Erschienen in: Proceedings of the 3rd IFAC Workshop on Distributed Estimation and Control in Networked Systems. - Laxenburg, Austria : IFAC, 2012. - (IFAC Proceedings Volumes ; 45,26). - S. 240-245. - ISSN 1474-6670. - ISBN 978-3-902823-22-9
https://dx.doi.org/10.3182/20120914-2-US-4030.00024

Bio-inspired Source Seeking with no Explicit Gradient Estimation ⋆

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Abstract:
Inspired by behaviors of fish groups seeking darker (shaded) regions in environments with complex lighting variations, we develop distributed source-seeking algorithms for a group of sensing agents with no explicit gradient estimation. We choose a baseline for agent groups and decompose the velocity of each agent into two parts. The first part, which is perpendicular to the baseline, is chosen to be proportional to the measurements, agreeing with observations from fish groups. The second part, which is parallel to the baseline, can be designed to control the relative distances among the agents. This decomposition is leveraged to implement formation-maintaining strategies and source seeking behaviors for the entire group. We prove that the moving direction of a group will converge towards the gradient direction while the formation is maintained.

Keywords: Cooperative sensing, Cooperative control, Source-seeking.

1. INTRODUCTION

Autonomous sensing agents that are capable of localizing sources are of great importance in various scenarios such as locating chemical spills, searching for survivors after a disaster, and detecting fire in its early stage. Various studies have developed source-seeking algorithms, many of which are inspired by behaviors of different animal species. For example, Keller et al. (2003) introduced gradient-free source-seeking algorithms inspired by blue crabs, and Russella et al. (2003) and Pyk et al. (2006) presented approaches inspired by the silkworm moth. These algorithms mainly focus on source-seeking using one agent.

Collective behaviors have been observed in a broad range of species. As addressed in Clark (1986), because of the effectiveness and robustness, collective behaviors are proved to be beneficial to other members in the group and profitable for the survival of the entire group. Therefore, researchers in engineering have been studying collective behaviors of animal groups and gaining inspirations and insights for controlling multi-robot systems. Bachmayer and Leonard (2002), Cortes (2007), and Wu and Zhang (2012) introduced collective gradient estimating and tracking algorithms in distributed scalar fields, Farrell et al. (2003), Liu and Passino (2004), Russell (2004), and Torney et al. (2010) presented plume-tracing algorithms in turbulent flow, and Zhang and Leonard (2010) and Wu and Zhang (2011) discussed level curve tracking in two- and three-dimensional spaces.

Couzin's group Berdahl et al. (2012) observed that fish groups are able to perform gradient tracking to locate darker (shaded) regions in complex light environments even if the field is time-varying. However, it is conjectured that each fish in a group have very poor or no gradient estimates. They principally measure the intensities of the light field and respond to the positions of other fish within their view. Based on the measurements, a fish in a group speeds up when the light intensity at its current position is relatively high and slows down as the light intensity decreases. In this way, the group is capable of aligning its trajectory with gradient directions and moves towards the shade as described in Berdahl et al. (2012). Once the group reaches the shade, the forward motion of the group becomes circular, in which some fish in the group reverse their directions of movement. The group circles around the shade until the position of the shade changes. Then, the group resumes the forward motion.

These data inspire us to investigate source-seeking for a group of sensing agents in a distributed fashion with no explicit gradient estimation. We consider a group of sensing agents that track gradients to seek a local minimum of a field. We choose a baseline for the group and decompose the velocities of the agents in the group into two parts, which decouples the control laws for the motion and for the formation. One part of the velocity, which is perpendicular to the baseline, is chosen to be proportional to the measurements. When the measurement of an agent increases, this velocity increases, and when the mea-

⋆ The research work is supported by ONR grants N00014-08-1-1007, N00014-09-1-1074, and N00014-10-10712 (YIP), and NSF grants ECCS-0841195 (CAREER), CNS-0931576, and ECCS-1056253.

surement decreases, this velocity decreases. If the agents seek a local maximum instead of minimum, this velocity can simply be reversed. We prove the convergence of the moving direction of the group towards gradient directions, which agrees with the numerical and analytical results presented in Berdahl et al. (2012). The other part of the velocity, which is parallel to the baseline, can be designed to control the relative distances among the agents. We propose formation-maintaining strategies for a group of more than two agents remaining in a desired formation, which may be verified by future biological experiments, and prove the convergence of the formation control laws using shape variables as described in Zhang (2010).

Our results reveal a strong connection with the well-known Braitenberg-style differential drive vehicles as introduced in Braitenberg (1984), which have similar properties in that the movement of a wheel is directly controlled by the measurement of the sensor connected to it. Braitenberg-style source-seeking algorithms have been proposed in studies as Kazadi et al. (2000) and Lilienthal and Duckett (2004). However, these algorithms are developed for one agent. The approaches we develop are for multi-agent systems. Our results suggest that by knowing only measurement information and the relative distances to other agents, a group of agents tend to behave like a Braitenberg-style vehicle.

The rest of the paper is organized as follows. Section 2 introduces the problem formulation. Section 3 presents source-seeking control for two-agent groups. Section 3 discusses different formation-maintaining strategies for N-agent groups. Section 5 introduces the experimental results, and Section 6 provides the conclusions.

2. PROBLEM FORMULATION

Consider a group of N sensing agents that are seeking a minimum of an unknown scalar field $z(\mathbf{r})$, in which $\mathbf{r} \in R^2$ denotes a location in the field. Let $\mathbf{r}_i$ represent the position and $\mathbf{v}_i$ represent the velocity of the ith agent. Suppose the motion of each agent in the group satisfies $\dot{\mathbf{r}}_i = \mathbf{v}_i, i = 1, \cdots, N$. Denote the position and velocity of the group center as $\mathbf{r}_c$ and $\mathbf{v}_c$, respectively. Then, we derive $\mathbf{r}_c = \frac{1}{N}\sum_{i=1}^{N} \mathbf{r}_i$, and $\mathbf{v}_c = \frac{1}{N}\sum_{i=1}^{N} \mathbf{v}_i$.

Suppose the field value satisfies $z_{\min} \leq z(\mathbf{r}) \leq z_{\max}$, in which $z_{\min} \geq 0$. The gradient of the field at location $\mathbf{r}$ is denoted as $\nabla z(\mathbf{r})$. Along their trajectories, the sensing agents take measurements of the field, which can be written as $y(\mathbf{r}_i) = z(\mathbf{r}_i) + v(\mathbf{r}_i), i = 1, \cdots, N$, in which $v(\mathbf{r}_i)$ is the noise term that may come from measuring process or the field. If we consider only linear approximation of the field, then, we derive

$$z(\mathbf{r}_i) = z(\mathbf{r}_c) + \nabla z(\mathbf{r}_c) \cdot (\mathbf{r}_i - \mathbf{r}_c) + H.O.T, \qquad (1)$$

where H.O.T represent higher order terms in the above Taylor expansion. In addition to the measurements, we assume that the agents have knowledge of their relative positions to neighboring agents.

The problem is to design controls for the velocities of the agents so that the group can move close to a local minimum in the field without explicit gradient estimation while maintaining a desired formation. More specifically, the goal is to design $\mathbf{v}_i$ so that: (1) $\frac{\mathbf{v}_c}{\|\mathbf{v}_c\|} \cdot \frac{\nabla z(\mathbf{r_c})}{\|\nabla z(\mathbf{r_c})\|}$ converges to -1, but without gradient estimation, and (2) the relative displacement between agents $\mathbf{r}_i - \mathbf{r}_j$, where $j \neq i$, converges to a desired vector. Note that the first goal is invalid when $\| \nabla z(\mathbf{r_c}) \| = 0$, which indicates a singular point or saddle point in the field. In this paper, we consider fields with no saddle points, and we will control the agents to switch to circular motion once they approach a singular point.

3. CONTROL OF TWO-AGENT GROUPS

We start with $N = 2$. We control the two agents to converge asymptotically to a constant formation with distance a between each other in steady state. Therefore, they can be considered as a rigid body with the center of mass being at $\mathbf{r}_c$. Define the inertial frame as X_I and Y_I. Let $\mathbf{q} = \mathbf{r}_2 - \mathbf{r}_1$, and define $\mathbf{q}^\perp$ to be the vector perpendicular to $\mathbf{q}$ that forms a right handed frame with $\mathbf{q}$. $\mathbf{q}$ and $\mathbf{q}^\perp$ intersects at $\mathbf{r}_c$. Set the origin of the rigid body frame at $\mathbf{r}_c$, and select X_B and Y_B to be aligned with $\mathbf{q}$ and $\mathbf{q}^\perp$. Denote the angle between X_B and X_I as $\theta \in [-\pi, \pi]$. Then, we obtain a rigid body rotation matrix $g = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, and the angular velocity $\Omega = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}$. For each agent, we decompose its velocity into two parts: $\mathbf{v}_i^\perp$, which is perpendicular to $\mathbf{q}$ and proportional to the measurements $y(\mathbf{r}_i)$, and $\mathbf{v}_i^{//}$, which is aligned with $\mathbf{q}$ and maintains formation. Then, $\mathbf{v}_i = \mathbf{v}_i^\perp + \mathbf{v}_i^{//}$. We will design $\mathbf{v}_i^\perp$ and $\mathbf{v}_i^{//}$ separately.

Fig. 1 illustrates the desired motion of the two-agent group. The two agents move in the same direction when they are seeking a source, as shown in Fig. 1 (a). At this stage, the group is performing forward motion. Once the group approaches a local minimum of the field, one of the agents reverse its moving direction. Then, the group performs circular motion around the source, as shown in Fig. 1 (b).

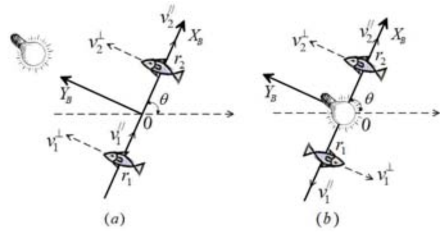

Fig. 1. Desired motion of the two-agent group when seeking a source. (a) Forward motion. (b) Circular motion.

Let $\phi_i, i = 1, 2$, be the angles between velocity $\mathbf{v}_i^\perp$ and X_I. We can write the perpendicular velocities as $\mathbf{v}_i^\perp = v_i^\perp \begin{pmatrix} \cos\phi_i \\ \sin\phi_i \end{pmatrix}$, where $v_i^\perp$ is the magnitude of $\mathbf{v}_i^\perp$. Similarly, $\mathbf{v}_i^{//} = v_i^{//} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$. If the agents are performing forward motion, then, $\phi_1 = \phi_2 = \theta + \frac{\pi}{2}$. If the agents are performing circular motion, then, $\phi_1 = \theta + \frac{3\pi}{2}$ and $\phi_2 = \theta + \frac{\pi}{2}$. Inspired by the behaviors of fish groups, we design

$$v_i^{\perp} = ky(\mathbf{r}_i) + C, i = 1, 2, \quad (2)$$

where k and C are constants selected by design. In the direction that is aligned with $\mathbf{q}$, we design $v_i^{//}$, $i = 1, 2$, as

$$v_1^{//} = k_p((\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{q} - a), \quad (3)$$

$$v_2^{//} = -k_p((\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{q} - a). \quad (4)$$

Define $s = (\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{q}$. Based on Equations (3) and (4), we calculate that $\dot{s} = 2(v_2^{//} - v_1^{//}) = -4k_p(s - a)$, where $s = a$ is an asymptotically stable equilibrium. Therefore, the two agents will converge to a constant formation with a distance a between each other. Once $\mathbf{v}_i^{\perp}$ and $\mathbf{v}_i^{//}$ are determined, the velocities of the ith agent can be calculated as $\mathbf{v}_i = \mathbf{v}_i^{\perp} + \mathbf{v}_i^{//}$, which produces

$$\mathbf{v}_i = (ky(\mathbf{r}_i) + C) \begin{pmatrix} \cos\phi_1 \\ \sin\phi_1 \end{pmatrix} - k_p((\mathbf{r}_i - \mathbf{r}_j) \cdot \mathbf{q} - a) \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad (5)$$

where $j = 1$ or 2, and $j \neq i$.

3.1 Forward Motion

We first discuss the forward motion of the group and prove that the first goal, the convergence of the moving direction of the group towards the gradient direction, can be achieved. In this case, $\phi_1 = \phi_2 = \theta + \frac{\pi}{2}$ are always satisfied, as illustrated in Fig. 1 (a). In this section, we investigate the situation in which noise in the measurements can be ignored, that is, $v(\mathbf{r}) = 0$. We will discuss the situation that the measurements are noisy in Section 3.3.

If there is no noise, the velocity of the formation center can be written as

$$\mathbf{v}_c = (\frac{1}{2}k(z(\mathbf{r}_1) + z(\mathbf{r}_2)) + C) \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}. \quad (6)$$

The angular velocity of the formation is $\dot{\theta} = \omega = \frac{v_2^{\perp} - v_1^{\perp}}{\|\mathbf{q}\|} = \frac{k(z(\mathbf{r}_2) - z(\mathbf{r}_1))}{\|\mathbf{q}\|}$. Denote the angle between the gradient direction $\nabla z(\mathbf{r}_c)$ and the inertial frame X_I as $\alpha \in [-\pi, \pi]$. From the linear approximation of the field, we derive

$$\begin{aligned} \dot{\theta} &\cong \frac{k}{\| \mathbf{q} \|}(\nabla z(\mathbf{r}_c) \cdot (\mathbf{r}_2 - \mathbf{r}_1)) = \frac{k}{\| \mathbf{q} \|}(\nabla z(\mathbf{r}_c) \cdot \mathbf{q}) \\ &= k \| \nabla z(\mathbf{r}_c) \| (\frac{\nabla z(\mathbf{r}_c)}{\| \nabla z(\mathbf{r}_c) \|} \cdot \frac{\mathbf{q}}{\| \mathbf{q} \|}) \\ &= -k \| \nabla z(\mathbf{r}_c) \| \sin(\theta - \alpha - \frac{\pi}{2}). \end{aligned} \quad (7)$$

Choose the state to be $\theta - \alpha$, then we obtain

$$\dot{\theta} - \dot{\alpha} = -k \| \nabla z(\mathbf{r}_c) \| \sin(\theta - \alpha - \frac{\pi}{2}) - \dot{\alpha}. \quad (8)$$

When $\| \nabla z(\mathbf{r}_c) \| \neq 0$, the above system has a stable equilibrium $\theta - \alpha = \frac{\pi}{2}$ and an unstable equilibrium $\theta - \alpha = -\frac{\pi}{2}$. Given the above system, we have the following proposition.

Proposition 1. If the gradient direction α is constant, that is, $\dot{\alpha} = 0$, then, as $t \to \infty$, $lim_{t\to\infty}\theta(t) = \alpha + \frac{\pi}{2}$. If the rate of change $\dot{\alpha} \neq 0$ is considered as an input to the system (8), then $\theta - \alpha = \frac{\pi}{2}$ is an equilibrium of (8) that is input-to-state stable (ISS).

Proof. If $\dot{\alpha} = 0$, we choose a Lyapunov candidate function as

$$V = -\ln(\cos(\frac{\theta - \alpha - \frac{\pi}{2}}{2})). \quad (9)$$

We calculate

$$\begin{aligned} \dot{V} &= \tan(\frac{\theta - \alpha - \frac{\pi}{2}}{2})(\dot{\theta} - \dot{\alpha}) \\ &= -2k \| \nabla z(\mathbf{r}_c) \| \sin^2(\frac{\theta - \alpha - \frac{\pi}{2}}{2}) - \tan(\frac{\theta - \alpha - \frac{\pi}{2}}{2})\dot{\alpha} \\ &= -2k(1-\epsilon) \| \nabla z(\mathbf{r}_c) \| \sin^2(\frac{\theta - \alpha - \frac{\pi}{2}}{2}) \\ &\quad - 2k\epsilon \| \nabla z(\mathbf{r}_c) \| \sin^2(\frac{\theta - \alpha - \frac{\pi}{2}}{2}) - \tan(\frac{\theta - \alpha - \frac{\pi}{2}}{2})\dot{\alpha} \\ &\leq -2k(1-\epsilon) \| \nabla z(\mathbf{r}_c) \| \sin^2(\frac{\theta - \alpha - \frac{\pi}{2}}{2}), \end{aligned} \quad (10)$$

when $|\dot{\alpha}| \leq k\epsilon \| \nabla z(\mathbf{r}_c) \| |\sin(\theta - \alpha - \frac{\pi}{2})|$ and $0 < \epsilon < 1$. Therefore, according to Theorem 4.19 in Khalil (2001), if $\dot{\alpha}$ is considered as the input, the system (8) is input-to-state stable (ISS). If the input $\dot{\alpha} = 0$, θ converges to the equilibrium point $\alpha + \frac{\pi}{2}$. If the rate of change $\dot{\alpha}$ is bounded, then at the steady state, the deviation $|(\theta - \alpha - \frac{\pi}{2})|$ is also bounded.

Proposition 1 indicates that $\frac{\mathbf{v}_c}{\|\mathbf{v}_c\|} \cdot \frac{\nabla z(\mathbf{r_c})}{\|\nabla z(\mathbf{r_c})\|} = \cos(\theta + \frac{\pi}{2} - \alpha)$ converges to -1 as $t \to \infty$. The convergence of the moving direction of the group verifies the observations that fish groups are able to align their averaged motion with the gradient direction.

3.2 Circular Motion

When the two-agent group moves close to a local minimum of the field, it switches from forward motion to circular motion. The switching condition can be $\| \mathbf{v}_i^{\perp} \| < \varepsilon_1$, in which ε_1 is a positive constant. That is, when any agent senses that the forward speed is less than the threshold ϵ_1, it changes its moving direction. We will show that the circular motion can only be maintained around a point where $\| \nabla z(\mathbf{r}_c) \| = 0$.

In this case, the angles satisfy $\phi_1 = \theta + \frac{3\pi}{2}$ and $\phi_2 = \theta + \frac{\pi}{2}$. Then, we calculate

$$\mathbf{v}_c = \frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) \quad (11)$$

$$\begin{aligned} &= \frac{1}{2}k(z(\mathbf{r}_1) + C) \begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix} + \frac{1}{2}k(z(\mathbf{r}_2) + C) \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \\ &= \frac{1}{2}k(\nabla z(\mathbf{r}_c) \cdot \mathbf{q}) \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \\ &= \frac{1}{2}k \| \mathbf{q} \| \| \nabla z(\mathbf{r}_c) \| \cos(\theta - \alpha) \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}. \end{aligned} \quad (12)$$

The angular velocity satisfies

$$\omega = \frac{v_2^{\perp} + v_1^{\perp}}{\| \mathbf{q} \|} = \frac{k(z(\mathbf{r}_2) + z(\mathbf{r}_1)) + 2C)}{\| \mathbf{q} \|}. \quad (13)$$

If $\mathbf{v}_c = 0$, we must have $\| \nabla z(\mathbf{r}_c) \| = 0$ or $\cos(\theta - \alpha) = 0$. Consider $\cos(\theta - \alpha) = 0$, which indicates $\theta = \alpha \pm \frac{\pi}{2}$. Since $\omega \neq 0$ if $C \neq 0$, this case will not occur since $\theta = \alpha + \frac{\pi}{2}$ cannot always be satisfied. Therefore, $\mathbf{v}_c = 0$ only when $\| \nabla z(\mathbf{r}_c) \| = 0$. This shows that circular motion can only sustain around a singular point.

3.3 Noisy Measurements

Usually, noise exists in the field or in the measuring process, which leads to uncertainties in the estimation of moving directions. If we consider $y(\mathbf{r}) = z(\mathbf{r}) + v(\mathbf{r})$, in which $v(\mathbf{r}) \neq 0$ represents the noise, then, Equations (6) and (7) become $\mathbf{v}_c = (\frac{1}{2}k(z(\mathbf{r}_1)+z(\mathbf{r}_2)+v(\mathbf{r}_1)+v(\mathbf{r}_2)))+C)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$, and $\dot{\theta} = \frac{k}{\|\mathbf{q}\|}(\nabla z(\mathbf{r}_c)\cdot\mathbf{q} + v(\mathbf{r}_2) - v(\mathbf{r}_1))$.

Assume that $v(\mathbf{r})$ is zero mean Gaussian noise with variance σ^2. To show the convergence of the moving direction of the group, we examine the expected value and variance of angle θ, which satisfies $\frac{d\theta}{dt} = \omega$. We have

$$\begin{aligned} d\theta =& -k \parallel \nabla z(\mathbf{r}_c) \parallel \sin(\theta - \alpha - \frac{\pi}{2})dt \\ &+ \frac{k\sigma}{\parallel \mathbf{q} \parallel}(d(v(\mathbf{r}_2)) - d(v(\mathbf{r}_1))), \end{aligned} \tag{14}$$

which yields $\frac{dE(\theta)}{dt} = -k \parallel \nabla z(\mathbf{r}_c) \parallel E(\sin(\theta - \alpha - \frac{\pi}{2}))$, where the expectation E is taken with respect to the noise term. Let $e = \theta - \alpha - \frac{\pi}{2}$ and $\theta_0 = \alpha + \frac{\pi}{2}$. Assume e is small. Then, from Taylor expansion $\sin e = e - \frac{e^3}{3!} + \frac{e^5}{5!} + \cdots$, we obtain

$$\frac{dE(e)}{dt} = -k \parallel \nabla z(\mathbf{r}_c) \parallel E(e), \tag{15}$$

which indicates that $E(e) = 0$ is a stable equilibrium. Therefore, as $t \to \infty$, $E(\theta) = \alpha + \frac{\pi}{2}$, which proves the convergence of the expectation of the moving direction. Now, let's calculate the variance of θ. Define $\psi(e) = e^2$. Since we have

$$de = -k \parallel \nabla z(\mathbf{r}_c) \parallel edt + \frac{k\sigma}{\parallel \mathbf{q} \parallel}(d(v(\mathbf{r}_2)) - d(v(\mathbf{r}_1))), \tag{16}$$

then, according to Ito's differentiation rule, we derive

$$\begin{aligned} de^2 &= 2ede + 2(\frac{k\sigma}{\parallel \mathbf{q} \parallel})^2 dt \\ &= -2k \parallel \nabla z(\mathbf{r}_c) \parallel e^2 dt + 2(\frac{k\sigma}{\parallel \mathbf{q} \parallel})^2 dt \\ &+ \frac{2ek\sigma}{\parallel \mathbf{q} \parallel}(d(v(\mathbf{r}_2)) - d(v(\mathbf{r}_1))), \end{aligned} \tag{17}$$

which yields

$$\frac{dE(e^2)}{dt} = -2k \parallel \nabla z(\mathbf{r}_c) \parallel E(e^2) + 2(\frac{k\sigma}{\parallel \mathbf{q} \parallel})^2. \tag{18}$$

As $t \to \infty$, $E(e^2) \to 2(\frac{k\sigma}{\|\mathbf{q}\|})^2$. Therefore, the variance of θ is $\frac{\sqrt{2}k\sigma}{\|\mathbf{q}\|}$, which indicates that as measurement noise increases, the variance of θ increases, and as the distance between the two agents $\parallel \mathbf{q} \parallel$ increases, the variance of θ decreases.

4. GENERALIZATION TO N-AGENT GROUPS

We consider a group of N agents in the field seeking a local minimum. Arbitrarily select $\mathbf{q}$ as an unit vector that forms an angle θ with the inertial frame X_I. As illustrated in Fig. 2, we also decompose the velocities of the agents into two parts: $\mathbf{v}_i^{\perp}$, which is perpendicular to $\mathbf{q}$ and $\mathbf{v}_i^{//}$, which is aligned with $\mathbf{q}$. $\mathbf{v}_i^{\perp}$ and $\mathbf{v}_i^{//}$ can be designed in different ways. In this section, we discuss different designs for the formation control, and show the convergence of group motion towards the gradient direction.

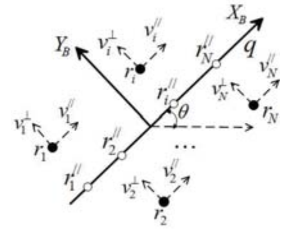

Fig. 2. Decomposition of the velocities of the agents in a N-agent group.

4.1 Non-rigid Body Motion

We first design velocities of the agents so that the agents maintain only the relative positions to other agents in direction $\mathbf{q}$. In this case, we keep $\mathbf{v}_i^{\perp}$ the same as in the two-agent case, which is $\mathbf{v}_i^{\perp} = (kz(\mathbf{r}_i)+C)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}, i = 1, \cdots, N$. Note that under this design, the relative positions among agents may change in direction $\mathbf{q}^{\perp}$, which is perpendicular to $\mathbf{q}$.

Along direction $\mathbf{q}$, let $\mathbf{r}_i^{//}$ be the projection of location $\mathbf{r}_i$ onto vector $\mathbf{q}$, as illustrated in Fig. 2. For agent i, we define set $\mathcal{N}_i$ to contain the closest agents to agent i to the right and to the left along direction $\mathbf{q}$. For example, as shown in Fig. 2, $\mathcal{N}_1 = \{2\}$, $\mathcal{N}_i = \{i-1, i+1\}, i \neq 1, N$, and $\mathcal{N}_N = \{N-1\}$. The goal is to design $\mathbf{v}_i^{//}$ so that the relative distance from $\mathbf{r}_i^{//}$ to $\mathbf{r}_j^{//}, i \neq j$, converges to a constant a_{ij}^0. Furthermore, we require that $\mathbf{v}_c^{//} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{v}_i^{//} = 0$. Therefore, we design $v_i^{//}$ as

$$v_i^{//} = k_p \sum_{j \in \mathcal{N}_i} ((\mathbf{r}_j - \mathbf{r}_i)\cdot\mathbf{q} - a_{j,i}^0), \tag{19}$$

where $a_{i,j}^0 = -a_{j,i}^0$. To prove the convergence of the control (19), we define shape variables $s_i = (\mathbf{r}_{i+1} - \mathbf{r}_i)\cdot\mathbf{q}$, in which $i = 1, \cdots, N-1$. Then, for $i \neq 1, N$, we derive

$$\begin{aligned} \dot{s}_i = (\dot{\mathbf{r}}_{i+1} - \dot{\mathbf{r}}_i)\cdot\mathbf{q} = v_{i+1}^{//} - v_i^{//} = k_p(s_{i-1} - a_{i,i-1}^0) \\ - 2k_p(s_i - a_{i+1,i}^0) + k_p(s_{i+1} - a_{i+2,i+1}^0). \end{aligned} \tag{20}$$

For $i = 1$, we have

$$\dot{s}_1 = -2k_p(s_1 - a_{2,1}^0) + k_p(s_2 - a_{3,2}^0), \tag{21}$$

and for $i = N$, we have

$$\dot{s}_N = k_p(s_{N-2} - a_{N-1,N-2}^0) - 2k_p(s_{N-1} - a_{N,N-1}^0). \tag{22}$$

Denote $\mathbf{s} = (\, s_1,\ s_2,\ \cdots,\ s_{N-1}\,)^T$ and $\mathbf{a}^0 = (\, a_{2,1}^0,\ a_{3,2}^0,\ \cdots,\ a_{N,N-1}^0, a_{N,N-1}^0\,)^T$. Then, from Equations (21), (20), and (22), we obtain

$$\dot{\mathbf{s}} = k_p A(\mathbf{s} - \mathbf{a}^0), \tag{23}$$

where $A = \begin{pmatrix} -2 & 1 & 0 & \dots & 0 \\ 1 & -2 & 1 & \dots & 0 \\ 0 & & \ddots & & \vdots \\ \vdots & & 1 & -2 & 1 \\ 0 & \dots & 0 & 1 & -2 \end{pmatrix}$. The eigenvalues of A are $\lambda_i = -2 + 2\cos(\frac{i\pi}{N}) < 0$ for $i = 1, \cdots, N-1$. Therefore, system (23) is asymptotically stable. The shape variable $\mathbf{s}$ converges to $\mathbf{a}^0$ as $t \to \infty$. Then, the formation is stabilized.

Since each agent now has its velocity given by $\mathbf{v}_i = (kz(\mathbf{r}_i)+C)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} + k_p \sum_{j\in\mathcal{N}_i}((\mathbf{r}_j-\mathbf{r}_i)\cdot\mathbf{q}-a^0_{j,i})\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$, and $\mathbf{v}_c^{//} = \frac{1}{N}\sum_{i=1}^N \mathbf{v}_i^{//} = 0$, we obtain the velocity of the formation center as $\mathbf{v}_c = (\frac{1}{N}k\sum_{i=1}^N z(\mathbf{r}_i)+C)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$.

4.2 Motion Under a Rigid Formation Controller

We can also design a rigid formation controller, which controls the relative distances of the agents in direction $\mathbf{q}^\perp$. For this purpose, we replace the constant C in Equation (5) by a feedback control term.

Let $\mathbf{r}_i^\perp, i = 1, \cdots, N$ be the projections of locations $\mathbf{r}_i$ onto vector $\mathbf{q}^\perp$. Fig. 3 illustrates the case that $N = 3$, in which $\mathbf{q}^\perp$ is chosen to start from location $\mathbf{r}_3$. Denote the desired distance between agent i and agent j in direction $\mathbf{q}^\perp$ as $b^0_{i,j}$, in which $b^0_{i,j} = -b^0_{j,i}$. Let $\mathcal{N}_i^\perp$ be the neighboring set of agent i along direction $\mathbf{q}^\perp$. Then we design

$$v_i^\perp = kz(\mathbf{r}_i) + k_d \sum_{j\in\mathcal{N}_i^\perp}((\mathbf{r}_j-\mathbf{r}_i)\cdot\mathbf{q}^\perp - b^0_{j,i}), \tag{24}$$

where $i = 1, \cdots, N$. Define shape variables $s_i^\perp = (\mathbf{r}_{i+1} - \mathbf{r}_i)\cdot\mathbf{q}^\perp$. Denote $\mathbf{s}^\perp = \left(s_1^\perp,\ s_2^\perp,\ \cdots,\ s_{N-1}^\perp\right)^T$ and $\mathbf{b}^0 = \left(b^0_{2,1},\ b^0_{3,2},\ \cdots,\ b^0_{N,N-1}, b^0_{N,N-1}\right)^T$. Let $\mathbf{z}$ be a column vector with the ith entry being $[z(\mathbf{r}_{i+1}) - z(\mathbf{r}_i)]$. Then, similar to the non-rigid body case, we obtain

$$\dot{\mathbf{s}}^\perp = k_p A(\mathbf{s}^\perp - \mathbf{b}^0) + k\mathbf{z}. \tag{25}$$

Starting from $t = 0$, the solution of the above system is $\mathbf{s}^\perp(t) = e^{k_pAt}(\mathbf{s}^\perp(0)-\mathbf{b}^0)+\mathbf{b}^0+\int_0^t ke^{k_pA(t-\tau)}\mathbf{z}(\tau)d\tau$. Since $z_{\min}\leq z(\mathbf{r})\leq z_{\max}$, $\mathbf{z}$ is bounded. Therefore, the solution satisfies $\| \mathbf{s}^\perp(t) - \mathbf{b}^0 \| \leq e^{\lambda k_p t} \| \mathbf{s}^\perp(0) - \mathbf{b}^0 \| + k \| \mathbf{z}(\tau) \| \, |\int_0^t e^{\lambda k_p(t-\tau)}d\tau| \leq e^{\lambda k_p t} \| \mathbf{s}^\perp(0) - \mathbf{b}^0 \| + \frac{k}{|\lambda|}\sup_{0\leq\tau\leq t} \| \mathbf{z}(\tau) \|$, in which λ is the maximum eigenvalue of matrix A. Therefore, the system (25) is input-to-state stable (ISS)(Khalil (2001)), which implies that for any bounded $\mathbf{z}$, the shape variable $\mathbf{s}^\perp$ will be bounded, and if the input $\mathbf{z}$ converges to zero as $t \to 0$, $\mathbf{s}^\perp - \mathbf{b}^0$ will converge to 0. For $\mathbf{v}_i^{//}$, we design it to be the same form as in Equation (19). Then, we calculate $\mathbf{v}_c = \frac{1}{N}k\sum_{i=1}^N z(\mathbf{r}_i)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$, in which the velocities that control the formation cancel out.

We observe an interesting fact that from the ISS property of system (25), even though a rigid formation controller is used, the agents may not stay in a rigid formation due to the nonzero term $\mathbf{z}$, which seems to coincide with real life observations that fish don't tend to maintain a rigid formation. This insight may hint further investigations.

4.3 Rotation of the Group

To calculate the angular velocity of the group, we consider only the motion of the vector $\mathbf{q}$. Given two locations $\mathbf{r}_i^{//}$ and $\mathbf{r}_j^{//}$ along $\mathbf{q}$, then, in the non-rigid body case, we derive $\omega = -k \| \nabla z(\mathbf{r}_c) \| \sin(\theta-\alpha-\frac{\pi}{2})$, the convergence of which has been proven in Proposition 1. Therefore, the moving

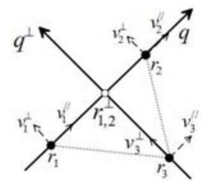

Fig. 3. Decomposition of the velocities of the three-agent group.

direction of the group converges to the gradient direction. In rigid body motion, the angular velocity is obtained by

$$\begin{aligned}\dot\theta - \dot\alpha = &-k \| \nabla z(\mathbf{r}_c) \| \sin(\theta - \alpha - \frac{\pi}{2}) \\ &+ \frac{k_d}{\| \mathbf{q} \|}\Big(\sum_{l\in\mathcal{N}_i^\perp}((\mathbf{r}_l - \mathbf{r}_i)\cdot\mathbf{q}^\perp - b^0_{l,i}) \\ &- \sum_{l\in\mathcal{N}_j^\perp}((\mathbf{r}_l - \mathbf{r}_j)\cdot\mathbf{q}^\perp - b^0_{l,j})\Big) - \dot\alpha, \end{aligned}\tag{26}$$

Since the second term of system (26) is bounded, then similar to Proposition 1, we can also prove that system (26) is input-to-state stable. Therefore, the formation may not align exactly with the gradient direction, but will nonetheless able to move in a direction to decrease their measurements.

5. SIMULATION AND EXPERIMENTAL RESULTS

We implement the source-seeking algorithm in a mobile-robot test-bed developed in our lab. The test-bed consists of a standard 40W incandescent light bulb that generates a light field and several Khepera III robots. We use IR sensors mounted on the robots to measure the ambient light intensity. The higher the light intensity is, the lower the sensor reading is. Therefore, seeking the maximum of the light field corresponds to finding the minimum of the measured field.

We deploy two Khepera III robots that perform light source-seeking in the field. The velocities of the robots are determined by Equation (5), and are translated into step sizes in the experiment, Once one robot detects that $\| \mathbf{v}_i^\perp \| < \varepsilon_1$, it changes direction so that the two-agent group starts circular motion. Fig. 4 shows the snapshots of two agents moving towards the light source, and Fig. 5 demonstrates the trajectories of the two robots. The two figures shows that the two robots maintain a desired distance and converge to the light source.

In addition to the experiments, we also simulate eight agents in a scalar field seeking a minimum of the field. Fig. 6 demonstrates the trajectories of the eight agents in non-rigid body motion, in which blue dots are the positions of the agents. The formation is plotted every 30 steps. Since the agents control only the relative distances from other agents along direction $\mathbf{q}$, which corresponds to the yellow vector in the figure, they do not maintain a solid formation. As suggested by the figure, since the velocities of the agents depend on the measurements of the field, when the field value is high, the agents move faster, resulting in a larger step size. Fig. 7 illustrates the relative distances between neighboring agents in direction $\mathbf{q}$ as the agent group moves in the field, which shows the convergence of the relative distances to constant values.

Fig. 4. Trajectories of two agents seeking a light source.

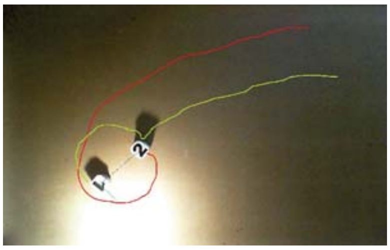

Fig. 5. Trajectories of two agents seeking a light source.

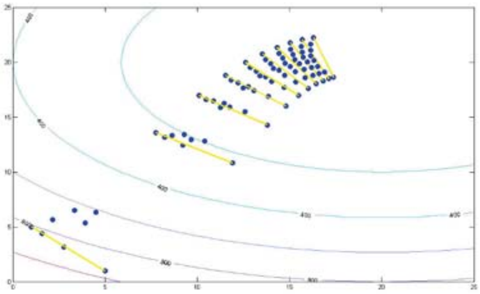

Fig. 6. Trajectories of an eight-agent group seeking a minimum in a field performing non-rigid body motion.

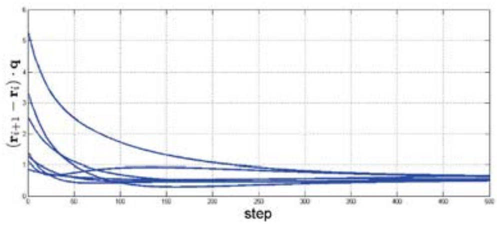

Fig. 7. Relative distances between neighboring agents in direction $\mathbf{q}$.

6. CONCLUSIONS

Inspired by the behaviors of fish groups, we develop source-seeking algorithms for a group of sensing agents with no explicit gradient estimation. By decomposing the velocity of each agent into two parts and designing each part as feedback control, we control the moving direction of the group to converge to gradient directions while formation is maintained. Our results show that a group of sensing agents are able to emulate the source seeking behaviors of a fish group.

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