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On the Exclusivity Implicature of 'Or' or on the Meaning of Eating Strawberries

Abstract. This paper is a contribution to the program of constructing formal representations of pragmatic aspects of human reasoning. We propose a formalization within the framework of Adaptive Logics of the exclusivity implicature governing the connective 'or'.

Keywords: exclusivity implicature, Adaptive Logics.

Implicatures

Even though one of the aims of logic is to explicate human reasoning, there remains a serious gap between derivations in classical and non-classical formal systems on the one hand and common sense reasoning in natural language on the other hand. The philosopher of language H.P. Grice claims that the discrepancy can be bridged by attending to the nature and importance of the pragmatic conditions governing conversation (see [6]). His central thesis is that the behaviour of participants in a conversation is ruled by a general principle, the so-called *Cooperative Principle.* This principle states that a conversational contribution should be in accordance with what is required (at the stage at which it occurs) by the accepted purpose or direction of the talk exchange in which the speaker is engaged. He distinguishes four specific categories (which he calls *Maxims*) to add substance to the Cooperative Principle:

- The *Maxim of Quantity* requires that each contribution should be made as *informative* as is required, but not more informative than is required —though he admits the latter to be open to debate.

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- The *Maxim of Quality* requires one to make true contributions, not to say things one believes to be false or for which one lacks adequate evidence.
- The *Maxim of Relation* states that each conversational contribution should be *relevant* to the subject of the conversation.
- The *Maxim of Manner* concerns the *way in which* things should be said. Things should be said perspicuously, obscurity of expression and ambiguity should be avoided, things should be said concisely (avoiding unnecessary prolixity), orderly, ...

Even if a conversation takes place with the kind and effective cooperation of all participants, it can still occur– and in fact regularly does occur– that these principles are not followed literally. In such cases, the speaker thinks (and would expect the hearer to think that he thinks) that it is within the competence of the hearer to work out the supposition needed to maintain that the speaker is still observing the Cooperative Principle. This then allows the hearer to infer statements which do not follow logically from what is said, but which follow pragmatically from the assumption that the Principle of Cooperation is maintained in the conversation. The pragmatic rules governing the generation of these 'weak' inferences are called *conversational implicatures*. From a logical point of view, the most tractable conversational implicatures are those that only depend on what is said and do not depend on the extra-linguistic context. These are called *generalized* conversational implicatures (**GCI**). In the present contribution, we model one particular generalized implicature, namely one which is associated with the interpretation of the connective 'or'. The so-called quantitative scalar implicature associated with 'or' says that if in a conversation $A \vee B$ is asserted, this assertion is to be interpreted as:

$$(A \vee B) \wedge \neg(A \wedge B).$$

In other words, when a participant in a conversation asserts $A \vee B$, it is conversationally implicated that he / she conveys an *exclusive* disjunction.[1]

At first sight, it seems that this implicature is purely an aspect of *informativeness* (Maxim of Quantity). But we will show that an adequate logical modeling of this implicature must also take matters of *relevance* into consideration (Maxim of Relation). It will turn out that in order to adequately

[1]Not all authors agree with Grice's general view on the relation between the meaning and pragmatics of 'or'. In [8] it is argued that the meaning of 'or' cannot be explicated in truth-functional terms.

model the exclusivity-implicature of 'or', one must take into account that it is conversationally inappropriate to assert a disjunctive statement if one knows of one of its disjuncts that it is true.

Adaptive Logics

One of the most important features of **GCI**'s is *cancellability*: in particular situations the application of the implicature may be cancelled because it no longer suits the context when additional information is provided. Non-monotonicity is the logical equivalent of the feature of cancellability. For this reason Levinson in [9] suggests that general conversational implicatures should be modelled as non-monotonic inference rules. Non-monotonic rules require a dynamics in which conclusions can in certain circumstances be rejected and revised. This means that the derivations corresponding with the implicatures should be made *conditionally*. As soon as in view of the interpretation of further information it becomes clear that the application of the implicature can no longer be supported, it must be renounced. Also dynamics in the other direction should be represented. As soon as good reasons for restoring a withdrawn application become available, the implicature should be restored. This internal dynamics is exactly the kind of dynamics incorporated in *adaptive logics*.[2] Characteristic of the proof theory of an adaptive logic is that some inference rules (and in some cases also premise rules) are conditional. The validity of the conclusion of a conditional rule is in each individual case dependent upon the acquired insight in the information contained in the premises. The effectiveness of a conditional rule is not global: the validity of an application adapts itself to the specific context of the acquired insight in the information contained in the premises. Therefore, the framework of adaptive logics (see [3]) is extremely suitable for the implementation of **GCI**'s.

The inference rules for a dynamic proof can be classified into two sorts: (i) the *unconditional rule(s)* and (ii) the *conditional rule(s)*. All can be applied at any time in the proof, but the latter only conditionally. The unconditional rules form the so-called *lower limit logic* and supplemented with the conditional ones, they form the *upper limit logic*. The invalidity of an inference is indicated by a mark on the line of that inference. The *marking definitions* regulate the marking and unmarking of lines in a dynamic proof. For each adaptive logic, a set of *abnormalities* is defined. When an

[2]For a detailed description of adaptive logics see [2] and [3], for more on a tableau method for some Adaptive Logics see [4].

abnormality (or a disjunction of abnormalities) is derived, this is taken as an indication that the upper limit logic can no longer be followed with respect to the formulas involved in the abnormality (or disjunction of abnormalities). The marking definitions create an adaptive logic somewhere in between the lower limit logic and the upper limit logic. Two important sorts of marking are the reliability marking and the minimal abnormality marking (see [3]).[3] Following the reliability marking, as soon as an abnormality (or a disjunction of abnormalities) comes to the surface, the logic locally reverts to the lower limit logic, i.e. all applications of conditional rules whose condition is not fulfilled in view of the abnormality (or of an abnormality in the disjunction of abnormalities) are marked. All other applications of conditional rules valid until then remain valid. The minimal abnormality marking on the other hand realizes the approach to the upper limit logic as close as possible, i.e. only those situations are considered in which the set of verified abnormalities is minimal. A variation on the minimal abnormality marking that is used in this paper, is the *counting marking*: only those situations are considered in which the number of verified abnormalities is minimal.

A line in a dynamic proof generally consists of five elements: (i) the line number, (ii) the formula derived on that line, (iii) the numbers of the lines used to derive the second element, (iv) the rule applied to derive the second element and (v) the condition on which the second element is derived.

As the derivation of formulas in these proofs has a provisional or conditional character, also an absolute notion of derivability has to be specified. The new concept is called *final derivability*. The second elements of lines that have an empty fifth element are *finally derived* on that line. For conditionally derived statements it does not suffice that the line is unmarked to be finally derivable, because in general there is no guarantee that it stays unmarked in all extensions of the proof. Therefore, in order for a second element of an unmarked line to be acceptable as finally derived, the following is required (by definition). For every extension E of the proof in which the line is marked, there is a further extension E' of E in which it is unmarked again.

The Meaning of the Disjunction

In the first part of [7] the implementation of the 'or'-implicature in the context of Classical Propositional Logic is investigated. It is shown that a

[3] In the *Artificial Intelligence* literature, default logics based on minimizing abnormalities have also been considered. See for instance [10].

default-style formulation of the implicature and a definition of a strong consequence relation (regarding all possible extensions) similar to the notion of final derivability in the framework of adaptive logics, is not sufficient. The rule corresponding with the implicature concerning the exclusive interpretation of the disjunction, is essentially defeasible in the context of Classical Propositional Logic. (We will abbreviate Classical Propositional Logic as **CL**.) The problem is the following. Whenever a weakening $A \vee B$ of a formula A is introduced by the *Rule of Addition*, the implicature may be applied. This results in $\neg(A \wedge B)$ and together with A in $\neg B$. In this way, *every* formula compatible with the premises turns out to be a weak consequence, i.e., an element of at least one extension. Obviously therefore **CL** supplemented with a default-style rule for the implicature blocks any new result of the implicature to be a strong consequence (to be an element of all possible extensions). It is suggested that imposing further restrictions on the application of the implicature rule, like not to apply it in cases of weakening by Addition, could solve this problem.

What is really the essence of the problem is the meaning of the disjunction in Classical Logic. Traditionally the disjunction is given a *truth-functional* meaning:

$$A \vee B \text{ is true iff } A \text{ is true or } B \text{ is true.}$$

In the Gricean framework the conversational meaning of 'or' is needed. This amounts to the following. On the assumption that the Cooperative Principle is followed, $A \vee B$ can be asserted whenever the speaker knows that A is true or B is true, but does not know which is true. In most situations, asserting a disjunction while one knows one particular disjunct to be true is almost as bad as lying. For example, when one says that Peter will come today or tomorrow while one knows that he will come tomorrow but not today, is withholding relevant information. The hearer may indeed rightfully consider the latter as deceit on the part of the speaker. The fault can be seen as a breach of the Maxim of Relation: as soon as the speaker knows that A, it is *irrelevant* to assert $A \vee B$.[4] There are precedents of giving a logical treatment of relevance considerations. Famously, Anderson and Belnap argued that due to the irrelevance of the antecedent for the consequent, theorems of the form $(A \wedge \neg A) \rightarrow B$ are judged to be unacceptable.[5] Schurz elaborated another

[4] Implicatures concerning Relevance are examined in detail in [12].

[5] See [1]. Note that for intensional disjunction, which is defined in terms of relevant implication, Addition is an invalid inference too.

approach to relevance, namely relevance as a filter on deduction in general.[6] Here we bring relevance to bear on the logical treatment of disjunction.[7]

Classical Logic should not be taken as the lower limit logic.[8] What is more appropriate is a system that interprets the disjunction according to its conversational meaning. The point is that $A \vee B$ loses its conversational value as soon as one of the disjuncts is known to be true. To express this in a natural way, an alternative semantical explication seems necessary. Indeed, a disjunction $A \vee B$ that is verified by a traditional valuation, can never be fulfilled in conversational terms. On the one hand, the verification of $A \vee B$ (by a traditional valuation) is precisely determined truth-functionally: $A \vee B$ is verified iff A is verified or B is verified. On the other hand, in order to have $A \vee B$ fulfilled in conversational terms, A and B may not be more than mere epistemic possibilities: neither A nor B may be *known* to be true.

In a nutshell, our strategy is roughly as follows. Relevance considerations entail the pragmatic invalidity of the Rule of Addition. And this allows us to formulate the exclusivity implicature of 'or' in a straightforward way.

The Lower Limit Logic RAD

The Semantics

The aim is to define an alternative system to **CL** which realizes a relevantly assertable disjunction[9], i.e. a system in which the connectives have the same meaning as in **CL** except that Addition is invalid. To understand the construction of the system **RAD**, it is helpful to look at an alternative formulation of **CL** first, let us call it **L**. Let M range over the classical propositional truth value assignments (classical propositional models) and let S range over the sets of classical propositional truth value assignments.

Reformulation of the Semantics of CL

1. $S \models_{\mathrm{L}} A$ iff for all $M \in S$, $M \models_{\mathrm{CL}} A$ for A a propositional letter.
2. $S \models_{\mathrm{L}} \neg A$ iff for all $M \in S$, $M \models_{\mathrm{CL}} \neg A$ for A a propositional letter.

[6] See [11].

[7] In [13] it is shown how the present theory can be extended to obtain a theory of a relevant (or relevantly assertable) implication.

[8] Even though, due to the possibility to attribute the feature of conditionality to logical rules in the framework of Adaptive Logics, it is technically possible to work with Classical Logic as lower limit logic here, philosophically it makes more sense not to do so.

[9] We will abbreviate the system of the Relevantly Assertable Disjunction as **RAD**.

3. $S \models_{\mathrm{L}} A \wedge B$ iff $S \models_{\mathrm{L}} A$ and $S \models_{\mathrm{L}} B$.
4. $S \models_{\mathrm{L}} A \vee B$ iff there are S_1 and S_2 for which $S_1 \cup S_2 = S$ and $S_1 \cap S_2 = \emptyset$, such that:
   - $S_1 \models_{\mathrm{L}} A$,
   - $S_2 \models_{\mathrm{L}} B$.
5. $S \models_{\mathrm{L}} \neg(A \wedge B)$ iff $S \models_{\mathrm{L}} \neg A \vee \neg B$.
6. $S \models_{\mathrm{L}} \neg(A \vee B)$ iff $S \models_{\mathrm{L}} \neg A \wedge \neg B$.
7. $S \models_{\mathrm{L}} \neg\neg A$ iff $S \models_{\mathrm{L}} A$.

Note that in case of $S = \emptyset$, $S \models_{\mathrm{L}} A$ for all formulas A. The material implication can be defined in terms of the negation and the disjunction or equivalently can be introduced by an appropriate clause.

THEOREM 1. *$S \models_{\mathrm{L}} A$ iff for all $M \in S$, $M \models_{\mathrm{CL}} A$.*

PROOF. The proof is left as an exercise to the reader. ■

DEFINITION 1. $S \models_{\mathrm{L}} \Gamma$ iff $S \models_{\mathrm{L}} A$ for all $A \in \Gamma$.

DEFINITION 2. $S_{\mathrm{L}}(\Gamma) =$ the maximal S for which $S \models_{\mathrm{L}} \Gamma$.

COROLLARY 1. $S_{\mathrm{L}}(\Gamma) = \{M \mid M \models_{\mathrm{CL}} \Gamma\}$.

PROOF. In view of Definition 2 and Theorem 1. ■

DEFINITION 3. $\Gamma \models_{\mathrm{L}} A$ iff $S_{\mathrm{L}}(\Gamma) \models_{\mathrm{L}} A$.

COROLLARY 2. *$\Gamma \models_{\mathrm{L}} A$ iff $\Gamma \models_{\mathrm{CL}} A$.*

PROOF. In view of Definition 3, Corollary 1, Theorem 1 and the standard definition of semantical consequence relation for **CL**. ■

Hence we write **CL** instead of **L**.

Definition of the Semantics of RAD

1. $S \models_{\mathrm{RAD}} A$ iff for all $M \in S$, $M \models_{\mathrm{CL}} A$ for A a propositional letter.
2. $S \models_{\mathrm{RAD}} \neg A$ iff for all $M \in S$, $M \models_{\mathrm{CL}} \neg A$ for A a propositional letter.
3. $S \models_{\mathrm{RAD}} A \wedge B$ iff $S \models_{\mathrm{RAD}} A$ and $S \models_{\mathrm{RAD}} B$.

4. $S \models_{\mathrm{RAD}} A \vee B$ iff there are S_1 and S_2 for which $S_1 \cup S_2 = S$, $S_1 \cap S_2 = \emptyset$,

   - $S_1 \models_{\mathrm{CL}} A$,
   - $S_2 \models_{\mathrm{CL}} B$,

   and for all such S_1 and S_2,

   (a) • $S_1 \neq \emptyset$,
       • $S_2 \neq \emptyset$,

   (b) • $S_1 \models_{\mathrm{RAD}} A$,
       • $S_2 \models_{\mathrm{RAD}} B$.

5. $S \models_{\mathrm{RAD}} \neg(A \wedge B)$ iff $S \models_{\mathrm{RAD}} \neg A \vee \neg B$.
6. $S \models_{\mathrm{RAD}} \neg(A \vee B)$ iff $S \models_{\mathrm{RAD}} \neg A \wedge \neg B$.
7. $S \models_{\mathrm{RAD}} \neg\neg A$ iff $S \models_{\mathrm{RAD}} A$.

To obtain an unaltered implication as the one in **CL**, an appropriate clause can be added. An alternative material implication based on the relevantly assertable disjunction is studied in [13].

Definition 4. $\Gamma \models_{\mathrm{RAD}} A$ iff $S_{\mathrm{CL}}(\Gamma) \models_{\mathrm{RAD}} A$.[10]

The crucial difference with Classical Propositional Logic is located in clause 4. In view of clauses 5, 6 and 7 of **RAD**, we restrict our attention to formulas in which negations are pushed inward to the level of propositional letters (by the Rules of De Morgan and Double Negation). The conditions in (a) require that neither disjunct is **CL**-verified by S, i.e. verified by all members of S.[11] The conditions in (b) introduce a recursion in case one of the disjuncts is in turn a disjunction. Essentially, clause 4 realizes the feature that as soon as $S \models_{\mathrm{RAD}} A$ or $S \models_{\mathrm{RAD}} B$ holds, it follows that $S \not\models_{\mathrm{RAD}} A \vee B$. In other words, it expresses the unvalidity of the semantical equivalent of the Rule of Addition and, what is more, it even brings about the opposite: *every* single application of the semantical equivalent of the Rule of Addition is wrong.

[10]Note that the selection of models is determined by **CL**.

[11]If there are S_1 and S_2 for which $S_1 \cup S_2 = S$, $S_1 \cap S_2 = \emptyset$, $S_1 \models_{\mathrm{CL}} A$, $S_2 \models_{\mathrm{CL}} B$ and $S_1 = \emptyset$, resp. $S_2 = \emptyset$, then $S \models_{\mathrm{CL}} B$, resp. $S \models_{\mathrm{CL}} A$, as in that case $S = S_2$, resp. $S = S_1$.

Example

For the premises $\Gamma = \{(p \vee q) \vee r,\ \neg q,\ \neg q \vee s\}$, we have $\Gamma \models_{\text{RAD}} p \vee r$ and $\Gamma \models_{\text{RAD}} \neg q$, but $\Gamma \not\models_{\text{RAD}} (p \vee q) \vee r$ and $\Gamma \not\models_{\text{RAD}} \neg q \vee s$. Note that this illustrates the non-monotonicity of RAD, as $\neg q \vee s \models_{\text{RAD}} \neg q \vee s$ and $(p \vee q) \vee r \models_{\text{RAD}} (p \vee q) \vee r$, but $\neg q \vee s,\ \neg q \not\models_{\text{RAD}} \neg q \vee s$ and $(p \vee q) \vee r,\ \neg q \not\models_{\text{RAD}} (p \vee q) \vee r$.

The Proof Theory

The derivations possible in an **RAD**-proof are the ones that are validated by **CL**, but always– even for the premises– on the condition that the derived formula is 'relevant' with respect to the other formulas derived in the proof. If that condition is not or no longer fulfilled, the derivation is marked as irrelevant, which means that the formula is no longer derived. In fact, **RAD** is on itself already an adaptive logic.

To check whether a formula A meets the relevance criterion, we use a normal form called the *negation normal form* $N(A)$. It is constructed by elaborating all negations up to the level of propositions by the rules of *De Morgan* and *Double Negation*. For example, for the formula $\neg(p \wedge \neg q) \vee (r \wedge (s \vee t))$, we obtain the following form: $(\neg p \vee q) \vee (r \wedge (s \vee t))$. If for two formulas A and B, $N(A)$ can be obtained by replacing in $N(B)$ some disjunction by only one of its disjuncts, the formula A should be evaluated as 'more relevant' than the formula B. The latter relation can be extended by recursion to formulas which are the first and last element of a sequence in which each formula is more relevant than the next one. For a formula A, $\rho(A)$ will be a measure to evaluate the 'relevance' of A with respect to other formulas B with associated $\rho(B)$. We give a recursive algorithm for ρ that first computes the negation normal form and then produces multisets for disjunctions (noted by $[]$) and sets for conjunctions (noted by $\{\}$).

$$\rho(A) =_{\text{def}} \delta(N(A))$$

where $\delta(A)$ is defined as follows:

1. $\delta(A) = A$ for A an atom.
2. $\delta(A \wedge B) = \{\delta(A), \delta(B)\}$.
3. $\delta(A \vee B) = [\delta(A), \delta(B)]$.

For example, for the formula $\neg(p \wedge \neg q) \vee (r \wedge (s \vee t))$, that is transformed into negation normal form $(\neg p \vee q) \vee (r \wedge (s \vee t))$, we obtain $[[\neg p, q], \{r, [s, t]\}]$

and for the formula $(p \vee q) \wedge (r \vee (s \wedge t))$ we obtain $\{[p, q], [r, \{s, t\}]\}$. The relevance order of the formulas corresponds with a (partial) ordering on the associated ρ's:

$\rho_1 < \rho_2$ iff

- $\rho_1 \in \rho_2$ and ρ_2 is a multiset

  or
- $\rho_1 = [\alpha, \beta_1]$

  $\rho_2 = [\alpha, \beta_2]$

  and $\beta_1 < \beta_2$

  or
- $\rho_1 = \{\alpha, \beta_1\}$

  $\rho_2 = \{\alpha, \beta_2\}$

  and $\beta_1 < \beta_2$

  or
- there is a ρ_3 such that $\rho_1 < \rho_3$ and $\rho_3 < \rho_2$.

For example $\neg(p \wedge \neg q) \vee (r \wedge t)$ is more relevant than $\neg(p \wedge \neg q) \vee (r \wedge (s \vee t))$ and correspondingly $[[\neg p, q], \{r, t\}] < [[\neg p, q], \{r, [s, t]\}]$. Another example of a comparison of relevance is: $(p \vee q) \wedge (s \wedge t)$ is more relevant than $(p \vee q) \wedge (r \vee (s \wedge t))$ and for the corresponding ρ's we have $\{[p, q], \{s, t\}\} < \{[p, q], [r, \{s, t\}]\}$. As we have the tool now to evaluate the relevance of a formula, we can proceed with the proof theory.

A stage s of a proof is the stage at which the proof has developed from line 1 up to line s. For each stage of a proof, the rules and marking definitions are then given as follows.

Prem If $A \in \Gamma$, one can write down the following line: (i) the appropriate line number, (ii) A, (iii) $\emptyset$, (iv) 'Prem' and (v) $\rho(A)$.

RAD If $A_1, \ldots, A_n \vdash_{\mathrm{CL}} B$ and $A_1, \ldots, A_n$ occur unmarked[12] on line $i_1, \ldots, i_n$, one can make the following derivation: (i) the appropriate line number, (ii) B, (iii) $i_1, \ldots, i_n$ (iv) 'RAD' and (v) $\rho(B)$.

[12]The requirement of being unmarked is unnecessary for the functioning of the logic as the I-marking definition blocks the irrelevant derivations anyway. It is for heuristical purposes that the latter requirement is introduced.

Definition 5. A line on which a formula A is derived is I-marked at stage s of the proof iff there is another line, marked[13] or unmarked at stage s of the proof, on which a formula B is derived such that $\rho(B) < \rho(A)$.

The definition of final derivability can be simplified for **RAD**-proofs, as a line once marked can never be unmarked again in a **RAD**-proof.

Definition 6. A formula A is finally derivable in a **RAD**-proof from Γ iff it occurs unmarked in any extension of that proof.

Definition 7. $\Gamma \vdash_{\mathrm{RAD}} A$ iff A is finally derived in a **RAD**-proof from Γ.

The soundness and completeness of **RAD** are proved in [13]. The proof is founded on the following theorem that reflects the relation between **RAD** and **CL**.

For any formula F: $S \models_{\mathbf{RAD}} F$ iff

1. $S \models_{\mathbf{CL}} F$,
2. for any $N(F)'$ which is the result of replacing in the negation normal form $N(F)$ of F a disjunction by one of its disjuncts, $S \not\models_{\mathbf{CL}} N(F)'$.

This ensures the decidability of the propositional logic **RAD**. But in general, for predicate logical adaptive systems, the notion of final derivability is not effective. The reason is, roughly, that the requirement of considering all extensions of a given proof essentially amounts to a consistency check.[14]

Note in this context also that both the conjunction and the disjunction are associative in **RAD**.

The Upper Limit Logic RAED

The aim is to add the exclusivity implicature for the disjunction as a rule of inference to the Lower Limit Logic **RAD**. We will call the resulting logic **RAED**. Semantically the implicature can be expressed as a selection operator Σ. This selection operator Σ operates on the sets S and excludes those models from a set S that prevent a disjunction **RAD**-verified by S from being an exclusive disjunction.

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[13] As the marking here can only be an I-marking and the relation $<$ is transitive, for a marked line on which a formula B is derived such that $\rho(B) < \rho(A)$, one can always find an unmarked line on which a formula C is derived such that $\rho(C) < \rho(B) < \rho(A)$.

[14] See [3, p. 63] for a discussion of this issue.

DEFINITION 8. $\Sigma(S) = S \setminus \{M \in S \mid$ for some formulas A and B, it holds that $S \models_{\mathrm{RAD}} A \vee B$ and $M \models_{\mathrm{CL}} A \wedge B\}$.

DEFINITION 9. $\Gamma \models_{\mathrm{RAED}} A$ iff $\Sigma(S_{\mathrm{CL}}(\Gamma)) \models_{\mathrm{RAD}} A$.

Examples

For the premise set $\Gamma = \{p \vee q\}$, we have $S_{\mathrm{CL}}(\Gamma) = \{M \mid M \models_{\mathrm{CL}} p \vee q\}$ and hence $\Gamma \not\models_{\mathrm{RAD}} \neg(p \wedge q)$ but also $\Gamma \not\models_{\mathrm{RAD}} p \wedge q$. After the operation by Σ, we obtain $\Sigma(S_{\mathrm{CL}}(\Gamma)) = \{M \mid M \models_{\mathrm{CL}} p \vee q$ and $M \models_{\mathrm{CL}} \neg(p \wedge q)\}$ and hence $\Gamma \models_{\mathrm{RAED}} p \vee q$ and $\Gamma \models_{\mathrm{RAED}} \neg(p \wedge q)$.

The problem with **RAED** is that in some cases it leads to trivial results, namely when it is not suitable to apply all implicatures. For example[15], for the premises $p \vee q$, $q \vee r$ and $r \vee p$, to apply all implicatures $\neg(p \wedge q)$,$\neg(q \wedge r)$ and $\neg(r \wedge p)$, would amount to triviality[16]! The upper limit logic is clearly too strong for this example.

To obtain a formalization of the implicature that is more generally applicable than **RAED**, the feature of cancellability needs to be incorporated. As mentioned above, adaptive strategies can be used to realize this. They establish a state of balance in between the lower limit logic and the upper limit logic in case the latter is too strong, i.e. leads to triviality. Such situations are recognized by means of a certain syntactical form derivable from the premises by the lower limit logic that indicates that application of rules of the upper limit logic would lead to triviality. Those syntactical forms are referred to as the *abnormalities* of the adaptive logic.

The Abnormalities

An abnormality of the present adaptive logics should indicate that a certain application of the exclusivity implicature may lead to triviality. At first sight, the general form of an abnormality here should be a disjunction in conjunction with the contradiction of its exclusivity implicature: $(A \vee B) \wedge (A \wedge B)$. As a formula of the latter form is not **RAD**-satisfiable, we should look for another candidate. That is because $S \models_{\mathrm{RAD}} (A \vee B) \wedge (A \wedge B)$ iff

- $S \models_{\mathrm{RAD}} A \vee B$,

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[15]We owe this example to Kristof De Clercq.

[16]In the case of inconsistent Γ, $S_{\mathrm{CL}}(\Gamma) = \emptyset$ and $\Gamma \models_{\mathrm{RAD}} A$ for all A whose negation normal form is disjunction-free.

i.e. there are S_1 and S_2 for which $S_1 \cup S_2 = S$ and $S_1 \cap S_2 = \emptyset$, such that

$$S_1 \models_{\mathrm{CL}} A \text{ and } S_2 \models_{\mathrm{CL}} B,$$

and for all such S_1 and S_2,

$$S_1 \neq \emptyset \text{ and } S_2 \neq \emptyset,$$

$$S_1 \models_{\mathrm{RAD}} A \text{ and } S_2 \models_{\mathrm{RAD}} B,$$

and

- $S \models_{\mathrm{RAD}} A \wedge B$,

  i.e. $S \models_{\mathrm{RAD}} A$ and $S \models_{\mathrm{RAD}} B$, which is a contradiction.[17]

Obviously, isolated abnormalities do not occur here. From the example of the previous section, we know that connected abnormalities do occur. Now what do they look like? Formulas of the form $((A_1^1 \vee A_2^1) \wedge (A_1^1 \wedge A_2^1)) \vee \ldots \vee ((A_1^n \vee A_2^n) \wedge (A_1^n \wedge A_2^n))$ are always **RAD**-invalid for the same reason as those of the form $(A \vee B) \wedge (A \wedge B)$ are. Somehow this would-be disjunction of abnormalities should be modified a little to a form that is classically equivalent but that can be **RAD**-valid. Let us look at the example of the previous section. There we have $S \models_{\mathrm{RAD}} p \vee q$, $S \models_{\mathrm{RAD}} q \vee r$ and $S \models_{\mathrm{RAD}} r \vee p$, but we do not have $S \models_{\mathrm{RAD}} p \wedge q$ nor $S \models_{\mathrm{RAD}} q \wedge r$ nor $S \models_{\mathrm{RAD}} r \wedge p$. What we do have is that $S \models_{\mathrm{RAD}} (p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$. All this put in conjunction gives $S \models_{\mathrm{RAD}} (p \vee q) \wedge (q \vee r) \wedge (r \vee p) \wedge ((p \wedge q) \vee (q \wedge r) \vee (r \wedge p))$. This leads us to the following general form of connections of abnormalities:

$$(A_1^1 \vee A_2^1) \wedge \ldots \wedge (A_1^n \vee A_2^n) \wedge ((A_1^1 \wedge A_2^1) \vee \ldots \vee (A_1^n \wedge A_2^n))$$

for $n > 1$ (as isolated abnormalities do not occur).

Semantics

Reliability

For the reliability approach, we have to define the unreliable formulas for a given premise set Γ. These are the formulas for which the application of the implicature may lead to trivial results when combined with applications of

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[17] $S \models_{\mathrm{RAD}} A$, resp. $S \models_{\mathrm{RAD}} B$, admits to choose $S_1 = S$ and $S_2 = \emptyset$, resp. $S_1 = \emptyset$ and $S_2 = S$.

the implicature on other formulas. They can be extracted from the connections of abnormalities that are **RAD**-derivable from Γ. We call each formula that is of the form (after possibly leaving out some brackets)

$$(A_1^1 \vee A_2^1) \wedge \ldots \wedge (A_1^n \vee A_2^n) \wedge ((A_1^1 \wedge A_2^1) \vee \ldots \vee (A_1^n \wedge A_2^n))$$

for $n > 1$ a *Cab*-formula and denote this as $Cab(\{A_1^1 \vee A_2^1, \ldots, A_1^n \vee A_2^n\})$.

DEFINITION 10. $U(\Gamma) = \{A \mid A \in \theta$ for some $Cab(\theta)$ for which $\Gamma \models_{\text{RAD}} Cab(\theta)\}$.

The reliability approach selects those models from $S_{\text{CL}}(\Gamma)$ that verify the implicatures of formulas that are not unreliable.

DEFINITION 11. $S_{\text{RADE1}}(\Gamma) = \{M \mid M \in S_{\text{CL}}(\Gamma)$ and for all formulas A and B for which $S \models_{\text{RAD}} A \vee B$ and $A \vee B \notin U(\Gamma)$ it holds that $M \models_{\text{CL}} \neg(A \wedge B)\}$.

DEFINITION 12. $\Gamma \models_{\text{RADE1}} A$ iff $S_{\text{RADE1}}(\Gamma) \models_{\text{RAD}} A$.

Note that in contrast with **RAED**, **RADE1** has the following property.

If Γ is consistent, $S_{\text{RADE1}}(\Gamma) \neq \emptyset$.

Example

For the premises $\Gamma = \{p \vee q,\ q \vee r,\ r \vee p,\ s \vee t\}$ we have $U(\Gamma) = \{p \vee q,\ q \vee r,\ r \vee p\}$ and as $\Gamma \models_{\text{RAD}} (p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$, $S_{\text{RADE1}}(\Gamma) =$ the intersection of

- $\{M \mid M \models_{\text{CL}} \Gamma\} \cap \{M \mid M \models_{\text{CL}} \neg(s \wedge t)\}$
- $\{M \mid M \models_{\text{CL}} \neg((p \wedge q) \wedge ((q \wedge r) \vee (r \wedge p)))\}$
- $\{M \mid M \models_{\text{CL}} \neg(((p \wedge q) \vee (q \wedge r)) \wedge (r \wedge p))\}$
- $\{M \mid M \models_{\text{CL}} \neg(((p \wedge q) \vee (r \wedge p)) \wedge (q \wedge r))\}$,

which is the union of

- $\{M \mid M \models_{\text{CL}} \Gamma$ and $M \models_{\text{CL}} \neg(s \wedge t)$ and $M \models_{\text{CL}} p \wedge q \wedge \neg r\}$
- $\{M \mid M \models_{\text{CL}} \Gamma$ and $M \models_{\text{CL}} \neg(s \wedge t)$ and $M \models_{\text{CL}} \neg p \wedge q \wedge r\}$
- $\{M \mid M \models_{\text{CL}} \Gamma$ and $M \models_{\text{CL}} \neg(s \wedge t)$ and $M \models_{\text{CL}} p \wedge \neg q \wedge r\}$.

As results we have $\Gamma \models_{\text{RADE1}} \neg(s \wedge t)$, $\Gamma \not\models_{\text{RADE1}} \neg(p \wedge q)$, $\Gamma \not\models_{\text{RADE1}} \neg(q \wedge r)$, $\Gamma \not\models_{\text{RADE1}} \neg(r \wedge p)$, but $\Gamma \models_{\text{RADE1}} (p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$ and $\Gamma \models_{\text{RADE1}} (\neg p \vee \neg q \vee \neg r)$.

Counting

The semantics of the counting approach is much more straightforward. It selects those models M from $S_{\text{CL}}(\Gamma)$ that have the least number of abnormalities.

Definition 13. $Ab(M, S) = \{A \vee B \mid S \models_{\text{RAD}} A \vee B \text{ and } M \models_{\text{CL}} A \wedge B\}$.[18]

Definition 14. $S_{\text{RADE2}}(\Gamma) = \{M \mid M \in S_{\text{CL}}(\Gamma)$ and there is no $M' \in S_{\text{CL}}(\Gamma)$ such that $\sharp Ab(M', S_{\text{CL}}(\Gamma)) < \sharp Ab(M, S_{\text{CL}}(\Gamma))\}$.[19]

Definition 15. $\Gamma \models_{\text{RADE2}} A$ iff $S_{\text{RADE2}}(\Gamma) \models_{\text{RAD}} A$.

Note that also **RADE2** preserves consistency.

$$\text{If } \Gamma \text{ is consistent, } S_{\text{RADE12}}(\Gamma) \neq \emptyset.$$

Example

For the premises $\Gamma = \{p \vee q,\ q \vee r,\ r \vee p,\ q \vee s,\ s \vee p\}$ we have

$$S_{\text{RADE2}}(\Gamma) = \{M \mid M \models_{\text{CL}} \Gamma \text{ and } Ab(M, S_{\text{CL}}(\Gamma)) = \{p \vee q\}\}$$

and hence $S_{\text{RADE2}}(\Gamma) \models_{\text{CL}} p \wedge q$, $S_{\text{RADE2}}(\Gamma) \models_{\text{CL}} \neg(q \wedge r)$, $S_{\text{RADE2}}(\Gamma) \models_{\text{CL}} \neg(r \wedge p)$, $S_{\text{RADE2}}(\Gamma) \models_{\text{CL}} \neg(q \wedge s)$ and $S_{\text{RADE2}}(\Gamma) \models_{\text{CL}} \neg(s \wedge p)$, but $\Gamma \not\models_{\text{RADE2}} \neg(q \wedge r)$, $\Gamma \not\models_{\text{RADE2}} \neg(r \wedge p)$, $\Gamma \not\models_{\text{RADE2}} \neg(q \wedge s)$ and $\Gamma \not\models_{\text{RADE2}} \neg(s \wedge p)$ as $\Gamma \models_{\text{RADE2}} p$, $\Gamma \models_{\text{RADE2}} q$, $\Gamma \models_{\text{RADE2}} \neg r$ and $\Gamma \models_{\text{RADE2}} \neg s$.

Proof Theory

As the lower limit logic **RAD** is in itself already an adaptive logic, two dynamics are present in the proof theory. A line contains the four first elements as mentioned above. The fifth element contains the condition corresponding to the lower limit logic **RAD**, i.e., the relevance condition with respect to the disjunction, and the sixth element contains the condition corresponding with the applicability of the exclusivity implicature for the disjunction. The **RAD**-marking of irrelevant formulas is split up into two markings I_1 and I_2. The I_1-marking indicates irrelevance with respect to the information contained in the premises, interpreted literally. The I_2-marking indicates

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[18]In view of this definition, $S_{\text{RADE1}}(\Gamma) = \{M \mid M \in S_{\text{CL}}(\Gamma)$ and $Ab(M, S_{\text{CL}}(\Gamma)) \subseteq U(\Gamma)\}$ is equivalent to definition 11.

[19]We note $\sharp V$ to denote the cardinality of the set V.

irrelevance with respect to the information obtained by an interpretation of the premises that uses the exclusivity implicature.

The marking definitions must be applied in order of presentation. The rules Prem and RAD together with the I_1-marking definition compose the 'unconditional part' of the proof theory with respect to the dynamics of the exclusivity implicature.

Prem If $A \in \Gamma$, one can write down the following line: (i) the appropriate line number, (ii) A, (iii) $\emptyset$, (iv) 'Prem', (v) $\rho(A)$ and (vi) $\emptyset$.

RAD If $A_1, \ldots, A_n \vdash_{\mathrm{CL}} B$ and $A_1, \ldots, A_n$ occur I_1-unmarked[20] on line $i_1, \ldots, i_n$ on second conditions $\Delta_1, \ldots, \Delta_n$, one can make the following derivation: (i) the appropriate line number, (ii) B, (iii) $i_1, \ldots, i_n$, (iv) 'RAD', (v) $\rho(B)$ and (vi) $\Delta_1 \cup \ldots \cup \Delta_n$.

Definition 16. A line on which a formula A is derived is I_1-marked at stage s of the proof iff there is a line, unmarked at stage s of the proof, on which a formula B is derived on an *empty* second condition such that $\rho(B) < \rho(A)$.

The inference rule E performs the exclusivity implicature and is the only 'conditional rule' with respect to the dynamics concerning the exclusivity implicature, i.e., it is the only rule that creates a 'new' condition in the sixth element instead of just taking the union of occurring ones. The exclusivity implicature may only be applied to formulas that are **RAD**-derivable from the premises. On the one hand this means that only the rules Prem and RAD are needed for their derivation, i.e. they should be derived on an empty second condition. On the other hand this means that these formulas are relevant with respect to the information contained in the premises, i.e. they are I_1-unmarked.

E If $A \vee B$ occurs I_1-unmarked[21] on a line i, on an empty second condition, one can conditionally derive the exclusivity implicature as follows: (i) the appropriate line number, (ii) $\neg(A \wedge B)$, (iii) i, (iv) E, (v) $\rho(\neg(A \wedge B))$ and (vi) $\{A \vee B\}$.

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[20] The requirement of being I_1-unmarked is unnecessary for the functioning of the logic as the I_1-marking definition blocks the irrelevant derivations anyway. It is for heuristical purposes that the latter requirement is introduced.

[21] Again, the requirement of being I_1-unmarked is unnecessary for the functioning of the logic. Here it is the C_1-marking definition that blocks the consequences of applications of the exclusivity implicature on irrelevant derivations. Here also, the requirement is introduced for heuristical purposes.

The C_1-marking definition regulates the interplay between the dynamics of **RAD** and that concerning the applicability of the exclusivity implicature. The exclusivity implicature is cancelled when the original disjunction is I_1-marked.

DEFINITION 17. A line on second condition Δ is C_1-marked at stage s iff there is an $(A \vee B) \in \Delta$ for which its derivation is I_1-marked at stage s.

The C_2^r-and C_2^c-marking definitions perform the property of cancellability of the exclusivity implicature for connected disjunctions: C_2^r-marking is reliability marking and C_2^c-marking is counting marking.

Reliability

The unreliable formulas corresponding to the stages of a proof are defined as follows.

DEFINITION 18. $U_s(\Gamma) = \{A \mid A \in \theta$ for some $Cab(\theta)$ that is derived at stage s on an empty second condition and unmarked at that stage$\}$.

DEFINITION 19. A line with second condition Δ is C_2^r-marked at that stage iff $\Delta \cap U_s(\Gamma) \neq \emptyset$.

As the exclusivity implicature gives rise to additional information, it can occur that derivations that were relevant before, become irrelevant. This irrelevance of second order is regulated by the I_2-marking definition.

DEFINITION 20. A line on which a formula A is derived is I_2-marked at stage s of the proof iff there is a line, unmarked at stage s of the proof, on which a formula B is derived on a *non-empty* second condition such that $\rho(B) < \rho(A)$.

DEFINITION 21. A formula A is finally derivable in an **RADE1**-proof iff it occurs unmarked in the proof and for any extension of that proof in which it is marked, there is an extension of that extension in which it is unmarked again.

DEFINITION 22. $\Gamma \vdash_{\text{RADE1}} A$ iff A is finally derived in an **RADE1**-proof from Γ.

Counting

Let $\Phi^0_s(\Gamma)$ be the set of all sets that contain one member of θ for each $Cab(\theta)$ derived at stage s on an empty second condition and unmarked at that stage. Let $\Phi_s(\Gamma)$ contain those members of $\Phi^0_s(\Gamma)$ that have no more elements than other members of $\Phi^0_s(\Gamma)$.[22]

Definition 23. A line with second condition Δ is C^c_2-marked at stage s iff (i) there is no $\phi \in \Phi_s(\Gamma)$ such that $\phi \cap \Delta = \emptyset$, or (ii) for some $\phi \in \Phi_s(\Gamma)$, there is no line at which A is derived on a second condition Δ' for which $\phi \cap \Delta' = \emptyset$.

For irrelevance of second order, we have the I_2-marking definition.

Definition 24. A line on which a formula A is derived is I_2-marked at stage s of the proof iff there is a line, unmarked at stage s of the proof, on which a formula B is derived on a non-empty second condition such that $\rho(B) < \rho(A)$.

Definition 25. A formula A is finally derivable in an **RADE2**-proof iff it occurs unmarked in the proof and for any extension of that proof in which it is marked, there is an extension of that extension in which it is unmarked again.

Definition 26. $\Gamma \vdash_{\text{RADE2}} A$ iff A is finally derived in an **RADE2**-proof from Γ.

Example

Let $\Gamma = \{p \vee q,\ q \vee r,\ r \vee p,\ q \vee s,\ s \vee p\}$. We give a proof from Γ that illustrates the premise rule, the implicature rule and the marking definitions and that illustrates the difference between the reliability approach and the counting approach. For both approaches, it looks as follows at stage 4:

1	$p \vee q$	-	Prem	$[p, q]$	$\emptyset$
2	$q \vee r$	-	Prem	$[q, r]$	$\emptyset$
3	$\neg(p \wedge q)$	1	E	$[\neg p, \neg q]$	$\{p \vee q\}$
4	$\neg(q \wedge r)$	2	E	$[\neg q, \neg r]$	$\{q \vee r\}$

[22] If no Cab-formulas are derived, $\Phi^0_s(\Gamma) = \{\emptyset\}$.

At stage 7, $Cab(\theta_1)$ is derived, where we choose the form $((p \vee q) \wedge ((q \vee r) \wedge (r \vee p))) \wedge ((p \wedge q) \vee ((q \wedge r) \vee (r \wedge p)))$. The letter ϑ_1 at line 8 stands for the subformula $(p \wedge q) \vee ((q \wedge r) \vee (r \wedge p))$. Both approaches give the same results up till then, namely the cancellations of the exclusivity implicatures on lines 3, 4 and 6:

1	$p \vee q$	-	Prem	$[p, q]$	$\emptyset$	
2	$q \vee r$	-	Prem	$[q, r]$	$\emptyset$	
3	$\neg(p \wedge q)$	1	E	$[\neg p, \neg q]$	$\{p \vee q\}$	$C_2^r(7)$
4	$\neg(q \wedge r)$	2	E	$[\neg q, \neg r]$	$\{q \vee r\}$	$C_2^r(7)$
5	$r \vee p$	-	Prem	$[r, p]$	$\emptyset$	
6	$\neg(r \wedge p)$	5	E	$[\neg r, \neg p]$	$\{r \vee p\}$	$C_2^r(7)$
7	$Cab(\theta_1)$	1,2,5	RAD	$\{\{[p, q], \{[q, r], [r, p]\}\},$ $[\{p, q\}, [\{q, r\}, \{r, p\}]]\}$	$\emptyset$	
8	ϑ_1	1,2,5	RAD	$[\{p, q\}, [\{q, r\}, \{r, p\}]]$	$\emptyset$	

1	$p \vee q$	-	Prem	$[p, q]$	$\emptyset$	
2	$q \vee r$	-	Prem	$[q, r]$	$\emptyset$	
3	$\neg(p \wedge q)$	1	E	$[\neg p, \neg q]$	$\{p \vee q\}$	$C_2^c(7)$
4	$\neg(q \wedge r)$	2	E	$[\neg q, \neg r]$	$\{q \vee r\}$	$C_2^c(7)$
5	$r \vee p$	-	Prem	$[r, p]$	$\emptyset$	
6	$\neg(r \wedge p)$	5	E	$[\neg r, \neg p]$	$\{r \vee p\}$	$C_2^c(7)$
7	$Cab(\theta_1)$	1,2,5	RAD	$\{\{[p, q], \{[q, r], [r, p]\}\},$ $[\{p, q\}, [\{q, r\}, \{r, p\}]]\}$	$\emptyset$	
8	ϑ_1	1,2,5	RAD	$[\{p, q\}, [\{q, r\}, \{r, p\}]]$	$\emptyset$	

At stage 13, $Cab(\theta_2)$ is derived, where we choose the form $((p \vee q) \wedge ((q \vee s) \wedge (s \vee p))) \wedge ((p \wedge q) \vee ((q \wedge s) \vee (s \wedge p)))$. The letter ϑ_2 at line 14 stands for the subformula $(p \wedge q) \vee ((q \wedge s) \vee (s \wedge p))$. By reliability marking, the exclusivity implicatures on line 11 and 12 are cancelled too:

1	$p \vee q$	-	Prem	$[p, q]$	$\emptyset$	
2	$q \vee r$	-	Prem	$[q, r]$	$\emptyset$	
3	$\neg(p \wedge q)$	1	E	$[\neg p, \neg q]$	$\{p \vee q\}$	$C_2^r(7{,}13)$
4	$\neg(q \wedge r)$	2	E	$[\neg q, \neg r]$	$\{q \vee r\}$	$C_2^r(7)$
5	$r \vee p$	-	Prem	$[r, p]$	$\emptyset$	
6	$\neg(r \wedge p)$	5	E	$[\neg r, \neg p]$	$\{r \vee p\}$	$C_2^r(7)$
7	$Cab(\theta_1)$	1,2,5	RAD	$\{\{[p, q], \{[q, r], [r, p]\}\},$ $[\{p, q\}, [\{q, r\}, \{r, p\}]]\}$	$\emptyset$	
8	ϑ_1	1,2,5	RAD	$[\{p, q\}, [\{q, r\}, \{r, p\}]]$	$\emptyset$	
9	$q \vee s$	-	Prem	$[q, s]$	$\emptyset$	

10	$s \vee p$	-	Prem	$[s, p]$	$\emptyset$	
11	$\neg(q \wedge s)$	9	E	$[\neg q, \neg s]$	$\{q \vee s\}$	$C_2^r(13)$
12	$\neg(s \wedge p)$	10	E	$[\neg s, \neg p]$	$\{s \vee p\}$	$C_2^r(13)$
13	$Cab(\theta_2)$	1,9,10	RAD	$\{\{[p, q], \{[q, s], [s, p]\}\},$ $[\{p, q\}, [\{q, s\}, \{s, p\}]]\}$	$\emptyset$	
14	ϑ_2	1,9,10	RAD	$[\{p, q\}, [\{q, s\}, \{s, p\}]]$	$\emptyset$	

By counting marking in contrast, the exclusivity implicatures on line 4 and 6 are restored, because only in the situation where all exclusivity implicatures are applied except for the exclusivity implicature on line 3, the number of abnormalities is minimal:

1	$p \vee q$	-	Prem	$[p, q]$	$\emptyset$	
2	$q \vee r$	-	Prem	$[q, r]$	$\emptyset$	
3	$\neg(p \wedge q)$	1	E	$[\neg p, \neg q]$	$\{p \vee q\}$	$C_2^c(7{,}13)$
4	$\neg(q \wedge r)$	2	E	$[\neg q, \neg r]$	$\{q \vee r\}$	
5	$r \vee p$	-	Prem	$[r, p]$	$\emptyset$	
6	$\neg(r \wedge p)$	5	E	$[\neg r, \neg p]$	$\{r \vee p\}$	
7	$Cab(\theta_1)$	1,2,5	RAD	$\{\{[p, q], \{[q, r], [r, p]\}\},$ $[\{p, q\}, [\{q, r\}, \{r, p\}]]\}$	$\emptyset$	
8	ϑ_1	1,2,5	RAD	$[\{p, q\}, [\{q, r\}, \{r, p\}]]$	$\emptyset$	
9	$q \vee s$	-	Prem	$[q, s]$	$\emptyset$	
10	$s \vee p$	-	Prem	$[s, p]$	$\emptyset$	
11	$\neg(q \wedge s)$	9	E	$[\neg q, \neg s]$	$\{q \vee s\}$	
12	$\neg(s \wedge p)$	10	E	$[\neg s, \neg p]$	$\{s \vee p\}$	
13	$Cab(\theta_2)$	1,9,10	RAD	$\{\{[p, q], \{[q, s], [s, p]\}\},$ $[\{p, q\}, [\{q, s\}, \{s, p\}]]\}$	$\emptyset$	
14	ϑ_2	1,9,10	RAD	$[\{p, q\}, [\{q, s\}, \{s, p\}]]$	$\emptyset$	

The formulas that are derived at stage 14 following the reliability marking, are all finally derived. The latter is not the case for the counting marking, as the following extension illustrates:

1	$p \vee q$	-	Prem	$[p, q]$	$\emptyset$	$I_2(16)$
2	$q \vee r$	-	Prem	$[q, r]$	$\emptyset$	$I_2(17)$
3	$\neg(p \wedge q)$	1	E	$[\neg p, \neg q]$	$\{p \vee q\}$	$C_2^c(7{,}13)$
4	$\neg(q \wedge r)$	2	E	$[\neg q, \neg r]$	$\{q \vee r\}$	$I_2(18)$
5	$r \vee p$	-	Prem	$[r, p]$	$\emptyset$	$I_2(16)$
6	$\neg(r \wedge p)$	5	E	$[\neg r, \neg p]$	$\{r \vee p\}$	$I_2(18)$
7	$Cab(\theta_1)$	1,2,5	RAD	$\{\{[p, q], \{[q, r], [r, p]\}\},$ $[\{p, q\}, [\{q, r\}, \{r, p\}]]\}$	$\emptyset$	

8	ϑ_1	1,2,5	RAD	$[\{p,q\},[\{q,r\},\{r,p\}]]$	$\emptyset$	$I_2(15)$
9	$q \vee s$	-	Prem	$[q,s]$	$\emptyset$	$I_2(17)$
10	$s \vee p$	-	Prem	$[s,p]$	$\emptyset$	$I_2(16)$
11	$\neg(q \wedge s)$	9	E	$[\neg q,\neg s]$	$\{q \vee s\}$	$I_2(19)$
12	$\neg(s \wedge p)$	10	E	$[\neg s,\neg p]$	$\{s \vee p\}$	$I_2(19)$
13	$Cab(\theta_2)$	1,9,10	RAD	$\{\{[p,q],\{[q,s],[s,p]\}\},$ $[\{p,q\},[\{q,s\},\{s,p\}]]\}$	$\emptyset$	
14	ϑ_2	1,9,10	RAD	$[\{p,q\},[\{q,s\},\{s,p\}]]$	$\emptyset$	$I_2(15)$
15	$p \wedge q$	4,6,8	RAD	$\{p,q\}$	$\{q \vee r, r \vee p\}$	
16	p	15	RAD	p	$\{q \vee r, r \vee p\}$	
17	q	15	RAD	q	$\{q \vee r, r \vee p\}$	
18	$\neg r$	4,15	RAD	$\neg r$	$\{q \vee r, r \vee p\}$	
19	$\neg s$	11,15	RAD	$\neg s$	$\{q \vee r, r \vee p\}$	

Except for those on line and 7 and 13, all formulas that are derived at stage 19 following the counting marking, are finally derived.

Open Problem

As explained above, we did not want the exclusivity implicature to be applied on weakenings by Addition. Because **RAD** verifies only the disjunctions that are in a sense minimal, we chose it as the lower limit logic. In the resulting adaptive logics the exclusivity implicature can only be applied to minimal disjunctions.

The following example[23] shows that there are cases in which the premises might require an application of the exclusivity implicature to a disjunction that is *not* minimal. Let $\Gamma = \{p,\ p \vee q\}$:

1	p	-	Prem	p	$\emptyset$		
2	$p \vee q$	-	Prem	$[p,q]$	$\emptyset$	$I_1(1)$	
3	$\neg(p \wedge q)$	2	E	$[\neg p,\neg q]$	$\{p \vee q\}$	$C_1(2)$	$I_2(4)$
4	$\neg q$	1,3	RAD	$\neg q$	$\{p \vee q\}$	$C_1(2)$	

To interpret such premises adequately, a further differentiation is needed. Disjunctions that are not minimal, though relevant in another way– for example because they were minimal at some earlier moment in time or because they are minimal for some particular information source– should not be rejected. Solutions to this problem will be proposed in [5] and [14].

[23]We owe this example to Kristof De Clercq.

Conclusion

The aim of this article was to contribute to the task of formalizing pragmatics. In particular, we have attempted to model the exclusivity-interpretation of 'or' as a non-monotonic rule of inference in the framework of Adaptive Logic[24]. It emerged that taking into account other pragmatic aspects connected with 'or' is a precondition for adequately modelling the exclusivity interpretation. Specifically, *relevance* constraints on the assertion of disjunctions have to be taken into account. Therefore we started from a regimentation of these relevance constraints and formalized the exclusivity-interpretation on top of this regimentation.

The two pragmatic aspects of 'or' that were considered in this article belong to different categories. The exclusivity-interpretation is usually seen as an implicature related to the *Maxim of Quantity*, in Gricean terms, whereas the relevance constraint that is needed for the exclusivity-interpretation can also be seen as an implicature related to the *Maxim of Relation*. That these implicatures are of a different nature emerged in our formalization. Concerning the exclusivity interpretation of 'or', we took it to be (albeit defeasibly) part of what is said when one asserts $A \vee B$, that $\neg(A \wedge B)$.[25] In other words, the exclusivity-interpretation is of a *semantical* nature. The relevance constraint, in contrast, is of a more thoroughly pragmatic nature. For it is not (normally) *false* to assert $A \vee B$ when one has already asserted A, it is just being irrelevant. Because their natures are so different, it comes as something of a surprise that the exclusivity implicature and the relevance constraint are inextricably intertwined.

An advantage of Adaptive Logics is that their proof theories are close to the proof theory of Classical Logic. However, one might have the feeling that the proof theory that is proposed here to formalize pragmatic implicatures relating to 'or' is rather complicated. It must be admitted that

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[24] We have loosely used the terminology of adaptive logics. The logic **RAD** is in the strict sense of the word not a lower limit logic for **RADE1** or **RADE2**, because in general it is not the case that all **RAD**-consequences of an arbitrary premise set are **RADE1**- and **RADE2**-consequences of that premise set. Neither is **RAED** an upper limit logic in the strict sense of the word, there are premise sets for which some **RADE1**- or **RADE2**-consequences are not **RAED**-consequences. It would be interesting to study which logics can be and which logics can not be a lower limit logic or an upper limit logic (in the *strict* sense of the word) for **RADE1**, respectively **RADE2**. It seems that several constructions are possible and that they may provide new perspectives that could be useful for other problems.

[25] There is some discussion of this in the literature. See [9, p.77-79], where also epistemic and doxastic interpretations of the exclusivity implicature are discussed.

when one wants to draw out the implicature of a collection of long and interrelated disjunctions, this becomes a complicated matter. However, in ordinary life we only assert in a conversation very small collections of interrelated disjunctions of virtually never more than two or three members. In such situations, it does not seem unreasonable to claim that we actually do -albeit unconsciously- go through something like the computations which we have sketched. For more complex situations (which only very rarely arise), people simply cannot manage the complexity of the situation. We do claim that our proof theory gives the logically correct predictions even for those cases. Our theory aims at giving an extrapolation to arbitrarily complex sets of assertions of the logical mechanisms that are present in our processing of small classes of simple disjunctive statements.

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