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Martin Gubisch

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Well-posedness and asymptotic behaviour for linear magneto-thermo-elasticity with second sound

Martin Gubisch
Universität Konstanz, FB Mathematik und Statistik
Universitätsstraße 10, 78457 Konstanz, Germany

Abstract. We consider the Cauchy problem of magneto-thermo-elasticity with second sound in $\mathbb{R}^3$. After proving the existence of a unique solution, we use Fourier transform and multiplier methods to show polynomial decay rates for suitable initial data. We compare the qualitative and quantitative asymptotic behaviour of magneto-thermo-elasticity with second sound with that of the classical system.

1. Introduction. We study the reciprocal effects between temperature, elasticity and magnetism in some thermoconductive, elastic and magnetic 3D-medium.

Let $\theta = \theta(t,x) \in \mathbb{R}$ denote the temperature difference to some fixed reference temperature, $u = u(t,x) \in \mathbb{R}^3$ the displacement vector with respect to some reference configuration, and $H = H(H_0, h)$ the (total) magnetic field, consisting of the primary field $H_0 \in \mathbb{R}^3$ and the induced field $h = h(t,x) \in \mathbb{R}^3$.

The classical coupled system then reads as

$$\left\{\begin{array}{rcccccl} M(x)u_{tt} - Eu & + & \gamma\nabla\theta & - & \alpha(\nabla\times h)\times H_0 & = & 0 \\ \mu_0(x)h_t - \nabla^T\Lambda(x)\nabla h & & & - & \beta\nabla\times(u_t\times H_0) & = & 0 \\ c(x)\theta_t - \nabla^T K(x)\nabla\theta & + & \gamma\nabla^T u_t & & & = & 0. \end{array}\right. \tag{1.1}$$

The matrices $M(x)$ (mass density), $K(x)$ (thermal conductivity) and $\Lambda(x)$ (magnetic coefficients) are assumed to be symmetric and positive definite uniformly in the variable x, $c(x) \geq c > 0$ denotes the specific heat capacity, $\mu_0(x) \geq \mu_0 > 0$ the magnetic permeability and E the elasticity operator:

$$Eu := \frac{1}{2}\sum_{j,k,l=1}^{3} \frac{\partial}{\partial x_j} s_{\cdot jkl}(x)\left(\frac{\partial}{\partial x_k}u_l + \frac{\partial}{\partial x_l}u_k\right) = \mathcal{D}^T S\mathcal{D}u,$$

depending on the matrix $S(x) = (s_{ijkl}(x))_{ijkl}$ of the elastic coefficients which shall fullfill the symmetries $s_{ijkl} = s_{jikl} = s_{klij}$ and be positive definite uniformly in x, too, and on the natural gradient $\mathcal{D}$ corresponding to E which is defined in (2.1)

The classical (parabolic) model for the temperature,

$$c(x)\theta_t(t,x) - \nabla^T K(x)\nabla\theta(t,x) = 0, \tag{1.2}$$

implies the paradox of infinite propagation speed. Therefore, we split this heat equation in the transport equation for the heat flux $q = q(t,x) \in \mathbb{R}^3$ and Fourier's law of heat conduction and introduce a small delay parameter $\tau > 0$:

$$\left\{\begin{array}{rcl} c(x)\theta_t(t,x) + \nabla^T q(t,x) & = & 0 \\ q(t+\tau,x) + K(x)\nabla\theta(t,x) & = & 0. \end{array}\right. \tag{1.3}$$

Taylor expansion of the delay term $q(t+\tau)$ up to first order yields

$$\left\{\begin{array}{rcl} c(x)\theta_t + \nabla^T q & = & 0 \\ \tau q_t + q + K(x)\nabla\theta & = & 0. \end{array}\right. \tag{1.4}$$

Hereby, Fourier's law is replaced by the so-called Cattaneo law of heat conduction which respects the finite propagation speed of the heat flux; we say that the second sound effect occurs in the model. Notice that any θ which solves this system with delay also is a solution of the damped wave equation

$$c(x)\tau\theta_{tt} + c(x)\theta_t - \nabla^T K(x)\nabla\theta \;=\; 0, \tag{1.5}$$

i.e. the system (1.4) is of hyperbolic type. Our coupled system with relaxation now reads as

$$\begin{cases} M(x)u_{tt} - Eu \;+\; \gamma\nabla\theta \;-\; \alpha(\nabla\times h)\times H_0 \;=\; 0 \\ \mu_0(x)h_t - \nabla^T\Lambda(x)\nabla h \;-\; \beta\nabla\times(u_t\times H_0) \;=\; 0 \\ c(x)\theta_t + \nabla^T q \;+\; \gamma\nabla^T u_t \;=\; 0 \\ \tau q_t + q + K(x)\nabla\theta \;=\; 0. \end{cases} \tag{1.6}$$

Additionally, we postulate initial conditions

$$(u, u_t, h, \theta, q)(0) \;=\; (u_0, u_1, h_0, \theta_0, q_0) \tag{1.7}$$

with some regularity which is determined in section 2.

A detailed derivation of the system and an overview of the results for classical magneto-elasticity and magneto-thermo-elasticity in the sixties was made by Paria [18]. The polynomial stablity of the magneto-elastic model, neglecting the thermal effects in (1.1) by choosing $\gamma = 0$ and $\theta \equiv 0$, has been analyzed by Andreou & Dassios [01] using spectral analysis and techniques provided from thermo-elasticity. The equations of pure thermo-elasticity, i.e. neglecting the influence of the magnetic field in (1.1) resp. (1.6), choosing $\alpha = 0$, $\beta = 0$ and $h \equiv 0$, have recently been discussed in details by Jiang & Racke [11], [22], [23], Irmscher [09], [10], Weinmann [26] and others, using different space dimensions and both bounded domains in $\mathbb{R}^d$ and the full space ($d = 1, 2, 3$).

Under the additional assumption that the rotation of the initial data $V_0 = (S\mathcal{D}u_0, u_1, \theta_0)$ is constantly zero and the condition

$$\int_{\mathbb{R}^3} \frac{1}{C_3(\xi)^m} |\mathcal{F}V(0,\xi)| \,\mathrm{d}\xi < \infty, \qquad C_3(\xi) := \frac{|\xi|^2}{1+|\xi|^2} \tag{1.8}$$

holds, the solution $V = (S\mathcal{D}u, u_t, \theta)$ to the classical system of thermo-elasticity in $\mathbb{R}^3$ decays polynomially with rate m in the Lebesgue space $\mathcal{L}^2$ (a proof using methods of the Fourier analysis can be found in [24], for example). For the system with delay, qualitatively equal results are shown. However, this result does not hold for all thermo-elastic models: Fernández & Racke [06] showed that solutions to classical damped Timoshenko systems are exponentially stable and that the second sound effect destroys this stability. Since Muñoz Rivera and Racke [16] proved polynomial stability for the classical equations of magneto-thermo-elasticity if a condition corresponding to (1.8) holds, it is therefore natural to investigate if this behaviour inherits on the equations with delay parameter.

The paper is structured as follows: In the first section, the well-posedness of the Cauchy problem is shown by using standard methods of semigroup theory. In the second section, a corrected version of the decay rates for the classical system is presented which can be received by slightly modifying the original proof of Muñoz Rivera and Racke given in [16]. On this basis, decay rates for the second sound system are derived afterwards. The third section observes the asymptotic behaviour of solutions for vanishing delay parameter τ_0 and the short-time behaviour of solutions. In the forth section, usual simplifying assumptions on the magnetic field are dropped; in this case, the coupling between elastic and magnetic effects becomes stronger. The techniques used in section three are modified and decay rates are given under additional assumptions on the initial data. Finally, in the last section, those initial conditions which refer to the Fourier-transformed data are retranslated into regularity assumptions on the original data.

2. Well-posedness. We apply methods presented in [11] for the well-posedness of the initial value problem in thermo-elasticity. Reduction of (1.6) to a first order systems yields

$$\begin{cases} V_t + AV &= 0 \\ V(0) &= V_0 \end{cases} \tag{2.1}$$

where

$$V := \begin{pmatrix} S\mathcal{D}u \\ u_t \\ h \\ \theta \\ q \end{pmatrix}, \qquad V_0 := \begin{pmatrix} S\mathcal{D}u_0 \\ u_1 \\ h_0 \\ \theta_0 \\ q_0 \end{pmatrix}, \qquad \Gamma := \begin{pmatrix} \gamma \\ \gamma \\ \gamma \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \mathcal{D} := \begin{pmatrix} \partial_1 & 0 & 0 \\ 0 & \partial_2 & 0 \\ 0 & 0 & \partial_3 \\ 0 & \partial_3 & \partial_2 \\ \partial_3 & 0 & \partial_1 \\ \partial_2 & \partial_1 & 0 \end{pmatrix}$$

and the differential operator in x is given as

$$A := \underbrace{\begin{pmatrix} S & 0 & 0 & 0 & 0 \\ 0 & M^{-1} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\mu_0}\frac{\beta}{\alpha}\mathrm{Id} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{c} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\tau}K \end{pmatrix}}_{=:Q} \begin{pmatrix} 0 & -\mathcal{D} & 0 & 0 & 0 \\ -\mathcal{D}^T & 0 & -\alpha(\nabla\times\cdot)\times H_0 & \mathcal{D}^T\Gamma & 0 \\ 0 & -\alpha\nabla\times(\cdot\times H_0) & -\frac{\alpha}{\beta}\nabla^T\Lambda\nabla & 0 & 0 \\ 0 & \Gamma^T\mathcal{D} & 0 & 0 & \nabla^T \\ 0 & 0 & 0 & \nabla & K^{-1} \end{pmatrix}.$$

Notice that the canonical domain of A,

$$\mathcal{D}(A) = \{V \in \mathcal{H} \mid \exists W \in \mathcal{H} : \forall \Phi \in \mathcal{C}_0^\infty : \langle V, A^T\Phi\rangle_{\mathcal{L}^2} = -\langle W, \Phi\rangle\}_{\mathcal{L}^2}$$

is dense in the Hilbert space $\mathcal{H} := \mathcal{L}^2$ and $A : \mathcal{D}(A) \to \mathcal{H}$ defines a closed operator. We provide $\mathcal{H}$ with the modified $\mathcal{L}^2$-scalar product

$$\langle\cdot,\cdot\rangle_{\mathcal{H}} := \langle\cdot, Q^{-1}\cdot\rangle_{\mathcal{L}^2}$$

which is equivalent to $\langle\cdot,\cdot\rangle_{\mathcal{L}^2}$, then A is dissipative with respect to $\langle\cdot,\cdot\rangle_{\mathcal{H}}$:

Let $V \in \mathcal{D}(A)$. Since Λ and K are positive definite, we get

$$\begin{aligned} \mathrm{Re}\langle AV, V\rangle_{\mathcal{H}} &= \frac{\alpha}{\beta}\mathrm{Re}\langle\Lambda\nabla V^3, \nabla V^3\rangle_{\mathcal{L}^2} + \mathrm{Re}\langle K^{-1}V^5, V^5\rangle_{\mathcal{L}^2} \\ &\geq \frac{\alpha}{\beta}\lambda_0||\nabla V^3||^2_{\mathcal{L}^2} + k_0||K^{-1}V^5||^2_{\mathcal{L}^2} \quad \geq \quad 0. \end{aligned} \tag{2.2}$$

Therefore, for any $\lambda \in (-\infty, 0)$ and $V \in \mathcal{D}(A)$ the following inequality holds:

$$\begin{aligned} ||(A-\lambda)V||_{\mathcal{H}}||V||_{\mathcal{H}} &\geq \mathrm{Re}\langle (A-\lambda)V, V\rangle_{\mathcal{H}} \\ &\geq \lambda_0 \frac{\alpha}{\beta}||\nabla V^3||^2_{\mathcal{L}^2} + k_0||K^{-1}V^5||^2_{\mathcal{L}^2} - \lambda||V||^2_{\mathcal{H}} \end{aligned}$$

and we get $||(A-\lambda)V||_{\mathcal{H}} \geq -\lambda||V||_{\mathcal{H}}$ for any $V \neq 0$. Especially, $A-\lambda$ is injective and $(A-\lambda)^{-1} : \mathrm{im}(A-\lambda) \to \mathcal{H}$ is continous.

Finally, since the adjoint operator A^* of A is defined on $\mathcal{D}(A)$ and has the form

$$(A^*)_{ij} = (-1)^{1+\delta_{ij}} A_{ij} \qquad (i,j = 1,...,5)$$

where δ_{ij} is the Kronecker Delta, $A^* - \lambda$ is injective, too, and the partition

$$\mathcal{H} = \overline{\mathrm{im}(A-\lambda)} \oplus \ker(A^*-\lambda) = \overline{\mathrm{im}(A-\lambda)} \oplus \{0\}$$

implies that the image of $A-\lambda$ is dense in $\mathcal{H}$. Therefore, any $\lambda < 0$ is an element of the resolvent set $\varrho(A)$ and the corresponding resolvent fullfills the estimate

$$||(A-\lambda)^{-1}|| \leq -\frac{1}{\lambda}.$$

The Hille & Yosida Theorem implies that A is the generator of the $\mathcal{C}_0$-semigroup $(e^{tA})_{t\geq 0}$, i.e. the Cauchy Problem (2.1) is well-posed:

Theorem 2.1 (existence and uniqueness) *For any initial value $V_0 \in \mathcal{H}$, the initial value problem (2.1) has a unique solution $V \in \mathcal{C}^0([0,\infty), \mathcal{H})$ which depends continously on the data.*

Additionally, if $V_0 \in \mathcal{D}(A)$, we get $V \in \mathcal{C}^0([0,\infty), \mathcal{D}(A)) \cap \mathcal{C}^1([0,\infty), \mathcal{H})$.

3. Asymptotic behaviour. From now on, we assume that our medium is homogenous and isotropic, i.e. we take $M = \mathrm{Id}$, $K = \varkappa\mathrm{Id}$, $\Lambda = \mathrm{Id}$, $c = 1$, $\mu_0 = 1$ and

$$E = \mu\Delta + (\mu+\lambda)\nabla\nabla^T$$

for some constants $\mu, \lambda > 0$ which are called the "Lamé moduli". Furthermore, we take $H_0 := (0,0,H)^T$, $H > 0$ (which is not assumed without loss of generality as we will see in section 5).

Let $\mathcal{F} = \widehat{(\cdot)}$ denote the Fourier transform on $\mathcal{L}^2$ and $v := \hat{u}$, $w := \hat{h}, \vartheta := \hat{\theta}$, $r := \hat{q}$, then the transformed system (1.6) reads as

$$\left\{ \begin{aligned} v_{tt} + \mu|\xi|^2 v + (\mu+\lambda)(\xi\cdot v)\xi - i\gamma\vartheta\xi - i\alpha H w^3\xi + i\alpha H\xi_3 w &= 0 \\ w_t + |\xi|^2 w + i\beta H\xi_3 v_t - i\beta H(0,0,\xi\cdot v_t)^T &= 0 \\ \vartheta_t - i(\xi\cdot r) - i\gamma(\xi\cdot v_t) &= 0 \\ \tau r_t + r - i\varkappa\vartheta\xi &= 0. \end{aligned} \right. \tag{3.1}$$

According to the Plancherel Theorem, the energy associated to (3.1),

$$\mathcal{E}^\tau(t) := \frac{1}{2}\int_{\mathbb{R}^3} \left(|v_t|^2 + \mu|\xi|^2|v|^2 + (\mu+\lambda)(\xi\cdot v)^2 + \frac{\alpha}{\beta}|w|^2 + |\vartheta|^2 + \frac{\tau}{\varkappa}|r|^2\right)(t,\xi)\ \mathrm{d}\xi,$$

is equal to the energy of the original system (1.6):

$$\mathcal{E}^\tau(t) = \frac{1}{2}\int\limits_{\mathbb{R}^3} \left(|u_t|^2 + \mu|\nabla u|^2 + (\mu+\lambda)|\nabla^T u|^2 + \frac{\alpha}{\beta}|h|^2 + |\theta|^2 + \frac{\tau}{\varkappa}|q|^2 \right)(t,x)\ \mathrm{d}x.$$

We show that the integrand

$$\hat{\mathcal{E}}^\tau(t,\xi) := \frac{1}{2}\left(|v_t|^2 + \mu|\xi|^2|v|^2 + (\mu+\lambda)(\xi\cdot v)^2 + \frac{\alpha}{\beta}|w|^2 + |\vartheta|^2 + \frac{\tau}{\varkappa}|r|^2 \right)(t,\xi)$$

decays exponentially in t for any fixed $\xi \in \mathbb{R}^3$. For suitable initial data, this yields to polynomial decay rates for $\mathcal{E}^\tau$ where the decay rate depends on the data. To construct a Lyapunov functional $\mathcal{L}^\tau = \mathcal{L}^\tau(t,\xi)$ which is equivalent to $\hat{\mathcal{E}}^\tau$ and decays exponentially in t, we modify the methods used in [16] for the classical system (1.1).

In the following, we present a corrected version of the proof for polynomial decay rates in classical magneto-thermo-elasticity. Let (v, w, ϑ) the Fourier-transformed solution to (1.1). With multiplier methods, the following estimate can be shown, replacing Lemma 2.1, 2.2 & 2.3 in [16]:

Define

$$\begin{aligned} \Phi_1(t) &:= \mathrm{Re}\left(\frac{i}{\xi_3}\left(v_t^1\overline{w^1} + v_t^2\overline{w^2}\right)\right)(t); & \Phi_4(t) &:= \mathrm{Re}\left(v_{tt}^2\overline{v_t^3}\right)(t);\\ \Phi_2(t) &:= \mathrm{Re}\left(\frac{i}{\xi_3}\left(v_{tt}^1\overline{w_t^1} + v_{tt}^2\overline{w_t^2}\right)\right)(t); & \Phi_5(t) &:= \mathrm{Re}\left(v_t\overline{v}\right)(t);\\ \Phi_3(t) &:= \mathrm{Re}\left(v_{tt}^1\overline{v_t^3}\right)(t); & \Phi_6(t) &:= \mathrm{Re}\left(v_t^1\overline{v^1} + v_t^2\overline{v^2}\right)(t) \end{aligned}$$

and let $a := |\xi|^2\left(\frac{1}{\xi_1^2} + \frac{1}{\xi_3^2}\right)$, then the function

$$\begin{aligned} \Phi(t) \;:=\;& (1+a)|\xi|^2\Phi_1(t) + a\Phi_2(t) + \delta\left(\frac{\xi_1}{\xi_3} + \frac{\xi_3}{\xi_1}\right)\Phi_3(t)\\ &+\delta\frac{\xi_2}{\xi_3}\Phi_4(t) + \frac{\delta}{2}|\xi|^2\Phi_5(t) + \frac{2\mu}{\mu+\lambda}\frac{\delta}{8}\frac{|\xi|^4}{\xi_3^2}\Phi_6(t) \end{aligned}$$

fullfills the inequality

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\Phi(t) \;\le\;& \frac{c}{\delta}a^2|\xi|^2(1+|\xi|^2)|w|^2 + \frac{c}{\delta}a^2(1+|\xi|^2)|w_t|^2 + ca|\xi|^2|\vartheta|^2\\ &+c\delta a|\vartheta_t|^2 - \frac{\beta H}{4}a(|v_{tt}^1|^2 + |v_{tt}^2|^2) - \frac{\delta}{16}|\xi|^2\hat{\mathcal{E}}^0(t) \end{aligned} \tag{3.2}$$

for some $c > 0$ which depends only on the coefficients of the differential equation and some $\delta = \delta(c) > 0$.

Since the dissipation of the classical system reads as

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathcal{E}}^0(t) \;=\; -\frac{\alpha}{\beta}|\xi|^2|w|^2 - \varkappa|\xi|^2|\vartheta|^2, \tag{3.3}$$

it is easy to see (compare Theorem 3.3 in this work) that for sufficiently large constants $N_1, N_2 > 0$,

$$\mathcal{L}^0(t) := \Phi(t) + N_1 a^2(1+|\xi|^2)\hat{\mathcal{E}}_1^0(t) + N_2 a^2 \frac{1+|\xi|^2}{|\xi|^2}\hat{\mathcal{E}}_2^0(t) \tag{3.4}$$

is a Lyapunov funcional with the properties described above where the energy terms of first, second and third order (which we need in section 4) are defined as

$$\begin{aligned}\hat{\mathcal{E}}_i^\tau(t,\xi) &:= \frac{1}{2}\Big(|\partial_t^i v|^2 + \mu|\xi|^2|\partial_t^{i-1}v|^2 + (\mu+\lambda)(\xi\cdot\partial_t^{i-1}v)^2 \\ &\quad + \frac{\alpha}{\beta}|\partial_t^{i-1}w|^2 + |\partial_t^{i-1}\vartheta|^2 + \frac{\tau}{\varkappa}|\partial_t^{i-1}r|^2\Big)(t,\xi) \qquad (i=1,2,3;\ \tau\geq 0).\end{aligned}$$

The proof for the main decay result of Muñoz Rivera and Racke in [16] then reads as follows:

THEOREM 3.1 (decay rates for classical magneto-thermo-elasticity) *Let* $\tau_0 = 0$ *and let* $m \in \mathbb{N}_0$ *arbitrary. If the Fourier-transformed initial data* $(\hat{u}_0, \hat{u}_1, \hat{h}, \hat{\theta})$ *of classical magneto-thermo-elasticity fullfill*

$$\int_{\mathbb{R}^3} (1+|\xi|^2)\left(\frac{\xi_1^4\xi_3^4}{(\xi_1^2+\xi_3^2)|\xi|^2(1+|\xi|^2)^2}\right)^{-m}\hat{\mathcal{E}}^0(0,\xi)\,\mathrm{d}\xi < \infty$$

where

$$\hat{\mathcal{E}}^0(0,\xi) := \frac{1}{2}\left(|\hat{u}_1|^2 + \mu|\xi|^2|\hat{u}_0|^2 + (\mu+\lambda)(\xi\cdot\hat{u}_0)^2 + \frac{\alpha}{\beta}|\hat{h}_0|^2 + \frac{\gamma}{\delta}|\hat{\theta}_0|^2\right)(\xi),$$

then the energy of the solution (u,h,θ),

$$\mathcal{E}^0(t) = \frac{1}{2}\int_{\mathbb{R}^3}\left(|u_t|^2 + \mu|\nabla u|^2 + (\mu+\lambda)|\mathrm{div}\, u|^2 + \frac{\alpha}{\beta}|h|^2 + \frac{\gamma}{\delta}|\theta|^2\right)(t,x)\,\mathrm{d}x,$$

decays polynomially with rate m: $\mathcal{E}^0(t) = \mathcal{O}(t^{-m})$.

Now let (v,w,ϑ,r) the Fourier-transformed solution of the second sound system. To transfer the result above, we just have to estimate the terms $|\vartheta|^2$ and $|\vartheta_t|^2$ in (3.2) towards $|r|^2$, $|r_t|^2$, $|w|^2$ and $|w_t|^2$ since the dissipation of the system with second sound is, according to (2.2),

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathcal{E}}^\tau(t) = \frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}||V(t)||_{\mathcal{H}}^2 = -\mathrm{Re}\langle AV,V\rangle_{\mathcal{H}} = -\frac{\alpha}{\beta}|\xi|^2|w|^2 - \frac{1}{\varkappa}|r|^2. \tag{3.5}$$

Notice that (3.2) also holds for the second sound system since only the first two differential equations of (1.1) or (1.6), respectively, are needed for the proof. Therefore, we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t) &\leq \frac{c}{\delta}a^2|\xi|^2(1+|\xi|^2)|w|^2 + \frac{c}{\delta}a^2(1+|\xi|^2)|w_t|^2 + ca|\xi|^2|\vartheta|^2 + c\delta a|\vartheta_t|^2 \\ &\quad - \frac{\beta H}{4}a(|v_{tt}^1|^2 + |v_{tt}^2|^2) + \frac{\delta}{32}\frac{\tau}{\varkappa}|\xi|^2|r|^2 - \frac{\delta}{16}|\xi|^2\hat{\mathcal{E}}^\tau(t). \end{aligned} \tag{3.6}$$

Using the last differential equation of (3.1), we can estimate $|\vartheta|^2$ directly:

$$|\vartheta|^2 \leq \frac{c}{|\xi|^2}(|r_t|^2 + |r|^2). \tag{3.7}$$

Lemma 3.2 *For any $0 < \epsilon < 1$ the functional*

$$\Phi_7(t) \quad := \quad a\mathrm{Re}\left(\frac{i}{\xi_3} r_t^3 \overline{\vartheta_t}\right)(t)$$

fullfills the estimate

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\Phi_7 \quad &\le \quad -\frac{\varkappa}{2\tau}a|\vartheta_t|^2 + \frac{c}{\epsilon}a^3\left(1+\frac{1}{|\xi|^2}\right)|r_t|^2 \\ &\quad + \frac{\beta H}{8}a(|v_{tt}^1|^2 + |v_{tt}^2|^2) + \frac{\epsilon}{2}\left(\mu|\xi|^4|v|^2 + |\xi|^2|\vartheta|^2\right).\end{aligned}$$

Proof. Multiplication of the fourth resp. third equation of (3.1) with ϑ_t resp. r_t^3 yields

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\Phi_7 \quad &= \quad a\mathrm{Re}\left(\frac{i}{\xi_3}\left(r_{tt}^3\overline{\vartheta_t} + r_t^3\overline{\vartheta_{tt}}\right)\right) \\ &= \quad -\frac{1}{\tau}\frac{1}{\xi_3}a\mathrm{Re}\left(i r_t^3 \overline{\vartheta_t}\right) - \frac{\varkappa}{\tau}a\mathrm{Re}\left(\vartheta_t\overline{\vartheta_t}\right) \\ &\quad + \frac{1}{\xi_3}a\mathrm{Re}\left(r_t^3(\xi\overline{r_t})\right) + \gamma\frac{1}{\xi_3}a\mathrm{Re}\left(r_t^3(\xi\overline{v_{tt}})\right) \\ &\le \quad \frac{\epsilon_1}{2}a|\vartheta_t|^2 + \frac{c}{2\epsilon_1}a\frac{|\xi|^2}{\xi_3^2}\frac{|r_t|^2}{|\xi|^2} - \frac{\varkappa}{\tau}a|\vartheta_t|^2 + ca\frac{|\xi|}{|\xi_3|}|r_t|^2 \\ &\quad + \frac{\epsilon_2}{2}a(|v_{tt}^1|^2 + |v_{tt}^2|^2) + \frac{c}{2\epsilon_2}a\frac{|\xi|^2}{\xi_3^2}|r_t|^2 + \frac{\epsilon_3}{2}|v_{tt}^3|^2 + \frac{c}{2\epsilon_3}a^2\frac{|\xi|^2}{\xi_3^2}|r_t|^2 \\ &\le \quad -\frac{\varkappa}{2\tau}a|\vartheta_t|^2 + \frac{c}{\epsilon}a^3\left(1+\frac{1}{|\xi|^2}\right)|r_t|^2 + \frac{\beta H}{8}(|v_{tt}^1|^2 + |v_{tt}^2|^2) \\ &\quad + \frac{\epsilon}{2}\left(\mu|\xi|^4|v|^2 + |\xi|^2|\vartheta|^2\right)\end{aligned}$$

with $\epsilon_1 = \dfrac{\varkappa}{\tau}$, $\epsilon_2 = \dfrac{\beta H}{4}$ and some $\epsilon \in (0,1)$ which will be determined in the next proof. □

Theorem 3.3 *We define the functional*

$$\mathcal{L}^\tau(t) \quad := \quad \Phi(t) + \Phi_7(t) + N_1 a^2(1+|\xi|^2)\hat{\mathcal{E}}_1^\tau(t) + N_2 a^3\left(1+\frac{1}{|\xi|^2}\right)\hat{\mathcal{E}}_2^\tau(t). \tag{3.8}$$

If $N_1, N_2 > 0$ are large enough, then $\mathcal{L}^\tau$ is equivalent to $\hat{\mathcal{E}}^\tau$ and decays exponentially, i.e. for any $\xi \in \mathbb{R}^3$ there are constants $C_1(\xi), C_2(\xi), C_3(\xi) > 0$ such that for any $t \ge 0$ the following holds:

$$C_1(\xi)\hat{\mathcal{E}}^\tau(t,\xi) \le \mathcal{L}^\tau(t,\xi) \le C_2(\xi)\hat{\mathcal{E}}^\tau(t,\xi), \qquad \mathcal{L}(t,\xi) \le \mathcal{L}(0,\xi)e^{-C_3(\xi)t}.$$

Proof. Putting (3.6), (3.7) and Lemma 3.2 together, we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{L}^\tau(t) \quad &\le \quad \left(\frac{\epsilon}{2} - \frac{\delta}{16}\right)|\xi|^2\hat{\mathcal{E}}^\tau(t) + \left(c\delta - \frac{\varkappa}{2\tau}\right)|\vartheta_t|^2 + \beta H a\left(\frac{1}{8} - \frac{1}{4}\right)(|v_{tt}^1|^2 + |v_{tt}^2|^2) \\ &\quad + \left(\frac{c}{\delta} - N_1\frac{\alpha}{\beta}\right)a^2|\xi|^2(1+|\xi|^2)|w|^2 + \left(\frac{c}{\delta} - N_2\frac{\alpha}{\beta}\right)a^3(1+|\xi|^2)|w_t|^2 \\ &\quad + \left(c - \frac{N_1}{\varkappa}\right)a^2(1+|\xi|^2)|r|^2 + \left(\frac{c}{\delta} - \frac{N_2}{\varkappa}\right)a^3\left(1+\frac{1}{|\xi|^2}\right)|r_t|^2.\end{aligned}$$

Choosing ϵ, δ small enough and N_1, N_2 large enough, this implies

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{L}^\tau(t) \;\leq\; -\frac{\delta}{32}|\xi|^2\hat{\mathcal{E}}^\tau(t). \tag{3.9}$$

Using the differential equations (3.1), we estimate

$$|\Phi| \;\leq\; C\left(a(1+|\xi|^2)\hat{\mathcal{E}}_1^\tau + a^2\frac{1+|\xi|^2}{|\xi|^2}\hat{\mathcal{E}}_2^\tau\right); \tag{3.10}$$

$$\hat{\mathcal{E}}_2^\tau \;\leq\; C(1+|\xi|^2)^2\hat{\mathcal{E}}_1 \tag{3.11}$$

which yields

$$\begin{aligned}
\mathcal{L}^\tau &\leq |\Phi| + N_1a^2(1+|\xi|^2)\hat{\mathcal{E}}_1^\tau + N_2a^3\left(1+\frac{1}{|\xi|^2}\right)\hat{\mathcal{E}}_2^\tau \\
&\leq \left((C+N_1)a^2(1+|\xi|^2) + (C+N_2)a^3\left(1+\frac{1}{|\xi|^2}\right)C(1+|\xi|^2)^2\right)\hat{\mathcal{E}}_1^\tau \\
&\leq d_1a^3\left(|\xi|^4+\frac{1}{|\xi|^2}\right)\hat{\mathcal{E}}_1^\tau, \\
\mathcal{L}^\tau &\geq -|\Phi| + N_1a^2(1+|\xi|^2)\hat{\mathcal{E}}_1^\tau + N_2a^3\left(1+\frac{1}{|\xi|^2}\right)\hat{\mathcal{E}}_2^\tau \\
&\geq (N_1-C)a^2(1+|\xi|^2)\hat{\mathcal{E}}_1^\tau + (N_2-C)a^3\left(1+\frac{1}{|\xi|^2}\right)\hat{\mathcal{E}}_2^\tau \\
&\geq d_2a^2(1+|\xi|^2)\hat{\mathcal{E}}_1^\tau
\end{aligned}$$

with $d_1, d_2 > 0$ if N_1, N_2 are large enough. We choose

$$C_1(\xi) \;:=\; d_2a^2(1+|\xi|^2), \tag{3.12}$$

$$C_2(\xi) \;:=\; d_1a^3\left(|\xi|^4+\frac{1}{|\xi|^2}\right) \tag{3.13}$$

and it remains to show the existence of $C_3(\xi)$. (3.9) implies

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{L}^\tau(t) \;\leq\; -\frac{\delta}{32}\frac{|\xi|^4}{d_1a^3(1+|\xi|^6)}\mathcal{L}^\tau(t) \;=:\; -C_3(\xi)\mathcal{L}^\tau(t) \tag{3.14}$$

and, usings the Lemma of Gronwall, we get the exponential decay of $\mathcal{L}^\tau$:

$$\mathcal{L}^\tau(t) \;\leq\; \mathcal{L}^\tau(0)e^{-C_3(\xi)t}.$$ □

To deduce the polynomial decay of $\mathcal{E}^\tau(t)$ from the exponential decay of $\hat{\mathcal{E}}^\tau(t,\xi)$ pointwise in ξ, we need the following lemma:

LEMMA 3.4 *For all $m \in \mathbb{N}_0$ there exists some $c(m) > 0$ such that, for any $t \geq 0$, the following holds:*

$$\int_0^t s^m e^{-C_3(\xi)s}\,\mathrm{d}s \;\leq\; \frac{c(m)}{C_3^{m+1}(\xi)}$$

Proof. Via induction. Let $m = 0$, then

$$\int_0^t e^{-C_3(\xi)s}\,\mathrm{d}s \quad = \quad \frac{1}{C_3(\xi)}(1 - e^{-C_3(\xi)t}) \quad \leq \quad \left(\frac{1}{C_3(\xi)}\right)^1.$$

Now assume that the formula holds for m, then we receive by partial integration:

$$\begin{aligned}\int_0^t s^{m+1}e^{-C_3(\xi)s}\,\mathrm{d}s &= -\frac{1}{C_3(\xi)}\left(t^{m+1}e^{-C_3(\xi)t}\right) + \frac{m+1}{C_3(\xi)}\int_0^t s^m e^{-C_3(\xi)s}\,\mathrm{d}s \\ &\leq c(m)(m+1)\left(\frac{1}{C_3(\xi)}\right)^{m+2}.\end{aligned}$$

□

Theorem 3.5 (decay rates for magneto-thermo-elasticity with second sound) *For any $m \in \mathbb{N}_0$, let the initial energy*

$$\hat{\mathcal{E}}^\tau(0,\xi) := \frac{1}{2}\left(|v_1|^2 + \mu|\xi|^2|v_0|^2 + (\mu+\lambda)(\xi\cdot v_0)^2 + \frac{\alpha}{\beta}|w_0|^2 + |\vartheta_0|^2 + \frac{\tau}{\varkappa}|r_0|^2\right)(t,\xi)$$

fullfill

$$\int_{\mathbb{R}^3} \frac{C_2(\xi)}{C_1(\xi)C_3^m(\xi)}\hat{\mathcal{E}}_1^\tau(0)\,\mathrm{d}\xi < \infty. \tag{3.15}$$

Then the energy associated to (u,h,θ,q) decays polynomially with order m: $\mathcal{E}^\tau(t) = \mathcal{O}(t^{-m})$.

Proof. Let $m \geq 1$. Using the dissipation (3.5) and Lemma 3.3 & 3.4, we get

$$\begin{aligned} t^m\mathcal{E}^\tau(t) &= \int_{\mathbb{R}^3}\int_0^t \frac{\mathrm{d}}{\mathrm{d}s}s^m\hat{\mathcal{E}}_1^\tau(s,\xi)\,\mathrm{d}s\,\mathrm{d}\xi \\ &\leq m\int_{\mathbb{R}^3}\int_0^t s^{m-1}\hat{\mathcal{E}}_1^\tau(s,\xi)\,\mathrm{d}s\,\mathrm{d}\xi \\ &\leq m\int_{\mathbb{R}^3}\frac{C_2(\xi)}{C_1(\xi)}\hat{\mathcal{E}}_1^\tau(0,\xi)\int_0^t s^{m-1}e^{-C_3(\xi)s}\,\mathrm{d}s\,\mathrm{d}\xi \\ &\leq m\cdot c(m-1)\int_{\mathbb{R}^3}\frac{C_2(\xi)}{C_1(\xi)C_3(\xi)^m}\hat{\mathcal{E}}_1^\tau(0,\xi)\,\mathrm{d}\xi \\ &< \infty. \end{aligned}$$

□

4. The behaviour of the energy for vanishing delay parameter $\tau \to 0$. Let $(u^0, h^0, \theta^0, q^0)$ resp. $(u^{\tau_0}, h^{\tau_0}, \theta^{\tau_0}, q^{\tau_0})$ the solution to (1.6) for $\tau = 0$ resp. $\tau = \tau_0$. Our objective is to estimate the difference of the corresponding energies against a function depending on τ_0.

Let $(u^d, h^d, \theta^d, q^d) := (u^{\tau_0} - u^0, h^{\tau_0} - h^0, \theta^{\tau_0} - \theta^0, q^{\tau_0} - q^0)$ the difference of the solutions. If the compability condition $q_0^0 = -\varkappa\nabla\theta_0^0$ holds for the classical system, then $(u^d, h^d, \theta^d, q^d)$ solves

$$\left\{\begin{array}{rcccccl} u_{tt}^d - \mu\Delta u^d - (\mu+\lambda)\nabla\nabla^T u^d & + & \gamma\nabla\theta^d & - & \alpha(\nabla\times h^d)\times H_0 & = & 0 \\ h_t^d - \Delta h^d & & & - & \beta\nabla\times(u_t^d\times H_0) & = & 0 \\ \theta_t^d + \nabla^T q^d & + & \gamma\nabla^T u_t^d & & & = & 0 \\ \tau q_t^d + q^d + \varkappa\nabla\theta^d & & & & & = & -\tau_0 q_t^0 \end{array}\right. \tag{4.1}$$

to the initial condition

$$(u^d, u_t^d, h^d, \theta^d, q^d)(0) = (0,0,0,0,0). \tag{4.2}$$

The energy term corresponding to the Fourier transform $(v^d, w^d, \vartheta^d, r^d)$ of $(u^d, h^d, \theta^d, q^d)$ fullfills

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathcal{E}}^d(t,\xi) \quad = \quad -\frac{\alpha}{\beta}|\xi|^2|w^d|^2 - \frac{1}{\varkappa}|r^d|^2 - \tau_0\mathrm{Re}(i\vartheta_t^0(\xi\cdot\overline{r^d})) \quad \le \quad \tau_0^2\frac{\varkappa}{2}|\xi|^2|\vartheta_t^0|^2. \tag{4.3}$$

The dissipation of the classical system yields for the energy term of second order:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathcal{E}}_2^0(t) \quad = \quad -\frac{\alpha}{\beta}|\xi|^2|w_t^0|^2 - \varkappa|\xi|^2|\vartheta_t^0|^2. \tag{4.4}$$

By integration over $[0,\infty)$ and over $\mathbb{R}^3$ we get

$$\mathcal{E}_2^0(0) \ge \mathcal{E}_2^0(0) - \lim_{t\to\infty}\mathcal{E}_2^0(t) = \frac{\alpha}{\beta}\int_0^\infty ||\nabla h_t^0||_{\mathcal{L}^2}^2\ \mathrm{d}s + \varkappa\int_0^\infty ||\nabla\theta_t^0||_{\mathcal{L}^2}^2 \tag{4.5}$$

and with (4.3) and the initial condition (4.2) it follows, again by intergration over $[0,\infty)$, that

$$\mathcal{E}^d(t) = \int_{\mathbb{R}^3}\int_0^\infty \frac{\mathrm{d}}{\mathrm{d}s}\hat{\mathcal{E}}^d(s,\xi)\ \mathrm{d}s\ \mathrm{d}\xi \le \tau_0^2\frac{\varkappa}{2}\int_0^\infty ||\nabla\theta_t^0||_{\mathcal{L}^2}^2\ \mathrm{d}t \le \frac{\tau_0^2}{2}\mathcal{E}_2^0(0). \tag{4.6}$$

Theorem 4.1 (comparison of energies) *Let $\tau_0 \to 0$, then the energy $\mathcal{E}^{\tau_0}$ corresponding to the system with second sound converges quadratically towards the energy $\mathcal{E}^0$ of the classical system.*

Sometimes it is useful to get sharper results for the short-time behaviour of energies: In physical applications like the laser cleaning [14] or the pulsed laser heating [25], very small time periodes are considered. In the following we apply methods introduced in [10]:

Lemma 4.2 (Irmscher, 06) *The second order energy of a dissipative system satisfies*

$$\mathcal{E}_2^0(0) - \mathcal{E}_2^0(t) \le E_2^0(t) := \left\{\begin{array}{ll} 2t\sqrt{\mathcal{E}_2^0(0)\mathcal{E}_3^0(0)} - t^2\mathcal{E}_3^0(t) & \text{if} \quad 0 \le t \le \sqrt{\dfrac{\mathcal{E}_2^0(0)}{\mathcal{E}_3^0(0)}}; \\ \mathcal{E}_2^0(0) & \text{else} \end{array}\right.$$

Inequality (4.6) then reads as

$$\begin{aligned}\mathcal{E}^d(t) &\leq \tau_0^2\frac{\varkappa}{2}\int_0^t\int_{\mathbb{R}^3}|\xi|^2|\vartheta_t^0|^2\,\mathrm{d}\xi\,\mathrm{d}s \quad\leq\quad -\frac{\tau_0^2}{2}\int_{\mathbb{R}^3}\int_0^t\frac{\mathrm{d}}{\mathrm{d}s}\hat{\mathcal{E}}_2^0(s,\xi)\,\mathrm{d}s\,\mathrm{d}\xi\\ &= \frac{\tau_0^2}{2}(\mathcal{E}_2^0(0)-\mathcal{E}_2^0(t)) \quad\leq\quad \frac{\tau_0^2}{2}E_2^0(t).\end{aligned}\tag{4.7}$$

5. A generalized model for the stationary magnetic field H_0. At the beginning of section 3, we mentioned that technical difficulties arise if we do not assume $H_1, H_2 = 0$ for the stationary field $H_0 = (H_1, H_2, H_3)$ – this assumption is made in most works about magneto-elastic and magneto-thermo-elastic models. Notice that the following results can be applied for classical magneto-thermo-elasticity (taking $\tau = 0$) and magneto-elasticity (taking $\gamma = 0$ and $\theta \equiv 0$), too.

Initially, we take H_1, H_2 and $H_3 \geq 0$ arbitrary. Then the first two Fourier-transformed equations of (1.6) in components have the form

$$\begin{aligned}v_{tt}^1+\mu|\xi|^2v^1+(\mu+\lambda)(\xi\cdot v)\xi_1-i\gamma\vartheta\xi_1&\\ -i\alpha H_3(\xi_1w^3-\xi_3w^1)+i\alpha H_2(\xi_2w^1-\xi_1w^2) &= 0;\end{aligned}\tag{5.1}$$

$$\begin{aligned}v_{tt}^2+\mu|\xi|^2v^2+(\mu+\lambda)(\xi\cdot v)\xi_2-i\gamma\vartheta\xi_2&\\ -i\alpha H_1(\xi_2w^1-\xi_1w^2)+i\alpha H_3(\xi_3w^2-\xi_2w^3) &= 0;\end{aligned}\tag{5.2}$$

$$\begin{aligned}v_{tt}^3+\mu|\xi|^2v^3+(\mu+\lambda)(\xi\cdot v)\xi_3-i\gamma\vartheta\xi_3&\\ -i\alpha H_2(\xi_3w^2-\xi_2w^3)+i\alpha H_1(\xi_1w^3-\xi_3w^1) &= 0;\end{aligned}\tag{5.3}$$

$$w_t^1+|\xi|^2w^1-i\beta H_1(\xi_2v_t^2+\xi_3v_t^3)+i\beta H_2\xi_2v_t^1+i\beta H_3\xi_3v_t^1 = 0 \tag{5.4}$$

$$w_t^2+|\xi|^2w^2+i\beta H_1\xi_1v_t^2-i\beta H_2(\xi_1v_t^1+\xi_3v_t^3)+i\beta H_3\xi_3v_t^2 = 0 \tag{5.5}$$

$$w_t^2+|\xi|^2w^2+i\beta H_1\xi_1v_t^3+i\beta H_2\xi_2v_t^3-i\beta H_3(\xi_1v_t^1+\xi_2v_t^2) = 0 \tag{5.6}$$

In the situation of section 3, i.e. if $H_1 = 0$ and $H_2 = 0$, the derivative of $\Phi_1(t)$ can be estimated towards

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\Phi_1(t) &= \mathrm{Re}\left(\frac{i}{\xi_3}\left(v_{tt}^1\overline{w^1}+v_t^1\overline{w_t^1}+v_{tt}^2\overline{w^2}+v_t^2\overline{w_t^2}\right)\right)\\ &\leq -\frac{\beta H}{2}(|v_t^1|^2+|v_t^2|^2)+\frac{c}{\varepsilon}\frac{|\xi|^2}{\xi_3^2}\left(1+|\xi|^2\right)|w|^2+\varepsilon(|\xi|^2|v|^2+|\vartheta|^2)\end{aligned}\tag{5.7}$$

by multiplication of (5.1), (5.2) with $\overline{w^1}, \overline{w^2}$ and $\overline{(5.4)}, \overline{(5.5)}$ with v_t^1, v_t^2 where $\varepsilon > 0$ can be chosen as small as desired, compare [16]. Therefore, we can deal with the following mixed terms:

$$\mathrm{Re}\left(v_t^1\overline{v_t^3}\right)+\mathrm{Re}\left(v_t^2\overline{v_t^3}\right) \leq \frac{1}{2\varepsilon}(|v_t^1|^2+|v_t^2|^2)+\varepsilon|v_t^3|^2 \tag{5.8}$$

arising in the derivatives of $\Phi_2(t)$ and $\Phi_3(t)$.

On the other hand, if we choose $H_2 > 0$, the derivative of the counterpart $\tilde{\Phi}_1(t)$ to $\Phi_1(t)$,

$$\tilde{\Phi}_1(t) \;:=\; -\underbrace{\frac{1}{H_2\xi_2 + H_3\xi_3}}_{=:\zeta} \mathrm{Re}\left(iv_t^1\overline{w^1}\right), \tag{5.9}$$

just creates the negative term $-|v_t^1|^2$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}_1(t) \;\leq\; -\frac{\beta}{2}|v_t^1|^2 + \frac{c}{\varepsilon}(1+|\xi|^4)(1+|\zeta|^2)|w|^2 + \varepsilon(|\xi|^2|v|^2 + |\vartheta|^2) \tag{5.10}$$

which does not suffice to estimate $\mathrm{Re}\left(v_t^2\overline{v_t^3}\right)$.

THEOREM 5.1 *Define the functionals*

$$\tilde{\Phi}_1(t) \;:=\; -\mathrm{Re}\left(i\zeta v_t^1\overline{w^1}\right)(t); \qquad \tilde{\Phi}_4(t) \;:=\; \mathrm{Re}\left(\frac{H_2}{H_3}\frac{v_{tt}^1\overline{v_t^2}}{\xi_1\xi_3} + \frac{H_3}{H_2}\frac{v_{tt}^1\overline{v_t^2}}{\xi_1\xi_2}\right)(t);$$

$$\tilde{\Phi}_2(t) \;:=\; -\mathrm{Re}\left(i\zeta v_{tt}^1\overline{w_t^1}\right)(t); \qquad \tilde{\Phi}_5(t) \;:=\; \mathrm{Re}\left(v_t\overline{v}\right)(t);$$

$$\tilde{\Phi}_3(t) \;:=\; -\mathrm{Re}\left(\frac{iv_t^2\overline{w^2}}{H_3\xi_3} + \frac{iv_t^3\overline{w^3}}{H_2\xi_2}\right)(t); \qquad \tilde{\Phi}_6(t) \;:=\; \frac{\tau}{\varkappa}\mathrm{Re}\left(\frac{i}{\xi_1}r_t^1\overline{\vartheta_t}\right)(t)$$

and, for $\tilde{c}$ sufficiently large, $\xi_1 \neq 0,\ \xi_2\xi_3 > 0$ and $a := |\xi|^2\left(\frac{1}{\xi_1^2} + \frac{1}{\xi_3^2}\right)$,

$$\begin{aligned}\tilde{\Phi}(t) \;:=\;& \frac{2}{\beta}\left(\frac{7}{4}\frac{64}{\beta^2}\tilde{c}^2 a + \frac{7}{4}\frac{64}{(\mu+\lambda)^2}\tilde{c}^2\frac{|\xi|^2}{\xi_1^2}a + 8\right)\tilde{\Phi}_1(t) + \frac{8}{\beta}\tilde{\Phi}_3(t)\\ &+\frac{2}{\beta}\frac{8(\mu^2+(\mu+\lambda)^2)}{\mu}\frac{64}{(\mu+\lambda)^2}\left(\frac{H_2}{H_3}+\frac{H_3}{H_2}\right)^2\frac{a}{\xi_1^2}\tilde{\Phi}_2(t) + \frac{8}{\mu+\lambda}\tilde{\Phi}_4(t)\\ &+4\tilde{\Phi}_5(t) + \left(\frac{7}{4}\frac{128}{(\mu+\lambda)^2}\left(\frac{H_2}{H_3}+\frac{H_3}{H_2}\right)^2\gamma^2\frac{a}{\xi_1^2} + 1\right)\tilde{\Phi}_6(t).\end{aligned}$$

Then there is some $c > 0$ such that the following estimate holds:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}(t) \;\leq\;& ca^4\frac{|\xi|^2+|\xi|^4+|\xi|^6}{\xi_1^4}|w|^2 + ca^4\frac{1+|\xi|^2+|\xi|^4}{\xi_1^4}|w_t|^2\\ &+c\frac{1+|\xi|^2}{|\xi|^2}|r|^2 + ca^2\frac{|\xi|^2+|\xi|^4+|\xi|^6+|\xi|^8}{\xi_1^6}|r_t|^2 - \hat{\mathcal{E}}(t).\end{aligned}$$

PROOF. Analogously to (5.10), multiplying the derivatives of (5.1), (5.4) with $\overline{w_t^1}, \overline{v_{tt}^1}$, we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}_2 \;\leq\;& c|\zeta||\xi|^2|v_t||w_t| + c|\zeta||\xi||\vartheta_t||w_t|\\ &+c|\zeta||\xi||w_t|^2 + c\zeta^2|\xi|^4|w_t|^2 - \frac{\beta}{2}|v_{tt}^1|^2.\end{aligned} \tag{5.11}$$

Multiplication of (5.2), (5.3) with $\overline{w^2}, \overline{w^3}$ and (5.5), (5.6) with $\overline{v_t^2}, \overline{v_t^3}$ yields

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}_3(t) &= -\mathrm{Re}\left(\frac{i}{H_3\xi_3}\overline{v_{tt}^2}w^2 + \frac{i}{H_3\xi_3}\overline{v_t^2}w_t^2 + \frac{i}{H_2\xi_2}\overline{v_{tt}^3}w^3 + \frac{i}{H_2\xi_2}\overline{v_t^3}w_t^3\right) \\ &\le c|\xi|^2|w|\left(\frac{|v^2|}{|\xi_3|} + \frac{|v^3|}{|\xi_2|}\right) + c\left(\frac{|\xi_2|}{|\xi_3|} + \frac{|\xi_3|}{|\xi_2|}\right)|\xi||w||v| + c\left(\frac{|\xi_2|}{|\xi_3|} + \frac{|\xi_3|}{|\xi_2|}\right)|w||\vartheta| \\ &\quad + c\left(\frac{|\xi_2|}{|\xi_3|} + \frac{|\xi_3|}{|\xi_2|}\right)|w|^2 + c|\xi|^2|w|\left(\frac{|v_t^2|}{|\xi_3|} + \frac{|v_t^3|}{|\xi_2|}\right) + c|\xi||v_t^1|\left(\frac{|v_t^2|}{|\xi_3|} + \frac{|v_t^3|}{|\xi_2|}\right) \\ &\quad - \alpha(|w^2|^2 + |w^3|^2) - \beta(|v_t^2|^2 + |v_t^3|^2) + \beta\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)\mathrm{Re}\left(\overline{v_t^2}v_t^3\right). \end{aligned} \tag{5.12}$$

To annulate the mixed term $\mathrm{Re}(v_t^2\overline{v_t^3})$, we multiply the derivative of (5.1) with $\overline{v_t^2}$ resp. $\overline{v_t^3}$ and estimate $\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}_4(t)$ as follows:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}_4(t) &= \mathrm{Re}\left(\frac{H_2}{H_3}\frac{v_{ttt}^1\overline{v_t^2}}{\xi_1\xi_3} + \frac{H_2}{H_3}\frac{v_{tt}^1\overline{v_{tt}^2}}{\xi_1\xi_3} + \frac{H_3}{H_2}\frac{v_{ttt}^1\overline{v_t^3}}{\xi_1\xi_2} + \frac{H_3}{H_2}\frac{v_{tt}^1\overline{v_{tt}^3}}{\xi_1\xi_2}\right) \\ &\le c\left(\frac{|\xi|^2}{|\xi_1|} + |\xi|\right)|v_t^1|\left(\frac{|v_t^2|}{|\xi_3|} + \frac{|v_t^3|}{|\xi_2|}\right) - (\mu+\lambda)\left(\frac{H_2}{H_3}\frac{\xi_2}{\xi_3}|v_t^2|^2 + \frac{H_3}{H_2}\frac{\xi_3}{\xi_2}|v_t^3|^2\right) \\ &\quad - (\mu+\lambda)\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)\mathrm{Re}\left(v_t^2\overline{v_t^3}\right) + \gamma\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)|\vartheta_t|\left(\frac{|v_t^2|}{|\xi_3|} + \frac{|v_t^2|}{|\xi_2|}\right) \\ &\quad + c\frac{|\xi|}{|\xi_1|}|w_t|\left(\frac{|v_t^2|}{|\xi_3|} + \frac{|v_t^3|}{|\xi_2|}\right) + \left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)\frac{|v_{tt}^1|}{|\xi_1|}\left(\frac{|v_{tt}^2|}{|\xi_3|} + \frac{|v_{tt}^3|}{|\xi_2|}\right). \end{aligned} \tag{5.13}$$

At this point, we need the assumption that ξ_2 and ξ_3 have the same sign as postulated in the assumptions of Theorem 5.1 to get rid of the second summand in (5.13). The two remaining functionals create the missing components of the negative energy in $\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}(t)$:

Multiplying (5.1), (5.2), (5.3) with $\overline{v^1}, \overline{v^2}, \overline{v^3}$ yields

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}_5(t) &= \mathrm{Re}\left(v_{tt}\overline{v} + v_t\overline{v_t}\right) \\ &\le -\mu|\xi|^2|v|^2 - (\mu+\lambda)(\xi v)^2 + c|\vartheta|^2 \\ &\quad + \frac{\mu}{4}|\xi|^2|v|^2 + c|w|^2 + \frac{\mu}{4}|\xi|^2|v|^2 + |v_t|^2 \end{aligned} \tag{5.14}$$

and multiplying the derivatives of the third and fourth equation of (3.1) with $\overline{\vartheta_t}$ resp. $\overline{r_t^1}$:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}_6(t) &= \mathrm{Re}\left(\frac{i\tau_0}{\varkappa\xi_1}r_{tt}^1\overline{\vartheta_t} + \frac{i\tau_0}{\varkappa\xi_1}r_t^1\overline{\vartheta_{tt}}\right) \\ &\le -\frac{1}{2}|\vartheta_t|^2 + \frac{c}{\xi_1^2}|r_t|^2 + c\frac{|\xi|}{|\xi_1|}|r_t||v_t| + c\frac{|\xi|^2}{|\xi_1|}|r_t|\frac{|v_{tt}|}{|\xi|}. \end{aligned} \tag{5.15}$$

Finally, using the differential equations (3.1) directly, we get

$$\begin{aligned} |v_{tt}|^2 &\le 8|\xi|^2\left(\mu^2|\xi|^2|v|^2 + (\mu+\lambda)^2(\xi\cdot v)^2 + \gamma^2|\vartheta|^2 + 2\alpha^2|H_0|^2|w|^2\right); \\ |\vartheta|^2 &\le \frac{c}{|\xi|^2}\left(|r|^2 + |r_t|^2\right) \end{aligned} \tag{5.16}$$

and receive, summing up the inequalities (5.10)-(5.15), weighted with the ξ-dependent terms given in Theorem 5.1,

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\Phi}(t) \leq\ & \frac{c}{2\epsilon(v)}a^2\frac{|\xi|^4}{\xi_1^4}\zeta^2|\xi|^2|w|^2 + \frac{\epsilon(v)}{2}|\xi|^2|v|^2 + ca^2\frac{|\xi|^4}{\xi_1^4}\zeta^2|\xi|^2|w|^2 + c|\vartheta|^2 \\
& + ca\frac{|\xi|^2}{\xi_1^2}|\zeta||\xi||w|^2 + ca\frac{|\xi|^2}{\xi_1^2}\zeta^2|\xi|^4|w|^2 - \frac{\tilde{c}^2}{2\epsilon(v_t)}\frac{64}{\beta^2}a|v_t^1|^2 \\
& - \frac{\tilde{c}^2}{2\epsilon(v_t)}\frac{64}{(\mu+\lambda)^2}a\frac{|\xi|^2}{\xi_1^2}|v_t^1|^2 - 8|v_t^1|^2 \\
& \\
& + \frac{c}{2\epsilon(v_t)}\frac{a^2}{\xi_1^4}\zeta^2|\xi|^4|w_t|^2 + \frac{\epsilon(v_t)}{2}|v_t|^2 + \frac{c}{2\epsilon(\vartheta_t)}\frac{a^2}{\xi_1^4}\zeta^2|\xi|^2|w_t|^2 + \frac{\epsilon(\vartheta_t)}{2}|\vartheta_t|^2 \\
& + c\frac{a}{\xi_1^2}|\zeta||\xi||w_t|^2 + c\frac{a}{\xi_1^2}\zeta^2|\xi|^4|w_t|^2 - \frac{1}{2\epsilon(v_{tt})}\frac{64}{(\mu+\lambda)^2}\frac{a}{\xi_1^2}|v_{tt}^1|^2 \\
& \\
& + \frac{c}{2\epsilon(v)}a|w|^2 + \frac{\epsilon(v)}{2}|\xi|^2|v|^2 + \frac{c}{2\epsilon(v)}a|w|^2 + \frac{\epsilon(v)}{2}|\xi|^2|v|^2 + ca|w|^2 \\
& + c|\vartheta|^2 + ca|w|^2 + \frac{c}{2\epsilon(v_t)}a|\xi|^2|w|^2 + \frac{\epsilon(v_t)}{2}|v_t|^2 + \frac{\tilde{c}^2}{2\epsilon(v_t)}\frac{64}{\beta^2}a|v_t^1|^2 \\
& + \frac{\epsilon(v_t)}{2}|v_t|^2 - 8(|v_t^2|^2 + |v_t^3|^2) + 8\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)\mathrm{Re}\left(\overline{v_t^2}v_t^3\right) \\
& \\
& - 8\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)\mathrm{Re}\left(\overline{v_t^2}v_t^3\right) + \frac{\tilde{c}^2}{2\epsilon(v_t)}\frac{64}{(\mu+\lambda)^2}\frac{|\xi|^2}{\xi_1^2}a|v_t^1|^2 + \frac{\epsilon(v_t)}{2}|v_t|^2 \\
& + \frac{1}{2\epsilon(v_t)}\frac{64}{(\mu+\lambda)^2}\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)^2\gamma^2\frac{a}{\xi_1^2}|\vartheta_t|^2 + \frac{\epsilon(v_t)}{2}|v_t|^2 + \frac{c}{2\epsilon(v_t)}\frac{a}{\xi_1^2}|w_t|^2 \\
& + \frac{\epsilon(v_t)}{2}|v_t|^2 + \frac{1}{2\epsilon(v_{tt})}\frac{64}{(\mu+\lambda)^2}\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)^2\frac{a}{\xi_1^2}|v_{tt}^1|^2 + \frac{\epsilon(v_{tt})}{2}\frac{|v_{tt}|^2}{|\xi|^2} \\
& \\
& - 2\mu|\xi|^2|v|^2 - 4(\mu+\lambda)(\xi\cdot v)^2 + c|\vartheta|^2 + c|w|^2 + 4|v_t|^2 \\
& \\
& - \frac{1}{2\epsilon(v_t)}\frac{64}{(\mu+\lambda)^2}\left(\frac{H_2}{H_3} + \frac{H_3}{H_2}\right)^2\gamma^2\frac{a}{\xi_1^2}|\vartheta_t|^2 - \frac{\epsilon(\vartheta_t)}{2}|\vartheta_t|^2 \\
& + \frac{c}{\xi_1^2}\left(\frac{a}{\xi_1^2} + 1\right)|r_t|^2 + \frac{c}{2\epsilon(v_t)}\left(\frac{a}{\xi_1^2} + 1\right)^2\frac{|\xi|^2}{\xi_1^2}|r_t|^2 + \frac{\epsilon(v_t)}{2}|v_t|^2 \\
& + \frac{c}{2\epsilon(v_{tt})}\left(\frac{a}{\xi_1^2} + 1\right)^2\frac{|\xi|^4}{\xi_1^2}|r_t|^2 + \frac{\epsilon(v_{tt})}{2}\frac{|v_{tt}|^2}{|\xi|^2} \\
& \\
& - \frac{\alpha}{\beta}|w|^2 - \frac{\gamma}{\delta}|\vartheta|^2 - \frac{\tau_0\rho\gamma}{\varkappa\delta}|r|^2 + \frac{\alpha}{\beta}|w|^2 + \frac{\gamma}{\delta}|\vartheta|^2 + \frac{\tau_0\rho\gamma}{\varkappa\delta}|r|^2
\end{aligned}
$$

$$
\begin{aligned}
\leq \quad & \left(\frac{3}{2}\epsilon(v) - \frac{\mu}{2}\right)|\xi|^2|v|^2 + \left(4 + \frac{7}{2}\epsilon(v_t) - 5\right)|v_t|^2 \\
& + \left(8\epsilon(v_{tt})(\mu^2 + (\mu+\lambda)^2) - \frac{\mu}{2}\right)|\xi|^2|v|^2 \\
& - \hat{\mathcal{E}}(t) + c\frac{|\xi|^2 + |\xi|^4 + |\xi|^6}{\xi_1^4}a^4|w|^2 + c\frac{1 + |\xi|^2 + |\xi|^4}{\xi_1^4}a^4|w_t|^2 \\
& + c\frac{1 + |\xi|^2}{|\xi|^2}|r|^2 + ca^2\frac{|\xi|^2 + |\xi|^4 + |\xi|^6 + |\xi|^8}{\xi_1^6}|r_t|^2. \qquad (5.17)
\end{aligned}
$$

Choosing

$$
\epsilon(v) := \frac{\mu}{3}, \qquad \epsilon(v_t) := \frac{2}{7}, \qquad \epsilon(v_{tt}) := \frac{\mu}{16(\mu^2 + (\mu+\lambda)^2)}, \qquad \epsilon(\vartheta_t) := 1,
$$

the first terms of (5.17) sum up to zero which finishes the proof. □

To justify the artificial assumption "$\xi_2\xi_3 > 0$", we restrict ourself on data with fitting support:

$$
\operatorname{supp}\left(\hat{u}_0, \hat{u}_1, \hat{h}_0, \hat{\theta}_0, \hat{q}_0\right) \subseteq \{\xi \in \mathbb{R}^3 \mid \xi_2 \cdot \xi_3 > 0\}. \qquad (5.18)
$$

Defining appropriate constants $\tilde{C}_1(\xi), \tilde{C}_2(\xi), \tilde{C}_3(\xi) > 0$, Theorem 3.5 can be applied then and we get polynomial decay rates for the energy corresponding to system (1.6) with $H_0 = (0, H_2, H_3)$, $H_2, H_3 > 0$ if the data additionally fullfill the integrability condition (3.15).

We finish this section with some remarks on the condition for the support of the transformed data. It is easy to see (cp. [15], Theorem 2.6) that for any non-zero $f \in \mathcal{L}^2(\mathbb{R}^n)$ with $\operatorname{supp}(\mathcal{F}f) \subseteq \mathbb{R}^{n-1} \times [-\hat{S}, \hat{S}]$ there is no interval $[a, b] \subseteq \mathbb{R}$ with $f = 0$ on $\mathbb{R}^{n-1} \times [a, b]$. It follows that if the transformed data have a *compact* support in $\{\xi \in \mathbb{R}^3 \mid \xi_2\xi_3 > 0\}$ (and therefore fullfill condition (5.18)), the data themselves vanish on no interval in no space direction. Using this, we contruct initial data which guarantee the desired decay. Let

$$
E_0 : \mathbb{R}^3 \to \mathbb{C}^3 \times \mathbb{C}^3 \times \mathbb{C}^3 \times \mathbb{C} \times \mathbb{C}^3, \qquad E_0(\xi) = (E_{u_0}, E_{u_1}, E_{h_0}, E_{\theta_0}, E_{q_0})(\xi)
$$

with $\operatorname{supp}(E_0) \subseteq \{\xi \in \mathbb{R}^3 \mid \xi_1\xi_3 > 0\}$. Then the equations of magneto-thermo-elasticity with second sound, given some initial data $F_0 := \mathcal{F}^{-1}(E_0)$, have a unique, complex-valued solution

$$
F \in \mathcal{C}^1([0,\infty), \mathcal{H}) \cap \mathcal{C}^0([0,\infty), \mathcal{D}(A))
$$

such that $\mathcal{E}_F(t) = \mathcal{O}(t^{-m})$ holds for the corresponding energy if

$$
\int_{\mathbb{R}^3} \frac{C_2(\xi)/C_1(\xi)}{C_3(\xi)^m}\hat{\mathcal{E}}_{F_0}(\xi)\,\mathrm{d}\xi < \infty
$$

applies for F_0. Especially, $G := \operatorname{Re}(F)$ is a real-valued solution to the system with initial condition $G_0 := \operatorname{Re}(F_0)$ and the energy $\mathcal{E}_G$ corresponding to G decays polynomially with rate m, too: $\mathcal{E}_G(t) \leq \mathcal{E}_F(t) = C_0 t^{-m}$. However, C_0 does not depend on $\mathcal{E}_{G_0}$ as before, but on $\mathcal{E}_{F_0}$.

Generally, we are interested in initial values $f_0 \in \mathcal{L}^1 \cap \mathcal{L}^2$ for which $\operatorname{supp}(\hat{f}_0)$ is compact: The Dominated Convergence Theorem implies that $\hat{f}_0$ is continous, i.e.

$$\xi \mapsto \frac{C_2(\xi)}{C_1(\xi)C_3(\xi)^m}|\mathcal{F}f_0(\xi)|^2$$

takes its maximum on $\operatorname{supp}(\hat{f}_0)$ and therefore the condition (3.15) holds for any m.

6. Some remarks on the stability condition for the initial data. We want to translate the condition (3.15)

$$\int_{\mathbb{R}^3} \frac{C_2(\xi)}{C_1(\xi)C_3^m(\xi)} \hat{\mathcal{E}}_1^\tau(0)\ \mathrm{d}\xi < \infty$$

for the energy of the transformed initial data into a condition for the initial data themselves. Without loss of generality we restrict ourself on the magneto-thermo-elastic sytem with second sound, i.e. $C_1(\xi)$, $C_2(\xi)$ and $C_3(\xi)$ are the functions given in (3.12)-(3.14), and we assume $m = 1$; it is easy to see how the following method can be applied for $m > 1$ and other models of (magneto-)thermo-elasticity (as long as $C_i(\xi)$ are rational functions in $\xi_j^2,\ i = 1, 2, 3,\ j = 1, ..., n$ where n is the space dimension; here: $n = 3$).

In this situation, the stability condition of Theorem 3.5 holds if any $V_0 \in \{u_0^\tau, u_1^\tau, h_0^\tau, \theta_0^\tau, q_0^\tau\}$ fullfills

$$\int_{\mathbb{R}^3} \frac{a^4(1+|\xi|^6)^2}{|\xi|^6}|\hat{V}_0(\xi)|^2\ \mathrm{d}\xi < \infty \tag{6.1}$$

resp. if the following, more restrictive condition holds which is easier to handle:

$$\int_{\mathbb{R}^3} \frac{(1+|\xi|^2)^8(\xi_1^2+\xi_3^2)^4}{\xi_1^8\xi_3^8}|\hat{V}_0(\xi)|^2\ \mathrm{d}\xi < \infty. \tag{6.2}$$

We can read (6.2) as

$$\exists W_0 \in \mathcal{L}^2(\mathbb{R}^3) : (1+|\xi|^2)^8(\xi_1^2+\xi_3^2)^4\hat{V}_0 = \xi_1^8\xi_3^8\hat{W}_0.$$

Retransformation yields the regularity condition

$$\exists W_0 \in \mathcal{L}^2(\mathbb{R}^3) : \underbrace{\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}\right)^4(1-\Delta)^8}_{=:\mathrm{D}} V_0 \quad = \quad \underbrace{\left(\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_3}\right)^8}_{=:\mathrm{d}} W_0. \tag{6.3}$$

Now, if we choose an element f in the Schwartz space

$$\mathcal{S}(\mathbb{R}^3) := \left\{\varphi \in \mathcal{C}^\infty(\mathbb{R}^3) \mid \forall\alpha,\beta \in \mathbb{N}^3 : \sup_{x\in\mathbb{R}^3}|x^\alpha\nabla^\beta\varphi(x)| < \infty\right\} \subseteq \mathcal{L}^2(\mathbb{R}^3),$$

then $V_0 := \mathrm{d}f$ and $W_0 := \mathrm{D}f$ are Schwartz functions, too, and obviously the required condition $\mathrm{D}V_0 = \mathrm{d}W_0$ holds. Finally, for any $f \in \mathcal{L}^2(\mathbb{R}^3)$ and any $\epsilon > 0$ we can choose a smooth function $\tilde{f} \in \mathcal{C}^\infty(\mathbb{R}^3)$ such that $||f - \tilde{f}||_{\mathcal{L}^2} < \epsilon$ by using Friedrichs mollifier [07]:

$$\tilde{f}(x) := (\Psi_k f)(x) := (\psi_k * f)(x), \qquad \psi_k(x) := \begin{cases} c_0 k^3 e^{-\frac{1}{1-|kx|^2}} & |x| < 1 \\ 0 & |x| \geq 1 \end{cases} \tag{6.4}$$

where c_0 is a normalisation constant such that $||\psi_1||_{\mathcal{L}^1} = 1$ and $k = k(\epsilon)$ is chosen sufficiently large. Then (3.15) holds for

$$V_0 := \mathrm{d}\tilde{f} = (\mathrm{d}\psi_k) * f. \tag{6.5}$$

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Martin Gubisch
Department of Mathematics and Statistics
University of Konstanz (Germany)
e-mail: martin.gubisch@uni-konstanz.de

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