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Full Counting Statistics of a Superconductor/Ferromagnet Entangler

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We consider the production of spatially separated entangled electrons in transport between a superconductor and ferromagnets: Spin singlet Cooper pairs are filtered into different leads by spin dependent tunneling. We find the full counting statistics depending on the magnetizations and identify the probabilities for productions of entangled electrons taking into account the superconducting proximity effect. Positive contribution to cross correlations are due to coherent transport of two electrons from the superconductor and negative contribution caused by Fermi statistics. We finally show the conditions for a violation of a Bell inequality as experimental test of entanglement.

A solid state entangler is an electronic analog of the optical setups used for experimental Bell inequality tests, quantum cryptography and quantum teleportation [1]. Ideally, such a device should produce separated currents of entangled electrons. Superconductors are suitable candidates as sources in solid state entanglers since Cooper pairs consitute spin-entangled states. This prospect has resulted in several papers addressing the properties of hybrid superconductor and normal metal systems [2]. One of the challenges is to prevent processes where pairs of entangled particles reach the same lead, i.e. are not spatially separated. Separation of entangled particles into different leads using ferromagnets or quantum dots has been suggested [3]. Here we consider separation by ferromagnets. Electrons from Cooper pairs are entangled in spin and energy space. Upon filtering, the spin part of the two-particle wave function collapses and energy entanglement remains [4].

Solid state entanglers have been analyzed in Refs. [2, 3, 4] in terms of currents, noise and cross correlations. We will consider the full counting statistics (FCS) which encompasses these properties and provides the complete statistics of charge transfer [5, 6]. The FCS determines the probability that a given number of charges are transferred in a time interval. Fluctuations are due to quantum-mechanical uncertainty and statistics. The first two moments of the FCS are the average current and the current noise. Schemes to measure the third and higher moments are currently developed [7]. FCS for several superconductor/normal metal (S/N) [8, 9] and also normal metal/ferromagnet (N/F) [10] hybrid structures have been calculated. Studies of noise [11] and FCS [12] for a beam of entangled electrons show that entanglement gives qualitatively different noise characteristics as compared to transport of non-entangled electrons.

In this Letter, we calculate the FCS of the superconductor/ferromagnet (S/F) system shown in Fig. 1. The superconducting proximity effect and spin-active ferromagnetic interfaces are taken into account. A supercurrent flowing from S is converted into an electron-hole quasiparticle current by Andreev reflection, and subsequently separated into ferromagnetic leads according to the spin of the quasiparticles by means of spin active interfaces. We calculate the current of entangled particles into separated leads which is enhanced by spin dependent tunneling in the antiparall magnetization configuration. We then study the noise, cross correlation and higher moment noise dependence on magnetization alignment and identify the origin of the positive and negative cross correlations. Crossed Andreev reflections, where simultaneously transferred electrons and holes go to separate leads, give positive contribution. Negative contribution results from a separation of the current according to the Pauli exclusion principle if Andreev reflection events are correlated. With ferromagnetic leads as spin detectors we show that a Bell inequality can be violated as test of entanglement.

We utilize the circuit theory of mesoscopic superconductivity [13]; a finite-element technique for semiclassical Green's functions. This formalism was generalized in Refs. [14, 15] to account for spin-active interfaces which arise at contacts to ferromagnetic leads in superconducting heterostructures. Recent applications in Ref. [16] study the interplay between ferromagnetism and superconductivity using this formalism. Circuit theory is formulated in terms of matrix currents with arbitrary structure and allows to derive the FCS by introducing counting fields in these currents [8]. Taking into account matrix currents through spin active interfaces, we calculate the FCS for the three terminal "beam splitter" in Fig. 1(a). This generic structure could be fabricated by metals or semiconductors. The source of entangled electrons is a singlet superconducting terminal. The drains (F_1 and F_2) are ferromagnetic terminals at bias voltage V. Terminals are described in circuit theory by equilibrium quasiclassical Green's function matrices $\check{G}_i$ determined by electrochemical potential and temperature. Our notation for matrix subspace is: $\hat{}$ for spin, $\bar{}$ for Nambu and $\check{}$ for Keldysh, and Pauli matrices are denoted τ_i. At zero temperature $\check{G}_{1(2)}$ is $\bar{\tau}_3 + (\check{\tau}_1 + i\check{\tau}_2)$ for $|E| \leq eV$ and $\bar{\tau}_3 + \mathrm{sgn}(E)\bar{\tau}_3(\check{\tau}_1 + i\check{\tau}_2)$ for $|E| > eV$. The Green's function of S is $\check{G}_\mathrm{S} = \bar{\tau}_1$ where we assume $E \ll \Delta$, Δ being

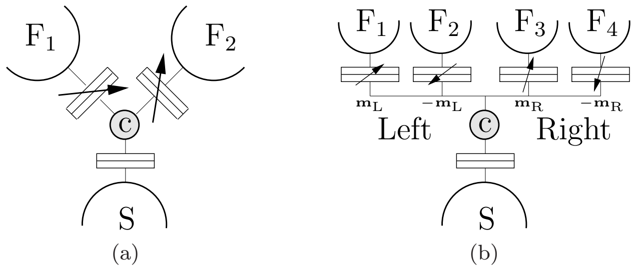

FIG. 1: a) Beamsplitter: Cavity c connects terminals F_1, F_2 and S. Magnetization of spin active tunnel barriers is depicted by arrows. b) Bell test system.

the gap of S and E the quasiparticle energy. To calculate the FCS we introduce counting fields by the transformation

$$\check{G}_i \to \mathrm{e}^{i\chi_i \check{\tau}_\mathrm{K}} \check{G}_i \mathrm{e}^{-i\chi_i \check{\tau}_\mathrm{K}} \text{ for } i\text{=1,2,S}; \; \check{\tau}_\mathrm{K} = \bar{\tau}_3 \bar{\tau}_1. \tag{1}$$

The three terminals are connected to a cavity c described by the Green's function $\check{G}_\mathrm{c}$, assumed isotropic due to chaotic or diffusive scattering. We assume that c is large enough so that charging effects can be neglected, and small enough so that $\check{G}_\mathrm{c}$ is spatially homogeneous.

The connectors can be spin active due to spin dependent transmission and reflection amplitudes $t^i_{n,\sigma}$ and $r^i_{n,\sigma}$ for particles incident on the interface i from the cavity side in channel n with spin σ. The matrix current $\check{I}_i$ through a spin active tunnel barrier between c and reservoir i evaluated at the cavity side is [14, 15]

$$\check{I}_i = \big[\frac{g_i}{2}\check{G}_i + \frac{g_{\mathrm{MR}i}}{4}\left\{\mathbf{m}_i \hat{\boldsymbol{\tau}}\bar{\tau}_3, \check{G}_i\right\} + i\frac{g_{\phi i}}{2}\mathbf{m}_i \hat{\boldsymbol{\tau}}\bar{\tau}_3, \check{G}_\mathrm{c}\big]. \tag{2}$$

Here, $g_i = g_\mathrm{Q}\sum_{n,\sigma}\left|t^i_{n,\sigma}\right|^2$ is the tunnel conductance where $g_\mathrm{Q} = e^2/h$ is the conductance quantum. The unit vector $\mathbf{m}_i$ points in the direction of the magnetization of the spin polarizing contact, $g_{\mathrm{MR}i} = g_\mathrm{Q}\sum_n(|t^i_{n,\uparrow}|^2 - |t^i_{n,\downarrow}|^2)$, and $g_{\phi i} = 2g_\mathrm{Q}\sum_n \mathrm{Im}\left\{r^i_{n,\uparrow}r^{i*}_{n,\downarrow}\right\}$. The matrix current into S is $\check{I}_\mathrm{S} = g_\mathrm{S}[\check{G}_\mathrm{S}, \check{G}_\mathrm{c}]/2$ [13].

The conductance through a polarizing contact is higher for a particle with spin parallel to the magnetization than for a particle with spin antiparallel to the magnetization. The entangler in Fig. 1(a) therefore separates spins most efficiently into F_1 and F_2 when $\mathbf{m}_1$ and $\mathbf{m}_2$ are antiparallel since electron pairs from S are in spin singlet states, i.e. with opposite spin. The cavity Green's function $\check{G}_\mathrm{c}$ is determined by matrix current conservation $\sum_i \check{I}_i = 0$ where dependence on $g_{\phi i}$ appear as a sum $g_{\phi 1}\mathbf{m}_1 + g_{\phi 2}\mathbf{m}_2$, when using Eq. (2). This means that for similar ferromagnetic contacts in antiparallel alignment ($g_{\phi 1} = g_{\phi 2}$ and $\mathbf{m}_1 = -\mathbf{m}_2$), the dependence on $g_{\phi i}$ cancels and only $g_{\mathrm{MR}i}$-terms give magnetization dependent transport.

The FCS can be quantified by the cumulant-generating function (CGF) $\mathcal{S}$ of the probability distribution for charge transport. It was shown in Ref. [9] that matrix current conservation in an S/N beam splitter allows exact solution for $\mathcal{S}$. Due to the explicit spin space structure of matrix currents and Green's functions in S/F-systems, it is not obvious that the same result applies. We have derived that similar calculations as in Ref. [9] are possible using Eq. (2) to take spin active contacts into account. To the best of our knowledge, this is the first time such a calculation is done. Thus the CGF can be computed $\mathcal{S} = -t_0/(4e^2)\int \mathrm{d}E \sum_n \lambda_n$, where $\{\lambda_n\}$ is the set of eigenvalues of the matrix $\check{M}$ defined by writing matrix current conservation $\sum_i \check{I}_i \equiv [\check{M}, \check{G}_c] = 0$. We now evaluate $\mathcal{S}$ when $g_{\mathrm{MR}i}$ is the relevant contribution from the polarization of the magnetic contacts, which is the case for the entangler geometry considered here. We consider transport in the linear response regime, and $eV \ll \Delta$. Defining $g_\Sigma = [g_\mathrm{S}^2 + (\sum_i g_i)^2 + (\sum_i g_{\mathrm{MR}i}\mathbf{m}_i)^2]^{1/2}$ we obtain

$$\mathcal{S} = -\frac{t_0 V g_\Sigma}{\sqrt{2}e}\sqrt{1 + \sqrt{1 + s_\mathrm{MR}}}, \tag{3a}$$

$$\begin{aligned} s_\mathrm{MR} = &\sum_i \left(p_i^2 - \mathbf{p}^2_{\mathrm{MR}i}\right)\left(\mathrm{e}^{2i(\chi_\mathrm{S} - \chi_i)} - 1\right) \\ &+ 2\left(p_1 p_2 - \mathbf{p}_{\mathrm{MR}1}\cdot\mathbf{p}_{\mathrm{MR}2}\right)\left(\mathrm{e}^{i(2\chi_\mathrm{S} - \chi_1 - \chi_2)} - 1\right) \\ &- \left[1 + \left(\frac{\sum_i g_i}{g_\mathrm{S}}\right)^2\right]\left(\sum_i \mathbf{p}_{\mathrm{MR}i}\right)^2, \; i = 1,2 \end{aligned} \tag{3b}$$

introducing $p_i = 2g_\mathrm{S} g_i/g^2_\Sigma$, $\mathbf{p}_{\mathrm{MR}i} = 2g_\mathrm{S} g_{\mathrm{MR}i}\mathbf{m}_i/g^2_\Sigma$. The counting factor $\exp(2i\chi_\mathrm{S} - i\chi_1 - i\chi_2) - 1$ corresponds to events where two charges leave S and one charge is counted at both F_1 and F_2 [9], i.e. crossed Andreev (CA) reflection. Counting factors $\exp(2i\chi_\mathrm{S} - 2i\chi_{1(2)}) - 1$ correspond to tunneling of two electrons into terminal $\mathrm{F}_{1(2)}$, direct Andreev (DA) reflection. CA reflections provide the desired current of entangled particles into spatially separated terminals. Prefactors $p_i^2 - \mathbf{p}^2_{\mathrm{MR}i}$ and $2(p_1 p_2 - \mathbf{p}_{\mathrm{MR}1}\cdot\mathbf{p}_{\mathrm{MR}2})$ are related to the probability for DA and CA reflections respectively. The magnetization dependence of these probabilities can be understood as follows: DA has reduced probability in the presence of magnetic interfaces since one particle of the singlet must tunnel through a low conductance barrier due to antiparallel spin and magnetization. When the F_1 and F_2 magnetizations are antiparallel, CA has increased probability since both particles of the singlet encounter a high conductance barrier, i.e. with magnetization parallel to its spin. The χ-independent term on the last line of Eq. (3b) is due to spin accumulation in the cavity. When $g_{\mathrm{MR}1} = g_{\mathrm{MR}2}$ and the magnetizations of F_1 and F_2 are antiparallel this term vanishes since there is no spin accumulation.

Let us now consider the magnetization dependence of the transport properties following from $\mathcal{S}$. Currents into the terminals are obtained from derivatives of the CGF: $I_i = -(ie/t_0)\partial\mathcal{S}/\partial\chi_i|_{\chi_1=\chi_2=\chi_S=0}$. These currents are the sum of current due to CA (I_i^CA) and DA (I_i^DA). We consider the case $g_{\rm MR1} = g_{\rm MR2}$ in the following. In Fig. 2(a) we plot the conductance as a function of conductance asymmetry $g_{\rm S}/(g_1+g_2)$. The conductance in a parallel alignment ($\mathbf{m}_1 = \mathbf{m}_2$) is reduced with respect to the S/N system due to spin accumulation. The effect is of second order in the polarization. In the antiparallel alignment ($\mathbf{m}_1 = -\mathbf{m}_2$), we calculate the CA- and DA-contributions separately and find that $g_{{\rm MR}i}$-terms control the separation of the total current according to ($I_i = I_i^{\rm CA} + I_i^{\rm DA}$)

$$\frac{I_i^{\rm CA}}{I_i^{\rm DA}} = \frac{g_1 g_2 + g_{\rm MR1}g_{\rm MR2}}{g_i^2 - g_{{\rm MR}i}^2};\ I_i = \frac{V g_{\rm S}^2 g_i}{g_\Sigma^3}(g_1+g_2). \quad (4)$$

These relations show how $g_{{\rm MR}i}$ increases the CA current and decreases the DA current. However, the total current into terminal $\rm F_i$ is independent on $g_{{\rm MR}i}$ and corresponds to the current in an S/N structure [9] where $g_\Sigma \to \sqrt{g_{\rm S}^2 + (g_1+g_2)^2}$ in this case. Thus in the antiparallel alignment of this system, the effect of spin filtering through $g_{{\rm MR}i}$ cannot be detected by measuring the average currents.

The noise measured in the system will depend on the increased fraction of entangled particles flowing into different terminals $\rm F_1$ and $\rm F_2$ according to Eqs. (4). Noise and cross correlation is calculated from second derivatives of the CGF: $P_{ij} = (2e^2/t_0)\partial^2\mathcal{S}/\partial\chi_i\partial\chi_j|_{\chi_1=\chi_2=\chi_S=0}$. We define Fano factors $F_{ij} = P_{ij}/2eI_{\rm S}$. The autocorrelation noise F_{11} will be reduced in antiparallel alignment with respect to S/N system due to enhancement of CA (not shown). Let us now consider the cross correlation F_{12}. Depending on the parameters, F_{12} shown in right panels of Fig. 2 can be both positive and negative. The positive part is the signature of spatially separated entangled currents. In the antiparallel alignment, we find $F_{12} = (g_1g_2 + g_{\rm MR1}g_{\rm MR2})/(g_1+g_2)^2 - 5g_{\rm S}^2 g_1 g_2/g_\Sigma^4$. The positive contribution from $g_{{\rm MR}i}$ to F_{12} shows that spin filtering in antiparallel alignment enhances the cross correlations, i.e, enhances correlation between currents in $\rm F_1$ and $\rm F_2$ with respect to the S/N system. Note that for the values of $g_i, g_{{\rm MR}i}$ chosen, F_{12} is only positive. F_{12} in parallel alignment is equal to the result for S/N system when $g_{\rm S}/(g_1+g_2)$ is either very small or very large. In Fig. 2(b) we show the third cumulant of the total current: $C_3 = (ie/I_{\rm S})\partial^3\mathcal{S}/\partial\chi_{\rm S}^3|_{\chi_1=\chi_2=\chi_S=0}$. Similar to the behavior of the conductances, it is equal to the S/N case when the magnetizations are in the antiparallel alignment, but enhanced by the spin accumulation in parallel alignment.

Now we will identify the physical origin of positive and negative contributions to F_{12}. Electron-hole pairs in spin singlet states are transferred pairwise into the cavity by Andreev reflection, which leads to positive cross correlations in CA reflection (bunching behavior) [11]. A different, negative contribution (antibunching) is induced by the fermion exclusion principle upon splitting a correlated electron-hole pair current into $\rm F_1$ and $\rm F_2$ in DA:

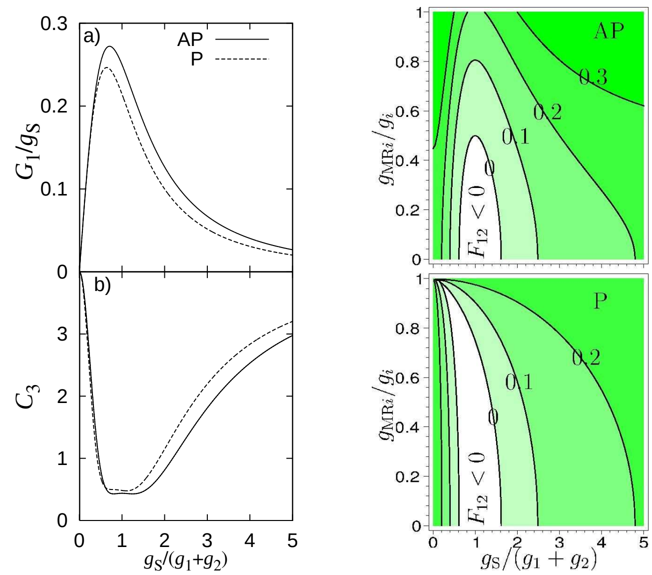

FIG. 2: Left panels: Conductance a) and third cumulant b) for a symmetric structure ($g_1 = g_2$, $g_{\rm MR1} = g_{\rm MR2}$, $g_{\rm MR1}/g_1 = 0.5$). Antiparallel alignment (AP) in solid line, parallel (P) alignment in dashed line. Right panels: Contourplots of F_{12}. Positive regions shown in color.

The transfer of one electron-hole pair from a "train" of pairs into $\rm F_{1(2)}$ prevents the simultaneous transfer of another pair into $\rm F_{2(1)}$. However, if the pair current is very noisy (Poissonian) the exclusion principle cannot induce negative cross correlation because there is no correlation between one transport event and the next. The transfer of pairs is uncorrelated when the conductance asymmetry $g_{\rm S}/(g_1+g_2)$ is either very small or very large. The total cross correlation is determined by the competing positive and negative contributions, which can be identified in the structure of P_{12}: The double root in Eq. (3a) gives $P_{12} \sim \alpha\partial^2 s_{\rm MR}/\partial\chi_1\partial\chi_2 - \beta(\partial s_{\rm MR}/\partial\chi_1)(\partial s_{\rm MR}/\partial\chi_2)$ where α and β are constants. We will show that the positive contribution corresponds to the cross correlation of an asymmetric structure (denoted $\tilde{P}_{12}$). The negative contribution has structure $\propto I_1 I_2$, same as in an N/F beamsplitter. This motivates the notation $P_{12} = \tilde{P}_{12} - \kappa I_1 I_2$. We now discuss these two contributions.

For systems where $g_{\rm S}/(g_1+g_2)$ is not close to 1, we can expand the double root in Eq. (3a) in the parameters $p_i, |\mathbf{p}_{{\rm MR}i}| \ll 1$. This gives the CGF

$$\mathcal{S} = -\frac{t_0 V g_{\rm S}^2}{2e g_\Sigma^3}\Big(\sum_{i=1,2}[g_i^2 - g_{{\rm MR}i}^2][{\rm e}^{2i(\chi_{\rm S}-\chi_i)} - 1] \quad (5)$$
$$+ 2[g_1g_2 - g_{\rm MR1}g_{\rm MR2}\mathbf{m}_1\cdot\mathbf{m}_2][{\rm e}^{i(2\chi_{\rm S}-\chi_1-\chi_2)} - 1]\Big).$$

Sums of CGFs of independent statistical processes are additive, thus the CGF in Eq. (5) can be interpreted as the sum of independent Poisson processes. Only CA contributes (positively) to cross correlations, and with $g_{\rm MR1} = g_{\rm MR2}$ and antiparallel alignment, this contribution is exactly $\tilde{P}_{12}$ defined above. The Fano factors following from Eq. (5) are $F_{12} = 1/4$ in the parallel alignment and $F_{12} = (1 + [g_{\rm MR1}/g_1]^2)/4$ for the antiparallel alignment. With S replaced by an incoherent normal metal, $\tilde{P}_{12}$ vanishes [9] since particles emanate from the source with no pairwise correlation.

The negative contribution to P_{12} vanishes in the limit $p_i, |\mathbf{p}_{{\rm MR}i}| \ll 1$ considered above since pairwise charge transfers are uncorrelated. For general values of $p_i, |\mathbf{p}_{{\rm MR}i}|$, the exclusion principle induces negative cross correlations from correlated DA events. With $g_{\rm MR1} = g_{\rm MR2}$ and antiparallel alignment this contribution becomes $-\kappa I_1 I_2$ with $\kappa = 10e/(V g_\Sigma)$. If we replace S by a normal metal terminal, correlated transport of single electrons through the cavity induces a contribution with same structure and $\kappa = 2e/(V[g_{\rm S} + g_1 + g_2])$. Scattering theory gives similar expressions for the negative part of F_{12}, as well as enhanced positive contributions for an asymmetric system [17].

We have seen that in the asymmetric limit, F_{12} is a direct measure of the degree of entanglement between the currents in $\rm F_1$ and $\rm F_2$, as only CA contributes. In the general case, the sign of F_{12} is determined a competition between the positive contribution from CA and induced negative correlations from DA due to the exclusion principle. The sum of these contributions depends on magnetization configuration. A negative F_{12} in parallel alignment, can for certain parameter values be switced to positive value in the antiparallel alignment, as shown in Fig. 2 at $g_{\rm S}/(g_1+g_2) \sim 1$. This is because the number of DA events is reduced with respect to the CA events when going from parallel to antiparallel alignment, see Eq. (4).

Utilizing ferromagnetic drains as detectors for spin rather than filters, we can demonstrate the violation of a Bell-Clauser-Horne-Shimony-Holt inequality [18]. In the circuit Fig. 1(b) the S source is connected to four ferromagnetic drains $\rm F_i$ $(i = 1..4)$ (we follow the analysis of Ref. [10]). Drains 1,2 and 3,4 have pairwise equally large and antiparallel magnetizations, $g_{\rm MR1}\mathbf{m}_1 = -g_{\rm MR2}\mathbf{m}_2 = g_{\rm MR\,L}\mathbf{m}_{\rm R}$, $g_{\rm MR3}\mathbf{m}_3 = -g_{\rm MR4}\mathbf{m}_4 = g_{\rm MR\,R}\mathbf{m}_{\rm R}$. We assume that $g_1 = g_2$ and $g_3 = g_4$. The left and right pairs of drains each form a spin detector with respect to the magnetization $\mathbf{m}_{\rm L,R}$: Spins up in left detector are measured by the current in $\rm F_1$ etc. Two experiments are performed with each polarization taking directions $\mathbf{m}_{\rm L,R}$ and $\mathbf{m}'_{\rm L,R}$. We discard DA events by normalizing the probabilities to go to different detectors. Thus, the probability to measure e.g. spin up in the left and right detectors becomes $P_{++} = p_{1,3}/(p_{1,3}+p_{1,4}+p_{2,3}+p_{2,4})$ where $p_{i,j}$ is the probability to measure simultaneously an electron into $\rm F_i$ and $\rm F_j$. The Bell parameter is defined $\mathcal{E} = |E(\mathbf{m}_{\rm L}, \mathbf{m}_{\rm R}) + E(\mathbf{m}'_{\rm L}, \mathbf{m}_{\rm R}) + E(\mathbf{m}_{\rm L}, \mathbf{m}'_{\rm R}) - E(\mathbf{m}'_{\rm L}, \mathbf{m}'_{\rm R})|$ where the correlators are given by $E(\mathbf{m}, \mathbf{m}') = P_{++} + P_{--} - P_{+-} - P_{-+}$. From the probabilities implicit in Eq. (3b) we obtain $E = -g_{\rm MR\,L}g_{\rm MR\,R}\mathbf{m}_{\rm L}\cdot\mathbf{m}_{\rm R}/g_1g_2$. Thus the Bell parameter is $\mathcal{E} = g_{\rm MR\,L}g_{\rm MR\,R}\mathcal{E}_0/g_1g_2$ where $\mathcal{E}_0 = |\mathbf{m}_{\rm L}\cdot\mathbf{m}_{\rm R} + \mathbf{m}'_{\rm L}\cdot\mathbf{m}_{\rm R} + \mathbf{m}_{\rm L}\cdot\mathbf{m}'_{\rm R} - \mathbf{m}'_{\rm L}\cdot\mathbf{m}'_{\rm R}|$ is the expression for fully efficient detectors. The largest possible value of $\mathcal{E}$ in a local theory is 2. Since the maximum of $\mathcal{E}_0$ is $2\sqrt{2}$ in the optimum orientation of the magnetizations, violation of Bell's inequality $\mathcal{E} \leq 2$ can occur provided the $g_{\rm MR\,L,R}/g_{\rm L,R} \geq 2^{-1/4}$. This condition on the efficiency of the detectors can be satisfied with half-metallic ferromagnets or magnetically engineered magnetic tunnel junctions [19]. Similar results were computed in Ref. [10] applying the matrix current Eq. (2) to detectors for electron singlets from a ballistic normal conductor. However, in this system only a fraction of the current is carried by entangled pairs since there is an additional process of single electron conduction.

In conclusion, we have calculated the full counting statistics of a superconductor/ferromagnet entangler. Spin filtering with spin active interfaces spatially separates electrons in singlet states, and gives rise to spatially separated currents of entangled particles. The effect of entanglement can be seen in the noise and higher moments of the charge transport probability distribution. In the noise cross correlations, we have identified the physical origin of positive and negative contributions. We finally show how our setup can be used to demonstrate the violation of a Bell inequality.

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