

## On the Intergenerational Pareto Efficiency of Pay-as-you-go Financed Pension Systems

by

FRIEDRICH BREYER \*

### *1. Introduction*

In pay-as-you-go financed pension plans, pension payments to the retired are not drawn from a capital fund accumulated during their working lives but financed directly by the contributions of the current workers/contributors (see e.g. AARON [1982, 7]). As SAMUELSON [1958] and AARON [1966] have shown, this financing method can provide every generation of workers with a better internal rate of return on their contributions than a capital funded system *if* the sum of the growth rates of the population (Samuelson's "biological rate of interest") and of the income per worker exceeds the rate of interest on productive investment.

This so-called "Aaron condition" does not, however, give a complete and final answer to the question of the efficiency of financing methods, or, to put it more precisely, the question under what circumstances an intergenerationally Pareto-efficient allocation of consumption is reached via one or the other method for financing public pensions.

The limitations pertain to three different problems. First, both SAMUELSON [1958] and AARON [1966] assumed exogenous wages and interest rate so that the result stated above strictly applies only to a "small" open economy. Secondly, the comparison referred to a pure funded and a pure pay-as-you-go system with constant contribution rate over time, so that the possibility of reaching a Pareto improvement via mixed systems or fluctuating contribution rates was excluded. Thirdly, both authors considered mature systems, i.e., the windfall benefits to the generation founding a pay-as-you-go system did not play a role in their analysis.

The first aspect was taken up by SAMUELSON [1975], who showed in a neoclassical growth model for a closed economy that, in the presence of a pay-as-you-go financed pension system, the economy can reach a steady state with higher

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per-capita consumption than without *if* the economy is “over-capitalized”. Again, the analysis was confined to a comparison of steady states, and transition problems were not addressed. In fact, SAMUELSON [1975, 543] explicitly warned of the conclusion that the non-golden-rule steady state is Pareto-inefficient.

The second limitation was overcome by SPREMANN [1984], who compared funded pension plans and pay-as-you-go plans with freely variable contribution rates (up to some given maximum rate) and showed that the funded plan can *always* be Pareto-dominated by some pay-as-you-go plan *unless* the sum of population and wage growth is permanently smaller than the interest rate (which in his model is assumed to be exogenous).

Again, this does not answer the opposite question: provided that the rate of return on pay-as-you-go contributions is indeed permanently lower than the rate of interest, does this render the pay-as-you-go financing method Pareto-inefficient? In other words, can the first pensioners, who would clearly benefit from the introduction of a pay-as-you-go plan because they receive payments without having contributed, be compensated by later generations for doing without the system?

This question is of particular interest in view of the literature on the political economy of pension systems (e.g. GREENE [1974], BROWNING [1975], TOWNLEY [1981], HU [1982]), where it is shown that if there is a referendum on the introduction of mandatory old-age insurance and voters believe the outcome to be valid indefinitely, then the first-generation benefits will bias the decision towards a pay-as-you-go system even if the Aaron condition mentioned above fails to hold.<sup>1</sup>

Surprisingly, the possibility of compensating the first generation of pensioners for the “loss” due to the nonintroduction of a pay-as-you-go pension system has not so far been thoroughly examined. An exception is the paper by TOWNLEY [1981] who considers an economy with constant wages and an exogenous and constant interest rate, which exceeds the rate of population growth. He proposes roughly the following compensation scheme. The retired generation gets pension payments equivalent to the ones they would have received under a new pay-as-you-go system.

Simultaneously, the working generation pays contributions to a funded pension plan. To close the gap, the government incurs a debt. In the following periods, the contributions to the funded plan earn a return equal to the population growth rate rather than the interest rate, the difference being taken to repay the government debt. Thus all generations are as well off as in a pay-as-you-go system *until* the government debt is completely paid off. All subsequent generations can then earn the full interest rate on their pension fund contribu-

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<sup>1</sup> It should be noted that this conclusion does not follow if instead of each individual a whole family (i.e. a pensioner and all his descendants of working age) is considered as a decision-making unit. For an analysis based on this viewpoint see BREYER and SCHULENBURG [1987].

tions and hence are better off than under pay-as-you-go. Although Townley gives a few numerical results as to the number of periods it takes to pay off the initial debt depending upon the parameters of the model, his calculations are not entirely transparent, and a general proof is missing.

This paper is devoted to a more detailed analysis of the compensation question. In Section 2 we shall re-examine the Townley proposal within a simple model of overlapping generations in an open economy. Section 3 extends the analysis to the case of a closed economy, and in Section 4 some conclusions are drawn.

## *2. Efficiency in the Open Economy*

### *2.1 Assumptions and Notation*

We consider a model of overlapping generations, which is a simplification of the model analyzed by TOWNLEY [1981] insofar as Townley distinguishes cohorts by year of birth and we do so only by generation. Every person lives one standard period as worker and one period as pensioner. In period 0 there are  $N_0$  workers. The reproduction rate of the society,  $G (= 1 + g)$ , is assumed to be exogenous and constant. Hence the number of workers in any given period  $t$  ("generation  $t$ ") can be calculated as

$$(2.1) \quad N_t = N_0 \cdot G^t.$$

There is only one good in the economy, and each worker is endowed with an exogenous and constant income of one unit of this good. Capital markets are perfect, so that saving and borrowing are both possible without limits at the exogenous and constant rate  $r$ . Further, let  $R (= 1 + r)$  denote the interest factor.

Consumption at working age of each member of generation  $t$ ,  $c_t$ , is given by the equation

$$(2.2) \quad c_t = 1 - b_t^K - b_t^P,$$

where  $b_t^P$  denotes contributions to a mandatory pay-as-you-go financed public pension system and  $b_t^K$  denotes contributions to a funded pension plan or, equivalently, private saving. In the absence of transaction costs, the corresponding pension payments for the same generation at retirement age are

$$(2.3) \quad z_{t+1}^P = G \cdot b_{t+1}^P$$

$$(2.4) \quad z_{t+1}^K = R \cdot b_t^K.$$

Including a (positive or negative) government subsidy of  $\psi_{t+1}$  per pensioner in period  $t + 1$ , and ignoring a bequest motive, yields the consumption per pensioner

$$(2.5) \quad z_{t+1} = z_{t+1}^K + z_{t+1}^P + \psi_{t+1} = R \cdot b_t^K + G \cdot b_{t+1}^P + \psi_{t+1}.$$

Discounted lifetime consumption (or, equivalently, discounted lifetime income) of a member of generation  $t$  is denoted by  $V_t$  and can be calculated from (2.2) and (2.5) as

$$(2.6) \quad V_t = c_t + \frac{z_{t+1}}{R} = 1 - b_t^P + \frac{G \cdot b_{t+1}^P + \psi_{t+1}}{R}.$$

It is mathematically straightforward and economically intuitive that borrowing and saving does not alter total lifetime income  $V_t$  but only serves to shift consumption back or forth over the lifecycle.

Finally, if  $D_t$  denotes the size of government debt at the end of period  $t$ , this debt develops according to

$$(2.7) \quad D_{t+1} = R \cdot D_t + \psi_{t+1} \cdot N_t = R \cdot D_t + \psi_{t+1} \cdot N_0 \cdot G^t.$$

## 2.2 Pay-as-you-go Versus Funded Pensions With Government Debt

In the following, the intergenerational Pareto-efficiency of a pay-as-you-go pension system will be examined. For simplicity, we assume that in this pension plan (which we call "plan  $P$ ") the contribution  $b^P$  is constant over time so that the period index can be dropped. The plan  $P$  is introduced in period  $t = 1$ , and government subsidies are set equal to zero. As members of generation 0 never pay contributions to plan  $P$ , their discounted lifetime income is given by

$$(2.8) \quad V_0^P = 1 + \frac{G}{R} \cdot b^P,$$

while for all other generations ( $t = 1, 2, \dots$ )

$$(2.9) \quad V_t^P = 1 + \left( \frac{G}{R} - 1 \right) \cdot b^P.$$

Clearly, in the presence of perfect capital markets the position of the lifetime budget line of a member of a generation  $t$  is uniquely determined by the value of  $V_t$ , such that his or her maximum utility is a monotonic function of  $V_t$ . Therefore, a change in the financing method of the public pension system can bring about a Pareto improvement if and only if lifetime income of no generation is decreased and that of at least one generation increased.

Now Townley effectively claims that if the condition  $g < r$  holds, i.e. if the population grows more slowly than a unit of the consumption good invested at the rate of interest, then generation 0 can be compensated for the non-introduction of plan  $P$  through a government subsidy  $\psi_1$  and that the resulting government debt can be repaid later by siphoning off the benefits of all future generations. If we call this alternative “plan  $D$ ” and assume without loss of generality that all (positive and negative) subsidies are paid to pensioners, full compensation to generation 0 implies

$$(2.10) \quad 1 + \frac{\psi_1}{R} = V_0^D \stackrel{!}{=} V_0^P = 1 + \frac{G}{R} \cdot b^P$$

and hence

$$(2.11) \quad \psi_1 = G \cdot b^P.$$

Thus with plan  $D$ , government debt at the end of period 1 is

$$(2.12) \quad D_1^D = 0 + \psi_1 \cdot N_0 = N_0 \cdot G \cdot b^P.$$

In order to keep any other generation  $t$  ( $t \geq 1$ ) indifferent as well, the compensations  $\psi_t$  paid (or, if negative, collected) can be calculated from the equation

$$(2.13) \quad V_t^D = 1 + \frac{\psi_{t+1}}{R} \stackrel{!}{=} V_t^P = 1 + \left( \frac{G}{R} - 1 \right) \cdot b^P, \quad (t = 1, 2, \dots)$$

and hence

$$(2.14) \quad \psi_{t+1} = (G - R) \cdot b^P = (g - r) \cdot b^P \quad (t = 1, 2, \dots).$$

The crucial question for the Pareto dominance of plan  $D$  as compared to plan  $P$  is whether the government debt is run down to zero in finite time. To answer this question we evaluate the size of the debt at the end of an arbitrary period  $t$ . This is composed of the original debt  $D_1^D$  plus all net compensation payments to intermediate generations, both including interest at the rate  $r$  per period:

$$(2.15) \quad \begin{aligned} D_t^D &= R^{t-1} \cdot D_1^D + \sum_{\tau=2}^t R^{t-\tau} \cdot G^{\tau-1} \cdot N_0 \cdot \psi_\tau \\ &= R^{t-1} \cdot N_0 \cdot b^P \cdot \left[ G + (G - R) \cdot \sum_{\tau=1}^{t-1} \left( \frac{G}{R} \right)^\tau \right]. \end{aligned}$$

The sum of the geometric progression in the last term of (2.15) can be replaced by

$$(2.16) \quad \sum_{\tau=1}^{t-1} \left(\frac{G}{R}\right)^{\tau} = \frac{G}{R} \cdot \frac{\frac{G^{t-1} - R^{t-1}}{R^{t-1}}}{\frac{G - R}{R}} = \frac{G \cdot (G^{t-1} - R^{t-1})}{(G - R) \cdot R^{t-1}}$$

so that (2.15) reduces to

$$(2.17) \quad D_t^D = R^{t-1} \cdot N_0 \cdot b^P \cdot \left[ \frac{G \cdot R^{t-1} + G \cdot (G^{t-1} - R^{t-1})}{R^{t-1}} \right] = N_0 \cdot b^P \cdot G^t.$$

Consequently, per-capita debt,

$$(2.18) \quad \frac{D_t^D}{N_t} = \frac{D_t^D}{N_0 \cdot G^t} = b^P$$

is constant over time. Thus government debt may shrink in absolute value – this is the case when population growth is negative – but it can never become zero because per-capita debt remains constant. Therefore there will never be a generation that will benefit from the higher internal yield of the funded pension system, because the inherited burden originating from the initial compensation payments can never be completely thrown off without making at least one intermediate generation worse off than under pay-as-you-go.<sup>2</sup>

This not only disproves the Townley assertion but shows that under the assumptions made there is no possibility of improving the position of *every* generation as compared to the existence of a pay-as-you-go financed pension system.

### 3. Efficiency in the Closed Economy

#### 3.1 The Model

In a closed economy, matters become slightly more complicated because both income per worker and the rate of interest become endogenous. Therefore, the production sector of the economy must be explicitly modelled. Also, with an endogenous interest rate, it cannot simply be *assumed* that the interest rate is larger than the rate of population growth. However, this relation will be taken into account in our analysis by choosing a production technique which ensures a positive marginal productivity of capital and by placing no restrictions on the value of the population growth rate.

<sup>2</sup> The same point was recently made by VERBON [1989]. His analysis, however, was restricted to a particular type of utility functions.

The latter will be assumed to be exogenous but not necessarily constant over time.  $N_t$  denotes, as before, the number of workers in period  $t$  and  $G_t (= 1 + g_t)$  the corresponding growth factor, i.e.

$$(3.1) \quad N_t = G_t \cdot N_{t-1}.$$

The analysis is based on a one-sector growth model which was first developed by DIAMOND [1965] and was subsequently used by SAMUELSON [1975] and several other authors. There is only one good, which is produced in each period  $t$  according to the (twice differentiable) constant-returns-to-scale production function

$$(3.2) \quad Y_t = F(K_t, N_t) \quad \text{with} \quad F(K_t, 0) = F(0, N_t) = 0,$$

where  $K_t$  denotes the stock of the only good at the beginning of period  $t$ , which is completely used up in production.<sup>3</sup> Using small letters for per-capita variables, the production function (3.2) can be written in the form

$$(3.3) \quad y_t = F(k_t, 1) = f(k_t),$$

where  $f$  is strictly monotonic and strictly concave:

$$(3.4) \quad f'(k_t) > 0, \quad f''(k_t) < 0 \quad \text{for all } k_t > 0.$$

Perfect competition in the factor markets ensures that in equilibrium the interest factor and the income per worker are equal to the respective marginal products, i.e.

$$(3.5) \quad R_t = 1 + r_t = f'(k_t),$$

$$(3.6) \quad w_t = f(k_t) - k_t \cdot f'(k_t).$$

In this model of two overlapping generations and no bequest motive, the only reason for saving is to provide for retirement age, so that in each period the capital stock (plus interest) is transformed into consumption by the old, and a new capital stock is formed by the savings of the young. Therefore, if  $S_t$  denotes total savings and  $s_t$  savings per worker, the latter determines the capital intensity of the following period via

$$(3.7) \quad k_t = \frac{K_t}{N_t} = \frac{S_{t-1}}{N_{t-1} \cdot G_t} = \frac{s_{t-1}}{G_t}.$$

<sup>3</sup> Note that our model deviates in this point from Diamond's, in which capital is durable.

If we use again the symbols  $c_t$  and  $z_{t+1}$  for per-capita consumption of a representative member of the generation  $t$  in the two periods of his (economic) life, the values of these variables are determined by

$$(3.8) \quad c_t = w_t - s_t - b,$$

$$(3.9) \quad z_{t+1} = R_{t+1} \cdot s_t + b \cdot G_{t+1},$$

where  $b$  denotes the contributions to a pay-as-you-go financed public pension system. Clearly,  $b$  equals zero if no such system exists.

The representative worker in period  $t$  chooses  $s_t$  so as to maximize the twice differentiable and strictly quasi-concave utility function

$$(3.10) \quad U_t = U(c_t, z_{t+1})$$

under the constraint, derived from (3.8) and (3.9),

$$(3.11) \quad c_t + \frac{z_{t+1}}{R_{t+1}} = w_t - b \cdot \frac{R_{t+1} - G_{t+1}}{R_{t+1}}.$$

The optimal choice of  $s_t$  is characterized by the necessary first-order condition

$$(3.12) \quad \frac{\partial U / \partial c_t}{\partial U / \partial z_{t+1}} = R_{t+1}.$$

### 3.2 Efficiency Results

In the following, we shall compare two time paths of consumption, saving, production and income determination, beginning with period  $t = 1$ :

- A) the one which results when there exists a pay-as-you-go financed public pension system with constant contribution (and benefit) rate  $b = b^* > 0$  and individuals adjust optimally to this regime,
- B) the one which corresponds to the absence of a public pension system in all periods  $t$  ( $t \geq 1$ ), i.e.  $b = b^0 = 0$ .

To distinguish easily between the two time paths, the values of all variables will be characterized by the superscripts  $*$  or  $^0$ , respectively. Both time paths, however, start from the same values of the capital stock in period 1 (i.e.  $k_1^* = k_1^0 = k_1$ ), and thus also per-capita production and income per worker are initially identical ( $y_1^* = y_1^0 = y_1$ ,  $w_1^* = w_1^0 = w_1$ ).

Note that we do not specify whether the pay-as-you-go financed system has existed *before* period 1 or is introduced in period 1 for the first time. The analysis therefore covers both the transition from a pay-as-you-go financed public pen-

sion system to a funded plan and the decision *not* to initiate such a system when the population is not growing.

We ask for the intergenerational Pareto-efficiency of type A development in this model. So the question is whether – at least for some sequence  $\{g_1, g_2, \dots\}$  of population growth rates – the time path of consumption in case A is Pareto-dominated by the time path in case B, i.e. whether

$$(3.13) \quad U_t^\circ = U(c_t^\circ, z_{t+1}^\circ) \geq U(c_t^*, z_{t+1}^*) = U_t^* \quad \text{for } t = 0, 1, \dots$$

is fulfilled with “>” for at least one  $t \geq 0$ .

First, it is well-known (see, e.g., FELDSTEIN [1977]) that in a partial equilibrium – i.e. taking the interest rate as given – savings per worker are higher in the absence of a pay-as-you-go financed pension system provided that consumption at working age is a “normal” good, i.e. increases with the income  $w$ . Here, however, we have to employ a general equilibrium framework since – by (3.5), (3.7) and (3.12) –  $s_t$  and  $R_{t+1}$  are determined simultaneously.<sup>4</sup> Nevertheless, it can be shown (see the Appendix) that our assumption  $b^* > b^\circ$  implies

$$(3.14) \quad s_t^\circ > s_t^* \quad \text{for all } t \geq 1$$

which in turn, by (3.7) and the strict monotonicity of the per-capita production function  $f$ , (3.4), implies that case B leads to higher capital intensity and higher per-capita production in every period than does case A.<sup>5</sup>

In case A, however, all members of generation 0 receive pension payments of size  $b^* \cdot G_1$  in their retirement age (period 1). In order to fulfil condition (3.13) for  $t = 0$ , therefore, a compensation payment of size  $b^* \cdot G_1$  must be paid in case B in period 1 to each member of generation 0. Since in this period only generations 0 and 1 are alive, the compensation payment must be financed by the members of generation 1 ( $b^*$  per capita), who in turn can be compensated only in their retirement age by the members of generation 2 and so forth.

Clearly, regime B Pareto-dominates regime A if there is a sequence  $\{\varphi_t\}_{t=1, 2, \dots}$  of compensation payments, each from the active generation  $t$  to the retired generation  $t - 1$ , so that (3.13) is fulfilled for

$$(3.15) \quad c_t^\circ = w_t^\circ - s_t^\circ - \varphi_t,$$

$$(3.16) \quad z_{t+1}^\circ = R_{t+1}^\circ \cdot s_t^\circ + G_{t+1} \cdot \varphi_{t+1}$$

<sup>4</sup> This aspect was already taken into account by HU [1979], who, however, confined his welfare analysis to a comparison of steady states.

<sup>5</sup> VERBON [1988, 65] claims that a Pareto-improving transition from a pay-as-you-go to a funded system is impossible. His argument is based upon the consideration that such a transition would actually *lower* per-capita saving and raise the rate of interest. Our analysis is broader in that it starts from the assumption that people indeed save more even in the presence of compensation payments to be described below.

and  $\varphi_t$  drops to zero for a finite value of  $t$ . This latter requirement is necessary to make sure that, for some later generation, (3.13) holds as a strict inequality.<sup>6</sup>

That such a vanishing sequence of compensation payments exists was recently claimed by HOMBURG [1987, 89f.]. His "proof" is based on the consideration that the losses from regime  $B$  for generation 0, which must be compensated for, are finite, and that the gains for all later generations due to the additional production are not infinitesimally small. He concludes that there must be a period  $T < \infty$  so that the cumulated gains of all generations up to period  $T$  outweigh the losses of generation 0 and thus generations 1 through  $T$  alone are able to fully compensate generation 0.

This kind of reasoning is erroneous since it simply adds gains and losses which accrue in different periods so that Homburg's assertion is not yet established. On the contrary, we shall prove in the following that any sequence of compensation payments which fulfils (3.13) is non-vanishing in the sense that all  $\varphi_t$ 's are bounded from below by a positive number. More precisely, we shall prove the following theorem:

**Proposition 1:** Given the assumptions of the model presented in Section 3.1, any sequence of compensation payments that fulfils condition (3.13) must have the property

$$(3.17) \quad \varphi_t > b^* \quad \text{for all } t = 2, 3, \dots$$

The *proof* is established via induction: In Step 1, we show that the inequality

$$(3.18) \quad \varphi_t > b^* + (R_t^* - R_t^\circ) \cdot s_{t-1}^\circ / G_t,$$

holds for  $t = 2$ , and in Step 2 we show that if (3.18) is true for some period  $t = v$ , then it holds for  $t = v + 1$  as well. We note that (3.14), (3.7), (3.5) and (3.4) imply  $R_t^* > R_t^\circ$  and, therefore, (3.18) for all  $t \geq 2$  is sufficient for (3.17).

**Step 1:** We know that each member of generation 1 pays a compensation of  $\varphi_1 = b^*$  to generation 0. Hence the compensation  $\varphi_2$  (per capita of generation 2) necessary to make generation 1 indifferent between the regimes  $A$  and  $B$  can be derived from (3.13) for  $t = 1$ :

$$(3.19) \quad \begin{aligned} & U[w_1 - s_1^\circ - b^*, R_2^\circ \cdot s_1^\circ + G_2 \cdot \varphi_2] \\ & = U[w_1 - s_1^* - b^*, R_2^* \cdot s_1^* + G_2 \cdot b^*]. \end{aligned}$$

<sup>6</sup> However, these compensation payments must *not* be taken into account by individuals when they determine their saving in the first period of their life, because if they did, they would reduce their saving below  $s_t^\circ$  for the same reason that they reduce it under a pay-as-you-go system. So we leave the question open as to how people can be induced to disregard the compensation payments and go on saving  $s_t^\circ$ , but we show that *even* if this could somehow be accomplished, the resulting allocation would not Pareto-dominate the pay-as-you-go solution.

Due to the strict quasi-concavity of  $U$ , (3.12) and (3.14) we must have

$$(3.20) \quad z_2^\circ - z_2^* > -R_2^* \cdot (c_1^\circ - c_1^*) \quad \text{or} \\ R_2^\circ \cdot s_1^\circ - R_2^* \cdot s_1^* + G_2 \cdot (\varphi_2 - b^*) > -R_2^* \cdot (-s_1^\circ + s_1^*)$$

or, after appropriate simplification,

$$(3.21) \quad \varphi_2 - b^* > s_1^\circ \cdot (R_2^* - R_2^\circ) / G_2,$$

which establishes (3.18) for  $t = 2$ .

**Step 2:** Suppose (3.18) holds for some period  $v \geq 2$ . So we have

$$(3.22) \quad \varphi_v > b^* + (R_v^* \cdot s_{v-1}^\circ - R_v^\circ \cdot s_{v-1}^\circ) / G_v.$$

In regime  $B$ , income per worker in period  $v$  is determined by

$$(3.23) \quad w_v^\circ = f\left(\frac{s_{v-1}^\circ}{G_v}\right) - \frac{s_{v-1}^\circ}{G_v} \cdot f'\left(\frac{s_{v-1}^\circ}{G_v}\right) = f\left(\frac{s_{v-1}^\circ}{G_v}\right) - \frac{s_{v-1}^\circ}{G_v} \cdot R_v^\circ$$

due to (3.6), (3.7) and (3.5). Hence consumption at working age can be expressed, using (3.15), (3.23) and (3.22), by

$$(3.24) \quad c_v^\circ = w_v^\circ - s_v^\circ - \varphi_v^\circ \\ < f\left(\frac{s_{v-1}^\circ}{G_v}\right) - \frac{s_{v-1}^\circ}{G_v} \cdot R_v^\circ - s_v^\circ - b^* - (R_v^* - R_v^\circ) \cdot \frac{s_{v-1}^\circ}{G_v} \\ = f\left(\frac{s_{v-1}^\circ}{G_v}\right) - s_v^\circ - b^* - R_v^* \cdot \frac{s_{v-1}^\circ}{G_v}.$$

From the strict concavity of  $f$ , (3.2), (3.14) and (3.5) we obtain (see Figure 1):

$$(3.25) \quad f\left(\frac{s_{v-1}^\circ}{G_v}\right) - f\left(\frac{s_{v-1}^*}{G_v}\right) < \frac{1}{G_v} \cdot (s_{v-1}^\circ - s_{v-1}^*) \cdot f'\left(\frac{s_{v-1}^*}{G_v}\right) \\ = (s_{v-1}^\circ \cdot R_v^* - s_{v-1}^* \cdot R_v^*) / G_v$$

and, by inserting (3.25) into (3.24)

$$(3.26) \quad c_v^\circ < f\left(\frac{s_{v-1}^*}{G_v}\right) - R_v^* \cdot \frac{s_{v-1}^*}{G_v} - s_v^\circ - b^* = w_v^* - s_v^\circ - b^*.$$

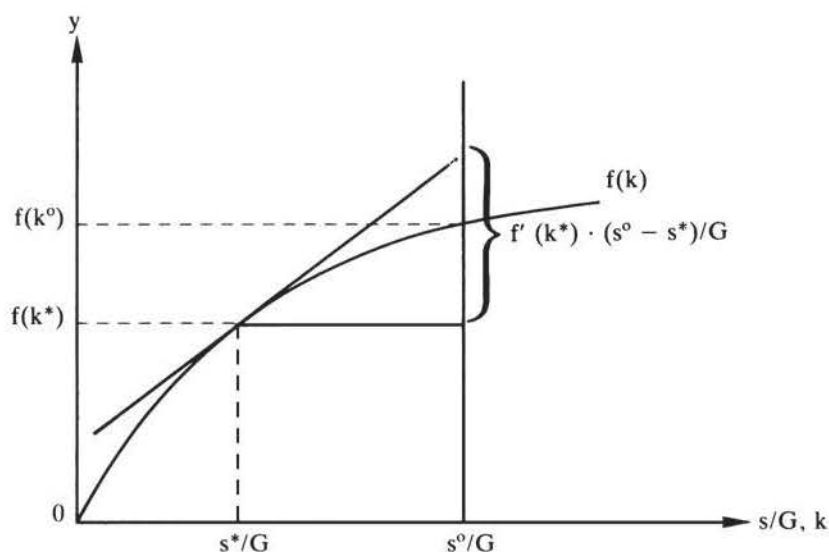


Figure 1

In order to fulfil (3.13) for generation  $v$ , we must have (due to the strict quasi-concavity of  $U$  and (3.12))

$$(3.27) \quad z_{v+1}^{\circ} - z_{v+1}^{*} > -R_{v+1}^{*} \cdot (c_v^{\circ} - c_v^{*}) \quad \text{or}$$

$$R_{v+1}^{\circ} \cdot s_1^{\circ} - R_{v+1}^{*} \cdot s_1^{*} + G_{v+1} \cdot (\varphi_{v+1} - b^{*}) > -R_{v+1}^{*} \cdot (-s_v^{\circ} + s_v^{*})$$

or, after some simplifications,

$$(3.28) \quad \varphi_{v+1} - b^{*} > s_v^{\circ} \cdot (R_{v+1}^{*} - R_{v+1}^{\circ}) / G_{v+1},$$

which establishes (3.18) for  $t = v + 1$  and completes Step 2 of the proof of Proposition 1.

The economic intuition behind these inequalities is the following. In the closed economy, the population as a whole cannot borrow. Therefore, in order to benefit from the increased production in case  $B$ , each working generation must save more than in case  $A$ . As it also has to make side-payments to the pensioners, which are compensated for only in its own retirement age, the allocation of consumption between the two periods of life is no longer optimal, and the increase in the size of total consumption is not sufficient to make up for this suboptimal division.

#### 4. Conclusions

This paper has addressed the question whether a pay-as-you-go financed public pension system is intergenerationally Pareto-efficient even if it yields a lower internal rate of return on the contributions than a capital funded system for all generations except the first. In contrast to opposing claims in the literature it was shown that, when the pay-as-you-go system is replaced by a funded system or by private savings, it is generally impossible to compensate the first generation of pensioners for the loss incurred without making at least one later generation worse off than under pay-as-you-go. Therefore the choice between financing systems for social security cannot be reduced to efficiency criteria but is a distributional matter: the problem of equity among generations inevitably arises.

This conclusion is valid under quite general circumstances. First, it holds both for an open economy with exogenous interest rate and perfect capital markets and for a closed economy, where the return on capital is determined by domestic supply and demand conditions. In the first case, our analysis was based on the assumption of a steady-state development of all exogenous variables, but the proof can be straightforwardly extended to any non-steady-state development.

Finally, it was not specified which financing system for social security exists in the *status quo* situation. Thus the above mentioned conclusion is valid both for the transition from pay-as-you-go to a funded system and for the opposite measure: in the first case, the first generation of pensioners incurs a loss for which they cannot be compensated, and in the second case they make a gain which cannot be reached through side-payments, when capital funding remains the single financing system for social security.

For Western European countries, who experienced a marked decline in birth rates during the last decades, the implications from our analysis are at the same time comforting and disappointing. On the one hand, the decision to introduce the pay-as-you-go system after the second world war was not wrong on efficiency grounds *even* if the present population development could have been foreseen at that time. On the other hand, following a recently quite popular proposal (e.g. by NEUMANN [1986]), and shifting to a funded system now cannot provide *every* generation with a better yield on their social security contributions than the low one they can expect from the pay-as-you-go system in the near future.

#### Summary

When a pay-as-you-go financed public pension system is replaced by a funded system or by private savings, it is generally impossible to compensate the pensioners in the transition generation for the loss incurred without making at least one later generation worse off. This is shown in a standard one-sector

overlapping-generations model, both for a “small” open and for a closed economy. Our findings imply that the choice between financing methods for social security can not be reduced to the intergenerational Pareto-criterion but necessarily involves distributional conflicts among generations.

### *Zusammenfassung*

Beim Übergang von einem umlagefinanzierten staatlichen Rentensystem zu einem System mit Kapitaldeckung oder zu privater Alterssicherung ist es generell unmöglich, die erste betroffene Rentnergeneration für ihre Verluste zu kompensieren, ohne wenigstens eine der späteren Generationen schlechter zu stellen als im Umlageverfahren. Dies wird im Rahmen eines üblichen Ein-Sektoren-Modells mit überlappenden Generationen bewiesen, und zwar sowohl für eine „kleine“ offene als auch für eine geschlossene Volkswirtschaft. Diese Ergebnisse implizieren, daß die Auswahl zwischen verschiedenen Finanzierungsmethoden in der Alterssicherung nicht allein auf Überlegungen der intergenerationalen Pareto-Effizienz gründen kann, sondern daß sie notwendigerweise einen intergenerationalen Verteilungskonflikt in sich birgt.

### *Appendix*

In order to prove inequality (3.14) in the text, we begin with period  $t = 1$ . By inserting (3.7) for  $t = 2$  into (3.5), and (3.8) and (3.9) for  $t = 1$  into (3.12) we get a system of two equations for the simultaneous determination of  $s_1$  and  $R_2$ , where the exogenous variables are  $w_1$  and  $b$ . To avoid notational clutter we drop all time indices and obtain

$$(A.1) \quad R(w, b) = f'[s(w, b)]$$

$$(A.2) \quad \begin{aligned} &U_c[w - b - s(w, b), R(w, b) \cdot s(w, b) + G \cdot b] \\ &= R(w, b) \cdot U_z[w - b - s(w, b), R(w, b) \cdot s(w, b) + G \cdot b]. \end{aligned}$$

Differentiating this system with respect to  $w$  and inserting the first equation into the second yields

$$(A.3) \quad R_w = f'' \cdot s_w$$

$$(A.4) \quad \begin{aligned} &U_{cc} \cdot (1 - s_w) + U_{cz} \cdot R \cdot s_w + U_{cz} \cdot s \cdot R_w \\ &= R_w \cdot U_z + R \cdot U_{cz} \cdot (1 - s_w) + R \cdot U_{zz} \cdot s \cdot R_w + R^2 \cdot U_{zz} \cdot s_w \end{aligned}$$

$$(A.5) \quad s_w = (R \cdot U_{cz} - U_{cc})/H \quad \text{with}$$

$$(A.6) \quad H = -U_{cc} + (2R + s \cdot f'') \cdot U_{cz} - (R^2 - R \cdot s \cdot f'') \cdot U_{zz} - f'' \cdot U_z > 0$$

if the equilibrium is stable. Similarly, by differentiating with respect to  $b$  one obtains

$$(A.7) \quad R_b = f'' \cdot s_b$$

$$(A.8) \quad \begin{aligned} & -U_{cc} \cdot (1 + s_b) + U_{cz} \cdot R \cdot s_b + U_{cz} \cdot s \cdot R_b + U_{cz} \cdot G \\ & = R_b + U_z - R \cdot U_{cz} \cdot (1 + s_b) + R \cdot U_{zz} \cdot s \cdot R_b + R^2 \cdot U_{zz} \cdot s_b + R \cdot G \cdot U_{zz} \end{aligned}$$

$$(A.9) \quad \begin{aligned} s_b &= [R \cdot G \cdot U_{zz} + U_{cc} - (R + G) U_{cz}] / H \\ &= -s_w - (1 - s_w) \cdot G/R = -z_w/R - c_w \cdot G/R, \end{aligned}$$

where the last equality follows from differentiating (3.8) and (3.9) with respect to  $w$ . We conclude that if consumption in both periods of life is "normal" (in *total* equilibrium, i.e. taking account of the endogenous nature of the interest rate),<sup>7</sup> then savings decrease when social security contributions are raised. We thus have shown that

$$(A.10) \quad s_1(w_1, b^\circ) > s_1(w_1, b^*)$$

which implies  $k_2^\circ > k_2^*$ ,  $y_2^\circ > y_2^*$  and, because of (3.4) and (3.6),  $w_2^\circ > w_2^*$ . Again using the positive income elasticity of consumption at retirement age, which implies  $s_w > 0$ , and (A.9) for period 2 we get

$$(A.11) \quad s_2(w_2^\circ, b^\circ) > s_2(w_2^*, b^\circ) > s_2(w_2^*, b^*),$$

and by induction we obtain equation (3.14) in the text.

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<sup>7</sup> We note that the condition of normality of consumption at retirement age in total equilibrium is *weaker* than the same condition in partial equilibrium as long as  $\partial s_t / \partial R_{t+1}$  is negative, which is true if the substitution effect of an increase in the interest rate on savings is outweighed by the corresponding income effect. For the two conditions on the normality of consumption at working age the opposite is true.

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*Prof. Dr. Friedrich Breyer*  
*Fern-Universität Hagen*  
*FB Wirtschaftswissenschaft*  
*Feithstr. 140/AVZ II*  
*D-5800 Hagen*