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# Income Risks, Education, and Taxation as Public Insurance

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# Income Risks, Education, and Taxation as Public Insurance

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# Income Risks, Education, and Taxation as Public Insurance*

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June 08, 2007

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# 1 Introduction

In 1789, Benjamin Franklin stated in a letter to Jean-Baptist Leroy that "*in this world nothing can be said to be certain, except death and taxes*." Households are in fact (and still) exposed to multiple risks in their lives – and among these, apart from the risk of falling seriously ill, wage and capital income risks probably are the most important risk factors for well-being.

Another feature affecting well-being is education (respectively human capital). Whilst the importance of education is emphasized by many branches in the economic literature – very prominent is the one on human capital and growth (e.g., Bils and Klenow, 2000), – it has ambiguous and interdependent effects on income risk: studies show that human capital on the one hand acts as insurance against unemployment (Chapman, 1993), while on the other hand it amplifies other income risks (Mincer, 1974). However, all these risks are not only due to human capital investment, but simultaneously have feedback effects on investment behavior of households.

Levhari and Weiss (1974) are first to analyze the effect of a variety of wage risks onto human capital investment, while Williams (1978) extends the analysis onto multiple wage and capital income risk. Both papers show that these risks have a major impact on household behavior. Furthermore, the latter points out that investment in human capital and portfolio choice in real capital assets are strongly linked, if returns to both investments are risky. These papers do, however, neither deal with public policy nor with insurance possibilities. Thus, what can be done in order to make life safer?

Almost 200 years after Franklin, it turned out in economic literature that the certainty of taxes can also have a welfare improving effect, because taxation and its revenue can provide risk insurance (i.e., Eaton and Rosen, 1980a,b). However, the literature – to the best of our knowledge – restricts to only one aspect of risk per model and to a limited set of governmental instruments. Thus, it neglects combined effects of multiple income risk, faced by households and the fact that the government can use a wider set of instruments for public policy, such as progressive taxation and educational policy.

If then the scope of such optimal taxation models is widened to multiple income risk and a full set of governmental instruments, the questions therefore emerging and being tackled in this paper, are: (i) What is the optimal tax structure in order to cope with multiple income risk, and in which way is public policy challenged by multiple risk? (ii) Which effects on the optimal trade-off between efficiency and insurance will emerge? (iii) Are direct or indirect instruments better to counter inefficiencies caused by the insurance effects of taxation, hence, is it better to use education fees or capital taxation?

Eaton and Rosen (1980a,b) are first, showing that income taxation can have an insurance effect on household's utility *and* that this insurance effect can overcompensate induced distortions in labor supply or human capital formation.

Meanwhile, there is a broader literature on optimal taxation in case of risky income, but this literature focuses either on risk in human capital formation (i.e., Eaton and Rosen, 1980b, Hamilton, 1987, Anderberg and Andersson, 2003), or on risk in capital income (i.e., Varian, 1980, Christiansen, 1993), or on some unspecified income risk in general (Eaton and Rosen, 1980a). Moreover, the literature is focused mostly on standard income taxes at proportional rates[1] and on idiosyncratic risk, which can be eliminated by pooling.[2] Thus, the government can bear risk at no costs, and households can be modeled as risk-neutral in public consumption.

Therefore, the literature on optimal taxation in case of risky tax bases neglects both the occurrence of multiple risks in several income types and – in most papers – the fact that the government has control over a broad set of instruments: It can apply both progressive taxation in wage taxation and capital taxes, which are tailored to specific parts of return – i.e., the excess return, which can be seen as the market price of aggregate capital risk. Moreover, it can use its educational policy, as there are tuition fees or education subsidies, as direct instrument in order to correct for induced distortions in human capital investment and in order to control private educational investment indirectly, instead of having mandatory levels of enrollment.

[1] Anderberg and Andersson (2003) combine this taxation with public education policy, but focus on mandatory enrollment levels, set by the government. Varian (1980) incorporates non-linear income taxation in a model of unspecified capital income risk, whereas García-Peñalosa and Wälde (2000) examine the effects of a highly stylized graduate taxation.

[2] An exception is Christiansen (1993), who derives optimal tax rules in case of aggregate risk.

In order to incorporate the effects of multiple income risks and the optimal response of a benevolent government, which has access to a wide range of instruments, we set up an OLG-model, where households live for two periods.

In their first period of life, they decide on educational investment, on overall savings and on portfolio choice between a risky and a riskless asset. Furthermore, they supply unskilled labor. In their second period, households are faced by risk in both return to real capital and wage income, they receive as skilled workers. The risks are due to stochastic technological shocks, which can increase the productivity of real capital, but can also cause depreciations in the capital stock. Labor productivity is affected indirectly by the change in capital productivity and directly by the fact that technological progress can either be complementary to households skills or depreciate their stock of human capital, as households cannot handle the new technology.

The government is supposed to provide funding for the educational sector and to supply a public consumption good. It turns out that the government can provide efficient diversification of both types of risk onto private and public consumption, if leisure demand is inelastic, and if the government has access to differentiated wage taxation, tuition fees/education subsidies and a capital tax, which is only levied on the excess return in the risky asset.

Whilst the exact tax structure depends on risk aversion with respect to wage risks, riskless interest income should not be taxed in any case. Endogenizing leisure demand will complicate the analysis very much, and explicit solutions can hardly be derived. Instead, we provide some intuitive conjectures, based on results in simplified models.

The paper is organized as follows: In section 2 we provide a short discussion on income risks and on the possibilities of the government to insure them. Section 3 then presents the model and is followed by the description of household behavior. In section 5 we derive the optimal public policy in case of inelastic leisure demand, whereas extensions and omissions are discussed in section 6. Section 7 closes with some conclusions.

## 2 Income Risks and Public Insurance by Taxation

Sources of capital risk are manifold: it can be caused by business fluctuations and therefore by an uncertain profitability of the firm, by technological change, which can increase capital productivity, but also may cause extraordinary depreciations in the capital stock, and, for financial assets, risk can be due to speculative shocks within financial capital markets. However, capital risk can still be traded in these capital markets, and each household can adjust its exposure to risk. Moreover, unsystematic capital risk can be entirely diversified.

This differs from wage risk, and the sources of the latter are even more various: wage risk can also be caused by business cycles and technological progress may both increase or decrease the productivity of (skilled) labor, as well as depreciate the stock of skills. However, wage risk can neither be (fully) insured nor traded in private markets because of moral hazard and the fact that manpower cannot serve as collateral – at least as long as slavery is precluded.[3] It can therefore also hardly be diversified.

This implies that households must additionally bear idiosyncratic (success) risk in human capital formation and firm-specific risk at their employers. The latter implies that an employee additionally has to bear the idiosyncratic risk that its employer either goes bankrupt or cuts wages in order to avoid mass layoffs. These ex-post wage cuts have been very popular, e.g., in Germany for the last decade.[4] Taken together, there is large variety in wage risk, ranging from unemployment risk to productivity risk – and even wages, which are fixed ex ante, can be risky due to the mentioned wage renegotiations.

Higher education is often recommended as substitute for wage insurance. However, it cuts both ways: On the one hand human capital is in fact an insurance against the risk of getting unemployed and empirical data shows that unemployment is significantly higher among unskilled workers. On the other hand human capital investment is accompanied by the risk to fail in graduation as well as it

[3] See e.g., Eaton and Rosen (1980b), pp. 707. Even public unemployment insurance does not offer full coverage.

[4] Wage cuts have been mostly implemented by reducing gratifications like Christmas or vacation bonus. Wage reduction options are meanwhile a common tool in contracts between trade unions and employers in Germany.

promotes, i.e., occupational risk and the risk of having highly specialized knowledge, which can only be used in few sectors – consequently, exposing its owner to sector-specific risks.[5]

Another possibility for improving the allocation of risk and for providing some insurance even in those cases, where the private sector will not supply insurance against income risk (due, i.e., to moral hazard and market failure) is to rely on public policy: By reducing the variance in ex-post incomes and by redistributing tax revenue as deterministic transfers in case of idiosyncratic risk respectively by diversifying aggregate risk onto private and public consumption, taxation can insure these risks. In the former case, the government can eliminate risk by pooling, thus it bears the risk at no costs – as long as we abstain from induced distortions.[6] The latter case is somewhat more complicated.

It appears somehow odd that the government should be able to deal better with aggregate risk than the private market, as long as one restricts to public projects, which could (in principle) also be realized by the private market. There has been a lively debate on that issue, and meanwhile it seems widely accepted that for such projects, it is reasonable to assume that private and social valuation and discount factors should be equal.[7]

Nevertheless, the government can improve the allocation of aggregate risk, by supplying a public good, which is not provided by the market,[8] and therefore implements a public project, which is not contained in the private sector.[9] This holds true even when households are entirely diversified in all *private* assets, because the public good augments the number of *social* assets and therefore allows to spread risk onto more securities.

Optimal risk diversification then implies that aggregate risk is balanced on private and public consumption. In a first-best optimum, public insurance guarantees

[5]See Chapman (1993) vs. Mincer (1974) and Wildasin (2000). A short overview on the interdependency of human capital and various kinds of risk is to be found in Anderberg and Andersson (2003).

[6]See Varian (1980) for a detailed discussion.

[7]See, e.g., Arrow and Lind (1970) vs. Hirshleifer (1966), Sandmo (1972) or Bailey and Jensen (1972), whereby the latter denote the assumption of risk neutrality in this case as ‘nirvana approach,’ because of comparing apples and oranges due to different institutional settings.

[8]See Kaplow (1994), p. 795

[9]See Myles (1995), pp. 210.

that the ex-post realized marginal utilities of private and public consumption are identical in each state of the world, what can be ensured by using state-dependent lump-sum taxes.[10]

If state-dependent lump-sum taxes are not available, a trade-off is emerging between risk diversification and potential distortions. There are several studies, characterizing second-best optima for different kinds of risk and for a limited set of public policies:

As Eaton and Rosen (1980a,b) as well as Hamilton (1987) point out in case of idiosyncratic wage risk, proportional income taxation and lump-sum transfers show the mentioned welfare-enhancing insurance effects.[11] In a second-best optimum, these insurance effects are balanced against induced distortions in labor supply and human capital formation. In Hamilton (1987), moreover, capital taxation can also serve as indirect instrument to correct for distortions in human capital investment.

In case of business risk, Kanbur (1980) models the occupational choice decision between working as an employee for a deterministic wage, or becoming entrepreneur and being faced by risk. In the second-best optimum, partial public income insurance by differentiated taxation of both types of workforce is balanced against distortions in occupational choice.

For (idiosyncratic) risky human capital formation, García-Peñalosa and Wälde (2000) examine a broader range of instruments. Basically, they show that a graduate tax, accompanied by some direct education subsidies, is optimal in order to insure individuals. However, they restrict to a binary risk model, where students are either successful in investing or not and model the graduate tax as a lump-sum payment of all graduated households.

A more detailed linkage between wage risk, distortionary taxation, and education policy provide Anderberg and Andersson (2003), examining the effect of several types of wage risk onto tax revenue and welfare. They state that it is optimal to overprovide education, if human capital has an insurance function. However, in their model the government can control all human capital investment by

[10] See, e.g., Christiansen (1995) or Gollier (2001), who relates the sensitivity of consumption to absolute risk tolerance (pp. 313 and Proposition 80).

[11] Varian (1980) shows similar results in a model with risky return to capital investment and generalizes the result to non-linear income taxation.

mandatory education, and there is no private investment decision.

Turning to capital risk, a methodologically corresponding framework to Kanbur (1980) can be applied in case of portfolio choice. As shown in Christiansen (1993), there is an optimal trade-off between distorting investment in risky and riskless assets and the diversification of aggregate risk on private and public consumption by implementing differentiated asset-specific tax rates.

However, it is neglected to the best of our knowledge that households face simultaneously capital and wage risks for different reasons. The only study focusing on this issue seems to be the work by Williams (1978). The optimal public policy in such a case has never been examined. Additionally, the cited studies restrict to a limited set of public instruments. This must have effects both on the ability to diversify risk and on the efficiency costs.

Modeling the effect of multiple risk and enlarging public instruments for progressive wage taxation and tuition fees as well as capital taxes focusing on the excess return is the challenge to be tackled in the section to come.

# 3 The Model

We assume a small open economy with overlapping generations. In each generation there is a continuum of homogenous households. Each household lives for two periods, supplies unskilled labor in its first period of life and invests in real and human capital. Real capital is internationally perfectly mobile, whereas labor force is entirely immobile.

As each individual lives for two periods, overall population in period $t$ is equal to $N_t^{t-1} + N_t^t$. Superscript $t-1$ indicates the old generation, born in period $t-1$, whereas superscript $t$ represents the actual young generation in period $t$. Furthermore, we assume constant and exogenous population growth at rate $\eta$, which is equal to the riskless interest rate $r$. $\eta = r$ guarantees the 'golden rule' of real capital accumulation and avoids – without loss of generality – any intertemporal fiscal externality stemming from dynamical inefficiency (see, e.g, Atkinson and Sandmo, 1980, or Sandmo, 1985, p. 292).

**Production Sector** The domestic industry produces a homogenous consumption good $y$, whose price is normalized to unity. Production can take place in two sectors: sector 0, exhibiting both deterministic output and costs, and sector 1, which uses a risky production technology.[12]

In the deterministic sector 0, the representative firm issues riskless bonds $I^0$, which pay out return $r$ in order to attract real capital, $K^0$, and the firm demands unskilled labor, $L^0$. It uses a constant-returns-to-scale production function, $y^0 = F^0(K^0, L^0)$. The riskless interest rate is then determined by perfect capital mobility and the production function as $F_K^0 = r$, where $F_K^0$ is the marginal productivity of real capital in sector 0. Moreover, international capital flows enforce a wage rate for unskilled labor of $F_L^0 = W^0$.

The risky sector 1 utilizes always the latest production technology, which depends on a stochastic technology parameter $\theta$. In each period, there is a capital-augmenting technological shock, which can on the one hand increase the productivity of capital, but on the other hand also affects depreciations $\delta$ either positive or negative. Moreover, this technology requires skilled labor $H$ to be used. The production function then takes the form $y^1 = F^1(K^1, H, \theta)$.

The representative firm issues stocks $I^1$, which deliver a stochastic return $\tilde{x}$ in order to attract venture capital for production. Employment of capital follows from marginal productivity equal to capital costs. This can be rearranged to $F_K^1(K^1, H, \theta) - \delta(\theta) = \tilde{x}$. In the good states of the world, capital productivity is increased by the technological shock and depreciations are low, resulting in a high return to venture capital. In the bad states of the world, however, capital productivity is unaffected or even lowered by the shock, and it turns out that the capital stock has fully depreciated at the end of the production process. If this happens, the return to capital turns out to be negative or capital is even lost entirely. Taken together, the return to venture capital has in principle support $\tilde{x} \in [-1; \infty]$.

Accordingly, we get the optimal demand for human capital from $F_H^1(K^1, H, \theta) = \tilde{W}^1$. The marginal productivity of human capital depends twofold on the technological shock: First, there is an indirect effect via the productivity of capital. If the utilization of real capital changes, this should also affect the produc-

[12]The basic set-up equals Stiglitz (1972), and extends his model for both skilled and unskilled labor as well as endogenous, but risky human capital formation.

tivity of and the demand for human capital. Second, there is also a direct effect, which is independent of the productivity change in real capital. The productivity of human capital is directly affected by the capability to utilize the new technology. It may turn out that the qualifications of skilled workers are not sufficient in order to handle the new technology properly, or it might happen that the new technology is easier to cope with given a certain type of qualification. Thus, even if the shock increases (decreases) real capital productivity worldwide, it may occur in some countries that human capital productivity decreases (increases). This direct effect is a country-specific shock and is driven by differentiated education systems, where different skills might be acquired across countries. If marginal productivity of skilled workers gets too low, however, they can supply their labor force in the riskless sector. In the riskless sector, human capital is useless, and the skilled just imitate the unskilled. Taken together, the wage rate of skilled labor has support $\tilde{W}^1 \in [\frac{W^0}{g(E)}; \infty]$.

**Households** The risk averse households are provided with one unit of time per period. In their first period of life, they decide to spend time $e$ at university in order to accumulate human capital. Time $1-e_{t-1}$ is supplied at wage rate $W^0$ as unskilled labor. Hence, pre-tax income in that period is $W^0 \cdot (1-e_{t-1})$. First-period income is split on consumption, $C_{t-1}$, and savings, $S_{t-1}$. Hereby, savings can be allocated in two assets: the amount $A^0_{t-1}$ is invested in riskless bonds, which deliver a return $r$ before capital taxation; the amount $A^1_{t-1}$ is invested in a risky asset, which supplies the risky production sector with real capital. It pays out a stochastic pre-tax return $\tilde{x}$, being due to aggregate risk. Overall savings can be written as $S_{t-1} = A^0_{t-1} + A^1_{t-1}$.[13]

In their second period of life, labor supply of households is inelastic and they supply one unit of time. If they are employed in the risky sector, their effective labor supply in units of skilled labor depends on the amount of human capital acquired. Human capital is accumulated according to a concave production function $g(e)$ and increases in the time spent at university, that is $g'(e) > 0$, $g''(e) < 0$ and $\tilde{g}(0) = 1$. Thus, effective human capital supply is $g(e)$, labor market equilib-

[13] Using $m$ as a country index, world capital market equilibrium then implies $\sum_m A^0_m = \sum_m I^0_m$ and $\sum_m A^1_m = \sum_m I^1_m$ in each period of time.

rium implies $H_t = g(e_{t-1})$, and pre-tax labor income in the second period equals $\tilde{W}^1 \cdot g(e)$. The latter is risky in aggregate, due to a stochastic wage rate $\tilde{W}^1$. The lower bound of labor income is the unskilled wage income $W^0$, because if the marginal productivity in human capital and skilled labor income gets too low, $g(e) \cdot \tilde{W}^1 < W^0$, the skilled households decide to work in the riskless sector 0. Here they cannot utilize their human capital and supply one unit of labor at the unskilled wage rate $W^0$.

In any case, consumption when old, $C_t$, has to be financed from two risky earnings bases, namely stochastic labor income and risky capital income.

**Government** The government on the one hand provides a pure public consumption good $P_t$. On the other hand, the government also has to provide a public higher education system, which causes real resource costs $\bar{B}$ per student. This expenditure is assumed to be fixed per student and independent from time investment $e$. The government charges, however, a price $p_B$ per semester and can exclude students, who are not willing to pay $p_B$ per unit of time spent at university, $e$. This price for education can be seen as tuition fees per semester, if $p_B > 0$, or it will turn into education subsidies, if $p_B < 0$. The overall net public expenditure for education in period $t$ is then given by

$$B_t^{net} = N_t^t \cdot (\bar{B} - p_B \cdot e_t)\,. \tag{1}$$

Taken together, overall public expenditure in period $t$ is

$$R_t = P_t + B_t^{net} = P_t + N_t^t \cdot (\bar{B} - p_B \cdot e_t)\,. \tag{2}$$

In order to finance its expenditure, the government can use a set of labor and capital income taxes. For labor taxation, we use a two-bracket tax schedule as in Nielsen and Sørensen (1997): All labor income until a threshold $X = W^0$ is liable to the labor tax rate $t_1^L$. The part of labor income, exceeding this threshold, consequently the skill premium $\tilde{W}^1 \cdot g(e) - W^0$, is liable to the labor tax rate $t_2^L$. Therefore, unskilled workers are only faced by the tax rate $t_1^L$, whereas the marginal tax rate of the skilled ones is equal to the surtax rate $t_2^L$.

Capital taxation is also differentiated: Riskless capital income in *both* assets is taxed at rate $t_0^K$. The excess return in the risky asset, $\tilde{x}-r$, thus the price received for incurring risk, is taxed instead at rate $t_1^K$. In the latter tax base, full loss offset is guaranteed. This implies a refund of $t_1^K \cdot (\tilde{x}-r)$ per unit of risky capital investment, $A^1$, if $\tilde{x}-r$ turns ex-post out to be negative.

**Risk in the Economy and Timing Structure** There are two different income risks in the economy, which depend both on the technology shock. First, this shock can be seen as capital-augmenting technological progress. However, it is ex-ante uncertain, whether production is really enhanced and what the effects on depreciation costs are. We assume that this shock strikes all firms in the risky sector in all countries at the same time and in the same manner. Hence, the shock cannot be insured and it translates into aggregate income risk for stock holders.

Second, the technological shock affects human capital in the risky sector twofold: (i) There is an indirect effect via the productivity of venture capital. It seems reasonable to assume that the productivity of skilled labor is ceteris paribus increased (decreased), if the productivity of real capital increases (decreases). (ii) There is also a direct impact of the technological shock. We assume that the capability of skilled labor to utilize the new technology depends on the skills acquired at university and differs across the countries. The reasoning behind this is the implicit assumption that there are international differences in the educational systems. Accordingly, this corresponds to an asymmetric shock. In some countries human capital productivity may be enhanced, whereas, in extremum, in some other, few, countries, the skilled workers cannot use the new technology at all. In the latter case, there will be no production in the risky sector and all skilled workers will supply one unit of unskilled labor in the deterministic sector. As labor force is internationally immobile, and, following standard assumptions, human capital risk cannot be insured because of moral hazard, the effects of the technological shock translate into aggregate labor income risk for skilled workers as well.

From the government point of view, both the labor income tax base and the capital income tax base are partly risky, and, thus, overall tax revenue is stochastic, too.

The timing structure and the realization of risk is as follows: First, the benevolent government sets welfare-maximizing tax rates and tuition fees. Second, the young generation decides for its human capital investments, optimal savings and portfolio allocation. Next, the impact of the technological shock $\theta$ on venture and human capital realizes, real capital is allocated worldwide, and the skilled workers decide to work either in the risky sector or in the deterministic one. Then, production takes place, and the real value of depreciation in venture capital, $\delta(\theta)$ realizes. Finally, all incomes and taxes are paid, and private as well as public consumption take place.

# 4 Household Choice

An individual, born in period $t-1$, maximizes its von-Neumann-Morgenstern expected utility function

$$Z = \mathrm{E}[U(c_{t-1}, c_t)] + \mathrm{E}[V(P_t)] \tag{3}$$

by choosing its optimal educational investment $e_{t-1}$, its consumption $c_{t-1}$ and its investments in the riskless respective the risky financial asset, $A^S_{t-1}$ and $A^R_{t-1}$.

We assume the utility function to be additively separable in private and public consumption. Moreover, the individual does not anticipate any effects of its behavior on the level or the riskiness of the public good, because each household is arbitrary small.

The budget constraint of the household under consideration is in period $t-1$ given by

$$(1-t^L_1) \cdot W^0_{t-1} \cdot (1-e_{t-1}) = c_{t-1} + p_B \cdot e_{t-1} + A^S_{t-1} + A^R_{t-1}, \tag{4}$$

and human capital formation and savings translate into second-period-of-life consumption[14]

$$\begin{aligned} \tilde{c}_t &= (1-t^L_2) \cdot \left[\tilde{W}^1 \cdot g(e_{t-1}) - W^0\right] - t^L_1 \cdot W^0 + (1-t^K_1)(\tilde{x}-r) \cdot A^R_{t-1} \\ &\quad + [1+r(1-t^K_0)] \cdot \left[A^S_{t-1} + A^R_{t-1}\right]. \end{aligned} \tag{5}$$

[14] All variables indicated with a tilde depend on the realization of $\theta$ and are stochastic.

Consolidating these expressions leads to the intertemporal budget constraint

$$\begin{aligned}\tilde{c}_t &= (1-t_2^L)\cdot\left[\tilde{W}^1\cdot g(e_{t-1})-W^0\right]-t_1^L\cdot W^0+(1-t_1^K)(\tilde{x}-r)\cdot A_{t-1}^R\\ &\quad+[1+r(1-t_0^K)]\cdot\left[(1-t_1^L)\cdot W_{t-1}^0\cdot(1-e_{t-1})-c_{t-1}+p_B\cdot e_{t-1}\right], \qquad (6)\end{aligned}$$

whereby $(1-t_1^L)\cdot W_{t-1}^0\cdot(1-e_{t-1})-c_{t-1}+p_B\cdot e_{t-1}=A_{t_1}^S+A_{t-1}^R=s_{t-1}$ are overall savings.

Thus, the household solves

$$\max_{c_{t-1},A_{t-1}^R,e_{t-1}} \quad \mathrm{E}[U(c_{t-1},\tilde{c}_t)]+\mathrm{E}[V(P_t)] \quad \text{s.t.} \quad (6). \qquad (7)$$

First order conditions are

$$\mathrm{E}[U_{c_{t-1}}]-p\cdot\mathrm{E}[U_{c_t}] = 0 \qquad (8)$$

$$(1-t_1^K)\cdot\mathrm{E}[U_{c_t}\cdot(\tilde{x}-r)] = 0 \qquad (9)$$

$$\mathrm{E}\left[U_{c_t}\cdot\left\{(1-t_2^L)\cdot\tilde{W}_t^1\cdot g'(e_{t-1})-p\cdot\left((1-t_1^L)\cdot W^0+p_B\right)\right\}\right] = 0, \qquad (10)$$

where $p=1+r(1-t_0^K)$.

From (8) we infer the usual condition that the marginal rate of time preferences, $\rho=\mathrm{E}[U_{c_{t-1}}]/\mathrm{E}[U_{c_t}]-1$, must be equal to the riskless after-tax interest rate, accordingly $\rho=r(1-t_0^K)$.

First order condition (9) implies that the risk tax $t_1^K$ on the excess return in the risky financial asset only has a Musgrave-substitution effect[15]

$$\frac{\partial A_{t-1}^R}{\partial t_1^K}=\frac{A_{t-1}^R}{1-t_1^K}, \qquad (11)$$

and does not affect welfare from private consumption and therefore has neither effect on consumption $c_{t-1}$ nor on educational investment $e_{t-1}$. Thus, we have $\frac{\partial c_{t-1}}{\partial t_1^K}=\frac{\partial e_{t-1}}{\partial t_1^K}=0$. All of this can easily be understood by using the optimal investment function $A_{t-1}^R(t_1^K)=\frac{A_{t-1}^R}{1-t_1^K}$ in the household budget constraint (6).

Last, but not least, we draw from (10) that the effective risk-adjusted marginal

[15] This effect is well-known in the literature on risk taking and taxation. See, e.g., Mossin (1968), Sandmo (1969, 1977).

return to human capital will be equalized to the after-tax marginal return in riskless real capital and

$$\frac{(1-t_2^L)\cdot\left[\bar{W}^1 - RP_C(W^1)\right]\cdot g'(e_{t-1})}{(1-t_1^L)\cdot W^0 + p_B} - 1 = r(1-t_0^K), \tag{12}$$

whereby we have been using the certainty equivalent

$$W_{ad_C}^1 = \frac{\mathrm{E}[U_{c_t}\cdot\tilde{W}^1]}{\mathrm{E}[U_{c_t}]} = \mathrm{E}[\tilde{W}^1] + \frac{\mathrm{Cov}(U_{c_t},\tilde{W}^1)}{\mathrm{E}[U_{c_t}]}, \tag{13}$$

and $\bar{W}^1 = \mathrm{E}[\tilde{W}^1]$, as well as $RP_C(W^1) = -\frac{\mathrm{Cov}(U_{c_t},\tilde{W}^1)}{\mathrm{E}[U_{c_t}]}$. $RP_C(W^1) > 0$ is the risk premium demanded in private consumption in order to bear the wage risk of an high-skilled worker.

From (13) and the first order condition (9) we can also infer an effect of the fact that human capital risk cannot be traded, whilst risk in real capital can be sold and bought via the risky asset. Equation (9) implies that the household is perfectly diversified in all real capital assets, because in the optimum the risk adjusted return of another marginal unit in the risky asset equals exactly the return in the riskless asset. By rearranging the optimality condition, we receive

$$\mathrm{E}[\tilde{x}-r] = -\frac{\mathrm{Cov}(U_{c_t},\tilde{x})}{\mathrm{E}[U_{c_t}]} = RP_C(\tilde{x}). \tag{14}$$

The certainty equivalent is given by the riskless market return. The household's risk premium in real capital can therefore be inferred from market data, $\mathrm{E}[\tilde{x}-r]$, and taxing the excess return $\tilde{x}-r$ allows to tax the risk premium itself.

Transforming (13), the risk premium in human capital is equal to

$$RP_C(\tilde{W}^1) = -\frac{\mathrm{Cov}(U_{c_t},\tilde{W}^1)}{\mathrm{E}[U_{c_t}]} = \bar{W}^1 - W_{ad_C}^1, \tag{15}$$

but market data does not give any information about the certainty equivalent $W_{ad_C}^1$.

The skill premium $\tilde{W}^1\cdot g(e) - W^0$ can be seen as a possible approximation for tax purposes, but it still mixes up the expected return to human capital and its risk premium. Thus, it seems not to be possible to tax the risk premium in wage income alone.[16] Moreover, it indicates that the household is not able to diversify the wage risk entirely.

Optimal household behavior determines the indirect utility function

$$\Omega(t_1^L, t_2^L, t_0^K, t_1^K, p_B) = \mathrm{E}[U(c_{t-1}^*, c_t^*)] + \mathrm{E}[V(\tilde{G})], \tag{16}$$

and applying the Envelope-theorem leads to

$$\frac{\partial \Omega}{\partial t_1^L} = -W^0 \cdot [1 + p \cdot (1 - e_{t-1}^*)] \cdot \mathrm{E}[U_{c_t}] \tag{17}$$

$$\frac{\partial \Omega}{\partial t_2^L} = \mathrm{E}[U_{c_t} \cdot (W^0 - \tilde{W}^1 \cdot g(e_{t-1}^*))] = -(\tilde{W}_{ad_C}^1 \cdot g(e_{t-1}^*) - W^0) \cdot \mathrm{E}[U_{c_t}] \tag{18}$$

$$\frac{\partial \Omega}{\partial t_0^K} = -r \cdot s_{t-1}^* \cdot \mathrm{E}[U_{c_t}] \tag{19}$$

$$\frac{\partial \Omega}{\partial t_1^K} = -A_{t-1}^{R*} \cdot \mathrm{E}[U_{c_t} \cdot (\tilde{x} - r)] = 0 \tag{20}$$

$$\frac{\partial \Omega}{\partial p_B} = -p \cdot e_{t-1}^* \cdot \mathrm{E}[U_{c_t}], \tag{21}$$

where the second equality in equation (20) stems from the household first order condition (9) and confirms our arguments given above for the effects of $t_1^K$ in comparative statics.

## 5 Optimal Public Policy

The government provides a pure public good, $P_t = N_t^{t-1} \cdot G_t$, and also has to provide a higher education system, publicly financed, which causes fixed costs $\bar{B}$ per student. Whilst the level of the public good can vary, dependent on tax revenue, the education system must be fully funded in each state of nature.

Subtracting revenue from tuition fees, the overall (net) public expenditure for education in period $t$ is given by

$$B_t^{net} = N_t^t \cdot (\bar{B} - p_B \cdot e_t^*) \,. \tag{22}$$

[16] Of course, it is possible to solve equation (12) for the risk premium $RP_C(\tilde{W}^1)$, but this will not deliver a suitable tax base.

Summed up, overall public net expenditure in period $t$ is

$$\tilde{R}_t = N_t^{t-1} \cdot \tilde{G}_t + N_t^t \cdot (\bar{B} - p_B \cdot e_t^*)\,, \tag{23}$$

whereby $\tilde{G}_t$ are the units of the public good per member of the old generation.

In order to finance its expenditure, the government can use the set of wage and capital income taxes stated in section 3. Labor income up to a threshold $W^0$ is liable to the wage tax rate $t_1^L$. The part of labor income, exceeding this threshold, is liable to the wage tax rate $t_2^L$. Riskless capital income in both assets is taxed at rate $t_0^K$, whereas the excess return in the risky asset, $\tilde{x} - r$, is taxed at rate $t_1^K$. In the latter tax base, full loss offset is guaranteed.

All together, the government receives in each period $t$ wage tax revenue $N_t^t \cdot t_1^L \cdot W^0 \cdot (1 - e_t^*)$ from the young generation. The old generation pays wage taxes $N_t^{t-1} \cdot t_1^L \cdot W^0$ at the standard rate and, additionally, has to pay $N_t^{t-1} \cdot t_2^L \cdot [\tilde{W}^1 \cdot \tilde{g}(e_{t-1}^*) - W_0]$ under the surcharge tax rate. The latter tax base is risky in aggregate, but as the income of a skilled worker cannot be lower than the wage paid in the riskless and unskilled sector, $W^0$, this tax base cannot be negative, thus $[\tilde{W}^1 \cdot g(e_{t-1}^*) - W^0] \geq 0$.

The governmental budget restriction for period $t$ is therefore given by

$$\begin{aligned} & N_t^{t-1} \cdot \left\{ t_2^L \cdot \left[\tilde{W}^1 \cdot g(e_{t-1}^*) - W^0\right] + t_1^L \cdot W^0 \right\} + N_t^t \cdot t_1^L \cdot W^0 \cdot (1 - e_t^*) + \\ & N_t^{t-1} \cdot \left\{ t_1^K \cdot (\tilde{x} - r) \cdot A_{t-1}^R + t_0^K r \cdot \left[(1 - t_1^L) \cdot W^0 \cdot (1 - e_{t-1}^*) - p_B \cdot e_{t-1}^* - c_{t-1}^*\right] \right\} \\ = & \tilde{R}_t = N_t^{t-1} \cdot \tilde{G}_t + N_t^t \cdot \left(\bar{B} - p_B \cdot e_{t-1}^*\right). \end{aligned} \tag{24}$$

Rearranging and transforming into a per-capita constraint results in

$$\begin{aligned} & t_2^L \cdot \left[\tilde{W}^1 \cdot g(e_{t-1}^*) - W^0\right] + t_1^L \cdot W^0 + (1 + r) \cdot \left[t_1^L \cdot W^0 \cdot (1 - e_t^*) + p_B \cdot e_t^*\right] \\ & + t_1^K \cdot (\tilde{x} - r) \cdot A_{t-1}^{R*} + t_0^K r \cdot s_{t-1}^* - (1 + r) \cdot \bar{B} = \tilde{G}_t, \end{aligned} \tag{25}$$

where we used $N_t^t / N_t^{t-1} = 1 + \eta = 1 + r$ and $s_{t-1}^* = (1 - t_1^L) \cdot W^0 \cdot (1 - e_{t-1}^*) - p_B \cdot e_{t-1}^* - c_{t-1}^*$. As the education system is always fully funded, the consumption of the public good $\tilde{G}_t$ turns risky, as it is financed by risky tax revenue.

The government maximizes expected utility of a representative steady-state generation, born at $t-1$.[17] Using the indirect utility function (16), the optimization problem can be stated as

$$\max_{t_1^L, t_2^L, t_0^K, t_1^K, p_B} N_t^{t-1} \cdot \Omega(t_1^L, t_2^L, t_0^K, t_1^K, p_B, \bar{B}) + \mathrm{E}\left[V(N_t^{t-1} \cdot \tilde{G}_t)\right], \tag{26}$$

where $\tilde{G}_t$ is subject to the budget restriction (25). Given the steady-state assumption, we are going to drop the superscripts for generations and time indices, whenever possible without causing confusion, in order to simplify the notation.

The first order conditions are

$$-W^0 \cdot [1 + p \cdot (1-e^*)] \cdot \mathrm{E}[U_{c_t}] \quad + \tag{27}$$

$$\mathrm{E}\left\{V_G \cdot \left[W^0\,[1 + p \cdot (1-e^*)] + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_1^L} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_1^L} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_1^L}\right]\right\} \quad = \quad 0,$$

$$-(\tilde{W}^1_{ad_C} \cdot g(e^*) - W^0) \cdot \mathrm{E}[U_{c_t}] \quad + \tag{28}$$

$$\mathrm{E}\left\{V_G \cdot \left[\tilde{W}^1\, g(e^*) - W^0 + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_2^L} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_2^L} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_2^L}\right]\right\} \quad = \quad 0,$$

$$-r \cdot s^* \cdot \mathrm{E}[U_{c_t}] \quad + \tag{29}$$

$$\mathrm{E}\left\{V_G \cdot \left[r\,s + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_0^K} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_0^K} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_0^K}\right]\right\} \quad = \quad 0,$$

$$\mathrm{E}\left\{V_G \cdot \left[(\tilde{x} - r)\,A^R_{t-1} + \tilde{\alpha} \cdot \frac{\partial e}{\partial t_1^K} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial t_1^K} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_1^K}\right]\right\} \quad = \quad 0, \tag{30}$$

$$-p \cdot e^* \cdot \mathrm{E}[U_{c_t}] \quad + \tag{31}$$

$$\mathrm{E}\left\{V_G \cdot \left[p\,e^* + \tilde{\alpha} \cdot \frac{\partial e}{\partial p_B} + t_1^K \cdot (\tilde{x} - r) \cdot \frac{\partial A^R}{\partial p_B} - t_0^K r \cdot \frac{\partial c_{-1}}{\partial p_B}\right]\right\} \quad = \quad 0,$$

whereby $\tilde{\alpha} = t_2^L \cdot \tilde{W}^1 \cdot g'(e^*) - (1+r)\left[t_1^L \cdot W^0 - p_B\right] - t_0^K r \cdot \left[(1-t_1^L) \cdot W^0 + p_B\right]$ and $c_{-1}$ indicates consumption in the first period of life and where we have already inserted the envelope-effects (17) – (21) for the derivatives of the indirect utility function.

As we have $\frac{\partial e}{\partial t_1^K} = \frac{\partial c_{-1}}{\partial t_1^K} = 0$ and $\frac{\partial A^R}{\partial t_1^K} = \frac{A^R}{1-t_1^K}$ from (11) and comparative-statics,

[17] This approach is compatible with a Pareto-improving tax reform as in Nielsen and Sørensen (1997), if we redefine expenditure $\bar{B}$ and add debt payments necessary in order to keep the utility of the transition generation constant.

first order condition (30) simplifies to

$$\mathrm{E}\left[V_G \cdot (\tilde{x}-r)\right] \cdot \frac{A_1}{1-t_1^K} = 0 \quad \Leftrightarrow \quad \mathrm{E}\left[V_G \cdot (\tilde{x}-r)\right] = 0. \tag{32}$$

Consequently, a marginal increase in the tax rate $t_1^K$ will create additional tax revenue of $\tilde{x}-r$, however, in the optimum the risk adjusted value of this (additional) marginal tax revenue must be zero.

Next, we define analogous to $W^1_{ad_C}$

$$W^1_{ad_G} = \frac{\mathrm{E}\left[V_G \cdot \tilde{W}^1\right]}{\mathrm{E}\left[V_G\right]} = \frac{\mathrm{E}\left[V_G\right] \cdot \mathrm{E}[\tilde{W}^1]}{\mathrm{E}\left[V_G\right]} + \frac{\mathrm{Cov}(V_G, \tilde{g}(E))}{\mathrm{E}\left[V_G\right]} = \bar{W}^1 - RP_G(\tilde{W}^1). \tag{33}$$

$W^1_{ad_G}$ is the risk adjusted skilled wage, whereby the adjustment is now based on public consumption. It is equal to the expected skilled wage, $\mathrm{E}[\tilde{W}^1] = \bar{g}$, minus the risk premium measured in *public* consumption, $RP_G(\tilde{W}^1)$.

Using equations (32) and (33) in the other first order conditions, we obtain

$$\begin{aligned}
W^0 \cdot [1+p\cdot(1-e^*)] \cdot \frac{\mathrm{E}\left[U_{C_t}-V_G\right]}{\mathrm{E}\left[V_G\right]} &= \beta \cdot \frac{\partial e}{\partial t_1^L} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_1^L}, && (34)\\
\left[W^1_{ad_C}\cdot g(e^*) - W^0\right] \cdot \frac{\mathrm{E}\left[U_{C_t}\right]}{\mathrm{E}\left[V_G\right]} - \left[W^1_{ad_G}\cdot g(e^*) - W^0\right] &= \beta \cdot \frac{\partial e}{\partial t_2^L} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_2^L}, && (35)\\
r \cdot s^* \cdot \frac{\mathrm{E}\left[U_{C_t}-V_G\right]}{\mathrm{E}\left[V_G\right]} &= \beta \cdot \frac{\partial e}{\partial t_0^K} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial t_0^K}, && (36)\\
p \cdot e^* \cdot \frac{\mathrm{E}\left[U_{C_t}-V_G\right]}{\mathrm{E}\left[V_G\right]} &= \beta \cdot \frac{\partial e}{\partial p_B} + t_0^K r \cdot \frac{\partial c_{-1}}{\partial p_B}, && (37)
\end{aligned}$$

where $\beta = t_2^L \cdot \left[\bar{W}^1 - RP_G(\tilde{W}^1)\right] \cdot g'(e^*) - p \cdot \left(t_1^L \cdot W^0 - p_B\right) - t_0^K r$.

Dividing equation (37) by $p \cdot e^*$, inserting the new expression in (36), respectively (34), and rearranging them, reveals

$$\left(p \cdot e^* \cdot \frac{\partial e}{\partial t_0^K} - r \cdot s^* \cdot \frac{\partial e}{\partial p_B}\right) \cdot \beta = t_0^K r \cdot \left(p \cdot e^* \cdot \frac{\partial c_{-1}}{\partial t_0^K} - r \cdot s^* \cdot \frac{\partial c_{-1}}{\partial p_B}\right) \tag{38}$$

$$\begin{aligned}
\left(p\, e^* \, \frac{\partial e}{\partial t_1^L} - W^0\, [1+p\,(1-e^*)]\, \frac{\partial e}{\partial p_B}\right) \cdot \beta = t_0^K r \cdot \Bigg(& p\, e^*\, \frac{\partial c_{-1}}{\partial t_1^L} - \\
& W^0\, [1+p\,(1-e^*)]\, \frac{\partial c_{-1}}{\partial p_B}\Bigg)
\end{aligned} \tag{39}$$

We can now state a first result:

**Proposition 1.** *It is not optimal to tax the riskless rate of return in financial assets. $t_0^K = 0$ also implies that capital taxation is not used as indirect instrument to correct for labor-tax induced distortions in education demand.*

*Proof.* We start by restating the household budget constraint as

$$\begin{aligned} p \cdot c_{-1}^* + \tilde{c}_t^* + p_e^{eff} \cdot e^* &= (1-t_2^L) \cdot \left[\tilde{W}^1 \cdot g(e^*) - W^0\right] - t_1^L \cdot W^0 \\ &\quad + (1-t_1^K)(\tilde{x}-r) \cdot A_1^{R*} + p \cdot (1-t_1^L) \cdot W^0, \end{aligned} \tag{40}$$

where $p_e^{eff} = p \cdot p_e = p \cdot \left[(1-t_1^L) \cdot W^0 + p_B\right]$ is the effective (inflated) price of education and where the RHS of (40) mirrors total income, which has to be considered for the endowment effects, when the Slutsky decomposition is applied.

The required Slutsky decompositions are therefore

$$\frac{\partial c_{-1}}{\partial p_B} = \left[S_{c_{-1}e} - e^* \cdot \frac{\partial c_{-1}}{\partial I}\right] \cdot p, \tag{41}$$

$$\frac{\partial c_{-1}}{\partial t_1^L} = (-W^0) \cdot S_{c_{-1}e} \cdot p - W^0 \cdot [1 + p \cdot (1-e^*)] \cdot \frac{\partial c_{-1}}{\partial I}, \tag{42}$$

$$\begin{aligned} \frac{\partial c_{-1}}{\partial t_0^K} &= \frac{\partial c_{-1}}{\partial p} \cdot \frac{\partial p}{\partial t_0^K} + \frac{\partial c_{-1}}{\partial p_e^{eff}} \cdot \frac{\partial p_e^{eff}}{\partial t_0^K} \\ &= \left[S_{c_{-1}c_{-1}} + p_e \cdot S_{c_{-1}e} + s^* \cdot \frac{\partial c_{-1}}{\partial I}\right] \cdot (-r), \end{aligned} \tag{43}$$

$$\frac{\partial e}{\partial p_B} = \left[S_{ee} - e \cdot \frac{\partial e}{\partial I}\right] \cdot p, \tag{44}$$

$$\frac{\partial e}{\partial t_1^L} = (-W^0) \cdot S_{ee} \cdot p - W^0 \cdot [1 + p \cdot (1-e)] \cdot \frac{\partial e}{\partial I}, \tag{45}$$

$$\begin{aligned} \frac{\partial e}{\partial t_0^K} &= \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial t_0^K} + \frac{\partial e}{\partial p_e^{eff}} \cdot \frac{\partial p_e^{eff}}{\partial t_0^K} \\ &= \left[S_{c_{-1}e} + p_e \cdot S_{ee} + s^* \cdot \frac{\partial c_{-1}}{\partial I}\right] \cdot (-r). \end{aligned} \tag{46}$$

Thereby, $S_{ij}$ represents the substitution effect in demand for good $i$, if price $j$ changes, and the partial derivative with respect to $I$ indicates the corresponding income/endowment effect.

By replacing all derivatives in equations (38) and (39) by the expressions above, all income effects cancel out, and further simplification leaves us in (39) with

$$S_{c_{-1}e} \cdot \beta = S_{c_{-1}c_{-1}} \cdot t_0^K\, r. \tag{47}$$

Using (47) in order to simplify (38) even more, we end up with equation (48) as

$$S_{ee} \cdot \beta = S_{c_{-1}e} \cdot t_0^K\, r. \tag{48}$$

Combining (47) and (48) by substituting for $\beta$, results in

$$\left(S_{ee} \cdot S_{c_{-1}c_{-1}} - S_{c_{-1}e} \cdot S_{ec_{-1}}\right) \cdot t_0^K\, r = 0. \tag{49}$$

The first term in (49) is a principal minor of the substitution matrix, which is known to be negative (semi-)definite. As long as we rule out semi-definiteness, this expression cannot be equal to zero, and consequently we receive $t_0^K = 0$ from the second term in (49). □

As it will turn out later, insurance and risk diversification is carried out by differentiated wage taxation and the risk tax on the excess return in risky assets. Furthermore, educational investment can be controlled by using tuition fees / education subsidies. Hence, there is no reason for distorting intertemporal consumption choice and no need for (riskless) capital taxation. Accordingly, we have $\rho = r$.

That the optimal tax system on the one hand actually does not create distortions, but on the other hand is still able to ensure the diversification of wage risk on private and public consumption in a very efficient manner, can be seen in Proposition 2:

**Proposition 2.** *Optimal public policy ensures (i) ex-ante efficiency in allocation and (ii) an ex-post wage-risk sharing rule, which equates the (wage) risk premia in private and public consumption,* $RP_C(\tilde{W}^1) = RP_G(\tilde{W}^1)$.

*Proof.* For $t_0^K = 0$ follows from equation (47) that $\beta = 0$, because $S_{c_{-1}c_{-1}}, S_{c_{-1}e} < 0$. Applying then $t_0^K = \beta = 0$, and $e^* < 1$ in any of the equations (34), (36) or (37), results in

$$\mathrm{E}[U_{c_t}] = \mathrm{E}[V_G], \tag{50}$$

being exactly the definition of ex-ante efficiency.

Part (ii) can be proven by substituting $t_0^K = \beta = 0$ in equation (35), where we obtain

$$\left[W^1_{ad_C} \cdot g(e^*) - W^0\right] \cdot \frac{\mathrm{E}\left[U_{C_t}\right]}{\mathrm{E}\left[V_G\right]} - \left[W^1_{ad_G} \cdot g(e^*) - W^0\right] = 0. \tag{51}$$

Applying $\mathrm{E}[U_{c_t}] = \mathrm{E}[V_G]$ as well as the definition of $W^1_{ad_a} = \bar{W}^1 - RP_a(\tilde{W}^1)$, $a = C, G$ and collecting terms then leads to

$$RP_C(\tilde{W}^1) - RP_G(\tilde{W}^1) = 0. \tag{52}$$

Thus, the wage risk premia in private and public consumption are equalized for an optimal public policy. □

Accordingly, marginal utilities in private and public consumption are linked by a risk sharing rule, which guarantees that marginal utilities fluctuate in a similar way, but which does not cause efficiency costs. Although this optimal policy is better than in standard optimal taxation models, featuring wage risk (i.e., Eaton and Rosen, (1980b, Kanbur, 1980), the risk sharing rule cannot guarantee a first-best solution, because the risk is diversified in a linear manner, due to a constant surtax rate $t_2$.

However, it allows to transfer risk from the household to the government and attains therefore a twofold improvement. First, the household gets enabled to "trade" a part of its wage risk. Second, the government increases the number of (social) assets, onto which aggregate risk can be diversified, by providing a public good. The public good can be seen as an additional asset, which cannot be provided by the private markets.[18] This result does neither imply any assumption, whether the government can deal better with risk than private markets, nor does it require a statement, what the correct social discount rate should be.

From $\beta = t_0^K = 0$ and $RP_C(\tilde{W}^1) = RP_G(\tilde{W}^1) = RP(\tilde{W}^1)$, we can then derive the optimal wage tax structure and the optimal tuition fees, which are necessary to ensure Proposition 2.

[18] See Kaplow (1994), p. 795.

In the social optimum, we have

$$\beta^* = t_2^L \cdot \left[\bar{W}^1 - RP(\tilde{W}^1)\right] \cdot g'(e^*) - (1+r) \cdot \left(t_1^L \cdot W^0 - p_B\right) = 0, \tag{53}$$

and can add the first order condition (10) from the household's optimization problem in order to receive

$$g'(e^*) \cdot \left[\bar{W}^1 - RP(\tilde{W}^1)\right] = (1+r)W^0. \tag{54}$$

The LHS of equation (54) gives (social) marginal revenue of optimal educational investment, whereas the RHS shows its (social) marginal costs, which are equal to the wage forgone by attending university and bringing forward these costs into the second period of life.

Note that there is no direct tax term, distorting marginal revenue and marginal costs, and that in case of aggregate risk society is not risk neutral, because risk cannot be eliminated by pooling. As the term $RP(\tilde{W}^1)$ mirrors optimal wage risk diversification on private and public consumption, equation (54) can be seen as stating production efficiency under uncertainty, because optimal human capital production is not distorted.[19]

Substituting (54) into (53) leaves us with

$$t_2^L - t_1^L = -\frac{p_B}{W^0}. \tag{55}$$

**Proposition 3.** *The differentiated wage tax and tuition fees are used in order to guarantee the optimal risk diversification without distorting educational investment. Consequently,*

*(i)* *optimal wage taxation implies* either *progressive wage taxation and education subsidies* or *regressive wage taxation and tuition fees.*

*(ii)* *ceteris paribus, the surtax rate $t_2$ is increasing in the risk aversion in private consumption and decreasing in the risk aversion in public consumption.*

[19]Production efficiency to be desirable in second-best models dates back to the analysis of Diamond and Mirrlees (1971). However, they restrict to the case of certainty.

*Proof.* The first part of Proposition 3 is directly taken from (55) and the fact that any distortive wage taxation is fully compensated by either tuition fees, $p_B > 0$, or education subsidies, $p_B < 0$. Remind that production efficiency thereby implies $\frac{g'(e^*)\cdot[\bar{W}^1 - RP(\tilde{W}^1)]}{1+r} = W^0$ from (54).

For part (ii), we can apply that $RP_C(\tilde{W}^1) = RP_G(\tilde{W}^1)$ requires

$$\mathrm{Cov}(U_{c_t}, \tilde{W}^1) = \mathrm{Cov}(V_G, \tilde{W}^1). \tag{56}$$

These covariances indeed measure the ex-post sensitivity of marginal utility with respect to fluctuations in the skilled wage rate, which makes the argumentation simpler.

Taking the derivative of marginal utilities with respect to an ex-post variation of the wage rate $\tilde{W}^1$, we get

$$\frac{\partial U_{c_t}}{\partial \tilde{W}^1} = (1 - t_2^l) \cdot g(e^*) \cdot U_{c_t c_t}, \tag{57}$$

$$\frac{\partial V_G}{\partial \tilde{W}^1} = t_2^l \cdot g(e^*) \cdot V_{GG}. \tag{58}$$

Requiring that the sensitivities in equations (57) and (58) must be equal, we end up with

$$\frac{t_2^L}{1 - t_2^L} = \frac{U_{c_t c_t}}{V_{GG}}. \tag{59}$$

The optimal tax rate $t_2^L$ is increasing in the curvature of the (sub-)utility function $U_{c_t c_t}$ and decreasing in $V_{GG}$. We now obtain part (ii) in Proposition 3 by recognizing that higher risk aversion implies that $U_{c_t c_t}$, respectively $V_{GG}$, increase. Obviously, $t_2^L = 1$, if $V_{GG} = 0$, thus if there is risk neutrality in public consumption, and $t_2^L \to 0$, if $U_{c_t c_t} \to 0$, thus if the household is nearly risk neutral in private consumption. □

The diversification depends on the strength of the risk aversion in private consumption relative to that in public consumption. Therefore, the tax rate $t_2^L$ depends on this relative strength: The higher the risk aversion in private consumption relative to that in public consumption, the higher the tax rate on the skill premium. The intuition is as follows: The more disutility in private consumption is caused by risk, relative to disutility in public consumption, the more risk should be transferred to public consumption.

If the risk aversion in private (public) consumption is sufficiently high (low), only a progressive taxation $t_2^L > t_1^L$ ensures optimal risk diversification. However, this must be complemented by an education subsidy $p_B < 0$ in order to avoid disincentive effects on human capital investment. Thereby, the tax differential in percent should equal the ratio between the subsidy per semester, $p_B$ and wage earnings per unit of time, $W^0$.

If the household is little risk averse in private consumption, or the optimal tax rate $t_1^L$ is very high because of the need to finance large public spending, the optimal wage tax structure can turn out to be regressive. In this case tuition fees $p_B > 0$ are required to secure efficiency in allocation.

Proposition 3 fits to the results of optimal wage taxation in case of idiosyncratic risky human capital formation. If the risk is idiosyncraticly distributed, the society itself is risk neutral in public consumption. Thus, the optimal surtax rate would be equal to one, if skilled labor was inelastic, and all disincentive effects could be controlled by education subsidies (see Schindler and Yang, 2007).

In a nutshell, a strong linkage between wage taxation and tuition fees/educations subsidies is once more needed in order to improve or even restore efficiency, while the differentiated wage tax allows to follow another aim. This principle is well-known as 'Siamese Twins'- concept by Bovenberg and Jacobs (2005).[20]

Turning to risk in real capital investment, we conclude that an equivalent risk diversification rule applies as implied by Propositions 2 and 3. It is not necessary, however, to amend the diversification of capital risk by another policy instrument.

**Proposition 4.** *The optimal capital-risk sharing rule ensures that the (capital) risk premia in private and public consumption are equalized in equilibrium,* $RP_C(\tilde{x}) = RP_G(\tilde{x})$.

[20] In Bovenberg and Jacobs (2005) this second aim is income redistribution, whilst in our paper the differentiation is required in order to secure an optimal diversification of risk onto all social assets.

*Proof.* We infer from equations (9) and (32) that

$$\mathrm{E}\left[U_{c_t} \cdot (\tilde{x} - r)\right] = 0 = \mathrm{E}\left[V_G \cdot (\tilde{x} - r)\right]. \tag{60}$$

Applying Steiner's Rule for covariances and $\mathrm{E}\left[U_{c_t}\right] = \mathrm{E}\left[V_G\right]$ from (50), as well as rearranging, we come to

$$RP_C(\tilde{x}) = -\frac{\mathrm{Cov}(U_{c_t}, x)}{\mathrm{E}\left[U_{c_t}\right]} = -\frac{\mathrm{Cov}(V_G, x)}{\mathrm{E}\left[V_G\right]} = RP_G(\tilde{x}) \tag{61}$$

□

The risk in capital income is also diversified on private and public consumption in order to ensure that utility in private and public consumption are ex-post fluctuating in a desirable way – this time dependent on the realization of the risky return $\tilde{x}$. This diversification is achieved by the tax rate $t_1^K$ onto the excess return, and the level of the tax rate depends – analogous to the reasoning given for the level of tax rate $t_2^L$ – on the strength of the risk aversion in private consumption, relative to that in public consumption.

However, there are two differences between diversifying wage risk and capital risk. First, optimal capital risk sharing can be implemented without any distortions in the household's behavior and in private consumption. The reason for it is that it is possible to tax the risk premium directly, which causes only a Musgrave-substitution effect and leaves utility in private consumption unaffected (see, i.e., equation (20)). Second, the households are already entirely diversified in capital risk, therefore, the government cannot improve private risk allocation. However, it can again provide an increased diversification of risk, because the provision of the public good, which is not provided by the capital market, increases the number of *socially* available assets. Again, the diversification result does not imply any assumption concerning the social discount rate. As the tax revenue is not redistributed as income, Gordon's (1985) neutrality result does not apply as well.[21]

Combining our results, optimal risk-sharing in wage and capital income im-

[21] See also the intuition given for Proposition 2 and section 2.

plies

$$\frac{RP_C(\tilde{W}^1)}{RP_G(\tilde{W}^1)} = 1 = \frac{RP_C(\tilde{x})}{RP_G(\tilde{x})}, \tag{62}$$

and we end up with equal proportions of risk premia in human and real capital. Accordingly:

**Corollary 1.** *There is an indirect risk diversification effect, equating the relative risk premia* $\frac{RP_C(b)}{RP_G(b)}$*,* $b = \tilde{W}^1, \tilde{x}$ *between human and real capital investment.*

Thus, as long as leisure demand is inelastic, a tax system, incorporating a differentiated wage tax combined with either tuition fees or education subsidies and a risk tax on excess returns in real capital assets, can ensure efficient risk diversification of aggregate income risk onto private and public consumption, whereby all risk premia are equated. Moreover, such a tax system does not cause any inefficiencies or distortions.

Hence, although human capital and real capital investment under risk are strongly linked, as shown by Williams (1978), and the risk premium in wage income cannot be taxed separately, the presence of multiple aggregate risk does not cause major problems for an optimal tax and public policy – as long as the government has access to sufficient and suitable instruments.

However, this might change, if one introduces endogenous labor supply. In this case it is of importance that only the entire skill premium, but not the compensation for risk alone can be taxed.

# 6 Extensions and Omissions

To assume that leisure demand is exogenously given, is of course a very restrictive assumption. It is helpful to derive explicit solutions, because this is hardly possible, if there are multiple income risks and endogenous leisure demand, but we must be aware that we neglect substantial costs of taxation. Therefore, we are going to provide some intuition on which effects should be expected in an extended setting, and we will thereby refer to results in other papers, using simplified models.

It is well-known from Atkinson and Sandmo (1980) that it is optimal to balance overall excess-burden on distortions in labor supply and savings – except for special cases, where there is weak separability in leisure. Jacobs (2005) shows that endogenous educational investment increases the elasticity of labor supply, and therefore efficiency costs of labor taxation. Thus, implementing this in Atkinson and Sandmo (1980) should decrease the tax burden on labor.

Next, Jacobs and Bovenberg (2005) state that, even in the case of weak separability in leisure, capital taxation can be useful as indirect instrument in order to mitigate distortions in human capital taxation.

However, all these papers focus on deterministic incomes. The model, most similar to the present one, is Schindler (2006). He examines the optimal tax structure, using a proportional labor tax and the same capital tax system as we do, but focuses on a model, where only capital returns are risky in aggregate. While using endogenous labor supply, he neglects human capital investment and labor supply in the second period of life.

Schindler (2006) shows that the risk tax on the excess return allows to separate the risk issue and that in this case a twofold trade-off is emerging. First, the equivalent optimal taxation rule for deterministic labor and capital income like in Atkinson and Sandmo (1980) applies. This will most likely lead to underprovision of the public good. The latter determines the second trade-off. Underprovision can be countered by increasing the tax rate onto the excess return above the rate, which equates the risk premia in private and public consumption. This will generate more tax revenue in expected value and mitigate expected underprovision. However, this has to be paid with increased risk in public consumption. Thus, the second trade-off is optimal risk diversification versus underprovision; there are no additional effects on private consumption.

Embedding all these results above into the results of this paper, allows to state a conjecture, how endogenous leisure demand will influence our conclusions:

**Conjecture 1.** *If leisure demand is endogenous in both periods of life, it is most likely that, compared to the results in section 5,*

*(i) deterministic interest income will be taxed at a positive rate.*

*(ii)* *the tax rate* $t_2^L$ *on the skill premium will be decreased, and there is only suboptimal risk shifting to public consumption.*

*(iii)* *progressivity of the wage tax will be increased (decreased), if unskilled labor supply is more (less) elastic than skilled labor supply. The opposite holds true in case of regressive taxation. Moreover, in case of progressivity, capital taxation acts as indirect subsidy to education, and education subsidies* $p_B$ *should be expected to decrease. If labor taxation is regressive, instead, positive capital taxation should increase tuition fees.*

*(iv)* *the tax rate* $t_1^K$ *on the excess return should be increased, in order to generate more expected tax revenue and to mitigate the underprovision with the public good. This will be repaid by increased risk in public consumption, thus there will be too much social risk than compared to Proposition 4.*

The intuition for that conjecture is as follows: If distortions in labor supply cannot be avoided, it is optimal to balance the distortions over labor supply, savings and human capital investment. As taxation gets now more expensive, this will shift the trade-off away from risk diversification and towards efficiency. The major problem here is that there is no equivalent to the risk tax in capital, which only targets the risk premium. Any wage tax will not only shift risk, but also cause disincentives, which cannot be fully controlled by educational policy.

Moreover, the more elastic a tax base is, the less should be its tax burden, then. This explains the first set of effects in part (iii). As capital taxation subsidizes human capital investment, direct subsidies can decrease even more, as they would anyway. However, if the wage tax is regressive and tuition fees are used, the latter should be increased, in case there is positive capital taxation.

The result in (iv) follows directly from the shift in the trade-off between efficiency and risk diversification in capital risk and the discussion of Schindler (2006).

Whereas introducing endogenous leisure demand seems to have strong effects on the results, it is straightforward to introduce several risky assets. This can be done by assuming several sectors employing both a risky technology and skilled labor. As long as the Markowitz-case can be applied, each household will then hold a fully diversified, identical market portfolio of risky assets. Taxing the excess return in each risky asset with the risk tax rate $t_1^K$ will have the same effects as in the present model, where the risky asset can be interpreted as the market portfolio of all risky assets (see also Schindler, 2006, relying on Sandmo, 1977). In a nutshell, several risky assets should not change optimal public policy.

Another neglected item is unemployment risk. In fact, households are faced either with substantial unemployment risk or with risky income as unskilled worker. Due to competitive labor markets, our model cannot give any information about unemployment and education as insurance device. Of course, it is possible to model the flip side of the coin, stochastic unskilled labor income, but in our setting this is also of limited use, as households are unskilled in the first period only – before acquiring education. Although the absence of wage risk in the unskilled sector is on the one hand a deficiency of the model, it allows on the other hand for clear-cut results on optimal tax systems for skilled households.

# 7 Conclusions

We have shown that the government can provide efficient risk diversification on private and public consumption and that it can create an institution to 'trade' a part of uninsurable wage risk by using differentiated wage and capital taxes and relying on adjusted educational policies.

The simultaneous presence of wage and income risk does not challenge public policy very much, if it has access to a full set of instruments and leisure demand is inelastic. This is in contrast to the challenge in the privat sector, where the effects of wage and capital risk differ substantially.

Our results fit into a growing literature, which emphasizes a strong linkage between optimal tax systems and educational policies. It turns out that in the presence of risk and sufficient risk aversion in private consumption, it is better to have ex-post tuition fees, thus, progressive wage taxation, which has to be accompanied by educational subsidies in order to stabilize human capital investment. This can be seen as a potential justification for most European education systems, traditionally not (very much) relying on tuition fees, but on progressive taxation – and sometimes even tending to offer public scholarships (i.e., the Nordic countries).

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