Erschienen in: Sports Engineering ; 22 (2019), 1. - 6
https://dx.doi.org/10.1007/s12283-019-0294-5

Adaptive feedback system for optimal pacing strategies in road cycling

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Abstract
In road cycling, the pacing strategy plays an important role, especially in solo events like individual time trials. Nevertheless, not much is known about pacing under varying conditions. Based on mathematical models, optimal pacing strategies were derived for courses with varying slope or wind, but rarely tested for their practical validity. In this paper, we present a framework for feedback during rides in the field based on optimal pacing strategies and methods to update the strategy if conditions are different than expected in the optimal pacing strategy. To update the strategy, two solutions based on model predictive control and proportional–integral–derivative control, respectively, are presented. Real rides are simulated inducing perturbations like unexpected wind or errors in the model parameter estimates, e.g., rolling resistance. It is shown that the performance drops below the best achievable one taking into account the perturbations when the strategy is not updated. This is mainly due to premature exhaustion or unused energy resources at the end of the ride. Both the proposed strategy updates handle those problems and ensure that a performance close to the best under the given conditions is delivered.

Keywords Optimal pacing · Road cycling · Pacing control · Feedback

1 Introduction

In recent years, the phrase "aggregation of marginal gains" has become popular in professional road cycling. Dave Brailsford developed this concept in 2010 for cycling with great success [1]. The idea is that an improvement of only 1% in every area related to cycling will add up to a significant performance gain. Starting with improvements in training, nutrition, and bike technology, other less obvious areas were also investigated, such as sleep and some aspects of health. In our work, we contribute to one piece of the puzzle, about which not much information and practical experience is available: pacing strategy during the race. During individual time trials, where the rider is completely on his own, pacing strategy plays a major role in winning or losing. For example, the 2016 time trial world champion Tony Martin finished only 9th in the 2017 world championship. One reason for the disappointing result was that he had not enough energy reserves for the final climb, on which he lost a significant amount of time compared to his competitors. A better pacing strategy would most probably have resulted in a better overall standing.

Not much is known about the best pacing strategy for races. In the context of this paper, "pacing strategy" means the distribution of power over the course of the race. 'Therefore, a good pacing strategy defines, where in the course and how hard, the rider has to push and where he has to save energy to finish with a good overall time. Abbiss and Laursen [2] provided an overview of the different general pacing strategies on flat courses. The most relevant for most cycling events is even pacing. For tasks over 2 min, a constant speed is considered to be optimal on a flat course. More successful track cyclists competing in 1000 m time trials used a more constant power output during the race than did those who were less successful. And also, the nearly constant speed during a successful 1-h track cycling world record attempt supports this hypothesis.

This research was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)—247721022.

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Nevertheless, in road cycling, the courses are rarely completely flat, and therefore, different strategies seem beneficial. It was shown theoretically as well as experimentally that varying the rider's power output according to the slope or wind improves the race performance [3–5]. On uphill- or headwind sections the power output should be increased, while on downhill- or tailwind sections, the power output should be decreased. With this type of strategy, a course is completed faster than with a constant strategy with the same mean power output.

Gordon [6] was the first who systematically evaluated the best power variation. Incorporating a model for exertion, the power distribution was optimised mathematically on courses with a piecewise constant slope. Similar strategies emerged with this approach, having a higher power output on the climbs and a lower one on the descends. The time savings depended on the steepness of the course: for steeper courses, a larger time gain was achieved. This approach was expanded by including acceleration, which allowed to investigate the optimal starting behavior, and by considering complex slope profiles, which allowed to optimize pacing strategies for real-world courses [7]. In a showcase comparison between a self-paced ride on the Schienerberg in South Germany and the corresponding calculated optimal ride, a time improvement of 1.8% was achieved.

Most of the recent studies investigating optimal pacing strategies have focused on the underlying physiological model [8–12]. Unfortunately, they provide not much information with regard to practical applications, since they are mainly based on theoretical arguments and none of them provide experimental evidence for the validity of the resulting strategies. Just lately, it was shown in a laboratory experiment, that optimal pacing strategies, based on mathematical methods, can actually improve performance in practice [13]. During several rides of a real-world course on a bicycle simulator, the subjects performed with self-paced strategies and with forced precalculated strategies. It was shown that the optimal pacing strategies were valid, in a way that all subjects were able to follow the optimal power output and that the race time with optimal feedback was improved between 0.8% and 3.2% compared to the subjects self-paced rides.

Another interesting application of optimizing pacing strategies has arisen from the increased availability of electric bicycles. In this case, not only does the rider propel the bike, but also an additional electric motor. Therefore, similar optimal control problems can be defined, which additionally incorporate a model for changes in the battery charge. Minimum time problems [14] as well as minimum rider fatigue problems [15] have been investigated so far. As for pacing strategies for individual time trials, it remains uncertain how well these approaches work in practice.

In this paper, we present a framework to apply feedback based on optimal pacing strategies in the field and investigate the errors induced by defects of the physical model, like unknown wind conditions or inaccurate model parameters, as well as errors due to the inability of the rider to follow the suggested pacing strategies precisely. To handle those defects, two control approaches are presented to update the optimal strategy in real time during the ride: model predictive control (MPC) and proportional–integral–derivative control (PID). In the following, the underlying mathematical models and the two control approaches are explained, an overview over the feedback loop is provided, and simulated results are presented to evaluate the performance of the implemented methods.

2 Methods

2.1 Physical and physiological model

The physical and physiological models describe mathematically the relation between power output and speed and between power output and effort, respectively. The physical model was developed by Martin et al. [16] and validated under realistic conditions by Dahmen et al. [17]. It describes the equilibrium of the rider's pedal power P and the power induced by aerodynamic drag P_{air}, rolling resistance P_{roll}, bearing friction P_{bear}, gravitation P_{pot} and inertia P_{kin}, which is given by

$$\begin{aligned}\eta P = \underbrace{mgsv}_{P_{\text{pot}}} + \underbrace{\mu mgv}_{P_{\text{roll}}} + \underbrace{\beta_0 v + \beta_1 v^2}_{P_{\text{bear}}} + \underbrace{\left(m + \frac{I_w}{r_w^2}\right)\dot{v}v}_{P_{\text{kin}}} \\ + \underbrace{\frac{1}{2}c_d\rho A v^3}_{P_{\text{air}}}\end{aligned} \tag{1}$$

with parameters presented in Table 1.

While in this formulation, speed and power are functions of time, it is advantageous for the optimization to transfer the problem into the spatial domain. Formulating Eq. (1) in the spatial domain, with the traveled distance being the independent variable, and choosing the explicit representation of the ordinary differential equation results in

$$\begin{aligned}v' &= \frac{1}{Mv^2}\left(\eta P - mgsv - \mu mgv - \beta_0 v - \beta_1 v^2 - 0.5c_d\rho A v^3\right) \\ &=: F_{\text{physical}}(v, P)\end{aligned} \tag{2}$$

which is used in the following.

Note that the space derivative of the speed v' is related to the time derivative of the speed $\dot{v}$ by

$$\dot{v} = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x}v = v'v.$$

Table 1 Parameters of the physical and physiological model and their default values used in the simulations

Description	Variable	Unit	Default value
Power	P	W	–
Speed	v	m/s	–
Total mass	m	kg	80
Acceleration of gravity	g	m/s^2	9.81
Slope of the course	s	–	Fig. 4
Friction factor	μ	–	0.004
Bearing factor	β_0	N m	0.091
Bearing factor	β_1	N m s	0.0087
Wheel inertia	I_w	kgm^2	0.14
Wheel radius	r_w	m	0.33
Drag coefficient	c_d	–	0.7
Air density	ρ	kg/m^3	1.2
Cross-sectional area	A	m^2	0.4
Chain efficiency	η	–	0.975
Remaining resources	e_{an}	J	–
Maximum capacity	E_{an}	J	25,000
Critical power	P_C	W	300
Recovery reduction	α	–	0.1

The physiological model is based on the critical power concept [18]. It assumes a limited reservoir of anaerobic energy E_{an}, which depletes or replenishes when the rider is performing with a power output above or below a certain threshold, called critical power P_C. The rider's state of exhaustion is associated with the remaining anaerobic resources e_{an}: if the remaining anaerobic resources are depleted, the rider is fully exhausted, while the reservoir is completely filled in a rested state. Here, we additionally reduce the rate of recovery by a constant factor α as proposed in [13]. The dynamic system in the spatial domain describing the change of the remaining anaerobic resources per meter (J/m) is given by

$$e'_{an} = \underbrace{\left(\frac{1-\alpha}{2}\tanh\left(-\frac{P_C - P}{20}\right) + \frac{1+\alpha}{2}\right)}_{B} \cdot \underbrace{(P_C - P)}_{A} \cdot \underbrace{\frac{1}{v}}_{C} =: F_{fatigue}(v, P) \tag{3}$$

and is illustrated in Fig. 1.

It combines three multiplicative terms: (A) the work done per second above or below P_C; (B) a sigmoid function reducing the recovery rate below P_C by a constant factor $\alpha < 1$ and providing a smooth transition to the full exertion rate above P_C; and (C) the translation to the spatial domain by dividing by the speed.

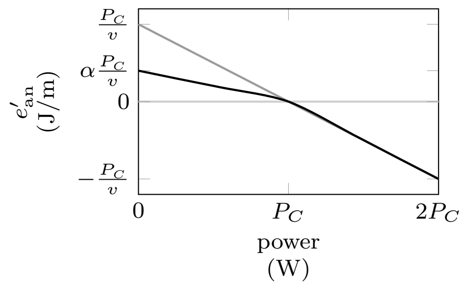

Fig. 1 Illustration of the change in the remaining anaerobic resources: the original critical power model (grey) and the extended model (black) with reduced recovery

This model provided a good prediction performance for optimal pacing strategies: subjects were able to follow the optimal strategy without premature exhaustion and with an improved race time compared to self-paced rides [13]. Another advantage is the small number of model parameters, which allowed estimating them with only three simulator tests [13].

2.2 Optimal pacing strategy

Based on the two mathematical models described in Sect. 2.1, an optimal control problem to minimize the time needed to finish a course is defined. To avoid numerical problems and to ensure a certain smoothness of the power output, its variation $Q(x) := P'(x)$ is reduced by introducing a penalty term. This leads to the following optimal control problem. Minimize the cost functional

$$J = \int_{x_0}^{x_f} \frac{1}{v(x)} + \epsilon\, Q(x)^2 \mathrm{d}x,$$

subject to the dynamic constraints

$$\begin{aligned} P'(x) &= Q(x), \\ v'(x) &= F_{physical}(v(x), P(x)), \\ e'_{an}(x) &= F_{fatigue}(v(x), P(x)), \end{aligned}$$

the path constraints

$$0 \le e_{an}(x) \le E_{an},$$

and the initial conditions

$$\begin{aligned} v(x_0) &= v_0, \\ e_{an}(x_0) &= E_{an}, \end{aligned}$$

where P, v, and e_{an} are the states, Q is the control, x_0 and x_f are the start and end distance on the course, and v_0 is the speed at the start x_0. It is assumed that the rider is fully recovered at the beginning of the ride, and therefore, the remaining anaerobic resources are equal to the maximum anaerobic capacity E_{an}. No restrictions are made for the final conditions. The problem is solved by the numerical state-of-the-art optimal control solver FALCON.m [19] with a spatial discretization of 10 m. This solver works very well for this optimal control problem. For other formulations, other solvers might be superior, e.g., GPOPS-II [20] for an equivalent time-domain formulation.

2.3 Strategy update

Environmental factors like wind or inaccuracies in the physical model can make a ride less or more demanding than assumed in the off-line calculation of the optimal strategy. Therefore, it may happen that, based on the physiological model, the rider is not fully exhausted when reaching the finish or already completely exhausted before getting to the finish. Both situations result in suboptimal rides. Therefore, two approaches were developed to update the strategy during the ride and avoid premature or insufficient exhaustion: (1) a PID controller to stay close to the trajectory of the remaining anaerobic resources and (2) a more sophisticated MPC approach.

2.3.1 Proportional–integral–derivative controller (PID)

During the ride, the remaining anaerobic resources are calculated by Eq. (3) using the measurements of a powermeter. The PID controller adjusts the displayed reference power value to ensure that the remaining anaerobic resources stay close to their reference trajectory. For example, if the current remaining anaerobic resources are above their reference value due to tailwind, the rider is less fatigued than assumed in the optimal pacing strategy without wind and the demanded power output is increased. The reference power is updated by adding the control u, which is calculated by

$$u = K\left(E + \frac{1}{T_{\mathrm{i}}}\int_{x_0}^{x} E(\xi)\mathrm{d}\xi + T_{\mathrm{d}}E'(x)\right),$$

where E is the difference between the remaining anaerobic resources calculated during the ride and the corresponding reference value in the optimal strategy. K, T_{i}, and T_{d} are the controller parameters for the proportional, integral, and derivative terms, which were set to $K = 0.1$, $T_{\mathrm{i}} = 100$, and $T_{\mathrm{d}} = 0$. These parameters were tuned manually by performing a series of simulations (see Sect. 2.5) avoiding overshoot and oscillations.

2.3.2 Model predictive control (MPC)

While the PID controller calculates a single control value which is only valid for the next time step, i.e., does not include any preview of the future state evolution on a longer horizon, the MPC approach calculates a new control on a spatial window into the future. In addition, not only the remaining anaerobic resources are considered, but several goals should be reached on this segment: (1) the updated power output should be smooth; (2) the corresponding remaining anaerobic resources should be close to their reference values on the whole segment; (3) the time needed to ride the segment should be minimized; (4) the speed; and (5) the power should be equal to their reference values at the end of the segment. The above was formulated as an optimal control problem with a weighted sum of costs (i.e., multi-objective optimization), which is used to derive an updated control Q starting at the current position $\bar{x}$: Minimize the cost functional

$$\begin{aligned} J = &\int_{\bar{x}}^{\bar{x}+l} w_1 Q(x)^2 + w_2\left(e_{\mathrm{an}}(x) - \hat{e}_{\mathrm{an}}(x)\right)^2 + w_3\frac{1}{v(x)}\mathrm{d}x \\ &+ w_4(v(\bar{x}+l) - \hat{v}(\bar{x}+l))^2 \\ &+ w_5(P(\bar{x}+l) - \hat{P}(\bar{x}+l))^2, \end{aligned}$$

subject to the dynamic constraints

$$\begin{aligned} P'(x) &= Q(x), \\ v'(x) &= F_{\mathrm{physical}}(v(x), P(x)), \\ e'_{\mathrm{an}}(x) &= F_{\mathrm{fatigue}}(v(x), P(x)), \end{aligned}$$

and the initial conditions

$$\begin{aligned} P_0 &= \bar{P}, \\ v_0 &= \bar{v}, \\ e_{\mathrm{an},0} &= \bar{e}_{\mathrm{an}}, \end{aligned}$$

where $w_1, \ldots, w_5$ are the weights for the different optimization goals; $\hat{v}$, $\hat{P}$, and $\hat{e}_{\mathrm{an}}$ are the speed, power, and remaining anaerobic resources of the precalculated optimal strategy; $\bar{v}$, $\bar{P}$, and $\bar{e}_{\mathrm{an}}$ are their currently measured values; and l is the length of the optimization horizon, which was set to $l = 300$ m.

The weights $w_1, \ldots, w_5$ were derived by simulating rides in different conditions without strategy update to get an estimate of the magnitudes of the error terms in the cost functional. More details on the simulation environment are provided in Sects. 2.4 and 2.5. Setting the weights to the reciprocal of these estimates, provided an even balance of all optimization goals and simultaneously scaled the objective function, to achieve good numerical properties. In total, 1134 simulations were performed with different combinations of perturbations by wind and changes in the model parameters. Weight w_1 is an exception, since it was tuned manually: if it is defined too small, the problem is singular and the solution tends to oscillate; and if it is defined too large, the problem is too restricted by only allowing very smooth solutions. This resulted in the following weights:

$$w_1 = 3.3 \times 10^{-3},$$
$$w_2 = 1.2 \times 10^{-4},$$
$$w_3 = 3.7 \times 10^{-4},$$
$$w_4 = 1.7,$$
$$w_5 = 6.2 \times 10^{-4}.$$

The model predictive control problem was solved with Acado [21] using a grid step size of 10 m and employing Acado's Code Generation Tool.

2.4 Feedback loop

Figure 2 shows a flow chart of the feedback loop for a field ride. Prior to the ride, the reference trajectories for speed, power, and remaining anaerobic resources are derived by calculating the optimal pacing strategy. In the first step, after beginning the ride, the current speed, power, and GPS position are measured. Based on the GPS and speed measurements, the current position on the course is estimated employing a Kalman filter [22]. It was shown in a field experiment under realistic cycling conditions, that this approach achieved an accuracy of less than 1 m, which should be sufficiently good for this application. With the power measurements, the remaining anaerobic resources are updated employing the time-domain equivalent of Eq. (3). If the rider is in the segment on which the optimal strategy is available, i.e., the rider passed the start and did not reach the finish yet, the reference strategy is updated based on the previously described PID or MPC controller. With the updated strategy, feedback for the rider is generated and visually presented via a mobile device, like a handlebar mounted mobile phone or a programmable cycling computer. This procedure is repeated, until the rider reaches the finish.

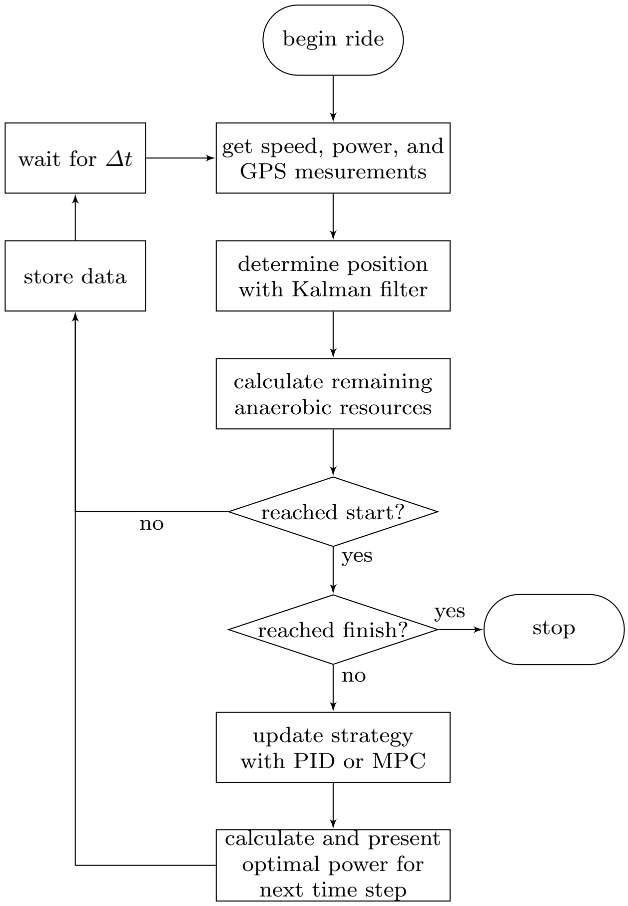

Fig. 2 Feedback loop for a field ride

2.5 Simulation

To test the developed methods, the feedback loop was passed through with time steps $\Delta t = 1$ s, while the real ride was simulated by forward integration of the speed of Eq. (2). This allowed to easily test different perturbations of the physical model like wind or inaccuracies in the model parameters. The perturbations investigated in this work are listed in Sect. 3. With the power feedback provided to the rider, the speed on the course was calculated by Eq. (2). Since the rider cannot follow the feedback precisely, random, normally distributed noise with a standard deviation of 24 W was added to the power feedback value. If the anaerobic resources are depleted and the power feedback value is above critical power, the rider cannot follow the feedback and the speed was calculated with critical power instead of the proposed power feedback value. To account for measurement noise, random, normally distributed noise with a standard deviation of 0.04 m/s was added to the resulting speed. The magnitude of these perturbations was estimated in secondary experiments. Wolf and Saupe [23] showed that the slope of the course is only estimated with an absolute accuracy of about 0.9% using ordinary cycling computers. Therefore, random, normally distributed noise with a standard deviation of 0.9% was added to the slope s in Eq. 2. Figure 3 provides an overview over the simulation loop.

3 Results

In this section, simulated rides are presented for different conditions. The reference optimal pacing strategy was calculated assuming that there is no wind and with the model parameters, as presented in Table 1. The simulated track was a hill-climb from Ermatingen to Helsighausen in Switzerland. It has a length of 3.7 km, a total climb of 213 m,

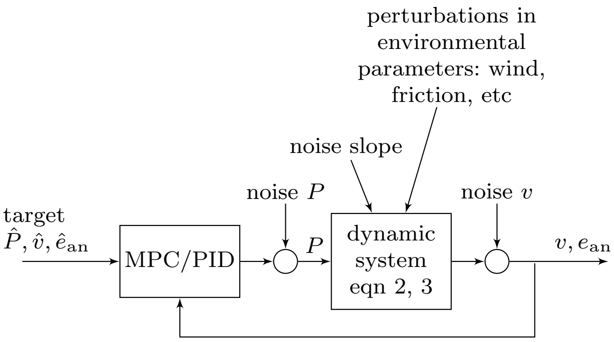

Fig. 3 Closed-loop simulation of the field ride

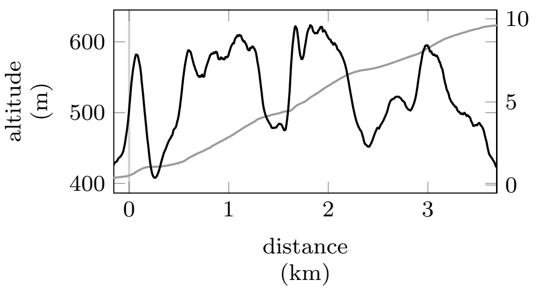

Fig. 4 Slope (black) and altitude (grey) of the simulated track near Ermatingen, Switzerland

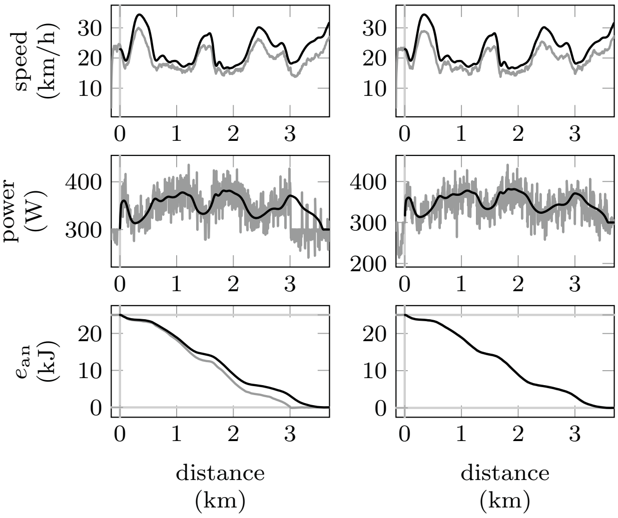

Fig. 5 Simulated ride with headwind without update (left) and with MPC update (right). Black curves represent the reference trajectories and grey curve the simulated ride

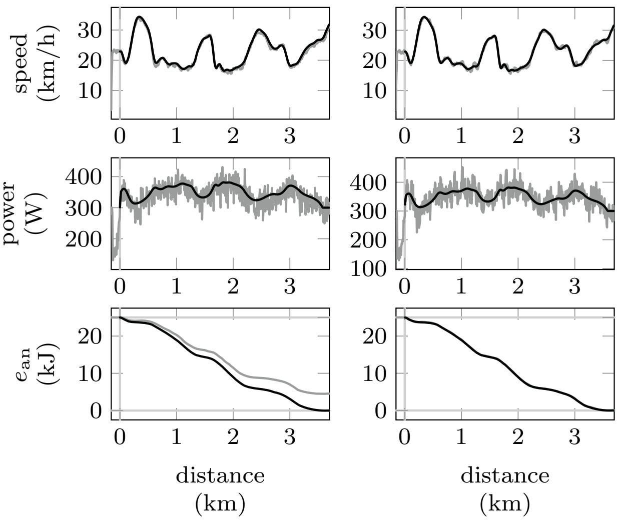

Fig. 6 Simulated ride (grey), where the rider stays below the reference (black) on average without update (left) and with MPC update (right)

and a mean slope of 5.7% (see Fig. 4). Due to the random perturbations introduced during the simulations described in Sect. 2.5, the resulting rides vary. Therefore, the race times in the following are provided as < mean value > ± < standard deviation > over 100 simulations.

Figure 5 shows a simulated ride with a constant headwind of 4 m/s without strategy update (left) and with (right). Calculating the optimal strategy taking into account the conditions during the ride, i.e., considering the headwind, resulted in a total race time of 11 min : 36 s. The race times of the simulated rides resulted in 11 min : 37 s ± 02 s with strategy update and 11 min : 38 s ± 02 s without. Figure 6 shows a simulated ride, where the rider was performing below the feedback value on average by 10 W without strategy update (left) and with (right). The optimal race time is 10 min : 01 s. The race times of the simulated rides resulted in 10 min : 02 s ± 02 s with strategy update and 10 min : 16 s ± 02 s without.

Figure 7 shows an overview over the simulated race times in different conditions compared to the race time of the optimal pacing strategy in the corresponding condition. For reference, a simulation with a precisely known physical model was performed (none). In the following use-cases, wind was introduced during the simulation: constant head- and tailwind (constHead, constTail), 1.85 km headwind followed by 1.85 km tailwind and vice versa (headTail, tailHead), and two short strong blasts of headwind and tailwind along the course (blastsHead, blastsTail). An overview over the wind conditions is provided in Fig. 8. In addition, deviations from the assumed model parameters are presented: deviations of ± 20% in the rolling and air resistance parameters μ and c_dA (roll− 20%, roll+ 20%, air− 20%, air+ 20%), deviations of − 10% and + 1% in the chain efficiency parameter η (eff− 10%, eff+ 1%), and deviations of ± 10% in the estimate of the mass of the bike and the rider m (mass− 10%, mass+ 10%). The last two use-cases show the effects of the

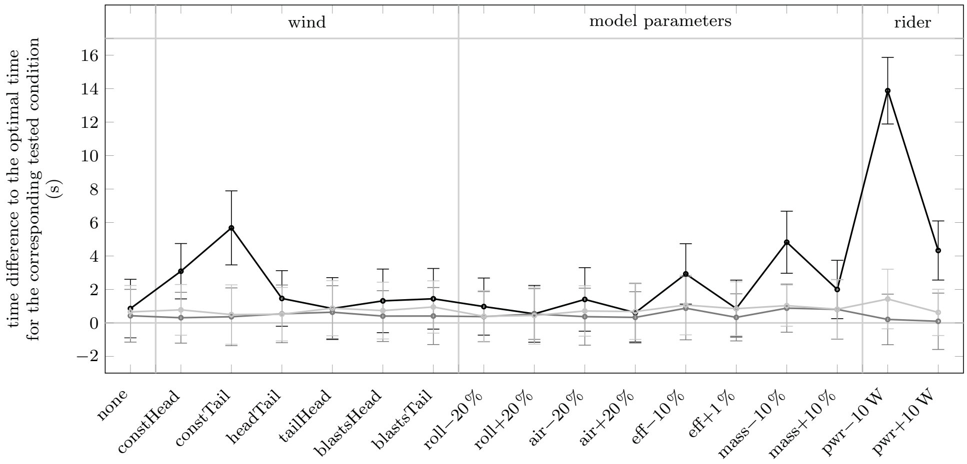

Fig. 7 Mean race times over 100 simulations under various conditions without strategy update (black), with MPC update (light grey), and with PID update (dark grey). The error bars show the corresponding standard deviations

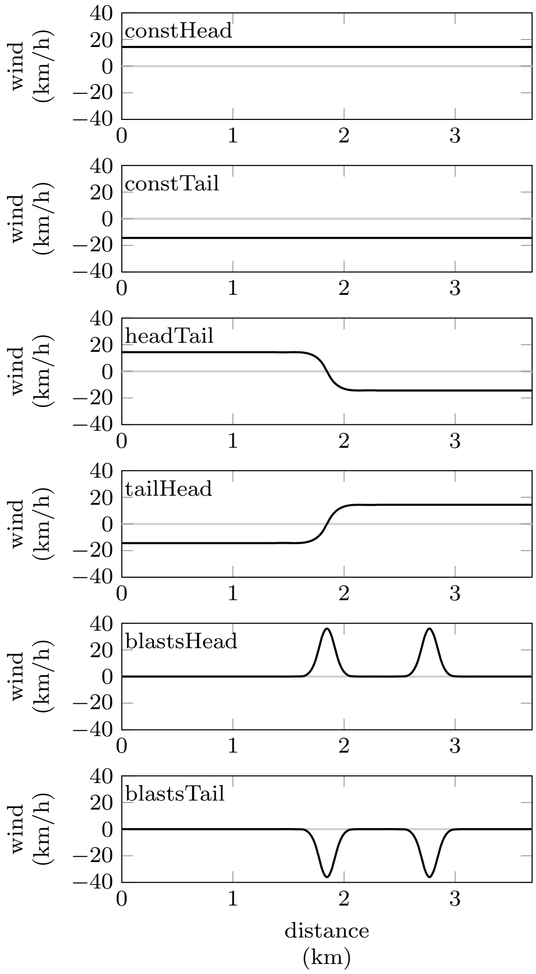

Fig. 8 Overview over the different simulated wind conditions

rider not following the feedback on average: during the simulated ride, the feedback was followed with an offset of ± 10 W (pwr− 10 W, pwr+ 10 W).

4 Discussion

The results reveal an outcome that may be unexpected: in some cases, the simulated rides were faster than the race time of the optimal strategy. If a strategy is optimal in terms of the final time, there should be no other strategy that is faster. However, this holds only if the assumptions about the conditions made for calculating the optimal strategy are identical to the conditions during the simulation. As described in Sect. 2.5, the slope of the course is changed randomly by adding Gaussian noise during the simulation. Therefore, the underlying conditions between simulation and optimal strategy may vary, explaining the seemingly implausible results. This also reveals that different final times than predicted are expected during field rides, since the parameters are not exactly known, and it can be determined for which parameters the accuracy should be improved for a better time prediction.

When facing unexpected headwind during the ride, the resulting speed is lower at the same power output. Therefore, the ride takes longer than calculated, such that the anaerobic resources are depleted before reaching the finish line. This behavior is shown in Fig. 5 (left). Around 3 km, the resources are depleted and the rider can only perform around critical power for the rest of the ride. With an MPC strategy update, this can be avoided. The controller reduces the reference power output during the ride and prevents from premature exhaustion [Fig. 5 (right)]. The race time is hardly affected: the time difference between optimal time and mean simulated time is less than 2 s in both cases, with and without strategy update. Nevertheless, without strategy update, the rider needs to perform in a completely exhausted state for about 700 m, which should be avoided.

If the rider follows the strategy with a reduced mean power, less anaerobic energy is required and the rider is not exhausted at the end of the ride [Fig. 6 (left)]. With the MPC strategy update, the reference power is increased according to force the rider to follow the initial strategy [Fig. 6 (right)]. In this case, a large time difference is observed: without strategy update, the rider needs about 14 s longer to complete the course than with strategy update. This is due to the rider's unused remaining anaerobic resources during the ride without strategy update.

The race time comparisons in Fig. 7 show that in some situations, a strategy update may not be required in terms of a time loss on the optimal time. However, there are also situations, where a large time loss is observed, especially if the simulated ride is easier than expected, e.g., pwr−10 W. In this case, not all anaerobic energy resources are spent and a condition adjusted strategy holds a significant advantage. Both controlling schemes work well in all tested conditions. Since the PID controller is as good as the MPC approach, easier to implement, and faster to calculate, it is preferred for an implementation on a mobile device.

The main focus of the presented control techniques is to compensate for unknown or imprecisely known parameters of the physical model, to finish a ride in an exhausted state while avoiding premature exhaustion. Thereby, the state of exhaustion is associated with the remaining anaerobic resources in the physiological model. Obviously, this model is a strong simplification of reality and together with the inability to test the actual level of anaerobic capacity, the accuracy of the model is disputable.

The quality of computed optimal pacing strategies depends on the validity of the physiological model and the precision of the estimation of its parameters just like on the validity of the physical model and its parameters. Validation of the physiological model is more challenging than validation of the physical model, since it is not based on a well-grounded theory like classical mechanics, and because it is a meta model in which parameters (E_{an}, P_{C}, α) are abstract and only indirectly accessible. Following a computed optimal pacing strategy, errors in the physiological model will lead to deviations of the remaining anaerobic energy E_{an} from the predicted one. If a direct measurement of E_{an} would be available (as for power, speed, or distance), we could incorporate this difference in E_{an} into the calculation of the adaptive feedback, thereby compensating for the deficiencies of the inaccurate physiological model, at least to some degree. However, the lack of precise physiological sensors prevents an objective evaluation.

Our approach is an indirect one: the physiological model is suitable for our application if it does not overestimate the riders capabilities (resulting in premature exhaustion), if the parameters can be estimated accordingly in a few tests, and if performance can be increased with optimal pacing strategies. All these properties were fulfilled in the previously executed laboratory study [13], and therefore, we are optimistic that this simple model can also be applied to improve performance in practice. Hopefully, in the future, also more sophisticated models are practically applicable and performance can be further improved.

Such more sophisticated models, which include up-to-date knowledge in human bioenergetics, are already available [24], but rarely used in practice. One reason for this is that they incorporate a large number of parameters which must be derived for each athlete individually. A main difficulty is the lack of precise and reliable markers regarding fatigue. Available markers like heart rate or oxygen consumption tend to saturate towards the point of exhaustion which makes it hard to distinguish between level of fatigue and day-to-day variability. Therefore, each single test provides only little information and athletes need to perform a large number of tests to tune the model.

Since the focus of this work is towards utilizing optimal strategies in practical applications, we decided against these more complicated models and prioritized to have a small number of parameters. This allowed to estimate all parameters in just a few tests. Despite the simple nature of the model, it performed sufficiently well to improve rides on a simulator in the laboratory [13]. Therefore, time improvements are also expected for field rides if the deficiencies of the physical model are handled by the presented controllers.

Another restriction that we made is the choice of the course. Our approach is limited to uphill courses, since downhill segments introduce additional complexity to the optimization that should be handled in subsequent research. The physical as well as the physiological model must be extended to approach downhill segments realistically. Braking as well as speed constraints during turns must be considered in the physical model. While models for the energy expenditure of the athlete are available but still quite imprecise, even less information is available regarding modelling recovery. Simple models like the one we use [13] may provide a first approximation of recovery with short periods of cycling at sub-critical power, but they are very hard to validate empirically. Since athletes may recover to a larger degree on descents, the physiological model should be extended and validated for longer recovery periods to be applicable for optimal pacing strategies including downhill segments.

Nevertheless, the proposed method for adaptive feedback itself is not restricted to uphill courses. The advantage of a ride with adaptive feedback over one without adaptive feedback is expected to be even larger on downhill segments. Due to higher speed, errors in the $c_{\mathrm{d}}A$ estimate will induce a large error in rides without adaptive feedback which is handled by the proposed techniques. Therefore, we expect similarly good results for downhill segments assuming suitable physical as well as physiological models are available.

For field experiments, the feedback loop is implemented on a stand alone device. One question that remains uncertain

Fig. 9 Proposed mobile feedback based on a tachometer and the rider's power output. The number on the left shows the current power output, the one on the right the reference value, and in the middle the difference between both. This difference is also shown by the needle. If the needle is in the middle, the rider follows the reference perfectly, if it is on the left, the rider should increase the power output, and if it is on the right, the rider should decrease the power output. This display can also be adapted to other parameters like speed or remaining anaerobic resources

is how well the riders can follow the feedback and how this feedback should look exactly. First pilot studies with a real time feedback based on the methods presented in this paper showed that cyclists can follow the feedback to a satisfying degree [25, 26]. The feedback was presented to the rider on a handle bar-mounted mobile phone via a tachometer-like display, as shown in Fig. 9. These studies indicate that applying optimal pacing strategies together with the presented controlling techniques will improve performance of recreational and perhaps also professional riders.

5 Conclusions

Based on simulated rides, it was shown that updating a precalculated strategy may be necessary, if the conditions are different during the ride. PID, as well as MPC controllers proved suitable to update the strategy in real time to react to changing conditions and avoid premature exhaustion or insufficient exhaustion at the end of the ride. Due to an easier implementation and faster runtime, the PID controller is preferred over the MPC approach on a mobile device. The final race times during rides with strategy update were comparable to the optimal time in the corresponding condition, while significant time losses were observed without strategy update.

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