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Electrons on Liquid Helium in a Resonator

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An investigation of the microwave absorption for 2–dimensional electron layers in a resonator cavity are presented. The difference in the eigenmodes of the resonator in case of an empty cavity and in presence of a 2-dimensional electron layer on a helium film within the cavity are calculated. When introducing electrons into the cavity a pronounced frequency dependence is found. The expected shift in the resonance frequency is compared to previous and new data of resonance response measurements.

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1. INTRODUCTION

Two-dimensional electron systems (2DES), localized on the surface of liquid helium, have been studied widely by resonance absorption techniques. Resonance data are quite important for the investigation of the dynamics of a 2DES. Using this technique provided the first demonstration of 2D behaviour for electrons on helium [1], the essential details of the momentum time relaxation [1–8], the heating phenomena [9–11], the resonance frequency shift measurements [3,12] etc.

The interpretation of the existing experimental data is as simple as possible from the setup contribution point of view. For e.g., in all absorption estimations either the 'half width over half peak' rule is used or the conventional transmission line description with an additional assumption with respect to the lorentzian line shape and the weak coupling between the 2DES and the resonator degrees of freedom. So far the most elaborate method of fitting activity is an utilization of the 2D free electron equations of motion (classical or quantum) to calculate both the real and imaginary parts of the electron currents in an uniform electric field, $E_{\|}$ with frequency ω, [13,14]

$$\mathrm{Re}j_x(\omega,\omega_c) = \frac{n_s e\tau E_{\|}}{m_e}\frac{(1+\omega^2\tau^2+\omega_c^2\tau^2)}{(1-\omega^2\tau^2+\omega_c^2\tau^2)^2+4\omega^2\tau^2} \quad (1)$$

$$\mathrm{Im}j_x(\omega,\omega_c) = -(\omega\tau)\frac{n_s e\tau E_{\|}}{m_e}\frac{(1+\omega^2\tau^2-\omega_c^2\tau^2)}{(1-\omega^2\tau^2+\omega_c^2\tau^2)^2+4\omega^2\tau^2}. \quad (2)$$

Here τ is the elastic time relaxation, n_s the free electron density, m_e the electron mass, and ω_c the cyclotron frequency.

Such a way shows that an absorption line shape is not necessarily lorentzian. Eqs. (1–2) are also useful to demonstrate that in some cases the 'half width over half peak' rule is too crude and so the real absorption line shape fitting has to be realized [14]. Nevertheless, even then Eqs. (1–2) are not enough to fully describe the electron behaviour in the cavity. Especially the question comes about the shift of the resonance frequency which follows from Eqs. (1–2) and which reflects the real and imaginary contributions only of the 2D conductivity.

The main goal of the presented calculations is to give an accurate formulation in some reliable configurations of the self-consistent eigenmode problem for 2D electrons in a resonator. The investigation of this problem helps to understand how realistic the existing fitting simplifications are, both in absorption and frequency shift measurements. This problem is also interesting as a good example of the eigenmodes behaviour of two resonators in the presence of a flexible coupling between them.

2. THEORETICAL DESCRIPTION

First we consider the resonator cavity as simple as possible, yet with geometrical properties similar to experiments [3] (and similar conditions are presented in Refs. [4–8]). The resonator has two infinite parallel metallic plates with the coordinates: $z=0$, $z=h$. The bottom of the resonator is covered by a liquid helium film with thickness d. 2D electrons with density n_s are distributed along this film. For the investigations here we do not apply any external magnetic field.

The E-modes with $E_z=0$, $E_y=0$, $E_x(z)\neq 0$ and the conditions at the walls

$$E_x(0)=0, \quad E_x(h)=0 \quad (3)$$

have the structure

$$E_x(z)=E_o\sin(\pi z/h), \quad \omega_o/c=\pi/h \quad (4)$$

where c is the speed of light and ω_o the lowest eigenfrequency. Without electrons ω_o in such a resonator is proportional to h^{-1}.

In the presence of 2DES along the helium film we have to equate (in addition to Eq. (3)) the electric fields in the vacuum and the helium film. Also the ac-magnetic field becomes important. In particular one has to take the jump of this field, caused by the electrons along the helium film, and so

$$H_y^{(vac)}(d) - H_y^{(He)}(d) = 4\pi j_x(d)/c \tag{5}$$

where $H_y^{(vac)}$ and $H_y^{(He)}$ are the resulting magnetic components of the applied electric field within the vacuum and the helium film, respectively. Then

$$j_x(d) = \sigma_{xx} E_x(d), \quad \sigma_{xx} = \frac{n_s e^2 \tau}{m_e^*(i\omega\tau + 1)}\,, \tag{6}$$

where σ_{xx} is the diagonal conductivity of the 2D system and m_e^* the electron effective mass, where for electrons on liquid helium $m_e^* \approx m_e$. The solution of the corresponding eigenmode problem leads to

$$\tan(kh) - i\sigma \sin(kd)[\sin(kd) - \cos(kd)\tan(kh)] = 0, \tag{7}$$

with $\sigma = \frac{4\pi\sigma_{xx}}{c}$ and $k = \frac{\omega}{c}$ where σ is the important coupling constant, which reflects the coupling between the electromagnetic and the free electron motion degrees of freedom. Some estimations about σ: say $n_s \approx 10^8 \mathrm{cm}^{-2}$ and $\tau \approx 10^{-7}$s then $\sigma = \frac{4\pi}{c}\frac{n_s e^2 \tau}{m_e} \approx 1$. Here the coupling is about one and so is not an exotic value. Variations in n_s, ω, and τ can make σ quite flexible and cause crossing '1' in both directions.

The general properties of Eq. (7) are quite evident. In the limit, $\sigma \to 0$, condition (7) is reduced to

$$\tan(k_o h) \simeq 0, \quad \text{with} \quad k_o = \pi/h. \tag{8}$$

This result coincides with ω_o from Eq. (4). In the opposite limit, $\sigma \to \infty$, condition (7) becomes $\sin[(k_1(h-d)] \simeq 0$ or $\sin(k_2 d) = 0$. So

$$k_1 = \pi/(h-d) \quad \text{or} \quad k_2 = \pi/d. \tag{9}$$

The results show, that when $\sigma \to \infty$ the charged helium surface is equivalent to the bottom (or top) of two metallic resonators, and we just have the change of the vertical size in the resonantor. One of this resonators has the height $(h-d)$, the other one d.

Most interesting, of course, is the intermediate situation, i.e. when $0 < \sigma < \infty$. In this case the correction of the eigenmode has both real (frequency shift) and imaginary (line broadening) contributions. For e.g.

$$\sigma \ll 1, \quad \text{with} \quad \sigma = \sigma' + i\sigma'' \tag{10}$$

where

$$\sigma' = \frac{4\pi}{c}\,\frac{n_s e^2 \tau^2 \omega}{m_e(\omega^2\tau^2+1)} < 1 \quad \text{and} \quad \sigma'' = -\frac{4\pi}{c}\,\frac{n_s e^2 \tau}{m_e(\omega^2\tau^2+1)} = -\omega\tau\sigma' < 0$$

and

$$\omega = \omega_o + \delta\omega, \quad \delta\omega = \delta\omega' + i\delta\omega'' \tag{11}$$

where ω_o is from (3) and $\delta\omega'$ and $\delta\omega''$ are the real and imaginary part, respectively. Then in this case we have from (8)

$$\frac{\delta\omega'}{c}h = -\sigma''(\omega_o)\sin^2(k_o d) \quad \text{and} \quad \frac{\delta\omega''}{c}h = \sigma'(\omega_o)\sin^2(k_o d) \tag{12}$$

and σ' and σ'' are from Eq. (10). Both perturbations are linear functions of n_s. In the limit $\omega\tau \gg 1$ the ω shift $\delta\omega'$ is not sensitive to τ and is positive (as it was indicated above). It is also evident that the shift $\delta\omega'$ (12) is not only controlled by the 2D conductivity (as indicated from Eqs. (1–2)) but also by the parameters of the cavity.

In the opposite limit $\sigma \gg 1$ the perturbation presentation exists near the eigenmode k_1 (8) or k_2 (9). So for k_1 from Eq. (7) we get

$$\sin[k(h-d)] = \frac{i\sin(kh)}{\sigma\sin(kd)}\,. \tag{13}$$

If again $k = k_1 + \delta k_1$ with $k_1\,(h-d) = \pi$ then from Eq. (13) it follows

$$\delta k_1'(h-d) = \frac{\sigma''(\omega_1)}{(\sigma')^2+(\sigma'')^2} < 0 \quad \text{and} \quad \delta k_1''(h-d) = \frac{\sigma'(\omega_1)}{(\sigma')^2+(\sigma'')^2}\,. \tag{14}$$

The behaviour $\delta k_1', \delta k_1'' \propto 1/n_s$ from (14) does not exist in Eqs. (1–2), and this is the main reason for the special investigation of the electron motion in a resonator. Nevertheless, description (1) is useful at least in the limiting cases $\sigma \ll 1$ or $\sigma \gg 1$. In these limits the ratio

$$\delta\omega'/\delta\omega'' = -\omega\tau \tag{15}$$

has the same structure as $(\mathrm{Re}j_x/\mathrm{Im}j_x)_{\omega_c\to 0}$ from Eqs. (1–2).

3. EXPERIMENTAL OBSERVATIONS

To describe the experiment the developed model is modified, for details see Fig. 1. Using the same approach as above we have the following eigenmode equation:

$$\tan(qd) - \frac{\sin(kd)+\cos(kd)\cot(kh)}{\cos(kd)-\sin(kd)\cot(kh)} = i\frac{4\pi}{c}\sigma_{xx} \tag{16}$$

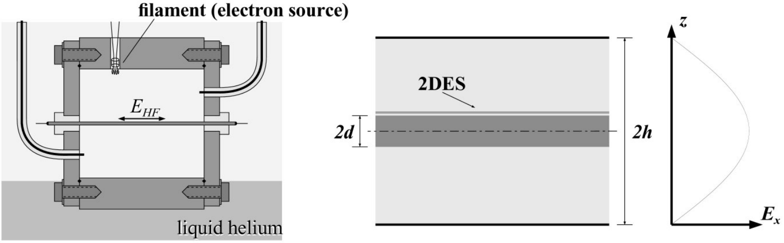

Fig. 1: *Shown is on the left a schematic picture of the cavity, which is used in the experiments. On the right is the presented simplified model, this time with cylindrical symmetry around the cavities axis in x-direction.*

where $k = \omega/c$, $q^2 = \varepsilon\, \omega^2/c^2$, and $q > k$. Here ε is the dielectric constant of the strip with thickness $2d$. The thickness of the helium film along this strip is not important for the calculation of the eigenmode as long as $d_{He} \ll d$. So the 2DES is distributed just along the dielectric strip.

Some details of the eigenmode behaviour, Eq. (16): in the empty cavity, when $d \to 0$ and $\sigma_{xx} \to 0$ the eigenmode k_o follows from the condition

$$\cot(k_o h) = 0 \quad \text{or} \quad k_o h = \pi/2 \, . \tag{17}$$

The dielectric ($\varepsilon > 1, d > 0$) and conductivity ($\sigma_{xx} \neq 0$) contributions in the behaviour of the eigenmode with a comparison to experimental data are presented briefly in Fig. 2, using the limit $\tau \to \infty$ and dimensionless values:

$$\omega^* = \frac{\omega}{\omega_o} \, , \quad \sigma = \frac{4\pi n_s e^2 h}{m_e c^2} \, , \quad \delta = \frac{d}{h} \, .$$

So we get from Eq. (17) line 1 (see Fig. 2): $\omega^*(\sigma)$ for $\delta = 0$. ω^* grows from the vacuum value (18) to $2\omega_o$ if $\sigma \gg 1$. Lines 2, 3, and 4 correspond to an increasing dielectric constant ε, respectively.

This shows the change of the lowest eigenmode as function of the dielectric contribution and an interesting saturation effect as function of σ (the increase of the eigenmode for the cases 2 to 4 in Fig. 2 is limited by some frequencies $\omega_\delta < \omega_o$, sensitive to ε).

4. CONCLUSIONS

We have calculated the influence of a dielectric substrate and an additional 2DES in a resonator. The results are supported by experimental

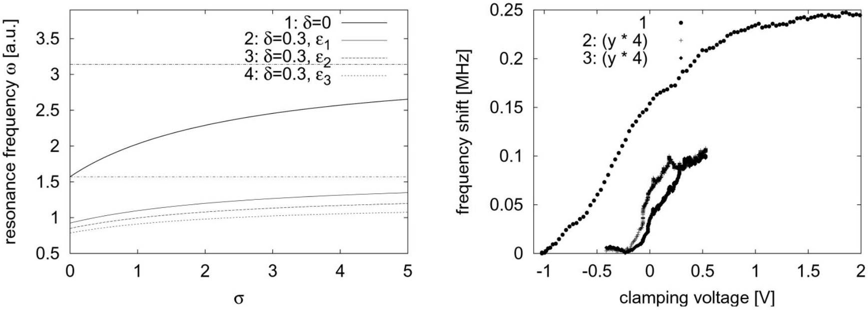

Fig. 2: *Shown on the left is the dependence of ω versus σ from Eq. (16). The frequency goes down with increasing δ and dielectric constant ε of the substrate. On the right some experimental data of the frequency shift $\delta\omega$ versus the clamping voltage $U_{\rm clamp} \propto n_e \propto \sigma$ are shown. The three datalines are measured on $d_{\rm He} \approx$ 400 Å on differently prepared Si-substrates.*

data. So it is possible to explain at least part of the behaviour of the frequency shift in microwave measurements of 2DES due to intrinsic effects of the setup. With this model it will be easier to give a better interpretation of forthcoming experiments.

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