First publ. in: Physical Review E 60 (1999), 5, pp. 5202-5207

Depinning transition and thermal fluctuations in the random-field Ising model

L. Roters,* A. Hucht,† S. Lübeck,‡ U. Nowak,§ and K. D. Usadel‖
Theoretische Tieftemperaturphysik, Gerhard-Mercator-Universität Duisburg, 47048 Duisburg, Germany
(Received 20 May 1999)

We analyze the depinning transition of a driven interface in the three-dimensional (3D) random field Ising model (RFIM) with quenched disorder by means of Monte Carlo simulations. The interface initially built into the system is perpendicular to the [111] direction of a simple cubic lattice. We introduce an algorithm which is capable of simulating such an interface independent of the considered dimension and time scale. This algorithm is applied to the 3D RFIM to study both the depinning transition and the influence of thermal fluctuations on this transition. It turns out that in the RFIM characteristics of the depinning transition depend crucially on the existence of overhangs. Our analysis yields critical exponents of the interface velocity, the correlation length, and the thermal rounding of the transition. We find numerical evidence for a scaling relation for these exponents and the dimension d of the system. [S1063-651X(99)03011-1]

PACS number(s): 68.35.Rh, 75.10.Hk, 75.40.Mg

I. INTRODUCTION

Driven interfaces in systems with quenched disorder display with increasing driving force a transition from a phase where no interface motion takes place to a phase with a finite interface velocity. This so-called depinning transition is caused by a competition of driving force and quenched disorder. While the driving force tends to move the interface, the motion is hindered by the disorder (see, e.g., Ref. [1]).

Depinning transitions are found in a large variety of physical problems, such as fluid invasion in porous media [2], depinning of charge density waves [3,4] or field-driven motion of domain walls in ferromagnets [5]. In magnetic systems a domain wall separates regions of different spin orientations. With the assumption that the corresponding interface shows properties of an elastic membrane, it has been argued [5] that the depinning of the interface can be described by an Edwards-Wilkinson equation [6] with quenched disorder. While the interface motion in a system with quenched disorder near the critical threshold is theoretically often investigated in the absence of thermal fluctuations, these fluctuations affect the experimental study of the depinning transition [7–9]. The crucial point is that energy barriers which are responsible for a trapping of the interface in a metastable state at zero temperature can always be overcome due to thermal fluctuations. For driving fields far below the transition field this yields a thermally activated creep motion (see Ref. [9], and references therein). This behavior changes approaching the transition point, where finite temperatures cause a rounded depinning transition (for experimental evidence see, for instance, Fig. 2 in [9]). To describe the dependence of the interface velocity on driving force and temperature near the transition point, a scaling ansatz has been proposed [4]. This ansatz which is based on an equation of motion for sliding charge density waves predicts its characteristic velocity to be a power law of temperature at the critical threshold. This scaling ansatz has been shown to be a valid description for the depinning of a domain wall in the 2D random field Ising model (RFIM) with quenched disorder [10].

The outline of our paper is as follows. Section II describes the RFIM and reflects properties of [111] interfaces in this model. In Sec. III we discuss the depinning transition from a microscopic point of view, analyzing the mechanisms of interface motion near the depinning transition. Also, we determine numerically the exponents of the interface velocity and of the correlation length, allowing an estimation of the universality class of the 3D RFIM. In Sec. IV we analyze the influence of temperature on the depinning transition. By assuming the interface velocity to be a generalized homogenous function, our analysis is based on applying standard concepts of critical equilibrium phenomena. The ansatz allows the characterization of the thermal rounding of the depinning transition by a critical exponent δ. We determine δ for the depinning transition in the 3D RFIM and find numerical evidence for a scaling relation among certain critical exponents characterizing this transition. This scaling relation also holds in the 2D RFIM analyzed previously [10].

II. INTERFACES IN THE RFIM

We investigate the 3D RFIM with quenched disorder on a simple cubic lattice. The Hamiltonian of the system is given by

$$\mathcal{H}=-\frac{J}{2}\sum_{\langle i,j\rangle} S_iS_j-H\sum_i S_i-\sum_i h_iS_i, \qquad (1)$$

where the first sum is restricted to nearest neighbors. H denotes the driving field and h_i quenched random fields which are uniformly distributed within an interval $[-\Delta,\Delta]$. Both fields as well as the temperature T are given in units of the coupling constant J. We carry out Monte Carlo simulations with single-spin-flip dynamics and we use transition probabilities $p(S_i\to -S_i,T)$, where T denotes the temperature,

*Electronic address: lars@thp.uni-duisburg.de
†Electronic address: fred@thp.uni-duisburg.de
‡Electronic address: sven@thp.uni-duisburg.de
§Electronic address: uli@thp.uni-duisburg.de
‖Electronic address: usadel@thp.uni-duisburg.de

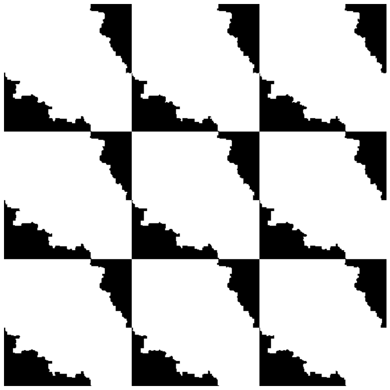

FIG. 1. Periodic images of a moving interface in $d=2$. Antiperiodic boundary conditions are applied. Black areas correspond to $S_i<0$ and white areas to $S_i>0$.

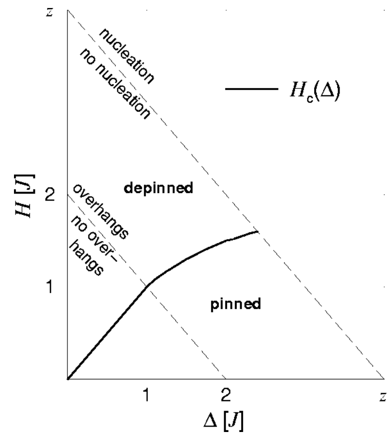

FIG. 2. Dependence of magnetization reversal processes on system parameters in the RFIM at $T=0$. The dimension enters only through the number z of nearest neighbors. The bold line indicates the dependence of the critical field H_c on the strength of the disorder Δ. For $\Delta<J$ the critical field equals Δ, while for $\Delta>J$ the bold line is only a sketch. The dashed lines indicate the regions where overhangs and island growth appear.

according to a heat-bath-Algorithm (see e.g., Ref. [11], and references therein). At zero temperature these transition probabilities reduce to

$$p(S_i \rightarrow -S_i,0) = \begin{cases} 1: \delta\mathcal{H}<0, \\ 1/2: \delta\mathcal{H}=0, \\ 0: \delta\mathcal{H}>0, \end{cases} \tag{2}$$

where $\delta\mathcal{H}=\mathcal{H}(-S_i)-\mathcal{H}(S_i)$. We investigate three-dimensional cubic systems of linear extension from $L=12$ to $L=162$.

An initially flat interface is built into the system separating regions of up and down spins. The applied field H drives the interface. Within the Monte Carlo simulation spins adjacent to the interface flip causing a movement of the interface. Also, nucleation may occur, i.e., a spin initially parallel to all of its neighbors may turn. Since we are interested in the scaling behavior of the interface motion in the vicinity of the depinning transition, it is essential that within the observation time nucleation does not occur. The minimum energy needed for isolated spin flips is $2(zJ-H-\Delta)$. As long as this quantity is large as compared to temperature, the time scales on which nucleation and interface motion occur are separated, and within the observation time no nucleation takes place [10]. In particular, there is no need to suppress artificially nucleation or isolated spin flips during the simulation.

The analysis of interface motion on simple cubic lattices considers usually [100] interfaces. However, investigating [100]-interfaces in the limit of vanishing disorder means that the interface motion is restricted to driving fields $H/J>z-2$ (see Ref. [12]). To avoid this, we consider [111] interfaces which move in the absence of disorder at arbitrarily small driving fields increasing the separation of time scales for interface motion and nucleation even further [10].

We have found that the most convenient way to implement [111] interfaces in the numerics is the introduction of antiperiodic boundary conditions. This implementation is illustrated in Fig. 1. For simplicity, periodic images of a snapshot of an interface in $d=2$ are shown. As can be seen from Fig. 1, the orientations of up and down are exchanged when passing the boundaries of the system. Of course, the exchange of up and down also affects the driving field whose sign has to be chosen in an appropriate manner. Our implementation will work as long as the different parts of the interface do not interact. An interaction takes place if the interface width $w\sim L^{\zeta}$ is of the same magnitude as the typical distance a between two neighboring parts of the interface. This distance is proportional to the linear extension of the system $a\propto L$ independent of the considered dimension. Our implementation is therefore applicable to situations where $\zeta<1$. Despite this restriction antiperiodic boundary conditions have the advantages that they can be applied to any dimension and generalized to other orientations of the interface. They are a natural choice for interfaces, because the moving interface can be investigated without any time limit. This is especially an advantage close to the depinning transition, where the critical slowing down effect causes large relaxation times [13].

In our implementation the moving interface visits the same spatial position after finite time intervals. Therefore, we have to update the random fields in a spatial area visited by the interface before it moves again into this area. This is easily done by drawing new random fields at sites away from the interface. Thus, the average over samples necessary in Monte Carlo simulations is obtained by averaging over large times. However, if the interface is trapped a usual sample average has to be done.

III. ZERO TEMPERATURE

In the RFIM with an interface initially built into the system, there are in general two magnetization reversal processes: interface motion and nucleation. Without thermal fluctuations the second process does not occur as long as $(H+\Delta)/J$ does not exceed the number z of nearest neighbors. The corresponding threshold is shown in Fig. 2 (upper broken curve). Above this threshold nucleation processes take place and interfere with the interface motion.

In the following we are interested in the influence of overhangs on the value of the critical field $H_c(\Delta)$ at which the transition takes place. Close to the depinning transition, there

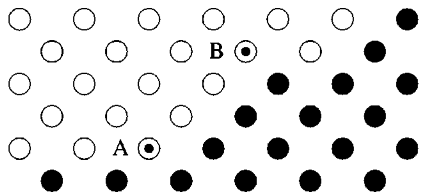

FIG. 3. Part of a diagonal interface in $d=2$. Different orientations of the spins S_i are denoted by black circles (favored by the driving field) and white circles, respectively. To cause a spin flip at the A, B sites, different driving fields are necessary (see text).

are two important kinds of spin flips (see Fig. 3). All spins of type A with

$$\sum_{\langle j\rangle} S_A S_j = 0 \quad \text{will flip if} \quad H \geqslant H_0 = \Delta, \tag{3}$$

while the first spin of type B with

$$\sum_{\langle j\rangle} S_B S_j = -2 \quad \text{can flip if} \quad H \geqslant H_{-2} = 2J - \Delta. \tag{4}$$

Here, the sum is taken over nearest neighbors of A and B, respectively. If the strength of disorder Δ is smaller than the exchange energy J, then it follows that $H_0 < H_{-2}$. The critical field at which the transition takes place is given by $H_c(\Delta < J) = \Delta$. Hence, no overhangs occur in the vicinity of the transition point. Taking into account the transition probabilities given by Eq. (2), this value of the critical field means that the interface velocity depends neither on the driving field nor on the strength of disorder as long as no overhangs occur ($H < H_{-2}$). In particular, in the absence of overhangs the interface velocity observed in a disordered system coincides with that of a nondisordered system ($\Delta = 0$). Figure 4 and its inset show numerical data which confirm this scenario for the 3D RFIM within the error bars.

Next we investigate the depinning transition occurring in the RFIM for $\Delta > J$. In this case the transition takes place at a certain field $H_c < \Delta$ as can be understood from the following consideration: For $H_0 < H_{-2}$, not all spins of type A can flip if $H < \Delta$. But because of the existence of overhangs, a second growth mechanism is possible to the interface: If a spin of type A cannot flip due to its large random field h_i, an overhang created elsewhere can cause an avalanche by which additional neighbors of A are flipped. Thus the interface can be kept moving. Contrary to the regime $\Delta < J$, the interface motion now is based on the existence of overhangs. Note that our considerations do not depend on the dimension d of the system, because Eqs. (3) and (4) are independent of d.

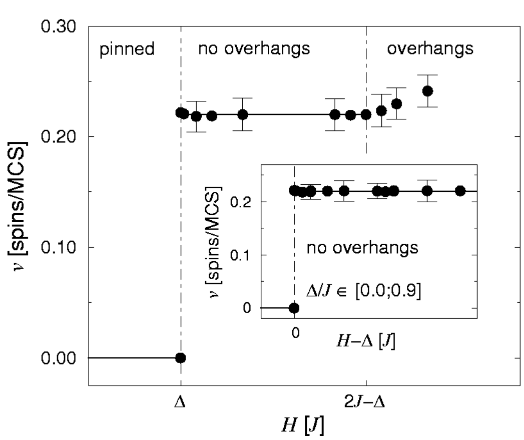

FIG. 4. Interface velocity v and its dependence on the driving field H for $\Delta/J = 0.7$. The depinning transition takes place at $H = \Delta$. The inset shows interface velocities for different system sizes $L \in \{30,42\}$ and ratios $\Delta/J < 1$. For reasons of clearness not all error bars are shown.

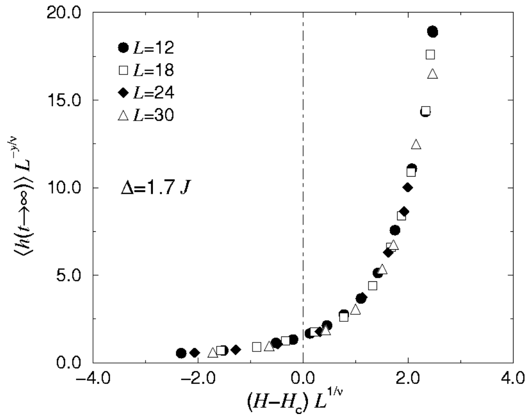

FIG. 5. Scaling plot of the distance $\langle h(t\to\infty)\rangle$ traveled by the interface before pinning occurs as a function of the driving field H according to Eq. (6). The data collapse yields $1/\nu = 1.31 \pm 0.07$, $y/\nu = 0.98 \pm 0.4$, and $H_c = (1.371 \pm 0.003)J$.

We start examining the regime $\Delta > J$ in the 3D RFIM numerically by investigating the depinning transition from below. We analyze the disorder-averaged distance $\langle h(t \to \infty)\rangle$ traveled by an initially flat interface before pinning occurs. This quantity is closely related to the total volume invaded by a growing domain which was analyzed in Refs. [14,15]. However, while in Refs. [14,15] the driving force is increased step by step to allow for relaxation processes in between, we focus our attention to driving fields which remain unchanged during the interface motion.

Below the depinning transition $\langle h(t\to\infty)\rangle$ is finite. Approaching the transition point with increasing driving field, the distance traveled before pinning occurs increases and finally diverges at the transition point. We assume that in the vicinity of the transition point $\langle h(t\to\infty)\rangle$ diverges algebraically, characterized by some exponent y,

$$\langle h(t\to\infty)\rangle \sim (H_c - H)^{-y}, \tag{5}$$

where H_c denotes the critical field observed in a system of infinite extension. In a finite system with linear dimension L finite-size scaling is assumed. The corresponding scaling ansatz reads

$$\langle h(t\to\infty)\rangle = L^{y/\nu} f[(H-H_c)L^{1/\nu}], \tag{6}$$

with $f(x) \sim |x|^{-y}$ for $x \to -\infty$. Note that $\langle h(t\to\infty)\rangle$ also diverges in any finite system which means that $f(x)$ should diverge at a finite value of $x^\star$. The corresponding driving field defines a size dependent critical field $H_c(L)$ given by $[H_c(L) - H_c]L^{1/\nu} = x^\star$. A scaling plot of the data according to Eq. (6) is shown in Fig. 5. The divergence of $f(x)$ occurs at $x^\star \approx 2.5$ showing that in a finite system the threshold field is always shifted to fields larger that H_c.

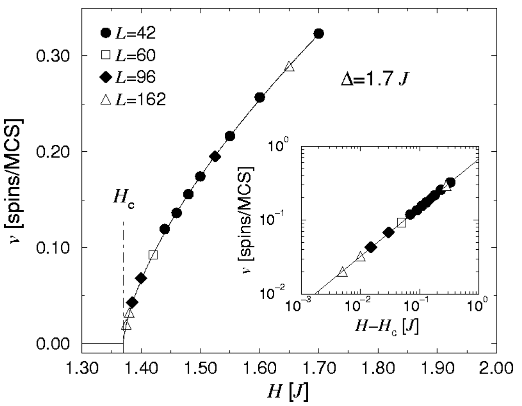

FIG. 6. Dependence of the interface velocity v on the driving field H in the vicinity of the transition point. Approaching the critical field H_c, the system size L is increased in order to avoid finite-size effects. Fitting the data according to Eq. (7) (solid line) yields $\beta = 0.66 \pm 0.04$, $H_c = (1.37 \pm 0.01)J$, and $A = 0.67 \pm 0.03$.

The critical exponent of the correlation length parallel to the interface is given by $1/\nu = 1.31 \pm 0.07$ and the critical field turns out to be $H_c = (1.371 \pm 0.03)J$. The value of ν coincides with Ref. [15], where an [100] interface in the self-affine growth regime corresponding in our case to $\Delta > J$ has been investigated. This suggests that the behavior of the correlation length at the depinning transition does not depend on the orientation of the interface in the RFIM.

In the following we consider the disorder averaged interface velocity $v = \langle dh/dt \rangle$ above the transition point in the limit of large times. This quantity can be interpreted as the order parameter of the depinning transition. Approaching a continuous phase transition the order parameter vanishes in leading order according to

$$v(H) = A(H - H_c)^{\beta}. \tag{7}$$

The corresponding data are shown in Fig. 6. The prefactor A is a nonuniversal constant which can be used to compare the results obtained at zero temperature with those presented in the next section. Since in the vicinity of the depinning transition finite-size effects may become important, we calculated each interface velocity $v(H)$ in systems of different linear extension L. For sufficiently large L we observed no significant dependence on the system size from which we concluded that the data shown in Fig. 6 correspond within negligible errors to those of the limit $L \to \infty$. As can be seen from the data, Eq. (7) is fulfilled and we obtain $A = 0.671 \pm 0.03$, $\beta = 0.66 \pm 0.04$, and $H_c = (1.37 \pm 0.01)J$.

The values of β and ν obtained from our analysis coincide within the error bars with those of the Edwards-Wilkinson equation with quenched disorder in $d = 2+1$, $\beta_{EW} = 2/3$, and $\nu_{EW} = 3/4$. These values are obtained by an ϵ expansion within a functional renormalization group scheme (see Refs. [1,16]). While the value of β_{EW} is obtained to first order of ϵ, there are arguments that ν_{EW} is exact in all orders to ϵ [1,17]. Taking this into account, our results suggest that the depinning transition of a domain wall in the 3D RFIM with quenched disorder is in the same universality class as the depinning transition of the corresponding Edwards-Wilkinson equation which also coincides with Refs. [18,19].

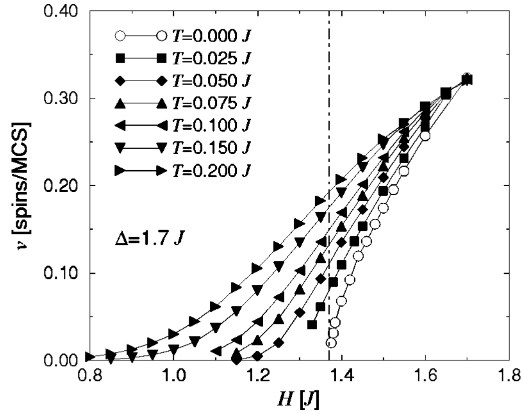

FIG. 7. Dependence of the interface velocity on the driving field for different temperatures, as indicated. The open symbols are from Fig. 6. The vertical line denotes the critical field obtained at $T = 0$.

IV. FINITE TEMPERATURES

In this section we study the influence of finite temperatures on the depinning transition. For $T > 0$ the interface velocity does not vanish for finite driving fields since the energy needed to overcome local energy barriers is provided by thermal fluctuations at any finite T. This results in a rounded depinning transition. The rounding can be seen in Fig. 7, where interface velocities for different driving fields and temperatures are presented. As expected, the rounding of the transition increases with increasing temperature. Again, we ensured that the interface velocities presented in this and the following figures correspond within negligible errors to those of the thermodynamic limit.

To analyze the thermal rounding of the depinning transition quantitatively, we first note that the depinning transition can be described in terms of a continuous nonequilibrium phase transition. This is suggested by the divergence of the correlation length (see determination of ν and Fig. 5) and the dependence of the interface velocity on the driving field near the transition point (Fig. 6). In the standard theory of critical phenomena a continuous phase transition is characterized by critical exponents (see, for instance, Ref. [20], and references therein). Beside β describing the field dependence of the order parameter and ν characterizing the divergence of the correlation length near the transition point, the rounding of a phase transition is characterized by the critical exponent δ. In magnetic systems, for instance, β and δ determine the magnetic equation of state. We now apply this approach to the depinning transition by assuming its order parameter to be a generalized homogenous function of temperature and driving field,

$$v[T, H - H_c] = \lambda v[\lambda^{a_T} T, \lambda^{a_H}(H - H_c)]. \tag{8}$$

Choosing $\lambda = T^{-1/a_T}$ we obtain the scaling ansatz

$$v(T,H) = T^{1/\delta} f_T[(H - H_c) T^{-1/\beta\delta}], \tag{9}$$

with $f_T(x \to 0) = \text{const}$. In particular, this equation corresponds to the magnetic equation of state [20]. From an equation of motion of sliding charge density waves a scaling form corresponding to Eq. (9) has been obtained [4]. Note that

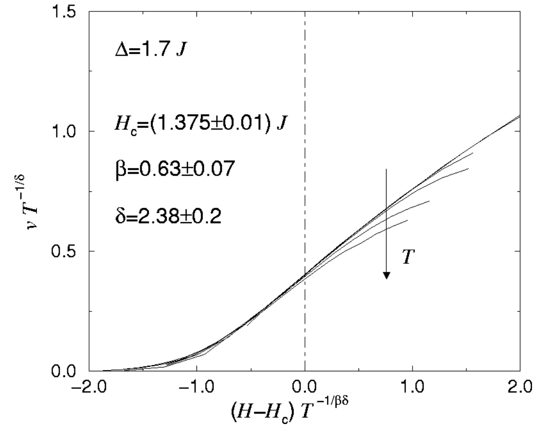

FIG. 8. Dependence of the interface velocity on H for given values of T. The data shown are identical to those in Fig. 7 for $T>0$ and rescaled according to Eq. (9).

contrary to Ref. [4] our ansatz which is based on Eq. (8) yields no predictions on the values for β and δ.

It has been shown previously that Eq. (9) is valid in the 2D RFIM with quenched disorder [10]. We have tested this scaling ansatz in the present situation for the 3D RFIM with the interface velocities shown in Fig. 7. As can be seen from Fig. 8, the scaling ansatz leads to a data collapse for $\beta=0.63\pm0.07$, $\delta=2.38\pm0.2$, and $H_c=(1.375\pm0.01)J$. Thus at $H=H_c$ the influence of temperature on the interface velocity can be described by a power law $v\sim T^{1/\delta}$. To support this value for δ we can determine δ from a different scaling function obtained from Eq. (8) by choosing $\lambda=|H-H_c|^{-1/a_H}$:

$$v(T,H)=(H-H_c)^{\beta}f_H[(H-H_c)^{-\beta\delta}T], \qquad (10)$$

with $f_H(x\rightarrow 0)=\text{const}$. This ansatz is valid above the transition point and it is closely related to Eq. (9). It corresponds to a different formulation of the magnetic equation of state. Interface velocities rescaled according to Eq. (10) are shown in Fig. 9. One obtains $\beta=0.67\pm0.03$, $\delta=2.55\pm0.37$, and $H_c=(1.37\pm0.05)J$. This result confirms within the error bars the value of δ determined by Eq. (9). Beside these quantities the data collapse also allows a determination of the prefactor $A=f_H(x\rightarrow 0)$ [see Eq. (7)] which turns out to be $A=0.685\pm0.025$.

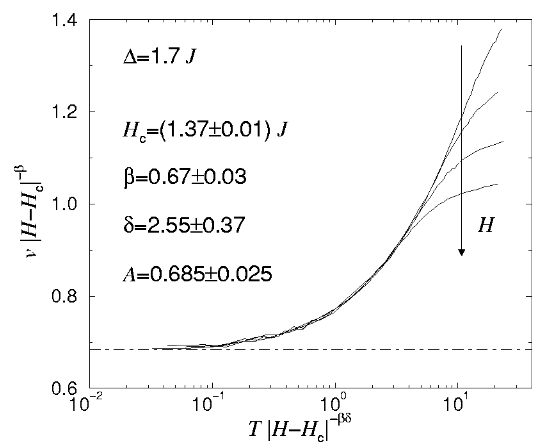

FIG. 9. Dependence of the interface velocity on T for given values of H. The data are rescaled according to Eq. (10). The horizontal line marks the value of A which is given by Eq. (7).

The values of A, H_c, and β found for $T>0$ coincide within sufficient accuracy with those values obtained at $T=0$. We have demonstrated that Eqs. (9) and (10) are valid confirming that the interface velocity is a generalized homogenous function in the vicinity of the transition point [21]. Thus, the influence of temperature on the depinning transition can be described within well-established concepts.

The knowledge of β and δ allows a test of the scaling relation $\delta=2+1/\beta$ proposed by Tang and Stepanow [22]. This scaling relation was shown to be fulfilled in the 2D RFIM [10]. For $\beta\approx 0.67$ the scaling relation suggests $\delta\approx 3.5$ which is not supported by our results. On the other hand, standard theory of critical phenomena predicts relations among critical exponents. For instance, combining the Rushbrooke, the Widom, and the hyperscaling relation yields in equilibrium physics

$$\delta=\frac{d\nu}{\beta}-1. \qquad (11)$$

This scaling relation is valid in dimensions d below the upper critical dimension d_c due to the restriction of the hyperscaling relation to $d<d_c$. We have tested the scaling relation (11) with the numerically evaluated exponents at the depinning transition and found out that both the exponents in the present case $d=3$ as well as the exponents for $d=2$ ($\nu_{2D}\approx 1.0$, $\beta_{2D}\approx 0.33$, and $\delta_{2D}\approx 5.0$; see Ref. [10]) fulfill Eq. (11) within the error bars. Unfortunately however, a firm foundation of this scaling relation in the present situation for nonequilibrium phase transitions is unknown.

V. CONCLUSION

We investigated the motion of a driven interface in a magnetic system with quenched disorder. To improve the efficiency of our numerics we applied antiperiodic boundary conditions. These boundary conditions allow to investigate the interface motion on any time scale. At zero temperature a depinning transition occurs at a finite driving field. We discussed the influence of overhangs and avalanches on this transition. If the strength of disorder exceeds the coupling constant, the interface motion is based on the existence of overhangs. Under these circumstances the depinning transition can be characterized by critical exponents, both below and above the critical threshold. Our results suggest that the depinning transition of a domain wall in the 3D RFIM with quenched disorder and the depinning transition of the corresponding Edwards-Wilkinson equation are in the same universality class.

Different distributions of random numbers are possible but so far we investigated interface motion only in the presence of uniformly distributed noise. The dependence of the interface motion on the particular choice of the disorder distribution like the bimodal or Gaussian distribution remains an open question and further investigations are needed to clarify this point.

Experimental investigations of domain wall motion in magnetic systems take place at finite temperatures. Both in experimental investigations, for instance on ultrathin magnetic film structures [9], and in our simulations it turns out that the interface velocity does not vanish below the critical threshold due to thermal fluctuations. In Ref. [9] creep motion was analyzed which occurs for $H \ll H_c$ but we are not aware of experimental measurements very close to the critical field H_c. Measurements which allow an estimation of the universality class of the depinning transition are possible (see, e.g., Ref. [23] where the roughness exponent of a domain wall in CoPt is analyzed) and future work in this context is desirable.

We assume the interface velocity to be a generalized homogenous function of temperature and driving field. The validity of this approach is confirmed by the fact that at the threshold field the interface velocity vanishes with decreasing temperature according to a power law characterized by an exponent δ. We have tested a scaling relation [Eq. (11)] among different exponents characterizing the depinning transition and found numerical evidence that the scaling relation is valid both in the 2D and the 3D RFIM.

ACKNOWLEDGMENTS

This work was supported by the Deutsche Forschungsgemeinschaft through the Graduiertenkolleg *Struktur und Dynamik heterogener Systeme* at the University of Duisburg, Germany.

[1] H. Leschhorn, T. Nattermann, S. Stepanow, and L.-H. Tang, Ann. Phys. (Leipzig) **6**, 1 (1997).
[2] M. Cieplak and M. Robbins, Phys. Rev. Lett. **60**, 2042 (1988).
[3] D. S. Fisher, Phys. Rev. Lett. **50**, 1486 (1983).
[4] D. S. Fisher, Phys. Rev. B **31**, 1396 (1985).
[5] R. Bruinsma and G. Aeppli, Phys. Rev. Lett. **52**, 1547 (1984).
[6] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. London, Ser. A **381**, 17 (1982).
[7] F. Ladieu, M. Sanquer, and J. P. Bouchaud, Phys. Rev. B **53**, 973 (1996).
[8] U. Nowak, J. Heimel, T. Kleinefeld, and D. Weller, Phys. Rev. B **56**, 8143 (1997).
[9] S. Lemerle, J. Ferré, C. Chappert, V. Mathet, T. Giamarchi, and P. Le Doussal, Phys. Rev. Lett. **80**, 849 (1998).
[10] U. Nowak and K. D. Usadel, Europhys. Lett. **44**, 634 (1998).
[11] K. Binder and D. W. Heermann, *Monte Carlo Simulation in Statistical Physics*, Vol. 80 of Springer Series in Solid-State Sciences, 3rd ed. (Springer, Heidelberg, 1997).
[12] B. Koiller, H. Ji, and M. O. Robbins, Phys. Rev. B **46**, 5258 (1992).
[13] K. Binder, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, London, 1976), Vol. 5b.
[14] N. Martys, M. O. Robbins, and M. Cieplak, Phys. Rev. B **44**, 12 294 (1991).
[15] H. Ji and M. O. Robbins, Phys. Rev. B **46**, 14 519 (1992).
[16] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leschhorn, J. Phys. II **2**, 1483 (1992).
[17] O. Narayan and D. S. Fisher, Phys. Rev. B **48**, 7030 (1993).
[18] L. A. N. Amaral, A. L. Barabási, and H. E. Stanley, Phys. Rev. Lett. **73**, 62 (1994).
[19] L. A. N. Amaral, A. L. Barabási, H. A. Makse, and H. E. Stanley, Phys. Rev. E **52**, 4087 (1995).
[20] H. E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Oxford University Press, Oxford, 1971).
[21] A. Hankey and H. E. Stanley, Phys. Rev. B **6**, 3515 (1972).
[22] L.-H. Tang and S. Stepanow (unpublished).
[23] M. Jost, J. Heimel, and T. Kleinefeld, Phys. Rev. B **57**, 5316 (1998).