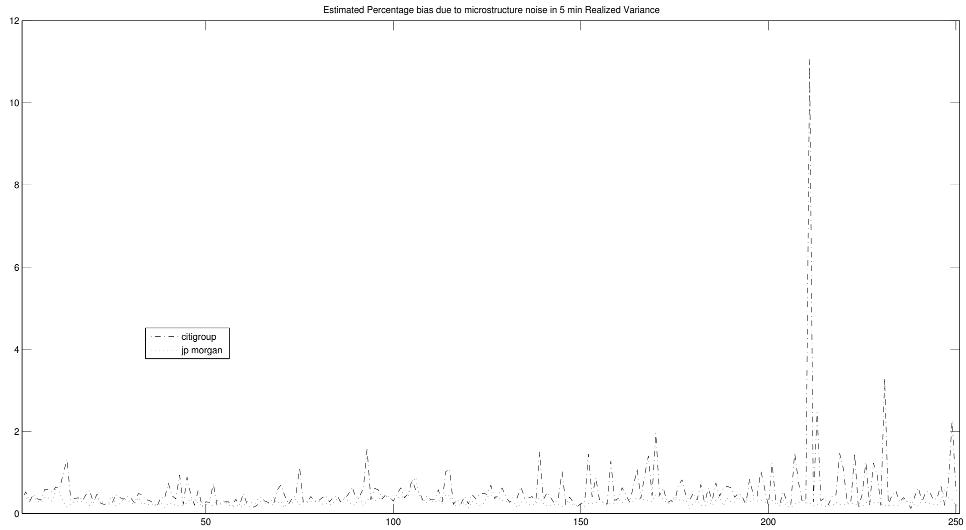


Figure 3.9: Estimated percentage bias due to microstructure noise in realized variances for Citigroup (dash-dotted) and JP Morgan (dotted) for the year 2007. From the results, a 12% upper bound, λ, on noise levels is obtained.

the bounds under corrupted sampling (solid lines) and contaminated sampling (dash-dotted lines). As expected, the bounds under contaminated sampling lie within the bounds under corrupted sampling.

Estimated overall bounds

Figure 3.15 in the Appendix graphs the overall bounds due to asynchronous data and microstructure noise for RC under corrupted (top) and contaminated sampling (bottom). The previous-tick RC (solid lines) and subsampled RC ($ssRC$, dash-dot lines) estimators are also graphed. It shows that the RC estimators lie closer to the upper bounds during the second half of 2007 as compared to the first half. Figure 3.11 shows the corresponding graph for realized correlations. It appears that when contaminated sampling is assumed, the $RCorr$ estimators lie rather close to the maximum bounds.

Table 3.1 gives the percentage coverage rate of the bounds on the RC, $ssRC$, $RCorr$ and $ssRCorr$ estimators for the first half of 2007 up to 30 June (day 1-124) and the second half of 2007 up to 31st December (day 125-251). The second half of 2007 will later be the out-of-sample period that is forecasted, while the first half of

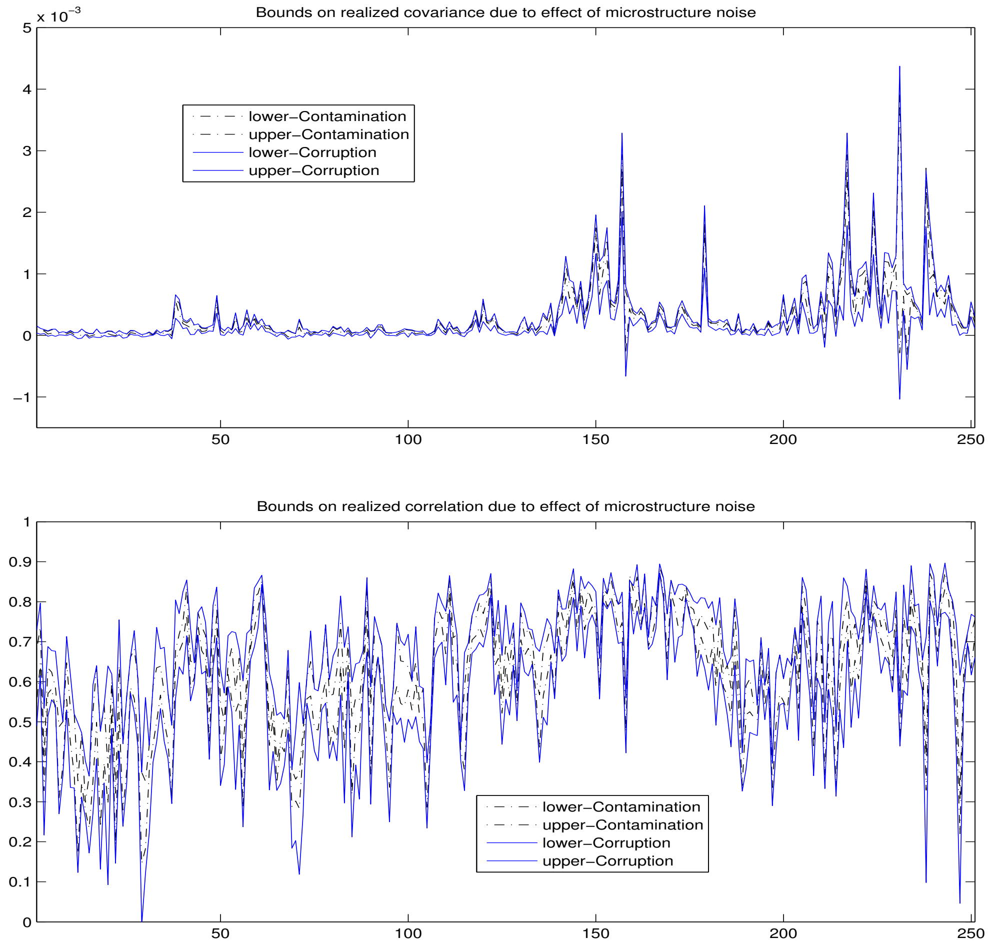


Figure 3.10: Bounds on realized covariance RC (top) and correlations $RCorr$ (bottom) due to the effect of microstructure noise. Solid lines show bounds obtained under corrupted sampling scheme while dash-dotted lines show bounds obtained under contamination sampling.

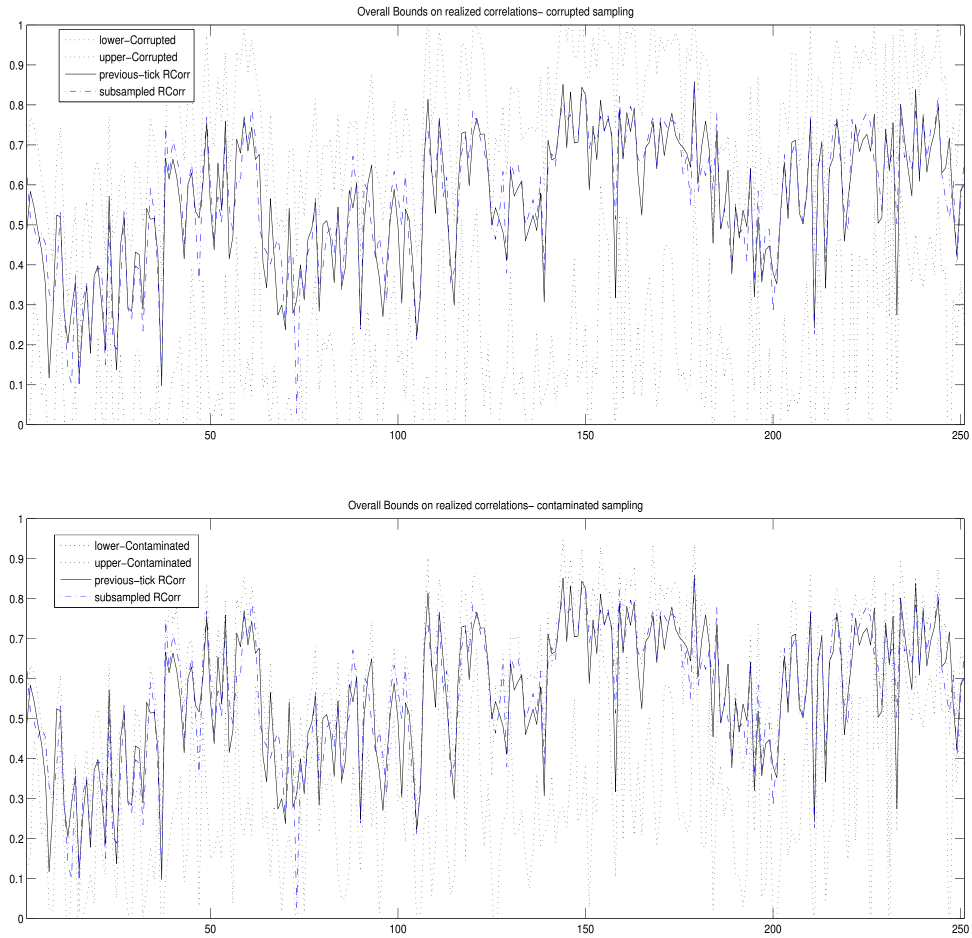


Figure 3.11: Overall Bounds on Realized Correlations: Dotted lines show the overall bounds under corrupted sampling (top) and contaminated sampling (bottom) for microstructure noise. $RCorr$ is given by the solid lines and $ssRCorr$ is given by the dash-dot lines.

2007 serves as the initial in-sample period. The coverage of the bounds under the corrupted sampling on the estimators in the first half of 2007 is 100% but the coverage of the bounds under contaminated sampling is inadequate especially for the $ssRCorr$ which has a coverage rate of 82%. This implies that microstructure noise cannot be regarded as statistically independent of the efficient price process, as was also concluded by Phillips and Yu (2006) and Barndorff-Nielsen et al. (2008). Market microstructure theory also predicts the noise process to be correlated with the efficient price process (see Kalnina and Linton (2006)), hence our results here are not only unsurprising but also indicate that the bounds are tight and that the estimate of the upper bound of microstructure noise is reasonable. The coverage of the bounds during the volatile second half of 2007 crisis period is less than 100%, with some observations exceeding the upper bounds. Coverage is however still above the 95% level for the corrupted sampling scheme.

	Realized Covariance				Realized Correlations			
% coverage	1st 2007		2nd 2007		1st 2007		2nd 2007	
overall bounds	RC	$ssRC$	RC	$ssRC$	$RCorr$	$ssRCorr$	$RCorr$	$ssRCorr$
corrupted	100	100	99.21	95.28	100	99.19	95.28	96.85
contaminated	100	98.39	96.06	83.46	90.32	82.26	87.40	83.46

Table 3.1: Percentage coverage by overall bounds of the previous-tick realized covariance and correlation (RC and $RCorr$) and subsampled realized covariance and correlation ($ssRC$ and $ssRCorr$) under corrupted and contaminated sampling.

Forecasted bounds

Table 3.2 shows the initial in-sample estimates of the HAR model for $RCorr$, $ssRCorr$ and their bounds. The Newey-West standard deviations using four lags are given in brackets. The coefficients for weekly and monthly correlations are insignificant due to the small sample size used (e.g. for monthly correlations there are only six observations) such that the effects of long memory are harder to be captured by the model.

To obtain out-of-sample 1- and 10-step forecasts for the second half of 2007, we use rolling windows of 124 observations (approximately 6 months) to estimate the parameters. Iterated forecasts are used to obtain 10-step ahead forecasts. The predictive mean square error (PMSE) of the 1- and 10-steps forecasts are shown in Table 3.2. PMSE for the bounds are larger than PMSE for $RCorr$ and $ssRCorr$. However for multistep forecasting, the worsening in terms of PMSE as compared to the 1-step forecast is less severe for the bounds than $RCorr$ and $ssRCorr$. This feature suggests that bounds can be effectively forecasted for longer periods ahead with greater certainty as compared to point estimators. In terms of coverage, coverage at 1-step forecast is equally good as the actual coverage (see Table 3.1) and at 10-step forecast, although there is a slight reduction in percentage coverage, it remains above 90%.

In-sample HAR coefficient estimates								
	$RCorr$		$ssRCorr$		Lower Bound		Upper Bound	
c	0.1835	(0.0759)	0.1756	(0.0713)	0.0494	(0.0272)	0.2356	(0.1039)
$\beta^{(d)}$	0.2396	(0.1162)	0.3451	(0.1235)	0.1639	(0.0987)	0.3509	(0.1007)
$\beta^{(w)}$	0.4398	(0.2105)	0.2228	(0.2200)	-0.0369	(0.2469)	0.2578	(0.1869)
$\beta^{(m)}$	-0.0312	(0.2217)	0.1016	(0.2269)	0.5738	(0.4054)	0.0718	(0.2161)
PMSE (Out-of-sample)								
	$RCorr$		$ssRCorr$		Lower Bound		Upper Bound	
	1 step	10 step	1 step	10 step	1 step	10 step	1 step	10 step
	0.0172	0.0263	0.0143	0.0222	0.0444	0.0487	0.0225	0.0328
Percentage coverage by forecasted bounds (Out-of-sample)								
			1 step	10 step			1 step	10 step
	$RCorr$		96.85	93.22	$ssRCorr$		96.85	91.53

Table 3.2: HAR in-sample estimations for realized correlations and bounds and out-of-sample forecast evaluations. Newey-West standard errors with 4 lags are given in brackets.

Figure 3.12 plots the HAR 1-step (top) and 10-step (bottom) forecasts for $RCorr$ and the bounds. Solid lines marked with $(\cdot)$ and $(*)$ indicate the $RCorr$ and $ssRCorr$ respectively, while the middle solid line is the forecasted $RCorr$. The dotted lines are the estimated bounds that are realized while the thin solid lines at the bounds indicate the forecasted bounds. From the graphs, the forecasted bounds are smoother than the realized bounds and provide tight predictive bounds for $RCorr$ and $ssRCorr$.

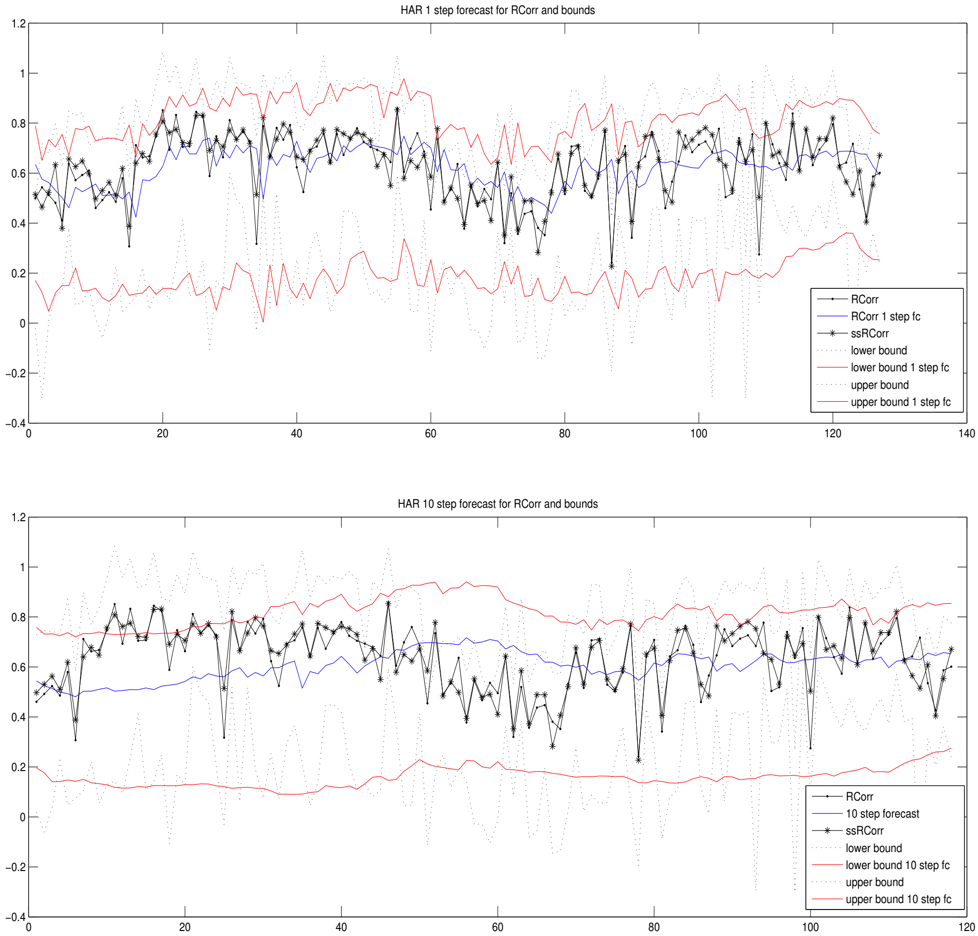


Figure 3.12: HAR 1-step (top) and 10-step (bottom) forecasts for $RCorr$ and its bounds. Actual estimated bounds are in dotted lines and solid lines are the forecasted bounds. $RCorr$ is marked with $(\cdot)$ and $ssRCorr$ is marked with $(*)$. Forecasts of $RCorr$ are given by the solid line in the middle.

3.6 Conclusion

Estimating realized covariance and correlations is problematic due to data problems of asynchronous observations and microstructure noise. While different bias-correction methods exist, they often involve making assumptions about the latent noise process and quote/trade arrival-time process. This paper posits that rather than attempting to obtain point identification of the estimators that have to be bias corrected, a more robust approach can be used by way of partial identification (Manski (1995)).

We identify bounds due to the presence of asynchronicity by using the partial identification approach of Horowitz and Manski (2006) for missing data and the bounds due to microstructure noise by using the approach of Horowitz and Manski (1995) for treatment of contaminated and corrupted data to estimate the identification region. The bounds due to microstructure noise are estimated under two different error models: the contaminated sampling and the corrupted sampling schemes. The contaminated sampling scheme assumes that the error process is random and independent of the sampling process while the corrupted sampling scheme does not make such assumptions and is the more general case. To obtain bounds due to microstructure noise, an upper bound of the percentage of noise is assumed to be known a priori. We use the asymptotic bias due to noise of Bandi and Russell (2008) to estimate the upper bound of microstructure noise.

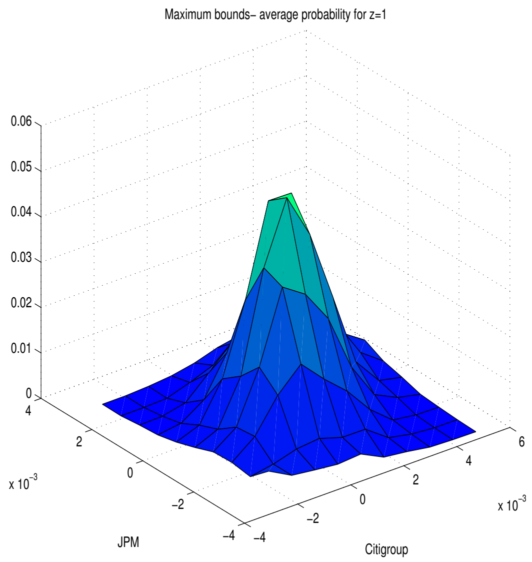
Our simulation study shows that bounds provide good coverage of the RC and $RCorr$ estimators and their bias-corrected estimators via subsampling. Furthermore, we show how the tuning parameters in the estimation (namely the tolerance used to assign missingness and the assumed upper bound of noise levels) influence the widths of the estimated bounds.

We show via an empirical application that the overall bounds under the corrupted sampling scheme provide a high degree of coverage of the estimators and the subsampled estimators both in the pre-crisis and in the crisis period. However under the contaminated scheme, the coverage is unsatisfactory, which indicates that the noise process cannot be assumed to be independent of the efficient price process. This result is expected under market microstructure theory and in line with findings of Phillips and Yu (2006) and Barndorff-Nielsen et al. (2008). This gives indication that the estimated bounds are tight, and that the estimate of the upper bound of microstructure noise is reasonable.

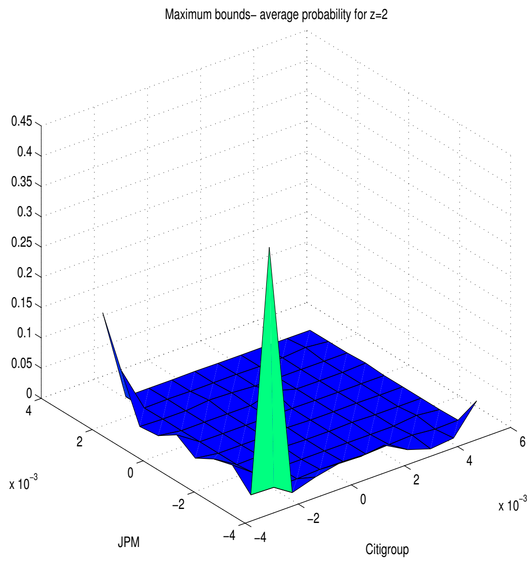
We forecast the bounds of the realized correlations using the HAR model (Corsi (2003)) for 1-step and 10-step periods, and find that the forecasted bounds are tight with excellent coverage despite dealing with data during the volatile crisis period. While the accuracy of point estimators declines greatly under the 10-step forecast, the tightness and coverage of the bounds remain stable under multistep forecasting.

Future applications of these bounds can be used in financial risk management, such as for forecasting of the Value-at-risk, and in portfolio management, where best- and worst-case scenarios can be more reliably drawn.

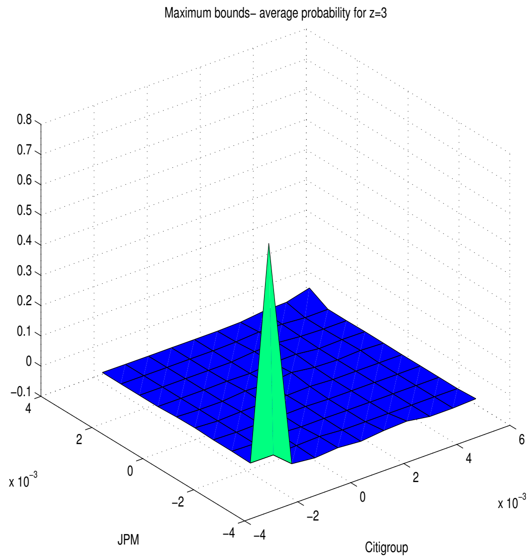
Appendix A:



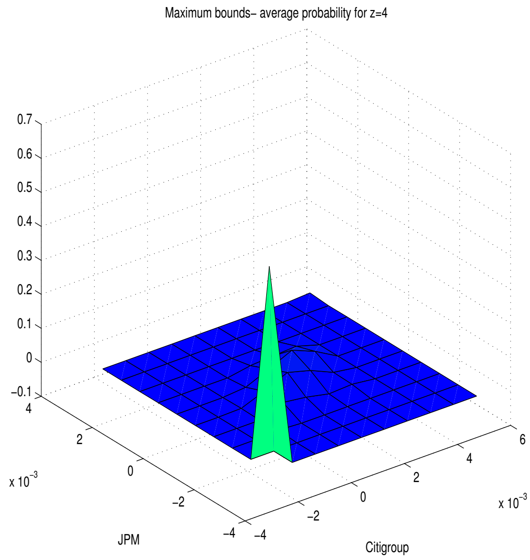
(a) $\hat{p}_{1j}$ (both jpm and c returns are observed)



(b) $\hat{p}_{2j}$ (c returns observed, jpm returns missing)

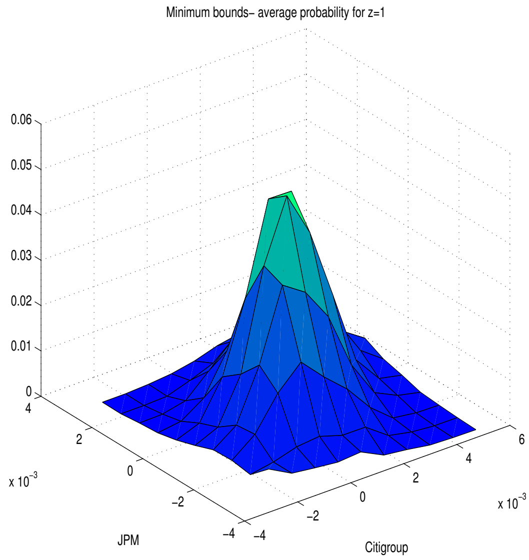


(c) $\hat{p}_{3j}$ (jpm returns observed, c missing)

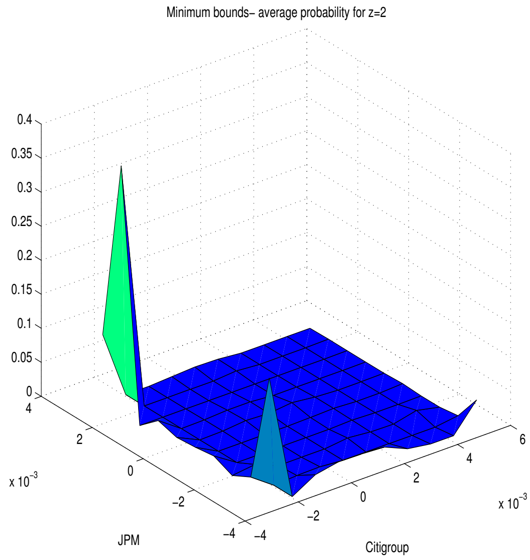


(d) $\hat{p}_{4j}$ (both c and jpm returns missing)

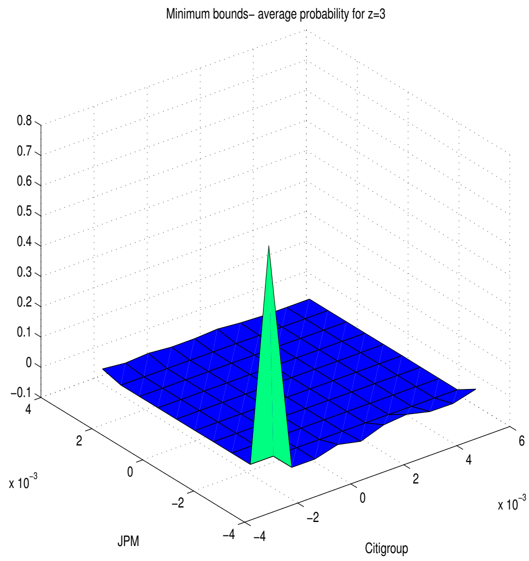
Figure 3.13: Average estimated probabilities $\hat{p}_{zj}$ for obtaining maximum bounds of realized correlations.



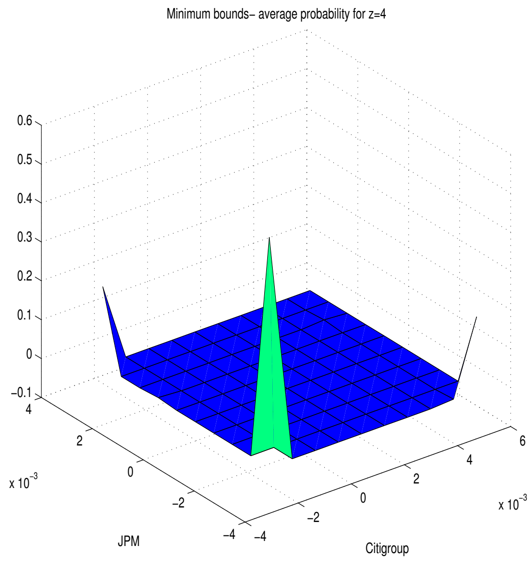
(a) $\hat{p}_{1j}$ (both jpm and c returns are observed)



(b) $\hat{p}_{2j}$ (c returns observed, jpm returns missing)



(c) $\hat{p}_{3j}$ (jpm returns observed, c missing)



(d) $\hat{p}_{4j}$ (both c and jpm returns missing)

Figure 3.14: Average estimated probabilities $\hat{p}_{zj}$ for obtaining minimum bounds of realized correlations.

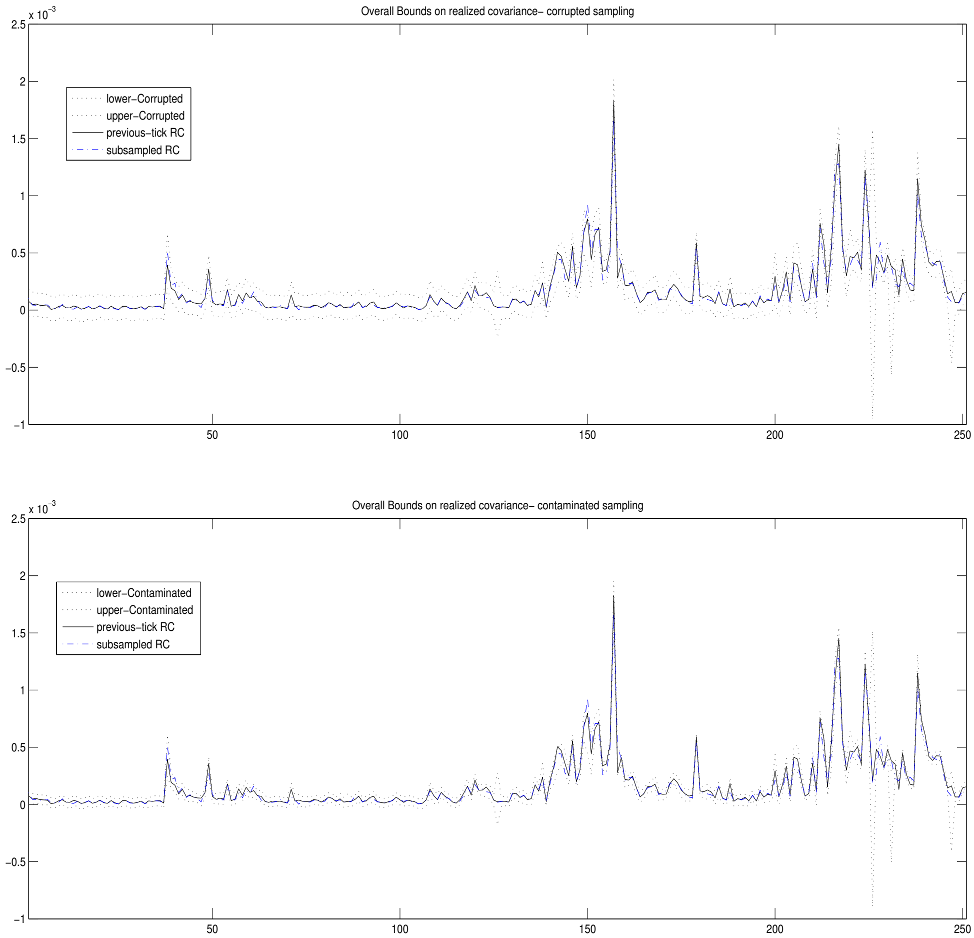


Figure 3.15: Overall Bounds on Realized Covariance: Dotted lines show the overall bounds under corrupted sampling (top) and contaminated sampling (bottom) for microstructure noise. Previous-tick RC is given by solid lines and subsampled RC ($ssRC$) is given by dash-dotted lines.

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Erklärung

Ich erkläre hiermit eidesstattlich: Die vorliegende Arbeit mit dem Thema

Three Essays on Using High Frequency Data in Financial Risks Measurement

habe ich ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt. Die aus anderen Quellen direkt oder indirekt übernommenen Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Weitere Personen, insbesondere Promotionsberater, waren an der inhaltlich materiellen Erstellung dieser Arbeit nicht beteiligt. Die Arbeit wurde bisher weder im In- noch Ausland in gleicher oder ähnlicher Form einer anderen Prüfbehörde vorgelegt.

Konstanz, den 11. August 2012 Lidan Großmaß

Abgrenzung

Kapitel 1 entstammt einer gemeinsamen Arbeit mit Herrn Hao Liu (Universität Konstanz). Meine individuelle Leistung bei der Erstellung dieser Arbeit beträgt 90%.

Kapitel 2 entstammt einer gemeinsamen Arbeit mit Frau Prof. Ser-Huang Poon (Manchester Business School). Meine individuelle Leistung bei der Erstellung dieser Arbeit beträgt 85%.

Ich versichere hiermit, dass ich Kapitel 3 der vorliegenden Arbeit ohne Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe.