Wavelets and their Applications in Databases

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Introduction

- Wavelets are

  „the (re)discovery of the last decade
  in Computer Graphics“

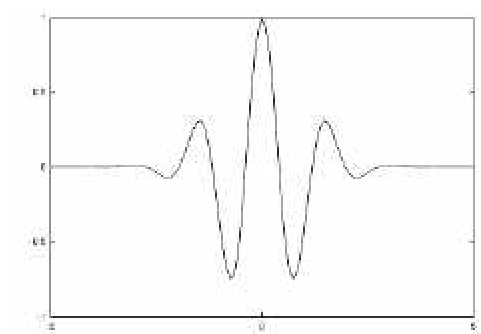
- Wavelets have
  - firm mathematical basis
  - nice theoretical properties
  - many practical CS applications
    - Data Compression
    - Computer Graphics & Visualization
    - Databases ...

Overview

1. Introduction
2. Foundations of Wavelet Theory
3. Standard Applications
4. Applications in Database Area
5. Summary and Conclusion

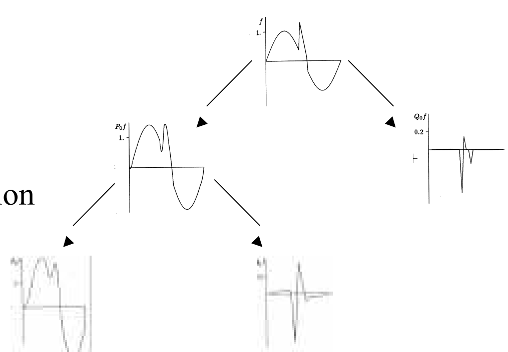
Introduction

- Notion *Wavelet* comes from seismology
- Wavelets describe of a wave spreading an impulse



Introduction

- <u>*Basic Idea:*</u>
  Hierarchical Decomposition of a function into a set of *Basis Functions* and *Wavelet Functions*

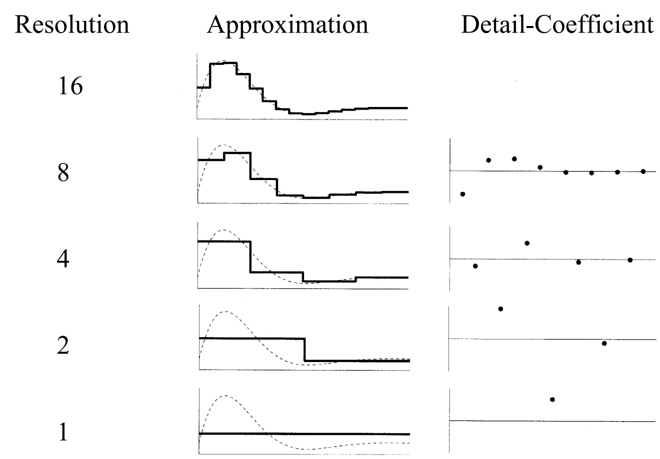


basis function

wavelet function

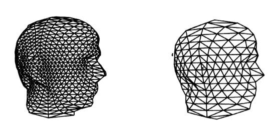
Introduction

- *Example*

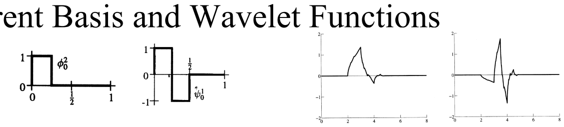


Advantages of Wavelet Transformations

- Space and Time Efficiency
  –> Low Complexity of DWT



- Multiresolution Properties
  –> Hierarchical Representation & Manipulation
- Generality & Adaptability
  –> Different Basis and Wavelet Functions



History

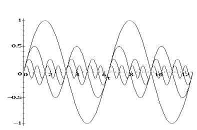
- 1873 - Karl Weierstraß [Wei95]
  Family of scaled overlapping copies of a Basis Function
- 1910 - Alfred Haar [Haa10]
  Orthonormal system of compact functions (Haar-basis)
- 1946 - Dennis Gabor:
  Non-orthogonal Wavelet basis with unlimited support
- 1980s - A. Grossman, J. Morlet and I. Daubechies:
  Signal Analysis with Wavelets
- 1989 - Stephane Mallat [Mal89] & Yves Meyer:
  Multiresolution Analysis
- 1990+ : Rediscovery of Wavelets in Computer Science

Wavelets versus Fourier

Wavelet and Fourier

- signal decomposition in the frequency domain
- efficiency of FFT and DWT

Fourier



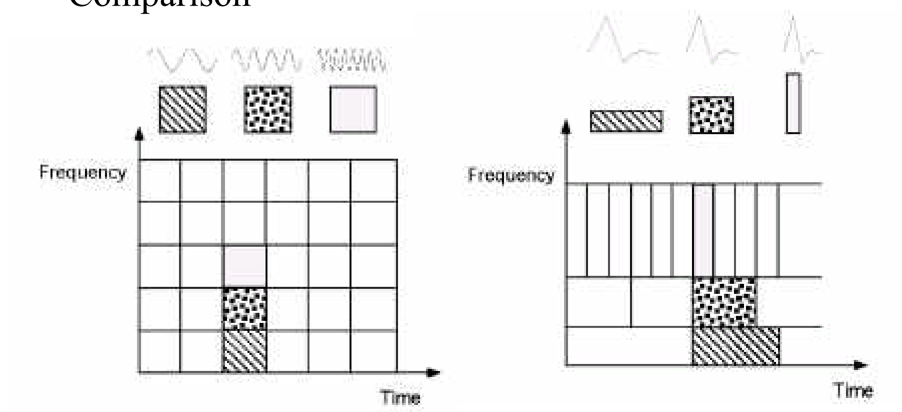
- unlimited support
- sinus/cosinus functions (different frequencies)
- same resolution

Wavelet

- compact support (–> locality, discontinuities)
- different basis functions (scaling & translation)
- multiresolution (higher frequencies => higher resolution)

Wavelets versus Fourier

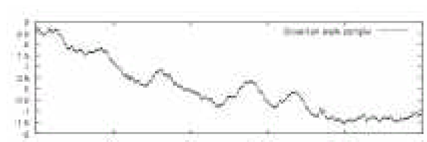
Comparison



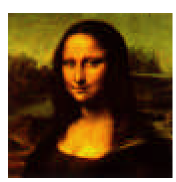
Fourier Wavelet

Applications in Computer Science

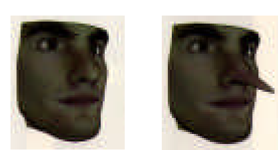
- Signal Processing
  - Time Series Analysis
  - Noise Reduction



- Data Compression
  - Image
  - Video



- Computer Graphics
  - Multiresolution Data Representation
  - Multiresolution Rendering
  - Multiresolution Data Manipulation



Overview

1. Introduction
2. Foundations of Wavelet Theory
   - 2.1 Basics of Wavelet Transformations
   - 2.2 Multiresolution Analysis
   - 2.3 Advantages of Wavelet Transformation
3. Standard Applications
4. Applications in Database Area
5. Summary and Conclusion

Basics of Wavelet Transformations

(example: Haar Wavelet Transformation)

- Data objects can often be described as

  piecewise linear functions

- *Haar Wavelet Transformation* is a hierarchical decomposition based on the vector space of piecewise linear functions on the interval [0,1)

  Let V^j be defined as the vector space of all 2^j functions over the intervals

$$[0,1/2^j), \ \ldots, \ [(2^j-1)/2^j, 1)$$

Basics of Wavelet Transformations
(example: Haar Wavelet Transformation)

Example:

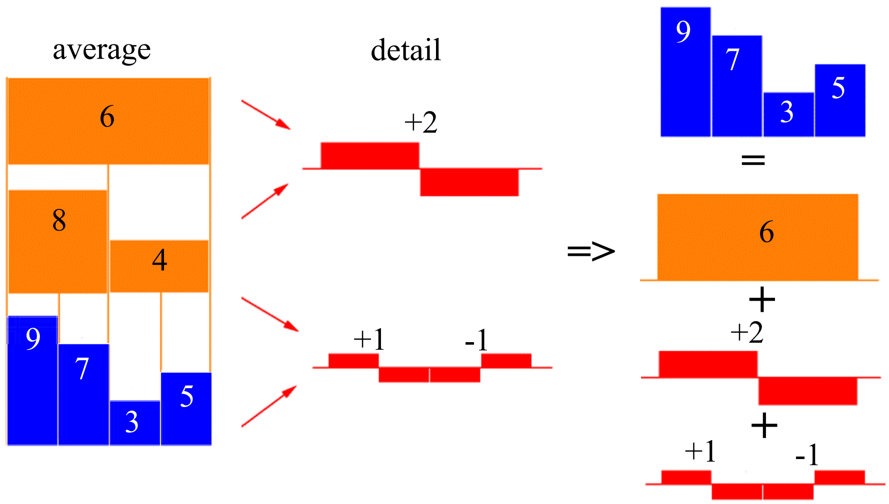
- 1 data value ⇔ piecewise constant function over [0, 1)
  in vector space V^0
- 2 data values ⇔ function constant over [0, ½) and [½,1)
  in vector space V^1
- 2^j data values ⇔ function constant over 2^j subintervals
  of [0,1) in vector space V^j
- Observation: $V^0 \subset V^1 \subset V^2 \subset V^3 \subset \dots$

Wavelet decomposition
Simple Example

- First step: sequence (9 7 3 5)
  pair-wise average (8 4)
  lost detail information (1 -1)
  [8+1=9 8-1=7 4+(-1)=3 4-(-1)=5]
- Next step: sequence (8 4)
  average (6)
  detail (2)

–> wavelet transformation (6 2 1 -1)

Wavelet decomposition
Simple Example



2. Foundations of Wavelet Theory

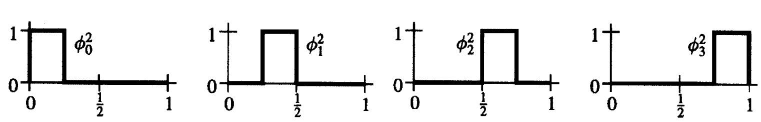
Basics of Wavelet Transformations
(example: Haar Wavelet Transformation)

- Haar Basis Function (for vector space V^j)

$$f_i^j(x) := f(2^j x - i) \qquad i = 0, \ldots, 2^j - 1$$

$$with \quad f(x) := \begin{cases} 1 & if\ 0 \le x < 1 \\ 0 & otherwise \end{cases}$$

- Example: Haar basis for V^2



2. Foundations of Wavelet Theory - 2.1 Basics of the Wavelet Transformation

Basics of Wavelet Transformations

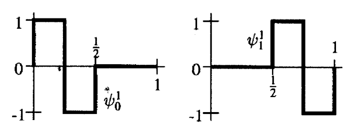
(example: Haar Wavelet Transformation)

- Haar Wavelet Function (for vector space W^j)

$$y_i^j(x) := y(2^j x - i) \qquad i = 0, \ldots, 2^j - 1$$

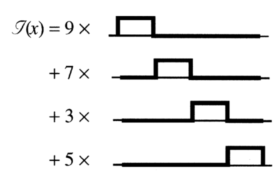
$$\text{with} \quad y(x) := \begin{cases} 1 & \text{if } 0 \le x < 0.5 \\ -1 & \text{if } 0.5 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Example: Haar wavelet for W^1

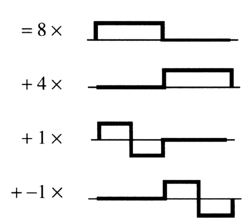


Example: (9 7 3 5)

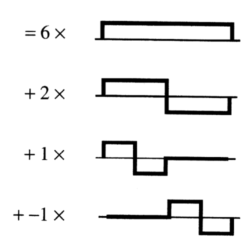
$$\mathcal{I}(x) = c_0^2\,\phi_0^2(x) + c_1^2\,\phi_1^2(x) + c_2^2\,\phi_2^2(x) + c_3^2\,\phi_3^2(x)$$



$$\mathcal{I}(x) = c_0^1\,\phi_0^1(x) + c_1^1\,\phi_1^1(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$



$$\mathcal{I}(x) = c_0^0\,\phi_0^0(x) + d_0^0\,\psi_0^0(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$

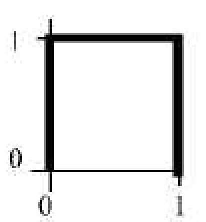


Orthogonal Complement

- W^j is orthogonal complement of V^j in V^{j+1}
  $\Rightarrow$ all $f \in W^j$ are orthogonal to all $g \in V^j$
- linear independent y_i^j spanning W^j
  are the ***wavelets***

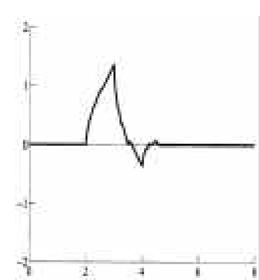
=> - basis of V^j and W^j form a basis for V^{j+1}
   - basis functions of V^j and W^j are orthogonal

Scaling Functions



Haar basis

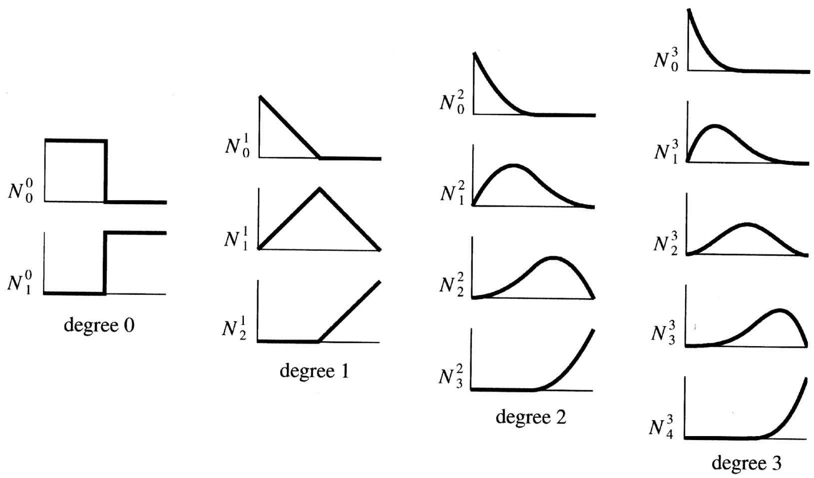
cubic B-spline basis



Daubechies basis

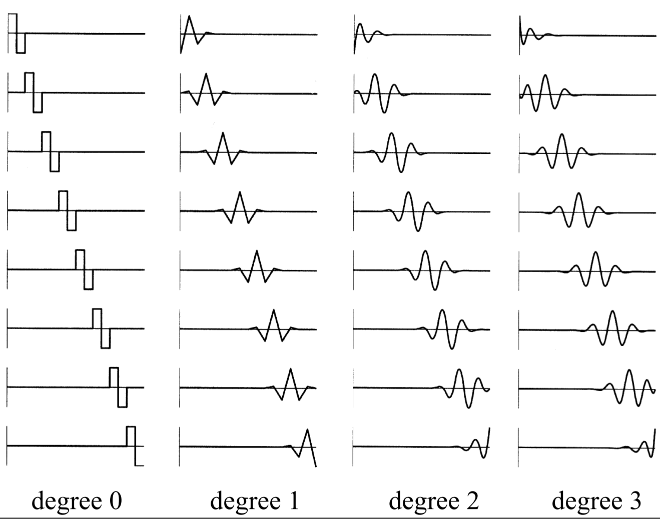
2. Foundations of Wavelet Theory - 2.1 Basics of the Wavelet Transformation

B-Spline Scaling Functions



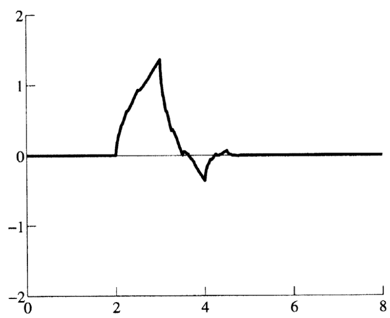
2. Foundations of Wavelet Theory - 2.1 Basics of the Wavelet Transformation

B-spline Wavelet Functions

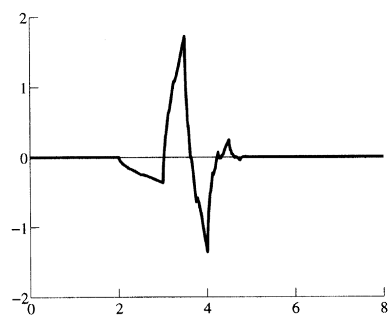


2. Foundations of Wavelet Theory - 2.1 Basics of the Wavelet Transformation

Daubechies Wavelets

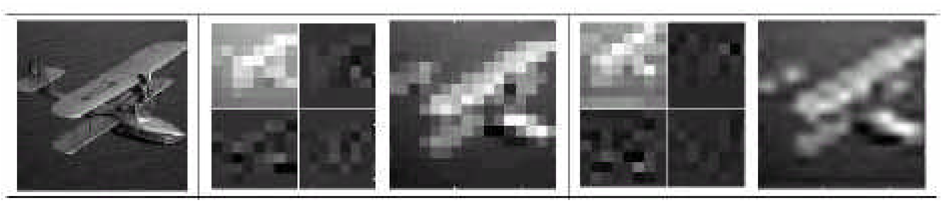


Daubechies Basis

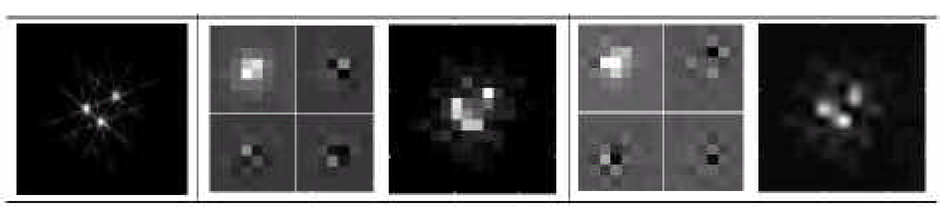


Daubechies Wavelet

Comparison Haar - Daubechies



Original | Haar | Image Reconstr. | Daubechies-8 | Image Reconstr.



Original | Haar | Image Reconstr. | Daubechies-8 | Image Reconstr.

Properties

- Orthogonality
  - wavelets y and basis f are orthogonal if

$$\forall i, j, l: \left\langle f_k^j \mid y_l^j \right\rangle = 0$$

$$\forall j, k, l: \left\langle f_k^j \mid f_l^j \right\rangle = \begin{cases} c & if\ k = l \\ 0 & otherwise \end{cases} \wedge \left\langle y_k^j \mid y_l^j \right\rangle = \begin{cases} c & if\ k = l \\ 0 & otherwise \end{cases}$$

  - wavelets y are semi-orthogonal if

$$\forall j, k, l: \left\langle f_k^j \mid y_l^j \right\rangle = 0$$

  - wavelets y are bi-orthogonal if (~ indicates dual basis)

$$\forall i, j, l: \left\langle f_k^j \mid \tilde{y}_l^j \right\rangle = 0 \quad \wedge \quad \left\langle y_k^j \mid \tilde{f}_l^j \right\rangle = 0$$

Properties

- Normalization
  - vector u is normalized if $\| u \| = 1$
- Orthonormality
  - basis $u_1, u_2, \ldots$ is orthonormal if

$$\forall i, j: \left\langle u_i \mid u_j \right\rangle = d_{i,j} \quad with\ \ d_{i,j} = 1\ \ if\ \ i = j\ \ and\ \ 0\ otherwise$$

(orthonormality = orthogonality + normalization)

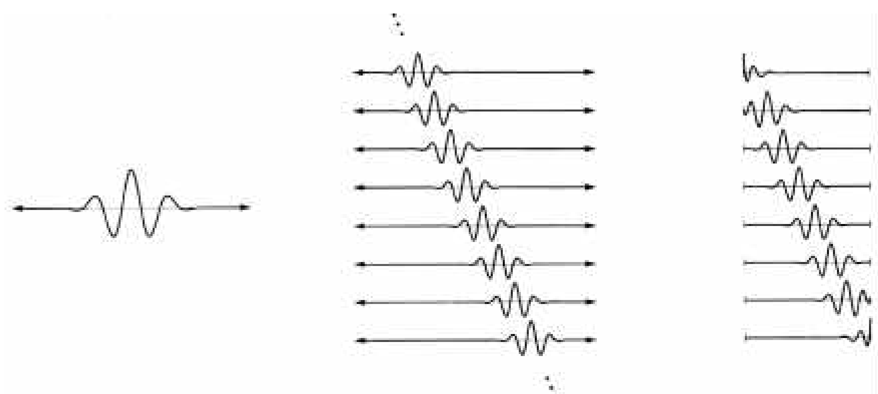
Properties

- **Symmetry** of Scaling and Wavelet function (about their center)
- **Compact Support**
- **Smoothness / Differentiability** of the Scaling and Wavelet Functions

  –> compact support and smoothness are conflicting goals
- **Continuous** Wavelet Transformation (CWT) versus **Discrete** Wavelet Transformation (DWT)

Properties

Continuous versus Endpoint-Interpolating Wavelets

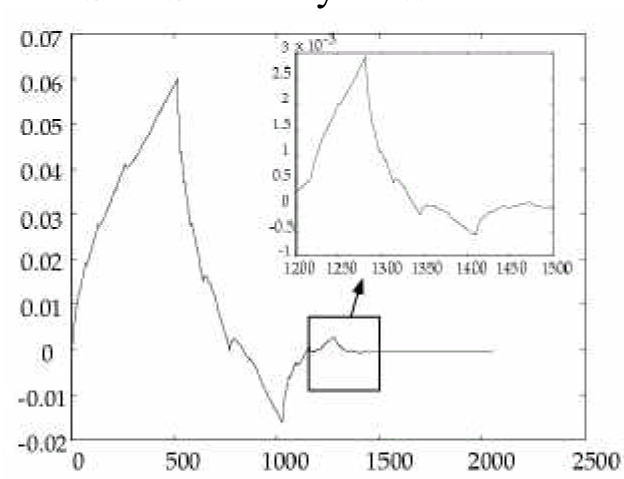


mother wavelet | shift-invariant wavelets | shift-variant wavelets

2. Foundations of Wavelet Theory - 2.1 Basics of the Wavelet Transformation

Properties

Self-Similarity of Wavelets



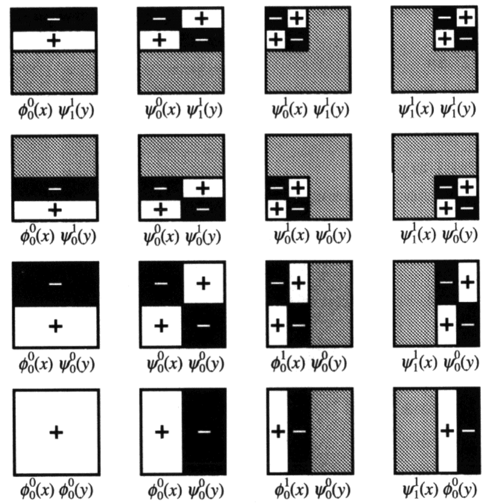
Extensions to Higher Dimensions

Construction of the basis function

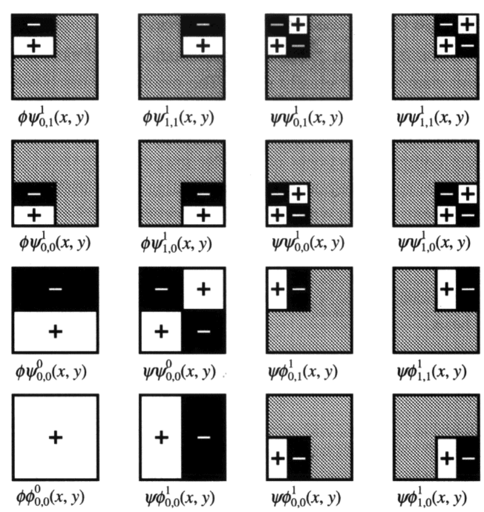
– **Standard**
corresponds to a transformation of first the rows and then the columns (basis functions corresponds to the Tensor product of $f_0^0 y_0^0 y_0^1 y_1^1$)

– **Non-Standard**
corresponds to a mutual transformation of the rows and the columns

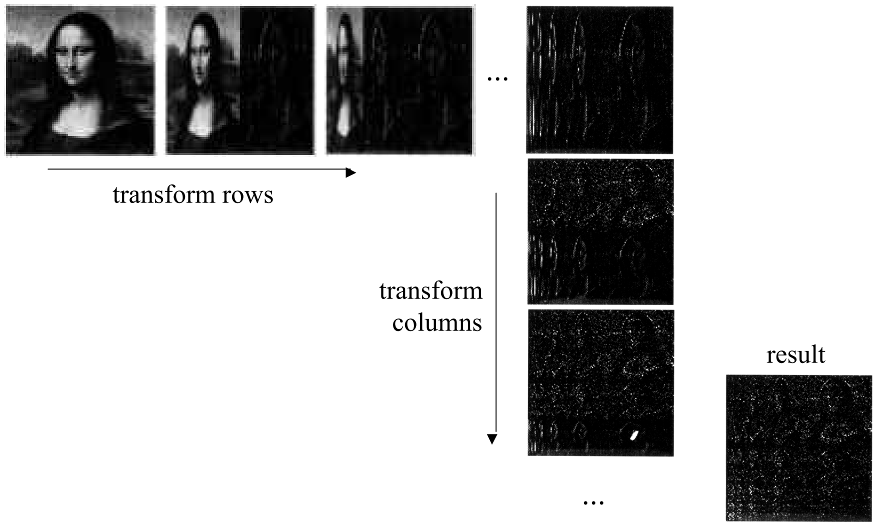
Standard Construction
(2-dim. Haar wavelet)



Non-Standard Construction
(2-dim. Haar wavelet)



Standard Decomposition



2. Foundations of Wavelet Theory - 2.1 Basics of the Wavelet Transformation

Non-Standard Decomposition

2. Foundations of Wavelet Theory - 2.1 Basics of the Wavelet Transformation

2.2 Multiresolution Analysis

- Scaling Function and Wavelets can be used to *decompose* data into components of *multiple resolutions*
- Transformation is *efficient* since it can be performed by Matrix Operations (for bounded domain)
- Transformation is reversible

<u>*in the following:*</u>
wavelets on bounded domain $\Rightarrow V^j$ has a finite basis

Multiresolution Analysis

- Basis of Multiresolution Analysis:
  nested set of linear function spaces

$$V^0 \subset V^1 \subset V^2 \subset V^3 \subset \ldots$$

- Wavelet spaces W^j
  are the complement of V^j in V^{j+1}

  (orthogonality not required)

Multiresolution Analysis

- Matrix Notation of Scaling and Wavelet Functions

$$\Phi^j = \left[f_0^j \cdots f_{v(j)-1}^j\right] \qquad \Psi^j = \left[y_0^j \cdots y_{w(j)-1}^j\right]$$

- nested function spaces
  $\Rightarrow$ Scaling and Wavelet Functions are refinable

$$\Phi^{j-1}(x) = \Phi^j(x) \cdot P^j \qquad \Psi^{j-1}(x) = \Phi^j(x) \cdot Q^j$$

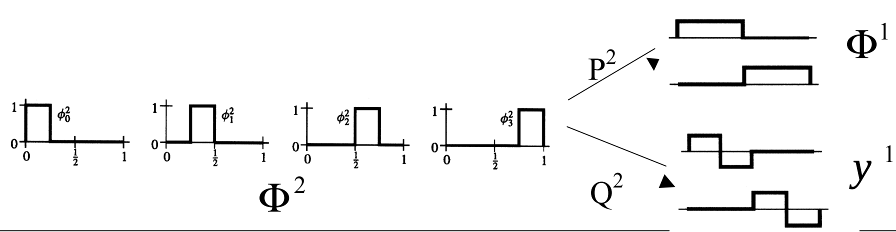
- two-scale relation for scaling functions and wavelets:

$$\left[\Phi^{j-1} \mid \Psi^{j-1}\right] = \Phi^j \left[P^j \mid Q^j\right]$$

Two-Scale Relation for Haar Basis

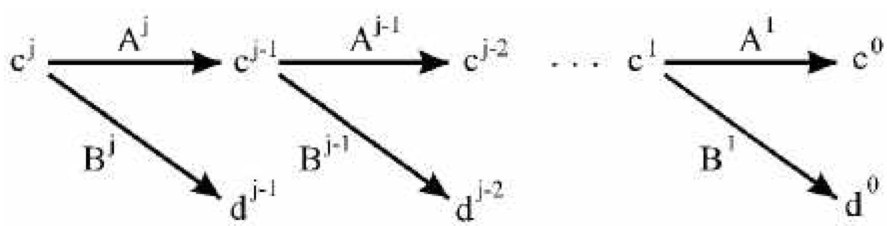
$$\left[\Phi^1 \mid \Psi^1\right] = \Phi^2 \left[P^2 \mid Q^2\right]$$

$$\Rightarrow \quad \left[f_0^1 f_1^1 y_0^1 y_1^1\right] = \left[f_0^2 f_1^2 f_2^2 f_3^2\right] \cdot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$



Analysis Filter

- Matrices (A^j and B^j) can be used to decompose the data
- Data c^j can be decomposed by A^j and B^j into low-resolution part c^{j-1} and detail part d^{j-1}
- Decomposition can be applied recursively to c^{j-1}



Analysis and Synthesis Filters

- Analysis Filters: $c^{j-1} = A^j c^j \qquad d^{j-1} = B^j c^j$

satisfying: $$\left[\Phi^{j-1} \mid \Psi^{j-1}\right] \cdot \left[\frac{A^j}{B^j}\right] = \Phi^j$$

- Synthesis Filters: $c^j = P^j c^{j-1} + Q^j d^{j-1}$

$$\left[\Phi^{j-1} \mid \Psi^{j-1}\right] = \Phi^j \left[P^j \mid Q^j\right] \quad \Rightarrow \quad \left[\frac{A^j}{B^j}\right] = \left[P^j \mid Q^j\right]^{-1}$$

Analysis Filter for Haar Basis

Analysis Filter: $A^2 = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$$B^2 = \frac{1}{2} \cdot \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

<u>*Example:*</u>

$$[9 \quad 7 \quad 3 \quad 5]^T \xrightarrow{A^2} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$[9 \quad 7 \quad 3 \quad 5]^T \xrightarrow{B^2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Semi- & Bi-orthogonal Wavelets

- *Orthogonal Basis* is no requirement for Multiresolution Analysis
- *Semi-orthogonal* or *Bi-orthogonal Basis* are sufficient, sometimes even better since they can be constructed to be sparse

<u>Definitions:</u>

– wavelets y are semi-orthogonal if

$$\forall j,k,l: \left\langle f_k^j \mid y_l^j \right\rangle = 0$$

– wavelets y are bi-orthogonal if (~ indicates dual basis)

$$\forall i,j,l: \left\langle f_k^j \mid \tilde{y}_l^j \right\rangle = 0 \quad \wedge \quad \left\langle y_k^j \mid \tilde{f}_l^j \right\rangle = 0$$

2.3 Advantages of the Wavelet Transformation

- Generality of the Transformation
  (Generalization of other Transformations)
- Adaptability of the Transformation
  (Different Basis Functions allow different Properties of the Transformation)
- Transformation is Hierarchical
  (Multiresolution - Properties)
- Transformation is Loss-Free
- Efficiency of the Transformation
  (Linear Time and Space Complexity for Orthogonal Wavelets)

⇒ Advantages translate into specific advantages in the different applications

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   3.2 Data Compression
   3.3 Computer Graphics
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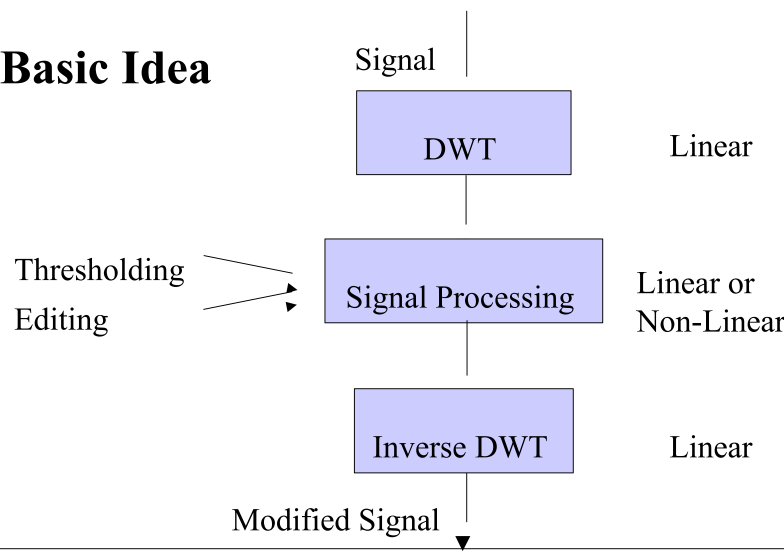
3. Standard Applications

3.1 Signal Processing

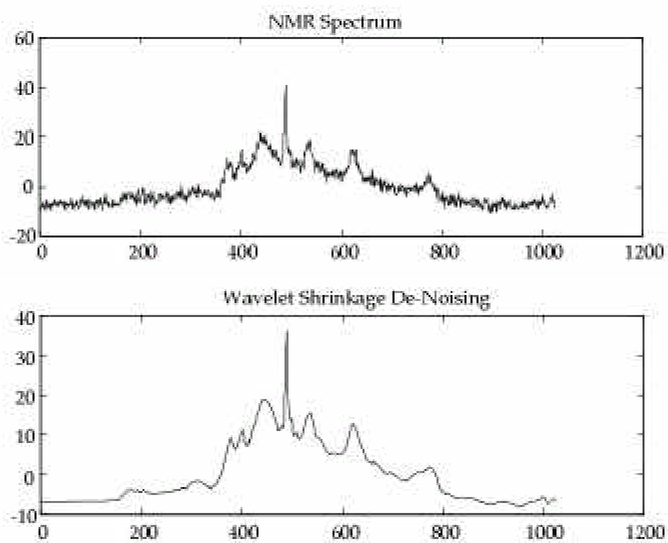
- Signal Filtering (Linear / Non-Linear)
  - Noise-Reduction (De-Noising)
  - Speckle-Reduction (De-Speckling)
- Edge-Detection
- Multi-Resolution Editing

3.1 Signal Processing

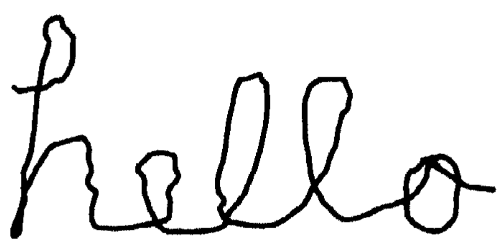
Basic Idea



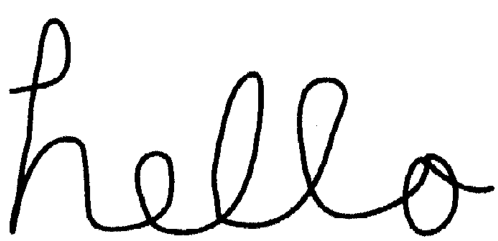
Noise-Reduction



Noise-Reduction

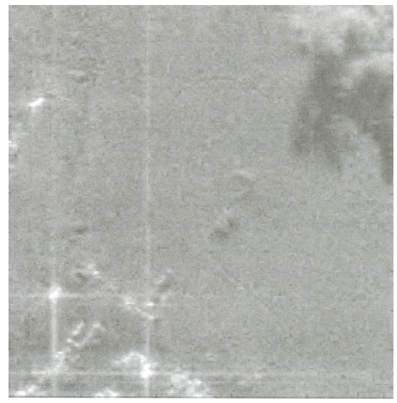


before
noise-reduction



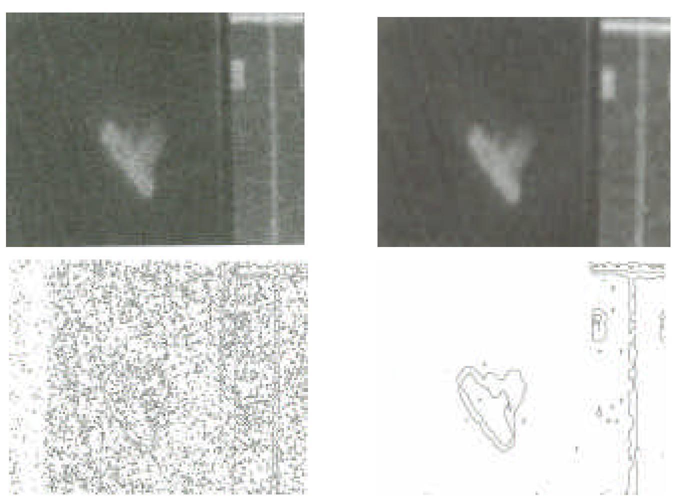
after
noise-reduction

Speckle-Reduction



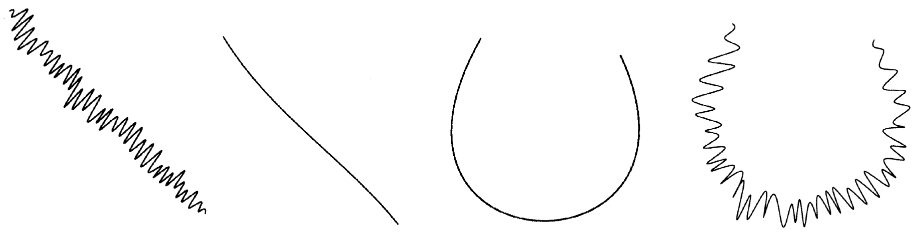
before reduction after reduction

Edge-Detection



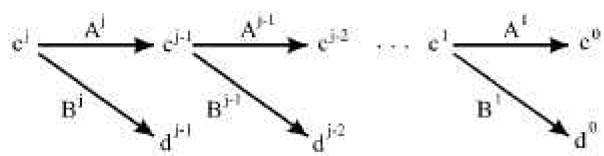
without wavelet transformation with wavelet transformation

Multiresolution Editing



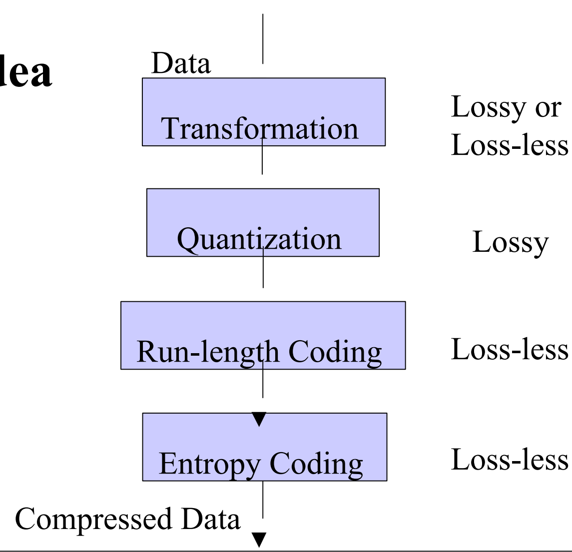
Original | Wavelet Filtering of Low-Res.Signal | Editing of Low-Res. Signal | Re-Application of Detail Signal

Principle Idea



3.2 Data Compression

Basic Idea

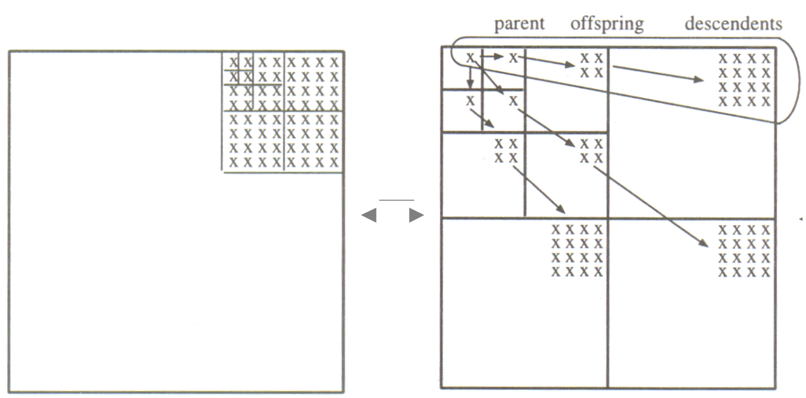


Data Compression

- Transformation
  - Discrete Cosine Transformation (JPEG)
  - Wavelet Transformation (JPEG2000)
- Lossy Compression
  - Scalar Quantization
  - Vector Quantization
- Loss-less Compression
  - Run-Length Coding
  - Entropy Coding: Huffman or Arithmetic Coding

Data Compression

- Data Storage



3. Standard Applications - 3.2 Data Compression

Data Compression

- Data Storage

Example:
Wavelet Image Compression Algorithm

```
procedure Compress(c: array [1 . . m] of reals; ε: real)
  τ_min ← min{ |c[i]| }
  τ_max ← max{ |c[i]| }
  do
    τ ← (τ_min + τ_max)/2
    s ← 0
    for i ← 1 to m do
      if |c[i]| < τ then s ← s + |c[i]|^2
    end for
    if s < ε^2 then τ_min ← τ else τ_max ← τ
  until τ_min ≈ τ_max
  for i ← 1 to m do
    if |c[i]| < τ then c[i] ← 0
  end for
end procedure
```

c[i]: coefficients of Wavelet Transformation

t : threshold with error e

Wavelet Image Compression

Original

1:50

1:100

1:200

Wavelet Image Compression

Original

1:6 Compression

3. Standard Applications - 3.2 Data Compression

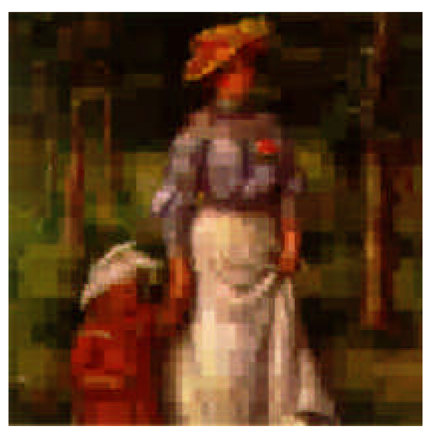
Wavelet Image Compression

Original

1:42 Compression

Wavelet Image Compression

Original



1:222 Compression

Example: FBI Fingerprint Compression

Original | 1:26 Compression

Wavelet Image Compression

Wavelet Compression | Fourier Compression

Wavelet Image Compression

Advantages

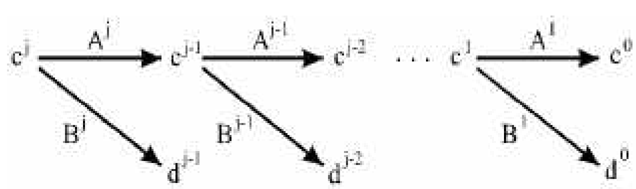
- loss-free (up to 1:5) or lossy (up to 1:200)
- improved image quality (at same compression ratio)
- flexible image format
  (different resolutions, partial password protection)
- fast image preview, successive image loading
  (–> important feature for internet applications!)

Example

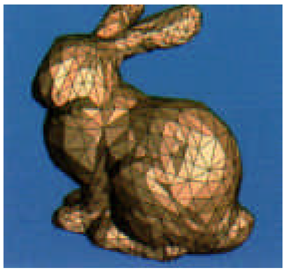
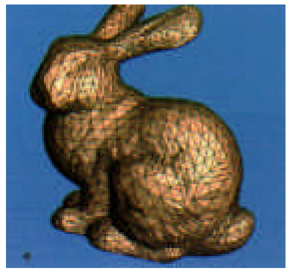
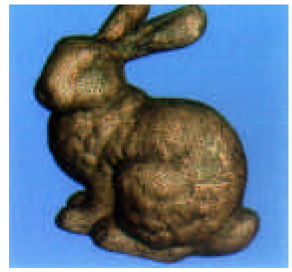
3.3 Computer Graphics

- Approximation of Curves and Surfaces
- Multiresolution Data Representation
- Multiresolution Manipulation of Images, Curves, and Surfaces

Principle Idea



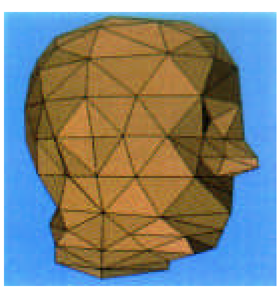
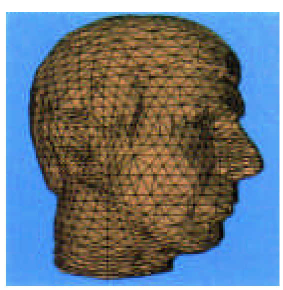
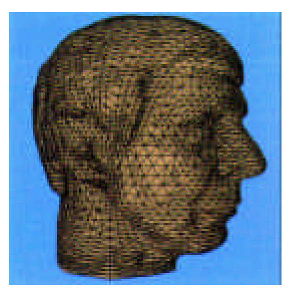
Approximation of Surfaces

952 Wavelet Coefficients   2268 Wavelet Coefficients   18636 Wavelet Coefficients

3. Standard Applications - 3.3 Computer Graphics

Multiresolution Data Representation

952 Wavelet Coefficients   2268 Wavelet Coefficients   18636 Wavelet Coefficients

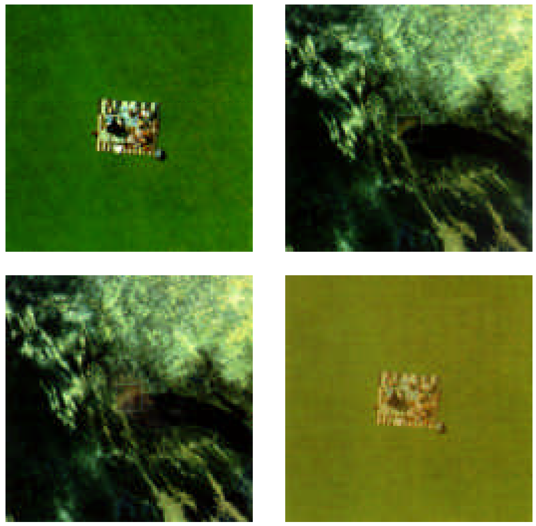
In rendering applications, approximations with different resolutions are used depending on the distance from the viewer.

3. Standard Applications - 3.3 Computer Graphics

Multiresolution Image Manipulation

Original

Zoom out by a Factor of 100,000



Adding Smog

Changed Original

Multiresolution Curve Manipulation

Original Spline Surface

Changes at narrow Scale

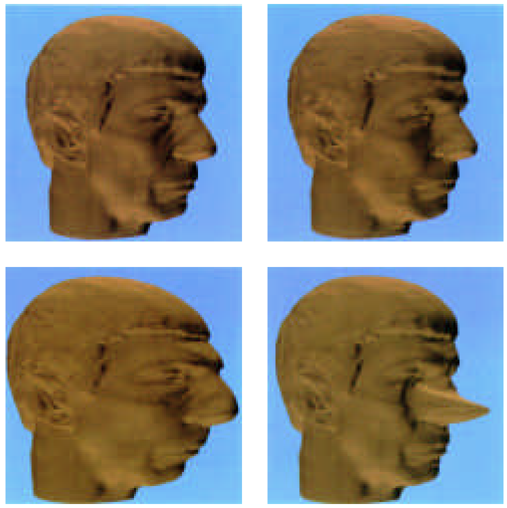
Changes at intermediate Scale

Changes at broad Scale

Multiresolution Surface Manipulation

Original

Compressed to 16%



Changed at Coarse Level

Changes at Fine Level

Overview

1. Introduction
2. Foundations of Wavelet Theory
3. Standard Applications
4. Applications in Database Area
   - 4.1 Efficient Data Processing
   - 4.2 Similarity Search
5. Summary and Conclusion

4. Applications in Databases

4.1 Efficient Data Processing

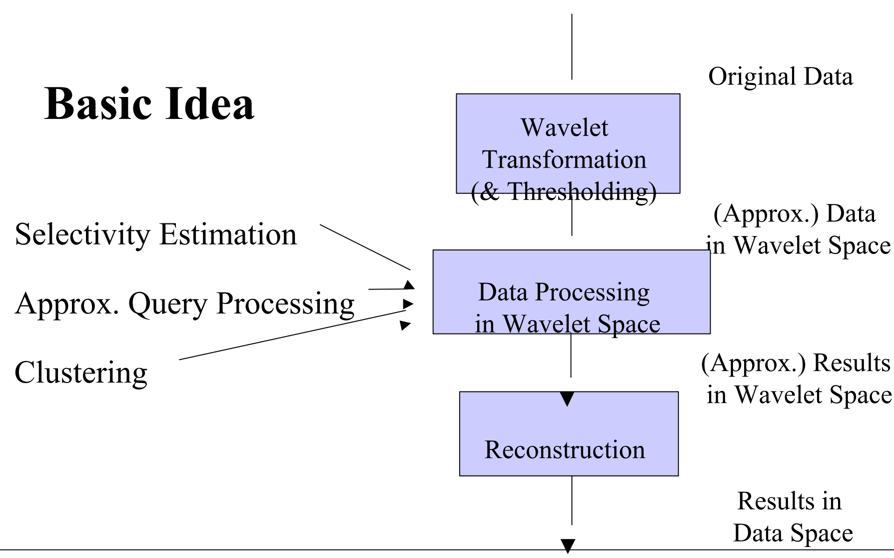
– Selectivity Estimation
  - Wavelet-based Histograms [MVW 98, MVW 00]

– Approximate Query Processing
  - Data Cubes [VWI 98, VW99]
  - Relational Databases [CGRS00]

– Clustering Techniques
  - WaveCluster [SCZ98, SCZ00]

4.1 Efficient Data Processing

Basic Idea



Selectivity Estimation:

Wavelet-based Histograms [MVW98, MVW00]

– **Goal**
  - Compact histograms
  - Accurate estimation of the data distribution

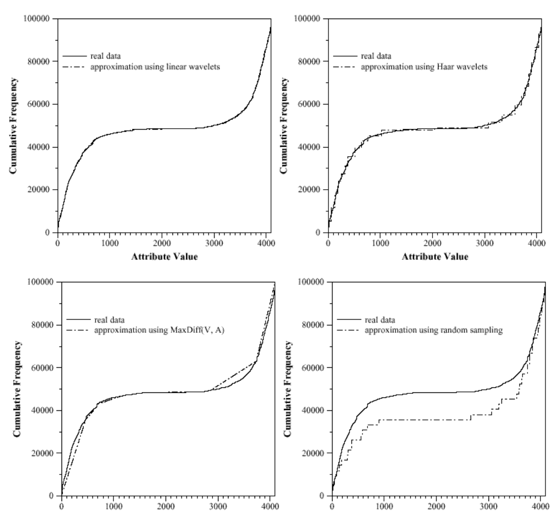
– **Approach**
  - compute extended cumulative data distribution (cumulative data distribution with zero values)
  - compute Haar (or linear) wavelet transformation
  - thresholding methods:
    1. Take largest wavelet coefficients
    2. Take wavelet coefficients which lead to a large error reduction
    3. Throw away wavelet coefficients whose deletion lead to the small error increase

Selectivity Estimation:

Wavelet-based Histograms [MVW98, MVW00]

linear wavelet

Haar wavelet



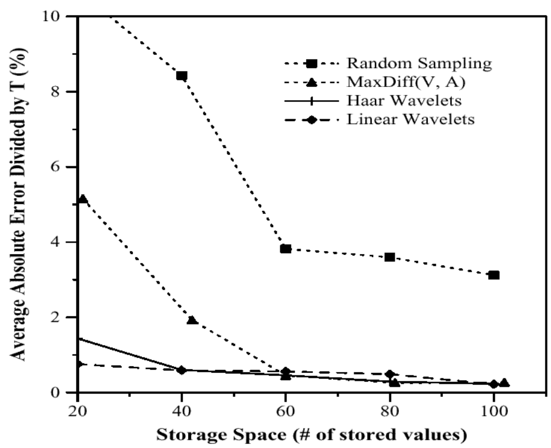
MaxDiff

random sampling

Selectivity Estimation:

Wavelet-based Histograms [MVW98, MVW00]

Average Absolute Error Results (depending on storage size)



Approximate Query Processing:

Data Cube [VWI98, VW99]

Goal

- compact representation of data cube
- efficient support of partial range-sum queries
- I/O-efficient construction of the wavelet decomposition
  –> construction directly from the sparse representation of orig. cube

Approach

1. Haar wavelet transformation of the data cube
2. thresholding according to storage and accuracy requirements
3. inverse wavelet transformation to determine the results

Approximate Query Processing:
Data Cube [VWI98]

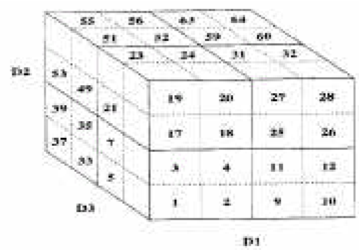
Ideas

– partial sum cube

<u>Motivation:</u> partial sum cube is monotonously increasing
⇒ wavelet decomposition provides better results

– cell-wise logarithmic transformation of the cube

– chunked data processing



Approximate Query Processing:
Data Cube [VW99]

– **Haar Wavelet Decomposition**

- based directly on the sparse repres. of the original cube
- result: C′ coefficients (~ non-zero entries of the cube)

– **Thresholding and Ranking**

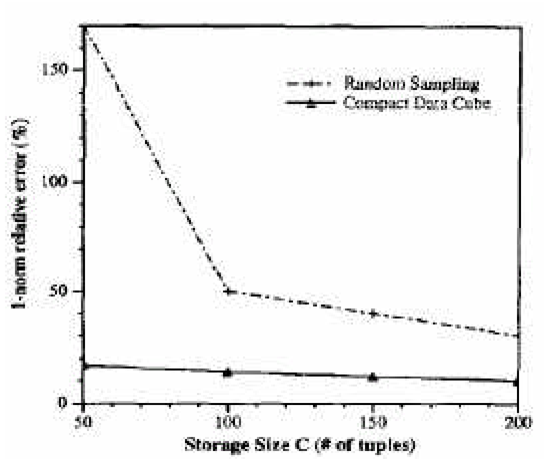
- Threshold to reduce coefficients to C << C′
- Ranking according to importance for answering typical (range) queries by weighting the coefficients

– **Reconstruction**

- Determining the k < C′ coefficients which are most important for answering the aggregation query
- k determines the accuracy of the answer

Approximate Query Processing:
Data Cube [VW99]

Relative Error Results (depending on storage size)



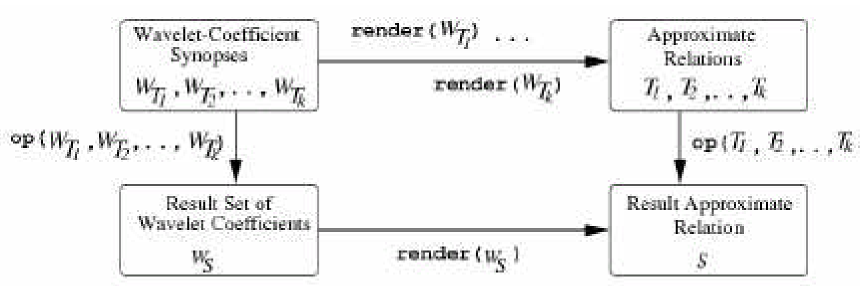
Approximate Query Processing:
Relational Databases [CGRS00]

- **Wavelet Decomposition**
  - Non-Standard Haar-Wavelet Decomposition of the d-dim. array of attribute value combinations (joint frequency distribution)
  - Thresholding to retain highest coefficients
- **Processing of Relational Algebra Operations in the Wavelet-Coefficient Domain**
  - Select-Operation
  - Project-Operation
  - Join-Operation
- **Determining the Approximate Results („Rendering“)**
  - Mapping Results in Wavelet-Domain to Relational Tuples

Approximate Query Processing

Relational Databases [CGRS00]

Schema of Wavelet-based Approximate Query Processing

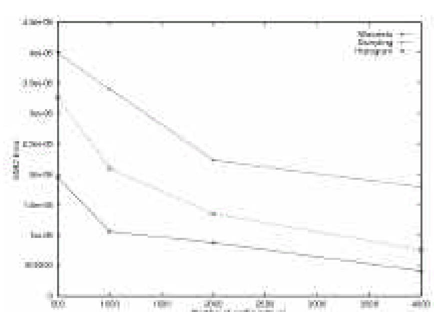


Approximate Query Processing

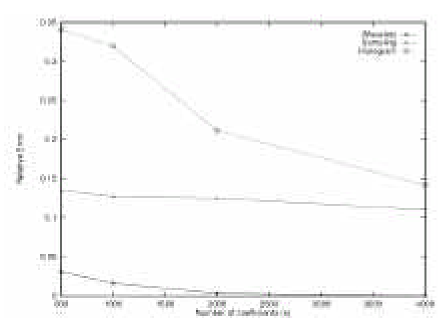
Relational Databases [CGRS00]

Relative Error Results

(depending on number of coefficients used)



Select Query



Select-Sum Query

Clustering Techniques:
WaveCluster [SCZ98, SCZ00]

Basic Idea

- Partitioning the data space by a grid reduces the number of data objects but induces a small error
- Application of the wavelet-transformation to the reduced feature space provides multiresolution data representation
- Finding the connected components can be performed at different resolutions
- Compression of the grid is crucial for the efficiency

  (→ Does not work in high dimensional space!)

4. Applications in Databases - 4.1 Efficient Data Processing

Clustering Techniques:
WaveCluster [SCZ98, SCZ00]

Clustering based on a wavelet approximation of the data

Algorithm:

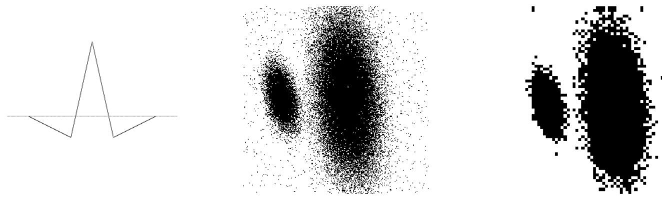
Input: Multidimensional data objects' feature vectors
Output: clustered objects

1. Quantize feature space, then assign objects to the units.
2. Apply wavelet transform on the feature space.
3. Find the connected components (clusters) in the subbands of transformed feature space, at different levels.
4. Assign label to the units.
5. Make the lookup table.
6. Map the objects to the clusters.

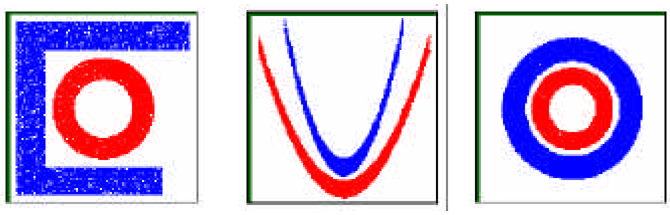
4. Applications in Databases - 4.1 Efficient Data Processing

Clustering Techniques:
WaveCluster [SCZ98, SCZ00]

- Effect of Wavelet Transformation

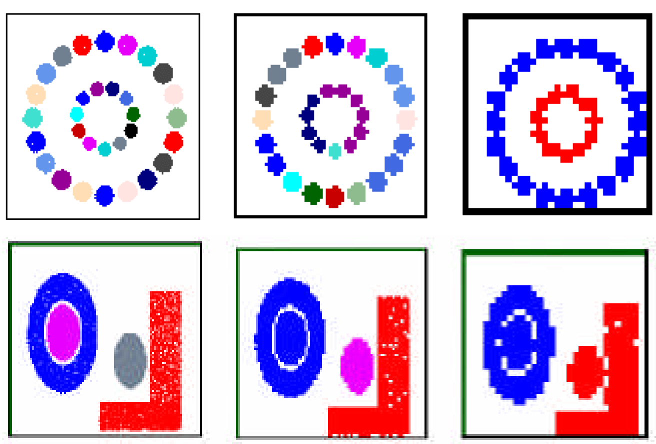


- Arbitrary shaped clusters found by WaveCluster



Clustering Techniques:
WaveCluster [SCZ98, SCZ00]

Results From Clustering on different Resolution Levels



4.2 Similarity Search

- Similarity Search in Time-Series Databases
  [CF99, SS99a, SS99b]
- Similarity Search in Image Databases
  – WBIIS [WWFW97a, WWFW97b]
  – WALRUS [NRS99]
  – Windsurf [ABP99]
  – Multiresolution Search [Hec99]
  – Similarity Measure Learning [BVGS99]
  – WIPE [WWF97]

Similarity Search in Time Series Databases

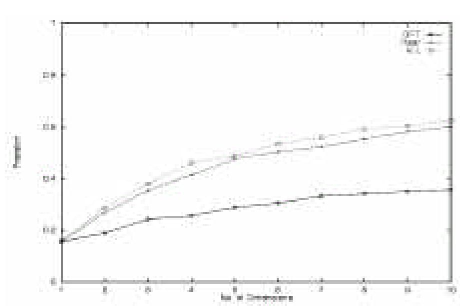
- **Goal**
  – time series matching and retrieval
  – new similarity measures (–> effectiveness)
  – fast computation of the similarity measure (–> efficiency)
- **General Approach**
  – Haar wavelet transformation
  – Similarity measure defined in Wavelet Space

Similarity Search in Time Series Databases [CF99]

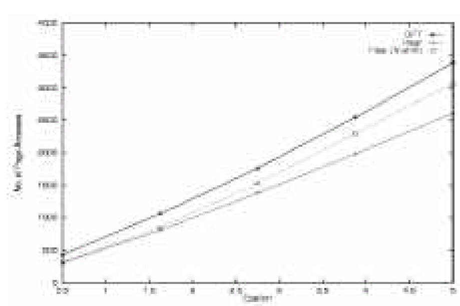
- **Specific Goals**
  - efficient support for range and k-NN queries
  - allow for vertical shifts
- **Approach**
  - wavelet coefficients of sliding window are stored in an index
  - Range and k-NN queries are computed based on the index
- **Results**
  - approach outperforms discrete Fourier transform

Similarity Search in Time Series Databases [CF99]

Results



Accuracy



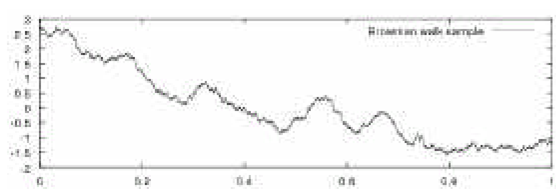
Page Accesses

Similarity Search in Time Series Databases [SS99a,b]

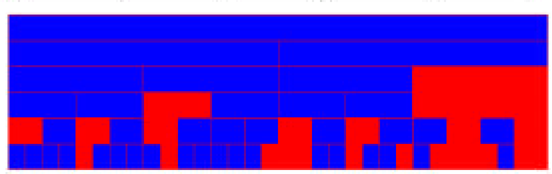
- **Goal**
  - new similarity measure for data mining applications
  - effectiveness and efficiency
- **Approach**
  - only sign change and maximum information (Hölder exponent) of the wavelet transformed data is stored
  - step-wise hierarchical comparison for correlations
  - time shifts are considered

Similarity Search in Time Series Databases [CF99]

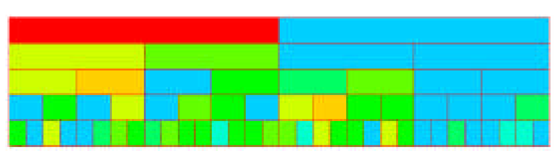
Time Series Data



Sign Information
(6 levels)



Hölder Decomposition
(5-levels)



Similarity Search in Image Databases

Content-based Similarity Search

- WBIIS [WWFW97a, WWFW97b]
- WALRUS [NRS99]
- Windsurf [ABP99]
- Multiresolution Search [Hec99]
- Similarity Measure Learning [BVGS99]
- WIPE [WWF97]

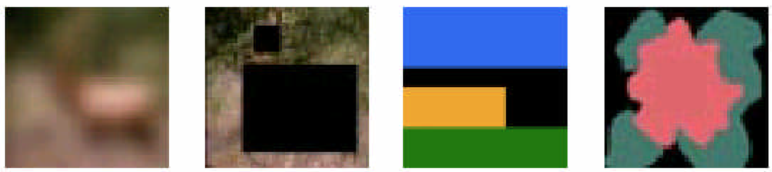
Similarity Search in Image Databases

- **Goal**
  - Content-based Similarity Search in Large Image Databases
  - Improved Recall without explicit Object Recognition
- **General Approach**
  - Wavelet Transformation to extract compact Feature Vectors (possibly more than one per image)
  - Post-processing in Feature Space (e.g., Clustering of Feature Vectors)
  - Search on Feature Vectors using Index Structure / Linear Scan
  - Support of different Similarity Measures

4. Applications in Databases - 4.2 Similarity Search

Similarity Search in Image Databases WBIIS [WWFW97a,b]

- **Goal**
  - Improved Content-based Retrieval:
    Partial Image Retrieval & Sketch Retrieval



low resolution | partial image | block sketch | hand-drawn sketch

Similarity Search in Image Databases WBIIS [WWFW97a,b]

Approach

- Wavelet transformation for each color component using Daubechies-8 Wavelets

- Low Frequency Wavelet Coefficients and their Variance are stored as Feature Vectors
- 2-step retrieval:
  - pre-selection (filtering) based on variance (–> candidates)
  - similarity computation based on full feature vectors of candidates
- Extension: Two-level Multi-Resolution Similarity Search

Similarity Search in Image Databases
WBIIS [WWFW 97a,b]

Partial Match Retrieval Examples

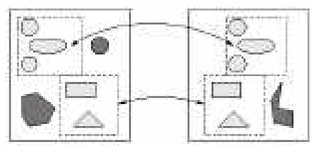
Similarity Search in Image Databases
WBIIS [WWFW 97a,b]

- Retrieval by Sketch Example

Similarity Search in Image Databases WALRUS [NRS99]

- **Goal**
  - Content-based Similarity Search in Large Image Databases:

    Invariance w.r.t. Translation and Scaling of Regions in Image

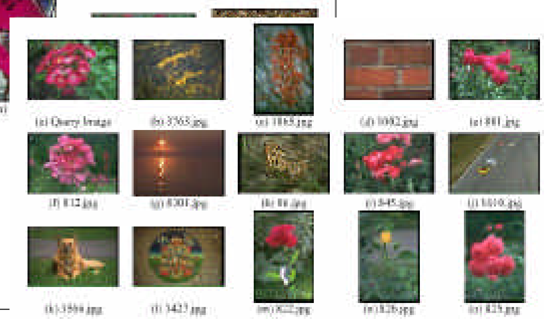


- **Approach**
  - Haar Wavelet Transformation of sliding window of varying size
  - Clustering of Signatures in Wavelet Space (BIRCH)

    => variable Number of Signatures per Image
  - Storage of Centroids of Clusters in Index Structure (R*-Tree)
  - Similarity Search: Matching Pairs of Signatures (largest overlap)

Similarity Search in Image Databases WALRUS [NRS99]

WALRUS Results

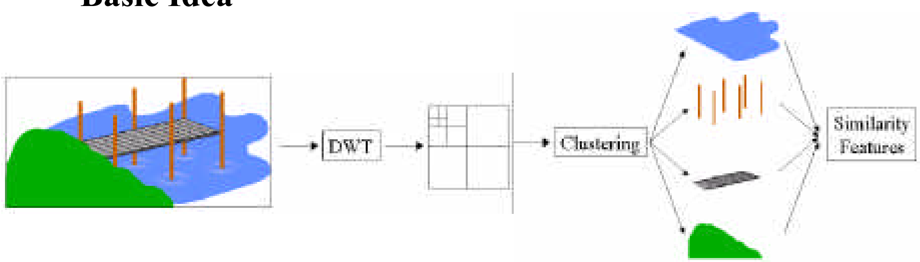


WBIIS Results

Similarity Search in Image Databases
Windsurf [ABP99]

- **Goal**
  – Content-based Similarity Search in Large Image Databases: Partial Similarity based on Image Regions

- **Basic Idea**

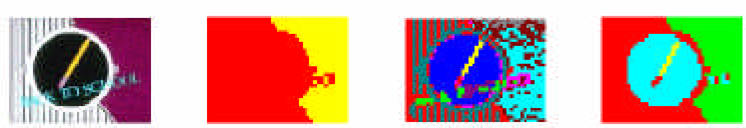


Similarity Search in Image Databases
Windsurf [ABP99]

Approach

– Haar Wavelet Transformation of each color channel

– Partitioning of the Image based on Clustering the three color coefficients (k-means clustering on 3rd subband wavelet coefficients)



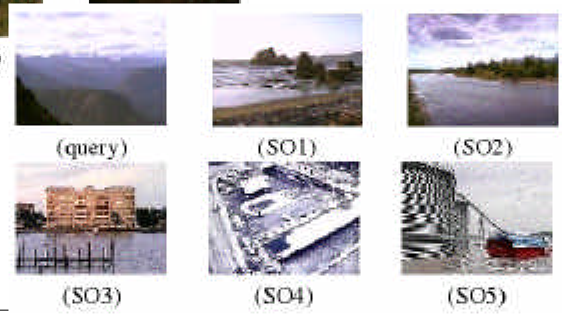
Original k-means k=2 k=10 k=4

– Feature Vectors correspond to Regions found in Clustering Step:
  (Size, Centroids, Covariance Matrix of Pixels in Region)

– Similarity Retrieval based on Matching Regions

Similarity Search in Image Databases
Windsurf [ABP99]

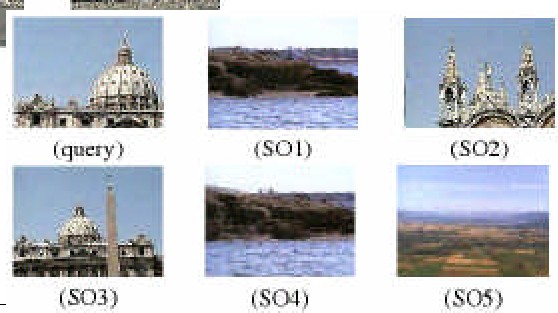
Windsurf Results



Results of comparative System

Similarity Search in Image Databases
Windsurf [ABP99]

Windsurf Results



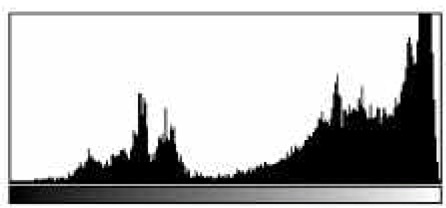
Results of comparative System

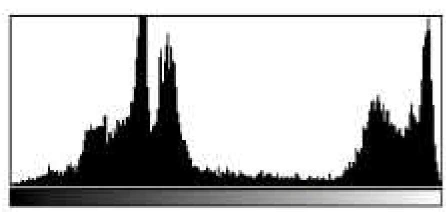
Similarity Search in Image Databases
Multiresolution Search [Hec99]

- **Goal**
  - Content-based Similarity Search in Large Image Databases: Flexible Similarity Search which allows
    - global / detail matching
    - texture / structure matching
- **Approach**
  - Haar Wavelet Transformation of Color Histograms
  - Storage of Wavelet Coefficients on different resolutions as Feature Vectors
  - Search based on arbitrary Combinations of Wavelet Coefficients

Similarity Search in Image Databases
Multiresolution Search [Hec99]

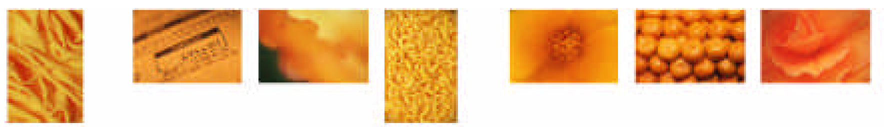
Color Histogram Example





Similarity Search in Image Databases
Multiresolution Search [Hec99]

Search on different level Wavelet Coefficients

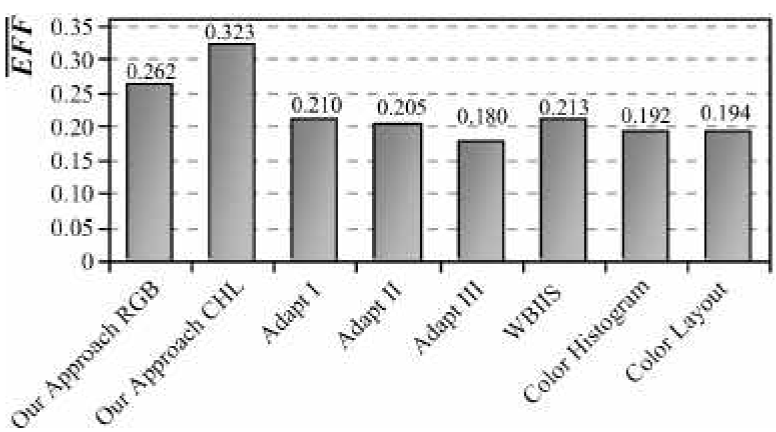


Query Image Results of Search on Low-Frequency Coefficients

Query Image Results of Search on High-Frequency Coefficients

Similarity Search in Image Databases
Multiresolution Search [Hec99]

Comparison to other Approaches: Effectiveness Results



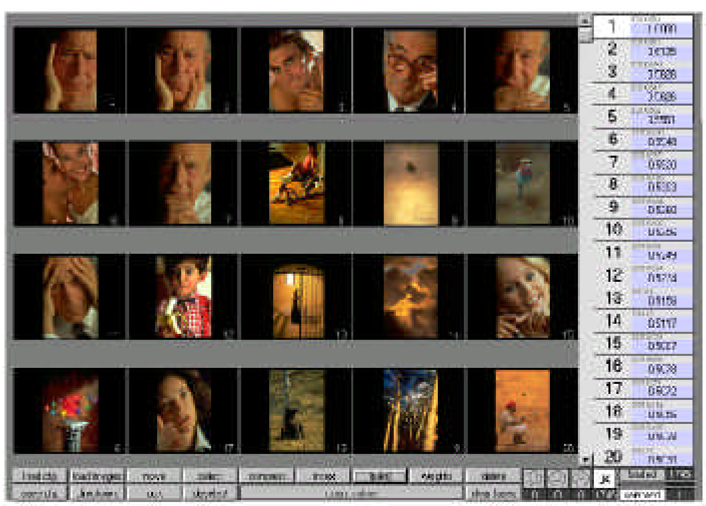
4. Applications in Databases - 4.2 Similarity Search

Similarity Search in Image Databases
Similarity Measure Learning [BVGS99]

- **Goal**
  - Content-based Similarity Search in Large Image Databases: Interactive Learning of the Similarity Measure
- **Approach**
  - Vector median Filtering (to remove noise)
  - Haar Wavelet Transformation
  - Storage of 128 largest coefficients (quantized to +1 / -1)
  - Supervised learning to find similarity measure to find weighting for feature vector comparison

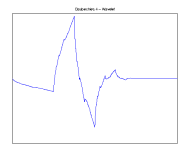
Similarity Search in Image Databases
Similarity Measure Learning [BVGS99]

Example Results



Similarity Search in Image Databases
WIPE [WWF97]

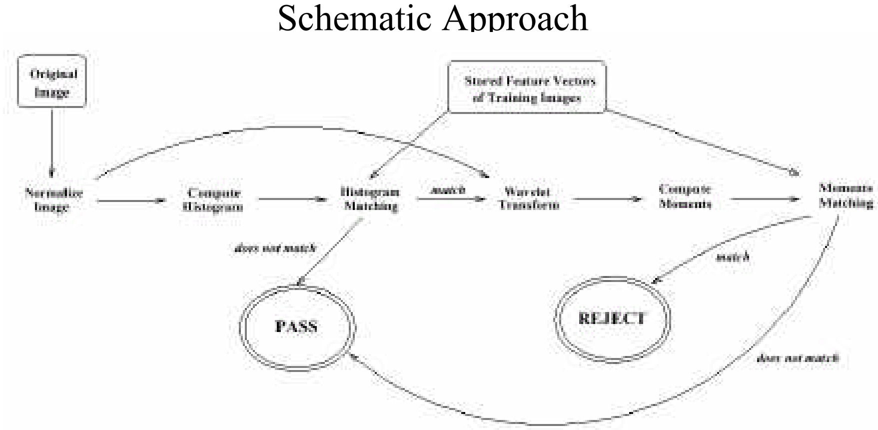
- **Goal**
  - WIPE: Wavelet Image Pornography Elimination
    –> Fast Special Purpose Image Filtering



- **Approach**
  - Normalization of Images to Standard Size
  - Wavelet Transformation using Daubechies-3 Wavelets
  - Edge Detection in Different Subbands of Wavelet Transformation
  - Feature Vectors used for Similarity Matching:
    Central Moments & Invariant Moments & Color Histograms
  - Filtering based on Training the Search for the desired filtering

Similarity Search in Image Databases
WIPE [WWF97]

Schematic Approach



Results: 95% Correct Images found with 10% Wrong Rejects

5. Summary and Conclusion

- Wavelet Transformations
  - improve the Efficiency of existing Methods (–> faster)
  - improve the Effectiveness of existing Methods (–> better)
  - enable new Applications (–> new)
- Successful Applications
  - Signal Processing (Noise Reduction, Edge Detection, ...)
  - Data Compression (Image & Video Compression)
  - Multi-Resolution Data Representation and Manipulation (Computer Graphics)

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5. Summary and Conclusion

- Database Applications
  - Wavelets often only used for an efficient pre-processing
  - often only wavelet coefficients of one resolution are used
- Future Research Directions
  - Loss-free Database Compression
  - Approximate Query Results
  - Multi-Resolution Data Analysis
- Potential New Application
  - New Similarity Search Applications (Video, ...)
  - Fast Approximate Data Mining
  - Fast Approximate Information Retrieval

5. Summary and Conclusion

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