

Non-exponential relaxation in diluted antiferromagnets

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February 1, 2008

Abstract

Diluted Ising antiferromagnets in a homogenous magnetic field have a disordered phase for sufficiently large values of the field and for low temperatures. Here, the system is in a domain state with a broad size-distribution of fractal domains. We study the relaxation dynamics of this domain state after removing the external field for two and three dimensions. Using Monte Carlo simulation techniques, we measure the decay of the remanent magnetization. Its temperature dependence can be understood as thermal activation. All data can be described by a unique generalized power law for a wide range of temperatures in two and three dimensions. The question whether the exponent of the generalized power law is universal remains open.

Keywords: Ising-Models, Random Magnets, Numerical Methods

1 INTRODUCTION

The dynamics of phase ordering of pure systems is quite well understood. Starting from a disordered state and quenching the system to temperatures well below the critical one, the growth of order is characterized by a length $L(t)$ which scales with $t^{1/2}$ for any spatial dimension d (for a review see Bray, 1993). However, the effect of quenched disorder on the dynamics of

phase ordering is less well understood. It is generally accepted that the dynamics of disordered systems is driven by thermal activation following $t = \tau \exp(\delta E/T)$, where t is the time necessary for a physical process that has to overcome energy barriers of height δE and τ a microscopic time scale of the system. Within a Monte Carlo simulation, this time scale can be expected to be of the order of 1 Monte Carlo Step (MCS). This leads to a natural scaling variable of $T \ln(t/\tau)$. In theoretical works the focus is usually laid on the characteristic length scale of the system such as the average domain size (Fisher, 1993). However, it is not always clear, in which way physical quantities like the magnetization of a system can be related to such length scales, since the complete distribution of domain sizes might influence them (Nowak, Esser, and Usadel, 1996). In experiments, usually the magnetization or the dynamic susceptibilities are measured. In order to get results which are comparable to experimental situations, we analyzed the decay of the remanent magnetization of diluted Ising antiferromagnets, a model which is a prototype for strongly disordered systems.

2 MODEL AND SIMULATION

We simulated two- and three-dimensional diluted antiferromagnets in an external magnetic field. The Hamiltonian of the Ising system ($\sigma_i = \pm 1$) is given by

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \varepsilon_i \varepsilon_j \sigma_i \sigma_j - B \sum_i \varepsilon_i \sigma_i, \quad (1)$$

where the summation runs over the nearest neighbors on a square ($d = 2$) and cubic ($d = 3$) lattice, respectively. The interaction J is set to -1 . The dilution is given by the quenched variables ε_i , which are 0 with a probability p and 1 otherwise.

The DAFF is in the same universality class as the Random Field Ising Model (RFIM) (Fishman and Aharony, 1979). Hence, for $D = 2$ the system is antiferromagnetically ordered only for $B = 0$ below the critical temperature T_N (Aizenman and Wehr, 1989). In $D = 3$ there is long range antiferromagnetic order also for external fields $B > 0$. In both $D = 2$ and $D = 3$ the system is in a domain state for sufficiently large B . This is also true for $T = 0$ as can be shown by exact ground state calculations (Esser, Nowak, and Usadel, 1997). Here, the domains of the DAFF are highly fractal, with a complex shape. Their size distribution is broad, following a power-law with an exponential cut-off which depends on the strength of the external field. Experimental realizations of this model are e. g. $\text{Fe}_{1-p}\text{Zn}_p\text{F}_2$ ($D = 3$) and $\text{Rb}_2\text{Co}_{1-p}\text{Mg}_p\text{F}_4$ ($D = 2$) (for a review of the experimental work, see

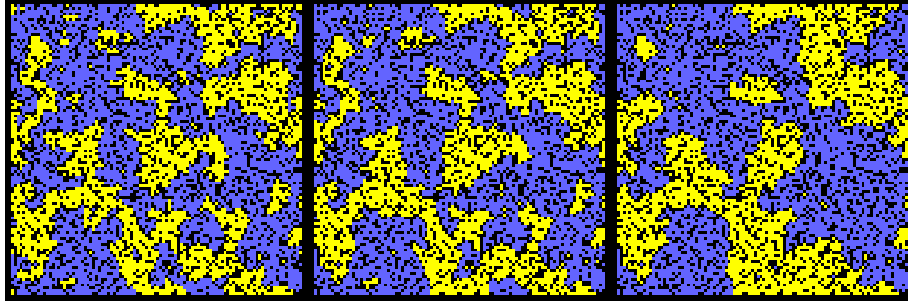


Figure 1: Domain configurations during the relaxation after 1, 1023, and 262143 MCS (from left). The two antiferromagnetic phases are represented in yellow and blue, vacancies are black.

Kleemann, 1993).

We investigate the dynamics of the DAFF with Monte Carlo techniques using the heat-bath algorithm which simulates Glauber dynamics. Spins are updated in a random sequential way. For simulation we averaged over 20 runs ($D = 2$) with different realizations of the dilution. The lattice size is 400×400 spins in 2D and $50 \times 50 \times 50$ in 3D with periodical boundary conditions.

We prepare the system starting with a disordered spin configuration in a region of the phase diagram where the equilibrium state of the system is a domain state. We performed a large number of Monte Carlo updates until the homogeneous magnetization does not change significantly any more. Due to the slow dynamics, the system will not have reached thermal equilibrium within our simulation time. However, this is not important here. The only important aspect here is that the system is in a typical domain state. After this preparation, the external field is switched off and the relaxation of the magnetization is measured for different temperatures. The system will relax into its long range ordered ground state with zero magnetization - which it cannot reach within reasonable time scales due to the extremely slow dynamics. These non-exponential dynamics stems from the fact that the relaxation of the system is due to domain wall movement. The domain walls are pinned to those sites with vacancies and due to this pinning effect, energy barriers have to be overcome by thermal activation. Hence, exponentially large time scales are involved in the dynamics of the relaxation. Figure 1 shows three snap shots of the 2D DAFF during the relaxation, at the beginning of the relaxation, for an intermediate time, and at the end of our simulation time. The size of the system in Figure 1 is 100×100 . As one can see, and as was shown earlier quantitatively (Esser, Nowak, and Usadel, 1997) the initial do-

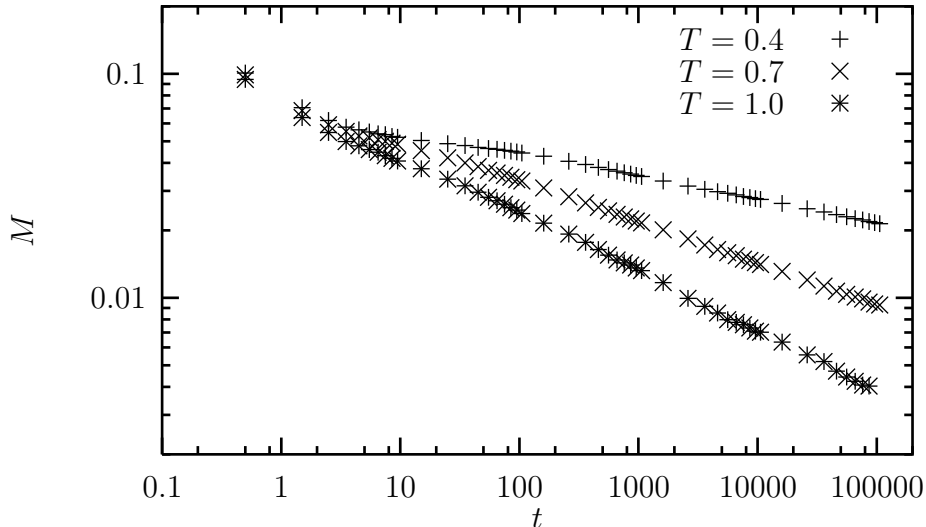


Figure 2: Magnetisation versus time t for the relaxation of a 2D DAFF for three different temperatures.

main state of the system consists of domains on all length scales. During the relaxation, the smaller domains vanish first (except for thermal fluctuation, which are also visible since the pictures are snap-shots). At later times only large domains still exist. Also, the domain walls are flattened. The integral change of magnetization at some time t is due to the sum of all changes that happened during time - small changes for shorter time and larger changes for longer times (Nowak, Esser, and Usadel, 1996). The largest length scale involved in the dynamics can be expected to be connected to the corresponding time scale t by thermal activation $t = \tau \exp(\delta E(L)/T)$.

3 ANALYSIS OF SIMULATION DATA

Figure 2 shows the remanent magnetization versus time for a two dimensional DAFF with an initial field of $B = 2.0$ and a dilution of $p = 0.25$. Data for three different temperatures are shown. As the semi-logarithmic plot suggests, the data follow roughly a power-law with a temperature dependent exponent. However, a slight curvature is visible. Therefore, also other relaxation-laws which are usually used to describe slow relaxation should be taken into account. To improve the accuracy of our analysis, we analyzed the data in two steps. First, we consider for fixed external field B and fixed

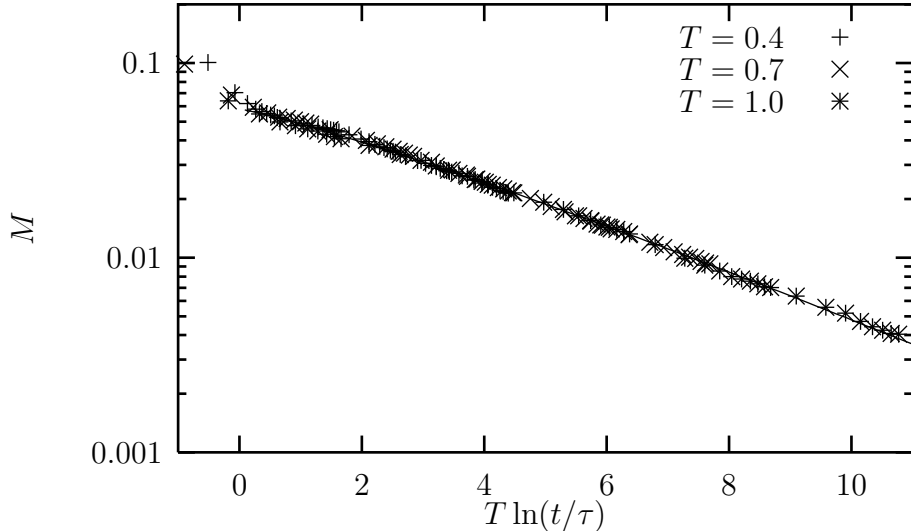


Figure 3: Magnetisation versus scaling variable $T \ln(t/\tau)$ for the data of Figure 2.

dilution p all data for different temperatures. If, as we expect, $T \ln(t/\tau)$ is the correct scaling variable all these data should collapse on the same universal curve by a suitable choice of τ . Figure 3 demonstrates that this works very well for the 2D system yielding $\tau = 1.8$. The same analysis was done for the remanent magnetization of a 3D DAFF with an initial field of $B = 2.0$ and a dilution of 50%. The corresponding scaling plot is shown in Figure 4 yielding $\tau = 1.5$. Note, that we determined the values of τ without any assumption about the decay law at this stage of the analysis.

Having established that the decaying magnetization scales with $T \ln(t/\tau)$ we now discuss its scaling function. It is not expected that the magnetization decays as a simple power of the scaling length as does presumably the characteristic domain size since there is no obvious relation between the maximum relaxed domain size for a certain time and the decay of magnetization during that time. A more elaborate analysis (Nowak, Esser, and Usadel 1996) for the same systems but for the dynamics of domain growth in a field starting with long range order comes to the result that the change of magnetization in the limit of long times decays according to

$$\Delta M(t) = a e^{-b(T \ln(t/\tau))^y}. \quad (2)$$

Therefore, we expect a similar behavior in our simulation. Apart from that, experimental results (Han, Belanger, Kleemann, and Nowak, 1992) as well as

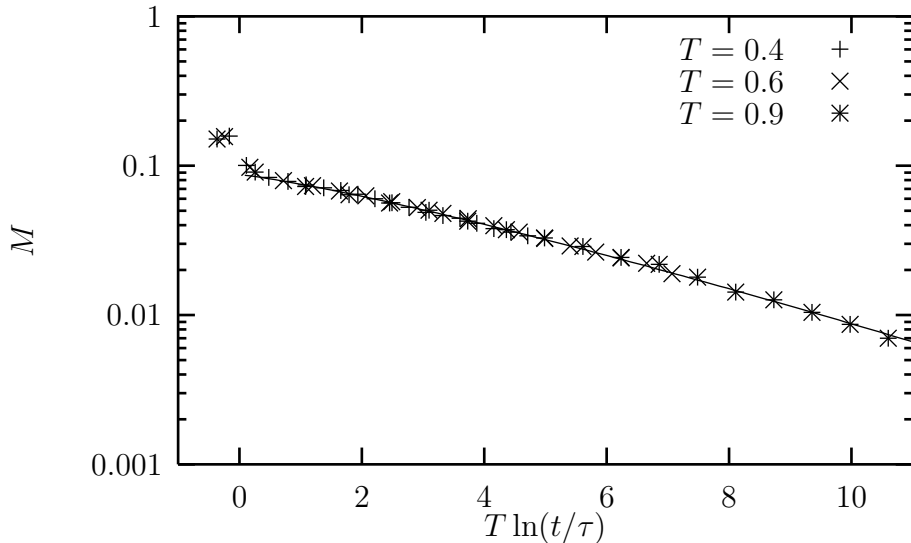


Figure 4: Magnetisation versus scaling variable $T \ln(t/\tau)$ for the relaxation of a 3D DAFF.

earlier simulations (Han and Belanger, 1992) for three dimensional systems have also been fitted by this function. With this function a convincing fit of our time scaled data can be done as is shown in Figures 3 and 4 by solid lines. The resulting values for the parameters τ and the exponent y are summarized in Table 1. We did not find any significant influence of the initial field B on the relaxation.

Through the introduction of the scaling variable $x = T \ln(t/\tau)$ we enhanced the range of data which can be analyzed. Hence, we can differentiate the quality of several relaxation laws without any fitting procedure. E. g., in terms of the scaling variable x , a pure logarithmic law (see e. g. Nattermann

Table 1: Constants of the generalized power law for different dilutions in two and in three dimensions

	2D					3D
p	0.15	0.2	0.25	0.3	0.35	0.5
y	≈ 1	≈ 1	1.15	1.3	1.5	1.2
τ	5	3	1.8	1	0.4	1.2

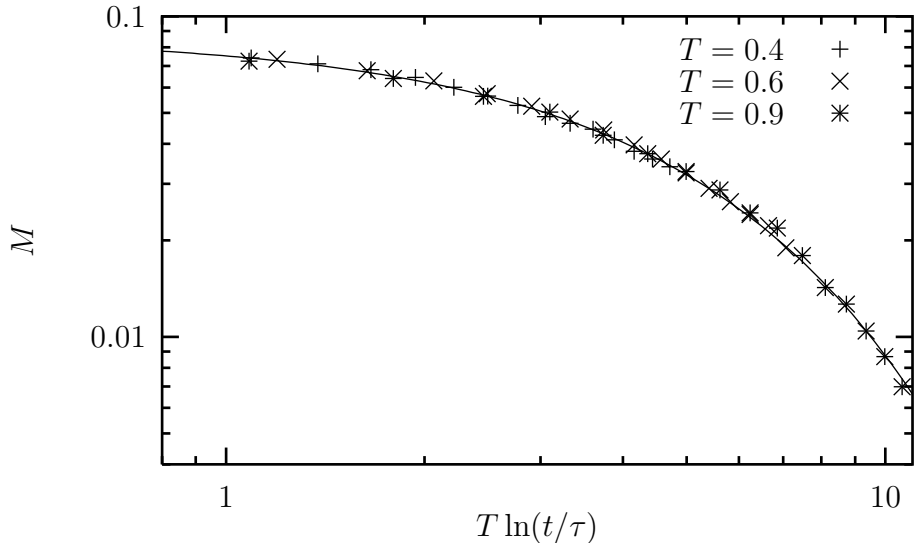


Figure 5: Magnetisation versus scaling variable $T \ln(t/\tau)$ for the same data as in Figure 3 on a logarithmic scale.

and Vilfan, 1988) has the form of a power law, $M(T, t) = M_0(p, B)x^{-\Psi}$. The corresponding plot in Figure 5 demonstrates, that this description is not appropriate - otherwise we had a straight line in the logarithmic plot.

4 CONCLUSIONS

We analyzed the non-exponential time dependence of the remanent magnetization of diluted antiferromagnets. By introducing a scaling variable $T \ln(t/\tau)$ we showed that the basic mechanism of the relaxation is thermal activation. Also, we found that the relaxation can very well be described by a generalized power-law for two as well as for three dimensional systems. We did not find any significant dependence of the relaxation on the initial magnetic field.

For two dimensional systems, we varied the dilution p of the system. The exponent y of the generalized power law and the time constant τ depend on the dilution of the system. However, we cannot exclude the possibility that y is a universal exponent and that the p dependence in our data stems from a crossover phenomenon due to the percolation problem, because close to the percolation threshold p_c a crossover to a new kind of dynamics can be expected, since for concentrations below p_c no long-range order can be

established in the system.

Acknowledgments: This work was in part supported by the Deutsche Forschungsgemeinschaft through Sonderforschungsbereich 166.

5 References

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