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Elementary events of electron transfer in a voltage-driven quantum point contact

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We show that the statistics of electron transfer in a coherent quantum point contact driven by an arbitrary time-dependent voltage is composed of elementary events of two kinds: unidirectional one-electron transfers determining the average current and bidirectional two-electron processes contributing to the noise only. This result pertains at vanishing temperature while the extended Keldysh-Green's function formalism in use also enables the systematic calculation of the higher-order current correlators at finite temperatures.

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The most detailed description of the charge transfer in coherent conductors is a statistical one. At constant bias, the full counting statistics (FCS) of electron transfer [1] can be readily interpreted in terms of elementary events independent at different energies. The FCS approach is readily generalized to the case of a time-dependent voltage bias [2, 3]. The current fluctuations in coherent systems driven by a periodic voltage strongly depend on the shape of the driving [4], this dependence being frequently concealed in average current [5]. The noise power, for instance, exhibits at low temperatures a piecewise linear dependence on the dc voltage with kinks corresponding to integer multiples of the ac drive frequency and slopes which depend on the shape and the amplitude of the ac component. This dependence has been observed experimentally in normal coherent conductors [6] and diffusive normal metal–superconductor junctions [7].

What are elementary events of charge transfer driven by a general time-dependent voltage? The time dependence mixes the electron states at different energies [5] which makes this question both interesting and nontrivial. First step in this research has been made in [8] for a special choice of the time-dependent voltage. The authors have considered a superposition of overlapping Lorentzian pulses of the same sign ("solitons"), with each pulse carrying a single charge quantum. The resulting charge transfer is unidirectional with a binomial distribution of transmitted charges. The number of attempts per unit time for quasiparticles to transverse the junction is given by the dc component of the voltage, independent of the overlap between the pulses and their duration [9]. It has been shown that such superposition minimizes the noise reducing it to that of a corresponding dc bias. A microscopic picture behind the soliton pulses has been revealed only recently [10]. In contrast to a general voltage pulse which can in principle create a random number of electron-hole pairs with random directions, a soliton pulse at zero temperature always creates a single electron-hole pair with quasiparticles moving in opposite directions. One of the quasiparticles (say, electron) comes to the contact and takes part in the transport while the hole goes away. Therefore, soliton pulses can be used to create minimal excitation states with "pure" electrons excited from the filled Fermi sea and no holes left below. The existence of such states can be probed by noise measurements [10, 11, 12].

In this Letter, we identify the independent elementary events for an arbitrary time-dependent driving. It is enough to give those for a single conduction channel of transmission T, implying that in multichannel conductors the statistics in different channels are independent. The answer is surprisingly simple. There are two kinds of such events: We call them *bidirectional* and *unidirectional*. In the course of a *bidirectional* event k an electron-hole pair is created with probability $\sin^2(\alpha_k/2)$, with α_k being determined by the details of the time-dependent voltage [Fig. 1(a,b)]. The electron and hole move in the *same* direction reaching the scatterer. The charge transfer occurs if the electron is transmitted and the hole is reflected [Fig. 1(a)], or vice versa [Fig. 1(b)]. The probabilities of both outcomes, TR (R being reflection coefficient), are the same. Therefore, the bidirectional events do not contribute to the average current and odd cumulants of the charge transferred although they do contribute to the

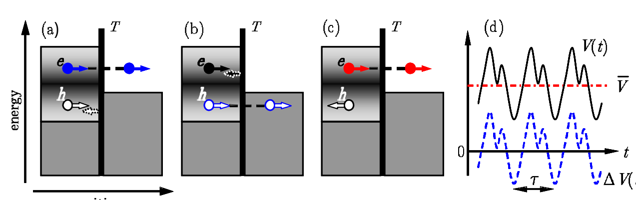

FIG. 1: Schematic representation of elementary events: bidirectional (a, b) and unidirectional (c). Shifts of the effective chemical potential in the left lead due to time-dependent voltage drive are indicated by shading. For periodic drive, the dc voltage component [panel (d), dash-dotted line] describes unidirectional charge transfer, while the ac component (dashed curve) describes bidirectional events affecting the noise and higher-order even cumulants.

noise and higher-order even cumulants. A specific example of a bidirectional event for a soliton-antisoliton pulse was in fact given in [9].

The *unidirectional* events are the same as for a constant bias or a soliton pulse. They are characterized by chirality $\kappa_l = \pm 1$ which gives the direction of the charge transfer. An electron-hole pair is always created in the course of the event, with electron and hole moving in opposite directions [Fig. 1(c)]. Either electron ($\kappa_l = 1$) or hole ($\kappa_l = -1$) passes the contact with probability T, thus contributing to the current.

Mathematically, the above description corresponds to the following cumulant generating function $S(\chi) = S_1(\chi) + S_2(\chi)$, where

$$S_1(\chi) = 2\sum_k \ln\left[1 + TR\sin^2\left(\frac{\alpha_k}{2}\right)(e^{i\chi} + e^{-i\chi} - 2)\right] \quad (1)$$

presents the contribution of the bidirectional events and

$$S_2(\chi) = 2\sum_l \ln[1 + T(e^{-i\kappa_l\chi} - 1)] \quad (2)$$

that of the unidirectional ones. The sum in both formulas is over the set of corresponding events [13]. Here χ is the counting field, and α_k and κ_l are the parameters of the driving to be specified later. The probability that N charges are transmitted within the time of measurement is given by $P(N) = (2\pi)^{-1}\int_{-\pi}^{\pi} d\chi \exp[S(\chi) - iN\chi]$. Higher-order derivatives of S with respect to χ are proportional to the cumulants of transmitted charge, or equivalently, to higher-order current correlators at zero frequency.

The elementary events have been inferred from the form of the cumulant-generating function, as it has been done in [14, 15]. Below we present the microscopic derivation of the Eqs. (1) and (2).

The approach we use is the nonequilibrium Keldysh-Green's function technique, extended to access the full counting statistics [16, 17, 18, 19]. We neglect charging effects and assume instantaneous scattering at the contact with quasiparticle dwell times much smaller than the characteristic time scale of the voltage variations. It is advantageous to use the abstract operator notation which is representation invariant. In this notation, the Green's functions of the left (1) and right (2) leads are given by [18, 19]

$$\check{G}_1 = e^{-i\chi\check{\tau}_1/2}\begin{pmatrix}1 & 2\tilde{h}\\ 0 & -1\end{pmatrix}e^{i\chi\check{\tau}_1/2} \quad \text{and} \quad \check{G}_2 = \begin{pmatrix}1 & 2h\\ 0 & -1\end{pmatrix}. \quad (3)$$

Here $\check{\tau}_1 = \left(\begin{smallmatrix}0 & 1\\ 1 & 0\end{smallmatrix}\right)$ is a matrix in Keldysh($\check{\ }$) space and $\check{G}_2$ is the equilibrium quasiclassical Keldysh-Green's function with $h = \tanh(\mathcal{E}/2T_e)$. The quasiparticle energy $\mathcal{E}$ is measured with respect to the chemical potential in the absence of the bias and T_e is the temperature. The effect of the applied voltage $V(t)$ across the junction is taken into account by the gauge transformation $\tilde{h} = UhU^\dagger$. In the time representation, the unitary operator U is given by $U(t', t'') = f(t')\delta(t' - t'')$, where $f(t') = \exp[-i\int_0^{t'} eV(t)dt]$. The cumulant generating function $S(\chi)$ of the charge transfer through the junction is given by [19, 20]

$$S(\chi) = \mathrm{Tr}\ln\left[\check{1} + \frac{T}{2}\left(\frac{\{\check{G}_1, \check{G}_2\}}{2} - \check{1}\right)\right]. \quad (4)$$

Here the trace is taken both in Keldysh and in time (energy) indices and the convolution over internal indices is assumed. For the anticommutator of the Green's functions we find $\{\check{G}_1, \check{G}_2\}/2 - \check{1} = -2\sin(\chi/2)(\check{\mathcal{A}} + \check{\mathcal{B}})$, where

$$\check{\mathcal{A}} = \begin{pmatrix}1 & b\\ 0 & 0\end{pmatrix} \otimes A, \qquad \check{\mathcal{B}} = \begin{pmatrix}0 & -b\\ 0 & 1\end{pmatrix} \otimes B, \quad (5)$$

$$A = (1 - h\tilde{h})\sin(\chi/2) + i(h - \tilde{h})\cos(\chi/2), \quad (6)$$

$$B = (1 - \tilde{h}h)\sin(\chi/2) + i(h - \tilde{h})\cos(\chi/2), \quad (7)$$

$b = -i\cot(\chi/2)$, and $\otimes$ is the tensor product. Since $\check{\mathcal{A}}\check{\mathcal{B}} = \check{\mathcal{B}}\check{\mathcal{A}} = 0$, the operators $\check{\mathcal{A}}$ and $\check{\mathcal{B}}$ commute and satisfy for integer n

$$(\check{\mathcal{A}} + \check{\mathcal{B}})^n = \begin{pmatrix}1 & b\\ 0 & 0\end{pmatrix} \otimes A^n + \begin{pmatrix}0 & -b\\ 0 & 1\end{pmatrix} \otimes B^n. \quad (8)$$

Therefore, $S(\chi)$ given by Eq. (4) reduces to

$$S(\chi) = \mathrm{Tr}\ln[1 - T\sin(\chi/2)A] + \mathrm{Tr}\ln[1 - T\sin(\chi/2)B]. \quad (9)$$

A further simplification of $S(\chi)$ is possible in the zero temperature limit, in which the hermitian h-operators are involutive: $h^2 = \tilde{h}^2 = 1$. The operators $h\tilde{h}$ and $\tilde{h}h$ are mutually inverse and commute with each other. Because $h\tilde{h}$ is unitary, it has the eigenvalues of the form $e^{i\alpha_k}$ with real α_k, and possesses an orthonormal eigenbasis $\{\mathrm{v}_{\alpha_k}\}$. The *typical* eigenvalues of $h\tilde{h}$ (or $\tilde{h}h$) appear in pairs $e^{\pm i\alpha}$ with the corresponding eigenvectors v_α and $\mathrm{v}_{-\alpha} = h\mathrm{v}_\alpha$. In the span$(\mathrm{v}_\alpha, \mathrm{v}_{-\alpha})$ operators $h\tilde{h}$ and $\tilde{h}h$ are diagonal and given by $h\tilde{h} = \mathrm{diag}(e^{i\alpha}, e^{-i\alpha})$ and $\tilde{h}h = \mathrm{diag}(e^{-i\alpha}, e^{i\alpha})$. Because $[h, \{h, \tilde{h}\}] = [\tilde{h}, \{h, \tilde{h}\}] = 0$, the operators h and $\tilde{h}$ reduce in the eigensubspaces span$(\mathrm{v}_\alpha, \mathrm{v}_{-\alpha}) \leftrightarrow e^{i\alpha} + e^{-i\alpha}$ of $\{h, \tilde{h}\}$. In the basis $(\mathrm{v}_\alpha, \mathrm{v}_{-\alpha})$ they are given by

$$h = \begin{pmatrix}0 & e^{i(\alpha+\varphi)/2}\\ e^{-i(\alpha+\varphi)/2} & 0\end{pmatrix}, \quad (10)$$

$$\tilde{h} = \begin{pmatrix}0 & e^{-i(\alpha-\varphi)/2}\\ e^{i(\alpha-\varphi)/2} & 0\end{pmatrix}, \quad (11)$$

where φ is a real number. The operator A given by Eq. (6) also reduces in span$(\mathrm{v}_\alpha, \mathrm{v}_{-\alpha})$ and can be diagonalized in invariant subspaces. Substituting Eqs. (10) and (11) in Eq. (6) we find the typical eigenvalues of A:

$$\mathrm{ev}\, A = 2\sin(\alpha/2)\bigg(\sin(\alpha/2)\sin(\chi/2) \pm i\sqrt{1-\sin^2(\alpha/2)\sin^2(\chi/2)}\bigg), \quad (12)$$

independent of φ. Substitution into Eq. (7) gives the same eigenvalues: $\mathrm{ev}\, B = \mathrm{ev}\, A$. From Eqs. (12) and (9) we recover the generating function $S_1(\chi)$ given by Eq. (1), which is associated with the paired eigenvalues $e^{\pm i\alpha_k}$.

There are, however, some *special* eigenvectors of $h\tilde{h}$ which do not appear in pairs. The pair property discussed above was based on the assumption that v_α and $h\mathrm{v}_\alpha = \mathrm{v}_{-\alpha}$ are linearly independent vectors. In the special case, these vectors are the same apart from a coefficient. Therefore, the special eigenvectors of $h\tilde{h}$ are the eigenvectors of both h and $\tilde{h}$ with eigenvalues ± 1. This means that the special eigenvectors posses *chirality*, with positive (negative) chirality defined by $h\mathrm{v} = \mathrm{v}$ and $\tilde{h}\mathrm{v} = -\mathrm{v}$ ($h\mathrm{v} = -\mathrm{v}$ and $\tilde{h}\mathrm{v} = \mathrm{v}$). From Eqs. (6), (7), and (9) we obtain the generating function $S_2(\chi)$ given by Eq. (2), where l labels the special eigenvectors and κ_l is the chirality.

The cumulant generating function given by Eqs. (1) and (2), together with the interpretation, is the main result of this Letter. It holds at zero temperature only: since the elementary events are the electron-hole pairs created by the applied voltage, the presence of thermally excited pairs will smear the picture. Equations (1) and (2) contain the complete χ-field dependence in explicit form which allows for the calculation of higher-order cumulants and charge transfer statistics for arbitrary time-dependent voltage. The details of the driving are separated from the χ-field dependence and contained in the set of parameters $\{\alpha_k\}$. This opens an interesting possibility to excite the specific elementary processes and design the charge transfer statistics by appropriate time dependence of the applied voltage, with possible applications in production and detection of the many-body entangled states [14, 21].

In the following we focus on a periodic driving $V(t+\tau) = V(t)$ with the period $\tau = 2\pi/\omega$, for which the eigenvalues of $h\tilde{h}$ can be easily obtained by matrix diagonalization. The operator $\tilde{h}$ in the energy representation is given by

$$\tilde{h}(\mathcal{E}', \mathcal{E}'') = \sum_{n,m} \tilde{f}_n \tilde{f}^*_{n+m}\, h(\mathcal{E}' - n\omega - e\bar{V}) \times 2\pi\delta(\mathcal{E}'' - \mathcal{E}' - m\omega). \quad (13)$$

Here $h(\mathcal{E}) = \tanh(\mathcal{E}/2T_e)$,

$$\tilde{f}_n = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} dt\; e^{-i\int_0^t dt'\, e\Delta V(t')}\, e^{in\omega t}, \quad (14)$$

$\bar{V} = (1/\tau)\int V(t)dt$ is the dc voltage offset and $\Delta V(t) = V(t) - \bar{V}$ is the ac voltage component. The coefficients $\tilde{f}_n$ given by Eq. (14) satisfy $\sum_k \tilde{f}_{n+k}\tilde{f}^*_{m+k} = \delta_{nm}$ and $\sum_n n|\tilde{f}_n|^2 = 0$. The operator $\tilde{h}$ couples only energies which differ by an integer multiple of ω, which allows to map the problem into the energy interval $0 < \mathcal{E} < \omega$ while retaining the discrete matrix structure in steps of ω. Therefore, the trace operation in Eq. (4) becomes an integral over $\mathcal{E}$ and the trace in discrete matrix indices. In this way we obtain that $S_1(\chi)$ at zero temperature consists of two terms, $S_1 = S_{1L} + S_{1R}$, where

$$S_{1L}(\chi) = \frac{t_0}{\pi}(e\bar{V} - N\omega)\sum_k \ln[1 + TR\sin^2(\alpha_{kL}/2) \times (e^{i\chi} + e^{-i\chi} - 2)] \quad (15)$$

and

$$S_{1R}(\chi) = \frac{t_0}{\pi}[(N+1)\omega - e\bar{V}]\sum_k \ln[1 + TR \times \sin^2(\alpha_{kR}/2)(e^{i\chi} + e^{-i\chi} - 2)]. \quad (16)$$

Here t_0 is the total measurement time which is much larger than τ and the characteristic time scale on which the current fluctuations are correlated. Parameters $\alpha_{kL(R)}$ are related to the eigenvalues $e^{\pm i\alpha_k(\mathcal{E})}$ of the matrix $(h\tilde{h})_{nm}(\mathcal{E})$ which is given by

$$(h\tilde{h})_{nm}(\mathcal{E}) = \mathrm{sign}(\mathcal{E} + n\omega)\sum_k \tilde{f}_{n+k}\tilde{f}^*_{m+k} \times \mathrm{sign}(\mathcal{E} - k\omega - e\bar{V}). \quad (17)$$

The matrix $(h\tilde{h})_{nm}(\mathcal{E})$ is piecewise constant for $\mathcal{E} \in (0, \omega_1)$ and $\mathcal{E} \in (\omega_1, \omega)$, where $\omega_1 = e\bar{V} - N\omega$ and $N = \lfloor e\bar{V}/\omega \rfloor$ is the largest integer less than or equal $e\bar{V}/\omega$. The parameters $\alpha_{kL(R)} \equiv \alpha_k(\mathcal{E})$ are calculated for $\mathcal{E} \in (0, \omega_1)$ [$\mathcal{E} \in (\omega_1, \omega)$].

The special eigenvectors all have the same chirality which is given by the sign of the dc offset $\bar{V}$. For $e\bar{V} > 0$, there are $N_1 = N + 1$ special eigenvectors for $\mathcal{E} \in (0, \omega_1)$ and $N_2 = N$ for $\mathcal{E} \in (\omega_1, \omega)$. Because $e\bar{V} = N_1\omega_1 + N_2(\omega - \omega_1)$, the effect of the special eigenvectors is the same as of the dc bias

$$S_2(\chi) = \frac{t_0 e\bar{V}}{\pi}\ln[1 + T(e^{-i\chi} - 1)]. \quad (18)$$

Comparing Eqs. (2) and (18) we see that unidirectional events for periodic drive are uncountable. The summation in Eq. (2) stands both for the energy integration in the interval ω and the trace in the discrete matrix indices. In the limit of a single pulse $\omega \to 0$ unidirectional events remain uncountable for a generic voltage, while being countable, e.g., for soliton pulses carrying integer number of charge quanta [9].

Equations (14)–(18) determine the charge transfer statistics at zero temperature for an arbitrary periodic voltage applied. The generating function consists of a binomial part (S_2) which originates from the dc offset $\bar{V}$, and a contribution of the ac voltage component (S_1) [Fig. 1(d)]. The latter is the sum of two terms which depend on the number of unidirectional attempts per period $e\bar{V}/\omega$. The simplest statistics is obtained for an integer number of attempts for which S_{1L} vanishes [2]. For optimal Lorentzian pulses of width τ_L given by $V_L(t) = (2\tau_L/e)\sum_k[(t-k\tau)^2+\tau_L^2]^{-1}$ we find that $S_1 = 0$ and the statistics is *exactly* binomial with one electron-hole excitation per period, in agreement with Refs. [9, 10].

Higher-order cumulants of current fluctuations at finite temperatures can be obtained from Eqs. (6), (7) and (13), by expanding the generating function given by Eq. (9) in the counting field. For the current noise power $P_I = -(2e^2/t_0)\partial_\chi^2 S|_{\chi=0}$ and the third cumulant $C_I = -i(e^3/t_0)\partial_\chi^3 S|_{\chi=0}$ we find

$$P_I = \frac{2e^2}{\pi}\Bigg[T^2\, 2T_e + T(1-T) \times \sum_{n=-\infty}^{\infty} |\tilde{f}_n|^2 (e\bar{V}+n\omega)\coth\left(\frac{e\bar{V}+n\omega}{2T_e}\right)\Bigg], \quad (19)$$

and

$$C_I = \frac{e^3}{\pi}\Bigg\{e\bar{V}T(1-T^2) + 3T^2(1-T) \times \sum_{n=-\infty}^{\infty} |\tilde{f}_n|^2 \Bigg[2T_e\coth\left(\frac{e\bar{V}+n\omega}{2T_e}\right) - (e\bar{V}+n\omega)\coth^2\left(\frac{e\bar{V}+n\omega}{2T_e}\right)\Bigg]\Bigg\}. \quad (20)$$

Equation (19) describes photon-assisted noise (the non-stationary Aharonov-Bohm effect) [4, 6, 7] for arbitrary periodic driving. Lorentzian pulses $V_L(t)$ (with $\tau_L > 0$) are characterized by $\tilde{f}_{-1} = -e^{-2\pi\tau_L/\tau}$, $\tilde{f}_n = e^{-2\pi n\tau_L/\tau} - e^{-2\pi(n+2)\tau_L/\tau}$ for $n \geq 0$, and $\tilde{f}_n = 0$ otherwise. In this case Eq. (19) accounts for the crossover from the dc noise in the minimal excitation state with $e\bar{V}/\omega = 1$ to the thermal noise as the temperature increases. For general driving, the effect of the ac component in the odd-order cumulants exists at finite temperatures and vanishes in the zero-temperature limit [cf. Eqs. (1) and (20)].

In conclusion, we have studied the statistics of the charge transfer in a quantum point contact with an applied time-dependent voltage using the extended Keldysh-Green's function technique. We have obtained an analytical result for the cumulant generating function at zero temperature as a function of the counting field and the parameters of the driving. The generating function consists of a binomial part which is given by the dc voltage offset and a contribution of the ac voltage component. The dc (ac) part can be interpreted in terms of electrons and holes which move in opposite (the same) directions. Whereas the dc component of the generating function accounts for the unidirectional net charge transfer, the ac component has no net effect on the odd-order cumulants at zero temperature. However, it depends on the number of attempts per period for quasiparticles to transverse the junction and assumes the simplest form for an integer number of attempts. This results in photon-assisted effects in even-order cumulants as a function of a dc offset. The approach we have used also allows for the systematic calculation of higher-order cumulants at finite temperatures by expansion of the generating function in the counting field. As an example we have found the current noise power and the third cumulant of current fluctuations at finite temperatures and for arbitrary periodic driving.

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