Liquid drops attract or repel by the inverted Cheerios effect

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Edited by Kari Dalnoki-Veress, McMaster University, Hamilton, ON, Canada, and accepted by Editorial Board Member John D. Weeks May 4, 2016 (received for review January 27, 2016)

Solid particles floating at a liquid interface exhibit a long-ranged attraction mediated by surface tension. In the absence of bulk elasticity, this is the dominant lateral interaction of mechanical origin. Here, we show that an analogous long-range interaction occurs between adjacent droplets on solid substrates, which crucially relies on a combination of capillarity and bulk elasticity. We experimentally observe the interaction between droplets on soft gels and provide a theoretical framework that quantitatively predicts the interaction force between the droplets. Remarkably, we find that, although on thick substrates the interaction is purely attractive and leads to drop-drop coalescence, for relatively thin substrates a short-range repulsion occurs, which prevents the two drops from coming into direct contact. This versatile interaction is the liquid-on-solid analog of the “Cheerios effect.” The effect will strongly influence the condensation and coarsening of drops on soft polymer films, and has potential implications for colloidal assembly and mechanobiology.

elastocapillarity | wetting | soft matter | mechanosensing | droplets

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he long ranged interaction between particles trapped at a fluid interface is exploited for the fabrication of microstructured materials via self assembly and self patterning (1–5) and occurs widely in the natural environment when living organisms or fine particles float on the surface of water (6, 7). In a certain class of capillary interactions, the particles deform the interface because of their shape or chemical heterogeneity (8–10). In this case, the change in interfacial area upon particle–particle approach causes an attractive capillary interaction between the particles. In the so-called Cheerios effect, the interaction between floating objects is mainly due to the change in gravitational potential energy associated to the weight of the particles, which deform the interface while being supported by surface tension (11), and the same principle applies when the interface is elastic (12–14). The name “Cheerios effect” is reminiscent of breakfast cereals floating on milk and sticking to each other or to the walls of the breakfast bowl.

Here, we consider a situation opposite to that of the Cheerios effect, liquid drops deposited on a solid. The solid is sufficiently soft to be deformed by the surface tension of the drops, resulting in a lateral interaction. Recent studies have provided a detailed view of statics of single drop wetting on deformable surfaces (15–19). The length scale over which the substrate is deformed is set by the ratio of the droplet surface tension γ and the substrate shear modulus G. The deformation can be seen as an elastocapillary meniscus, or “wetting ridge,” around the drop (Fig. 1 A and B). Interestingly, the contact angles at the edge of the drop are governed by Neumann’s law, just as for oil drops floating on water. In contrast to the statics of soft wetting, its dynamics has only been explored recently. New effects such as stick slip motion induced by substrate viscoelasticity (20, 21) and droplet migration due to stiffness gradients (22) have been revealed. The possibility that elastocapillarity induces an interaction between neighboring drops is of major importance for applications such as drop condensation on polymer films (23) and self cleaning surfaces (24–27). The interaction between drops on soft surfaces might also provide insights into the mechanics of cell locomotion (28–30) and cell–cell interaction (31).

Here, we show experimentally that long ranged elastic deformations lead to an interaction between neighboring liquid drops on a layer of cross linked polydimethylsiloxane (PDMS). The layer is sufficiently soft for significant surface tension induced deformations to occur (Fig. 1). The interaction we observe can be thought of as the inverse Cheerios effect, because the roles of the solid and liquid phases are exchanged. Remarkably, the interaction can be either attractive or repulsive, depending on the geometry of the gel. We propose a theoretical derivation of the interaction force from a free energy calculation that self consistently accounts for the deformability of both the liquid drop and the elastic solid.

Experiment: Attraction Versus Repulsion

Here, the inverted Cheerios effect is observed with submillimeter drops of ethylene glycol on a PDMS gel. The gel is a reticulated polymer network formed by crosslinking small multifunctional prepolymers contrary to hydrogels, there is no liquid phase trapped inside the network. The low shear modulus of the PDMS

Significance

The Cheerios effect is the attraction of solid particles floating on liquids, mediated by surface tension forces. We demonstrate experimentally that a similar interaction can also occur for the inverse case, liquid particles on the surface of solids, provided that the solid is sufficiently soft. Remarkably, depending on the thickness of the solid layer, the interaction can be either purely attractive or become repulsive. A theoretical model, in excellent agreement with the experimental data, shows that the interaction requires both elasticity and capillarity. Interactions between objects on soft substrates could play an important role in phenomena of cell–cell interaction and cell adhesion to biological tissues, and be exploited to engineer soft smart surfaces for controlled drop coalescence and colloidal assembly.


The authors declare no conflict of interest.

This article is a PNAS Direct Submission. K.D.-V. is a guest editor invited by the Editorial Board.

Freely available online through the PNAS open access option.

See Commentary on page 7294.

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This article contains supporting information online at www.pnas.org/cgi/doi/10.1073/pnas.1601411113/-/DCSupplemental.
gel gives an elastocapillary length \( \ell = \gamma / G = 0.17 \text{ mm} \) sufficiently large to be measurable in the optical domain.

The interaction between two neighboring liquid drops is quantified by tracking their positions while they are sliding under the effect of gravity along the surface of a soft solid held vertically. The interaction can be either attractive (Fig. 1A) or repulsive (Fig. 1B); drops on relatively thick gel layers attract each other, whereas drops on relatively thin layers experience a repulsion.

The drop drop interaction induces a lateral motion that can be quantified by measuring the horizontal component of the relative droplet velocity, \( \Delta v_x \) (\( \Delta v_x > 0 \) implies repulsion). In Fig. 1C, we report \( \Delta v_x \) as a function of the separation \( d \), defined as the shortest distance between the surfaces of the drops. The drops (\( R \approx 0.5 \sim 0.8 \text{ mm} \)) exhibit attraction when sliding down a thick layer (\( h_0 = 8 \text{ mm} \), black curve), whereas they repel on a thin layer (\( h_0 = 0.04 \text{ mm} \), red curve). The value of \( \Delta v_x \) is larger at close proximity, signaling an increase in the interaction force. Spontaneous merging occurs where drops come into direct contact. Importantly, these interactions provide a new mechanism for droplet coarsening (or ordering) by coalescence (or its suppression) that has no counterpart on rigid surfaces.

The interaction force \( F \) can be inferred from the relative velocities between the drops, by using an effective “drag law,” where the drag is due to sliding on the gel. We first calibrate this drag law by considering drops that are sufficiently separated, so that they do not experience any mutual interaction. The motion is purely downward and driven only by a gravitational force \( F_g = mg \) (inertia is negligible). Fig. 1D shows that the droplet velocity \( v \) approximately scales as \( F_g^{0.2} \). This force velocity calibration curve is in good agreement with viscoelastic dissipation in the gel, based on which one expects the scaling law (21):

\[
\nu \sim \frac{\ell}{\tau} \left( \frac{F}{2\pi R} \right)^{1/n}.
\]

Here, \( n \) is the rheological exponent that emerges from the scale invariance of the gel network (32–34), and \( \tau \) is a characteristic timescale. The parameter values \( n \approx 0.61 \) and \( \tau \approx 0.68 \text{ s} \) are calibrated in a rheometer (SI Materials and Methods). Eq. 1 is valid for \( v \) below the characteristic rheological speed, \( \ell/\tau \approx 0.25 \text{ mm/s} \) for the silicone gel, whereas the reported speeds reach at most \( \approx 100 \text{ mm/s} \). The large viscoelastic dissipation in the gel exceeds the dissipation within the drop by orders of magnitude, and explains these extremely slow drop velocities observed experimentally (21, 35). In this case, it was also shown that all of the dissipation occurs in a very narrow region around the wetting ridge (21). Therefore, the dynamic substrate deformation approaches the corresponding static deformation beyond a distance \( \nu \tau \lesssim 60 \text{ nm} \) from the contact line. The force distance relation for the inverse Cheerios effect can now be measured directly using the independently calibrated force velocity relation (Fig. 1D and Eq. 1). By monitoring how the trajectories are deflected with respect to the downward motion of the drops, we obtain \( F \) (see Materials and Methods for additional details). Despite the different origins of calibration and interaction forces, both are balanced by the same dissipative mechanism—cause the dissipative viscoelastic force is nearly perfectly localized at the contact line (21), which corroborates the validity of our calibration routine.

The key result is shown in Fig. 2, where we report the interaction force \( F \) as a function of distance \( d \). Fig. 2A shows experimental data for the attractive force (\( F < 0 \)) between drops on thick layers (black dots), together with the theoretical prediction outlined below. Movie S1 shows an example of attractive drop drop interaction. The attractive force is of the order of microneutrons, which is comparable to both the capillary force scale \( \gamma R \) and the elastic force scale \( GR^2 \). The force decreases for larger distance and its measurable influence was up to \( d \sim R \).

Fig. 2B shows the interaction force between drops on thin layers. The dominant interaction is now repulsive (\( d \gtrsim h_0 \)) (Movie S2). Intriguingly, we find that the interaction is not purely repulsive, but displays an attractive range at very small distance. It is possible to access this range experimentally in case the motion of the individual drops is sufficiently closely aligned (Movie S3). The “neutral” distance for which the interaction force changes sign appears when the separation is comparable to the substrate thickness \( h_0 \), suggesting that the key parameter governing whether the drops attract or repel is the thickness of the gel.

Mechanism of Interaction: Rotation of Elastic Meniscus

We explain the attraction versus repulsion of neighboring drops by computing the total free energy \( \tilde{\epsilon} \) of drops on gel layers of different thicknesses. The interaction force between the drops is equal to the energy gradient with respect to the separation, \( -\partial \tilde{\epsilon} / \partial d \), which in the experiment is balanced by the forces due to viscoelastic dissipation in the vicinity of the contact line. In contrast to the normal Cheerios effect, which involves two rigid particles, both the droplet and the elastic substrate are deformable, and their shapes will change upon varying the distance \( d \). Hence, the interaction force involves both the elastic and the surface tension contributions to the free energy. The free energy emerges from self consistently computed shapes of the drops and elastic deformations.
To reveal the mechanism of interaction, we first consider 2D drops, for which the free energy can be written as follows:

$$\mathcal{E}[h] = \mathcal{E}_0[h] + \int_0^h \, dx \left( \gamma \sqrt{1 + x^2} \langle h \rangle \right) + \int_0^h \, dx \left( \gamma \sqrt{1 + x^2} \frac{\partial \gamma}{\partial x} \langle h \rangle \right).$$

The geometry is sketched in Fig. 3B and C, and further details are given in Supporting Information and Fig. S1. The elastic energy $\mathcal{E}_0$ is a functional of the profile $h(x)$ describing the shape of the elastic solid: the functional explicitly depends on the layer thickness and is ultimately responsible for the change from attraction to repulsion. The function $\gamma(h)$ represents the shape of the liquid vapor interface. The integrals in Eq. 2 represent the interfacial energies; they depend on the surface tensions $\gamma, \gamma_{SL}, \gamma_{SV}$ associated with the liquid vapor, solid liquid, and solid vapor interfaces, respectively. For the sake of simplicity, we ignore here the possibility of a dependence of surface energy on the elastic strain. In absence of the Shuttleworth effect (36), surface stress and surface energy are equal.

At equilibrium, the droplet shapes can be found by analyzing the changes in the free energy upon variations of the functions $h(x)$ and $\gamma(x)$. The variation of the contact line positions provide the relevant boundary conditions (19). However, when two drops are separated by a finite distance $d$, the drops are not at equilibrium: the gradient in free energy results in an overdamped motion in which changes in free energy are dissipated in the solid. To compute the interaction force $f$ (per unit length in the 2D model), one therefore needs to consider the work done by the dissipative force $-\partial \mathcal{E}/\partial h$ that we can assume to be localized near the inner contact line. This allows one to determine the interaction force $f = -\partial \mathcal{E}/\partial d$, with the convention that attractive forces correspond to $f < 0$ (see Supporting Information and Fig. S2 for details).

The energy minimization reveals the mechanism of drop drop interaction: the interaction force $f$ appears in the boundary condition for the contact angles,

$$f = \gamma \cos \theta + \gamma_{SL} \cos \theta_{SL} - \gamma_{SV} \cos \theta_{SV},$$

where the angles are defined in Fig. 3. Eq. 3 can be thought of as an “imbalance” of the static Neumann boundary condition. The resulting interaction force due to the elastocapillary energy gradient is balanced by the dissipation due to the viscoelastic nature of the substrate. For a single droplet, the contact angles satisfy Neumann’s law, which is Eq. 3 with $f = 0$ (Fig. 3A). On a thick elastic layer, the overall shape of the wetting ridge is of the following form (18, 19):

$$h(x) \sim \frac{\gamma}{G} \psi \left( \frac{x}{\gamma \sqrt{G}} \right),$$

where the horizontal scale is set by the elastocapillary length based on the solid surface tension $\gamma$. The origin of $f$ can be understood from the principle of superposition. Due to the substrate deformation of a single drop, a second drop approaching the first one will see a surface that is locally rotated by an angle $\phi \sim h \gamma / \gamma_{SL}$. The elastic meniscus near the inner contact line of this approaching drop will correspondingly rotate by an angle $\phi (\gamma \gamma_{SL})$. Importantly, changes in the liquid angle $\theta$ scale as $\sim h / (R \gamma / (GR))$, which for large drops can be ignored. As a consequence, this meniscus rotation induces a net resultant surface tension forces according to Eq. 3, which is balanced by the dissipative force $f$ due to the viscoelastic nature of the substrate (21). For small rotations, one obtains $f \approx \gamma \varphi$, where $\varphi$ follows from the single drop deformation (Eq. 4). There is no resultant interaction force from the stress below the drop, which, due to deformability of the drop, results only in a uniform pressure on the solid liquid interface.

The inverted Cheerios effect is substantially different from the Cheerios effect between two particles floating at the surface of a liquid. Apart from the drop being deformable, we note that the energy driving the interaction is different in the two cases: whereas the liquid interface shape is determined by the balance between gravity and surface tension in the Cheerios effect, the solid shape is determined by elastocapillarity in the inverted Cheerios effect. Another difference is the mechanism by which the interaction is mediated. The Cheerios effect is primarily driven by a change in gravitational potential energy, which implies a vertical displacement of particles: a heavy particle slides downward, like a bead on a string, along the deformation created by a neighboring particle (11). A similar interaction was recently
discussed for rigid cylinders that deform an elastic surface due to gravity (14). In contrast, the inverted Cheezer effect discussed here does not involve gravity and can be totally ascribed to elastocapillary tilting of the solid interfaces, as shown in Fig. 3.

The rotation of contact angles explains why the drop drop in interaction can be either attractive or repulsive. On a thick substrate, the second drop experiences solid contact angles that are rotated counterclockwise, inducing an attractive force (Fig. 3B). In contrast, on a thin substrate, the elastic deformation induced by the second drop has a nonmonotonic profile \( h(x) \). This is due to volume conservation of the substrate: lifting the gel near the contact line creates a depression at larger distances (Fig. 3C). The rotation of the contact angles thus changes sign, and, accordingly, the interaction force changes from attractive to repulsive. The relevant length scale for this phenomenon is set by the layer thickness \( h_0 \).

**Three-Dimensional Theory**

The extension of the theory to three dimensions is straightforward and allows for a quantitative comparison with the experiments. For the 3D case, we compute the shape of the solid numerically, by first solving for the deformation field induced by a single drop using an axisymmetric elastic Green’s function (18). Adding a second drop on this deformed surface gives an intricate deformation that is shown in Fig. 4 and Fig. S3. The imbalance of the Neumann law applies everywhere around the contact line: the background deformation induces a rotation of the solid contact angles around the drop. According to Eq. 3, these rotations result in a distribution of force per unit length contact line \( f = f(\beta)\hat{e}_\beta \), where \( \hat{e}_\beta \) is the radial unit vector and \( \beta \) is the azimuthal angle (Fig. 4). The resultant interaction force \( \vec{F} \) is obtained by integration along the contact line, \( \vec{F} = R \int d\beta f(\beta) \) (see Supporting Information and Fig. S4 for details).

By symmetry, this force is oriented along the line connecting the two drops.

The interaction force obtained by the 3D theory is indicated by the red curves in Fig. 2 A and B. The theory gives an excellent description of the experimental data without adjustable parameters. The quantitative agreement indicates that the interaction mechanism is indeed caused by the rotation of the elastic meniscus.

**Discussion**

In summary, we have shown that liquid droplets can exhibit a mutual interaction when deposited on soft surfaces. The interaction is mediated by substrate deformations, and its direction (repulsive versus attractive) can be tuned by varying the thickness of the layer. The measured force-distance relation is in quantitative agreement with the proposed elastocapillary theory. The current study reveals that multiple “pinchings” of an elastic layer by localized tractions \( \gamma \) lead to an interaction having a range comparable to \( R/G \). The key insight is that interaction emerges from the rotation of the elastic surface, providing a generic mechanism that should be applicable to a wide range of objects interacting on soft media.

Our model provides general concepts that are applicable to a wide range of experimental settings, whenever objects exert dipolar or quadrupolar forces on their substrate (the integral force must vanish in this case, however, as is the case, e.g., for cells (37)). The length scale of interaction is governed by the ratio of two quantities: the force (per unit length) \( \gamma \), and the substrate shear modulus \( G \). This can range from nanometers for small forces or stiff substrates, to hundreds of microns for strong forces or soft substrates. In biological settings, elastocapillary interactions may play a role in cell cell interactions, which are known to be sensitive to substrate stiffness (31). One example would be stem cell aggregates that interact with their extracellular matrix (38). In addition, the elastic interaction could also play a role in cell extracellular matrix interactions, as a purely passive force promoting aggregation between anchor points on the surface of adhered biological cells. For example, it has been demonstrated that a characteristic distance of about 70 nm between topographical features enables the clustering of integrins. These transmembrane proteins are responsible for cell adhesion to the surrounding matrix, mediating the formation of

![Fig. 3](image)

**Fig. 3.** Mechanism of interaction between two liquid drops on a soft solid. (A) Deformation \( h(x) \) induced by a single droplet on a thick substrate. The zoom near the contact line illustrates that the contact angles satisfy the Neumann condition. (B) A second drop placed on a thick substrate experiences a background profile due to the deformation induced by the neighboring drop on the right. This background profile is shown in red. As a consequence, the solid angles near the elastic meniscus rotate by an angle \( \phi \) (see zoom). This rotation perturbs the Neumann balance, yielding an attractive force \( f \). In the experiment, this force is balanced by the dissipative force due to the viscoelastic deformation of the wetting ridge. (C) On a thin substrate, the single drop profile yields a nonmonotonic elastic deformation. The zoom illustrates a rotation \( \phi \) of the Neumann triangle in the opposite direction, leading to a repulsive interaction.

![Fig. 4](image)

**Fig. 4.** Three dimensional calculation of deformation for a pair of axisymmetric drops. The elastocapillary meniscus between the two drops is clearly visible, giving a rotation of the contact angle around the drop. The total interaction force \( \vec{F} \) is obtained by integration of the horizontal force \( f \) (indicated in red) and is related to the free energy gradient associated with a change in separation between the drops. Parameter values are \( \ell / R = 0.1, \gamma / \gamma_s = 1 \).
strong anchor points when cells adhere to substrates (39, 40). Assuming that the topographical features “pinch” the cell with a force likely comparable to the cell’s cortical tension, which takes values in the range 0.1–1 mN/m (41–44), and an elastic modulus of 10^3–10^4 Pa in the physiological range of biological tissues (45), one predicts a range of interaction consistent with observations.

More generally, substrate mediated interactions could be dynamically programmed using the responsiveness of many gels to external stimuli (pH, temperature, electric fields). Possible applications range from fog harvesting and cooling to self cleaning or anti fouling surfaces, which rely on controlling drop migration and coalescence. The physical mechanisms revealed here, in combination with the fully quantitative elastocapillary theory we propose, pave the way for new design strategies for smart soft surfaces.

Materials and Methods

Supporting Information

provides further technical information, the derivation underlying Eq. 3, and the numerical scheme used for the calculations of Figs. 2 and 4. Movies S1–S3 show typical experiments of drop interactions.

Substrate Preparation. The two prepolymer components (Dow Corning; CY5276 A and B) were mixed in a ratio of 1:1.5 (A:B). Thick elastic layers (~8 mm) were prepared in Petri dishes (diameter, ~90 mm). Thin layers (~40 μm) were prepared by spin coating the gel onto silicon wafers. The thickness was determined by color interferometry. See SI Materials and Methods for details on substrate curing and rheology (Fig. S5).

Determining the Interaction Between Drops. Droplets of ethylene glycol (V = 0.3–0.8 μL) were pipetted onto a small region near the center of the cured substrate.

The sample was then mounted vertically so that gravity acts along the surface normal (z direction; compare Fig. 1 A and B). The droplets were observed in transmission (thin layers) or reflection (thick layers) with confocal microscopy, using a telecentric lens (Zeiss苦门 1x) and a digital camera (pco 1200). Images were taken every 10 s. The contours of the droplets were determined by a standard correlation technique.

At large separation, droplets move downward due to gravity. The gravitational force on each droplet is proportional to its volume. The relation between force and velocity follows the same power law as the rheology, as was explained recently (21).

Individual droplets have different volumes and move with different velocities. Thus their distances change with time. Whenever two droplets approach each other, their trajectories change due to their interaction. Drops on thick substrates (Fig. 1C, black) attract and eventually adhere by capillary forces (21). On a rigid surface these droplets would not have merged. The opposite holds for droplets on thin layers (red): the droplets repel each other, which prevents coalescence.

To determine the interaction forces, we first evaluate the velocity vector of each individual droplet. The droplets move in a quasi stationary manner, and the total force vector acting on each droplet is aligned with its velocity vector. The magnitude of the total force is obtained through calibration from the data shown in Fig. 1D. The interaction force is obtained by subtracting the gravitational force from the total force. Fig. 2A shows data from nine individual droplet pairs, corresponding to different volumes and different locations on the substrate. Fig. 2B shows data from 18 different droplet pairs. The raw data have been averaged over distance bins, taking the SD as error bar.

ACKNOWLEDGMENTS. L.B. acknowledges useful discussion on mechanobiology with Dr. Nuria Gavara, and financial support from European Union Grant 615987. A.P. acknowledges financial support from NWO through VICI Grant 113040. A.P. and J.H.S. acknowledge financial support from European Research Council Consolidator Grant 616918.

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