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Dissipative Frustration in a One Dimensional Josephson Junction Chain †

Dominik Maile [1,2], Sabine Andergassen [2], Wolfgang Belzig [1] and Gianluca Rastelli [1,*]

1 Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany; dominik.maile@uni-konstanz.de (D.M.); wolfgang.belzig@uni-konstanz.de (W.B.)
2 Institut für Theoretische Physik and Center for Quantum Science, Universität Tübingen, 72076 Tübingen, Germany; sabine.andergassen@uni-tuebingen.de
* Correspondence: gianluca.rastelli@uni-konstanz.de

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Abstract: We study the influence of dissipative frustration on the one dimensional Josephson junction chain. In particular we analyze the dissipative quantum phase transition between the chain being superconducting or insulating, the purity as a measure of quantum—environment correlation and the logarithmic negativity as an entanglement measure. The dissipative frustration is provided by coupling two non-commuting operators to the environment. A possible realization of these environmental couplings are shunt resistances between the superconducting islands and resistances to the ground. Using a self-consistent harmonic approximation we determine the critical line separating superconducting and insulating phases and find a nonmonotonic behavior as a function of the dissipative strength. The interplay between both environmental couplings is also reflected in the purity and the logarithmic negativity. We find a change in their behavior depending whether a second bath is present or not.

Keywords: quantum phase transition; dissipation; Josephson junction chain

1. Introduction

Mesoscopic quantum many-body systems combine two very important attributes, they are small enough to show quantum behavior and large enough to make their quantum properties experimental accessible. These systems are realized for example by superconducting circuits [1,2], atoms in optical lattices [3] or arrays of coupled QED cavities [4]. They are of major importance in the exploration of fundamental questions of quantum mechanics, as well as in the growing interest in quantum applications like quantum simulators or quantum computers.

Owing the fact that in this mesoscopically large systems undesired couplings to the surrounding environment can not be avoided experimentally one has to take dissipative effects into account in their theoretical description. Studying the variety of dissipative effects on quantum many-body systems leads on the one hand to a better understanding of the quantum to classical transition and on the other to the possibility of using these effects via engineering dissipation.

As a main source of decoherence the effect of dissipation is often considered to destroy quantum behavior. However, recent research acitivties deal with much more implications of environmental couplings on quantum many-body systems. Coupling the whole system to one global dissipative bath can for example also entangle subsystems, i.e., enhance quantum effects [5]. The Heisenberg uncertainty leads to finite fluctuations of the quantum ground state. The effect of dissipation on these quantum fluctuations crucially depends on the observable the bath is coupled to. The environment can enhance or squeeze them. Further, coupling different baths to canonically conjugate observables leads to dissipative frustration in the system and the fluctuations can become non monotonic as a function of the dissipative coupling strength [6,7].

The mentioned effects on quantum fluctuations can also lead to dissipative quantum phase transitions in many body systems. An often discussed problem is the superconductor—insulator transition in a chain of Josephson junctions (JJC). At zero temperature one can map this one dimensional quantum chain to the classical XY - model showing a Berezinski-Kosterlitz-Thouless (BKT) phase transition [8]. The individual effect of different dissipative couplings on this transition have been studied in the past [9,10], while their interplay and the resulting frustration have been considered only recently [11]. In this proceeding we give an overview on the work [11], providing a theoretical analysis of the quantum phase transition in presence of dissipative frustration together with a proposal of an experimental realization. Furthermore, we study the effect on the purity as a measure of system environment correlation and on the logarithmic negativity as a measure of entanglement in the system.

2. System and Model

We consider a JJC with a large superconducting gap and neglect quasiparticle excitations. Further we consider the self-charging limit where we disregard capacitances between the islands. Each superconducting island has a Cooper pair charge $\hat{Q}_n = i2e(\partial/\partial\hat{\varphi}_n)$ and a superconducting phase $\hat{\varphi}_n$. These observables satisfy the commutation relation $\left[\hat{\varphi}_n, \hat{Q}_m\right] = 2ei\delta_{n,m}$ and the Hamiltonian reads

$$\hat{H}_S = \sum_{n=0}^{N-1} \left[-\frac{E_C}{2}(\partial^2/\partial\hat{\varphi}_n^2) - E_J \cos\left(\Delta\hat{\varphi}_n\right) \right], \tag{1}$$

where $E_C = 4e^2/C_0$ is the charging energy, C_0 the capacitance to the ground and E_J the Josephson coupling. We set $\Delta\hat{\varphi}_n = (\hat{\varphi}_{n+1} - \hat{\varphi}_n)$ and periodic boundary conditions $\hat{\varphi}_n \equiv \hat{\varphi}_0$. The intrinsic zero temperature (quantum) fluctuations of the system are governed by the ratio $g = \sqrt{E_J/E_C}$ (increasing g the phase fluctuations decrease). The one dimensional JJC shows a BKT phase transition at a critical g_{BKT}. For $g < g_{BKT}$ the phases are uncorrelated and the JJC is an insulator. For $g > g_{BKT}$ the chain is superconducting and the phases are quasi ordered in the BKT sense (the fluctuations of the phase difference $\langle\Delta\hat{\varphi}_n^2\rangle$ are finite in the thermodynamic limit while the fluctuations of $\langle\hat{\varphi}_n^2\rangle$ diverge). We include dissipation via the Caldeira Leggett model and consider one ohmic bath coupled to each phase difference $\Delta\hat{\varphi}_n$ and one ohmic bath coupled to each charge $\hat{Q}_n$ of the superconducting islands. Figure 1a) shows a possible realization of such a system. The shunt resistance between the superconducting islands R_s leads to (conventional) dissipation affecting the phase difference and the resistance to the ground R_g to (unconventional) charge dissipation. Using the imaginary time path integral formalism we obtain the partition function $\mathcal{Z}_{\text{eff}} = \prod_{n=0}^{N-1} \oint_c \mathcal{D}[\varphi_n(\tau)]\, e^{-\mathcal{S}[\{\varphi_n(\tau)\}]/\hbar}$, with $\beta = \hbar/(k_B T)$ and the Euclidean action

$$\mathcal{S} = \mathcal{S}_d + \int_0^\beta d\tau \left[\frac{\hbar}{2e} \frac{\dot{\varphi}_n^2(\tau)}{E_C} - E_J \cos\left(\Delta\varphi_n(\tau)\right) \right], \tag{2}$$

where the dissipative and electrostatic part, with the kernel $F(\tau)$ affecting the phase difference and the unconventional kernel $\widetilde{F}(\tau)$ affecting the total charge, reads

$$\mathcal{S}_d = \sum_{n=0}^{N-1} \left[-\frac{1}{2} \int_0^\beta\!\!\int_0^\beta d\tau d\tau'\, F(\tau-\tau') \left|\Delta\varphi_n(\tau) - \Delta\varphi_n(\tau')\right|^2 + \frac{1}{2} \int_0^\beta\!\!\int_0^\beta d\tau d\tau'\, \widetilde{F}(\tau-\tau')\, \dot{\varphi}_n(\tau)\, \dot{\varphi}_n(\tau') \right]. \tag{3}$$

We diagonalize Eq. (3) via a transformation to Matsubara frequencies $\omega_l = 2\pi l/\beta$. The dissipative kernel $F(\tau)$ transforms to $F_l \propto (R_q/R_s)|\omega_l|$ and the kernel including charge dissipation to $\widetilde{F}_l \propto -R_g C_0|\omega_l|/(1 + R_g C_0|\omega_l|)$. Further we define a dissipative parameter for conventional dissipation

$\alpha = R_q/R_s$ and for unconventional dissipation $\widetilde{\alpha} = R_g/R_q$, with $R_q = h/2e^2$ being the quantum of resistance. As the partition function is not exactly solvable due to the cosine interaction, we use a self-consistent harmonic approximation (SCHA) first introduced by Feynman [12] and used in literature [13]. In this approach the cosine is approximated by an effective parabolic interaction with variational interaction strength. The SCHA allows to determine a qualitative phase diagram for the JJC. By varying the parameters $\alpha, \widetilde{\alpha}$ and g we can determine a critical value for a phase transition [11].

3. Quantum Phase Diagram with Dissipative Frustration

The quantum fluctuations of a system affected by dissipative couplings to non-commuting observables can exhibit a non-monotonic behavior as a function of the total damping [6]. We analyze this dissipative frustration by fixing the ratio $\widetilde{\alpha}/\alpha$. If the quantum fluctuations are too large the phase transition from a superconductor to an insulator occurs. Consequently, this non-monotonic behavior of the phase fluctuations in presence of dissipative frustration can lead to the situation that the system is superconducting for small and large dissipation, but in the intermediate regime insulating. Hence, a non-monotonic phase transition line emerges in the phase diagram in Figure 1b).

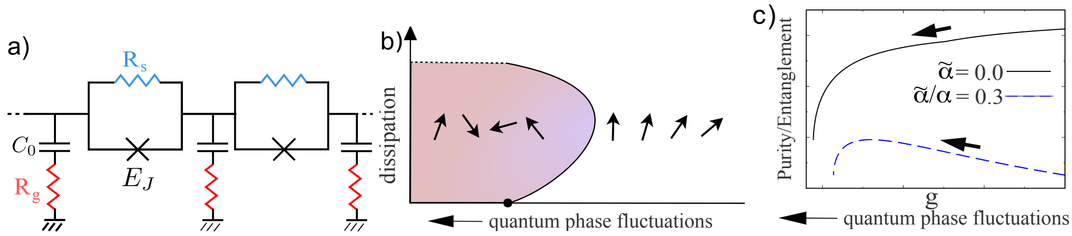

Figure 1. (**a**) 1D chain of superconducting islands with Josephson coupling of energy E_J and a capacity to the ground C_0. The shunt resistance R_s corresponds to a dissipative coupling for the difference of the superconducting phases φ_n, whereas the resistance to the ground R_g yields a dissipative coupling in the charge Q_n. (**b**) Qualitative quantum phase diagram with dissipative frustration. (**c**) Qualitative results for the purity and the logarithmic negativity as function of the parameter g for conventional dissipation (solid line) and frustrated dissipation (dashed line).

4. Purity and Entanglement

Besides affecting the phase diagram dissipation also influences quantum correlations. To study these effects we calculate the purity as a measure of system - bath correlations and the logarithmic negativity as a measure of entanglement in the JJC. As the qualitative behavior of both quantities is the same we summarize the results in a single schematic plot in Figure 1c).

The purity is defined as $\mathcal{P} = \text{Tr}\left(\rho_{sc}^2\right)$, where ρ_{sc} is the reduced density matrix describing the one dimensional JJC within the SCHA. In the non-dissipative case the chain remains in a pure state and $\mathcal{P} = 1$. Dissipation mixes the density matrix and we generally find $\mathcal{P} < 1$. Figure 1c) shows $\mathcal{P}$ as a function of the parameter g. For conventional dissipation ($\widetilde{\alpha} = 0$) the purity decreases for lower values of g (i.e., enhancing phase fluctuations). Here conventional dissipation favors a more ordered state of the phases and hence complies with the effect of the interaction. In the strong interaction limit the system tends to a pure classical state. Remarkably, in presence of both dissipative couplings ($\widetilde{\alpha} \neq 0$) the purity increases by decreasing g. The unconventional part of the dissipation acts in favor of an enhancement of phase fluctuations. This corresponds to a quenching of momentum (charge) fluctuations and leads to a more pure (classical) state in this basis. In the conventional and the frustrated dissipative case the purity decreases close to the phase transition.

We also compute the bipartite entanglement for two subsystems A and B of the JJC, from which we extract the logarithmic negativity via a partial transposition of the density matrix $\rho_{sc} \to \rho_{sc}^{T_A}$. The superscript T_A denotes the transposition of the part of ρ_{sc} corresponding to subsystem A. We calculate the covariance matrix $\sigma[\rho_{sc}^{T_A}]$ and with this the logarithmic negativity for Gaussian states $E_{\mathcal{N}}[\hat{\rho}_{sc}] = -\sum_k \log_2\left(\min[1,(2c_k)]\right)$, where c_k represent the symplectic eigenvalues of the covariance matrix [14]. The amount of entanglement generally depends on the partition of the JJC. We calculate $E_{\mathcal{N}}$ for a chain containing $N = 9$ sites and subsystems $N_A = 4$ and $N_B = 5$. Analyzing $E_{\mathcal{N}}$ in the same way as the purity we find qualitatively the same behavior (see also Figure 1c)).

5. Summary

In this work we studied the influence of dissipative frustration on the superconductor—insulator phase transition in a chain of Josephson junctions by coupling the environment to two non-commuting observables. We find that the critical line is non-monotonic when fixing the ratio of the two dissipative coupling strengths. This peculiar behavior can be traced back to the non-monotonic zero temperature fluctuations emerging from dissipative frustration. Because dissipative frustration is a pure quantum effect we study the influence of dissipation on system—environment and correlations and on the internal entanglement. For this we analyze the purity and the logarithmic negativity as a function of the intrinsic quantum fluctuation parameter g and find a change of slope by increasing the interaction strength with the unconventional bath. Further we find a non-monotonic behavior approaching the critical point associated to the phase transition.

Our results demonstrate that dissipative frustration can lead to interesting effects and novel features in the physics of open quantum many body systems.

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