Three Essays in Labor Economics

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Summary

This dissertation has been written as part of my studies in the "Doctoral Programme in Quantitative Economics and Finance" at the University of Konstanz. It comprises three stand-alone research papers. All three deal with labor market phenomena; they can be divided into two main areas of research: intergenerational wage mobility in chapter 1 and joint search in chapters 2 and 3. Chapter 1 gives a new explanation for the high intergenerational correlation of wages in the US. In an Overlapping Generations (OLG) Model, I analyze the impact of the progressivity of the income tax system on intergenerational mobility, assuming that consumption aspirations of children are linked to disposable income of the parents. Chapters 2 and 3 cover one area of research: joint search theory. They both address a model in which couples have a joint utility function and are searching for a job in a stochastic environment. Chapter 2 is the structural estimation of a partial search model for couples (Guler et al. [2009]). Chapter 3 develops an equilibrium model for couples with random search and wage posting firms. This work is motivated by several shortcomings of the previous literature. Search theory of the labor market has been successful at explaining various labor market phenomena: equilibrium unemployment, unemployment durations, the wage offer distribution etc. However almost all models developed in the last 40 years deal with the case of a single agent. The study of couples and families has been studied in the field of family economics, focusing among others on consumption, savings, migration and labor supply. It is surprising that little work has been done on the intersection of family economics and search theory of the labor market. In both chapters 2 and 3, I consider a model in which the main decision making unit is a couple which consists of two agents that have a common utility function over their pooled income (and possibly their individual leisure). Chapter 2 is empirical work on a partial model, while chapter 3 studies a theoretical general equilibrium model (i.e. modelling firms and hence the wage setting mechanism explicitly).

Chapter 1 studies intergenerational mobility as measured by the correlation of fathers' and sons' wage income. The main transmission mechanism of intergenerational inequality are differences in the choices of education between children of high income and low income parents. The model I use to study this problem is a three period OLG model. Individuals choose their education when young (going to college or not), consumption and savings when adult and consume their savings when retired. The central idea is the following: suppose individuals have a utility function with a reference point (the so called catching up with the Joneses formulation). I assume that in the first period (youth) agents form their reference point, e.g. by observing the lifestyle of their parents (which I proxy by the after-tax income of the parents). When choosing whether to acquire tertiary education or not, the young weigh the potential gains in income against the disutility of studying. A lower reference point decreases the marginal utility of (future) consumption and thereby makes going to college less advantageous. Hence the lower the earnings of the parents, the lower the consumption aspirations of the child and hence the lower the probability of going to college. Since going to college results in higher lifetime earnings, the model creates a positive correlation of father's and son's earnings. The calibration shows that the model can replicate stylized facts for the US economy. The model also gives predictions for international differences in intergenerational mobility. A more progressive income tax system "cools down" aspirations for the children of high-income parents and hence education choices of children from high- and low-income families become more similar. I illustrate this idea by decomposing the difference in intergenerational mobility between Sweden and the US in a component that is due to more equal schooling in Sweden (as measured by differences in PISA test scores between children of high- and low-income families) and one that is due to more progressive income taxes. This illustrates a trade-off between efficiency (output) and equality (intergenerational mobility). A more progressive income tax system increases intergenerational mobility but also reduces output by making education less attractive.

Chapter 2 estimates the partial joint search equilibrium model of Guler et al. [2009] structurally on French data. It focuses on two main predictions of their work. The first prediction concerns couples where one spouse is working at a (relatively) low wage while the other spouse is searching. When offered a higher wage for the searching spouse, the couple accepts the second job and the first spouse quits his job. Guler et al. [2009] call this the "breadwinner cycle". The second prediction is that the reservation wage of the searching spouse is an increasing function of the partner's wage (under the assumption of DARA utility). I choose the French Labor Force Survey (Enquête Emploi) to test the predictions and estimate the parameters of the model structurally. The results show that there are very few couples who change in the breadwinner cycle sense. Furthermore, the occurrence of the breadwinner cycle is not (negatively) linked to the income of the quitting spouse, as predicted by the theoretical model. Second, the dependence of the reservation wage on the partner's wage is U-shaped and has little influence on the estimation of the structural parameters.

In chapter 3, I propose a wage posting random search model for risk-averse couples. The resulting equilibrium wage offer distribution depends heavily on the assumptions about the couple's preferences. In a homogeneous setting with zero utility of leisure, a Diamond [1971] solution arises, i.e. the unique equilibrium is characterized by a discrete wage offer distribution where firms only pay the workers' reservation wage. If there is heterogeneity in unemployment income between the spouses, the resulting equilibrium is of the Albrecht and Axell [1984] type: two wages are offered, the respective reservation wages of the two types. Introducing leisure into the utility function leads to an equilibrium where the wage offer distribution consists of a continuous part for lower wages and a mass point at the upper bound of the support. If there is heterogeneity in leisure, the type of equilibrium that emerges (i.e. the number of wages offered and the indifference conditions characterizing the wages) depends heavily on the different parameters: firm productivity, job offer arrival and job destruction rate. A complete analytical solution is not feasible, I only characterize three different equilibria without wage dispersion analytically. As an application of the model, I present a numerical example with equilibrium wage dispersion for the German economy. I show that the model can explain the unemployment rates of men and women, the marriage premium of men, and partially the gender wage gap and the observed differences in reservation wages between men and women.

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Zusammenfassung

Diese Dissertation ist als Teil des Promotionsprogramms "Quantitative Economics and Finance" an der Universität Konstanz entstanden. Sie umfasst drei eigenständige Artikel, die sich mit Fragen des Arbeitsmarktes beschäftigen. Die drei Artikel können weiterhin in zwei Themenbereiche unterteilt werden: intergenerationelle Lohnmobilität in Kapitel 1 und Suchmodelle des Arbeitsmarktes für Paare in Kapiteln 2 und 3. In Kapitel 1 wird eine neue Erklärung für die relativ hohe Korrelation von Löhnen von Vätern und Söhnen in den Vereinigten Staaten (als Maß für die intergenerationelle Mobilität) vorgeschlagen. Im Rahmen eines Modells mit überlappenden Generationen untersuche ich den Einfluss der Progressivität der Einkommenssteuer auf die intergenerationelle Mobilität unter der Annahme, dass Kinder "Konsumaspirationen" haben, die vom verfügbaren Einkommen der Eltern beeinflusst werden. Kapitel 2 und 3 haben einen gemeinsamen Forschungsfokus: "Joint Search Theory". Damit sind Modelle gemeint, in denen das Suchverhalten von Paaren in einem dynamischen und stochastischen Arbeitsmarkt untersucht wird. In Kapitel 2 schätze ich ein partielles Gleichgewichtsmodell (Guler et al. [2009]) strukturell. In Kapitel 3 entwickle ich ein Allgemeines Gleichgewichtsmodell für Paare, in dem Firmen und Arbeitnehmer zufällig zueinander finden[1] und Firmen ihre Löhne öffentlich ankündigen (und an die Ankündigungen gebunden sind, "wage posting"). Die Arbeit an "Joint Search" Modellen steht noch am Anfang und versucht, wichtige Fragen, die in der bisherigen Forschung vernachlässigt wurden, zu beantworten. Suchtheorie des Arbeitsmarks war erfolgreich bei der Erklärung von u.v.a. Arbeitslosigkeit im Gleichgewicht, Dauer der Arbeitslosigkeit oder der endogenen Bestimmung der Lohnangebotsverteilung. Aber die diskutierten Modell untersuchen

[1]Es handelt sich also um ein sogenanntes "random search" Modell. Im Gegensatz dazu gibt es Modelle, in denen Arbeitnehmer sich nur bei einem Teil der Unternehmen bewerben (nachdem sie bestimmte Charakteriska der Unternehmen beobachtet haben), die sogenannten "directed search" Modelle.

fast ausnahmslos einen alleinstehenden Arbeiter. Die Familienökonomik beschäftigt sich andererseits mit den ökonomischen Entscheidungen von Paaren, wie etwa der Konsum-, der Spar-, der Migrations-, oder der Arbeitsangebotsentscheidung. Es ist erstaunlich, dass es wenig Forschung in der Schnittmenge von Suchtheorie und Familienökonomik gibt. In Kapiteln 2 und 3 beschäftige ich mich mit Modellen, in denen die Entscheidungen (auf der Arbeiterseite) von Paaren getroffen werden. Diese Paare bestehen aus zwei Individuen, die eine gemeinsame Nutzenfunktion über die Summe ihrer Einkommen (und ihre Freizeit) haben. Kapitel 2 präsentiert empirische Arbeit für ein partielles Gleichgewichtsmodell, während Kapitel 3 sich theoretisch mit einem allgemeinen Gleichgewichtsmodell beschäftigt (in dem die Entscheidung von Firmen und die Lohnsetzung explizit modelliert werden).

In Kapitel 1 untersuche ich intergenerationelle Mobilität, gemessen durch die Korrelation zwischen dem Arbeitseinkommen von Vätern und Söhnen. Der wichtigste Transmissionsmechanismus von intergenerationeller Ungleichheit sind die unterschiedlichen Entscheidungen bei der Wahl der tertiären Bildung zwischen Kindern von Eltern mit hohem und niedrigem Einkommen. Ich untersuche diese Frage in einem Model überlappender Generationen mit drei Perioden. Am Ende der Jugend wählen Individuen, ob sie tertiäre Ausbildung erwerben möchten, als Erwachsene werden Konsum- und Sparentscheidungen getroffen, und in der dritten Periode werden die Ersparnisse konsumiert. Die zentrale Annahme des Modells ist, dass die Nutzenfunktion einen Referenzpunkt enthält. Dieser Referenzpunkt wird während der Kindheit gebildet, wenn Kinder das Konsumniveau der Eltern erleben. Bei der Wahl der tertiären Bildung werden die zukünftigen Einkommensvorteile mit dem negativen Nutzen des Studiums abgewägt. Je niedriger der Referenzpunkt, desto niedriger der Grenznutzen des zukünftigen Konsums und desto geringer der Nutzen durch die zusätzliche Bildung. Durch diesen Mechanismus beeinflusst ein niedriges Einkommen der Eltern die Konsumaspirationen der Kinder und reduziert dadurch deren Wahrscheinlichkeit zu studieren. Da ein Studium mit höherem Einkommen verbunden ist, wird durch diesen Mechanismus eine positive Korrelation zwischen dem Einkommen von Vater und Sohn erzeugt. Eine Kalibrierung des Modells zeigt, dass dessen Vorhersagen in vielerlei Hinsicht mit US-amerikanischen Daten im Einklang stehen. Das Modell ermöglicht außerdem, Vorhersagen zu internationalen Unterschieden bei der intergenerationellen Mobilität zu machen. Eine progressivere

Einkommensbesteuerung kühlt die Aspirationen von Kindern, deren Eltern ein hohes Einkommen haben, ab und sorgt so dafür, dass die Konsumaspirationen bei der Wahl der tertiären Bildung weniger entscheidend sind, so dass die Unterschiede bei der Bildungsentscheidung weniger stark vom Einkommen der Eltern abhängen. Dieser Abkühlungseffekt wird deutlich, wenn man Schweden und die USA vergleicht. Ich zerlege den Unterschied bei der intergenerationellen Mobilität zwischen den beiden Ländern in einen Effekt, der durch die Ungleichheit im Schulsystem (primär und sekundär, gemessen an PISA Testergebnissen) entsteht, und einen, der durch die progressiveren Steuern in Schweden entsteht. Dieses Beispiel zeigt, dass es einen trade-off zwischen Effizienz (Output) und Gleichheit (in diesem Fall intergenerationelle Mobilität) gibt. Je progressiver die Einkommensbesteuerung, desto größer die intergenerationelle Mobilität, desto niedriger aber gleichzeitig auch das Ausbildungsniveau und damit der Gesamtoutput der Volkswirtschaft.

In Kapitel 2 wird das partielle, also mit exogen gegebener Lohnangebotsverteilung, "Joint Search" Gleichgewichtsmodell von Guler et al. [2009] strukturell mit französischen Daten geschätzt. Der Fokus liegt hier auf zwei Vorhersagen des Modells. Die erste Vorhersage betrifft Paare bei denen ein Partner bei einem relativ niedrigen Lohn arbeitet während der andere auf der Jobsuche ist. Wenn so ein Paar ein Jobangebot mit einem höheren Lohn für den suchenden Partner bekommt, drehen sich die Rollen innerhalb des Paares um: der suchende wird zum "breadwinner", der arbeitende Partner kündigt seinen Job, um nach einem höher bezahlten Angebot zu suchen. Dieser simultane Wechsel wird von Guler et al. [2009] der "breadwinner cycle" genannt. Die zweite Hauptvorhersage des Modells ist, dass - unter der Annahme einer DARA Nutzenfunktion - der Reservationslohn des suchenden Partners mit dem Lohn des arbeitenden Partners ansteigt. Um diese Aussagen zu überprüfen und die Parameter des Modell strukturell zu schätzen, bediene ich mich des französischen Datensatzes Enquête Emploi. Die empirischen Ergebnisse zeigen, dass nur sehr wenige Paare im "breadwinner cycle" Sinn ihre Rollen tauschen, und dass der "breadwinner cycle" auch nicht häufiger beobachtet wird bei Paaren, in denen der Partner einen eher niedrigen Lohn verdient (wie es vom Modell vorhergesagt wird). Des Weiteren ist die Abhängigkeit des Reservationslohn vom Lohn des Partners U-förmig und hat nur einen sehr geringen Einfluss auf die Schätzergebnisse der strukturellen Parameter.

In Kapitel 3 stelle ich ein Gleichgewichts-Suchmodel des Arbeitsmarkts für risikoaverse Paare vor, in dem Firmen ihre Löhne ankündigen ("wage posting") und Arbeiter und Firmen zufällig aufeinander treffen ("random matching"). Die Lohnangebotsverteilung ist endogen und hängt stark von den Annahmen über die Präferenzen der Paare ab. Bei homogenen Präferenzen (und ohne Nutzen von Freizeit) ergibt sich eine Diamond [1971] Lösung, das heißt das einzige Gleichgewicht besteht aus nur einem Lohn, dem Reservationslohn von arbeitslosen Paaren. Wenn sich die beiden Partner im Hinblick auf ihr Einkommen bei Arbeitslosigkeit (staatliche Unterstützung und Heimarbeit) unterscheiden, ergibt sich ein Albrecht and Axell [1984] Gleichgewicht: zwei Löhne werden angeboten, die Reservationslöhne von Männern und Frauen. Führt man Nutzen von Freizeit ein, so ergibt sich ein Gleichgewicht, dessen Arbeitsangebotsverteilung aus einem stetigen Teil für niedrige Löhne und einem Massepunkt am oberen Rand der Verteilung besteht. Wenn Unterschiede zwischen Männern und Frauen bei dem Wert der Freizeit angenommen werden, hängt die Art des Gleichgewichts stark von den gewählten Parametern ab, vor allem der Produktivität der Unternehmen, aber auch der Rate, mit der Arbeiter Jobangebote bekommen, und der Rate, mit der Arbeitsverhältnisse exogen zerstört werden. Eine vollständige analytische Lösung ist in diesem Fall nicht möglich. Ich charakterisiere drei ausgewählte Gleichgewichte analytisch. Als Anwendung des Modells präsentiere ich ein numerisches Beispiel für Deutschland. Das Modell kann die Arbeitslosenraten für Männer und Frauen, die "Heiratsprämie" für Männer und teilweise den "Gender Wage Gap" und die Unterschiede zwischen den Geschlechtern bei den Reservationslöhnen erklären.

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Chapter 1

Consumption Aspirations and Intergenerational Mobility

1.1 Introduction

At the end of the 1980s, most studies showed a rather low correlation of fathers' and sons' earnings (see the references in Solon [1992]). This view is summarized in Becker's presidential address to the American Economic Association 1988: "low earnings as well as high earnings are not strongly transmitted from fathers to sons" (Becker [1988]). New evidence - overcoming previous measurement errors and using better data - challenged this conclusion: the empirical findings about intergenerational income mobility show that there is a substantial correlation between fathers' and sons' wage income (≥ 0.4 for the US, see Solon [2002]).

Education is central to understanding labor market outcomes. Accepting this premise, there are several models that have been used to explain the low observed intergenerational mobility. Assuming some credit market imperfection, the accumulation of human capital could be constrained for less wealthy people. However, for college education, credit constraints do not seem to be critical empirically (Mulligan [1997], Cunha et al. [2006]). At the earlier age of the child, investment decisions of the parents are important. That is why, parental altruism has been explored in the literature: Better educated parents may be able to spend more money on the education of their children (tutor them, buying books or engaging private tutors), which makes their children more successful in school. But there is neither a theoretical consensus on how to model altruism, nor can the basic altruism model find much empirical support (see e.g. Altonji et al. [1997]). Genetic inheritance or learning during childhood (for both, cognitive and non-cognitive abilities) is a transmission mechanism through which highly productive parents influence the labor market outcome of their children. Research in this area, especially concerning the importance and the exact channel through which non-cognitive skills are transmitted, is still actively ongoing. In this article, I argue that the transmission of abilities alone is not enough to explain the low level of intergenerational mobility in the US. It is only in conjunction with consumption aspirations in the utility function that the true size of the correlation of fathers' and sons' income can be explained.

In my model, I control for the transmission of cognitive ability using PISA data. In addition to that, I depart from the standard preference formulation: I present a model where consumption aspirations that are formed during youth affect college choice and labor market success. The basic idea is that for the same level of cognitive ability, children of unskilled parents have lower aspirations and hence gain lower value from going to college than children of skilled parents. This assumes that aspirations of children depend on the (after-tax) income of their parents. I show that this when taken together with differences in income tax schemes, is an important factor in explaining the cross-country differences in intergenerational mobility. The more progressive an income tax scheme is, the closer the aspirations between children of skilled and unskilled parents are, and hence their choices become more similar. A very progressive income tax scheme hence reduces the intergenerational wage correlation by "cooling down" aspirations.

The paper is structured as follows: First, I will outline the motivation for studying intergenerational mobility and show its quantitative importance with a simple numerical example. I then discuss different theories that have been used in the literature to explain intergenerational mobility, and use recent empirical evidence to show that there is still substantial ignorance in understanding intergenerational mobility. My model will be presented in a basic form to highlight the ground mechanism before coming to the calibration of an extended model.

1.2 Related literature

Intergenerational mobility has been studied for decades and in different disciplines. While sociologists focus on "classes" and transition probabilities between the parent's and the child's class, most economists analyze the correlation between parents' and children's wages or wealth since it offers a simple and widely available measure of intergenerational mobility (Erikson and Goldthorpe [2002]). Sticking to the tradition of economics, my interest is in how labor market success - as measured by the wage - is correlated between two generations[1]. More precisely, I am interested in ρ, the correlation coefficient between fathers' and sons' wage income[2].

The following numerical example gives an intuition of what different values of ρ mean for individual earnings perspectives. Assume that wages are log-normally distributed, i.e. that log-wages are normal for the father's and the son's generation and that this distribution does not vary over time. Assume for the distribution of log wages $\mu = 7.5$ and $\sigma = 1$[3]. Then, the 25%-, 50%- and 75%-quantiles of the wage distribution are $921, 1,808$ and $3,549$ respectively. In this example, moving the father of an individual from the 25%- to the 75%-quantile increases the expected wage of the individual by 15% if $\rho = 0.1$, 50% if $\rho = 0.3$ and almost doubles it if $\rho = 0.5$.

Bearing this example in mind, we can have a look at empirical estimates of ρ. The early estimates ($\rho < 0.2$ for the US, e.g. Behrman and Taubman [1985]) were mainly based on unrepresentative data sets and prone to measurement errors. Thus Gary Becker came to the above mentioned conclusion that "low earnings as well as high earnings are not strongly transmitted from fathers to sons". Since then, better panel data (PSID, SOEP, etc.) became available for longer time spans and the empirical evidence resulted in different conclusions. The challenges with regards to ρ's estimation are already outlined in Becker and Tomes [1986] and described in detail in Solon [1992]. The first reliable estimates for the US indicate a ρ of

[1]Throughout the paper, I will consider only stationary wage distributions. Then, there is no difference between the correlation coefficient ρ and the intergenerational income elasticity between father and son (the percentage increase of the son's income for a one percent increase in the father's income, given that log wages are considered). Both differ by a multiplicative constant, the ratio of the standard deviation of the father-generation's wage distribution over the standard deviation of the son-generation's wage distribution, for details see e.g. Bowles and Gintis [2002]

[2]Due to data availability, most studies consider correlation between fathers and sons, an exception is Chadwick and Solon [2002] who investigate how women's success is correlated to their parents' success in the labor market.

[3]As long as every individual works the same amount of hours during his life-span and wages are constant over time, it is of no importance if we take monthly wages or discounted lifetime earnings. Monthly wages seem to be more telling.

around $0.4 - 0.5$ (Solon [1992], Mulligan [1997]) which is high in the international comparison: Corak [2006] reports 0.15 for Denmark, Solon [2002] 0.28 for Sweden and 0.34 for Germany[4]. Returning to the numerical example: A child with a father at the 75% quantile of the income distribution has almost double the expected income of a child whose father is at the 25% earnings quantile in the United States, whereas the difference is only about 15% in Denmark.

So how can we explain that in a country that is supposed to give equal opportunities to everybody such as the United States, the success of a newborn depends so heavily on the success of his father? Why are European societies socially more mobile? Let us first have a look at the existing theory.

Theories of intergenerational links in wealth and income

There is a large amount of microeconomic literature trying to explain intergenerational links in economic status. I will briefly present the most prominent ones. For a survey of some theoretical models, see Piketty [2000], and for a description of empirical facts, refer to chapters 6-8 of Mulligan [1997]. In the following, I look at theories where "something" is passed on from one generation to the next.

First of all, wealth can be passed on (bequests and inter-vivos transfers), increasing consumption and (capital) earnings for the offspring of the well-off. Obviously, those with richer parents can inherit more wealth than the less well-off can. It is therefore not surprising that the correlation between fathers' and sons' income (or consumption) is larger empirically than their earnings correlation (.7 compared to .5 for the US, according to Mulligan [1997]). But this still leaves an important factor unexplained: what is it that children of wealthier, high earning families have that gives them a substantial advantage in the labor market?

Wealth transmission with credit constraints may influence education choices and other investment opportunities. Supposing that credit markets are not perfect, the transmission of wealth could be important for many investment decisions. Arguably the most important investment decision is education choice, from the early age (private lessons, books, etc.) to college (living expenses, tuition fees, etc.). Given

[4]Since the data sets and the estimation methods differ from country to country and study to study, the resulting estimates can only be compared up to a certain degree. But the evidence seems strong enough to conclude that there are large differences between the US and the Nordic countries, see for example the discussion in Solon [2002].

that human capital is rarely accepted as collateral for loans (due to moral hazard problems for example), less wealthy people cannot make the efficient amount of investment into their children at the early age and their children cannot get sufficient loans for their college education. This argument has a lot of theoretical appeal; but it does not seem that credit constraints are binding for many individuals when it comes to paying for college education. Conditional on ability, adolescents from minority groups are more likely to be admitted to college, and estimates suggest that less than 8% of American adolescents are credit constrained on their college investment (see Cunha et al. [2006] and the references therein). A further argument against wealth as a major factor for intergenerational immobility can be found in Mulligan [1997]: it is shown that the intergenerational correlation of economic status is the same for people that expect to inherit little or nothing as compared to those who expect large inheritances.

Not only wealth can be passed on. Transmission - genetic or through upbringing - of cognitive and non-cognitive ability may raise productivity in the labor market directly as well as through the education channel. For econometric evidence on this, see e.g. Cunha et al. [2006] for the discussion and references therein. They find that the role of parents is important at the early age, both in boosting cognitive and non-cognitive ability. Cognitive ability seems to have greater tendency to be genetically transmitted, and non-cognitive ability through upbringing. Hence we have one important transmission mechanism that may explain intergenerational links in wages: ability causes labor market success and ability is passed on to children. This is one channel that has been successfully explored by Restuccia and Urrutia [2004] for the US. In the model presented here, intergenerational ability transmission will also be controlled for. However, suppose that the transmission of abilities is similar in every country, how can we explain the huge differences between countries with respect to the intergenerational correlation of wages? There seem to be other important factors that have not yet been explored.

Preferences may be inherited as well (e.g. the subjective discount factor) and may influence education and labor market choices. There is indeed experimental evidence that preferences are linked between parents and children (see Dohmen et al. [2011] and Cesarini et al. [2009]). In a macroeconomic (long run) model, it is unclear where the differences come from in the first place, hence there have been few attempts to model preference links across generations. One exception is

Krusell and Smith [1998], where the discount factor is partially passed from parents to children[5]. Furthermore, empirical evidence suggests that cognitive abilities and preferences are linked (Dohmen et al. [2010]), which makes it difficult to find the underlying, determining variable. Is it cognitive ability that is passed on and causes preferences to be correlated or the other way round? Hence the question remains: Where do differences in preferences across individuals originate? Or, more generally, how are preferences formed? It is important to stress that the treatment I adopt in this paper stipulates the same preferences for every individual in every generation, i.e. parameters of the utility function are the same across all agents. The aspirations formulation of preferences uses one more input - the reference point - which differs across family backgrounds. Naturally, the distinction between these two is not clear cut. The important point is that in the model presented here its only assumed that there is a small difference in income between two groups which is then able to generate the differences in preferences. This compares to models where different groups have to have different preferences right from the start in order to generate differences in income between groups.

Moreover, there may be explanations for the behavior of children being related to their parents' behavior other than direct transmission of whatever it might be. For example, one might think of an underlying variable that determines both parents' and children's success in the labor market, e.g. because they live in the same peer group or neighborhood (Calvó-Armengol and Jackson [2009]).

The macroeconomics of the wealth and income distribution

In the last decade, much macroeconomic research has focused on the wealth and income distribution within a society and in the international comparison (intragenerational mobility). One of the puzzles that is addressed is the fat right tail of the wealth distribution, i.e. the fact that most wealth is held by a small proportion of the society. Examples of this strand are De Nardi [2004] or Krusell and Smith [1998]. Less attention has been paid to models about intergenerational mobility. There is at least one exception: Restuccia and Urrutia [2004] construct a four period OLG model with ability transmission to explain the US intergenerational wage correlation, focusing on early education and college dropout probabilities. I will adopt a different methodology by using consumption aspirations formed during youth as an

[5] An empirical example of preference transmission is Altonji and Dunn [2000].

explanatory factor of why children of high income parents are more likely to go to college. These aspirations are part of a theory of habits that I outline below.

Consumption aspirations

There has been much recent research exploring the formation of habits or aspirations, in the macroeconomic as well as in the experimental literature[6]. Habits are normally modelled in the following way: The utility function is assumed to be logarithmic or CES and is given for period t by:

$$u(c_t, h_t, a_t) = \frac{(c_t - \alpha a_t)^{1-\gamma} - 1}{1-\gamma} - \beta h_t$$

where c_t is individual consumption, h_t is labor supplied and $\beta \geq 0$. a_t are aspirations and $\alpha \in [0, 1)$ measures the strength of the aspiration.

In general, there are two different concepts, the so called keeping-up-with-the-Joneses (external habit) or catching-up-with-the-Joneses (internal habit) formulations. This refers to the modeling of a_t. In the keeping-up-with-the-Joneses formulation, a_t is supposed to stand for average consumption in the economy, by catching-up, it is meant that a_t is past consumption of the individual in question. Ljungqvist and Uhlig [2000] may serve as an example to illustrate the different versions. They construct an RBC model and analyze the effect of aspirations on income tax policy. The main finding is that demand management (procyclical taxes on income redistributed as a lump sum transfer) can improve welfare by "cooling down" aspirations in the catching-up case (optimality depends on timing of externality, for the Catching up case, this externaility is a period in advance). In contrast, in the keeping-up case, the externality is in the same period as the shock, so the optimal tax rate does not depend on the shock. There are other articles that utilize habits: for example, Constantinides [1990] or more recently Boldrin et al. [2001] address asset pricing puzzles in the RBC literature.

The concept of habits can be seen in a broader context. The most prominent alternative to expected utility theory is prospect theory. And one of the main ingredients of prospect theory is that the evaluation of a "prospect" depends on the so called "anchor" which is a sort of status quo. The same is true for habits in the catching-up-with-the-Joneses case: the anchor is consumption of the last period.

[6] Habits can also be axiomatically formalized, see Rozen [2010].

The paper which is closest to my contribution is de la Croix [2001] (an earlier version without human capital is de la Croix [1996]), a growth model with human capital depending on the human capital of the parents and with aspirations inherited from the parents. The main difference to my model is that aspirations play no direct role in the accumulation of human capital. In my model, this is the key for understanding why the family background determines labor market success. Other papers using the OLG framework analyze the effect of aspirations on the pension system (see Caballé and Moro-Egido [2008]) or address welfare issues arising due to aspirations (Alonso-Carrera et al. [2006] and Alonso-Carrera et al. [2005]).

The following section gives an intuition of my basic mechanism. It uses a simple model with exogenous wages that permits an analytical solution of the unique steady state. In section 1.4, I will calibrate a more realistic model with endogenous wages, capital accumulation and intergenerational transmission of cognitive ability to the US economy. The computation of the steady state will then be limited to a numerical solution.

1.3 An illustrative model

This section presents a simple model with exogenous wages and equal ability across social backgrounds. The education choice is driven by wage differences, and the children's choice depends on their consumption aspirations. These aspirations can be thought of as a standard-of-living idea that children experience during their youth. Aspirations make children of wealthier families (where the income of the parent is higher) more likely to go to college, for a given ability level. It is shown that the more progressive an income tax scheme is, the greater the level of intergenerational mobility. This is due to a "cooling down" of aspirations[7]. Higher tax rates on skilled labor narrow the standard-of-living aspirations.

Model setup and steady state

In every generation, there is a continuum of individuals of mass one. Time is discrete and goes from 0 to ∞. Every individual lives two periods. The first period is youth (including college education), and the second is adulthood where individuals work,

[7] The mechanism studied here is similar to Ljungqvist and Uhlig [2000] and hence I borrow their expression of "cooling down" aspirations.

either as skilled or unskilled labor depending on the decision on education in the first period. There are two labor markets that pay different (exogenous) wages $w_t^s > w_t^n$ (skilled and unskilled). Studying has some disutility, which is decreasing in ability and known to the person before going to college. There is no population growth. Every individual has one parent and one child. If skilled, a typical individual born in period $t-1$ has lifetime utility of:

$$\log((1-\tau)w_t^s - \alpha a_t) - v(A)$$

where consumption equals (after-tax) earnings. Aspirations a_t equal the after-tax wage of the parent (his consumption). A is ability for education and $v'(A) < 0$, i.e. smart people need less effort (less foregone leisure) to study. Individuals are heterogeneous with respect to A and in the following I assume that A follows a continuous distribution. The income tax rate τ reflects the tax rate that skilled people have to pay in excess of the unskilled tax rate. Since only the progressivity of the income tax matters for the results, the tax rate of the unskilled is set to 0. The government uses taxes to produce a public good that enters the utility function additively (it is disregarded in the following as it appears in both skilled and unskilled utilities). If an individual decides not to get an education, the utility is given by:

$$\log(w_t^n - \alpha a_t).$$

Hence the education problem for the individual is

$$\begin{aligned} & \log((1-\tau)w_t^s - \alpha a_t) - v(A) \geq \log(w_t^n - \alpha a_t) \\ \Rightarrow \quad & \tilde{A} := e^{-v(A)} \geq \frac{w_t^n - \alpha a_t}{(1-\tau)w_t^s - \alpha a_t} =: T. \end{aligned}$$

In the following, I will always assume that in the case of indifference individuals will acquire education. Furthermore, I will denote the distribution function of $\tilde{A}$ by F. From the above equation, one can see that T is decreasing in a_t if $(1-\tau)w_t^s > w_t^n$, which is assumed to hold in the following, hence children of skilled parents are more likely to get education (for fixed A). Furthermore, smarter children are more likely to get educated, $\tilde{A}$ is increasing in A. There are two thresholds (since there are only two aspiration levels: parents had either a skilled or an unskilled job):

$$T^h = \frac{w_t^n - \alpha w_{t-1}^n}{(1-\tau)w_t^s - \alpha w_{t-1}^n} \qquad \text{and} \qquad T^l = \frac{w_t^n - \alpha(1-\tau)w_{t-1}^s}{(1-\tau)w_t^s - \alpha(1-\tau)w_{t-1}^s}$$

where $T^h > T^l \Leftrightarrow \alpha > 0$, i.e. the ability thresholds for getting education of children of skilled and unskilled parents differ if and only if the habit parameter is strictly positive. Everybody with $\tilde{A} \geq T^h$ gets education, nobody with $\tilde{A} < T^l$. If $T^l \leq \tilde{A} < T^h$, only children of skilled parents get educated.

Assume that wages are constant over time. Denote n_t the share of skilled workers in period t. The dynamics of n_t are given by:

$$\begin{aligned} n_{t+1} &= n_t \mathbb{P}(\tilde{A} > T^l) + (1 - n_t)\mathbb{P}(\tilde{A} > T^h) \\ &= n_t \mathbb{P}(T^l < \tilde{A} < T^h) + \mathbb{P}(\tilde{A} > T^h) \\ &= n_0 \mathbb{P}(T^l < \tilde{A} < T^h)^{t+1} + \mathbb{P}(\tilde{A} > T^h) \sum_{s=0}^{t} \mathbb{P}(T^l < \tilde{A} < T^h)^s. \end{aligned}$$

Thus, n_t converges to a unique steady state n^* determined by the two thresholds T^l and T^h:

$$n^* = \frac{\mathbb{P}(\tilde{A} > T^h)}{\mathbb{P}(\tilde{A} < T^l) + \mathbb{P}(\tilde{A} > T^h)}.$$

Now to compute the intergenerational correlation of wages, suppose that the ability of a newborn individual is stochastic (the transformed ability $\tilde{A}$ has distribution function F). Then the coefficient of intergenerational correlation of wages is given by

$$\rho_t := \rho(w_{t+1}, w_t) = \frac{\mathrm{Cov}(w_{t+1}, w_t)}{\sqrt{\mathrm{Var}(w_t)\mathrm{Var}(w_{t+1})}}.$$

Straightforward calculations yield

$$\rho_t = \frac{n_t(1 - n_t)\mathbb{P}(T^l < \tilde{A} < T^h)}{\{n_t(1 - n_t)n_{t+1}(1 - n_{t+1})\}^{\frac{1}{2}}}$$

and in the steady state:

$$\rho = \mathbb{P}(T^l < \tilde{A} < T^h)$$

which means that as long as $\alpha > 0$ and the support of the distribution F of $\tilde{A}$ is large enough, we have positive intergenerational wage correlation in the steady state.

"Cooling down" aspirations with a progressive income tax scheme

In this simple model, there is only one tax rate τ which determines the progressivity of the tax system and which has two effects: the higher τ is, the less attractive it is to go to college, since this decreases the benefits of doing so. The second effect is important for the intergenerational correlation of wages: the higher τ is, the more similar aspirations across individuals are, and the smaller the difference across families is. To see this, recall the two thresholds:

$$T^h = \frac{w_t^n - \alpha w_{t-1}^n}{(1-\tau)w_t^s - \alpha w_{t-1}^n} \qquad \text{and} \qquad T^l = \frac{w_t^n - \alpha(1-\tau)w_{t-1}^s}{(1-\tau)w_t^s - \alpha(1-\tau)w_{t-1}^s}.$$

T^h and T^l are increasing in τ (obvious, since taxes reduce the incentives to study and hence decrease the steady state mass of skilled in the economy). But straightforward differentiation shows that $T^h - T^l$ is decreasing in τ. $T^h - T^l$ decreasing in τ is not sufficient to ensure that $\mathbb{P}(T^l < \tilde{A} < T^h)$ is decreasing in τ as well, as this also depends on the distribution F of $\tilde{A}$. But assuming that the density F' is decreasing $\forall \tilde{A} > T^l$, the condition is sufficient. This assumption is for example fulfilled when F is a symmetric distribution and $n < 0.5$.

In summary: the steeper the income tax scheme, the lower the proportion of skilled in the population and the lower the intergenerational correlation of wages due to a "cooling down" of aspirations for children of high earning parents.

1.4 A general equilibrium model

The simple model of the previous section highlights the key mechanism. Nonetheless, interesting aspects are left out, in particular the savings decision of the households and the wage determination mechanism. The main extension in this section therefore consists of endogenizing wages by introducing a production function for the skilled labor market that uses capital as input. Furthermore, cognitive ability will be transmitted from one generation to the next. These extensions would permit the calibration of the model to the US economy.

Model description There are three periods, youth and working adulthood (as before) plus retirement. Education takes place at the end of the youth period (after the reference point is formed): individuals can choose whether to go to college. Working individuals either receive a skilled or unskilled (constant) wage, conditional on education. They save for their retirement (third period). The problem of the working **adult agent** is given by:

$$\max_{c_t^i, d_{t+1}^i} U(c_t^i, d_{t+1}^i; a_t) = \frac{(c_t^i - \alpha a_t)^{1-\gamma} - 1}{1-\gamma} + \beta \frac{(d_{t+1}^i - \alpha a_{t+1}^o)^{1-\gamma} - 1}{1-\gamma}$$

$$\text{s.t.} \quad \begin{cases} c_t^i = (1-\tau^i)w_t^i - s_t^i \\ d_{t+1}^i = (1-\theta+r_{t+1})s_t^i \\ a_{t+1}^o = c_t^i \end{cases}$$

where $i = s, n$ if the individual is skilled or unskilled. θ denotes the depreciation rate, r_{t+1} the marginal product of capital between t and $t+1$. Let us define $R_{t+1} := 1-\theta+r_{t+1}$. Aspirations a_{t+1}^o for the old age equal consumption when adult. One could also model the old age aspirations as a_t, which is the aspirations developed during youth (passed on from the parents). Another assumption would be that aspirations are nullified at retirement age (as in de la Croix [2001]). None of these assumptions would change the main results (they have little impact quantitatively). First order conditions imply

$$\begin{cases} s_t^i(a_t) & = \frac{(1-\tau^i)w_t^i[1+\alpha\beta^{-1/\gamma}(R_{t+1}+\alpha)^{-1/\gamma}]-\alpha a_t}{1+\beta^{-1/\gamma}(R_{t+1}+\alpha)^{1-1/\gamma}} \\ c_t^i(a_t) & = (1-\tau^i)w_t^i - s_t^i = \frac{(1-\tau^i)w_t^i\beta^{-1/\gamma}(R_{t+1}+\alpha)^{-1/\gamma}R_{t+1}+\alpha a_t}{1+\beta^{-1/\gamma}(R_{t+1}+\alpha)^{1-1/\gamma}} \\ d_{t+1}^i(a_t) & = R_{t+1}s_t^i = R_{t+1}\frac{[(1-\tau^i)w_t^i-\alpha a_t]+[\beta(R_{t+1}+\alpha)]^{-1/\gamma}[\alpha(1-\tau^i)w_t^i]}{1+[\beta(R_{t+1}+\alpha)]^{-1/\gamma}(R_{t+1}+\alpha)}. \end{cases}$$

For the **young agent**, suppose that there is some disutility of studying $v(A)$, where v is strictly decreasing in ability A. A is drawn from some distribution function, i.i.d. across individuals and generations. Then a young individual in t will get education if the utility of being skilled minus the disutility of studying is higher than the utility of being unskilled, i.e. if

$$U^{\text{diff}}(a_t) := U(c_t^s(a_t), d_{t+1}^s(a_t); a_t) - U(c_t^n(a_t), d_{t+1}^n(a_t); a_t) \geq v(A).$$

As in the previous section, the solution to the young's problem can be summarized by two thresholds, T_t^h and T_t^l which are the minimum ability levels for children of unskilled and skilled respectively for acquiring tertiary education.

On the **firm's side**, in the spirit of Galor and Zeira [1993], assume a Cobb-Douglas production function for the skilled sector and a linear-in-labor production function for the unskilled sector producing both one homogeneous good (n_t is again the proportion of skilled labor in the population):

$$Y_t^s = BK_t^\delta n_t^{1-\delta} \quad \text{and} \quad Y_t^n = w_t^n(1-n_t).$$

First order conditions for the skilled sector can be straightforwardly written as:

$$\begin{aligned} r_{t+1} &= \delta Y^s / K_t \\ w_t^s &= (1-\delta)Y^s / n_t. \end{aligned}$$

We can now define the **equilibrium** in this economy. A competitive equilibrium is defined by sequences $\{c_t^i, d_{t+1}^i, s_t^i\}_{t\geq 1}^{i=s,n}$, $\{T_t^h, T_t^l\}_{t\geq 1}$, $\{a_t^i\}_{t\geq 1}^{i=s,n}$, $\{K_t, n_t\}_{t\geq 1}$ and factor prices $\{r_{t+1}\}_{t\geq 1}$, $\{w_t^s\}_{t\geq 1}$ such that, for given K_0, n_0, a_0^s, a_0^n the following holds, $\forall t \geq 1$:

1. $(c_t^i, d_{t+1}^i, s_t^i)^{i=s,n}$ solves the adult's optimization problem.
2. (T_t^h, T_t^l) characterizes the solution of the young's optimization problem.
3. (K_t, n_t) solves the firm's problem.
4. The following market clearing conditions hold:
   (a) The capital market clears:
   $$K_{t+1} = s_t$$
   with
   $$s_t = s_t^{ss} n_t n_{t-1} + s_t^{sn} n_t (1 - n_{t-1}) + s_t^{ns} (1 - n_t) n_{t-1} + s_t^{nn} (1 - n_t)(1 - n_{t-1})$$
   where the notation is $s_t^{ij} = s_t^i(a_t^j), i, j = s, n$.
   (b) From the education decision and labor market clearing:
   $$n_{t+1} = n_t \mathbb{P}(v(A) < U^{\text{diff}}(a_t^s)) + (1 - n_t)\mathbb{P}(v(A) < U^{\text{diff}}(a_t^n)).$$
   Expectations about n_{t+1} are assumed to be correct in equilibrium (perfect foresight). This is important since $U^{\text{diff}}(a_t)$ depends on w_{t+1} and R_{t+2}, which both depend on K_{t+1} and n_{t+1}. K_{t+1} is determined in t (see above) but n_{t+1} depends on the choices of the other young agents.
   (c) The goods market clears (the redundant condition):
   $$c_t + d_t + T_t + s_t = Y_t^s + Y_t^n + (1 - \theta)K_t$$
   where:
   $$\begin{cases} c_t &= c_t^{ss} n_t n_{t-1} + c_t^{sn} n_t (1 - n_{t-1}) \\ & \qquad + c_t^{ns} (1 - n_t) n_{t-1} + c_t^{nn} (1 - n_t)(1 - n_{t-1}) \\ d_t &= d_t^{ss} n_{t-1} n_{t-2} + d_t^{sn} n_{t-1} (1 - n_{t-2}) \\ & \qquad + d_t^{ns} (1 - n_{t-1}) n_{t-2} + d_t^{nn} (1 - n_{t-1})(1 - n_{t-2}) \\ s_t &= s_t^{ss} n_t n_{t-1} + s_t^{sn} n_t (1 - n_{t-1}) \\ & \qquad + s_t^{ns} (1 - n_t) n_{t-1} + s_t^{nn} (1 - n_t)(1 - n_{t-1}) \\ T_t &= \tau^s w_t^s n_t^s + \tau^n w_t^n (1 - n_t). \end{cases}$$

5. $\forall i = s, n$ (parent skilled or not), aspirations are:

$$a^i_{t+1} = (1 - \tau^i) w^i_t.$$

   Hence children's aspirations are supposed to equal the after-tax wage of the parents. The after-tax wage proxies for the consumption level of the parents and thereby the consumption level of the children experienced during youth. This assumption simplifies the analysis substantially: if aspirations depended on consumption of the parents, then - through savings - they would also depend on consumption of the grandparents, and so on. This implies that the distribution of aspirations is far more complicated than the two levels we use here, without adding further important insights. By assuming that aspirations depend on the after-tax wage, I assume that they only depend on the total capital stock in the economy (and hence total savings) and not the individual savings. In the calibration, it turns out that the after-tax wage premium is around 2.1 for the US, whereas the consumption premium for the skilled is around 2. Thus, ex post in the steady state, assuming aspirations to be after-tax wage or consumption of the parents makes little difference.

6. Abilities of skilled and unskilled are passed on from one generation to the next. Details follow in the next section.

The numerical strategy to compute the steady state is discussed in the appendix.

Calibration

The notation for the steady state quantities is given in Table 1.1.

Name	Letter	Calculation
Total consumption (of young and old)	C	$c+d$
Savings	S	See above
Production and capital	Y	$Y^s+Y^n+(1-\theta)K$
Taxes	T	$\tau^s w^s n+\tau^n w^n(1-n)$
Intergenerational correlation of Wages	ρ	$\mathbb{P}(v(A)>U^{\text{diff}}(a^n))-\mathbb{P}(v(A)>U^{\text{diff}}(a^s))$

Table 1.1: Steady state notation

Calibration strategy to the US economy In order to calibrate the model, the incorporation of the ability transmission must first be specified. Furthermore there is the issue of choice of the parameters of the utility function, in particular the aspirations parameter α, the discount rate β and the elasticity of substitution γ. For the production technology, there are δ and the scale parameter B, as well as the depreciation rate θ and the unskilled wage w^n. I now describe in detail how I choose these parameters.

PISA data[8] is used to calibrate the distribution of ability and its transmission across generations. Assume first that $A\sim\mathcal{N}(\mu,\sigma)$. Let the mean depend on the education of the father, i.e. $\mu=\mu^s$ for the son of a skilled father and $\mu=\mu^n<\mu^s$ for the son of an unskilled. In order to pin down σ, I use the fact that the average test score from PISA has mean of 500 and standard deviation of 100 and hence I fix σ by $\mu^n/\sigma=5$. To obtain μ^s, I calculate $(\mu^s-\mu^n)/\mu^n$ from the data set, the advantage in the average PISA test score for the child of a skilled father compared to a child whose father is unskilled. Define skilled to mean that the father has completed ISCED 5 or above[9]. This advantage is 12.7% for the US. Furthermore, I assume $v(A)=-\eta A$ for the utility cost of going to college. In summary there are four parameters to choose for the ability distribution $(\mu^s,\mu^n,\sigma,\eta)$ and only one

[8]PISA stands for Programme for International Student assessment, an OECD test of the 15 year old. Results can be accessed at http://www.pisa.oecd.org/.

[9]ISCED is the International Standard Classification of Education, a UNESCO measure that tries to make education systems comparable over countries. ISCED 5 is the first stage of tertiary education.

target n, which is the proportion of skilled labor in the economy. μ^s and σ are chosen as described above. This leaves two parameters of which one is to be normalized. I fix $\eta = 1$ and then use n to pin down μ^n [10].

Assume that one period in the model represents 25 years. The subjective discount rate is then fixed to $\beta = 1/(1+0.02)^{25} \approx 0.6$ and corresponds to the endogenously determined interest rate of 2% per year. In order to make my results comparable to Restuccia and Urrutia [2004], I choose $\gamma = 1.5$ (see their discussion). Empirical estimates of yearly depreciation rates for physical capital are between 0.034 and 0.126 (Nadiri and Prucha [1996]). The assumption that one period is equivalent to 25 years means $\theta \in [0.5, 1]$ approximately. I choose $\theta = 1-(1-0.059)^T = 0.78$. In the calibration, it turns out that the choice of θ in this range has little impact on the predictions.

There are still five unknown parameters: B, δ, μ^n, w^n and α. I use the following targets to pin them down: n, the proportion of skilled in the economy is used to pin down μ^n. The central parameter α determines the level of ρ. The skill premium w^s/w^n can be targeted using the scale parameter B of the production function. rK/Y, the capital income share, can be used to obtain δ, the elasticity parameter of the production function. And finally, w^n gives C/Y, i.e. consumption over GDP. Table 1.2 recapitulates the parameter choices.

[10] The other possibility would be to normalize μ^n, for example by choosing $\mu^n = 1$, letting η vary freely (and redefining $\sigma' = \eta^2\sigma$). This would mean that instead of having $\eta A \sim \mathcal{N}(\mu^n, \sigma)$ for the children of the unskilled, we would have $\eta A \sim \mathcal{N}(\eta, \eta^2\sigma)$ which is equivalent to the approach I assume.

	Description	Choice
μ^s	Mean of ability distribution for skilled	$1.127\mu^n$
μ^n	Mean of ability distribution for unskilled	To target n
σ	Standard deviation of ability distribution	$0.2\mu^n$
$v(A)$	Disutility of going to college	- Id
α	Aspirations parameter	To target ρ
β	Discount rate	$1/(1+0.02)^{25} \approx 0.6$
γ	Elasticity of substitution	1.5
θ	Depreciation rate	$1-(1-0.059)^T = 0.78$
δ	Elasticity (production function)	To target rK/Y
B	Scale parameter of production function	To target w^s/w^n
w^n	Unskilled wage	To target C/Y
r	Interest rate	Endogenous

Table 1.2: Parameter choices for the calibration

Data Data on taxes, the wage premium, the capital income share, GDP and consumption is taken from OECD statistics[11]. More precisely, the wage premium is defined as the .9-quantile over the .5-quantile of the gross earnings distribution. It turns out that using this definition of the wage premium, my value for the US wage premium (2.34) is equal to what Restuccia and Urrutia [2004] find for the premium of college graduates compared to high school graduates (2.33). The OECD provides four income levels for which taxes are calculated: at $67\%, 100\%, 133\%$ and 167% of average annual gross earnings (for singles). I choose the tax rate at 67% of average earnings for τ^n and 167% of average earnings for τ^s. This implies that the wage level is 2.5 times higher for the skilled compared to the unskilled and is close to the skill premium of 2.34. When comparing the results between the US and Sweden later, the progressivity of the income tax scheme is more important than the absolute values. Hence the results are robust to the specification of taxes, as long as the progressivity of the US is smaller than in other countries. The values chosen for the US are then $\tau^n = 0.168$ and $\tau^s = 0.228$. The capital income share rK/Y is pinned down by total labor costs over nominal GDP. GDP (Y) is calculated by

[11] Available at http://stats.oecd.org.

Target	Calibrated	US Data
n	0.39	0.39
ρ	0.23	0.47
w^s/w^n	2.34	2.34
rK/Y	0.33	0.34
C/Y	0.71	0.69
r	1.64	

Target	Calibrated	US data
n	0.40	0.39
ρ	0.46	0.47
w^s/w^n	2.34	2.34
rK/Y	0.32	0.34
C/Y	0.71	0.69
r	1.66	

Table 1.3: Calibration to the US economy, left with $\alpha = 0$, right with $\alpha = 0.07$.

the expenditure approach[12], consumption (C) is final consumption expenditure of households, both figures are for 2007, in US Dollars per head at current prices and current PPPs. The share of skilled workers n is the share of the population aged 25-64 that has attained tertiary education and taken from the OECD's "Education at a Glance 2008". The data sources for ρ are found in Solon [2002] and Corak [2006].

Calibration results for the steady state First I try to calibrate the model while forcing α to zero. The result (Table 1.3, left hand side) shows that the model without consumption aspirations cannot reproduce the high value for ρ that we observe in reality. The right hand side of Table 1.3 shows that when allowing for aspirations in the preference formulation, the fit improves substantially. It turns out that the best choices for the unknown parameters are: $\delta = 0.44, w^n = 0.096, \mu^n = -5.9$ and $B = 1.08$. If we suppose that one period corresponds to 25 years, then r corresponds to 2% annually (and hence is equal to the subjective discount factor of 2%, reflected in β).

The calibration result shows that the model is able to reproduce the data appropriately. With five parameters (including α) and six targets (including the interest rate) it is possible to fit this (nonlinear) model convincingly to the US economy. Limitations of this approach are discussed at the end of the paper.

[12] Y is not exactly GDP but if considering 25 periods, $(1-\theta)K$ becomes negligible before 25 years of GDP since the capital to GDP ratio is around 3 for the US.

US calibration

	Data US	Cal.
n	0.39	0.39
ρ	0.47	0.46

US, Swedish taxes

n	0.32
ρ	0.43
Y	-16%

US, Swedish schools

n	0.38
ρ	0.32
Y	-3%

US, Swedish taxes and schools

	Data SWE	Cal.
n	0.31	0.3
ρ	0.28	0.29
Y/Y_{US}	0.78	0.82

Table 1.4: Decomposing the difference in the share of the population with tertiary education n, the intergenerational correlation of wages ρ and GDP Y between the US and Sweden.

Comparing the US to Sweden Using this calibration to the US economy, it is possible to compare the results to a hypothetical country that shares the technology and preference parameters with the US but has taxes and education system that is akin to Sweden, rather like a thought experiment as in Prescott [2004]. Sweden is characterized by higher and more progressive income taxes and a primary and secondary education system that provides more equal outcomes across social backgrounds. Sweden and the US spend a similar proportion of GDP on primary and secondary education (Sweden 4.1%, US 3.9%)[13], hence differences in the outcomes of the education system are unlikely to be driven by more investment of the Swedish compared to the US. The fact that the US school system has a higher share of private institutions compared to Sweden will be reflected in differences in equality of PISA scores.

The calibration for the US predicts n to be 0.4 and $\rho = 0.46$ which are both close to actual levels. There are now two possibilities. Increasing US taxes to the Swedish level, i.e. τ^s from 22.8% to 32.5% ($\tau^n = 16.8\%$ is the same in both countries) has the following effects: the proportion of skilled in the economy drops from 39% to 32% and subsequently reduces GDP by 16%. The effect on ρ is rather small (from 0.46 to 0.43). If one supposes that the US has the same equality in the school system as in

[13]Again from the OECD data base: the measure is primary, secondary and post-secondary non-tertiary education expenditures from public, private or international sources as a percentage of GDP.

Sweden (2.8% advantage for the son of the skilled in the PISA test scores compared to 12.7%), ρ is reduced to 0.32. Combining both - higher taxes and more equality in the school system - predicts for Sweden: $n = 0.3, \rho = 0.29$ (actual values are $n = 0.31, \rho = 0.28$) and that the Swedish GDP is at 82% of the US (actual value: 78%). The results are summarized in Table 1.4. This simple example illustrates the two forces that separate intergenerational mobility between the two countries. It also stresses that differences in the progressivity of the income tax system are important for GDP but small for intergenerational mobility.

The US with a flat tax Another interesting example is to consider the case where the US has a flat income tax system. For comparison purposes, the total amount of taxes should be constant. This implies $\tau^s = \tau^n = 19.4\%$. The effect is positive for n (increase by 7 percentage points) and for ρ (increase by 1 percentage point). Thus a flat income tax has a small effect on intergenerational mobility but a considerable effect on GDP (increase by 4%).

Discussion of the results

The approach followed here assumed a particularly simple structure: there are only two wages. Furthermore, I assume that there are also only two aspiration levels. Such assumptions simplify the analysis substantially as the wage and the aspirations distribution can be modelled by a discrete two state distribution instead of a continuous distribution. This is however also restrictive.

For example, ability is only relevant for the education decision and not for the wage in the model. In reality cognitive ability also influences productivity and that should - in a competitive labor market - be reflected in the wage. But this would mean having more than two wage levels and hence would increase the number of aspiration levels, which gives a much more complex model. In such a complex model there would be two additional (and countervailing) effects. On the one hand, the lower ability level of children of skilled parents needed to enter college causes the average ability level of skilled children with skilled parents to be lower than the average ability level of skilled children with unskilled parents. This should depress the wage of children of skilled parents. On the other hand, since cognitive ability is passed from skilled parents to their children, children with skilled parents are on average smarter. These two effects influence ρ in two different directions. First, ρ

should decrease since the children of the skilled are less productive in the skilled labor market. The second effect should increase ρ, since the children of the skilled are smarter on average. Thus it is not clear what the overall effect would be.

1.5 Conclusion

I show that a model where children form consumption aspirations during their youth that influence college choice can be convincingly calibrated to the US economy. Without these aspirations and using only the transmission of cognitive ability, the coefficient of intergenerational correlation is predicted to be much lower than in reality. Steeper income taxes weaken the dependence of the college decision on the background of the parents and hence increase intergenerational mobility. However, more progressive income taxes discourage individuals to go to college and hence reduce output. The comparison of the United States to Sweden shows that increasing the equality within the school system has a larger positive impact on intergenerational mobility than steeper income taxes and less negative consequences on GDP.

The mechanism proposed here is one way in which education influences the intergenerational correlation of wages. As the quantitative part of the paper indicates, a considerable amount of inequality is generated by the primary and secondary school system, therefore, while this work focuses on the preferences of young adults when choosing their tertiary education, future work could concentrate on parents' preferences and their choices for the early education of their children and how these differ across parents' income.

Appendix

This appendix describes the numerical procedure to compute the steady state of the model as well as how the model is calibrated. Uniqueness of the steady state cannot be guaranteed. For the parameter values used in the text and the method described here, convergence was fast and there were no apparent discontinuities in the calculation of the steady state.

The equilibrium is calculated numerically with the following steps: the starting values that have to be chosen are K_0, n_0, a_0^s and a_0^n, i.e. the capital stock, the share of the population with tertiary education and the two aspirations levels (skilled and unskilled). K_0 and n_0 then determine Y_0^s and Y_0^n and hence the factor prices w_0^s and R_1. Together with the aspirations a_0^s and a_0^n, these determine the solution of the adult agent's problem, i.e. s_0, c_0 and d_1. The next period's share of the population with tertiary education n_1 is solution to the young agent's problem of whether to acquire education or not.

The starting value for K_0 is chosen such that the first skill premium is 2. Significantly higher or lower skill premia created convergence problems. The other starting values are taken as $n_0 = 0.3$, $a_0^s = a_0^n = 0$.

Convergence to the steady state is achieved for t such that $n_t - n_{t-1} < 0.0001$.

The calibration of the model then calculates the steady state for each step of the optimization. The problem is optimized over the parameter vector $(\alpha, \delta, w^n, \mu^n, B)$ in order to minimize the distance between the model's prediction and the actual US data for the following five variables: the proportion of skilled n, the coefficient of intergenerational wage correlation ρ, the skill premium w^s/w^n, the capital income share rK/Y and consumption over GDP C/Y.

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Chapter 2

Structural Estimation of a Joint Search Model

2.1 Introduction

Labor search models have traditionally considered individual agents, not couples, as their main decision making unit. In reality most people are in a relationship or married and marital status and income of the spouse do matter empirically, as some examples from the literature on female labor supply show. For the relationship between marital status and labor force participation see e.g. Killingsworth and Heckman [1986], for the impact of husband's income on the labor force participation of married women see e.g. Eckstein and Wolpin [1989]. There is also evidence on the link between the reservation wage of women and their husbands' earnings (for a recent contribution see Brown et al. [2011]). Nonetheless, there are very few theoretical search models that try to analyze the joint decision making of a couple and the subsequent labor supply decisions. One attempt is the recent model by Guler et al. [2009]: "Joint-Search Theory: New Opportunities and New Frictions". New opportunities arise since the couple may be thought of as a consumption smoothing device in periods where one is searching for a job. New frictions may arise as compared to the single agent if job offers come from different locations and it is costly for couples to live apart.

In this paper, I analyze empirical evidence of "new opportunities" in a joint search model. Demands on the size and structure of the data set are especially important since one has to have a panel of couples with their precise wage information for every transition between the different employment states (both searching, one searching, the other one working and both working). Since most data sets that can be used are household based surveys, it is extremely difficult to test the multiple locations case, since couples are not identified anymore within most surveys when they move apart. The new opportunities part however offers predictions that can be tested: Guler et al. [2009] argue that a "breadwinner cycle" may exist, which means that when the partner of an employed person finds a job, the employed may quit his job in order to search again while the other becomes the breadwinner. The second prediction is that the wage of the partner has an influence on the reservation wage. I attempt to verify these predictions using French data (Enquête Emploi). The results are as such: First, I find no evidence for the breadwinner cycle. Second, the dependence of the reservation wage on the partner's wage has little influence on the estimation of the structural parameters. Possible explanations for these results are discussed at the end of the paper.

I will first present the theoretical model, and then describe the data set and show how I come to the conclusion that the breadwinner cycle is of little empirical importance. Thereafter, I estimate a structural model without the breadwinner cycle in order to see if the dependence of the reservation wage on the partner's wage changes estimates of the structural parameters. I do not find this to be the case. This is confirmed by some robustness checks (different specification of the estimation procedure, different data sets). Finally I conclude.

2.2 The theoretical model of joint search

In this section, I give the main results of the basic model in Guler et al. [2009][1]. Extensions like on-the-job search, job destruction, non-participation or job offers from multiple locations are not discussed.

The decision making unit in the model is a couple consisting of two workers. There are no singles and marriage is exogenous (the marriage market is not considered). Continuous time is assumed and all workers are infinitely lived. At the beginning of time, all couples are searcher-searcher couples (meaning that both spouses are unemployed and searching for a job, there is no labor force participation

[1] In order to make all chapters of this thesis self-contained, I describe this partial model in both chapters 2 and 3, to different degrees of detail.

margin in the model). Job offers are drawn from a continuous distribution with distribution function F which is given exogenously. Jobs and workers are matched together randomly. From the perspective of a worker, wage offers arrive according to a Poisson process with intensity λ. Once a worker draws a wage offer, he can either accept the job and start working for the given wage or reject the wage offer. In the simplest version of the model, jobs are not destroyed, so a worker can decide to keep a job forever (he can however quit the job voluntarily). The couple pools income, i.e. a unitary utility function over pooled income is assumed. There is neither borrowing nor saving, hence income equals consumption. The couple maximizes the expected, discounted lifetime utility. There are infinitely many states for the worker (since the wage offer distribution is continuous) but they can be summarized in three main states: both members of the couple are working (worker-worker state), both are searching (searcher-searcher state) or one is working and the other one searching (worker-searcher state). The Bellman equations for these three states are given by:

$$rT(w_1, w_2) = u(w_1 + w_2) \tag{2.1}$$

$$rU = u(2b) + 2\lambda \int \max\{\Omega(w) - U, 0\} dF(w) \tag{2.2}$$

$$r\Omega(w_1) = u(w_1 + b) + \lambda \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \tag{2.3}$$

where $T(w_1, w_2), \Omega(w)$ and U are the value functions for the worker-worker, the worker-searcher and the searcher-searcher state respectively. u is the instantaneous utility function of the couple, r is the discount rate and b is unemployment income. λ denotes the job offer arrival rate and F the exogenously given wage offer distribution function.

The solution to this maximization problem is given by a reservation wage for each state. By symmetry, a searcher-searcher couple has a single reservation wage for both spouses. In the worker-searcher state, there is one reservation wage for the searching spouse for each wage of the partner. The main difference to the standard model is what Guler et al. [2009] call the breadwinner cycle. This cycle emerges when the employed spouse quits his or her job once the unemployed finds a job, i.e. when $\Omega(w_2) - \Omega(w_1)$ is the maximum under the integral in equation (2.3). The solution to the model depends crucially on the assumptions on risk aversion. For a risk-neutral couple, the solution of the joint problem is the same as in the single agent context. Intuitively, if both individuals are risk-neutral, there is no consumption smoothing gain from being in a couple. If, however, individuals are assumed to be risk-averse, the breadwinner cycle exists. Furthermore, depending on the type of risk aversion (CARA, IARA or DARA are discussed), the reservation wage function, depicted as $\tilde{w}$ in Figure 2.1 may be flat (CARA), increasing (DARA, as shown) or decreasing (IARA). The notation is the following: w^{**} is the reservation wage of a searcher-searcher couple, $\hat{w}$ denotes the double indifference point for a worker-searcher couple, i.e. where we have indifference between working and searching (for the unemployed spouse), and indifference between quitting and staying (for the employed spouse). The so-called breadwinner cycle exists between w^{**} and $\hat{w}$, i.e. if the employed spouse has earnings within the interval $(w^{**}, \hat{w})$, the searching spouse accepts every job with a wage higher than the current wage in which case the employed spouse quits his or her job. $\tilde{w}$ is the reservation wage and is a function of the partner's wage.

Intuitively, one can understand the decision of the couple in Figure 2.1 by thinking about the option value of searching and about the role of risk-aversion. Every time a spouse accepts a job, the couple is foregoing the option of searching and possibly getting a higher paying job in the future. The lower the wage of the current job, the more the relative loss of this search value matters. This is why the breadwinner cycle emerges for low wages of the employed spouse: the couple prefers to keep the option of getting a higher paying job in the future while at the same time takes the wage offer that is available for the searching spouse that pays a higher wage than the current job. The higher the wage of the employed spouse is, the less likely it is (the longer it takes) to find a job that pays even more. Hence, at some point the couple prefers to switch from the worker-searcher to the worker-worker state.

The fact that the reservation wage function $\tilde{w}$ is increasing for wages greater than $\hat{w}$ is due to the assumption of DARA utility. Decreasing absolute risk aversion means in particular that a worker-searcher couple is less risk-averse as wages become higher. With risk aversion decreasing, the couple becomes more choosy about the wages that they will accept. Hence the upward sloping reservation wage function.

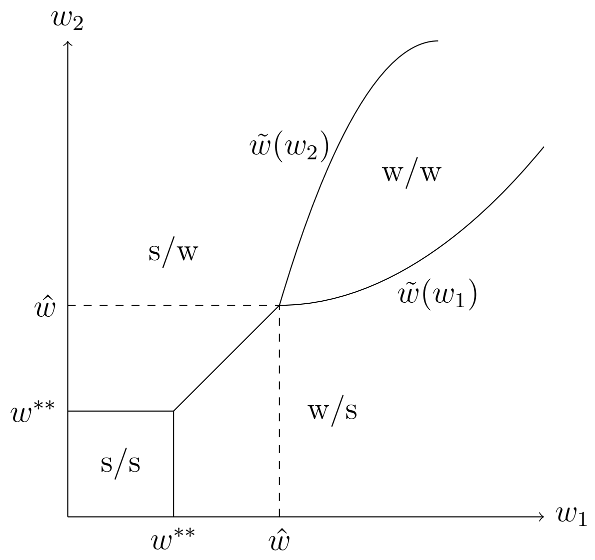

Figure 2.1: The reservation wage strategy of the couple with DARA preferences

2.3 Data

One of the main challenges in testing the model is to find a panel data set that includes the personal relationship of the individuals and wages. Two different types of data sources could potentially be used for that. Household Panel Surveys (like German SOEP, UK Family Expenditure Survey, US CES or PSID) identify who lives with whom and report wages. The drawback is that these are mostly smaller surveys - normally less than 10,000 households in any year - and that households are only rarely removed (in contrast to rotating panels, see below). This is a problem since there has to be a sufficient number of changers from each labor market state to any other. In particular, there need to be changers in the breadwinner cycle sense, which are, as can be seen below, around half a percent of the total number of spells. As an example: if the original data set contains 10,000 observations, that means - since there are some singles - 3,000 couples. Furthermore we impose restrictions on age, that wage information and couple identifier should be provided, and there remain only approximately 2,000 spells. Of these only around 10 transitions (0.5%) are in the breadwinner cycle sense which is not adequate for drawing valid conclusions.

The second possible data source are rotating panels on labor market issues. These are typically larger data sets. Examples include the German Mikrozensus, the US

CPS or the French Enquête Emploi. The Mikrozensus provides only classified data on wages. The CPS has problems matching individuals between different periods and hence is mostly used in its CPS MORG cross-section version for analyses on wages and labor market statistics. For the Enquête Emploi (from 2003 on), every household is removed from the sample after 6 quarters. There are 35,000 households for every cross-section. The wage is given at the first and sixth interview, i.e. with 1.25 years in between (if you assume that the survey is always done in the same month for each quarter). This gives a small panel structure ($T = 2$) which is sufficient to estimate a parametric model (we only need one quantile to identify the exponential distribution of job and unemployment spell lengths). Thus the Enquête Emploi 2003-2007 is a suitable data set for testing the model.

The data set contains 26,150 spells (i.e. couples). These observations fulfil general criteria that are critical: individuals have to declare a partner and this partner has to be identifiable in the data set[2], both members of the couple have to declare their employment status and if employed their wage. Furthermore, both partners have to be between 20 and 60 years old. The wage variable used here is the monthly net salary. Hours worked are only provided in 30% of the spells and hence not taken into account here since this would reduce the size of the data set substantially.

To identify the different transitions that are possible for a couple between working and searching, I use the labor market status in the first and sixth interview. Searching is defined as having no employment or being only marginally employed. A person is considered to be working if a wage is observed and he or she does not declare himself or herself to be marginally employed. Table 2.1 gives the absolute and relative frequencies of the transitions between the different employment states. The notation is as follows: the first letter corresponds to the husband's labor market state (w working for wage w, s searching), the second letter to the wife's labor market state. A transition to w' means that the individual had a job at the first interview (denoted by $t = 0$) and changed to a job with a different wage at the sixth interview (denoted $t = 1$ and being 1.25 years after the first interview). I define a wage change as being in absolute terms more than 5% different from the first one. This is of course a rough treatment of the data and only used for the first description

[2] In the following, I will use the words partner or wife and husband interchangeably. It is always to be understood as being a self-identified partnership between a man and a woman who are not necessarily married.

of the data set. A problem is that the variable indicating if the job is changed between the first and the sixth interview is often missing, so it would reduce the data set substantially when only individuals with a non-missing variable are included. This treatment of the data leaves out some important possibilities. First, promotions are considered a job change. Second, changes from job to unemployment and from unemployment to a new job between the two interviews are recorded as direct transitions from job to job.

To illustrate how to read the table, consider line 1. A change from ww to ww' means that both partners are employed at the beginning and that the wife has found a different job at the end. The second column gives the total number of this type of transition. Columns 3 and 4 give the absolute and relative frequency of job to job transitions with wage cuts. Columns 5-14 give the descriptive statistics for the different state changes: the mean age of men and women, the mean initial log wage if any (i.e. at $t = 0$), the mean log wage if any at $t = 1$, the proportion of men and women who have obtained at least a high-school degree (baccalauréat) and the proportion of men with French nationality and of men without children[3].

This "raw" data set contains only 120 observations (0.5% of all observations) that display some form of breadwinner cycle. Reducing the data set any further would not permit statistical analyses. Therefore, I continue with this data set, with 26,150 observations.

[3]To conserve space, only the respective proportions for men are presented here since the results for women are similar.

State Change	N (26,150)	With wage cut[1] Total	%	Age^M	Age^F	w_0^M	w_0^F	w_1^M	w_1^F	HS^M	HS^F	$French^M$	$Childless^M$
ww to ww'	2,340	819	35.0	42.59	40.60	7.45	6.96	7.46	7.03	0.35	0.45	0.97	0.37
ww to w'w	2,231	782	35.1	41.84	40.01	7.45	7.17	7.49	7.17	0.38	0.51	0.97	0.39
ww to ws	285			41.38	39.20	7.43	6.75	7.44	-	0.32	0.40	0.96	0.41
ww to sw	185			46.59	43.82	7.35	7.11	-	7.12	0.30	0.43	0.94	0.58
ws to ww	337			38.32	36.20	7.38	-	7.38	6.62	0.31	0.41	0.96	0.26
ws to w's	3,007	1,152	38.3	42.63	40.49	7.47	-	7.50	-	0.35	0.35	0.92	0.34
ws to sw	62			39.47	35.89	7.32	-	-	6.76	0.32	0.53	0.92	0.32
ws to ss	420			47.83	45.03	7.27	-	-	-	0.21	0.23	0.89	0.56
sw to ww	127			39.09	37.31	-	7.19	7.16	7.19	0.39	0.57	0.94	0.38
sw to ws	58			38.22	36.24	-	6.63	7.06	-	0.31	0.29	0.86	0.36
sw to sw'	1,978	729	36.9	47.20	44.51	-	6.96	-	7.02	0.35	0.45	0.95	0.49
sw to ss	298			45.97	43.62	-	6.76	-	-	0.33	0.35	0.93	0.55
ss to ws	263			39.84	37.59	-	-	7.11	-	0.29	0.31	0.83	0.28
ss to sw	263			43.06	39.96	-	-	-	6.61	0.32	0.42	0.96	0.36
ww to w'w'	5,810	3,211	55.3	41.98	40.08	7.46	7.01	7.51	7.09	0.40	0.49	0.97	0.39
ww to w's	626	234	37.4	39.65	37.66	7.38	6.72	7.43	-	0.39	0.42	0.96	0.38
ww to sw'	461	193	41.9	46.99	43.90	7.31	6.93	-	6.97	0.26	0.35	0.95	0.59
ww to ss	97			44.73	42.67	7.27	6.84	-	-	0.28	0.33	0.92	0.61
ws to w'w	839	307	36.6	37.72	35.65	7.38	-	7.44	6.66	0.40	0.45	0.94	0.25
sw to ww'	313	106	33.9	39.89	38.02	-	6.96	7.10	7.02	0.37	0.49	0.90	0.37
ss to ww	100			38.12	35.46	-	-	7.22	6.79	0.51	0.50	0.88	0.36
no change: ww	1,305			43.07	41.10	7.44	7.16	7.45	7.17	0.37	0.47	0.97	0.41
no change: ws	1,172			43.29	41.08	7.43	-	7.44	-	0.28	0.28	0.92	0.36
no change: sw	739			48.45	45.67	-	7.15	-	7.15	0.28	0.43	0.95	0.56
no change: ss	2,834			48.52	45.97	-	-	-	-	0.29	0.31	0.91	0.55

[1] A wage cut for the transition ww to w'w' means that at least one of the spouses changes with a wage cut.

Table 2.1: Transitions between the different states and means for different variables: Age, initial log wage, log wage in period $t = 1$, proportion of high school (baccalauréat) graduates (all for men (^{M}) and women (^{F})) and proportion of men with French nationality and of childless men.

2.4 The breadwinner cycle

This section checks if the breadwinner cycle idea can be confirmed empirically. This means we are interested in transitions from ws to ww and sw and from sw to ww and ws. Suppose we look at ws couples, i.e. those where the man is working and the woman is not. The interesting transitions from this state are to states sw (the breadwinner switches) and to ww (both spouses hold a job). If the breadwinner cycle is observed then there should be an interval for the man's wage ranging from the reservation wage w^{**} to some constant wage $\hat{w}$ where all jobs accepted by the woman are followed by the man quitting his job. Above $\hat{w}$, the husband should not quit his job and the couple would become a worker-worker couple.

Figure 2.2 plots the accepted woman log wages as a function of the initial log wage of the man. Figure 2.3 is the analogous plot for couples starting in the sw state, i.e. where the man accepts a new wage and whether or not the woman quits her job. The point plots show that it is hard to make a clear cut distinction between regions where the partner quits or stays (or in other words, the estimation of this cut-off point $\hat{w}$ seems not feasible). We first look at the quitting decision of men (Figure 2.2). Many men keep their low-paid job even if the woman finds a high paying job (the crosses to the left of the graph). To the right of the graph, highly paid men quit their job, when the woman finds a new one. A point to note is that I do not differentiate between quits initiated by the worker or the firm, it could be the case that these men were laid-off and did not quit their job voluntarily. In the middle, both quitting and staying occur. The picture is a little more telling for the quitting decision of women (Figure 2.3): to the left, low paid women mostly quit their jobs if the husband finds a new one. But there is again a range in the middle (log wages between 6.5 and 7.5) where both quitting and staying occurs. Hence the breadwinner cycle finds some support for the quitting decision of women. But it is difficult to come up with an estimation technique for $\hat{w}$, the double indifference point.

Figures 2.2 and 2.3 show the raw data, i.e. assuming all workers to be homogeneous ex ante. Introducing heterogeneity by using the residual of a Mincer earnings regression instead of the log wage does not change these plots substantially (see Figure 2.9 and Figure 2.10 of the appendix). For the Mincer earnings equation, I regress the initial ($t = 0$) log wage on five education dummies and an age polynomial of order 2. Second period ($t = 1$) log wages are predicted using the estimated equation.

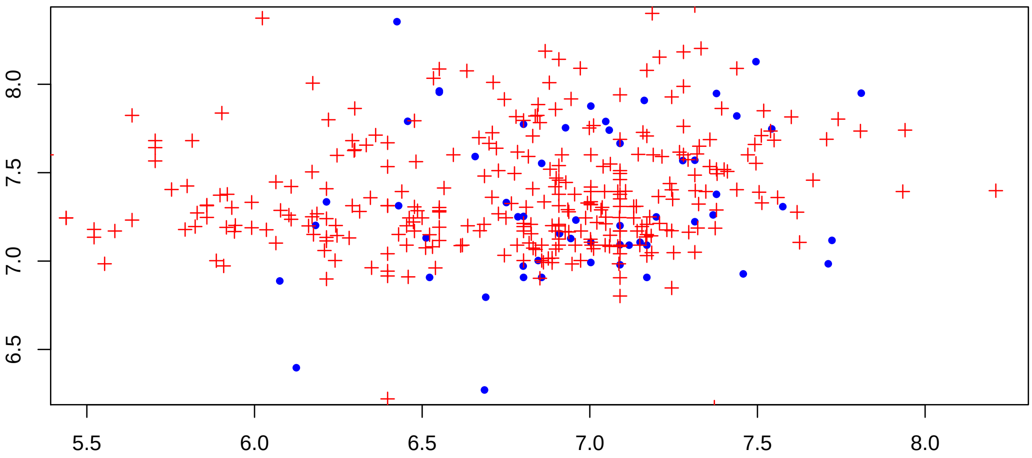

Figure 2.2: Accepted wages for women as a function of the initial wage of the husband; for transition from man working/woman searching to man working/woman working (crosses) and to man searching/woman working (breadwinner cycle, points)

The residual of this regression is then assumed to be only due to firm heterogeneity (which is captured by the exogenous wage offer distribution F). Also, the number of children in the household could play a role, especially for the reservation wage. Figures 2.11 and 2.12 in the appendix show the same data as Figures 2.9 and 2.10 for only those couples without children. The graphs are similar just with less than half of the observations than before.

2.5 Structural estimation

In this section, I estimate a joint search model without breadwinner cycle, i.e. only trying to capture the idea that the reservation wages should depend on the partner's wage. I estimate the model to see if this dependence has a sizeable effect on the estimates of the structural parameters. To do this I first present the estimation idea of structural job search models and estimate a single agent job search model with the same data set.

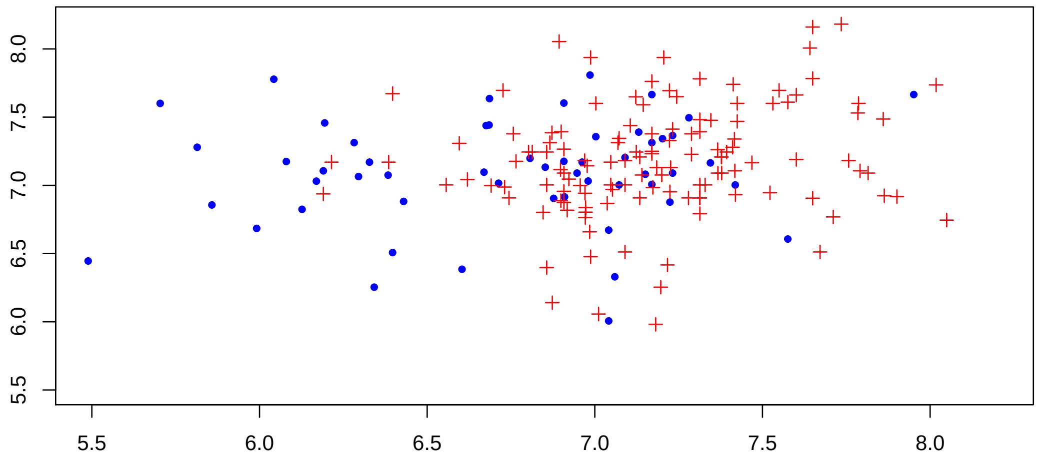

Figure 2.3: Accepted wages for men as a function of the initial wage of the wife; for transition from man searching/woman working to man working/woman working (crosses) and to man working/woman searching (breadwinner cycle, points)

2.5.1 Structural estimation of single agent models

I now briefly describe the Maximum Likelihood estimation for single agent search models which closely follows the survey by Eckstein and van den Berg [2007]. For a recent survey on different techniques used for structural estimation, see Aguirregabiria and Mira [2010].

Suppose we have data on wages (w_i) and unemployment spell lengths (τ_i). All individuals are unemployed at the beginning. There is neither on-the-job search nor are there job losses. Denote the job offer arrival rate by λ and the wage offer distribution function by F. Suppose that we have estimated the reservation wage w^* beforehand (e.g. by the minimum of observed wages; the reservation wage has to be estimated before, since obviously it would not be possible to identify λ and the reservation wage in $\lambda(1 - F(w^*))$). In order to construct the likelihood, first have a look at the probability of observing a specific (τ_i, w_i):

$$\mathbb{P}(w = w_i, \tau = \tau_i) = \underbrace{\underbrace{\lambda(1 - F(w^*))}_{\text{Hazard rate}} e^{-\lambda(1-F(w^*))\tau_i}}_{\text{(Exponential) Density of } \tau_i} \underbrace{\frac{f(w_i)}{1 - F(w^*)}}_{\text{Cond. density of } w_i} = \lambda e^{-\lambda(1-F(w^*))\tau_i} f(w_i).$$

The density is conditional on the fact that the accepted wage is larger than the reservation wage. Assuming that wages are log normally distributed with (μ, σ), density function φ and cumulative density function Φ (it is not possible to identify F nonparametrically in this simple setting), the likelihood becomes

$$\log \mathcal{L} = \log \prod_{i=1}^{N} e^{-\lambda(1-\Phi(\ln w^* - \mu)/\sigma)))\tau_i} \frac{\lambda}{w_i \sigma} \varphi\left(\frac{\ln w_i - \mu}{\sigma}\right) = \sum_{i=1}^{N} l_i$$

where l_i is the individual contribution to the likelihood. In the following, the basic structure of the likelihood is unaltered: the first part consists of the exponential density of τ (shocks are Poisson). The intensity parameter depends on the Poisson arrival rates of the different shocks and on the reservation wage. The second part is the probability of observing a specific wage w_i, conditional on the fact that the spell ended with a transition to the working state (i.e. that a wage offer arrived that was "high enough" for the individual). In the following, the notation w_i will always refer to log wages (whenever the residuals of a Mincer regression are used, the regression will have the log wage as dependent variable).

The data set used only allows identification on whether a person changed the labor market state (uncensored) or not (censored) before $\bar{\tau} = 1.25$ years, but this information is sufficient to identify the distribution of τ as an exponential distribution is identified by just one quantile: we only need to know which proportion of the population changed before a fixed $\bar{\tau}$. To see this consider $F^{\exp}(\bar{\tau}) = q \;\; \Rightarrow \;\; 1 - e^{-\lambda\bar{\tau}} = q$ which obviously has only one solution for λ given $q \in [0, 1]$. For this setting, the individual contribution to the log likelihood can be expressed up to a constant as:

$$\begin{aligned} l_i = e_{1i} \Bigg\{ & \log[1 - e^{-P\bar{\tau}}] - \log\left[1 - \Phi\left(\frac{w^* - \mu}{\sigma}\right)\right] \\ & - \log[\sigma] + \log\left[\varphi\left(\frac{w_{1i} - \mu}{\sigma}\right)\right]\Bigg\} - (1 - e_{1i})P\bar{\tau} \end{aligned}$$

where e_{1i} is a dummy for being employed in period 1 and is the censoring indicator: $e_{1i} = 1$ means that the person found a job before $\bar{\tau}$. $e_{1i} = 0$ are the censored observations where the labor market status did not change. $P = \lambda(1 - \Phi(\frac{w^* - \mu}{\sigma}))$ is the intensity parameter of the exponential distribution of the unemployment spell length. w_{1i} is the residual of a Mincer earnings regression, again of the accepted log wage of period $t = 1$ on dummies for 5 different schooling levels and a second order age polynomial. The analysis is carried out for men and women separately.

	N	Age	w_1	HS	French	Nochild
Men finding job	1,733	33.90	7.00	0.44	0.92	0.56
Men staying searching	9,800	43.82	-	0.35	0.94	0.64
Women finding job	2,645	34.62	6.67	0.48	0.95	0.41
Women staying searching	12,785	42.75	-	0.33	0.92	0.55

Table 2.2: Descriptive statistics for single model; HS, French and Nochild are the proportion of high school graduates, French citizens and childless persons.

Table 2.2 gives descriptive statistics of the data set. Only individuals who declare themselves searching are considered unemployed. In the previous section this restriction was not made since that would have reduced the size of the data set substantially. In the case of singles, this restriction yields a sufficient number of observations for the estimation. The descriptive statistics show, that the young, better educated and those with children are more likely to find and accept a job offer. This heterogeneity in the job offer arrival rate has to be taken into account. Allowing for heterogeneity in the job offer arrival rate λ can for example be done by using the approach in Ridder and van den Berg [1998]. The parameter is assumed to be a log-linear function of observable worker characteristics, i.e. $\lambda = \exp(X'\beta)$. In my estimation, X includes an intercept, a second order polynomial for age and two education dummies: edu_2 means that the indivdual completed a vocational training degree (below high school graduation: "CAP" or "BEP", around 29%), edu_3 stands for middle-school diploma ("brevet de collège") or no diploma (around 34%) and the reference category is high school graduation ("baccalauréat", around 36%). Using more education categories (as in the Mincer regressions) created convergence problems with the likelihood. To make the exposition coherent, I assume this log-linear specification that will be used later on. Note that it is not possible to include the number of children or the nationality in X since this might change preferences about leisure/home-production but does not define a separate labor market (see Ridder and van den Berg [1998]). Taking the number of children into account would require a different model with different predictions (e.g. differences in reservation wages).

Table 2.3 gives the result of the estimation procedure. The reservation wage is calculated as the 5th fixed order statistic (i.e. the fifth lowest wage observed; this has little importance for the estimation of the Poisson arrival rate of job offers, λ). The first two rows of the table give the point estimates and confidence intervals for the parameters of the log normal distribution, separately for men and women. The wages used are residuals of a Mincer regression of the log wage. In rows 3-8, the results for the regression of the job offer arrival rate λ are given. As noted above, λ is modelled as a log-linear function of a second order age polynomial and two education dummies. From this regression, one can also calculate the average estimated job offer arrival rate, which is quite low (on average .13 for men and .15 for women), compared e.g. to estimates for the US. For France, Jolivet et al. [2006] find $\lambda = 0.56$ using the European Community Household Panel. However Cahuc et al. [2006], also using the Enquête Emploi (in a more complicated equilibrium model) obtain values in the range of my estimates (see their discussion of the estimate of λ and the references that they provide). Depending on industries and different skill categories, their estimate of λ is between 0.05 and 0.32 on an annual basis[4]. Although the results are comparable to at least one other prominent study using the Enquête Emploi, it could be that the estimate is low because workers switch from employment to unemployment and back between the two time points which cannot be captured in our treatment.

Furthermore it is interesting to note that the job offer arrival rate is higher for women than for men (which is not explained by a higher male reservation wage). The estimated job offer arrival rate is first an increasing function of age (until the age of 25 approximately) then it becomes decreasing. A difference between men and women is in the influence of education on the estimate of λ. Being in the lowest education category (edu_3) has for both sexes a negative influence (as one would expect). However women in the second category (vocational training: edu_2) have higher arrival rates compared to high school graduates. Men in the second education category on the other hand have lower job offer arrival rates than even those with the lowest education level. The results for the estimation of the job offer arrival rate are depicted in Figure 2.4.

2.5.2 Structural estimation of a joint search model

In this section, I construct the likelihood of the joint search model of Guler et al. [2009] under the assumption that $\hat{w} = w^{**}$. This is the outcome of the Burdett and Mortensen [1977] model of which the relevant parts are depicted in Figure 2.5.

[4] Note that also their estimates for the job destruction rates are quite low, between 0.03 and 0.1, again depending on industries and workers' skills.

	Men			Women		
	Estimate	Confidence Interval		Estimate	Confidence Interval	
μ	0.0375	0.0105	0.0645	0.0112	-0.0145	0.0369
σ	0.5707	0.5512	0.5903	0.6739	0.6557	0.6921
intercept	-2.3168	-2.8214	-1.8122	-2.2569	-2.6834	-1.8304
age	0.9923	0.6934	1.2911	0.9599	0.7109	1.2088
age^2	-0.2116	-0.2521	-0.1711	-0.2032	-0.2368	-0.1696
edu$_2$	-0.2965	-0.4245	-0.1685	0.100	0.0017	0.1982
edu$_3$	-0.0921	-0.2058	0.0216	-0.2356	-0.3274	-0.1438

Table 2.3: Estimation results, estimated separately on men (superscript M) and women (superscript F). $N^M = 11,528$ and $N^F = 15,425$. Age is divided by 10.

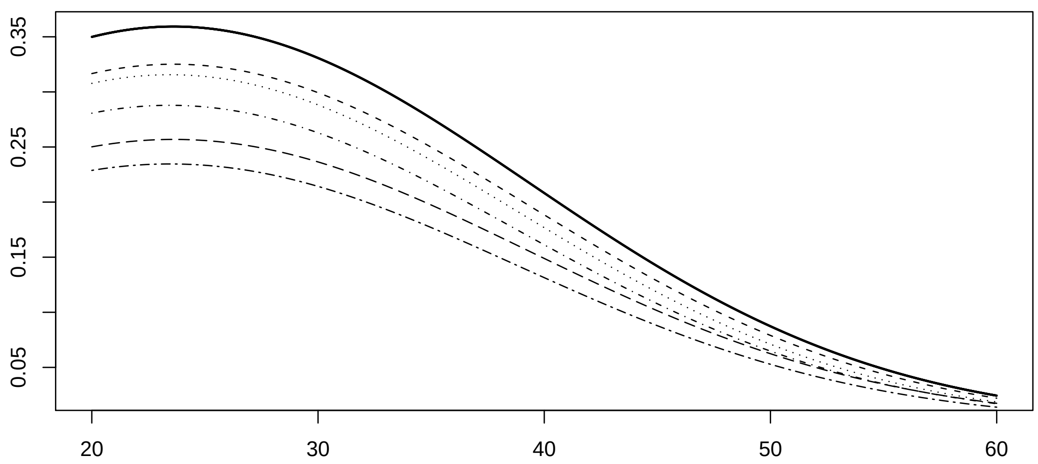

Figure 2.4: Estimates of the job offer arrival rate λ for different education levels, as a function of age. The depicted curves are from top to bottom: 1) Women with medium education; 2) Women with high education; 3) Men with high education; 4) Men with low education: 5) Women with low education and 6) Men with medium education

There is no breadwinner cycle due to on-the-job search and the job offer arrival rates are the same for unemployed and employed. In contrast to the relevant section of Guler et al. [2009] however (where the joint model collapses to the single model), the model of Burdett and Mortensen [1977] features additionally a search intensity margin. The upward sloping reservation wage function then comes from the fact that the higher the wage of the first spouse, the more leisure is enjoyed by the second spouse (the second spouse is searching less) and hence the higher has to be the wage offer in order to induce the second spouse to work.

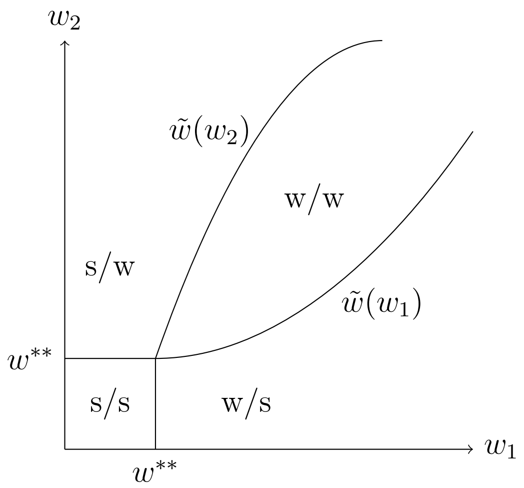

Figure 2.5: The couple's reservation wage strategy in the Burdett and Mortensen [1977] model

The vector of shock parameters is denoted: (λ^M, λ^F), where superscripts refer to the sex of the partner (M for men, F for women), and λ is the job offer arrival rate for the unemployed. Define $\bar{F} = 1 - F$, where F is the wage offer distribution. The notation for the variables is explained in the following Table 2.4.

	Description
w_{i0}^k	Wage, if initially employed
w_{i1}^k	Second wage if any
e_{i0}^k	Initial Employment status, 1 for employed, 0 for searching
e_{i1}^k	Second employment status
$\tilde{w}^k(.)$	Reservation wage, calculation details, see text
c_i	Censoring dummy, 1 if both members are censored
fc_i	Dummy; who changes first the employment status, 1 for man
τ_i	Spell length of the first changer, constant equal to 1.25 years

Table 2.4: Notation for variables used in the likelihood. Superscript k can be either M (Male) or F (Female)

Suppose that the reservation wage functions $\tilde{w}^k(.), k = M, F$ and the two thresholds $w^{**k}, k = M, F$ have been estimated beforehand. Table 2.5 gives the flow probabilities between the different employment states. If state 1 is ws, that means that the husband is employed, the wife is searching and sw is the reverse case.

State 1	State 2	Description	Flow probability
sw	ww	man finds job	$\lambda^M \bar{F}^M(\tilde{w}^M(w_0^F))$
ws	ww	woman finds job	$\lambda^F \bar{F}^F(\tilde{w}^F(w_0^M))$
ss	ws	man finds job	$\lambda^M \bar{F}^M(w^{**M})$
ss	sw	woman finds job	$\lambda^F \bar{F}^F(w^{**F})$

Table 2.5: The transition probabilities

For the likelihood, one needs to calculate the expected length of every employment spell (ws, sw, ss). Since shocks are Poisson, the spell length is exponentially distributed. The respective intensity parameters for the different states are:

$$P^{ws} := \lambda^F \bar{F}^F(\tilde{w}^F(w_0^M))$$
$$P^{sw} := \lambda^M \bar{F}^M(\tilde{w}^M(w_0^F))$$
$$P^{ss} := P^{ss1} + P^{ss2} = \lambda^M \bar{F}^M(w^{**}) + \lambda^F \bar{F}^F(w^{**}).$$

To write down the likelihood l_i of the observed vector of variables x_i for a couple

i, denote

$$x_i = (e_{i0}^M, e_{i1}^M, e_{i0}^F, e_{i1}^F, w_{i0}^M, w_{i1}^M, w_{i0}^F, w_{i1}^F, \tau_i, c_i, fc_i)$$
$$\theta = (\lambda^M, \mu^M, \sigma^M, \lambda^F, \mu^F, \sigma^F)$$

where - recalling the notation - e_{it}^k is the employment status (1 for working, 0 for unemployed) of individual $k = M, F$ in couple $i \in \{1, \ldots, n\}$ at time $t = 0, 1$; w_{it}^k is the wage, sub- and superscripts as for e_{it}^k; τ_i is the spell length of the individual within the couple who first changes its employment state (which is constant for this particular data set, either one changes before 1.25 years or after); fc_i is an indicator that is 1 if it is the husband who first changes his labor market state and 0 if it is the wife.

Assume $F^i, i = M, F$ known: log wages are normal and the two moments of the distribution are μ^i, σ^i for men ($i = M$) and women ($i = F$). Note that the normal (cumulative) density function is given by φ (Φ). The log likelihood can be written as:

$$\log \mathcal{L} = \sum_{i=1}^{n} \log l_i(x_i | e_{i0}^M, e_{i0}^F, w_{i0}^M, w_{i0}^F, c_i, fc_i; \theta, w^{**M}, w^{**F}, \tilde{w}^M(.), \tilde{w}^F(.)).$$

Up to a multiplicative constant, $\log l_i$ can be expressed as:

$$(1 - c_i)\Big\{ e_{0i}^M (1 - e_{0i}^F) e_{1i}^F (1 - fc_i)(D^1(P^{ws}) + D^2(F, \tilde{w}^F(w_{i0}^M), w_{i1}^F)) \tag{2.4}$$

$$+ (1 - e_{0i}^M) e_{0i}^F e_{1i}^M fc_i (D^1(P^{sw}) + D^2(M, \tilde{w}^M(w_{i0}^F), w_{i1}^M)) \tag{2.5}$$

$$+ (1 - e_{0i}^M)(1 - e_{0i}^F) \times \Big[fc_i (D^1(P^{ss1}) + D^2(M, w^{**M}, w_{i1}^M)) \tag{2.6}$$

$$+ (1 - fc_i)(D^1(P^{ss2}) + D^2(F, w^{**F}, w_{i1}^F))\Big]\Big\} \tag{2.7}$$

$$+ c_i \times \Big\{ - e_{0i}^M (1 - e_{0i}^F) P^{ws} \tau_i - (1 - e_{0i}^M) e_{0i}^F P^{sw} \tau_i - (1 - e_{0i}^M)(1 - e_{0i}^F) P^{ss} \tau_i \Big\} \tag{2.8}$$

where

$$D^1(P) = \log(1 - e^{-P\tau})$$
$$D^2(d, w_1, w_2) = -\log\left[1 - \Phi\left(\frac{w_1 - \mu^d}{\sigma^d}\right)\right] - \log[\sigma^d] + \log\left[\varphi\left(\frac{w_2 - \mu^d}{\sigma^d}\right)\right].$$

Line (2.8) corresponds to the censored observations, where the spell length is only known to be equal to or greater than the one that we observe, hence the terms are the probability of observing a spell equal to or greater than the observed one of 1.25 years. Lines (2.4) to (2.7) are for uncensored observations where the spell length is at most 1.25 years. Line 1 corresponds to the initial worker-searcher couples (man employed, woman not) and line 2 to the initial searcher-worker couples (woman employed, man not). Lines (2.6) and (2.7) represent the initial searcher-searcher couples.

Up to now, I have assumed that the reservation wages are already estimated. To estimate the constant reservation wages (for men and women not in the searcher-searcher state) and the reservation wage functions (out of the worker-searcher state), I construct a method analogous to Flinn and Heckman [1982]. They show that any fixed order statistic is a consistent estimator of the reservation wage. In the following, I use fixed order statistics that depend on the number of observations. Robustness checks however show that this has little impact on the estimation results. More precisely, I estimate the constant reservation wage by the fifth lowest accepted wage out of unemployment. For the reservation wage functions (for worker-searcher couples), I divide the range of the employed spouse's wage into several bins (I used at most 7 bins during the estimation). For each bin, I estimate the reservation wage by the second lowest accepted wage. The results of this procedure can be seen in Figures 2.6 and 2.7: the reservation wage functions $\tilde{w}^M$ and $\tilde{w}^F$ are step functions of the wage of the employed spouse[5]. These step functions seem to be U-shaped. Burdett and Mortensen [1977] predicted them to be increasing functions of the wage of the employed spouse. Here (see Figure 2.7), the reservation wage seems to depend on the wage of the partner and the differences are substantial for women wages: they range between almost -1.4 and -0.1 (these numbers refer to residuals from a Mincer earnings regression with estimated standard deviation of around 0.4). Therefore the reservation wage functions should make a difference in the estimation of the job offer arrival rate.

[5]If taking instead of this fixed order statistic approach, a nonparametric quantile regression, the results do not change qualitatively.

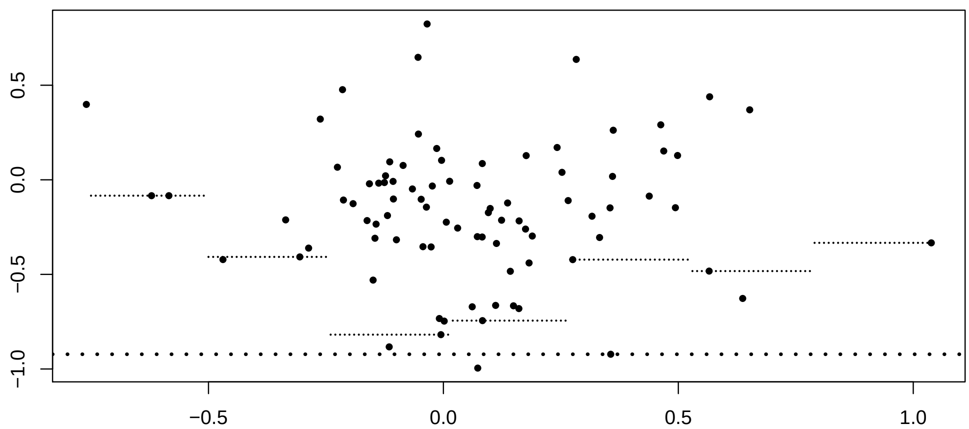

Figure 2.6: Accepted men wages (residual of Mincer regression) as a function of the initial wage of the wife; the step function is the second lowest accepted wage for each of the 7 bins. The dashed line is the constant reservation wage for men in searcher-searcher couples

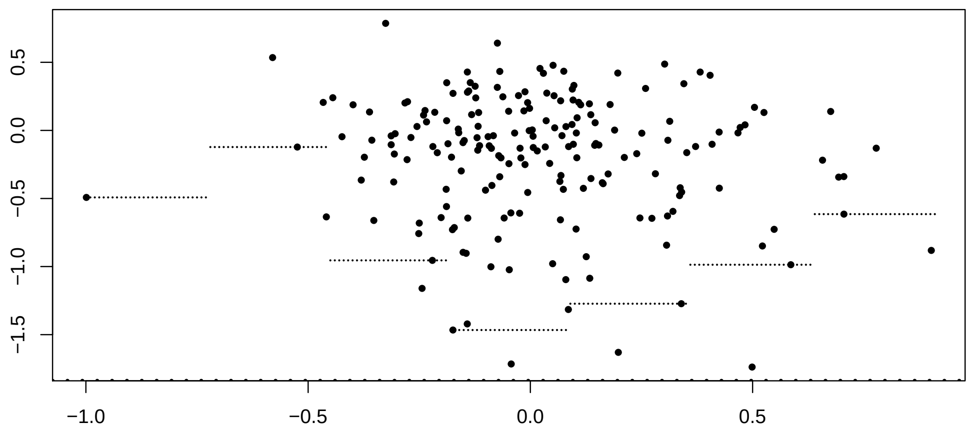

Figure 2.7: Accepted women wages (residual of Mincer regression) as a function of the initial wage of the husband; the step function is the second lowest accepted wage for each of the 7 bins. The dashed line is the constant reservation wage for women in searcher-searcher couples

Change	N	Age^M	Age^F	w_0^M	w_0^F	w_1^M	w_1^F	HS^M	HS^F	Fr^M	NC^M
sw to ww	89	39.11	37.29	-	7.21	7.21	7.21	0.42	0.58	0.96	0.36
ws to ww	200	38.12	35.98	7.39	-	7.39	6.79	0.32	0.42	0.95	0.24
ss to ws	59	39.24	36.78	-	-	7.07	-	0.42	0.41	0.88	0.27
ss to sw	119	41.49	38.46	-	-	-	6.81	0.33	0.51	0.96	0.31
ws cens.	218	41.42	39.05	7.40	-	7.40	-	0.37	0.39	0.94	0.35
sw cens.	425	45.60	43.38	-	7.16	-	7.16	0.34	0.51	0.96	0.42
ss cens.	989	45.88	43.50	-	-	-	-	0.32	0.41	0.96	0.45

Table 2.6: Descriptive statistics: mean values for age, period 0 and period 1 log wages, high school (baccalauréat) dummy (all for men and women), French nationality and childless dummies (only for men)

2.5.3 Estimation results

The data set I use for the estimation contains only individuals that are search unemployed. Descriptive statistics can be found in Table 2.6. Again, there seems to be heterogeneity in the offer arrival rate. Using the same specification as for the estimation of the single agent model, see Table 2.3, a flat likelihood gives rise to convergence problems in the optimization algorithm. Thus, instead of a second order age polynomial, I use two dummy variables for ages 25 to 34 and 35 to 60 (the reference category is 20 to 24 years of age). I also redefine the education dummies: one dummy is for having a vocational training, another one for no diploma at all which means that the reference category is at least two years of education above the high school diploma. The redefinition of education dummies is aimed at obtaining some influence of education on the job offer arrival rate (the influence remains however minor even with this specification, see below). To compare the results of the joint model to the single model, I estimate a single model with the newly defined variables (Table 2.8). I estimate two different reservation wage specifications. The first with only constant reservation wages which is equivalent to the single agent model estimated before, the second reservation wage specification includes the reservation wage functions (estimated as described above). If the theoretical model is empirically relevant, then these two specifications should give different results.

Table 2.7 presents the results. First, one observes that confidence intervals are much larger as compared to the single agent model (Table 2.8). This is due to the smaller number of observations used here (only couples). Secondly, as one would expect, age has a negative influence on the job offer arrival rate. Education does not seem to play an important role and only the male vocational training group has lower λ estimates due to education influences.

The specification of the reservation wages only changes the estimation results slightly (given the confidence intervals). The overall conclusion is thus that the introduction of reservation wage functions $\tilde{w}(.)$ for men and women does not alter the estimation result of the structural parameters of the model (given this particular data set and its limitations, as discussed in section 2.3).

	Only const. reserv. wages			With reserv. wage functions		
	Estim.	Conf. Interval		Estimate	Conf. Interval	
μ^M	-0.1913	-0.2805	-0.0917	-0.2000	-0.2932	-0.1029
σ^M	0.2892	0.2289	0.3579	0.2930	0.2263	0.3573
μ^F	-0.2029	-0.2770	-0.1324	-0.2071	-0.2772	-0.1358
σ^F	0.6297	0.5285	0.7386	0.6337	0.5337	0.7417
Intercept Men	-0.5555	-1.6544	0.1094	-0.5361	-1.6578	0.0788
Age 25-34 Men	-0.7021	-1.3915	0.4221	-0.7152	-1.4284	0.3940
Age 35-60 Men	-2.0849	-2.7437	-1.0224	-2.0832	-2.7074	-0.9833
Voc. training Men	-0.4981	-0.8524	-0.1377	-0.4972	-0.8588	-0.1508
No diploma Men	0.0179	-0.4199	0.4979	0.0058	-0.4277	0.4404
Intercept Women	-0.6116	-1.1372	-0.1704	-0.6190	-1.0696	-0.2446
Age 25-34 Women	-0.4805	-0.8757	-0.0150	-0.4873	-0.8408	-0.0793
Age 35-60 Women	-1.3951	-1.7835	-0.9096	-1.3879	-1.7443	-0.9962
Voc. training Women	-0.0190	-0.2627	0.2333	-0.0086	-0.2485	0.2461
No diploma Women	0.0003	-0.2997	0.3345	0.0128	-0.2804	0.3342

Table 2.7: Estimation results for the structural parameters; 95% bootstrapped (500 replications) confidence intervals, average number of observations is around 2,072 (left) and 2,060 (right).

	Men			Women		
	Estimate	Confidence Interval		Estimate	Confidence Interval	
μ	0.0121	-0.0142	0.0384	-0.0025	-0.0281	0.0231
σ	0.5580	0.5395	0.5764	0.6716	0.6535	0.6897
Intercept	-1.2400	-1.3611	-1.1188	-1.1422	-1.2449	-1.0394
Age 25-34	-0.0050	-0.1327	0.1227	0.0542	-0.0545	0.1628
Age 35-60	-1.2495	-1.3660	-1.1330	-0.7886	-0.8902	-0.6869
Voc. training	-0.0944	-0.2122	0.0235	-0.2005	-0.2921	-0.1088
No diploma	-0.1021	-0.2439	0.0397	-0.7267	-0.8414	-0.6120

Table 2.8: Estimation results for the single agent model with the same specification as for the joint search model. $N^M = 11,528$ and $N^F = 15,425$.

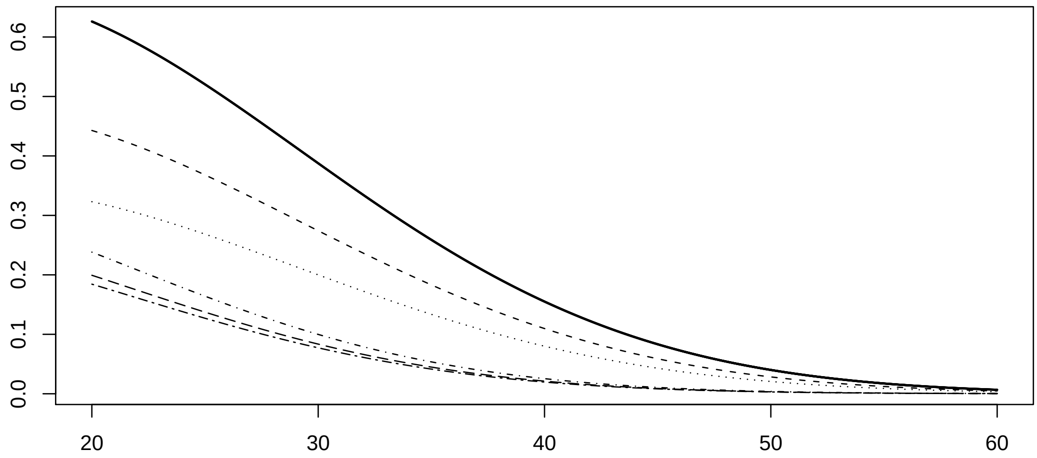

Figure 2.8: Estimates of the job offer arrival rate λ for different education levels, as a function of age. The depicted curves are from top to bottom: 1) Men with low education; 2) Men with high education; 3) Men with medium education; 4) Women with low education: 5) Women with high education and 6) Women with medium education

2.6 Conclusion and discussion

Using Enquête Emploi data, I find little empirical evidence for the two main predictions of the "new opportunities" part of the Guler et al. [2009] model. There are very few couples where the breadwinner changes and these couples do not behave as predicted. Secondly, the reservation wage function is not a monotone function as predicted but U-shaped. Taking this into account does not alter the estimates of the structural parameters. What do these results signify? In this model setting it must be that individuals are completely risk-neutral which implies that the spouses' decisions are not influenced by each other. Yet complete risk neutrality is at odds with empirical evidence on risk attitudes.

It is more plausible that the limitations of this study resulted in such conclusions. It is possible that the breadwinner cycle plays a role for specific groups of couples. For the breadwinner cycle to be observed however, the data set should not be reduced further, thereby losing too many observations. Another problem with the definition of the data set is that one does not know if a job is changed in-between the interviews when the wage is reported: information on the labor market change is often missing. Trying a different data set also yields similar results for the existence of the breadwinner cycle. With the German Socio-Economic Panel for the period 1994-2007, there are basically no changes in the breadwinner cycle sense (< 10 out of 2,125 spells). Perhaps when more detailed, more precise and larger data sets become available in the future, empirical evidence of the breadwinner cycle can then be found.

Appendix

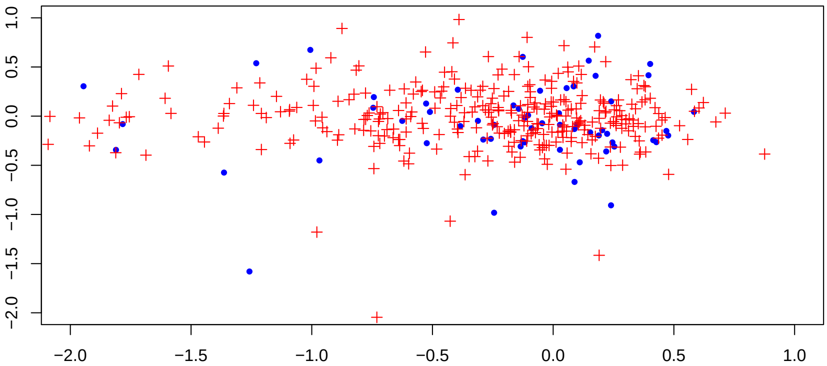

Figure 2.9: Accepted wages as the residual of a Mincer earnings regression for women as a function of the initial wage of the husband; for transition from man working/woman searching to man working/woman working (crosses) and to man searching/woman working (breadwinner cycle, points)

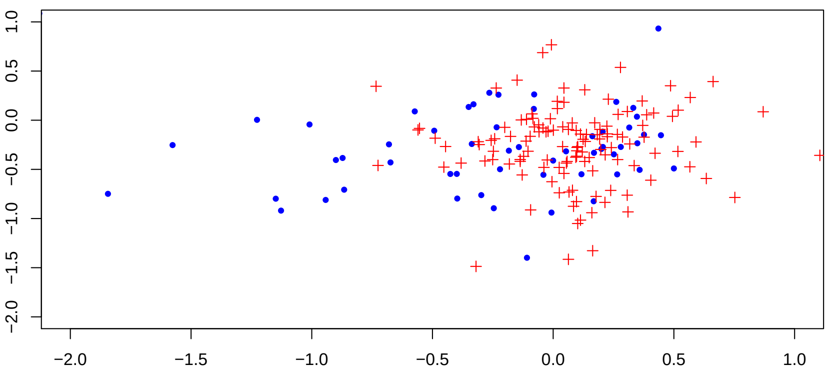

Figure 2.10: Accepted wages as the residual of a Mincer earnings regression for men as a function of the initial wage of the wife; for transition from man searching/woman working to man working/woman working (crosses) and to man working/woman searching (breadwinner cycle, points)

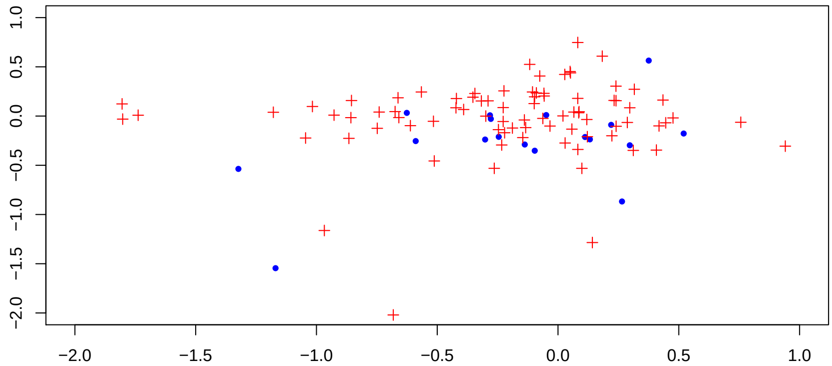

Figure 2.11: Only couples without children. Accepted wages as the residual of a Mincer earnings regression for women as a function of the initial wage of the husband; for transition from man working/woman searching to man working/woman working (crosses) and to man searching/woman working (breadwinner cycle, points)

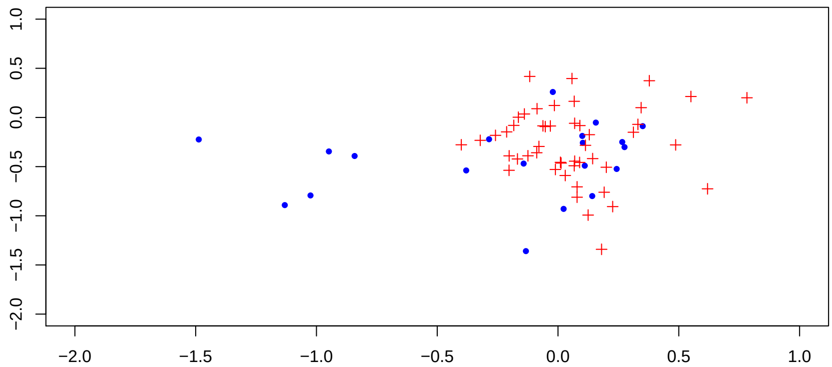

Figure 2.12: Only couples without children. Accepted wages as the residual of a Mincer earnings regression for men as a function of the initial wage of the wife; for transition from man searching/woman working to man working/woman working (crosses) and to man working/woman searching (breadwinner cycle, points)

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Chapter 3

Joint Search and Wage Posting

3.1 Introduction

Partnership or family are a central part of most people's lives. In many economic problems, singles and couples behave differently when they make decisions about their consumption, savings or migration and - which will be the focus of this paper - labor supply. How does one's partner's employment status alter one's labor supply decisions? In which way does the partner's income influence one's reservation wage? Questions like these have been addressed in the field of family economics, giving rise to models of unitary and collective household decision making. On the empirical side, the bulk of the literature has focused on female labor supply, often taking the husband's and firm's decisions as given. But important questions remain: how can firms utilize knowledge about the joint decision making process of a couple when setting wages? Does this in turn translate into wage differentials between men and women and between married and single persons? How does this then affect the equilibrium unemployment rates of the two sexes (i.e. given the workers' and the firms' optimal behavior)? To discuss these questions, equilibrium models of search theory would give the appropriate framework. Given the importance of these questions, it is surprising that little work has been done on the intersection of search theoretical models and family economics.

In this paper, I use a random search, wage posting approach to address some of these questions. The model features risk-averse couples who maximize their joint, expected lifetime utility, i.e. couples have a unitary utility function over the pooled income. As in the basic search theoretical setup, wage offers arrive exogenously and the worker's only decision is whether to accept a given offer or to reject it and continue searching. Once a job is accepted it can be exogenously destroyed or the couple may choose to quit the job once the employment situation of the other spouse changes. This is the central difference to the single model where voluntary quits never occur (in the absence of shocks). Firms post wages in order to maximize the expected value of a filled job. By doing so, they trade-off the speed with which they can hire against the wage they have to pay. Since firms are randomly matched to workers with possibly different reservation wages, offering a higher wage might make it more likely that a given worker accepts the job offer.

The worker's side is identical to the partial model of Guler et al. [2009]. They show that under risk aversion, the reservation wage of a worker depends on the spouse's wage, given a non-degenerate wage offer distribution. In particular, they show that a "breadwinner cycle" exists: a couple where one partner is employed at a low wage quits his or her job once the other partner finds a new job at a wage higher than the wage of the currently employed partner. In equilibrium however, the question is if firms are willing to offer wages above the reservation wage of the unemployed and under which circumstances multiple wages will be offered in equilibrium (i.e. there is wage dispersion). As the classic result of Diamond [1971] suggests, as long as there is no on-the-job search, no leisure or some type of heterogeneity, the model will have a single equilibrium in which firms offer exactly worker's unemployment income. The question that I want to address is under which assumptions about leisure and about the heterogeneity between men and women we do have equilibrium wage dispersion.

The starting point (section 3.5) is a homogeneous model with zero utility of leisure. Men and women have the same unemployment income and hence they also have the same reservation wage. I show that firms have no incentive to offer a wage higher than the reservation wage of the unemployed. As a consequence, couples are indifferent between all states (all states giving the same income) and hence the model has a Diamond [1971] solution.

In section 3.6, I consider a model where there is heterogeneity in unemployment income between men and women (and still zero utility of leisure). I interpret unemployment income in a broad sense as comprising government transfers as well as home production. More specifically, I assume a utility function $u(w^M + w^F)$, where w^M, w^F are drawn from the same wage offer distribution if working and where

$w^i = b^i, i = M, F$ if unemployed with $b^F \neq b^M$. In this case there is an equilibrium that consists of two wages, the respective reservation wages of the two types when unemployed. Hence with differences only in unemployment income, the joint search model is the same as the classic wage dispersion model of Albrecht and Axell [1984].

In section 3.7, I introduce leisure into the utility function. I show that there is an equilibrium in which the wage offer distribution consists of a continuous part and a mass point. In the continuous part, there are only couples that change in the breadwinner cycle sense whenever offered a strictly greater wage. Hence the continuous part is similar to models of on-the-job search (e.g. Burdett and Mortensen [1998]). Paying higher wages increases the probability for the firm to hire the searching spouse in a worker-searcher couple. At the same time, a higher wage increases the length of the employment spell, since the higher the wage, the less likely it is that the searching spouse finds a wage that is higher than the wage of the currently employed spouse (and hence triggers a breadwinner cycle). The mass point is set at the lowest wage for which both spouses choose to simultaneously work.

In section 3.8, I analyze heterogeneity in leisure between men and women. The utility function of the couple is assumed to be $l^M + l^F + u(w^M + w^F)$ where the wages are drawn from the same wage offer distribution and unemployment compensation is identical for men and women. But if unemployed, men enjoy leisure of l^M and women leisure of l^F, where I assume, without loss of generality, that $l^F < l^M$. I show that depending on the productivity of the firm, different equilibria may arise. I characterize the equilibria that exist if the firms' productivity y is low or high. All these equilibria consist of only one wage. In an intermediate range of y, analytical solutions are not feasible. As an application of the model, I present a numerical example of a two-wage equilibrium for the German economy in section 3.9. I show that the model can quantitatively explain the unemployment rates of men and women, the marriage premium of men, and partially the gender wage gap and observed differences in reservation wages between men and women.

3.2 Related literature

The most closely related work is the model by Ek and Holmlund [2010]. They consider an equilibrium joint search model with Nash-bargaining. The key difference in the results is that due to their bargaining solution concept there are no differences in unemployment rates across sexes. Every job is accepted and the only source of wage differences is the differences in threat points for workers with a working or non-working spouse. In my model, firms do not know the type of worker they are matched to and hence may end up with a worker whose reservation wage is higher than the offered wage. Since wages are posted on a take-it-or-leave-it basis, some worker-firm matches do not result in a job.

Both Ek and Holmlund [2010] and my work build on the partial joint search model of Guler et al. [2009] (the worker's side of all models is identical). This model is described below. Guler et al. [2009] in turn rely on a much older sketch of a joint search model in Burdett and Mortensen [1977].

My work also builds on and expands the literature on wage dispersion (for a survey see e.g. Rogerson et al. [2005]). The main question this area of literature addresses is why homogeneous workers (as to their observable characteristics) are paid different wages, as empirical evidence suggests. The work of Albrecht and Axell [1984] rationalizes the fact that firms pay different wages by assuming two different types of leisure values (and hence reservation wages) in the population. Firms know that this heterogeneity exists and hence may post two different wages, the two respective reservation wages. The lower wage is only accepted by the low reservation wage workers while the higher wage is accepted by both workers (and hence allows faster to recruit). Burdett and Mortensen [1998] introduce on-the-job search into the search model and show that this gives rise to a continuous wage offer distribution. The idea is simple: suppose workers earn different wages and hence have different reservation wages. As in Albrecht and Axell [1984], this difference in reservation wages entices firms to offer different wages. On the one hand, a higher wage reduces the profit of a filled job. On the other hand, a higher wage makes it more likely that a randomly matched worker accepts the offer and also increases the employment spell length.

The theoretical literature on family economics started with the unitary model, i.e. the assumption that although each member of the household has his or her own utility function, the household somehow agrees to maximizes a joint utility function (e.g. by reaching some kind of consensus as in Samuelson [1956]). The unitary model finds little support empirically (see e.g. Thomas [1990] or Browning and Chiappori [1998]) and is also opposed to the methodological individualism that economists favor. Efforts have hence been made to construct models that are underpinned by rational individuals making individual decisions. The first class of models considers Nash-bargaining within the household (starting with Brown and Manser [1980]). Another route has been the collective approach introduced by Chiappori who derives testable assumptions on household behavior based on the condition that household decisions are Pareto optimal (see Chiappori [1992]).

In my model, I restrict myself to the unitary model. The unitary model may not be the best description of household behavior, it is nonetheless a good starting point. More realistic assumptions like bargaining models within the family or a collective approach would render the model intractable.

On the empirical side, there is much evidence on differences in labor market outcomes between the sexes. One example is the German labor market, where the literature has consistently found a gender wage gap (see e.g. Hirsch et al. [2010]), a marriage premium for men (see e.g. Barg and Beblo [2007]) but not for women, while unemployment rates are comparable between men and women.

3.3 The model environment

A couple consists of two persons, denoted by $i = M, F$. Couples have a linearly separable instantaneous utility function over their pooled income and leisure:

$$u(w^M + w^F) + L(s^F, s^M)$$

where w^M, w^F are the respective wages of husband and wife and $s^M, s^F \in \{0, 1\}$ are indicators that take the value of 0 if husband or wife are working, 1 if searching (and enjoying leisure). $u(.)$ is strictly increasing, strictly concave and of the DARA type (this choice is explained below). $L(., s^M)$ and $L(s^F, .)$ are increasing. If unemployed, men and women receive unemployment income consisting of a government transfer and home production. In this paper, I will study homogeneity and heterogeneity of unemployment income assuming $L(s^F, s^M) \equiv 0$ (chapters 3.5 and 3.6) and homogeneity and heterogeneity in leisure for $L(s^F, s^M) \geq 0$ (chapters 3.7 and 3.8).

Couples maximize the expected present discounted utility over their infinite lifetime. The discount rate for individuals (as well as for firms) is denoted by r. There is a mass 1 of couples. There is neither saving nor stocking, i.e. consumption equals earnings.

Some additional assumptions about the behavior of the couple when being indifferent between multiple states have to be made. The couple may be in three different states: both spouses working and not searching for a job (the *worker-worker state*), one spouse working, the other spouse unemployed and searching (the *worker-searcher state*) and both unemployed and searching (the *searcher-searcher state*). Standard literature on single models assumes that an unemployed will accept a job offer when indifferent. In models with on-the-job search, it is assumed that a worker only quits his current job when he is strictly better off at the new one, i.e. that a job is not quit at indifference. Following this literature, I assume the following. If a couple is indifferent between the searcher-searcher state and a worker-searcher state, the job is accepted. If it is indifferent between a worker-searcher state and a worker-worker state, the second job is accepted. But a couple that is indifferent between two worker-searcher states does not accept the job (no quits at indifference).

For the firms, I follow the literature on wage posting. Firms post wages in order to maximize the expected, present discounted value of a job. When doing so, firms cannot observe the gender of an applicant, the employment status or wage of the partner. These anonymity assumptions can be justified by a lack of information of the firm (the wage of the partner is rarely communicated during the application process). Furthermore, there are legal constraints prohibiting for example sex discrimination. Another reason are fairness concerns that make it costly for the firm (in terms of low work effort) to discriminate with respect to sex or the wage of the partner. All firms have the same productivity level y which is independent of the worker. There is a mass 1 of firms.

Workers and firms are randomly matched together. From the worker's perspective, offers arrive according to the exogenous Poisson rate of λ. There is no bargaining, i.e. workers can only accept or reject the posted wage upon meeting a firm. Matches are exogenously destroyed at rate δ. Finally, only symmetric Nash equilibria are considered.

3.4 The wage posting equilibrium

Workers

The workers' side of my model is identical to the partial equilibrium model of Guler et al. [2009], except that I consider an endogenous wage offer distribution and heterogeneity between men and women. The Bellman equations are given by:

$$\begin{aligned}
rU &= u(b^M + b^F) + L(1,1) + \lambda \int \max\{\Omega^M(w) - U, 0\} dF(w) \\
&\quad + \lambda \int \max\{\Omega^F(w) - U, 0\} dF(w) \\
rT(w^F, w^M) &= u(w^M + w^F) + L(0,0) + \delta(\Omega^M(w^M) - T(w^F, w^M)) \\
&\quad + \delta(\Omega^F(w^F) - T(w^F, w^M)) \\
r\Omega^M(w^M) &= u(w^M + b^F) + L(1,0) + \delta(U - \Omega^M(w^M)) \\
&\quad + \lambda \int \max\{T(w, w^M) - \Omega^M(w^M), \Omega^F(w) - \Omega^M(w^M), 0\} dF(w) \\
r\Omega^F(w^F) &= u(w^F + b^M) + L(0,1) + \delta(U - \Omega^F(w^F)) \\
&\quad + \lambda \int \max\{T(w^F, w) - \Omega^F(w^F), \Omega^M(w) - \Omega^F(w^F), 0\} dF(w)
\end{aligned} \tag{3.1}$$

where λ is the job finding rate, δ is the separation rate and F is the wage offer distribution. While men and women may differ in their preferences, they draw wages from the same - endogenous - wage offer distribution F and share the same labor market parameters (job offer arrival rate, separation rate, discount rate). The model has (potentially) infinitely many states, depending on the wage offer distribution. First, a couple can be in the searcher-searcher state, i.e. both partners are searching for a job (the value function is denoted by U). Couples leave this state whenever one of the two partners draws a wage offer (above the reservation wage). The second state is the worker-searcher state where either the man or the woman is working (value functions are denoted by $\Omega^M(w)$ and $\Omega^F(w)$ respectively). Transitions from this state are either to the worker-worker state or the worker-searcher state if the unemployed spouse finds a job or to the searcher-searcher state if the working spouse loses the job. Transitions from the worker-worker state (the value function is $T(w^F, w^M)$) can only be to the worker-searcher state when one of the spouses loses the job (since this is a continuous model, the probability that both lose the job simultaneously is zero). The main difference to the standard model concerns the possible optimal choices of a worker-searcher couple. If a worker-searcher couple draws a wage offer for the searching spouse, the couple has three alternatives: accept the job offer and become a worker-worker couple, accept the job offer and quit the first job to stay a worker-searcher couple but with reversed roles, or to reject the offer. The second option is what Guler et al. [2009] term as the breadwinner cycle: the employed spouse quits his or her job as the unemployed finds a job.

Before discussing the case with an endogenous wage offer distribution, it is helpful to present the results of the partial model, i.e. with an exogenous wage offer distribution (this section closely follows Guler et al. [2009]).

Guler et al. [2009] show that the solution of the partial model depends crucially on assumptions on risk aversion. For a risk-neutral couple, the solution of the joint problem is the same as in the single agent context, i.e. the breadwinner cycle never emerges and the couple acts as two singles. Intuitively, if individuals are risk-neutral, there is no consumption smoothing gain from being in a couple. With risk aversion however, couples are better off since they can share the risk (of losing the job) and hence partners can at least partially insure each other. In this case, the breadwinner cycle exists and the joint model differs from the single model. Here I will restrict myself to utility functions that are linearly separable in leisure and consumption ($u(w^M + w^F) + L(s^F, s^M)$) and where u is of the DARA type (decreasing absolute risk aversion, one member of this class is CRRA utility)[1]. As such, the reservation wage strategies can be represented as in Figure 3.1.

[1]The policy functions are different for IARA (increasing absolute risk aversion) or CARA (constant absolute risk aversion) cases. For a discussion of the implications of the type of the utility function, see Guler et al. [2009].

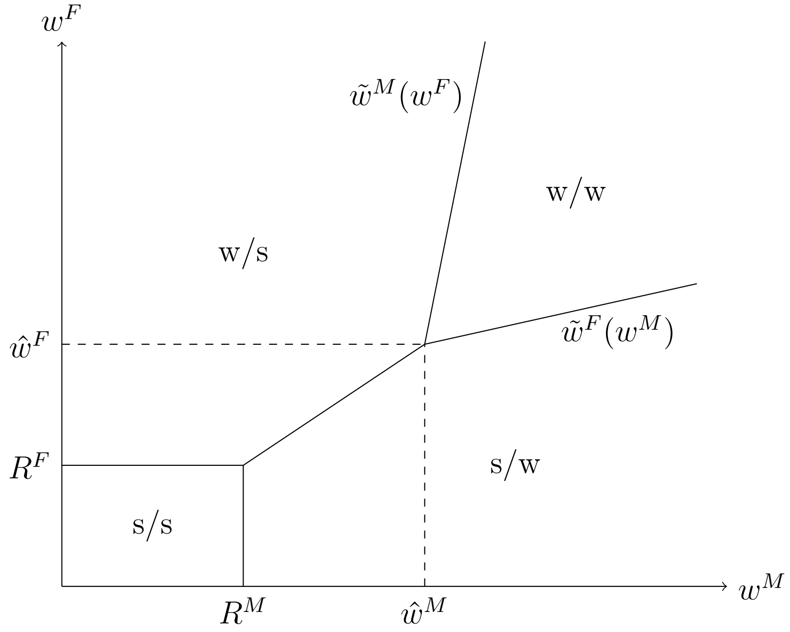

Figure 3.1: The reservation wage strategy of the couple

Figure 3.1 reads as follows. There are two reservation wages for a searcher-searcher couple: R^M for wage offers to the man and R^F for the woman. A searcher-searcher couple accepts every job greater than R^M or R^F and switches to the worker-searcher state. If the wage of the worker-searcher couple - where the man is working - is between R^M and $\hat{w}^M$, the couple switches in the breadwinner cycle sense whenever a sufficiently high wage is offered to the (unemployed) wife. If however the current wage of the worker-searcher couple, call it w^M, is greater than $\hat{w}^M$, the couple switches to the worker-worker state when offered a wage greater than $\tilde{w}^F(w^M)$ and switches in the breadwinner cycle sense whenever the wage is strictly greater than $\tilde{w}^{M^{-1}}(w^M)$. (The reasoning is analogous for worker-searcher couples where the woman is working.)

Hence the solution of the workers' problem in (3.1) is characterized by two reservation wages of searcher-searcher couples R^M and R^F and by two reservation wage functions of worker-searcher couples, denoted by $\tilde{w}^M(.)$ and $\tilde{w}^F(.)$ (these functions being defined on $[R^F, \infty)$ and $[R^M, \infty)$ respectively).

Firms

Firms maximize the expected, present discounted value of a filled job π:

$$\max \pi(w) = p_1^M(w)J_1^M(w) + p_1^F(w)J_1^F(w) + p_2^M(w)J_2^M(w) + p_2^F(w)J_2^F(w) \qquad (3.2)$$

where $J_1^M(w)$ is the value of a job filled with a man whose wife is searching, $J_1^F(w)$ the value of a job filled with a woman whose husband is searching, $J_2^M(w)$ the value of a job filled with a man whose wife is working, and $J_2^F(w)$ the value of a job filled with a woman whose husband is working. Note that $J_2^M(w)$ and $J_2^F(w)$ are independent of the wage of the partner since separations in the worker-worker state only occur exogenously (the acceptance probability however depends on the partner's wage).

The probabilities faced by the firm are as follows: $p_1^M(w)$ is the probability to be matched to a man who accepts the wage offer and starts to work in a worker-searcher couple. $p_2^M(w)$ is the probability to be matched to a man who accepts the wage offer and starts to work in a worker-worker couple. $p_1^F(w)$ and $p_2^F(w)$ are the analogous probabilities for women.

In order to see why it is important to distinguish between having a worker whose spouse is working and one who is searching, consider the Bellman equations for the different states:

$$\begin{aligned}
rJ_1^M(w) &= y - w - (\delta + p_q^M(w))J_1^M(w) + p_a^M(w)(J_2^M(w) - J_1^M(w)) \\
rJ_1^F(w) &= y - w - (\delta + p_q^F(w))J_1^F(w) + p_a^F(w)(J_2^F(w) - J_1^F(w)) \\
rJ_2^M(w) &= y - w - \delta J_2^M(w) + \delta(J_1^M(w) - J_2^M(w)) \\
rJ_2^F(w) &= y - w - \delta J_2^F(w) + \delta(J_1^F(w) - J_2^F(w))
\end{aligned}$$

where $p_q^M(w)$ is the voluntary quit probability, i.e. the probability that the match ends because the male worker quits his job as his wife finds a job. $p_a^M(w)$ is the probability that the wife of the male worker accepts a job without the man quitting his job. These equations imply that firms are seeking to have a worker whose spouse is working as long as $p_q^M(w)$ is strictly positive because a worker in a worker-worker couple does not quit his or her job voluntarily.

The equations also illustrate the challenges of this model: one has to determine the different probabilities that do not only depend on the wage w but also on equilibrium outcomes, i.e. the distribution of couples over the different states and the workers' policy. Hence the proofs are constructed by conjecturing plausible wage offer distributions, deriving in turn the worker's optimal decision and showing that indeed, the wage offer distribution is profit maximizing.

Equilibrium definition

An equilibrium in this model is defined by a tuple $(R^M, R^F, \tilde{w}^M(.), \tilde{w}^F(.), \bar{\pi}, F)$ such that:

1. Couples maximize the expected present discounted value of lifetime utility taking the wage offer distribution as given, i.e. $(R^M, R^F, \tilde{w}^M(.), \tilde{w}^F(.))$ solves (3.1).

2. Firms set wages as to maximize the expected present discounted value of a filled job (see equation 3.2), i.e. F is such that $\forall w \in \text{supp}(F), \bar{\pi} = \pi(w)$ and $\forall w' \notin \text{supp}(F), \bar{\pi} \geq \pi(w')$.

3.5 A homogeneous wage posting joint search equilibrium without utility of leisure: A Diamond solution

First consider a model without utility of leisure and with homogeneous workers, i.e. where husband and wife have the same parameters concerning unemployment income, home production, etc. Suppose that the couple has the following utility function (the same as in Guler et al. [2009]):

$$u(w^M + w^F) = \begin{cases} u(w^M + w^F) & \text{if both spouses are working} \\ u(w^i + b) & \text{for a worker-searcher couple, } i = M, F \text{ working} \\ u(2b) & \text{for a searcher-searcher couple} \end{cases}$$

where w^M, w^F are wages and b is unemployment income.

Again firms are assumed to post wages as to maximize the value of a filled job taking the workers' decision as given. The solution to this model is summarized in the following proposition.

Proposition 1. *In the model without heterogeneity in the parameters of the members of the couple and zero utility of leisure, the joint search model exhibits a Diamond solution. The only wage offered equals the unemployment income b.*

Proof. See appendix. □

This result might be surprising given the complex solution to the partial equilibrium model. The reason is that the reservation wage of one spouse does not depend on the wage of the other spouse given that only one wage b is offered: the reservation wage of a worker-searcher couple is equal to the reservation wage for a searcher-searcher couple. Hence there is no incentive for firms to offer a higher wage, since all couples accept b when offered.

The dynamics are the following, starting with a searcher-searcher couple. If one of the spouses draws a wage offer (offering b), it is accepted. This worker-searcher couple has still the reservation wage of b and will hence accept all wage offers (of b) and become a worker-worker couple.

3.6 An equilibrium with heterogeneity in unemployment compensation and without utility of leisure: An Albrecht-Axell solution

If men and women are homogeneous and there is no utility of leisure, only one wage will be offered. In reality however, men and women may differ in several aspects. I will focus on differences in home production and differences in leisure. Of course, one could also assume that job offer arrival rates or separation rates differ, e.g. that men find jobs faster and keep them longer (i.e. that $\lambda^M > \lambda^F$ or $\delta^M < \delta^F$) which for example could be due to differences in unobserved characteristics between men and women or to discrimination against women. The reason I abstract from these differences is simple: they do not change the Diamond result from the previous section. To understand this intuitively, start again by assuming that firms offer one wage, the unemployment income of workers. This means that the flow utility in each state is equal to $u(2b)$, which in turn implies that the search value of being unemployed has to be zero (workers do not gain from starting to work). Thus a change in the job offer arrival rate does not alter the result of Proposition 1: firms have no incentive to offer a higher wage than the reservation wage (which is equal to unemployment income)[2].

[2] Analytically, one can proceed analogously to the proof of Proposition 1, see Appendix.

In what follows I will always assume that men have higher reservation wages which is in line with the empirical literature (see Pannenberg [2010] or Brown et al. [2011]). In order to have higher reservation wages, I assume that men have higher values of home production (this section) or higher values of leisure (section 3.8). The first step and the focus of this section is on differences in home production between men and women (and there is no utility of leisure). Suppose that the joint utility function is given by:

$$\begin{cases} u(w^M + w^F) & \text{for a worker-worker couple} \\ u(w^M + b^F) & \text{for a worker-searcher couple, man working} \\ u(b^M + w^F) & \text{for a worker-searcher couple, woman working} \\ u(b^M + b^F) & \text{for a searcher-searcher couple} \end{cases}$$

where $w^M, w^F \sim F$ and $b^M > b^F$ i.e. men and women sample job offers from the same endogenous wage offer distribution but have different values of unemployment income (government transfers and home production).

In this model where spouses differ in their unemployment income and there is zero utility of leisure, the joint search model exhibits an Albrecht-Axell solution. Albrecht and Axell [1984] considered a model with wage posting with two types of workers in the population, one with a high value of leisure compared to the other type. Their resulting equilibrium wage offer distribution depends on firm productivity. When firm productivity is low, only the reservation wage of the low type is offered, when firm productivity is high, only the reservation wage of the high type is offered. But in an intermediate range of firm productivities, the equilibrium consists of two wages, the respective reservation wages of the two types. This is also true in my model, as summarized in the following proposition.

Proposition 2. *For a low value of productivity, only one wage is offered, the reservation wage of women b^F. For an intermediate level of productivity, two wages w_1, w_2, where $b^F < w_1 < w_2$, are offered which are characterized by:*

$$w_2 = b^M$$
$$(r + \delta + \lambda(1 - \gamma))u(w_1 + b^M) = (r + \delta)u(b^F + b^M) + \lambda(1 - \gamma)u(2b^M)$$

where γ is the proportion of firms offering w_1. If the productivity is high, only one wage is offered, the reservation wage of men b^M.

Proof. See appendix. □

The dynamics of this equilibrium are as follows: women accept both wages, men only w_2. This is true for searcher-searcher couples but also for worker-searcher couples, i.e. in this setting it is still true that the reservation wage does not depend on the wage of the partner (given the wage offer distribution). So, worker-searcher couples switch to the worker-worker state whenever the woman draws one of the wages or when the man draws w_2.

This is exactly the solution of Albrecht and Axell [1984] where half of the population is of type 1 and the other half of type 2, i.e. differences in unemployment income are not generating qualitatively new equilibria. In the next section, I introduce leisure into the utility function (section 3.7) and later heterogeneity in leisure (section 3.8). I will then compare the different specifications and explain why the small change has substantial influence on the equilibrium outcome.

3.7 Leisure in the utility function: the homogeneous case

In this section, I introduce leisure into the utility function. I consider the homogeneous case, i.e. there is no difference whether the man or the woman is searching (and enjoying leisure). In the next section, I will then look at the heterogeneous leisure case, i.e. where one spouse values leisure more than the other.

In this section I do not attempt a complete characterization of possible equilibria. Instead, I present particular equilibria, i.e. for specific sets of parameters. There might be other equilibria and I will discuss which characteristics these equilibria would have to fulfil.

Suppose that the utility function is given by

$$\begin{cases} u(w^M + w^F) & \text{for a worker-worker couple} \\ l^{ws} + u(w^M + b) & \text{for a worker-searcher couple, man working} \\ l^{ws} + u(b + w^F) & \text{for a worker-searcher couple, woman working} \\ l^{ss} + u(2b) & \text{for a searcher-searcher couple} \end{cases}$$

where $l^{ss} > l^{ws}$. The notation of this section is as follows: The endogenous wage offer distribution is denoted by $F(x)$, the reservation wage of searcher-searcher couples by R. $\hat{w}$ is the double indifference point characterized by the indifference condition $\Omega(\hat{w}) = T(\hat{w}, \hat{w})$. For all wages above $\hat{w}$, the reservation wage of a worker-searcher couple is given by $\tilde{w}(w)$. The reservation wage strategy of a couple can be represented as in Figure 3.2.

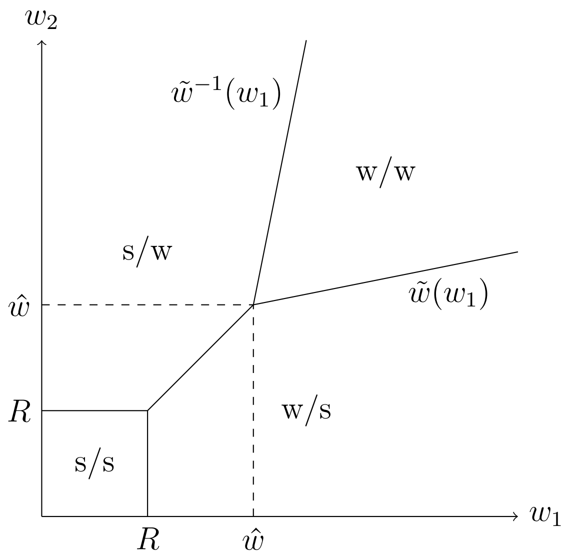

Figure 3.2: The reservation wage strategy of the couple

It is first instructive to ask whether there can be a Diamond [1971] solution in this case, i.e. an equilibrium in which only the reservation wage of searcher-searcher couples is offered, as was the case with zero leisure utility ($l^{ss} = l^{ws} = 0$) in section 3.5. The answer is no in general, there is however a particular parameter constellation for which indeed we have only one wage in equilibrium. Proposition 3 summarizes this.

Proposition 3. *The model with homogeneous leisure has a Diamond [1971] equilibrium with wage R, defined by*

$$l^{ss} + u(2b) = l^{ws} + u(R + b)$$

if and only if the following equation holds:

$$u(2R) = u(R + b) + l^{ws}.$$

Proof. See appendix. □

The reason why we have this result is simple. The reservation wage of a searcher-searcher couple is determined by trading off the loss of leisure for the partner who starts to work with the additional consumption gain provided by the job. The reservation wage of a worker-searcher couple (working at R) is determined in the same way, but due to the concavity of the utility function, the level of consumption gained if the second spouse starts to work is smaller than for the first spouse. Hence there is exactly one level of leisure for which gains in utility of consumption and the loss of leisure for the second spouse balance. In that case, we have the indifference between all states (i.e. $U = \Omega(R) = T(R, R)$) and the Diamond [1971] solution is stable.

For the rest of this section, I will assume that parameters are such that

$$u(2R) \neq u(R + b) + l^{ws}$$

and consequently $\Omega(R) \neq T(R, R)$, given that R, the reservation wage of searcher-searcher couples, is the only offered wage. As shown in Proposition 3, offering R with mass 1 cannot be an equilibrium. Furthermore, there cannot be any equilibrium where R and a finite number of other wages are offered since whenever there is a positive mass at R, the result of Proposition 3 holds and firms have a profitable deviation. Hence the equilibrium has to have a continuous part (whenever R is offered). This is also intuitive if thinking about joint search as one way of on-the-job search. As in the Burdett and Mortensen [1998] model, firms have an incentive to offer a higher wage because the acceptance probability of a worker increases and the quitting probability decreases. This is the same here: worker-searcher couples switch in the breadwinner cycle sense whenever offered a higher wage. The difference to on-the-job search is that it is not the worker who accepts the job but his or her partner.

Now the question is if the whole distribution can be continuous or if there have to be mass points. For any continuous distribution, the results of Guler et al. [2009] apply, i.e. at some point, couples start to accept wages to become worker-worker couples. Hence firms will set their wages accordingly. An important role is played in this case by the double indifference wage, denoted by $\hat{w}$ in Figure 3.2. This wage is the lowest wage for which a worker-worker couple can be observed. Formally, it is defined by $\Omega(\hat{w}) = T(\hat{w}, \hat{w})$.

The question is now if it can be profitable to offer a wage higher than $\hat{w}$. Suppose the upper bound w^u of the support was strictly greater than $\hat{w}$. Firms offering w^u would attract all workers, first of all of course the unemployed and the worker-searcher couples earning less than $\hat{w}$ (who would accept the job and change in the breadwinner cycle sense). They would also attract worker-searcher couples working at a wage of $\hat{w}$ or higher: the ones who work at a wage lower than $\tilde{w}^{-1}(w^u)$ would change in the breadwinner cycle sense, while those earning between $\tilde{w}^{-1}(w^u)$ and w^u would accept and become worker-worker couples. Now, could the firm attract all these workers with a lower wage? All workers who switch to become worker-worker couples (earning currently between $\tilde{w}^{-1}(w^u)$ and w^u) have reservation wages lower than w^u (the slope of $\tilde{w}$ is smaller than one, see Guler et al. [2009], Proposition 3). Hence the firm could reduce the wage and still get the same number of workers[3]. Thus there cannot be an equilibrium in which wages above $\hat{w}$ are offered.

Proposition 4 defines and characterises the equilibrium for the above utility function with homogeneous leisure:

Proposition 4. *Consider the limiting case $r \to 0$. Suppose that the firm productivity is in some intermediate range $y \in [y^0, y^\infty]$. Let the endogenous wage offer distribution function F be given by:*

$$F(x) = \begin{cases} 0 & x < R \\ F_1(x) & R \leq x < \bar{w} \\ 1-\gamma & \bar{w} \leq x < \hat{w} \\ 1 & x \geq \hat{w} \end{cases}$$

where F_1 is strictly increasing. Denote the reservation wage of searcher-searcher couples by R and the double indifference point by $\hat{w}$. Let $(F_1(x), R, \bar{w}, \hat{w}, \gamma)$ be determined by the following five conditions (in the appendix, conditions (i)-(v) are expressed in terms of parameters and the utility function only, i.e. eliminating value functions):

[3]If firms reduce the wage in this case, they would attract less couples who change in the breadwinner cycle sense but more who become worker-worker couples, but the total mass of couples the firm can attract stays constant. Since it is assumed that the firm offers w^u (the highest wage in the model), it is not important for the firm if it attracts worker-worker or worker-searcher couples. Both types will not quit their job endogenously (only exogenously with shock rate δ).

(i) $F_1(x)$ as a function of γ and R is determined by the indifference of firms:

$$\pi(R) = \pi(w), \quad \forall w \in [R, \bar{w}].$$

(ii) $\bar{w}$ as a function of γ and R is determined by

$$F(\bar{w}) = 1 - \gamma.$$

(iii) R as a function of γ and $\hat{w}$ is determined by the indifference condition of workers:

$$U = \Omega(R).$$

(iv) $\hat{w}$ as a function of γ is determined by the indifference condition of workers:

$$\Omega(\hat{w}) = T(\hat{w}, \hat{w}).$$

(v) γ is determined by the firm's indifference condition:

$$\pi(R) = \pi(\hat{w}).$$

If $(F_1(x), R, \bar{w}, \hat{w}, \gamma)$ fulfil (i)-(v) and in addition $\hat{w} > R$ holds, then $(F_1(x), R, \bar{w}, \hat{w}, \gamma)$ is an equilibrium.

Proof. See appendix. □

Graphically, the wage offer distribution can be depicted as in Figure 3.3.

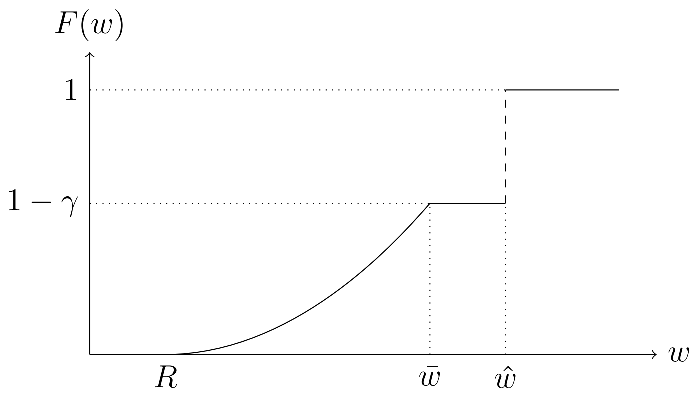

Figure 3.3: The endogenous wage offer distribution (homogeneity in leisure case)

The intuition behind the wage offer distribution is the following. Obviously, no firm will offer a lower wage than R. F consists of a continuous part with mass $1-\gamma$ (offering wages where couples will change in the breadwinner cycle sense) and a mass point at $\hat{w}$ (with mass γ), the double indifference point characterized by the indifference condition $\Omega(\hat{w}) = T(\hat{w}, \hat{w})$. Why is there no possible deviation in this case? A firm that offers $\hat{w}$ has an acceptance probability of one: it attracts all couples where one spouse is earning less than $\hat{w}$ (breadwinner cycle) and all those worker-searcher couples working at $\hat{w}$ (who become worker-worker couples). Hence there is no way to increase the acceptance probability. By reducing the wage, firms would reduce the acceptance probability discretely (all couples working $\hat{w}$ would not accept anymore), hence the hole in the distribution between $\bar{w}$ and $\hat{w}$.

To understand why there are tuples $(F_1(x), R, \bar{w}, \hat{w}, \gamma)$ that fulfil equations (i)-(v) of Proposition 4 but that have $R > \hat{w}$, consider the Bellman equations at $\hat{w}$:

$$rT(\hat{w}, \hat{w}) = u(2\hat{w})$$
$$r\Omega(\hat{w}) = l^{ws} + u(\hat{w} + b) + \delta(U - \Omega(\hat{w})).$$

Hence $\hat{w}$ is such that:

$$u(2\hat{w}) = l^{ws} + u(\hat{w} + b) + \delta(U - \Omega(\hat{w})). \tag{3.3}$$

Since $U - \Omega(\hat{w}) < 0$, we have necessarily $u(2\hat{w}) < l^{ws} + u(\hat{w} + b)$. Since u is strictly increasing, we know that $u(2\hat{w}) > u(\hat{w} + b)$. Hence l^{ws} has to be sufficiently large in order for $\hat{w} > b$ to exist in equation (3.3). Furthermore, $\hat{w}$ also has to fulfil $\hat{w} > R$. Since R depends positively on l^{ss}, we can choose l^{ss} relatively small (i.e. close to l^{ws}, but still $l^{ss} > l^{ws}$) in order to indeed get $\hat{w} > R$. This is also intuitive: a worker-searcher couple has to have a relatively high value of leisure in order to be indifferent between being a worker-searcher couple (where job losses and hence a change to a less valuable state occur often) and a worker-worker couple (where changes to the searcher-searcher state are relatively less likely). On the other hand, if the loss of leisure is large when changing from searcher-searcher to worker-searcher state (meaning l^{ws} and l^{ss} are not close to each other), then R is relatively large and it might be the case that even though $\hat{w} > b$ we still have $\hat{w} < R$.

In order to illustrate the equilibrium, I choose a numerical example with parameters values as given in Table 3.1.

Parameter	Signification	Value
λ	Job offer arrival rate	0.2
δ	Separation rate	0.05
r	Interest rate	0
σ	CRRA parameter	1.5
l^{ss}	Leisure of searcher-searcher couples	0.3
l^{ws}	Leisure of worker-searcher couples	0.25
b	Unemployment income	1.1
y	Firm productivity	2

Table 3.1: Parameters for the equilibrium with homogeneous leisure

The resulting reservation wage for searcher-searcher couples is $R = 1.756$. The wage offer distribution has the mass point at $\hat{w} = 1.99$, with mass $\gamma = 0.194$. The upper bound of the continuous part of F is given by $\bar{w} = 1.946$. Hence we can check ex post that indeed $\hat{w} > R$.

9.35% of couples are searcher-searcher couples, 33.99% are worker-searcher couples earning less than $\hat{w}$, 40.84% are worker-searcher couples earning $\hat{w}$ and 15.82% are worker-worker couples (both earning $\hat{w}$).

Figure 3.4 gives the wage offer distribution, Figure 3.5 the earnings distribution of worker-searcher couples and Figures 3.6 and 3.7 give the value of a filled job $J(w)$ and the acceptance probability of a worker $h(w)$ respectively.

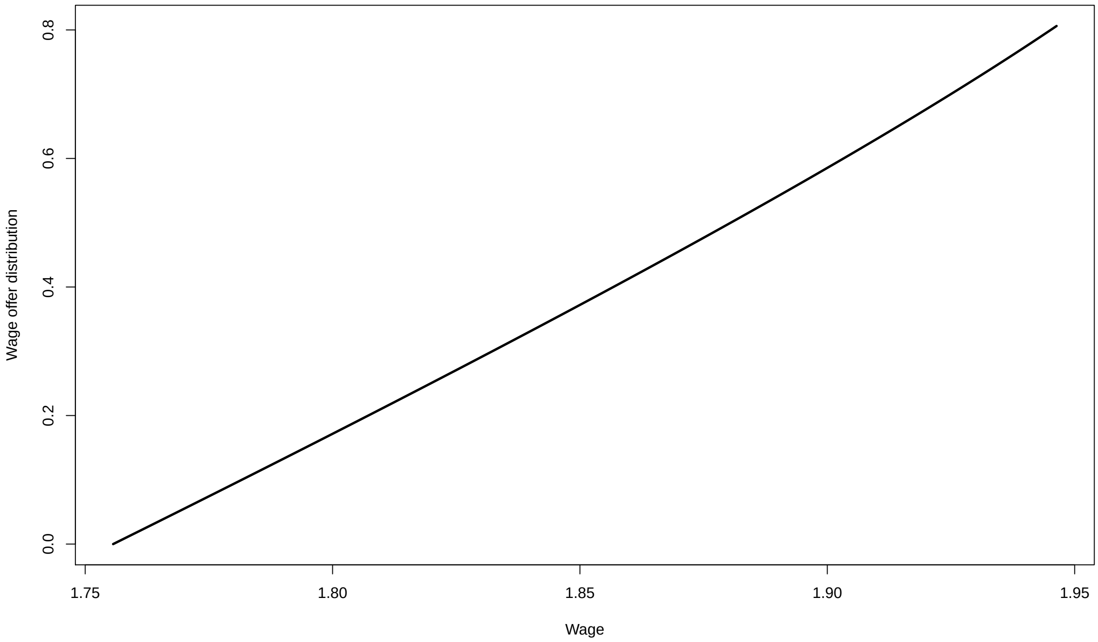

Figure 3.4: Wage offer distribution between R and $\bar{w}$

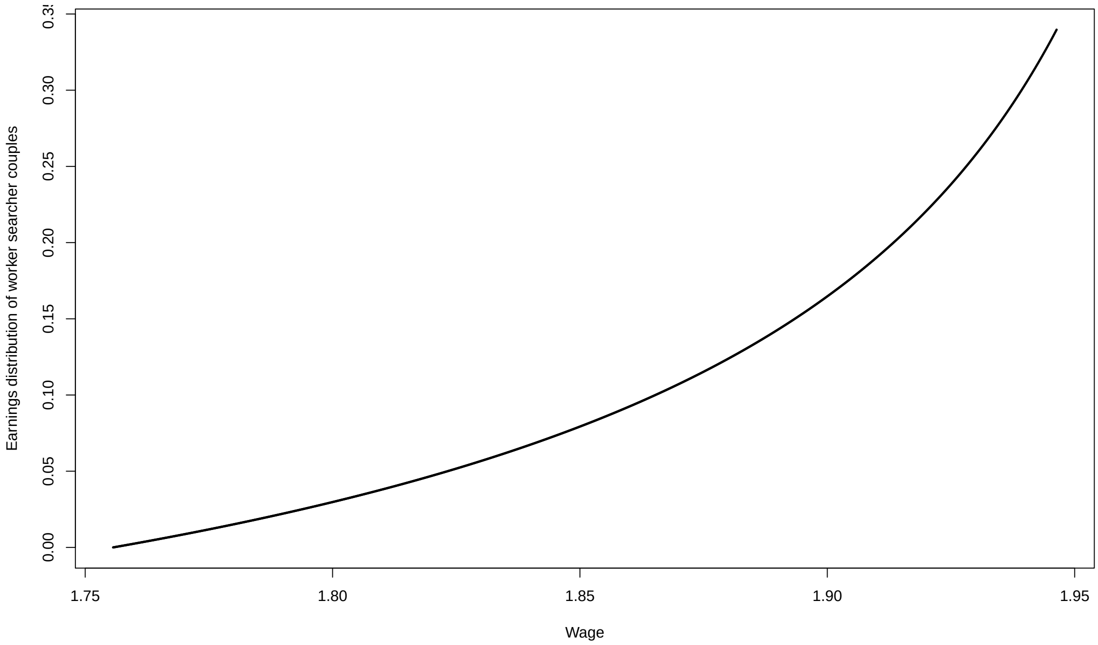

Figure 3.5: Earnings distribution of worker-searcher couples between R and $\bar{w}$

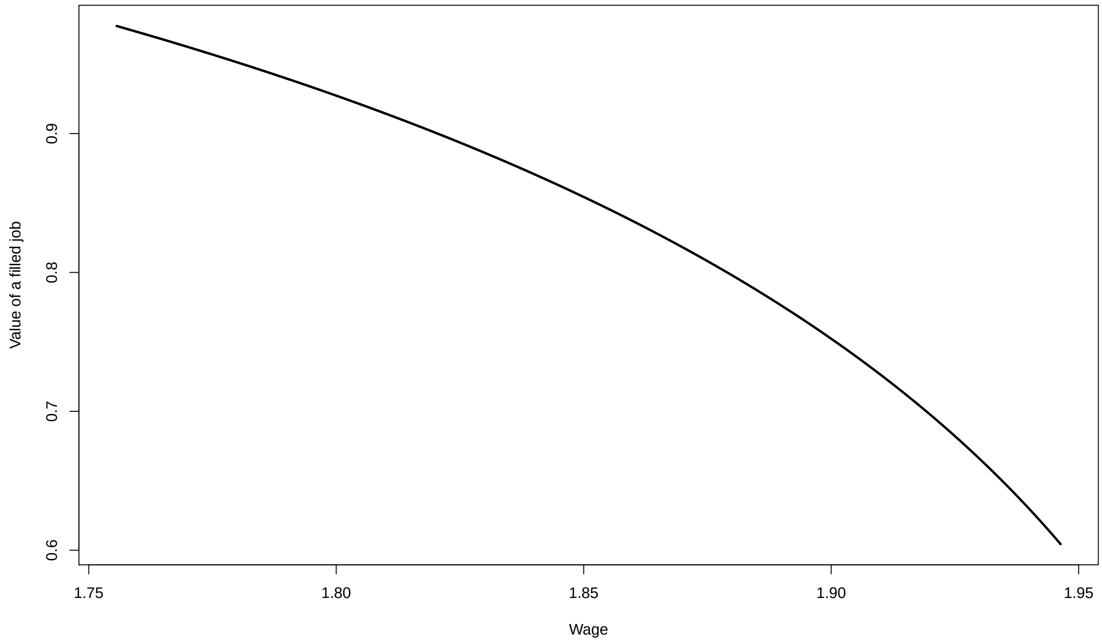

Figure 3.6: Value of a filled job between R and $\bar{w}$

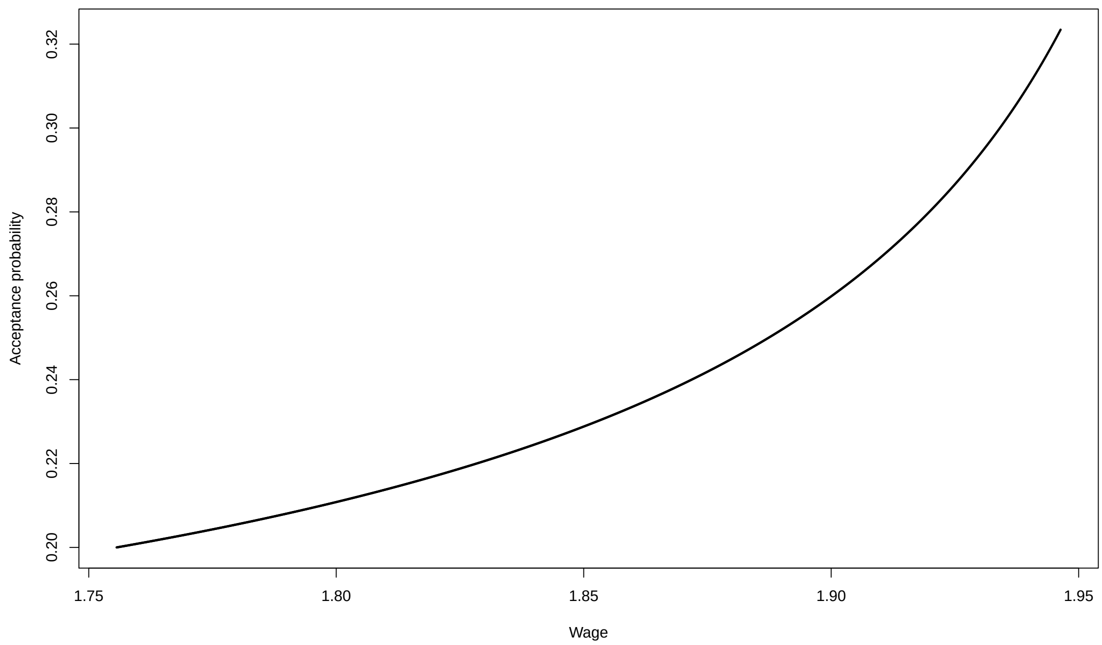

Figure 3.7: Acceptance probability of a worker between R and $\bar{w}$

The equilibrium in Proposition 4 can only occur if $y \in [y^0, y^\infty]$. Obviously, if the productivity is too low and even R cannot be offered with positive profit, then the equilibrium cannot exist. What is the intuition behind y^∞? If y is increased, then more and more mass is put on $\hat{w}$. This is because the higher the productivity, the less important is the wage and the more important becomes the acceptance probability (which is one for offering $\hat{w}$ and strictly smaller for all wages below). Hence at some point, only $\hat{w}$ will be offered. This is summarized in Proposition 5.

Proposition 5. *Consider the limiting case $r \to 0$. The following two equations determine $\hat{w}$ (equation (3.4)) and R (equation (3.5)):*

$$2\lambda(l^{ws} + u(\hat{w} + b)) + \delta(l^{ss} + u(2b)) = (2\lambda + \delta)u(2\hat{w}) \tag{3.4}$$

$$u(2\hat{w}) - u(R + b) = u(R + b) - u(2b) + 2l^{ws} - l^{ss}. \tag{3.5}$$

Suppose that $2l^{ws} - l^{ss}$ is large enough such that the solution to equations (3.4) and (3.5) fulfils $R < \hat{w}$. In this case, $\exists y^{\infty}$ such that $\forall y > y^{\infty}$, offering $\hat{w}$ characterized by the indifference condition $\Omega(\hat{w}) = T(\hat{w}, \hat{w})$ (equation (3.4)) is an equilibrium.

Proof. See appendix. □

For the parameter values in Table 3.1, we get the following results for the variables in Proposition 5: $y^{\infty} = 6.85, R = 2.60$ and $\hat{w} = 6.68$ (for the limiting case $r \to 0$). Hence for all firm productivities that are greater than 6.85, only $\hat{w} = 6.68$ is offered in equilibrium.

3.8 Leisure in the utility function: the heterogeneous case

In the previous section, I analyzed a model with homogeneous leisure, i.e. men and women had the same value for leisure, creating wage dispersion but the roles of men and women were symmetric (and hence their labor market outcomes). In this section, I am interested to see if gender differences in leisure can generate wage dispersion but also explain differences in wages between men and women.

3.8.1 Introduction and model setup

In this section, I discuss a model where the utility function of the couple is given by:

$$\begin{cases} u(w^M + w^F) & \text{for a worker-worker couple} \\ l^F + u(w^M + b) & \text{for a worker-searcher couple, man working} \\ l^M + u(b + w^F) & \text{for a worker-searcher couple, woman working} \\ l^M + l^F + u(2b) & \text{for a searcher-searcher couple} \end{cases}$$

where $w^M, w^F \sim F$, b is unemployment income (government transfers and home production) and l^M, l^F are the respective leisure values for men and women. Without loss of generality, I assume $l^M > l^F$. This utility function is more general than one might think at first glance. For the couple there are only four different values of leisure: both searching, man working, woman working or both working. Leisure is normalized to zero for working individuals. The utility function is linearly separable in leisure and consumption with a linear specification for leisure and a concave one for consumption. Compared to the previous section 3.7, I only consider here the special case of linear utility in leisure, i.e. where the joint utility of leisure is equal to the sum of the individual leisure values of the couple.

3.8.2 Analytical results

This section gives analytical results for two special cases: low and high firm productivity y. These cases can be characterized analytically since the equilibrium wage offer distribution is simple and only a few other candidate equilibria have to be ruled out. To illustrate an equilibrium in the intermediate range of y, I present a numerical example for the German economy in section 3.9.

Equilibrium for low firm productivity y

Obviously, if firms' productivity is too low (lower than the reservation wage of any agent in the model), then no wage will be offered since firms would make negative profits. It is also obvious that for a low level of productivity, only the reservation wage of women will be offered (since $l^F < l^M$, the reservation wage of women is lower than the reservation wage of men in searcher-searcher couples). As in Albrecht-Axell, there is also an interval in which only the reservation wage of men is offered. The results are summarized in the following proposition.

Proposition 6. *(i) $\exists \bar{y}^0, \bar{y}^1$ such that $\forall y \in [\bar{y}^0, \bar{y}^1]$ the equilibrium is characterized by a single wage, R^F, defined by:*

$$u(R^F + b) = l^F + u(2b)$$

and $\bar{y}^0 = R^F$.

(ii) $\exists \bar{y}^2, \bar{y}^3$, such that $\forall y \in [\bar{y}^2, \bar{y}^3]$ the equilibrium is characterized by a single wage, R^M, defined by:

$$u(R^M + b) = l^M + u(2b).$$

(iii) A sufficient condition for $\bar{y}^1 < \bar{y}^2$ is $\delta(r + \delta) < \lambda(\delta + 2\lambda)$.

Proof. See appendix. □

If only the reservation wage of women R^F is offered, then there are only searcher-searcher couples and worker-searcher couples in the model. Couples only change their employment state if the woman accepts the offered wage or loses her job.

If only the reservation wage of men R^M is offered, the dynamics are as follows: R^M is accepted by men and women in searcher-searcher couples, which then become worker-searcher couples. Once a woman has accepted a job (at R^M), a state change can only be due to an exogenous separation. In contrast, a couple where the man is working (at R^M) and the woman is searching changes in the breadwinner cycle sense whenever the woman draws the R^M wage. In this setting, no worker-worker couples may be observed.

There is not a standard Albrecht-Axell solution anymore because the reservation wage of the woman is higher when the man is working at R^M than if the man is searching. This is summarized in the following proposition.

Proposition 7. *In the model with heterogeneity in leisure, there cannot be an Albrecht-Axell equilibrium, i.e. an equilibrium where both the reservation wage of women and the reservation wage of men in searcher-searcher couples are offered, for $y \in [\bar{y}^1, \bar{y}^2]$.*

Proof. See appendix. □

The intuition is similar to why there was no Diamond [1971] solution in the case of homogeneous leisure (section 3.7). Suppose that firms offer R^M and $\tilde{R}^F$ where these two wages are defined as the reservation wages of respectively men and women in searcher-searcher couples. I denote $\tilde{R}^F$ the reservation wage of women in searcher-searcher couples given that R^M is offered with probability one. R^F denotes the reservation wage of these women given that only R^F is offered. Since the reservation wage is increasing in the offered wage, we have $\tilde{R}^F > R^F$. By offering $\tilde{R}^F$ firms attract all women in searcher-searcher couples, if offering R^M firms attract all women and all men from searcher-searcher couples. But can that be an equilibrium? Suppose that a firm offers $R^M + \varepsilon$. Then it also attracts men from worker-searcher couples where the woman is working at $\tilde{R}^F$ and hence starts the breadwinner cycle (the proof of Proposition 7 in the appendix shows that couples do not change to become a worker-worker couple). Put differently, offering $R^M + \varepsilon$ compared to R^M increases the acceptance probability discretely while increasing the wage continuously and therefore increases profit. Hence there cannot be an equilibrium where $\tilde{R}^F$ and R^M are offered.

Heterogeneity in unemployment income vs heterogeneity in leisure

To fully understand the difference between heterogeneity in unemployment income and heterogeneity in leisure, compare the value functions of the two cases. Some simple algebra gives the following expressions for the values of the different states for the case with heterogeneous unemployment income and zero leisure utility (i.e. when we have an Albrecht-Axell solution):

$$U = \frac{u(b^F + b^M)}{r} + \frac{\lambda(1-\gamma)}{r}\frac{u(b^M + w_2) - u(b^F + b^M)}{r + \delta + \lambda(1-\gamma)} \tag{3.6}$$

$$\Omega^M(w_2) = \frac{u(b^F + w_2)}{r} + \frac{\lambda(1-\gamma)}{r}\frac{u(2w_2) - u(b^F + w_2)}{r + \delta + \lambda(1-\gamma)} \tag{3.7}$$

$$\Omega^F(w_1) = \frac{u(w_1 + b^M)}{r} \tag{3.8}$$

$$T(w_1, w_2) = \frac{u(w_1 + w_2)}{r} \tag{3.9}$$

$$\Omega^F(w_2) = \frac{u(b^M + w_2)}{r} + \frac{\delta}{r}\frac{u(b^F + b^M) - u(b^M + w_2)}{r + \delta + \lambda(1-\gamma)} \tag{3.10}$$

$$T(w_2, w_2) = \frac{u(2w_2)}{r} + \frac{\delta}{r}\frac{u(b^F + w_2) - u(2w_2)}{r + \delta + \lambda(1-\gamma)}. \tag{3.11}$$

The first part of each line is the discounted value of the respective state. The second part gives the gain (or loss) of discounted utility associated with a state change (to a state with a different value, given the assumptions about the indifference). Comparing lines pairwise determines the reservation wage for a woman that is offered w_2 given that the couple is a searcher-searcher couple (lines (3.6) and (3.7)), a worker-searcher couple where the man is working at w_1 (lines (3.8) and (3.9)) and a worker-searcher couple where the man is working at w_2 (lines (3.10) and (3.11)). In lines (3.6) and (3.7) and in lines (3.10) and (3.11), the second part of the equation gives the gain of an added man at wage w_2 or the loss if the man's job at w_2 is destroyed. One can immediately see that the only wage fulfilling the conjectured indifference conditions is $w_2 = b^M$ (w_1 is then determined by the indifference condition $U = \Omega^F(w_1)$).

We can compare this to the case where there is heterogeneity in leisure. To simplify, I set $b = 0$. Denote all value functions with a bar. Suppose that we still have an equilibrium with two wages and that the indifference conditions hold as before, i.e.

$$\bar{U} = \bar{\Omega}^F(w_1) = \bar{\Omega}^M(w_2).$$

Then, we can write the value functions as:

$$\begin{aligned}
\bar{U} &= \frac{l^M + l^F}{r} + \frac{\lambda(1-\gamma)}{r}\frac{l^M + u(w_2) - l^M - l^F}{r + \delta + \lambda(1-\gamma)} \\
\bar{\Omega}^M(w_2) &= \frac{l^F + u(w_2)}{r} + \frac{\lambda(1-\gamma)}{r}\frac{u(2w_2) - l^F - u(w_2)}{r + \delta + \lambda(1-\gamma)} \\
\bar{\Omega}^F(w_1) &= \frac{u(w_1) + l^M}{r} \\
\bar{T}(w_1, w_2) &= \frac{u(w_1 + w_2)}{r} \\
\bar{\Omega}^F(w_2) &= \frac{l^M + u(w_2)}{r} + \frac{\delta}{r}\frac{l^F + l^M - l^M - u(w_2)}{r + \delta + \lambda(1-\gamma)} \\
\bar{T}(w_2, w_2) &= \frac{u(w_2 + w_2)}{r} + \frac{\delta}{r}\frac{l^F + u(w_2) - u(w_2 + w_2)}{r + \delta + \lambda(1-\gamma)}.
\end{aligned}$$

As before w_1 and w_2 are defined by the indifference conditions $\bar{U} = \bar{\Omega}^F(w_1)$ and $\bar{U} = \bar{\Omega}^M(w_2)$. One can then show the following relationships between the Bellmann equations (see the proof of Proposition 7):

$$\begin{aligned}
\bar{\Omega}^F(w_1) &> \bar{T}(w_1, w_2) \\
\bar{\Omega}^F(w_2) &> \bar{T}(w_2, w_2).
\end{aligned}$$

The main difference between heterogeneity in leisure and heterogeneity in unemployment income is that the reservation wage is increasing in the partner's wage even if only the two reservation wages of a searcher-searcher couple are offered. The two inequalities $\bar{\Omega}^F(w_1) > \bar{T}(w_1, w_2)$ and $\bar{\Omega}^F(w_2) > \bar{T}(w_2, w_2)$ show this dependence of the reservation wage on the partner's wage. A men in a searcher-searcher couple has a reservation wage of w_2 but once his wife earns w_1 or w_2, the man's reservation wage is higher than w_2. This means that in the case of heterogeneity in leisure, a couple in the worker-searcher state where the man is working (at w_2) will change in the breadwinner cycle sense when offered the wage w_1 for the woman (while in the heterogeneity in unemployment income case, they become a worker-worker couple). But this also means - as argued above - that firms have an incentive to deviate. The wage that firms have to offer to a couple where the man is working at w_2 is infinitesimally greater than w_1. By offering some wage $w_1 + \varepsilon$, firms could attract a positive mass of couples while increasing the wage continuously.

This shows that the solution to the model changes substantially if there is heterogeneity in leisure instead of heterogeneity in unemployment income.

Equilibrium for large firm productivity y

Another equilibrium can be analytically characterized if the firm's productivity is large. It is the analogous equilibrium to the one in Proposition 5 in the homogeneous leisure case. It is summarized in the next proposition:

Proposition 8. *$\exists y^\infty : \forall y > y^\infty$, only one wage R^{F2} is offered in equilibrium, characterized by:*

$$\begin{aligned}(r+\delta)(r+\delta+2\lambda)u(2R^{F2}) = \{&(r+2\delta)(r+\delta+\lambda)+\lambda r\}l^M \\ &+ \{r(r+\delta+2\lambda)+2\lambda\delta\}u(R^{F2}+b)+\delta(r+\delta)u(2b).\end{aligned}$$

This wage is solution to the indifference condition $\Omega^F(R^{F2}) = T(R^{F2}, R^{F2})$.

Proof. See appendix. □

In this equilibrium, every worker accepts the offered wage. Hence the dynamics can be described as follows: searcher-searcher couples accept the wage and become worker-searcher couples. Any worker-searcher couple accepts the wage for the second spouse and becomes a worker-worker couple.

To understand this equilibrium intuitively, one has to concentrate on the state-change of a worker-searcher couple where the woman is working at wage R^{F2} to the worker-worker state. Throughout the paper, I assume that leisure is lower for women than for men ($l^F < l^M$). As a direct consequence, a couple always prefers to be a worker-searcher couple where the woman is working to a worker-searcher couple where the man is working at the same wage. Put differently, the reservation wage of men in worker-searcher couples is higher than the reservation wage of women in worker-searcher couples. Thus the reservation of men in worker-searcher couples where the woman earns the highest offered wage is also the highest reservation wage in the model. Now consider only wage offer distributions with a single wage. If this wage is rather low, worker-searcher couples where the woman is working do not accept the second wage since the gain in income is not compensated by the (rather) huge loss in the man's leisure. But once this single wage offered by the firms is high enough, even worker-searcher couples where the woman is working are ready to accept the offered wage and become worker-worker couples. Once this threshold of productivity is passed, there is no possible other equilibrium any more. Firms offer the wage R^{F2} that is accepted by worker-searcher couples. That is to say that the wage is high enough to satisfy even the most choosy of all couples, i.e. the ones where the woman is working and the man is searching and enjoying leisure. The acceptance probability of a firm conditional on being matched to the worker is 1.

3.8.3 Equilibria without analytical characterization

In the previous sections, we have seen that there is a unique wage offered (a Diamond [1971] solution) in the case with zero utility of leisure and homogeneity of preferences. If there is heterogeneity in unemployment income (and still zero utility of leisure), we get an Albrecht and Axell [1984] solution, i.e. the reservation wages of both, men and women in searcher-searcher couples, are offered. Introducing leisure into the utility function shows that joint search can be similar to on-the-job search (as e.g. in Burdett and Mortensen [1998]). It also shows that it cannot be the case (except for a knife-edge constellation) that the reservation wage of a searcher-searcher couple is offered with probability 1.

In this section, with heterogeneity in leisure, I find that for some parameter values it can be the case again that only one reservation wage (of a searcher-searcher couple) is offered, either of men or of women. There might however be other equilibria. One can think of equilibria for example that are adapted to heterogeneity in leisure but are basically similar to the equilibrium in the case of homogeneous leisure, consisting of a continuous distribution and a mass point. These equilibria might occur whenever searcher-searcher couples prefer to switch in the breadwinner cycle sense and it is profitable for firms to offer wages slightly above the reservation wage (of searcher-searcher couples). However, numerically, there are also equilibria with a discrete distribution where multiple wages are offered. It depends strongly on the parameters if we find these equilibria. In the next section, I present one particular equilibrium of this type (consisting of two wages) to illustrate why we can find these equilibria with heterogeneous but not with homogeneous leisure.

3.9 Numerical example for Germany

In this section, I explore the model with heterogeneous leisure quantitatively with respect to gender differences in unemployment rates, marriage premia and mean wages. I choose an equilibrium with wage dispersion (in the intermediate range of firms' productivity y). In order to compare the marriage premium between men and women, I obviously need singles in the model. The introduction of singles in the model is discussed first and then I come to the choice of the parameter values.

Introducing singles

I assume here that a single has the same utility function as a couple, i.e.

$$u(w;l^i) = \begin{cases} l^i + u(b) & \text{if searching} \\ u(w) & \text{if working} \end{cases}$$

where $i = M, F$ and as before $l^M > l^F$. The utility functions of couples and singles then imply that there is no rivalry in consumption for couples. (This assumption is certainly a simplifying starting point, but the aim of introducing singles is only to show how the model can generate a positive marriage premium for men. For further insights, one certainly would have to be more specific about how utility functions differ between couples and singles.)

The utility function for singles gives the standard problem in search theory, characterized by a reservation wage strategy. Singles will accept every job offering more than the reservation wage and reject every wage below. I denote the two reservation wages by R_s^M for single men and R_s^F for single women (where $R_s^M > R_s^F$).

Parameter Values for monthly German data

The parameter of the CRRA utility function is chosen in a standard way as $\sigma = 1.5$. The period length is one month. The interest rate r (discount rate for couples and firms) is set to 0.005, i.e. around 6% on an annual basis.

I denote the proportion of singles in the population by η. According to the German statistics agency[4], singles constituted 27% of all working age adults and hence $\eta = 0.27$.

I choose gender specific values for job offer arrival rate and separation rate. The separation rates are taken from Azmat et al. [2006] and they are very similar for men and women: $\delta^M = \delta^F = 0.006$ per month. λ^M and λ^F are chosen in order to match the gender specific unemployment rates. According to the German Employment Agency, unemployment rates in Germany between 2000-2009 averaged 9.7% for men and 9.8% for women[5]. This implies $\lambda^M = 0.106$ and $\lambda^F = 0.056$, i.e. the job offer arrival rate for men has to be almost twice the one for women because low wage job offers are rejected by men in worker-searcher couples. This is because the higher reservation wage of men in worker-searcher couples and the fact that unemployment rates differ only slightly between men and women can only be reconciled if the job offer arrival rates differ substantially (in this particular equilibrium).

The other parameters (for the couple's preferences) are l^M and l^F, the respective values of leisure for men and women and b, the level of unemployment income. The numerical values attached to these are: $l^M = 0.25, l^F = 0.05, b = 0.55$. Due to discontinuities in the profit of the firm (see Figure 3.9 below), the model is difficult to calibrate. The strategy I apply here is to match roughly three quantities: the difference between reservation wages between men and women, the gender wage gap and the marriage premium for men and women. According to Pannenberg [2010], the reservation wage of men is between 6% and 16% higher than the reservation wage of women (where the reservation wage is a survey variable, the size of the effect depends on the specification of the econometric model, data is from the GSOEP). A similar result is from Brown et al. [2011] who find that the reservation wage of

[4]"Leben in Deutschland, Bevölkerung und Erwerbstätigkeit", 2008.

[5]Bundesagentur für Arbeit, Jahresberichte.

men is around 10% higher than for women in the UK. In this numerical example, I find that reservation wages of male singles are 9% higher than for single women and for couples that men in searcher-searcher couples have a 21% higher reservation wage than women in searcher-searcher couples. The conditional gender wage gap in Germany is around 12% (Hirsch et al. [2010]) using standard covariates in a Mincer-type earnings regression. The male marriage premium for Germany is around 9% (Barg and Beblo [2007][6]), for women the marriage premium is close to 0. In the model I find a marriage premium for men of 8.6% and 0 for women. The gender wage gap is 6.3% (0 for singles, 8.6% for couples). This validates the choice of the preference parameters insofar as the resulting predictions are not too far off reality. A closer calibration is not feasible due to the discrete nature of the model. Table 3.2 summarizes the parameter choices.

Parameter	Signification	Value	Source/Target
η	Proportion of singles	0.27	Destatis
λ^M, λ^F	Job offer arrival rate	0.106, 0.056	Target u^M, u^F
δ^M, δ^F	Separation rate	0.006	Azmat et al. (2006)
r	Interest rate	0.005	Yearly rate of 6%
σ	CRRA parameter	1.5	Standard
l^F	Leisure of women	0.05	Match gender gap in
l^M	Leisure of men	0.25	res. wages, mean wages,
b	Unemployment income	0.55	marriage premium for
y	Firm productivity	1.6	men and women

Table 3.2: Parameters chosen, numerical example for the German economy

[6] 9% is the marriage premium without taking selection into marriage into account. Barg and Beblo [2007] argue in a matching framework that around 4% are due to specialization in marriage and the other 5% are due to selection into marriage. Their paper only considers the first two years after marriage, hence it is not surprising that selection into marriage is that important (specialization or in my case, the dependence of the reservation wage on the partner's wage are more likely to matter more years into the marriage). I hence take the raw data of 9% as an upper bound.

Wage offer distribution, workers and firms

Given the above parameters (Table 3.2), two wages are offered, w_1 and w_2, characterized by the worker's indifference conditions:

$$U = \Omega^{M,\text{single}}(w_1)$$
$$\Omega^F(w_2) = T(w_2, w_2)$$

i.e. the lower wage is chosen such that single men are indifferent between working and searching[7] and the higher wage is the wage from Proposition 8. The numerical values for the wage offer distribution are:

$$w_1 = 1.20 \text{ (mass 0.55)} \quad \text{and} \quad w_2 = 1.40 \text{ (mass 0.45)}$$

Given this wage offer distribution, the behavior of the singles is easily described. Since women's reservation wages are always lower than men's, single women also accept w_1. Hence all singles accept all wages offered and switch from search unemployment to working upon meeting a firm. The dynamics of couples are more complex since the reservation wage increases in the partner's wage. Figure 3.8 depicts the dynamics for couples.

[7] The fact that an indifference condition of a single defines the equilibrium is of minor importance. With slightly different parameters, the equilibrium would be based on the wage w given by the indifference condition: $\Omega^M(w_2) = T(w, w_2)$.

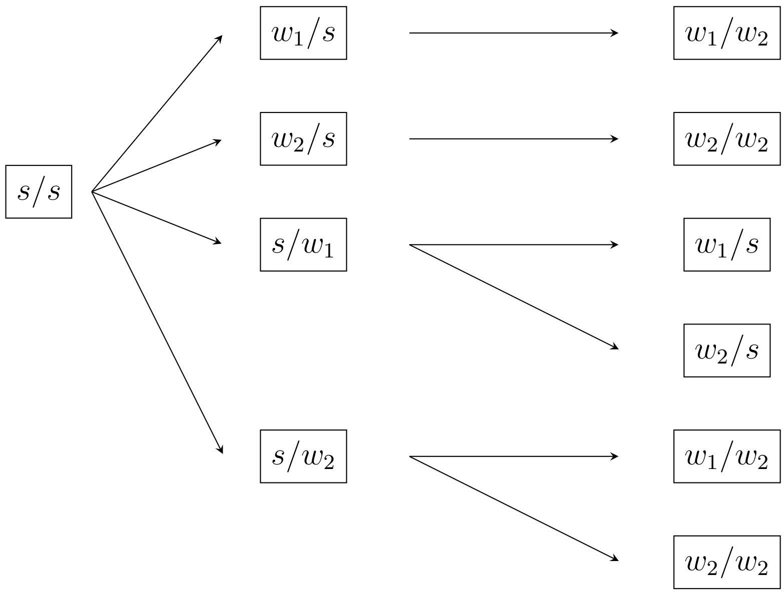

Figure 3.8: The dynamics for couples. Notation: w_1/s means that the woman is working at wage w_1 while the man is searching

The graph can be read as follows, from left to right. Searcher-searcher couples (s/s) accept all wages and upon being offered one of the wages they switch to one of the four different worker-searcher states. Worker-searcher couples where the woman is working at the lower wage (w_1/s) reject w_1 for the man but accept w_2 in which case the couple becomes a worker-worker couple. In the next line, worker-searcher couples (woman working at w_2) accept only w_2 for the man. A couple where the husband is working at the lower wage (s/w_1) always changes in the breadwinner cycle sense if offered a job for the woman (because of the high leisure value of the man). From the s/w_2 state, the couple chooses to become a worker-worker couple if offered w_1 or w_2.

Turning to the firm's side, it is easiest to describe the strategy by having a look at the firm's profit (Figure 3.9). The two vertical lines give the two offered wages. To the left of w_1, on can see at around $y = 0.95$ the reservation wage of women in searcher-searcher couples R^F. At around $y = 1.1$, there is the reservation wage of single women R_s^F. The reservation wage of men in searcher-searcher couples (R^M) is around 1.15. At w_1, there is not only R_s^F but also w_1^c such that $\Omega^M(w_2) = T(w_1^c, w_2)$.

Between w_1 and w_2, there are two different wages that are reservation wages of worker-searcher couples, both around 1.37. The left one of these two - w_2^c -

is given by the indifference condition: $\Omega^M(w_1) = \Omega^F(w_2^c)$, the right one w_3^c - by $\Omega^F(w_1) = T(w_1, w_3^c)$.

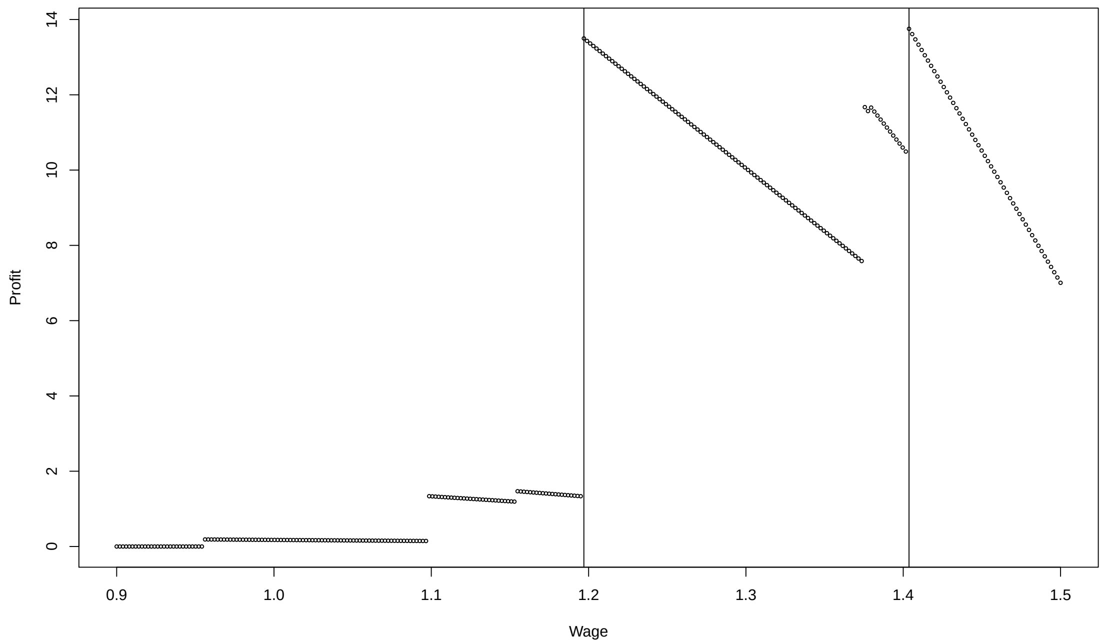

Figure 3.9: Firms' profit as a function of offered wages; the profit is calculated given that all other firms follow the equilibrium strategy (offering w_1 and w_2 as defined above).

Distribution of couples over the different labor market states

Given these dynamics, one can calculate the numerical values for all equilibrium quantities. The statistics for male singles reflect the wage offer distribution and the rather high job offer arrival rate: 5.3% are unemployed, 51.5% work at w_1 and 43.2% work at w_2. Female singles have a higher unemployment rate of 9.7% due to the lower job offer arrival rate of women. The distribution over the two wages is analogous to men (working at w_1: 49.1%, working at w_2: 41.2%).

For couples we get the following distribution over the different states (in percent):

Both searching	Woman working		Man working		Both working	
0.6	10.6		9.2		79.6	
s/s	w_1/s	w_2/s	s/w_1	s/w_2	w_1/w_2	w_2/w_2
0.6	5.7	4.9	0.6	8.7	43.0	36.6

One can see that there are very few couples in which both members are unemployed and very few couples where the man is working at the lower wage (s/w_1). Both states are left immediately once a new job offer arrives. Furthermore, in single earner households where only the man is working, the man almost always works at the high wage while in female single earner couples the lower wage is more likely. The majority of double earner couples consists of a low earning women and a high earning man. Nonetheless, a sizeable proportion of the population (36.6%) consists of double earner couples where both spouses earn the high wage w_2.

3.10 Conclusion

This paper studied a wage posting, random search equilibrium model for couples under different assumptions on the couple's preferences. It has been shown that without utility of leisure and with homogeneous preferences, a Diamond solution arises. Under heterogeneous unemployment income (and still zero utility of leisure), an Albrecht-Axell solution emerges. With leisure in the utility function, there is a wage dispersion equilibrium where the wage offer distribution has a continuous part and a mass point. Assuming heterogeneity in leisure between the two spouses can again give discrete wage offer distributions (with or without wage dispersion). The numerical example shows that a model with heterogeneity in leisure may explain the male marriage premium (of around 9%) and around half of the gender wage gap (12% for Germany, model predicts 6%).

The model in itself is not very tractable, hence further analytical extensions seem to be extremely tedious (one could for example think of letting unemployment income be dependent on the wage of the spouse). Instead, it would be interesting to explore a different type of matching model. In a directed search model, one could for example analyze the application decision of both spouses: most probably the trade-off between low risk/low wage and high risk/high wage firms would be different for couples than for singles (under risk aversion).

Appendix

Proof of Proposition 1. Suppose firms offer one wage w. Then, the Bellman equations of the couple are:

$$\begin{aligned} rU &= u(2b) + 2\lambda(\Omega(w) - U) \\ rT(w,w) &= u(2w) + 2\delta(\Omega(w) - T(w,w)) \\ r\Omega(w) &= u(w+b) + \lambda \max\{T(w,w) - \Omega(w), 0\} + \delta(U - \Omega(w)). \end{aligned}$$

Conjecture that $U = \Omega(w) = T(w,w)$. Then,

$$\begin{aligned} rU &= u(2b) \\ rT(w,w) &= u(2w) \\ r\Omega(w) &= u(w+b). \end{aligned}$$

It is obvious that $w = b$ makes workers indifferent between all states (and shows that the conjectured relationship $U = T(w,w) = \Omega(w)$ is correct). In this equilibrium searcher-searcher as well as worker-searcher couples accept the offered wage of b. There is no incentive for firms to increase the wage since the acceptance probability of a randomly matched worker is 1.

□

Proof of Proposition 2. Suppose that men and women have different values for unemployment income ($b^F < b^M$). Suppose that firms offer two wages: $w_1 < w_2$, w_1 with probability γ, w_2 with $1-\gamma$. First I conjecture an equilibrium and the corresponding dynamics, then solve the firm's problem and derive the wages (corresponding to the conjecture). Then I prove that the conjectured dynamics are correct given the wages derived under the conjecture.

Suppose that the following relationships between the different value functions hold (which determines the dynamics for the couples):

$$U = \Omega^F(w_1) = \Omega^M(w_2) = T(w_1, w_2) < \Omega^F(w_2) = T(w_2, w_2). \tag{3.12}$$

Suppose that the breadwinner cycle never emerges, i.e. a worker-searcher couple that is offered a job (paying a high enough wage) for the second spouse will always accept the second job without quitting the first one. Then, the system of value functions simplifies to (omitting the states that are never chosen):

$$rU = u(b^F + b^M) + \lambda(1-\gamma)(\Omega^F(w_2) - U)$$

$$
\begin{aligned}
rT(w_1, w_2) &= u(w_1 + w_2) \\
rT(w_2, w_2) &= u(w_2 + w_2) + \delta(\Omega^M(w_2) - T(w_2, w_2)) \\
r\Omega^F(w_1) &= u(w_1 + b^M) \\
r\Omega^F(w_2) &= u(w_2 + b^M) + \delta(U - \Omega^F(w_2)) \\
r\Omega^M(w_2) &= u(w_2 + b^F) + \lambda(1-\gamma)(T(w_2, w_2) - \Omega^M(w_2)).
\end{aligned}
$$

The Bellman equations are for all possible states: searcher-searcher (U), worker-worker ($T(w_1, w_2)$ and $T(w_1, w_2)$) or worker-searcher ($\Omega^F(w_1), \Omega^F(w_2), \Omega^M(w_2)$). The two wages correspond to the reservation wages of the two spouses, w_1 will be the reservation wage of women and w_2 the reservation wage of men (in a searcher-searcher couple). The only difference in value between the states is when the woman gets the high wage (w_2, the reservation wage of men). Solving this system of equations gives:

$$
\begin{aligned}
U &= \frac{u(b^F + b^M)}{r} + \frac{\lambda(1-\gamma)}{r}\frac{u(b^M + w_2) - u(b^F + b^M)}{r + \delta + \lambda(1-\gamma)} \\
\Omega^M(w_2) &= \frac{u(b^F + w_2)}{r} + \frac{\lambda(1-\gamma)}{r}\frac{u(2w_2) - u(b^F + w_2)}{r + \delta + \lambda(1-\gamma)} \\
\Omega^F(w_1) &= \frac{u(w_1 + b^M)}{r} \\
T(w_1, w_2) &= \frac{u(w_1 + w_2)}{r} \\
\Omega^F(w_2) &= \frac{u(b^M + w_2)}{r} + \frac{\delta}{r}\frac{u(b^F + b^M) - u(b^M + w_2)}{r + \delta + \lambda(1-\gamma)} \\
T(w_2, w_2) &= \frac{u(2w_2)}{r} + \frac{\delta}{r}\frac{u(b^F + w_2) - u(2w_2)}{r + \delta + \lambda(1-\gamma)}.
\end{aligned}
$$

From $\Omega^F(w_1) = T(w_1, w_2)$ it is obvious that $w_2 = b^M$. $U = \Omega^F(w_1)$ gives the equation that determines w_1:

$$
(r + \delta + \lambda(1-\gamma))u(w_1 + b^M) = (r + \delta)u(b^M + b^F) + \lambda(1-\gamma)u(2b). \tag{3.13}
$$

Now we still have to check that the assertions about the relationship of the different value functions in equation (3.12) hold. From the system of equations above, it is obvious - by the definition of $w_2 = b^M$ - that $U = \Omega^M(w_2) = \Omega^F(w_1) = T(w_1, w_2)$ and that $\Omega^F(w_2) = T(w_2, w_2)$. From the definition of w_1, we know that $U = \Omega^F(w_1)$. By the fact that $\Omega^{F\prime} > 0$, $\Omega^{M\prime} > 0$ and $T_1' > 0, T_2' > 0$ hold, the assertions $\Omega^F(w_1) < \Omega^F(w_2), \Omega^M(w_1) < U, \Omega^F(w_1) > T(w_1, w_1)$ and $\Omega^F(w_2) > T(w_2, w_2)$ follow. To see why the derivatives of the value functions are strictly positive, one can refer to the proof of Guler et al. [2009] (Proof of Lemma 1, p. 38).

Firms In this setting, it is obvious that firms only offer at most two wages, i.e. that there is no incentive to deviate: when offering w_2 firms attract every worker, so offering $w_2 + \varepsilon$ reduces profits without increasing the inflow of workers. If offering $w_2 - \varepsilon$ firms will no longer attract the mass of male workers, so this cannot be optimal either. Obviously no firm will offer a wage below w_1 because no worker would accept this. Offering $w_1 + \varepsilon$ does not attract more workers than offering w_1, so firms do not deviate to this either.

Now I show that the model has exactly the Albrecht-Axell solution. To calculate the equilibrium value of γ, first note that the flows between the different states can be written as follows, where u denotes the mass of searcher-searcher couples, e^{1M} the mass of worker-searcher couples (man working), e^{1F} the mass of worker-searcher couples (woman working) and e^2 the mass of worker-worker couples:

$$\begin{aligned}
\lambda u + \lambda(1-\gamma)u &= \delta e^{1M} + \delta e^{1F} \\
\delta e^{1F} + \lambda(1-\gamma)e^{1F} &= \lambda u + \delta e^2 \\
\delta e^{1M} + \lambda e^{1M} &= \lambda(1-\gamma)u + \delta e^2 \\
u + e^{1M} + e^{1F} + e^2 &= 1.
\end{aligned}$$

This can be solved as

$$\begin{aligned}
e^2 &= \frac{\lambda^2(1-\gamma)}{\delta^2 + \lambda(1-\gamma)\delta + \lambda\delta + \lambda^2(1-\gamma)} \\
u &= \frac{\delta^2}{\delta^2 + \lambda(1-\gamma)\delta + \lambda\delta + \lambda^2(1-\gamma)} \\
e^{1F} &= \frac{\lambda\delta}{\delta^2 + \lambda(1-\gamma)\delta + \lambda\delta + \lambda^2(1-\gamma)} \\
e^{1M} &= \frac{\lambda(1-\gamma)\delta}{\delta^2 + \lambda(1-\gamma)\delta + \lambda\delta + \lambda^2(1-\gamma)}
\end{aligned}$$

and hence:

$$2u + e^{1F} + e^{1M} = \frac{2\delta^2 + \lambda(1-\gamma)\delta + \lambda\delta}{\delta^2 + \lambda(1-\gamma)\delta + \lambda\delta + \lambda^2(1-\gamma)}.$$

Finally, the acceptance probabilities for offering w_1 is (for w_2 it is of course 1):

$$\tilde{u} := \frac{u + e^{1F}}{2u + e^{1M} + e^{1F}} = \frac{\delta^2 + \lambda(1-\gamma)\delta}{2\delta^2 + \lambda(1-\gamma)\delta + \lambda\delta} = \frac{\delta + \lambda(1-\gamma)}{2\delta + \lambda(2-\gamma)}.$$

Then the expected discounted profit for offering w_1 and w_2 are:

$$\Pi(w_1) = \alpha\tilde{u}\frac{y - w_1}{r + \delta}$$

$$\Pi(w_2) = \alpha \frac{y - w_2}{r + \delta}$$

and we get exactly the Albrecht and Axell [1984] result (where both types make up half of the population). γ is then determined by:

$$\Pi(w_1) = \Pi(w_2).$$

The Albrecht and Axell [1984] model is standard in the literature, for the solution of the model see for example the Rogerson et al. [2005] survey (for the case of linear utility). There, one can also find an explicit solution for γ.

□

Proof of Proposition 3. Suppose firms offer only one wage, the reservation wage of searcher-searcher couples R. Then, the Bellman equations are:

$$\begin{aligned} rU &= l^{ss} + u(2b) + 2\lambda(\Omega(w) - U) \\ rT(w,w) &= u(2w) + 2\delta(\Omega(w) - T(w,w)) \\ r\Omega(w) &= l^{ws} + u(w+b) + \lambda \max\{T(w,w) - \Omega(w), 0\} + \delta(U - \Omega(w)). \end{aligned}$$

R is characterized by $U = \Omega(R)$. Now two cases have to be considered, either $T(R,R) \leq \Omega(R)$ or $T(R,R) > \Omega(R)$.

Conjecture first that $T(R,R) \leq \Omega(R)$. The Bellman equations then simplify to:

$$\begin{aligned} rU &= l^{ss} + u(2b) \\ r\Omega(R) &= l^{ws} + u(R+b) \\ rT(R,R) &= u(2R) + 2\delta(\Omega(R) - T(R,R)). \end{aligned}$$

Hence the wage R is characterized by $l^{ss} + u(2b) = l^{ws} + u(R+b)$. Furthermore we get:

$$\begin{aligned} T(R,R) &= \frac{u(2R) + 2\delta\Omega(R)}{r + 2\delta} = \frac{u(2R) + 2\delta\frac{u(R+b)}{r}}{r + 2\delta} \\ &= \frac{u(2R) + 2\delta\frac{u(R+b)+l^{ws}}{r}}{r + 2\delta} = \frac{ru(2R) + 2\delta u(R+b) + 2\delta l^{ws}}{r(r + 2\delta)}. \end{aligned}$$

Thus the initial conjecture $T(R,R) \leq \Omega(R)$ is correct iff

$$u(2R) \leq l^{ws} + u(R+b). \tag{3.14}$$

The last equation (3.14) shows that there is exactly one parameter constellation for which offering R is an equilibrium, i.e. if

$$u(2R) = l^{ws} + u(R+b)$$

Now it is easy to see why if l^{ss}, l^{ws} are such that we have "<" in equation (3.14), offering R cannot be an equilibrium anymore. In this case, couples do not change to become a worker-worker couple when offered R. That means that offering R cannot be an equilibrium, since there is a possible deviation for firms: by offering $R+\varepsilon$, firms can attract a positive mass of workers (all workers whose spouse is currently working at R in addition to the unemployed), by only paying infinitesimally more.

The second possibility is that we have $T(R,R) > \Omega(R)$. The Bellman equations then are:

$$r\Omega(R) = l^{ws} + u(R+b) + \lambda(T(R,R) - \Omega(R))$$
$$rT(R,R) = u(2R) + 2\delta(\Omega(R) - T(R,R))$$

and we get that the conjecture $\Omega(R) < T(R,R)$ is correct iff

$$\frac{l^{ws} + u(R+b)}{r+\lambda} < \frac{r}{r+\lambda}\frac{(r+\lambda)u(2R) + 2\delta(l^{ws} + u(R+b))}{r(r+2\delta+\lambda)}$$
$$\Leftrightarrow \quad (r+2\delta+\lambda)(l^{ws} + u(R+b)) < (r+\lambda)u(2R) + 2\delta(l^{ws} + u(R+b))$$
$$\Leftrightarrow \quad l^{ws} + u(R+b) < u(2R).$$

It is also easy to see that offering R alone cannot be an equilibrium in this case. Since $\Omega(R) < T(R,R)$ firms could offer a wage $R^- < R$, characterized by $\Omega(R) = T(R^-, R)$ and would attract all worker-searcher couples.

Therefore there is a knife-edge solution: offering R with mass 1 is an equilibrium if and only if:

$$u(2R) = l^{ws} + u(R+b).$$

□

Proof of Proposition 4. Suppose F is such as in Proposition 4. Start with the firm's profit. Denote $h(w)$ the acceptance probability and $J(w)$ the value of a filled job. Then the expected profit is:

$$\pi(w) = h(w)J(w)$$

where

$$rJ(w) = y - w - \{\delta + \lambda[1 - F(w)]\}J(w)$$

and hence

$$J(w) = \frac{y - w}{r + \delta + \lambda[1 - F(w)]}$$

since the flow probability with which a person leaves is given by the exogenous separation rate δ and the endogenous quit probability $\lambda[1 - F(w)]$ (the probability that a job offer for the spouse arrives and that the offered wage is higher than the current wage of the employed spouse). If offering $\hat{w}$, the probability that a worker quits endogenously is 0, since all wages lower than $\hat{w}$ are rejected by the searching spouse and if offered $\hat{w}$, the couple becomes a worker-worker couple.

Now determine the earnings distribution. In the steady state, $E(w)$, the mass of worker-searcher couples earning less than w is constant and given by, $\forall w \in [R, \bar{w}]$:

$$2\lambda F(w)u = E(w)[\delta + \lambda(1 - F(w))]$$

i.e. the flows into the state from unemployment have to be equal to the flows out of the state, either by exogenous separation or by a change in the breadwinner cycle sense. Note that changes in the breadwinner cycle sense where one partner earned less than w before and after the change are comprised in $E(w)$. Hence,

$$E(w) = \frac{2\lambda F(w)u}{\delta + \lambda(1 - F(w))}.$$

In order to calculate the flows between the different states, denote u the mass of unemployed couples, e^1 the mass of worker-searcher couples earning less than $\hat{w}$, $\hat{e}^1$ the mass of worker-searcher couples earning $\hat{w}$ and e^2 the mass of worker-worker couples (both earning $\hat{w}$). Then the flows between the different states are given by (inflow = outflow):

$$\begin{aligned}
\delta e^1 + \delta \hat{e}^1 &= 2\lambda u \\
2\lambda(1 - \gamma)u &= (\lambda\gamma + \delta)e^1 \\
2\lambda\gamma u + \lambda\gamma e^1 + 2\delta e^2 &= (\lambda\gamma + \delta)\hat{e}^1 \\
\lambda\gamma\hat{e}^1 &= 2\delta e^2.
\end{aligned}$$

Using,

$$\begin{aligned}
e^2 &= \frac{\lambda\gamma}{2\delta}\hat{e}^1 \\
u &= \frac{\delta}{2\lambda}e^1 + \frac{\delta}{2\lambda}\hat{e}^1
\end{aligned}$$

$$e^1 = \frac{(1-\gamma)\delta}{(\lambda+\delta)\gamma}\hat{e}^1$$

and the fact that $u + e^1 + \hat{e}^1 + e^2 = 1$, we get:

$$\frac{\lambda\gamma}{2\delta} + \frac{\delta}{2\lambda}\frac{(1-\gamma)\delta}{(\lambda+\delta)\gamma} + \frac{\delta}{2\lambda} + \frac{(1-\gamma)\delta}{(\lambda+\delta)\gamma} + 1 = 1/\hat{e}^1.$$

$u, \hat{e}^1, e^2$ can then be computed as:

$$\begin{aligned}
\hat{e}^1 &= 2\lambda\delta(\lambda+\delta)\gamma N \\
e^1 &= 2\lambda\delta^2(1-\gamma)N \\
e^2 &= \lambda^2(\lambda+\delta)\gamma^2 N \\
u &= \delta^2(\delta+\lambda\gamma)N
\end{aligned}$$

where

$$\begin{aligned}
N &= \frac{1}{\lambda^2(\lambda+\delta)\gamma^2 + (1-\gamma)\delta^3 + \delta^2(\lambda+\delta)\gamma + 2(1-\gamma)\delta^2\lambda + 2\lambda\delta(\lambda+\delta)\gamma} \\
&= \frac{1}{(\lambda+\delta)^3 - \lambda(1-\gamma)[\lambda(\lambda+\delta)(\gamma+1)+\delta^2]}.
\end{aligned}$$

We can also compute:

$$2u + e^1 + \hat{e}^1 = 2\delta(\lambda+\delta)(\lambda\gamma+\delta)N$$

and hence:

$$\frac{2u}{2u+e^1+\hat{e}^1} = \frac{\delta}{\delta+\lambda}.$$

Now we can calculate the acceptance probabilities of a worker that is randomly matched to a firm, $\forall w < \hat{w}$:

$$\begin{aligned}
h(w) &= \frac{2u + e^1 E(w)}{2u+e^1+\hat{e}^1} = \frac{2u + e^1\frac{2\lambda F(w)u}{\lambda(1-F(w))+\delta}}{2u+e^1+\hat{e}^1} \\
&= 2u\frac{\lambda(1-F(w)) + \delta + \lambda F(w)e^1}{(2u+e^1+\hat{e}^1)(\lambda(1-F(w))+\delta)} \\
&= \delta\frac{\lambda(1-F(w)) + \delta + 2F(w)\lambda^2\delta^2(1-\gamma)N}{(\lambda+\delta)(\lambda(1-F(w))+\delta)}
\end{aligned}$$

and $h(\hat{w}) = 1$. Then, we get that the profit for offering R is:

$$\pi(R) = h(R)J(R) = \frac{\delta}{\delta+\lambda}\frac{y-R}{r+\delta+\lambda}.$$

The profit of offering an intermediate wage $w \in (R, \bar{w}]$ is

$$\begin{aligned}\pi(w) &= h(w)J(w)\\ &= \delta\frac{\lambda(1-F(w))+\delta+2F(w)\lambda^2\delta^2(1-\gamma)N}{(\lambda+\delta)(\lambda(1-F(w))+\delta)}\frac{y-w}{r+\delta+\lambda[1-F(w)]}.\end{aligned}$$

Now setting $\pi(R) = \pi(w)$, we get:

$$\frac{y-R}{r+\delta+\lambda} = \frac{\lambda(1-F(w))+\delta+2F(w)\lambda^2\delta^2(1-\gamma)N}{\lambda(1-F(w))+\delta}\frac{y-w}{r+\delta+\lambda[1-F(w)]}$$

and hence

$$\begin{aligned}&[y-R][\delta+\lambda(1-F(w))][r+\delta+\lambda[1-F(w)]]\\ &\quad= [r+\delta+\lambda][\lambda(1-F(w))+\delta+2F(w)\lambda^2\delta^2(1-\gamma)N][y-w].\end{aligned}$$

Thus we get a quadratic equation in $F(w)$ (again for $w \in [R, \bar{w}]$):

$$aF(w)^2 + bF(w) + c = 0$$

where

$$\begin{aligned}a &= \lambda^2\\ b &= -[(r+\delta+\lambda)\lambda+(\delta+\lambda)\lambda)] - \frac{y-w}{y-R}[r+\delta+\lambda]\lambda\{2\lambda\delta^2(1-\gamma)N-1\}\\ c &= [r+\delta+\lambda][\lambda+\delta]\left(1-\frac{y-w}{y-R}\right).\end{aligned}$$

The limiting case is $r \to 0$ which simplifies the wage offer distribution to the solution of a quadratic equation with coefficients:

$$\begin{aligned}a &= \lambda^2\\ b &= -2\lambda(\delta+\lambda)\left[1+\{\lambda\delta^2(1-\gamma)N-1/2\}\frac{y-w}{y-R}\right]\\ c &= [\lambda+\delta]^2\left(1-\frac{y-w}{y-R}\right)\end{aligned}$$

and hence:

$$\begin{aligned}F_1(w) = \frac{\delta+\lambda}{\lambda}\Bigg\{&\left[1+\{\lambda\delta^2(1-\gamma)N-1/2\}\frac{y-w}{y-R}\right]\\ &-\sqrt{\left[1+\{\lambda\delta^2(1-\gamma)N-1/2\}\frac{y-w}{y-R}\right]^2-\left(1-\frac{y-w}{y-R}\right)}\Bigg\}.\end{aligned}$$

$\bar{w}$ as a function of R is then characterized by $F(\bar{w}) = 1 - \gamma$.

Now calculate the profit of offering the double indifference wage $\hat{w}$. Since every worker accepts and no worker leaves endogenously, we get:

$$\pi(\hat{w}) = h(\hat{w})J(\hat{w}) = \frac{y - \hat{w}}{r + \delta}.$$

Going back to the workers' side, the Bellman equations are

$$rU = l^{ss} + u(2b) + 2\lambda \int_R \Omega(w) - U dF(w) \tag{3.15}$$

$$r\Omega(w) = l^{ws} + u(w + b) + \delta(U - \Omega(w)) + \lambda \int_w \Omega(w') - \Omega(w) dF(w'). \tag{3.16}$$

R is defined by $\Omega(R) = U$. Hence we get:

$$u(R + b) + l^{ws} - l^{ss} - u(2b) = \lambda \int_R \Omega(w) - U dF(w).$$

Using partial integration, we get:

$$\begin{aligned}
\int_R \Omega(w) - U dF(w) &= \int_R^{\bar{w}} \Omega(w) - U dF(w) + \gamma[\Omega(\hat{w}) - U] \\
&= [\Omega(w) - U]_R^{\bar{w}} - \int_R^{\bar{w}} \Omega'(w) F(w) dw + \gamma[\Omega(\hat{w}) - U] \\
&= [\Omega(\bar{w}) - \Omega(R)] - \int_R^{\bar{w}} \Omega'(w) F(w) dw + \gamma[\Omega(\hat{w}) - U] \\
&= \int_R^{\bar{w}} \Omega'(w)(1 - F(w)) dw + \gamma[\Omega(\hat{w}) - U].
\end{aligned}$$

Differentiation of equation (3.16) yields:

$$\begin{aligned}
r\Omega'(w) &= u'(w + b) - \delta\Omega'(w) - \lambda \int_w \Omega'(w) dF(w') \\
\Rightarrow \qquad \Omega'(w) &= \frac{u'(w + b)}{r + \delta + \lambda(1 - F(w))}.
\end{aligned}$$

Thus R as a function of γ and $\hat{w}$ is determined by (again for $r = 0$):

$$u(R + b) + l^{ws} - l^{ss} - u(2b) = \lambda \int_R^{\bar{w}} \frac{u'(w + b)(1 - F(w))}{\delta + \lambda(1 - F(w))} dw + \lambda\gamma[\Omega(\hat{w}) - U].$$

Define

$$I = \int_R^{\bar{w}} \frac{u'(w + b)(1 - F(w))}{\delta + \lambda(1 - F(w))} dw.$$

Then we get:

$$\Omega(\hat{w}) - U = \frac{l^{ws} + u(\hat{w} + b) + \delta U}{r + \delta} - U$$

$$
\begin{aligned}
&= \frac{l^{ws} + u(\hat{w}+b)}{r+\delta} - \frac{r}{r+\delta}U \\
&= \frac{l^{ws} + u(\hat{w}+b)}{r+\delta} - \frac{1}{r+\delta}[l^{ss} + u(2b) + 2\lambda \int_R \Omega(w) - UdF(w)] \\
&= \frac{l^{ws} + u(\hat{w}+b)}{r+\delta} - \frac{1}{r+\delta}[l^{ss} + u(2b) + 2\lambda^2[I + \gamma[\Omega(\hat{w}) - U]]] \\
&= \frac{l^{ws} + u(\hat{w}+b)}{r+\delta} - \frac{1}{r+\delta}[l^{ss} + u(2b) + 2\lambda^2 I] - \frac{1}{r+\delta}2\lambda^2\gamma[\Omega(\hat{w}) - U].
\end{aligned}
$$

Hence,

$$
\Omega(\hat{w}) - U = \frac{l^{ws} + u(\hat{w}+b) - [l^{ss} + u(2b) + 2\lambda^2 I]}{2\lambda^2\gamma + r + \delta}.. \tag{3.17}
$$

Hence R is determined by

$$
u(R+b) + l^{ws} - l^{ss} - u(2b) = \lambda \int_R^{\bar{w}} \frac{u'(w+b)(1-F(w))}{\delta + \lambda(1-F(w))} dw + \lambda\gamma[\Omega(\hat{w}) - U]
$$

where $\Omega(\hat{w}) - U$ is determined by equation (3.17). The Bellman equations at $\hat{w}$ (characterized by $T(\hat{w},\hat{w}) = \Omega(\hat{w})$) are:

$$
\begin{aligned}
rT(\hat{w},\hat{w}) &= u(2\hat{w}) \\
r\Omega(\hat{w}) &= l^{ws} + u(\hat{w}+b) + \delta(U - \Omega(\hat{w})).
\end{aligned}
$$

Hence we have that $\hat{w}$ is characterized by:

$$
u(2\hat{w}) = l^{ws} + u(\hat{w}+b) + \delta(U - \Omega(\hat{w}))
$$

where $\Omega(\hat{w}) - U$ is determined by equation (3.17).

Finally, γ is determined by:

$$
\begin{aligned}
&\pi(\hat{w}) = \pi(R) \\
\Leftrightarrow \qquad &\frac{y - \hat{w}}{r+\delta} = \frac{\delta}{\delta+\lambda}\frac{y-R}{r+\delta+\lambda}.
\end{aligned}
$$

This equilibrium is only feasible if the firm's productivity is neither too low nor too high, i.e. $y \in [y^0, y^\infty]$. The lower bound y^0 is obvious: if firms cannot make a strictly positive profit by offering the reservation wage of searcher-searcher couples, this equilibrium cannot exist. The upper bound y^∞ is characterized in Proposition 5.

□

Proof of Proposition 5. Suppose that $\hat{w}$ (characterized by $T(\hat{w},\hat{w}) = \Omega(\hat{w})$) is offered with mass 1. The Bellman equations are:

$$\begin{aligned} rU &= l^{ss} + u(2b) + 2\lambda(\Omega(\hat{w}) - U) \\ rT(\hat{w},\hat{w}) &= u(2\hat{w}) \\ r\Omega(\hat{w}) &= l^{ws} + u(\hat{w}+b) + \delta(U - \Omega(\hat{w})) \\ r\Omega(R) &= l^{ws} + u(R+b) + \lambda(\Omega(\hat{w}) - \Omega(R)). \end{aligned}$$

Hence,

$$\begin{aligned} U &= \frac{l^{ss} + u(2b) + 2\lambda\Omega(\hat{w})}{r+2\lambda} \\ r\Omega(\hat{w}) &= l^{ws} + u(\hat{w}+b) + \delta(\frac{l^{ss} + u(2b) + 2\lambda\Omega(\hat{w})}{r+2\lambda} - \Omega(\hat{w})). \end{aligned}$$

And then:

$$\Omega(\hat{w}) = \frac{1}{r(r+2\lambda+\delta)}[(r+2\lambda)(l^{ws} + u(\hat{w}+b)) + \delta(l^{ss} + u(2b))].$$

Furthermore,

$$\Omega(R) = \frac{l^{ws} + u(R+b) + \lambda\Omega(\hat{w})}{r+\lambda}.$$

Then one can determine the wage $\hat{w}$ using the indifference condition $T(\hat{w},\hat{w}) = \Omega(\hat{w})$:

$$\begin{aligned} & \frac{1}{r(r+2\lambda+\delta)}[(r+2\lambda)(l^{ws} + u(\hat{w}+b)) + \delta(l^{ss} + u(2b))] = \frac{u(2\hat{w}))}{r} \\ \Rightarrow \quad & (r+2\lambda)(l^{ws} + u(\hat{w}+b)) + \delta(l^{ss} + u(2b)) = (r+2\lambda+\delta)u(2\hat{w}). \end{aligned}$$

For given $\hat{w}$, one can solve for R, using $U = \Omega(R)$:

$$\begin{aligned} & \frac{l^{ss} + u(2b) + 2\lambda\Omega(\hat{w})}{r+2\lambda} = \Omega(R) \\ \Rightarrow \quad & \frac{l^{ss} + u(2b) + 2\lambda\Omega(\hat{w})}{r+2\lambda} = \frac{l^{ws} + u(R+b) + \lambda\Omega(\hat{w})}{r+\lambda} \\ \Rightarrow \quad & \frac{\lambda r\Omega(\hat{w})}{(r+2\lambda)(r+\lambda)} = \frac{l^{ws} + u(R+b)}{r+\lambda} - \frac{l^{ss} + u(2b)}{r+2\lambda} \\ \Rightarrow \quad & \frac{\lambda r[(r+2\lambda)(l^{ws} + u(\hat{w}+b)) + \delta(l^{ss} + u(2b))]}{r(r+2\lambda+\delta)(r+2\lambda)(r+\lambda)} = \frac{l^{ws} + u(R+b)}{r+\lambda} - \frac{l^{ss} + u(2b)}{r+2\lambda} \\ \Rightarrow \quad & \frac{\lambda r[(r+2\lambda)(l^{ws} + u(\hat{w}+b)) + \delta(l^{ss} + u(2b))]}{r(r+2\lambda+\delta)} \end{aligned}$$

$$= [l^{ws} + u(R+b)](r+2\lambda) - [l^{ss} + u(2b)](r+\lambda)$$

$$\Rightarrow \quad \lambda(r+2\lambda)(l^{ws} + u(\hat{w}+b)) + \lambda\delta(l^{ss} + u(2b)) + (r+\lambda)(r+2\lambda+\delta)[l^{ss} + u(2b)]$$
$$= (r+2\lambda)(r+2\lambda+\delta)[l^{ws} + u(R+b)].$$

Going to the limit $r \to 0$, we get a system of two equations that determines R and $\hat{w}$:

$$2\lambda(l^{ws} + u(\hat{w}+b)) + \delta(l^{ss} + u(2b)) = (2\lambda+\delta)u(2\hat{w})$$
$$2\lambda(l^{ws} + u(\hat{w}+b)) + \delta(l^{ss} + u(2b)) = (2\lambda+\delta)[2l^{ws} + 2u(R+b) - l^{ss} - u(2b)].$$

Hence we get an equation that determines whether $R < \hat{w}$:

$$u(2\hat{w}) - u(R+b) = u(R+b) - u(2b) + 2l^{ws} - l^{ss}.$$

From this equation, one can see that as long as $2l^{ws} - l^{ss} \geq 0$, we must have $\hat{w} > R$. However if $2l^{ws} - l^{ss}$ becomes large and negative, there might be situations in which $\hat{w} > R$ is not fulfilled anymore.

The profit of offering both wages is

$$\pi(R) = \frac{\delta}{\delta+\lambda}\frac{y-R}{r+\delta+\lambda}$$
$$\pi(\hat{w}) = \frac{y-\hat{w}}{r+\delta}.$$

As y grows large and under the assumption that $\hat{w} > R$, there must be a y^{∞} such that $\pi(R) = \pi(\hat{w})$. For all $y > y^{\infty}$, deviating to R cannot be optimal. Offering a higher wage is not a possible deviation either since no worker leaves and all workers accept at $\hat{w}$ (which cannot be improved by increasing the wage). Furthermore, deviating to a lower wage other than R cannot be optimal either. The reservation wage of all worker-searcher couples is $\hat{w}$ (given that only $\hat{w}$ is offered). Hence, by reducing the wage, firms could only attract searcher-searcher couples which they could also attract by offering only R. Thus offering $\hat{w}$ with probability 1 is an equilibrium $\forall y > y^{\infty}$.

□

Proof of Proposition 6. (i). Suppose that y is lower than some threshold, i.e. $y < \bar{y}^1$ and that only R^F is offered. Only women accept jobs, only worker-searcher couples exist. Conjecture that

$$U = \Omega^F(R^F) > \Omega^M(R^F), T(R^F, R^F). \tag{3.18}$$

Suppose that the conjectured behavior of the couples is correct given the offered wage. The reservation wage of women is:

$$\begin{aligned} & U = \Omega^F(R^F) \\ \Rightarrow \qquad & l^F + u(2b) = u(R^F + b). \end{aligned}$$

To show that the conjecture in equation (3.18) about the relationship is correct, calculate the two other values of states:

$$\begin{aligned} \Omega^M(R^F) &= \frac{1}{r(r+\delta+\lambda)}[r(l^F + u(R^F+b)) + (\lambda+\delta)(l^M + u(R^F+b))] \\ &< \Omega^F(R^F) = \frac{l^M + u(R^F+b)}{r} \\ T(R^F, R^F) &= \frac{u(2R^F)}{r+\delta} + \frac{\delta}{r+\delta}\frac{1}{r(r+\delta+\lambda)}[r(l^F + u(R^F+b)) + (\lambda+\delta)(l^M + u(R^F+b))] \\ &< \Omega^F(R^F). \end{aligned}$$

The first inequality is obvious, the second one can be shown by using the concavity of u by noting that:

$$\begin{aligned} & u(R^F + b) = l^F + u(2b) \\ \Rightarrow \qquad & u(2R^F) < l^F + u(R^F + b) \\ \Rightarrow \qquad & u(2R^F) < l^M + u(R^F + b). \end{aligned}$$

The reservation wage of men is strictly greater than R^F. The profit of firms when offering R^F is strictly positive as long as productivity is strictly greater than the offered wage, i..e. as long as $y > R^F$. Thus there exists $\bar{y}^1$ such that for $R^F \leq y < \bar{y}^1$ only R^F is offered. Firms deviating to the reservation wage of men in this interval of y would either make a negative profit or a lower profit then offering R^F and hence only R^F is offered.

One can calculate the profit if offering the reservation wage of women. First, the acceptance probability of a randomly matched worker is:

$$\frac{u}{2u + e^{1M}} = \frac{\delta}{2\delta + \lambda}.$$

Then, firms' profit is

$$\Pi(R^F) = \alpha \frac{\delta}{2\delta + \lambda} \frac{y - R^F}{r + \delta}.$$

(ii). Suppose that for $\bar{y}^2 < y < \bar{y}^3$ only R^M is offered. The relevant relationships for the Bellman equations are:

$$U = \Omega^M(R^M) < \Omega^F(R^M) \tag{3.19}$$

$$T(R^M, R^M) < \Omega^F(R^M). \tag{3.20}$$

Firms only offer the reservation wage of men in searcher-searcher couples and the following Bellman equations hold:

$$\begin{aligned} rU &= l^F + l^M + u(2b) + \lambda(\Omega^F(R^M) - U) \\ r\Omega^F(R^M) &= l^M + u(R^M + b) + \delta(U - \Omega^F(R^M)) \\ r\Omega^M(R^M) &= l^F + u(R^M + b) + \lambda(\Omega^F(R^M) - \Omega^M(R^M)) \end{aligned}$$

and hence:

$$\begin{aligned} \Omega^M(R^M) &= \frac{l^F + u(R^M + b) + \lambda\Omega^F(R^M)}{r + \lambda} \\ U &= \frac{l^F + l^M + u(2b) + \lambda\Omega^F(R^M)}{r + \lambda} \\ r\Omega^F(R^M) &= l^M + u(R^M + b) + \delta\left(\frac{l^F + l^M + u(2b) + \lambda\Omega^F(R^M)}{r + \lambda} - \Omega^F(R^M)\right) \end{aligned}$$

and hence

$$\begin{aligned} \Omega^M(R^M) &= \frac{l^F + u(R^M + b) + \lambda\Omega^F(R^M)}{r + \lambda} \\ U &= \frac{l^F + l^M + u(2b) + \lambda\Omega^F(R^M)}{r + \lambda} \\ \Omega^F(R^M) &= \frac{r + \lambda}{r(r + \lambda + \delta)}\left(l^M + u(R^M + b) + \delta\frac{l^F + l^M + u(2b)}{r + \lambda}\right) \end{aligned}$$

and R^M is determined by: $\Omega^M(R^M) = U$, i.e.

$$\begin{aligned} \frac{l^F + u(R^M + b) + \lambda\Omega^F(R^M)}{r + \lambda} &= \frac{l^F + l^M + u(2b) + \lambda\Omega^F(R^M)}{r + \lambda} \\ \Rightarrow u(R^M + b) &= l^M + u(2b). \end{aligned}$$

Now we still have to prove that the conjecture in equations (3.19) and (3.20) about the relationship between the different Bellman equations is correct. The first proof of the assertion that $\Omega^F(R^M) > \Omega^M(R^M)$ is obvious due to the higher leisure values of men. What remains to be shown is that $\Omega^F(R^M) > T(R^M, R^M)$, i.e. that a couple that is offered R^M for the wife switches in the breadwinner cycle sense and does not become a worker-worker couple. To show this note first that

$$\begin{aligned} T(R^M, R^M) &= \frac{u(2R^M) + \delta\Omega^M(R^M) + \delta\Omega^F(R^M)}{r + 2\delta} \\ &= \frac{u(2R^M) + \delta\frac{l^F + u(R^M+b) + \lambda\Omega^F(R^M)}{r+\lambda} + \delta\Omega^F(R^M)}{r + 2\delta}. \end{aligned}$$

Hence

$$\begin{aligned} & \Omega^F(R^M) > T(R^M, R^M) \\ \Leftrightarrow \quad & \left(1 - \frac{\delta}{r + 2\delta} - \frac{\delta\lambda}{(r + 2\delta)(r + \lambda)}\right)\Omega^F(R^M) > \frac{u(2R^M)}{r + 2\delta} + \frac{\delta(l^F + u(R^M + b))}{(r + 2\delta)(r + \lambda)} \\ \Leftrightarrow \quad & r(r + \lambda + \delta)\Omega^F(R^M) > (r + \lambda)u(2R^M) + \delta(l^F + u(R^M + b)). \end{aligned}$$

Plugging in for $\Omega^F(R^M)$ and using $u(R^M + b) = l^M + u(2b)$ yields:

$$\begin{aligned} & (r + \lambda)\left(l^M + u(R^M + b) + \delta\frac{l^F + l^M + u(2b)}{r + \lambda}\right) \\ & \qquad > (r + \lambda)u(2R^M) + \delta(l^F + u(R^M + b)) \\ \Leftrightarrow \quad & (r + \lambda)(l^M + u(R^M + b)) + \delta(l^F + l^M + u(2b))) \\ & \qquad > (r + \lambda)u(2R^M) + \delta(l^F + u(R^M + b)) \\ \Leftrightarrow \quad & l^M + u(R^M + b) > u(2R^M). \end{aligned}$$

The equation defining R^M is: $l^M + u(b + b) = u(R^M + b)$ from which by strict concavity of u and the fact that $R^M > b$ immediately follows that indeed we have: $l^M + u(R^M + b) > u(R^M + R^M)$.

Now calculate the value of a vacancy if offering the reservation wage of men R^M. First, the flows are characterized by:

$$\begin{aligned} u + e^{1M} + e^{1F} &= 1 \\ 2\lambda u &= \delta e^{1M} + \delta e^{1F} \\ \delta e^{1F} &= \lambda u + \lambda e^{1M} \\ \delta e^{1M} + \lambda e^{1M} &= \lambda u \end{aligned}$$

which gives:

$$u = \frac{\delta}{2\lambda + \delta}$$

$$e^{1M} = \frac{\lambda}{\lambda+\delta}\frac{\delta}{2\lambda+\delta}$$
$$e^{1F} = \frac{\lambda}{\lambda+\delta}.$$

Hence the acceptance probabilities are:

$$\text{Men:} \quad \frac{u}{2u+e^{1M}+e^{1F}} = \frac{\delta}{2(\lambda+\delta)}$$
$$\text{Women:} \quad \frac{u+e^{1F}}{2u+e^{1M}+e^{1F}} = \frac{(2\lambda+\delta)\delta}{2(\lambda+\delta)^2}.$$

One can now calculate the profit of the firm when offering R^M:

$$\Pi(R^M) = \frac{\alpha}{2}\frac{\delta}{\lambda+\delta}\frac{y-R^M}{r+\delta+\lambda} + \frac{\alpha}{2}\frac{(2\lambda+\delta)\delta}{(\lambda+\delta)^2}\frac{y-R^M}{r+\delta}.$$

The first part of the equation concerns men. The expected profit for hiring a man consists of the flow probability with which the firm meets a worker, times the acceptance probability of a man times the profit of having a male worker to the firm, over the employment spell. The second part is the analogous expression for hiring a woman. The two differences are the acceptance probabilities (women are more likely to accept) and the discounting of the flow profit (men are more likely to leave that is why there is an additional λ in the denominator). The expression above can be simplified to:

$$\Pi(R^M) = \frac{\alpha\delta}{2(\lambda+\delta)}(y-R^M)\frac{(\lambda+\delta)(r+\delta)+(2\lambda+\delta)(r+\delta+\lambda)}{(r+\delta+\lambda)(\lambda+\delta)(r+\delta)}.$$

In (iii) I derive the thresholds $\bar{y}^1$ and $\bar{y}^2$. In order to characterize the threshold $\bar{y}^3$, one has to determine when deviating from offering only R^M becomes profitable. Here, I only sketch the proof. There are two possible candidate wages w^{c1} and w^{c2}, the two indifference conditions being:

$$\Omega^F(R^M) = T(R^M, w^{c1}) \quad \text{or} \quad \Omega^F(R^M) = \Omega^M(w^{c2}),$$

i.e. the man of a worker-searcher couple accepts a new job at a higher wage and the woman either quits her job or stays at her job. Why no other wages? Assume that R^M is offered with probability one. Then there are two types of worker-searcher couples, either the man or the woman working at R^M. We know that the reservation wage of a woman in a worker-searcher couple where the man is working (at R^M) is $\tilde{R}^F$. Hence the relevant threshold $\bar{y}^3$ is determined by an indifference for men, either for a change to the worker-searcher state (the woman quits her job, the man accepts a new one) or to the worker-worker state (the woman keeps her job as the man accepts a new one). We have the following two additional Bellman equations:

$$rT(R^M,w) = u(w+R^M) + \delta(\Omega^M(w) - T(R^M,w)) + \delta(\Omega^F(R^M) - T(R^M,w))$$
$$r\Omega^M(w) = l^F + u(w+b) + \delta(U - \Omega^M(w)) + \lambda(\Omega^F(R^M) - \Omega^M(w)).$$

Next, one has to calculate the wages w^{c1} and w^{c2} from the above indifference conditions and calculate the profits for firms. By equating the profits of each of the two candidate wages with the profit of offering R^M, one determines $\bar{y}^3_{c1}$ and $\bar{y}^3_{c2}$, the thresholds for y given that either w^{c1} or w^{c2} are offered. Then $\bar{y}^3 = \min\{\bar{y}^3_{c1}, \bar{y}^3_{c2}\}$.

(iii). In order to derive the thresholds for the R^F and the R^M equilibria ($\bar{y}^1$ and $\bar{y}^2$), suppose that firms only offer R^F. The question is which is the lowest value for y for which deviation becomes profitable. For that suppose that one firm sets a negligible mass on $R^M + \varepsilon$, where R^M is defined as above. By offering R^M - the reservation wage of men in searcher-searcher couples - firms attract all women and all men from searcher-searcher couples (since $R^M > R^F$). If offering $R^M + \varepsilon$, firms also attract men from worker-searcher couples (where the woman is working at R^F) and hence start the breadwinner cycle. Hence the equilibrium of R^F becomes unstable if y is greater than $\bar{y}^1$ which is defined by:

$$\alpha\frac{\delta}{2\delta+\lambda}\frac{\bar{y}^1 - R^F}{r+\delta} = \alpha\frac{\bar{y}^1 - R^M}{r+\delta}.$$

On the left hand side, there is the profit of offering R^F if all firms offer only this wage. On the right hand side, there is the profit of offering R^M. Here, the acceptance probability is 1. There are only searcher-searcher and worker-searcher couples where the woman is working. For all men and women in searcher-searcher couples, R^M is sufficient. For worker-searcher couples, firms have to offer a little bit more than the indifference to make them change in the breadwinner cycle sense. Furthermore, no worker leaves voluntarily. This is true because, if all others offer R^F, the probability of finding a higher wage is 0.

On the other hand, assume that firms offer R^M with mass 1. Now the question is when firms will deviate to offering the reservation wage of women. This reservation wage will attract all women in searcher-searcher and worker-searcher couples. Denote this wage by $\tilde{R}^F$ which is characterized by the indifference condition:

$$\Omega^F(\tilde{R}^F) = U,$$

where

$$r\Omega^F(\tilde{R}^F) = l^F + u(\tilde{R}^F + b).$$

Using this and the equation for U from above, one finds that the wage that has to be offered to women is:

$$\tilde{R}^F = u^{-1}\left(\frac{(r+\delta)l^F + \lambda l^M}{r+\delta+\lambda} + u(2b)\right) - b.$$

Then the threshold $\bar{y}^2$ is defined by:

$$\alpha\frac{\delta}{2(\delta+\lambda)}\frac{\bar{y}^2 - R^M}{r+\delta+\lambda} + \alpha\frac{\delta(\delta+2\lambda)}{2(\delta+\lambda)^2}\frac{\bar{y}^2 - R^M}{r+\delta} = \alpha\frac{\delta(\delta+2\lambda)}{2(\delta+\lambda)^2}\frac{\bar{y}^2 - \tilde{R}^F}{r+\delta}.$$

On the left hand side, we have the profit of offering R^M (given that all firms offer only R^M) as argued above. On the right hand side, there is the profit of offering the reservation wage of women $\tilde{R}^F + \varepsilon$. This wage is accepted by women in searcher-searcher couples and by women in worker-searcher couples where the man is working (at R^M). Since couples are indifferent between the searcher-searcher state and the worker-searcher state where the man is working at R^M, the reservation wages of women in worker-searcher and searcher-searcher couples coincide. Only women accept $\tilde{R}^F + \varepsilon$ and they will never voluntarily quit their job.

In the following I show that $\bar{y}^1 < \bar{y}^2$. First, one can calculate:

$$\bar{y}^1 = \frac{2\delta+\lambda}{\delta+\lambda}\left[R^M - \frac{\delta}{2\delta+\lambda}R^F\right]$$
$$\bar{y}^2 = R^M + \frac{(\delta+2\lambda)(r+\delta+\lambda)}{(\delta+\lambda)(r+\delta)}(R^M - \tilde{R}^F).$$

Hence $\bar{y}^1 < \bar{y}^2$ is equivalent to:

$$\lambda(2\lambda + 3\delta + 2r)R^M + \delta(r+\delta)R^F > (r+\delta+\lambda)(2\lambda+\delta)\tilde{R}^F.$$

Hence,

$$u^{-1}\left(\frac{(r+\delta)l^F + \lambda l^M}{r+\delta+\lambda} + u(2b)\right) < \theta u^{-1}(l^F + u(2b)) + (1-\theta)u^{-1}(l^M + u(2b))$$

where

$$\theta = \frac{\lambda(2\lambda+3\delta+2r)}{(r+\delta+\lambda)(2\lambda+\delta)}.$$

By strict convexity of u^{-1}, we have:

$$u^{-1}\left(\frac{(r+\delta)l^F + \lambda l^M}{r+\delta+\lambda} + u(2b)\right) < \frac{r+\delta}{r+\delta+\lambda}u^{-1}(l^F + u(2b)) + \frac{\lambda}{r+\delta+\lambda}u^{-1}(l^M + u(2b)).$$

Hence a sufficient condition for $\bar{y}^1 < \bar{y}^2$ is

$$\begin{aligned} \frac{r+\delta}{r+\delta+\lambda} &< \frac{\lambda(2\lambda+3\delta+2r)}{(r+\delta+\lambda)(2\lambda+\delta)} \\ \Rightarrow \qquad \delta(r+\delta) &< \lambda(\delta+2\lambda) \end{aligned}$$

which will be fulfilled in most applications since in general $\lambda > \delta$ and $\lambda > r$.

□

Proof of Proposition 7. Suppose that both R^F and R^M are offered, with probability γ and $1-\gamma$ respectively. Men and women accept jobs out of unemployment. The conjectures for the relevant relationships for the Bellman equations are:

$$U = \Omega^M(R^M) = \Omega^F(R^F) \tag{3.21}$$

$$T(R^F, R^M) < \Omega^F(R^F) = \Omega^M(R^M) < T(R^M, R^M) < \Omega^F(R^M). \tag{3.22}$$

Then, the Bellman equations simplify to:

$$\begin{aligned} rU &= l^F + l^M + u(2b) + (1-\gamma)\lambda(\Omega^F(R^M) - U) \\ r\Omega^F(R^F) &= l^M + u(R^F + b) \\ r\Omega^M(R^M) &= l^F + u(R^M + b) + \lambda(1-\gamma)(\Omega^F(R^M) - \Omega^M(R^M)) \\ r\Omega^F(R^M) &= l^M + u(R^M + b) + \delta(U - \Omega^F(R^M)) \end{aligned}$$

with indifference conditions $U = \Omega^M(R^M) = \Omega^F(R^F)$. Hence:

$$\begin{aligned} \Omega^F(R^M) &= \frac{l^M + u(R^M+b) + \delta U}{r+\delta} \\ U &= \frac{r+\delta}{r(r+\delta)+r(1-\gamma)\lambda}\left[l^F + l^M + u(2b) + (1-\gamma)\lambda\frac{l^M + u(R^M+b)}{r+\delta}\right] \end{aligned}$$

which gives:

$$\begin{aligned} \Omega^F(R^M) = {} & \frac{l^M + u(R^M+b)}{r+\delta} \\ & + \frac{\delta}{r(r+\delta)+r(1-\gamma)\lambda}\left[l^F + l^M + u(2b) + (1-\gamma)\lambda\frac{l^M + u(R^M+b)}{r+\delta}\right] \end{aligned}$$

and

$$\Omega^F(R^F) = \frac{l^M + u(R^F+b)}{r}$$

$$\Omega^M(R^M) = \frac{l^F + u(R^M + b) + \lambda(1-\gamma)\Omega^F(R^M)}{r + \lambda(1-\gamma)}.$$

Hence $\Omega^M(R^M) = U$ determines R^M by:

$$\begin{aligned} & \frac{l^F + u(R^M + b) + \lambda(1-\gamma)\Omega^F(R^M)}{r + \lambda(1-\gamma)} = U \\ \Rightarrow \quad & (l^F + u(R^M + b))(r+\delta) + \lambda(1-\gamma)(l^M + u(R^M + b)) \\ & \qquad = (r+\delta)[l^F + l^M + u(2b)] + (1-\gamma)\lambda(l^M + u(R^M + b)) \\ \Rightarrow \quad & u(R^M + b) = l^M + u(2b). \end{aligned}$$

R^F is then determined by $\Omega^F(R^F) = U$:

$$\begin{aligned} & (l^M + u(R^F + b))((r+\delta) + \lambda(1-\gamma)) \\ & \qquad = (r+\delta)[l^F + l^M + u(2b)] + (1-\gamma)\lambda(l^M + u(R^M + b)) \\ \Rightarrow \quad & ((r+\delta) + \lambda(1-\gamma))u(R^F + b) \\ & \qquad = (r+\delta)[l^F + u(2b)] + (1-\gamma)\lambda u(R^M + b). \end{aligned}$$

Now we still have to prove that the conjectured dynamics in equations (3.21) and (3.22) are correct given the wages.

Proof of assertion $\Omega^F(R^M) > T(R^M, R^M)$. From above:

$$\begin{aligned} \Omega^F(R^M) = & \frac{l^M + u(R^M + b)}{r+\delta} \\ & + \frac{\delta}{r(r+\delta) + r(1-\gamma)\lambda}\left[l^F + l^M + u(2b) + (1-\gamma)\lambda\frac{l^M + u(R^M + b)}{r+\delta}\right] \end{aligned}$$

and

$$T(R^M, R^M) = \frac{u(2R^M) + \delta\Omega^M(R^M) + \delta\Omega^F(R^M)}{r + 2\delta}$$

where

$$\Omega^M(R^M) = \frac{l^F + u(R^M + b) + \lambda(1-\gamma)\Omega^F(R^M)}{r + \lambda(1-\gamma)}.$$

Hence,

$$\begin{aligned} & \Omega^F(R^M) > T(R^M, R^M) \\ \Leftrightarrow \quad & \left(1 - \frac{\delta}{r+2\delta} - \frac{\delta\lambda(1-\gamma)}{(r+2\delta)(r+\lambda(1-\gamma))}\right)\Omega^F(R^M) > \frac{u(2R^M)}{r+2\delta} + \frac{\delta(l^F + u(R^M + b)))}{(r+2\delta)(r+\lambda(1-\gamma)} \\ \Leftrightarrow \quad & r(r + \lambda(1-\gamma) + \delta)\Omega^F(R^M) > (r + \lambda(1-\gamma))u(2R^M) + \delta(l^F + u(R^M + b)). \end{aligned}$$

Plugging in for $\Omega^F(R^M)$ and using $u(R^M+b) = l^M + u(2b)$ yields:

$$
\begin{aligned}
& r(r+\lambda(1-\gamma)+\delta)\frac{l^M+u(R^M+b)}{r+\delta} + \delta\left[l^F + l^M + u(2b) + (1-\gamma)\lambda\frac{l^M+u(R^M+b)}{r+\delta}\right] \\
& \qquad > (r+\lambda(1-\gamma))u(2R^M) + \delta(l^F + u(R^M+b)) \\
\Leftrightarrow \quad & r(r+\lambda(1-\gamma)+\delta)\frac{l^M+u(R^M+b)}{r+\delta} + \delta\left[(1-\gamma)\lambda\frac{l^M+u(R^M+b)}{r+\delta}\right] \\
& \qquad > (r+\lambda(1-\gamma))u(2R^M) \\
\Leftrightarrow \quad & l^M + u(R^M+b) > u(2R^M).
\end{aligned}
$$

The equation defining R^M is: $l^M + u(b+b) = u(R^M+b)$ from which by strict concavity of u and the fact that $R^M > b$ immediately follows that indeed we have: $l^M + u(R^M+b) > u(R^M+R^M)$.

Proof of assertion $\Omega^F(R^F) > T(R^F, R^M)$. From above:

$$\Omega^F(R^F) = \frac{l^M + u(R^F+b)}{r}$$

and

$$T(R^F, R^M) = \frac{u(R^F+R^M) + \delta\Omega^F(R^F) + \delta\Omega^M(R^M)}{r+2\delta}.$$

Hence using $\Omega^F(R^F) = \Omega^M(R^M)$:

$$
\begin{aligned}
& \Omega^F(R^F) > T(R^F, R^M) \\
\Leftrightarrow \quad & \Omega^F(R^F) > \frac{u(R^F+R^M) + 2\delta\Omega^F(R^F)}{r+2\delta} \\
\Leftrightarrow \quad & r\Omega^F(R^F) > u(R^F+R^M) \\
\Leftrightarrow \quad & l^M + u(R^F+b) > u(R^F+R^M)
\end{aligned}
$$

which holds again by strict concavity of u and the fact that $R^F > b$.

The profits (value of a filled job) can be calculated as follows. First, calculate the flows:

$$
\begin{aligned}
u + e^{1M} + e^{1F} &= 1 \\
\lambda u + \lambda(1-\gamma)u &= \delta e^{1M} + \delta e^{1F} \\
\delta e^{1M} &= \lambda u + \lambda(1-\gamma)e^{1F} \\
\delta e^{1F} + \lambda(1-\gamma)e^{1F} &= \lambda(1-\gamma)u
\end{aligned}
$$

which gives:

$$\begin{aligned}
u &= \frac{\delta}{\delta + \lambda(2-\gamma)} \\
e^{1F} &= \frac{\lambda(1-\gamma)}{\delta + \lambda(1-\gamma)} \frac{\delta}{\delta + \lambda(2-\gamma)} \\
e^{1M} &= \frac{\lambda[\delta + \lambda(1-\gamma)(2-\gamma)]}{(\delta + \lambda(1-\gamma))(\delta + \lambda(2-\gamma))}
\end{aligned}$$

and hence

$$\begin{aligned}
2u + e^{1F} + e^{1M} =& \left(2 + \frac{\lambda(1-\gamma)}{\delta + \lambda(1-\gamma)} + \frac{\lambda(\delta + \lambda(1-\gamma)(2-\gamma))}{\delta(\delta + \lambda(1-\gamma))}\right) u \\
=& \frac{2\delta(\delta + \lambda(1-\gamma)) + \lambda(1-\gamma)\delta + \lambda(\delta + \lambda(1-\gamma)(2-\gamma))}{\delta(\delta + \lambda(1-\gamma))} u \\
=& \frac{2\delta + \lambda(2-\gamma)}{\delta} u
\end{aligned}$$

and hence the acceptance probabilities are:

$$\begin{aligned}
\frac{u}{2u + e^{1M} + e^{1F}} &= \frac{\delta}{2\delta + \lambda(2-\gamma)} \\
\frac{u + e^{1F}}{2u + e^{1M} + e^{1F}} &= \delta \frac{\delta + 2\lambda(1-\gamma)}{[2\delta + \lambda(2-\gamma)][\delta + \lambda(1-\gamma)]}.
\end{aligned}$$

Then, profit is for wage R^F

$$\Pi(R^F) = \alpha \frac{u}{2u + e^{1M} + e^{1F}} \frac{y - R^F}{r + \delta}$$

and if offering R^M:

$$\Pi(R^M) = \alpha \frac{u + e^{1F}}{2u + e^{1M} + e^{1F}} \frac{y - R^M}{r + \delta} + \alpha \frac{u}{2u + e^{1M} + e^{1F}} \frac{y - R^M}{r + \delta + \lambda(1-\gamma)}.$$

One can now show that

$$\partial \Pi(R^F)/\partial \gamma > 0$$

which means that the profit is decreasing if γ is moving away from 1. That means that for any γ, there is always a profitable deviation possible which is to increase γ. Hence there cannot be any $\gamma \in (0,1)$ in equilibrium.

To show that $\partial \Pi(R^F)/\partial \gamma > 0$, note that

$$\frac{\partial \Pi(R^F)}{\partial \gamma} = \frac{\alpha\delta}{r+\delta} \left\{ \frac{\lambda}{2\delta + \lambda(2-\gamma)} y - \frac{1}{2\delta + \lambda(2-\gamma)} \frac{\partial R^F}{\partial \gamma} - \frac{\lambda}{2\delta + \lambda(2-\gamma)} R^F \right\}$$

and hence

$$\partial\Pi(R^F)/\partial\gamma > 0 \qquad \Rightarrow \qquad \lambda(y - R^F) - \frac{\partial R^F}{\partial\gamma} > 0.$$

Since obviously $\frac{\partial R^F}{\partial\gamma} < 0$ (if less mass is put on the higher wage then the lower wage will decrease), and $y > R^F$ if firms offer R^F, this completes the proof.

□

Proof of Proposition 8. Conjecture the following relationship if only R^{F2} is offered:

$$U < \Omega^M(R^{F2}), \Omega^F(R^{F2})$$
$$\Omega^M(R^{F2}) < \Omega^F(R^{F2}) = T(R^{F2}, R^{F2})$$

which means that the value functions are given by

$$\begin{aligned}
rU &= l^F + l^M + u(2b) + \lambda(\Omega^F(R^{F2}) - U) + \lambda(\Omega^M(R^{F2}) - U)\\
rT(R^{F2}, R^{F2}) &= u(2R^{F2}) + \delta(\Omega^M(R^{F2}) - T(R^{F2}, R^{F2}))\\
r\Omega^F(R^{F2}) &= l^M + u(R^{F2} + b) + \delta(U - \Omega^F(R^{F2}))\\
r\Omega^M(R^{F2}) &= l^F + u(R^{F2} + b) + \delta(U - \Omega^M(R^{F2})) + \lambda(T(R^{F2}, R^{F2}) - \Omega^M(R^{F2})).
\end{aligned}$$

One can show that R^{F2} can be characterized by:

$$K(u(2R^{F2}) - l^M - u(R^{F2} + b)) = \bar{U}(R^{F2}) - \bar{\Omega}^M(R^{F2})$$

where

$$\begin{aligned}
K &= \frac{(r+\delta)(r+\delta+\lambda) + \lambda r}{\delta(r+\delta)}\\
\bar{U}(R^{F2}) &= l^F + l^M + u(2b) + \frac{\lambda(l^M + u(R^{F2} + b))}{r+\delta}\\
\bar{\Omega}^M(R^{F2}) &= l^F + u(R^{F2} + b) + \frac{\lambda u(2R^{F2})}{r+\delta}
\end{aligned}$$

and hence:

$$\begin{aligned}
&\{(r+\delta)(r+\delta+\lambda) + \lambda r\}(u(2R^{F2}) - l^M - u(R^{F2} + b))\\
&\quad = \delta[(r+\delta)(l^F + l^M + u(2b)) + \lambda(l^M + u(R^{F2} + b))]\\
&\qquad - \delta[(r+\delta)(l^F + u(R^{F2} + b)) + \lambda u(2R^{F2})].
\end{aligned}$$

Finally, R^{F2} is characterized by:

$$(r+\delta)(r+\delta+2\lambda)u(2R^{F2}) = \{(r+2\delta)(r+\delta+\lambda) + \lambda r\}l^M$$

$$+ \{r(r + \delta + 2\lambda) + 2\lambda\delta\}u(R^{F2} + b) + \delta(r + \delta)u(2b).$$

This equation has a unique solution by strict concavity of u using $R^{F2} > b$. The profit is then (since every wage offer is accepted and quits occur only exogenously):

$$\Pi(R^{F2}) = \alpha \frac{y - R^{F2}}{r + \delta}.$$

It is obvious that there cannot be a higher acceptance probability than 1. Also, the discounting part $(1/(r + \delta))$ cannot be higher either, since there are only exogenous separations. Hence there is no higher wage that increases profits.

On the other hand, can a lower wage increase profits? A lower wage will have a strictly smaller acceptance probability (since all men in worker-searcher couples where the woman is working at R^{F2} will not accept). As y goes to infinity, at some point, the difference in wages will become negligible before the difference in acceptance probabilities. Thus a deviation to a lower wage cannot be profitable either.

□

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