Prices, Public Goods and Politics
Three Essays in Public Economics

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2. Referent: Herr Prof. Michael Becher, Ph.D.
3. Referent: Herr Prof. Dr. Heinrich Ursprung

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Chapter 1

Introduction

1.1 Summary

This dissertation consists of three independent research papers. Whereas the specific topics of the individual papers are quite distinct, they all share a common motive. Starting with empirical observations that clash with findings from theoretical research, each paper attempts to explain the observed phenomena within the existing frameworks and to realign them with the theoretical concepts. After a concise description of this general theme, each paper is summarized in this introduction.

With the help of models, theoretical research tries to explain how different mechanisms work in reality. The form of these models and the insights derived from them shape the thinking in the related areas in academia and also the general public. This is exemplified by three coherencies that are commonly accepted as true: first, the functioning of markets improves if entry barriers are lowered, if information becomes better and easier to access, and if transaction costs decrease (see Chapter 2); second, local preferences can be better addressed whenever public goods are provided on a regional level, however, this limits the possibility to internalize externalities (see Chapter 3); third, the design of electoral rules determines how well democracies achieve certain goals: Compared to proportional systems, majoritarian systems create clear political responsibilities but often fail to represent societal preferences (see Chapter 4). Now, whenever empirical assessments yield evidence against these expectations, the same questions arise: What is the cause for the observed discrepancy? Are the conceptions that underlie theoretical modeling wrong? Is the explanatory power of theoretical models lower than commonly presumed?

These questions are of particular relevance if actual policy- and decision-making is based on theoretical considerations. The research papers that constitute this dissertation show that, contrary to the initial impression, there is no discrepancy in the regarded cases and that the seemingly inconsistent empirical findings do not pose contradictions to theory. In this context, it is shown that basic theoretical conceptions of real-world aspects are in general accurate and not the main causes of the inconsistencies. Rather, it are common pervasive assumptions and the specific selection of elements that are included in theoretical models, which drive the model results. In all three papers the inconsistencies are resolved by dropping common simplifications and by combining aspects that have so far only been analyzed in isolation.

Chapter 2: Prices

The paper *Endogenous Search Behavior on Posted Offer Markets with Capacity Constrained Sellers* in Chapter 2 is joint work with Michal Marenčák. It regards the development of so called posted offer markets related to changing market conditions. Especially decreasing costs of participation and transaction costs as well as cheaper and better accessible information is crucial in this context. Examples for posted offer markets are the labor market and the housing market. All posted offer markets share the feature that a transaction can only occur if a potential buyer signals her interest in the offer of the respective offerer. Due to the relocation of the mentioned markets to Internet platforms, the cost of market participation and the cost of signaling interest have decreased substantially. Empirical investigations have shown that this led to an increase in the number of applications. However, contrary to theoretical predictions, the number of inappropriate and uninformed applications has risen greatly at the same time. This means more people are signaling interest in offers, despite they do not meet the offerers requirements, or they are in fact not interested in the offer, what they could have learned after a more thorough examination of the offer. A transaction is impossible in both scenarios. This observation is especially counterintuitive, since the shift of these markets to the Internet allows to attach more information to offers in a highly structured way. Chapter 2 rationalizes why these wrong applications can occur. This is achieved by combining all relevant characteristics of the analyzed markets, namely multiple applications and capacity constrained sellers, for the first time in one model. It can be optimal for buyers to signal interest in several offers uninformedly, if the cost of applying decrease and even though the cost of information has fallen as well. The central mechanism of the paper shares similarities to classical problems of common goods. It is costly for offerers to process applications such that they only want to consider a small number of suitable applications. However, due to the standardized way of signaling interest and applying, offerers can hardly infer whether applicants have examined offers carefully or not. This implies that an informed decision to apply cannot be rewarded by the offerer in the selection of the applications, which opens the possibility to send uninformed applications. Necessary information can be obtained later if such an uninformed application is considered by the offerer. This behavior is individually rational. It allows potential buyers to save the cost of information whenever their application is not considered by an offerer. Yet, it leads to a decrease in market efficiency and implies a welfare loss. The respective market's ability to generate matches between offerers and buyers is impaired and decreases in the number of uninformed applications. The analysis in Chapter 2 shows that falling transaction costs and decreasing costs of information do not lead to the expected improvements concerning market efficiency by default. Caused by the implied change in the behavior of agents, which can be observed in reality and is explained in our model, these potential improvements are undone. Since this central mechanism is of simple structure, potential improvements are obvious: the cost of applying have to be increased. Exemplary, there are some few operators of Internet marketplaces that restrict the communication between users such that only a limited number of signals of interest can be sent. In this case, opportunity cost of applications increase and offerers no longer have to cope with vast quantities of "SPAM"-applications. However, since operators of Internet job or housing platforms mainly create profit from usage of their services and not from the generation of transactions, it is unlikely that improvements like this will be initiated by operators in general. This creates room for targeted policies that may help to overcome this situation.

Chapter 3: Public Goods

Chapter 3 features the work titled *Local Public Goods as Perfect Substitutes: Centralization versus Decentralization*. It analyzes the provision of public goods under different forms of government. There exists a rich literature related to this topic in the fields of political economy and public administration. Earlier contributions that were mostly theoretical have established the following basal trade-off: Local, decentralized structures of governance allow a targeted provision of public goods. This is much harder to achieve in superordinate, centralized structures. However, under centralized structures it is possible to account for and internalize positive externalities that are related to public goods. In turn, this is hardly possible under decentralization. This insight was considered in the design of structures of governance. Due to the presumed superiority of decentralized structures over centralized ones, the process of decentralization can be, together with the economization of former governmental dominions, considered as one of the main transformations of the last century. However, findings from recent empirical evaluations of reforms of decentralization stand in contrast to the anticipated effects such that a reconsideration of the topic is in order. Either, the analyzed reforms toward decentralization have not created measurable improvements, or led to inconsistent findings. The paper presented in Chapter 3 provides a simple explanation for these observations and the apparent discrepancy between theoretical and empirical findings.

Using an established model framework, I show that centralized and decentralized provision structures can lead to identical outcomes. This equivalence is generated by rethinking one essential element of previous models. The critical assumption of former models, which led to the presumed trade-off as sketched above, was the so called *assumption of separability*. This assumption greatly simplifies how individual provision levels of a public good are aggregated: it implies that individual provisions can be treated as separate goods in the utility function. This is illustrated by the following example: National reductions of CO_2 emissions help to counteract global warming. In this context, the public good is the reduction of the global concentration of greenhouse gases in the atmosphere. It seems reasonable that citizens evaluate local efforts of national states according to their specific contribution to this overall goal. This relationship is advocated by the literature on the aggregation of public goods but destroyed by the assumption of separability, since only individual provision levels matter in the utility function. The analytical simplification related to the solution of the models is obvious, however, the additional implications of this assumption are not clear. Dropping this standard assumption leads to the observed equivalence of centralized and decentralized provision structures. A crucial element related to this finding is the aspect of delegation. The well-established free-rider dilemma is transmitted to the stage of delegation in representative democracy. Electorates tend to elect representatives that exhibit a lower preference for the public good than the respective median citizens in the electorates. This behavior of strategic delegation emerges, since the election of an appropriate representative constitutes a binding commitment to provide a low amount of the public good. In this way, one region can pressure another region to provide a larger amount of the public good itself. The degree of strategic delegation is different in centralized and decentralized structures, yet leads, in interplay with the behavior of the elected representatives, to identical provision levels. Thereby, Chapter 3 reintegrates empirical findings into the established theoretical models, while preserving basic distinctions between the two forms of government. In a wider sense, the work raises the question which elements separate the concepts of centralization and decentralization. It is apparent that differences are present if the two concepts imply different scopes of action and information situations by assumption. However, this is open for debate. Assuming that politicians are subject to different political procedures may not be sufficient to generate different outcomes: rational voters anticipate different behavior of politicians. This happens via strategic delegation and ultimately leads to the equivalence of the two forms of government.

Chapter 4: Politics

The paper presented in Chapter 4 is titled *Accountability or Representation? Democracy and Trade-Offs in Electoral Engineering*. It is joint work with Michael Becher and addresses a well-established topic of political science. It examines how varying electoral rules achieve different fundamental goals of representative democracy. In this regard, the focus lies on two goals: representation and accountability. Representation captures the fit of enacted policies and voter preferences. Accountability characterizes the ability to deselect incompetent politicians, irrespective of ideological aspects. The two electoral rules analyzed in this context are proportional representation and majority voting. A large body of literature addresses the complex of electoral rules, politics and parties. Yet, the question as to which electoral rule is the best one allows no clear answer. A presumed pattern is that some electoral rules are better able to foster certain goals than others. Therefore, the choice and design of electoral rules always implies a trade-off. According to the generally held view, the trade-off implied by the two analyzed electoral rules is the following: Majority voting fails to represent voter preferences as good as proportional representation. In contrast, it is better able to generate accountability compared to proportional representation.

A reassessment and revision of this connection is in order to explain the large heterogeneity within empirical and theoretical research on the topic. In addition to the notion sketched above, there exist minority views, that deny the presence of a trade-off and presume majority voting or proportional representation as superior. All these views can be backed by empirical findings. Some studies support the sketched trade-off, others provide evidence in favor of the minority views and some lead to inconclusive findings. The analysis in Chapter 4 is based on a formal model that features voters with different preferences for policy, such that a conflict over policy is implied and the aspect of representation is relevant. Accountability is included in the model, since politicians exhibit different ability. Since abilities of legislators are linked to the performance of the government, all voters have an incentive to select able representatives, irrespective of the different preferences for policy. The theoretical model in Chapter 4 suggests that trade-offs between electoral rules are not always present and more context-dependent than commonly assumed. The relevant context in this regard is given by the distribution of voter preferences and the political polarization within the electorate. One main message of the analysis is scenarios exist, characterized by voter preferences and polarization, where neither of the two electoral rules lead to representation and accountability. These scenarios are denoted as electoral *sour spots*. Analogously, the analysis identifies scenarios in which both electoral rules generate representation and accountability, these are labeled electoral *sweet spot*. Furthermore, there exist scenarios that imply a trade-off or render one electoral rule superior over the other. Amongst others, the following conclusions can be drawn: On the one hand, the heterogeneity of research findings can be expected and explained by the identified context dependence. On the other hand, the choice of an electoral rule has no effect on representation and accountability in electoral sour spots and electoral sweet spots. Chapter 4 concludes with a discussion of related aspects (e.g., the number of active parties) and the assumptions of the model. It is unclear, for instance, whether voters' behavior is driven by strategic or rather expressive considerations. Strategic voters can be characterized as outcome-oriented and cast votes with the intention to induce a certain outcome. In this context, a possible outcome is the enactment of a certain policy. In contrast, expressive voters use their votes as a means of articulation and vote for the candidate who conveys the desired message best. However, the voting behavior that is implied by the two different behavioral assumptions is identical in many scenarios. Hence, attempts to determine the motivation of voters with the help of observed voting behavior are not promising.

1.2 Zusammenfassung

Diese Dissertation besteht aus drei unabhängigen Forschungsarbeiten. Die Themenfelder der einzelnen Arbeiten sind unterschiedlich, alle drei Arbeiten verbindet jedoch die selbe Intention. Ausgehend von empirischen Befunden, welche im Gegensatz zu etablierten Erkenntnissen theoretischer Forschung stehen, wird der Versuch unternommen, diese innerhalb des bestehenden theoretischen Modellrahmens zu erklären und wieder mit diesem in Einklang zu bringen. Nach einer knappen Ausführung dieser Grundthematik werden die einzelnen Papiere in diesem Kapitel zusammengefasst.

Theoretische Wissenschaft versucht die Wirkungsweise von verschiedenen Mechanismen in der Realität anhand von Modellen zu verstehen. Diese Modelle und die abgeleiteten Erkenntnisse beeinflussen gesellschaftliches Denken und Handeln in allen betroffenen Bereichen. Drei Beispiele hierfür, angelehnt an die Themen der Arbeiten in dieser Dissertation, stellen folgende Erwartungshaltungen dar: Märkte funktionieren besser, wenn der Eintritt vereinfacht wird, es umfangreichere und bessere Möglichkeiten der Information gibt und Transaktionskosten verringert werden (siehe Kapitel 2). Die Bereitstellung von öffentlichen Gütern auf regionaler Ebene ermöglicht die Berücksichtigung von lokalen Bedürfnissen, erschwert jedoch den Interessenausgleich in einem übergeordneten Rahmen (siehe Kapitel 3). Die Ausgestaltung eines Wahlsystems beeinflusst, wie gut Demokratien verschiedene Ziele erreichen. Im Vergleich zum Verhältniswahlrecht führen Mehrheitssysteme zu klaren Zuständigkeiten, bilden jedoch gesellschaftliche Präferenzen oftmals deutlich schlechter ab (siehe Kapitel 4). Falls nun jedoch eine empirische Überprüfung der Realität solchen lange kultivierten Erkenntnissen zuwider läuft, wirft dies stets die selben Fragen auf: Woher kommt die vorliegende Diskrepanz? Ist die Aussagekraft theoretischer Modelle doch begrenzter als angenommen? Diese Fragen wiegen besonders schwer, wenn auf Basis der theoretischen Grundlagen konkrete Entscheidungen getroffen wurden. Die in dieser Dissertation enthaltenen Arbeiten zeigen auf, dass in vielen Fällen keine Diskrepanz vorliegt und die scheinbar widersprüchlichen Befunde durchaus ihre Entsprechungen in Modellergebnissen finden. Hierbei zeigt sich, dass die grundlegenden theoretischen Konzeptionen von realen Zusammenhängen im Allgemeinen zutreffend sind und nicht, in erster Linie, zu den Widersprüchen führen. Vielmehr sind es weitverbreitete und vereinfachende Annahmen sowie die Auswahl der im Modell enthaltenen Elemente, welche die inkonsistenten Modellergebnisse hervorbringen. Indem übliche Vereinfachungen in der Modellierung aufgegeben werden und dadurch, dass Modellaspekte verbunden werden, die bisher nur einzeln betrachtet wurden, kann das Spannungsfeld zwischen Theorie und Empirie aufgelöst werden.

Kapitel 2: Prices

Die Arbeit *Endogenous Search Behavior on Posted Offer Markets with Capacity Constrained Sellers* in Kapitel 2 ist eine gemeinsame Arbeit mit Michal Marenčák. Sie betrachtet die Entwicklung von sogenannten *posted offer markets* (zu deutsch etwa *öffentliches-Angebot-* oder *Inserat-Märkte*) im Zusammenhang mit fallenden Teilnahme- und Transaktionskosten sowie gesunkenen Kosten von Information und deren bessere Verfügbarkeit. Beispiele für solche Märkte sind der Arbeits- oder auch der Wohnungsmarkt. Ein gemeinsames Element ist, dass ein veröffentlichtes Angebot, z.B. ein Stellenangebot oder ein Wohnungsinserat, nur in eine Transaktion münden kann, wenn ein Interessent dem Anbieter sein Interesse signalisiert. Im Rahmen der Verlagerung der angesprochenen Märkte in das Internet sind die Kosten der Marktteilnahme und der Interessensbekundung deutlich gesunken. In diesem Zusammenhang haben empirische Untersuchungen aufgezeigt, dass dies zu einer Erhöhung der Anzahl von Bewerbungen geführt hat. Im Widerspruch zu theoretischen Überlegungen steht jedoch, dass ebenso die Anzahl von aussichtslosen und falsch informierten Bewerbungen ansteigt: Es gibt mehr Bewerber, die ihr Interesse an einem Angebot signalisieren, obwohl es klar ist, dass sie die im Angebot geforderten Eigenschaften nicht aufweisen, oder sie eigentlich gar kein Interesse an dem Angebot haben, was sie bei einer genaueren Prüfung des Angebotes auch hätten wissen können. In beiden Fällen kann keine Transaktion zustande kommen. Diese Beobachtung ist insbesondere kontraintuitiv, da die Verlagerung der Märkte ins Internet die Menge der Informationen, die in einem Angebot enthalten sind, sowie deren Darstellung erheblich verbessert hat. Kapitel 2 erklärt, warum Interessenten solche falschen Interessensbekundungen abgeben. Dies gelingt, indem erstmalig alle relevanten Eigenschaften der analysierten Märkte, Mehrfachbewerbungen, Anbieter mit begrenztem Angebot und weitere, in einem Modell vereint werden. Falls die Kosten der Interessensbekundung fallen, ist es, trotz des günstigeren Zugriffs auf Informationen, eine optimale Strategie für Käufer, uninformiert Interesse an mehreren Angeboten zu signalisieren. Der herausgearbeitete Mechanismus weist Parallelen zu klassischen Allmende Problemen auf. Das Bearbeiten von Interessensbekundungen ist für Anbieter mit Kosten verbunden, sodass sie nur eine begrenzte Auswahl von geeigneten Bewerbern in Betracht ziehen wollen. Aufgrund von standardisierten Interessensmeldungen und Bewerbungen können Anbieter jedoch keine Rückschlüsse ziehen, ob ein Interessent sich mit dem Angebot auseinander gesetzt hat oder nicht. Da die Informationsbeschaffung vor der Bewerbung nicht bei der Selektion der Bewerbungen honoriert werden kann, dann bietet dies jedem potentiellen Interessenten die Möglichkeit einer uniformierten Bewerbung. Falls diese Bewerbung dann berücksichtigt wird, können relevante Informationen zu einem späteren Zeitpunkt eingeholt werden. Dieses Verhalten ist individuell rational, es erlaubt potentiellen Interessenten die Kosten der Informationsbeschaffung zu vermeiden, falls ihre Bewerbung nicht berücksichtigt wird, führt im Aggregat jedoch zu Effizienz- und Wohlfahrtsverlusten. Die Funktionsfähigkeit des entsprechenden Marktes Anbieter mit Nachfragern zusammenzubringen, nimmt mit der Zahl der uninformierten Bewerbungen ab. Die Analyse in Kapitel 2 arbeitet somit heraus, dass fallende Transaktions- und Informationskosten nicht notwendigerweise zu den erhofften Effizienzgewinnen auf Märkten führen müssen. Aufgrund der induzierten Verhaltensänderung der Akteure, die in der Realität zu beobachten ist und in unserem Modell erklärt wird, wird diese potentielle Verbesserung zunichte gemacht. Da der zugrundeliegende Mechanismus einfach ist, liegen mögliche Verbesserungen auf der Hand: Die Kosten von Bewerbungen müssen erhöht werden. Beispielsweise gibt es einige wenige Betreiber von Internet-Marktplätzen, welche die Kommunikationsmöglichkeiten der Teilnehmer einschränken und nur noch eine limitierte Anzahl von Interessensbekundungen zulassen, sodass Anbieter nicht mit Unmengen von „SPAM"-Bewerbungen konfrontiert werden. Da jedoch Betreiber von Job- oder Immobilienportalen im Internet ihren Gewinn hauptsächlich aufgrund der Nutzung ihrer Dienste, anstatt vorrangig durch das Generieren von Transaktionen, erzielen, erscheint es unwahrscheinlich, dass Verbesserungen wie diese im Allgemeinen von Plattformbetreibern ausgehen. Dies eröffnet einen Raum für Politikmaßnahmen, welche helfen können diese Situation zu überwinden.

Kapitel 3: Public Goods

Das Forschungspapier *Local Public Goods as Perfect Substitutes: Centralization versus Decentralization* in Kapitel 3 analysiert die Bereitstellung von öffentlichen Gütern unter verschiedenen politischen Verwaltungsformen. Dieses Themengebiet hat eine reichhaltige Literatur im Bereich der Politischen Ökonomie und Verwaltungswissenschaften hervorgebracht. Frühere, häufig theoretische Arbeiten in diesem Bereich haben in ihrer Gesamtheit die folgende, grundlegende Abwägung etabliert: Lokale, dezentrale Verwaltungsstrukturen erlauben eine zielgerichtete Bereitstellung von öffentlichen Gütern. In übergeordneten, zentralen Systemen ist dies schwerer möglich. Hier können jedoch die mit öffentlichen Gütern verbundenen positiven externen Effekte berücksichtigt und internalisiert werden. Diese Erkenntnis fand durchaus Beachtung bei der Ausgestaltung von Verwaltungssystemen. Aufgrund der angenommenen Überlegenheit von dezentralen Strukturen, kann die Dezentralisierung von Verwaltungen, gemeinsam mit der Ökonomisierung von vormals staatlichen Herrschaftsbereichen, als eine der grundlegenden Transformationen des vergangenen Jahrhunderts verstanden werden. Neuere Erkenntnisse von empirischen Auswertungen von Verwaltungsreformen führen jedoch zu gegenteiligen Befunden, sodass eine Neubetrachtung dieser Thematik erforderlich ist. Diese empirischen Studien ergeben insgesamt keineswegs ein einheitliches Bild. Die analysierten Reformen hin zu dezentralen Strukturen haben entweder zu keinen messbaren Verbesserungen geführt oder es kam zu widersprüchlichen Befunden. Das in Kapitel 3 präsentierte Papier liefert eine einfache Begründung für diese

Beobachtung und die scheinbare Diskrepanz zwischen theoretischen und empirischen Erkenntnissen. Es zeigt auf, dass die Äquivalenz von zentralen und dezentralen Bereitstellungssystemen durchaus im Rahmen einer etablierten Modellstruktur zu erwarten ist. Diese Äquivalenz wird gefunden, indem ein zentrales Element bisheriger Modellierungen neu konzeptualisiert wird. Der Aspekt, welcher in theoretischen Modellen die oben skizzierte und vermutete Abwägung herbeigeführt hat, ist die sogenannte *assumption of separability* (zu deutsch *Annahme der Trennbarkeit*). Diese vereinfachende Annahme betrifft die Art und Weise, wie individuelle Bereitstellungsmengen eines öffentlichen Gutes aggregiert werden. Diese Fragestellung hat selbst eine große Zahl von Forschungsarbeiten hervorgebracht, die Annahme der *Trennbarkeit* erlaubt es jedoch, diesen Aspekt zu vernachlässigen, sodass einzelne Bereitstellungsmengen eines öffentlichen Gutes als individuelle Güter betrachtet werden können. Folgendes Beispiel verdeutlicht dies: Verringerungen von CO_2 Emissionen tragen dazu bei, die Erwärmung der Erde zu verlangsamen. Als das öffentliche Gut in diesem Beispiel kann daher die Verringerung der Konzentration von Treibhausgasen in der Atmosphäre verstanden werden. Es erscheint sinnvoll, dass Individuen, abhängig von ihrer Präferenz für Umweltschutz, nationale Anstrengungen der Emissionsreduktion danach beurteilen, inwiefern sie zum globalen Ziel des Klimaschutzes beitragen. Diese Sichtweise findet sich in der Literatur über die Aggregation von lokalen öffentlichen Gütern wieder. Die angesprochene Annahme löst diesen Zusammenhang jedoch auf und impliziert, dass Akteure in theoretischen Modellen ihren Fokus nicht auf das Aggregat, sondern auf die individuellen Bereitstellungsmengen legen. Die implizierten technischen Vereinfachungen in Bezug auf die Modelllösungen liegen auf der Hand, darüber hinaus sind die Implikationen dieser Annahme jedoch unklar. Wird diese, in der Literatur übliche, Vereinfachung innerhalb eines etablierten Modellrahmens aufgegeben, führt dies zu der beobachteten Äquivalenz von zentralen und dezentralen Bereitstellungsmechanismen. Ein entscheidendes Element hierbei ist der Aspekt der Delegation. Das bekannte Trittbrettfahrerproblem bei der Bereitstellung von öffentlichen Gütern setzt sich in der Bestimmung von politischen Repräsentanten fort und es kommt zur Wahl von Repräsentanten, die selbst eine geringere Präferenz für das öffentliche Gut aufweisen als der entsprechende Bevölkerungsmedian. Im Wettbewerb zwischen Verwaltungsregionen kommt dieses Wahlverhalten, das als strategische Delegation begriffen werden kann, zustande, da die Festlegung auf einen Repräsentanten ein bindendes Signal darstellt, selbst möglichst wenig von dem öffentlichen Gut bereit zu stellen. Auf diese Weise kann eine Region Druck auf andere Verwaltungseinheiten aufbauen, sodass diese einen Anreiz haben, selbst eine größere Menge des öffentlichen Gutes bereitzustellen. Dieser Aspekt der strategischen Delegation tritt in zentralen und dezentralen Strukturen unterschiedlich stark zutage, führt in Verbindung mit dem Verhalten der gewählten Repräsentanten jedoch zu identischen Bereitstellungsmengen. Kapitel 3 integriert also die empirischen Erkenntnisse in den etablierten theoretischen Modellrahmen, wobei jedoch eine grundlegende Unterscheidbarkeit zwischen den Systemen erhalten bleibt. Im weiteren Sinne wird die Frage nach der Unterscheidung der Konzepte Zentralisierung und Dezentralisierung aufgeworfen. Es ist klar, dass Unterschiede vorliegen, falls diese Konzepte inhärent unterschiedliche Handlungsspielräume und Informationssituationen implizieren. Dies ist jedoch alles andere als gesichert. Wird hingegen lediglich angenommen, dass gewählte Politiker an einem anderen Ort anhand eines unterschiedlichen politischen Modus entscheiden, ist durchaus zu erwarten, dass rationales Wählerverhalten dieser nachgelagerten Stufe Rechnung trägt. Genau dies geschieht über die strategische Delegation und führt zur Äquivalenz der beiden politischen Systeme.

Kapitel 4: Politics

In Kapitel 4 befasst sich das Papier *Accountability or Representation? Democracy and Trade-Offs in Electoral Engineering* mit einem klassischen Thema aus dem Bereich der Politikwissenschaft. Es ist eine gemeinsame Arbeit mit Michael Becher und untersucht, wie gut unterschiedliche Wahlsysteme verschiedene grundlegende Ziele repräsentativer Demokratie erreichen. Die Ziele, auf denen hierbei der Fokus liegt, sind Repräsentanz und politische Verantwortlichkeit. Repräsentanz beschreibt in diesem Zusammenhang wie gut Politik die Präferenzen der Wählerschaft widerspiegelt. Politische Verantwortlichkeit bezieht sich darauf, ob es gelingt offensichtlich ungeeignete Politiker, unabhängig von ideologischen Aspekten, abzuwählen. Die beiden Wahlsysteme, die in diesem Spannungsfeld analysiert werden, sind das Verhältnis- und das Mehrheitswahlrecht. Eine umfassende Literatur beschäftigt sich mit dem Themenkomplex von Wahlsystemen, Politik und Parteien. Die Frage nach dem bestmöglichen Wahlsystem kann jedoch nicht eindeutig beantwortet werden. Als gesichert gilt lediglich, dass manche Ziele besser in gewissen Wahlsystemen erreicht werden können als in anderen. Bei der Auswahl bzw. der Ausgestaltung eines Wahlsystems liegt daher stets ein Zielkonflikt vor. Die Abwägung, die gemäß der herrschenden Mehrheitsmeinung den beiden analysierten Wahlsystemen zugrunde liegt, lautet wie folgt: Mehrheitswahlrecht spiegelt die Verteilung von Wählerpräferenzen nicht so getreu wider wie das Verhältniswahlrecht. Im Gegensatz dazu befördert es jedoch politische Verantwortlichkeit besser als das Verhältniswahlrecht.

Eine Neubetrachtung und Überprüfung dieses Zusammenhangs ist angebracht, um die hohe Heterogenität innerhalb empirischer und theoretischer Forschung begreifbar zu machen. In Bezug auf theoretische Arbeiten gibt es zusätzlich zur oben skizzierten Anschauung durchaus Minderheitenmeinungen, die einen Zielkonflikt verneinen und Mehrheitswahlrecht bzw. Verhältniswahlrecht für generell überlegen erachten. Für alle diese Abstufungen finden sich empirische Belege: Einige Studien bestätigen den skizzierten Zielkonflikt, andere geben den Minderheitenmeinungen recht und wieder andere liefern uneindeutige Befunde. Die Analyse in Kapitel 4 geschieht anhand eines formalen Modells, das auf Wählern mit unterschiedlichen Politikpräferenzen aufbaut, sodass ein Konflikt zwischen Wählern über Politik vorliegt und die Frage der Repräsentanz relevant ist. Politische Verantwortlichkeit spiegelt sich in dem Modell wider, da Politiker unterschiedliche Befähigungen aufweisen. Da diese Befähigungen die Leistungsfähigkeit der Regierung und der Verwaltung bestimmt, haben alle Wähler, unabhängig von ihrem ideologischen Standpunkt, einen gemeinsamen Anreiz fähige Repräsentanten zu finden. Die Ergebnisse des Modells legen nahe, dass Zielkonflikte zwischen Wahlsystemen nicht zwangsläufig vorliegen und stärker kontextabhängig sind als allgemein angenommen. Der relevante Kontext ist hierbei durch die Verteilung der Wählerpräferenzen und die politische Polarisierung der Wählerschaft gegeben. Eine Hauptaussage der Analyse ist, dass es Szenarien, charakterisiert durch Wählerpräferenzen und Polarisierung, gibt, in denen beide Wahlsysteme weder zu Repräsentanz noch zu politischer Verantwortlichkeit führen. Diese Szenarien werden als *electoral sour spots* bezeichnet. Analog dazu lassen sich Szenarien identifizieren, in denen beide Wahlsysteme Repräsentanz und politische Verantwortlichkeit herbeiführen, sodass ein *electoral sweet spot* vorliegt. Innerhalb des Modellrahmens finden sich ebenso Szenarien, die einen Zielkonflikt implizieren oder in denen ein Wahlsystem dem anderen überlegen ist. Es lassen sich zwei Schlussfolgerungen ziehen: Zum einen ist die Heterogenität in den Forschungsergebnissen aufgrund der aufgezeigten Kontextabhängigkeit erklärbar. Andererseits hat die Wahl des Wahlsystems in *electoral sour spots* und *electoral sweet spots* keinen Einfluss auf die relevanten Zielgrößen. Abschließend werden in Kapitel 4 einige verwandte Aspekte (z.B. die Anzahl der im Parlament vertretenen Parteien) und Modellannahmen diskutiert. Beispielsweise ist in Bezug auf die Motivation von Wählern unklar, ob deren Verhalten eher strategisch oder eher expressiv geprägt ist. Strategische Wähler können als ergebnisorientiert beschrieben werden und treffen ihre Wahlentscheidung mit der Intention ein bestimmtes Ziel herbeizuführen. Ein mögliches Ziel wäre in diesem Kontext die Umsetzung einer bestimmten Politik. Expressive Wähler hingegen nutzen ihre Stimmabgabe als Artikulationsmöglichkeit und wählen die Kandidatin, welche die gewünschte Aussage bestmöglich transportiert. Das Wahlverhalten, dass sich durch die verschiedenen Wählermotivationen im Modellrahmen ergibt, ist jedoch in den meisten Szenarien identisch. Somit erscheinen Versuche, die Motivation von Wählern anhand von beobachtetem Wahlverhalten zu bestimmen, als wenig erfolgversprechend.

Chapter 2

Endogenous Search Behavior on Posted Offer Markets with Capacity Constrained Sellers

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Abstract

Many markets that are essential for an economy have shifted to online platforms, e.g. the labor market. This shift can be associated with decreasing costs of participation and information. In this context, empirical research identifies a significant increase in the number of inappropriate applications. We explain this observation by combining concepts that have not been analyzed in interplay before but are relevant on the markets of interest. We model a posted offer market with capacity constrained sellers in which buyers decide about information acquisition and can apply to multiple sellers. We find that buyers may cease to acquire information that would allow directed search if the cost of applying is low, even if the cost of information acquisition has decreased as well. This leads to equilibria where buyers signal interest uninformedly and thereby to decreased market efficiency and lower welfare. We argue that falling transaction costs and the usage of online tools do not necessarily reduce market frictions and enhance efficiency by default. Our model offers clear policy implications such that the improvements that are commonly associated with falling costs of market participation and information may still be achieved on posted offer markets.

2.1 Introduction

The Internet has brought new market-places, promising far greater efficiency, based on the net's ability to gather in the same virtual place, at hardly any cost, lots of information and processing power and vast numbers of potential buyers and sellers.

The Economist, *How to be perfect*, February 10, 2000

The notion that more information and cheaper interaction of market participants lead to more efficient market outcomes is taken as given in academic society, as well as by the general public. Yet it was understood early on that falling transaction costs might also affect the behavior of market participants and thereby lead to adverse effects: "A natural consequence of lowering the cost of application is that many workers will apply for many more jobs. In fact, excess application appears to be the norm for on-line job postings, with employers reporting [...] unmanageable numbers of resumes from both under- and overqualified candidates [...]" (Autor, 2001, p. 30 f.).[1] The paper at hand takes up this idea and presents a dynamic game of incomplete information to investigate equilibrium behavior and outcomes when signaling interest in offers is cheap. We show that low costs of market participation can cause market efficiency and social welfare to deteriorate.

In general, posted offer markets require that buyers signal interest for offers they want to accept. Yet these signals can be inappropriate in two ways. Either an interested buyer does not meet the seller's requirements or, after learning more about the offer, she loses interest. For the first example, empirical evidence from labor markets, e.g., Kuhn (2014), shows that the submitted applications are often indeed inappropriate.[2] The straightforward explanation that decreased costs of applying cause inappropriate applications is not satisfactory. The cost of information has fallen substantially as well, such that it is also cheaper to identify a potential application as inappropriate and pointless. Thus, one of the main objectives of this paper is to provide a convincing rationale for the large number of inappropriate signals.

We develop a simple theoretical model and show that the significant number of inappropriate signals observed in the data can be rationalized as an equilibrium outcome. Our model features a posted offer market with capacity constrained sellers. Buyers determine their search behavior endogenously and are allowed to apply to more than one seller. The works of Lester (2011) and Albrecht et al. (2006) exhibit many similarities to our approach, as they combine some of the aspects mentioned. However, especially the inclusion of endogenous information acquisition into the framework of posted offer markets with capacity constrained sellers constitutes a novelty in this literature. The main mechanism developed in this paper can be described as follows. When observing a posted offer, a potential buyer can do research to learn whether it meets her preferences and if she meets the requirements of the seller. Hence she can learn whether a deal is possible. Yet, if she does not engage in research, she can nevertheless signal her interest

[1] Also, the Economist seems to have surrendered their hopeful sentiment: "In the online job market it's trivially 'cheap' to submit one more CV for one more role, so employers receive hundreds of unsuitable suitors for every open position. Online apartment-hunters and apartment-owners face similar levels of inundation and frustration" (Bram, 2016).

[2] The terms *buyer* and *seller* refer to actors on posted offer markets. *Sellers* post offers and *buyers* can react to these offers. Note that this denomination reverses the usual naming used in the context of labor markets.

uninformedly. In this case a deal is not always possible such that her signal is inappropriate with some probability. We show that buyers' behavior can be explained by cost avoidance, and it depends on the size of the cost of signaling interest compared to the cost of research. Hence, a switch from an equilibrium in which buyers conduct research before signaling their interest to an equilibrium with uninformed signaling can occur even if both of these costs fall. Signaling interest uninformedly can be individually rational for a buyer, yet the implied joint behavior can be considered as a coordination failure and shares traits with problems of common resources.

The basic structure of our model is the following. On a market, sellers post offers to which buyers can react. Each offer constitutes the option to buy one good for the posted price. Sellers are capacity constrained such that each seller posts only one offer. Furthermore, each buyer wants to buy only one good. The goods offered are differentiated: either downtown two-room apartments, vacant positions as an accountant in a large company or used cars with five seats and less than fifty thousand kilometers. If a deal between a buyer and a seller is made, the buyer pays the price posted by the seller. The surpluses of the involved counterparts are the price for the seller and the willingness to pay for the offer less the price in the case of the buyer.

In short, our findings are the following: It is rational for buyers not to acquire information and to signal interest uninformedly if the cost of applying is low compared to the cost of information. The prices posted by the sellers are linked to the buyers' information decision. Sellers try to induce buyers to inform themselves by countering the falling cost of application with higher prices. When this becomes impossible, sellers adjust to the uninformed buyers and demand lower prices. Equilibria where buyers do not acquire information lead to market congestion, are not constrained efficient and imply welfare losses.

The paper is structured as follows: after the introduction, Section 2 provides an overview of the related literature. Section 3 introduces and solves a version of the model that is based on a single seller. Section 4 interprets and discusses the implications derived from this model. Section 5 introduces and solves an extended model with several sellers. After this, Section 6 discusses the obtained results and Section 7 features a general discussion touching issues like policy implications and alternative interpretations. Section 8 concludes the paper.

2.2 Related literature

In this work we provide an analysis of posted offer markets with capacity constrained sellers where buyers can determine their information acquisition and are allowed to apply to more than one seller.[3] Thus, the paper at hand is situated in the nexus of information, prices and the implied efficiency and welfare.

Our work is linked to the vast literature concerning information and prices, Baye et al. (2006) offer a comprehensive overview of this strand of literature. Yet, in contrast to seminal contributions like those of Varian (1980) and Burdett and Judd (1983), we do not seek to explain

[3] Furthermore, we allow sellers to approach interested buyers repeatedly. Hence the matching procedure between buyers and sellers is not a one-shot game where sellers unlucky in their first try remain unmatched.

distributions of prices as outcomes shaped by the distribution of information but are interested in the reciprocal connection of the two concepts. Given the abundance of information that is accessible online nowadays, this point of view is relevant, since information acquisition and usage by market participants resembles a choice that is shaped by market conditions. It becomes apparent that this stance is an interesting starting ground, since more recent works that incorporate aspects of online marketplaces lead to reversed findings when compared to earlier contributions. Most notably, Lester (2011) considers a posted offer market and capacity constrained sellers but does not allow multiple applications and assumes exogenous shares of informed and uninformed buyers. Focusing on the link between the information buyers have and market prices, his work leads up to Lester's paradox: A larger share of informed buyers leads to higher market prices. This pattern is caused by the introduction of capacity constraints. Buyers turn their interest to offers with higher prices as they expect fewer competing buyers for these offers. We enrich this debate by identifying that the relation between prices and information works in both directions, such that sellers can affect the information acquisition of buyers by setting prices. Also Burdett et al. (2001) focus on frictions arising in posted offer markets where sellers may face capacity constraints.

A large share of real-world markets are in fact posted offer markets and feature capacity constraints, different search strategies and multiple applications. This situation is acknowledged by many recent papers that include some or all of these aspects in theoretical models. Due to the shape of the labor market, a large part of the literature on posted offer markets stems from the field of labor economics. Important topics are how actors engage in search and matching in different scenarios, the characterization of the underlying matching functions and issues of matching quality and efficiency. For example, Albrecht et al. (2004, 2006) and Galenianos and Kircher (2009) all consider posted offer markets and capacity constrained sellers, while allowing multiple applications. Albrecht et al. (2004) derive a matching function under these assumptions, and Albrecht et al. (2006) assess the impact of multiple applications on wages and matching efficiency and identify multiple applications as inefficient. Whereas these two works focused on undirected search, meaning uniformed buyers and random applications, Galenianos and Kircher (2009) look at direct search and let applicants choose their number of applications. In contrast to previous findings, they conclude that multiple applications can even impair efficiency given directed search, which was previously associated with efficient matching. A further relaxation of some assumptions is found in Kircher (2009). He discards the previous one-shot models and allows sellers to approach buyers in their queues one after another. While assuming directed search, he also lets buyers decide about the number of applications.[4] Kircher (2009) finds that outcomes converge to a social optimum if application costs vanish, but they are also constraint efficient given positive application costs. The driving element of this finding is that sellers can process all applicants in their queues. This factor eliminates the chances that vacancies cannot be filled due to coordination frictions. Our work adds to this scenario, as it suggests that this positive finding is no longer true if buyers decide to engage in undirected search.

[4]Kircher (2009) assumes continuums of agents in his model. Working with a finite number of discrete agents complicates the analysis, since utility and probabilities become detailed. Yet, results under the assumption of infinitesimal agents normally correspond to the limit case of discrete models. See Galenianos and Kircher (2012) for a work on this transition.

The articles by Cheremukhin et al. (2013, 2016) are linked to the literature of rational inattention and motivate search costs by costly processing of information. They extend the concept of directed search to cases in which buyers decide about the intensity and location of their search efforts. Our work can be considered to incorporate the extreme cases of this continuum of targeted search: Either buyers do not search at all or they examine every offer with an intensity such that every piece of information is revealed. In contrast to the two mentioned works we allow multiple applications. Similar to the models mentioned so far, dynamic models can model repeated interaction, see Guerrieri (2008) and especially Garcia (2017). The latter paper constitutes a dynamic version of our model without endogenous search behavior. Coles and Smith (1998) also made an early contribution to the understanding of marketplaces in a dynamic framework in which buyers can decide to acquire additional information.[5]

A large body of empirical literature exists that assesses the impact of the Internet on problems of matching, which have previously been characterized by analogue communication and marketplaces. In line with the prediction of Autor (2001, p. 30 f.) presented above, Kuhn (2014, p. 1) emphasizes negative aspects and finds that "the low cost of applying for jobs online can result in large numbers of inappropriate applications being submitted". Kuhn and Skuterud (2004) and Kuhn and Mansour (2014) focus on online job search and effects on job markets outcomes such as unemployment duration. Their findings suggest that online job search has changed and evolved in the years since its emergence. In their early assessment using data from 1998 to 2000, they found that unemployment duration and other outcomes cannot be improved by the use of online job platforms. In contrast, with data from 2008 and 2009, they show that unemployment duration is significantly reduced when using online tools, and wage gains from job switches are larger. Kroft and Pope (2014) take the inclusion of US cities on the platform craigslist.com as a quasi-experimental setting and produce mixed findings alike. Inclusion on the platform led to fewer ads for housing in local newspapers and lowered vacancy rates in rental and apartment markets, but it had no effect on the labor market. The presented findings suggest that the designs of online marketplaces affect transaction costs and thereby shape behavior and achieved efficiency. Importantly, these design features have surpassed tools that are neutral with respect to past behavior, such as basic search and filter functions. Examples are Google and the hospitality platform Airbnb. Both condition search results on a user's search history and evaluate past behavior, such as viewed offers and bookings. See Varian (2010) for more examples and for issues of "computer mediated transactions". For the case of Airbnb, the algorithms employed improve the chances of a match greatly (Fradkin, 2015, 2017). Only the preselection made by these algorithms renders (a subset of) the vast amount of offers manageable and comprehensible. Staying in the realm of computer-based communication, the connection to the literature on spam is noteworthy. Excessive use of undirected search with multiple applications can be understood as spam, which leads to coordination frictions and impairs efficiency. Rao and Reiley (2012) estimate that the costs not borne by senders and burdened on receivers of spam emails exceed the profits of the senders by a hundredfold.

A body of literature addresses small markets with capacity constrained sellers with the help of experiments. Cason and Noussair (2007) construct a market in which coordination frictions

[5]Note that the comparison of dynamic and static models is not straightforward. Decentralized outcomes are usually constraint efficient in static models, but dynamic models can lead to inefficient market outcomes.

arise because buyers that approach identical sellers compete for limited goods, while some sellers fail to sell their goods. Further works investigate the nexus of information and prices in an experimental setting and focus on the paradoxical pattern established in Lester (2011), that a larger share of informed buyers leads to higher prices. Whereas Helland et al. (2017) find support for this finding, Anbarci and Feltovich (2014) are unable to find evidence for it. Recent works enrich the basic model by including behavioral concepts like communication (Momsen, 2016) or social segregation (Chen et al., 2015).

The literature on two-sided markets (Rochet and Tirole, 2006) constitutes another field with ties to the analysis of posted offer markets. Most such markets are based on online platforms that serve as meeting places for sellers and buyers. Usually, these platforms are neither operated by buyers nor sellers but by third parties pursuing their own economic interests. For example, placing an offer on an online housing platform may cost a fee for sellers or, what is usually more important, the platform generates profits by selling advertisement slots on their website.[6]

2.3 The single seller model (SSM)

2.3.1 Description of the SSM

This first model includes one representative seller (he) who demands the price P in his offer. The assumption that the reactions of buyers (she) to offers are not strategically connected can serve as a foundation for this simplification. Announcing a price is binding for the seller. In addition, there are B potential buyers. Let χ be a random variable with cumulative density function F and $\chi \in [0, \overline{\chi}]$. For clarity, we denote χ as the willingness of one buyer to pay for the offer. We assume that a deal between a buyer and the seller is possible if $\chi \geq P$. Note that this framework is very general. As a straightforward example, consider sellers of used cars. Here, a seller will sell to any buyer who is ready to pay the demanded price. Therefore, the probability $pr(\chi \geq P) = 1 - F(P)$ characterizes the chance that a deal between a buyer and the seller is possible.[7] In the case of the labor market, P is the wage that is offered for a certain job and χ is the inverse of an potential applicant's reservation wage.[8] If a deal is achieved, the involved buyer realizes the surplus $\chi - P$ and the seller the surplus P.[9]

Before a seller/buyer pair is found, the following decisions are necessary. First, the seller decides

[6]Typically, the more price-sensitive group of users is not charged a price for usage or even subsidized (Eisenmann et al., 2006). This feature can often be observed when looking at the pricing policy of online posted offer markets.

[7]Lester (2011) points to uncertainty about the prices posted by sellers. We introduce uncertainty via χ to make the model more tractable. This approach resembles the idea that buyers are initially unaware of some characteristics of the offers, for example the proximity of an offered flat to the nearest childcare facility or bus stop. Note that our model can be also interpreted exactly as in Lester (2011).

[8]In a wider sense, χ corresponds to a person's willingness to work and her ability to fulfill the tasks described in the job description given the salary, while P characterizes the reservation performance demanded by the employer. Keep in mind that this example reverses the typical naming. A buyer as introduced here is actually selling her workforce to the seller.

[9]The term *surplus* captures the benefits from achieving a deal for the seller and the respective buyer, disregarding the costs they have incurred so far. In contrast, the term *payoff* is used to refer to the surplus less the costs.

about the price P. After observing P, every potential buyer chooses whether to do research or not. A buyer who does research learns her realization of χ and incurs cost of research c_R. Doing research means that buyers learn the hidden characteristics of an offer (with the help of Google Maps, reviews of previous customers or employees or otherwise). c_R (and any other cost parameters) may be monetary figures or opportunity costs, since research is time consuming. If no research is done, potential buyers gain no new knowledge and are unaware of their realization of χ. Last, every potential buyer can signal interest in the offer, which is a prerequisite to be part of a deal and leads to costs c_I. Obviously, buyers who have undertaken research can condition their behavior on the acquired information. Potential buyers who do not signal interest drop out of the game with all costs incurred so far.

The seller observes the notifications of interest but is unable to distinguish between interested buyers and cannot infer whether such a buyer has done research or not. Therefore the seller will randomly approach an interested buyer.[10] This implies cost c_A for the seller who learns this buyer's realization of χ. Furthermore, the buyer will also learn her realization of χ and incur cost c_R if she has not done research before.[11] If $\chi \geq P$, a deal is achieved and the game ends. For $\chi < P$, the seller continues and approaches a different interested buyer randomly (at the cost of c_A). This process continues until a deal is reached or $\chi < P$ is revealed for all interested buyers. For the rest of this paper we set $\overline{\chi} = 1$. Hence, all cost parameters and P can be interpreted as fractions of the maximal willingness to pay. Furthermore, due to the domain $[0, 1]$ the β distribution can be employed in all numerical approaches that come in the following.

Lastly, note that our model includes only the two states in which a buyer has no information or is perfectly informed. This implies the assumption that buyers can learn all characteristics of an offer such that they will know whether a deal is possible with certainty. This assumption appears rather strong. However, the qualitative findings derived in the following are unaffected if it is relaxed. Let there be some idiosyncratic risk that a deal is not possible despite $\chi \geq P$, the chance for this is given by $1 - q$. Importantly, this risk cannot be resolved by the buyer by doing research. The overall chance of a deal is now given by $(1 - F(P))q$. Exchanging $1 - F(P)$ by this newfound composite probability in the coming calculus is straightforward and does not yield different qualitative findings.[12]

[10] This supposition can be interpreted such that the seller uses a heuristic that does not depend on the characteristics of the interested buyers (like first-come first-served or indeed random selection). Feenberg et al. (2017) find that papers listed on top of the NBER newsletter are downloaded and cited more frequently than papers in the bottom part. Thus, people do indeed focus attention on early signals, which allows the interpretation of c_R as opportunity cost.

[11] Given $\chi \geq P$, a deal is possible and realized if the seller asks the buyer to accept the offer and the buyer does so. Asked buyers can only make sound decisions if they know χ. Thus, the assumption is made that interested buyers who have not done research learn χ and pay c_R upon being asked.

[12] Earlier versions of this work were based on this formulation. In a wider sense, the discarded approach highlights distinctions of different posted offer markets: Sellers of used cars only care about a buyer's willingness and ability to pay (implying $q = 1$). The same is true for landlords, yet, they probably have additional requirements for potential tenants (non smoker, neatness, etc.), implying $q < 1$.

2.3.2 Equilibrium concept and strategies in the SSM

The equilibria determined in the following are subgame-perfect Nash equilibria. The buyers' actions must form Nash equilibria in all subgames, meaning given any price P, such that buyers cannot force a certain price. We focus on symmetric equilibria where all buyers play according to the same strategy.[13]

Players maximize their expected payoffs by choosing strategies. The seller's strategy is the price P that can be set to any weakly positive number. The buyers face the decisions about research and signaling interest that allow more sophisticated strategies. Two strategies of the buyers can be ruled out here since obvious deviations exist that yield a greater expected payoff. First, doing research and never signaling interest definitely leads to the cost c_R with no chance to realize a surplus, as the buyer cannot be part of a deal. Second, doing research and signaling interest irrespective of the learned realization of χ implies that a buyer pays c_I even in the cases where $\chi < P$ is known. This approach implies that buyers have to utilize any additional information they have obtained and condition their signaling behavior on it. The strategy of never doing research and never signaling interest is optimal for all $P > \overline{\chi}$, if the seller demands a prohibitive price. Yet, it cannot constitute an equilibrium in the SSM, as will become clear. Thus buyers are left with two viable strategies: $UNINFORMED$ (abbreviated by U), where no research is conducted and interest is always signaled, and $INFORMED$ (abbreviated by I) where buyers do research and signal interest if they exhibit a high willingness to pay.

2.3.3 Solution of the SSM

The model is solved by backward induction. First, the buyers' stage is solved given an arbitrary price P. Second, the price is determined by maximizing the seller's expected payoff, given the buyers' established behavior.

The buyers' stage in the SSM

Let P be the price set by the seller. Consider strategy I first. Buyers learn their willingness to pay, χ, and we assume that they will signal interest if $\chi > P$. We impose this rule since it renders the main insights of the paper more comprehensible. However, the findings we provide are not driven by the assumption (see the discussion in Appendix A.1). Assume that all buyers play I. The expected payoff of one individual buyer is given by

$$E\Pi_B^{I,I} = -c_R + (1 - F(P)) \left(-c_I + PA^{I,I} \left(E(\chi|\chi \geq P) - P\right)\right). \tag{2.1}$$

A buyer definitely incurs cost c_R. $\chi \geq P$ is true with probability $1 - F(P)$, and in this case she will signal interest and incur cost c_I. With probability $PA^{I,I}$, she will be approached and asked by the seller to accept the offer (since $\chi \geq P$ is ensured at this point) and realize the expected

[13] The standard justification in the related literature for focusing on symmetric equilibria is that asymmetric equilibria require too much coordination among players and are therefore unlikely. Following this argumentation, we show that the model yields only symmetric solutions for a vast majority of scenarios.

surplus $E(\chi|\chi \geq P) - P$. Keep in mind that all buyers play I. Therefore $\chi \geq P$ is true for all buyers who signal interest. Hence, the seller is able to achieve a deal with the first interested buyer he approaches, which implies that an individual buyer's probability of being asked by the seller equals the probability of being asked first. When all buyers play I, an individual buyer knows that each of the $B-1$ remaining buyers learns $\chi \geq P$ with probability $1 - F(P)$ and signals interest in this case. Thus, from the perspective of a buyer who has learned $\chi \geq P$ and signaled interest, the expected number of interested buyers is given by $(B-1)(1-F(P)) + 1$. Summing up, the probability $PA^{I,I}$ is given by

$$PA^{I,I} = \frac{1}{(B-1)(1-F(P))+1}. \tag{2.2}$$

Holding the strategies of all other buyers fixed, the individual buyer can deviate and play U instead of I. The deviating buyer's expected payoff is

$$E\Pi_B^{U,I} = -c_I + PA^{U,I}\left(-c_R + (1-F(P))\left(E(\chi|\chi \geq P) - P\right)\right). \tag{2.3}$$

The deviating buyer will always incur c_I. If she is approached, she has to do research implying cost c_R. Given $\chi \geq P$, she buys the good from the seller and realizes the expected surplus. Note that all other buyers still play I. Therefore, the deviating buyer can also only buy the good if she is approached first, since the seller will achieve a deal with any other interested buyer at first try. Thus $PA^{U,I} = PA^{I,I} \equiv PA^I$. Simplifying $E\Pi_B^{I,I} \geq E\Pi_B^{U,I}$ leads to the inequality

$$c \equiv \frac{c_I}{c_R} \geq \frac{1 - PA^I}{F(P)} \equiv \Omega^I. \tag{2.4}$$

Note that we set $\frac{c_I}{c_R} = c$ for simplicity. Condition (2.4) characterizes when it is optimal for an individual buyer to play I, given all other buyers play I. Thus, it constitutes the condition for a symmetric equilibrium on the buyers' stage where all buyers play I.

Next, assume that all buyers play U. The expected payoff of one individual buyer in this case and the deviation payoff when only that buyer switches to strategy U are given by

$$E\Pi_B^{U,U} = -c_I + PA^{U,U}\left(-c_R + (1-F(P))\left(E(\chi|\chi \geq P) - P\right)\right) \text{ and} \tag{2.5}$$

$$E\Pi_B^{I,U} = -c_R + (1-F(P))\left(-c_I + PA^{I,U}\left(E(\chi|\chi \geq P) - P\right)\right). \tag{2.6}$$

In the case of $E\Pi_B^{U,U}$, cost c_I has to be paid always. c_R occurs when being approached, and only with probability $pr(\chi \geq P)$ is the expected surplus realized. Deviating to I implies that c_R is always incurred. Interest is signaled if $\chi \geq P$, and the expected surplus may be realized if the buyer is approached by the seller (since $\chi \geq P$ is ensured at this point). As before, $PA^{I,U} = PA^{U,U}$. Here however, the possibility exists that no deal is reached with a random interested buyer, since $\chi < P$ is possible. So buyers can actually be approached as the second, third, and so on. Being approached second requires three things. The particular buyer must not have been asked first, the willingness to pay of the buyer who was asked first must have been lower than the price, and the buyer in question must have been actually approached second. Similar conditions are required to be approached third. Since the remaining buyers play U by assumption, there will be B interested buyers if the individual buyer signals interest. Thus, a buyer might even be approached as the Bth by the seller. Let $PA^{I,U} = PA^{U,U} \equiv PA^U$ and

$$\begin{aligned} PA^U &= \frac{1}{B} + \frac{B-1}{B} F(P) \frac{1}{B-1} + \frac{B-1}{B} \frac{B-2}{B-1} (F(P))^2 \frac{1}{B-2} + \ldots \\ &= \frac{1}{B} \sum_{n=0}^{B-1} (F(P))^n = \frac{1 - F(P)^B}{B\,(1 - F(P))}. \end{aligned} \tag{2.7}$$

Simplifying $E\Pi_B^{U,U} \geq E\Pi_B^{I,U}$ yields the inequality below, which characterizes a symmetric equilibrium on the buyers' stage in which every buyer plays U.

$$c \leq \frac{1 - PA^U}{F(P)} \equiv \Omega^U. \tag{2.8}$$

Equations (2.4) and (2.8) allow an intuitive interpretation. Given the strategies of the remaining buyers, an individual buyer cannot affect her chance of being approached by the seller with the choice of her strategy (recall $PA^{I,I} = PA^{U,I}$ and $PA^{U,U} = PA^{I,U}$). Since $pr(\chi \geq P)$ is also independent of a buyer's strategy, the probability of realizing the surplus is only determined by the behavior of the other buyers. Thus, a buyer minimizes expected costs. Equation (2.4) holds if c is large (whenever c_I is rather large and/or c_R rather small). In this case, playing I is optimal, since saving the cost of signaling interest in the cases where $\chi < P$ outweighs the cost of research. Equation (2.8) holds if c is small. Now U is optimal, since saving the cost of research c_R in the cases where a buyer is not approached by the seller outweighs the cost of signaling interest unconditionally.

In Equations (2.4) and (2.8), the identity of the individual buyer is arbitrary. Thus, the strategy profile where every buyer plays I constitutes a symmetric Nash equilibrium in every subgame where P satisfies $c \geq \Omega^I$. Analogously, the strategy profile where every buyer plays U constitutes a symmetric Nash equilibrium in every subgame where P satisfies $c \leq \Omega^I$. Note that $\Omega^I, \Omega^U \geq 0$ since $PA^I, PA^U \leq 1$. Furthermore, $\Omega^I \geq \Omega^U$ is always true.[14] Hence, there is no issue of equilibrium selection. However, for $c \in (\Omega^U, \Omega^I)$, no symmetric equilibrium in pure strategies exists. We can show that a symmetric mixed strategy equilibrium exists for each $c \in (\Omega^U, \Omega^I)$, and that asymmetric pure strategy equilibria exist for some $c \in (\Omega^U, \Omega^I)$.[15] Consider the mixed strategy σ_r where buyers play action I with probability r and action U with probability $1 - r$. Obviously $\sigma_1 = I$ and $\sigma_0 = U$ are implied. A r^* exists that characterizes a symmetric Nash equilibrium in the buyers' stage for every $c \in \left[\Omega^U, \Omega^I\right]$. Figure 2.1 below exhibits the equilibrium values r^* that characterize the equilibria in the buyers' stage. The mixed strategy equilibria lie between the indicated thresholds. Some of these mixed strategy equilibria correspond to asymmetric pure strategy equilibria. For example, the highlighted point $(0.128, 0.2)$ in Figure 2.1 for $B = 5$ characterizes a mixed strategy equilibrium that exists if $c = 1.28$ and where each of the five buyers plays action I with probability 0.2. The corresponding asymmetric pure strategy equilibrium for $c = 1.28$ is given if one of the five buyers plays the pure strategy I and the remaining buyers the pure strategy U. Lastly, the interval $[\Omega^U, \Omega^I]$ is arguably small

[14] Refer to Appendix A.2.

[15] See Appendix A.3 for the formal parts of the discussion on mixed strategy equilibria and asymmetric equilibria.

compared to the whole domain of the price ratio c, that is $[0, \infty)$. In addition both thresholds Ω^I and Ω^U converge to $\frac{1}{F(P)} \equiv \Omega$ if the number of buyers is sufficiently large for the mixed strategy domain to vanish; see Appendix A.2. For the following derivation of the seller's behavior, we focus on this limit case with the single boundary Ω and assume that $c = \Omega$ induces buyers to play I.

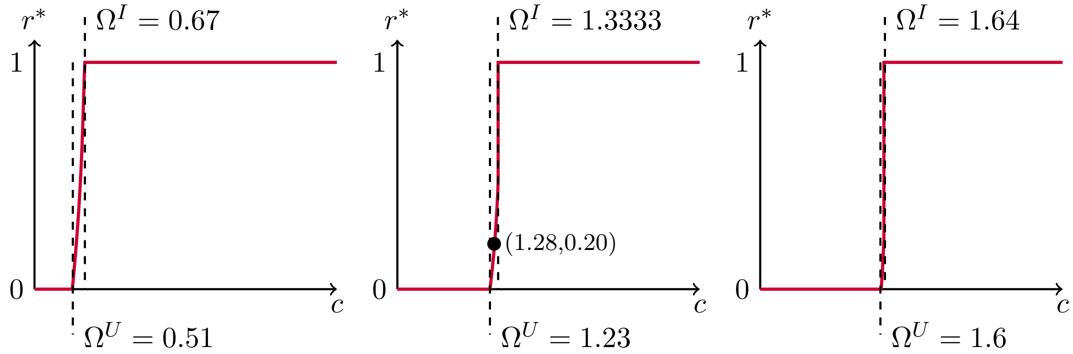

Figure 2.1: Mixed strategy equilibria
Equilibrium values r^* of the mixed strategy $\sigma_r = \{I, r; U, 1-r\}$ for different c and B. Underlying parameters: $F(P) = 0.5$ and $B = 2$ (left), $B = 5$ (middle), $B = 10$ (right).

The seller's stage in the SSM

In the second step of the solution, the price set by the seller will be determined. Given all buyers playing I, the seller will be able to achieve a deal if there is at least one interested buyer, since strategy I ensures $\chi \geq P$. If this is true, the seller picks a random interested buyer, pays c_A once and receives the surplus P. The seller's expected payoff if all buyers play I is given by

$$E\Pi_S^I = PD^I \left(-c_A + P\right), \tag{2.9}$$

where PD^I, the probability of achieving a deal given all buyers play I, is given by $1 - F(P)^B$. At least one buyer must learn $\chi \geq P$ to signal interest. The seller's expected payoff if all buyers play U can be constructed accordingly. If all buyers play U, all B buyers signal interest. This implies that the seller pays c_A at least once, since he will approach at least one interested buyer. With probability $1 - F(P)$, $\chi \geq P$ holds and a deal is achieved with the buyer approached first and the surplus P is realized. With the counter probability $F(P)$, no deal is possible. Thus the seller picks another interested buyer, incurs cost c_A again and has the same chance of achieving a deal with this new interested buyer. In the worst case the seller will not be able to achieve a deal with any of the B interested buyers. In this case he pays c_A B times. Following from this logic, the seller's expected payoff given all buyers play U is

$$\begin{aligned} E\Pi_S^U = -c_A + (1-F(P))P + F(P)\Big(&-c_A + (1-F(P))P \\ &+F(P)\Big(-c_A + (1-F(P))P \\ &+ F(P)\left(\ldots\right)\Big)\Big) \end{aligned}$$

and can be simplified to

$$E\Pi_S^U = \frac{1-(F(P))^B}{1-F(P)}\big(P(1-F(P))-c_A\big). \tag{2.10}$$

Comparing $E\Pi_S^I$ and $E\Pi_S^U$ given by Equations (2.9) and (2.10) reveals that $E\Pi_S^I \geq E\Pi_S^U$ is always true.[16] This is quite intuitive: Whenever buyers play U, the seller needs to identify appropriate buyers, which is costly. If buyers play I, this is done by the buyers. Furthermore,

$$E\Pi_S^I\Big|_{P=0} = -c_A \ , \qquad\qquad E\Pi_S^I\Big|_{P=1} = 0 \ , \tag{2.11}$$

$$E\Pi_S^U\Big|_{P=0} = -c_A \qquad \text{and} \qquad E\Pi_S^U\Big|_{P=1} = -Bc_A \tag{2.12}$$

hold. The seller is always able to give the good away for free to the first approached buyer. $P = 0$ also ensures that all potential buyers signal interest if all play I. Demanding a prohibitive price, he will not receive any notifications of interest if the buyers play I. However, all B buyers declare interest if they play U. Then the seller will approach each of them and pay c_A every time, but he will not be able to achieve a deal. Evaluating the slopes of the two expected payoffs of the seller at $P = 0$ and $P = 1$ reveals

$$\frac{\partial E\Pi_S^I}{\partial P}\Big|_{P=0} = 1 > 0 \ , \qquad \frac{\partial E\Pi_S^I}{\partial P}\Big|_{P=1} = -Bf(1)(1-c_A) < 0 \ , \tag{2.13}$$

$$\frac{\partial E\Pi_S^U}{\partial P}\Big|_{P=0} = 1 - f(0)c_A > 0 \quad \text{and} \quad \frac{\partial E\Pi_S^U}{\partial P}\Big|_{P=1} = -Bf(1)\left(1 - c_A\frac{B-1}{2}\right) < 0 \tag{2.14}$$

where f is the pdf of χ. A marginal rise in price from $P = 0$ will translate to an equal rise in expected payoff if buyers play I. If buyers play U, the seller now faces the risk of approaching a buyer with $\chi < P$. This effect is strong if probability mass lies on small realizations of χ. The positive slopes at $P = 0$ and the negative ones at $P = 1$ imply that both expected payoffs exhibit at least one interior maximum.[17] The prices that maximize $E\Pi_S^I$ and $E\Pi_S^U$ are denoted by P^{*I} and P^{*U}. Lastly, $\frac{\partial E\Pi_S^I}{\partial P} \geq \frac{\partial E\Pi_S^U}{\partial P}$ is true for all P. For any P that implies $f(P) \geq 0$, the inequality is strict. This aspect is crucial and implies the two findings $P^{*I} > P^{*U}$ and $E\Pi_S^I \geq E\Pi_S^U$ for all P. See Figure 2.2 for examples of the two payoff functions.[18]

Given the single threshold Ω, the conditions $c < \Omega$ and $c \geq \Omega$ can be interpreted as incentive compatibility constraints and the maximization problem of the seller reads

$$\max_P \ E\Pi_S = \begin{cases} E\Pi_S^U = \frac{1-(F(P))^B}{1-F(P)}\big(P(1-F(P))-c_A\big), & \text{for } c < \Omega \\ E\Pi_S^I = \left(1-F(P)^B\right)(P-c_A)\,, & \text{for } \Omega \leq c \end{cases} . \tag{2.15}$$

Since $\Omega = \frac{1}{F(P)}$, the seller can provide incentives for buyers to behave in a certain way. Note that Ω (and both Ω^I and Ω^U) fall in P; see Appendix A.5. Therefore, the seller will induce buyers to play I if he sets a sufficiently large P; conversely, he can induce buyers to play U

[16] See Appendix A.4 for the formal parts of the discussion here.

[17] The normalization $\chi = 1$ obviously implies $P \in [0, 1]$. Thus $c_A \leq 1$ such that $1 - f(0)c_A > 0$ follows.

[18] Note that the solution of the seller's stage does not require f to be single-peaked. See Appendix A.6 for an example.

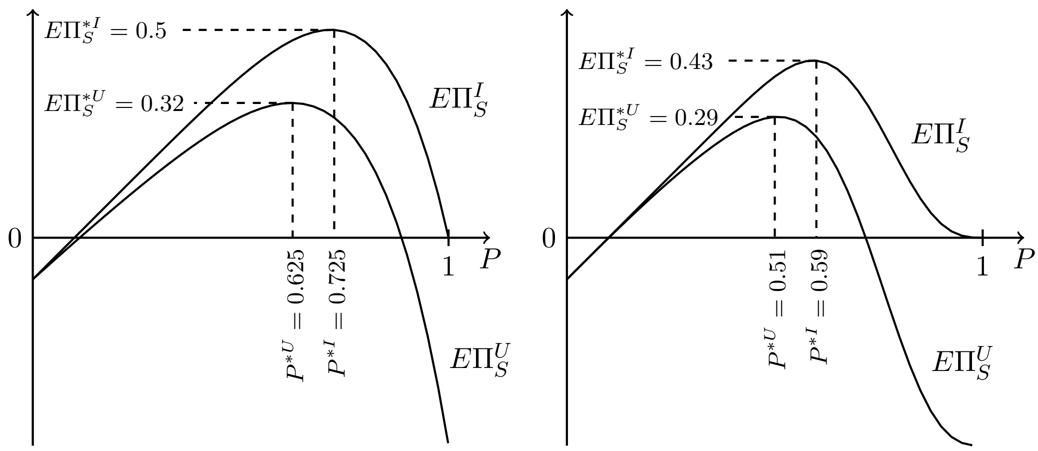

Figure 2.2: The shape of the seller's expected payoff
Curvature of $E\Pi_S^I$ and $E\Pi_S^I$ for different F. Underlying parameters: $c_A = 0.1$, $B = 5$ and willingness to pay $\chi \sim U[0,1] \hat{=} \beta(1,1)$ (left), $\chi \sim \beta(3,3)$ (right).

for low prices. The seller's influence on the buyers' behavior is shaped by the relative costs c. One critical price $\widetilde{P}$ exists for each c that implies $\frac{1}{F(\widetilde{P})} = c$. For any $P < \widetilde{P}$ the buyers play U and for $P \geq \widetilde{P}$ buyers play I. See Figure 2.3 for an illustration of the logic presented in the following. The figure displays the expected payoffs obtained before (left) and the resulting optimal prices P^* (right). Due to $E\Pi_S^I \geq E\Pi_S^U$, setting the price P^{*I} is the best choice if $P^{*I} \geq \widetilde{P}$ (e.g. the case of $\widetilde{P}^1$ in Figure 2.3). Here the incentive compatibility constraint is not constraining the seller effectively. The seller faces a binding constraint whenever $\widetilde{P} > P^{*I}$. He can no longer induce the buyers to play U by setting P^{*I}. Furthermore, since $E\Pi_S^I$ falls for all $P > P^{*I}$, the price that maximizes the seller's payoff while inducing buyers to play I is the lowest possible price, $\widetilde{P}$. Two scenarios must be distinguished. First, if $\widetilde{P}$ is sufficiently small (e.g. $\widetilde{P}^2$ in Figure 2.3), the seller obtains a higher expected payoff by setting $P = \widetilde{P}^2$ than by setting the price P^{*U}. Note that $P = P^{*U}$ is always feasible and incentive compatible if $\widetilde{P} > P^{*I}$ since $P^{*U} < P^{*I}$. Second, if $\widetilde{P}$ is sufficiently large (e.g. $\widetilde{P}^3$), the seller will no longer try to induce buyers to play I and will set $P = P^{*U}$. The critical $\widetilde{P}$ where this change occurs is the one that sets the seller indifferent and satisfies $E\Pi_S^I(\widetilde{P}) = E\Pi_S^U(P^{*U})$. For the parametrization underlying Figure 2.3, this price is $P = 0.89$. For simplicity we denote $c(\widetilde{P})$ as the cost ratio that implies the critical price $\widetilde{P}$ and $\widetilde{P}(c)$ vice versa.

Concluding the solution of the SSM, the optimal prices P^* that are set by the single seller for different values of the cost ratio c are given by

$$P^*(c) = \begin{cases} P^{*U} & \text{for } c \text{ that imply } P^{\text{indif}} < \widetilde{P}(c) \textit{ (low values of c)}, \\ \widetilde{P} & \text{for } c \text{ that imply } P^{*I} < \widetilde{P}(c) \leq P^{\text{indif}} \textit{ (medium values of c)}, \\ P^{*I} & \text{for } c \text{ that imply } \widetilde{P}(c) \leq P^{*I} \textit{ (high values of c)} \end{cases} \tag{2.16}$$

where P^{indif} is the highest price that is set by the seller. It is given by $E\Pi_S^U(P^{*U}) = E\Pi_S^I(P^{\text{indif}})$, since it renders the seller indifferent. Note that buyers play U for $P^* = P^{*U}$ and I in all other cases.

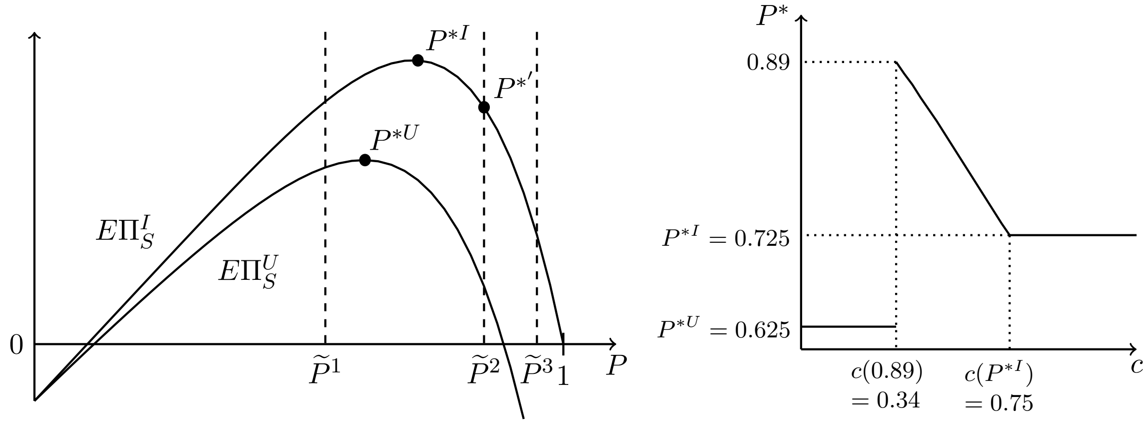

Figure 2.3: Optimal pricing scheme of the seller
Derivation of the optimal prices P^* given different critical prices $\widetilde{P}$. The expected payoffs (left) are from the left graph of Figure 2.2. Underlying parameters: $c_A = 0.1$, $B = 5$ and $\chi \sim U[0,1] \widehat{=} \beta(1,1)$.

2.4 Findings of the SSM

2.4.1 Pricing in the SSM and Lester's paradox

Figure 2.4 displays the function $P^*(c)$ given by Equation (2.16) for different values of B.[19] The general structure of P^* does not depend on the number of buyers; hence, two conclusions follow. First, the single seller tries to provide incentives for buyers to acquire information. As argued before, it falls upon the seller to identify appropriate buyers whenever buyers signal interest uninformedly, and this is costly for him. By raising the price, the seller can induce buyers to play I. Given any c, the seller can always induce buyers to play I by demanding a price that is sufficiently close to $\overline{\chi} = 1$; see Equations (2.4) and (2.8). However, raising the price lowers the chance that a buyer learns $\chi \geq P$ and declares interest. Thus, the seller faces an increasing probability of being unable to sell his good. When c is low such that the lowest price P that would induce buyers to play I is very large (formally $P = P^{\text{indiff}}$; see Equation (2.16)), this negative effect dominates the positive one. In these cases the seller forfeits his objective to induce informed buyers and adapts his pricing to the uninformed buyers by lowering his price to $P^* = P^{*U} < P^{*I}$. This sacrifice of potential surplus is profitable, since a lower price limits the expected number of instances he has to approach a buyer to achieve a deal. This process leads to the second observation. Lester's paradox states that a higher share of informed buyers who can engage in directed compared to undirected search may lead to higher prices on posted offer markets with capacity constrained sellers. The function $P^*(c)$ (Equation (2.16)) exhibits exactly these features, although the causation here is reversed. The seller deliberately sets higher prices to ensure that buyers become informed and is not reacting to an exogenous increase in the share of informed buyers.

[19] To account for the small numbers of buyers, the graphs were constructed using the thresholds Ω^I and Ω^U instead of the limit case. Thus, the implied critical prices $\widetilde{P}$ as shown in Figure 2.3 are now split into a lower and upper boundary. We still restrict buyers to play pure strategies such that no pure strategy equilibrium exists on the buyers' stage for prices between these boundaries. For these we set $E\Pi_S = 0$ such that the seller never chooses such a price.

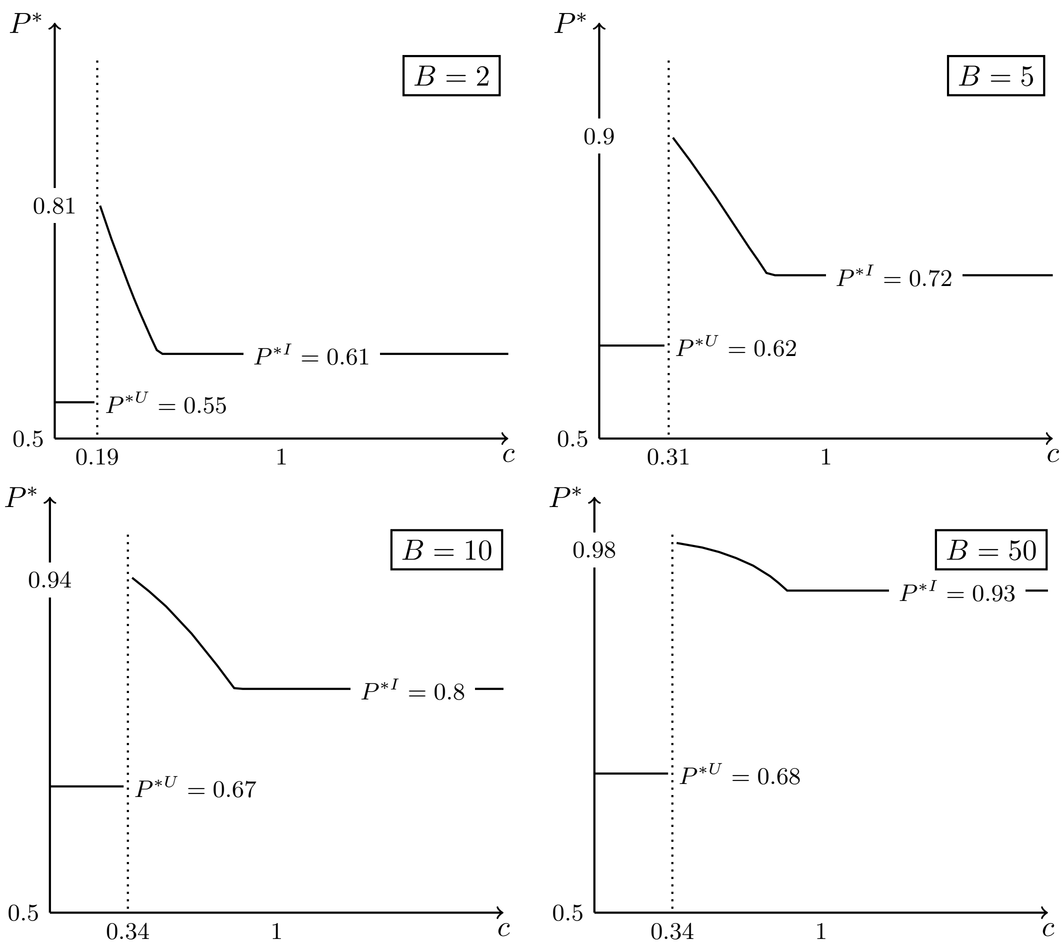

Figure 2.4: Equilibrium prices
Prices P^* in the unique symmetric equilibria given the cost ratio c for different numbers of buyers. Underlying parameters: $c_A = 0.1$ and $\chi \sim U[0,1] \hat{=} \beta(1,1)$.

Comparing the specific shapes of the functions $P^*(c)$ for different numbers of buyers (Figure 2.4) reveals standard features. For given thresholds, the seller is able to charge a higher price if he is confronted with a larger number of buyers. Yet, note that the thresholds where the behavior of the buyers changes depend on the number of buyers. An increase in B raises the thresholds Ω^I and Ω^U, since the probabilities of being asked fall. Thus it becomes harder to induce the buyers to play I. However, this effect can be neglected for $B \geq 10$, as becomes apparent in Figure 2.4.

2.4.2 Efficiency and welfare in the SSM

See Figure 2.5 for the expected payoffs of the seller, an individual buyer and the sum of all payoffs that are implied by the prices P^* displayed in Figure 2.4. Clearly, the shapes of $E\Pi_S$ and $E\Pi_B$ exhibit the features identified above. The seller is worse off given uninformed buyers. He induces buyers to become informed for some c such that his expected payoff declines smoothly. An individual buyer obtains an expected payoff around zero. This result is implied by the model, since buyers are price takers. However, she is better off if she can play U instead of I. In particular, the buyers are hurt by the seller's attempt to keep them from playing U. This finding is best visible for $B = 2$ but generally true. Lastly and disregarding the transition from I to U, $E\Pi_B$ and therefore also the sum of the expected payoffs rises if c falls. Caused by the falling costs, this increase is best visible in the graph for $B = 50$.[20] Knowing this, one can understand the different slopes of the sum of the payoffs: Whenever buyers play U, all buyers signal interest unconditionally. Thus, for small c in these instances, all buyers will profit from the falling cost ratio compared to only those who learn $\chi \geq P$ whenever buyers play I.

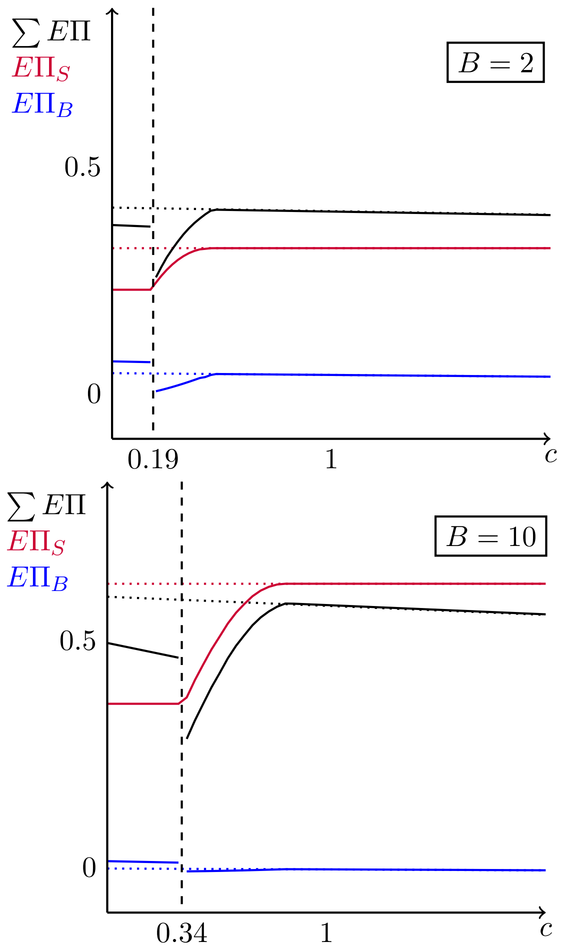

Figure 2.5: Individual expected payoffs and welfare
Expected payoffs of the seller (red), an individual buyer (blue) and the sum of all expected payoffs $E\Pi_S + B \cdot E\Pi_B$ (black) given the cost ratio c for various numbers of buyers. Dotted graphs assume that buyers always play I. Underlying parameters are the same as in Figure 2.4, furthermore $c_R = 0.01$.

Next we focus on efficiency and welfare. Concerning welfare we take a utilitarian stance and consider the sum of all expected payoffs as well as the Pareto criterion. An appropriate measure of the efficiency of the reached equilibria is introduced later. Two viewpoints exist when approaching the issue of efficiency and welfare. First, the question of how a falling cost ratio c affects welfare (a move along the x-axis) is addressed. For this exercise only the solid lines in Figure 2.5 are of interest. Whenever the seller is able to set $P^* = P^{*I}$, a marginal decrease in c implies higher welfare. This is straightforward: The behavior of the agents is unaffected, the same price is posted and only (at least one) cost parameter falls. However, given $P^* = \widetilde{P}$ (see Equation (2.16)) the seller induces buyers to play I and a falling cost ratio results in lower

[20] The term *falling cost ratio* implies that neither c_R nor c_I rise. Note that Figure 2.5 sets $c_R = 0.01$ such that c_I is varied to imply different cost ratios.

welfare. This outcome is due to the increase in the posted price, since the corner solution price $P^* = \widetilde{P}$ rises if c falls. Notably, the decline of welfare constitutes a worsening according to the Pareto criterion. For $P^* > P^{*I}$ (e.g., $P^* = 0.9$ at $c = 0.31$ for $B = 5$ in Figure 2.4) that imply action I, both the seller and all buyers profit if the cost ratio is increased by some exogenous mechanism such that the seller can set $P^* = P^{*I}$. This result holds for all numbers of buyers. If c is low and the seller accommodates with buyers playing U and sets $P^* = P^{*U}$, welfare increases if c falls. For the cost ratios that lead to the equilibria where buyers do not acquire information, the buyers' expected payoff is larger than in all other scenarios, and buyers profit from a falling cost ratio. As before, there occurs no more change in equilibrium behavior if the cost ratio falls such that the seller's payoff is flat. A large decline in c such that a transition from $P^* = P^{*I}$ to $P^* = P^{*U}$ is implied results in falling welfare for most scenarios, even if c falls to zero. The gain of an individual buyer from this transition is small compared to the loss of the seller. Therefore, an equilibrium where buyers play U can only be welfare maximizing at $c = 0$ and if the number of buyers is sufficiently large. Here, only the graph for $B = 50$ exhibits this feature.

The second view on efficiency concerns the behavior of the agents given a certain cost ratio. Imagine the problem of a social planner in this context. Our measure for efficiency follows the usual approach in the related literature and is based on the concept of constrained efficiency, meaning that the planner's information mirrors that of the agents in the model. This approach is chosen because it constitutes an appropriate comparison to the equilibrium outcomes of the model. The planner who is constrained in his information has no way to tackle inefficiencies caused by information problems, therefore we let the planner choose buyer strategies such that any occurring coordination problems can be resolved.[21] The dotted lines correspond to the scenario in which the buyers' strategy is exogenously fixed such that they play action I always. Whenever the black dotted lines lie above the solid black lines, the equilibrium behavior is not constrained efficient, since a planner, even without an advantage in information, can improve welfare. Similarly as argued before, buyers face a commitment problem for $P^* > P^{*I}$, and a Pareto improvement is possible. Buyers have an incentive to commit to action I publicly before the seller sets his price. This is not possible here since no device exists to enforce the commitments, and the buyers fall prey to their individual utility maximization. This scenario resembles a public goods dilemma, it that buyers over-exploit a common resource (i.e., the seller's attention). Each of the buyers would be better off by constraining themselves, but they fail to do so.[22] As before, if c is sufficiently small such that action U is implied, the buyers' incentive to commit to I is erased. However, if the number of buyers is not too large, the seller's solution still constitutes an improvement in terms of utilitarian welfare. Only $B = 50$ renders the obtained equilibrium behavior constrained efficient.

[21] Importantly, the planner is – like the buyers themselves – unaware of the realizations of χ. Hence, the planner is unable to improve the quality of the achieved matches.

[22] See Bram (2016), who provides a similar interpretation for the messaging behavior of men on dating platforms. In his case, the attention of women constitutes the public resource. Ultimately, women are driven off the platform due to excessive spam.

2.4.3 Choice of the equilibrium concept

Consider values of c where the buyers would profit from committing to action I before the seller chooses P (e.g., c close to but above 0.19 for $B = 2$ as exhibited in Figure 2.5). In these cases the results presented here are obviously driven by the focus on Nash equilibria, as this equilibrium concept does not grant any commitment power to the players. Employing the concept of strong Nash equilibria instead allows for deviations of coalitions such that groups of players can bind themselves to a certain behavior. The straightforward rationale why this concept is disregarded here as well as in the related literature is that it requires too much coordination between players. This argument seems convincing since buyers do not even share a common goal, as they are competing for a single good. Yet, the idea of coordination in not totally implausible, especially for low values of B. However, granting buyers the capacity to coordinate probably comes in hand with the ability to anticipate the behavior of fellow buyers. This dynamic is reflected by the concept of coalition-proof Nash equilibrium. Buyers are free to form coalitions and deviate in a coordinated way, as with a strong Nash equilibrium, but any coalition deviation must be coalition-proof itself. This stricture rules out deviations like the one where all buyers commit to action I, since each individual buyer who is a member of the deviating coalition faces an incentive to switch back to action U immediately after the deviation. Thus, the presented solutions are in line with the concept of coalition-proof Nash equilibrium.

2.5 The multiple sellers model (MSM)

The assumption of a single seller helps to clarify the price setting mechanism and the influence of the price on the buyers' behavior. Furthermore, many findings can be drawn from the SSM without considering the extended model. However, the assumption of only one seller constitutes a serious restriction. The extended model presented in this section overcomes this limitation and can therefore be considered as a robustness check. Furthermore, we allow buyers to react to more than one offer. This extension is worth the effort, since real-world posted offer markets (e.g. the labor or housing markets) clearly allow multiple applications of buyers. Sellers now face the danger that buyers who have a sufficiently large willingness to pay might nevertheless turn down the offer, since they have already accepted the offer of another seller. However, this extension renders an analytical solution of the model impossible. This fact seems surprising at first glance, yet even the simplest scenario that includes two sellers and two buyers is too complicated to solve for a closed form solution. Appendix A.7 illustrates the complexity that emerges in this simple setting. The combination of multiple applications and the sellers' ability to move along their queues and ask interested buyers sequentially leads to expected payoffs of buyers that are difficult to handle. As a result, the buyers' information decision cannot be identified properly, which in turn implies that the sellers' stage cannot be modeled accordingly.

2.5.1 Description of the MSM

Instead of one, there are now S sellers, each offering one good. Seller j posts price P_j. Each of the B buyers again seeks to buy one of the goods. We assume that all buyers have to react to all posted offers in the same way – either by playing I or U. This assumption is not uncommon (e.g., see (Bulow and Levin, 2006)) and is made for two reasons. First, it keeps the solution of the model tractable.[23] Second, it excludes ex ante competition between sellers in prices that arises if buyers have to make a portfolio choice when deciding where to apply. Notably, this assumption does not eliminate all competition for buyers among sellers, as it still allows ex post competition if a buyer is approached by several sellers. In a wider sense, the assumption reflects our focus on symmetric equilibria, since "vacancies [offers] are equally attractive ex ante" (Albrecht et al., 2006, p. 885).

Again, sellers receive notifications of interest from buyers. Similarly as before, these notifications carry no information on what action the respective buyers have played. All sellers try to achieve a deal with interested buyers at the same time. We assume the following mechanism for this process:

(1) Each active seller approaches a random interested buyer who has not been approached before by the respective seller. A seller is considered active as long as he has not yet sold his good and is confronted with a positive number of interested buyers.

(2) The buyers approached decide about accepting the offers of the respective sellers. A buyer approached by one or more sellers will accept the offer that yielding the largest weakly positive surplus. A buyer will decline all offers that lead to negative surpluses. Approached buyers who have signaled interest uninformedly incur c_R and learn their willingness to pay.

(3) All sellers eliminate the buyer they have approached from their queues of interested buyers if she has declined the offer.

These three steps are repeated as long as there are active sellers. No active sellers are left if either all sellers have sold their good or the sellers who have not yet sold it have no interested buyers left to approach. This mechanism implies three assumptions. First, buyers will opt for the offer that yields the largest surplus. Second, buyers do not wait strategically and turn down offers to wait for further approaches of other sellers that may lead to a larger surplus. Third, buyers who have already accepted an offer and are approached by a seller have to pay c_R again if they have signaled interest unconditionally to this seller. There are two justifications for this assumption. It puts the MSM closer to the SSM where approached uninformed buyers surely incurred c_R, and it reduces the complexity of the calculus greatly. Yet the assumption can be debated, since it seems unintuitive in some of the introduced examples. However, the cost c_R can be viewed as a loss that comes with turning down an offer. Exemplary, a buyer who is

[23]It will become clear that this assumption allows one to determine the buyers' behavior analytically, even with an endogenous decision of information acquisition. Giving buyers a portfolio choice of where to apply renders this impossible. Furthermore, the implied computational solution becomes much more complex, since the size of the joint strategy space multiplies.

offered a job or a place in a degree program is required to take some action to turn the offer down.

2.5.2 Strategies and equilibrium concept in the MSM

Strategies of buyers are mappings of prices to actions, whereas the former strategies I and U are now actions. The strategies of the sellers are the prices they demand for their respective goods.

The equilibria are again subgame-perfect Nash equilibria, meaning that the buyers' strategies must imply Nash equilibria in the buyers' stage for any set of prices $\{P_j\}_{j=1}^{S}$. Furthermore, the set of prices implied by the sellers' strategies must be Nash as well, such that every seller plays a best response to the prices set by the other sellers and anticipates the buyers' equilibrium behavior. In the equilibrium, all buyers play the same strategy. In addition, we also assume symmetry in the sellers' stage and disregard all asymmetric equilibria.

2.5.3 Solution of the MSM

The solution is again done by backward induction. To distinguish the calculations that follow from the solution of the preceding model, the capital Greek letters are replaced by their lower case counterparts.

The buyers' stage in the MSM

The buyers' behavior is derived in a similar way as above. Let the prices posted by the sellers be $\{P_j\}_{j=1}^{S}$. If an individual buyer plays I for all offers – she signals interest if her willingness to pay exceeds the price of an offer – and the remaining buyers play the same strategy, the individual buyer's expected payoff is given by

$$E\pi_B^{I,I} = -c_R S - c_I \sum_{j=1}^{S} (1 - F(P_j)) + Pr_{deal}^{I,I} E_j \left(E(\chi|\chi \geq P_j) - P_j \right). \tag{2.17}$$

The interpretation of the expected payoff is simple. Always playing action I implies that c_R is incurred S times. c_I is paid if the buyer signals interest. In expectation, this happens $\sum 1 - F(P_j)$ times. With the probability of a deal from the perspective of the buyer ($Pr_{deal}^{I,I}$), the expected surplus is realized. If the buyer changes her strategy and plays U for all offers, she realizes the expected payoff

$$E\pi_B^{U,I} = -c_I S - c_R E(\#asked, I) + Pr_{deal}^{U,I} E_j \left(E(\chi|\chi \geq P_j) - P_j \right). \tag{2.18}$$

It is given that c_I is paid S times. c_R is incurred each time the buyer is asked by a seller. The expected number of times the individual buyer is asked if all other buyers play action I for all offers is characterized by $E(\#asked, I)$. With the probability of a deal the expected surplus is realized. As in the simpler model, $Pr_{deal}^{I,I} = Pr_{deal}^{U,I}$ holds, which is quite intuitive. Achieving a deal requires being asked by a seller and a willingness to pay more than the offered price. If all other buyers always play I, then the individual buyer is not able to affect the probabilities that these two prerequisites are met by her choice of strategy. Therefore, the last terms can be dropped and the comparison of the two expected payoffs, $E\pi_B^{I,I} \geq E\pi_B^{U,I}$, boils down to the inequality

$$\frac{c_I}{c_R} \geq \frac{S - E(\#asked, I)}{\sum_{j=1}^{S} F(P_j)} \equiv \omega^I. \tag{2.19}$$

Next, let all other buyers play action U for all offers. The expected payoffs of an individual buyer who also always plays U or deviates to I are given by

$$E\pi_B^{U,U} = -c_I S - c_R E(\#asked, U) + Pr_{deal}^{U,U} \left(E(\chi|\chi \geq P_j) - P_j\right) \text{ and} \tag{2.20}$$

$$E\pi_B^{I,U} = -c_R S - c_I \sum_{j=1}^{S} (1 - F(P_j)) + Pr_{deal}^{I,U} \left(E(\chi|\chi \geq P_j) - P_j\right) \tag{2.21}$$

where $E(\#asked, U)$ characterizes the expected number of instances the buyer is asked by sellers if the remaining buyers play U. By the logic presented above, $Pr_{deal}^{U,U} = Pr_{deal}^{I,U}$. The comparison $E\pi_B^{U,U} \geq E\pi_B^{I,U}$ yields

$$\frac{c_I}{c_R} \leq \frac{S - E(\#asked, U)}{\sum_{j=1}^{S} F(P_j)} \equiv \omega^U. \tag{2.22}$$

Note that both thresholds ω^I and ω^U are the multi-seller counterparts of Ω^I and Ω^U derived in model one. Accordingly, the thresholds characterize symmetric pure strategy equilibria in the buyers' stage. In the limit case with infinitely many buyers,

$$\lim_{B\to\infty} \omega^I = \lim_{B\to\infty} \omega^U = \frac{S}{\sum F(P_j)} \equiv \omega \tag{2.23}$$

is also valid here, since in both inequalities the expected number of instances a buyer is asked converges to zero for large values of B. In the following, we focus on this limit case.[24] Assuming $\chi \sim U[0,1]$ implies $F(P_j) = P_j$ and therefore $\sum F(P_j) = SF(\overline{P})$ where $\overline{P}$ is the average posted price $\overline{P} = \frac{1}{S}\sum P_j$. Hence, the assumption of the uniform distribution implies $\omega = \frac{1}{F(\overline{P})}$. This condition is exactly the same as in model one, with $\overline{P}$ replacing P. It allows the interpretation that buyers base their action on an observed market signal given by the average price. This simplification is not possible if $\chi \sim \beta(a,b)$ with any $a \neq 1$ and $b \neq 1$.[25] In these cases the market signal is given by a modified mean price $\overline{P}'$ that is obviously given by $\overline{P}' = F^{-1}\left(\sum F(P_j)/S\right)$, depending on the curvature and skewness of F.[26] As before, a unique critical (average) price

[24] Apart from simplicity, one reason is that in the limit case the buyers' behavior does not depend on the expected number of times a buyer is asked. This dynamic is crucial, since $E(\#asked, I)$ and $E(\#asked, U)$ in Equations (2.19) and (2.22) can only determined ex post using the decision rule based on the limit case. Appendix A.7 illustrates that a general analytical solution for the two expressions cannot be included in the model, even in the simplest case of $S = B = 2$.

[25] Recall the normalization $\overline{\chi} = 1$, such that the use of the beta distribution is appropriate.

[26] $\overline{P}'$ follows from $\frac{S}{\sum F(P_j)} = \frac{1}{F(\overline{P}')}$.

$\widetilde{P}$ exists that implies $\frac{c_I}{c_R} = \frac{1}{F(\widetilde{P})}$. If we let $\omega = \frac{1}{F(\overline{P}')}$, then all buyers will play U as a response to all offers if $\overline{P}' < \widetilde{P}$ (since this implies $\frac{c_I}{c_R} < \frac{1}{F(\overline{P}')}$) and I for $\widetilde{P} < \overline{P}'$.

Formally, the strategy followed by the buyers (σ_B) is given by the assignment

$$\sigma_B :\ a\left(\overline{P}'\right) = \begin{cases} U, & \text{for } \overline{P}' < \widetilde{P} \\ I, & \text{for } \widetilde{P} \leq \overline{P}' \end{cases} \qquad \text{with} \qquad \widetilde{P} = F^{-1}\left(\frac{c_R}{c_I}\right) \tag{2.24}$$

where a characterizes the action that as a response to each offer and $\overline{P}'$ is the market signal, as introduced above. Note the assumption that buyers play I whenever $\overline{P}' = \widetilde{P}$.

The sellers' stage in the MSM

First, note that an individual seller's influence on the behavior of the buyers is limited, since buyers evaluate the market signal $\overline{P}'$. Given $P_{-m} = \{P_j\}_{j \neq m}$, the prices of all other sellers, buyers might play the same action for all $P_m \in [0,1]$ – every possible price that can be set by seller m. Second, P_m, the strategy played by seller m, can only be an element of an equilibrium if it is a best response to $P_{-m} = \{P_j\}_{j \neq m}$, the prices of all other sellers, and considering the optimal behavior of the buyers characterized by strategy σ_B^*. As stated, we focus on symmetric equilibria where all sellers play the same strategy and thus choose the identical price P^*. We determine this price numerically. Let all sellers except seller m play the same strategy, such that P_{-m} also characterizes the price set by all sellers $j \neq m$. Now, $P_m^*(P_{-m})$ is seller m's best response function to the uniform behavior of the remaining sellers. As the identity of seller m is arbitrary, the price P^* of a symmetric equilibrium is obviously a fixed point of this best response function $P_m^*(P_{-m})$. We obtain the solutions of the MSM with a computational approach. The ensuing steps and the underlying logic are described in the following.[27] Although the model described is simple in its structure, it gives rise to highly complex and potentially discontinuous utility functions. The two main drivers of this complexity are the endogenous decisions of information acquisition by buyers and the mechanism by which sellers try to achieve a deal with interested buyers. An analytical solution is complicated but possible in the case of multiple applications and directed search (by informed buyers): for example, as in Galenianos and Kircher (2009). However, these authors look at one-shot matching attempts only. Sellers who were rejected by the buyer they have approached cannot approach another buyer in their queue and are left unmatched. Kircher (2009) includes this feature in his model but has to assume continuums of agents.[28] Lastly, we are unaware of any analytical solutions of undirected search with multiple applications and sequential matching of sellers and interested buyers. Thus, since we want to find out when buyers decide to switch from acquiring information (directed search) to remaining uninformed (undirected search), the computational approach is chosen.

Concerning the shape of the best response function, two features are certain. First, $P_m^* > 0$ for

[27] For a complete description of the algorithm employed and further remarks see Appendix A.8.

[28] This implies that entry of additional sellers does not alter an individual buyer's utility. Thus, the *market utility property* holds (refer to Galenianos and Kircher (2012), which focuses on the microfoundations of this assumption). This assumption is only true in the limit; therefore, we abstain from making it and focus on "small markets" (Lester, 2011).

all $P_{-m} \in [0,1]$. Giving the good away for free is never optimal, since marginally raising the price does not alter the probability of selling it and leads to a strictly larger payoff. Second, $P_m^* < 1$ for all P_{-m}. Demanding a prohibitively high price eliminates seller m's chances to sell the good. Thus, whenever $P_m^*(P_{-m})$ is continuous, the existence of at least one fixed point is guaranteed. Bear in mind that the best response function may be discontinuous if seller m is able to change the behavior of the buyers. Recall the definition of the market signal $\overline{P}'$ in the preceding section and the threshold $\widetilde{P}$. Utilizing the symmetric price P_{-m} and solving $\overline{P}' = \widetilde{P}$ for P_m yields

$$\widetilde{P}_m = F^{-1}\left(S\frac{c_R}{c_I} - (S-1)F(P_{-m})\right). \tag{2.25}$$

Here $\widetilde{P}_m$ is seller m's critical price that allows him to change the buyers' behavior given the price that is set by all other sellers. If $\widetilde{P}_m \notin [0,1]$, then seller m cannot influence the buyers' behavior. This is obviously true for $\frac{c_I}{c_R} \leq 1$. The numerical determination of the best response function is based on a Monte Carlo approach to determine the expected payoffs of seller m given P_m and P_{-m}.

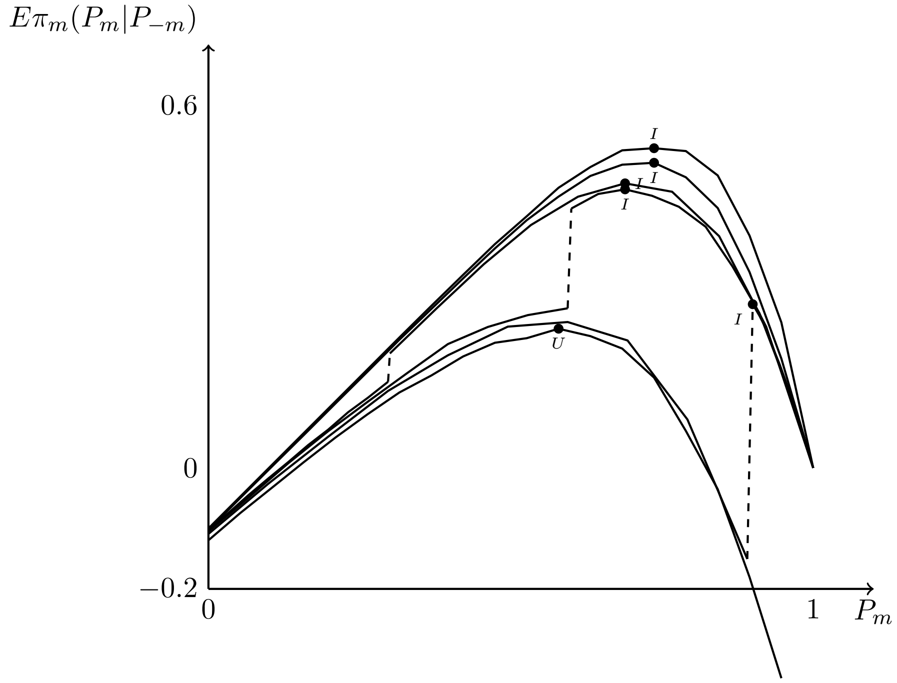

Figure 2.6: Expected payoff of seller m given different P_{-m}
Graphs $E\pi_m(P_m|P_{-m})$ for different values of P_{-m}. From top to bottom: $P_{-m} = 1$, 0.8, 0.6, 0.45, 0.3 and 0. Maxima and the implied buyers' behavior are highlighted. Underlying parameters: $B = 6$, $S = 3$, $c_A = 0.1$ as well as $\chi \sim U[0,1]$ and $c_I = 0.02$, $c_R = 0.01$, $c = 2$. MC-configuration: The P_M grid includes 20 values, $P_{-m} \in \{0, 0.05, \ldots, 1\}$, some graphs omitted for clarity. 5000 simulations for each point in the joint strategy space.

Figure 2.6 above features different graphs of expected payoffs of seller m for varying values of P_{-m}. It becomes visibly clear that different values of P_{-m} lead to different critical prices $\widetilde{P}_m$. The maxima are highlighted; furthermore, at each maximum the behavior of the buyers implied by the respective prices is indicated. Obviously the location of the maxima depends to some extent on the grid of prices used.[29] This limitation can be overcome with a polynomial approximation of the obtained graphs of expected payoffs $E\Pi_m(P_m|P_{-m})$ and by considering the maxima of these approximations. Note that for each P_{-m} that implies $\widetilde{P}_m \in (0,1)$ the expected payoff is discontinuous and the polynomial fitting has to be split in two parts to account for this. The expected payoffs in Figure 2.6 have not been smoothed by a polynomial approximation. The shape of the graphs exhibited there highlights that the procedure of polynomial approximation does not impact the findings, apart from the desired smoothing to overcome the limitations of a price grid.

The best response function that emerges from the expected payoffs displayed in Figure 2.6 is shown in Figure 2.7. Overall, the best response function obtained resembles the structure of the equilibrium prices of the SSM shown in Figures 2.3 and 2.4. Given the price of the other sellers, seller m will, to some extent, provide the incentives for buyers to play I instead of U. If P_{-m} becomes too small, providing these incentives becomes impossible or too expensive. The gap in the best response correspondence indicates where this change occurs. Keep in mind that our interest lies in the intersection of the best response correspondence with the 45 degree line, the fixed point. For the costs $c_I = 0.02$ and $c_R = 0.01$, this fixed point lies at 0.7. The location of the fixed point can be found with greater precision by repeating the first step of determining the expected payoffs $E\Pi_m(P_m|P_{-m})$ for a smaller grid of P_{-m} values around the obtained fixed point candidate 0.7.

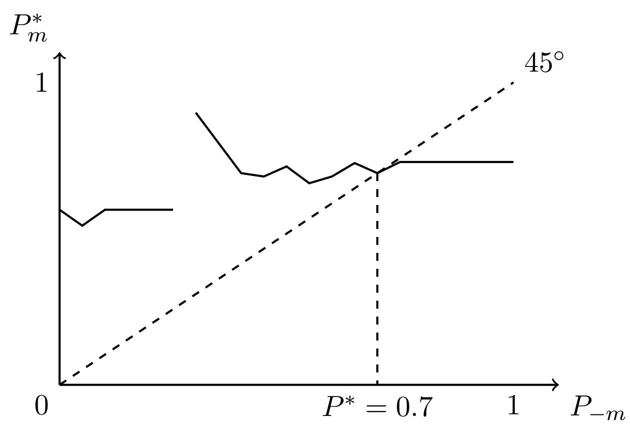

Figure 2.7: Best response function of seller m
Function $P^*(P_{-m})$ implied by the expected payoffs in Figure 2.6.

Assuming different cost parameters or changing the numbers of sellers and buyers obviously implies different best response functions. Theoretically three outcomes are possible. First, for some parameters the best response function is continuous, since the considered seller cannot alter the buyers' behavior. In these cases the existence of a fixed point is guaranteed. If this is not the case and the function exhibits a discontinuity, the overall shape of the best response function does not depend on the chosen parameters. Second, if the location of the discontinuity overlaps with the 45 degree line, there may be no fixed point.[30] Third, best response functions may exhibit two fixed points. Increasing the cost ratio c moves the discontinuity to the left,

[29]For each P_{-m} that implies $\widetilde{P}_m \in (0,1)$, the price grid of seller m includes $\widetilde{P}_m$ (where all buyers play I) and a price slightly lower than this critical price.
[30]This outcome is highly unlikely. However, see Appendix A.8 for a discussion.

since it becomes easier to induce buyers to play I. The opposite is true if the cost ratio falls. In this context changing the cost ratio to $c = 1.3$ and holding all other assumptions fixed gives rise to the best response function displayed in Figure 2.8, which exhibits two fixed points.[31]

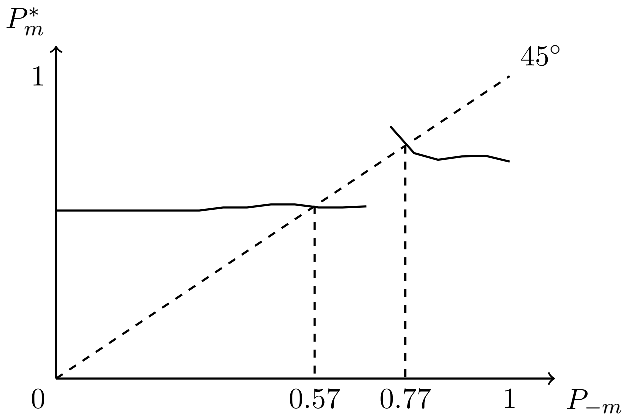

Figure 2.8: Best response function of seller m (two fixed points)
Function $P^*(P_{-m})$. Underlying parameters: $B = 6$, $S = 3$, $c_A = 0.1$ as well as $\chi \sim U[0, 1]$ and $c_I = 0.013$, $c_R = 0.01$, $c = 1.3$. MC-configuration: The P_M grid includes 20 values, $P_{-m} \in \{0, 0.05, \ldots, 1\}$, some graphs omitted for clarity. 5000 simulations for each point in the strategy space.

The multiplicity in fixed points implies that two Nash equilibria of the game exist for the model parameters as given under Figure 2.8. Note that the buyers always play the same strategy σ_B, as given by Equation (2.24). Due to this fact and because buyers are the second movers, the assumption of subgame perfection rules out incredible threats and a possible second mover advantage. The first stage of the game can be interpreted as a coordination game between sellers. The realized expected payoffs of seller m are $E\pi_m(P_m = P_{-m} = 0.57) = 0.2705$ and $E\pi_m(P_m = P_{-m} = 0.77) = 0.4848$. Hence, the equilibrium where $P_* = 0.77$ is set by all sellers is payoff dominant over $P_* = 0.57$. This pattern is persistent for different model parameters. Given there are two fixed points, there is always one that induces buyers to play U and one where they play I. As argued, the latter one is always preferred by all sellers, since sellers are better off when confronted with informed buyers. Furthermore, let sellers randomize and play $P = 0.57$ and $P = 0.77$ with equal probability: $\sigma_S = \{0.57, 0.5; 0.77, 0.5\}$. The expected payoffs of seller m who plays a pure strategy and faces only randomizing counterparts are given by $E\pi_m(P_m = 0.57, P_{-m} = \sigma_S) = 0.4326$ and $E\pi_m(P_m = 0.77, P_{-m} = \sigma_S) = 0.4603$. Therefore, the equilibrium with $P^* = 0.77$ even risk dominates $P^* = 0.57$. Whenever equilibrium selection is required (these cases are rare), we follow Harsanyi (1995) and choose the risk dominant equilibrium to resolve multiplicity in the fixed points. Hence, the solution of the model is given by $P^* = 0.77$ and the buyers' strategy σ_B.[32] It is worth mentioning that concepts of payoff and risk dominance coincide in the majority of cases. Furthermore, given multiplicity in fixed points, risk dominance identifies the price that induces buyers to play I as the relevant equilibrium in the nearly all cases. Importantly, since welfare is presumably larger there (recall the results

[31] Here the underlying expected payoffs $E\pi_m(P_m|P_{-m})$ have been smoothed by a polynomial approximation (compared to the example given by Figures 2.6 and 2.7).

[32] In addition, $P^* = 0.77$ (and following equilibrium play by the buyers) is the unique symmetric pure strategy coalition proof Nash equilibrium in the given example (see the discussion of the equilibrium concept in Section 2.4.3). Given the equilibrium $P^* = 0.57$, sellers can coordinate on a joint deviation that is coalition proof itself.

derived in the SSM), our findings concerning welfare are not driven by the equilibrium selection.

As has been tested in the SSM, one can show that non single-peaked distributions of χ do not render the solution of the MSM impossible.[33] To conclude the solution of the MSM, note that the algorithm sketched here must be used to obtain the symmetric equilibrium price for all cost ratios c of interest, to produce the desired correspondence of c and P^*.

2.6 Findings from the MSM

2.6.1 Pricing in the MSM

Figure 2.9 exhibits the equilibrium prices that are reached for different numbers of buyers and sellers. The thresholds displayed (vertical dashed lines) at the discontinuities indicate the cost ratios at which the buyers' behavior changes. Buyers play I for those larger than the thresholds and U for the ones lower than the thresholds. The prices obtained exhibit standard properties. Lowering the number of buyers (rows, from right to left) or increasing the number of sellers (columns, from top to bottom) while holding the respective other number fixed leads to falling prices. Furthermore, whenever there is excess demand, the critical cost ratios at which buyers change their behavior from I to U are lower compared to cases with excess supply. This dynamic resembles the higher market power of sellers in these scenarios.

Most important, the shape that was identified in the SSM is confirmed here. Prices are higher if buyers are informed, and sellers try to induce buyers to inform themselves by increasing the price if the cost ratio falls. Compared to the price set by the single seller (Figure 2.4), the most prominent difference is that the critical thresholds are much larger here, due to the competition between sellers that is introduced in the MSM.[34] Effectively, this competition creates a coordination problem and impairs sellers' ability to provide incentives for buyers to stay informed. Collectively raising the posted prices increases the incentives for individual sellers to deviate and set a lower price. Related to this mechanism, and forestalling some aspects concerning payoffs, note the following. As long as sellers are able to raise their prices together (e.g., for c close to but above 1.35 for $S = 12$ and $B = 12$), every seller enjoys an increase in his expected payoff (as captured by the upward bent solid red lines in Figure 2.10). This contrast to the findings in the SSM can be rationalized when considering the competition between sellers, which prevents cartel behavior on their part. In this sense, a falling cost ratio can help to limit competition, since the increased likelihood of inducing buyers to play U disciplines sellers.

The competition between sellers seems to be more fierce if buyers are uninformed. Facing only uninformed buyers might result in numerous attempts by an individual seller to sell his good. The number of these costly approaches can be lowered by demanding a lower price, such that the chance $pr(\chi \geq P) = F(P)$ is increased. If all sellers follow this logic, nobody is better

[33] See Appendix A.9.

[34] We have assumed that buyers have to react to all offers. Thus, sellers do not compete for signals of interest and competition means "ex post competition" as characterized in Galenianos and Kircher (2009, p. 455). We are able to obtain equilibriums without wage dispersion, since buyers face no portfolio choice and posted prices are binding.

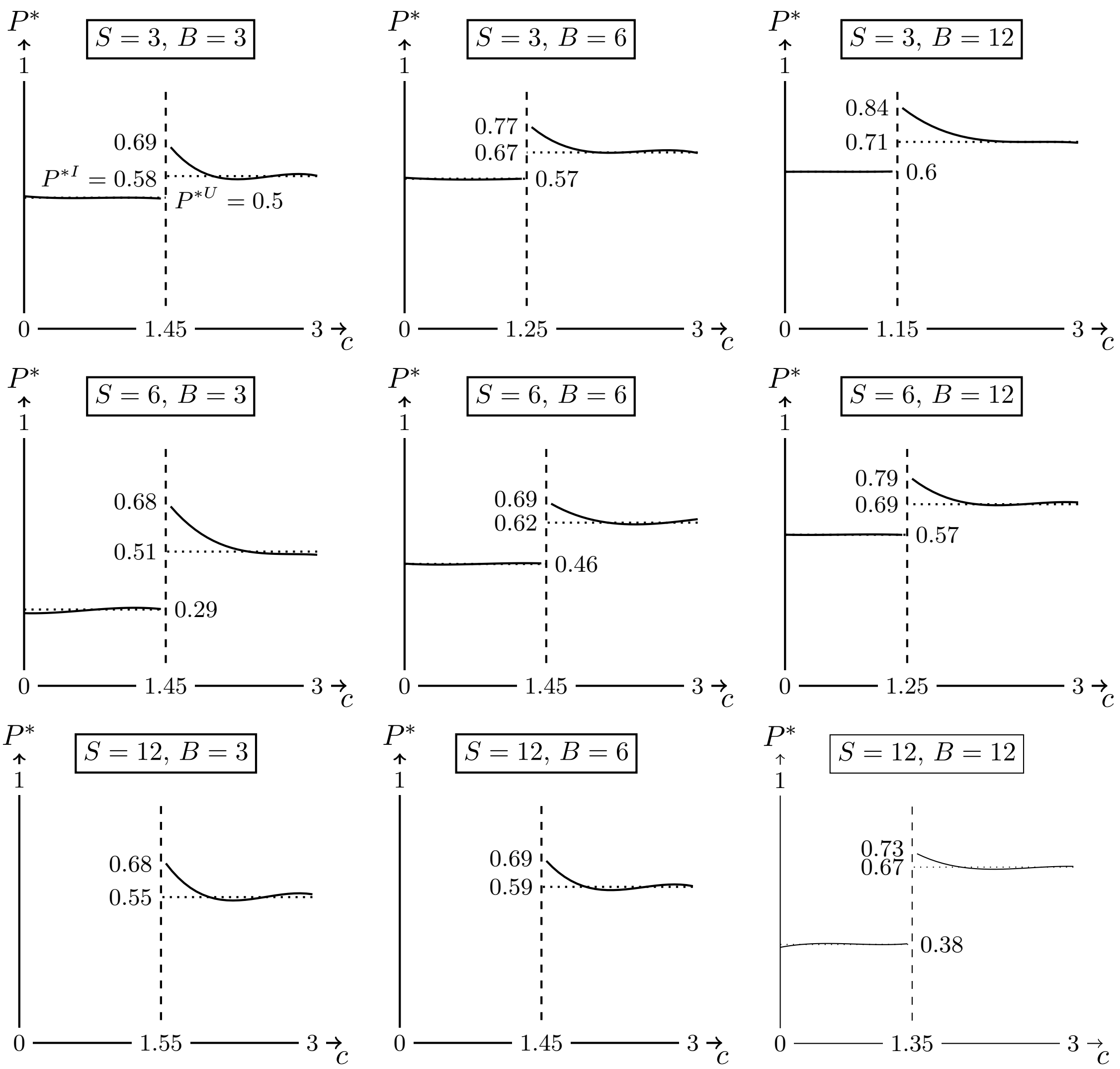

Figure 2.9: Equilibrium prices
Symmetric equilibrium prices P^* given the cost ratio c for various numbers of buyers and sellers. Underlying parameters: $c_A = 0.1$ and $\chi \sim U[0,1] \hat{=} \beta(1,1)$. MC-configuration: For each combination of S and B both grids of P_M and P_{-m} include 16 values. 1000 simulations for each point in the strategy space.

off (except the buyers), and prices deteriorate. Cases of excessive supply (e.g. for $S = 12$, $B = 3$ and $S = 12$, $B = 6$), result in Bertrand competition, and the equilibrium prices given uninformed signaling are zero.[35] Bertrand competition does not unfold if buyers are informed. Here it is sufficiently unlikely to lose an interested buyer to another seller. Due to the higher price, the probability $pr(\chi \geq P)$ is smaller, such that an interested buyer standing in the queue of a particular seller is unlikely to have expressed interest to another seller. A final insight is yielded in the scenario $S = 6$, $B = 3$. Despite its featuring the same ratio of sellers over buyers as $S = 12$, $B = 6$, the equilibrium price with uninformed sellers does not fall to zero. Therefore, it seems that both S and B matter in terms of relative and absolute size. The same inference can be drawn by comparing the scenarios on the diagonals, $S = B = 3$, $S = B = 6$ and $S = B = 12$. Here prices are affected by a proportional increase of S and B.

[35]Obviously, another factor that favors this sharp decline in prices is the assumption that sellers cannot drop out of the game and have to sell their good.

2.6.2 Efficiency and welfare in the MSM

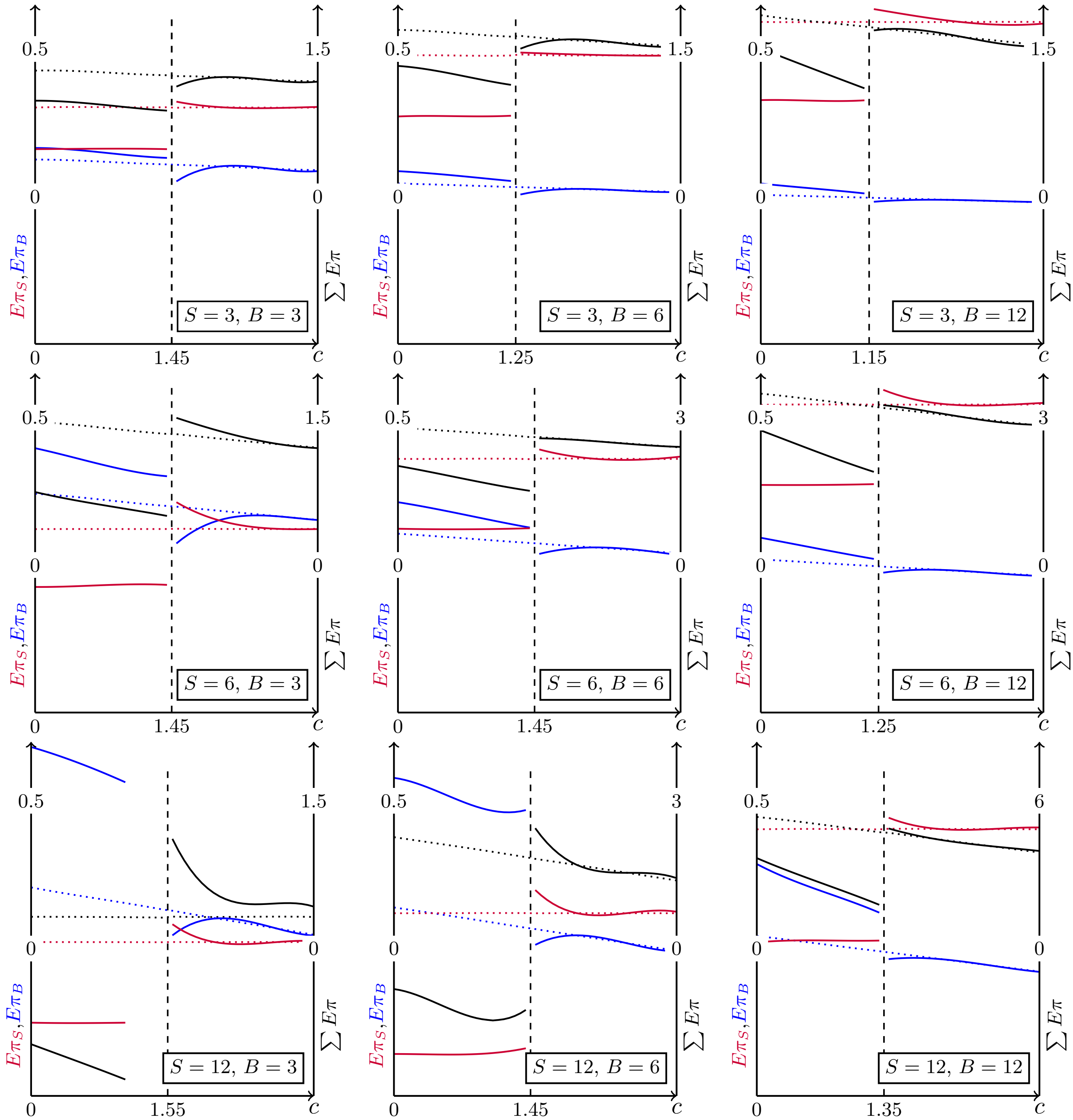

Figure 2.10: Individual expected payoffs and welfare
Expected payoffs of an individual seller (red), buyer (blue) and the sum of all expected payoffs $E\pi_S + B \cdot E\pi_B$ (black, right y-axis) for different B and S. Dotted graphs assume that buyers always play I. Underlying eq. prices and parameters stem from Figure 2.9, furthermore $c_R = 0.01$.

Refer to Figure 2.10 for the expected payoffs of an individual buyer and seller that result from the equilibrium prices of Figure 2.9. Furthermore, the sum of all expected payoffs is displayed for each scenario. Note that the sum of expected payoffs, utilitarian welfare, is measured on the right y-axis due to the different magnitude. However, values of the transformation $\frac{\sum E\pi}{\min\{S,B\}}$ can be taken from the left y-axis. Dividing the sum of expected payoffs by the number of potential matches allows a comparison of welfare for varying numbers of buyers and sellers.

This modification is required, since the number of potential matches and the maximal achievable welfare differs in the scenarios.

As in the analysis of the results of the SSM, one can take two viewpoints here. First, we focus on the development of individual payoffs and welfare if the cost ratio c changes, Obviously the discontinuities at the thresholds are of special interest here. Concerning the expected payoffs of an individual buyer (the solid blue lines), the same pattern as obtained in the SSM can be found (compare to Figure 2.5). An individual buyer experiences decreasing expected payoff if the sellers counteract falling cost ratios and raise prices to induce buyers to stick to I. Furthermore, every buyer is always better off for cost ratios where the equilibrium action of the buyers is U, since prices are smaller there. As argued above, a falling cost ratio can act as a commitment device for sellers, and an individual seller's expected profit (the solid red lines) increases due to the price increase close to but above the threshold cost ratios. Again, every seller obtains a strictly smaller payoff whenever buyers are uninformed, compared to cost ratios that lead to informed buyers. Recall that the results discussed here were derived by employing the limit case threshold ω. This choice constitutes a slight variation compared to the SSM, where the results were based on the true boundaries Ω_I and Ω^U. This mismatch is negligible, however, as the boundaries ω^I and ω^U are sufficiently close, even for low numbers of buyers and sellers (consider Figure 2.1 in the discussion of the SSM).[36] The development of the sum of the expected payoffs (the solid black lines) at the thresholds is shaped by the opposing effects concerning buyers and sellers. With more buyers than sellers, welfare falls due to the price increase above the thresholds. Adversely, if there are more sellers than buyers and prices rise, the sellers' gains offset the loss of the buyers and welfare increases. Without exception, welfare is impaired for cost ratios when buyers do not engage in research and signal interest uninformedly.

The second point of view allows for statements concerning efficiency. For this exercise, assume that buyers acquire information continually. Thus, sellers no longer can and need to affect the behavior of buyers. The equilibrium prices for this setting are the ones of the standard setting, in which sellers compete but are not constrained by a low cost ratio that disciplines them (e.g., for $S = B = 3$ as indicated in Figure 2.9, the unique equilibrium price for all cost ratios is $P = 0.58$ if buyers will play I with certainty). As above, forcing buyers to play I constitutes the solution of the constrained planner. The expected payoffs and welfare in the planner's solution (the dotted lines) are compared to their counterparts that arise under decentralized decision-making. Notably, this exercise reiterates the findings of the SSM: Welfare is larger in the planner's solution, and the market solution is not constrained efficient. Again, buyers face an incentive to bind themselves to action I to prevent the rise in prices, which occurs for cost ratios slightly larger than the thresholds. In contrast to the results of the SSM, this does not constitute a Pareto improvement, since sellers profit from the increase in prices.

Two aspects remain before concluding the analysis. First, Figure 2.11 exhibits equilibrium pricing and implied efficiency if both costs c_I and c_R fall jointly. Apparently, the findings generated by this exercise resemble the established features and confirm the central claim of

[36] Note that an individual seller's expected payoff exhibits a discontinuity here that is not true in the SSM. The discontinuity is not caused by the focus on the limit threshold ω. It is created by the competition between sellers, which prevents them from coordinating on high prices (e.g., an individual seller is able to demand the price P^{indif}, see Equation (2.16), this is not feasible here).

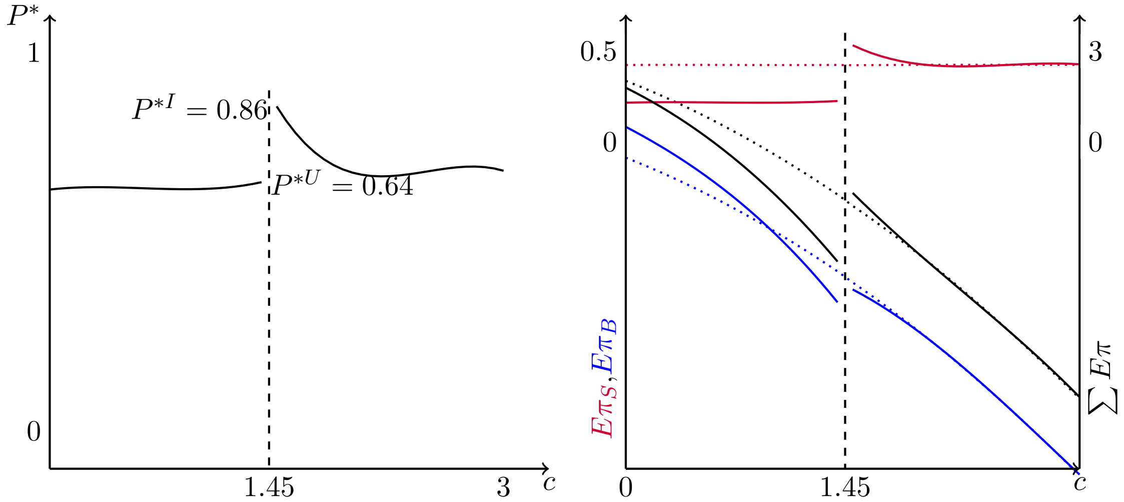

Figure 2.11: Prices and efficiency if c_I and c_R fall jointly
Symmetric equilibrium prices P^* (left) and expected payoffs and welfare (right, colors as above) for $S = B = 6$. Dotted graphs assume that buyers always play I. Underlying parameters: $c_A = 0.1$ and $\chi \sim U[0,1] \hat{=} \beta(1,1)$. Furthermore, c_I falls linearly from 0.45 to 0 and c_R linearly from 0.15 to 0.05, giving rise to the same cost ratios $c = 3, \ldots, 0$ as in all figures above. MC-configuration: Both grids of P_M and P_{-m} include 16 values. 1000 simulations for each point in the strategy space.

the paper. Impaired efficiency and deteriorating welfare due to a fall of the cost ratio c and the implied uninformed signaling are even present if both components of c fall. Thus, the simplification of fixing c_R and considering falling values of c_I as done above is without loss of generality.

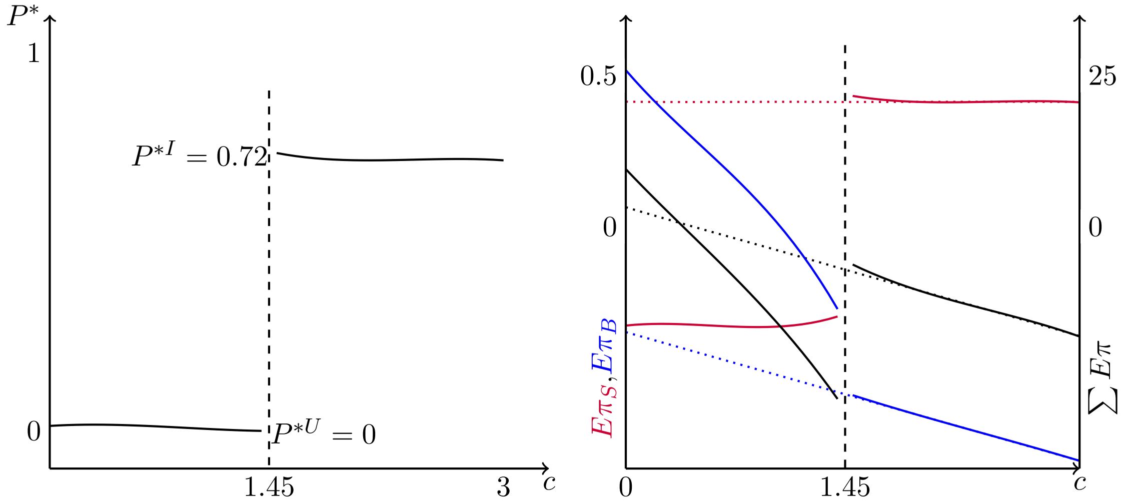

Figure 2.12: Prices and efficiency in a larger market
Symmetric equilibrium prices P^* (left) and expected payoffs and welfare (right, colors as above) for $S = B = 50$. Dotted graphs assume that buyers always play I. Underlying parameters and MC-configuration as in Figure 2.11.

Second and last, consider the outcomes displayed in Figure 2.12. Here we address a larger market with $S = B = 50$.[37] Most striking for sufficiently small cost ratios, the market outcomes in

[37] We deliberately refrain from labeling the market as "large", since the market utility assumption (Galenianos and Kircher, 2012) is not fulfilled. For a discussion of the difference between "small" and "large" markets, see the next section.

terms of welfare are larger than the achieved welfare if buyers are forced to play I. Thus there are settings where uninformed signaling can be constrained efficient within the framework of our model. Obviously, forcing buyers to inform themselves may not be efficient if the cost of information is high and the cost of applying nil. This paradigm constitutes a contrast to the notion that "Typically, random search leads to inefficient outcomes [...], while directed search leads to efficient outcomes" (Marinescu and Wolthoff, 2016, p. 23). However, we do not want to put too much emphasis on this finding. To keep the model tractable, we assume that buyers have to assess all offers and sellers have to keep approaching buyers until there are none left or they have sold the good. Thus, we force the maximal amount of (costly) communication, and this assumption becomes more unrealistic if the number of market participants rises. A simple modification of the model might allow a better understanding of market outcomes for small cost ratios if S and B are large. Buyers obviously assess only a subset of offers. Kircher (2009) includes this choice for buyers, but only for directed search.[38] Another point of importance is the decline of prices to zero if buyers play U. As hinted before, endogenous market entry and exit of sellers may yield additional insights. Note that if buyers play I, the equilibrium prices are largely identical to those introduced in Figure 2.9 for the scenarios $S = B = 3$, $S = B = 6$ and $S = B = 12$. The single difference is that more sellers are less able to impose higher prices as coordination becomes more difficult. Having made these caveats, the results with respect to the parts where buyers play I and the location of the threshold are thus robust, even in regard to larger markets.

2.7 General discussion and extensions

2.7.1 Policy implications

Having identified uninformed signaling as welfare reducing, our model speaks a clear language concerning policy improvements. Due to uninformed signaling, applications carry no information if they are very cheap. Therefore, the issue can be resolved by making these signals costly. This option resembles a notion in the seminal contribution by Spence (1973): With costly degrees, which are easier to obtain for intelligent persons, applicants can send signals that carry some information. We are not interested in matching quality; thus, absence of information is not related to characteristics of the applicant but rather to the application itself. The dilemma in many posted offer markets is that applications carry little information concerning the applicants' determination to accept the job when given the chance.

Intuitively, efficiency gains can be realized by increasing the cost ratio $\frac{c_I}{c_R}$. Whether this is achieved by raising c_I, lowering c_R or any other way is not crucial from a theoretical perspective. However, as argued e.g. by Eisenmann et al. (2006), for example, price sensitivity on platforms of posted offer markets may inhibit charging higher prices. Thus, an increase in c_I is probably

[38] In the framework of the MSM, there are S sellers and offers. If we impose symmetry and assume that every buyer asses an offer with the same probability, the expected number of offers assessed by an individual buyer can be any real number between 0 and S. Thus, the proposed modification increases the joint strategy space (that is so far determined by the assumed grids of prices P_m and P_{-m}) by times S. To put things in perspective: The solution of the MSM with the parametrization underlying Figure 2.12 takes more than a day in Matlab.

best done by increasing opportunity costs, which corresponds to the measures discussed in Bram (2016). Newly emerging dating platforms allow users to send a limited number of "special" signals or charge prices for sending signals in some on-platform currency.[39] Measures can even be drastic. In one instance only women, the providers of the scarce goods, were allowed to initiate contact. Another way in which signals can be costly – and may even carry additional information – is to demand preference orderings from applicants. Visibly placing a company on the first place of your ordering may credibly signal that it is your preferred choice.[40] Furthermore, if all potential employers can observe the full list, companies not ranked on top know and can assess their competitors.[41] Abstracting from the labor market, the standard example in the literature on two-sided matching is the admittance of high school students to universities. Consider the work of Roth and Oliveira Sotomayor (1992) for an introduction to this large body of literature.

Lowering c_R is equally fitting for raising the crucial cost ratio c. Effectively, this can be done by making information more accessible and easier to process. Note that this task can be accomplished by sellers within the boundaries of the respective platform where they post their offer. Our model is based on the price as the instrument that allows sellers to influence the behavior of buyers. Yet, recalling the buyers' decision rules given by Equations (2.4), (2.8), (2.19) and (2.22), the same results can be obtained if sellers can affect c_R. Obviously, this fine tuning of offers has to be costly for sellers. Otherwise they can steer the buyers' behavior without any negative effects. In reality, posting offers on online platforms has become much more standardized since the emergence of these services. Sellers of apartments are required to provide details on square meters, facilities, energy efficiency and much more. This standardized information can also be filtered much better with improved built-in search tools.[42] Another method for lowering c_R can be found when regarding c_R as opportunity costs. Feenberg et al. (2017) provide evidence that top items in lists where the ordering contains no information at all receive significantly more attention than bottom items. Due to this behavioral bias, 15 minutes of researching an offer can be quite costly if the seller has already received an abundance of signals of interest in the meantime. Behavioral biases like this one can be overcome by submitting all notifications of interest at a certain time of the day in randomized order.

In conjunction with the improvements of platforms of posted offer markets, the operators of these and other private companies have started to provide extensive services in their areas of expertise. Today, the labor market in particular is characterized by intensive outsourcing.

[39] A less obvious issue that is also important here is that the informativeness of signals (with respect to the interest of the sender in the receiver's offer) can suffer if the signal is presumed to carry additional information. Despite being interested in a job, it seems universally accepted the applicants should not contact employers two hours after having applied. Users of dating platforms do not respond to messages immediately to avoid signaling "over-enthusiasm" (Bram, 2016). These patterns can be fixed by setting maximal response times or structuring the communication such that the signals are cleared of any (presumed) additional content.

[40] Recall the discussion at the end of Section 2.3. The top position in the personal preference list does not mean that the applicant will accept a job offer from the respective company. It indicates rather that the company constitutes the best element in the applicant's set of options.

[41] An example for this is the (informal) hiring procedure for faculty positions. Despite an applicant's preference ordering is not publicly announced, senior faculty members and networks often help to collect information concerning this list.

[42] This feature may also provide opposing incentives. Entering your search parameters once on the housing platform immobilienscout24 will notify you instantly whenever there is a new offer that fits these parameters. Having specified all parameters that seem important, it is questionable whether notified users examine the offer thoroughly.

More details and information about the share of firms that engage in outsourcing activities is provided in the discussion by Isenhour (2018, p. 201 ff.). While delegating tasks like filtering applicants or organizing and conducting assessment centers to experts may indeed be efficiency enhancing, information technology has left its mark on this industry as well. The number of applications and the standardized queries in application processes led to the widespread use of so called applicant tracking systems that allow electronic processing and evaluation of resumes and applications. This in turn enables applicants to play this system by dropping appropriate buzzwords; see Weber (2012). Lastly, another example for improved platforms and procedures and a more far-reaching measure that relates to the findings of this paper are internal algorithms used by Airbnb, as evaluated in Fradkin (2017). These algorithms counteract market congestion by limiting the universe of offers that can be observed by a buyer to a subset that fits this buyer's characteristics as approximated by past behavior.

2.7.2 Posted offer markets as two-sided markets

Posted offer markets require some medium or platform where offers can be posted. Formerly these were public bulletin boards or newspapers, but nowadays these platforms are websites. Like their old analog counterparts, the operators of these websites often pursue their own economic interests. Thus, the marketplaces and platforms that facilitate posted offer markets cater to two-sided markets. The definition of Rochet and Tirole (2006, p. 646) that "the volume of transactions between end-users depends on the structure and not only on the overall level of the fees charged by the platform" is obviously met for the introduced examples of real estate sale and rental platforms.

Although the presented model is not framed as a model of a two-sided market with an active provider of a platform, it can nevertheless be regarded from this perspective. We argue that the assumed cost structure (c_R, c_I and c_A) is present, since the associated tasks are inherently costly even if the use of the platform and any activity there is free. Considering the posted offer market as being two-sided, these cost parameters can be interpreted as the fees set by the platform. The cost c_R is due when potential buyers wish to view the details of an offer, c_I is the fee that comes with contacting a seller and c_A must be paid by the sellers when reacting to contacts by potential buyers.[43] Importantly, operators of platforms in two-sided markets generate profit from the use of their platform. Producers of videogame consoles, the example introduced by Rochet and Tirole (2006, p. 645), achieve use by satisfying the preferences of both groups of users. Players are supplied with quality videogames and publishers are given access to a large community of players. This quid pro quo is not true for the examples given here, such as the labor and housing markets, or especially dating platforms. Satisfying the preferences in these examples means the generation of a match and the loss of two customers. Figuratively speaking, having obtained the formula for true love will probably enable a researcher to earn some money, but any dating platform based on these insights will quickly run out of customers.[44] Thus, two-

[43] The second part of the definition by Rochet and Tirole (2006, p. 646) requires that "the two sides do not negotiate away the corresponding usage and membership externalities". This means that communication between buyers and sellers can only happen via the platform, or the counterparts can only access other means of communication after having paid c_I and c_A.

[44] Obviously, operators of posted offer market platforms have to weigh various effects against each other. Users

sided markets exist where the incentives of users and operators of the marketplaces are more misaligned than in other two-sided markets. Going one step further, this interpretation provides an explanation of why the undesirable features that may arise if costs fall might even play into the hands of platform operators. First, passing on lowered costs to users helps to attract more customers and to raise the attractiveness of the platform. Second, lowered costs may change the behavior of users, hindering coordination and prolonging the time of buyers and sellers on the platform.[45] This is crucial since the profit of platform operators usually stems form the sale of targeted advertisement opportunities.

2.7.3 Model extensions

Several possible extensions are imaginable. We force sellers and buyers to participate in the posted offer market. Furthermore, buyers are forced to assess all offers. As has been addressed, an extension in the spirit of Kircher (2009), where buyers assess an optimal subset of offers, can help to overcome these limitations. Whether this modification limits coordination problems is not clear. Sellers probably face lower risks of losing an interested buyer to another seller, since buyers signal interest to fewer sellers. However, this supposition implies that each seller receives fewer signals of interest, thus increasing the chances that sellers will be unable to sell their goods. Supposedly, the findings presented in this work survive the proposed extension, if one keeps the focus on symmetric equilibria. Dropping this assumption and allowing price dispersion obviously complicates the model to a great extent. However, it would allow buyers to make a portfolio choice, and selecting offers with high prices might be beneficial for buyers as shown by Lester (2011).

On the other hand, sellers cannot leave the market or just throw their good away in our model. This approach leads to certain losses for the sellers in the scenarios where they are unable to post positive prices due to excessive supply. Arguably, these scenarios seem unlikely. As shown in Figure 2.10, all scenarios with excessive supply result in negative expected payoffs for individual sellers. Yet, a more comprehensive model including this feature may help to provide insights concerning platform competition. The effects of endogenous exit and entry of sellers is of interest concerning the interplay with the extension discussed above. Excess supply might be a stable market situation if sellers can attract more buyers by posting higher prices.

The assumption that buyers react similarly to all offers and evaluate a market signal (given by $\overline{P}'$; recall Equation (2.24)) seems rather strong. Exchanging the market signal by the individual prices of the offers enables buyers to react individually to offers. The extended strategy

$$\sigma_B^{\text{ext}}:\ a_j\left(P_j\right)=\begin{cases}U, & \text{for } P_j<\widetilde{P}\\ I, & \text{for } \widetilde{P}\leq P_j\end{cases}\qquad \text{with}\qquad \widetilde{P}=F^{-1}\left(\frac{c_R}{c_I}\right) \tag{2.26}$$

expresses that an individual buyer responds to the offer of seller j by playing U if P_j is smaller

will also abandon any platform that is unable to create any matches between buyers and sellers. Again, see the discussion in Bram (2016) on dating platforms.

[45] As discussed in Eisenmann et al. (2006), price sensitivity of users plays a crucial role when charging ideal fees (subsidizing the more price-sensitive group is often optimal). Thus, passing on lowered cost to users might be forced by platform competition. See Rochet and Tirole (2003) for an in-depth analysis.

than the critical price $\widetilde{P}$ and by choosing action I in all other cases. The direct implication of this change is that sellers have greater control over the response of buyers to their offers. An earlier version of the paper was based on strategy σ_B^{ext} but led to similar findings.[46]

Lastly, the computational solution of the MSM allows for putting different policy improvements discussed earlier to a test. For example, limiting the total number of signals of interest is easy to implement in the code used to generate the solutions. Similarly, it is straightforward that many of our findings can be tested in an experimental setting. Works by authors such as Helland et al. (2017) take posted offer markets to the lab. Since the basic structure of our model is simple, an extension of existing frameworks should be feasible. We believe that a test is promising as to what extent the selection of buyers' search strategies is shaped by the cost structure and how the buyers' search behavior can be affected by sellers.

2.7.4 Small and large markets, the market utility assumption

The characterization of buyers and sellers as infinitesimally small agents allows the use of continuums of agents, implies that the market utility assumption is fulfilled. This assumption states that entry of one additional agent does not affect the situation of those already present; consider Galenianos and Kircher (2012) for more information. This approach is commonly assumed to capture "large" markets and labeled accordingly. In contrast, the market utility assumption is violated when assuming finite numbers of discrete agents, as done here. Accordingly, this way of modeling is denoted as a "small" market. Naturally, both approaches offer advantages and disadvantages. Working with agents that have zero mass allows more elegant and concise formulations, but it may pose problems for incorporating aspects such as the repeated attempts of sellers to achieve a deal with interested buyers. Models with discrete agents can incorporate aspects like this, yet they tend to become complicated and cumbersome to handle.[47]

In reality, many markets are large. The labor market surely can be described as large and single job openings do not improve the situation of unemployed workers. We do not want to engage in the discussion that small markets are relevant when compared to large markets. If asked, we take the position of Lester (2011, p. 1600) and consent to the statement. Yet, we prefer to emphasize that the conceptual distinction between large and small markets may not be that severe. In the framework of our model, a buyer can achieve a deal if $\chi \geq P$ is true and she is approached by the respective seller. The probability that she is approached obviously depends on the number of remaining buyers. Yet for the parametrization underlying the solution presented in Figure 2.12, namely $S = B = 50$, the market utility assumption is practically true. The same can be argued for the scenario $S = B = 12$ exhibited in Figures 2.9 and 2.10. The transition from small to large markets is surely gradual. Hence, when splitting

[46] This approach was discarded to provide a correct solution of the buyers' stage in the MSM. The strategy σ_B^{ext} can only be derived by assuming an identical response to all offers (meaning strategy σ_B) and dropping this assumption afterwards, as done here. Deriving optimal behavior while assuming strategy σ_B^{ext} from the beginning is not feasible.

[47] In the words of Serene Tan, who identified the error in Albrecht et al. (2003) and helped resolve it in Albrecht et al. (2004): "At this point, the keen reader may have intuited that the general expression for the matching function [...] is going to be horrendous. And it is." (From *Matching with Multiple Applications: A Correction*, 2003. This work can be obtained via Google Scholar).

markets into small and large by evaluating the market utility assumption, we may find that markets are identified as “large” despite exhibiting only few market participants. We may have to acknowledge also that we are rather ignorant where – in terms of market participants – the border between small and large markets lies.

2.8 Conclusion

This paper analyzes a posted offer market with capacity constrained sellers and multiple applications such that buyers can express their interest in more than one offer. Buyers can acquire costly information that enables them to identify offers that allow a match (direct search). If they decide not to acquire information they contact all sellers, including those where a deal is impossible (undirected search). We derive our findings with a simple model that yields closed-form solutions and generalize them by considering a model that includes more sellers. Due to its complexity, the extended model is solved with computational methods. The main findings of this paper are as follows:

(1) The buyers' behavior is shaped by the ratio of the cost of signaling interest over the cost of information. Buyers acquire information and send informed signals if this ratio is high. If it is low, they do not acquire information and send uninformed signals. Compared to the usual notion that low costs of applying cause inappropriate applications, our findings imply a more thorough explanation. Despite lowered cost of information it can be rational to send uninformed signals if the cost of applying have fallen by a larger extent.

(2) Sellers are always put at a disadvantage when they are confronted with uninformed buyers compared to informed buyers. If the crucial cost ratio (c_I/c_R) is falling and buyers cease information acquisition, sellers can to some extent provide incentives for buyers to stay informed by demanding higher prices. This allows a new interpretation of Lester's paradox where the causation is reversed. Sellers proactively increase prices to induce buyers to acquire information.

(3) The transition to an equilibrium where buyers send uninformed applications leads to market outcomes that are not constrained efficient. Signaling interest uninformedly leads to a market congestion that is costly to resolve. Uninformed signaling can be interpreted as overexploitation of the attention of sellers, a public resource. In this light, buyers would like to publicly commit to acquire information or use another mechanism that helps them to coordinate their behavior.

(4) Scenarios in which sellers induce information acquisition by demanding higher prices even allow a Pareto improvement.

(5) Market outcomes with uninformed buyers can only be constrained efficient if the cost of applying are practically zero and the market is sufficiently large.

We combine elements that are central in the study of posted offer markets but have been studied only in isolation or combined in parts. In doing so, one can develop a better understanding of posted offer markets. This understanding is important since many markets, such as the labor market, are in fact posted offer markets and are pivotal for any economy. Furthermore, the platforms where the related market activity is conducted have been affected greatly by the emergence and development of online services. In line with our main findings, we believe that one of the key tasks related to these markets is to ensure that market participants obtain and use available information, especially since (or although) the range of available information is constantly growing. Our model provides options for policy improvements that may help to achieve this goal.

A Appendix of Chapter 2

A.1 Optimal threshold $\widetilde{\chi}$ for strategy I

If an individual buyer knows χ, there exists a critical willingness to pay $\tilde{\chi}$, at which the buyer is indifferent between signaling interest or not. In the SSM, this threshold can be determined as shown below.

$$0 = -c_I + PA^{I,I}\left(\widetilde{\chi} - P\right) \qquad \rightarrow \qquad \widetilde{\chi} = \frac{c_I}{PA^{I,I}} + P$$

Trivially, $\widetilde{\chi} > P$. The willingness to pay must cover the price and the weighted cost of signaling interest. As explained in Section 2.3, $PA^{I,I}$ obviously depends on $\widetilde{\chi}$ as well since all other buyers are assumed to play the same strategy (I). Thus $\widetilde{\chi}$ is defined by

$$\widetilde{\chi} = \frac{c_I}{PA^{I,I}(\widetilde{\chi})} - P = c_I\left((B-1)\left(1 - F(\widetilde{\chi})\right) + 1\right) - P.$$

Different distributional assumptions of χ affect the critical willingness to pay differently. For $\chi \sim U[0,1]$, that will be in large parts of this paper, an analytical solution is possible since $F(\chi) = \chi$ and

$$\widetilde{\chi} = \frac{c_I B}{c_I(B-1)+1} + \frac{P}{c_I(B-1)+1}.$$

The impact of imposing $\widetilde{\chi} = P$ can be assessed by looking at the difference

$$\widetilde{\chi} - P = \frac{c_I B}{c_I(B-1)+1} + \frac{P}{c_I(B-1)+1} - P = \frac{c_I}{c_I(B-1)+1}\left(B - (B-1)P\right).$$

It reveals that the difference is minuscule for sufficiently small values of c_I, relative to $\overline{\chi}$. In the parameterizations of the model that are introduced after Section 2.3, $c_I \approx 0.01$. Additionally, the difference if smaller if P is large (meaning in the domain where a buyer plays I). For other distributions, $\widetilde{\chi}$ can be determined computationally. Last and most important, the assumption $\widetilde{\chi} = P$ works against the findings of our paper. It implies that buyers decide to signal interest, while knowing their willingness to pay, too long, meaning for too low willingnesses to pay and thus also for lower prices. Yet, this is exactly what the seller wants to achieve, as will be carved out.

A.2 Boundaries Ω^I and Ω^U

Proposition: $\Omega^I \geq \Omega^U$ is always true for Ω^I and Ω^U as defined in Equation (2.4) and 2.8.

Proof:

$$\Omega^I = \frac{1 - PA^I}{F(P)} \geq \frac{1 - PA^U}{F(P)} = \Omega^U \qquad \rightarrow \qquad PA^I \leq PA^U.$$

Using a computational approach reveals that the maximum of $\psi = PA^I - PA^U$ is zero and reached for $B = 1$ or $P = \overline{\chi}$. Recall

$$PA^I = \frac{1}{(B-1)(1-F(P))+1} \qquad \text{and} \qquad PA^U = \frac{1-F(P)^B}{B\,(1-F(P))}$$

given by Equations (2.2) and (2.7). Hence $PA^I = PA^U$ for either $B = 1$ or $P = \overline{\chi}$ (implying $F(\overline{\chi}) = 0$). □

Proposition: $\lim_{B\to\infty} \Omega^U = \lim_{B\to\infty} \Omega^I = \frac{1}{F(P)} \equiv \Omega$.

Proof: Recall

$$\Omega^I = \frac{1-PA^I}{F(P)} \qquad \text{and} \qquad \Omega^U = \frac{1-PA^U}{F(P)}.$$

$\lim_{B\to\infty} \Omega^U = \lim_{B\to\infty} \Omega^I = \frac{1}{F(P)}$ is implied by $\lim_{B\to\infty} PA^I = \lim_{B\to\infty} PA^U = 0$. This is quite intuitive since due to increased demand, the probability of being approached in both pure strategy equilibriums is decreasing in the number of buyers. □

A.3 Equilibria for $c \in (\Omega^U, \Omega^I)$

Proposition: On the buyers' stage, there exists a unique mixed strategy equilibrium for all $c \in [\Omega^U, \Omega^I]$.

Alternative proposition: For each $c \in [\Omega^U, \Omega^I]$ and given the mixed strategy σ_r (as introduced in Section 2.3.3), there exists a mixture r^* that sets the last buyer indifferent between playing I or U if all other buyers play σ_{r^*}. Formally: $\exists r^* \in [0,1]$ s.t. $E\Pi_B^{I,\sigma_{r^*}} = E\Pi_B^{U,\sigma_{r^*}}$ $\forall$ $c \in [\Omega^U, \Omega^I]$.

Proof: As argued in Section 2.3.3, $PA^{I,\sigma_r} = PA^{U,\sigma_r} \equiv PA^{\sigma_r}$. Thus

$$\begin{aligned} E\Pi_B^{I,\sigma_r} &= E\Pi_B^{U,\sigma_r} \\ -c_R + (1-F(P))\left(-c_I + PA^{I,\sigma_r}(\dots)\right) &= -c_I + PA^{U,\sigma_r}\left(-c_I + (1-F(P))(\dots)\right) \\ c &= \frac{1-PA^{\sigma_r}}{F(P)} \equiv \Omega^r. \end{aligned}$$

PA^{σ_r} is derived in the following. From the perspective of a buyer who has signaled interest, the expected total number of buyers that signal interest is given by $1+(B-1)(r(1-F(P))+1-r) \equiv \eta$. In expectation, each of the remaining $B-1$ buyers plays I with probability r and will signal interest with probability $1-F(P)$. If a buyer plays action U (with probability $1-r$) she will always signal interest. Trivially, η converges to the (expected) numbers of buyers that signal interest in the pure strategy cases for $r \to 1$ and $r \to 0$. The same logic as presented in the derivation of PA^U (Equation (2.7)) applies. As above, being asked second (third, ...) requires $\chi < P$ for the buyer(s) approached before. Here, this is only possible if the buyer(s) approached before has played U (with probability $1-r$). Therefore

$$PA^{\sigma_r} = \frac{1}{\eta} + \frac{\eta-1}{\eta}(1-r)F(P)\frac{1}{\eta-1} + \frac{\eta-1}{\eta}\frac{\eta-2}{\eta-1}(1-r)^2(F(P))^2\frac{1}{\eta-2} + \dots\ ,$$

what can be simplified to

$$PA^{\sigma_r} = \frac{1-((1-r)F(P))^{\eta}}{\eta\,(1-(1-r)F(P))}.$$

As hinted above, $\lim_{r\to 1} PA^{\sigma_r} = PA^I$ and $\lim_{r\to 0} PA^{\sigma_r} = PA^U$. Thus $c = \Omega^I$ requires $r^* = 1$ and $c = \Omega^U$ requires $r^* = 0$ for $E\Pi_B^{I,\sigma_{r^*}} = E\Pi_B^{U,\sigma_{r^*}}$ to hold. Easily to see, PA^{σ_r} and thus also Ω^r is smooth for all $r \in [0,1]$. Hence for every $c \in [\Omega^U, \Omega^I]$ there exists at least one r^* that implies $c = \frac{1-PA^{\sigma_{r^*}}}{F(P)}$. □

Proposition: On the buyers' stage, there exists a corresponding asymmetric pure strategy equilibrium for each symmetric mixed strategy equilibrium in strategy σ_{r^*} where $B \cdot r^* \in \mathbb{N}$.

Proof: The proof of the existence of mixed strategy equilibria above can be easily rewritten and interpreted in a frequentist rather than a probabilistic way. This step is easy if one assumes a large number of buyers. If B buyers play strategy σ_r, there are in expectation $B \cdot r$ buyers who play action I. For B being sufficiently large, one can directly assume that the share r of the B buyers plays the pure strategy I and the remaining share plays the pure strategy U. The calculus presented in the proof above applies without restriction for this interpretation that is based on the assumption that B is sufficiently large. Yet, this is of limited importance since the domain where symmetric mixed strategy equilibria and asymmetric pure strategy equilibria can occur vanishes due to $\lim_{B\to\infty} \Omega^I = \lim_{B\to\infty} \Omega^U = \Omega$.

If B is small, some equilibrium values of r^* cannot be reproduced appropriately by assigning pure strategies. This is not an issue whenever $B \cdot r^* \in \mathbb{N}$. Consider the following example: Given $B = 5$, the value of c that requires $r^* = 0.2$ such that the mixed strategy $\sigma_{0.2}$ constitutes an symmetric equilibrium (e.g. $c = 1.29$ for $B = 5$ (middle graph) in Figure 2.1) can be generated if one of the five buyers plays pure strategy I and the remaining four buyers play U. Given this behavior, every buyer is indifferent between the pure strategies I and U and has no incentive to change her strategy.[48] □

A.4 Curvature of $E\Pi_S^I$ and $E\Pi_S^U$

Lemma: $E\Pi_S^I \geq E\Pi_S^U$ (defined in Equations (2.9) and (2.10)) holds for all $P \in [0,1]$ and any F, B and $c_A > 0$.

Proof:

$$E\Pi_S^I = (1-F(P)^B)\,(-c_A + P) \geq \frac{1-(F(P))^B}{1-F(P)}\big(P(1-F(P)) - c_A\big) = E\Pi_S^U$$

$$(1-F(P))(P-c_A) \geq (1-F(P))P - c_A$$

$$c_A F(P) \geq 0$$

[48] This highlights that the proven proposition is rather strict and not exhaustive. Pure strategy equilibria obviously do not require indifference. Therefore, an asymmetric pure strategy equilibrium that corresponds to a symmetric mixed strategy equilibrium for a certain $c' \in (\Omega^U, \Omega^I)$ and an implied r^* is very likely also an equilibrium for other $c \in [c' - \varepsilon, c' + \varepsilon]$.

Lemma: $E\Pi_S^I = -Bc_A$ for $P = 1$.

Proof: From L'Hôspital's rule follows

$$\lim_{P\to\overline{\chi}} E\Pi_S^U = \lim_{P\to\overline{\chi}} \frac{1-(F(P))^B}{1-F(P)}\big(P(1-F(P))-c_A\big) = \lim_{P\to\overline{\chi}} BF(P)^{B-1}(-c_A) = -Bc_A$$

such that $E\Pi_S^I > E\Pi_S^U$ is true for all $P > 0$ (see above).

Lemma: $\frac{\partial E\Pi_S^I}{\partial P} \geq \frac{\partial E\Pi_S^U}{\partial P}$ for all P and B, f, F.

Proof: First obtain

$$\begin{aligned}
\frac{\partial E\Pi_S^I}{\partial P} =& -BF(P)^{B-1}f(P)\,(P-c_A) + \big(1-F(P)^B\big) \quad \text{and} \\
\frac{\partial E\Pi_S^U}{\partial P} =& 1-F(P)^B + P\left(-BF(P)^{B-1}f(P)\right)\ldots \\
& - c_A\frac{-BF(P)^{B-1}f(P)(1-F(P)) - \big(1-F(P)^B\big)\,(-f(P))}{(1-F(P))^2}.
\end{aligned}$$

Second, evaluating the derivatives at $P = 0$ and $P = 1$ is simple. Only determining

$$\begin{aligned}
\left.\frac{\partial E\Pi_S^U}{\partial P}\right|_{P=1} =& \lim_{P\to 1}\ 1-F(P)^B + P\left(-BF(P)^{B-1}f(P)\right)\ldots \\
& - c_A\left(\frac{-BF(P)^{B-1}f(P)}{1-F(P)}\right) - c_A f(P)\frac{1-F(P)^B}{(1-F(P))^2} \\
=& -Bf(1) - c_A f(1)\left(\lim_{P\to 1}\ \frac{B(B-1)F(P)^{B-2}}{-2}\right) \\
=& -Bf(1) + c_A f(1)\frac{B(B-1)}{2}
\end{aligned}$$

requires some rearrangement and the use of L'Hôspital's rule twice for the last summand. Thus the proposition holds strictly at $P = 1$. Only $f(0) = 0$ implies the weak inequality at $\frac{\partial E\Pi_S^I}{\partial P} = \frac{\partial E\Pi_S^U}{\partial P}$ at $P = 0$.

For any $P \in (0,1)$, consider and rearrange the inequality $\frac{\partial E\Pi_S^I}{\partial P} \geq \frac{\partial E\Pi_S^U}{\partial P}$:

$$\begin{aligned}
-BF(P)^{B-1}f(P)\,\big(\cancel{P}-c_A\big) + \cancel{1-F(P)^B} \geq \cancel{1-F(P)^B}\ldots \\
\cancel{-PBF(P)^{B-1}}f(P) + c_A\frac{BF(P)^{B-1}f(P)(1-F(P)) - \big(1-F(P)^B\big)\,f(P)}{(1-F(P))^2}
\end{aligned}$$

$$\begin{aligned}
BF(P)^{B-1}f(P)\,(1-F(P))^2 &\geq f(P)BF(P)^{B-1}\,(1-F(P)) - \big(1-F(P)^B\big)\,f(P) \\
BF(P)^{B-1}f(P)\,(1-F(P))\,(1-F(P)-1) &\geq -\big(1-F(P)^B\big)\,f(P) \\
-BF(P)^B f(P)\,(1-F(P)) - F(P)^B f(P) &\geq -f(P) \\
F(P)^B f(P)\,(B(1-F(P))+1) &\leq f(P) \\
f(P)\big(\underbrace{F(P)^B\,(B(1-F(P))+1)}_{\equiv\psi} - 1\big) &\leq 0
\end{aligned}$$

For all P that imply $f(P) = 0$, the inequality is strict. In these cases a marginal raise of the price cannot alter the expected payoff since no buyers are affected by the change. Lastly, a numerical approach reveals that ψ is maximal for $P = 1$ (for all values of B) and takes on the value one. □

A.5 Curvature of Ω^I and Ω^U

Proposition: $\frac{\partial \Omega}{\partial P} < 0$, $\frac{\partial \Omega^I}{\partial P} < 0$ and $\frac{\partial \Omega^U}{\partial P} < 0$ for $P \in [0, 1]$.

Proof: For Ω the exercise is simple. Rearrange Ω^I and receive

$$\Omega^I = \frac{1 - PA^I}{F(P)} = \frac{1 - \frac{1}{(B-1)(1-F(P))+1}}{F(P)} = 1 - \frac{1}{(B-1)(1-F(P))+1}.$$

Since $\frac{\partial \Omega^I}{\partial P} = \frac{\partial \Omega^I}{\partial F(P)} \frac{\partial F(P)}{\partial P}$ and $\frac{\partial F(P)}{\partial P} \geq 1$, only $\frac{\partial \Omega^I}{\partial F(P)}$ matters. For brevity, denote $F(P)$ by F.

$$\frac{\partial \Omega^I}{\partial F} = -\left(-\frac{1}{((B-1)(1-F)+1)^2}(-(B-1))\right) = -\frac{B-1}{(\dots)^2}$$

For Ω^U, $\Omega^U \xrightarrow{F \to 0} \infty$ is true due to $PA^U \xrightarrow{F \to 0} \frac{1}{B}$ and $\Omega^U \xrightarrow{F \to 1} 0$ is true since $PA^U \xrightarrow{F \to 1} 1$ (using L'Hôspital's rule). With a computational approach, one can show that Ω^U is strictly monotonically decreasing in F for $F \in [0, 1]$. □

A.6 $E\Pi_S^I$, $E\Pi_S^U$ and P^* for non single-peaked distributions of χ

Let $\chi \sim \beta(0.25, 0.25)$. Figure 2.13 shows the two expected payoffs of the seller and the optimal prices P^* for several cost ratios $\frac{c_I}{c_R}$.

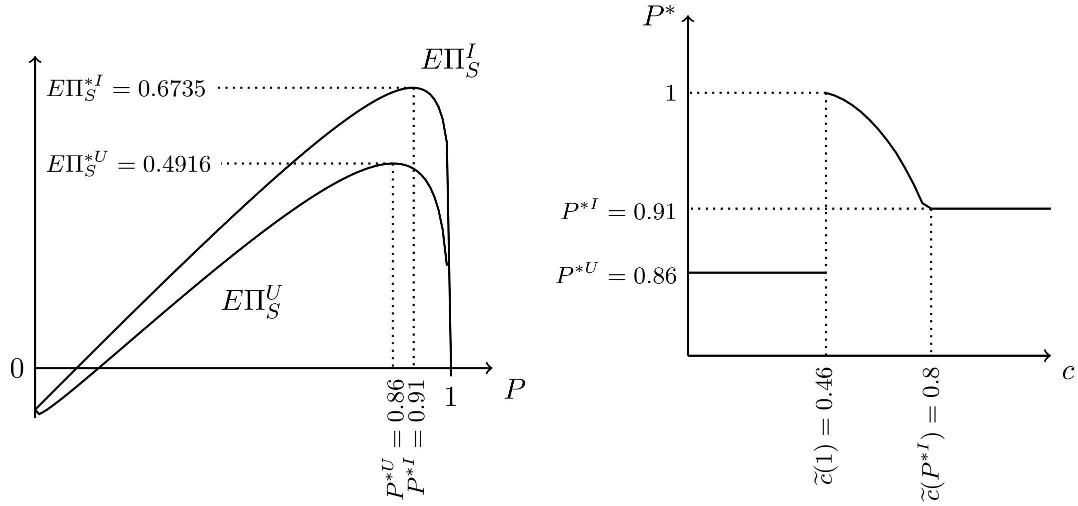

Figure 2.13: Outcomes in the SSM for non single-peaked χ
Curvature of $E\Pi_S^I$ and $E\Pi_S^I$ (left) and the optimal price P^*. Underlying parameters: $c_A = 0.1$, $B = 5$ and $\chi \sim \beta(0.25, 0.25)$.

A.7 The MSM with $S = 2$ and $B = 2$

All assumptions made in Section 2.5 hold. Furthermore, set $S = 2$ and $B = 2$. Note that this scenario is even simpler than $S = 2$ and $B = 3$ as analyzed in Lester (2011). The two sellers post prices P_1 and P_2. For brevity, let $F(P_s) \equiv F_s$ and simplify the expected surpluses $E(\chi|\chi \geq P_s) - P_s \equiv ESP_s$, both for $s = 1, 2$. First, let both buyers play I. In this case a single buyer's expected payoff is given by Equation (2.27) below.

$$
\begin{aligned}
E\tilde{\pi}_B^{I,I} = & -2c_R + (1-F_1)\cdot F_2\Big[-c_I + F_1 \cdot ESP_1 + (1-F_1)\frac{1}{2}\cdot ESP_1 \ldots \\
& \quad + (1-F_1)\frac{1}{2}(1-F_2)I(1 = \operatorname{argmin} ESP_s)ESP_1\Big] \ldots \\
& + F_1 \cdot (1-F_2)\Big[-c_I + F_2 \cdot ESP_2 + (1-F_2)\frac{1}{2}\cdot ESP_2 \ldots \\
& \quad + (1-F_2)\frac{1}{2}(1-F_1)I(2 = \operatorname{argmin} ESP_s)ESP_2\Big] \ldots \\
& + (1-F_1)\cdot(1-F_2)\Bigg[-2c_I + F_1 \cdot F_2 \max_s ESP_s \ldots \\
& \quad + (1-F_1)\cdot F_2\left(\frac{1}{2}\max_s ESP_s + \frac{1}{2}ESP_2\right) \ldots \\
& \quad + F_1 \cdot (1-F_2)\left(\frac{1}{2}\max_s ESP_s + \frac{1}{2}ESP_1\right) \ldots \\
& \quad + (1-F_1)\cdot(1-F_2)\left(\frac{1}{4}\max_s ESP_s + \frac{1}{4}ESP_1 + \frac{1}{4}ESP_2 + \frac{1}{4}\min_s ESP_s\right)\Bigg]
\end{aligned}
\tag{2.27}
$$

Although the expression is enormous (even for $S = B = 2$), the logic behind it is simple: Playing I for both offers leads to costs $2c_R$ in all cases. The buyer signals interest only in the offer of seller one when learning $\chi \geq P_1$ and $\chi < P_2$ (with probability $(1 - F_1) \cdot F_2$). In this case she pays c_I once and obtains ESP_1 surely if the other buyer has not signaled interest in this offer (with probability F_1). If the other buyer has learned $\chi \geq P_1$ (with probability $1 - F_1$), seller one will randomly pick one of the two buyers. If the regarded buyer is not picked, she still has a chance to buy the good of seller one and realize ESP_1. For this the other buyer must signal interest in the offer of seller two as well (with probability $1 - F_2$), then the other buyer is asked by both sellers first and will accept the offer of seller two if $ESP_2 > ESP_1$. This last condition is captured by the indicator function $I(1 = \operatorname{argmin} ESP_s)$ that is equal to one if the argument is true. Next (the third row), the buyer might learn $\chi < P_1$ and $\chi \geq P_2$ (with probability $F_1(1 - F_2)$) and only signal interest in the offer of seller two. She incurs cost c_I once and the same logic as described applies. Last, the buyer pays c_I twice and signals interest in both offers after learning $\chi \geq P_s$ for $s = 1, 2$ (with probability $(1 - F_1) \cdot (1 - F_2)$). If the other buyer has not signaled interest in any offer (probability $F_1 \cdot F_2$), the regarded buyer is approached by both sellers and picks the better offer. If the other buyer has signaled interest in one of the two offers (probabilities $(1 - F_1) \cdot F_2$ and $F_1 \cdot (1 - F_2)$), the regarded buyer can nevertheless be approached by both sellers (probability $\frac{1}{2}$) and pick the better offer. With counter probability she is approached only by the seller where she alone has signaled interest and accepts this offer. In the case where the other buyer signals interest in both offers (probability $(1 - F_1) \cdot (1 - F_2)$), the regarded buyer can (with equal probabilities of $\frac{1}{4}$) be approached by both sellers (and pick the best offer), she can be approached by seller one (two) and accept his offer, or both sellers approach the other buyer first such that the regarded buyer can accept the poorer offer in the second round.

$$
\begin{aligned}
E\tilde{\pi}_B^{U,I} = -2c_I + F_1 \cdot F_2 \Big[& -2c_R + (1-F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1-F_2) \cdot ESP_2 \ldots \\
& + (1-F_1) \cdot (1-F_2) \cdot \max_s ESP_s \Big] \ldots \\
& + (1-F_1) \cdot F_2 \Big[-c_R + \frac{1}{2}\big(-c_R + (1-F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1-F_2) \cdot ESP_2 \ldots \\
& + (1-F_1) \cdot (1-F_2) \cdot \max_s ESP_s\big) + \frac{1}{2}(1-F_2) \cdot ESP_2 \Big] \ldots \\
& + F_1 \cdot (1-F_2) \Big[-c_R + \frac{1}{2}\big(-c_R + (1-F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1-F_2) \cdot ESP_2 \ldots \\
& + (1-F_1) \cdot (1-F_2) \cdot \max_s ESP_s\big) + \frac{1}{2}(1-F_1) \cdot ESP_1 \Big] \ldots \\
& + (1-F_1) \cdot (1-F_2) \Bigg[\frac{1}{4}\Big[-c_R + (1-F_1) \cdot ESP_1 \Big] \ldots \\
& + \frac{1}{4}\Big[-c_R + (1-F_2) \cdot ESP_2 \Big] \ldots \\
& + \frac{1}{4}\Big[-2c_R + (1-F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1-F_2) \cdot ESP_2 \ldots \\
& + (1-F_1) \cdot (1-F_2) \cdot \max_s ESP_s \Big] \ldots \\
& + \frac{1}{4}\Big[I(1 = \operatorname{argmin} ESP_s) \cdot (1-F_1) \cdot ESP_1 \ldots \\
& + I(2 = \operatorname{argmin} ESP_s) \cdot (1-F_2) \cdot ESP_2 \Big] \Bigg]
\end{aligned} \tag{2.28}
$$

A similar expression can be derived if one considers a deviation of the regarded buyer to strategy U while the other buyer keeps playing I. Refer to Equation (2.28) above. Again, the expression is tremendous but the interpretation straightforward: The cost c_I occur with certainty twice when playing U. The other buyer does not signal interest to any seller if she learns $\chi < P_s$ for both $s = 1, 2$ (with probability $F_1 \cdot F_2$). Thus the regarded seller is approached by both sellers and she pays c_R twice. She realizes ESP_1 if she learns $\chi \geq P_1$ and $\chi < P_2$ (with probability $(1-F_1) \cdot F_2$). Vice versa, she realizes ESP_2 with probability $F_1 \cdot (1-F_2)$. If she learns $\chi \geq P_s$ for both $s = 1, 2$ (with probability $(1-F_1) \cdot (1-F_2)$) she picks the better offer. If the other buyer learns $\chi \geq P_s$ for only one $s = 1, 2$ and the opposite for the other offer (with probabilities $(1-F_1) \cdot F_2$ and $F_1(1-F_2)$, third and fifth row), the regarded buyer will surely be approached by the seller where she is the sole interested buyer. Thus cost c_R are incurred at least once. With probability $\frac{1}{2}$, the regarded buyer is also approached by the seller where the other buyer has signaled interest such that c_R is paid a second time. In this case, if the buyer learns $\chi \geq P_s$ for only one $s = 1, 2$ (with probability $(1-F_s)F_{-s}$), she accepts the offer of seller s and realizes ESP_s. The buyer is able to pick the best offer if she learns $\chi \geq P_s$ for both $s = 1, 2$ (probability $(1-F_1) \cdot (1-F_2)$). With counter probability of $\frac{1}{2}$, the buyer is asked only by the seller where she is the sole interested buyer and she realizes the respective surplus if $\chi \geq P_s$ is true. Last, the other buyer might learn $\chi \geq P_s$ for both $s = 1, 2$ (with probability $(1 - F_1) \cdot (1 - F_2)$, row seven). In this case, four outcomes (all with equal probability) are possible. First, the regarded buyer is only approached by seller one, she has to pay c_R once and realizes ESP_1 if $\chi \geq P_1$ is true (probability $1 - F_1$). Second, the same pattern can emerge if she is only approached by seller two. Third, she is approached by both sellers. In this case she realizes ESP_1 (EPS_2) only if $\chi \geq P_1$ ($\chi \geq P_2$) is true (probabilities $(1 - F_1) \cdot F_2$ and $F_1 \cdot (1 - F_2)$). Otherwise she can pick the best offer if $\chi \geq P_s$ is true for both $s = 1, 2$ (with probability $(1 - F_1) \cdot (1 - F_2)$). Fourth, both sellers approach the other buyer first. Here she can accept the offer that was declined by the other buyer is she learns that her willingness to pay exceeds the offer's price.

It is apparent that the comparison $E\tilde{\pi}_B^{I,I} \geq E\tilde{\pi}_B^{U,I}$ cannot be simplified to a clear-cut threshold that defines where equilibria in which both buyers play I exist. Note that the two similarly gargantuan expressions $E\tilde{pi}_B^{U,U}$ and $E\tilde{pi}_B^{I,U}$ need to be derived and compared to identify equilibria where both buyers play U. Furthermore, the stage of the sellers requires the identification of the symmetric best response function $P_s^*(P_{-s})$ (that takes the buyers' optimal behavior into account) to be able to determine the equilibrium price. At the latest at this point it becomes clear that even the simplest scenario $S = B = 2$ cannot be solved analytically. The expressions given by Equations (2.27) and (2.28) can be simplified to a very large extent by assuming that sellers post identical prices what implies $F_1 = F_2$ and $ESP_1 = ESP_2$. Yet, this simplification implies a different model. It eliminates the competition between the two sellers since the mentioned best response functions do not exist under this assumption. The optimal price that one may obtain given the simplification corresponds to the cartel price.

A.8 Fixed point search algorithm

To obtain the solution of the MSM we use Matlab. The used algorithm is structured as follows:[49]

A Define values the model parameters S, B, F and the cost parameters c_R, c_A.

B Define a grid of costs c_I denoted as $\mathbb{C}_I$.

C **For** $\text{LOOP}_{c_I} = 1, 2, \ldots, |\mathbb{C}_I|$

- C.1 Set $c_I = \mathbb{C}_I(\text{LOOP}_{c_I})$ implying the cost ratio $c = \frac{c_I}{c_R}$.
- C.2 Define a grid of prices P_{-m} denoted as $\mathbb{P}_{-m}$.
- C.3 **For** $\text{LOOP}_{P_{-m}} = 1, 2, \ldots, |\mathbb{P}_{-m}|$
  - C.3.1 Set $P_{-m} = \mathbb{P}_{-m}(\text{LOOP}_{P_{-m}})$.
  - C.3.2 Endogenously define a grid of prices P_m denoted as $\mathbb{P}_m$.
    In the first iteration $\mathbb{P}_m$ must include 0 and 1. If ($\widetilde{P}_m$ lay within the boundaries of $\mathbb{P}_m$, the set includes the values $(1 - \epsilon) * \widetilde{P}_m$ and $\widetilde{P}_m$ itself.)
  - C.3.3 **For** $\text{LOOP}_{P_m} = 1, 2, \ldots, |\mathbb{P}_m|$
    - C.3.3.1 Set $P_m = \mathbb{P}_m(\text{LOOP}_{P_m})$.

[49] All readers who have reached this point are more than welcome to contact us if they are interested in the Matlab files.

C.3.3.2 Simulate the model MC-times (given the current c_I, P_{-m}, P_m).

C.3.3.3 Average over the MC realized payoffs to get $E\pi_m(P_m|P_{-m})$.
[Polynomial approximation accounting for discontinuities]

C.3.4 Find and save $\underset{P_m}{\operatorname{argmax}} E\pi_m(P_m|P_{-m}) = P_m^*(\text{LOOP}P_{-m})$.

C.4 Combine the obtained P_m^* to get the correspondence $P_m^*(P_{-m})$.
[Polynomial approximation accounting for discontinuities.]

C.5 **If** THERE EXISTS AT LEAST ONE FIXED POINT.

C.5.t *true:* **If** THERE ARE MULTIPLE FIXED POINTS.

C.5.t.t *true:* Equilibrium selection by risk dominance, save the fixed point $P^*(\text{LOOP}_{c_I})$.

C.5.t.f *false:* Save the fixed point $P^*(\text{LOOP}_{c_I})$.

C.5.f *false:* No sym. eq. on the buyers' stage for c_I. Save $P^*(\text{LOOP}_{c_I}) = \text{nan}$.

D Combine the obtained P^* to get the correspondence $P^*(c_I)$.
[Polynomial approximation accounting for discontinuities]

The following remarks are due. First, the criterion to evaluate C.5 is a graphical check whether there exists at least one intersection of $P_m^*(P_{-m})$ with the 45° line. Second, the polynomial approximations at steps C.3.3.3, C.4 and D help to smooth the obtained functions and reduce noise from the simulations. Whenever the considered functions exhibit a discontinuity, the approximation has to be twofold to capture this. In addition one has to select an appropriate degree concerning the polynomial. Higher degrees preserve more variation (or noise) but may not be feasible due to the limited number of data points, lower degrees might actually destroy much of the shape of the underlying function. We choose a degree of 3 (2) if the number of data points is greater or equal than 6 (4) and a linear approximation in the remaining cases. Third, the relationship between the size of $\mathbb{P}_{-m}$ and $\mathbb{P}_m$, the number MC and the precision that is reached by the algorithm is of interest, especially as greater accuracy is favorable but requires (much) more time. Here an iterative approach can be employed. Rather coarse grids $\mathbb{P}_{-m}$ and $\mathbb{P}_m$ as well as fewer simulations can be set to determine the location of a fixed point candidate roughly in a first iteration. In further iterations the grids of prices can be finer and centered around this candidate and more simulations can be run to pinpoint the location of the fixed point. However, this approach comes at the risk of missing or mis-specifying a fixed point candidate. Fourth, a few basic rules can help to identify ill-defined best response correspondences. E.g. for $\widetilde{P}_m \notin [0,1]$ the best response function should be smooth since no change in the buyers' behavior occurs. Also, if there exists a discontinuity for a c_I and this discontinuity vanishes for the next c_I that is marginally larger than the previous one, there is probably something wrong. Lastly, whenever one is unable to determine a fixed point, this is probably caused by too much noise and can be resolved by increasing MC or the grid $\mathbb{P}_{-m}$ is too coarse. The reason for this is the following. If $P_m^*(P_{-m})$ exhibits a discontinuity, this can hardly result in the absence of an intersection with the 45° line since the prices on the right side of the discontinuity (where buyers play I) are in all tackled cases higher than the ones on the left side, see Figure 2.7.

The equilibrium expected payoffs of buyers and sellers for cost ratio c_I are obtained by setting $P_m = P_{-m} = P^*(c_I)$ and executing step C.3.3.2 with a large number of simulations and averaging over the realized payoffs. 10000 simulations guarantee sufficient precision.

A.9 Non single-peaked distributions of χ in the MSM

Let $\chi \sim \beta(0.25, 0.25)$. Figure 2.14 shows the solution of the MSM given by equilibrium prices and implied expected payoffs for the indicated parametrization.

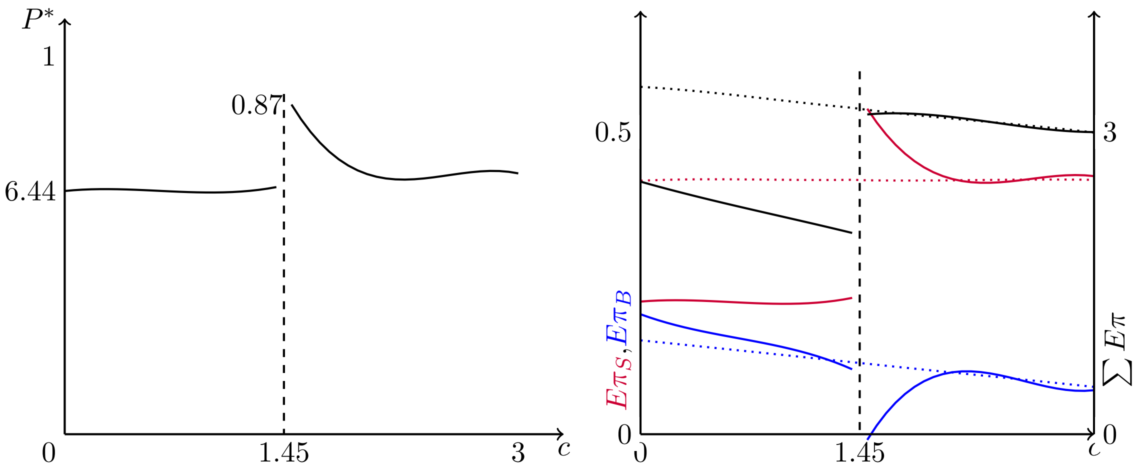

Figure 2.14: Non single-peaked distributions of χ in the MSM
Symmetric equilibrium prices P^* (left) and expected payoffs and welfare (right, color coding as in Figure 2.10) for $S = B = 6$. Dotted graphs assume that buyers always play I. Underlying parameters: $c_A = 0.1$ and $\chi \sim \beta(0.25, 0.25)$. MC-configuration: Both grids of P_M and P_{-m} include 16 values. 1000 simulations for each point in the strategy space.

Chapter 3

Local Public Goods as Perfect Substitutes: Centralization versus Decentralization

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Abstract

A rich body of theoretical literature on centralized and decentralized provision of public goods shows why decentralized administrative structures may promote welfare better than centralized structures. However, recent empirical evaluations of decentralization reforms find that they were on average unable to generate the predicted beneficial effects. In this paper, I develop a model in which local provisions of a public good are treated as perfect substitutes. This modeling resembles the initial conception of public goods more accurately, and leads, within a standard political economy model, to the finding that centralized and decentralized structures of governance induce identical provision levels of public goods. This result follows from the interaction between electorates and representatives. Although electorates and representatives behave differently under centralization and decentralization, the overall outcomes are identical due to the leveling effect of strategic delegation. Hence, the paper reintegrates the puzzling empirical findings into theoretical literature.

3.1 Introduction

The question as to where the power of decision should be located in a political system is an old one. The two natural counterparts in this debate are centralized and decentralized decision structures. On the one hand, allocating political power to a central (i.e., higher) tier of the political system can help internalize interregional effects such as spillovers. On the other hand, decentralized systems can better cater to different preferences. There is a trade-off in the presence of spillover effects and heterogeneous preferences, and it is necessary to weigh the advantages and disadvantages of the two systems. This is captured by Oates' decentralization theorem (Oates, 1972).

Because of the benefits of decentralization, there has occurred a "rapid spread of decentralization worldwide, which at its peak led to elections of local governments in 90% of countries around the world" (Malesky et al., 2014, p. 145). However, recent empirical investigations into these efforts have been unable to confirm whether these benefits have been achieved at large. On average, reforms to the structure of governance have not yielded the promised improvements, and the reasons may lie in the specific local peculiarities, which vary greatly across the different reforms. This paper takes a different approach and provides a much simpler answer. It argues that theoretical predictions are driven by certain pervasive assumptions and, given rigorous modeling based on theoretical conceptions, the commonly presumed trade-off might not be present. The crucial assumption that is discarded in this work is the assumption of separability with regard to the aggregation of local provisions of a public good. Discarding this assumption implies that local provisions are perfect substitutes and, within a model of a representative democracy in the spirit of Besley and Coate (2003), leads to identical provision levels of the public good under both regimes. The occurrence of strategic delegation plays a crucial role in this context. Different regions compete in actual local provision levels, and, as is well-established, this leads to inefficiently low provision levels. This competition is then transmitted to the election stage, and electorates select representatives who promote low provision levels to influence the competition between the representatives in their favor.

The sequence of this paper is as follows: after the introduction, Section 2 positions the paper within the context of existing literature. Section 3 introduces the model, and Section 4 determines the model's solutions under decentralization and centralization. The main result of the research follows from the comparison of the solutions under different regimes in Section 5. Section 6 features a discussion of related aspects and assumptions. The final section provides a short summary and concludes the work.

3.2 Related literature

This paper belongs to the field of fiscal federalism. The objects of study in this field include different forms of government or administration and how well they perform in their genuine tasks (e.g., the provision of public goods). The literature on this subject is vast and is usually split into two generations (Qian and Weingast, 1997). The first generation of fiscal federalism envisioned representatives and members of the political sphere as benevolent planners and therefore takes a rather technical stance. According to first-generation research, the public sector's main task is to identify and counter market shortcomings. Early influential contributions shifted attention toward public goods and government expenditures (Samuelson, 1954), the government's ability to counteract macroeconomic fluctuations (Musgrave, 1959) and market failure in the presence of externalities (Arrow, 1970). In line with this, Oates (2005, p.351) highlighted the importance of the "Arrow-Musgrave-Samuelson [...] perspective" in first-generation fiscal federalism. This technical approach was challenged as researchers found differences between the public agents' and the electorates' goals. Addressing the issue of the right size of the government sector, Brennan and Buchanan (1980) argued that public agents favor large budgets to gain influence and status. The second-generation theory of fiscal federalism that followed can be broadly associated with two main aspects (Oates, 2005, p.356). The first considers how political institutions shape the incentives and behaviors of public agents who maximize their private utility, while the second considers the role of information asymmetries. Weingast (2009, 2014) provides an overview of second-generation fiscal federalism and the current topics.

From a fundamental perspective, the basic trade-off (Oates, 1972) between the two standard forms of administration, centralized and decentralized, remains. Looking at real world developments, the promised advantages of decentralization – specifically a leaner and more efficient public sector – appear to have triumphed: "For anyone who might not yet have noticed, political decentralization is in fashion" (Treisman, 2007, p.1). Since the 1960s, the world has experienced a strong increase in elements of decentralization (Rodden, 2006; Malesky et al., 2014). The shift of power to regional authorities is taking place in both developing countries (Garman et al., 2001) and developed countries alike (Hooghe et al., 2010). However, there is no conclusive evidence that this "fashion" has in reality produced the expected benefits. Mansuri and Rao (2013) evaluated nearly 500 decentralization reforms, but on average, they were unable to identify positive effects. While these reforms were primarily located in developing countries, the evidence for developed countries is also inconclusive. Baskaran and Feld (2013) analyzed the link between fiscal decentralization and GDP growth in several OECD countries. They identified a negative link and raised doubts about the positive effects of fiscal decentralization. In line with these observations, Malesky et al. (2014, p. 145) concluded that efforts toward decentralization were "not met by real-world improvements in outcomes." Consequently, one can observe recentralization reforms that reduced regional authority (Dickovick, 2011). In this context, the study by Malesky et al. (2014) provides causal evidence that recentralization can improve the delivery of public services. In contrast Martinez-Vazquez et al. (2017) provide evidence that is mildly optimistic about the beneficial effects of decentralization. Altogether, the heterogeneity in these evaluations should not be interpreted blindly in favor of centralization. However, it does undermine the foundation of the worldwide shift toward decentralization.

This research belongs to the second-generation theory of fiscal federalism and explains the inconclusive empirical findings. It builds on a rather simple model that shares many elements with the frameworks used by Besley and Coate (2003) and Dur and Roelfsema (2005). In line with these two studies and many others (Roelfsema, 2007), this paper identifies strategic delegation as a driving mechanism. However, this research differs from these works in the aggregation of the public good and treats local provisions as perfect substitutes.[1] Thus, this paper adopts basic considerations, such as how individual donations or gifts contribute to a public good. In addition to the concepts of weakest-link and best-shot aggregation (Hirshleifer, 1983), a widely and long-used aggregation method is the summation of individual donations (see Olson 1965, for an early example). This method may introduce different weights, but individual provisions are perfect substitutes in this framework. This paper exhibits many similarities to the work by Bergstrom et al. (1986) in this regard. As will become clear in the following sections, adding politics to the model complicates the solution. To simplify the analysis, the large majority of papers treat individual provisions as separate goods. This assumption is not made without concern (Dur and Roelfsema, 2005, p. 399). These doubts are confirmed in the following analysis, which shows that the simplification has far-reaching implications and that the commonly presumed trade-off between centralization and decentralization may not be present.

3.3 Model description

To incorporate the aspects of spillovers and heterogeneous preferences, the model portrays a polity that is divided into two regions, indexed $j = 1, 2$. The populations in both regions are of equal size and normalized to unity. Citizens draw utility from the consumption of a private good (x) and a public good (G). The total amount of the public good depends on the local provision levels and citizens hold different preferences for the public good.

3.3.1 Aggregation of the public good

One of the main tasks of states and administrations is the provision of public goods. Straightforward examples are flood prevention, national defense, fire protection, a clean environment, and more. However, given the example of public fire protection, a state does not simply provide a certain quantity of the public good *fire protection*. A state constructs fire departments, provides them with personnel and maintains fire trucks and much more. This pattern applies to all mentioned examples. The quality and magnitude of a certain public good that can be enjoyed by citizens is shaped by individual and particulate provision efforts. The specific mechanisms how these particulate provisions contribute to the total quantity of a public good can be quite different. The effectiveness of flood prevention systems is determined by the weakest/lowest dike. The most powerful piece of weaponry in a nation's arsenal shapes this nation's deterrence potential (Hirshleifer, 1983). Yet, in line with a large body of literature, most public goods are envisioned to emerge from a summation of particulate provisions (Bergstrom et al., 1986).[2] From a theoretical perspective, the amount of fire protection available to a citizen at a certain

[1] Dur and Roelfsema (2005) consider this case briefly in their appendix. Their findings hint toward the central mechanism of this paper: strategic delegation to "conservative" representatives. They do not identify the equivalence of centralization and decentralization due to the general structure of their model and the focus on taxation issues.

[2] Refer to Cornes and Hartley (2007) for a generalization of the aggregation function which includes the aspects mentioned.

location is conceptualized best as a weighted sum of particulate, regional provisions. I follow this widely utilized approach such that, given the two-region framework with regional provisions g_j, the quantity of the public good available in region j is $G_j = g_j + \omega g_{-j}$, where ω is a weighting parameter; more on this aspect below. As standard in the literature, let citizens draw positive but falling marginal benefits from the public good. These features are captured by the concave function $b(\cdot)$, such that utility from the public good for a citizen located in region j is

$$u_j = \lambda b\left(G_j\right) = \lambda b\left(g_j + \omega g_{-j}\right). \tag{3.1}$$

The parameter λ characterizes the citizen's preference for the public good relative to a private good (that is omitted here for clarity).

The analysis of the paper at hand builds on the utility function given in Equation (3.1). This constitutes a distinguishing feature compared to the vast majority of research on fiscal federalism. Due to the "assumption of separability" (Dur and Roelfsema, 2005, p. 399), most research departs from theoretical considerations and treats individual provisions as individual goods. The conceptualization of public goods is diluted in exchange for computational convenience. The utility function implied by the simplifying assumption is

$$u_j = \lambda\big(b\left(g_j\right) + \omega\left(g_{-j}\right)\big). \tag{3.2}$$

Considering appearance, the difference between the utility function in Equations (3.1) and (3.2) seems minuscule. Yet, a model based on Equation (3.1) allows to assess the impact of assuming separability. Foremost, the public good G_j can no longer be identified under the assumption of separability and plays no role. Yet, the most severe implication of the assumption of separability concerns the substitutability of individual local provisions. A citizen's marginal rate of substitution between g_j and g_{-j} is given by the constant $\frac{1}{\omega}$ when evaluating the unmodified utility function in Equation (3.1). Hence, individual provisions are perfect substitutes, which is in line with the theoretical conceptualization: Citizens care about individual provisions insofar, as they add to the total accessible quantity G_j. In this regard, one unit of g_j can always be exchanged by the same amount of g_{-j}. In contrast to this, Equation (3.2) implies the marginal rate of substitution $\frac{1}{\omega}\frac{b'(g_j)}{b'(g_{-j})}$. This difference constitutes the core of this paper, since it reshapes the impact of spillovers and delegation of political power.

As noted above, Besley and Coate (2003) build on the assumption of separability and their model features a utility function as given in Equation (3.2). Following from this, Besley and Coate (2003, p. 2614) introduce g_j and g_{-j} as "two local public goods". The interpretation that the two provisions characterize different public goods is formally correct and implied by the assumption of separability. However, since citizens value both these goods similarly (by λ) and derive identical concave utility from g_j and g_{-j}, it appears that both these goods capture similar aspects such that the argumentation is not fully convincing. Dur and Roelfsema (2005) try to resolve this dichotomy by referring to both provisions as *public goods* and introduce the notion of g_j and g_{-j} being bundles of public goods. In contrast to this and in line with the theoretical groundwork introduced above, I argue that g_j and g_{-j} are constituent elements of the same public good and contribute to one aggregate.[3] Furthermore, since the aggregate G_j is not present in these formalizations, it is argued that citizens of region j "care" per se about g_{-j} to justify spillovers (Besley and Coate, 2003, p. 2614). In contrast, in the formalization employed in this paper, citizens care about g_{-j} as it adds to G_j. Thus, spillovers are independent from preferences and rather a technicality, which originates from the nature of the analyzed public good. Therefore, I argue that the approach of this paper can be considered as being closer to the initial conceptualization of public goods as weighted sums.

With respect to the weighting, I set $\frac{1}{1+d} = \omega$. This captures the notion that spillover effects may lessen with increased spatial distance, measured by d. Exemplary, a new fire department in a neighboring city increases the supply of fire protection, but not as effectively as a fire department that is erected in your immediate neighborhood. Spatial proximity is important in this context, since firemen and equipment need to be transported to fires. In the case of environmental protection, namely the reduction of carbon dioxide emissions, $d = 0$.[4] Concerning the concentration of greenhouse gases in the earth's atmosphere, it is irrelevant at which specific location emissions are reduced. A different interpretation of d is variation in the two individual provisions. The model focuses on the public good G, whatever it may be. Hence, an individual provision might not increase the accessible quantity of this public good one-to-one if it is of a different kind, but still related to G.

3.3.2 Heterogeneous preferences and utility

Citizens differ in their preference for the public good, which is captured by the parameter λ. This parameter is always positive, denote the type of median citizen in region j by m_j. Importantly, heterogeneity in preferences exists within and across districts. Without loss of generality, assume that the median citizen in region one exhibits a greater preference for the public good compared to the median citizen in region two, $m_1 \geq m_2$. For brevity, define $\mu \equiv \frac{m_1}{m_2}$, which can be considered as a measure of heterogeneity and is weakly larger than one. Furthermore, as in Besley and Coate (2003), I set $b(\cdot) = \ln(\cdot)$.[5] The utility of a citizen from region j who is of type λ_j is given by

$$u_j = x + \lambda_j \ln \left(g_j + \frac{g_{-j}}{1+d} \right), \tag{3.3}$$

where x refers to the consumed amount of the private good and g_j and g_{-j} are the provision levels of the public good. The public good is produced from the private good by a linear production function. One unit of the private good can be transformed into one unit of the public good. Let $\overline{x}$ be the identical endowment of the private good of both regions. I assume

[3]Many results in the two mentioned works are actually interpreted in this sense. In addition, it is unclear why models that analyze the provision of parks in one region and fire protection in the other region are more interesting than models where regions provide similar public goods.

[4]Buchholz et al. (2005) analyze this setting of global environmental policy without making the assumption of separability. Their results agree with the ones derived here but their model is limited to the decentralized scenario.

[5]Given $b(\cdot) = \ln(\cdot)$, both $g_j = 0$ and $g_{-j} = 0$ result in a utility of minus infinity in the standard specification. Hence, the issue of excessive free riding, where one region does not provide a positive amount of the public good, cannot arise there. This is possible here, since the assumption of separability is discarded.

$\frac{\lambda_j}{\overline{x}} \leq 1$ for every λ_j. This implies that regions are never constrained by their endowment and that every desired quantity of the public good can be provided in a region. The budget constraint reads $\overline{x} \geq g_j + x$ and holds with equality due to the positive marginal utility. Therefore, x can be replaced by $\overline{x} - g_j$ in Equation (3.3). The constant $\overline{x}$ never affects optimal policy and will be dropped in the following analysis.

3.3.3 Election of representatives

Both regions have to elect representatives. All citizens of a region cast a vote and all citizens of a region are eligible to represent the region. There are no benefits of office or costs related to candidacy. However, the elected representatives can choose a course of action and thereby determine the provision levels in the regions. The specific procedure for the political game depends on the form of governance – centralization or decentralization – and is introduced later. This approach deliberately ignores issues such as endogenous candidacy decisions, as I am purely interested in the effects that a specific design of the political system exerts on the incentives of representatives, and ultimately, voters. Representatives maximize private utility. Thus, the types of citizens can also be interpreted as publicly observable party affiliation.

The preferences implied by the utility function in Equation (3.3) are single peaked. Given a citizen's preference for the public good, there is a specific amount of the public good that is strictly preferred to all other provision levels. Moving away from this amount will result in monotonically declining utility on both sides. This argument is also valid when considering representatives, since the relationship between the type of regional representative and the implied provision levels is monotonic. Therefore, the median citizens of the regions, m_1 and m_2, will be decisive in the election of the representatives.

3.3.4 Strategies and equilibrium concept

The strategies of voters are given by voting behavior. The strategy of a representative of region j is an assignment that specifies the provision level g_j, given the representative's own type and the provision level in the other region, g_{-j}. Since every citizen is eligible, every citizen and voter is a potential representative and exhibits a strategy that specifies the behavior as representative. The established equilibria are subgame perfect Nash equilibria in pure strategies, henceforth referred to as equilibria. Two conditions must be met in equilibrium: first, the strategies of representatives constitute a Nash equilibrium in all subgames, meaning for all potential combinations of types of representatives; and second, the elected pair of representatives constitutes a Nash equilibrium itself and is mutually, in both districts, majority-preferred (mmp) (Besley and Coate, 2003, p. 2618).

Given a certain form of governance, the model is solved by backward induction. The optimal strategies of representatives of arbitrary types are determined and, in the next step, the equilibrium voting behavior, given the behavior of the representatives, is determined.

3.4 Solution of the model

3.4.1 Decentralization

Under decentralization, local governments decide the provision level in their region. For simplicity, these governments consist of a single representative for each region, and this representative can set the local provision level in her region.[6] Each region finances the respective provision level independently. If the type of region j's representative is λ_j and the amount provided in the other region g_{-j}, then the regarded representative maximizes her utility, given by $u_j(\lambda_j, g_{-j}) = \lambda_j \ln\left(g_j + \frac{g_{-j}}{1+d}\right) - g_j$, by her choice of the provision level g_j. The implied optimal provision level in region j is

$$g_j^* = \begin{cases} \lambda_j - \frac{g_{-j}}{1+d} & \text{for } g_{-j} < \lambda_j(1+d) \\ 0 & \text{for } g_{-j} \geq \lambda_j(1+d) \end{cases}. \tag{3.4}$$

The interpretation is straightforward: region j is idle if the other region provides a large amount relative to λ_j, the regarded representative's preference for the public good, and d, the spillovers. The representative sets a positive g_j if g_{-j} is rather small. Obviously, due to logarithmic utility, $g_1 = g_2 = 0$ can never be an equilibrium. Hence, given two representatives λ_1 and λ_2, three different types of equilibria can occur at the representatives' stage. Either region one or region two provides the public good alone, or both regions provide a positive amount of the public good. One of the former outcomes emerges if one region's representative has a much larger valuation of the public good compared to the other representative. The latter outcome is reached if the types of representatives are sufficiently similar. Depending on λ_1, λ_2 and d, the interaction of the representatives implies the provision levels shown in Figure 3.1 below.

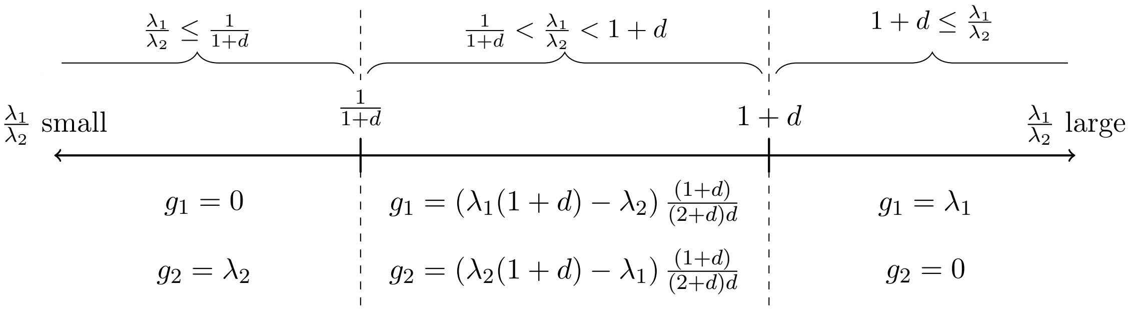

Figure 3.1: Outcomes on the representatives' stage under decentralization
Provision levels g_1 and g_2, given representatives λ_1 and λ_2 and spillovers d, under decentralization.

At the boundaries $\lambda_j = \lambda_{-j}(1+d)$, representatives are indifferent between provision alone by region j and joint provision. For clarity, I assume that region j provides the public good alone at these boundaries. Thus, a representative of slightly lower type, $\lambda_j = \lambda_{-j}(1+d) - \varepsilon$, definitely implies joint provision. The assumption is without loss of generality since the provision levels under joint provision converge to the provision levels in the two more extreme scenarios, $g_j = \lambda_j$

[6] Another interpretation is that local legislatures consist of several seats and the provision level is determined by majority voting. In this case, the median legislator is decisive while the median citizens in both regions can determine the identity of the median legislators.

and $g_{-j} = 0$, for $\varepsilon \to 0$. The model that is analyzed here exhibits similar features to many public goods provision models. Free riding is possible if both regions provide the public good and a race to the bottom occurs. This is more severe if spillovers are large (d is small), as free riding becomes easier. At the same time – and caused by this inefficiency – larger spillovers make joint provision more unlikely as an outcome.

In the first stage, citizens of region j vote for and determine the identity of their representative. In this process, they take the type of the other region's representative, λ_{-j}, as given and anticipate equilibrium play by the elected representatives. As argued, the median citizens of the districts are decisive. The median citizen of region j faces the optimization problem

$$\max_{\lambda_j} \; u_j = m_j \ln\left(g_j(\lambda_j, \lambda_{-j}) + \frac{g_{-j}(\lambda_j, \lambda_{-j})}{1+d} \right) - g_j(\lambda_j, \lambda_{-j}), \tag{3.5}$$

where $g_j(\lambda_j, \lambda_{-j})$ and $g_{-j}(\lambda_j, \lambda_{-j})$ are functions, as implicitly specified in Figure 3.1. Note that these two functions seem discontinuous, but are in fact smooth at the transitions between the different types of outcomes (on the representatives' stage); see Appendix B.1. However, given λ_{-j}, choosing a different representative may lead to slightly different provision levels while preserving the structure of the outcome, or it may induce a different type of outcome. This is not possible with the assumption of separability. A utility function like Equation (3.2) implies that each region will always provide a positive amount of the public good, since a region's optimal amount is independent of the other region's provision level.

Ignoring the boundaries specified in Figure 3.1 and considering a fixed outcome, the optimal types of the representative for region j from the perspective of the region's median citizen can be found easily:

any λ_j	if region $-j$ provides the public good alone,	(3.6)
$m_j \dfrac{(2+d)d}{(1+d)^2}$	if both regions provide the public good jointly, and	(3.7)
m_j	if region j provides the public good alone.	(3.8)

This provides a first impression of the emergence of strategic delegation. In the case of joint provision, the optimal representative for a region's median citizen exhibits a strictly lower type than the median citizen. The median citizen delegates power strategically to a representative of lower preference because this induces the other region's representative to provide a larger amount of the public good. Under joint provision, g_{-j} increases if λ_j decreases; refer to the provision levels in Figure 3.1.

The representatives given by Equations (3.6), (3.7) and (3.8) were derived by assuming a certain outcome on the representatives' stage. However, given the type of the other region's representative, the optimal representatives exhibited above do not necessarily induce the respective assumed outcome. Therefore, incentive compatibility constraints play a role: A median citizen must ensure that a representative who is selected to induce a certain outcome and certain provision levels, actually has an incentive to do so. Refer to Section B.1 in the appendix for more information on this aspect. The solution of the voting stage can be found in Appendix B.2.

Lemma 3.1 below characterizes the equilibria under decentralization that are established there.

Lemma 3.1 (Equilibria under decentralization)**.** *Under decentralization, the heterogeneity in median preferences, μ, and the spillovers, d, determine which of the following three equilibria can be reached. This is displayed in Figure 3.2 below.*

D1 *Both regions provide the public good jointly. This is an equilibrium if median preferences are rather similar and spillovers rather small. Formally,*

$$\text{for} \quad \mu \leq \frac{1}{(1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right)} : \quad \lambda_j = m_j \frac{(2+d)d}{(1+d)^2}, \quad g_j = \frac{m_j(1+d) - m_{-j}}{1+d} \tag{3.9}$$

D2 *Region one provides the public good alone. This is an equilibrium if median preferences are rather different and spillovers rather large. Formally,*

$$\text{for} \quad \frac{(2+d)d}{1+d} \leq \mu : \quad \lambda_1 = m_1, \quad \lambda_2 \text{ sufficiently small}, \quad g_1 = m_1, \quad g_2 = 0 \tag{3.10}$$

D3 *Region two provides the public good alone. This is an equilibrium if median preferences are very similar and spillovers very large. Formally,*

$$\text{for} \quad \mu \leq \frac{1+d}{(2+d)d} \quad \text{and} \quad \mu \leq \frac{\exp(1)}{1+d} \quad : \quad \lambda_1 \text{ s. s.}, \lambda_2 \quad = m_2 g_1 \quad = 0, g_2 \quad = m_2 \tag{3.11}$$

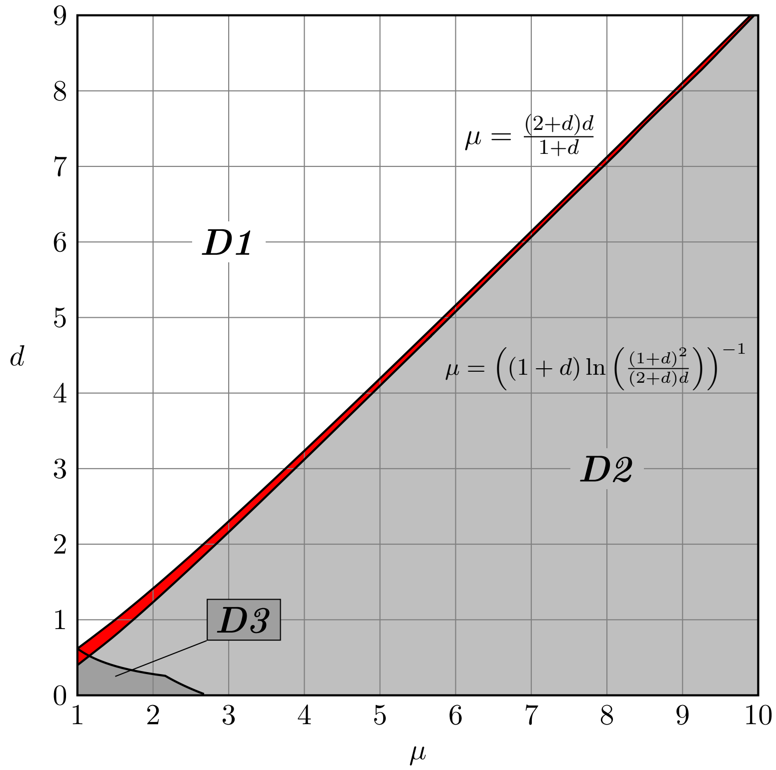

Figure 3.2: Model solution under decentralization
Graphical display of the equilibria under decentralization, as specified in Lemma 3.1. In the red area, either ***D1*** or ***D2*** (or even ***D3***, in intersection with the respective area) can occur.

The intuition behind the three different equilibria is simple. For $\mu < \frac{(2+d)d}{1+d}$, equilibrium ***D1*** can occur and both regions provide the public good jointly. In the relevant parameter space, both regions' median preferences are rather similar. There is no incentive to deviate from joint provision, since free riding and the implied decline in provision levels are not too severe, due to low spillovers. However, starting from any point in ***D1***, joint provision becomes less favorable if spillovers increase (d falls) and/or the difference in median preferences increases (μ increases). In particular, the high preference region is hurt by this change and will at some point be better off becoming the sole provider. This is the case for equilibrium ***D2***. Provision solely by region one is an equilibrium in the respective parameter space, since region one's median preference is much larger than region two's median preference. Spillovers are rather large, which impairs the provision levels under joint provision, such that this is not a viable alternative. It is possible that region two becomes the sole provider of the public good if spillovers are large and the median preferences are sufficiently similar (see equilibrium ***D3***).

3.4.2 Centralization

I now examine a central legislative body that determines both provision levels g_1 and g_2. This central government consists of two representatives, one from each region, and is best envisioned as a form of coalition. The representatives are again elected from within their home regions. The central government can set individual provision levels and the costs of provision are not shared but are incurred by the respective regions. The two representatives maximize joint utility. These assumptions are similar to those outlined by Besley and Coate (2003) and Dur and Roelfsema (2005) and are made for a number of reasons. Exemplary, bargaining aspects are deliberately disregarded since they are known to imply additional incentives for strategic delegation. The same applies to any form of cost sharing. In addition, different provision levels in the two regions are allowed to ensure good comparability of the two forms of governance. Both versions of the model only differ with respect to political decision-making. At the same time, the cooperative nature of the representatives' decision-making potentially allows the internalization of spillovers, which is generally associated with centralization.

The two arbitrary representatives, λ_1 and λ_2, maximize the sum of their private utility, and face the optimization problem

$$\max_{\lambda_1,\lambda_2}\ u_1 + u_2 = \lambda_1 \ln\left(g_1 + \frac{g_2}{1+d}\right) + \lambda_2 \ln\left(g_2 + \frac{g_1}{1+d}\right) - g_1 - g_2. \tag{3.12}$$

The implied optimal provision levels are displayed in Figure 3.3 below. The separating conditions of the three Nash equilibria are the same as under decentralization. Given representatives λ_1 and λ_2, the provision levels are strictly larger under centralization than under decentralization; compare to Figure 3.1. The problem of free riding is mitigated under centralization since representatives maximize joint utility. This can be seen in the provision levels if region j is idle, since the preferences of region j, when properly expressed via λ_j, are nevertheless reflected in the provision level of the public good, $g_{-j} = \lambda_{-j} + \lambda_j$.

As stated above for the case of decentralization, the median citizen in region j determines the identity of the region's representative, taking the other region's represenative as given, and maximizes private utility as exhibited in Equation (3.5). Again, both $g_j(\lambda_j, \lambda_{-j})$ and

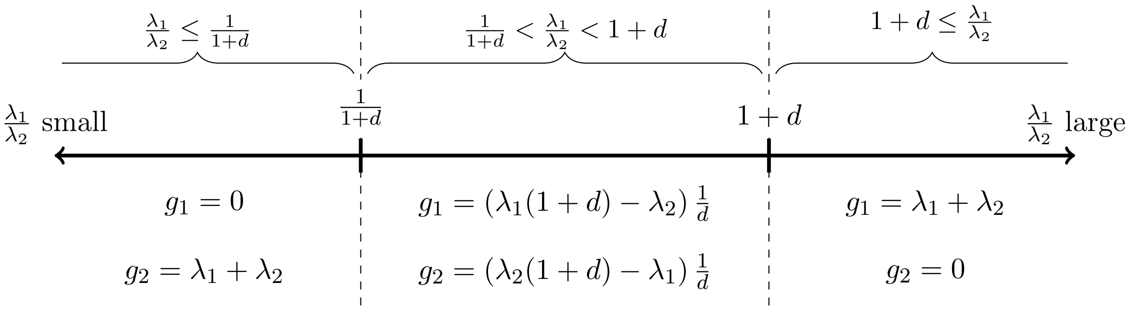

Figure 3.3: Outcomes on the representatives' stage under centralization
Provision levels g_1 and g_2, given representatives λ_1 and λ_2 and spillovers d, under centralization.

$g_{-j}(\lambda_j, \lambda_{-j})$ are functions as implicitly defined in Figure 3.3. As indicated in the figure, the three different types of outcomes – sole provision by one region or joint provision – are also present under centralization. Considering a fixed outcome, the optimal types of representatives for region j from the perspective of the region's median citizen are

$$\lambda_j \text{ as high as possible} \qquad \text{if region } -j \text{ provides the public good alone,} \tag{3.13}$$

$$m_j \frac{d}{1+d} \qquad \text{if both regions provide the public good jointly, and} \tag{3.14}$$

$$m_j - \lambda_{-j} \qquad \text{if region } j \text{ provides the public good alone.} \tag{3.15}$$

As argued above, the representatives in Equations (3.13), (3.14) and (3.15) are not necessarily incentive compatible in the sense that they induce the outcome which was assumed. Refer to Appendix B.3 for more information on this aspect. The determination of mmp representatives is shown in Appendix B.4. Lemma 3.2 below characterizes the equilibrium under centralization.

Lemma 3.2 (Equilibria under centralization)**.** *Under centralization, there exists only one unique equilibrium in pure strategies; this is joint provision.*

C1 *Both regions provide the public good jointly. This is an equilibrium if median preferences are rather similar and spillovers rather small. Formally,*

$$\textit{for} \quad \mu \leq \frac{1}{(1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right)} : \quad \lambda_j = m_j \frac{d}{1+d}, \quad g_j = \frac{m_j(1+d) - m_{-j}}{1+d} \tag{3.16}$$

As specified in Lemma 3.2, centralization as analyzed so far can only give rise to joint provision. Note that the separating condition where joint provision occurs is the same as for equilibrium ***D1*** under decentralization. The absence of equilibria where one region provides the public good alone can be easily explained. If region j provides the public good alone, $g_j = \lambda_j + \lambda_{-j}$, the provision level reflects both representatives' types. Hence, the idle region's median citizen has an incentive to elect a representative whose type is as high as possible (see Equation (3.13)). This is offset by the optimal reaction of the providing region's median citizen (see Equation (3.15)), but implies, nevertheless, that the idle region cannot elect an arbitrarily low type as representative. In turn, this opens the possibility that the providing region can escape the role of sole provider, and this is always profitable. As a result, there are no equilibria in pure strategies where a region provides the public good alone. As outlined, the nonexistence of the relevant equilibria follows from the absence of commitment possibilities in the election phase and should be seen as a technicality. This undesirable feature of the model can be overcome with a slight modification. Assume that region j's median citizen is able to bindingly commit to a certain representative, λ_j. The solution for the sequential model under centralization is shown in Appendix B.5. Lemma 3.3 characterizes the equilibria that arise in this scenario.

Lemma 3.3 (Equilibria under centralization in the sequential model)**.** *Under centralization and if region j elects representative λ_j first, all three established equilibria can occur. The identity of the first mover, the difference in median preferences, μ, and the spillovers, d, determine which unique equilibrium is reached. Refer to Section B.5 in the appendix for all relevant thresholds and see Figure 3.4 below.*

C1 *Both regions provide the public good jointly. This is an equilibrium irrespective of the identity of the first mover if median preferences are rather similar and spillovers rather small. Equilibrium representatives and provision levels are $\lambda_j = m_j \frac{d}{1+d}$ and $g_j = \frac{m_j(1+d)-m_{-j}}{1+d}$.*

C2 *Region one provides the public good alone. This is an equilibrium irrespective of the identity of the first mover if median preferences are rather different and spillovers rather large. Equilibrium representatives and provision levels are $\lambda_1 = m_1 \frac{(1+d)^2-d}{(1+d)^2}$, $\lambda_2 = m_1 \frac{d}{(1+d)^2}$ and $g_1 = m_1$, $g_2 = 0$.*

C3 *Region two provides the public good alone. This is an equilibrium only if region one moves first and if median preferences are very similar and spillovers very large. Equilibrium representatives and provision levels are $\lambda_1 = m_2 \frac{d}{(1+d)^2}$, $\lambda_2 = m_2 \frac{(1+d)^2-d}{(1+d)^2}$ and $g_1 = 0$, $g_2 = m_2$.*

The introduction of sequentiality can be interpreted such that the election in region j happens before the election in the other region, or that the political forecast in region j about the prospective winner is much clearer and more precise, what is known by all voters. The alteration implies that the first region can, with some limitations, by anticipating the other region's optimal response induce a certain outcome. The features that prevented the respective pure equilibria before are no longer present since the region that moves first has to ensure incentive compatibility.

The equilibria of the sequential model are specified in Lemma 3.3. Yet, depending on the identity of the first-mover, different boundaries between the equilibria arise. This is displayed in Figure 3.4. Apparently, the region that elects their representative first enjoys a first mover advantage. Both regions have an incentive to free-ride on the provision levels of the other region. This is most favorable if the respective other region provides the public good alone. In this case, provision levels do not suffer from strategic delegation and the race to the bottom, in contrast to joint provision. Hence, if region one moves first, it will force region two into the role of sole provider whenever it can. The opposite is true if region two moves first. In addition, all equilibria are unique in the sequential model. Whenever median citizens from first moving regions can induce several outcomes, they will always select the most favorable one.[7]

[7]Obviously, this finding is result of the introduced sequentiality. By drawing the identity of the first mover randomly, one obtains a structure of equilibria (put the two graphs in Figure 3.4 on top of each other) that is very similar to the case under decentralization; see Figure 3.2.

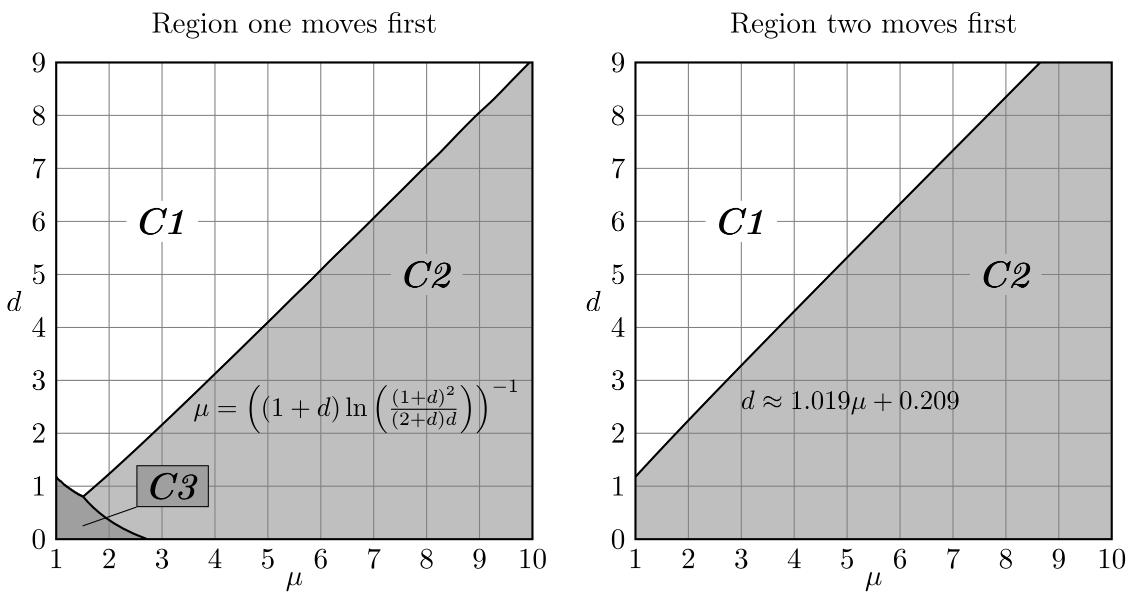

Figure 3.4: Model solution under centralization in the sequential model
Graphical display of the equilibria and relevant boundaries under sequential centralization as specified in Lemma 3.3.

Finally, note that the equilibrium characterized by joint provision is denoted by ***C1*** in the standard and the sequential model (see Lemmas 3.2 and 3.3). This is done for clarity. In addition, the separating condition of equilibrium ***C1*** is the same in both lemmas, if region one is the first mover. Furthermore, the consideration of a sequential model is easily applicable to the scenario of decentralized provision. This is not done here since it does not yield new insights. Hence, the shift to the sequential model does not affect the general outcomes, apart from establishing sole provision of a single region as an equilibrium outcome.

3.5 Comparing different forms of governance

The following comparison builds on the solutions given in Lemmas 3.1, 3.2 and 3.3, which are displayed in Figures 3.2 and 3.4. The measures introduced for heterogeneity (μ) and spillovers (d) withstand any quantitative interpretation. Therefore, the following simplifications can be made without loss of generality: First, disregard the equilibria ***D3*** and ***C3*** where region two provides the public good alone. This can be justified by an argument that relates to the concept of bargaining. It is unlikely that region one, the high preference region, exhibits a greater bargaining power than region two, which would be necessary to make region two the sole provider. Second, let

$$\mu = \frac{(2+d)d}{1+d} \tag{3.17}$$

be the single threshold that separates outcome ***D1*** from outcome ***D2*** under decentralization and ***C1*** from ***C2*** under centralization. The same argument as above applies here: Region two prefers being idle to joint provision, and, due to lower median preference, may be able to induce region one to provide the public good alone. Obviously, these two assumptions help to address multiplicity in outcomes. Yet, the solutions under centralization and decentralization are unique with respect to the identity of the providing regions and apparently identical for the large majority of the parameter space. Therefore, if the two assumptions above seem too restrictive, all the following comparisons hold, regardless, of whether or not one disregards the critical parameter combinations above the 45°-line, meaning around the assumed threshold given in Equation (3.17). Thus, following from the simplifications above, both regions will provide the public good jointly for all $\mu < \frac{(2+d)d}{1+d}$ (equilibria ***D1*** and ***C1***), and region one will provide the public good alone for all other parameter combinations (equilibria ***D2*** and ***C2***). Proposition 3.1 below characterizes the main finding of this paper.

Proposition 3.1 (Equivalence in provision levels)**.** *The individual provision levels and the overall level of the public good are identical under centralization and decentralization. This is true, irrespective of which equilibrium is reached (joint provision or provision alone by region one).*

Proposition 3.1 immediately follows from the solution of the model and the assumption of the single threshold above.[8] Joint provision will always lead to the provision levels $g_j = \frac{m_j(1+d)-m_{-j}}{1+d}$ (for both $j = 1, 2$), and sole provision by region one will always result in $g_1 = m_1$ and $g_2 = 0$. The representatives that are elected in the two different systems and that induce these provision levels are given by the three lemmas. Focusing on the case of joint provision, the representatives are

$$\textbf{\textit{D1}}: \qquad \lambda_j = m_j \frac{(2+d)d}{(1+d)^2}, \qquad \textbf{\textit{C1}}: \qquad \lambda_j = m_j \frac{d}{1+d}. \tag{3.18}$$

These representatives reveal two important features. First, the voting behavior of both regions is shaped by strategic delegation. The median citizens vote representatives into office whose types are strictly lower than their own types. This argument is true for all other voters. All voters would, when asked about their preferred representative, name a representative whose preference for the public good is strictly lower than their own. The reason for the occurrence of strategic delegation is straightforward. Both regions' representatives compete in provision levels, which leads to the typical race to the bottom and sub-optimally low provision levels. Now, voters can influence this competition in their favor by sending a representative who has a small preference for the public good. The election of this representative constitutes a binding signal that the region is not willing to provide a large or even positive amount of the public good and puts pressure on the other region to increase their provision level. Both regions engage in this behavior, and the competition between the representatives is mirrored in the election stage. The second feature concerns the degree of strategic delegation in the two different systems. The representatives that are elected under joint provision and decentralization exhibit a higher preference than their counterparts under centralization since $\frac{d}{1+d} < \frac{(2+d)d}{(1+d)^2}$. This can be understood, when considering the provision levels that are implied by the representatives

[8]The equivalence of provision levels specified in Proposition 3.1 also holds with respect to the equilibria ***D3*** and ***C3***. Hence the above simplification to a single threshold (Equation (3.17)) is only made to improve clarity, but not necessary to obtain the theorem.

under joint provision:

$$g_j = (\lambda_j(1+d) - \lambda_{-j})\,\frac{(1+d)}{(2+d)d} \quad \text{for dec.,} \qquad g_j = (\lambda_j(1+d) - \lambda_{-j})\,\frac{1}{d} \quad \text{for cen.} \tag{3.19}$$

Obviously, the same representatives will settle on larger provision levels under centralization than under decentralization. This is expected since, by assumption, the competition between representatives is less fierce under centralization than under decentralization. The novel feature of this framework of perfect substitutes is, that voters consider the different behavior of representatives and make different delegation decisions. This interplay levels the outcomes of the two systems and leads to the equivalence captured by Proposition 3.1.

Next, consider the case where region one provides the public good alone. The equilibrium representatives are:

$$\textbf{\textit{D2}}: \quad \lambda_1 = m_1, \quad \lambda_2 \text{ s.s.,} \qquad \textbf{\textit{C2}}: \quad \lambda_1 = m_1\frac{(1+d)^2 - d}{(1+d)^2}, \qquad \lambda_2 = m_1\frac{d}{(1+d)^2}. \tag{3.20}$$

As in Lemma 3.1, "s.s." stands for sufficiently small. There is no strategic delegation of power to below median representatives under decentralization. Under centralization, $\lambda_1 < m_1$, and yet this follows from a different logic than the case of strategic delegation in joint provision. The cooperative policy-making of representatives under centralization allows the idle region to increase provision levels while remaining idle. It is clear that this is done by region two, the idle region, but immediately countered by region one such that $g_1 = \lambda_1 + \lambda_2 = m_1$ is always true (see Equation (3.15)). Note that even $m_2 < \lambda_2$ can be true under centralization if region two is idle and whenever median preferences are very heterogeneous, meaning for large μ. Policy-making is less cooperative under decentralization; thus, if region j provides the public good alone, only λ_j determines g_j (see Equation (3.8)). Hence, as is the case for joint provision, the process of policy-making and the identities of the elected representatives differ between the two regimes. This is summarized by Proposition 3.2 below.

Proposition 3.2 (Different policy-making and elected representatives)**.** *The actual process of policy-making, meaning the provision levels specified in Figures 3.1 and 3.3, and the types of the elected representatives, given in Equations* (3.18) *and* (3.20)*, differ between centralization and decentralization.*

The finding that the two different systems, centralization and decentralization, play out differently confirms basic expectations that have shaped research for many years. However, the analysis at hand suggests that this distinction is rather superficial since both systems lead to identical enacted policies. Taking a broader perspective, this is not surprising: the representatives are only a means to an end. The incentives of electorates are unaffected by the mode of provision or the competition between regions, and electorates remain unchanged. Hence, rational voters use these means to reach the same goals and should be able to anticipate that they operate in slightly different ways in the two systems.

3.6 Discussion and extensions

This section provides a discussion of aspects that lie beyond the scope of the main findings. To begin, note that the cause for strategic delegation is different here than in the majority of works on fiscal federalism. As highlighted above, Buchholz et al. (2005) is a notable exception. Usually tax and cost schemes that imply a transfer between regions lead to the election of public good lovers or conservatives. Here, in the absence of cost sharing but with perfectly substitutable local provisions, it seems that spillovers alone are sufficient to generate incentives for strategic delegation. The addition of a policy stage in this context only provides voters with a commitment mechanism that can be utilized in the competition between the regions. Following from this, the addition of a delegation stage and the implied strategic delegation does not merely shift competition between the stages, but aggravates the issue of under-provision. This follows from the welfare maximizing provision levels (see Appendix B.6)

$$g_1^* = \begin{cases} \frac{(1+d)m_1 - m_2}{d} & \text{for } 1+d > \mu \\ m_1 + m_2 & \text{for } 1+d \leq \mu \end{cases} \quad \text{and} \quad g_2^* = \begin{cases} \frac{(1+d)m_2 - m_1}{d} & \text{for } 1+d > \mu \\ 0 & \text{for } 1+d \leq \mu \end{cases}. \tag{3.21}$$

Whenever the electorates in the regions hold votes over actual provision levels instead of representatives, the provision levels are closer to the optimal levels compared to the case with delegation. Obviously, median citizens are decisive in these plebiscites such that they can be treated as the representatives. Hence, $\lambda_j = m_j$ applies, and the provision levels follow from Figures 3.1 and 3.3. In this case, centralization will actually induce the social optimum, although this should not be taken literally as the assumption of cooperative representatives is forced onto the median citizens. It can be easily illustrated that the total degree of strategic delegation is unaffected if one considers a multistage government. In this extension electorates elect parliamentary members who, in turn, select single representatives who determine provision levels. The number of these sequential and intermediary boards is arbitrary; median citizens retain their decisive position, and the last representatives who hold decision-making power are of the same types as the ones identified here. This is not surprising: the act of delegation can only serve as a commitment if the delegating group actually delegates competencies. This is not true for the serial boards that emerge from each other and have identical competencies.

This paper advocates that centralization and decentralization should lead to identical outcomes. In contrast, the behavior of electorates and political representatives is supposedly different in the two systems. Due to the availability and quality of data, "there are very few studies that directly compare centralized and decentralized provision" (Gadenne and Singhal, 2014, p. 600). It is likely that this will be mitigated over the course of time and the aspect of equivalence can be further examined. Related to the latter aspect, it is important to boost the understanding of the operating principles of the two concepts on a smaller scale. In addition, the challenge of measuring abstract concepts such as the degree of decentralization cannot be easily overcome. Martinez-Vazquez and Timofeev (2010) argue that distinct dimensions of decentralization (e.g., the size of local budgets, regional authorities' rights to levy taxes and spending competencies) have different effects in empirical models. Yet, due to data availability, an analysis based on several of these explanatory variables is often not possible. Public good provision has been studied with the help of economic experiments for a long time. Generally, the design of the experiments followed theoretical research and were based on the assumption of separability. However, this can be easily modified. For example, the work of Hamman et al. (2011) provides an experimental framework that can be modified to test whether, and to what degree, voters engage in strategic delegation under varying provision structures. With respect to theoretical research, fiscal federalism has already addressed a large number of different aspects, such as rent-seeking politicians, party politics and more. Any extension that affects both centralization and decentralization similarly will probably not alter the main findings of this paper. However, it is difficult to assess whether certain extensions play out differently in the two systems, as past research relied on the assumption of separability. Therefore, it may be promising to introduce perfect substitutability into different models to obtain an indication of the robustness of the obtained results with regard to the assumption of separability.

Finally, this paper raises concerns as to whether the concepts of centralization and decentralization are fundamentally different. The notion that this is indeed true is deeply rooted in the literature on fiscal federalism and has shaped thinking in this field. However, it is unclear to what extent this notion was also fostered by this research. It is apparent that the two concepts can differ by assumption (for example, the assumptions that centralized governments can only provide a uniform level of the public good and that local administrations are unable to coordinate and internalize spillovers (Oates, 1972)). Although these assumptions can be justified, more recent research has forsaken them and moved to more realistic models. It is by no means implied that modern administrations have, depending on their degree of decentralization, fundamentally different capabilities to make politics or states of information. If this is true, one of the few differences between the two systems may be the political game of representatives. Yet, assuming different versions of this game in the present paper failed to yield different results.

3.7 Conclusion

Recent empirical evaluations of reforms of decentralization around the world indicate that these reforms were, on average, unable to improve the provision of public goods. This paper provides a simple explanation for this finding, which stands in contrast to the large body of theoretical research that addresses the trade-off of different forms of governance related to the provision of public goods. The model introduced in this paper argues that local provisions of a public good are perfect substitutes. This notion is well-established in the literature but relinquished for a simpler approach in the majority of theoretical papers because of analytical convenience. Staying true to this notion leads to identical provision levels under centralization and decentralization. The element that leads to this finding and follows from local provisions being perfect substitutes is strategic delegation. The main findings of this paper are as follows:

(1) Centralized and decentralized structures of government will result in identical provision levels if local provisions are modeled as perfect substitutes.

(2) The elected representatives in centralized and decentralized systems differ in their type, or in other words, in their preference for the public good. In addition, policy-making

differs in the two regimes.

(3) Strategic delegation occurs such that the elected representatives are of lower types than the regions' median citizens whenever the public good is provided jointly by both regions. In contrast to previous works, the emergence of strategic delegation is not caused by any form of cost-sharing.

(4) The degree of strategic delegation is larger under centralization than under decentralization. Under centralization, voters anticipate less intense competition between representatives in their delegation decision.

The question of whether local provisions of public goods are perfect substitutes, has no general answer. The truth probably lies somewhere between the two extremes, or it could by different for each imaginable public good. In light of this, the present model constitutes a counterbalance to the existing models that are based on imperfect substitutes. It provides valuable insights since it reintegrates puzzling empirical findings into the theoretical framework without rendering centralization and decentralization indistinguishable. Finally, by upholding the notion of public goods as weighted sums, it illustrates the gravity of the assumption of separation and demonstrates how modeling can be moved closer to the initial conception of public goods.

B Appendix of Chapter 3

B.1 Optimal representatives and utility under decentralization

With some abuse of notation, denote the representatives in Equations (3.6), (3.7) and (3.8) as *first-best* types, since a certain outcome is assumed to emerge. Yet, given λ_{-j}, the assumed outcome may not be implied by electing these first-best representatives. Accordingly, the optimal representative who induces a desired outcome when the respective first-best representative is not feasible is denoted as *second-best*. When accounting for the separating thresholds ($1+d$ and $\frac{1}{1+d}$), the representatives that maximize median utility while inducing a certain outcome are (given λ_{-j})

$$\lambda_j = \begin{cases} \text{region } j \text{ idle:} & \text{any } \lambda_j \text{ satisfying } \lambda_j \leq \frac{\lambda_{-j}}{1+d} \\ \text{joint provision:} & \begin{cases} \frac{\lambda_{-j}}{1+d} + \varepsilon & \text{for } m_j \frac{(2+d)d}{(1+d)^2} \leq \frac{\lambda_{-j}}{1+d} \\ m_j \frac{(2+d)d}{(1+d)^2} & \text{for } \frac{\lambda_{-j}}{1+d} < m_j \frac{(2+d)d}{(1+d)^2} < \lambda_{-j}(1+d) \\ \lambda_{-j}(1+d) - \varepsilon & \text{for } \lambda_{-j}(1+d) \leq m_j \frac{(2+d)d}{(1+d)^2} \end{cases} \\ \text{prov. alone by region } j\text{:} & \begin{cases} \lambda_{-j}(1+d) & \text{for } m_j < \lambda_{-j}(1+d) \\ m_j & \text{for } \lambda_{-j}(1+d) \leq m_j \end{cases} \end{cases} \tag{3.22}$$

Obviously, ε is infinitesimally small and can be neglected. Now, take an arbitrary λ_{-j}. Assume that the respective separating conditions from Equation (3.22) hold and region j's median citizen elects representatives as specified in Equation (3.22). The implied provision levels g_1 and g_2 can be found with the help of Figure 3.1. As noted, these levels are smooth at the transition from provision alone by one region to joint provision. Thus, the utility of region j's median citizen implied by λ_{-j} and λ_j as in Equation (3.22) is also smooth at these transitions and given by

$$u_j = \begin{cases} \text{r. } j \text{ idle:} & m_j \ln\left(\frac{\lambda_{-j}}{1+d}\right) \\ \text{j. prov.:} & \begin{cases} m_j \ln\left(\frac{\lambda_{-j}}{1+d}\right) & \text{for } m_j \frac{(2+d)d}{(1+d)^2} \leq \frac{\lambda_{-j}}{1+d} \\ m_j \ln\left(m_j \frac{(2+d)d}{(1+d)^2}\right) - m_j + \lambda_{-j} \frac{1+d}{(2+d)d} & \text{for } \frac{\lambda_{-j}}{1+d} < m_j \frac{(2+d)d}{(1+d)^2} < \lambda_{-j}(1+d) \\ m_j \ln\left(\lambda_{-j}(1+d)\right) - \lambda_{-j}(1+d) & \text{for } \lambda_{-j}(1+d) \leq m_j \frac{(2+d)d}{(1+d)^2} \end{cases} \\ \text{r. } -j \text{ idle:} & \begin{cases} m_j \ln\left(\lambda_{-j}(1+d)\right) - \lambda_{-j}(1+d) & \text{for } m_j < \lambda_{-j}(1+d) \\ m_j \ln\left(m_j\right) - m_j & \text{for } \lambda_{-j}(1+d) \leq m_j \end{cases} \end{cases} . \tag{3.23}$$

B.2 Proof of Lemma 3.1

Here, the voters' stage is solved for decentralization. First, it is shown that any outcome where region j provides the public good alone, can only be an equilibrium if region j's representative is the (first-best) type m_j. Obviously, being the sole provider with a second-best representative (see Equation (3.22)), while the first-best representative $\lambda_j = m_j$ is feasible, can never constitute an equilibrium. Therefore, assume that region j is the sole provider of the public good (requires $1+d \leq \frac{\lambda_j}{\lambda_{-j}}$) and $m_j < \lambda_{-j}(1+d)$. In this case, region j can only become the sole provider with the (second-best) representative $\lambda_j = \lambda_{-j}(1+d)$. The implied utility is $u_j = m_j \ln(\lambda_{-j}(1+d)) - \lambda_{-j}(1+d)$. Marginally lowering the representative's type induces joint provision and results in the same utility, see Equation (3.23). Since $m_j < \lambda_{-j}(1+d)$ was assumed, $m_j \frac{(2+d)d}{(1+d)^2} < \lambda_{-j}(1+d)$ is implied and the first-best representative for joint provision is surely of lower type than the currently elected representative. Median utility is single peaked for all outcomes. Therefore, lowering the type of the representative further will surely increase median utility. Hence, the median citizen of region j always faces an incentive to induce joint provision.

In the next step, I establish the outcomes introduced in Lemma 3.1. Start with the case where region two is the sole provider. Assume $\lambda_2 = m_2$ and $\lambda_1 \leq \frac{m_2}{1+d}$. This implies $g_1 = 0$, $g_2 = m_2$ and $u_1 = m_1 \ln\left(\frac{m_2}{1+d}\right)$, region two's median utility is not of relevance here. The elected representatives play best responses and they cannot engage in any profitable deviation. Yet, this is not guaranteed with respect to the decisive median citizens. Lowering λ_1 further cannot induce a different outcome. Hence, there are two possible deviations for (the median citizen in) region one. It can increase λ_1 and induce joint provision or even become the sole provider. Consider the deviation to joint provision first. Joint provision can always be induced by setting $\lambda_1 = \frac{m_2}{1+d} + \varepsilon$. As carved out in Equation (3.23), the median utility in region one resulting from this deviation is equal to the utility that was realized before, when region two was the sole provider. Again, utility in the joint provision scenario is single peaked. Thus, median utility in region one can be further increased whenever an increase on the representative's type moves the representative closer to the first-best type for joint provision, $m_1 \frac{(2+d)d}{(1+d)^2}$. Formally, for $\frac{m_2}{1+d} < m_1 \frac{(2+d)d}{(1+d)^2}$, what can be simplified to

$$\frac{1+d}{(2+d)d} < \mu. \tag{3.24}$$

Now, consider the next deviation: Region one can become the sole provider by sufficiently increasing λ_1. For $m_1 < m_2(1+d)$, the first-best representative $\lambda_1 = m_1$ will not induce provision alone by region one. In these cases, sole provision by region one can only be achieved by the second-best type $\lambda_1 = m_2(1+d)$. In all other cases $\lambda_1 = m_1$ is set. The median utility in region one that follows from these representatives is

$$u_1 = \begin{cases} m_1 \ln\left(m_2(1+d)\right) - m_2(1+d) & \text{for } m_1 < m_2(1+d), \text{ and} \\ m_1 \ln\left(m_1\right) - m_1 & \text{for } m_2(1+d) \leq m_1 \ . \end{cases} \tag{3.25}$$

Subtracting the currently realized utility ($u_1 = m_1 \ln\left(\frac{m_2}{1+d}\right)$) from this deviation utility gives rise to the expressions

$$\Delta u_1 = \begin{cases} m_1 \ln\left((1+d)^2 \exp\left(-\frac{1+d}{\mu}\right)\right) & \text{for } m_1 < m_2(1+d), \text{ and} \\ m_1 \ln\left(\frac{\mu(1+d)}{\exp(1)}\right) & \text{for } m_2(1+d) \leq m_1 \ . \end{cases} \tag{3.26}$$

Only μ and d determine whether there is an increase in utility. An increase occurs, if the arguments of the logarithms are larger than one. This in turn is true given

$$\frac{1+d}{2\ln(1+d)} < \mu \quad \text{for } m_1 < m_2(1+d), \quad \text{and} \quad \frac{\exp(1)}{1+d} < \mu \quad \text{for } m_2(1+d) \leq m_1. \tag{3.27}$$

Note that region two itself might want to lower λ_2 to induce joint provision or even make region one the sole provider. However, any incentive of region two to deviate in this way can be eliminated if region one's representative exhibits a sufficiently small type. E.g. $\lambda_1 = 0$ will always guarantee that region two has an incentive to remain the sole provider of the public good. Thus, the initially assumed representatives are mmp for all parameter combinations that are not ruled out by any of the two established conditions, given by Equations (3.24) and (3.27). This is exhibited in Figure 3.5 below. With regard to the condition in Equation (3.27), it is apparent that only the partial condition for $1 + d \leq \mu$ is of relevance.

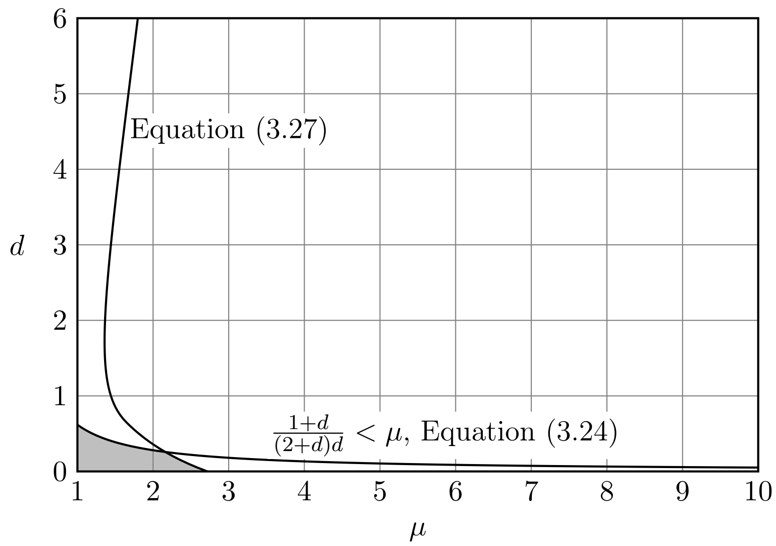

Figure 3.5: Equilibrium ***D3***
Sole provision by region two can occur under decentralization for all μ and d that do not fulfill Equations (3.24) and (3.27), meaning in the gray area. Furthermore, $\lambda_2 = m_2$ is guaranteed and λ_1 must be sufficiently low.

Next, consider the case where region one is the sole provider of the public good. As above, $\lambda_1 = m_1$ and $\lambda_2 \leq \frac{m_1}{1+d}$. This implies $g_1 = m_1$, $g_2 = 0$ and $u_2 = m_2 \ln\left(\frac{m_1}{1+d}\right)$. Representatives play best responses and the focus lies on the regions, or rather the regions' median citizens. Lowering λ_2 further will not affect the outcome. However, region two can deviate by inducing joint provision by electing the representative $\lambda_2 = \frac{m_1}{1+d} + \varepsilon$. Similar as argued above, this is profitable for region two whenever $\frac{m_1}{1+d} < m_2 \frac{(2+d)d}{(1+d)^2}$. This simplifies to

$$\mu < \frac{(2+d)d}{1+d} \tag{3.28}$$

what constitutes the a similar requirement for this case as (3.24) above. Yet, this requirement is harder to fulfill than the counterpart given by Equation (3.24) and resembles region two's lower median preference for the public good. Last, region two might even want to become the sole provider of the public good. Note that this is never possible with the first-best representative

$\lambda_2 = m_2$. $m_2 \leq m_1$ by assumption such that $m_1(1+d) \leq m_2$ can never be true. Therefore, $\lambda_2 = m_1(1+d)$ is required if region two seeks to provide the public good alone. This implies median utility

$$u_2 = m_2 \ln(m_1(1+d)) - m_1(1+d).$$

Subtracting the initial median utility and rearranging yields the inequality

$$\mu < \frac{\ln(1)}{1+d} = 0. \tag{3.29}$$

Obviously, this can never be true. As before, λ_2 must be sufficiently small such that region one itself cannot deviate in a profitable way. Figure 3.6 below displays when sole provision by region one is an equilibrium.

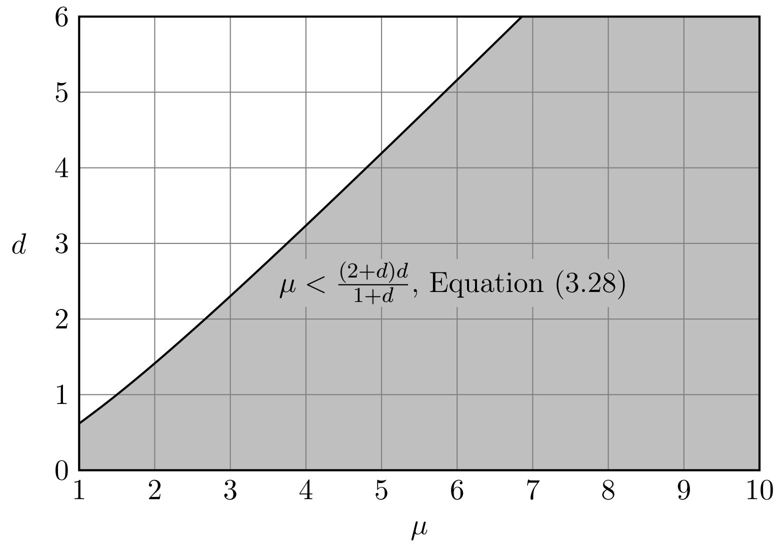

Figure 3.6: Equilibrium ***D2***
Sole provision by region one can occur under decentralization for all μ and d that do not fulfill Equation (3.28), meaning in the gray area. Furthermore, $\lambda_1 = m_1$ is guaranteed and λ_2 must be sufficiently low.

Last, consider the case of joint provision. Joint provision is implied if $\frac{1}{1+d} < \frac{\lambda_1}{\lambda_2} < 1+d$. Assume that both regions elect the first-best representatives $\lambda_j = m_j \frac{(2+d)d}{(1+d)^2}$ (for $j = 1,2$) such that $\frac{\lambda_1}{\lambda_2} = \mu$. By assumption, the lower bound is always true and joint provision is induced if $\mu < 1+d$. The provision levels $g_j = \frac{m_j(1+d)-m_{-j}}{1+d}$ and median utility levels $u_j = m_j \ln\left(m_j \frac{(2+d)d}{(1+d)^2}\right) - \frac{m_j(1+d)-m_{-j}}{1+d}$ are implied. Again, representatives play best responses and one needs to focus on median citizens. The assumed representatives are the first-best representatives for joint provision. Thus, no median citizen has an incentive to elect a different representative who also induces joint provision. This means that there are only two potential deviations possible: Either a region becomes the sole provider of the public good or forces the other region into this role. Start with the first aspect. Region j will become the sole provider with a representative $\lambda_j \geq \lambda_{-j}(1+d) = m_{-j}\frac{(2+d)d}{1+d}$. If $m_{-j}\frac{(2+d)d}{1+d} \leq m_j$, region j can become the sole provider with the first-best representative $\lambda_j = m_j$. Otherwise, the second-best representative

$\lambda_j = m_{-j}\frac{(2+d)d}{1+d}$ is required. The following median utility is implied:

$$u_j = \begin{cases} m_j \ln(m_j) - m_j & \text{for } m_{-j}\frac{(2+d)d}{1+d} \leq m_j \text{, and} \\ m_j \ln\left(m_{-j}\frac{(2+d)d}{1+d}\right) - m_{-j}\frac{(2+d)d}{1+d} & \text{for } m_{-j}\frac{(2+d)d}{1+d} > m_j. \end{cases} \tag{3.30}$$

Subtracting the initial median utility and rearranging leads to the differences

$$\Delta u_j = \begin{cases} m_j \ln\left(\frac{(1+d)^2}{(2+d)d} \exp\left(-\frac{m_{-j}}{m_j(1+d)}\right)\right) & \text{for } m_{-j}\frac{(2+d)d}{1+d} \leq m_j \text{, and} \\ m_j \ln\left(\frac{m_{-j}}{m_j}(1+d) \exp\left(1 - \frac{m_{-j}(1+d)}{m_j}\right)\right) & \text{for } m_{-j}\frac{(2+d)d}{1+d} > m_j. \end{cases} \tag{3.31}$$

Let $j = 1$. The arguments of the natural logarithms reduce to $\frac{(1+d)^2}{(2+d)d} \exp\left(-\frac{1}{\mu(1+d)}\right)$ (in the first-best case) and to $\frac{1+d}{\mu} \exp\left(1 - \frac{1+d}{\mu}\right)$ (in the second-best case). By maximizing or with any computational approach, one can easily show that the argument of the second-best case is bounded above by one. The first-best argument is strictly larger than one if

$$\frac{1}{(1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right)} < \mu. \tag{3.32}$$

The first-best deviation is only feasible for region one if $\frac{(2+d)d}{1+d} \leq \mu$, what is guaranteed by Equation (3.32). Hence region one's median citizen will induce sole provision of the public good by his region for all μ as defined in Equation (3.32). Let $j = 2$. The argument of the logarithms read $\frac{(1+d)^2}{(2+d)d} \exp\left(-\frac{\mu}{1+d}\right)$ (first-best) and $\mu(1+d)\exp(1 - \mu(1+d))$ (second-best). The second-best argument can never exceed one. Yet, becoming the sole provider with the first-best representative is profitable whenever

$$\mu < (1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right). \tag{3.33}$$

and this implies $\mu \leq \frac{1+d}{(2+d)d}$. Hence the analyzed deviation $\lambda_2 = m_2$ is feasible whenever it is profitable and joint provision cannot be an equilibrium for all μ as defined in Equation (3.33). However, as exhibited in Figure 3.7, the related parameter space is included in the one specified by Equation (3.32) where region one deviates from joint provision. To conclude the case of joint provision, consider the regions' incentives to make the other region the sole provider. Region j can achieve this with a representative $\lambda_j \leq \frac{\lambda_{-j}}{1+d} = m_{-j}\frac{(2+d)d}{(1+d)^3}$. This deviation is always feasible and results in median utility $u_j = m_j \ln\left(m_{-j}\frac{(2+d)d}{(1+d^3)}\right)$. After rearranging, the implied surplus in median utility from this deviation is

$$\Delta u_j = m_j \ln\left(\frac{m_{-j}}{m_j(1+d)} \exp\left(1 - \frac{m_{-j}}{m_j(1+d)}\right)\right). \tag{3.34}$$

If can be shown easily that neither $j = 1$, nor $j = 2$ allow that this difference can be strictly positive. Thus, no region has an incentive to make the other region the sole supplier.

Finally, recall that the first-best representatives ($\lambda_j = m_j\frac{(2+d)d}{(1+d)^2}$) for joint provision were initially assumed in this case of joint provision. There are many other combinations of representatives that also induce joint provision. However, none of these combinations can constitute a pair

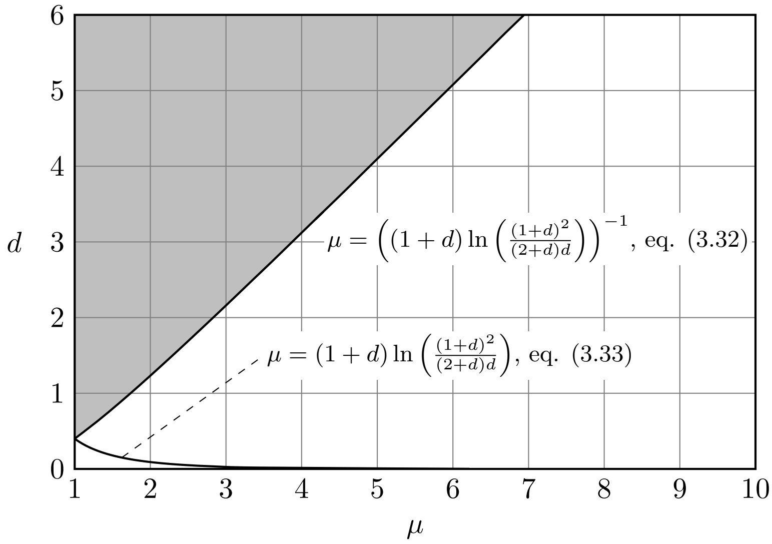

Figure 3.7: Equilibrium ***D1***
Joint provision can occur under decentralization for all μ and d that do not fulfill Equation (3.32), meaning in the gray area. The condition in Equation (3.33) must not be true as well, but this is implied by Equation (3.32). Furthermore, $\lambda_j = m_j \frac{(2+d)d}{(1+d)^2}$ is guaranteed.

of mmp representatives according to the following logic: The unique condition that pins down where joint provision with the first-best representatives is an equilibrium is given by the inverse of Equation (3.32). Yet, the parameter space where the assumed first-best representatives induce joint provision is given by $\mu < 1 + d$ and this space is actually larger than the one where they can constitute an equilibrium. Hence, ignoring the condition $\mu < 1 + d$ and assuming that joint provision will always follow, the first-best representatives for joint provision do not constitute an equilibrium for all μ as specified in Equation (3.32). Thus, no other pair of representatives (that may actually induce joint provision) can be an equilibrium there since the first-best representatives lead to the highest median utility under joint provision for sure. This concludes the proof of Lemma 3.1.

B.3 Optimal representatives and utility under centralization

As in Appendix B.1, I use the labels of first-best and second-best representatives here to refer to the issue of incentive compatibility.

The representatives in Equations (3.13), (3.14) and (3.15) are first-best types as a certain outcome is assumed to emerge. Given λ_{-j}, a certain outcome my not be implied by them. When accounting for the separating thresholds ($1 + d$ and $\frac{1}{1+d}$), the representatives that maximize median utility while inducing a certain outcome are:

$$\lambda_j = \begin{cases} \text{region } j \text{ idle:} & \lambda_j = \frac{\lambda_{-j}}{1+d} \\ \text{joint provision:} & \begin{cases} \frac{\lambda_{-j}}{1+d} + \varepsilon & \text{for } m_j \frac{d}{1+d} \leq \frac{\lambda_{-j}}{1+d} \\ m_j \frac{d}{1+d} & \text{for } \frac{\lambda_{-j}}{1+d} < m_j \frac{d}{1+d} < \lambda_{-j}(1+d) \\ \lambda_{-j}(1+d) - \varepsilon & \text{for } \lambda_{-j}(1+d) \leq m_j \frac{d}{1+d} \end{cases} \\ \text{prov. alone by region } j\text{:} & \begin{cases} \lambda_{-j}(1+d) & \text{for } m_j - \lambda_{-j} < \lambda_{-j}(1+d) \\ m_j - \lambda_{-j} & \text{for } \lambda_{-j}(1+d) \leq m_j - \lambda_{-j} \end{cases} \end{cases} \tag{3.35}$$

ε is infinitesimally small and can be neglected. The most striking difference to the optimal representatives under decentralization, see Equation (3.22), is that an idle region can affect provision levels. Therefore, an idle region has an incentive to elect a representative with the highest feasible preference for the public good. Take an arbitrary λ_{-j}. Assume that region j elects representatives as specified in Equation (3.35) and the thresholds indicated there apply. The implied utility of region j's median citizen is as in Equation (3.36) below. The corresponding provision levels can be found with the help of the formulas in Figure 3.3.

$$u_j = \begin{cases} \text{reg. } j \text{ idle:} & m_j \ln\left(\lambda_{-j} \frac{2+d}{(1+d)^2}\right) \\ \text{j. prov.:} & \begin{cases} m_j \ln\left(\lambda_{-j} \frac{2+d}{(1+d)^2}\right) & \text{for } m_j \frac{d}{1+d} \leq \frac{\lambda_{-j}}{1+d} \\ m_j \ln\left(m_j \frac{(2+d)d}{(1+d)^2}\right) - m_j + \frac{\lambda_{-j}}{d} & \text{for } \frac{\lambda_{-j}}{1+d} < m_j \frac{d}{1+d} < \lambda_{-j}(1+d) \\ m_j \ln\left(\lambda_{-j}(2+d)\right) - \lambda_{-j}(2+d) & \text{for } \lambda_{-j}(1+d) \leq m_j \frac{d}{1+d} \end{cases} \\ \text{reg. } -j \text{ idle:} & \begin{cases} m_j \ln\left(\lambda_{-j}(2+d)\right) - \lambda_{-j}(2+d) & \text{for } m_j - \lambda_{-j} < \lambda_{-j}(1+d) \\ m_j \ln\left(m_j\right) - m_j & \text{for } \lambda_{-j}(1+d) \leq m_j - \lambda_{-j} \end{cases} \end{cases} . \tag{3.36}$$

B.4 Proof of Lemma 3.2

Under centralization, all settings where one region is the sole provider but is unable to achieve this with the first-best representative $\lambda_j = m_j - \lambda_{-j}$, cannot constitute an equilibrium. If region j is the sole provider with a representative $\lambda_j \neq m_j - \lambda_{-j}$, but this first-best representative is actually feasible given λ_{-j}, the choice of the second-best representative is clearly suboptimal. Thus, assume $m_j - \lambda_{-j} < \lambda_{-j}(1+d)$ and region j elects $\lambda_j = \lambda_{-j}(1+d)$ and is the sole provider. The assumed inequality can be rewritten as $m_j \frac{1+d}{2+d} < \lambda_{-j}(1+d)$ what obviously implies $m_j \frac{d}{1+d} < \lambda_{-j}(1+d)$. Therefore, given λ_{-j}, the first-best representative of joint provision is of a strictly lower type than the current representative. This implies that region j's median citizen can strictly increase his utility by switching to joint provision with some $\lambda_j = \lambda_{-j}(1+d) - \varepsilon$.

The following procedure is the same as in the proof of Lemma 3.1 in Section B.2. First, consider provision alone by region j. As established above, this region must be able to elect the first-best representative $\lambda_j = m_j - \lambda_{-j}$. Obviously $\lambda_{-j} = \frac{\lambda_j}{1+d}$, as the idle region must elect a best response to λ_j, see Equation (3.35). These two conditions can de solved and $\lambda_j = m_j \frac{1+d}{2+d}$ and $\lambda_{-j} = m_j \frac{1}{2+d}$ follow. In contrast to decentralization, the idle region can increase the provided amount by electing a high type representative. However, $\lambda_{-j} = m_j \frac{1}{2+d}$ opens the possibility that region j itself can escape the role of sole provider. This could be ruled out under decentralization as the idle region was able to elect a representative of arbitrarily low type. Region j can always deviate and induce joint provision with $\lambda_j = \lambda_{-j}(1+d) - \varepsilon = m_j$ (ε can be ignored). Now, the same argument as above, where sole provision with a second-best representative was ruled out, can be applied. There exists a profitable deviation to joint provision for region j's median citizen if $m_j \frac{d}{1+d} < m_j$, meaning whenever the first-best representative of joint provision is of a lower type than the representative who is elected in the deviation and exactly induces joint provision. This is always true and no region will ever be the sole provider of the public good.

Second, consider the case of joint provision. Both regions set $\lambda_j = m_j \frac{d}{1+d}$. These representatives actually induce joint provision if $\frac{\lambda_1}{\lambda_2} = \mu < 1 + d$. The lower bound $\frac{1}{1+d} < \mu$ is always fulfilled. As under decentralization, the median utility $u_j = m_j \ln \left(m_j \frac{(2+d)d}{(1+d)^2} \right) - \frac{m_j(1+d) - m_{-j}}{1+d}$ is implied. Again, a region can decide to become the sole provider or make the other region the sole provider. Start with the first deviation. With a representative $\lambda_j \geq \lambda_{-j}(1+d) = m_{-j}d$, region j becomes the sole supplier. The first-best representative for sole provision, $\lambda_j = m_j - \lambda_{-j} = m_j - m_{-j}\frac{d}{1+d}$, is feasible if $m_{-j}d \leq m_j - m_{-j}\frac{d}{1+d}$. This condition can be rewritten as $m_{-j}\frac{(2+d)d}{1+d} \leq m_j$ what is the same condition as under decentralization above. If it is not true, region j can only become the sole provider with the second-best representative $\lambda_j = m_{-j}d$. The resulting levels of median utility in both cases are exactly the same as in Equation (3.30). Since the initial utility is also the same here as above under decentralization, the same differences arise, see Equation (3.31). The analysis is exactly the same as above and not reiterated here. Region one's median citizen will benefit from becoming the sole provider with the first-best representative if

$$\frac{1}{(1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right)} < \mu. \tag{3.37}$$

All the parameter combinations specified in Equation (3.37) imply that the first-best representative is feasible. There exists the possibility that region two's median citizen wants to deviate from joint provision. As above, this parameter space is always included in Equation (3.37). Considering the second possible deviation, region j can make region $-j$ the sole provider, reveals the same symmetry as just carved out. Forcing the other region into the role of sole provider can always be done optimally with $\lambda_j = \frac{\lambda_{-j}}{1+d} = m_{-j}\frac{d}{1+d}$. The median utility implied by this deviation is $u_j = m_j \ln \left(m_{-j} \frac{(2+d)d}{(1+d^3)} \right)$. This is the same as for the same deviation under decentralization and joint provision. Hence, recall that initial median utility is identical here, the same differences as given in Equation (3.34) arise. As shown under decentralization, they can never be positive. Figure 3.7 above also depicts the case of joint provision under centralization. Only the types of the representatives for joint provision under centralization are $\lambda_j = m_j \frac{d}{1+d}$. Lastly, the same argument as above applies here: No pair of representatives that induces joint provision but where at least one is not the first-best type for joint provision, can be an equilibrium.

B.5 Proof of Lemma 3.3

Let region j be the first mover that elects the representative λ_j first. Given that region j wants to induce a certain outcome, representative λ_j must be incentive compatible such that the other region's best response, the representative λ_{-j}, induces the desired outcome. Furthermore, λ_j must not only be incentive compatible but additionally maximize utility for region j's median citizen while inducing the required outcome. Much of the intuition of the preceding proofs also applies in this context and can be used. The following three paragraphs determine how region j's median citizen can induce a certain outcome optimally. These findings are later compared to determine the overall best behavior.

First, assume that region j's median citizen wants to induce that the other region becomes the sole provider of the public good. The proof of Lemma 3.2 in Section B.4 above led to various important insights: It can only be optimal for region $-j$ to induce sole provision, if this can be achieved with the respective first-best representative, $\lambda_{-j} = m_{-j} - \lambda_j$. Furthermore, it is likely that region $-j$ fares better with joint provision if λ_j is too large. This was clearly the case for $\lambda_j = \frac{1}{2+d}$ and prevented the existence of a pure strategy equilibrium under centralization in the simultaneous version of the model. Given λ_j, joint provision follows for $\lambda_{-j} = \lambda_j(1+d) - \varepsilon$ (neglect ε). Exploiting the flat transition between the two outcomes with respect to median utility leads to the known condition $\lambda_j(1+d) \leq m_{-j}\frac{d}{1+d}$. If this is fulfilled, the other region's median citizen prefers sole provision to any form of joint provision. Now the second aspect comes into play: Region j profits from a high type representative when being idle. Thus, the obtained inequality constitutes a restriction on λ_j that guarantees incentive compatibility and will hold with equality. $\lambda_j = m_{-j}\frac{d}{(1+d)^2}$ follows. This type allows that region $-j$ elects the first-best representative $\lambda_{-j} = m_{-j}\frac{(1+d)^2-d}{(1+d)^2}$ as an optimal response. The two representatives satisfy $\lambda_j(1+d) \leq \lambda_{-j}$ and imply provision alone by region $-j$. Furthermore, $g_j = 0$ and $g_{-j} = m_{-j}$. The levels of median utility are

$$u_j = m_j \ln\left(\frac{m_{-j}}{1+d}\right) \qquad \text{and} \qquad u_{-j} = m_{-j}\ln(m_{-j}) - m_{-j}. \tag{3.38}$$

Region $-j$ can always make region j the sole provider (optimally) with $\lambda_{-j} = \frac{\lambda_j}{1+d} = m_{-j}\frac{d}{(1+d)^3}$, what leads to $g_j = \lambda_j + \lambda_{-j} = m_{-j}\frac{(2+d)d}{(1+d)^3}$. Region $-j$'s median utility from this behavior is $u_{-j} = m_{-j}\ln\left(m_{-j}\frac{(2+d)d}{(1+d)^4}\right)$. Subtracting u_{-j} as given in Equation (3.38) and rearranging gives

$$\Delta u_{-j} = m_{-j}\ln\left(\frac{(2+d)d}{(1+d)^4}\exp(1)\right),$$

what is negative for all $d \geq 0$. Hence, $\lambda_{-j} = m_{-j}\frac{(1+d)^2-d}{(1+d)^2}$ is the best response to $\lambda_j = m_{-j}\frac{d}{(1+d)^2}$ and sole provision by region $-j$ will follow.

Second, assume that region j wants to become the sole provider. This case is more complicated to resolve than the previous one. Region j's objective requires $\lambda_{-j}(1+d) \leq \lambda_j$. We know from Section B.4 that the outcome for region j's median citizen is suboptimal if region j is the sole provider but he was unable to elect the first-best representative $\lambda_j = m_j - \lambda_{-j}$. For the moment assume that the other region complies and elects a representative who induces sole provision by region j. In this case, since region $-j$ moves second, this other region will surely elect $\lambda_{-j} = \frac{\lambda_j}{1+d}$. The only representative of region j who will, given this optimal response of the other region, still be the first-best representative is $\lambda_j = m_j \frac{1+d}{2+d}$. This must be assumed and $\lambda_{-j} = m_j \frac{1}{2+d}$ follows. Now, there are two potential deviations for region $-j$: The other region may want to induce joint provision or become the sole provider itself. Start with the first deviation. Here, the usual approach can be applied. $\lambda_{-j} = \lambda_j(1+d) - \varepsilon = \frac{m_j}{2+d}$ (neglect the ε) induces joint provision. Median utility at this transition is flat, see Equation (3.36). Hence, raising λ_{-j} further will surely increase median utility if this moves λ_{-j} closer to the first-best representative under joint provision, $m_{-j}\frac{d}{1+d}$. Meaning whenever $\frac{m_j}{2+d} < m_{-j}\frac{d}{1+d}$, what can be rewritten as

$$\frac{m_j}{m_{-j}} < \frac{(2+d)d}{1+d}. \tag{3.39}$$

Consider the second deviation. Region $-j$ might want to become the sole provider. This can only be profitable if it can be achieved with the first-best representative $\lambda_{-j} = m_{-j} - \lambda_j = m_{-j} - m_j\frac{1+d}{2+d}$. However, this first-best representative is only feasible whenever $\lambda_j(1+d) \leq \lambda_{-j}$. Inserting the representatives and rearranging gives $1 + d < \frac{m_{-j}}{m_j}$. Sole provision without the first-best representative is always worse than any form of joint provision. This was considered before and is covered by Equation (3.39). It is left to analyze if region $-j$ has an incentive to become the sole provider if this can be done with the first-best representative. Doing so leads to median utility $u_{-j} = m_{-j}\ln(m_{-j}) - m_{-j}$. Subtracting the utility if region $-j$ complies as sketched above, $u_{-j} = m_{-j}\ln\left(\frac{m_j}{1+d}\right)$, leads to

$$\Delta u_{-j} = m_{-j}\ln\left(\frac{m_{-j}}{m_j}(1+d)\frac{1}{\exp(1)}\right). \tag{3.40}$$

Let $j = 1$. Obviously, $1 + d < \frac{m_{-j}}{m_j} = \frac{1}{\mu}$ can never be true and region two can never, given $\lambda_1 = m_1\frac{1+d}{2+d}$, become the sole provider itself with the first-best representative. Hence Equation (3.40) is not of relevance here. But region two might want to induce joint provision rather than comply and make region one the sole provider. This is the case for $\mu < \frac{(2+d)d}{1+d}$, refer to Equation (3.39). Figure 3.6 above displays the boundary that was derived. The gray area indicates parameter combinations where region one can induce region two to comply and become the sole provider with $\lambda_1 = m_1\frac{1+d}{2+d}$. Let $j = 2$ such that region two aims to become the sole provider. By Equation (3.39), region one will not comply for all $\frac{1+d}{(2+d)d} < \mu$ and induce joint provision. In addition and given $\lambda_2 = m_2\frac{1+d}{2+d}$, region one can become the sole provider with the first-best representative for $1 + d < \mu$. Doing this is profitable whenever $\frac{\exp(1)}{1+d} < \mu$, since the figure in Equation (3.40) is positive in these cases. Note that the conditions that determine when region two can become the sole provider are exactly the same as above in the proof of Lemma 3.1 in Section B.2. See Equations (3.24) and 3.27. Hence, Figure 3.5 also applies here. The gray area characterizes parameter values where region one complies and region two can become the sole provider with $\lambda_2 = m_2\frac{1+d}{2+d}$.

Third and last, consider the case where region j wants to induce joint provision. This requires $\frac{1}{1+d} < \frac{\lambda_j}{\lambda_{-j}} < 1 + d$. Assume that $\lambda_j = m_j\frac{d}{1+d}$, the first-best representative of joint provision.

If region $-j$ complies and elects a representative who induces joint provision, the first-best representative is $\lambda_{-j} = m_{-j}\frac{d}{1+d}$ what is feasible if $\frac{1}{1+d} < \frac{m_j}{m_{-j}} < 1+d$. Assume that it is feasible. As will become clear in the following, this is the case in the parameter space that is derived here. If region $-j$ elects $\lambda_{-j} = m_{-j}\frac{d}{1+d}$, the median utility $u_j = m_j \ln\big(m_j \frac{(2+d)d}{(1+d)^2}\big) - \frac{m_j(1+d)-m_{-j}}{1+d}$ is reached in both regions (exchange j by $-j$). Now, since $\lambda_{-j} = m_{-j}\frac{d}{1+d}$ is feasible, becoming idle is never optimal for region $-j$. However, the other region may be better off if it becomes the sole provider, what must be achievable with the first-best type $\lambda_{-j} = m_{-j} - \lambda_j$ for this to be possible. Hence if $\lambda_j(1+d) \leq m_{-j} - \lambda_j$, insert λ_j and rearrange to $m_j\frac{(2+d)d}{1+d} \leq m_{-j}$. From sole provision with the first-best representative follows utility $u_{-j} = m_{-j}\ln(m_{-j}) - m_{-j}$. The initial utility as indicated above must be subtracted and rearranging leads to

$$\Delta u_{-j} = m_{-j} \ln\left(\frac{(1+d)^2}{(2+d)d}\frac{1}{\exp\left(\frac{m_j}{m_{-j}(1+d)}\right)}\right).$$

The argument of the logarithm is strictly positive if $\frac{m_j}{m_{-j}} < (1+d)\ln\big(\frac{(1+d)^2}{(2+d)d}\big)$. Let $j = 1$ such that region one wants to induce joint provision with $\lambda_1 = m_1\frac{d}{1+d}$. The following three conditions

$$\mu < 1+d, \qquad \mu \leq \frac{1+d}{(2+d)d} \qquad \text{and} \qquad \mu < (1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right) \tag{3.41}$$

are of importance in this case. The first inequality in Equation (3.41) secures that $\lambda_2 = m_2\frac{d}{1+d}$ is feasible and induces joint provision. This also implies that it is not profitable for region two to make region one the sole supplier. The second condition characterizes the parameters where region two can become the sole provider with the first-best representative and the last condition indicates where this is more profitable than compliance and joint provision. Figure 3.8 below shows these conditions and the implied area, where joint provision can be induced by region one as assumed.

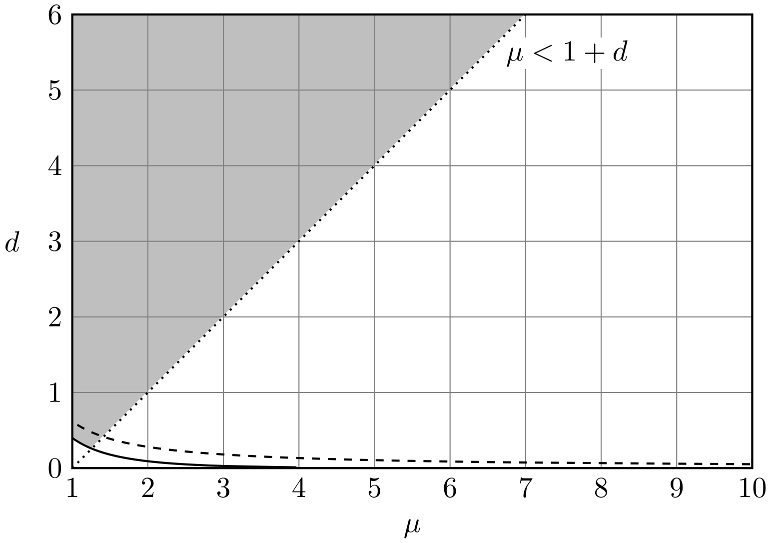

Figure 3.8: Region one induces joint provision
With $\lambda_1 = m_1\frac{d}{1+d}$, region one can induce $\lambda_2 = m_2\frac{d}{1+d}$ and joint provision in the gray area. The dashed (solid) line is the second (third) condition from Equation (3.41).

For $j = 2$, region two wants to induce joint provision. Similar conditions as given in Equation (3.41) for $j = 1$ exist in this scenario and read

$$\mu < 1 + d, \qquad \frac{(2+d)d}{1+d} \geq \mu \qquad \text{and} \qquad \left((1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right)\right)^{-1} < \mu. \tag{3.42}$$

The interpretation is identical as above. The first condition in Equation (3.42) implies that region one can comply with the first-best representative $\lambda_1 = m_1\frac{d}{1+d}$ and joint provision is induced. The second indicates where the first-best representative of sole provision by region one will actually make region one the sole provider. The last condition indicates where this is more profitable than compliance. Figure 3.9 displays these conditions and the area where region two can induce joint provision as assumed.

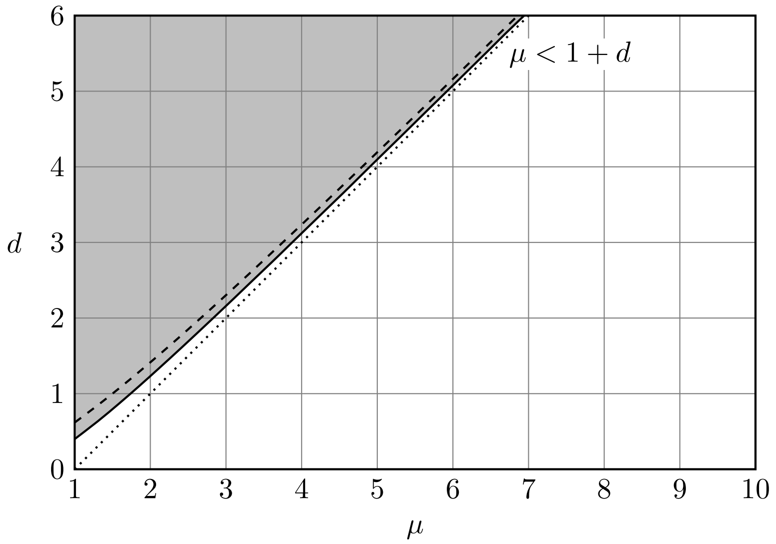

Figure 3.9: Region two induces joint provision
With $\lambda_2 = m_2\frac{d}{1+d}$, region two can induce $\lambda_1 = m_1\frac{d}{1+d}$ and joint provision in the gray area. The dashed (solid) line is the second (third) condition from Equation (3.42).

At this point, a summary of the derived features is due. From the perspective of a region's median citizen, the following strategies are possible:

Aj Region j can always make the other region the sole provider of the public good. This can be done with $\lambda_j = m_{-j}\frac{d}{(1+d)^2}$, what will induce $\lambda_{-j} = m_{-j}\frac{(1+d)^2-d}{(1+d)^2}$. Implied median utility is $u_j = m_j \ln\left(\frac{m_{-j}}{1+d}\right)$.

B1 Region one can become the sole provider with $\lambda_1 = m_1\frac{1+d}{2+d}$ for all $\frac{(2+d)d}{1+d} < \mu$. In this case, $\lambda_2 = m_1\frac{1}{2+d}$ is the best response of region two. Median utility is $u_1 = m_1\ln(m_1) - m_1$.

B2 Region two can become the sole provider with $\lambda_2 = m_2\frac{1+d}{2+d}$ only for parameter combinations that satisfy both $\mu \leq \frac{1+d}{(2+d)d}$ and $\mu < \frac{\exp(1)}{1+d}$ (this is the gray area in Figure 3.5). Region one will set $\lambda_1 = m_2\frac{1}{2+d}$ in these cases and $u_2 = m_2\ln(m_2) - m_2$ follows.

C1 Region one can induce joint provision with $\lambda_1 = m_1\frac{d}{1+d}$ what induces $\lambda_2 = m_2\frac{d}{1+d}$ for parameter combinations that satisfy both $\mu < 1 + d$ and $(1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right) < \mu$. Median utility $u_1 = m_1 \ln\left(m_1\frac{(2+d)d}{(1+d)^2}\right) - \frac{m_1(1+d)-m_2}{1+d}$ follows.

C2 Region two can induce joint provision with $\lambda_2 = m_2 \frac{d}{1+d}$ what leads to $\lambda_1 = m_1 \frac{d}{1+d}$ for all $\mu < \left((1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right)\right)^{-1}$. Median utility $u_2 = m_2 \ln\left(m_2 \frac{(2+d)d}{(1+d)^2}\right) - \frac{m_2(1+d)-m_1}{1+d}$ follows.

Now one can determine which course of action is the most profitable for the median citizen in the region that moves first. Three trade-offs must be considered. Strategy **Aj** (being idle) is strictly preferred to strategy **Bj** (provision alone) by the median citizen in region j, if

$$m_j \ln\left(\frac{m_{-j}}{1+d}\right) > m_j \ln(m_j) - m_j, \qquad \text{what is true if} \qquad \frac{1+d}{\exp(1)} < \frac{m_{-j}}{m_j}.$$

For $j = 1$, the obtained threshold is the same as the one that defined where strategy **B1** is feasible. Region j's median citizen strictly prefers strategy **Bj** (provision alone) to strategy **Cj** (joint provision), if

$$m_j \ln(m_j) - m_j > m_j \ln\left(m_j \frac{(2+d)d}{(1+d)^2}\right) \frac{m_j(1+d) - m_{-j}}{1+d},$$
$$\text{what is true if} \qquad \frac{m_{-j}}{m_j} < (1+d)\ln\left(\frac{(1+d)^2}{(2+d)d}\right).$$

As above, these are exactly the same thresholds that define where the strategies **C1** and **C2** are feasible. Region j's median citizen strictly prefers strategy **Aj** (being idle) to strategy **Cj** (joint provision), if

$$m_j \ln\left(\frac{m_{-j}}{1+d}\right) > m_j \ln\left(m_j \frac{(2+d)d}{(1+d)^2}\right) \frac{m_j(1+d) - m_{-j}}{1+d},$$
$$\text{what is true if} \qquad 1 < \frac{m_{-j}}{m_j} \frac{1+d}{(2+d)d} \exp\left(1 - \frac{m_{-j}}{m_j(1+d)}\right).$$

The last comparison yields no closed form solution. This can be overcome with any computational approach.

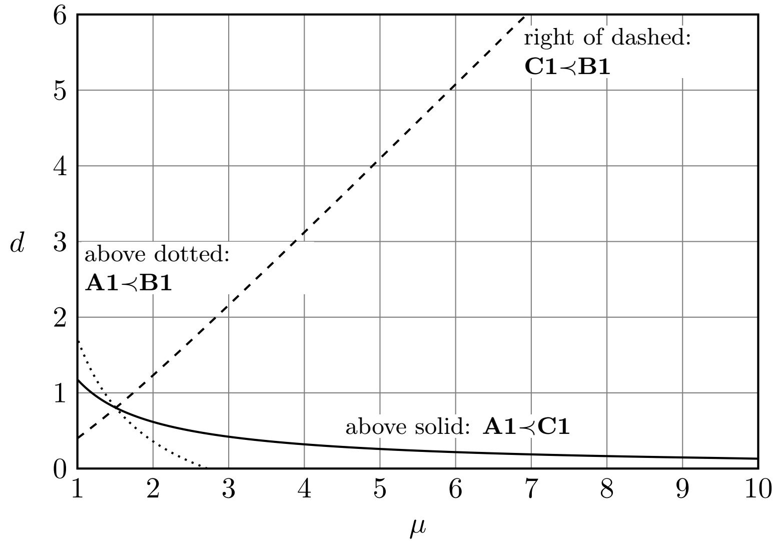

Figure 3.10: Delegation decision of region one's median citizen
Implied preference ordering over the strategies **A1**, **B1** and **C1** of region one's median citizen if region one moves first.

First, consider the incentives of region one's median citizen and let $j = 1$. Figure 3.10 above displays the thresholds that emerge from the three comparisons for $j = 1$, hence it displays the preference ordering of the median citizen in region one. Aligning these preferences with the thresholds that render a certain strategy feasible uniquely pins down the behavior of region one's median citizen: For all parameter combinations where strategy **C1** (joint provision) is the best strategy for region one's median citizen (above the solid and left of the dashed line), it is feasible and chosen. Similarly, all parameter combinations where strategy **B1** (provision alone) is the best strategy (right of the dotted and right of the dashed line), it is feasible and chosen. Strategy **A1** (being idle) is always feasible and thus chosen whenever it is the best choice (below the solid and left of the dotted line).

For $j = 2$, the same analysis has to be conducted. It is slightly more complicated since the comparison of strategies **A2** and **C2** defines not a single threshold, but rather a corridor, where **A2** (being idle) is preferred to **C2** (joint provision). The incentives of the median citizen of region two are displayed in Figure 3.11 below.

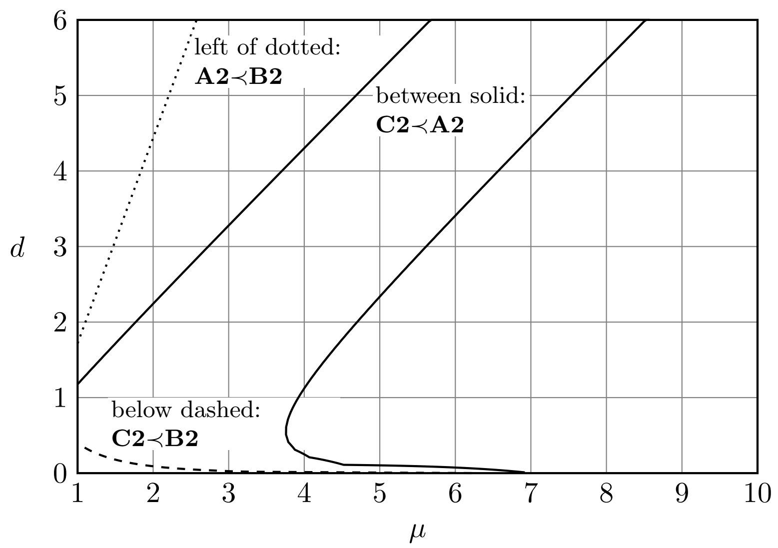

Figure 3.11: Delegation decision of region two's median citizen
Implied preference ordering over the strategies **A2**, **B2** and **C2** of region two's median citizen if region two moves first.

For some parameter combinations, the overall best strategy is not feasible and the median citizen in region two has to use the next best and feasible strategy. Intuitively, region two will never want to become the sole provider itself. The threshold that separates where region two's median citizen induces joint provision and where he induces provision alone by region one is the left of the solid lines in figure. 3.11. This threshold exhibits no closed from solution, but can be precisely approximated using polynomials. This leads to the threshold $d = 1.019\mu + 0.209$. This concludes the proof of Lemma 3.3. The equilibria that result from the optimal strategies of the respective first movers are displayed in Figure 3.4. An interpretation of the findings can be found there.

B.6 Social optimum

Assume that the preference for the public good, λ, is symmetrically distributed in both regions. This eliminates the known issue of misrepresentation if decisive median voters do exhibit average preferences, what is not of interest here. The welfare maximizing provision levels follow from the optimization problem

$$\max_{g_1,g_2} W = m_1 \ln\left(g_1 + \frac{g_2}{1+d}\right) + m_2 \ln\left(g_2 + \frac{g_1}{1+d}\right) - (g_1 + g_2)$$

that can be solved with the standard Kuhn-Tucker approach. They read

$$g_1^* = \begin{cases} \frac{(1+d)m_1 - m_2}{d} & \text{for } 1+d > \mu \\ m_1 + m_2 & \text{for } 1+d \leq \mu \end{cases} \quad \text{and} \quad g_2^* = \begin{cases} \frac{(1+d)m_2 - m_1}{d} & \text{for } 1+d > \mu \\ 0 & \text{for } 1+d \leq \mu \end{cases}.$$

Chapter 4

Accountability or Representation? Democracy and Trade-Offs in Electoral Engineering

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Abstract

Does the design of electoral systems entail a choice between political accountability and representation? While a significant body of research in comparative politics and political economy suggests that the choice of electoral institutions entails a steep trade-off between these two fundamental goals of representative democracy, we argue that trade-offs are contingent and can be weak. We clarify our argument using a formal model that combines conflicts between voters over policy and agency problems between voters and politicians under widely used electoral systems, plurality voting and proportional representation. We show that under common conditions, political polarization and certain distributions of voter preferences, the quality of democracy can be equally bad under both institutions. We label this constellation an electoral sour spot. The model also highlights conditions for trade-offs and electoral sweet spots. Our analysis has implications for the design of electoral systems. With respect to the empirical research agenda, the analysis clarifies the large scope of causal heterogeneity as well as neglected endogeneity and measurement problems.

4.1 Introduction

A fundamental choice for any democracy concerns the design of its electoral system, and several generations of scholars have studied how electoral systems influence (or not) democratic politics and the quality of democracy. One important question in this long-standing debate is to what degree the choice of electoral institutions entails a choice between competing democratic values. A significant body of research in comparative politics and political economy suggests that the design of electoral institutions matters and requires confronting a trade-off between two fundamental goals of representative governance: accountability and representation (Carey and Hix, 2011; Persson and Tabellini, 2000; Powell, 2000). On the one hand, majoritarian electoral systems are thought to be better than systems of proportional representation at fostering political accountability, understood as the ability of voters to throw out bad politicians. On the other hand, proportional systems are deemed to be better at representation, not only descriptively but also by generating policy that better represents the views of the national electorate. Establishing the existence of this trade-off in electoral engineering has recently been described as a "principal conclusion" of the literature (Persson and Tabellini 2008; also see Htun and Powell 2013).

While the idea of trade-offs in the design of electoral institutions is intuitive and has powerful influence on the literature, the academic consensus should not be overstated. For instance, some scholars argue that one electoral system is best on multiple dimensions (Beath et al., 2016; Lijpart, 1994). Several other empirical studies find that electoral reforms, while changing the translation of votes into seats, have surprisingly small or insignificant effects on the quality of democracy (Bowler and Donovan, 2013; Lupu et al., 2017).[1] These divergent views and empirical findings raise important questions about the relevance of electoral institutions. When do they matter for accountability and representation? Under which conditions does the choice of an electoral system imply a steep trade-off between these goals? The institutional theory we develop addresses these issues and helps to reconcile seemingly competing views. It suggests that institutional effects and trade-offs are more contingent than accounted for by existing theoretical approaches and many cross-national studies on the topic. It highlights that electoral geography and political polarization jointly shape when and how electoral systems matter for accountability and representation. Thus, the theory suggests new testable implications about the effects of electoral system design. It also clarifies causality problems faced by empirical research.

We sharpen our argument by developing a game theoretic model of a representative democracy that combines conflicts between voters over policy and agency problems between voters and politicians under commonly used electoral systems, plurality voting (PV) and proportional representation (PR). The formal model allows for varying distributions of voter preferences across

[1] In line with this, an additional empirical motivation for our investigation is the observation that citizens in countries with different electoral systems are similarly (dis)satisfied with political representation and government performance. For instance, survey data from the Comparative Study of Electoral System, Module 2 (2001-2006), suggest that a majority of respondents agrees that the government has been performing well and voters are represented well in Australia, New Zealand and Sweden, which all use different electoral systems. The opposite is the case in Italy (which had just switched back from a mixed system to PR), Israel (PR), Canada (majoritarian) and Germany (mixed).

and within regions of a polity, to capture electoral geography. The number and type of politicians competing in the election is endogenous, as strategic entry decisions are usually thought to be a crucial channel through which electoral rules affect politics. Analyzing the model, we characterize the resulting political equilibrium – in terms of the re-election of demonstrably bad politicians as a crucial aspect of accountability and representation understood as the responsiveness of policy to the national median voter – for all feasible distributions of voter preferences and varying degrees of political polarization. Our main contribution is to highlight and formalize the context-dependent institutional effects. In particular, we identify four distinct patterns depending on political polarization among citizens and electoral geography. First, the quality of democracy can be equally bad under both electoral systems, such that demonstrably bad incumbents are not held accountable and policy is not responsive. We call this constellation an *electoral sour spot*. It implies that a large-scale electoral reform will be ineffectual in improving either accountability or representation. Second, electoral competition generates both accountability and representation under both electoral rules (we call this an *electoral sweet spot*).[2] Third, electoral system choice can entail a significant trade-off between accountability and representation, consistent with the standard view in the literature. Finally, in some contexts one electoral system is best both in terms of accountability and representation. Taken together, seemingly opposing arguments and contradictory empirical findings emerge as special cases in our stylized but instructive theoretical framework.

To clarify the importance of political geography and political polarization in shaping trade-offs in electoral engineering, the model considers an environment in which partisan labels are meaningful and there is common information about the distribution of voter preferences. The mechanisms identified by seminal theories of electoral systems are most likely to work in this setting of institutionalized electoral competition (Cox, 1997; Persson and Tabellini, 2000). By design, the model rules out that an electoral system's failure to promote accountability or representation is due lack of democratic experience or low institutionalization of party competition. Voters are informed and can be coordinated. In that sense, we consider a hard case for exploring the variability in institutional effects. Perhaps surprisingly, the model nonetheless highlights how sensitive trade-offs are to political context, and that the promise of electoral reform may be more limited than we like to admit as scholars of democratic institutions.

The paper is organized as follows: Section 2 discusses the related literature. Section 3 introduces the model, which is solved in Section 4. Section 5 compares the performance of PV and PR with regard to accountability and representation. It contains the main results. Section 6 discusses modifications of the model. Section 7 concludes.

4.2 The debate on electoral systems and trade-offs

A common idea in the comparative politics and political economy literature is that majoritarian electoral systems based on plurality voting in single-member districts foster electoral accountability, understood as the ability of voters to punish bad politicians at the ballot box,

[2] For a different notion of sweet spots, see Carey and Hix (2011).

by reducing party fragmentation or by creating stiff electoral competition that reduces selection and moral hazard problems, at least in swing districts. However, these systems may fare comparatively poorly in representing the policy preferences of the median (or average) citizen in the policy-making process, thus failing on a key dimension of democratic governance in many normative accounts (Dahl, 1971; Powell, 2000). One reading of the literature is that it provides a broad and coherent vision of electoral institutions that has informed real-world debates about constitutional design (Htun and Powell, 2013). The clear-cut implication is that citizens and policy-makers choosing an electoral system have to confront a stark choice between competing goals of democratic governance.

Electoral geography has been identified as one important factor driving failures of representation in majoritarian systems, as the median voter in the median district need not correspond to the median in the population (Calvo and Rodden, 2015; Morelli, 2004; Rodden, 2010). In contrast, electoral systems using a variant of PR with multi-member districts mechanically improve the translation of votes into sets and hence may do a better job at representing the median (or average) citizen in the country at large (Huber and Powell, 1994; Powell, 2000). At the same time, PR systems tend for perform less well in terms of accountability as measured, for instance, using voter responses to bad economic performance (Duch and Stevenson, 2008; Powell, 2000) or to shirking by individual politicians (Gagliarducci et al., 2011).

Formal political economy models highlight that the institutional trade-off is also present when it comes to the determination of economic policy; PV may be comparatively better than PR at fostering accountability, but worse at representing demands for broad public goods (Persson and Tabellini 1999; 2000, ch. 8-9).[3] Considering the case of two-party competition over fiscal policy with purely office-seeking politicians and probabilistic voting, this result stems from stiffer electoral competition in swing districts under plurality rule. On the one hand, the political equilibrium under plurality rule entails more popular control of rent-seeking by politicians. On the other hand, it leads to an overemphasis of geographically targeted spending at the expense of public goods. Note that our approach differs in several ways: politicians vary in both ideology and quality such that accountability is achieved by selecting high-quality politicians (as in Besley, 2007; Myerson, 1993), the number of active parties is endogenous, and electoral geography is treated in a more general way.[4] In equilibrium, trade-offs emerge in some contexts but not in others. Sometimes the polity is stuck in an electoral sour spot and in some contexts one electoral system is best.

Not all prior research is in line with the trade-off view. Theoretically, if voters are strategic and adept at coordination, one might expect that both majoritarian and proportional electoral system faithfully represent the preferences of the median voter (Cox, 1997, 225-237). Empirically, the cross-national evidence on whether proportional electoral systems produce better ideological congruence between voters and parliaments is mixed, with several recent studies finding that institutional differences vary across space, time or measurement approaches (Golder and

[3]Empirical research has found evidence consistent with this trade-off in fiscal policy, mostly based on cross-national data facing difficult endogeneity problems (Acemoglu, 2005; Persson and Tabellini, 2005). For recent studies exploiting rare natural experiments, see Funk and Gathmann (2013); Gagliarducci et al. (2011).

[4]In the majoritarian model of Persson and Tabellini (2000), equilibrium existence requires a restriction on electoral geography in terms of a sufficiently large ideological bias in a majority of safe districts.

Stramski, 2010; Lupu et al., 2017; Powell, 2009). Similarly, a series of case studies of major electoral reforms in developed democracies finds "no clear-cut and clean effects of electoral system change" (Bowler and Donovan, 2013, 78), beyond mechanical changes in the translation of votes into seats. Furthermore, in some game theoretic accounts there is no trade-off between accountability and representation because PR tends to dominate majoritarian systems on both dimensions (Beath et al., 2016; Myerson, 1993).[5] Relatedly, a recent study of political selection in Sweden using unique data on the competence of politicians shows that the selection of competent politicians in a PR system may go hand in hand with a broad representation of social classes (Dal Bó et al., 2017).[6] Our model can rationalize these different outcomes – null effects, trade-offs and best electoral rules – and explain them as a function of primitive features of the polity.

As a result, this paper also communicates with research on the heterogeneous effects of political institutions. Going back at least to Duverger's seminal contribution (Duverger, 1954), scholars have understood that the link between electoral rules and the number of viable competitors depends on context, especially pre-existing societal cleavages (Clark and Golder, 2006; Cox, 1997). In terms of substantive representation, more recent contributions have emphasized factors such as the age of democracy and the institutionalization of the party system (Moser and Scheiner, 2012; Scartascini and Tommasi, 2012).[7] While we agree that these factors are important, we make a complementary point that also has important normative and empirical implications. The effect of electoral institutions on accountability and representation vary with political polarization and electoral geography, and institutional reform may not be effective even in established democracies with institutionalized party systems.

4.3 A model

Our model captures a representative democracy that faces two fundamental problems of democratic governance, representation and accountability, and its purpose is to analyze how they are resolved under varying rules of the electoral game. First, there exists a problem of preference aggregation as citizens disagree over policy on a spatial (i.e., programmatic) dimension. This opens the possibility that the policy chosen in the elected legislature does not appropriately represent the electorate's wishes. There are various notions of representation in the literature. Following scholarship on the effect of electoral institutions on substantive representation, we conceptualize representation as how well the policy enacted in parliament actually represents the interests of the median voter in the population (Cox 1997, 227; Morelli 2004; Powell and Vanberg 2000). To capture the importance of political geography for representation (Calvo and Rodden, 2015; Rodden, 2010), it is necessary to study a model with multiple electoral districts.

[5]Focusing on a broader set of political institutions, Lijpart (1999, 302) makes the related empirical claim that there is no trade-off between representation and effective government.

[6]Some scholars have proposed that particular combinations of electoral institutions – such as PR with low district magnitude or mixed member systems – minimize the trade-off between accountability and representation (Carey and Hix, 2011; Shugart and Wattenberg, 2003), promising "the best of two worlds". The analysis of mixed systems is beyond the scope of this work, though it is not apparent that they can avoid the problems of electoral sour spots.

[7]For a review, see Ferree et al. (2014).

In this respect, we build on previous work in a multi-district setting, especially the model of Morelli (2004).

Second, there exists a principal-agent problem since politicians differ in their quality and the quality of the leading policy-maker is linked to the policy outcome on a valence dimension. Valence is at least partially separable from disagreements about policy (see Stokes, 1963), and all citizens value valence similarly such that they have a common interest in deselecting bad incumbents. The interpretation of quality is broad. One may think of integrity or not being corruptible (Myerson, 1993). One may also think of competence, including skills related to managing the economy (Alesina and Rosenthal, 1995), the provision of public goods (Galasso and Nannicini, 2011) or the quality of legislation more broadly (Londregan, 2000). The quality of politicians is unknown at the beginning of the game (also to the politicians themselves, as in Persson and Tabellini 2000). Yet, the incumbent's quality will be revealed during her first term in office. Accountability fails if a demonstrably low-quality politician is re-elected. This can happen because parties do not fully internalize the societal benefits of selecting high-quality politicians and voters may face a hard choice between quality and policy. As a result, bad politicians may remain in power and parties do not always have strong incentives to replace them.

In the model, politicians are driven by partisan and office motivations. Following Stokes (1963), the performance of an incumbent politician on the valence dimension affects the reputation of the whole party in the next election. This is captured by allowing the pool of politicians to differ in terms of expected quality across parties. Whereas there exists a party for each group in the society, the number of active parties that are actually competing in the political game is determined endogenously through politicians' entry decisions. In this setting, a trade-off between representation and accountability may occur: One candidate/party can be expected to move policy toward a voter's bliss point on the spatial dimension but has a comparatively low expected quality on the valence dimension. Another candidate/party may deliver higher expected quality at the cost of increased spatial distance.[8] The goal of the model is to re-investigate the common view in the literature that the trade-off will be resolved differently under alternative electoral systems, as the result of strategic interactions between voters and politicians, leading to a sharp trade-off in electoral engineering.

Voters and electoral geography

We consider a representative democracy game with two periods $t = 1, 2$. In each period, politicians decide about their candidacy for a seat in a unicameral parliament, citizens cast their votes, which are translated into seats according to the given electoral institution, and finally, elected members of parliament determine the policy x_t on a spatial policy dimension as well as the performance g_t on a separate valence dimension. Figure 4.1 illustrates the timing of events. Including a first period into the model ensures that the initial partisan distribution of legislative seats is generated by equilibrium play and allows us to assess the effect of institutions on representation when politicians are not distinguishable in terms of valence. The tension

[8]For evidence that voters respond to this trade-off, see Eggers (2014); Kayser and Wlezien (2011); Rundquist et al. (1977).

between valence and policy emerges in the second election.

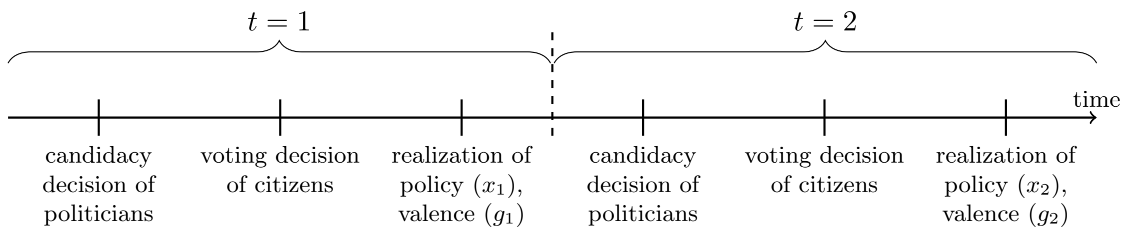


Figure 4.1: Time structure of the model.

To represent the conflict over policy, suppose that there are three groups of citizens that have distinct single-peaked policy preferences in a one-dimensional policy space. These time-invariant policy position are x_i, $i = L, M, H$. Without any loss in generality, we use the following normalization of the policy space: $0 = x_L < x_M < x_H$ and $x_M < x_H/2$. To capture electoral geography, let us further assume that the polity is divided into three districts, indexed by $l = 1, 2, 3$, of equal size that is normalized to one. The share of citizens in district l holding policy preference i is given by μ_i^l. We do not restrict feasible distributions of voters across districts. The set of all possible preference distributions is

$$D = \left\{ \{\mu_i^l\}_{i=L,M,H,\ l=1,2,3} : \sum_i \mu_i^l = 1 \ \forall l \text{ and } \mu_i^l \geq 0 \ \forall i \text{ and } \forall l \right\} \tag{4.1}$$

and $d \in D$ represents one specific distribution of preferences. Citizens do no relocate and d is public knowledge. The population share of citizens holding the policy preference i is denoted by $\mu_i \equiv \sum_l \mu_i^l$.

In addition to the implemented policy x_t, all citizens also care about the performance of the legislature measured on the valence dimension, g_t. To facilitate the exposition, we consider the simple case where performance can either be good ($g_t = 1$) or bad ($g_t = 0$). The parameter β captures the citizens' valuation of performance on the valence dimension relative to policy on the spatial dimension. Taken together, the utility of a citizen of policy preference x_i in period t is given by Equation (4.2) below.[9]

$$U_{i,t} = -|x_t - x_i| + \beta \cdot g_t \tag{4.2}$$

We assume that players maximize periodical utility. This assumption considerably simplifies notational complexity without affecting the substance of the argument.[10] Importantly, the simplification does not eliminate the crucial role of first-period performance for the second period. The revelation of the incumbent's quality allows voters to update their beliefs about the quality of politicians and parties and thereby affects voting behavior in the second period.

[9]The use of the linear functional form to represent voters' utility function in Equation (4.2) is for analytical clarity, but the conclusions do not depend on it.

[10]It is without loss of generality given the tie-breaking rules concerning the selection of the median legislator among several identical partisan types introduced below.

Politicians and policy-making

Politicians in the model care about policy and office. Politicians of type i share the policy preference of the group of citizens which exhibits the bliss point x_i and cannot credibly commit to any other policy: x_i is their observed partisan type. The quality of a politician is denoted by $\omega \in \{0, 1\}$, where high-quality (low-quality) is denoted by $\omega = 1$ ($\omega = 0$). Politicians derive a private benefit from being elected, π. They do not care about the production of valence as such (Besley et al., 2017; Galasso and Nannicini, 2011), which means there is a principal-agent problem between politicians and (like-minded) voters.[11]

In period t, the policy outcome x_t is determined by majority voting in a legislature consisting of three seats. Therefore, the politician with the median ideal point is decisive. Furthermore, the quality of this median legislator determines the government's performance on the valence dimension, g_t, and it is natural to think of the median legislator as the leader of the majority party or government (say the prime minister). A median legislator of high quality ($\omega = 1$) induces good performance of the government ($g_t = 1$) and a median legislator of low quality ($\omega = 0$) a poorly performing government ($g_t = 0$).[12]

Whenever the parliament includes two or more legislators of the same type, the following tie-breaking rule applies: The median legislator is drawn from a core district of her party, meaning a district where partisan supporters are the median. If there is none or more than one legislator with that feature, the leader is determined randomly. Moreover, if the incumbent median legislator and another politician with the same partisan type win seats in the legislature in $t = 2$, the incumbent will maintain her influence on g_2.

The model captures that the observed quality of the first-period median legislator changes the reputation of her party in the subsequent election (Stokes, 1963). It does so by allowing the pool of politicians to vary across parties. This also ensures that trade-offs faced by voters are not ruled out by assumption in several situations.[13] The probability that a politician of (partisan) type i is of high quality is given by θ_i, the complementary probability of being of low quality is $1 - \theta_i$. To let the quality of politicians differ systematically across parties, suppose $\theta_i \in \{\underline{\theta}, \overline{\theta}\}$ and $\underline{\theta} < \overline{\theta}$. Hence, $\theta_i = \overline{\theta}$ denotes a party with a high-quality pool and $\theta_i = \underline{\theta}$ denotes a party with a low-quality pool. Before the game starts, all ω and θ_i are unknown. Initially, parties have identical reputations concerning the expected quality of their politicians and all players hold the prior belief $pr(\theta_i = \overline{\theta}) = q$ about the likelihood of party i being a high quality party, with the counter probability being $pr(\theta_i = \underline{\theta}) = 1 - q$. Due to this, the expected quality of all politicians in the initial election is identical. However, observed performance g_1 reveals the quality of the median legislator in $t = 1$ and allows players to update q using Bayes' rule.

[11] We assume that β is negligibly small for politicians. An equivalent assumption is that the value of office is sufficiently high so that the following holds: $\pi > \tilde{q}_i^0\overline{\theta} + (1 - q_i^0)\underline{\theta} + c$, where $\tilde{q}_i^0$ is defined below.

[12] Nothing of interest changes if the relationship is probabilistic.

[13] What we will later call partial accountability.

Candidacy decisions

Prior to the election, there exists a potential party A_i representing citizens with policy preference i with branches in each district, that serves as an exogenous supply of politicians which may become candidates. What matters is that politicians strategically decide whether to compete in the election, and thus endogenously determine the number of active parties. In each district, each party is represented by one leading politician. This politician is determined by some process of within-party competition that is not modeled here. Politicians simultaneously decide whether to enter and becoming a candidate involves a cost c, with $0 < c < \pi/3$. The strategy of the politician of partisan type i from district l in period t is given by her candidacy decision, $I^l_{i,t}$, where $I^l_{i,t} = 1$ (0) indicates that she declares candidacy (does not declare candidacy).

In the second period, a demonstrably bad incumbent may choose not to run for re-election. In this situation, another politician of the same partisan type becomes the leader of the party in the district. This new politician is free to declare candidacy in the second period such that the incumbent's choice to step down does not deprive the incumbent's party of the chance to enter the competition and win a seat. To capture the principal-agent problem that exists even within a party, the model assumes that an incumbent median legislator who decides to run again in the second period cannot be dislodged by another candidate from her own party. This puts the burden on voters, in interaction with candidates from other parties, to vote a bad incumbent out of office. This assumption captures, in a stylized way, the formal and informal advantages of party elites in the nomination process that have been found or suspected in many democracies.[14] Note, however, that this view of party organization is not essential to our argument. As discussed in Section 4.6.2, the argument goes through under an alternative assumption about party organization, in which political parties decide whether to remove a bad incumbent subject to a (possibly small) political transaction cost.

Electoral institutions

We consider two common electoral systems: plurality voting in single-member districts (PV) and proportional representation (PR) with closed lists. Under PV, each district elects a representative. In line with this, the endogenous set of candidates under PV is given by $Y_t^{PV} = \{Y_t^1, Y_t^2, Y_t^3\}$, where Y_t^l is the set of politicians that have declared their candidacy in district l.[15] Naturally, the candidate who receives a plurality of votes in district l becomes the representative of this district and wins a seat in the legislature. The number of seats obtained by a party is simply determined by the number of districts it has won.

In the PR system, there is a polity-wide election. Candidates of the same partisan type i belong to a party list and voters allocate their votes to the eligible lists. Party A_i will not submit a

[14] For instance, re-selection of incumbent prime ministers was automatic during many decades in the British Labour Party. More generally, formal or informal vetting of candidates by incumbent party elites may preclude serious challengers, even where party members have a say on candidate selection (Katz, 2001). Perhaps as a result, successful leadership challenges are relatively rare (Gruber et al., 2015).

[15] To familiarize notation, the set $Y_t^{PV} = \{L, MH, LH\}$, for example, describes that only the politician of type L declared candidacy in the first district, the politicians of types M and H are candidates in the second district, and the politicians of types L and H are eligible in the last district.

list of candidates if no politician of type i declares candidacy. Y_t^{PR}, the endogenous set of lists in period t, is defined as $Y_t^{PR} = \{Y_{L,t}, Y_{M,t}, Y_{H,t}\}$, where $Y_{i,t}$ is the list of party A_i that includes all politicians of type i who have declared candidacy.[16] Note that politicians may still be associated with a certain district, however, this plays no role under PR and can be neglected.

Under PR, seats are allocated according to the Hare quota with the largest remainder method. Let v_i be the total number of votes for party i's list. The first seat is awarded to the party with the largest vote share v_i. This party's vote share is reduced by one, the quota for one seat (recall that the total size of the voting population is normalized to three). The next seat is then given to the party with the largest vote share/remainder. The winning party's vote share/remainder is (again) reduced by one. The third seat is distributed accordingly. The seats that are won by a party are allocated to the candidates on the party's list starting at the top. With closed lists, voters cannot change the order of candidates on the lists. In the first election, candidates of the same party are indistinguishable and so nothing is lost if we assume, following Morelli (2004), that candidates' order on the list is determined randomly. In line with the above discussion of the incumbency advantage of the first period median legislator in the candidate selection stage, this politician will always obtain the first place on the list of her party in the second period if she declares candidacy. Any other candidates of her party are allocated randomly to the lower ranks of the list.[17] As under PV, a new politician emerges whenever incumbents step down such that there are always three politicians of each party.

Strategic voting and the equilibrium concept

Each voter has to choose among the candidates in her district (under PV) or among party lists (under PR). In line with a large literature on the effects of electoral institutions, we focus on a scenario in which both politicians and voters are strategic (we later consider an alternative behavioral postulate about voters that may be called expressive voting). Following existing work, we also believe that it is realistic to assume that voters can coordinate or be coordinated during the campaign (Indridason, 2011; Morelli, 2004; Persson et al., 2007).

With a large (i.e., infinite) number of voters and given the possibility of multiple candidates, it is obvious that there are multiple Nash equilibria to the game under both electoral systems. This includes the worst-case scenario of a demonstrably bad politician being re-elected in the second period, even though a majority of voters (in a majority of districts) would prefer an alternative party with higher quality and a more aligned policy position. Many of these equilibria are due to a complete failure of electoral coordination among voters, even those in the same district or party. However, in real-world elections there are many coordination devices and the assumption that there is no coordination whatsoever is extreme. For instance, opinion polls, social networks, political campaigns, and tactical voting recommendations by opinion leaders or media outlets

[16] For instance, $Y_t^{PR} = \{LL, MMM\}$ indicates that two politicians of type L have declared candidacy and entered the list of party A_L. The list of party A_M includes three candidates. No politician of type H has declared candidacy such that party A_H is not participating in the election. For brevity, empty sets $Y_{i,t}$ are not included in Y_t^{PR}.

[17] The condition $\pi/3 > c$ ensures that a party that expects to win at least one seat will have a full list, if there is no incumbent who can occupy the top spot. If an incumbent occupies the top spot, both remaining politicians of the party join the list (behind the incumbent) if there are at least two seats to be won.

can coordinate voters. In the case of politicians, party hierarchies and political campaigns facilitate coordination. Moreover, an important point we want to highlight with the model is that electoral systems may fail to generate accountability and/or representation despite the capacity for electoral coordination in the polity, based on institutionalized party competition and sufficient experience with the electoral game.

Thus, we analyze strategic behavior that is robust to credible deviations by any coalition of voters in an environment where they can coordinate within and across districts. Formally, we employ the solution concept of perfectly coalition-proof Nash equilibrium in pure strategies (henceforth simply referred to as equilibrium). Analytically, this means that we can treat citizens of each group i as one voter with multiple votes. Under PV, the unity of voters of preference i chooses a triplet of candidates, one from the endogenous candidates in each district, denoted by $z_{i,t} = \{z^l_{i,t}\}_{l=1,2,3}$. In this electoral institution, the strategy of i voters, $s^{PV}_{i,t}$ maps to any combination of candidates (Y^{PV}_t) a vote choice $z_{i,t}$.[18] Under PR, voters vote for lists. Strategy $s^{PR}_{i,t}$ maps sets of lists Y^{PR}_t to vote choices $z'_{i,t}$, that are given by a single list that is an element of Y^{PR}_t. In sum, voter strategies are

$$s^{PV}_{i,t} : Y^{PV}_t \to z_{i,t} \qquad \text{and} \qquad s^{PR}_{i,t} : Y^{PR}_t \to z'_{i,t} \; . \tag{4.3}$$

The strategy profile S_t describes the strategies of all players in period t and under a certain electoral rule. Exemplary for PV (this is arbitrary and can be exchanged by PR):

$$S_t = \Big\{ \{I^l_{i,t}\}_{l=1,2,3,\ i=L,M,H} \ , \ \{s^{PV}_{i,t}\}_{i=L,M,H} \Big\} \tag{4.4}$$

A strategy profile S^*_t is an equilibrium if and only if there exists no mutually beneficial and self-enforcing joint deviation from it for any coalition of players (Bernheim et al., 1987). Given the sequential nature of the interactions, perfection requires that S^*_t must also lead to equilibrium play in subgames that are not reached on the equilibrium path. This solution concept is a natural fit for the environment studied in the model and it serves to highlight the importance of electoral geography and political polarization. In contrast to the concept of strong Nash equilibrium, coalition-proofness requires that joint deviations are self-enforcing. As a result, we do not assume that equilibria are Pareto-efficient. Rather, we analyze under what conditions strategic interactions lead to an efficient outcome. A coalition-proof equilibrium exists even when there is no strong Nash equilibrium.[19]

Finally, recall that political uncertainty in this setting is not strategic: all players have the same beliefs about the quality of politicians, and so there is no signaling. Whenever necessary, posterior beliefs after observing g_1 are consistent with Bayes' rule.

[18]Concerning notation, the tuple $z_{M,2} = \{L, M, H\}$ specifies that the voters of preference M vote for the candidate of type L in the first district, for the candidate of type M in the second district and so on.

[19]As discussed in Bernheim et al. (1987), the self-enforceability of a mutually beneficial deviation of a coalition is not impaired, if this deviation creates opportunities to deviate for players outside the coalition (or for any coalitions that include players that are not a member of the initially deviating coalition).

4.4 Equilibrium policy and performance

In this section, we describe the solution of the model for each electoral system. The next section compares equilibrium representation and accountability across electoral systems and thus characterizes institutional (non-)effects.

4.4.1 First election

Equilibrium play and policy x_1 in the initial period $t = 1$ are identical to the case of pure spatial policy conflict previously analyzed in the literature. This is so by construction because valence g_1 is only realized after the initial election. Ex-ante, parties have the same expected quality and expected performance is simply $E(g_1) = q\overline{\theta} + (1-q)\underline{\theta}$. Denote this baseline expected valence as $\bar{g}$. Hence, voting behavior in $t = 1$ is rationally only shaped by considerations about x_1.

Lemma 4.1 (First period median legislator and valence)**.** *In the initial period ($t = 1$) under PV, the partisan type i of the median legislator corresponds to the median citizen in the median district. Under PR, the type of the median legislator is $i \in \{L, H\}$ if the group of policy preference i is sufficiently large ($\mu_i = \max\{\mu_L, \mu_M, \mu_H\}$ and $\mu_i - 1 > \min_{j\neq i} \mu_j$) and M otherwise. The expected outcome on the valence dimension is $E(g_1) = \bar{g}$.*

Lemma 4.1 describes the identity of the median legislator, and thereby equilibrium policy, and the expected valence in the first period. A formal proof is omitted as it is nearly identical to the one in Morelli (2004).[20] Under PV, equilibrium policy corresponds to the bliss point of the median voter in the median district. Whether it corresponds to the median in the population and how many parties are actively competing in the election depends on electoral geography, as captured by d. Under PR, by contrast, electoral geography is not important. What matters is the relative size of each group of voters in the national electorate. Intuitively, a group i can ensure the selection of a majority of i politicians if it is large enough to independently secure the election of a majority of legislators. This is the case if $i = \arg\max \mu_i$ and $\mu_i - 1 > \min_{j\neq i} \mu_j$.[21] Otherwise, the median legislator is of type M and $x_1 = x_M$ follows.

4.4.2 Second election after $g_1 = 0$

After observing the outcomes of the initial legislative term (x_1, g_1), the quality of the incumbent leading politician becomes a salient issue in the subsequent election, in addition to policy conflict between citizens. Using the realized performance on the valence issue, g_1, voters and politicians update their beliefs about the quality of the incumbent median legislator and her party to make an inference about $E(g_2)$ for different voting strategies. The fortunate and theoretically straightforward case occurs after $g_1 = 1$, when the incumbent was demonstrably a

[20] The model of Morelli (2004) also considers party mergers. Mergers often do not occur in equilibrium and in strong equilibria they never occur under either system.

[21] The min in the last inequality guarantees that preference group i can determine the identity of the third legislative seat, though not necessarily the second.

high-quality type. In this case, re-electing the incumbent would lead to high future performance, $g_2 = 1$, and, consistent with Bayes' rule, the reputation of her party A_i strictly improves: $Pr(\theta_i = \overline{\theta}|g_1 = 1) = \frac{q\overline{\theta}}{q\overline{\theta}+(1-q)\underline{\theta}} \equiv q_i^1 > q$. Hence, the coalition of voters that previously brought i to power maximizes its payoffs by ensuring that the incumbent median legislator retains her position, and so she will run again and win. As a result, policy will be as in the initial period, $x_2 = x_1$, and performance will be $g_2 = 1$.

The normatively more difficult and theoretically more interesting case occurs after $g_1 = 0$, when the incumbent was demonstrably a low-quality type. In this situation, re-electing the incumbent parliament as is will definitely lead to $g_2 = 0$. The reputation of the incumbent's party A_i also deteriorates: $Pr(\theta_i = \overline{\theta}|g_1 = 0) = \frac{q(1-\overline{\theta})}{q(1-\overline{\theta})+(1-q)(1-\underline{\theta})} \equiv q_i^0 < q$.[22] This means that the electoral coalition that previously brought i to power now faces a trade-off between compromising on prospective quality ($E(g_2)$) or policy (x_2). For voters with ideal policy x_i, the optimal outcome – electing a parliament with high expected valence controlled by their preferred partisan type – is no longer feasible.

Three different accountability scenarios can emerge in equilibrium in this case, depending on electoral institutions (PV or PR), electoral geography (d) and political polarization. The latter can be parameterized as the ideological distance between M and L, $x_M - x_L \equiv \Delta_L \equiv \Delta_M$, and between H and M, $x_H - x_M \equiv \Delta_H$, and by assumption $\Delta_H > \Delta_L, \Delta_M$.[23] First, the bad incumbent is re-elected and the policy and valence outcomes are as before, $x_2 = x_1$ and $g_2 = 0$. This is a failure of accountability. Second, the bad incumbent of partisan type i does not run again and a different member of her party becomes median legislator. This is partial accountability, as this entails throwing the bad incumbent out but keeping a party in power that exhibits a lower probability of providing a competent politician than its competitors: $E(g_2) = q^0\overline{\theta} + (1-q^0)\underline{\theta} < \bar{g}$. We denote this discounted expected valence after a bad incumbent in $t = 1$ by $\bar{g}^0$. Again, $x_2 = x_1$ follows for partial accountability. Third, the bad incumbent and her party are punished in the second election so that a new partisan type with unharmed reputation is elected as median legislator, this is full accountability. Expected valence is $E(g_2) = \bar{g}$ and pivotal voters incur the cost of accepting a change in policy, since $x_2 \neq x_1$.

Plurality voting

Under PV, the distribution of voter preferences within and across districts matters for the equilibrium. To capture how electoral geography shapes accountability and representation, it is useful to distinguish the following cases. First, each group of voters is clustered in a different region and constitutes the median there. This implies that each group of voters can, when voting sincerely, send one representative of their preferred partisan type into the legislature. This set of geographies is denoted by D_1^{PV}. Second, the group holding policy preference x_i constitutes the median in two out of three districts. Hence, when voting sincerely, voters of group i can send two legislators of corresponding partisan type into the legislature. These

[22] Since both q_i^1 and q_i^0 do not depend on i, the subscript can be omitted. Thus, q^1 and q^0 always refer to the party of the median legislator of the first period.

[23] This somewhat redundant labelling Δ_L, Δ_M and Δ_H is chosen to simplify further notation. Note that these distances in the policy space correspond to the policy costs that are incurred by the groups of policy preference L, M and H, if they vote a bad incumbent median legislator out of office in $t = 2$.

geographies belong to set D_{2i}^{PV}. Third, group i is large enough and distributed such that it is the median in all districts. It can determine the allocation of all three seats in the legislature. Accordingly, this set of geographies is labeled D_{3i}^{PV}. Formally, these mutually exclusive and jointly exhaustive geographies are defined as follows:[24]

$$D_1^{PV} \equiv \Big\{d : \mu_L^l, \mu_H^{l'} > 0.5 \text{ for exactly one } l \in \{1,2,3\} \text{ and } l' \neq l\Big\} \tag{4.5}$$

$$D_{2M}^{PV} \equiv \Big\{d : \mu_L^l < 0.5, \mu_H^l < 0.5 \text{ for two } l \in \{1,2,3\}\Big\} \tag{4.6}$$

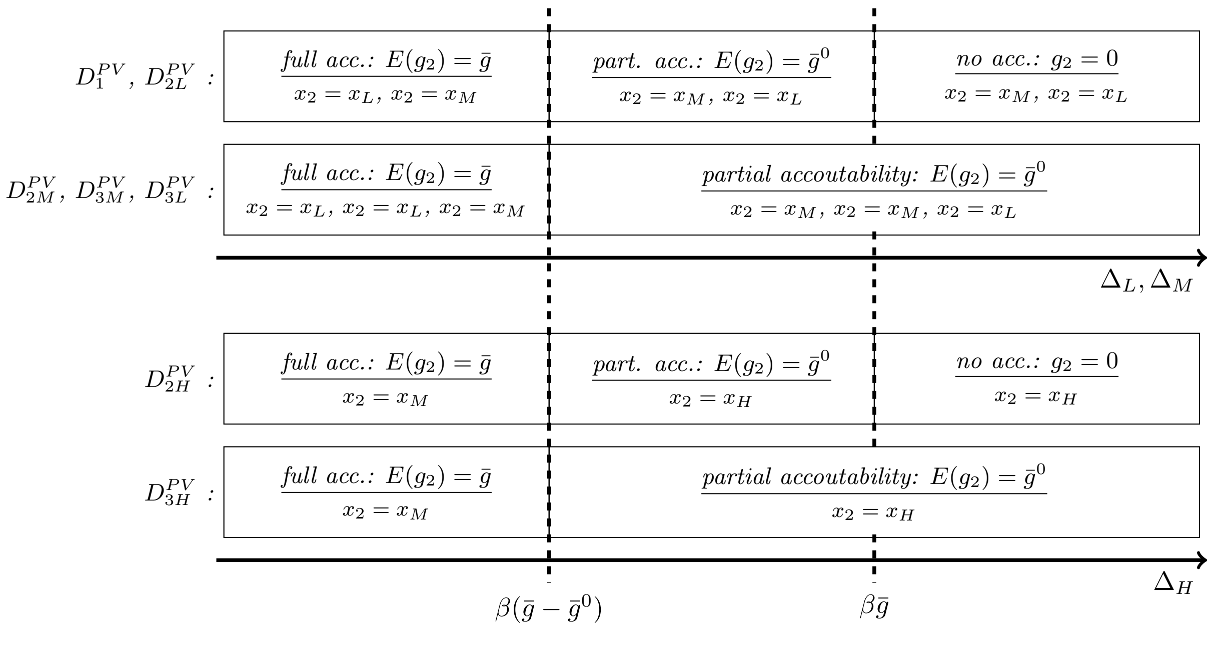
$$D_{2i}^{PV} \equiv \Big\{d : \mu_i^l > 0.5 \text{ for two } l \in \{1,2,3\}\Big\} \qquad \text{for } i \in \{L,H\} \tag{4.7}$$

$$D_{3M}^{PV} \equiv \Big\{d : \mu_L^l < 0.5, \mu_H^l < 0.5 \text{ for all } l \in \{1,2,3\}\Big\} \tag{4.8}$$

$$D_{3i}^{PV} \equiv \Big\{d : \mu_i^l > 0.5 \text{ for all } l \in \{1,2,3\}\Big\} \qquad \text{for } i \in \{L,H\} \tag{4.9}$$

Lemma 4.2 below characterizes the equilibrium outcomes in PV as a function of electoral geography and political polarization after bad performance in the first period. A formal proof can be found in Appendix C.1.

Lemma 4.2 (Outcomes under PV in $t = 2$ after $g_1 = 0$)**.** *If the first period median legislator was revealed to be of low quality, under PV second period policy and expected valence depend on electoral geography (rows, y-axis) and political polarization (x-axis, columns) as displayed below:*



Clearly, polarization on the policy dimension works against accountability.[25] However, polarization and geography interact in shaping accountability (as indicated by the expected second-period valence) and policy. If polarization is low, the bad incumbent and her party will be

[24] In the definition of sets of preference distributions relevant under PV (Equations (4.5) to (4.9)), the cases $i = M$ and $i \in \{L, H\}$ must be distinguished. The group of voters of preference $i \in \{L, H\}$ can only constitute a district's median if it holds a majority in the district; this is not necessary for $i = M$.

[25] The parameter β characterizes the valuation of high valence relative to the spatial policy preference. Therefore, in an alternative interpretation high polarization ($\beta\bar{g} < \Delta_i$) can also be caused by a low relative preference β instead of large differences between the bliss points x_L, x_M and x_H.

held accountable and removed from office. The decisive group puts an alternative politician whose type is closest (in terms of its policy preference) into office and full accountability is reached. Intuitively, low polarization eases the cost of throwing out a bad incumbent. Less obviously, the effect of polarization on accountability depends on the geographic distribution of voter preferences. While it is well-established that geography matters for representation, its effect on accountability and selection has been less well understood. High political polarization may either lead to no accountability or to partial accountability. Partial accountability exists despite high polarization if the geographic distribution of voter preferences generates some slack by allowing voters to credibly commit to punish the bad incumbent without changing the policy outcome. This induces the bad incumbent to step down and allows her party to compete with a new politician of higher expected quality q^0. In the absence of such geographically induced slack, the threat to vote a bad incumbent out of office is not credible, as it entails a decrease in the utility of the decisive group of voters due to high polarization. The result is no accountability. This argument implies that less competitive electoral geographies can foster accountability even if political polarization is high.

The logic underlying these results can be conveyed by considering two cases. To begin with, consider the case where L and H are majorities in $l = 1$ and $l = 3$ and M is the median group in $l = 2$, hence $d \in D_1^{PV}$. In this case, $x_1 = x_M$ (recall Lemma 4.1) and let $g_1 = 0$. In $t = 2$, the median type in the median district (M) is decisive when it comes to the identity of this period's median legislator. If political polarization is sufficiently large, there is no accountability, meaning that the bad incumbent is retained as the median legislator. The reason is that M voters cannot credibly commit to vote for a higher quality candidate of a different partisan type and, due to ideological disagreements, L and H voters are unable to form an electoral coalition for higher valence types. Formally, this requires $\beta\bar{g} < \Delta_M$. This condition ensures that M voters are willing to re-elect a parliament with the bad incumbent as the median legislator ($x_2 = x_M$) over a parliament with a majority for party A_L ($x_2 = x_L$) and higher expected valence. It also implies that there will be no successful L and H coalition to oust the incumbent. In equilibrium, only one candidate enters (and wins) in each district.[26] Due to high polarization, M voters cannot commit to vote for an L candidate in the incumbent's district to replace her with another politician of the same party (who can emerge after the incumbent is discouraged from candidacy). Collusion between M voters and another group of voters to throw out the bad incumbent but maintain party M in power by supporting it in another district would be generally welfare-enhancing. Yet, it is not a credible deviation as the non-M voters who initially support a new M candidate would be better off reneging, thereby helping their preferred party to win a majority of seats. Hence, the resulting political equilibrium is not Pareto-efficient.[27]

The threat to throw out the incumbent becomes credible as political polarization becomes more moderate, $\beta(\bar{g} - \bar{g}^0) < \Delta_M < \beta\bar{g}$. If the bad incumbent were to enter, she will be defeated by

[26] There are multiple outcome-equivalent strategy profiles that induce different parliamentary seat distributions in political equilibrium, all with the outcome $x_2 = x_M$ and $g_2 = 0$.

[27] Formally, any coalitional deviation as sketched is ruled out by the requirement of self-enforceability imposed by the equilibrium concept. Furthermore, it becomes apparent that the scenario $d \in D_1^{PV}$ and $\beta\bar{g} < \Delta_M$ cannot exhibit an equilibrium in pure strategies when focusing on strong Nash equilibria (since every strong Nash equilibrium must be located on the Pareto-frontier).

an L candidate as pivotal voters of preference M are willing to absorb the resulting loss on the policy dimension in exchange for an increase in expected valence. Hence, the bad incumbent does not run again. Instead, another politician from the same party takes her place and wins. Although the former incumbent's party has suffered a decline in reputation about candidate quality ($q^0 < q$), the expected quality is sufficiently high to prefer this outcome over partisan turnover. This is a case of partial accountability. Intuitively, the incumbent party will be thrown out of power if political polarization is reduced further to $\Delta_M < \beta(\bar{g} - \bar{g}^0)$. Then L candidates will compete and win a majority of districts, improving the expected performance on the valence dimension to the baseline level, $E(g_2) = \bar{g}$, at the cost of policy divergence Δ_M.

There are distributions of voter preferences d that do not give rise to a failure of accountability even if political polarization is high. To illustrate this, consider the case in which preference M citizens constitute the median in two districts and voters of preference L (or H, this is arbitrary) constitute the majority in the last district. This scenario was denoted by $d \in D^{PV}_{2M}$. We know that $x_1 = x_M$ from Lemma 4.1. After $g_1 = 0$, in equilibrium the incumbent median legislator never runs again. Suppose $\beta\bar{g} < \Delta_M$. In the previously considered electoral geography this level of polarization leads to a re-election of the bad incumbent. This is not true in the present case due to the slack enjoyed by the M voters. M voters are politically more dominant across districts. This puts them in the position to vote against the bad incumbent in her district, while being able to ensure that an M type politician will be the median in the new parliament. In this case, the trade-off faced by M voters between x_2 and g_2 is less steep. In equilibrium, the bad incumbent does not run again, as she would lose, and will be replaced by another M type politician and so policy does not change and expected performance is $E(g_2) = \bar{g}^0$ and (partial) accountability is reached.

Proportional representation

Under PR, the relative size of the groups of like-minded voters matters for the political equilibrium. There are two cases. First, the distribution of group sizes in the population is such that each group can unilaterally ensure the election of one candidate of their preferred party. This subset of distributions is denoted by D^{PR}_1. Given the electoral system, this requires that the largest group, with population share μ_i, is not too large relative to the smallest group. Formally, $\mu_i - 1$ is the remainder of party A_i's seat quota after having been assigned the first seat. As formally defined below, each group can determine one legislator if this residual is smaller than size of the smallest group.

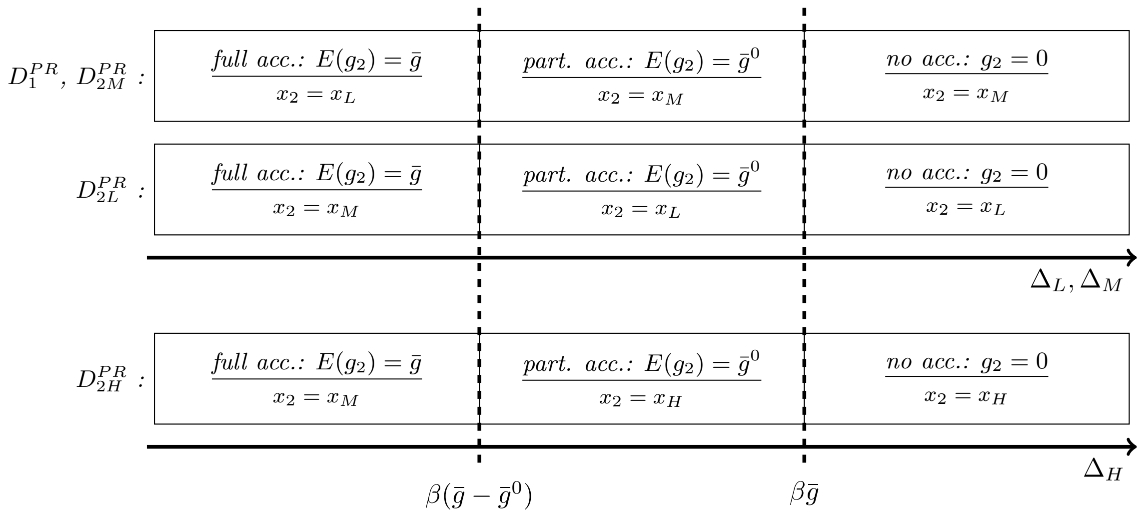
$$D^{PR}_1 \equiv \Big\{ d : \mu_i - 1 < \mu_j \text{ for all } i \in \{L, M, H\} \text{ and } j \neq i \Big\} \tag{4.10}$$

Second, one group i is sufficiently large to unilaterally determine a majority of seats. These distributions are denoted by D^{PR}_{2i}. Formally, this requires

$$D^{PR}_{2i} \equiv \Big\{ d : i = \arg\max \ \mu_i \text{ and } \mu_i - 1 > \min_{j \neq i} \ \mu_j \Big\}. \tag{4.11}$$

Lemma 4.3 characterizes the equilibrium outcomes in the proportional system as a function of relative group size and political polarization after bad performance in the first period (see Appendix C.2 for the proof). As before, the focus is on the interesting case in which the incumbent has been revealed to be a bad type at the end of the first period. This means that voters that initially brought her to power face a tough choice in the subsequent election between compromising on policy or expected quality. In this situation, other politicians will gauge whether there is a new chance to challenge the incumbent.

Lemma 4.3 (Outcomes under PR in $t = 2$ after $g_1 = 0$)**.** *If the first period median legislator was revealed to be of low quality, under PR second period policy and expected valence depend on group size (rows, y-axis) and political polarization (x-axis, columns) as displayed below:*



Similar to PV, Lemma 4.3 shows that polarization undermines accountability under PR. However, it is clear that the adverse effect of polarization on accountability is more pronounced under PR than under PV. If voters can credibly commit to vote against the party of the bad incumbent, she will resign and allow a new politician with a better reputation to take her place. Under PR, this threat is only credibly for low or median levels of polarization. In contrast to PV, accountability is never possible with high levels of polarization. This is because voters vote for closed lists in one large district and incumbents can place themselves at the top of the respective list. As a result, voters under PR have no slack to cast votes based on considerations about quality, without affecting equilibrium policy. This reduces the incentives for bad incumbents to resign and thus undermines accountability. In this institutional setting, high polarization always leads to no accountability, moderate polarization to partial accountability and low polarization to full accountability.

Intuitively, just as in the first period, the distribution of voter preferences over policy (denoted here by *electoral geography*) in the population is crucial for the policy outcome. In contrast to the majoritarian system, it does not affect accountability. In the next section, we will provide several examples that illustrate the underlying logic.

4.5 Comparing electoral institutions

Now we turn to the evaluation of the two electoral systems with respect to representation and accountability. We start by characterizing scenarios, based on electoral geography and polarization, where both PV and PR imply an equally bad or good performance on both these dimensions. Then we consider scenarios, more common in the literature, where PV and PR lead to different outcomes with respect to representation and accountability, suggesting the possibility of electoral engineering.

4.5.1 Electoral sour spots

In an electoral sour spot, representation and accountability fail jointly under both PR and PV. It implies that electoral reform may not improve the quality of democracy on these two important dimensions. Our theoretical framework shows that a sour spot can occur despite the existence of an institutionalized party system in which partisan labels are meaningful and voters are rational and can be coordinated. Proposition 4.1 characterizes the existence of sour spots based on electoral geography and political polarization. It follows directly from Lemmas 4.2 and 4.3 (see Appendix C.3 for a proof).

Proposition 4.1 (Electoral sour spots)**.** *Both PV and PR fail to achieve representation and accountability in the following two scenarios, characterized by electoral geography and polarization:*

- ***A*** *Political polarization is high and electoral geography is such that while corner group L or H constitutes the overall median, each preference group is the median in one district and the smallest group in the population is larger than group i minus the PR seat quota. Formally:*

$$d \in \Big\{ d: \ \mu_i > 1.5, \ \text{for } i \in \{L, H\}, \ \text{but } \mu_L^l > 0.5 \ \text{and } \mu_H^{l'} > 0.5 \ \text{for } l, l' \in \{1,2,3\}, \\ \text{and } \mu_i - 1 < \min_{j \neq i} \mu_j \Big\}, \tag{4.12}$$

$$\beta \bar{g} < \Delta_M \tag{4.13}$$

- ***B*** *Political polarization is high and electoral geography such that center group M constitutes the overall median but another group j is the median in two districts and the smallest group in the population is smaller than the largest group j′ (that may coincide with j) minus the PR seat quota. Formally:*

$$d \in \Big\{ d: \ \mu_j < 1.5, \ \text{for all } j \neq M, \ \text{but } \mu_j^l > 0.5 \ \text{in two } l \in \{1,2,3\} \ \text{for one } j \neq M, \\ \text{and } \mu_{j'} - 1 > \min_{k \neq j'} \mu_k \ \text{for one } j' \neq M \Big\}, \tag{4.14}$$

$$\beta \bar{g} < \Delta_j, \Delta_{j'} \tag{4.15}$$

A failure of accountability means that an evidently low-quality incumbent is re-elected. We have already seen that high political polarization may prevent accountability under PV and that it always does so under PR. Polarization undermines voters' commitment to vote against bad politicians of their preferred party. Recall that representation as defined here concerns whether the policy preferences of the median group are reflected in the policy outcome (Cox, 1997; Morelli, 2004; Powell, 2000). We say that a failure of representation occurs if the elected median legislator does not correspond to the overall median in times when valence considerations do not justify such a deviation.[28] In the model, this implies that the equilibrium policy will deviate from the preferences of the median group. Particular distributions of voter preferences are inimical to achieving representation.

Electoral geography, as classified in Equations (4.5) to (4.11), shapes the influence of voters in the composition of the legislature. There are distributions of voter preferences that lead to a failure of representation in both electoral systems. This can occur in two different constellations which, in conjunction with high polarization, constitute scenarios **A** and **B** formalized in Proposition 4.1. In Scenario **A**, group $i \in \{L, H\}$ constitutes the median but is not too large and heavily clustered in one district. The two other groups are of comparable size and each is clustered in one of the remaining districts. In this case, voters of group i are neither decisive under PV nor under PR, since each group can determine the identity of one legislator under both electoral rules.[29] In Scenario **B**, the group of M voters constitutes the median, but another group $j \in \{L, H\}$ is larger than the group of M voters. If j voters are clustered in two districts and the third group is sufficiently small, group M is indecisive under PV and PR as group j can determine the identity of two legislators.

Table 4.1 provides an example of an electoral sour spot based on condition **A** in Proposition 4.1. The electoral geography is such that voters of preference L are the majority in the population. They are heavily concentrated in the first district and fail to achieve a majority in a majority of districts. As a consequence, representation fails in the PV system. The equilibrium policy in the first period is $x_1 = x_M$. Generally, an electoral geography where some groups of voters are less efficiently distributed in space than others, puts these groups at a disadvantage under PV. The literature on electoral geography has documented that this issue is a common feature in many democracies, resulting from processes of economic transformation such as urbanization and industrialization (Rodden, 2010). Representation also fails under PR, however, because the largest group faces two similarly sized smaller groups of voters, which can manage to win seats in the parliament independently. Accountability fails because polarization is sufficiently high ($\beta(\bar{g}) < \Delta_M$) such that voters face a steep trade-off between policy and quality that does not enable them to incentivize a bad incumbent to step down. While PV can sometimes produce accountability despite high polarization, this requires that there is some slack in political competition. This does not exist in this context, as changing the partisan color of any district changes partisan control in the legislature.

Table 4.2 provides a different example of an electoral sour spot, in line with condition **B** in Proposition 4.1. The voters of preference M constitute the median but are inefficiently distributed in space. In contrast, the distribution of voters of preference L is highly efficient as they can win districts one and two under PV. As the L voters are the largest group and there

[28] After $g_1 = 0$, full accountability and representation exclude each other. Hence, representation is assessed by focusing on the first period (because all parties have the same expected valence) or the second period, given representation is achieved and after $g_1 = 1$ (since the party representing the national median has a valence advantage).

[29] It is also possible that preference group M can win two districts under PV despite group L or H constitutes the overall median. However, this case is ruled out by Equation (4.12) in Proposition 4.1 since it implies slack and thus partial accountability despite high polarization.

	1	2	3	μ_i
L	0.8	0.4	0.4	1.6
M	0.1	0.55	0.05	0.7
H	0.1	0.05	0.55	0.7

Table 4.1: Electoral sour spot (1)
The median group L cannot unilaterally determine a majority of seats. Given $\beta(\bar{g}) < \Delta_M$, an incompetent median legislator is re-elected under both electoral rules and the equilibrium policy $x_t = x_M$ deviates from preferences of the median group M in both periods. See **A** in Proposition 4.1.

exist few voters of policy preference H, representation also fails under PR, since party A_L will win (at least) two seats. Again, high polarization and the electoral geography imply that a low-quality L type incumbent is re-elected.[30]

It is instructive to note that condition **B** in Proposition 4.1 allows that the policy outcome can differ between PV and PR in an electoral sour spot, as long as they deviate from the population median. To make this point, see the example in table 4.8 in Appendix C.7. Hence, under a different definition, one may also say that the electoral system that leads to a lower degree of misrepresentation, given equally bad accountability, is best; in this case PV.

	1	2	3	μ_i
L	0.6	0.6	0.2	1.4
M	0.3	0.3	0.7	1.3
H	0.1	0.1	0.1	0.3

Table 4.2: Electoral sour spot (2)
The minority group L can unilaterally determine a majority of seats in both electoral systems. Given $\beta(\bar{g} - \bar{g}^0) < \Delta_L$, an incompetent median legislator is re-elected and the equilibrium policy $x_t = x_L$ deviates from preferences of the median group M in both periods. See **B** in Proposition 4.1.

4.5.2 Electoral sweet spots

In an electoral sweet spot both electoral systems deliver accountability and representation. This suggests that while electoral reform will be ineffectual with respect to these two goals, there is also less apparent need to consider a reform. Proposition 4.2 below characterizes the joint conditions on polarization and the distribution of voters that give rise to sweet spots, based on Lemma 4.2 and 4.3 (see Appendix C.4 for a proof). In an electoral sweet spot, political polarization is not too high. This allows bad incumbents to be replaced in any of the two electoral systems. In addition, sweet spots require a particular distribution of voter preferences, which will be explained below, to guarantee that the equilibrium policy corresponds to the preferences of the median group.

Note that the characterization of sweet spots does not assume that full accountability is nec-

[30]By switching M and H in the electoral geography given by table 4.2, one obtains an electoral sour spot where the spatial distance between the enacted policy and the preference of a considerably large share of voters is even greater.

essarily better than partial accountability. If the distribution of voter preferences is conducive to represent the median group's preferences, opting for the highest possible expected valence in $t = 2$ and punishing the bad incumbent's party implies a failure of representation. In this context, both full and partial accountability result in outcomes on the Pareto-frontier and only the preferences of the electorate determine which outcome is more desirable. Hence, Proposition 4.2 includes both outcomes as long as there is policy representation in the first period.[31]

Proposition 4.2 (Electoral sweet spots)**.** *Both PV and PR achieve representation and accountability in the following two scenarios, characterized by electoral geography and polarization:*

C *Political polarization is not too high and electoral geography such that corner group L or H constitutes the overall median, is the median in at least two districts and is larger than the smallest group plus the PR seat quota. Formally:*

$$d \in \Big\{ d: \ \mu_i > 1.5, \ \textit{for } i \in \{L, H\}, \ \textit{and } \mu_i^l > 0.5 \ \textit{for at least two } l \in \{1,2,3\}, \quad \textit{and } \mu_i - 1 > \min_{j \neq i} \mu_j \Big\}, \tag{4.16}$$

$$\Delta_i < \beta \bar{g} \tag{4.17}$$

D *Political polarization is not too high and electoral geography is such that center group M constitutes the overall median, the two other groups are the median in at most one district and none of them is larger than group M plus the PR seat quota. Formally:*

$$d \in \Big\{ d: \ \mu_j < 1.5 \ \forall j \neq M, \ \textit{and } \mu_j^l > 0.5 \ \textit{in at most one } l \in \{1,2,3\} \ \forall j \neq M, \quad \textit{and } \max_{j \neq M} \mu_j - 1 < \min\{\mu_L, \mu_M, \mu_H\} \Big\}, \tag{4.18}$$

$$\Delta_M < \beta \bar{g} \tag{4.19}$$

Clearly, sweet spots require that political polarization is limited to ensure accountability. With respect to the distribution of voter preferences, there are two kinds of sweet spots. First, the electoral geographies that lead to Scenario **C** are described by Equation (4.16): Preference group L or H constitutes the median. Representation is fulfilled whenever this group is large enough such that it can determine the identity of two legislators under PV and PR. An example for this scenario is given in table 4.3. Note that a relatively small shift in the distribution of voter preferences in district three from L to M is sufficient to destroy representation, as depicted in table 4.2.

	1	2	3	μ_i
L	0.6	0.6	0.4	1.6
M	0.3	0.3	0.5	1.0
H	0.1	0.1	0.1	0.4

Table 4.3: Electoral sweet spot (1)
In the first-period, equilibrium policy corresponds to the preferences of the median group, $x_1 = x_L$, and, given $\Delta_L < \beta \bar{g}$, an incompetent median legislator is replaced under both electoral rules. See **C** in Proposition 4.2.

[31]This general stance on accountability is not affecting the comparison between the two electoral rules. Both PV and PR will always lead to the same form of accountability, full or partial, in a sweet spot.

In the second Scenario **D** (see Equation (4.18)), the moderate preference group M constitutes the median. Representation follows if no other preference group can unilaterally determine the identity of more than one legislator under PV and PR. Refer to the electoral geography shown in table 4.4 for an illustration.

	1	2	3	μ_i
L	0.6	0.2	0.1	0.9
M	0.2	0.6	0.6	1.4
H	0.2	0.2	0.3	0.7

Table 4.4: Electoral sweet spot (2)
In the first-period, equilibrium policy corresponds to the preferences of the median group, $x_1 = x_M$, and, given $\Delta_M < \beta\bar{g}$, an incompetent median legislator is replaced under both electoral rules. See **D** in Proposition 4.2.

4.5.3 When electoral institutions matter

The model also identifies contexts in which electoral institutions clearly matter for representation and accountability. The outcomes, summarized by Proposition 4.3, correspond to different positions in the scientific debate on the impact of electoral institutions. Consistent with a common view in the literature, the choice of electoral institutions can entail a trade-off between these two goals. It may also be the case that one institution is better in both dimensions. Context crucially shapes the impact of electoral institutions.

Proposition 4.3 (Electoral institutions matter)**.** *A context, characterized by electoral geography and polarization, that is neither included in* ***A*** *or* ***B*** *in Proposition 4.1, nor in* ***C*** *or* ***D*** *in Proposition 4.2, implies the existence of a*

- ☐ ***trade-off****, where the electoral institution that performs better on accountability performs comparatively worse on representation;*
- ☐ *or of a* ***best electoral rule****, which performs strictly better on at least one dimension and worse on none.*

The example depicted in table 4.5 illustrates a scenario in which PV leads to higher accountability than PR but PR leads to better representation. Because political polarization is relatively high, a bad median legislator of partisan type L, corresponding to the population median, is re-elected under PR (no accountability and representation). Because of an uneven electoral geography, PV leads to an election of median legislator of party M in the first period (no representation). If the incumbent is revealed to be of bad quality, she will resign and a different M-type with higher expected quality will take her place (partial accountability). The reason is that under the given electoral geography M voters in the plurality voting system with multiple districts have more slack to vote based on quality without changing equilibrium policy. This incentivizes bad incumbents to step down. In this example, the choice of electoral institutions entails a significant trade-off. The nature of the trade-off in this situation is consistent with existing theoretical work emphasizing that while PV fosters more accountability, PR leads to better representation understood as policy responsiveness (Persson and Tabellini, 2000, 2008). Noteworthy, there exist scenarios that reverse this commonly assumed trade-off. These scenarios highlight how the distribution of voter preferences and polarization interact in shaping accountability. Exemplary, PV achieves representation if the overall median, given by group H, is the majority in two districts. However, PR can lead to misrepresentation by group M if the two remaining groups are large and of equal size (each party wins a seat in the legislature); refer to Table 4.9 in the appendix. With regard to accountability, the asymmetric distribution of bliss points allows $\Delta_M < \beta\bar{g} < \Delta_H$ such that PV will fail in terms of accountability, which is not the case for PR.[32]

	1	2	3	μ_i
L	0.8	0.4	0.4	1.6
M	0.1	0.4	0.4	0.9
H	0.1	0.2	0.2	0.5

Table 4.5: Electoral trade-off (1)
PV implies equilibrium policy $x_1 = x_M$ and misrepresentation. PR leads to $x_1 = x_L$ and representation. Given $\beta\bar{g} < \Delta_L = \Delta_M$, an incompetent median legislator is re-elected under PR but replaced by a new politician of type M under PV. See **E** in Proposition 4.5 in the appendix.

There exist contexts where one electoral rule should be preferred over the other because one electoral institution leads to strictly better outcomes in terms of both accountability and representation or it generates a better outcome in accountability (representation) compared to the alternative rule and implies equally good or bad performance with regard to representation (accountability). Tables 4.6 provides an example where PR is best, broadly consistent with arguments about the merits of PR made, for instance, by Lijpart (1994) or, more recently, by Beath et al. (2016). Intuitively, PR leads to a more proportional translation of group size to parliamentary seats and thereby better substantive representation, avoiding problems of misrepresentation created by electoral geography under PR. At the same time, accountability works in both systems as long as political polarization is not too high and fails otherwise.

	1	2	3	μ_i
L	0.8	0.4	0.4	1.6
M	0.2	0.3	0.05	0.55
H	0.0	0.4	0.55	0.95

Table 4.6: Best electoral rule (1)
While accountability is the same across electoral systems, representation fails under PV, but not under PR.

However, it can also be the case that PV constitutes the best electoral rule because it provides (partial) accountability even when political polarization is high and performs equally well in terms of representation as PR. Table 4.7 illustrates this pattern. It is well understood in the literature that electoral geography is the Achilles heel of majoritarian systems when it comes

[32]This scenario is formally defined in **G** in Proposition 4.5 in the appendix.

to representation (Calvo and Rodden, 2015; Morelli, 2004). However, if the heterogeneity of preferences across electoral districts is relatively small, a large number of districts approximates the national distribution of voter preferences and so our framework as well as standard Downsian theories predict that PV represents the national median voter (Callander, 2005). Similarly, models of election and policy-making under PR with multiple parties suggest that the final policy outcome will be sensitive to the national median (Austen-Smith and Banks, 1988; Cho, 2014). At the same time, a familiar argument is that PV leads to more electoral control of politicians compared to closed-list PR when it comes to common value issues (Powell and Whitten, 1993).[33]

	1	2	3	μ_i
L	0.6	0.1	0.2	0.9
M	0.2	0.6	0.4	1.2
H	0.2	0.3	0.4	0.9

Table 4.7: Best electoral rule (2)
Representation is achieved under both electoral systems, yet, given high polarization ($\beta\bar{g} < \Delta_M$), an incompetent median legislator is re-elected under PR but replaced by a new politician of type M under PV.

This section has shown that institutional effects previously identified in the literature are possible outcomes of our model as well, in addition to the possibility of non-effects, including the most problematic case of electoral sour spots, discussed before. Taken together, our model suggests that there is no general answer to the question how electoral systems shape accountability and representation. Institutional effects are contingent on political polarization and electoral geography.

4.5.4 The number of parties

The number of parties actively competing in the election is endogenous. While the focus of our investigation is not on analyzing the effects of electoral institutions on the number of viable competitors per se, a topic that has been widely studied in the literature, the analysis has noteworthy implications about the relationship between the number of parties and the quality of democracy in terms of accountability and representation. The main insight is that institutional effects on the number of parties do not neatly map onto institutional effects on accountability and representation. More specifically, there are electoral sour spots as well as sweet spots in which the endogenous number of parliamentary parties may vary across electoral systems. There are also contexts where electoral systems matter for accountability or representation but the number of parliamentary parties is the same. This disjuncture between different outcomes matters because the empirical literature sometimes conflates the number of parties with goals like accountability and representation.

For example, reconsider the distribution of voter preferences in table 4.4. In this scenario, the three groups of voters are of similar size and M voters are the median of the population and

[33] Examples for additional cases can be found in Section C.7 in the appendix.

the median in each electoral district. If polarization is sufficiently low, the polity is situated in a sweet spot, in which both electoral systems provide accountability and representation. Yet, the active number of parties may vary across electoral systems. Under PR, in equilibrium the first-period parliament may include three distinct parties, (L, M, H) (though pay-off equivalent parliaments with M as the median may have less than three parties). Under PV, in equilibrium one possible first-period parliament is (M, M, M) (again, pay-off equivalent parliaments may have more parties). In this case, there can be a maximal difference in the number of parties across electoral institutions despite no differences in terms of accountability and representation. Note that the number of parties is not uniquely pinned down in equilibrium. Doing so requires additional assumptions that are not needed to characterize accountability and representation. If non-pivotal voters vote sincerely, for instance, there are two active parties under PV (A_L and A_M in the first period) and three under PR, consistent with Duverger's law (Duverger, 1954).[34]

Table 4.5 illustrates that electoral institutions may matter for accountability and representation despite yielding the same number of parties. Recall that in this configuration of high polarization, PV delivers partial accountability but fails to represent the median voter whereas PR leads to representation but no accountability. Under the additional assumption that non-pivotal groups of voters support their most-preferred party, the same two parties compete and are elected under PV and PR in each period (A_L and A_M), with A_M being the majority party in the former and A_L in the latter.

In short, these examples clarify analytically that effects of electoral institutions on the party system and effects of electoral institutions on accountability and representation can be quite distinct. This insight is relevant for the empirical literature. Studies of the effect of electoral institutions on the the number of parties are often motivated by the underlying question of how institutions affect substantive representation. However, more parties do not always lead to better policy representation. Furthermore, some empirical studies take the effective number of parties as a proxy for accountability (e.g., Carey and Hix, 2011), based on the logic that more parties make it more difficult for voters to identify who is responsible (Powell and Whitten, 1993). We do not contest that the clarity of responsibility is a channel through which electoral rules influence accountability. Our framework analyzes a complementary channel: whether voters can credibly commit to vote against a demonstrably bad incumbent given political polarization. As a result, the straightforward link between the number of parties and accountability breaks down.

4.6 Modifications of the model

4.6.1 Non-strategic voters

While several canonical theories of electoral systems assume that voters are strategic, there is an ongoing empirical debate about the degree to which voters in the real world behave

[34] A similar argument applies to electoral sour spots and the reader may wish to refer to the example in table 4.1 to verify.

strategically.[35] Following a behavioral literature and some models of electoral competition, an alternative behavioral postulate assumes that voters are not strategic but sincere or expressive (Callander, 2005). Non-strategic voters do not use their votes instrumentally to influence outcomes but simply vote for a party or candidate that they like best in terms of their political attributes. Adopting this alternative behavioral postulate does not change our main results, though it leads to some interesting additional insights about the relevance of voting behavior for electoral systems design.

Assume that sincere voters value two attributes of a candidate or party list, depending on the electoral system. First, voters like ideological proximity. Second, voters like high-quality politicians. Specifically, the utility of the voter combines a spatial component that reflects his ideological distance to the district candidate (party) and a valence component, captured by the (posterior) belief that the district candidate (first-ranked candidate on the list) exhibits high quality.

Under this alternative assumption, in equilibrium there is the same pattern of electoral sour spots, electoral sweet spots and contexts where institutions matter, just as there is with strategic voters. The reason is that because politicians are strategic and the active number of competitors endogenous, strategic entry often leads to the same results even if voters are sincere. It is also true in this framework, as has been pointed out in previous work, that in equilibrium strategic voters often vote for their first-ranked party or candidate, which makes them empirically hard to distinguish from sincere voters (Alvarez et al., 2006; Kawai and Watanabe, 2013).

Proposition 4.4 (Sincere voters)**.** *The assumption of non-strategic voting implies that the parameter space, given by electoral geography and polarization,*

- ☐ *that characterizes electoral sour spots becomes larger;*
- ☐ *that characterizes electoral sweet spots is unaffected;*

That said, replacing strategic by sincere voters changes the comparison of electoral systems in an interesting way. As stated by Proposition 4.4, the probability that an electoral sour spot exists increases, where the term probability is used losely to indicate that something occurs for a larger range of parameter values.[36] The reason is that sincere voting reduces the capacity of PV to generate accountability despite high levels of polarization. Strategic voters can use slack in the distribution of voter preferences to coordinate against bad incumbents without changing the equilibrium policy. Incentivizing bad incumbents to resign requires voters to be willing to vote against a candidate they prefer to the alternative. This is no longer possible if all voters are sincere. As a consequence, electoral sour spots become more prevalent. This comes at the expense of cases where PV is better than PR in terms of accountability.[37]

[35]Accounting for incentives to vote strategically, several studies find that strategic voting is more prevalent than previously documented (Alvarez et al., 2006; Kawai and Watanabe, 2013).

[36]A proof can be found in Appendix C.6.

[37]Another difference is that with sincere voters, the number of actives parties in equilibrium is unique. Outcome-equivalent equilibria with different sets of politicians are eliminated. This leads to sharper predictions about the number of parties.

4.6.2 Party organization

In the model we make the simplifying assumption about party organization that a demonstrably bad incumbent that wants to run for re-election cannot be prevented from doing so by her party, though voters can of course decide to not support her. In equilibrium, this can lead to bad incumbents strategically resigning. While the assumption about party organization is not essential, the resulting behavior is consistent with the observation that resignations are by far the most common mode through which leadership tenures end in established democracies. A study based on a comparative dataset of 464 party leadership spells in 80 parties finds that "eight out of ten leaders leave through some form of resignation" (Gruber et al., 2015, 138). In the real world, formal leadership removals are of course possible, even if rare. But party scholars argue that this "is not only one of the most severe measures a party can take but also casts doubt on its internal integrity." In that sense, our simplifying assumption is quite realistic (Gruber et al., 2015, 139).

Allowing the party to remove a bad politician before the second election, at some cost (possibly small), does not alter our conclusions. To clarify this, consider an alternative model of party organization, similar in spirit to models of costly candidate selection with observable quality (Galasso and Nannicini, 2011). Rather than analyzing the entry decisions of individual politicians, the focus in this framework is on the decision of an extra-parliamentary party body (referred to as party for simplicity) to replace a bad incumbent before the second election with another candidate of the same partisan type (probability q^0 of being a good type). In line with the motivations of individual politicians, each party cares about the policy outcome x_t and values each parliamentary seat for its office value (π) and does not internalize the value of valence (beyond any effects candidate quality has on office and policy). To capture the cost of removing a party leader, it is sufficient to assume lexicographic preferences such that a party indifferent between maintaining or removing a bad incumbent, based on policy and office considerations, will maintain the leader. This approach makes the opposite assumption of the baseline model in that it is very easy for the party to remove a bad incumbent. Nonetheless, the equilibrium outcomes, in terms of policy and valence, are as before, and so the same comparative results follow. The reason is that resignations in our basic model are strategic. Informally speaking, they only occur under credible political pressure - exactly the same situation when a party is willing to incur the cost of removing a bad incumbent.

4.6.3 Legislative bargaining

Decision-making in legislatures tends to be majoritarian, including legislatures that are elected by PR (Indridason, 2011). Thus, we believe that the assumption that policy and valence are a function of the policy preferences and quality of the median legislator is a plausible approximation of this feature. However, it obviously neglects dynamics of legislative bargaining or government formation. Providing a full account of these features is beyond the scope of the paper. Though we think that making the legislative post-electoral bargaining game more complex is unlikely to fundamentally alter the results of the model.

Established models of legislative bargaining and agenda-setting focus on the dimension of policy. Our intuition of valence incorporates aspects of reputation, such that accountability may become a salient issue in the election. To preserve this intuition, we propose that valence depends solely on the quality of a government leader, who can be envisioned as the manager of the government. In contrast to this, modelling valence as an aggregate, of all members of the government coalition, for example, captures a slightly different notion. Furthermore, this approach greatly complicates voters' inferences about quality of legislators. Now, when thinking about different models of legislative bargaining or coalition formation, the question at hand is whether legislatures lead to different outcomes than under our assumption of policy-making.

Legislative bargaining is often conceptualized as potential departure from a status quo by accepting the proposition of a proposer (Romer and Rosenthal, 1978; Baron and Ferejohn, 1989; Cho, 2014). In our context, the outcome of the first period can, with some imagination, constitute a status quo as a caretaker government stays in power if no agreement on a proposition can be reached.[38] This heightens the importance of first-period outcomes and strengthens the position of legislators whose partisan types correspond to the status quo in the second period. Irrespective of the identity of the proposer, these legislators can, by preventing consent, induce their preferred policy. Only if extreme legislators share a common incentive to overcome the status quo (characterized by $g_1 = 0$), this advantage vanishes. As in our model, this is the case if polarization is low. Hence, we conjecture that our conclusions are robust in this regard. The model by Indridason (2011) overcomes the requirement to specify a status quo. His model specifies a formateur, who proposes a coalition and approached parties to accept automatically if the proposed coalition implies a majority.[39] It is apparent that little changes if the median legislator becomes the formateur: the optimal coalition includes all parties. In this case, the policy outcome is closest to the formateur's policy preference and it is optimal for extreme parties to accept this coalition proposal. Yet, cases where an extreme party becomes the formateur are less clear.

4.7 Conclusion

Representative democracy requires the choice of an electoral system. While scholars and electoral engineers are trained to think about the normative trade-offs that may be entailed by the choice of an electoral system, the theory we have developed suggests that when it comes to two fundamental democratic goals – accountability and policy representation – trade-offs and institutional effects more broadly are context-dependent and can be weaker than usually acknowledged. It implies that the effects of electoral institutions depend on political polarization and the distribution of voter preferences in the population and across districts.

Most disconcertingly, in an electoral sour spot both accountability and representation fail under alternative electoral institutions. Its existence suggests that an electoral reform based on two

[38] One may argue that, in the period before $t = 1$, legislators have served for the maximal number of legislative terms allowed and there is no status quo in $t = 1$, such that our median rule applies in this period.

[39] In Indridason (2011), this is the largest fraction in the legislature. In our model, the only legislature that is interesting in the context of other rules of policy-making is $\{L, M, H\}$, in which all parties win one seat.

common electoral systems, proportional representation and plurality voting, may be ineffective to address these failures. Our theoretical model shows that this outcome can occur even when voters are rational and in the context of institutionalized electoral competition, where there is no coordination failure. In particular, high political polarization undermines accountability and certain distributions of voter preferences undermine representation. If voters are not strategic, the conditions for sour spots become more permissive.

Fortunately, there are contexts that are conducive to better outcomes under proportional and/or majoritarian electoral institutions. In an electoral sweet spot, alternative electoral institutions generate accountability and representation. Consistent with previous work, there are also contexts in which electoral engineering entails a sharp trade-off between accountability and representation (Persson and Tabellini, 2000; Powell, 2000). Similar, there are contexts in which one electoral rule is best (Beath et al., 2016; Lijpart, 1994). Altogether, there is no general answer to the question how electoral systems shape accountability and representation.

While recent empirical research has made impressive progress in understanding how context shapes the effects of electoral institutions (Clark and Golder, 2006; Moser and Scheiner, 2012; Scartascini and Tommasi, 2012), it has paid relatively little attention to electoral geography and political polarization. Our model suggests that accounting for these two factors should fruitfully complement the more common focus on past democratic experience and party system institutionalization. It is clear that doing so requires a significant data collection effort. The potential payoff is two-fold. First, it provides a better understanding of the heterogeneity of institutional effects. This is important for theoretical reasons and because it allows our discipline to give context-sensitive advice. Second, accounting for these factors also addresses a largely ignored endogeneity problem highlighted by the model. By failing to control for polarization and electoral geography researchers may misattribute the effects of context to the electoral system (e.g., if they implicitly compare an electoral sour spot with an electoral sweet spot). The model also indicates that the effective number of parties does not neatly map onto substantive outcomes in terms of accountability and representation, thus, cautioning against normative interpretations of this frequently used variable.

Intriguingly, there may be electoral institutions that avoid the electoral sour spot characterized in our analysis. While investigating this possibility is an important task for future research, based on our framework we think that some obvious candidates - such as mixed electoral systems or open-list PR – are unlikely to do the trick.

C Appendix of Chapter 4

C.1 Proof of Lemma 4.2

Here we prove that the policy and valence outcomes in $t = 2$ under PV after $g_1 = 0$, specified by Lemma 4.2, are unique. Given an electoral geography (d) and a level of polarization ($\Delta_L = \Delta_M$, Δ_H), there may exist multiple equilibria, but as we will see they are outcome-equivalent and only differ with respect to the (partisan) identities of representatives from non-pivotal districts.

Following the cases defined in Lemma 4.2 based on electoral geography (rows) and polarization (columns), the proof considers which types of politician are (re-)elected as members of parliament. This in turn pins down x_2 and g_2. It is convenient to use the fact that in any pure strategy equilibrium of the game, there will be exactly one candidate in each district (candidacy is costly) and the endogenous set of candidates Y_2^{PV} corresponds to the set of elected politicians. It is sufficient to show which Y_2^{PV} can be part of an equilibrium. We refrain from fully describing voters' best-response functions, as solving the game by backward induction is notationally cumbersome and does not add additional insights.[40]

PV.A1 Let us start with the case in which each district has a different median voter, $d \in D_1^{PV}$, and polarization is high, $\beta\bar{g} < \Delta_M$.[41] Lemma 4.1 implies that the incumbent median legislator is of partisan type M. As we focus on the interesting case of bad first-period performance $g_1 = 0$, it is clear that she is a low quality type on the valence dimension. Arbitrarily denote the district with median voter L as $l = 1$, the district with with median voter M as $l = 2$ and that with median voter H as $l = 3$. In equilibrium, the only set of candidates (identical to the set of elected politicians) that can occur is

$$Y_2^{PV^*} = \{L, M^0, H\},$$

where M^0 denotes the bad incumbent. By the median voter theorem and the assumption that valence production is a function of the median legislator's quality, the equilibrium outcome is $x_2 = x_M$, $g_2 = 0$. To see why this is the case, first verify that in $Y_2^{PV^*}$ no player or coalition has an incentive to deviate. On the equilibrium path, the bad incumbent M^0 runs again, gets re-elected and her preferred policy is implemented. Clearly, she does not want to stay out of the election. The other elected politicians also do not benefit from withdrawing their candidacies, given what everybody else is doing. Given $\beta\bar{g} < \Delta_M$, M voters prefer $x_2 = x_M$, $g_2 = 0$ to any other outcome in which their party does not control the legislative median. For the same reason, there is no unilateral deviation that can make voters of type H or M better off. All voters would be better off by inducing M^0 to step down without changing the equilibrium policy. This could

[40] In particular, candidacy decisions of politicians can give rise to many different sets of eligible candidates. Disregarding the issue of reentry of the bad incumbent, there are three politicians in each district at all times. The set of candidates in district l and period t was defined as Y_t^l. An empty set is disregarded for obvious reasons. Apart from this, only one, two or all three politicians of the district can declare candidacy such that $Y_t^l \in \{L, M, H, LM, LH, MH, LMH\}$. Following from this, $Y_t^{PV} \in \{Y_t^1 \times Y_t^2 \times Y_t^3\}$ and $|Y_t^1 \times Y_t^2 \times Y_t^3| = 7^3 = 343$.

[41] A short note on polarization: $d \in D_1^{PV}$ puts the voters of group M in a decisive position. Speaking of *(high) polarization* is always linked to the identity of the decisive group (if not stated otherwise). This implies high polarization here for all voters here, due to $\Delta_L = \Delta_M < \Delta_H$. Yet, this is not true in all cases.

be achieved by a coalition of voters that agrees on M voters punishing M^0 in exchange for either L or H voters supporting enough M candidates elsewhere so that $x_2 = x_M$ is maintained and valence production reaches the Pareto frontier, $E(g_2) = \bar{g}^0$. However, such a deviation is not coalition proof. It requires that M voters in $l = 2$ vote against M^0 if she decides to run again, supporting either H or L instead. This provides the opportunity for a beneficial deviation by coalition member L or H. Also, the politicians that are not running play best responses in equilibrium. Staying out is optimal for them, given the candidacy decisions of the elected politicians, if entering entails no chance of winning a seat. This constitutes a requirement on the strategies of voters off the equilibrium path. It is met in equilibrium because no single group or coalition of voters can improve their utility by a self-enforcing deviation in support of another candidate. (In subsequent cases, this reasoning is not repeated for brevity. The shorthand that politicians who have not declared candidacy have no incentive to do otherwise is meant to imply the required condition on voting behavior.)

Next, check that no other sets of candidates $Y_2^{PV\prime} \neq Y_2^{PV^*}$ are consistent with equilibrium behavior. All other sets $Y_2^{PV\prime}$ that lead to a median legislator of type H, meaning with two or more candidates of type H, cannot be part of any equilibrium. Voters of preferences L and M prefer outcome $x_2 = x_M$, $g_2 = 0$ to $x_2 = x_H$, $E(g_2) = \bar{g}$ and, given $d \in D_1^{PV}$, either the coalition of L and M voters or one of the two groups alone is in a decisive position. Thus, for any set of candidates that implies a median legislator of type H, the decisive coalition/group likes to deviate to install the bad incumbent as median legislator. Furthermore, the coalitional deviation by L and M voters is always self-enforcing. Due to subgame perfection, the strategies of L and M voters must specify the execution of the deviation if it would be feasible (off the equilibrium path). This implies that at least one politician who has not declared candidacy, and thereby rendered the deviation unfeasible, is not playing a best response, which concludes the argument. This can be the bad incumbent herself or another politician of type M or L who would help to break the dominance of the H candidates or even both these politicians. Any set $Y_2^{PV\prime}$ that implies $x_2 = x_H$ cannot be an equilibrium in the present scenario.

Note that the voters of types M and H prefer outcome x_M, $g_2 = 0$ to x_L, $E(g_2) = \bar{g}$ and either the coalition of M and H voters or one of the two groups alone is in a decisive position. Thus, by the same logic as described above, all sets that lead to a median legislator of type L cannot constitute an equilibrium.

All sets $Y_2^{PV\prime}$ where the bad incumbent does not declare candidacy can be discarded. If the bad incumbent does declare candidacy, no other politician of type M can run in district two and the voters of preference M can only vote her out of office by electing a candidate of type L (H) in district two. This puts the voters of preference L (H) in a decisive position and equilibrium play will result in a median legislator of type L (H) and the implied outcome puts the voters of preference M in a worse position. Thus, not voting for the bad incumbent (this happens off the equilibrium path as the bad incumbent will not declare candidacy if she is not elected) violates subgame perfection.

Finally, the following sets that imply candidacy and reelection of the bad incumbent as median legislator are not part of any equilibrium:

$$\{L, M^0, M\},\quad \{M, M^0, H\},\quad \{M, M^0, M\},\quad \{H, M^0, M\},\quad \{M, M^0, L\},\quad \text{and}\quad \{H, M^0, L\}$$

In each of the sets, there exists one group of voters that could engage in a beneficial unilateral deviation, if the required politician had declared candidacy. For instance, take $\{M, M^0, H\}$. If politician of type L declares candidacy in $l = 2$, she will be elected by the voters of preference M there and improved valence $E(g_2) = \bar{g}^0$ (compared to $g_2 = 0$) is realized since the M candidate of district one becomes the median legislator. Whenever groups L or H (and both at the same time in the last set) can engage in a unilateral deviation, they can induce an improvement in valence to $E(g_2) = \bar{g}$ and an shift of policy to their respective bliss point. Note that a unilateral deviation is always self-enforcing as defined by the equilibrium concept. Hence, $Y_2^{PV} = \{L, M^0, H\}$ constitutes the unique set of candidates (and elected politicians) in equilibrium.

PV.A2 Keep $d \in D_1^{PV}$ (implies $x_1 = x_M$) and the labelling of districts as introduced under **PV.A1**. Let $\beta(\bar{g} - \bar{g}^0) < \Delta_M < \beta\bar{g}$ (medium polarization). In this case, the following sets of eligible candidates are the only ones that can occur in equilibrium ($Y_2^{PV^*}$):

$$\{L, M', H\},\qquad \{L, M', M\},\qquad \{M, M', H\}\qquad \text{and}\qquad \{M, M', M\},$$

where M' symbolizes the new politician of type M in district $l = 2$ after the bad incumbent steps down. All these sets are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}^0$.

First, none of the sets introduced above allows for a unilateral or jointly beneficial and self-enforcing coalitional deviation. The elected politicians clearly do not want to withdraw their candidacies. Given $\beta(\bar{g} - \bar{g}^0) < \Delta_M < \beta\bar{g}$, M voters realize the best feasible outcome and, by $d \in D_1^{PV}$, none of the remaining groups is decisive alone and because of ideological differences together they do not agree on a set of candidates to change the outcome. Politicians who do not run will not be elected if they do otherwise.

Second, no other sets can occur in equilibrium. All sets that lead to $x_2 = x_M$ but include the bad incumbent, generating $g_2 = 0$, cannot constitute equilibrium. Voters of type L and M prefer $x_2 = x_L$, $E(g_2) = \bar{g}$ to $x_2 = x_M$, $g_2 = 0$ and, given $d \in D_1^{PV}$, either the coalition of L and M or one of the two groups alone is in a decisive position. Whenever only the coalition is decisive, the deviation to induce outcome $x_2 = x_L$, $E(g_2) = \bar{g}$ is self enforcing. According to the logic introduced above in scenario **PV.A1**, the identification of this potential deviation allows to discard the related sets. Due to subgame perfection, some politicians who have not declared candidacy are surely elected when declaring candidacy such that the candidacy decisions cannot constitute an equilibrium. For brevity, we will not reiterate this second part of the argument each time in later parts of the proof and only identify a potential (mutually beneficial and self-enforing) deviation of a (coalition) group. We have established that the bad incumbent will not declare candidacy and a new politician, M', is available in district two.

M' must declare candidacy in district two in any equilibrium because subgame perfection requires the M voters to elect her. The argument is the same as under **PV.A1** related to the bad incumbent. Not electing the candidate M' after her candidacy decision will surely hurt M voters as they have to elect a different candidate of type L or H what puts the respective group of voters in a decisive position to induce $x_2 = x_L$ or $x_2 = x_H$.

Having established that politician M' will run and is elected in $l = 2$, one can easily rule out the sets that lead to $x_2 = x_L$ or $x_2 = x_H$ (there exists one each). The sets require that only politicians of type L (H) declare candidacy in districts one and three and clearly imply that one group of voters has to elect a candidate against their interests. L voters prefer $x_2 = x_M$, $E(g_2) = \bar{g}^0$ to $x_2 = x_H$, $E(g_2) = \bar{g}$ and H voters prefer $x_2 = x_M$, $E(g_2) = \bar{g}^0$ to $x_2 = x_L$, $E(g_2) = \bar{g}$. This implies that the median legislator in $t = 2$ must be of type M and thus is the new candidate from district two. She delivers discounted valence $E(g_2) = \bar{g}^0$ as median legislator.

As before, there exist some sets that lead to the outcome $x_2 = x_M$, $E(g_2) = \bar{g}^0$ but put one of the groups with policy preference L or H in a decisive position.[42] These sets cannot constitute an equilibrium.

PV.A3 Keep $d \in D_1^{PV}$ (implies $x_1 = x_M$) and the labelling of districts as introduced under **PV.A1**. Let $\Delta_M < \beta(\bar{g} - \bar{g}^0)$ (low polarization). In the assumed scenario, the following sets of eligible candidates

$$\{L, L, H\}, \qquad \{L, L, M\}, \qquad \{L, L, L\}, \qquad \{M, L, L\} \qquad \text{and} \qquad \{L, M^0, L\}$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}$.

First, it is easy to verify that no coalition of voters able to change the outcome prefers to do so and no politician has an incentive to change her candidacy decision, given what everybody else is doing.

Second, all sets that lead to $x_2 \neq x_L$ can be discarded. Both groups of M and L voters prefer the outcome $x_2 = x_L$, $E(g_2) = \bar{g}$ to all other outcomes. Hence the bad incumbent and the new politician of type M, who might run if the incumbent does not declare candidacy, can be successfully challenged by a politician of type L in $l = 2$. Therefore any set that leads to $x_2 \neq x_L$ implies that candidacy decisions cannot constitute a Nash equilibrium as L and M voters cannot induce their most desired outcome. The last two sets are possible because even H voters prefer $x_2 = x_L$, $E(g_2) = \bar{g}$ to $x_2 = x_M$, $E(g_2) = \bar{g}^0$ (and thus also to $x_2 = x_M$, $g_2 = 0$). As long as they cannot change the identity of the median legislator to H, they support an L candidate. This rules out the two sets $\{H, L, L\}$ and $\{L, H, L\}$, that lead to the only possible equilibrium outcome $x_2 = x_L$, $g_2 = \bar{g}$, but allow the voters of preference H to induce the outcome $x_2 = x_H$, $E(g_2) = \bar{g}$.

PV.B1 Next, consider the scenario $d \in D_{2L}^{PV}$ (implies $x_1 = x_L$) and $\beta\bar{g} < \Delta_L$ (high polarization). Arbitrarily, let districts one and two exhibit median L. The bad incumbent can be from one of these two districts. Without loss of generality, let the bad incumbent be stemming from

[42]The sets that fall into this category are $\{H, M', L\}$, $\{M, M', L\}$ and $\{H, M', M\}$.

district one. In the assumed scenario, the sets of eligible candidates

$$\{L^0, L, H\}, \qquad \{L^0, L, M\} \qquad \text{and} \qquad \{L^0, L, L\}$$

are the only ones that can occur in equilibria. All these sets are outcome-equivalent and imply $x_2 = x_L$ and $g_2 = 0$.

First, elected candidates in these sets have no incentive to change their candidacies. This blocks the entry of a new politician of type L in district one. L^0 is re-elected because L voters, who are decisive given $d \in D_{2L}^{PV}$, prefer the outcome $x_2 = x_L$, $g_2 = 0$ to $x_2 = x_M$, $E(g_2) = \bar{g}$ (and obviously to $x_2 = x_H$, $E(g_2) = \bar{g}$). They would not be better off supporting any new entrant from a different party.

Obviously, given any set of candidates that implies $x_2 \neq x_L$, the candidacy decisions cannot be part of an equilibrium. According to the standard argument, the bad incumbent and an arbitrary politician that is required to induce outcome $x_2 = x_L$, $g_2 = 0$ can ensure their (re-)election by declaring candidacy.

There exist sets that imply $x_2^* = x_L$ and $g_2 = 0$ but put the decisive voters from $l = 3$ in a decisive position. Obviously, these voters prefer any other outcome to the realized one and will utilize their decisiveness accordingly. These sets cannot be parts of equilibria.[43]

PV.B2 Keep $d \in D_{2L}^{PV}$ (implies $x_1 = x_L$) and the labelling of districts as introduced under **PV.B1**. Let $\beta(\bar{g} - \bar{g}^0) < \Delta_L < \beta\bar{g}$ (medium polarization). In the assumed scenario, the sets of eligible candidates

$$\{L', L, H\}, \qquad \{L', L, M\} \qquad \text{and} \qquad \{L', L, L\}$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}^0$.

First, in all the sets specified above the decisive L voters prefer the realized outcome to all other feasible outcomes and so there is no coalition willing to change it. The median group in district three is not pivotal and so, on the equilibrium path, it may support different candidates.

Second, all sets that imply the reelection of the bad incumbent can be discarded. Voters of preference L prefer the outcome $x_2 = x_M$, $E(g_2) = \bar{g}$ to $x_2 = x_L$, $g_2 = 0$ and can credibly commit to vote the incumbent out of office. Thus, she will not declare candidacy. Yet, L voters prefer $x_2 = x_L$, $E(g_2) = \bar{g}^0$ to all other outcomes and thereby all sets that lead to $x_2 \neq x_L$ can be discarded as well since the group of L voters is always decisive.

Voters of preference M and H prefer $x_2 = x_M$, $E(g_2) = \bar{g}$ to the equilibrium outcome. Therefore sets that imply the outcome $x_2^* = x_L$, $g_2 = \bar{g}^0$ but put the median voters in district three in a decisive position cannot be part of any equilibrium.[44].

PV.B3 Keep $d \in D_{2L}^{PV}$ (implies $x_1 = x_L$) and the labelling of districts as introduced under **PV.B1**. Let $\Delta_L < \beta(\bar{g} - \bar{g}^0)$ (low polarization). Furthermore, let the preference of the median

[43] The sets that fall into this category are $\{L^0, M, L\}$ and $\{L^0, H, L\}$, irrespective whether M is the median or H the majority in $l = 3$.

[44] The sets that fall into this category are $\{L', M, L\}$, $\{L', H, L\}$, $\{M, L, L\}$ and $\{H, L, L\}$.

voter in district three be i. In the assumed scenario, for both $i = M, H$ the sets of eligible candidates

$$\{L^0, M, M\}, \quad \{L^0, M, H\}, \quad \{M, L, M\}, \quad \{M, L, H\},$$
$$\{M, M, L\}, \quad \{M, M, M\}, \quad \{M, M, H\},$$

and additionally

for $i = M$: $\{H, M, M\}$, $\{H, M, L\}$, $\{M, H, M\}$, $\{M, H, L\}$, $\{H, L, M\}$ $\{L^0, H, M\}$,

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}$.

First, voters of preference L prefer the outcome $x_2 = x_M$, $E(g_2) = Eg^{base}$ to all other outcomes and are always decisive given the electoral geography. Furthermore, the outcome is either the best one for the median group in district three or this group cannot induce a more favorable outcome alone, due to the first aspect, there exist more equilibrium sets for $i = M$. All sets shown above are equilibria.

Second, all sets that lead to $x_2 \neq x_M$ can obviously be discarded. As argued above, there exist sets that lead to $x_2 = x_M$, $E(g_2) = \bar{g}$ but cannot occur in equilibrium for $i = H$, since an M politician in $l = 3$ can successfully challenge the running candidate there.[45]

PV.C1 Next, consider the scenario $d \in D_{2M}^{PV}$ (implies $x_1 = x_M$) and $\beta(\bar{g} - \bar{g}^0) < \Delta_M$ (high and medium polarization). Arbitrarily, let districts one and two exhibit median M. The bad incumbent can be from one of these two districts. Without loss of generality, let the bad incumbent be stemming from district two. Furthermore, let the preference of the median voter in district three be i. In the assumed scenario, for both $i = L, H$ the sets of eligible candidates

$$\{M, M', L\}, \quad \{M, M', M\} \quad \text{and} \quad \{M, M', H\},$$

and additionally

for $i = L$: $\{H, M', L\}$, $\{H, M', M\}$, $\{M, H, L\}$, $\{M, H, M\}$ and
for $i = H$: $\{L, M', M\}$, $\{L, M', H\}$, $\{M, L, M\}$, $\{M, L, H\}$,

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}^0$.

First, the sets listed above are equilibria since decisive group M prefers the induced outcome to all other possible outcomes and the median group of the last district is never in a position to induce a more favorable outcome for them. Elected politicians would be worse off not running and no politician not running prefers to enter, including the bad incumbent, see the argument below.

Second, M voters in district two can credibly threaten to vote the incumbent out of office if she

[45] The sets that fall into this category are $\{H, M, M\}$ and $\{M, H, M\}$.

runs. They are decisive and prefer outcome $x_2 = x_M$, $E(g_2) = \bar{g}^0$ to all other outcomes. Given any arbitrary candidacy and voting decision in district three, the incumbent can be successfully challenged by a candidate who exhibits a type that is not M and different from the type of the representative of district three. Electing a candidate of type M in district one still leads to improved valence $g_2 = \bar{g}^0$ and preserves $x_2^* = x_M$. This scenario is one where the decisive group enjoys slack in their voting decision. The incumbent is never declaring candidacy and a new politician of type M can run in district two.

Furthermore, equilibria that do not entail $x_2 = x_M$, $E(g_2) = \bar{g}^0$ can be discarded. The decisive group M wants to induce this, most favored, outcome and candidacy decisions cannot be optimal if they are unable to do this. Last, sets that lead to the established equilibrium outcome but put the median voter in district three in a decisive position such that he is able to deviate must be excluded.[46]

PV.C2 Keep $d \in D_{2M}^{PV}$ (implies $x_1 = x_M$) and the labelling of districts as introduced under **PV.C2**. Let $\Delta_M < \beta(\bar{g} - \bar{g}^0)$ (low polarization). Again, let the preference of the median voter in district three be i. In the assumed scenario, for both $i = L, H$ the sets of eligible candidates

$$\{L, L, L\}, \qquad \{L, L, M\}, \qquad \{L, L, H\}, \qquad \{L, M^0, L\} \qquad \text{and} \qquad \{M, L, L\},$$

and additionally

$$\text{for } i = L: \qquad \{L, H, L\} \qquad \text{and} \qquad \{H, L, L\},$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}$.

First, the sets introduced above are equilibria since the voters of preference M prefer the realized outcome to all other outcomes and the median group in $l = 3$ can never induce a more favorable outcome alone. This implies that candidacy decisions and voting behavior are optimal.

Second, the same logic allows to discard all sets that do not lead to $x_2 = x_L$, $E(g_2) = \bar{g}$. Voters of preference M want to induce this outcome at all times and being unable to do so renders candidacy decisions not optimal. All voters share the same objectives for $i = L$ and all sets that lead to the established equilibrium outcome are equilibrium sets. Obviously, for $i = H$, the two sets that only apply to $i = L$ must be excluded.

PV.D1 Next, consider the scenario $d \in D_{3M}^{PV}$ (implies $x_1 = x_M$) and $\beta(\bar{g} - \bar{g}^0) < \Delta_M$ (high and medium polarization). Arbitrarily, let the bad incumbent be stemming from district two. In the assumed scenario, the sets of eligible candidates

$$\{M, M', M\}, \quad \{L, M', M\}, \quad \{H, M', M\}, \quad \{M, L, M\}, \quad \{M, H, M\},$$
$$\{M, M', L\}, \quad \{M, M', H\}, \quad \{L, M', H\}, \quad \{H, L, M\}, \quad \{M, H, L\},$$
$$\{H, M', L\}, \quad \{L, H, M\} \quad \text{and} \quad \{M, L, H\},$$

[46] The sets that fall into this category are $\{L, M', M\}$, $\{L, M', H\}$, $\{M, L, M\}$ and $\{M, L, H\}$ for $i = L$ as well as $\{H, M', L\}$, $\{H, M', M\}$, $\{M, H, L\}$ and $\{M, H, M\}$ for $i = H$.

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}^0$.

The argument here is very similar to that in **PV.C1**, but even simpler. First, the sets shown above are equilibria since group M realizes the best outcome possible. Given $d \in D_{3M}^{PV}$, no other group of voters can affect the outcome and candidacy decisions are optimal. The electoral geography also allows M voters to vote against M^0 without changing $x_2 = x_M$, and so the bad incumbent stays out.

Second, all outcomes that differ from $x_2 = x_M$, $E(g_2) = \bar{g}^0$ can be discarded due to the standard logic. Since no other group of voters can be decisive, all sets that lead to the established outcome are equilibria.

PV.D2 Keep $d \in D_{3M}^{PV}$ (implies $x_1 = x_M$) and the labelling of districts as introduced under **PV.D1**. Let $\Delta_M < \beta(\bar{g} - \bar{g}^0)$ (low polarization). In the assumed scenario, the sets of eligible candidates

$$\{L, L, L\}, \quad \{L, L, M\}, \quad \{L, L, H\}, \quad \{L, M', L\}, \quad \{L, H, L\}, \quad \{M, L, L\} \quad \text{and} \quad \{H, L, L\},$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}$.

First, the sets above are equilibria since M voters, who are decisive given $d \in D_{3M}^{PV}$, realize the best outcome possible after $g_1 = 0$. They prefer $x_2 = x_L$, $E(g_2) = \bar{g}$ to all other outcomes and would not be better off supporting any new entrant.

Second, all sets that do not lead to the outcome $x_2 = x_L$, $E(g_2) = \bar{g}$ can be discarded according to the standard logic. No other group of voters can be decisive and all sets that lead to the established outcome are equilibria.

PV.E1 Next, consider the scenario $d \in D_{3L}^{PV}$ (implies $x_1 = x_L$) and $\beta(\bar{g} - \bar{g}^0) < \Delta_L$ (high and medium polarization). Without loss of generality, let the bad incumbent be stemming from district one. In the assumed scenario, the sets of eligible candidates

$$\{L', L, L\}, \quad \{M, L, L\}, \quad \{H, L, L\}, \quad \{L', M, L\}, \quad \{L', H, L\}, \quad \{L', L, M\} \quad \text{and} \quad \{L', L, H\},$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}^0$. The argument here is the same as under **PV.D1**. First, the sets obviously are equilibria since by $d \in D_{3L}^{PV}$ no group apart from L can be decisive and this group realizes the best outcome possible.

Second, all sets that do not lead to the outcome $x_2^* = x_L$, $E(g_2) = \bar{g}^0$ can be discarded according to the standard logic.

PV.E2 Keep $d \in D_{3L}^{PV}$ (implies $x_1 = x_L$) and the labelling of districts as introduced under **PV.E1**. Let $\Delta_L < \beta(\bar{g} - \bar{g}^0)$ (low polarization). In the assumed scenario, the sets of eligible candidates

(see the sets specified under **PV.D1**, and disregard the "primes") are the only ones that occur in equiibrium. All these sets are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}$. The argument here is the same as under **PV.D2**.

PV.F1 Next, consider the scenario $d \in D_{2H}^{PV}$ (implies $x_1 = x_H$) and $\beta\bar{g} < \Delta_H$ (high polarization). Arbitrarily, let districts two and three exhibit median H. The bad incumbent can be from one of these two districts. Without loss of generality, let the bad incumbent be stemming from district three. In the assumed scenario, the sets of eligible candidates

$$\{L, H, H^0\}, \qquad \{M, H, H^0\} \qquad \text{and} \qquad \{H, H, H^0\}$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_H$ and $g_2 = 0$. The reasoning in this scenario is equivalent to **PV.B1**.

PV.F2 Keep $d \in D_{2H}^{PV}$ (implies $x_1 = x_H$) and the labelling of districts as introduced under **PV.F1**. Let $\beta(\bar{g} - \bar{g}^0) < \Delta_H < \beta\bar{g}$ (medium polarization). In the assumed scenario, the sets of eligible candidates

$$\{L, H, H'\}, \qquad \{M, H, H'\} \qquad \text{and} \qquad \{H, H, H'\}$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_H$ and $E(g_2) = \bar{g}^0$. The reasoning in this scenario is equivalent to **PV.B2**.

PV.F3 Keep $d \in D_{2H}^{PV}$ (implies $x_1 = x_H$) and the labelling of districts as introduced under **PV.F1**. Let $\Delta_H < \beta(\bar{g} - \bar{g}^0)$ (low polarization). Furthermore, let the preference of the median voter in district one be i. In the assumed scenario, for both $i = L, M$ the sets of eligible candidates

$$\{L, M, M\}, \quad \{M, M, M\}, \quad \{H, M, M\}, \quad \{L, M, H^0\},$$
$$\{M, M, H^0\}, \quad \{L, H, M\}, \quad \{M, H, M\},$$

and additionally

for $i = M$: $\{M, M, L\}$, $\{H, M, L\}$, $\{M, L, M\}$, $\{H, L, M\}$, $\{M, H, L\}$ $\{M, L, H^0\}$,

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}$. The reasoning in this scenario is equivalent to **PV.B3**.

PV.G1 Next, consider the scenario $d \in D_{3H}^{PV}$ (implies $x_1 = x_H$) and $\beta(\bar{g} - \bar{g}^0) < \Delta_H$ (high and medium polarization). Arbitrarily, let the bad incumbent be stemming from district three. In the assumed scenario, the sets of eligible candidates

$$\{H, H, H'\}, \ \{L, H, H'\}, \ \{M, H, H'\}, \ \{H, L, H'\}, \ \{H, M, H'\}, \ \{H, H, L\} \ \{H, H, M\},$$

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_H$ and $E(g_2) = \bar{g}^0$. The reasoning in this scenario is equivalent to **PV.D1**.

PV.G2 Keep $d \in D_{3H}^{PV}$ (implies $x_1 = x_H$) and the labelling of districts as introduced under **PV.G1**. Let $\Delta_H < \beta(\bar{g} - \bar{g}^0)$ (low polarization). In the assumed scenario, the sets of eligible candidates

(see the sets specified under **PV.D1**, and disregard the "primes")

are the only ones that can occur in equilibrium. All these sets are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}$. The reasoning in this scenario is equivalent to **PV.D2**.

C.2 Proof of Lemma 4.3

We show that the policy and valence outcomes in $t = 2$ under PR after $g_1 = 0$, specified by Lemma 4.3, are unique. We follow the cases defined in Lemma 4.3 and characterize which types of politicians are selected as members of parliament in the second period.

PR.A1 Start with the scenario where each group can determine the identity of one representative, $d \in D_1^{PR}$, and let polarization be high, $\beta\bar{g} < \Delta_M$. Lemma 4.1 implies $x_1 = x_M$. Given $g_1 = 0$, it is clear that the incumbent median legislator is a low-quality type. In this case, the following legislatures

$$\{\underset{L}{L}, \underset{M}{M^0}, \underset{H}{H}\}, \qquad \{\underset{L}{L}, \underset{M,H}{M^0M}\}, \qquad \{\underset{L,M}{M^0M}, \underset{H}{H}\} \qquad \text{and} \qquad \{\underset{L,M,H}{M^0MM}\}$$

are the only ones that can occur in equilibrium. Recall that M^0 denotes the bad incumbent. The different fractions in the legislature are separated by commas. With some abuse of notation, the subscripts indicate which group of voters is supporting which party list (denoted as voting behavior). These parliaments imply the same outcome: $x_2 = x_M$, $g_2 = 0$. There are multiple outcome-equivalent parliaments because voters may play weakly dominated actions.

This outcome occurs for two reasons. As long as voters of preference type L and H are not willing to join forces and support the same non-centrist party, M voters cannot credibly commit to vote against a list of party A_M including the bad incumbent as the first candidate since, by $\beta\bar{g} < \Delta_M$, they prefer the outcome $x_2 = x_M$, $g_2 = 0$ to both $x_2 = x_L$, $g_2 = \bar{g}$ and $x_2 = x_H$, $g_2 = \bar{g}$. Thus, the bad incumbent declares candidacy again, occupying the top spot on her party's list, and is reelected. Given ideological polarization, L and H voters prefer outcome $x_2 = x_M$, $g_2 = 0$ to the one that entails better valence, $E(g_2) = \bar{g}$, and leads to the enactment of the policy that is preferred by the respective other group. Given $d \in D_1^{PR}$, the result is that A_M will be the median party and, because M^0 is elected, $g_2 = 0$ follows. No other outcome can occur in equilibrium.

Specifying equilibrium parliaments requires some additional restrictions on voting behavior and candidate decisions. The legislature $\{L, M^0, H\}$ is consistent with equilibrium if L (M) $[H]$ voters vote for the list of party A_L (A_M) $[A_H]$. This is indicated by the subscripts in the candidate sets. Any other assignment of votes implies that at least one group of voters is not casting an optimal vote. For instance, consider the outcome-equivalent parliament and corresponding voting behavior $\{\underset{M}{L}, \underset{L}{M^0}, \underset{H}{H}\}$. This is not an equilibrium because voters of preference L can induce the outcome $x_2 = x_L$, $E(g_2) = \bar{g}$ by supporting the list of party A_L instead of party A_M. The case for the other equilibrium parliaments is analogous. Implicit in the equilibrium parliaments listed above is that entry decisions by politicians are best responding to voters. By the assumption $0 < c < \pi/3$, all parties that can expect to win at least one seat will have a full list. The exception is the case where the party of M^0 only wins one seat. In this case, only M^0 declares her candidacy.

PR.A2 Keep $d \in D_1^{PR}$ (implies $x_1 = x_M$). Let $\beta(\bar{g} - \bar{g}^0) < \Delta_M < \beta\bar{g}$ (medium polarization). In the assumed scenario, the legislatures (with voting behavior as indicated by subscripts)

$$\{\underset{L}{L}, \underset{M}{M}, \underset{H}{H}\}, \qquad \{\underset{L}{L}, \underset{M,H}{MM}\}, \qquad \{\underset{L,M}{MM}, \underset{H}{H}\} \qquad \text{and} \qquad \{\underset{L,M,H}{MMM}\}$$

are the only ones that can occur in equilibrium.[47] All these legislatures are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}^0$.

By $\beta(\bar{g} - \bar{g}^0) < \Delta_M < \beta\bar{g}$, L and M voters prefer $x_2 = x_L$, $E(g_2) = \bar{g}$ to $x_2 = x_M$, $g_2 = 0$ and M voters prefer $x_2 = x_M$, $E(g_2) = \bar{g}^0$ to all other outcomes. Given $d \in D_1^{PR}$, a coalition of L and M voters is always decisive. As a result, bad incumbent M^0 will not be elected and so she refrains from running. M voters can always form a coalition with either L or H voters to overcome any legislature that implies $x_2 \neq x_M$. As above, ideological polarization prevents a joint deviation to a non-centrist majority by L and H voters. Given that M^0 does not enter, optimal candidacy decisions of the politicians require that each party's list is full, featuring three candidates.

PR.A3 Keep $d \in D_1^{PR}$ (implies $x_1 = x_M$). Let $\Delta_M < \beta(\bar{g} - \bar{g}^0)$ (high polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{L,M,H}{LLL}\}, \qquad \{\underset{L,M}{LL}, \underset{H}{M^0}\}, \qquad \{\underset{L,M}{LL}, \underset{H}{H}\}, \qquad \{\underset{L,H}{LL}, \underset{M}{M^0}\} \qquad \text{and} \qquad \{\underset{M,H}{LL}, \underset{L}{M^0}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}$.

Given $\Delta_M < \beta(\bar{g} - \bar{g}^0)$, voters L and M prefer the realized outcome to all other outcomes and, by $d \in D_1^{PR}$, are always decisive together. Hence, no seat distribution that leads to $x_2 \neq x_L$ and $E(g_2) \neq \bar{g}$ can occur in equilibrium. H voters have no profitable deviation if the vote assignments are as indicated above. Note that even H voters will support the list of party A_L if they have the choice between re-installing the bad incumbent as median legislator or a candidate of partisan type L. Optimal candidacy decisions require that the list of party A_L is full, the list of party A_H is full (whenever this party wins a seat) and only the bad incumbent declares candidacy in party A_M (whenever her party wins a seat).[48]

PR.B1 Next, consider case $d \in D_{2M}^{PR}$ (implies $x_1 = x_M$), which implies that M voters can determine the identity of at least two legislators, and let $\beta\bar{g} < \Delta_M$ (high polarization). In the

[47] Since the ordering of candidates is random, one of the M legislators may be the new politician who emerges after the bad incumbent resigns. Yet, this is not of importance and the notation M' is not used here.

[48] In the equilibrium legislatures (with vote assignments) where party A_M wins one seat, the bad incumbent can secure this seat by declaring candidacy and being listed first on the ballot. This behavior is optimal and must occur in equilibrium.

assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{i}{L}, \underset{M,M}{M^0M}\}, \qquad \{\underset{M,M,i}{M^0MM}\}, \qquad \{\underset{M,M}{M^0M}, \underset{i}{H}\},$$

and additionally

$$\text{for } i = L: \quad \{\underset{L}{L}, \underset{M}{M^0}, \underset{M}{H}\} \quad \text{and} \quad \{\underset{L,M}{M^0M}, \underset{M}{H}\},$$

$$\text{for } i = H: \quad \{\underset{M}{L}, \underset{M}{M^0}, \underset{H}{H}\} \quad \text{and} \quad \{\underset{M}{L}, \underset{M,H}{M^0}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_M$ and $g_2 = 0$.

By $\beta\bar{g} < \Delta_M$, M voters prefer outcome $x_2 = x_M$, $g_2 = 0$ to both $x_2 = x_L$, $g_2 = \bar{g}$ and $x_2 = x_H$, $g_2 = \bar{g}$. Given $d \in D_{2M}^{PR}$, they can ensure this outcome. Hence, M^0 runs and becomes the median legislator again. The case where group M can determine the identity of all three legislators is simple as no other group of voters can be decisive there. In this case all legislatures that lead to outcome $x_2 = x_M$, $g_2 = 0$ are equilibria with any distribution of votes (only group M can assign three votes).

PR.B2 Keep $d \in D_{2M}^{PR}$ (implies $x_1 = x_M$) and let $\beta(\bar{g} - \bar{g}^0) < \Delta_M < \beta\bar{g}$ (medium polarization). In the assumed scenario, the legislatures specified under **PR.B1**,replacing M^0 by M, are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}^0$.

Given $\beta(\bar{g} - \bar{g}^0) < \Delta_M < \beta\bar{g}$, M voters prefer $x_2 = x_L$, $E(g_2) = \bar{g}$ to $x_2 = x_M$, $g_2 = 0$. This outcome is the best for M voters and by $d \in D_{2M}^{PR}$ they can unilaterally ensure it. Hence a list containing M^0 would be defeated and so M^0 does not run. Instead, there M^0 is replaced by a new M type and party A_M runs a full list. Similarly, the two remaining parties offer a full list whenever they rationally expect to win one seat. Trivially, if group M can determine three legislators alone all legislatures that lead to outcome $x_2 = x_M$, $E(g_2) = \bar{g}^0$ can occur in equilibrium.

PR.B3 Keep $d \in D_{2M}^{PR}$ (implies $x_1 = x_M$) and let $\Delta_M < \beta(\bar{g} - \bar{g}^0)$ (low polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{M,M,i}{LLL}\}, \qquad \{\underset{M,M}{LL}, \underset{i}{M^0}\}, \qquad \{\underset{i,M}{LL}, \underset{M}{M^0}\} \qquad \{\underset{M,M}{LL}, \underset{i}{H}\},$$

and additionally

$$\text{for } i = L: \qquad \{\underset{L,M}{LL}, \underset{M}{H}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}$.

Given $\Delta_M < \beta(\bar{g} - \bar{g}^0)$, $x_2 = x_L$, $E(g_2) = \bar{g}$ is the best outcome for both M and L voters and, by $d \in D_{2M}^{PR}$, M voters alone are always decisive. No other outcomes can occur in equilibrium. It is easy to verify that voter assignments as indicated above are best responses. Due to low polarization H voters support the list of party A_L if they have to decide between outcomes $x_2 = x_M$, $E(g_2) = \bar{g}^0$ and $x_2 = x_L$, $E(g_2) = \bar{g}$. For candidacy decisions to be optimal, the list of party A_L must be full. Party A_H's list must be full whenever it can win one seat. Only the bad incumbent, who may become a member of parliament again, enters the list of her party whenever it wins a seat. The equilibrium legislatures if group M can determine three legislators alone are given by all legislatures that lead to outcome $x_2 = x_L$, $E(g_2) = \bar{g}$.

PR.C1 Next, consider the scenario $d \in D_{2L}^{PR}$ (implies $x_1 = x_L$) and let $\beta\bar{g} < \Delta_L$ (high polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{L}{L^0}\underset{L}{L}\underset{i}{L},\}, \qquad \{\underset{L}{L^0}\underset{L}{L}, \underset{i}{M}\}, \qquad \{\underset{L}{L^0}\underset{L}{L}, \underset{i}{H}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_L$ and $g_2 = 0$. The logic is identical to case **PR.B1**. Exchange i with L in the legislatures with vote assignments above for the case of a large group L (that can determine three seats alone).

PR.C2 Keep $d \in D_{2L}^{PR}$ (implies $x_1 = x_L$) and let $\beta(\bar{g} - \bar{g}^0) < \Delta_L < \beta\bar{g}$ (medium polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{L}{L}\underset{L}{L}\underset{i}{L},\}, \qquad \{\underset{L}{L}\underset{L}{L}, \underset{i}{M}\}, \qquad \{\underset{L}{L}\underset{L}{L}, \underset{i}{H}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}^0$. The argument is identical to case **PR.B2**. Exchange i with L in the legislatures with vote assignments above for the case of a large group L (that can determine three seats alone).

PR.C3 Keep $d \in D_{2L}^{PR}$ (implies $x_1 = x_L$) and let $\Delta_L < \beta(\bar{g} - \bar{g}^0)$ (low polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{L}{L^0}, \underset{L}{M}, \underset{i}{H}\}, \qquad \{\underset{L}{L^0}, \underset{L}{M}\underset{i}{M}\}, \qquad \{\underset{L}{M}\underset{L}{M}, \underset{i}{H}\}, \qquad \{\underset{L}{M}\underset{L}{M}\underset{i}{M}\},$$

and additionally

$$\text{for } i = M\text{:} \qquad \{\underset{L}{L^0}, \underset{i}{M}, \underset{L}{H}\}, \qquad \{\underset{i}{L^0}, \underset{L}{M}, \underset{L}{H}\} \qquad \text{and} \qquad \{\underset{L}{M}\underset{i}{M}, \underset{L}{H}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}$. The arguments concerning uniqueness and existence are identical to case **PR.B3**. Exchange i with L in the legislatures with vote assignments above for the case of a large group L (that can determine three seats alone).

PR.D1 Next, consider the scenario $d \in D_{2H}^{PR}$ (implies $x_1 = x_H$) and let $\beta\bar{g} < \Delta_H$ (high polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{H}{H^0}\underset{H}{H}\underset{i}{H},\}, \qquad \{\underset{i}{L}, \underset{H}{H^0}\underset{H}{H}\} \qquad \text{and} \qquad \{\underset{i}{M}, \underset{H}{H^0}\underset{H}{H}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_H$ and $g_2 = 0$. The arguments concerning uniqueness and existence are identical to case **PR.B1**. Exchange i with H in the legislatures with vote assignments above for the case of a large group H (that can determine three seats alone).

PR.D2 Keep $d \in D_{2H}^{PR}$ (implies $x_1 = x_H$) and let $\beta(\bar{g} - \bar{g}^0) < \Delta_H < \beta\bar{g}$ (medium polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{H,H,i}{HHH},\}, \qquad \{\underset{i}{L}, \underset{H,H}{HH}\} \qquad \text{and} \qquad \{\underset{i}{M}, \underset{H,H}{HH}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_L$ and $E(g_2) = \bar{g}^0$. The logic is identical to case **PR.B2**. Exchange i with H in the legislatures with vote assignments above for the case of a large group H (that can determine three seats alone).

PR.D3 Keep $d \in D_{2H}^{PR}$ (implies $x_1 = x_H$). Let $\Delta_H < \beta(\bar{g} - \bar{g}^0)$ (low polarization). In the assumed scenario, the legislatures (with voting behavior as indicated)

$$\{\underset{i}{L}, \underset{H}{M}, \underset{H}{H^0}\}, \qquad \{\underset{i}{L}, \underset{H,H}{MM}\}, \qquad \{\underset{H,H}{MM}, \underset{i}{H^0}\}, \qquad \{\underset{i,H,H}{MMM}\},$$

and additionally

$$\text{for } i = M\text{:} \qquad \{\underset{H}{L}, \underset{i}{M}, \underset{H}{H^0}\}, \qquad \{\underset{H}{L}, \underset{H}{M}, \underset{i}{H^0}\} \qquad \text{and} \qquad \{\underset{H}{L}, \underset{i,H}{MM}\}$$

are the only ones that can occur in equilibrium. All these legislatures are outcome-equivalent and imply $x_2 = x_M$ and $E(g_2) = \bar{g}$. The logic is identical to case **PR.B3**. Exchange i with H in the legislatures with vote assignments above for the case of a large group H (that can determine three seats alone).

C.3 Proof of Proposition 4.1

In an electoral sour spot, both PV and PR fail to achieve representation and accountability. First, consider part **B** of Proposition 4.1. As required, let the median citizen hold policy preference M. By Lemma 4.2 and 4.3, in this case representation fails in both systems if there is a group different from M that is the median in two districts under PV ($\mu_j^l > 0.5$ in two $l \in \{1, 2, 3\}$ for one $j \neq M$) and there is a group different from M that is able to win two seats under PR ($\mu_{j'} - 1 > \min_{k \neq j} \mu_k$ for one $j' \neq M$). These conditions on d are summarized by Equation (4.14) in Proposition 4.1. Under these conditions, high polarization $\beta\bar{g} < \Delta_j, \Delta_{j'}$ implies that neither group j nor group j' (whether they coincide or not) can credibly commit to vote a bad incumbent out of office and lead to no accountability.

Second, consider part **A** of Proposition 4.1 and assume that group $i \in \{L, H\}$ is the median in the population ($\mu_i > 1.5$). This implies that there can be no other group that is the median in all electoral districts. By Lemma 4.2 and 4.3, in this case representation fails in both systems if $d \in \left\{\{D_1^{PV} \cup D_{2j}^{PV}\} \cap D_1^{PR}\right\}$ for $j \neq i$. By Lemma 4.2, ruling out partial accountability requires discarding all $d \in D_{2M}^{PV}$. Hence $j = -i$ where $-i$ characterizes the respective other group of extreme policy preference. Note that $D_{2-i}^{PV} \cap D_1^{PR} = \varnothing$ given the initial assumption $\mu_i > 1.5$. This leaves $d \in \{D_1^{PV} \cap D_1^{PR}\}$. In these cases, there is a failure of representation and high polarization, $\beta\bar{g} < \Delta_M$, leads to a re-election of the bad incumbent. This covers all cases.

C.4 Proof of Proposition 4.2

A sweet spot exists if both PV and PR generate representation and accountability. Lemmas 4.2 and 4.3 show that $\Delta_i < \beta\bar{g}$ implies that at least partial accountability is reached for any d under both electoral rules. Under PR, accountability always fails for high polarization, what is not generally true for PV. Last, the threshold separating full from partial accountability, $\Delta_i = \beta(\bar{g} - \bar{g}^0)$, is identical in PV and PR. Hence, given $\Delta_i < \beta\bar{g}$, both rules will surely lead to the same degree of accountability (either full or partial).

To achieve representation, the electoral geography must put the same group i in a decisive position under both electoral rules and group i must be the overall median. Let group $i \in \{L, H\}$ be the median ($\mu_i > 1.5$). By Lemmas 4.2 and 4.3, this group is decisive under PV and PR if it is the median in at least two districts ($\mu_i^l > 0.5$ for at least two $l \in \{1, 2, 3\}$) and $\mu_i - 1 > \min_{j \neq i} \mu_j$ holds. These requirements are captured by Equation (4.16). Next, let group M be the median ($\mu_j < 1.5$ for all $j \neq M$). Lemmas 4.2 and 4.3 show representation holds if $\mu_j^l > 0.5$ in at most one $l \in \{1, 2, 3\}$ for all $j \neq M$ and $\max_{j \neq M} \mu_j - 1 < \min\{\mu_L, \mu_M, \mu_H\}$. See Equation (4.18) that features these requirements.

C.5 Characterization of trade-offs

Proposition 4.3 introduces trade-offs between the two electoral rules. Trade-offs require that different groups of voters are decisive under PV and PR. There exists the standard case where PV leads to misrepresentation but implies slack and thereby allows accountability. In addition, further cases arise since $\Delta_L = \Delta_M < \Delta_H$ allows different degrees of polarization for the two different decisive groups. Proposition 4.5 below formalizes all trade-off scenarios. Then intuition in these scenarios is the following:

E Group L constitutes the overall median and can determine the identities of two legislators under PR (representation), yet group M can determine the identities of two legislators under PV (misrepresentation). High polarization prevents accountability under PR. Partial accountability is achieved under PV due to slack.

F Group H constitutes the overall median and can determine the identities of two legislators under PR (representation), yet group M can determine the identities of two legislators under PV (misrepresentation). High polarization for group H prevents accountability under PR. (At least) partial accountability is achieved under PV due to slack. Even full accountability is possible under PV for sufficiently low Δ_M.

G Group H constitutes the overall median and can determine the identity of two legislators under PV (representation), yet each group can determine the identity of one legislator under PR (misrepresentation). High polarization for group H leads to no accountability under PV. Median (low) polarization for group M delivers partial (full) accountability under PR.

Proposition 4.5 (Electoral trade-offs). *One electoral rule performs strictly better than the other in one dimension but strictly worse in the respective other dimension. Four trade-off scenarios can occur and are characterized by electoral geography and polarization as follows:*

E

$$d \in \Big\{ d: \ \mu_L > 1.5, \ but \ \mu_L^l, \mu_H^l < 0.5 \ for \ two \ l \in \{1,2,3\}, \ and \ \mu_L - 1 > \min_{j \neq L} \mu_j \Big\}, \tag{4.20}$$

$$\beta \bar{g} < \Delta_L = \Delta_M \tag{4.21}$$

F

$$d \in \Big\{ d: \ \mu_H > 1.5, \ but \ \mu_L^l, \mu_H^l < 0.5 \ for \ two \ l \in \{1,2,3\}, \ and \ \mu_H - 1 > \min_{j \neq H} \mu_j \Big\}, \tag{4.22}$$

$$\beta \bar{g} < \Delta_H \tag{4.23}$$

G

$$d \in \Big\{ d: \ \mu_H > 1.5, \ and \ \mu_H^l > 0.5 \ for \ two \ l \in \{1,2,3\}, \ but \ \mu_H - 1 < \min_{j \neq H} \mu_j \Big\}, \tag{4.24}$$

$$\Delta_M < \beta \bar{g} < \Delta_H \tag{4.25}$$

C.6 Proof of Proposition 4.4

With regard to the first period outcomes established in Lemma 4.1, the assumption of sincere voters is ineffectual. Expressive voting behavior greatly reduces the multiplicity in outcome-equivalent legislatures, however, cannot yield different outcomes in terms of policy and valence. Related to second-period outcomes, Lemma 4.3 also applies for this modification of voter behaviour and, as argued above, only the number of outcome-equivalent legislatures is reduced. Reconsidering the cases in the proof of Lemma 4.3 reveals that the implied voter strategies do not hinge on the assumption of strategic voters and are also viable for sincere voters. Hence, the specified candidacy decisions are optimal as well and the obtained equilibrium outcomes survive the modification. Furthermore, considering sincere voters allows no additional equilibrium outcomes compared to the ones established in Lemma 4.3. This is not the case for PV. Lemma 4.2 is not generally true under the alternative voter motivation and some cases play out differently, since aspects of voters' strategies are no longer feasible. The scenarios in question are those where voters can utilize slack to overcome the re-election of a bad incumbent despite high polarization. Exemplary, reconsider case **PV.C1** from the proof of Lemma 4.2: voters of group M are the median in two districts ($d \in D_{2M}^{PV}$) and let polarization be high ($\beta \bar{g} < \Delta_M$). Strategically motivated M voters can, in the incumbent's district, credibly threaten to vote her out of office if she declares candidacy. This is possible since, due to slack, this behavior is neutral with respect to the policy outcome such that $x_2 = x_M$ is maintained. This threat cannot be made by expressive voters. From the perspective of an expressive M type voter in the bad incumbent's district, the bad incumbent is, compared to politicians of partisanship L or H (whose reputations are undamaged), still the most preferred candidate. Hence, the threat to deselect the bad incumbent is not credible (and violates the requirement of subgame perfection). This has implications on the candidacy decisions: in equilibrium, the bad incumbent declares her candidacy and retains her position as median legislator.

The other scenarios affected by sincere voting are, in interplay in high polarization, $d \in D_{3L}^{PV}$, $d \in D_{3M}^{PV}$ and $d \in D_{3H}^{PV}$. The argument which prevents that slack helps to overcome no accountability is the same as the one made above.

C.7 Additional examples

	1	2	3	μ_i
L	0.6	0.6	0.1	1.3
M	0.1	0.1	0.1	0.3
H	0.3	0.3	0.8	1.4

Table 4.8: Electoral sour spot (3)
This electoral geography and the polarization $\beta\bar{g} < \Delta_L$ constitute a sour spot as defined under **B** in Proposition 4.1. However, the electoral rules lead to a different non-congruent equilibrium policy: PV implies $x_1 = x_2 = x_L$ and PR implies $x_1 = x_2 = x_H$.

	1	2	3	μ_i
L	0.4	0.2	0.1	0.7
M	0.3	0.2	0.2	0.7
H	0.3	0.6	0.7	1.6

Table 4.9: Electoral trade-off (2)
PV implies $x_1 = x_H$ and representation, PR leads to $x_1 = x_M$ and misrepresentation. Given $\Delta_M < \beta\bar{g} < \Delta_H$, an incompetent median legislator is re-elected under PV, but not under PR.

	1	2	3	μ_i
L	0.8	0.4	0.2	1.4
M	0.2	0.4	0.7	1.3
H	0.0	0.2	0.1	0.3

Table 4.10: Best electoral rule (3)
PV implies $x_1 = x_M$ and representation, PR leads to $x_1 = x_L$ and misrepresentation. Given $\beta\bar{g} < \Delta_L, \Delta_M$, an incompetent median legislator is re-elected under PR but replaced by a new politician of type M under PV.

	1	2	3	μ_i
L	0.8	0.4	0.4	1.6
M	0.1	0.3	0.3	0.7
H	0.1	0.3	0.3	0.7

Table 4.11: Best electoral rule (4)
PV and PR lead to $x_1 = x_M$ and misrepresentation. Given $\beta\bar{g} < \Delta_L, \Delta_M$, an incompetent median legislator is re-elected under PR but replaced by a new politician of type M under PV.

	1	2	3	μ_i
L	0.7	0.6	0.5	1.6
M	0.1	0.2	0.4	0.7
H	0.2	0.2	0.3	0.7

Table 4.12: Best electoral rule (5)
PV implies $x_1 = x_L$ and representation, PR leads to $x_1 = x_M$ and misrepresentation. Accountability is the same across electoral systems.

	1	2	3	μ_i
L	0.1	0.1	0.4	0.6
M	0.2	0.3	0.5	1.0
H	0.7	0.6	0.1	1.4

Table 4.13: Best electoral rule (6)
PV implies $x_1 = x_H$ and misrepresentation, PR leads to $x_1 = x_M$ and representation. For $\Delta_M < \beta\bar{g} < \Delta_H$, the bad incumbent is re-elected under PV but replaced by a new politician of type M (or even L) under PR.

Abgrenzung

Die Idee für das Forschungspapier *Endogenous Search Behavior on Posted Offer Markets with Capacity Constrained Sellers* ist durch eigene Erfahrungen auf dem Immobilienmarkt im Internet entstanden. Die Herausarbeitung der Forschungsfrage sowie die Entwicklung und Lösung des Modells ist in gemeinsamer Arbeit mit meinem Koautor, Herrn Michal Marenčák, erfolgt. Die in Kapitel zwei präsentierte Version des Forschungspapiers entspringt ausschließlich meiner Hand.

Die Arbeit *Local Public Goods as Perfect Substitutes: Centralization versus Decentralization* in Kapitel drei wurde von mir eigenständig erarbeitet und verfasst.

Das Papier *Accountability or Representation? Democracy and Trade-Offs in Electoral Engineering*, das in Kapitel vier dargestellt wird, ist aus einer Idee meines Koautors, Herrn Michael Becher, entstanden. Die Erarbeitung der präsentierten Version ist gemeinsam erfolgt. Der Schwerpunkt meines Beitrags lag hierbei insbesondere auf der Ausgestaltung und Lösung des dem Papier zugrunde liegenden Modells.

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