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Comment on "Particle dispersal due to interplay of motions in the surface layer of a small reservoir" (by P. Okely, J. Imberger, and K. Shimizu) and "Processes affecting horizontal mixing and dispersion in Winam Gulf, Lake Victoria" (by P. Okely, J. Imberger, and J. P. Antenucci)

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Okely et al. (2010*a*,*b*) are concerned with horizontal dispersion and mixing in lakes and employ numerical modeling to estimate horizontal dispersion coefficients. In both papers, the dispersal of four numerical Lagrangian particles is employed to estimate dispersion coefficients, and the authors explicitly claim that "…this method measures horizontal dispersion due to large-scale horizontal shear only, and the influence of other processes, such as turbulent diffusion, is not accounted for" (Okely et al. 2010*b*, p. 591). However, we demonstrate here that without a diffusive process, e.g., molecular or turbulent diffusion, the dispersal rate estimated from Lagrangian particles as in Okely et al. (2010*a*,*b*) should be zero in principal for large-scale nondivergent horizontal flow fields, independent of the horizontal shear. The particle dispersal presented by Okely et al. (2010*a*,*b*) is not a measure of horizontal dispersion due to horizontal shear but depends on the initial position of the particles and the divergence of the simulated mean flow field and may be affected by undersampling of the flow field, since only four particles were considered. Whereas the study of Okely et al. (2010*b*) was entirely based on this Lagrangian particle technique, Okely et al. (2010*a*) compared horizontal dispersion coefficients estimated from this technique with coefficients determined from the spread of numerically simulated tracer distributions. According to Okely et al. (2010*a*), their horizontal dispersion coefficients agree reasonably well with an estimate based on the assumption that the effects of vertical shear and vertical turbulent mixing (K_z) determine horizontal dispersion (*see* Table 1 and Eq. 11 in Okely et al. 2010*a*). Okely et al. (2010*a*, p. 1875) state that "The vertical shear dispersion scaling was a good approximation for the magnitude and spatial variation of the average horizontal dispersion rates." However, the horizontal dispersion coefficients obtained from Eq. 11 using the observed vertical diffusivities are an order of magnitude smaller than the values presented in Table 1 of Okely et al. (2010*a*) and thus are substantially smaller than the horizontal dispersion coefficients obtained from the tracer simulations. Hence, the combined effect of vertical shear and vertical diffusion cannot explain the large horizontal dispersion in the simulation of Okely et al. (2010*a*). Finally, the Lagrangian particle technique and also the dispersion of the simulated tracer clouds appear to depend mainly on the horizontal divergence of the simulated flow field. Comparison of simulated and observed particle tracks and horizontal currents at a single location indicates that the quality of the simulated flow field is not sufficient to provide the divergence of the true flow field in the lakes studied. Hence, Okely et al. (2010*a*,*b*) may provide estimates of the spread of particles and tracers in their simulated flow field but cannot contribute information on horizontal particle dispersal or horizontal dispersion and mixing under field conditions in lakes.

Lagrangian particle technique and dispersion

In both papers, horizontal dispersion is estimated from the increase of the area enclosed by the hull of four Lagrangian particles propagating in a horizontal plane at a prescribed depth. The tracks of these Lagrangian particles were calculated numerically from the mean horizontal flow field obtained from the Estuary, Lake, and Coastal Ocean Model (ELCOM). Besides the limitation that the simulated particle tracks represent properties of the simulated rather than the true flow field, the temporal change of the area enclosed by the hull of the four particles does not provide information on the dispersion coefficient due to mean horizontal shear in principle.

Dispersion arises from the combined effect of small-scale, random motions described as turbulent or molecular diffusion and the mean flow field. Shear flow dispersion traditionally is understood as the spread of tracer distributions in shear flows and results from the interplay between the shear of the mean velocity field and random motions across the shear (Fischer et al. 1979). Without random motions, the particles never "forget" their initial position, and the Lagrangian timescale that has to elapse before the spread of particles can be described as dispersion (Fischer et al. 1979) becomes infinite. Hence, without turbulent or molecular diffusion, shear flow in a horizontal flow field does not lead to horizontal dispersion. A simple argument confirming this statement is the following: Consider a conservative dissolved tracer deployed with constant concentration (c [x,y,z]) far away from boundaries. Dispersion, i.e., the spread of a dissolved substance, implies that the space occupied by the tracer increases with

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time. Because the tracer mass remains constant, an increase in occupied space requires a decrease in the mean tracer concentration in the region occupied by the tracer. The mass balance of the tracer in incompressible flow with velocity $\vec{u} = (u,v,w)$ and diffusivity D (describing the effects of molecular diffusion, or alternatively of turbulent diffusion if $\vec{u}$ represents the large-scale mean flow field) is:

$$\frac{\partial c}{\partial t} + \vec{\nabla}\cdot(c\vec{u}) - \vec{\nabla}\cdot(D\vec{\nabla}c) = 0 \quad \text{with} \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \quad (1)$$

or because of continuity, $\vec{\nabla}\cdot\vec{u} = 0$, and

$$\frac{\partial c}{\partial t} + \vec{u}\cdot\vec{\nabla}c = \vec{\nabla}\cdot\left(D\vec{\nabla}c\right) \quad (2)$$

The left-hand side of Eq. 2 describes the temporal change of the concentration in the transported water, which is zero without diffusion. Hence, a diffusive process is required for dispersion to occur.

The numerical Lagrangian particles considered in Okely et al. (2010*a*,*b*) only propagate in a horizontal layer with the large-scale mean flow and are assumed not to be influenced by turbulent or molecular diffusion. Now consider large numbers of such particles that are deployed in a horizontal plane with uniform particle density ($c_P\,[x,y]$) in a region far away from boundaries. Because the particles are confined to the horizontal, the vertical flux of the particles is zero. The mass balance of the particles becomes

$$\frac{\partial c_P}{\partial t} + \vec{\nabla}_h\cdot(c_P\vec{u}_h) = 0 \quad (3)$$

$$\frac{\partial c_P}{\partial t} + \vec{u}_h\cdot\vec{\nabla}_h c_P = -c_P\vec{\nabla}_h\cdot\vec{u}_h \quad (4)$$

with

$$\vec{\nabla}_h = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) \quad (5)$$

and $\vec{u}_h = (u,v)$ is the velocity in the horizontal directions x and y. The left-hand side of Eq. 4 describes the change of the particle density in the water parcels transported by the velocities in the horizontal layer. If the divergence of the mean horizontal flow is not zero, this particle density may change with time, and thus also the area occupied by the particles, although diffusive processes are neglected. Continuity requires that $\vec{\nabla}_h\cdot\vec{u}_h = -\partial w/\partial z$. Hence, the temporal change in occupied area is due to the vertical gradient of the vertical velocity component w, i.e., it is due to sinks and sources of water in the horizontal layer. If the divergence of the mean flow in the horizontal is zero ($\vec{\nabla}_h\cdot\vec{u}_h = 0$), the particle density in the transported water in the horizontal layer remains constant, and hence the area occupied by the particles cannot increase in time, independent of the horizontal shear $\partial u/\partial y$ and $\partial v/\partial x$. In summary, the area of the hull enclosing the four numerical Lagrangian particles considered in Okely et al. (2010*a*,*b*) is constant in time for all flows without divergence in the horizontal layer, although dissolved substances may experience strong horizontal shear-dispersion or the area of the hull may increase with time although horizontal dispersion of dissolved substances due to horizontal shear is zero or very small in flows with horizontal divergence but no horizontal shear or small diffusivity. Clearly, the temporal change in area of the hull enclosing the four numerical Lagrangian particles does not provide a measure of horizontal dispersion due to large-scale mean horizontal shear but may arise from the divergence of the mean horizontal flow, i.e., from upwelling or downwelling water.

For a simple illustration of these statements, consider an unbounded horizontal flow field with constant shear $s_y = \mathrm{d}u/\mathrm{d}y$ as depicted in Fig. 1A. In the case of isotropic horizontal turbulence with diffusivity K_h, the dispersion of a tracer deployed instantaneously as point source is, for long diffusion times, proportional to K_h and s_y (Carter and Okubo 1965; Peeters et al. 1996). If four Lagrangian particles are positioned as a rectangle in this flow field, they propagate to the right, thereby forming a parallelogram where the angle of the paralellogram increases with time (Fig. 1A). Although the shear causes a change in the shape of the hull enclosing the four particles, the area of the hull is constant. Because the temporal change of the area of the hull is zero and thus independent of s_y and K_h, it clearly does not provide a measure of shear or dispersion.

As a second example, consider a horizontal flow field with nonlinear shear typical for channel flow. As above, the dispersion of a tracer distribution depends on the horizontal diffusion coefficient (K_h). In the classical case of channel flow with impermeable boundaries and constant diffusivity, the dispersion coefficient is inversely proportional to the diffusion coefficient for sufficiently long dispersion times (Fischer et al. 1979). Now consider again the spread of four Lagrangian particles in this flow field. The shape of the hull enclosing four particles initially positioned in a rectangle symmetric to the line of maximum flow (Fig. 1B) does not change with time. If the four particles are positioned in a rectangle asymmetric to the maximum flow line (Fig. 1C), the shape of the hull enclosing the four particles changes as in the case of constant shear flow (Fig. 1A). In both cases, the temporal change of the area of the hull ($\mathrm{d}A_\mathrm{H}/\mathrm{d}t$) enclosing the four particles is zero. However, if the four particles are positioned in the form of, e.g., a trapezoid (solid circles, Fig. 1D), $\mathrm{d}A_\mathrm{H}/\mathrm{d}t$ increases with time. Thus, for the same flow field, $\mathrm{d}A_\mathrm{H}/\mathrm{d}t$ depends on the initial positioning of the four particles. Note, however, that the increase in hull area with time suggested in the last case is an artifact of undersampling of the flow field, as can be demonstrated by introducing additional particles to the original four particles (open and solid circles, Fig. 1E). The area of the hull enclosing this larger ensemble of particles does not change with time. This example indicates that temporal changes in the area of the hull enclosing four particles may reflect undersampling of the flow field rather than true dispersal by the flow field.

In contrast to the situation in nondivergent horizontal flow, horizontal particle dispersal in flow fields with horizontal divergence leads to a change in the area occupied by the particles (Fig. 1F). The change in area per unit time, however, also depends on the initial position

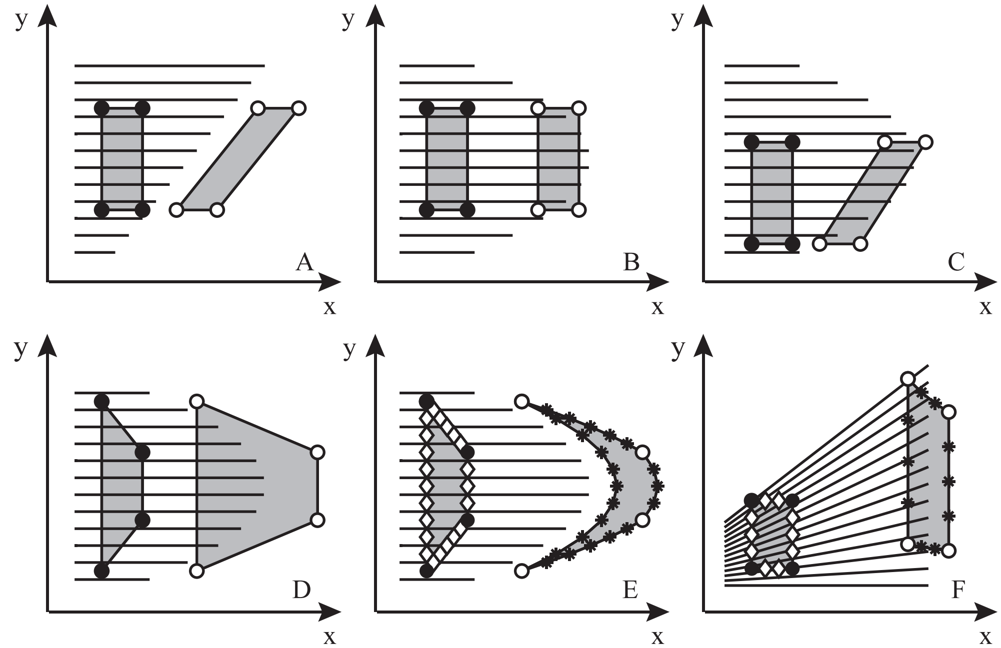

Fig. 1. Displacement of particles in horizontal flow fields with current shear. The black lines indicate the velocity vectors in the flow field; the length of the lines is proportional to the velocity. The particles are displaced by the flow field from their initial position (solid circles and open diamonds) to their final positions (open circles and asterisks). The area of the hull enclosing the initial and the displaced particle clouds, respectively, is depicted in gray.

of the particles; e.g., consider a shift of the initial particle arrangement in the y direction, a rotation by 90°, or different initial spacing between the particles. If all particles start at the same point, the temporal change in hull area is zero.

According to Ridderinkhof and Zimmerman (1992), chaotic advection in vertically integrated two-dimensional (2-D) flow may lead to transport that has a similar characteristic as dispersion. However, the flow field in Okely et al. (2010*a*,*b*) is dominated by oscillating motions due to internal seiching. Figure 2A,B illustrates the idealized flow field of the second horizontal and first vertical (H2V1) mode seiche (a mode identified to be important by Okely et al., 2010*b*) in a two-layer system within a narrow but long rectangular basin (i.e., Coriolis effects are neglected). Assuming inviscid and incompressible flow, the velocity field is divergence free in three dimensions, is not chaotic, and is not turbulent, and thus transport is entirely reversible. Hence, if molecular diffusion is neglected, dissolved substances are not mixed, and dispersion does not occur. However, the velocity gradients in the x direction, i.e., the divergence of the 2-D flow field in the x–y plane, cause particle spreading in the x direction (Fig. 2A,B). Therefore, the area of the hull enclosing particles deployed in the x–y plane changes with time, although there is no dispersion in the system. The change in hull area per unit time (dA_H/dt) oscillates and is highly dependent on the initial positioning of the particles (Fig. 2C). Also in the case of a H1V1-mode seiche, dA_H/dt oscillates and becomes negative during half of the seiche period. Clearly, in a flow field dominated by seiche motions, dA_H/dt is not equal to the dispersion coefficient.

These examples illustrate (1) that the temporal change in the area of the hull enclosing a Lagrangian particle distribution propagating at a fixed depth depends on the horizontal divergence of the flow field rather than being a measure of the "horizontal dispersion attributable to large-scale horizontal shear" as stated by Okely et al. (2010*a*, p. 1867; 2010*b*, p. 591), (2) that dA_H/dt is not a reliable measure of dispersion in seiche-dominated flow, and (3) that the change in hull area with time may be significantly influenced by the choice of the initial positions and the number of particles.

Importance of vertical shear and vertical mixing for horizontal dispersion

According to Okely et al. (2010*a*), the horizontal spread of simulated tracer distributions was insensitive to the choice of the horizontal turbulent diffusion coefficient considering values of K_h = 0, 0.1, and 1 m^2 s^{-1}. This insensitivity to K_h, and, in particular, the strong spread of the tracer clouds at $K_h = 0$, may suggest significant effects by numerical diffusion or strong positive divergence. However, Okely et al. (2010*a*, p. 1877) state that "the vertical shear dispersion process was, …, a dominant driver for horizontal dispersion in Winam Gulf." However, the vertical shear-diffusion process neither explains the horizontal spread of the tracer distribution in cross-current direction (*see* fig. 9 in Okely et al. 2010*a*) nor does it explain

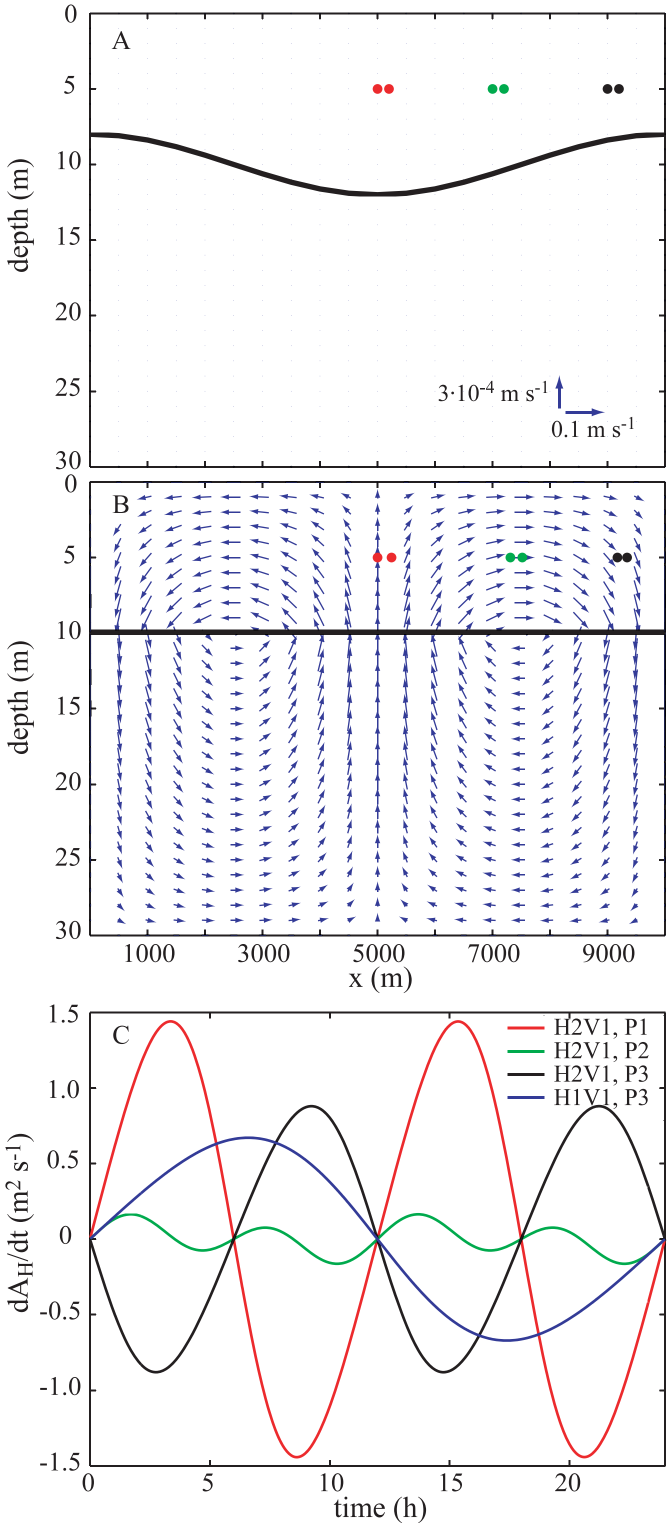

Fig. 2. Idealized seiche motion and dispersal of particles. (A,B) Divergence-free flow field and interface displacement in the *x-z* plane of an H2V1-mode seiche assuming a two-layer system in a narrow but long rectangular basin (A) at time $t = 0$ and (B) at t = T/4. The period T is 12 h. Scaling of the velocity vectors is shown in (A). The velocity in the y direction is zero. Groups of four particles propagating at 5-m depth were released at $t = 0$ in a rectangle with initial spacing of 200 m in the x and y directions at ← different locations along the x axis (symbols, y direction is perpendicular to the plane view). (C) Change per unit time of the area of the hull (dA_H/dt) enclosing the groups of four particles as a function of time for the different initial positions along the x axis: at 5000 m and 5200 m (P1), at 7000 m and 7200 m (P2), and 9000 m and 9200 m (P3). The blue line indicates the change in dA_H/dt for the fundamental horizontal mode H1V1 with a period of T = 24 h and particles initially positioned as at P3. Note that in all cases, dA_H/dt is proportional to the initial distance of the particles in the y direction.

the horizontal spread of the Lagrangian particle distributions, which, according to Table 1 in Okely et al. (2010*a*), agrees reasonably well with the dispersion coefficients obtained from the tracer simulations. Further, using Eq. 11 of Okely et al. (2010*a*) with the observed depth-averaged vertical diffusivities (K_z) given by Okely et al. (2010*a*) in fig. 4 ($K_z > 10^{-3}$ m^2 s^{-1}), we find horizontal dispersion coefficients that are 10 times smaller than the values presented in Table 1 of Okely et al. (2010*a*) and are thus typically more than one order of magnitude smaller than the dispersion coefficients obtained from the tracer simulations. Clearly, vertical shear combined with vertical turbulent diffusion cannot be the major cause of the horizontal dispersion of the simulated tracer distributions.

Conclusions on the relevance of the findings of Okely et al. (2010*a*,*b*) for field conditions

The conclusions of Okely et al. (2010*a*,*b*) on horizontal dispersion are entirely based on numerical simulations of particle dispersal and tracer spread and therefore depend not only on the validity of the interpretation of the particle spreading as dispersion, but also on the ability of the model to represent field conditions. We have demonstrated here that the Lagrangian particle technique employed in Okely et al. (2010*a*,*b*) does not provide reliable information on horizontal dispersion due to large-scale mean horizontal shear but depends on the combined effect of initial particle position and the divergence of the horizontal flow field. Furthermore, the simulated and observed particle tracks that were only compared by Okely et al. (2010*b*, *see* fig. 4) did not have much in common. In one of the two cases shown, the simulated particle moved in the opposite direction and at least at twice the speed of the drifter in the field. At 18 h after release, the distance of the simulated particle from the release point was eight times larger than that of the real particle. The poor agreement between simulated and measured particle tracks indicates that the change in the area of the hull enclosing the positions of the four simulated particles is unlikely to provide a reliable measure of the change in hull area of four particles drifting under field conditions.

The horizontal spread of simulated tracer distributions occurred even if $K_h = 0$ and was insensitive to the value of K_h (Okely et al. 2010*a*), suggesting that numerical diffusion may have contributed to tracer spreading. As explained already herein, the simulated horizontal tracer spreading cannot be explained by vertical shear diffusion using Eq. 11 with the observed K_z of fig. 4 from Okely et al. (2010*a*). Note further, that the vertical shear of the horizontal currents was about one order of magnitude larger in the simulations than in the data (*see* fig. 4 in Okely et al. 2010*a*). Hence, the consequence of vertical shear dispersion for the horizontal spread of a tracer distribution is at least about one order of magnitude larger in the simulations than in the field.

In both studies of Okely et al. (2010*a,b*), the direction of the horizontal currents differed by more than 45°, and the speed disagreed by more than 50% between simulations and data during about 50% of the time (*see* fig. 5 and fig. 2 in Okely et al. [2010*b*] and [2010*a*], respectively). Thus, simulated shear and divergence cannot agree reasonably well with shear and divergence of the velocity field in the lakes studied.

Considering these severe limitations of the model to adequately represent particle movement and flow under field conditions, the conclusions on horizontal shear dispersion based on the simulations of particle dispersal and tracer spreading by Okely et al. (2010*a*,*b*) may not have much in common with horizontal shear dispersion in the field. Considering further that the Lagrangian particle technique of Okely et al. (2010*b*) fails to be a measure of dispersion, especially in the oscillating flow caused by seiching typical for the studies of Okely et al. 2010(*a,b*), the papers by Okely et al. (2010*a*,*b*) do not provide reliable information on horizontal dispersion in lakes.

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