Three Essays on Structural Credit Risk Modelling

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Introduction

This dissertation consists of three essays which are all devoted to the credit risk of non-financial companies and follow the so-called “structural approach”. The roots of this approach can be easily traced back to the seminal papers of Black and Scholes (1973) and Merton (1974), which have been followed by a large body of subsequent theoretical and empirical papers. In a nutshell, these models postulate that some process drives the firm’s value or cash flows and define conditions on this process, under which the firm defaults. Some of the more important extensions to Merton’s model that have been suggested include default prior to maturity (Black and Cox 1976), endogenous capital structure and default boundary (Leland 1994), strategic default (Anderson and Sundaresan 1996), discontinuities in the firm-value process (Zhou 1997), and mean-reverting leverage ratios (Collin-Dufresne and Goldstein 2001). However, there is still no general consensus whether this approach can explain the observed credit spreads and default probabilities. As its alternative was proposed the “reduced-form approach”, which was pioneered by Jarrow and Turnbull (1992) and which does not model explicitly the default event, but rather postulates that an intensity function for the default arrival time (depending on observable or latent factors) exists. Some steps towards the unification of these two approaches have been suggested in Duffie and Lando (2001) and Jarrow and Protter (2004).

In the first chapter of this dissertation I propose a new structural model, which tries to focus on three specific aspects, which, I argue, are especially important for the credit risk of the firm, but have not been given their due attention in past research. These are namely the dynamic control of the capital structure and the risk level of the firm, and the fat-tailed distribution of firm-value returns. Unlike most of the earlier literature, I use a discrete-time setup, which is generally better suited to reflect reality, where it takes time to both implement and reverse decisions and which gives me the additional flexibility to incorporate these factors and their dynamic nature. The tractability of the model stems from a powerful numerical procedure, which can be applied to a wide range of dynamic-control problems.

Combining and endogenizing the above mentioned factors, I am able to analyze the marginal effects of any one of them on the firm’s credit risk. I show that only a model with all three of them can fit the observed default probabilities. Further, I recognize that ownership and control of the firm are not the same thing. In my model, I have a manager, who owns a share of the firm's equity; still, his interests do not necessarily align perfectly with those of equity holders. This introduces a manager-stockholder conflict in the model and relates my work to the strand of literature that analyses this agency conflict. The analysis of the managerial optimal choices with respect to the capital structure and the risk level brings a new perspective in the understanding of corporate decision.

Using several different specifications of the model, I gain also deeper insights into the driving forces behind the firm's credit spreads and default probabilities. I document considerable effects for the managerial dynamic risk-taking and the possibility to reduce the current debt level as well as for the underlying distribution. Then I investigate my model's predictions in two other research directions: default risk premiums and the impact of managerial compensation on the firm's credit risk. In both of them the model predictions conform to empirical evidence and are an improvement relative to the standard models in the structural literature.

Only few of the numerous models within the structural credit risk literature consider financial distress explicitly. Most of the models (in the spirit of Leland 1994) assume perfect financial markets so the firm can always issue new equity as long as it is not worthless. By incorporating important market frictions and imperfections such as transaction costs and division of ownership and control, I show in the next chapter of my dissertation that their modelling is of considerable importance and hence should not be overlooked. To this end, I analyze the credit spreads and default probabilities of non-financial firms in a setting with frictions. The special focus in the paper is on investigating the incentives of equity holders to infuse additional cash in the firm, which is not used in the firm's investment activities but is solely intended as a repayment of some of the outstanding debt. The financial flexibility that the firm gains in this way reduces the likelihood of its liquidation and the accompanying formal bankruptcy procedure and at the same time preserves the tax advantage of debt financing. A real world analogue can be also a debt repayment trigger – for a discussion, see Bhanot and Mello (2006). The main finding of the paper is that if equity holders can *ex-ante* commit to undertake this capital restructuring, the firm can actually borrow at much lower rates and the overall value of their claim can be increased.

There are three distinct market frictions that I introduce into the model. First, any changes in the firm's capital structure incur transaction costs. Second, I distinguish between the ownership and control in the firm. The equity holders cannot run the company themselves due to time constraints, so they delegate the control to a manager. Even though they try to align his interests with theirs (they include stock and stock options in his compensation package which is standard in reality), there can still be divergence of interests in particular states. Finally, I assume that there is a certain limit on equity issues in the markets as no underwriter would be willing to place huge amounts of shares and not many investors would be willing to buy them.

In Leland (1994), it makes no difference if the firm issues new equity or equity holders infuse additional money as the financial markets are assumed to be perfect. In my model, the equity issues are limited and come at a cost, which turns the equity holders' decision to infuse cash into an important parameter of the model. I show that by agreeing to contribute up to 5.5% of the initial total firm value for the sole purpose of debt repayment, equity holders can reduce (relative to the case in which they do not contribute anything) the firm's long term credit spreads from 295 to 235 basis points. The effect is even more pronounced when I consider the five-year default probability: it shrinks from 3.8% to less than 2%. The force that brings those reductions is best described as enhanced capital structure flexibility.

The evidence that expected losses can explain only a small fraction of actual credit spreads has been referred to as the "credit spread puzzle". One potential explanation is that there are sizable premiums related to the default event. These default risk premiums have been measured as the ratio of risk-neutral and actual default probabilities and have been attracting considerable attention in the last years, but the research has been mostly based on the reduced-form approach and very little has been done within a structural framework.

I try to fill this gap and in the third chapter of this dissertation I evaluate the default risk premiums from a theoretical structural perspective, focusing on the effects which the capital structure dynamics induce. The analysis covers all three possible assumptions with respect to the changes in the debt level which a structural model can make: constant debt level, debt level that can be only increased, and debt level that can be both increased and decreased. Berg (2009) claims that all structural models make similar predictions with respect to the default risk premiums. I show that this is not the case and the capital structure assumption alone can change the model-implied default risk premiums considerably. My results suggest that only a model that has both debt level increases and debt level decreases can capture the stylized facts about default risk premiums as documented by the reduced-form empirical research. This finding is interpreted as a strong point in favour of such modelling, which has been generally omitted in the structural credit risk literature. Further investigation suggests that other factors such as dynamic risk taking, managerial discretion with respect to the capital structure, and non-normal firm value returns may affect the values of default risk premiums, but leave their basic behaviour unaffected.

In this chapter I further quantify in a regression framework the effects of the time horizon and the firm's credit quality on the default risk premiums. In this way, the different models' predictions become numerically comparable and their statistical significance can be evaluated. Another reason for this exercise refers to a potential application of such an approach to interpolate or extrapolate default risk premiums both for a firm, which already has traded credit instruments with a different maturity, and for other firms.

Zusammenfassung

Diese Dissertation besteht aus drei eigenständigen theoretischen Aufsätzen, in denen das Kreditrisiko von Firmen, die nicht dem Finanzsektor zuzurechnen sind, analysiert wird. Dabei wird der so genannten "strukturellen Ansatz" gefolgt. Dieser Ansatz geht auf die bahnbrechenden Arbeiten von Black und Scholes (1973) und Merton (1974) zurück und beinhaltet aber auch zahlreiche spätere theoretische und empirische Beiträge. Eine unvollständige Liste von wichtigen Erweiterungen des Models von Merton (1974) beinhaltet die Möglichkeit eines Konkurses vor der Fälligkeit der Anleihen (Black und Cox 1976), eine endogene Kapitalstruktur und Konkursschwelle (Leland 1994), den strategischen Konkurs (Anderson und Sundaresan 1996), einen zugrundeliegenden Preisprozess mit Sprüngen (Zhou 1997) und einen zum Mittelwert zurücktendierenden Fremdfinanzierungsgrad (Collin-Dufresne und Goldstein 2001). Was alle Modelle innerhalb dieser Forschungsrichtung charakterisiert, ist eine Betrachtungsweise, die einen stochastischen Prozess für den Wert der Firma oder für ihren Cashflow annimmt. Darauf aufbauend werden passende Bedingungen definiert, unter denen ein Konkurs vorkommt. Im Gegensatz zu diesen Strukturmodellen stehen die Modelle reduzierter Form. Diese Modelle definieren keine exakten Bedingungen für den Konkurs, sondern nehmen an, dass eine Intensitätsfunktion abhängig von beobachtbaren oder latenten Faktoren existiert, die zu jedem Zeitpunkt einen Konkurs verursachen kann. Duffie und Lando (2001) und Jarrow und Protter (2004) haben die ersten Schritte zur Vereinigung dieser beiden oben genannten Ansätze gemacht.

Obwohl alle drei Aufsätze dem Kreditrisiko gewidmet sind, analysieren sie ganz unterschiedliche Aspekte davon. In meinem ersten Aufsatz schlage ich ein Modell vor, das den Akzent auf folgende Faktoren setzt:

- die dynamische Steuerung der Kapitalstruktur
- die dynamische Steuerung des Unternehmensrisikos
- die nicht normale Verteilung der Firmenrendite.

Ich möchte betonen, dass diese Faktoren von besonders großer Bedeutung für die Wahrscheinlichkeiten der Kreditausfälle der Firmen sind. Alle drei wurden aber in bisherigen Modellen mehr oder weniger vernachlässigt. Während der größere Teil der Strukturmodelle in einem stetigen Zeitrahmen entwickelt wurden, arbeitet mein Modell mit diskreten Zeitschritten. Das gibt mir die Flexibilität, die genannten Aspekte in dynamischer Hinsicht zu

analysieren. Der andere erhebliche Unterschied ist, dass die wichtigen Entscheidungen in der Firma (die Kapitalstruktur und die Risikosteuerung) in meinem Modell vom Manager getroffen werden, während sie in den anderen Modellen den Aktionären überlassen werden. Für die Entscheidungen spielt dann natürlich die Entlohnung des Managers eine große Rolle, da sie Anreize für sein Verhalten setzt.

Meine Untersuchung zielt auf die Beleuchtung der optimalen Entscheidungen des Managers ab. Dann können sowohl die Wahrscheinlichkeiten der Kreditausfälle und die risikobedingten Zinsaufschläge der Firmen, als auch die genauen Mechanismen, die sie beeinflussen, analysiert werden. Es stellt sich heraus, dass die Vorhersagen meines Modelles mit den früheren empirischen Befunden zu Kapitalstruktur, Risikosteuerung, Kreditausfallwahrscheinlichkeiten, risikobedingten Zinsaufschlägen und die Auswirkungen der Anreizvergütung des Managers auf diese Variablen übereinstimmen.

Im zweiten Kapitel meiner Dissertation erforsche ich den Einfluss einer oft benutzten und als harmlos betrachteten Annahme: perfekte Finanzmärkte. Diese Annahme impliziert, dass die Firma unbehinderten Zugang zu Refinanzierung hat. In Wirklichkeit gibt es aber zahlreiche Beschränkungen dieses Zugangs und ich versuche speziell drei davon zu modellieren. Zum einen haben alle Änderungen der Kapitalstruktur realistische Transaktionskosten. Ferner delegieren die Aktionäre die Steuerung des Unternehmens einem Manager. Und schließlich werden neue Aktien nur bis zu einer bestimmten Menge emittiert. Im Model von Leland (1994), in dem Finanzmärkte per Annahme perfekt sind, macht es keinen Unterschied, ob neue Aktien emittiert werden, oder die Aktionäre Privateinlagen tätigen. Mit den oben genannten Beschränkungen ist das nicht mehr der Fall und dieser Unterschied ist durchaus von Bedeutung.

Ich zeige, dass sowohl die risikobedingten Zinsaufschläge als auch die Kreditausfallwahrscheinlichkeit einer Firma durch diese Entscheidung beträchtlich beeinflusst werden. Darüber hinaus wird es gezeigt, dass es für die Aktionäre optimal sein kann, sich im Voraus zu einer Zuzahlung zu verpflichten, da das die Wahrscheinlichkeit finanziller Schwierigkeiten und damit den Preis des Fremdkapitals reduziert. Gleichzeitig wird aber der Steuervorteil des Fremdkapitals bewahrt. Ein Analogon dieser Verpflichtung zu einer Zuzahlung, welches in der Realität oft beobachtet wird, ist eine Fremdkapitaldeckungsauslösung – eine detailierte Diskussion bieten Bhanot und Mello (2006). Ich zeige, dass die Effekte dieser Fremdkapitaldeckungsauslösung von bedeutender Größe sind: für eine Zuzahlung von 5,5% des Firmenwertes sinken die langfristigen risikobedingten Zinsaufschläge um 60 Basispunkte, während die fünfjährige Ausfallwahrscheinlichkeit von 3,8 auf 1,8 Prozent sinkt. Die Mechanik dieser Reduzierung des Kreditrisikos kann am besten als eine erhöhte Kapitalstrukturflexibilität bezeichnet werden.

Früher war es sehr schwierig die Risikoprämien eines Konkurses zu messen, weil die Anleihen der Unternehmen die einzigen zugänglichen Daten waren und ihre risikobedingten Zinsaufschläge von vielen anderen Faktoren beeinflusst werden (z. B. Steuer und Liquidität – s. Elton, Gruber, Agrawal, und Mann 2001). Die rasante Entwicklung der Märkte für Derivate und besonders für Credit Default Swaps (CDS) haben dazu beigetragen, dass sich in letzter Zeit die Qualität der zugänglichen Daten stark verbessert hat. Das hat auch dazu geführt, dass diese Risikoprämien von Forschern häufiger berücksichtigt werden.

Im dritten Kapitel meiner Dissertation träge ich den ebenfalls Rechnung, indem ich die qualitativen und quantitativen Auswirkungen der Kapitalstrukturannahme in einem Strukturmodell auf diese Risikoprämien untersuche. Ich erforsche speziell den Einfluss von zwei Faktoren auf die Risikoprämien eines Konkurses: die Fälligkeit und die Kreditqualität. Meiner Meinung nach ist eine solche Untersuchung sowohl aus einem theoretischen, als auch aus einem praktischen Blickwinkel interessant. Für die Theorie ist wichtig, ob die Vorhersagen eines Strukturmodelles mit den empirischen Ergebnissen der Modelle reduzierter Form übereinstimmen, während die Auswirkungen der Preisfestsetzung dieser Prämien besonders wichtig für die Praxis sind.

In der Untersuchung werden alle drei möglichen Annahmen bezüglich der Kapitalstruktur erfasst: statische Kapitalstruktur, nur zunehmende Fremdfinanzierung und dynamische Kapitalstruktur. Die Ergebnisse deuten darauf hin, dass nur ein Strukturmodel mit dynamischer Kapitalstruktur das stilisierte Bild von Risikoprämien eines Konkurses widerspiegeln kann, welches die empirische Forschung basierend auf den Ansatz reduzierter Form dokumentiert hat. Dies interpretiere ich als einen triftigen Grund eine solche Modellierung zu wählen.

Darüber hinaus quantifiziere ich in einer linearen Regression die genauen Effekte auf Fälligkeit und Kreditqualität Effekte um die Vorhersagen unterschiedlicher Modelle numerisch vergleichen zu können und die statistische Signifikanz der Ergebnisse beurteilen zu können. Ein weiterer Grund für die Regression liegt darin, dass dann durch Inter- und Extrapolieren die Risikoprämien der Firmen bewertet werden können.

Chapter 1

Probability of Default and Optimal Control of Risk within a Firm

1.1 Introduction

A large body of literature has been dedicated to the pricing of defaultable corporate securities, especially within the so-called "structural approach", pioneered by Merton (1974). This approach has certain attractive features – for instance that it can provide insights about the capital structure of the firm in addition to securities prices – and intuitive appeal, as it stands on solid economic grounds and models explicitly the bankruptcy event. However, there is still no general consensus whether this approach can explain the observed credit spreads and capital structures.

I propose a new structural model, which tries to focus on three specific aspects, which, I argue, are especially important for the credit risk of the firm, but have not been given their due attention in past research. These are namely the dynamic control of the capital structure and the risk level of the firm, and the fat-tailed distribution of firm-value returns. The capital structure has been mostly assumed to be static or modelled under other restrictions, while the firm risk is usually assumed to be exogenous. Despite abundant evidence against normality of financial returns (see e.g. Liesenfeld 2001), the geometric Brownian motion remains the standard assumption for the firm value process. Using a mixture of two normal distributions instead of the normality assumption for the log-firm-value return, I incorporate the excess kurtosis and negative skewness observed in financial returns. An additional motivation for the use of such a distribution is that, unlike all models using normal distribution, it can reproduce quite closely the observed short-term default probabilities.

Combining and endogenizing the above mentioned factors, I am able to analyze the marginal effects of any one of them on the firm's credit risk. I show that only a model with all three of them can fit the observed default probabilities. Further, I recognize that ownership and control of the firm are not the same thing. In my model, I have a manager, who owns a share of the firm's equity; still, his interests do not necessarily align perfectly with those of equity holders. This introduces an agency conflict in the model and relates my work to a relatively new strand of the literature, which tries to explain the observed low leverage ratios with managerial discretion.

Many structural models suffer also from internal inconsistencies. One of them is that they model the tax advantage of debt as an inflow, while it is actually only a reduction of an outflow. The subtle difference is that these models predict the equity claim becomes more valuable when the corporate tax rate increases, which makes little economic sense. To overcome this and other delicate problems, I resort to the framework proposed in Goldstein, Ju, and Leland (2001), which uses the firm's EBIT as the underlying variable and models all claims on it in a consistent fashion. Unlike this model and almost all other models in the literature, I use a discrete-time setup, which is generally better suited to reflect reality, where it takes time to both implement and reverse decisions. The tractability of the model stems from a powerful numerical procedure, which can be applied to a wide range of dynamic-control problems.

My findings on the firm's credit spreads show that the dynamic control of the firm's risk level has a material impact on them. I also find that the possibility to adjust the debt level downwards (which is not present in the Goldstein et al. 2001 model) can have a serious effect on the firm's credit risk. Omitting this possibility generally gives good results for the credit spreads from a practical perspective, as it tends to increase them and the standard critique towards the structural models is that they give too low credit spreads. However, one should bear in mind the serious evidence for non-default components in the credit spreads, especially a liquidity component – for evidence see Longstaff, Mithal, and Neis (2005) or Ericsson, Reneby, and Wang (2005). Given that the structural models abstract from non-default components in the credit spreads, it would be quite natural that they underestimate them. Fitting better the data by excluding an often observed phenomenon from a model – the downward adjustment of the leverage ratio by debt retirement and stock issuance – gives an incorrect picture of what really drives the firm's credit risk and the actual credit spreads. The exclusion further causes an overprediction of default probabilities relative to the empirical observations. When calibrated to reasonable parameters, my model with adjustments of the debt level in both directions predicts default probabilities closer in line with actual data, while the Goldstein et al. (2001) model is shown to give too high default rate predictions over medium and long horizons. One still needs to account for the fat tails of the return distribution to match the short-term default probabilities, and I do this by assuming a mixture of normals as an underlying distribution instead of the conventional Gaussian distribution.

In addition to providing credit spreads and predicting default probabilities, my model gives deeper insights about both capital structure and risk-taking of the firm. I investigate the manager's optimal choices with respect to these variables and the patterns I find are similar to what earlier research suggests. The model also conforms to the empirical findings of Vasvari (2008), who documents that the managerial compensation's incentives provide important information for the firm's credit spreads. Traditional structural credit risk models abstract from this effect by assuming an exogenous process for the firm value. In my model, the relationship between managerial compensation and firm's credit spreads arises quite naturally, as the manager has discretion over the firm's risk level and more option-like compensation encourages him to normally take on more risks. Focusing on the managerial option holdings, which in recent decades have become the most important component of CEO compensation[1], I show that moving from the first to the third quartile of the empirical distribution of managerial option holdings changes the firm's credit spread by 5% in my main model specification – a value that is of economically significant magnitude and within the interval between 4 and 9%, suggested by the empirical results of Vasvari (2008).

Finally, I investigate the default risk premiums (as measured by the ratio of risk-neutral to actual default probabilities) suggested by my model and the structural model of Goldstein et al. (2001). I show that my model predicts an upward sloping term structure of default risk premiums, a pattern that has also been suggested by Berndt, Douglas, Duffie, Ferguson, and Schranz (2005). Unlike my model, Goldstein et al. (2001) model predicts an almost flat, slightly downward sloping term structure. The dynamic capital structure and, in particular, the possibility to reduce the debt level turns out to be the important factor which drives the increase of this ratio over longer horizons. Both models predict that default risk premiums are

[1] Only for the time between 1992 and 2002, the share of options in managerial total compensation has increased from 24 to 47% - see Jensen, Murphy, and Wruck (2004).

higher for riskier firms, which has been also found in the earlier literature (see Berndt et al. 2005).

The remaining part of the paper is structured as follows. Section 1.2 reviews the related literature and motivates my approach. The central for the paper section 1.3 presents the different specifications of the model, including the model's components one by one and discussing their marginal effects. Section 1.4 provides some concluding remarks. Details about the numerical implementation of the model are given in Appendix 1.

1.2 Related literature

By its setup, my model belongs to the large class of structural credit risk models. The roots of this strand of literature can be easily traced back to Black and Scholes (1973) and Merton (1974), who introduced formally the contingent claims approach. In a nutshell, these models postulate that some process drives the firm's value and define conditions on this process, under which the firm defaults. Merton's model has been criticized for abstracting from important factors, which were consequently introduced into the framework. To name few of the important contributions, Black and Cox (1976) introduced default prior to debt maturity, Leland (1994) endogenized the capital structure and the default boundary, Zhou (1997) allowed for discontinuities in the firm value process (the so-called "jumps"), and Collin-Dufresne and Goldstein (2001) modelled mean-reverting leverage ratios. Although the research has been quite extensive, the results have fallen short of expectations. Most empirical studies document that structural credit risk models significantly underestimate credit spreads, starting with Jones, Mason, and Rosenfeld (1984) to Huang and Huang (2003), while Eom, Helwege, and Huang (2004) somewhat surprisingly document big errors in both directions. More optimistic were the results of Longstaff et al. (2005) and Ericsson et al. (2005), who show that a large part of the underestimation of the credit spreads can be explained by liquidity factors, and in this way restore some of the belief in structural models.

The two structural models that are most closely related to my work are those of Goldstein et al. (2001) and Carlson and Lazrak (2006). As already mentioned, the former paper provides the general framework, in which I set up the model, and also provides some necessary inputs – namely the final values of the claims I need to implement my model. For details, see the Appendix 1. With regard to the model of Carlson and Lazrak (2006), it basically focuses on the same two choices as my model does – the manager's capital structure and risk-taking choices. Apart from using the internally consistent framework of Goldstein et al. (2001), my model improves additionally the realism compared to Carlson and Lazrak (2006) in several directions. First, I allow dynamic control of the capital structure, compared to the static one in their model. Second, I put an upper limit on the risk level, while in their model the manager can increase volatility to infinity. Third, I introduce an option component in the managerial compensation, which empirically has grown immensely in the last decades to eventually exceed the cash component. Last but not least, I explore the effect a fat-tailed distribution of firm value returns has on the model's results relative to the normal distribution case.

The dynamic capital structure in my model relates it also to the work of Gamba and Triantis (2008). They model explicitly both the investment and financing decisions in the firm (I focus only on the financing decisions), but do not consider the manager-shareholder conflict and the controlled dynamic risk taking. Instead, their focus is on the financial flexibility in the firm and its implications for the total firm value.

Another large literature which is closely related to my work is the part of the agency theory which focuses on the manager-shareholder conflict. It has been long known, that the interests of these two parties diverge (e.g. Easterbrook 1984), but many models still assume that the capital structure is set to maximize shareholders' wealth. A strong point against this assumption is raised by Lewellen (2006). She argues that in practice the manager has considerable discretion in the capital structure choice, because shareholders are not fully informed or face a coordination problem. She then shows in a regression analysis that the managers' incentives can predict quite well the changes in the capital structure. In my model, the capital structure and the risk-taking decisions are both made by the risk-averse manager and he sets them to maximize his own utility. This approach can potentially explain the observed differences in the capital structures of similar firms as stemming from the risk preferences of the manager and/or his compensation terms. It can also explain why most firms seem to be underleveraged compared to the predictions of models in which the capital structure is set to maximize shareholders' wealth. The fact that the manager has a significant part of his wealth in the firm makes him much less diversified than the shareholders. This would make him act in a more cautious way, which translated to the capital structure choice means he would take on less debt. The results of Lewellen (2006), as well of those of Morellec (2004) and Ju, Parrino, Poteshman, and Weisbach (2005) can be considered as strong support of this statement. My model also confirms these findings, extending them to the case where the manager can dynamically control the risk level of the firm.

As my model can be used to predict the manager's risk-taking behaviour and options are part of his compensation package, it is also related to the research on the relationship between the managerial option compensation and firm risk. It has been suggested in this literature (see Coles, Daniel, and Naveen 2006 and references therein) that undiversified, risk-averse managers would forgo profitable investment opportunities if their outcomes are relatively risky.[2] Introducing options in the compensation package of the manager would then mitigate this problem and align better his interests with those of the shareholders, as options have convex payout structure and hence encourage riskier behaviour. However, the results with respect to option-based compensation incentives are not as one-sided as it seems. Carpenter (2000) develops a model in which a risk-averse manager compensated with options would increase the risk for most of the strike-to-price ratios, but for deep in-the-money options would actually decrease the risk relative to the case in which he trades on his own account. Similar results are suggested by the work of Hodder and Jackwerth (2007) and the empirical results of Hjortshoj (2007). The latter results, however, suggest that the options have to be too deep in-the-money (price-to-strike ratio of 2 or higher) to discourage risk-taking. The intuition is that for those price-to-strike ratios option values actually behave like the stock; the convexity of the payoff is lost and they make the manager even less diversified. I find that my model yields similar, but not identical results. It predicts further an area for out-of-the-money options, in which the manager decreases the risk level due to bankruptcy fears. Only very close to the default boundary, when the manager has little more to lose, does he increase the risk level again.

The managerial discretion over the risk taking in the firm introduces a relationship between the managerial compensation scheme and the firm's credit spreads in my model, which is missing in traditional structural models as they assume an exogenous process for the firm value. In an empirical study, Vasvari (2008) finds that the managerial compensation indeed plays a role in a big sample of syndicated loans, so omitting it from a model would bias the model predicted credit spreads.[3] I investigate the effect of managerial option holdings in my model and compare them to the evidence in Vasvari (2008).

There exists strong evidence against normality of financial returns and stock returns in particular. They were shown to be non-normal at short-horizons (e.g. Fama 1965; Liesenfeld

[2] This is a very similar reasoning to the one applied to the capital structure choice.

[3] Thinking in terms of a linear regression, this would not be the case so long as the firm characteristics are not correlated with the managerial option holdings. However, there is little reason to believe that the manager's compensation scheme is not correlated with the firm's characteristics.

2001), but also at longer horizons (e.g. Duffee 2002). When it comes to total firm value returns distribution, there is no real research due to unavailable data.[4] I assume that the documented properties of other financial returns (negative skewness and excess kurtosis)[5] are valid also for them. Various approaches have been suggested to incorporate these properties in the model. As already mentioned, one way is to introduce jumps in the underlying process. An alternative approach, which I actually implement, is to discretize the underlying process and assume a theoretical distribution, which has fatter tails than the normal. Among the many suggested distributions (t-distribution, Laplace distribution, other stable distributions) I choose a mixture of two Gaussian distributions. Any alternative distribution could be easily utilized. Despite the serious evidence against it, the biggest part of the contemporary structural credit risk literature keeps on working with the normality assumption, mostly because of its tractability. The flexibility of my numerical procedures allows me to choose any distribution and I show that with reasonable parameters the short-term default probabilities can be fit as well as the longer-term ones.

I also build on earlier work of Elton, Gruber, Agrawal, and Mann (2001), who show that default risk is among the significant components in credit spreads, and Berndt et al. (2005), who investigate in detail the ratios of risk-neutral and actual default probabilities as a measure of default risk premiums for a sample of firms in three industries – broadcasting and entertaining, healthcare, and oil and gas. They document considerable time variation in them, as well as increasing term-structure and negative relationship to the firm's credit quality. Their approach, however, is the so-called "reduced-form" approach, which does not model explicitly the default event, but rather postulates that an intensity function for the default arrival time (depending on observable or latent factors) exists.[6] I investigate the theoretical predictions of my model and the Goldstein et al. (2001) model and compare them in light of the findings in Berndt et al. (2005). I demonstrate that my model can be easily reconciled with those findings, which is not quite true for the Goldstein et al. (2001) model.

[4] Hecht (2000) merges bond and equity data to construct firm value returns. However, he looks only at the expected first moments and not on the whole distribution.
[5] The excess kurtosis or the 'fat tails' is the much better documented fact about stock returns, skewness could be positive at short horizons (e.g. daily).
[6] Often cited reduced-form models are those of Jarrow and Turnbull (1995) and Duffie and Singleton (1999). See e.g. Lando (2004) for a review of both structural and reduced-form models.

1.3 Model setup and results

1.3.1 The model

As a starting point in my model, I use the general framework suggested in Goldstein et al. (2001), in which the underlying variable is the firm's EBIT, but in which there is a direct relationship between EBIT and the total claim on the firm (referred to later also as "total firm value"). It consists of four distinct claims: the claim of equity holders, debt holders, the government, and the claim to the bankruptcy costs.[7] The available cash flow in every period is distributed so that first the coupon payment and the associated tax rate are paid out and then the remaining part of the cash flow is paid out to equity holders, who also have a tax obligation. If the available cash flow is not enough to pay the coupon, equity holders normally infuse additional money in the firm and prevent bankruptcy. For low enough values of the cash flow, however, they prefer to let the firm default. Details on the default boundary follow shortly.

I introduce several important changes to the model of Goldstein et al. (2001). First, there is a final horizon, after which the firm keeps on operating under the assumptions of Goldstein et al. (2001) and the values of the claims are therefore the analytical solutions of the dynamic version of their model.[8] Second, I develop the model in a discrete-time setting. Third, I allow dynamic control of the firm's risk level, so instead of a geometric Brownian motion, the firm in my model follows a controlled stochastic process of the form

$$\Delta \log V_t = \xi_t(\mu_a(\kappa)\Delta t + \sigma_a(\kappa)\Delta W_{at}) + (1-\xi_t)(\mu_b(\kappa)\Delta t + \sigma_b(\kappa)\Delta W_{bt}) \ , \tag{1.1}$$

where W_{it} for $i=a,b$ are uncorrelated Wiener processes and the drift terms μ_i and the volatilities σ_i are functions of the risk level κ, while ξ_t are independent Bernoulli random variables that take on values 1 and 0 with probabilities p and $1–p$ respectively. In particular, I assume the following functional relationships:[9]

$$\bar{\mu} = p\bar{\mu}_a + (1-p)\bar{\mu}_b \tag{1.2}$$

[7] In fact there is a fifth claim on the firm when the capital structure is dynamic: the claim on the transaction costs for the adjustments of the debt level.

[8] I consider also an alternative assumption for the final values in my model based on an iteration procedure. For details see Appendix 1.

[9] Equation (1.2) only introduces some notation.

$$\mu_i(\kappa) = \kappa r + (1-\kappa) r_f - (r - \bar{\mu}_i) - \frac{1}{2}\kappa^2 \bar{\sigma}^2 \tag{1.3}$$

$$\sigma_i(\kappa) = \kappa \bar{\sigma}_i \, , \tag{1.4}$$

where $\bar{\mu}_i$ and $\bar{\sigma}_i$ are the expected instantaneous return and the instantaneous volatility for the normal risk level (κ equal to 1). Note that the firm always pays out (as coupon payments, taxes, and dividends) a fraction $r - \bar{\mu}$ of its current total value.

This specification can account for fatter tails than the normal distribution and left skewness even within a single time period, if the parameter p is between 0 and 1. In the first two model specifications, however, I restrict p to be 1, so the one-period returns are still normally distributed with a mean and variance which are chosen by the manager among a set of available alternatives to maximize his utility.[10] As equations (1.3) and (1.4) show, to increase the mean of the distribution, the manager has to accept also an increase in the variance.

Intuition for the process (1.1) is that the returns come from a mixture distribution of two normal distributions. To keep the model parsimonious, I consider only three possible risk levels: high risk, standard risk, and low risk. At the standard risk level, the volatility is equal to my baseline value. The high/low risk regimes correspond to a 25% increase/decrease of the risk level. The admissible values of κ are hence 0.75, 1, and 1.25. Additionally, the manager determines the debt level in every period.

The extensions I introduce to the framework require that I keep track of two additional variables: the amounts of shares and debt outstanding. Their values are essential for the determination of the manager's optimal choices. Together with the total firm value and time, they form a four-dimensional grid, on which all model calculations are performed.

The exact mechanism of the changes in the debt level requires some more comments. To abstract from the investment side of the firm and focus on the capital structure decisions, the proceeds from a debt issue are used to buy back shares and the necessary cash for a debt repayment comes from an equity issue. Given that I allow only a discrete set of values for the possible number of shares, I have to use some rule to determine the new number of shares for a given debt change. I use the smallest number of new shares outstanding which makes the overall cash flow from the debt restructuring non-negative. Appropriate modelling of the

[10] The multi-period returns are generally not normality distributed even when p is equal to 1 as the manager can change the risk level.

capital structure changes requires that a relatively big number of possible shares outstanding are allowed, so this is the variable I use the most precise steps in (0.5%). This makes the residual cash flow pretty small; I still compute it and it is distributed among the equity holders. For areas in the grid close to the highest debt level or close to the default boundary, the manager would like to issue shares and repurchase debt, but the equity he can issue can earn him less cash than the necessary for the debt repayment. In practice, firms hold some cash, so I still allow such debt repayments, as long as the necessary cash for the transaction does not exceed 10% of the initial total firm value. This fraction is only slightly less than 20% of the combined debt and equity claims[11] and is thus in line with the evidence in Bates, Kahle, and Stulz (2009) that firms hold on average about 20% of their assets in cash. In the framework my model is set up, this would mean that some of the increase of firm value in normal times is kept as a cash buffer against negative future earnings shocks. For all security issues, the firm pays reasonable transaction costs[12], which are ultimately borne by the equity holders too. All capital structure adjustments take place at the open market and are hence undertaken at the market prices of the debt and shares.

To determine the optimal choices in all nodes of the grid, I compute the manager's wealth in each state at the final date. Then I can calculate the expected utilities for all possible choices one period before the final horizon and store the optimal choice together with the associated indirect utility. Then I can proceed working backwards and determine the optimal choices at all nodes of the grid. Having the optimal decisions, I can also price all claims on the firm, using an adjustment of the actual measure to the risk-neutral one. More details on all these aspects are given in the Appendix.

A. Manager characteristics

I assume the manager has preferences governed by a power utility function and hence has a constant relative risk aversion coefficient γ:

$$u(w) = \frac{w^{1-\gamma} - 1}{1-\gamma}. \tag{1.5}$$

[11] The debt and equity claims in my model normally comprise together between 55 and 60% of total firm value.

[12] As in Gamba and Triantis (2008), a debt issue has transaction costs of 2%, while an equity issue has 6%. These values are also in line with the evidence in Eckbo, Masulis, and Norli (2008).

Any other utility function could be substituted with ease since the numerical solution does not depend on the exact structure of the utility function.

His compensation consists of stocks and options of the firm. The manager also has some outside wealth, which is to be interpreted more broadly to include expected income from future employment. As a simplification I assume that he consumes all his cash compensation, so that it does not show up in his wealth in any period. He is fired if the firm value hits the default boundary and he then suffers a loss in his outside wealth. His total wealth at time T is:

$$w = aE_T + b\max(E_T - K, 0) + W \qquad \text{if no default occurs and}$$

$$w = (1-\varphi)W \qquad \text{if the firm defaults,} \tag{1.6}$$

where a is the number of stocks he has, E_T is the stock price at date T, K is the strike price of the options he owns (they are struck approximately at-the-money), b is the number of his options, W is his outside wealth, and φ is the fraction of the manager's outside wealth which he loses in case of a default.

Having a risk-averse manager is of course suboptimal for the (risk-neutral) shareholders. However, it would not be realistic to assume that they run the firm as they face time constraints. That is why they have to delegate the responsibility on the firm's operations to a third party – the manager – and they try to align his interests to theirs by means of the above compensation scheme.[13]

B. Default boundary

In my model I allow a discrete set of debt values. Even in the specification in which I allow both upward and downward adjustments of the debt level, I assume that the firm has some long term debt which cannot be repaid, so the first level is not at zero debt, but at some non-trivial (e.g. 0.2 for an initial firm value of 1) value. I calibrate the model to produce an initial leverage ratio as close as possible to the observed average leverage ratio of a Baa-firm (which Leland (2004) reports to be 43.3%), which gives an initial debt level of 0.27. The debt step I set at 0.1, as for smaller steps the manager tends to skip particular debt levels; adding them

[13] The issue about the optimal managerial contract goes beyond the scope of this paper. That is why I only restrict my investigations to a typical managerial contract.

only slows down the implementation of the model without changing the results qualitatively, while the quantitative changes are negligible.

At the final date, the debt values are set so that they would be equal to the discrete set of values I have (0.27, 0.37, 0.47, 0.57, 0.67) if the debt were riskless.[14] As the debt is not riskless, the market price at each state should be lower but should approach the face value as the firm value grows. Given my assumption that after this date the firm is run according to the assumptions of Goldstein et al. (2001), I set all debt values at the analytical values from their model. I also follow their approach in setting the default boundary at the final date.[15] This default boundary is based on the assumption that equity holders have the control on the firm and choose the default boundary to maximize the value of their claim. This assumption is not quite in line with my model, in which equity holders do not have the decisive say on the important decisions in the firm like the capital structure and the risk level. I still consider this choice suitable as it fits the empirical observations that some firms keep on operating with negative net worth (Davydenko 2007) and it further enhances the comparability of my model with the model of Goldstein et al. (2001). Set in this way, the default boundary for a particular debt level is simply a fraction of the debt gross value (i.e. the debt value including the government claim on it) of the riskless debt.[16] For my baseline parameters, this fraction is approximately 91%. The interpretation here could be also that a debt covenant is set at this particular value.

Finally, I assume that the default boundary is at this level for all earlier periods in my model. I also assume that upon hitting the default boundary, the bankruptcy costs are incurred and the firm control is transferred to the debt holders. They own the new, now all-equity financed firm, which is run again according to the assumptions of Goldstein et al. (2001). For the calculation of the debt values, however, I assume that the old debt holders immediately sell their shares and calculate their recovery based on their fair price.

C. Baseline parameters

In Table 1.1 I present the set of standard parameter values I use in my model.

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[14] I interpret these as the debt face values.

[15] As I use a discrete framework, I put the default boundary at the first node below this analytical expression.

[16] The following relationship between the gross and the net debt value holds:

$$GrossDebt = \frac{NetDebt}{1-\tau_i},$$

where τ_i is the tax rate for interest payments.

Table 1.1

Baseline parameters.

Initial total firm value:	$V_0 = 1$	Final horizon:	$T = 5$
Upper/lower boundaries:	$V_u = 1/V_d = 6$	Time steps:	$m = 20$
Firm value step:	$\Delta log(V) = 0.045$	Possible up moves:	$n = 20$
Required return (total firm):	$r = 0.1$	Mean:	$\mu = 0.065$
Risk-free rate:	$r_f = 0.04$	Volatility:	$\sigma = 0.236$
Coupon rate:	$r_c = 0.05$	Initial debt level:	$d_0 = 0.27$
Debt step:	$\Delta d = 0.1$	Minimal debt level:	$d_{min} = 0.2$
Shares step:	$\Delta log(log(ST)) = 0.005$	Manager's risk-aversion:	$\gamma = 2$
Managerial share:	$a = 0.005$	Manager's loss in default:	$\varphi = 0.2$
Managerial option holdings:	$b = 0.01$	Manager's outside wealth:	$W = 0.01$
Manager's options strike price:	$K = 0.15$	Bankruptcy costs:	$\alpha = 0.3$

The parameters are chosen to represent a firm with a rating of Baa. For the volatility of the total firm I take the estimated parameter of Eom, Helwege, and Huang (2004) – 0.236 – as they have a sample in which the average firm is very close to the rating I have in mind, just a notch or so higher.[17] I additionally checked what equity volatility this value implies according to the Goldstein et al. (2001) model.[18] For the initial firm value and the leverage of a Baa-rated firm, it translates into 0.403, which is very much in line with the observed stock volatility of a median firm with this credit rating.[19] The risk premium is 6%, and the initial debt level is chosen to fit a leverage ratio of 43.3%, which is the average leverage ratio for a Baa-rated firm (Leland 2004). The managerial share and option holdings of the firm are

[17] Eom et al. (2004) discretize credit ratings by notches and the average rating in their sample is between A- and Baa+.

[18] I applied the delta method (see e.g. Greene 2000, p.72) to get the variance of a function of a random variable. In this case the value of the equity claim is a function of total firm value, whose variance I assume I know. Then, if

$E = f(V)$,

$Var(E) = Var(V)\,|\,f(V)'|^2$.

[19] In fact, Ericsson et al. (2005) document an average stock volatility of 0.46 in a sample of bond issuing companies predominantly from the Baa rating class. However, as the distribution of firm volatilities is right-skewed, the median is close to 0.4.

chosen so that the value of stocks and options are roughly equal as the sample of Vasvari (2008) suggests. The managerial loss in default of 20% is the cut in salaries documented by Fee and Hadlock (2004), which fired CEOs had to accept on the new job – if they managed to find a new job at all.

Additionally, I need three tax rates: the corporate tax-rate, the tax-rate on interest payments, and the tax-rate on the dividend payments. I follow Goldstein et al. (2001) and set the first two of them at 35% and the last one at 20%. Finally, I need some reasonable range in which the firm can issue/repurchase shares. For the debt region I allow, issuing and repurchasing about one third of the current amount of seem reasonable choices. Increasing the range did not change the results materially.

1.3.2 Benchmark case: only upward adjustments of the debt level and normal returns

To ensure better comparability with the work of Goldstein et al. (2001), I start with a benchmark model, in which I allow the manager to adjust the debt level only upwards and keep the normality assumption for the firm value log-returns (i.e. the parameter p is equal to 1). I will later allow both upward and downward adjustments of the debt level; in a last step, I will consider a leptokurtic negatively-skewed return distribution from the mixing of two normal distributions.

As I am not modelling explicitly the investment side of the company, I will consider only debt issues whose proceeds are used exclusively for stock repurchases; and in the later stages, when I allow also downward adjustments of the debt level, all debt retirements are financed by stock issuance. With my baseline parameters, the model of Goldstein et al. (2001) predicts a credit spread of 210.8 basis points. This credit spread, as well as the credit spreads predicted by my model, are to be understood as the spreads for long-term debt (e.g. with a maturity of 20 years).

Credit spreads are calculated in the following way: the price at T = 0 of a risk-free debt with the same coupon as the debt in my model is determined. If the price of this debt is $D_{rf,0}$ and the price, which my model predicts for the risky debt is D_0, then the credit spread is

$$CS = -10^{-4} \log(D_0 / D_{rf,0}) / T \,.$$

The results on the credit spreads and some comparative static analysis are presented in Table 1.2. The major reason for the increase of the credit spread in my baseline case relative to the

Goldstein et al. (2001) model is that my manager is able to increase the risk level of the company and this is indeed what he does for firm values close to the initial firm value. Another difference is that in Goldstein et al. (2001) the debt level is always increased at a fixed total firm value, while manager in my model increases the debt level at different firm values depending on the time remaining to the final date. This second difference accounts only for a small part of the increase in the credit spreads, while the first accounts for almost all of it.

Table 1.2

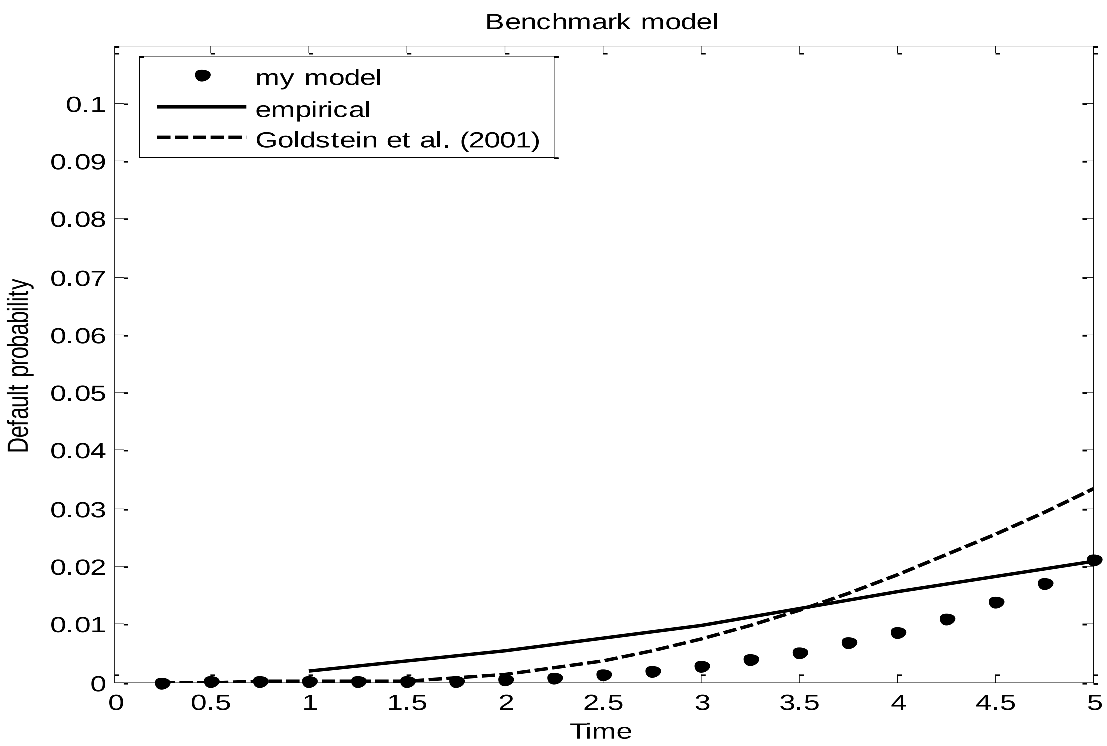
Credit spreads in the model specification allowing only upward debt adjustments

Parameters		Credit spread (bps)
Baseline case		300.6
Initial leverage	d_0 = 0.2 (leverage ratio: 32%)	214.1
	d_0 = 0.44 (leverage ratio: 65.7%)	505.3
Total firm value	V = 1.1	278.8
	V = 0.9	324.0
Volatility	σ = 0.216	260.4
	σ = 0.256	337.9
Managerial risk aversion	γ = 1.5	311.2
	γ = 2.5	292.2

The comparative statics of a set of parameters in my model yield intuitive results: the current leverage level plays an important role, as does the volatility of the firm. The changes in the leverage are such that they represent movements to an A-rated firm (leverage ratio of 32%) and a B-rated firm (leverage ratio of 65.7%), where the leverage ratios are the average ones for the respective rating categories – see Leland (2004). The managerial risk aversion also seems to play some role for the firm's credit risk, as more risk-averse managers would choose less aggressive risk-taking and leverage and hence reduce the firm's credit risk.

Figure 1.1 compares the empirical default probabilities and the predicted default probabilities from my first model specification and the Goldstein et al. (2001) model. For both models it is true that they underpredict the short-term default probabilities and overpredict the longer term default probabilities. Given the predicted credit spreads, it is somewhat surprising that my model actually predicts lower default probabilities. The explanation is in the risk taking pattern of the manager (Figure 1.2). For total firm values close to the initial firm value the manager chooses the high risk level. If the firm value increases, the manager's wealth becomes more concentrated in his equity positions and hence he becomes less and less diversified. This makes him gradually decrease the risk level and for high firm values (on average 2.5 times the initial one) his optimal choice is to set the risk level at the low value. If the firm value decreases from its initial value, there is a similar decrease in the risk level; that is because the manager fears the bankruptcy. This time, however, the decrease is less gradual. For values very close to the default boundary (basically on the verge of the bankruptcy), the manager is willing to gamble[20] once more, as his stock and options have a very low value, which puts him in a situation where he has little to lose.

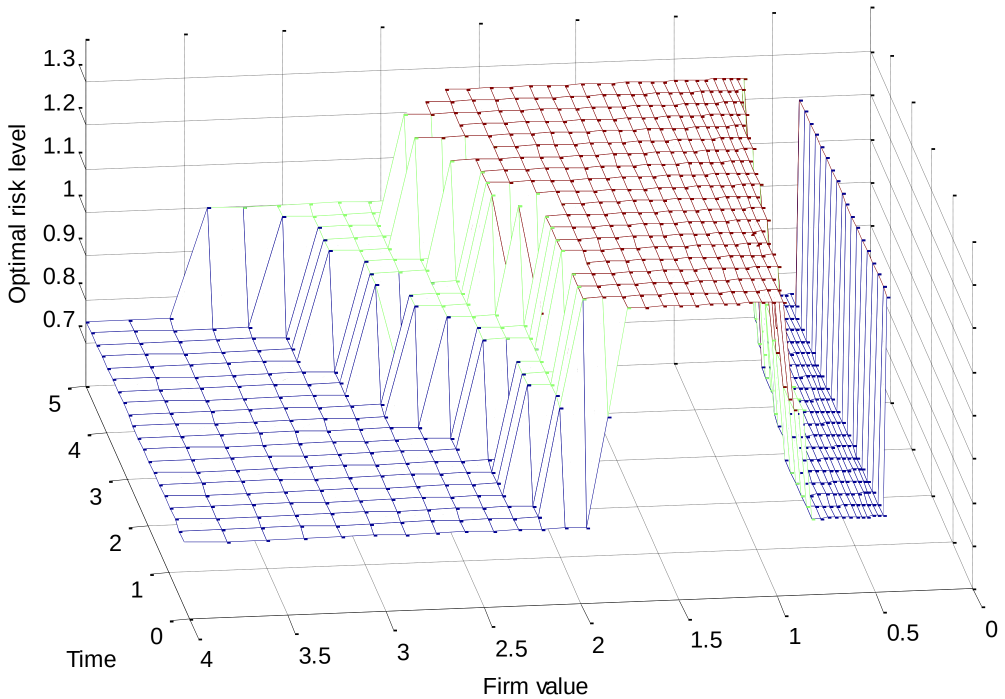
Fig. 1.1 Cumulative default probabilities predicted by my benchmark model with only upward adjustments of the debt level and normal distribution for the firm value returns, the Goldstein et al. (2001) model, and the empirical default rates as reported by Moody's (2005)



[20] This phenomenon is known in the literature as "gambling for resurrection" – see e.g. Dangl and Lehar (2004).

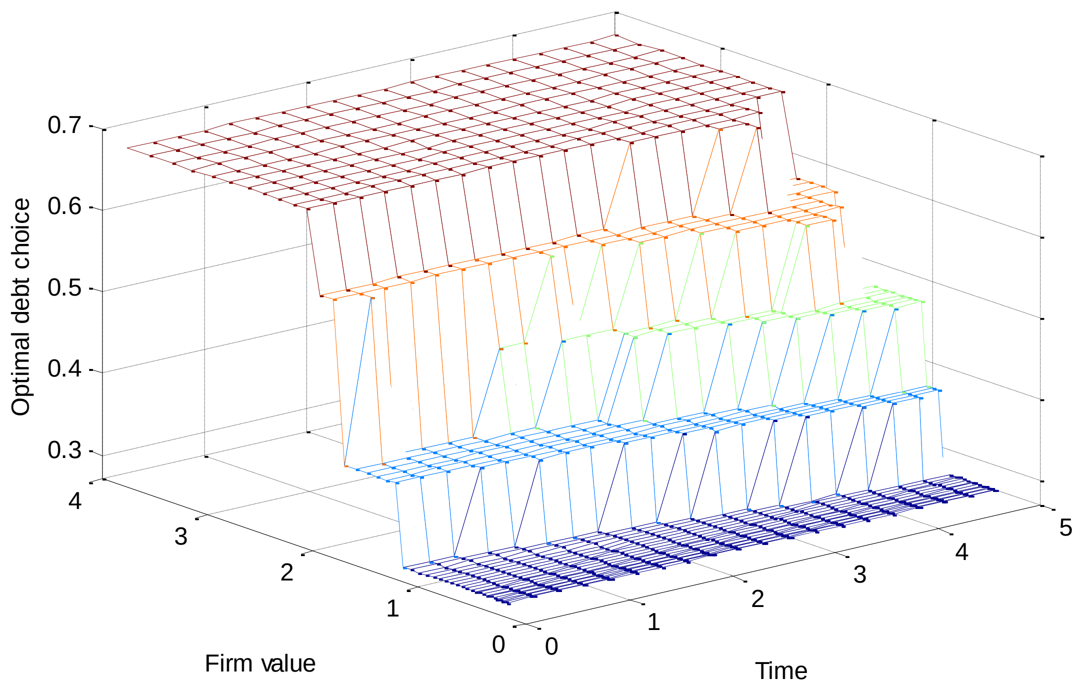
The risk-taking pattern is similar at higher debt levels, just the option ridge in the middle covers a smaller range, until it finally disappears for the debt level of 0.67 (which is reached with low probability and for firm values close to 3). Higher or lower risk-aversion coefficients relative to the baseline case do not change the results qualitatively, but have impact on the size of the option ridge. What can be also noticed in Figure 1.2 is that the manager's risk appetite generally grows with the passage of time and is highest shortly before the final date in the model. All these together create a distribution with more mass further from the mode (in both directions) relative to the geometric Brownian motion benchmark. As the gains bring little for the debt value, but the losses hurt it substantially, the long term debt values are decreased.

Fig. 1.2 Managerial risk taking at the lowest debt level for the benchmark model



The next step is to analyze the capital structure decision, which is best illustrated in Figure 1.3. Since the decision to increase the debt level is irreversible in this specification, the manager is willing to keep the current debt level initially for a relatively wide range of total firm values. As the final horizon approaches, the range reduces and the manager is willing to increase the debt level for a lower total firm value, because the probability to hit the (higher) default boundary at the higher debt level becomes smaller. A very similar pattern is observed at higher levels of debt, too. Overall, this model specification does not allow stating an interval, in which the manager does not change the capital structure as this interval is time-dependent.

Fig. 1.3 Optimal debt level choice of the manager in the baseline model given the firm is at the initial debt level



1.3.3 Upward and downward adjustments of the debt level and normal returns

In practice, firms adjust their capital structure in both directions. Leary and Roberts (2005) analyze the capital structure adjustments and find that 40% of them are debt issuances and 28% are debt retirements; stock issuances and stock repurchases account respectively for 17 and 14%, all this meaning that upward and downward adjustments of the capital structure are undertaken almost equally often (the fact that increases in the debt levels happen more often than decreases can be easily attributed to the overall growth of firms' assets). In view of these empirical observations, it is important to consider both upward and downward adjustments of the firm's debt level.

Table 1.3 documents my results for the credit spreads in this model specification. It is striking how much the credit spreads change once I allow downward adjustments of the debt level. Note that there is still a non-trivial proportion of the debt which cannot be repaid (0.2). Whenever I change the initial debt leverage, I change in the same proportion the amount of debt which cannot be repaid. These credit spreads are pretty close to what can be seen in the markets as the long-term credit spreads for a typical Baa-rated firm (in normal times) are in the neighbourhood of 200 bps. As regards the comparative statics, they do not change too much from the previous model specification.

Table 1.3

Credit spreads in the model specification allowing both upward and downward debt level adjustments.

Parameters		Credit spread (bps)
Baseline case		224.0
Initial leverage	$d_0 = 0.2$	171.4
	$d_0 = 0.44$	413.8
Total firm value	$V = 1.1$	213.6
	$V = 0.9$	242.5
Volatility	$\sigma = 0.216$	183.3
	$\sigma = 0.256$	263.8
Managerial risk aversion	$\gamma = 1.5$	229.3
	$\gamma = 2.5$	216.8

Figure 1.4 depicts the manager's debt level choice in this model specification. Here the total firm values, at which the manager switches to a higher debt level, are almost time-independent. This change in the manager's behaviour in comparison to the benchmark case owes primarily to the fact that the debt increase is now a reversible decision: the manager can issue equity to retire some of the debt; some proportional transaction costs are still incurred though. Overall, the manager would leave the capital structure unchanged for leverage ratios between 25 and 50%. Such leverage ratios are easily reconciled with real world observations.

For the dynamic risk-taking in this mode specification, changes are hardly discernible relative to the benchmark model and Figure 1.2.

Fig. 1.4 The manager's optimal debt level choice in the specification allowing both upward and downward adjustments of the debt level given he is currently at the initial debt level of 0.27

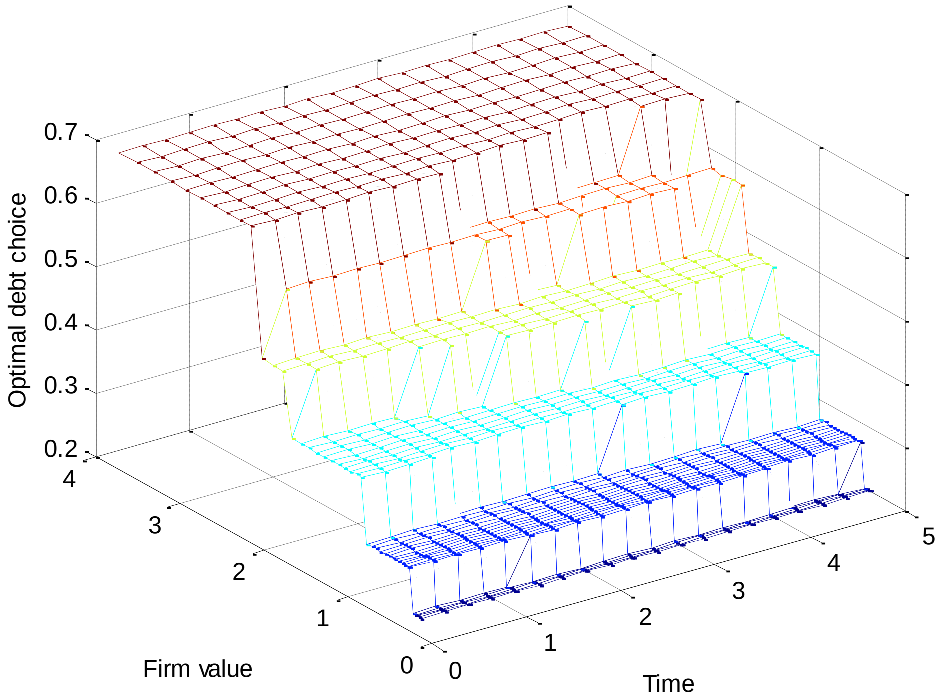


Fig. 1.5 Default probabilities predicted by my model with upward and downward adjustments of the debt level, the Goldstein et al. (2001) model and the empirical default rates as reported by Moody's (2005)

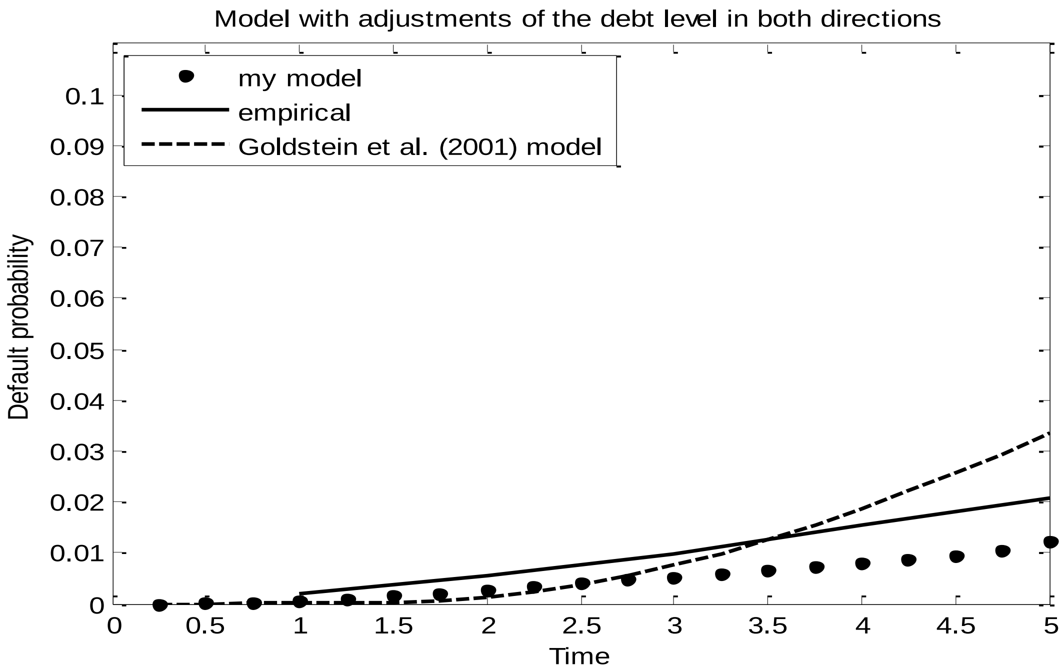


Figure 1.5 illustrates the effect of the possibility to reduce the amount of outstanding debt by issuing new equity on the default probabilities. Results generally appear to be quite plausible, as the probability to default between the third and the fifth year is captured relatively accurately, with only marginal underprediction. The main remaining problem is the underestimation of the short-term default probabilities.

The last component of the model I would like to introduce is the fat-tailed mixture distribution.

1.3.4 Main model specification: mixture distribution

The marked underprediction of short-term default probabilities has motivated me to explore alternative distributions, which could account for the empirically found excess kurtosis and negative skewness in financial returns (see e.g. Liesenfeld 2001; Duffee 2002). One parsimonious way to capture these stylized facts would be with a mixture of two normal distributions – one with a high mean and a low variance and one with a low mean and high variance. In an attempt to explore this possibility to account for the observed fat-tailed financial returns distributions, a mixture distribution of the above-mentioned type substitutes for the normal distribution in the previous model specifications by removing the restriction on the parameter p. I determine the exact mixture distribution to be used in a calibration exercise of fitting the empirical default probabilities reported by Moody's (2005). In so doing, I fix the mean and the volatility of the new mixture distribution to my baseline values of 0.065 and 0.236 respectively, and vary the parameters of the two compounding normal distributions.

Figure 1.6 presents the result of the calibrating procedure for the baseline credit rating Baa. As a robustness check I run the model for two additional credit ratings – A and B. The results for these credit ratings are presented in Figure 1.7. As can be seen, the empirical default probabilities can be matched pretty closely by the mixture distribution. The exact parameters of these distributions – means and volatilities of the mixed distributions, and the mixing parameter together with the volatility, skewness, and kurtosis of the resulting mixture – are given in Table 1.4. Table 1.5 presents the results for the credit spreads and their comparative statics, which are not very different from the previous model specification. The results for the risk-taking and debt level choices generally remain unaffected, apart from a hardly discernible shrinkage in the risk-taking ridge.

Fig. 1.6 Comparison of default probabilities predicted by my main model and the Goldstein et al. (2001) model with the empirical default probabilities for Baa-rated firms as reported by Moody's (2005)

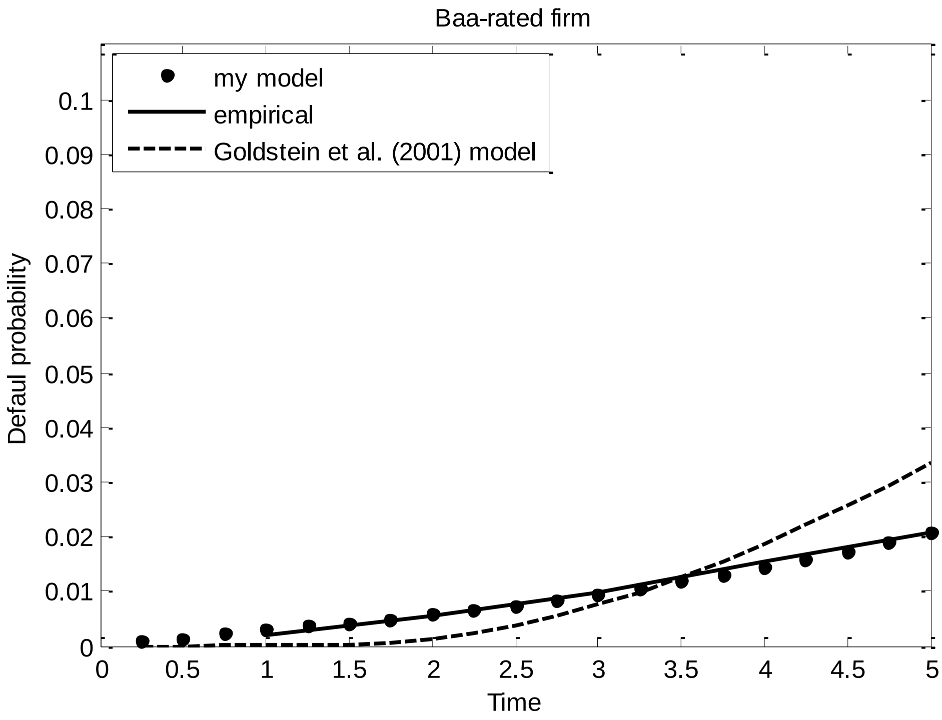


Table 1.4

Parameters for the mixed distributions and the second, third, and fourth standardized moments of the mixture distribution for different rating classes.

Parameters	Values
μ_a	0.08
μ_b	-0.02
σ_a	0.189
σ_b	0.4
Mixing parameter (p)	0.85
Skewness	-0.33
Kurtosis	5.09

Table 1.5

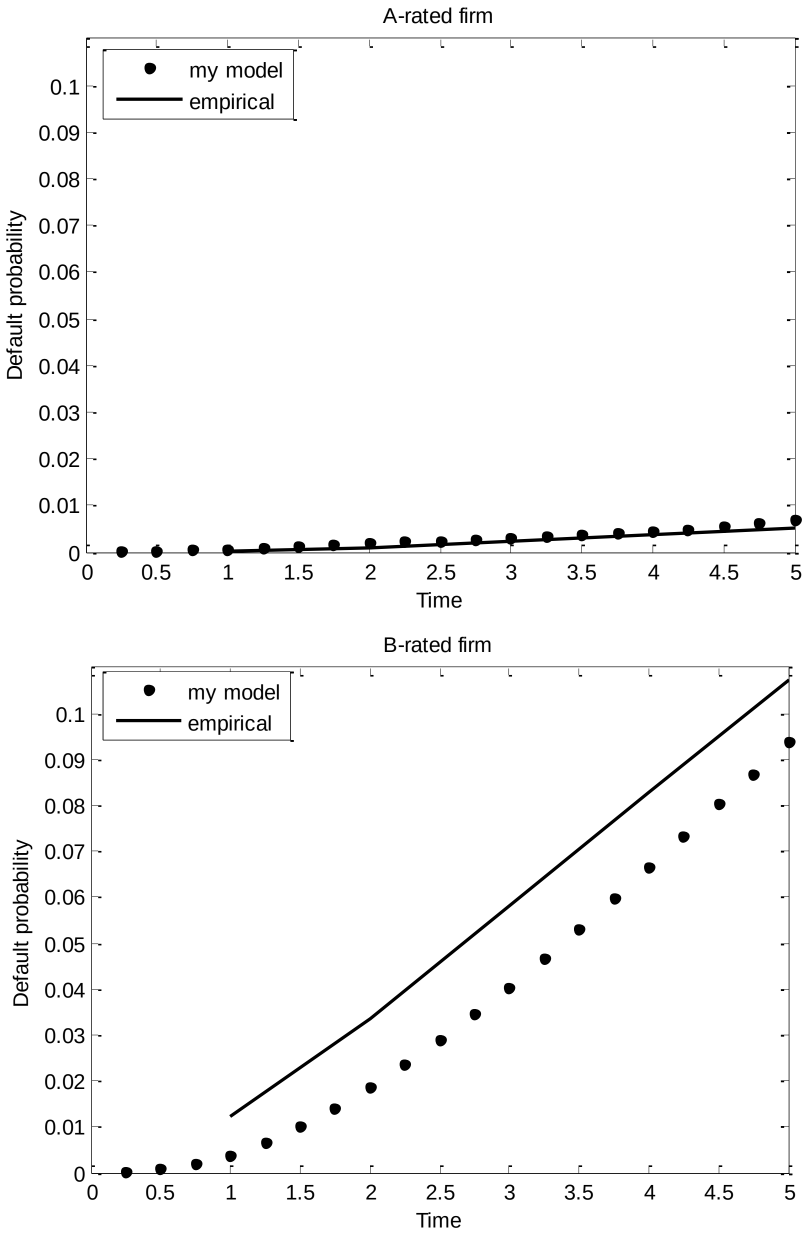
Credit spreads in the main model specification.

Parameters		Credit spread (bps)
Baseline case		214.3
Initial leverage	d_0 = 0.2	156.9
	d_0 = 0.44	401.3
Total firm value	V = 1.1	210.2
	V = 0.9	230.2
Volatility	σ = 0.216	174.0
	σ = 0.256	247.2
Managerial risk aversion	γ = 1.5	223.2
	γ = 2.5	201.9

In the robustness checks for different credit ratings, I only change the initial leverage to match the reported values in Leland (2004) for the average leverage ratios of A- and B-rated firms. As can be seen, the model tends to predict slightly higher default probabilities for the A-rated firms (and for that also slightly higher credit spreads than the empirically observed), while the predicted default probabilities for the B-rated firm are a bit too low. However, the overall shape is reasonably matched. I interpret the minor divergences as a sign that there can be some additional differences (apart from the leverage) between a Baa-rated firm, an A-rated firm, and a B-rated firm. The real patterns would look exactly like this if, for instance, more risk-averse managers tend to run safer firms.

Overall, these results are very encouraging. Even though it was not stressed until now, my model gives both actual and risk-neutral default probabilities. The precision in reproducing the former gives me confidence also in the latter. Therefore my model can extract with a very good precision the default components of credit spreads. Later I will look at the default premiums which my model suggests. The model could be further used to analyze the residual spreads and/or the pricing kernel for a particular company, which topics I leave for future research.

Fig. 1.7 Comparison between model-implied and empirically observed default probabilities for different credit ratings (an A-rated firm in the upper panel and a B-rated firm in the lower one) when a mixture distribution is used instead of the normal one



1.3.5 The effect of managerial option holdings

Having a risk-averse manager as the decision maker in the firm, my model incorporates an endogenous relationship between the managerial compensation and firm risk – a relationship, which is completely missing in the Goldstein et al. (2001) model. A recent work by Vasvari (2008) gives evidence that managerial incentives are indeed priced by market participants. In a sample of almost 7,000 syndicated loans, he documents an increase of credit spreads between 4 and 9% for a move of the first to the third quartile for managerial payment incentives depending on the proxy used (delta, gamma, share of equity-based compensation) after controlling for all other relevant factors.[21] These results are another confirmation that managers have an impact on the firm's riskiness and omitting the channel through which they exercise their control (which is done in the majority of structural credit risk models) leads to an incomplete picture.

To compare my model's predictions to these results, I investigate how the model-implied credit spreads react to similar changes in the managerial option holdings. In the sample of Vasvari (2008), the Black-Scholes value of option holdings of the top five executives increases about four times from the first to the third quartile. I decrease the option holdings by two times relative to my baseline scenario to represent the case of the first quartile and increase them by two times for the third quartile. These changes should match relatively closely the movement from the first to the third quartile of managerial pay incentives in the sample of Vasvari (2008). The resulting changes for the credit spreads are in the expected direction: 207.8 basis points for the low managerial option holdings and 218.5 bps for the high managerial option holdings. The overall change in the credit spread (10.7 bps or a bit more than 5% of the spread) is within the interval suggested by Vasvari (2008). I further investigated the exact relationship between the managerial option holdings and the firm's credit spreads. I find that it is concave: increasing option holdings has much bigger marginal impact for low initial holdings than for high. Increasing this parameter beyond 0.02 barely changes the credit spreads, as these changes have the effect of making the manager less and less diversified. The shape and the sensitivity of this relationship does not differ much between the different model specifications.

1.3.6 Default risk premiums

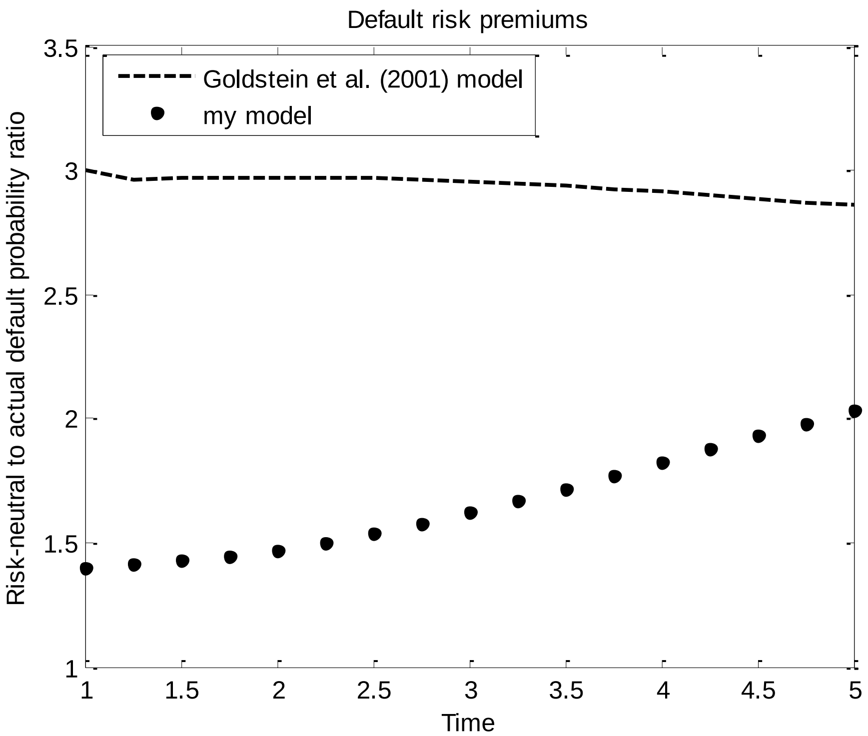
One of the proposed explanations for the structural models' credit spread underprediction has been the presence of a premium for default risk. Elton et al. (2001) show that a systematic factor can explain the bulk of the variation in residual spreads after they have accounted for the expected default loss and tax effects. In a reduced-form setting Berndt et al. (2005) measure the default risk premium as the ratio of risk-neutral to actual default probability.

[21] Vasvari (2008) considers the total compensation of the top five executives in the firm, while I consider a single manager. I acknowledge that major decision might not be taken by a single person in the firm, be it CEO or anybody else. However, I believe that my approach is only a simplifying assumption which approximates reasonably well the economic incentives of the board of executives.

For the intuitive appeal of this measure (it tells us by how much the actual default probability has to be multiplied in the credit spread formation so that investors are prepared to bear the default risk), I will also apply it for my analysis and compare my model's results with Berndt et al. (2005) and with the predictions that can be extracted from the Goldstein et al. (2001) model with respect to it.

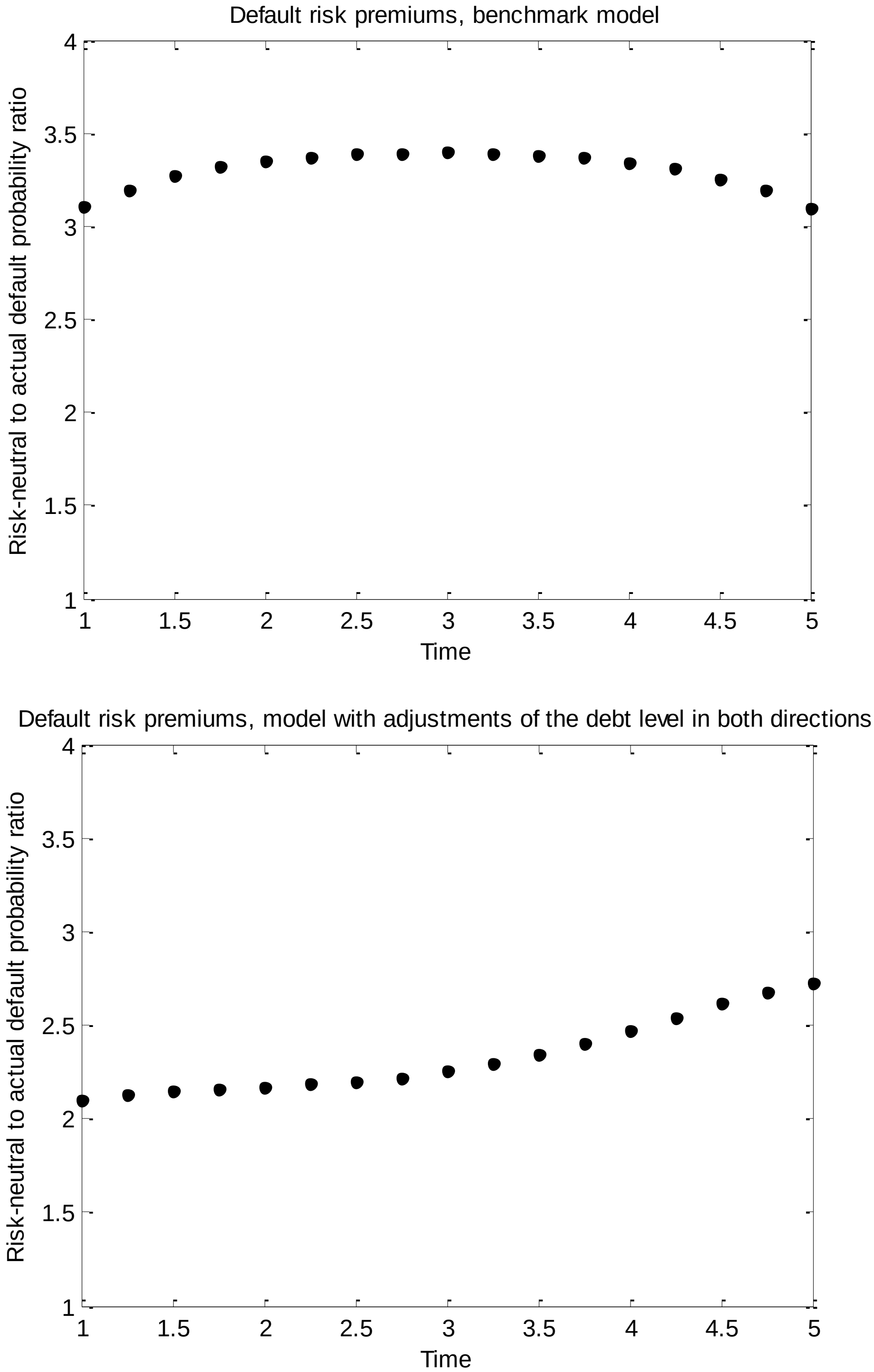
To this end, I measure the risk-neutral and actual default probabilities for every time step implied by my and Goldstein et al. (2001) model. For the initial firm value, these ratios are plotted in Figure 1.8 for both models. What can be seen is that my model predicts that this ratio is increasing over time, while Goldstein et al. (2001) model has it roughly constant, with a very slight decline over time. The evidence documented in Berndt et al. (2005) suggest strongly that this ratio increases over time and the authors interpret this result as a premium for the time-varying default intensity in their reduced-form model.

Fig. 1.8 Default risk premiums as measured by the ratio of risk-neutral to actual default probabilities



The magnitudes suggested by all three models are comparable, slightly higher in Berndt et al. (2005) with a median value in their sample between 3 and 4 in normal times (the value increases slightly at times of crises, when investors demand additional premiums for bearing default risk). It should be also noted that the ratio in all models is higher for safe firms and lower for risky firms – about 1.6 for the B-rated firm and close to 4 for the A-rated at the five-year horizon. In a further investigation, I find that the major factor for the upward slope of this curve in my model is the possibility to reduce the debt level, confirming once more the importance of this model component. Figure 1.9 shows the default risk premiums predicted by the first two model specifications.

Fig. 1.9 Default risk premiums as measured by the ratio of risk-neutral to actual default probabilities: my benchmark model specification (upper panel) and the model specification with normal distribution and upward and downward adjustments of the debt level (lower panel)



1.4 Conclusion

I present a structural credit risk model which focuses on the manager's optimal dynamic capital structure and risk-taking decisions, and also accounts for the non-normality of the total firm value returns. In my opinion these are the most important determinants of the firm's credit risk, but they have been modelled in an oversimplified fashion in previous research. I endogenize them in my model, which gives valuable insights with implications that extend beyond the traditional structural credit risk approach to topics such as agency theory and managerial option compensation.

With my different model specifications, I am able to show and analyze the effects of the separate components on the firm's credit spreads and default probabilities. I show that one of the major weaknesses of the structural credit models – namely, their inability to match short term credit spreads and default probabilities – can be overcome with more involved modelling. It takes, however, all three important components in my model – the dynamic capital structure with upward and downward adjustments of the debt level, the managerial risk-taking, and the fat-tailed distribution – to fit the empirical default probabilities. Additionally, I investigate the impact of the managerial compensation on the firm's credit spreads and the firm's default risk premiums. The model predictions conform to findings in previous empirical research.

Appendix 1

The underlying variable of the model is the *EBIT* or the payout flow from the firm's assets. Under the assumptions of a representative agent with a power utility function and a constant volatility of the payout flow on the firm, there is a one-to-one relationship between this payout flow and the total claim on the firm *V* (see Goldstein et al. 2001). This relationship is given by

$$V = EBIT/(r-\mu), \tag{A1.1}$$

where r is the required return on V and μ is the expected growth rate of *EBIT*. One way to think about this relationship is that on average a fraction μ/r of the cash flow is reinvested in the firm, while the remaining fraction is distributed among the firm's claimants.

The total claim on the firm consists of five distinct components: the claims of equity holders, debt holders, the government, the claim on the bankruptcy costs, and the claim on the restructuring costs.[22] The following relationship holds at any point in time:

$$V = E + D + G + BC + FC \tag{A1.2}$$

Here E is the value of the equity claim, D is the value of the debt claim, G – the government claim, BC – the claim on the bankruptcy costs, and FC – the claim on the flotation costs.

I assume that the relationship (A1.1) between *EBIT* and *V* holds, although in my model the manager can change the volatility of the payout flow. This is only a simplifying assumption which considerably enhances the tractability of the model. What actually happens is that my manager can reach higher/lower levels of μ at the expense of higher/lower volatility. Then to keep the relationship (A1.1), the required return for the total claim on the firm r should also change. I checked how this required returns should look like across the time and firm value dimensions so that the assumption for the constant relationship (A1.1) still holds and compared them to a benchmark constant volatility case.[23] I obtained required returns which are slightly higher than the benchmark case when the manager would choose higher than the standard risk, and slightly lower when the manager would choose lower than the standard risk. For a required return on the firm of 10%, the deviations were about 1% for a 25% increase in volatility. These intuitive results show that the assumption corresponds reasonably

[22] For details, see Goldstein et al. (2001).

[23] For a constant volatility, the required rates of return are constant across the firm value and time dimensions, which is the prerequisite for the one-to-one relationship.

well to a model with an endogenous relationship between the risk and the required return on the total firm claim.

I extend the framework proposed in Goldstein et al. (2001) by allowing the manager to dynamically control the risk level of the firm and to adjust the capital structure in both directions. Further, in my main model specification I assume a fat-tailed mixture distribution for the firm value returns, which can be considered as a realisation of two separate random variables: first, a Bernouli random variable determining which out of two Brownian motions will give the return; and second, the realisation of that particular Brownian motion.

These changes require several major adjustments. I use a discrete-time setting and a final horizon, at which the firm is sold to a third party. At the final horizon I set the values of the claims at all nodes based on the analytical results of Goldstein et al. (2001). An alternative interpretation of this approach is that from then on the firm is run according to the assumptions in their dynamic model. However, I consider also an alternative set of final values of the claim based on my model, which is going to be commented upon at the end of this appendix.

I further have to introduce two additional state variables: the debt outstanding and the number of shares. Together with time and total firm value, they form a four-dimensional grid, on which the model is implemented. I use quarterly time steps, constant steps in *Δ(logV)* of approximately 4.5%, debt steps of 0.1, and share steps in *Δ(log(log(ST))* of 0.5%. The constant steps in firm value ensure I can use the same transitional probabilities, conditional on the risk level, everywhere in the grid (detailed discussion how I obtain these probabilities follows shortly). The upper and the lower borders are set initially at 6 and 0.167 so that enough of the range of possible values is captured. The lower value actually depends on the debt level and is always higher than 0.167 (for the lowest debt level it is at approximately 0.27[24]) as I make all total firm values below the default boundary equal to the value exactly at the default boundary. I always put additionally an upper and a lower buffer of values for all claims, which are necessary for the optimal decisions close to the boundaries. The size of the buffers is determined by the maximal possible number of upward/downward moves in the model, which I signify by *n*. All values in these buffers are actually the values of the claims just one step above/below the maximal/minimal firm value in the grid. Those are not exactly the correct values, but they serve their purpose well. To check the effect of the boundaries, I

[24] The value is roughly 91% of $0.2/(1-\tau_i)$.

increase the boundaries further to 8 and 0.125. This changes the claim values at the initial node only after the fourth decimal.

The debt step of 0.1 I choose after some experimenting, which shows that for smaller debt steps the manager is sometimes skipping particular debt levels, which makes their introduction meaningless. I need very precise steps in the number of shares variable to capture well the trade-off the manager faces. Using less precise steps results in some cases of quite different managerial decisions with respect to the debt and risk level for similar firm values. The reason for this is that the manager's utility function might get similar values for different alternatives (e.g. an increase in the debt level and low risk and no change in the debt level and high risk). The use of equal steps in log-log number of shares further mitigates this numerical problem and in standard applications of the model, serious changes in managerial decisions appear only (if at all) in grid areas where significant changes of the capital structure are necessary, which are reached with extremely low probability.

Building the grid in equal log-steps of V means that moving i nodes up or down will be always associated with the same log-return (conditional on the risk level κ – details follow shortly), no matter where the firm value process is. The standard assumption of a geometric Brownian motion, which is also made for the payout flow in Goldstein et al. (2001) and which is modified here to account for the possibility to increase/decrease the firm's risk and to have fat tails, means that its log-returns over a given time horizon (one time step) will be always independently distributed, so my aim is to construct a vector of transition probabilities for each choice of the risk level. Then I can choose the risk level, which gives the highest expected utility to the manager, and store his optimal risk level at every point in the grid. In the following I discuss how to obtain the discretized distributions (normal and mixture) and how I use them to get values for the claims on the firm.

First, consider the normal distribution case. Let μ be the mean, and σ be the standard deviation of the normal distribution of the process for V. Both of them are actually functions of the risk level κ, which I allow to take the values 0.75, 1, and 1.25. The interpretation is that the manager can use different derivatives (e.g. forward contracts) to hedge part of the firm risk or to increase it. Alternatively, the manager could achieve similar results by investing in risky or safe projects. The functional form I assume for μ and σ is:

$$\mu(\kappa) = \kappa r + (1-\kappa) r_f - (r - \bar{\mu}) - \frac{1}{2}\kappa^2 \bar{\sigma}^2 \tag{A1.3}$$

$$\sigma(\kappa) = \kappa\bar{\sigma} \tag{A1.4}$$

where $\bar{\mu}$ and $\bar{\sigma}$ are the expected instantaneous return and the instantaneous volatility for the normal risk level (κ equal to 1). Note that the firm always pays out $r - \bar{\mu}$ of its current total value. For the probability to move i (i can take all integer values from $-n$ to n) moves away from the current value of V I use the following expression:

$$p_{i,\Delta t}(\kappa) = \frac{\frac{1}{\sqrt{2\pi}\sigma(\kappa)}\exp\left[-\frac{1}{2}\left(\frac{i\Delta\log V - \mu(\kappa)}{\sigma(\kappa)}\right)^2\right]}{\sum_{j=-n}^{n}\frac{1}{\sqrt{2\pi}\sigma(\kappa)}\exp\left[-\frac{1}{2}\left(\frac{j\Delta\log V - \mu(\kappa)}{\sigma(\kappa)}\right)^2\right]}. \tag{A1.5}$$

The numerator is simply the value of the normal density function I try to approximate. The denominator is a normalization constant which ensures that the probabilities for all *2n+1* possible moves sum up to one. Note that I need only the univariate normal distribution, although I have moves in firm value, debt level, and number of shares. This comes from the fact that the debt control is a perfect one and unlike the risk-level choice (the risk control only influences the probabilities to move to the *2n+1* possible one-step-ahead nodes) ensures that the process moves to a particular debt level, and the associated change in the number of shares comes from the minimal positive cash flow adjustment rule I described earlier. One could alternatively think of a degenerate distribution for the debt level, in which the movement for the optimal debt level from the manager's perspective has a probability of one, while all other moves have a probability of zero.

There are two ways in which the discretized distribution could deviate significantly from the normal distribution. Having few log-steps would result in covering insufficient part of the support of the distribution. Then some moves with meaningful probabilities will not be possible in the discrete model and even in the modelled range the approximation will be poor. Having steps which are too large would lead to coarse steps in the centre of the distribution where the majority of its mass is concentrated and eventually lead again to a poor approximation of the distribution. A good approximation of the normal distribution should then have enough steps of relatively small size. To have a meaningful way to test how well the discretized distribution matches the actual normal distribution, following Stuart and Ord (1987, p. 322), I construct a test statistic based on the first ten moments of the standardized discretized distribution (signified with hats) and the standard normal distribution:

$$\frac{1}{10}\sum_{j=1}^{10}\left(\frac{\hat{\mu}_j-\mu_j}{\frac{1}{const}\left(\mu_{2j}-\mu_j^2+j^2\mu_2\mu_{j-1}^2-2j\mu_{j-1}\mu_{j+1}\right)}\right)^2, \tag{A1.6}$$

where I set *const*=1 and μ_0=0.

After some experimenting, I set a critical value of the above statistic so that it requires the discretized distribution to match the analytical Black-Scholes solution for a one-year at-the money European call option up to 4 decimals in a simple two-dimensional grid (i.e. for a static capital structure). In my analysis, I consider only distributions, for which the test statistic has lower than the critical values, meaning that they are at least as close to the normal as the above mentioned distribution.

The procedure is similar for the mixture distribution. For it I create two normal distributions in the way I explained above. Then I choose weights of the two distributions which sum up to one. By construction this is a well-defined distribution, which, however, has skewness and kurtosis which deviate from the normal distribution. I put restrictions on the parameters of the distributions to match the first two moments of the normal distribution, but use the remaining degrees of freedom to fit the empirical default probabilities. Figures 1.10 and 1.11 illustrate the discretized distributions that are used in the model.

For valuation purposes, I would also need the risk-neutral distributions for any risk level choice. Then I simply adjust the mean term of the distribution in (A1.3) and also the mixture distribution so that the total claim on the firm has a mean equal to the risk-free interest rate. Of particular interest for me are the first two claims – the firm's debt and equity – as the manager can trade in them in any time period and his utility depends on their values. I set their values at the last period in time based on the values of *V* and the analytical expressions for them in Goldstein et al. (2001).

The manager faces in all periods two simultaneous choices. The first one is what risk level to choose. As discussed above, different risk levels are associated with different transition probabilities and therefore different expected utilities. The other choice which the manager faces in every period is the capital structure choice. He can issue more debt and use the proceeds to buy back equity, or issue new equity to repay some of the existing debt at its market price, or leave the current capital structure unchanged.[25] Any changes of the capital structure come at a given flotation cost, which is borne by the equity holders. The flotation

[25] In the benchmark specification I do not allow the manager to retire existing debt.

costs for issuing new debt and equity need not be the same. As baseline values, I use 2% for the debt issues and 6% for the equity issues.

Fig. 1.10 The discretized one-period normal distribution in the model for the different risk levels

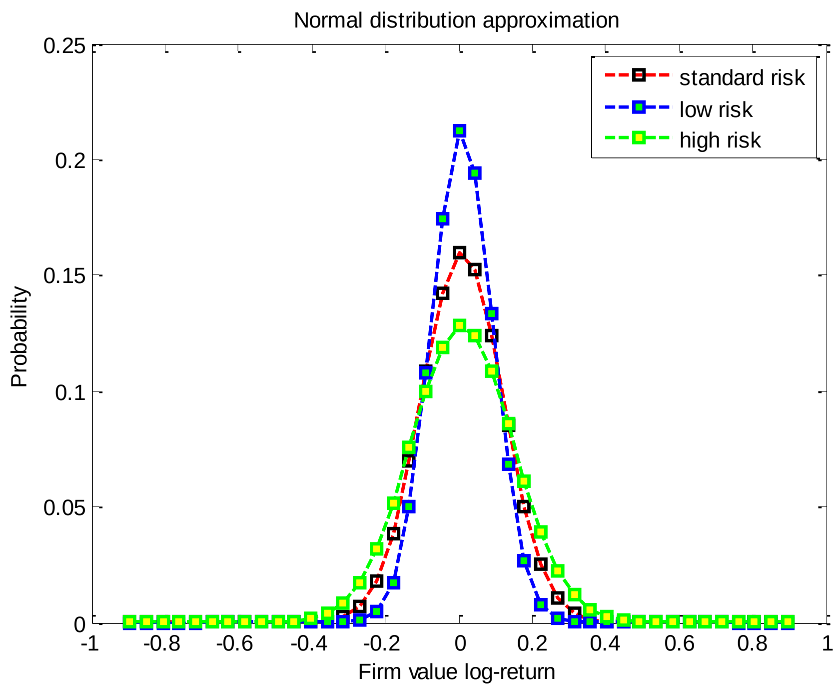
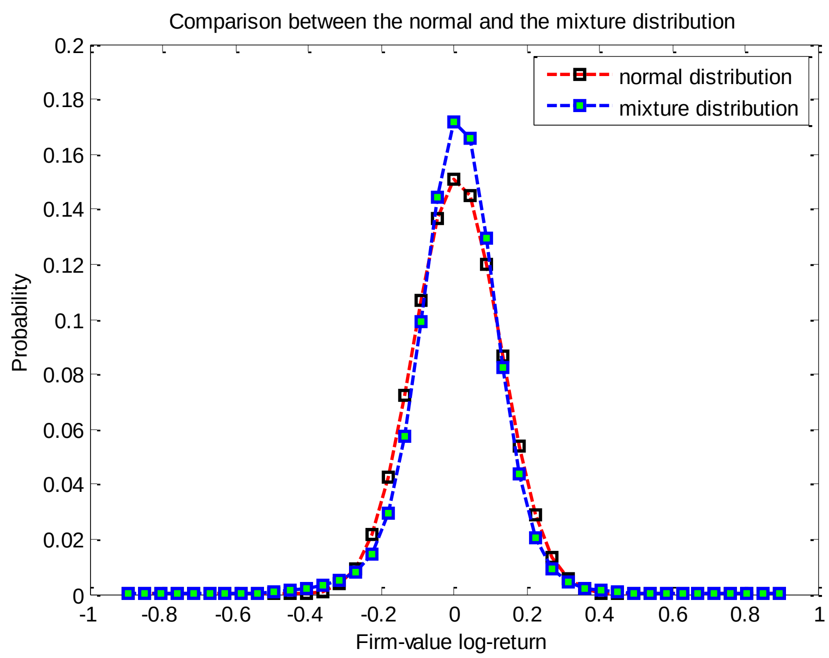


Fig. 1.11 Comparison between the mixture and the normal distribution for the standard risk level



I use a backward sweep on the grid to determine the manager's optimal decisions. As a first step, I compute his utility at any point in the grid in the last period based on his compensation package. If the manager can choose among *D* possible debt levels and *K* possible risk levels, then he has a total of *DK* possible alternatives. For any point of the grid at time $T\text{-}\Delta t$ I compute his expected utilities for each alternative and the associated certainty equivalent. Note that he cannot always issue debt to all possible debt levels as that could automatically trigger default. Note also that the current debt and equity values depend on the manager's risk-taking choice. I assume that in its valuation, the market correctly anticipates the manager's optimal choice and uses risk-neutral valuation to obtain the exact debt and equity values. I use the relationship between *V* and *EBIT* to determine the current cash flow and divide it among the claimants: debt holders receive the coupon and the equity holders receive dividends, while the government taxes them at the respective tax rates. The bankruptcy costs are incurred only if the firm value process hits the default boundary. The part of the cash flow which accrues to the manager is added to his certainty equivalent. I am then able to compute the manager's indirect utility at all points of the grid in time $T\text{-}\Delta t$ and always store the manager's choice. Repeating the same procedure in turn for all earlier periods, I find the manager's optimal choices and indirect utilities at all points of the grid.

To obtain values for all other claims, I use risk-neutral valuation in a backward sweep of the grid. Starting from the last period, in which the values of the claims are defined by the analytical expressions in Goldstein et al. (2001), I use the risk-neutral transition probabilities based on the optimal managerial choice in a particular node of *V* to determine the values of the claims step by step to the initial period. The default probabilities are determined in a subsequent forward sweep of the grid based on the actual probabilities after all optimal decisions have been identified.

To ensure that the assumption for the final values does not have a decisive influence on my results, I investigated an alternative set of final values based on an iteration of my model. As a first step, I ran the model with the analytical expressions of Goldstein et al. (2001). Having obtained values for all claims at the initial point of time, I inserted these values back as the final values of the claims and re-ran the model. As the results remained virtually unchanged, I stuck to the initial assumption that the firm fulfils the assumptions of Goldstein et al. (2001) at the final date as it saves me computational time.

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Chapter 2

Modelling Financial Distress in a Structural Framework: How Much Cash to Infuse?

2.1 Introduction

There are numerous models within the structural credit risk approach that address the issue of pricing corporate debt. However, only few of them consider financial distress explicitly, while the debt level is either assumed to be constant or only debt issues are considered. Most of the models (in the spirit of Leland 1994) assume perfect financial markets so the firm can always issue new equity as long as it is not worthless.

In this paper I model explicitly the capital restructurings that take place in a financially distressed firm. I incorporate also important market frictions and imperfections such as transaction costs and division of ownership and control and look at the credit spreads and default probabilities of non-financial firms in such a setting. The special focus in the paper is on investigating the incentives of equity holders to infuse additional cash in the firm, which is not used in the firm's investment activities but is solely intended as a repayment of some of the outstanding debt. The financial flexibility that the firm gains in this way reduces the likelihood of its liquidation and the accompanying formal bankruptcy procedure and at the same time preserves the tax advantage of debt financing. A real world analogue can be also a debt repayment trigger, which has recently become a wide-spread provision in debt contracts – see Bhanot and Mello (2006). The main finding of the paper is that if equity holders can *ex-ante* commit to undertake this capital restructuring, the firm can actually borrow at much lower rates and the overall value of their claim can be increased. The other important point the paper tries to make is that market frictions should not be overlooked as they can have sizable effects on a model's predictions with respect to a firm's credit risk.

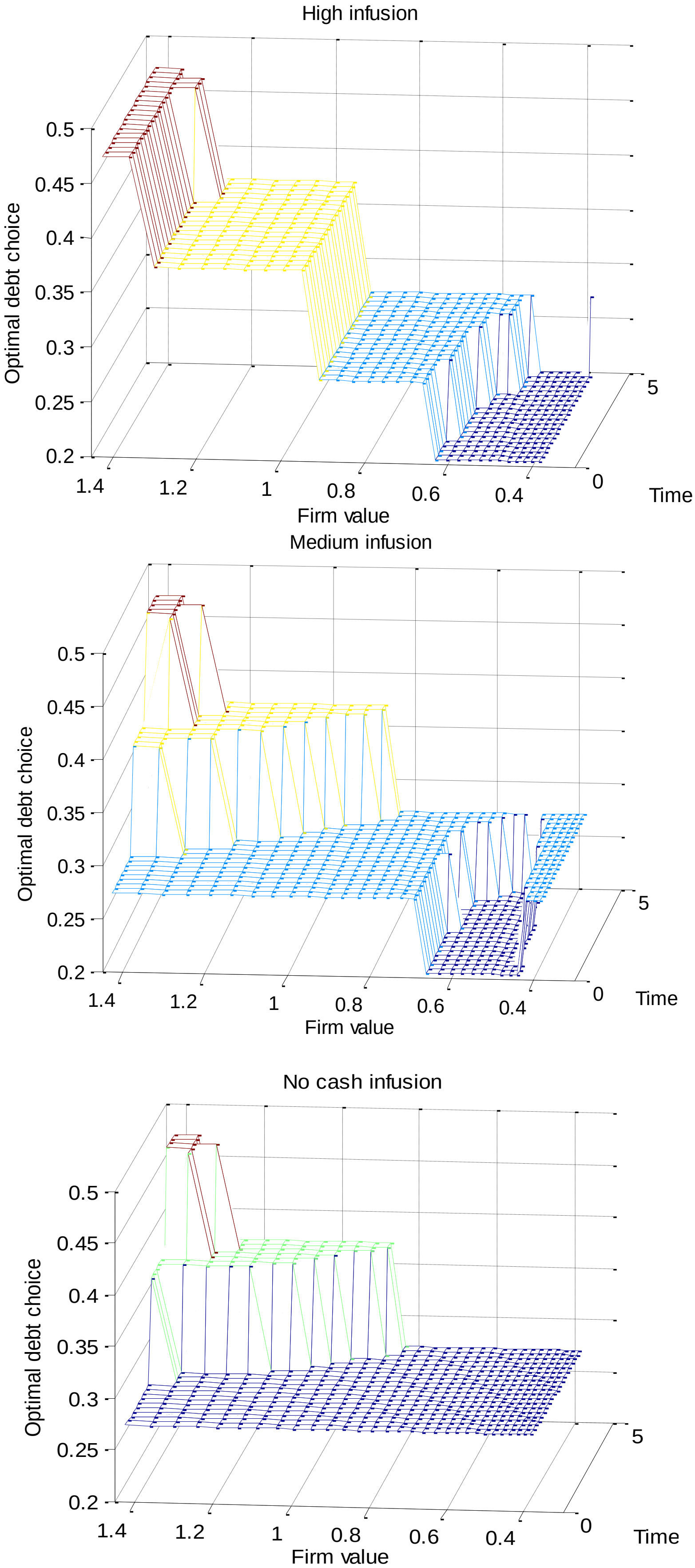
There are three distinct market frictions that I introduce into the model. First, all changes in the firm's capital structure incur transaction costs. Second, I distinguish between the ownership and control in the firm. The equity holders cannot run the company themselves due to time constraints, so they delegate the control to a manager. Even though they try to align his interests with theirs (they include stock and stock options in his compensation package which is standard in reality), there can still be divergence of interests in particular states. Finally, I assume that there is a certain limit on equity issues in the markets as no underwriter would be willing to place huge amounts of shares and not many investors would be willing to buy them.

Having these market frictions, I investigate the effects of one often considered innocuous assumption, namely the cash infusions into the firm from the equity holders. In Leland (1994), it makes no difference if the firm issues new equity or equity holders infuse additional money as the financial markets are perfect. In my model, the equity issues are limited and come at a cost, which turns the equity holders' decision to infuse cash into an important parameter of the model. I show that by agreeing to contribute up to 5.5% of the initial total firm value for the sole purpose of debt repayment, equity holders can reduce (relative to the case in which they do not contribute anything) the firm's long term credit spreads from 295 to 235 basis points. The effect is even more pronounced when I consider the five-year default probability: it shrinks from 3.8% to less than 2%.

The exact mechanism at work is best illustrated in the following figure. In all panels the optimal debt choices for a reasonable range of total firm values and for the initial debt level[26] in the model is depicted. In the upper-most panel equity holders have committed to infuse considerable additional amount of cash in the firm and repay part of the outstanding debt in case the firm experiences financial difficulties. Then for all firm values in the distressed region, the firm indeed repays some of its outstanding debt and alleviates the looming danger of default. In the middle panel, the equity holders commit to infuse only a moderate amount of cash for a debt repayment in case of a deterioration of the firm's financial situation. Still, upon entering the distressed region, the firm again repays some of its debt. In both cases the cash infusion happens only as a second resort after the firm's potential to issue new equity is

[26] In general, the optimal debt level choices look somewhat different if the firm has some other amount of debt outstanding, which is a natural consequence of the presence of transaction costs. They increase the likelihood of keeping the current debt level.

Fig. 2.1 The optimal debt level choice for the cases in which shareholders commit to make high, moderate, and no cash infusions



exhausted. In the latter case, however, if we observe an extreme shock in the firm value process, there can be a situation in which the equity issued and the equity holders' additional cash infusion are insufficient for a debt repayment.[27] Then the firm is in a situation in which even a minor negative shock can trigger bankruptcy, while a positive shock would increase the value of the equity claim and then the equity issue and the cash infusion can account together for the debt repayment, resulting in a reduction of the likelihood of a bankruptcy. In the last panel is illustrated the situation in which the equity holders do not commit to infuse any additional cash in the company. Even though the model still allows debt repayment in case an equity issue can yield the sufficient amount of cash, this is not observed due to the limited amount of new equity that can be issued.

As already mentioned, the effects of the cash infusions on the firm's credit spreads and default probabilities can be substantial. The force that brings those reductions is best described as enhanced capital structure flexibility.

The remaining part of the paper is structured as follows. In the next section I review the related literature. In section 2.3 the model and its assumption are presented, while in section 2.4 the model numerical results and predictions are documented. Section 2.5 concludes. Details on the numerical implementation of the model are given in Appendix 2.

2.2 Related literature

The paper is related to three directions in the big strand of structural credit risk literature: the models that focus on the capital structure decision, the models that focus on the default event, and the models with a manager-shareholder agency conflict.

The capital structure assumption has a serious influence on the quantification of a firm's credit risk in a structural framework. That is why researchers spent a lot of effort to improve realism relative to Merton's (1974) model, which assumes the firm has only one outstanding zero bond. Leland (1994) keeps to the static capital structure assumption, but analyzes the optimal level of debt in an infinite-horizon setup. Leland and Toft (1996) extend the analysis to finite maturity debt, but the dynamics are still predetermined: the firm issues and repays continuously the same amount of debt, keeping the overall level unchanged. Dynamic capital structure is present in the works of Fischer, Heinkel, and Zechner (1989) and Goldstein, Ju,

[27] If the firm repays some of its debt and moves to the lowest admissible debt level, however, there will be no subsequent increase of the debt level for the low firm values in the region next to the default boundary in the second panel.

and Leland (2001). In both models, however, only upward adjustments of the debt level are considered, which contradicts the evidence in Leary and Roberts (2005) that debt issues and repayments occur almost equally often. Collin-Dufresne and Goldstein (2001) have capital structure dynamics which include downward adjustments of the debt level, but they are modelled exogenously as a mean-reverting leverage ratio. Strebulaev (2007) models explicitly downward adjustments and in that sense his work is the closest to my paper. Both papers further share the same general framework suggested in Goldstein et al. (2001). However, the focus of his paper is on the tests of capital structure models and the downward debt adjustments are financed by fire asset sales, while in my paper the analysis is on the equity holders' cash infusions and their effects on the firm's credit spreads and default probability.

Modelling the default mechanism is the other very important choice in the structural credit risk literature. Merton (1974) allows default only at the debt's maturity, but Black and Cox (1976) fix this issue and consider the default as the first crossing of an exogenous default boundary. Leland (1994) endogenizes the default boundary by maximizing the equity holders' wealth with respect to it. Anderson and Sundaresan (1996) search motivation in the observed debt restructurings in which debt holders agree on smaller than contractual payments and introduce strategic defaults. In their model, equity holders make the decision whether to meet contractual payments fully, partly, or to completely skip them. In doing so, they always make debt holders indifferent between accepting the debt service and forcing liquidation for the firm, the latter leading to the incurrence of a fixed bankruptcy cost. Debt holders anticipate this behaviour from the very beginning and price the debt accordingly. This subgame-perfect equilibrium can be found via backward induction.

A separate direction of the literature is based on Chapter 11 in the U.S. Bankruptcy code, which is actually the more often applied liquidation procedure and in which the firm is allowed to keep running its business under the Bankruptcy Court supervision; the hope is that through different reorganizations a way out of the financial distress (without liquidation) can be found.[28] Francois and Morellec (2004) put forward a formal model in which liquidation occurs only after the firm value process spends continuously some exogenously given time below the default boundary. When this time is set to zero, the model is equivalent to Leland (1994). Moraux (2003) suggests that the time the firm value process spends below the default boundary should be measured cumulatively and not continuously, while Galai, Raviv, and

[28] In general, the U.S. legislation is considered to be "debtor-friendly" as opposed to the legislation in Germany, which is much more "creditor-friendly". For a discussion on bankruptcy legislation, see e.g. Hart (1995).

Wiener (2007) propose an alternative measure of financial distress that takes into account how much below the boundary the firm value process is, how long it spent there, and how long ago the distress was.

There have been only few attempts to model the default event based on liquidity trigger. Kim, Ramaswamy, and Sundaresan (1993) assume that the firm defaults at the moment it faces a cash shortage, but this is considered to be quite restrictive. Uhrig-Homburg (2005) relaxes this assumption so that the firm only enters a distressed region, in which all equity issues come at some cost. In essence, her model is an extension of Leland (1994) with this distressed region. The model generally allows for both overindebtness and liquidity as causes of default, but as the author shows, if firms structure optimally their debt, no liquidity default occurs without overindebtedness. Still, the costs in the distressed region determine the default boundary and therefore influence the default probability. In a sense the model is very similar to mine, as both introduce certain frictions in the issues of new equity, but the exact modelling choices differ as there is no debt repayment in her model. The other difference is that in her model the firm can keep issuing continuously new equity, so there is essentially no limit on the amount of shares, which cannot happen in mine.

There have been also some attempts to analyze the capital infusions in distressed firms from a normative perspective. Flannery (2005) suggests a new instrument called "reversed convertible debentures" that is essentially debt and as such has a tax advantage, but converts into equity upon the breeching of a trigger related to the firm's financial condition. Similar to the cash infusions in my model, this instrument can reduce significantly the potential bankruptcy costs. Kashyap, Rajan, and Stein (2008) consider instead a compulsory insurance that would pay in the bad states of the economy and increase the firm's equity capital. Both these papers, however, are dedicated to the financial companies and banks in particular. This predetermines also their focus to be the economy as a whole, while I am concerned only with the particular (non-financial) firm.

In another paper that is closely related to my work, Bhanot and Mello (2006) analyze the effects of debt triggers, searching motivation in the significant amount of such provisions observed among S&P 500 firms. The most widely observed trigger specifies conditions under which the firm is obliged to repay some of its debt. As these conditions are typically credit rating downgrades, such provision corresponds very closely to the cash infusions I explore. The results of Bhanot and Mello (2006) are also similar to mine: they find that firms may find it optimal to add such trigger to their debt and the source of the gains is reduction in the coupon payments and bankruptcy costs. The focus of their analysis, however, is on the optimal trigger conditions and on the agency costs stemming from the shareholder-debt holder conflict. My focus is on the effects on the credit spreads and default probabilities such triggers impose and on a different source of agency costs: the manager-shareholder conflict.

In all models discussed until now, it is assumed that the firm is run according to the equity holders' interests, while in reality the managers control the firm.[29] At the same time, it is long known that the separation of ownership and control of the firm generates certain agency costs (Jensen and Meckling 1976). Morellec (2004) proposes a contingent-claims model that builds on this insight and shows that managers would choose less leverage than optimal from the stockholders' perspective. Lewellen (2006) also shows theoretically that managers would be less aggressive in their leverage choice than shareholders as the former are much less diversified. She argues that shareholders are either not informed or face a coordination problem to enforce the optimal from their perspective leverage. She then provides empirical evidence that managerial incentives have considerable explanatory power with respect to future capital structure changes. Based on that evidence, in my model the manager has discretion over the choice of the debt level and the capital infusion timing. The paper that is most closely related in this respect is Carlson and Lazrak (2006). As in my model, the manager in theirs is controlling the risk level and the capital structure to maximize his expected utility from compensation. The differences are that they have static capital structure and allow volatility to go to infinity, while I have dynamic capital structure and only moderate deviations from the standard firm value return volatility.

2.3 The model

Here I use basically the same model as in Zahariev (2009), which is based on the framework proposed in Goldstein et al. (2001) but extends it in several important aspects. As shown in Zahariev (2009), this model can match pretty closely the default probabilities for different credit ratings. The exposition here follows closely that paper and extends it only with the introduction of capital infusions of the equity holders. A more detailed description of the basic model assumptions and implementation can be found in the Appendix 2.

In the general framework suggested in Goldstein et al. (2001) the underlying variable is the firm's EBIT, but there is a direct relationship between EBIT and the total claim on the firm

[29] There is actually one exception in the models discussed: Kashyap et al. (2008) consider the divergence of managerial and shareholders' interests.

(referred to later also as "total firm value"). It consists of four distinct claims: the claim of equity holders, debt holders, the government, and the claim to the bankruptcy costs.[30] The available cash flow in every period is distributed so that first the coupon payment and the associated tax rate are paid out and then the remaining part of the cash flow is paid out to equity holders, who also have a tax obligation. If the available cash flow is not enough to pay the coupon, equity holders normally infuse additional money in the firm and prevent bankruptcy. For low enough values of the cash flow, however, they prefer to let the firm default. Details on the default boundary follow shortly.

The extensions of Zahariev (2009) are the following. First, there is a final horizon, after which the firm keeps on operating under the assumptions of Goldstein et al. (2001) and the values of the claims are therefore the analytical solutions of the dynamic version of their model. Second, the model is developed in a discrete-time setting. Third, there is a dynamic control of the firm's risk level, so instead of a geometric Brownian motion, the firm in the model follows a controlled stochastic process of the form

$$\Delta \log V_t = \xi_t(\mu_a(\kappa)\Delta t + \sigma_a(\kappa)\Delta W_{at}) + (1-\xi_t)(\mu_b(\kappa)\Delta t + \sigma_b(\kappa)\Delta W_{bt}) \ , \qquad (2.1)$$

where W_{it} for $i=a,b$ are uncorrelated Wiener processes and the drift terms μ_i and the volatilities σ_i are functions of the risk level κ, while ξ_t is a Bernoulli random variable that takes on values 1 and 0 with probabilities p and $1-p$ respectively. In particular, I assume the following functional relationships:[31]

$$\bar{\mu} = p\bar{\mu}_a + (1-p)\bar{\mu}_b \qquad (2.2)$$

$$\mu_i(\kappa) = \kappa r + (1-\kappa)r_f - (r - \bar{\mu}_i) - \frac{1}{2}\kappa^2\bar{\sigma}^2 \qquad (2.3)$$

$$\sigma_i(\kappa) = \kappa\bar{\sigma}_i \ , \qquad (2.4)$$

where $\bar{\mu}_i$ and $\bar{\sigma}_i$ are the expected instantaneous return and the instantaneous volatility for the normal risk level (κ equal to 1). Note that the firm always pays out (as coupon payments, taxes, and dividends) a fraction $r - \bar{\mu}$ of its current total value.

[30] In fact there is a fifth claim on the firm: the claim on the transaction costs for the adjustments of the debt level.

[31] Equation (2.2) only introduces some notation.

This specification can account for fatter tails than the normal distribution and left skewness. The mean and variance are chosen by the manager among a set of available alternatives to maximize his utility. To increase the mean of the distribution, the manager has to accept also an increase in the variance.

Intuition for the process (2.1) is that the returns come from a mixture distribution of two normal distributions. To keep the model parsimonious, I consider only three possible risk levels: high risk, standard risk, and low risk. The high/low risk regimes correspond to a 25% increase/decrease of the risk level. The admissible values of κ are hence 0.75, 1, and 1.25. One possible interpretation of κ is that the manager can use different derivative contracts to hedge part of the company's risk or to take on more risks. Alternatively, one can think of investing in safer or riskier projects. In addition to κ, the manager determines the debt level in every period.

There are two additional variables which are necessary for the model implementation: the number of shares and the amount of debt outstanding. Their values are essential for the determination of the manager's optimal choices because they determine his wealth. Together with the total firm value and time, they form a four-dimensional grid, on which all model calculations are performed.

The exact mechanism of the changes in the debt level requires some more comments. To avoid influences that are not related to the capital structure decisions, the proceeds from a debt issue are used to buy back shares and the necessary cash for a debt repayment in general comes from an equity issue. The firm's assets are never sold in the process. In essence, I use a matrix to determine the exact amount of shares to be issued or repurchased for a particular debt issue and a risk choice. Every row of the matrix represents a particular choice of the manager with respect to both debt and risk levels, so if there are D possible debt level choices and 3 possible risk level choices, the matrix has $3D$ rows. Every column of the matrix corresponds to a particular number of shares outstanding, so if I allow S possible numbers of shares, the matrix has S columns. The limit on the number of shares that can be issued is imposed by this matrix. Within it, I compute all net cash flows associated with any possible capital structure change. These net cash flows include reasonable transaction costs[32] for securities issues. They are further strictly increasing within any row as every next cell on the row represents a higher number of shares. To every row (i.e. any possible choice) I match the

[32] As in Gamba and Triantis (2008), a debt issue has transaction costs of 2%, while an equity issue has transaction costs of 6%. Both are ultimately borne by the equity holders. These values are also in line with the evidence in Eckbo, Masulis, and Norli (2008).

number of shares that corresponds to the first cell with a non-negative value on this row. As all capital structure adjustments take place in the open market, they are undertaken at the market prices of the debt and shares.

The cash infusions can happen only when all elements on a certain row are negative. If the last element on the row is smaller in absolute value than the equity holders' commitment to infuse cash (which is constant within a single run of the model, but I generally vary from 0 to 7% of the initial total firm value at steps of 0.5%), the debt restructuring would still take place if it gives the manager higher expected utility than all other alternatives. Then the necessary additional amount of cash comes from the equity holders' pockets (the manager is also an equity holder so he also pays). The asset side of the company remains unaffected. I assume there are no transaction costs associated to the infusion, but checked also positive transaction costs that correspond to the equity issue; results were basically unchanged. The cash infusion is perfectly expected by everybody and all claims are priced with the expectation that it would occur in particular states. So the value of the equity claim (from the current point in time) is reduced in the states in which some payment would take place by the exact value of this payment. Still, there are very strong counter effects that result in an increase of the equity value after a capital restructuring. These effects are that after a debt repayment, a bigger cash flow would go in any period to equity holders and that the bankruptcy costs are significantly reduced.

Taking the model restrictions to the extreme, the commitment for the cash infusions might require that the stockholders do not sell their shares – something that is not very common. Otherwise, it has to be guaranteed that the transfer of the share transfers also this commitment. The infusions can be further considered as a relaxation of the limited liability principle. Still, this is not quite the case for two reasons. First, as already discussed, it is better for the equity holders to commit to those infusions; and second, I make sure that they do not pay more than the value of their shares after the infusion – which means that the shares never get negative value. If the model suggests a negative value for equity, shareholders simply declare default.

The commitment to infuse additional cash in the firm accomplishes two goals simultaneously. On the one hand, the bankruptcy costs are reduced. This effect is something close to increasing the firm's equity *ex-ante*. Unlike an *ex-ante* increase in the equity, the cash infusions preserve the tax advantage of debt. The trade-off from the equity holders' perspective comes in the fact that they cannot coordinate perfectly, so they have to delegate the decisions on the exact timing of the cash infusion to their agent – the manager.

Overall, the cash infusions are very similar to a new equity issue that is used to repay debt. Nothing changes on the asset side of the company, while on the liability side the debt shrinks and the equity increases. As Jostarndt (2007) documents, many of the financially distressed firms indeed issue equity to alleviate their financial condition. It is still realistic to assume that only a limited amount of new shares can be issued, after which only fresh capital from the equity holders can mitigate the firm's financial problems. Modelling explicitly a realistic friction in issuing new equity and then relaxing it gradually has the advantage that it illustrates the relevance of the rarely discussed assumption of unlimited equity issuance. This approach also introduces an observed phenomenon (cash infusions into a financially distressed firm) that generally remains outside the focus of the structural credit risk models. Finally, the approach allows an analysis of the optimal cash infusions, which I do in section 2.4.3.

Appropriate modelling of the capital structure changes further requires that many different possible numbers of shares outstanding are allowed, so this is the variable I use the most precise steps in (0.5%). This makes the residual cash flow really small; to minimize discretization distortions I still compute it and it is distributed among the equity holders.

To determine the optimal choices with respect to the debt and risk levels at all nodes of the grid, the manager's wealth in each state at the final date is computed. Then the expected utilities for all possible choices one period before the final horizon are calculated and the optimal choice and the associated indirect utility are stored. Working backwards, one can determine the optimal choices at all nodes of the grid. Having the optimal decisions, also the prices of all claims on the firm can be computed with an adjustment of the actual measure to the risk-neutral one. After all optimal choices are available, default probabilities can be computed in a forward sweep of the grid based on the actual probabilities. More details on all these aspects are given in the Appendix 2.

2.3.1 Manager characteristics

I assume the manager has preferences governed by a power utility function and hence has a constant relative risk aversion coefficient γ:

$$u(w) = \frac{w^{1-\gamma} - 1}{1-\gamma}. \tag{2.5}$$

Any other utility function could be substituted with ease since the numerical solution does not depend on the exact structure of the utility function.

His compensation consists of stock and options on the stock of the firm. The manager also has some outside wealth, which is to be interpreted more broadly to include also expected income from future employment. As a simplification I assume that he consumes all his cash compensation, so that it does not show up in his wealth in any period. He is fired if the firm value hits the default boundary and he then suffers a loss of φ of his outside wealth. His total wealth at time T is:

$$w = aE_T + b\max(E_T - K, 0) + W \qquad \text{if no default occurs and}$$

$$w = (1-\varphi)W \qquad \text{if the firm defaults,} \tag{2.6}$$

where a is the number of shares he has, E_T is the share price at date T, K is the strike price of the options he owns (they are struck at-the-money), b is the number of his options, W is his outside wealth, and φ is the fraction of the manager's outside wealth which he loses in case of a default.

2.3.2 Default boundary

A discrete set of debt values are allowed in the model. I assume that the firm has some long term debt which cannot be repaid, so the first level is not at zero debt, but at some non-trivial level (0.2 for an initial firm value of 1). The model is calibrated to produce an initial leverage ratio as close as possible to the observed average leverage ratio of a Baa-firm (which Leland (2004) reports to be 43.3%), which gives an initial debt level of 0.27. The debt step is set at 0.1, as for smaller steps the manager tends to skip particular debt levels; adding them only slows down the implementation of the model without changing the results qualitatively, while the quantitative changes are negligible.

At the final date, the debt values are set so that they would be equal to the discrete set of values I have (0.2 0.27, 0.37,… 0.77) if the debt were riskless.[33] As the debt is not riskless, the market price at each state should be lower but should approach the face value as the firm value grows. Given the assumption that after this date the firm is run according to the assumptions of Goldstein et al. (2001), debt values are set to the analytical values from their

[33] I interpret these as the debt face values.

model (i.e. the closed-form solutions they derive). I also follow their approach in setting the default boundary at the final date:[34] this default boundary is based on the assumption that equity holders have the control of the firm and choose the default boundary to maximize the value of their claim. This assumption is not quite in line with my model, in which equity holders do not have the decisive say on the important decisions in the firm like the capital structure and the risk level. I still consider my choice suitable as it fits the empirical observations that some firms keep on operating with negative net worth (Davydenko 2007). Set in this way, the default boundary for a particular debt level is simply a fraction of the debt gross value (i.e. the debt value including the government claim on it) of the riskless debt.[35] For my baseline parameters, this fraction is approximately 91%. The interpretation here could be also that a debt covenant is set at this particular value. I assume that the default boundary is at this level for all earlier periods in my model, given that the model predicts a positive value for the equity claim. In line with the limited liability of stockholders, I assume the firm defaults in all states, in which the model predicts a negative value of the equity claim. This, however, happens only for very few nodes in the grid and only when the cash infusions are at least 5% of the initial total firm value.

I also assume that upon hitting the default boundary, the bankruptcy costs are incurred and the firm control is transferred to the debt holders. They own the new, now all-equity financed firm, which is run again according to the assumptions of Goldstein et al. (2001). For the calculation of the debt values, however, I assume that the old debt holders immediately sell their shares and calculate their recovery based on their fair price.

As illustrated with an example, every increase of the potential cash infusions gives the manager some additional flexibility in changing the capital structure. It is the effect of this capital structure flexibility that I analyze in the next section.

[34] As I use a discrete framework, I put the default boundary at the first node below their analytical expression for the default boundary. Even though the lowest debt level of 0.2 in my model is always considered as one of the possible choices, it is reached only in a restructuring that includes cash infusion from the equity holders. As the manager has some shares, he also contributes to the infusion. I would like to stress that the effect of reducing the lowest admissible debt level further down on the model's results and predictions is actually very modest. The reason is that there are always two opposite effects. First, a lower debt level decreases the default boundary and hence also the credit spreads and default probabilities. However, a lower debt level is reached then in fewer states as the restructuring becomes less likely. For the lowest debt level of 0.2, the two effects are close to offsetting each other when the cash infusions are close to the optimal level from the equity holders' perspective.

[35] The following relationship between the gross and the net debt value holds:

$$GrossDebt = \frac{NetDebt}{1-\tau_i},$$

where τ_i is the tax rate for interest payments.

2.4 Results

2.4.1 Baseline parameters

Table 2.1 summarizes the baseline parameters, which are chosen to represent a firm with a rating of Baa.[36] For the volatility of the total firm I take the estimated parameter of Eom, Helwege, and Huang (2004) of 0.236 as they have a sample in which the average firm is very close to the rating I have in mind, just a notch or so higher.[37] I additionally checked what equity volatility this total firm volatility implies according to the Goldstein et al. (2001) model.[38] For the initial firm value and the leverage of a Baa-rated firm, it translates into 0.403, which is very much in line with the observed stock volatility of a median firm with this credit rating.[39]

The risk premium is 6%, and the initial debt level is chosen to fit a leverage ratio of 43.3%, which is the average leverage ratio for a Baa-rated firm (Leland 2004). The distribution parameters are the ones suggested in Zahariev (2009) and are based on a calibration exercise of fitting empirical default probabilities of Baa-rated firms. The managerial share and option holdings of the firm are chosen so that the value of stocks and options are roughly equal as the sample of Vasvari (2008) suggests. The managerial loss in default of 20% is the cut in salaries documented by Fee and Hadlock (2004), which dismissed CEOs had to accept on the new job – if they managed to find a new job at all.

Additionally, I need three tax rates: the corporate tax-rate, the tax-rate on interest payments, and the tax-rate on the dividend payments. I follow Goldstein et al. (2001) and set the first two of them at 35% and the last one at 20%. Finally, I need some reasonable range in which the firm can issue/repurchase shares. For the debt region I allow issuing and repurchasing

[36] The division of firms into rating classes throughout this paper is quite ad hoc. It simply means that the firm has the average leverage of the corresponding rating class. All other characteristics are kept constant.

[37] Eom et al. (2004) discretize credit ratings by notches and the average rating in their sample is between A- and Baa+.

[38] I applied the delta method (see e.g. Greene 2000, p.72) to get the variance of a function of a random variable. In this case the value of the equity claim is a function of total firm value, whose variance I assume I know. Then, if

$$E = f(V),$$

$$Var(E) = Var(V)\left| f(V)' \right|^2 .$$

[39] In fact, Ericsson et al. (2005) document an average stock volatility of 0.46 in a sample of bond issuing companies predominantly from the Baa rating class. However, as the distribution of firm volatilities is right-skewed, the median is close to 0.4.

about one third of the current amount of shares outstanding, which seems a reasonable choice over a five-year period. Increasing the range did not change the results materially.

Table 2.1

Baseline parameters

Initial total firm value:	$V_0 = 1$	Final horizon	$T = 5$
Upper/lower boundaries:	$V_u = 1/V_d = 6$	Time steps	$m = 20$
Firm value step:	$\Delta log(V) = 0.045$	Possible up moves:	$n = 20$
Required return (total firm):	$r = 0.1$	Means:	$\mu_a = 0.08$ $\mu_b = -0.02$
Risk-free rate:	$r_f = 0.04$	Volatilities:	$\sigma_a = 0.189$ $\sigma_b = 0.4$
Coupon rate:	$r_c = 0.05$	Mixing parameter:	$p = 0.85$
Debt step:	$\Delta d = 0.1$	Initial debt level:	$d_0 = 0.27$
Minimal debt level:	$d_{min} = 0.2$	Maximal number of shares:	$ST_{max} = 3.4$
Shares step:	$\Delta log(log(ST)) = 0.005$	Minimal number of shares:	$ST_{min} = 1.8$
Managerial share:	$a = 0.005$	Initial number of shares:	$ST_0 = 2.6$
Managerial option holdings:	$b = 0.01$	Manager's loss in default:	$\varphi = 0.2$
Manager's options strike price:	$K = 0.15$	Manager's outside wealth:	$W = 0.02$
Bankruptcy costs:	$\alpha = 0.3$	Manager's risk-aversion:	$\gamma = 2$

2.4.2 Credit spreads and default probabilities

In this section the effect of the cash infusions on the firm's credit spreads is analyzed. In the whole paper, credit spreads are very long-term credit spreads (e.g. 20 years) which is a result of the infinite time horizon in the Goldstein et al. (2001) model. To analyze also medium-term effects, the five-year default probabilities are considered too.

As can be seen on Figure 2.2, the long-term credit spreads can be decreased by more than 60 basis points if equity holders commit to infuse 5% of the initial total firm value for a debt repayment in case of financial distress, which translates into almost 10% of the equity claim in the unlevered firm. Still, 60 basis points is a very significant reduction that shows how important the modelling of the firm in distress for the value of its debt is.

Fig. 2.2 The firm's credit spreads as a function of the cash infusion equity holders commit themselves to

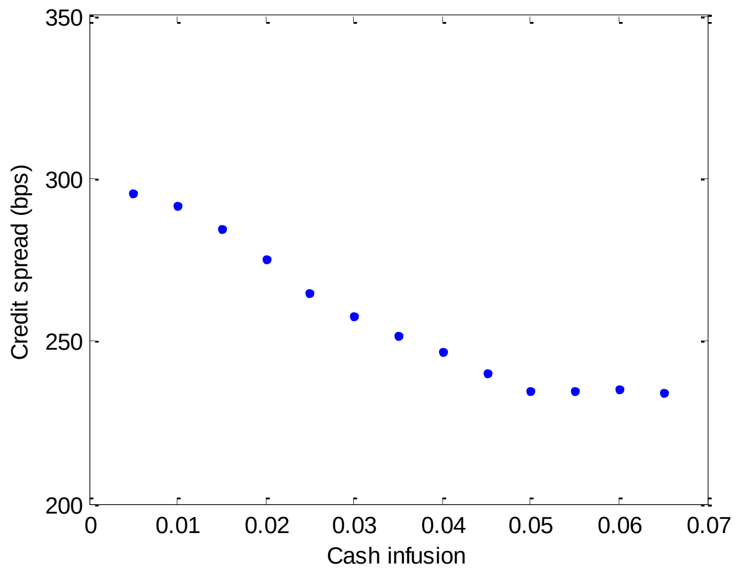
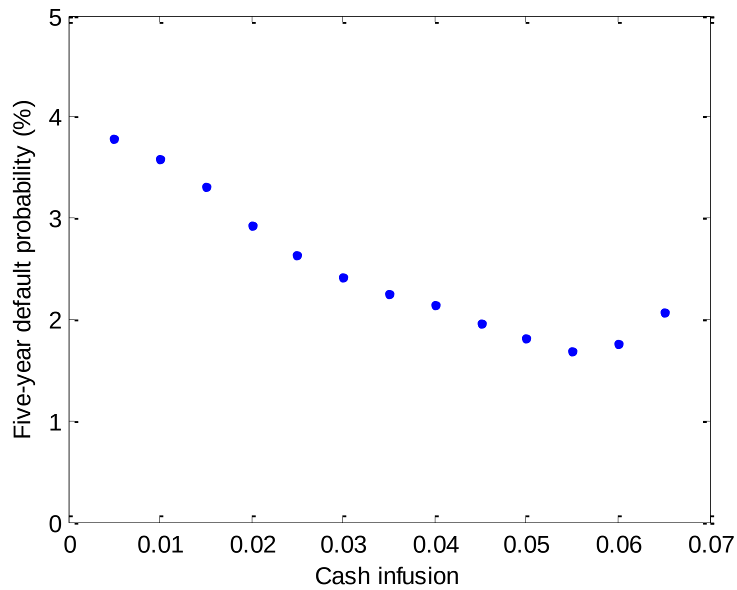


Fig. 2.3 The firm's five-year default probabilities as a function of the cash infusion commitment



It can be also noted that the negative relationship disappears for cash infusions between 5 and 7%. In an investigation aiming to explain this puzzling result, I figure out that as the manager obtains more capital structure flexibility, he increases his risk appetite, both with respect to the leverage and firm value volatility, which more than offsets the positive effect of the higher cash infusion commitment.

The next step is to analyze the effects of the cash infusions on the five-year default probability. The relationship is similar to the one with the credit spreads, the main difference being that now the default probability even increases for the highest cash infusions I consider. The mechanism is not different from the case of credit spreads. Overall, we can observe a reduction in the five-year default probability from 3.8 to 1.8%. The lowest default probability is observed for cash infusions of 0.055.

2.4.3 Share price maximization

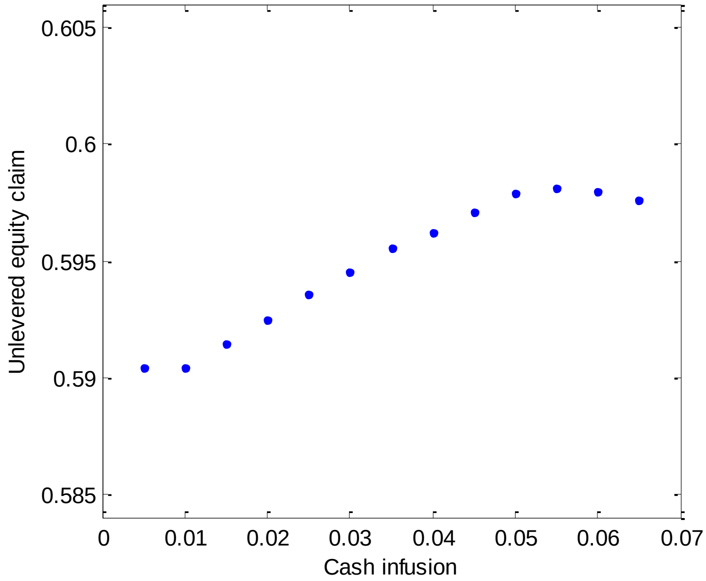
In the following graph the equity claim value is presented as a functions of the size of the cash infusion commitment. Committing to higher cash infusions initially increases the value of the equity claim[40] until it reaches its maximum for cash infusions of 0.055, after which the equity value goes down again.

The intuition for such dynamics is that equity holders' marginal gains from committing more cash are decreasing and they eventually start hurting their own interests. For low enough cash infusion commitment, the gains from the higher value of the issued debt and the additional flexibility with respect to the capital structure that was illustrated in Figure 2.1, make it worthwhile to commit for higher cash infusions, but after some level (0.055), this is no longer the case.

These results are generally in accordance with the conclusions in Bhanot and Mello (2006) that it can be wealth-increasing for equity holders to set a debt repayment trigger. However, it has to be noted that the equity value is not very sensitive to the actual size of the cash infusion commitment. Within the whole range that I analyze, the maximal absolute changes remain below 1.5%.

[40] The equity value is measured as the value of the equity claim of the unlevered firm just before it issues debt with a face value of 0.27 predicted by my model. When I consider the equity claim after the debt issue, the maximum value is still at a cash infusion of 0.055, but then this value is almost the same as the value when there are no cash infusions. In this latter case any benefits for equity holders come almost exclusively from the reduced likelihood of default, while in the former case the higher issue price of debt is more important.

Fig. 2.4 The value of the equity claim before the debt issue as a function of the cash infusion commitment



2.5 Conclusion

In the conclusions to their survey, Graham and Harvey (2001) claim that "[…] informal criteria such as financial flexibility and credit ratings are the most important debt policy factors." In this paper, I investigate the effects of the financial flexibility which equity holders' commitment to infuse additional cash in the firm in case of financial distress provides. I show that within the reasonable range of cash infusions, an investment-grade firm's long term credit spreads can go down by up to 60 basis points. The effect on the five-year default probability can be even more impressive: while it is 3.8% for the case of no cash infusions, it can be reduced to a value as low as 1.8%. The lowest default probability is not achieved for the maximal commitment for cash infusions I consider but at the level of 5.5% of the initial total value, which is just short of 10% of the equity claim in the unlevered firm. I investigate also the optimal level of the equity holders' commitment to infuse additional cash in the company and it should come as no surprise that it is exactly at 5.5% of the total firm value.

The documented effects suggests strongly that the standard structural framework, which assumes perfect financial markets and therefore abstracts from the effects I am interested in, might give an incomplete or even misleading picture of a firm's credit risk. To obtain more reliable results, structural models have to find ways to account for the various market frictions observed in reality.

The model might be also deemed as a normative prescription for a mechanism which ensures that fresh capital is infused in firms in case of financial distress, though the analysis in this direction is too superficial to allow a detailed proposal as in Kashyap et al. (2008). Without being the focus of the paper, the model also accounts for the manager-shareholder agency conflict with respect to the capital structure. Exact predictions in that direction, as well as the analysis of actual and optimal managerial contracts, remain exciting topics for future research.

Appendix 2

This Appendix repeats the Appendix in Zahariev (2009) with some insignificant changes.

The underlying variable of the model is the *EBIT* or the payout flow from the firm's assets. Under the assumptions of a representative agent with a power utility function and a constant volatility of the payout flow on the firm, there is a one-to-one relationship between this payout flow and the total claim on the firm *V* (see Goldstein et al. 2001). This relationship is given by

$$V = EBIT/(r-\mu), \tag{A2.1}$$

where r is the required return on V and μ is the expected growth rate of *EBIT*. One way to think about this relationship is that a fraction μ/r of the cash flow is reinvested in the firm, while the remaining fraction is distributed among the firm's claimants.

The total claim on the firm consists of five distinct components: the claims of equity holders, debt holders, the government, the claim on the bankruptcy costs, and the claim on the restructuring costs.[41] The following relationship holds at any point in time:

$$V = E + D + G + BC + FC\,. \tag{A2.2}$$

Here E is the value of the equity claim, D is the value of the debt claim, G – the government claim, BC – the claim on the bankruptcy costs, and FC – the claim on the flotation costs.

I assume that the relationship (A1) between *EBIT* and *V* holds, although in my model the manager can change the volatility of the payout flow. This is only a simplifying assumption which considerably enhances the tractability of the model. What actually happens is that my manager can reach higher/lower levels of μ at the expense of higher/lower volatility. Then to keep the relationship (A1), the required return for the total claim on the firm r should also change. I checked how this required returns should look like across the time and firm value dimensions so that the assumption for the constant relationship (A1) still holds and compared them to a benchmark constant volatility case.[42] I obtained required returns which are slightly higher than the benchmark case when the manager would choose higher than the standard risk, and slightly lower when the manager would choose lower than the standard risk. For a required return on the firm of 10%, the deviations were about 1% for a 25% increase in volatility. These intuitive results show that the assumption corresponds reasonably well to a

[41] For details, see Goldstein et al. (2001).

[42] For a constant volatility, the required rates of return are constant across the firm value and time dimensions, which is the prerequisite for the one-to-one relationship.

model with an endogenous relationship between the risk and the required return on the total firm claim.

I extend the framework proposed in Goldstein et al. (2001) by allowing the manager to dynamically control the risk level of the firm and to adjust the capital structure in both directions. Further, I assume a fat-tailed mixture distribution for the firm value returns, which can be considered as a mixture of two normal distributions. Finally, I assume the equity holders can precommit to infuse a given amount of cash and analyze the effect of this infusion.

These changes require several major adjustments. I use a discrete-time setting and a final horizon, at which the firm is sold to a third party. At the final horizon I set the values of the claims at all nodes based on the analytical results of Goldstein et al. (2001). An alternative interpretation of this approach is that from then on the firm is run according to the assumptions in their dynamic model.

I further have to introduce two additional state variables: the debt outstanding and the number of shares. Together with time and total firm value, they form a four-dimensional grid, on which the model is implemented. I use quarterly time steps, constant steps in $\Delta(logV)$ of approximately 4.5%, debt steps of 0.1, and share steps in $\Delta(log(log(ST))$ of 0.5%. The constant steps in firm value ensure I can use the same transitional probabilities, conditional on the risk level, everywhere in the grid (detailed discussion how I obtain these probabilities follows shortly). The upper and the lower borders are set initially at 6 and 0.167 so that enough of the range of possible values is captured. The lower value actually depends on the debt level and is always higher than 0.167 (for the lowest debt level it is at approximately 0.27[43]) as I make all total firm values below the default boundary equal to the value exactly at the default boundary. In this way I stick to the deterministic default boundary in Goldstein et al. (2001). I always put additionally an upper and a lower buffer of values for all claims, which are necessary for the optimal decisions close to the boundaries. The size of the buffers is determined by the maximal possible number of upward/downward moves in the model, which I signify by *n*. All values in these buffers are actually the values of the claims just one step above the maximal/minimal firm value in the grid. Those are not the exactly the correct values, but they serve their purpose well. To check the effect of the boundaries, I increase the boundaries further to 8 and 0.125. This changes the claim values at the initial node only after the fourth decimal.

[43] The value is roughly 91% of $0.2/(1-\tau_i)$.

The debt step of 0.1 I choose after some experimenting, which shows that for smaller debt steps the manager is sometimes skipping particular debt levels, which makes their introduction meaningless. I need very precise steps in the number of shares variable to capture well the manager's trade-off. Using less precise steps results in some cases of quite different managerial decisions with respect to the debt and risk level for similar firm values. The reason for this is that the manager's utility function might get similar values for different alternatives (e.g. an increase in the debt level and low risk and no change in the debt level and high risk). The use of equal steps in log-log number of shares further mitigates this numerical problem and in standard applications of the model, serious changes in managerial decisions appear only (if at all) in grid areas where significant changes of the capital structure are necessary, which are reached with extremely low probability.

Building the grid in equal log-steps of V means that moving i nodes up or down will be always associated with the same log-return (conditional on the risk level κ – details follow shortly), no matter where the firm value process is. The standard assumption of a geometric Brownian motion, which is also made for the payout flow in Goldstein et al. (2001) and which is modified here to account for the possibility to increase/decrease the firm's risk and to have fat tails, means that its log-returns over a given time horizon (one time step) will be always independently distributed, so my aim is to construct a vector of transition probabilities for each choice of the risk level. Then I can choose the risk level, which gives the highest expected utility to the manager, and store his optimal risk level at every point in the grid. In the following I discuss how to obtain the discretized distributions (normal and mixture) and how I use them to get values for the claims on the firm.

First, consider a single normal distribution. For the probability to move i (i can take all integer values from $-n$ to n) moves away from the current value of V, I use the following expression:

$$p_{i,\Delta t}(\kappa) = \frac{\dfrac{1}{\sqrt{2\pi}\sigma(\kappa)} \exp\left[-\dfrac{1}{2}\left(\dfrac{i\Delta \log V - \mu(\kappa)}{\sigma(\kappa)} \right)^2 \right]}{\sum_{j=-n}^{n} \dfrac{1}{\sqrt{2\pi}\sigma(\kappa)} \exp\left[-\dfrac{1}{2}\left(\dfrac{j\Delta \log V - \mu(\kappa)}{\sigma(\kappa)} \right)^2 \right]} . \tag{A2.3}$$

The numerator is simply the value of the normal density function I try to approximate. The denominator is a normalization constant which ensures that the probabilities for all *2n+1* possible moves sum up to one. Note that I need only the univariate normal distribution, although I have moves in firm value, debt level, and number of shares. This comes from the fact that the debt control is a perfect one and unlike the risk-level choice (the risk control only influences the probabilities to move to the *2n+1* possible one-step-ahead nodes) ensures that the process moves to a particular debt level, and the associated change in the number of shares comes from the minimal positive cash flow adjustment rule I described earlier. One could alternatively think of a degenerate distribution for the debt level, in which the movement for the optimal debt level from the manager's perspective has a probability of one, while all other moves have a probability of zero.

There are two ways in which this discretized distribution could deviate significantly from the normal distribution. Having few log-steps would result in covering insufficient part of the support of the distribution. Then some moves with meaningful probabilities will not be possible in the discrete model and even in the modelled range the approximation will be poor. Having steps which are too large would lead to coarse steps in the centre of the distribution where the majority of its mass is concentrated and eventually lead again to a poor approximation of the distribution. A good approximation of the normal distribution should then have enough steps of relatively small size. To have a meaningful way to test how well the discretized distribution matches the actual normal distribution, following Stuart and Ord (1987, p. 322), I construct a test statistic based on the first ten moments of the standardized discretized distribution (signified with hats) and the standard normal distribution:

$$\frac{1}{10}\sum_{j=1}^{10}\left(\frac{\hat{\mu}_j-\mu_j}{\frac{1}{const}\left(\mu_{2j}-\mu_j^2+j^2\mu_2\mu_{j-1}^2-2j\mu_{j-1}\mu_{j+1}\right)}\right)^2, \tag{A2.4}$$

where I set *const*=1 and μ_0=0.

After some experimenting, I set a critical value of the above statistic so that it requires the discretized distribution to match the analytical Black-Scholes solution for a one-year at-the money European call option up to 4 decimals in a simple two-dimensional grid (i.e. for a given capital structure). In my analysis, I consider only distributions, for which the test statistic has lower than the critical values, meaning that they are at least as close to the normal as the above mentioned distribution.

To obtain the mixture distribution, I create two normal distributions in the way I explained above. Then I choose weights of the two distributions which sum up to one. By construction this is a well-defined distribution, which, however, has skewness and kurtosis which deviate from the normal distribution. I put restrictions on the parameters of the distributions to match the first two moments of the normal distribution with the baseline parameters, but use the remaining degrees of freedom to fit the empirical default probabilities.

For valuation purposes, I would also need the risk-neutral distributions for any risk level choice. Then I simply adjust the mean term of the distribution in (1) so that the total claim on the firm has a mean equal to the risk-free interest rate.

Goldstein et al. (2001) state that the total claim on the firm consists of four distinct components: the claims of equity holders, debt holders, the government, and the claim on the bankruptcy costs. However, in both theirs and my model there is also a minor fifth claim – the claim on the restructuring costs. Of particular interest for my model are the first two claims, as the manager can trade in them in any time period and his utility depends on their values. It is important to stress that the debt reduction has always a positive effect to equity holders as it reduces the present value of the cash flows to debt holders and increases those to the equity holders.

I set their values of the claims at the last period in time based on the values of V and the analytical expressions for them in Goldstein et al. (2001). To ensure that this assumption for the final values does not have a decisive influence on my results, I investigated an alternative set of final values based on an iteration of my model. As a first step, I ran the model with the analytical expressions of Goldstein et al. (2001). Having obtained values for the claims of all claims for the initial point of time, I inserted these values back as the final values of the claims and re-ran the model. As the results remained virtually unchanged, I stuck to the initial assumption that the firm fulfils the assumptions of Goldstein et al. (2001) at the final date, which saves me computation time.

The manager faces in all periods two simultaneous choices. The first one is what risk level to choose. As discussed above, different risk levels are associated with different transition probabilities and therefore different expected utilities. The other choice which the manager faces in every period is the capital structure choice. He can issue more debt and use the proceeds to buy back equity, or issue new equity to repay some of the existing debt at its market price, or leave the current capital structure unchanged. Any changes of the capital structure come at a given flotation cost, which is borne by the equity holders. The flotation costs for issuing new debt and equity need not be the same. As baseline values, I use 2% for the debt issues and 6% for the equity issues. The results remain very similar when slightly higher or lower transaction costs are considered.

I use a backward sweep on the grid to determine the manager's optimal decisions. As a first step, I compute his utility at any point in the grid in the last period based on his compensation package. If the manager can choose among D possible debt levels and K possible risk levels, then he has a total of DK possible alternatives. For any point of the grid at time $T\text{-}\Delta t$ I compute his expected utilities for each alternative and the associated certainty equivalent. Note that he cannot always issue debt to all possible debt levels as that could automatically trigger default. Note also that the current debt and equity values depend on the manager's risk-taking choice. I assume that in its valuation, the market correctly anticipates the manager's optimal choice and uses risk-neutral valuation to obtain the exact debt and equity values. I use the relationship between V and $EBIT$ to determine the current cash flow and divide it among the claimants: debt holders receive the coupon and the equity holders receive dividends, while the government taxes them at the respective tax rates. The part of the cash flow which accrues to the manager is added to his certainty equivalent. Only then I am able to compute the manager's indirect utility at all points of the grid in time $T\text{-}\Delta t$ and always store the manager's choice. Repeating the same procedure in turn for all earlier periods, I find the manager's optimal choices and indirect utilities at all points of the grid.

To obtain values for all other claims, I use risk-neutral valuation in a backward sweep of the grid. Starting from the last period, in which the values of the claims are defined by the analytical expressions in Goldstein et al. (2001), I use the risk-neutral transition probabilities based on the optimal managerial choice in a particular node of V to determine the values of the claims step by step to the initial period. The default probabilities are determined in a subsequent forward sweep of the grid based on the actual probabilities after all optimal decisions have been identified.

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Chapter 3

Default Risk Premiums: Structural Approach

3.1 Introduction

A large number of empirical studies have been trying to quantify the components of corporate bonds' credit spreads. Often cited references such as Elton, Gruber, Agrawal, and Mann (2001), Driessen (2005), and Longstaff, Mithal, and Neis (2005) among others document that apart from the risk of default, several other factors influence the credit spreads: e.g. taxes, liquidity, and systematic risks. Exact quantification of all effects seems to be a demanding task and results vary slightly from one study to another. Some of the researchers speak also about a "credit spreads puzzle", but the meaning they attribute to it varies slightly among them. Amato and Remolona (2003) refer in this way to the small part of the credit spreads that is attributable to expected losses. Driessen (2005) regards as a puzzle the finding that models tend to predict too high default rates and too low credit spreads at the same time. For Chen, Collin-Dufresne, and Goldstein (2009), the puzzle is the large historical difference between the spreads of Aaa- and Baa-rated firms.

A potential explanation of all these results can be searched for in a sizable premium for the default event and this topic has recently attracted considerable attention. What has facilitated this research is the rapid development in derivatives markets and especially the credit default swaps (CDS). As they are supposed to be much less affected by non-default components, the CDS present an excellent opportunity to isolate the default risk premiums and researchers started addressing the issue. Berndt, Douglas, Duffie, Ferguson, and Schranz (2005), Hull, Predescu, and White (2005), Driessen (2005), and Saita (2006) are recent contributions, documenting the behaviour of default risk premiums.

These papers, however, rely on the so-called reduced-form approach in the credit risk literature, which was pioneered by Jarrow and Turnbull (1992). This approach postulates that the default event is unpredictable and can happen at any point in time. The probability of its occurrence is driven by an intensity function, which depends on observable or latent factors. The other major approach in the credit risk literature is the structural one. It originates from the path-breaking works of Black and Scholes (1973) and Merton (1974). In a nutshell, this approach assumes a particular process for the firm value and defines conditions on this process under which the firm defaults. Conclusions about the default risk premiums based on it, however, are relatively scarce, Berg (2009) and Chen, Collin-Dufresne, and Goldstein (2009) to my best knowledge being the only contributions.

The debate which approach in the credit risk literature is the more fruitful one is still ongoing. The aim of this paper is not to give an answer to it. Here I simply investigate from a theoretical structural perspective the driving forces behind the default risk premiums and illustrate the qualitative and quantitative effects of the most important factors for corporate bond pricing, namely the maturity of the bond and the credit quality of the issuing company. Then, I investigate what changes the different assumptions with respect to the capital structure dynamics in structural models can impose.

The capital structure dynamics is no doubt a major factor for the credit risk of a company. Still, in the structural literature it has not received its due attention and many benchmark models rule out changes in the initial debt level (e.g. Leland 1994). The reason for this simplifying treatment can be explained in the additional state variable that has to be introduced into the model to take care of more complicated dynamics, which has of course a serious impact on a model's tractability. A couple of papers do introduce dynamics in the debt level, but do it in a restrictive manner and allow only new issues of debt but no debt retirements (Goldstein, Ju, and Leland 2001; Fischer, Heinkel, and Zechner 1989 impose in effect the same restriction). Then, the models are solved with the help of a special scaling property. Goldstein et al. (2001) find considerable effects even in their treatment of the capital structure that reconcile some of the mismatches of model predictions and observed data. However, compelling empirical evidence that debt retirements do occur questions the realism of their assumption.

In this paper I apply three structural models that differ only in their assumptions with respect to the capital structure. By showing that both debt issues and retirements are necessary components of a model that matches the stylized behaviour of default risk premiums, I add to the evidence in favour of such a model. I also show that including further factors into the model changes the default risk premiums but preserves their behaviour qualitatively. Finally, my paper contributes to the literature by quantifying the credit quality and maturity effects on default risk premiums.

The paper is structured as follows. The next section reviews the related literature. Section 3.3 presents the models that are consequently applied in section 3.4 to predict default risk premiums over different time horizons and for safer and riskier firms. Section 3.5 concludes. Some details on the numerical implementations of the models are given in Appendix 3.

3.2 Related literature

My paper is related to a couple of research directions within the corporate credit risk literature. One such direction includes the attempts to quantify the relative importance of all components of credit spreads on corporate bonds, in which the default risk premiums are a major contributor. Elton et al. (2001) consider three components: expected losses, taxes[44], and systematic risks (as captured by the Fama-French three-factor model). They find that expected losses are only a minor fraction of credit spreads, especially for high-quality firms: for an A-rated firm, less than 20% of the credit spreads are due to the expected losses. Despite being important (about one third for the credit spread of an A-rated firm), tax effects cannot explain the observed credit spreads, either. The bulk of the remaining part of the spreads, however, can be explained by systematic risk factors. The authors do not consider a special premium for the default event. Amato and Remolona (2003) also emphasize the small part of credit spreads that is due to expected losses. In a discussion of the potential other components, apart from the standard tax and liquidity factors, they propose an additional explanation stemming from the highly skewed distribution of bond returns. They argue that even in portfolios of 300 different bond issuers (which are extremely difficult to attain in practice) there is still some undiversified risk. Given that investors are not able to diversify the risks in bond portfolios, they demand an additional risk premium.

Huang and Huang (2003) consider only default and non-default components in the credit spreads. They extract the default component by using different structural credit risk models and account for a default premium in the default component. Still, they find that only 20% of

[44] Corporate bonds' interest payments are taxed in the USA at the state level while government bonds are exempt from taxation.

the spread for A- or higher rated firms is due to default. This fraction increases to 30% for Baa-rated firms and is above 60% for junk bonds. Their results are often interpreted as a general failure of structural credit risk models. In similar studies Eom, Helwege, and Huang (2004) find that structural models can even predict too high credit spreads, especially for non-investment grade firms and the overall performance is not particularly good, while Ericsson, Reneby, and Wang (2005) find satisfactory performance of structural models in explaining CDS spreads and attribute the underprediction of bond credit spreads to a liquidity component in them. The authors of the latter study conclude that the major weakness of the theoretical structural models is that they do not incorporate the non-default factors in the credit spreads. This is not particularly important in the case of CDS as they are supposed to be free from non-default factors, but might be a serious complication for corporate bonds. The diversity of the results for the structural credit risk models, however, suggests the calibration procedure might be of particular importance. The other open question is how precise the models are in extracting the default risk premiums, which I address in this paper.

A number of studies relied on the reduced-form approach instead. Longstaff et al. (2005) find that more than a half of the credit spreads of all firms come from the default component. Similar to Huang and Huang (2003), they document on average a higher fraction for risky firms than for safe firms. They show that most of the non-default component in the spreads can be explained by a liquidity factor that is almost constant across credit ratings. Driessen (2005) tries to be more precise and investigates a larger set of six different components. As expected, his results are that the default and taxes constitute the major part of credit spreads, but liquidity, firm specific, and common risk factors can also contribute, especially for bonds with longer maturities.

As a result of all these papers it emerges that the part of credit spreads that compensates for the borne credit risk is an important component of credit spreads but is quite difficult to quantify. The credit rating provided by credit rating agencies like Moody's and Standard & Poor's, and the extensive data on observed default losses, which they gathered and analyzed, give a fairly reliable estimate for the expected actual losses. For pricing purposes, however, much more important are the expected losses under the risk-neutral measure.

There are some recent papers that follow the reduced-form approach and try to quantify the risk premium associated with default risk. The intuitively appealing measure they use is the ratio of the risk-neutral and actual default probabilities as it puts in a simple number without dimension the compensation required by investors for bearing default risk. As this measure is becoming a benchmark, I also use it and refer to it throughout the paper as "default risk premium". Note that this measure depends on the time horizon. To circumvent this dependence, in some papers the ratio of the risk-neutral to actual default intensity is used, which is simply the instantaneous default risk premium.

One of the authors that consider the ratio of the risk-neutral and actual default intensity is Driessen (2005). He finds a ratio close to two, but acknowledges the low precision of his estimate. Berndt et al. (2005) analyze the default risk premiums in much more detail, looking how they change over different time horizons (what I call the "maturity effect") and for different credit ratings (the "credit quality effect"). They find a very similar value for the default intensities ratio of the median firm in their sample and further document that default risk premiums increase over time and are higher for safer firms, but the whole term structure varies considerably across time. While investigating the high Sharpe ratios that can be obtained in the corporate bond market, Saita (2006) considers the default risk premiums too. He documents quite high premiums for very safe firms (those with default probabilities below one basis point per year) – well over twenty for both one- and five-year maturities – but excluding them, the median ratios he finds are 2.37 for one year and 2.94 for five years. Finally, Hull et al. (2005) show that the ratio of risk-neutral to actual default intensity can take on very different values. Averaging over credit ratings, they find ratios from 1.2 for B-rated firms to 16.8 for Aaa-rated firms. However, they assume that all the credit spread is due to credit risk when they calculate these values and agree real values should be lower.

While the interest on default risk premiums within a reduced-form setting has been considerable, the structural literature is surprisingly not paying a lot of attention on this topic. Among the few papers that discuss it is the work of Chen, Collin-Dufresne, and Goldstein (2009), who try to explain in a structural approach the big differential between the credit spreads of Aaa- and Baa-rated firms and document both actual and risk-neutral default probabilities for different maturities. The reason to focus on the spread differential is that if the spreads of Aaa- and Baa-rated firms are (hopefully) equally affected by non-default factors, then the difference is unaffected by them. They calibrate the default probabilities for both types of firms to the empirical default rates for two different maturities – four and ten years – and document that the Merton (1974) model generally predicts a spread differential which is only one-half of the actual. Then, they show that the actual differential can be matched with more sophisticated models that provide significant covariance between the pricing kernel and the default time or the pricing kernel and the loss rates. A particular feature of the model they advocate is a counter-cyclical default boundary, though they agree other features of the model that increase the above mentioned covariances can also explain this credit puzzle. For the default risk premiums, their model predicts serious cross-sectional variation, with a median value between three and four for the Baa-rated firms and above ten for the Aaa-rated ones, both values over a four-year horizon. Like Berndt et al. (2005), they find an increase of those values when the horizon increases.

The paper of Berg (2009) is the most closely related to my work, as he considers the predictions of several structural models with respect to the default risk premiums and concludes that the model choice is actually not so relevant. He also starts with the Merton (1974) model. Instead of calibrating it to match a particular default probability, he derives a functional relationship between the actual and the risk-neutral default probability in the model, providing also some numerical examples. In them, both the maturity and the credit quality effects are demonstrated. He then calibrates the model of Duffie and Lando (2001) to match the empirical default probabilities for different firm ratings and maturities, varying other parameters as well. In doing so, he also considers the Leland and Toft (1996) model, which is nested in Duffie and Lando (2001). He concludes that the simple Merton (1974) model gives very similar results to the other more complicated ones. However, there is a slight problem with his approach which he does not mention. The Merton (1974) model has been shown to predict too low default probabilities at all horizons as it does not include the possibility for default prior to the debt maturity (see Jones, Mason, and Rosenfeld 1984). Note that this is not the common critique to structural models that they predict very low default probabilities at short horizons due to the normality assumption. Given this underprediction, for reasonable parameters it will take a longer time to reach a particular cumulative default probability in the model than in reality. As time is present in the functional relationship between the risk-neutral and actual cumulative default probabilities which Berg (2009) considers, it is not so clear what the model-implied relationship for reasonable parameters is. I also document that the capital structure assumption can change considerably the default risk premiums, which contradicts Berg’s (2009) conclusion that all structural models give similar predictions for them.

The other strand of literature that is related to my work is the one that focuses on the firm’s capital structure. It is a widespread idea in the finance literature that firms choose their debt to balance the tax advantage of debt and the associated bankruptcy costs, and this idea – the so-called “trade-off theory”, is present in some of the most often cited structural models (e.g.

Leland 1994). In general, there are three possible assumptions with respect to the capital structure, which a structural credit risk model can make. The simplest is a constant debt level (also referred to as "static capital structure"). This was the usual assumption in the early literature, e.g. in Merton (1974) (in a finite maturity setting) and Leland (1994) (in an infinite maturity setting). An improvement is to allow an increase in the debt level, which is the case in Goldstein et al. (2001). Fischer et al. (1989) work in a similar framework to Goldstein et al. (2001), analyzing also downward adjustments. Depending on the firm characteristics, two cases are possible in their model: when downward adjustments are optimal and when they are not optimal. As all models discussed till now, they work in continuous time. As a result, if downward adjustments are optimal, the firm's debt becomes riskless. However, for all parameter constellations they explore, downward adjustments never take place and the debt is risky, so even though they try to be general, Fischer et al. (1989) use the same setup that is utilized later by Goldstein et al. (2001).

The predictions of static capital structure models with respect to the optimal debt level have been criticized for being too high. Introducing the possibility of future debt issues in the model already improves these results by lowering the optimal initial debt level and increasing the default probability and credit spreads at the same time (see Goldstein et al. 2001 or Dangl and Zechner 2004), which follows naturally from the potential for future adjustments. Whether the predicted leverage ratios in this setting are low enough, however, remains an open question. The approach could be reconciled also with the cross-sectional variations in the observed leverage ratios as restructuring costs induce firms to leave the current debt level unchanged before the firm value increases substantially. Still, allowing both debt issues and debt retirements is the most realistic assumption as both of them occur in reality and, as Leary and Roberts (2005) document, are observed almost equally often. An elegant way to introduce both upward and downward adjustments of the debt level without modelling them explicitly is proposed in Collin-Dufresne and Goldstein (2001), who postulate that firms act in a deterministic way to bring their leverage ratio to its target level. A structural model that explicitly considers both debt issues and retirements is proposed in Zahariev (2009), where a discrete-time framework facilitates their treatment in a dynamic setting. The debt is still risky, as not all of it can be repaid. There is also an agency conflict in this model, as the most important decisions are made by the firm's manager and not by its shareholders. The advantages of giving the capital structure choice to the manager are that the cross-sectional variation can be explained by the differences in managerial compensation and risk preferences

(Lewellen 2006) and that a manager-shareholders conflict reduces further the optimal leverage ratios (Morellec 2004).

As the models of Goldstein et al. (2001) and Zahariev (2009) are applied in this paper to extract the default risk premiums in a structural setting, they are explained in more details later in the paper.

On the empirical side, quite a few papers look for evidence for the trade-off theory. Among them, Fama and French (2002) find that the mean-reversion in leverage is quite slow and note that it can even be a spurious result in their regression analysis. Baker and Wurgler (2002) find evidence that firms try to time the market when they issue new securities and this has persistent results on the capital structure. This makes them speculate that the capital structure is actually a cumulative result of the firm's effort to time the market. Welch (2004) analyzes the shocks in firm value and also finds persistent effects there, which again speaks against the trade-off theory. Leary and Roberts (2005) investigate the effect of transaction costs if in reality the trade-off theory holds. They find support for this theory and show that transaction costs can explain the results of the studies cited above.

3.3 Models

In this section the different models that I use to predict the default risk premiums are briefly presented.

I start with a simple static version of the Goldstein et al. (2001) model. All models presented use its general framework, in which the firm's EBIT is the underlying variable and there are four separate claims on the firm: the equity claim, the debt claim, the claim on the taxes, and the claim on the bankruptcy costs. By modelling in a consistent way all claims on the firm, this framework solves the delicate issue found in other structural models that the equity claim increases with the increase in the corporate tax rate. As Goldstein et al. (2001) show, the assumed geometric Brownian motion for EBIT results in the same process for the total firm value *V*:

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t\,, \tag{3.1}$$

where μ is the drift term, σ^2 is the instantaneous variance, and W_t is a Wiener process. The model also assumes that the firm is run by the shareholders and sets the default boundary to maximize the value of their claim.[45,46] Upon hitting the default boundary, it is no more optimal from the equity holders' perspective to pay the debt's coupon. The bankruptcy costs are incurred and the ownership is transferred to the debt holders, who are now the new equity holders of the all-equity firm. Goldstein et al. (2001) work in an infinite-horizon continuous-time framework, but as for all applied models, I use a discretization in a finite-horizon setting.

The following table 3.1 summarizes the baseline parameters, which are taken from Zahariev (2009). The parameters are thought to represent an average Baa-rated firm. In particular, the debt level is chosen so that the leverage for a total firm value of 1 matches the average leverage of Baa-rated firms as documented in Leland (2004). The range of total firm values, however, should cover all possible credit ratings.

Table 3.1

Baseline parameters valid for all models

Mean:	$\mu = 0.065$	Volatility:	$\sigma = 0.236$
Upper/lower boundaries:	$V_u = 1/V_d = 6$	Time steps:	$m = 20$
Firm value step:	$\Delta log(V) = 0.045$	Possible up moves for 1 period:	$n = 20$
Required return (total firm):	$r = 0.1$	Risk-free rate:	$r_f = 0.04$
Debt level:	$d_0 = 0.27$	Final horizon:	$T = 5$
Coupon rate:	$r_c = 0.05$	Bankruptcy costs:	$\alpha = 0.3$

The second model is the dynamic version of the model of Goldstein et al. (2001) which introduces upward adjustments in the debt level. I assume the debt is always adjusted to the initial leverage at steps of 0.1. This can be thought of as an upper threshold, at which the firm issues new debt. The default boundary is adjusted accordingly. There are also proportional restructuring costs when the firm issues new debt, which I set at 2% as in Gamba and Triantis (2008). The maximal debt level I consider is 0.77. All other model features copy the static version.

[45] An alternative interpretation of the assumption is that the manager's interests are perfectly aligned with those of the shareholders.
[46] In Goldstein et al. (2001) also the debt level and the coupon are set to maximize the shareholders' wealth, which I change slightly to assure the average leverage ratio of a Baa-rated firm is matched for a total firm value of 1. The resulting change in the debt level is almost negligible and the model predictions are not affected by it.

As already discussed, we do not observe only upward adjustments of the debt level in practice, so in the third model I consider I use the additional flexibility which my discrete framework gives me to allow both upward and downward adjustments of the debt level. There is still a minimal debt level set at a non-trivial value (0.2), which cannot be repaid, and the costs for debt retirement are set at 6%, which is again the value in Gamba and Triantis (2008). Whenever the firm value reaches a certain lower threshold, there is a restructuring: the firm issues new shares and repays some of its outstanding debt. As a result the firm reverts to its original leverage ratio. Everything else remains as in the discretization of the dynamic version of Goldstein et al. (2001). This third model is referred to in the following as "dynamic capital structure model".

The three models form a sequence, in which one restriction is relaxed at a time: first the upward and then the downward adjustments of the debt level. As the models are embedded, I can analyze the marginal effect of the relaxation of both constraints as well as their cumulative effect. Overall, the models are quite standard for the structural credit risk literature and keep a simple and tractable setup.

Finally, I also consider the default risk premiums in the more complicated model suggested in Zahariev (2009). This model introduces a risk-averse manager who runs the company to maximize his expected utility. He controls dynamically both the risk level and the capital structure of the firm. As a result of the dynamic risk level control, the mean and volatility of the firm value process (3.1) are chosen by the manager among three possible sets: standard, low, and high risk regimes. Both debt issues and retirements are allowed, but unlike in the dynamic capital structure model, now they take place when the manager finds them optimal. All optimal decisions are determined in a backward sweep of the grid. This model incorporates also a non-normal distribution for the firm value returns. More details on this model can be found in Appendix 3. Please refer also to Zahariev (2009) for the exact model description.

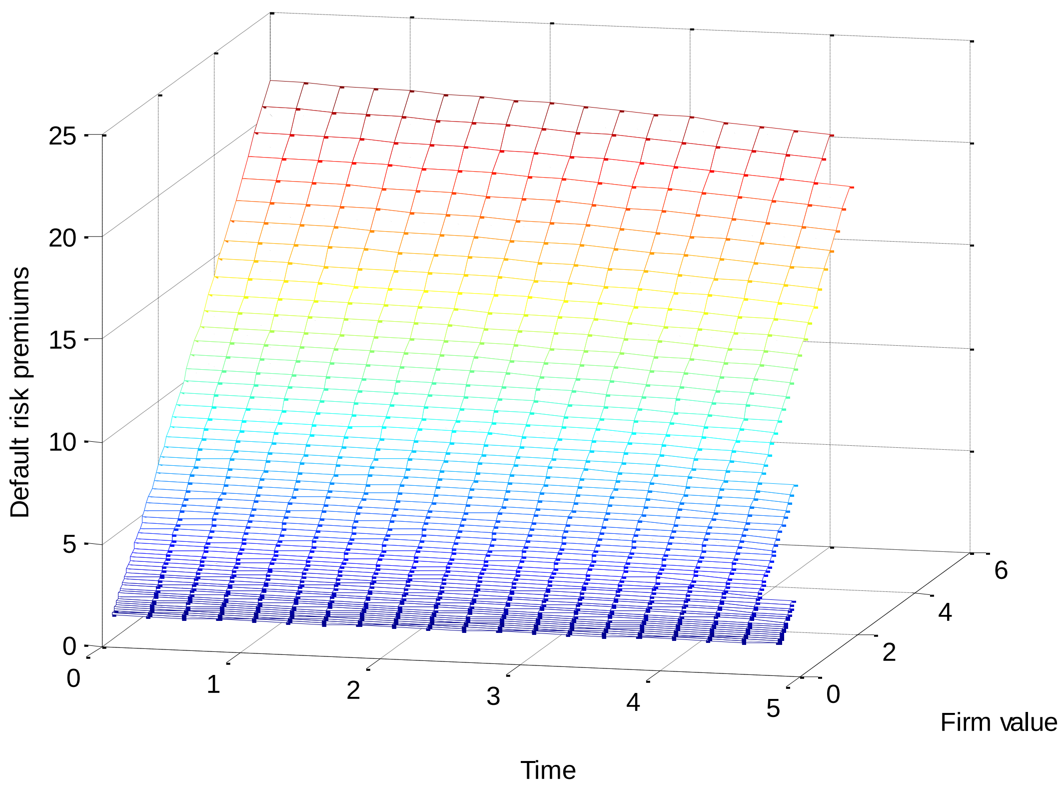
3.4 Results

3.4.1 Default risk premium surfaces

In the following, I plot the default risk premiums against firm values and time to maturity. The resulting surfaces are generated by discretizations of the models presented in the previous section. They are used to generate default probabilities under the risk-neutral (i.e. when the expected total firm return is the risk-free rate) and the actual measure from all nodes of the grid of firm values. Their ratio is then the default risk premium.

Figure 3.1 depicts the default risk premiums surface for the baseline structural model. In this simple setting, the default risk premiums increase almost linearly with the firm value, while the time to maturity does not seem to influence them. It is remarkable that for very safe firms, the risk-neutral default probability can be more than twenty times larger than the actual default probability. For those firms, both quantities tend to zero, so the absolute difference between them (unlike the ratio) is actually decreasing. These findings are simply a confirmation of the results of Berg (2009) for the Merton (1974) model, the only difference here being that default can happen prior to maturity. The ratios of more than twenty may seem odd, but they are not deviating too much from what Hull et al. (2005) or Amato and Remolona (2003) document empirically or what Chen et al. (2009) get as predictions from more complicated structural models.

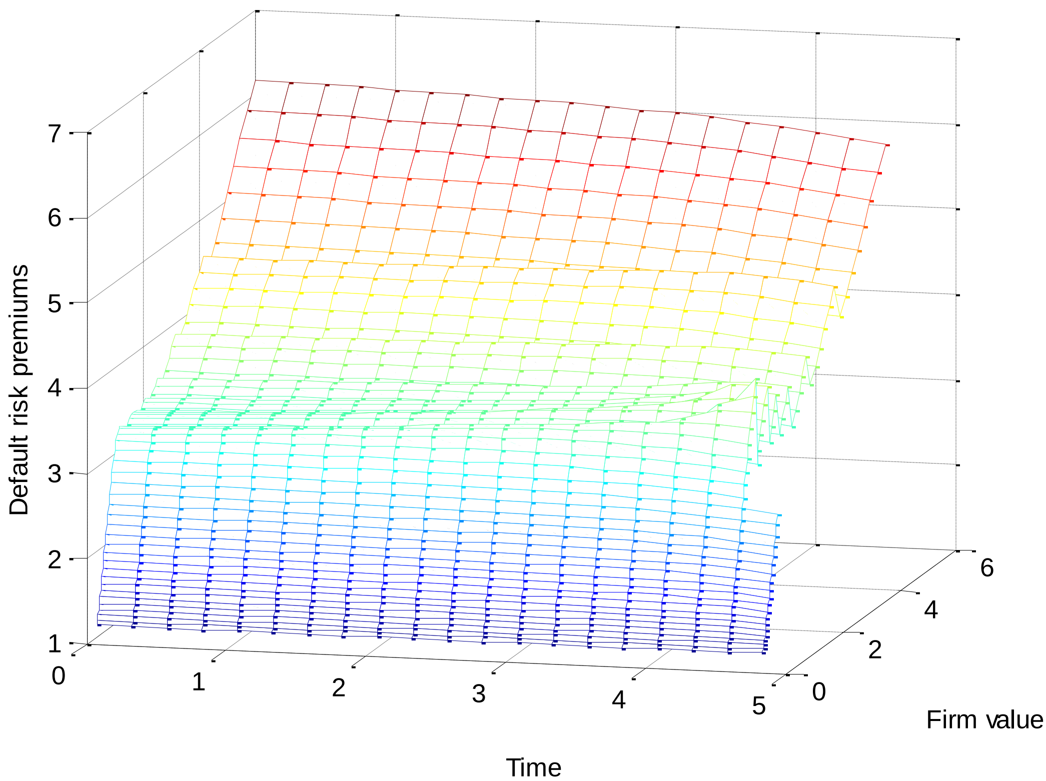
Fig. 3.1 Default risk premiums in the baseline model



My next step is to check how sensitive these results are to the assumption about the capital structure dynamics. The first extension follows the lines of Goldstein et al. (2001) and assumes capital recapitalizations at fixed firm values, at which the firm reverts to the initial leverage ratio. The following figure is a stylized representation of what we observe in this framework.

The smooth linear relationship between the default risk premiums and the firm value is now gone, replaced by some wave-like pattern. Only for low firm values do we observe a basically unchanged picture relative to the static capital structure case. Even there the potential future increases in the debt level affects both actual and risk-neutral default probabilities, but they grow in a similar fashion so that their ratio remains almost the same. For very high firm values, the pattern also seems to be strictly increasing as in the previous case, although the magnitudes of the default risk premiums are now about three times lower. The strictly increasing pattern, however, is only due to the model discretization. As the firm always reverts to a fixed leverage ratio (the standard one for a Baa-rated firm) and as we have a geometric Brownian motion for the firm value process, simple intuition suggests that default probabilities are the same every time the firm recapitalizes. As in the model discretization there is a limit in the debt level, once this limit is reached, we are back to the simple static case, in which default risk premiums grow in a linear fashion.

Fig. 3.2 Default risk premiums in the Goldstein et al. (2001) model

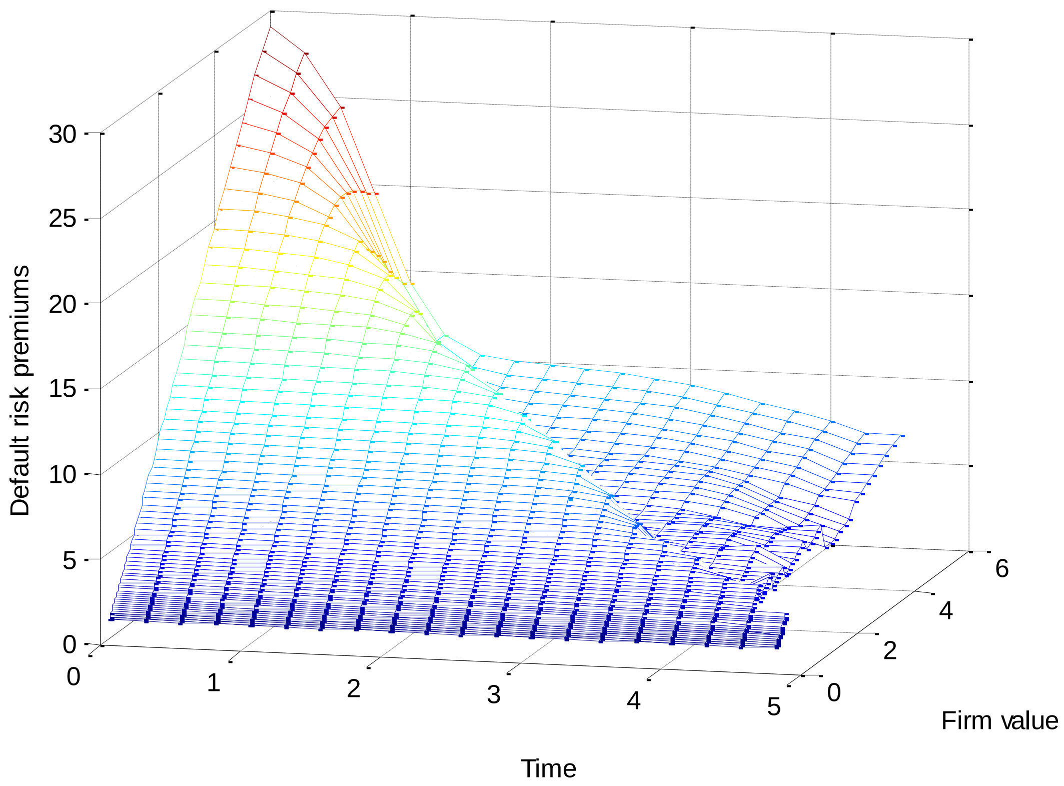


The other simplification in the default risk premiums surface in Figure 3.2 is the assumption that the firm currently is at the lowest admissible debt level. If the firm were at a higher debt level, the steep slope close to the default boundary would move to the new (higher) default boundary.

Overall, this is the model specification that is most severely affected by the discretization. What we should really observe in the model is one steep slope leading to the default boundary (which we still observe in the discretization) and an infinite sequence of identical waves for increasing firm values, with a typical default risk premium varying between three and four. If the firm has a higher (lower) target leverage ratio, which would be the case for a riskier (safer) firm, the typical value of the premium would be lower (higher).

The third model I consider is an extension of Goldstein et al. (2001) which incorporates debt reductions. As in the implementation of the original model, only a discrete set of debt levels is allowed. An additional lower debt level is introduced, but the minimal debt level is still a non-trivial fraction of the total firm value (20% of the standardized initial firm value). The default risk premium surface of this model is illustrated in Figure 3.3.

Fig. 3.3 Default risk premiums in the model with adjustments of the debt level in both directions

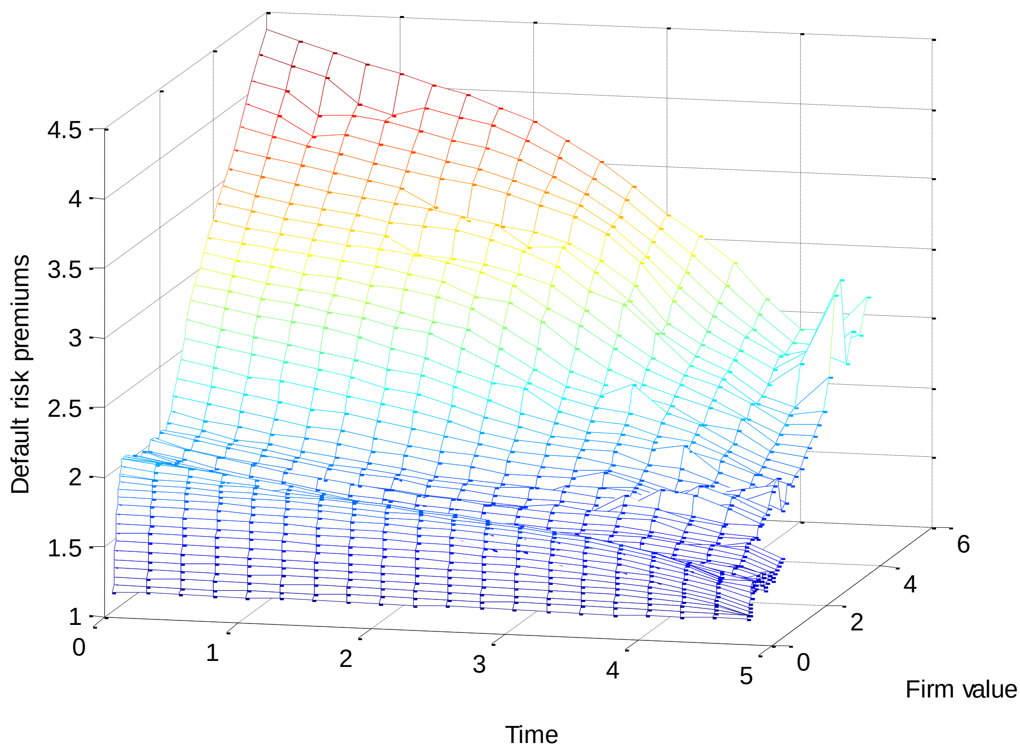


Two important comments are in place. First, as in the static model, the default risk premiums in this model can reach really high values (above 20), which, however, occur only for very high firm value, respectively for extremely safe firms, and over longer horizons (4-5 years to maturity). Second, we can now see very distinctly the maturity effect, that is, the increase of default risk premiums with the increase in debt maturities, which has been empirically documented by Berndt et al. (2005).

Having considered the three possible assumptions with respect to the debt level dynamics in the structural models, I can evaluate the impact of each of them on default risk premiums. The static capital structure produces a quite serious credit quality effect, but next to no maturity effect and is hence not able to match the empirical evidence. Upward adjustments of the debt level, even if able to explain the credit quality effect in the cross section as stemming from the target leverage ratio, cannot solve this problem either. To obtain meaningful results for the default risk premiums in a structural framework, one has to model the capital structure so that both upward and downward adjustments of the debt level are allowed. As shown in Zahariev (2009), this has also a sizable impact on the firm's credit spreads.

As an additional check of this conclusion I test the prediction of the model suggested in Zahariev (2009). Its predictions are depicted in Figure 3.4.

Fig. 3.4 Default risk premiums in the model of Zahariev (2009)



The shape of this surface does not differ considerably from the previous model but the size of the premiums is now relatively modest: over one period the average ratio is about 1.5 and increases to values slightly above 2 for the five-year horizon, while only for the safest firms ant the five-year maturity a default risk premium of 4 can be reached. Overall, the results suggests that even though this more complicated model was shown in Zahariev (2009) to be better than the model of Goldstein et al. (2001) at capturing both default probabilities and default risk premiums, the improvement with respect to the latter measure comes mostly from the more realistic debt level dynamics and not from the other extensions.

To check the validity of this last statement I also investigate if really none of the additional factors in the last model has a pronounced effect on the default risk premiums. My findings suggest that dynamic risk taking and capital structure decisions made by the manager (instead of the exogenous capital structure adjustments in the dynamic capital structure model) have only a minor quantitative effect. The introduction of a fat-tailed negatively-skewed mixture distribution brings most of the overall decrease in the default risk premiums in the last model. Its effect on the shape of the default risk premiums surface, however, is quite limited. As the results are not particularly interesting, I do not report them here.

3.4.2 Quantification of the credit quality and maturity effects

The figures in the previous subsection illustrated the general patterns that structural models with different capital structure dynamics predict for the default risk premiums. As the analysis there was only qualitative, the validity of the conclusions was not statistically tested. To make them more rigorous, in this subsection I resort to a bilinear smoothing to get numerical values for the credit quality and maturity effects. In this way, I can compare the different models' predictions numerically and get confidence intervals for the size of the effects. Another reason for this exercise refers to a potential application of such an approach to interpolate or extrapolate default risk premiums both for a firm, which already has traded credit instruments with a different maturity, and for other firms.

For all models, the following simple econometric relationship is assumed:

$$\ln DRP_{t,V_t} = \beta_0 + \beta_1 V_t + \beta_2 (T-t) + \varepsilon_{t,V_t}, \qquad (3.2)$$

where t is the time index, T is the final horizon, V_t is the firm value at all nodes of the grid, and β_i, $i=0,1,2$ are coefficients to be estimated. The error term is assumed to be with zero expectation, independent and identically distributed across time and firm values. A logtransformation of the default risk premium (*DRP*) is used to make the comparison between models easier, as in this case the interpretation of the coefficients is "a proportionate increase in the dependent variable for a unit increase of the independent variable". The regression equation is estimated by ordinary least squares. Note that all nodes are given in this way the same weight, while some parts of the grid might reflect the situation of more firms compared to other parts of the grid. Still, as the goal is to consider the whole universe of firms, the approach should not be a problem.

Table 3.2 presents the results for the different structural models. The sign of all coefficients is the expected one (positive for the maturity and negative for the credit quality effect) and all of them are very significant, apart from the maturity effect in the Goldstein et al. (2001) model, which has a *t*-statistic of less than one in absolute terms. Also the economical significance of the maturity effect in the baseline model is questionable, as 2% decrease of the default risk premium for every expired year is less than the empirical estimates in Berndt et al. (2005) and Saita (2006). These last two estimates confirm the initial impression of Figures 3.1 and 3.2 that the maturity effect is not really present. However, in the other two models, the effect is 15 and 8% for an additional year respectively, which figures can be reconciled with the empirical evidence. The R-squared in all regressions is quite high – above 0.65 – and even close to 0.9 in the baseline model.

The size of the credit quality effect varies considerably between the models and is, as expected, strongest in the baseline model. As already discussed, its size in the Goldstein et al. (2001) model could be misleading due to the discretization effect.

Table 3.2

Determinants of the default risk premiums. Standard errors of the estimates are given in brackets. * stands for statistically significant at 1% confidence level.

	Baseline model	Goldstein et al. (2001)	Dynamic capital structure model	Zahariev (2009)
β_0	0.5267* (0.0189)	0.6324* (0.0158)	1.1365* (0.0276)	0.4820* (0.0062)
β_1	0.5238* (0.0052)	0.2209* (0.0043)	0.3692* (0.0077)	0.1559* (0.0017)
β_2	-0.0169* (0.0056)	-0.0046 (0.0047)	-0.1481* (0.0084)	-0.0771* (0.0019)
R^2	0.8906	0.6806	0.6550	0.8775

3.5 Conclusion

In this paper I investigate the default risk premiums in a theoretical structural framework. I find that the assumption about the capital structure dynamics has a material impact on the model predictions. In particular, only a model that incorporates both debt issues and retirements can match the increase in default risk premiums for longer maturities that has been documented by Berndt et al. (2005) in an empirical study based on a reduced-form approach. This result together with simple observations on actual corporate capital structure decisions suggest that structural models should not overlook the dynamic leverage control that firms exercise in practice.

Having presented the default risk premiums surface qualitatively, I turn to a quantification exercise for the two main effects on them: the maturity and the credit quality effect. The practical application of this exercise can be in the pricing of bonds or CDS once some reliable information on similar liquid instruments is available.

One aspect of default risk premiums that remained outside the focus of this study is their considerable time series variation. It would be quite difficult to account for it in a structural setting without introducing some additional parameter governing either the macroeconomic conditions or time-varying risk-aversion of a representative agent. The introduction of such model characteristics together with a more realistic capital structure dynamics in a structural framework remain exciting topics for future research.

Appendix 3

In Zahariev (2009), I extend the framework proposed in Goldstein et al. (2001) by allowing the manager to dynamically control the risk level of the firm and to adjust the capital structure in both directions. Further, I assume a fat-tailed mixture distribution for the firm value returns, which can be considered as a realisation of two separate random variables: first, a Bernoulli random variable determining which out of two available processes will give the return; and second, the realisation of that particular process of the form given in equation (3.1).

I use a discrete-time setting and a final horizon, at which the firm is sold to a third party. At the final horizon I set the values of the claims at all nodes based on the analytical results of Goldstein et al. (2001). An alternative interpretation of this approach is that from then on the firm is run according to the assumptions in their dynamic model. However, I consider also an alternative set of final values of the claim based on my model, which is going to be commented upon at the end of this appendix.

I further have to introduce two additional state variables: the debt outstanding and the number of shares. Together with time and total firm value, they form a four-dimensional grid, on which the model is implemented. I use quarterly time steps, constant steps in *Δ(logV)* of approximately 4.5%, debt steps of 0.1, and share steps in *Δ(log(log(ST))* of 0.5%. The constant steps in firm log-value ensure I can use the same transitional probabilities, conditional on the risk level, everywhere in the grid (detailed discussion how I obtain these probabilities follows shortly). The upper and the lower borders are set initially at 6 and 0.167 so that enough of the range of possible values is captured. The lower value actually depends on the debt level and is always higher than 0.167 (for the lowest debt level it is at approximately 0.27) as I make all total firm values below the default boundary equal to the value exactly at the default boundary. I always put additionally an upper and a lower buffer of values for all claims, which are necessary for the optimal decisions close to the boundaries. The size of the buffers is determined by the maximal possible number of upward/downward moves in the model, which I signify by *n*. All values in these buffers are actually the values of the claims just one step above the maximal/minimal firm value in the grid. Those are not the exactly the correct values, but they serve their purpose well. To check the effect of the boundaries, I increase the boundaries further to 8 and 0.125. This changes the claim values at the initial node only after the fourth decimal.

The debt step of 0.1 I choose after some experimenting, which shows that for smaller debt steps the manager is sometimes skipping particular debt levels, which makes their introduction meaningless. I need very precise steps in the number of shares variable to capture well the trade-off the manager faces. Using less precise steps results in some cases of quite different managerial decisions with respect to the debt and risk level for similar firm values. The reason for this is that the manager's utility function might get similar values for different alternatives (e.g. an increase in the debt level and low risk and no change in the debt level and high risk). The use of equal steps in log-log number of shares further mitigates this numerical problem and in standard applications of the model, serious changes in managerial decisions appear only (if at all) in grid areas where significant changes of the capital structure are necessary, which are reached with extremely low probability.

Building the grid in equal log-steps of V means that moving i nodes up or down will be always associated with the same log-return (conditional on the risk level κ – details follow shortly), no matter where the firm value process is. The standard assumption of a geometric Brownian motion, which is also made for the payout flow in Goldstein et al. (2001) and which is modified here to account for the possibility to increase/decrease the firm's risk and to have fat tails, means that its log-returns over a given time horizon (one time step) and conditioning on the risk level choice, will be always independently distributed, so my aim is to construct a vector of transition probabilities for each choice of the risk level. Then I can choose the risk level, which gives the highest expected utility to the manager, and store his optimal risk level at every point in the grid. In the following I discuss how to obtain the discretized distributions (normal and mixture) and how I use them to get values for the claims on the firm.

First, consider a case where $\Delta log(V_t)$ is normally-distributed.[47] Let μ be the mean, and σ be the standard deviation of the normal distribution of the process for V. Both of them are actually functions of the risk level κ, which I allow to take the values 0.75, 1, and 1.25. The interpretation is that the manager can use different derivatives (e.g. forward contracts) to hedge part of the firm risk or to increase it. Alternatively, the manager could achieve similar results by investing in risky or safe projects. The functional form I assume for μ and σ is:

$$\mu(\kappa) = \kappa r + (1-\kappa) r_f - (r - \bar{\mu}) - \frac{1}{2}\kappa^2 \bar{\sigma}^2 \tag{A3.1}$$

$$\sigma(\kappa) = \kappa \bar{\sigma} \tag{A3.2}$$

[47] This discretization of the normal distribution is used in the first three models applied in the paper without the dynamic control of the risk level. For them I simply set κ to 1.

where $\bar{\mu}$ and $\bar{\sigma}$ are the expected instantaneous return and the instantaneous volatility for the normal risk level (κ equal to 1). Note that the firm always pays out $r - \bar{\mu}$ of its current total value. For the probability to move i (i can take all integer values from $-n$ to n) moves away from the current value of V I use the following expression:

$$p_{i,\Delta t}(\kappa) = \frac{\frac{1}{\sqrt{2\pi}\sigma(\kappa)} \exp\left[-\frac{1}{2}\left(\frac{i\Delta \log V - \mu(\kappa)}{\sigma(\kappa)}\right)^2\right]}{\sum_{j=-n}^{n} \frac{1}{\sqrt{2\pi}\sigma(\kappa)} \exp\left[-\frac{1}{2}\left(\frac{j\Delta \log V - \mu(\kappa)}{\sigma(\kappa)}\right)^2\right]}. \tag{A3.3}$$

The numerator is simply the value of the normal density function I try to approximate. The denominator is a normalization constant which ensures that the probabilities for all *2n+1* possible moves sum up to one. Note that I need only the univariate normal distribution, although I have moves in firm value, debt level, and number of shares. This comes from the fact that the debt control is a perfect one and unlike the risk-level choice (the risk control only influences the probabilities to move to the *2n+1* possible one-step-ahead nodes) ensures that the process moves to a particular debt level, and the associated change in the number of shares comes from the minimal positive cash flow adjustment rule I described earlier. One could alternatively think of a degenerate distribution for the debt level, in which the movement for the optimal debt level from the manager's perspective has a probability of one, while all other moves have a probability of zero.

There are two ways in which the discretized distribution could deviate significantly from the normal distribution. Having few log-steps would result in covering insufficient part of the support of the distribution. Then some moves with meaningful probabilities will not be possible in the discrete model and even in the modelled range the approximation will be poor. Having steps which are too large would lead to coarse steps in the centre of the distribution where the majority of its mass is concentrated and eventually lead again to a poor approximation of the distribution. A good approximation of the normal distribution should then have enough steps of relatively small size. To have a meaningful way to test how well the discretized distribution matches the actual normal distribution, following Stuart and Ord (1987, p. 322), I construct a test statistic based on the first ten moments of the standardized discretized distribution (signified with hats) and the standard normal distribution:

$$\frac{1}{10}\sum_{j=1}^{10}\left(\frac{\hat{\mu}_j-\mu_j}{\frac{1}{const}\left(\mu_{2j}-\mu_j^2+j^2\mu_2\mu_{j-1}^2-2j\mu_{j-1}\mu_{j+1}\right)}\right)^2, \tag{A3.4}$$

where I set *const*=1 and μ_0=0.

After some experimenting, I set a critical value of the above statistic so that it requires the discretized distribution to match the analytical Black-Scholes solution for a one-year at-the money European call option up to 4 decimals in a simple two-dimensional grid (i.e. for a given capital structure). I apply only normal distributions, for which the test statistic has lower than the critical values, meaning that they are at least as close to the normal as the above mentioned distribution.

The procedure is similar for the mixture distribution. For it I create two normal distributions in the way I explained above. Then I choose weights of the two distributions which sum up to one. By construction this is a well-defined distribution, which, however, has skewness and kurtosis which deviate from the normal distribution. The exact parameters of the two distributions are the same as in Zahariev (2009) and are obtained in a calibration exercise for the historical default rates of Baa-rated firms.

For valuation purposes, I would also need the risk-neutral distributions for any risk level choice. Then I simply adjust the mean term of the distribution in (A3.1) and also the mixture distribution so that the total claim on the firm has a mean equal to the risk-free interest rate.

The total claim on the firm consists of four distinct components: the claims of equity holders, debt holders, the government, and the claim on the bankruptcy costs. Of particular interest are the first two claims, as the manager can trade in them in any time period and his utility depends on their values. I set their values at the last period in time based on the values of *V* and the analytical expressions for them in Goldstein et al. (2001).

The manager faces in all periods two simultaneous choices. The first one is what risk level to choose. As discussed above, different risk levels are associated with different transition probabilities and therefore different expected utilities. The other choice which the manager faces in every period is the capital structure choice. He can issue more debt and use the proceeds to buy back equity, or issue new equity to repay some of the existing debt at its market price, or leave the current capital structure unchanged. Any changes of the capital structure come at a given flotation cost, which is borne by the equity holders. The flotation costs for issuing new debt and equity need not be the same. I use 2% for the debt issues and 6% for the equity issues.

I use a backward sweep on the grid to determine the manager's optimal decisions. As a first step, I compute his utility at any point in the grid in the last period based on his compensation package. The compensation I assume is the typically observed in practice: stocks, stock options, and cash. I use the same parameters as in Zahariev (2009). If the manager can choose among D possible debt levels and K possible risk levels, then he has a total of DK possible alternatives. For any point of the grid at time T-Δt I compute his expected utilities for each alternative and the associated certainty equivalent. Note that he cannot always issue debt to all possible debt levels as that could automatically trigger default. Note also that the current debt and equity values depend on the manager's risk-taking choice. I assume that in its valuation, the market correctly anticipates the manager's optimal choice and uses risk-neutral valuation to obtain the exact debt and equity values. I use the relationship between V and $EBIT$ in Goldstein et al. (2001) to determine the current cash flow and divide it among the claimants: debt holders receive the coupon and the equity holders receive dividends, while the government taxes them at the respective tax rates. The part of the cash flow which accrues to the manager is added to his certainty equivalent. I am then able to compute the manager's indirect utility at all points of the grid in time T-Δt and always store the manager's choice. Repeating the same procedure in turn for all earlier periods, I find the manager's optimal choices and indirect utilities at all points of the grid.

To obtain values for all other claims, I use risk-neutral valuation in a backward sweep of the grid. The default probabilities are determined in a subsequent forward sweep of the grid based on the actual probabilities after all optimal decisions have been identified. To obtain the ratio of risk-neutral to actual default probabilities in all points of the grid, I have to re-run the model as many times as the number of nodes I have. This is relatively time-consuming, but still manageable within an hour on a standard machine.

To ensure that the assumption for the final values does not have a decisive influence on my results, I investigated an alternative set of final values based on an iteration of my model. As a first step, I ran the model with the analytical expressions of Goldstein et al. (2001). Having obtained values for all claims at the initial point of time, I inserted these values back as the final values of the claims and re-ran the model. As the results remained virtually unchanged, I stuck to the initial assumption that the firm fulfils the assumptions of Goldstein et al. (2001) at the final date.

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Erklärung

Ich erkläre hiermit, dass ich die vorliegende Arbeit mit dem Thema

Three Essays on Structural Credit Risk Modelling

ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus den anderen Quellen direkt oder indirekt übernommenen Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Weitere Personen, insbesondere Promotionsberater, waren an der inhaltlich materiellen Erstellung dieser Arbeit nicht beteiligt. Die Arbeit wurde bisher weder im In- noch im Ausland in gleicher oder ähnlicher From einer anderen Prüfungsbehörde vorgelegt.

Konstanz, den 15. Oktober 2009 --

(Radoslav Zahariev)