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Local density of states in a dirty normal metal connected to a superconductor

W. Belzig and C. Bruder
Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

Gerd Schön
Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany
and Department of Technical Physics, Helsinki University of Technology, FIN-02150 Espoo, Finland
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A superconductor in contact with a normal metal not only induces superconducting correlations, known as the proximity effect, but also modifies the density of states at some distance from the interface. These modifications can be resolved experimentally in microstructured systems. We therefore study the local density of states $N(E,x)$ of a superconductor–normal-metal heterostructure. We find a suppression of $N(E,x)$ at small energies, which persists to large distances. If the normal metal forms a thin layer of thickness L_n, a minigap in the density of states appears which is of the order of the Thouless energy $\sim \hbar D/L_n^2$. A magnetic field suppresses the features. We find good agreement with recent experiments of Guéron *et al.*

I. INTRODUCTION

A normal metal in contact with a superconductor acquires partial superconducting properties. Superconducting correlations, described by a finite value of the pair amplitudes $\langle \psi_\downarrow(\boldsymbol{x})\psi_\uparrow(\boldsymbol{x})\rangle$, penetrate some distance into the normal metal. This *proximity effect* has been studied since the advent of BCS theory (see Ref. 1 and references therein). Recently, progress in low-temperature and microfabrication technology has rekindled interest in these properties.[2–6] Interference effects in dirty normal metals increase the Andreev conductance.[7,8] The effect of the superconductor on the level statistics of a small normal grain has been investigated.[9]

Whereas the order parameter penetrates into the normal metal, the pair potential $\Delta(\boldsymbol{x})$ vanishes in the ideal metal without an attractive interaction. Since Δ yields the gap in the single-particle spectrum of a bulk superconductor, the question arises as to how the spectrum of the normal metal is modified by the proximity to the superconductor. Recently, this question has been investigated experimentally by Guéron *et al.*[6] In their experiment, the local density of states of a dirty normal metal in contact with a superconductor was measured at different positions and as a function of an applied magnetic field.

In this paper, we evaluate the local density of states $N(E,x)$ of a superconductor–normal-metal (S-N) heterostructure with impurity scattering in a variety of situations. We generalize earlier theoretical work[10–13] by applying the quasiclassical Green's function formalism and by including the effect of a magnetic field. We compare with the experiment of Guéron *et al.*[6] and find good qualitative agreement with the experimental data both in the cases with and without a magnetic field.

II. MODEL

In the following we will consider geometries as shown in Fig. 1. The superconductor is characterized by a finite pairing interaction λ and transition temperature $T_c > 0$. In the normal metal we take $\lambda = T_c = 0$. Here we restrict ourselves to the dirty (diffusive) limit, $\xi \gg l_{\rm el}$, where $\xi = (D/2\Delta)^{1/2}$ is the superconducting coherence length at $T=0$ and $l_{\rm el}$ is the elastic mean free path. The latter is related to the diffusion constant via $D = \frac{1}{3} v_F l_{\rm el}$.

The density of states (DOS) of this inhomogeneous system can be derived systematically within the quasiclassical real-time Green's functions formalism.[14] In the dirty limit the equation of motion for the retarded Green's functions G_E and F_E reads[15]

$$\frac{D}{2}[G_E(\vec{\nabla} - 2ie\vec{A})^2 F_E - F_E \vec{\nabla}^2 G_E] = (-iE + \Gamma_{\rm in})F_E - \Delta G_E + 2\Gamma_{\rm sf} G_E F_E . \qquad (1)$$

The diagonal and off-diagonal parts of the matrix Green's function, G_E and F_E, obey the normalization condition

$$G_E^2 + F_E^2 = 1 , \qquad (2)$$

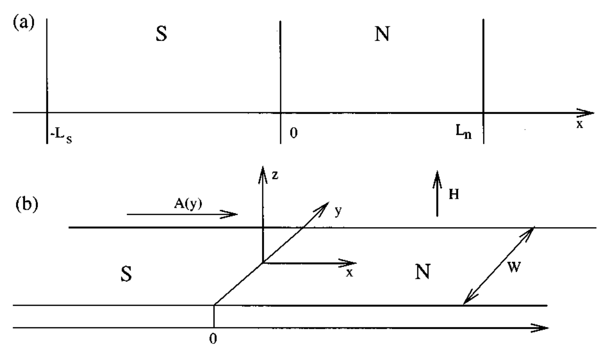

FIG. 1. Geometries considered in this article. (a) A strictly one-dimensional geometry. (b) A more realistic geometry similar to experimental setup.

which suggests to parametrize them by a function $\theta(E,x)$ via $F_E=\sin(\theta)$ and $G_E=\cos(\theta)$. Inelastic scattering processes are accounted for by the rate $\Gamma_{\rm in}=1/2\tau_{\rm in}$, while scattering processes from paramagnetic impurities are described by the spin-flip rate $\Gamma_{\rm sf}=1/2\tau_{\rm sf}$. At low temperatures the former is very small ($\Gamma_{\rm in}\sim 10^{-3}\Delta$), and will be neglected in the following.

For the geometry shown in Fig. 1 the order parameter can be taken to be real. On the other hand, in the vicinity of a N-S boundary the absolute value of the order parameter is space dependent, and has to be determined self-consistently. The self-consistency condition is conveniently expressed in the imaginary-time formulation, where

$$\Delta(x)\ln\left(\frac{T}{T_c(x)}\right)=2\pi T\sum_{\omega_\mu>0} F_{i\omega_\mu}(x)-\frac{\Delta(x)}{\omega_\mu}. \tag{3}$$

Here, $\omega_\mu=\pi T(2\mu+1)$ are Matsubara frequencies. The summation is cut off at energies of the order of the Debye energy. The coupling constant in S has been eliminated in favor of T_c, while the coupling constant in N is taken to be zero.

In the case where the interface between N and S has no additional potential, the boundary conditions are[16]

$$F_E(0_-)=F_E(0_+), \tag{4}$$

$$\frac{\sigma_s}{G_E(0_-)}\frac{d}{dx}F_E(0_-)=\frac{\sigma_n}{G_E(0_+)}\frac{d}{dx}F_E(0_+).$$

Here, $\sigma_{n(s)}$ are the conductivities of the normal metal and the superconductor, respectively. The complete self-consistent problem requires a numerical solution. Starting from a step-like model for the order parameter, self-consistency was typically reached within 10 steps. Finally the DOS is obtained from $N(E)=N_0{\rm Re}G_E(x)$, where N_0 is the Fermi level DOS in the normal state.

We will present now results for three different cases: (A) the DOS near the boundary of a semi-infinite normal metal and superconductor, (B) the DOS in a thin normal film in contact with a bulk superconductor, and (C) the effect of a magnetic field on the DOS in an experimentally realized N-S heterostructure. In the following sections energies and scattering rates will be measured in units of the bulk energy gap Δ and distances in units of the coherence length $\xi=(D/2\Delta)^{1/2}$.

III. RESULTS AND DISCUSSION

A. DOS in an infinite system

We assume that the normal metal and the superconductor are much thicker than the coherence length $L_s, L_n \gg \xi$ and investigate how the DOS changes continuously from the BCS form $N_{\rm BCS}(E)/N_0=|E|/(E^2-\Delta^2)^{1/2}$ deep inside the superconductor to the constant value $N_N(E)/N_0=1$ in the normal metal.

In a first approximation, neglecting self-consistency and paramagnetic impurities, we can solve Eq. (1) analytically, with the result

$$\theta(E,x)=\begin{cases} 4\arctan[\tan(\theta_0/4)\exp(-\sqrt{2\omega/D_n}x)], & x>0,\\ \theta_s+4\arctan\{\tan[(\theta_0-\theta_s)/4]\exp(\sqrt{2\sqrt{\omega^2+\Delta^2}/D_s}x)\} & x<0\,. \end{cases} \tag{5}$$

Here

$$\omega=-iE+\Gamma_{\rm in},$$

$$\theta_s=\arctan\left(\frac{\Delta}{-iE+\Gamma_{\rm in}}\right),$$

$$\sin\frac{\theta_0-\theta_s}{2}=\gamma\frac{(-iE+\Gamma_{\rm in})^{1/2}}{[(-iE+\Gamma_{\rm in})^2+\Delta^2]^{1/4}}\ \sin\frac{\theta_0}{2}.$$

Several material parameters combine into the parameter

$$\gamma=(\sigma_n\xi_s/\sigma_s\xi_n), \tag{6}$$

measuring the mismatch in the conductivities and the coherence lengths of the two materials. Furthermore, $\xi_{s(n)}$ is defined by $(D_{s(n)}/2\Delta)^{1/2}$, where $D_{s(n)}$ is the diffusion constant of the superconductor (normal metal).

The resulting DOS $N(E)$ in the normal metal at a distance $x=1.5\xi_n$ from the interface is shown in Fig. 2 for different values of the parameter γ. It shows a subgap structure with a peak below the superconducting gap energy $E<\Delta$ and a strong suppression at zero energy. The modification of the DOS is most pronounced at small values of γ and at small distances. The smaller the energy, the larger is the distance where the modifications are still visible. In particular at $E=0$ the DOS vanishes for all values of x. Pair-breaking effects lead to a finite zero-energy DOS, as will be shown later.

Next we solve the problem self-consistently and present some numerical results for the case $\gamma=1$. We first concentrate on the superconducting side of the boundary. As shown in Fig. 3 the peak in the DOS is strongly suppressed, changing from a singularity to a cusp, but it remains at the same position Δ as one approaches the boundary. On the other hand, the density of states increases for energies below Δ. The states with energies well below Δ decay over a characteristic length scale $\sqrt{D_s/(2\sqrt{\Delta^2-E^2})}$; see Eq. (5).

The DOS on the normal side at different distances from the N-S boundary is shown in Fig. 4. The pronounced subgap structure found in the approximate solution is still present in the self-consistent treatment. The figure shows how the peak height and position change with the distance. In the absence of pair-breaking effects the DOS vanishes at the Fermi level for all distances (dotted curves). Inclusion of a pair-breaking mechanism (solid curves) regularizes the DOS at the Fermi level, and also the peak height is sup-

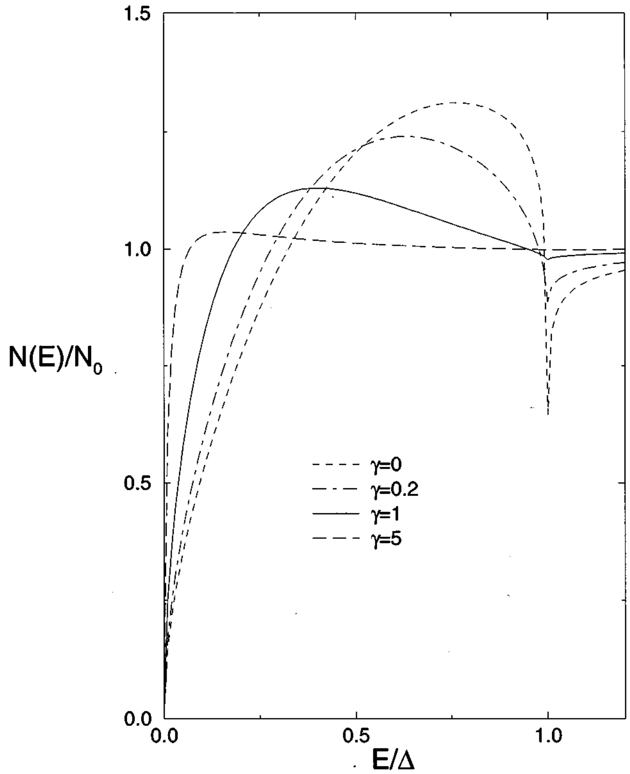

FIG. 2. DOS in the normal metal at $x=1.5\xi_n$.

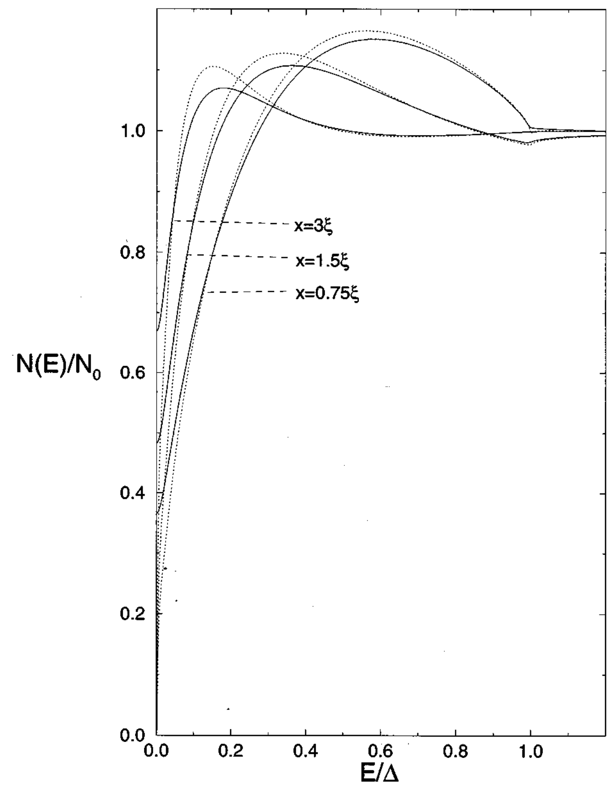

FIG. 4. Density of states on the normal side of a *N*-*S* boundary for two spin-flip scattering rates: $\Gamma_{sf}=0$ (dotted lines) and $\Gamma_{sf}=0.015\Delta$ (solid lines).

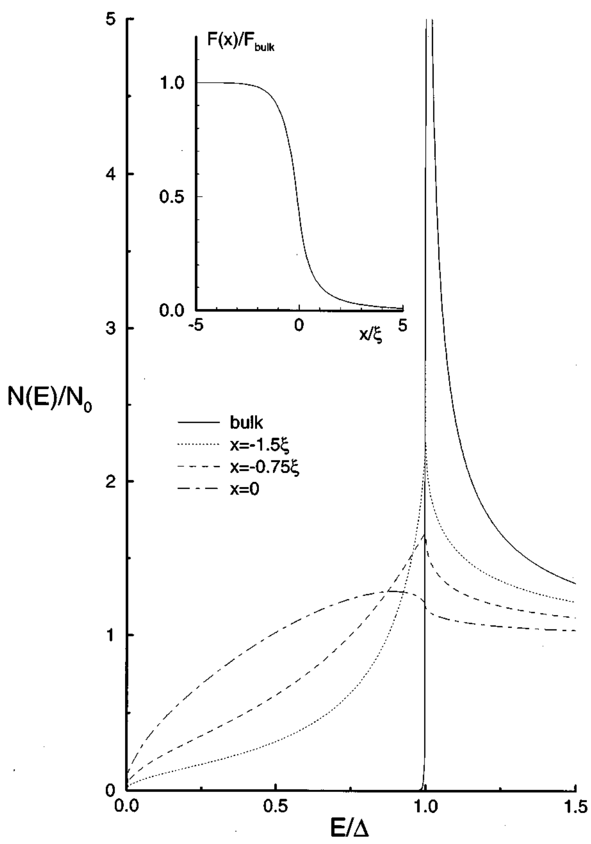

FIG. 3. Density of states on the superconducting side of the *N*-*S* boundary. The inset shows the self-consistent pair amplitude.

pressed. The curves are in qualitative agreement with the experimental data shown in Ref. 6. The self-consistent calculation presented here leads to a slightly better fit than the theoretical curves shown in Ref. 6 where a constant pair potential was used in the solution of the Usadel equation. In particular, the low-energy behavior of the experimental curves is reproduced correctly.

At finite temperatures (but $T \ll T_c$) we expect no qualitative changes in the behavior described above except that the structures in the DOS will be smeared out by inelastic scattering processes. Hence for an experimental verification temperatures as low as possible would be most favorable.

B. DOS in thin *N* layers

Next we consider a thin normal layer in contact with a bulk superconductor, $L_s \gg L_n \simeq \xi$. The boundary condition at $x=L_n$ is chosen to be $d\theta(E,x)/dx=0$; i.e., the normal metal is bounded by an insulator. In this case the DOS on the *N* side develops a minigap at the Fermi energy, which is smaller than the superconducting gap. If the thickness of the normal layer is increased, the size of this minigap decreases. Results obtained from the self-consistent treatment are shown in Fig. 5. Details of the shape of the DOS depend on the location in the *N* layer.[17] However, the magnitude of the minigap is space independent, as shown in the inset of Fig. 5. The magnitude of the gap is expected to be related to the Thouless energy D/L_n^2 which is the only relevant quantity which has the correct dimension. Of course the relation has to be modified in the limit $L_n \to 0$. Indeed as shown in Fig. 5

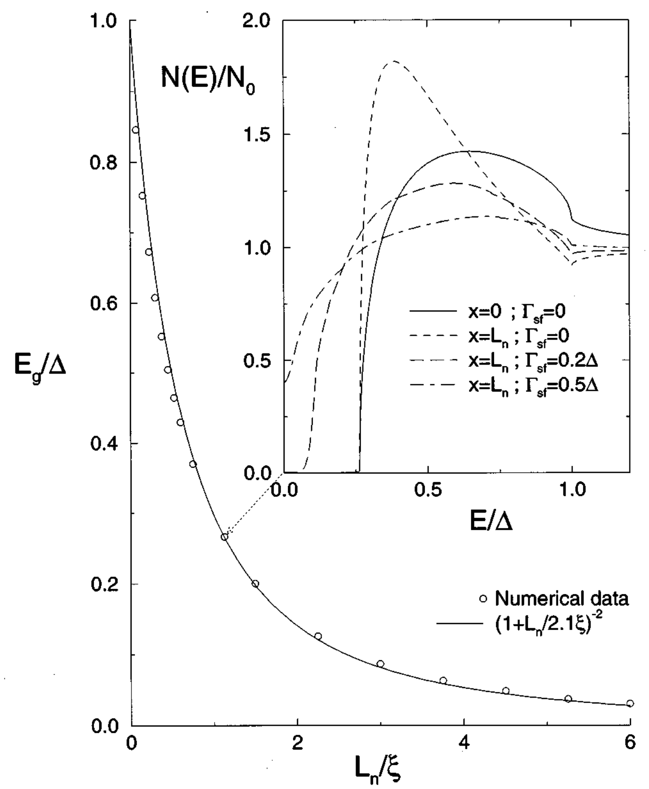

FIG. 5. Minigap E_g as a function of the normal-layer thickness. Inset: local DOS of a N layer of thickness $L_n = 1.1\xi$ in proximity with an bulk superconductor.

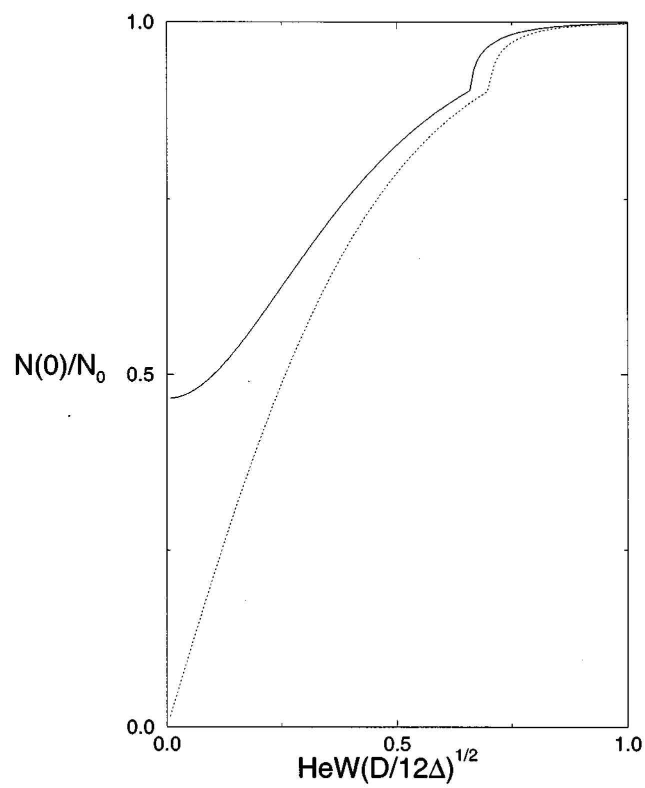

FIG. 6. Zero-energy DOS in the normal metal at $x = 1.5\xi$ as a function of the magnetic field. The spin-flip scattering rate is $\Gamma_{\rm sf} = 0$ (dashed curve) and $\Gamma_{\rm sf} = 0.06\Delta$ (solid curve).

a relation of the form $E_g \sim (\text{const} \times \xi + L_n)^{-2}$ fits quite well. The sum of the lengths may be interpreted as an effective thickness of the N layer since the quasiparticle states penetrate into the superconductor to distances of the order of ξ. The effect of spin-flip scattering in the normal metal on the minigap structure is also shown in the inset of Fig. 5. The minigap is suppressed as $\Gamma_{\rm sf}$ is increased until a gapless situation is reached at $\Gamma_{\rm sf} \approx 0.4\Delta$.

We would like to mention that a similar feature had been found before by McMillan[10] within a tunneling model ignoring the spatial dependence of the pair amplitude. We have considered here the opposite limit, assuming perfect transparency of the interface but accounting for the spatial dependence of the Green's functions. For $\Gamma_{\rm sf} = 0$ our results for the structure of the DOS agree further with previous findings of Golubov and Kupriyanov[11] and Golubov *et al.*[12] Recently, a minigap in a two-dimensional electron gas in contact to a superconductor has also been studied.[18]

C. Density of states in a magnetic field

An applied magnetic field suppresses the superconductivity in both superconductor and normal metal. To study the effect of the magnetic field on our system we consider the geometry shown in Fig. 1(b). Because in the experimental setup the thickness of the films is much smaller than the London penetration depth, we can neglect the magnetic field produced by screening currents. Therefore it is reasonable to assume a constant magnetic field, which is present in both S and N. The vector potential is then chosen to be

$$\vec{A} = A(y)\vec{e}_x, \quad A(y) = Hy. \tag{7}$$

Equation (1) can be considerably simplified in the case that the size of the system in the y direction is smaller or of the order of ξ. The system is limited to $-W/2 < y < W/2$, where $W \simeq \xi$. Therefore the Green's functions do not depend on y and the equation can be averaged over the width W. The equation reduces to the effective one-dimensional equation

$$\frac{D}{2}(G_E \partial_x^2 F_E - F_E \partial_x^2 G_E) = (-iE + \Gamma_{\rm in})F_E - \Delta G_E + 2\Gamma_{\rm eff} G_E F_E. \tag{8}$$

Here, $\Gamma_{\rm eff} = \Gamma_{\rm sf} + De^2H^2W^2/12$ acts as an effective pair-breaking rate, which depends on the transverse dimension and the applied magnetic field.

If we approximate the Green's functions in the superconductor by their bulk values, the DOS in the normal metal at zero energy can be calculated analytically:

$$\frac{N(0)}{N_0} = \begin{cases} \tanh(2\sqrt{\Gamma_{\rm eff}/D}\,x), & 2\Gamma_{\rm eff} < \Delta \\ (1-\alpha^2)/(1+\alpha^2), & 2\Gamma_{\rm eff} > \Delta, \end{cases} \tag{9}$$

where

$$\alpha = \frac{\Delta \exp(-2\sqrt{\Gamma_{\rm eff}/D}\,x)}{2\Gamma_{\rm eff} + \sqrt{4\Gamma_{\rm eff}^2 - \Delta^2}}. \tag{10}$$

In Fig. 6 the dependence of the DOS on the magnetic field at $x = 1.5\xi$ is shown for two different spin-flip scattering rates

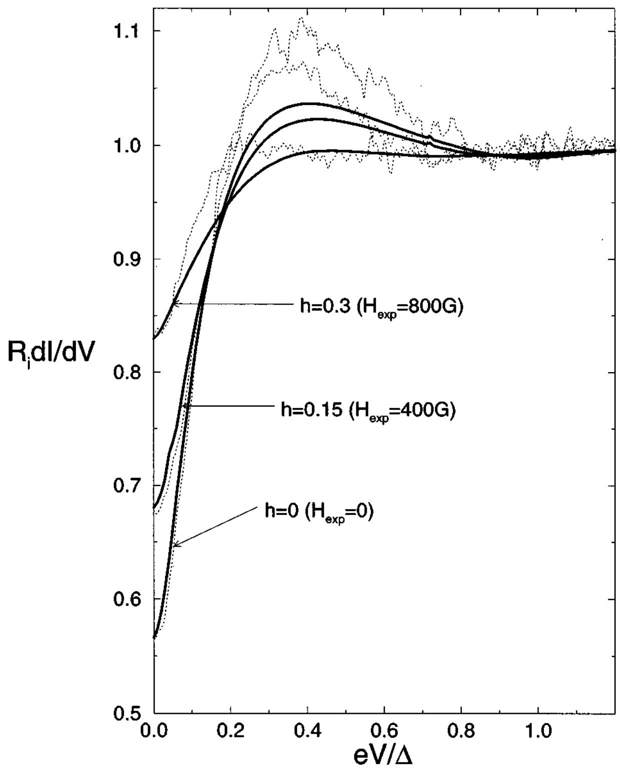

FIG. 7. Quantitative comparison of experiment (Ref. 19) (dotted lines) and theory (solid lines). The experimental magnetic fields are $H=0$, 400, and 800 G; $h=HeW(D/12\Delta)^{1/2}$. The theoretical curves have been normalized by $R_i \equiv dI/dV_{\rm expt}(eV/\Delta=1.5)$.

(equal rates for normal metal and superconductor). In the absence of paramagnetic impurities the DOS increases linearly with the field, whereas it starts quadratically if paramagnetic impurities are present. At the field defined by the relation $\Gamma_{\rm eff}=0.5\Delta$ the field dependence of the DOS shows a kink. This kink arises because above this value of $\Gamma_{\rm eff}$ the zero-energy DOS in the superconductor is nonzero (gapless behavior), which leads to an even stronger suppression of the proximity effect.

Figure 7 shows a quantitative comparison of our results with experimental data taken by the Saclay group.[19] In this experiment,[6] the differential conductance of three tunnel junctions attached to the normal metal part of the system was used to probe the DOS at different distances from the superconductor. Accordingly, we have calculated the self-consistent DOS in the presence of a magnetic field for all energies and determined the differential conductance.[20] We used $x=1.8\xi$, consistent with an estimate from a scanning electron microscope (SEM) photograph, and used a spin-flip scattering rate of $\Gamma_{\rm sf}=0.015\Delta$ in the normal metal as a fit parameter. This is necessary in the framework of our approach to explain the finite zero-bias conductance at zero field. We furthermore assumed ideal boundary conditions at the *N-S* interface, i.e., $\gamma=1$, the motivation being that great care was used in the experiment to produce a good metallic junction, and significant Fermi velocity mismatches are not to be expected.

At low and high voltages the agreement with the experimental data is good for all three field values. On the other hand, the maximum in the DOS is not reproduced well by our calculation. Including the effect of a nonideal boundary, i.e., $\gamma<1$, leads to an increase of the peak in the DOS but to a less satisfactory fit at low voltages. We cannot resolve this discrepancy, but we would like to point out that our theory is comparatively simple and does not include all the geometric details of the experiment (e.g., the geometry of the overlap junction is not really one dimensional and would be difficult to treat realistically). Our intention is to show that the theoretical treatment described here contains the physical ingredients to explain the basic features of the experimental data. The overall agreement between theory and experiment demonstrated in Fig. 7 shows this to be the case.

IV. CONCLUSIONS AND OUTLOOK

In conclusion, we have given a theoretical answer to the question asked in the Introduction; viz., what is—beyond the proximity effect—the effect of a superconductor on the spectrum of a normal metal coupled to it? Using the (real-time) Usadel equations, we have calculated the local density of states in the vicinity of a *N-S* boundary in both finite and infinite geometries. It shows an interesting subgap structure: If the normal metal is infinite, the density of states is suppressed close to the Fermi energy, but there is no gap in the spectrum. This is the behavior found in a recent experiment.[6] In thin normal metals we find a minigap in the density of states which is of the order of the Thouless energy. We have also investigated the suppression of these effects by an applied magnetic field and find good agreement with experiment.

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[20] The experimental differential conductance also shows single-electron effects caused by the small capacity of the tunnel junctions. It is necessary to include them for a quantitative comparison, and we have done so using the prescription given in Ref. 6.