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Lehrer Meets Ranking Theory

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Meets what? Ranking theory is, as far as I know, the only existing theory suited for underpinning Keith Lehrer's account of knowledge and justification. If this is true, it's high time to bring both together. This is what I shall do in this paper.*

However, the result of defining Lehrer's primitive notions in terms of ranking theory will be disappointing: justified acceptance will, depending on the interpretation, either have an unintelligible structure or reduce to mere acceptance, and in the latter interpretation knowledge will reduce to true belief. Of course, this result will require a discussion of who should be disappointed.

So, the plan of the paper is simple: In section 1 I shall briefly state what is required for underpinning Lehrer's account and why most of the familiar theories fail to do so. In section 2 I shall briefly motivate and introduce ranking theory. Basing Lehrer's account on it will be entirely straightforward. Section 3 proves the above-mentioned results. Section 4, finally, discusses the possible conclusions.

1. The basic notions of Lehrer's account of justification and knowledge

I shall base my considerations on Lehrer (2000), the most recent presentation of his theory. It indeed adds simplifications and clarifications to the first edition. For instance, the basic notions on which his theory of knowledge and justification builds

* I am indebted to Gordian Haas for various valuable remarks and for discovering bad faults in a first draft which required major corrections and to Erik Olsson for further helpful suggestions.

stand out more clearly. They are summarized in his definition of an evaluation system in Lehrer (2000, p. 170, D1)[1] which consists of three components: (a) an acceptance system, i.e., a set of accepted statements or propositions, (b) a preference system, i.e., a four-place relation among all statements or propositions saying for all A, B, C, D whether A is more reasonable to accept given or on the assumption that C than B given or on the assumption that D (I symbolize it here as $A|C \succ B|D$),[2] and (c) a reasoning system, i.e., a set of inferences each consisting of premises and a conclusion.

The idea behind (c) is that what a person accepts, and justifiedly accepts, depends also on the inferences she carries out. Here I understand Lehrer as referring to *deductive* inferences or rather to the inferences taken by the person to be deductively valid and sound.[3] The idea behind (b), by contrast, is to take care of inductive reasoning in the widest sense which is always a matter of weighing reasons and objections on the basis of some such preference system. We shall look in detail at Lehrer's specific proposal for this weighing of reasons.

The first step I would like to take in my present discussion is to fix the reasoning system once for all. The reason is that otherwise my comparative business could not even start. Of course, we shall have to discuss in the final section whether this is already the first misrepresentation of Lehrer that will entail all the other ones. How do I fix that system? Since Lehrer emphasizes again and again that he is interested in acceptance only insofar as it is governed by the aim of truth, I propose to extend this attitude to the objects of belief or acceptance and to conceive of them only insofar they can be true or false, i.e., as truth conditions or propositions. Thereby, I ignore all questions of syntactic structure, of logical equivalence, and of logical entailment, and assume that the rationality constraint of consistency and deductive closure of the acceptance system is trivially satisfied. This entails in particular the assumption that the reasoning system is maximal and has no independent role to play. I am well aware that I am taking this step very swiftly. My excuse is that I am convinced that a lengthy treatment of the issue would not reveal a viable constructive alternative.

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1 In the sequel, mere page numbers are always meant to refer to Lehrer (2000).

2 In D1, p. 170, Lehrer mentions only the unconditional preference relation, but it is clear that he requires the conditional one.

3 On p. 127, when introducing the reasoning system, Lehrer refers only to "cogent" reasoning, and generally the notion of validity makes clear sense only relative to deductive inference.

Having taken this step the task of underpinning reduces to accounting for the acceptance and the preference system. Concerning the latter, the first idea is, of course, to appeal to a probability measure P and to define that $A|C \succ B|D$ iff $P(A|C) > P(B|D)$. However, the relation between probability and acceptance is problematic, as is highlighted by the famous lottery paradox. I am not rejecting all attempts to solve this paradox out of hand, but the mere fact that they are debated heatedly and that all of them are contested shows that probability theory is, presently, not a good foundation for Lehrer's theory. Moreover, as I shall point out below, there is a particular feature in Lehrer's notion of neutralizing an objection which prevents any probabilistic interpretation. Olsson (1998a) discusses further difficulties of a purely probabilistic construal of justified acceptance.

For similar reasons Lehrer, too, has given up on finding purely probabilistic foundations, which he still hoped to build in Lehrer (1971, 1974). There he suggests, moreover, that the foundations may be construed as some kind of epistemic decision theory. The hint is still found in Lehrer (2000, pp.145ff.) and also used for a solution of the lottery paradox. However, I am doubtful because epistemic decision theory has remained a promise that has never been redeemed in a satisfying way in the last 35 years.

The basic difficulty, I believe, is this: Probability theory may claim, in a way, to offer a complete epistemology. If so, it is hard to complement or merge it with other epistemological ideas like acceptance, epistemic decisions, or whatever, and radical probabilism as Jeffrey (1992) has defended it seems unavoidable.

Hence, we should put probability theory aside and rather look at theories dealing directly with acceptance or belief. A large variety of such theories – such as default logic, AGM belief revision theory, Pollock's and other accounts of defeasible and non-monotonic reasoning, etc. – has been developed in the past 25 years. Maybe they provide an account of Lehrer's preference system as well. Alas, they don't. At least, I claim this with confidence with respect to AGM belief revision theory. There it is shown that the behavior of belief revisions is equivalent to the behavior of so-called entrenchment relations.[4] These could indeed fill the role of Lehrer's unconditional

[4] See Achourrón et al. (1985), which is considered as the foundation of AGM belief revision theory, or Gärdenfors (1988, ch. 4).

preference relation. Maybe entrenchment relations can be generalized so as to capture the special case $A|C \succ B|C$ of Lehrer's preference relation which refers twice to the same condition. However, Lehrer requires the full conditional relation, which cannot be accounted for in AGM belief revision theory. I suspect that essentially the same is true of all accounts of defeasible reasoning implicitly or explicitly appealing to some kind of epistemic ordering.

There is only one theory that is about belief or acceptance *and* provides a sufficiently powerful preference system: ranking theory. That's why I said it is the only existing theory suited for underpinning Lehrer's account. What does it look like?

2. Ranking theory

The basics are quickly told. Originally, ranking theory was developed[5] in order to overcome essential restrictions of AGM belief revision theory. As it turns out, AGM theory generally accounts only for one step of belief revision and thereafter returns to a static picture. But, of course, a full dynamics has to account for several or iterated belief changes. The problem is around since Harper (1976), and there have been quite a number of attempts to solve it within the confines of AGM theorizing.[6] However, I find these proposals inferior to the one provided by ranking theory.

Iterated belief revision is not our concern here. However, there exists a close connection between iterated belief change and full conditional epistemic preference.[7] It is for this reason that ranking theory, though addressed to the former, can also provide for the latter. So let's take a look:

Let's start with an exhaustive set W of possibilities (possible worlds, first-order valuations, or whatever). Subsets of W are propositions (let's not worry about their algebraic structure), W itself is the logically true and $\emptyset$ the logically false proposition. As explained above, I take such propositions as objects of epistemic attitudes.

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[5] In Spohn (1983, sect, 5.3) and (1988).

[6] Cf., e.g., Nayak (1994).

[7] The connection is not obvious and perhaps not cogent. In order to really understand it we would have to go much more deeply into issues of belief revision than we can and need to do here.

Ranks, then, are *grades of disbelief* (where I find it natural to take non-negative integers as grades, but other numbers would do so as well). These grades obey some fundamental laws summarized in

Definition 1: κ is a *ranking function* iff κ is a function from the power set of *W* into **N**∪{∞} such that κ(*W*) = 0, κ(*A*) = ∞ iff *A* = ∅, and κ(*A*∪*B*) = min{κ(*A*), κ(*B*)}. κ(*A*) is called the *rank of A*. The *rank of B given A* is defined as κ(*B*|*A*) = κ(*A*∩*B*) – κ(*A*).

Hence, κ(*A*) > 0 says that *A* is disbelieved (to some degree), and κ($\overline{A}$) > 0 says that *A* is believed.[8] κ(*A*) = 0 only expresses that *A* is not disbelieved and leaves open the possibility that $\overline{A}$ is not disbelieved as well. Since there is no point here in distinguishing between belief and acceptance, we thus have

Definition 2: *A* is *accepted* by κ iff κ($\overline{A}$) > 0. {*A* | *A* is accepted by κ} is the *acceptance system* of κ.

What are the fundamental laws according to Definition 1? κ(*W*) = 0 says that the logically true proposition is not disbelieved. The condition that κ(*A*) = ∞ iff *A* = ∅ says that it is most strongly believed, i.e., that the logically false proposition is more strongly disbelieved than any other. More substantial is the *law of disjunction* that κ(*A*∪*B*) = min {κ(*A*), κ(*B*)}. Clearly, the disjunction *A*∪*B* cannot be more firmly disbelieved than either of its disjuncts. Nor can it be less firmly disbelieved than both disjuncts, since this would entail the absurdity that given *A*∪*B* both, *A* and *B*, are disbelieved, though *A*∪*B* is not. An immediate consequence is the *law of negation* that either κ(*A*) = 0 or κ($\overline{A}$) = 0 (or both). *A* and $\overline{A}$ cannot be both disbelieved. Perhaps the most important law is the *law of conjunction* that κ(*A*∩*B*) = κ(*A*) + κ(*B*|*A*) which follows trivially from the definition of conditional ranks. It says that in order to arrive at the degree of disbelief in *A*∩*B* one has to sum up the degree of disbelief in *A* and the additional degree of disbelief in *B* given *A*. This, I believe, agrees with intuition.

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[8] $\overline{A}$ is the complement of *A* with respect to *W*. The thinking in negations will continue, though I am fully aware that it is cumbersome.

There is a surprisingly well working translation from probabilistic into ranking terms which almost automatically generates a large number of ranking theorems from probability theorems. This applies also to the account of belief change. The basic rule for probabilistic belief change is simple conditionalization according to which one moves to the probabilities conditional on the information received. This is generalized by Jeffrey's conditionalization[9] which is unrestrictedly performable and thus defines a full dynamics within the realm of strictly positive probability measures. In the corresponding way, such conditionalization with respect to ranks offers a full dynamics of belief or acceptance.[10]

This informal hint at belief revision may suffice. However, we should formally introduce *belief contraction* because Lehrer makes explicit use of it in what he calls the ultrasystem. Belief contraction is the operation of giving up some belief without adding new ones. It is extensively discussed in AGM belief revision theory because it is interchangeable with belief revision.[11] Within ranking theory it is easily defined as well (and turns out then to have all the properties described in AGM theory[12]):

Definition 3: The *contraction* $\kappa - A$ of κ by A is defined by $\kappa - A = \kappa$, if $\kappa(\overline{A}) = 0$. If not, it is defined by $(\kappa - A)(B) = \kappa(B)$ for $B \subseteq A$ and $(\kappa - A)(B) = \kappa(B) - \kappa(\overline{A})$ for $B \subseteq \overline{A}$; for other B the rank may be inferred from these conditions by applying the law of disjunction.

Hence, if A is not believed, anyway, the contraction by A does not have any effect at all. And if A is believed, i.e., $\overline{A}$ is disbelieved, then again $(\kappa - A)(\overline{A}) = 0$, i.e., $\overline{A}$ is no longer disbelieved after the contraction. However, the ranks conditional on A and on $\overline{A}$ are unaffected by the contraction.

So much for ranking theory as such. Let us now apply it to Lehrer's epistemology.

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9 Cf. Jeffrey (1965, ch. 11).
10 Cf. Spohn (1988, sect. 5).
11 Cf. Gärdenfors (1988, ch. 3).
12 Cf. Spohn (1988, p. 133, footnote 20).

3. Lehrer's account of justification and knowledge in ranking terms

Conditional ranks do not only allow us to account for (iterated) belief revision and contraction. They also offer what we are directly aiming at, namely an explanation of Lehrer's preference system, i.e., of his primitive four-place relation $\succ$:

Definition 4: *A* is *more reasonable to accept* given *C* than *B* given *D* relative to κ, i.e., $A|C \succ B|D$, iff $\kappa(\overline{A}|C) > \kappa(\overline{B}|D)$. Moreover, *A* is more reasonable to accept than *B*, $A \succ B$, iff $A|W \succ B|W$, i.e., $\kappa(\overline{A}) > \kappa(\overline{B})$. Similar notions are defined correspondingly.

Hence, a ranking function κ comprises all components of what Lehrer calls an evaluation system: an acceptance system (see Definition 2), a preference system (see Definition 4), and a reasoning system (as trivialized by me above). One may be tempted to say that Lehrer's account is wedded to ranking theory. The wedding is unhappy, however. On the basis of Definition 4, Lehrer's account of justification is translated in a straightforward way, and the fatal results are inescapable:

Definition 5 (= D4, p. 170): *B is an objection to A* iff $A|\overline{B} \succ A|B$.

Is being an objection a relation among accepted propositions or among propositions in general? The latter according to Definition 5, though Lehrer may intend the former. However, there is no issue here, since we shall have to stipulate that justified acceptance entails acceptance and shall thus consider only objections to accepted propositions. Otherwise, we would get nonsensical results.

Definition 6 (= D5, p. 170): The *objection B to A is answered* iff *B* is an objection to *A*, i.e., $A|\overline{B} \succ A|B$, *and* $A \succ B$.

Definition 7 (= D6, p. 170): *C neutralizes the objection B to A* iff *B* is an objection to *A*, i.e., $A|\overline{B} \succ A|B$, $B \cap C$ is not an objection to *A*, i.e, $A|\overline{B} \cup \overline{C} \preceq A|B \cap C$, and $B \cap C$ is at least as acceptable as *B*, i.e., $B \cap C \succeq B$.

Indeed, it is this last condition that prevents a probabilistic interpretation of Lehrer's preference system because it would be empty in this interpretation; no objection could then be neutralized. In ranking terms, however, this consequence need not be feared.

Thus, we arrive at

Definition 8 (= D3 and D7, pp. 170 f.): A is *justifiedly accepted*, or the acceptance of A is *personally justified*, relative to κ in the strong sense iff A is accepted and each objection to A is answered or neutralized by some C.

This is the literal translation of Lehrer's definition. My qualification "in the strong sense" indicates, however, that we shall have to consider a weaker sense as well.

Unfortunately, this chain of definitions yields strange results. What is justifiedly accepted according to Definition 8 is an unintelligible selection from the accepted propositions. In order to define this selection let $E_m = \bigcup\{D \mid \kappa(D) \geq m\}$; hence, E_m is the logically weakest proposition having at least rank m. Then we have:

Theorem 1: A is justifiedly accepted relative to κ in the strong sense iff, given $\kappa(\overline{A}) = n > 0$, for all $m \geq n$ with $E_m - E_{m+1} \neq \emptyset$ $\overline{A} \cap E_m - E_{n+m+1} \neq \emptyset$ holds true (or, equivalently, iff for all $m \geq n$ with $\kappa(E_m) = m$ there is a $D \subseteq \overline{A}$ with $m \leq \kappa(D) \leq n+m$).

Proof: Suppose there is an m for which this condition does not hold. Clearly, $m = n$ cannot be the exception. Hence, $m > n$. Now take any B such that $\kappa(\overline{B}) = m$. Since $\overline{A} \cap E_m - E_{n+m+1} = \emptyset$, $\kappa(\overline{A} \cap \overline{B}) \geq n+m+1$. In any case $\kappa(\overline{A} \cap B) = n$. So we have $\kappa(\overline{A} \mid \overline{B}) \geq n+1 > \kappa(\overline{A} \mid B)$, i.e., B is an unanswered objection to A. How could a C neutralize this objection? It should satisfy $\kappa(\overline{B} \cup \overline{C}) = m$ and thus $\kappa(\overline{C}) \geq m$. Hence, still $\kappa(\overline{A} \cap (\overline{B} \cup \overline{C})) \geq n+m+1$ and $\kappa(\overline{A} \cap B \cap C) = n$, and so $\kappa(\overline{A} \mid \overline{B} \cup \overline{C}) > \kappa(\overline{A} \mid B \cap C)$. Thus, there can be no neutralizer, and A is not justifiedly accepted relative to κ.

Suppose conversely that the condition holds true for all $m \geq n$. This unfolds into three cases:

First, it may be that $\kappa(\overline{A}) = \infty$, i.e., $A = W$. Since for all $m \geq \infty$ $E_m = \emptyset$, the condition is satisfied. However, in this case there can be no objection to A, and so A is justifiedly accepted.

Second, it may be that $\kappa(\overline{A}) = n < \infty$ and indeed $\overline{A} = E_n$. Suppose B is an objection to A. This requires at least $\kappa(\overline{A} \mid \overline{B}) > 0$. Suppose further that B is not answered. This requires $\kappa(\overline{A}) \leq \kappa(\overline{B})$. But since $\overline{A} = E_n$, this entails $\overline{B} \subseteq \overline{A}$ and thus $\kappa(\overline{A} \mid \overline{B}) = 0$. Contradiction. Hence, there is no unanswered objection to A, and A is justifiedly accepted.

Third, it may be that $\kappa(\overline{A}) = n < \infty$ and $\overline{A} \subset E_n$. Then there is a B with $\kappa(\overline{B}) = m$ and $\overline{B} \subseteq (E_n - \overline{A}) \cup E_{n+m+1}$, and exactly those B are unanswered objections to A, since exactly those B satisfy both, $m \geq n$, i.e., $\kappa(\overline{B}) \geq \kappa(\overline{A})$, and $\kappa(\overline{A} \cap \overline{B}) \geq n+m+1$, which is tantamount to $\kappa(\overline{A} \mid \overline{B}) > \kappa(\overline{A} \mid B)$, since $\kappa(\overline{A} \mid B) = n$. But we have supposed that $\overline{A} \cap E_m - E_{n+m+1} \neq \emptyset$. This secures that B can be neutralized. Take any C such that $\overline{A} \cap E_m - E_{n+m+1} \subseteq \overline{C} \subseteq E_m$. Hence $m \leq \kappa(\overline{C}) < n+m+1$. So, $\kappa(\overline{B} \cup \overline{C}) = m = \kappa(\overline{B})$, and $\kappa(\overline{A} \cap (\overline{B} \cup \overline{C})) < n+m+1$, and hence $\kappa(\overline{A} \mid \overline{B} \cup \overline{C}) \leq n = \kappa(\overline{A} \mid B \cap C)$. Therefore, any unanswered objection to A can be neutralized in this case, and, again, A is justifiedly accepted.

Now, we can see why we had to restrict justified acceptance to accepted propositions in Definition 8. If we had omitted this proviso, the proof of Theorem 1 would have to consider the case where $\kappa(\overline{A}) = 0$. However, for $n = 0$ every bit of the proof would go through in the same way. This would allow for many justifiedly accepted propositions not previously accepted. Indeed, even $\emptyset$, the logically false proposition, would turn out as justifiedly accepted! Simply because there can be no objections to $\emptyset$ according to Definition 5; $\emptyset$ is maximally disbelieved under all conditions. So, this had to be avoided.

Still, Theorem 1 is awkward. I have tried to express the necessary and sufficient condition for justified acceptance as perspicuously as possible (but I cannot circumvent the mathematical facts). However, this condition just makes no intuitive sense. If Lehrer's definitions force us to distinguish between justifiedly and unjustifiedly accepted propositions in *this* way, then there is something wrong with the definitions.

Indeed, there may be cause for suspicion. For instance, objections may be restricted to inductive objections, where B is an inductive objection to A iff $\kappa(\overline{A}|B) < \kappa(\overline{A}|\overline{B}) < \infty$. Or I was wondering whether Lehrer really meant Definition 7 as stated. Perhaps, the idea of neutralization is better expressed by the condition that *given* the neutralizer C the objection B to A is no longer an objection to A (i.e., $A|\overline{B}\cap C \preceq A|B\cap C$). However, as far as I have checked, this leads to nowhere. Theorem 1 thereby changes considerably, but does not improve.

No, a better cure is revealed by noticing that Theorem 1 implies that justified acceptance is not even deductively closed: if A is justifiedly accepted and logically implies B, B still need not be justifiedly accepted. This seems to be a flaw in Definition 8 as it stands. But then, of course, we have to correct Definition 8. Thinking about what one is personally justified to accept means also working through one's reasoning system. Given my fixation of the reasoning system, this leads us to the following weaker and more adequate sense of justified acceptance:

Definition 9: A is justifiedly accepted relative to κ in the weak sense iff A is logically implied by propositions justifiedly accepted in the strong sense.[13]

What is justifiedly accepted in this sense? This is answered by

Theorem 2: A is justifiedly accepted relative to κ in the weak sense iff A is accepted by κ.

Proof: The proof of Theorem 1 shows that $\overline{E}_1$ is justifiedly accepted. If A is accepted by κ, i.e., $\kappa(\overline{A}) > 0$, then $\overline{A} \subseteq E_1$, i.e., $\overline{E}_1 \subseteq A$. Thus A is logically implied by something justifiedly accepted.

In a sense, this looks much nicer, but it empties Lehrer's theory of justification. So, it looks undesirable as well. Theorem 2 would not change under the modifications of Lehrer's definitions mentioned above.

[13] Lehrer (1971, p.221) took recourse to the same move when observing that the rule of induction he proposed is not deductively closed.

These results also affect Lehrer's theory of knowledge. In order to see how we must first turn to undefeated justification and the ultrasystem. A person's ultrasystem is generated from her evaluation system by deleting from the latter all acceptances of, preferences for, and reasonings from falsehoods. This is immediately explicable in ranking terms.

Let $w^* \in W$ be the true or actual possibility in W, and let us consider any evaluation system, i.e., ranking function κ. If everything accepted in κ is true, then no falsehoods intrude into the justifications with respect to κ, anyway. In this case, κ is its own ultrasystem. That's the rare case, though. Usually, some false proposition will be believed in κ. This entails $\kappa(\{w^*\}) > 0$.

Hence, the logically weakest falsehood accepted by κ is $\overline{\{w^*\}}$. Now, if we contract κ by $\overline{\{w^*\}}$, not only this falsehood, but also all other, stronger falsehoods must go (since the contracted acceptance system is again deductively closed). Hence, in the resulting ranking function κ* exactly the true among the propositions accepted by κ are accepted. Moreover, if A is true and B is false, then $\kappa^*(A) = 0 \leq \kappa^*(B)$, and hence in the preference system provided by κ* no false proposition is ever preferred to a true one, as required by Lehrer in D8, p. 171. Finally, we do not worry about the reasoning system of κ*, for the familiar reason. All in all, the explication is remarkably smooth, and we may conclude with

Definition 10 (= D8, p. 171): The ultrafunction κ* of κ is $\kappa - \overline{\{w^*\}}$, the contraction of κ by $\overline{\{w^*\}}$.

This covers also the case where no falsehood is accepted in κ, i.e., where κ* = κ.

On this basis, the rest of Lehrer's definitions is immediately translated:

Definition 11 (= D9, p. 171): The justification for accepting A in κ is *undefeated* (or *irrefutable*) iff A is justifiedly accepted in κ*.

Definition 12 (= DK, pp. 169f.): *A is known* in κ iff (i) A is accepted in κ, (ii) A is true, i.e., $w^* \in A$, (iii) A is justifiedly accepted in κ, and (iv) the justification for accepting A in κ is undefeated.

Lehrer's claim that knowledge reduces to undefeatedly justified acceptance, i.e., to condition (iv) is now easily confirmed. But stronger results obtain. If knowledge is based on justified acceptance in the strong sense, just those propositions are known which satisfy the condition of Theorem 1 relative to κ*. Unpalatable knowledge! If knowledge is based on justified acceptance in the more adequate weak sense, we get

Theorem 3: *A* is known in κ iff *A* is true and *A* is accepted in κ.

Proof: Theorem 2 reduces condition (iii) to (i). But, Theorem 2 applies also to κ*. Hence, (iv) reduces to acceptance in κ*, and hence to (i) and (ii).

Thus, again, the sophisticated considerations relating to the ultrasystem seem empty, and knowledge reduces to true belief, a conclusion Lehrer definitely wants to avoid.

4. What to conclude?

It is not clear what to think of these results. I find that the structure of what I called justified acceptance in the strong sense is too weird to be worth discussing. Hence, I proceed on the assumption that it is the weak sense that is relevant. I have already indicated why I believe to agree with Lehrer on this point. But Theorems 2 and 3 look troublesome as well, though it is not clear where to locate the trouble. There are several ways, and more than one good way, to respond.

(1) One may point to various flaws in my translation. For instance, I have carelessly interchanged belief and acceptance, whereas Lehrer (pp. 12f.) emphasizes their difference. Likewise, I have defined an acceptance system as a set of accepted propositions, whereas for Lehrer it is a set of propositions of the form "I accept that *A*". But these kinds of flaws are insignificant. Also, I have neglected Lehrer's own foundations in terms of trustworthiness (pp.138ff.), but taking them into account would have no effect on the present considerations.

However, my fixation of Lehrer's reasoning system by rightaway assuming deductive closure is doubtlessly a major deviation from Lehrer. But, again, I don't think it does any harm. The acceptance system to start with could as well consist in an arbitrary, not deductively closed set of accepted statements, as long as it is consistent. Working out what is justified on the basis of such an acceptance system means working out the preference system, which is a more permanent disposition of the epistemic subject extending to all statements. And it means, as we have observed above, working out the reasoning system, i.e., accepting the logical consequences of what one has justifiedly accepted. Hence, we are back at deductive closure, at least as far as justified acceptance is concerned. Theorem 2 then says that the deductive closure of an acceptance system is justifiedly accepted relative to this system, and this is equally troublesome.

No, Theorems 1 and 2 are generated by the characteristics of the preference system as specified by a ranking function. The trouble lies there and not in my light way of dealing with deductive relations.

(2) This suggests another conclusion. If one proceeds from preference systems generated by ranking functions and arrives at such undesired results, then the two theories do not fit together, and one has to look for other preference systems. After all, nobody claims that ranking theory delivers the only legitimate kind of preference systems. So, the conclusion would be that ranking theory may have useful applications, but Lehrer's account of justification does not belong to them.

(3) However, I tend to the reverse conclusion. The more offers for an underpinning of Lehrer's account are rejected, the stronger the obligation to come up with some sound theoretical foundation. As for my part, I doubt that there is any better offer than the one made here. In any case, as long as the foundation is missing, there is not really any theory of justification and knowledge.

Perhaps this quest for a theoretical underpinning is too strong, though. Concerning the acceptance system the picture rather seems to be that it collects all the variegated items of information and inference, with the aim of truth, but without critical standards. Any arbitrary set of statements may be formed in this way. Then, it seems, there can be no theory about acceptance systems, and asking for one is asking too much.

This attitude, however, does not carry over to the preference system. We need not entertain the illusion that it is uniquely determined by rationality alone. Carnap was under this illusion with inductive logic, but soon woke up. The preference system and hence the standards of justification may well be subjective to some extent. However, this is not to say that anything goes. This would mean anarchy and throwing away the idea of rationality altogether. Some rationality standards should be set up and defended. This is just what ranking theory attempts to do, though perhaps in a debatable way. In any case, one cannot simply be silent on the structure of preference systems. Hence, if one thinks that the theorems above are undesirable, then, I think, Lehrer's account of justification is really in trouble.

(4) Perhaps, though, one need not think that the theorems are undesirable. When comparing ranking theory with Pollock's defeasible reasoning in Spohn (2002) I concluded, among other things, that ranking functions are best compared with what Pollock calls ideal warrant, which is, so to speak, the end product of his defeasible reasoning machinery.[14] Though Pollock's theory is quite different from Lehrer's, this suggests that acceptance according to ranking functions is already justified acceptance. This suggestion is supported by the fact that ranking acceptance cannot yield any arbitrary acceptance system, but is deductively closed, due to a correctly and maximally executed reasoning system. In this perspective, then, Theorem 2 would not at all be surprising, it should be expected. Accordingly, ranking theory and Lehrer's account of justification may indeed be seen as mutually supporting each other, since Theorem 2 shows that they reach the same result via entirely different considerations.

Too much harmony? Yes, I think so. First, the talk of ideal warrant is Pollock's, and it was helpful in the above-mentioned comparison. But there is nothing in ranking theory by itself forcing this comparison. Ranking theory is, as I prefer to say more neutrally, about rational belief and its dynamics. So, Theorem 2 rather shows either that ranking theory *is* about ideal warrant, despite my disclaimer, or that ideal warrant reduces to rational belief. This would be my preferred conclusion, but it reopens, it seems, a difference to Lehrer.

Secondly, even if Theorem 2 lends support to Lehrer's account of justification, it does so in an unfriendly way, because it renders it vacuous at the same time. The so-

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[14] Cf. Pollock (1995, sect. 3.10).

phisticated considerations about answering and neutralizing objections do not do any real work. This holds also when one starts, as Lehrer does, from an arbitrary, unorderly acceptance system, because it is simply its deductive closure that is justified relative to it.[15] This vacuity is certainly against Lehrer's intentions.

Thirdly, there remains a problem with Theorem 3. If, as stated above, ranking functions represent rational belief and nothing stronger, then the reduction of knowledge to true belief as represented by ranking functions appears doubtful, even though it has been defended by von Kutschera (1982, sect. 1.3), and Sartwell (1991, 1992). In any case, it is unacceptable to Lehrer. At this point, hence, harmony ceases at the latest.

So, as I said, it is not really clear what to conclude, though (3) would be my preferred conclusion. The case shows once more how difficult it often is to square different epistemological approaches.

However, there is, I think, a general lesson to learn. Justification is a central notion in epistemology, and hence it is rightly scrutinized in many discussions, from many perspectives, and with many examples and arguments. In all that literature, though, I find very little rigorous theory. However, the amenability to rigorous theorizing provides an important test. This test is usually not even sought.[16] But it is useful as a critical and as a constructive authority. At least I hope to have shown this here with respect to the paradigm of Lehrer's epistemology.

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[15] If the initial acceptance system should be inconsistent, the theoretical situation changes drastically. Then paraconsistent logic may help, or the theory of consolidation (cf. Hansson 1994 and Olsson 1998b), or whatever. But there is no evidence in Lehrer's writings that *this* is his problem.

[16] For instance, it springs to one's eyes that at least three of the five conditions BonJour (1985, pp. 95-99) offers for coherence are theoretically hardly explicable.

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