Robust Computation of Minimum-Time Pacing Strategies on Realistic Road Cycling Tracks

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Introduction

In a previous publication (Dahmen, 2012) we compared minimum-time pacing strategies for two endurance models: the 3-parameter critical power model (Morton, 1996) and an exertion model by Gordon (2005). For the former model, the problem is singular and only an approximate regularized solution for a synthetic continuously varying slope profile was computed. In this contribution, we improve the numerical methods to compute strategies on realistic tracks with complex height profiles.

Methods

We parametrized the problem using kinetic energy instead of speed. We used differential gps to record height profiles of real cycling tracks and approximated them with a cosine series in order to use automatic differentiation, which has recently improved the performance of GPOPS-II (Patterson & Rao, 2014). Moreover, we compute solutions for a combination of the two endurance models.

Results

The substitution vastly improves efficiency and robustness of the computations. Both the optimal power and speed function exhibit detailed variations that fit well to the height data and qualitative features of the used models. The regularization for the singular problem is no longer necessary.

Discussion

It seems that the substitution has improved the scaling of the optimization problem. Although automatic scaling methods were used in our previous publication, they rely on heuristics and may fail for a particular problem. Likewise, for singular problems no general method to guarantee the desired solution exists. However, the new combination of methods provides robust solutions for our problem class of optimal pacing strategies even on complex, realistic tracks.

Literature

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