Two atomic quantum dots interacting via coupling to BECs

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Abstract. - We consider a system of three weakly coupled Bose-Einstein condensates and two atomic quantum dots embedded in the barriers between the condensates. Each dot is coupled to two neighboring condensates by optical transitions and can be described as a two-state system, or a pseudospin 1/2. Although there is no direct coupling between the dots, an effective interaction between the pseudospins is induced due to their coupling to the condensate reservoirs. We investigate this effective interaction, depending on the strengths of the dot-condensate coupling T and the direct coupling J between the condensates. In particular, we show that an initially ferromagnetic arrangement of the two pseudospins stays intact even for large T/J. However, antiferromagnetically aligned spins undergo peculiar "breathing" modes for weak coupling $T/J < 1$, while for strong coupling the behaviour of the spins becomes uncorrelated.

Introduction. – Atomic quantum dots (AQDs) are a relatively new topic, referred to as nanobosonics in the context of cold atoms. An AQD constitutes a lowest atomic level of a tight trapping potential which can be maximally singly occupied due to a large repulsive interaction energy between the atoms. In analogy to nanoelectronics the question arises whether one can couple an atomic dot to large bosonic reservoirs, i.e. "leads" and investigate particle transport properties in such a combined system. Another interesting problem is the control and manipulation of the single spin behaviour, since such a quantum dot can be described in terms of a two-state system (the occupied state is equivalent to a "spin-up", and the empty state corresponds to a "spin-down").

An interesting variant of the optical coupling between an AQD and a uniform superfluid reservoir was considered by Recati *et al* [1]. Provided the dot atoms and the atoms of the superfluid reservoir are of different hyperfine species, one can (by an external laser) induce Raman transitions between dot atoms and those of the condensate. At low energies the spin degrees of freedom couple to the lowest excitation modes of a superfluid bath - phonons, and therefore the combined system can be mapped onto a spin-boson model, and the dissipative behaviour of the spin can be explored [1]. From quantum optical point of view this kind of Raman coupling and its consequences were numerically investigated by Zippilli and Morigi [2].

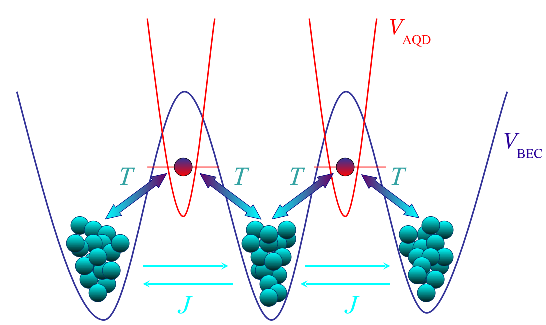

Fig. 1: System of two atomic quantum dots coupled to three Bose-Einstein condensates in a three-well potential $V_{\rm BEC}$. Inside the barriers between the wells atomic quantum dots (single atoms of a different from condensate bosons hyperfine species) are embedded. T is the optical coupling between the dots and condensate, and J is the Josephson tunneling between the wells.

In previous works by Bausmerth *et al.* [3] and Fischer *et al.* [4], a single AQD coherently coupled by optical transitions to *two* BEC-reservoirs with finite number of particles has been considered. A direct Josephson tunneling

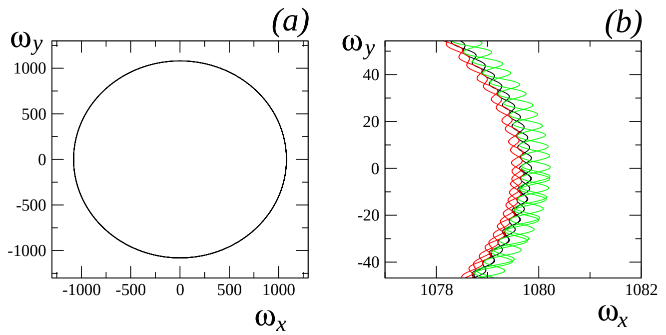

Fig. 2: Rotation frequencies (Eqs. (10) and (11)) of the pseudospins for symmetric initial conditions in the case of weak coupling. We took $N_1(0) = N_3(0) = 500$, $N_2(0) = 1000$, $U = 0$ and the relative coupling of the dot to the condensate is $T/J = 10$. The frequencies ω_x^j and ω_y^j are expressed in units of $2J$. It is clear that in both FM and AFM cases the spins nutate. Plot (b) shows a zoom in details of plot (a) on a scale, on which the differences between the three curves in plot (a) can be distinguished. Red and black curves show the frequencies of the two initially antiferromagnetically aligned spins, while the green line is for initially ferromagnetically aligned spins. The detuning energy is fixed to $\hbar\delta/2J = 1$.

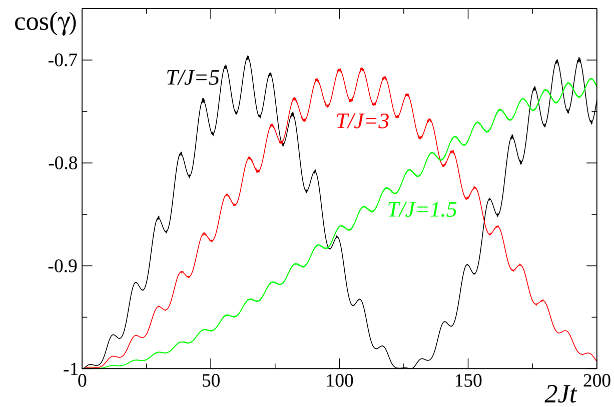

Fig. 3: Relative orientiaton $\vec{s}_1\vec{s}_2 = \cos(\gamma)$ versus time for different coupling constant T/J. The initial conditions are the same as in Fig. 2 and initial alignment of the two pseudospins is AFM. One can observe the "breathing" modes, whose period decreases for growing T/J.

between the two BECs has also been taken into account. One should note here, that a Bose Josephson junction exhibits an unusually rich dynamical behaviour [5–7], which was also confirmed experimentally [8].

In previous works [3] and [4] it was observed, that one should distinguish between the two important limiting cases: (i) the weak coupling regime, in which the coupling between dot and condensate T is smaller or of the same order as the direct Josephson coupling J between two condensates, and (ii) the strong coupling regime, in which the coupling between dot and condensate is dominating, so that the tunneling between the two condensates occurs predominantly through the dot channel [4].

Since in the weak coupling regime the effect of the AQD on the tunneling between the condensates is very small, it is the favorable regime in order to investigate the behaviour of the pseudospin under the influence of an effective time-dependent field, induced by the condensates dynamics. It turns out, that the pseudospin behaves in this case as a quantum top and undergoes multi-frequency rotations depending on the initial conditions and the mean-field interaction parameter [3].

In the strong coupling limit ($T \gg J$), it was shown that an atomic quantum dot, surprisingly, can cause large particle imbalance oscillations between the wells similar to the Josephson effect, which are characterized by the two frequencies. For non-interacting condensates these frequencies could be derived analytically [4].

In this work we discuss the three-well condensate with two AQDs embedded between the wells (fig. 1), coupled to the condensates by optical Raman transitions. We show that, although the AQDs do not directly interact with each other, their coupling to the condensates induces an effective coupling between the two pseudospins, associated with the two dots, which can be studied numerically. We show below that symmetric initial conditions favour the ferromagnetic (FM) coupling between the dots, while antiferromagnetic (AFM) spin alignment does not survive the increasing coupling T/J. These features are especially clearly observed in the strong coupling limit.

Model. – We investigate the setup depicted in Fig. 1. The condensate in a three-well potential $V_{\rm BEC}$ is described within a three mode approximation, which is just a generalization of the two-mode case [5, 6], so that the wavefunction is $\Psi(\mathbf{r},t) = \sum_{i=1}^{3} \phi_i(\mathbf{r})\Psi_i(t)$, where the $\phi_i(\mathbf{r})$'s are the normalized local mode solutions for well i, and

$$\Psi_i(t) = \sqrt{N_i(t)}e^{i\theta_i(t)}. \qquad (1)$$

Here N_i is the number of particles in the i-th condensate, and θ_i is its phase.

The Hamiltonian of our system reads

$$\begin{aligned} H &= \sum_{i=1}^{3} E_i|\Psi_i|^2 + \frac{U}{2}\sum_{i=1}^{3}|\Psi_i|^4 - J(\Psi_1^\dagger\Psi_2 + \Psi_2^\dagger\Psi_1) \\ &- J(\Psi_2^\dagger\Psi_3 + \Psi_3^\dagger\Psi_2) - \hbar\delta\sum_{i=1}^{2}\frac{1+\sigma_z^{(i)}}{2} \\ &+ T\sum_{i=1,2}\Big[(\Psi_i^\dagger + \Psi_{i+1}^\dagger)\sigma_-^{(i)} + h.c.\Big], \end{aligned} \qquad (2)$$

where $\sigma_\pm^{(j)} = (\sigma_x^{(j)} \pm i\sigma_y^{(j)})/2$, where $\sigma_x^{(j)}$, $\sigma_y^{(j)}$, and $\sigma_z^{(j)}$ are the Pauli matrices describing the pseudospin degrees of freedom of the first ($j = 1$) or the second ($j = 2$) atomic quantum dot. Each quantum dot is described by a state-vector $\vec{s}^{(j)} = \langle\Psi_d^j|\vec{\sigma}^{(j)}|\Psi_d^j\rangle$ with $j = 1, 2$ and Ψ_d^j being a two-state wave function of the j-th dot. This formalism is applicable in the limit of large interaction on the atomic dots.

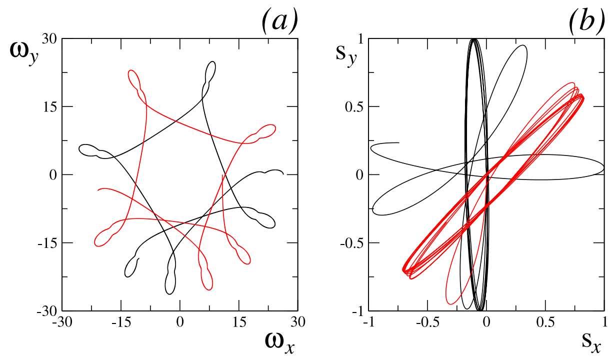

Fig. 4: Behaviour of the pseudospins for asymmetric initial condensate sizes in the case of weak coupling: $N_1(0) = 10000$, $N_2(0) = 1000$, $N_3(0) = 500$, interaction is fixed to $UN_0/2J = 1$, and the relative coupling of the dot to the condensate is $T/J = 0.1$. Plot (a) displays the frequency evolution in the ω_x-ω_y-plane, while in plot (b) the behaviour of the pseudospins is shown. The spins were initially FM aligned. The black curve denotes the left pseudospin, and red curves correspond to the right pseudospin.

For simplicity we consider a symmetric system: the Josephson couplings between the condensates are the equal: $J_{12} = J_{23} \equiv J$, defined through

$$J = -\int d\boldsymbol{r} \Big[\frac{\hbar^2}{2m}(\nabla\phi_i(\boldsymbol{r})\nabla\phi_{i+1}(\boldsymbol{r})) + \phi_i(\boldsymbol{r})V_{\rm BEC}(\boldsymbol{r})\phi_{i+1}(\boldsymbol{r})\Big]\,,\; i = 1,2\,. \quad (3)$$

All four dot-condensate couplings are expressed by a single parameter T

$$T = \hbar\Omega_R \int d\boldsymbol{r}\,\phi_j(\boldsymbol{r})\phi_d^j(\boldsymbol{r}) = \hbar\Omega_R \int d\boldsymbol{r}\,\phi_{j+1}(\boldsymbol{r})\phi_d^j(\boldsymbol{r}), \quad (4)$$

where $j = 1, 2$, and the spatial wave function of each dot $\phi_d^j(\boldsymbol{r})$ is normalized to unity. Ω_R is the Rabi frequency of the Raman transition. The Rabi frequency is an externally controllable parameter, that can be tuned to any desirable value. Spontaneous emission is suppressed by a large detuning from the excited electronic states, which is absorbed into the effective dot energy $\hbar\delta$ [1]. Another important parameter in the problem is the standard mean-field two-particle interaction between the condensate atoms

$$U = g\int d\boldsymbol{r}\,|\phi_i(\boldsymbol{r})|^4, \quad (5)$$

where $g = 4\pi a_s/m$ with a_s being the $s-$wave scattering rate. The zero-point energies are taken equal to each other $E_i = \int \Big[-\frac{\hbar^2}{2m} \mid \nabla\phi_i(\boldsymbol{r}) \mid^2 + |\phi_i(\boldsymbol{r})|^2 V_{\rm BEC}(\boldsymbol{r})\Big]\, d\boldsymbol{r} \equiv E$, and in the actual calculation we assume $E = 0$.

We can now derive equations of motion for the condensates wave functions, which take the following form

$$i\partial_t\Psi_1 = (E + U|\Psi_1|^2)\Psi_1 - J\Psi_2 + Ts_-^{(1)}\,, \quad (6)$$

$$\begin{aligned} i\partial_t\Psi_2 &= (E + U|\Psi_2|^2)\Psi_2 - J(\Psi_1 + \Psi_3) \\ &+ T(s_-^{(1)} + s_-^{(2)})\,, \end{aligned} \quad (7)$$

$$i\partial_t\Psi_3 = (E + U|\Psi_3|^2)\Psi_3 - J\Psi_2 + Ts_-^{(2)}\,. \quad (8)$$

The equations of motion for the pseudospin components of the two quantum dots are expressed as two Bloch equations

$$\hbar\partial_t\vec{s}^{(j)} = \vec{\omega}^{(j)} \times \vec{s}^{(j)}\,. \quad (9)$$

The Bloch equations for the two spins are coupled through the rotation frequencies

$$\vec{\omega}^{(1)} = \begin{pmatrix} 2T(\Re\Psi_1 + \Re\Psi_2) \\ -2T(\Im\Psi_1 + \Im\Psi_2) \\ -\hbar\delta \end{pmatrix}, \quad (10)$$

and

$$\vec{\omega}^{(2)} = \begin{pmatrix} 2T(\Re\Psi_2 + \Re\Psi_3) \\ -2T(\Im\Psi_2 + \Im\Psi_3) \\ -\hbar\delta \end{pmatrix}, \quad (11)$$

which are time-dependent due to a the coupling to the condensate dynamics Eqs. (6)-(8).

We solve this set of equations numerically for different initial conditions, and analyze the behaviour of the condensates and the dots in both regimes of weak and strong coupling. In case of the condensate we are interested in the behaviour of normalized particle imbalances

$$n_{12}(t) = \frac{N_1(t) - N_2(t)}{N_0}\,, \quad n_{23} = \frac{N_2(t) - N_3(t)}{N_0}, \quad (12)$$

where $N_0 = N_1(0) + N_2(0) + N_3(0)$. Note, that conserving quantities are $N_{tot} = N_0 + (s_z^{(1)} + 1)/2 + (s_z^{(2)} + 1)/2$ and $(\vec{s}^{(j)})^2 = (s_x^{(j)})^2 + (s_y^{(j)})^2 + (s_z^{(j)})^2$ for $j = 1, 2$.

Weak coupling limit. – The weak coupling limit implies that $T \leq J$ (we note that, since a quantum dot can be only singly occupied, T can be also order of magnitude larger than J, but we still refer to this realization as the weak coupling limit). In this limit the effect of the AQDs on the dynamics of the Josephson tunneling between the condensates is expected to be insignificant [3]. We concentrate therefore on the pseudospin behaviour.

It turns out, that symmetric initial conditions, which means that the two outer wells are equally populated $N_1(0) = N_3(0)$, favour nutations of the two pseudospins (Fig. 2(a)), which remain parallel if they were initially FM aligned, and antiparallel if they were AFM aligned at $t = 0$. This is true for a very weak coupling $T \ll J$, when the effective interaction between the spins is negligibly small.

However, with increasing coupling, a certain correlation between the two pseudospins develops, and their dynamical behaviour starts being dependent on their initial alignment. These features are to be observed in Fig. 2(b), in which part of Fig. 2(a) is enlarged. The two FM spins remain parallel and their frequencies follow the same (green) curve. However, initially antiparallel spins do not remain

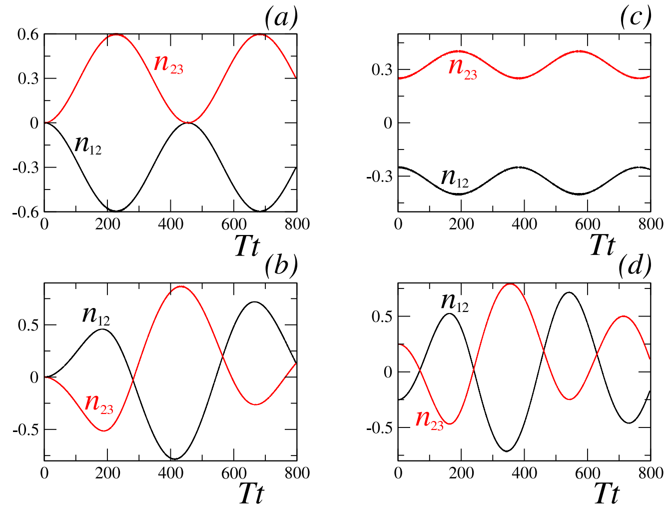

Fig. 5: Example of large amplitude particle oscillations between the wells n_{12} and n_{23} in case of strong coupling limit. Left column (a) is for initial conditions $N_1(0) = N_2(0) = N_3(0) = 1000$, right column (b) is for initial conditions $N_1(0) = N_3(0) = 500$, $N_2(0) = 1000$. For all plots the effective interaction $UN_0/T = 0.01$ and the detuning $\delta/T = 100$. Upper plots are for initial FM arrangement of the two AQDs, and lower plots are for initially AFM dots.

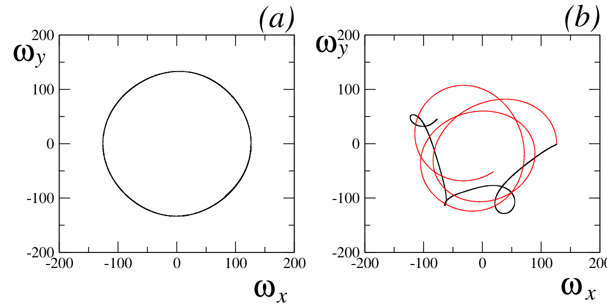

Fig. 6: Frequencies ω for the case of initial conditions of Fig. 5. Initially FM aligned spins remain FM and nutate as becomes clear from (a), while AFM aligned spins decohere. The black curve is the for left spin and the red curve is for the right spin of two initially AFM aligned quantum dots.

antiparallel with time. The red curve in fig. 2(b) corresponds to the left dot, while the black curve corresponds to the right dot.

In order to understand better this behaviour we analyze the time evolution of the relative angle γ between the spins, defined through

$$\vec{s}_1 \vec{s}_2 = \cos\gamma. \tag{13}$$

The time-dependence of $\cos\gamma$ for different coupling constants T/J is shown in Fig. 3. The angle γ oscillates between π and $\sim 0.75\pi$. In the following we refer to these oscillations as "breathing" modes. The breathing modes become slower for smaller coupling T/J and eventually disappear for $T \ll J$. We conclude that the breathing modes are indications of the induced correlation between the spins.

One can also consider asymmetric initial conditions: $N_1(0) \neq N_2(0) \neq N_3(0)$ as, for instance, in Fig. 4. In this case the dynamics of the two AQDs becomes very complicated, although not chaotic, as the Fourier transform of $(\cos\gamma)$ does not produce a typical white noise signal.

Strong coupling limit. – The peculiarities, which we found out in the dynamics of the initially AFM aligned spins in the weak coupling limit, are expected to become even more pronounced in the strong coupling regime, corresponding to $T \gg J$. In this case, the relevant time scale is set by T. It was previously shown that the ratio UN_0/δ plays an important role in the physical behaviour of the system [4]. Thus, the most interesting regime, when the AQD serves as a quantum "shuttle" of the atoms between the condensates, is achieved for $\delta \gg UN_0$. Naively one expects, that the oscillations induced solely due to the coupling of the condensates to the dot, would be always very small, however, surprisingly large amplitude Josephson-type oscillations between the wells were obtained [4].

In our case we also observe this property of AQDs (Fig. 5). For initially parallel spins we observe symmetric oscillations of the particle imbalances n_{12} and n_{23} (see Figs. (5a) and (5c)). These are similar to the oscillations in a triple well potential which are always symmetric for symmetric initial conditions. Note, that these oscillations are induced purely by the dots.

The pseudospins remain parallel, independent of the coupling strength between them, so that $\cos\gamma(t) = \cos\gamma(0) = 1$. The frequency $\vec{\omega}$ (which is the same for left and right spin) exhibits precession (Fig. 6(a)), which means that each of the pseudospins nutates.

The situation is very different for initially antiparallel pseudospins. First of all, the dynamics of interwell tunneling is of different character and seems to be not very sensitive to initial conditions for $N_i(0)$ (Figs. (5b) and (5d)). Second of all, one clearly observes asymmetric features, which we associate with the uncorrelated spin behaviour. This can be also seen from the behaviour of $\vec{\omega}^{(1)}$ and $\vec{\omega}^{(2)}$, which is shown in Fig. (6b). For instance, one can notice, that the right (red) pseudospins oscillates faster. Finally, the time-dependence of $\cos(\gamma)$ is depicted in Fig. (7), where the uncorrelated dynamics of the two initially antiparallel ($\cos(\gamma)(t=0) = -1$) pseudospins is presented. For small-amplitude oscillations the qualitative behaviour of the two AQDs is similar.

Conclusions and Discussion. – We have discussed the problem of an induced interaction between two atomic quantum dots which are embedded between three trapped Bose-Einstein condensates. As it is a multi-parameter problem, it helps to distinguish two different regimes: a weak coupling limit (when the direct Josephson coupling between the condensates is dominating) and a strong coupling case, in which the tunneling between the condensates

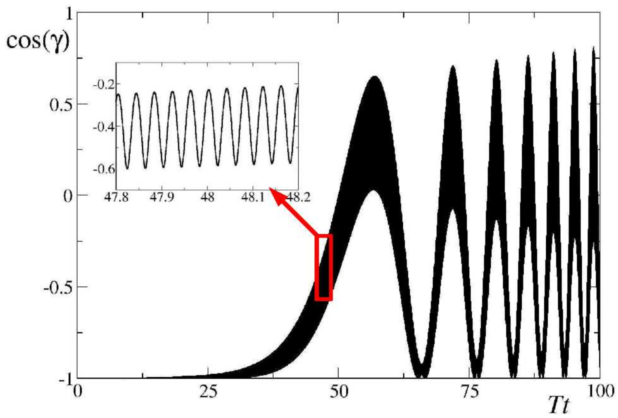

Fig. 7: Time-dependence of $\vec{s}_1\vec{s}_2 = \cos(\gamma)$ for the strong coupling case. Initial conditions are as in Fig. 5(b).

occurs predominantly through the additional channels mediated through the dots.

Our main conclusion is that indeed we clearly observe the signs of an effective interaction, induced between the two dots due to their coupling to the same condensate system. For this interaction to appear, T should be at least of the order of J. Interestingly, this induced interaction produces very different effects on the pseudospins, depending on whether they were initially FM or AFM aligned.

It turns out, that initially parallel spins are stable with respect to coupling between the dots and the condensates. They remain parallel even in the strong coupling limit, while undergoing two-frequency behaviour, as we can judge from the precessions of $\vec{\omega}$.

Initially antiparallel spins perform what we call "breathing" motion, which is characterized by small periodic deviations of the relative spin angle γ from π. The breathing modes, however, do not survive the strong coupling limit, when the effective interaction between the two spins becomes large. Instead, spins become rather uncorrelated.

Our analysis has important consequences for the periodic model of the system under consideration, which is the subject of future research. First of all, in a periodic system one shall be able to locally address and modify the interaction between the neighboring spins just by changing the occupation of each well. Second of all, by changing the coupling between AQDs and condensates (or optical lattice sites), one can control the different phase transitions in such a coupled system. For example, if an optical lattice was initially in the Mott state, a finite T should drive the system to a superfluid state once some critical value is exceeded. At the same time spins of the AQDs will become correlated, and the question will be if this correlaion is associated with a phase transition.

In the future it will be definitely worth studying quantum effects in such a model, as well as temperature effects, which would lead to decoherence phenomena and temperature-dependent phase transitions.

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REFERENCES

[1] Recati A., Fedichev P.O., Zwerger W., von Delft J., and Zoller P. , *Phys. Rev. Lett.*, **94** (2005) 040404.

[2] Zippilli S., and Morigi G.,*Phys. Rev. Lett.*, **95** (2005) 143001.

[3] Bausmerth I., Fischer U. R., and Posazhennikova A., *Phys. Rev. A*, **75** (2007) 053605.

[4] Fischer U. R., Iniotakis C., and Posazhennikova A., *Phys. Rev. A*, **77** (2008) 031602(R).

[5] Smerzi A., Fantoni S., Giovanazzi S., and Shenoy S. R.,*Phys. Rev. Lett.*, **79** (1997) 4950

[6] Milburn G. J., Corney J., Wright E. M., and Walls D.F.,*Phys. Rev. A*, **55** (1997) 4318

[7] Zapata I., Sols F., and Leggett A. J.,*Phys. Rev. A*, **57** (1998) R28

[8] Albiez M., Gati R., Föllig J., Hunsmann S., Cristiani M., and Oberthaler M.,*Phys. Rev. Lett.*, **95** (2005) 010402