

Three Essays on Financial Frictions and Macroeconomic Dynamics

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(Dr. rer. pol.)

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Summary

All three essays that comprise this dissertation contribute to the topic of macroeconomic dynamics in an environment with financial frictions. In recent decades and especially after the financial crises, macroeconomic researchers have been working on adding financial market frictions into business cycle models, in an effort to study how the financial sector works as an intermediary to transmit policies and shocks to macroeconomic dynamics. Among others, Bermanke and Gertler (1989) and Kiyotaki and Moore (1997) each provides an approach that is now considered as classic to incorporating financial frictions, and this dissertation finds its root in the latter. The main characteristic of the framework in Kiyotaki and Moore (1997) is the presence of a financial constraint, which requires debts to be collateralized so that lenders are able to recover at least a fraction of the collateral in bankruptcy. The first two chapters of this dissertation both employ financial constraints of this kind, and the last chapter is an empirical extension of the second.

The first chapter of this dissertation builds a mechanism through which the financial constraint works as an intermediary that links a technological change in the corporate sector to capital liquidation decisions. I try to understand how this technological change can explain the observed delay of capital liquidations with the help of the financial constraint. The past few decades have seen a highly pro-cyclical reallocation of capital from unproductive entities to productive entities. As capital liquidations which are supposed to occur intensively during recessions are actually observed more in normal times, the process of capital liquidations is delayed, hindering efficient reallocation of capital to more productive entities. Cui (2017) develops a mechanism in which adverse financial shocks tighten financial constraints and give rise to the delay. However, the observed persistent delay cannot be explained by solely exogenous financial shocks. During the same time period of the delayed process, a significant technological change has been documented: an increasing reliance on intangible capital as an input of production.

By incorporating the technological change into the model of Cui (2017), I create an endogenous tightening of the financial constraints and derive conditions under which the rising reliance on intangible capital indirectly provides incentives to delay capital liquidations. As has been mentioned, the technological change is linked to the capital liquidation

decision through their interactions with the financial constraints. In the model, the tangible capital is differentiated from the intangible capital assuming that only the former can be pledged as collateral for borrowing. Both analytic and quantitative result show that, as long as unproductive entrepreneurs are impatient enough and have sufficient net worth to sustain their business, a drop in the tangible share that worsens firms' borrowing capacities can raise entrepreneurs' inclination to postpone liquidation, as the lower borrowing means lighter future debt burden for them if the businesses continue. The delay of liquidation thus damps an efficient reallocation of capital to the more productive entities and impairs the aggregate investment, output and Total Factor Productivity (TFP). In addition, in terms of the relationship between firm productivity and firm leverage, the model reveals that financially constrained productive firms tend to have a larger leverage ratio than their unproductive peers. This finding is supported by micro-foundations and would not emerge if the tangible capital was not differentiated from the intangible.

The second chapter incorporates financial constraints into a classical two-country business cycle model developed by Backus, Kehoe, and Kydland (1994) and takes capital-control policy as a research objective, examining its effects on the Terms of Trade (TOT) and potentials to improve international risk sharing, as well as discussing its welfare implications. Capital-control policies, which depress the external borrowing capacity, enter the model as a restriction on the financial constraint. It can either be viewed as a control on capital inflows in the borrowing country or a control on capital outflows in the lending country.

Practices in this chapter show that capital-control policies can improve the welfare of the borrowing country by moving the TOT in favor of it. However, the implementation of capital-controls cannot be Pareto-improving in which case the welfare of the lending country is also a concern. The movement of the TOT works as an insurance against adverse shocks to a country. Impulse response exercises show that after the capital account is tightened, the TOT responses more intensely to a standardized shock than before, potentially improving the international risk-sharing between countries.

Due to the simplicity of the capital-control modeling, the second chapter fails to cover several features that might be important in analyzing the role of capital controls, for example the economic and social environment in which the capital-control policies are imposed. Thus I try to add to the study by focusing on some country-specific features which might influence the interaction between capital controls and economic growth. The underlying logic is that benefits of an open capital account can only be realized when some threshold conditions are satisfied, otherwise capital controls might be beneficial in sheltering the domestic framework from foreign impacts. In literature, most studies on this topic focus on one particular country-specific feature and report some interesting results. However, results reported by studies on multiple features vary a lot. Moreover,

as is pointed out by Prasad, Kose, and Taylor (2009), a majority of the studies use a simple linear interaction specification, leaving the statistical significance and confidence intervals of the threshold levels out of consideration.

Using a recent Structural Threshold Regression (STR) model, the third chapter of this dissertation investigates existence of certain threshold levels in the country-specific features, levels at which the effects of capital controls on growth reverse. The goal of this chapter is to identify intrinsic threshold levels in the country-specific features and to examine different effects of capital controls above and below the threshold levels, making use of the asymptotic theory and significance diagnoses of the STR methodology.

Several country-specific features are investigated. My practices identify a significant threshold level in the degree of financial development. When the degree of financial development is above this level capital controls significantly benefit growth. However a lower threshold level is not identified, a level at which the effect of capital controls turn from positive to negative. Another significant threshold level that is identified lies in the measurement of government size, in which case capital controls promotes economic growth when government is large enough. In the exercises on institutional quality, monetary policy and trade openness, no significant threshold levels are identified. However there are evidences that capital controls have significantly different impacts on growth at different levels of these country-specific features.

Zusammenfassung

Alle drei wissenschaftlichen Aufsätze, die diese Dissertation umfassen, handeln sich um die Dynamik der Makroökonomie in einem Umfeld mit Friktionen im Finanzmarkt. In den letzten Jahrzehnten und insbesondere nach der Finanzkrise haben makroökonomische Forscher daran gearbeitet, die Friktionen im Finanzmarkt in Konjunkturmodelle aufzunehmen, um zu untersuchen, wie der Finanzsektor als Vermittler fungiert, um Politik und Schocks mit der makroökonomischen Dynamik zu verbinden. Unter anderem bieten Bemanke and Gertler (1989) und Kiyotaki and Moore (1997) jeweils einen Ansatz, der heute als klassisch für die Einbeziehung finanzieller Friktionen gilt, und diese Dissertation findet ihre Wurzeln in letzteren. Das Hauptmerkmal des Rahmens von Kiyotaki and Moore (1997) ist das Vorhandensein einer finanziellen Einschränkung, die eine Besicherung der Schulden erfordert, so dass die Kreditgeber in der Lage sind, mindestens einen Bruchteil der Sicherheiten im Konkursfall einzuziehen. Die ersten beiden Kapitel dieser Dissertation beschäftigen beide finanzielle Zwänge dieser Art, und das letzte Kapitel ist eine empirische Erweiterung der zweiten.

Das erste Kapitel dieser Dissertation baut einen Mechanismus auf, in dem die finanzielle Einschränkung als Vermittler funktioniert, der einen technologischen Wandel mit Kapitalliquidationsentscheidungen verknüpft. Ich versuche zu verstehen, wie der technologische Wandel die beobachtete Verzögerung der Kapitalliquidation mit Hilfe der finanziellen Einschränkungen erklären kann. In den letzten Jahrzehnten fand eine sehr prozyklische Umverteilung des Kapitals von unproduktiven zu produktiven Unternehmen statt. Eine scheinbar logische Sichtweise vertritt die Ansicht, dass Kapitalumschichtungen in Rezessionen häufig stattfinden sollten, wenn viele Unternehmen unproduktiv werden. Allerdings werden Kapitalumschichtungen oder Liquidationen in normalen Zeiten eher beobachtet als in Rezessionen. In diesem Sinne verzögert sich der Prozess der Kapitalliquidationen, was eine effiziente Umverteilung des Kapitals auf produktivere Einheiten behindert. Cui (2017) entwickelt einen Mechanismus, in dem ungünstige finanzielle Schocks die finanziellen Zwänge verschärfen und zu einer Verzögerung führen. Die beobachtete anhaltende Verzögerung lässt sich jedoch nicht allein durch exogene finanzielle Schocks erklären.

Während des gleichen Zeitraums der beobachteten Verzögerung wurde ein signifikanter technologischer Wandel dokumentiert: eine zunehmende Abhängigkeit vom immateriellen Kapital als Input für die Produktion. Indem ich den technologischen Wandel in das Modell der Cui (2017) einbeziehe, schaffe ich eine endogene Verschärfung der finanziellen Beschränkungen und leite Bedingungen ab, unter denen die zunehmende Abhängigkeit vom immateriellen Kapital indirekt Anreize bietet, Kapitalliquidationen zu verzögern. Wie bereits erwähnt, ist der technologische Wandel mit der Kapitalliquidationsentscheidung durch ihre Wechselwirkungen mit den finanziellen Einschränkungen verbunden. Im Modell wird das Sachkapital vom immateriellen Kapital unterschieden, wobei davon ausgegangen wird, dass nur dieses als Sicherheit für die Kreditaufnahme verpfändet werden kann. Sowohl analytische als auch quantitative Ergebnisse zeigen, dass, solange unproduktive Unternehmer ungeduldig genug sind und über ein ausreichendes Vermögen verfügen, um ihr Geschäft aufrechtzuerhalten, ein Rückgang des materiellen Anteils, der die Kreditkapazitäten der Unternehmen verschlechtert, die Tendenz der Unternehmer erhöhen kann, die Liquidation zu verschieben, da die niedrigere Kreditaufnahme eine geringere zukünftige Schuldenlast für sie bedeutet, wenn die Unternehmen weitergehen. Die Verzögerung der Liquidation beeinträchtigt somit eine effiziente Umverteilung des Kapitals auf die produktiveren Einheiten und beeinträchtigt die gesamte Investitions-, Produktions- und Total Factor Productivity (TFP). Darüber hinaus zeigt das Modell in Bezug auf das Verhältnis zwischen Unternehmensproduktivität und Unternehmensverschuldung, dass finanziell angeschlagene Produktionsunternehmen in der Regel eine größere Leverage Ratio haben als ihre unproduktiven Konkurrenten. Diese Erkenntnis wird durch Mikrostiftungen gestützt und würde nicht entstehen, wenn das Sachkapital nicht vom immateriellen unterschieden würde.

Das zweite Kapitel beinhaltet finanzielle Einschränkungen in ein klassisches Zweiländer-Konjunkturmodell, das von Backus et al. (1994) entwickelt wurde, und nimmt die Kapitalkontrollpolitik als Forschungsziel, untersucht ihre Auswirkungen auf die Terms of Trade (TOT) und Potenziale zur Verbesserung der internationalen Risikoteilung und diskutiert ihre Auswirkungen auf den Wohlstand. Die Kapitalkontrollpolitik, die die externe Kreditaufnahmekapazität belastet, wird als Begrenzung der finanziellen Einschränkungen in das Modell aufgenommen. Sie kann entweder als Kontrolle der Kapitalzuflüsse im kreditaufnehmenden Land oder als Kontrolle der Kapitalabflüsse im ausleihenden Land angesehen werden.

Die Verfahren in diesem Kapitel zeigen, dass die Kapitalkontrollpolitik das Wohlergehen des kreditaufnehmenden Landes verbessern kann, indem sie die TOT zugunsten des Landes verschiebt. Die Durchführung von Kapitalkontrollen kann jedoch nicht Paretoverbessernd sein, in diesem Fall ist auch das Wohlergehen des kreditgebenden Landes ein Anliegen. Die Bewegung der TOT dient als Versicherung gegen ungünstige Schocks

in einem Land. Impulsreaktionsübungen zeigen, dass die TOT nach der Straffung der Kapitalbilanz intensiver als bisher auf einen standardisierten Schock reagiert und damit die internationale Risikoteilung zwischen den Ländern verbessert werden kann.

Aufgrund der Einfachheit der Modellierung der Kapitalverkehrskontrolle werden im zweiten Kapitel einige Aspekte nicht behandelt, die bei der Analyse der Rolle der Kapitalverkehrskontrolle von Bedeutung sein könnten, wie beispielsweise das wirtschaftliche und soziale Umfeld, in dem die Kapitalverkehrskontrollpolitik durchgesetzt wird. Daher versuche ich, die Studie zu ergänzen, indem ich mich auf die länderspezifischen Merkmale konzentriere, die die Wechselwirkung zwischen Kapitalverkehrskontrolle und Wirtschaftswachstum beeinflussen könnten. Die Logik ist, dass Vorteile eines offenen Kapitalkontos nur dann realisiert werden können, wenn bestimmte Schwellenwerte erfüllt sind, andernfalls könnten Kapitalkontrollen nützlich sein, um den inländischen Rahmen vor ausländischen Auswirkungen zu schützen. In der Literatur konzentrieren sich die meisten Studien zu diesem Thema auf ein bestimmtes länderspezifisches Merkmal und berichten über einige interessante Ergebnisse. Die von Studien über mehrere Merkmale berichteten Ergebnisse variieren stark. Darüber hinaus verwendet eine Mehrheit der Studien, wie von Prasad et al. (2009) hervorgehoben, eine einfache lineare Interaktionsspezifikation, wobei die statistische Signifikanz und die Konfidenzintervalle der Schwellenwerte außer Betracht bleiben.

Mit Hilfe eines aktuellen Structural Threshold Regressions (STR) untersucht das dritte Kapitel dieser Dissertation die Existenz bestimmter Schwellenwerte in den länderspezifischen Merkmalen, Niveaus, auf denen sich die Auswirkungen der Kapitalkontrolle auf das Wachstum umkehren. Ziel dieses Kapitels ist es, intrinsische Schwellenwerte in den länderspezifischen Merkmalen zu identifizieren und verschiedene Effekte von Kapitalkontrollen über und unter den Schwellenwerten zu untersuchen, wobei die asymptotische Theorie und Signifikanzdiagnosen der STR-Methodik genutzt werden.

Mehrere länderspezifische Merkmale werden untersucht. Meine Verfahren identifizieren einen signifikanten Schwellenwert für den Grad der finanziellen Entwicklung. Wenn der Grad der finanziellen Entwicklung über diesem Niveau liegt, profitiert das Kapital deutlich vom Wachstum. Ein niedrigerer Schwellenwert, bei dem die Wirkung der Kapitalkontrolle von positiv zu negativ wird, wird jedoch nicht identifiziert. Ein weiterer wichtiger Schwellenwert, der identifiziert wird, liegt in der Messung der Regierungsgröße, wobei Kapitalkontrollen das Wirtschaftswachstum fördern, wenn die Regierung groß genug ist. In dem Verfahren zur institutionellen Qualität, Geldpolitik und Handelsoffenheit werden keine signifikanten Schwellenwerte ermittelt. Es gibt jedoch Hinweise darauf, dass die Kapitalverkehrskontrolle auf verschiedenen Niveaus dieser länderspezifischen Merkmale signifikant unterschiedliche Auswirkungen auf das Wachstum hat.

Chapter 1

Financial Constraints, Rising Reliance on Intangible Capital and Delayed Capital Liquidation

1.1 Introduction

The past few decades have seen a highly pro-cyclical reallocation of capital from unproductive entities to productive entities in the U.S., as opposed to a seemingly logical viewpoint that efficiency requires more liquidations to occur during recession, a time when more firms turn unproductive, than in normal time.¹ In the sense that capital liquidations are in fact observed more in normal times, the process of capital liquidations is delayed. To account for this delay, Cui (2017) develops a mechanism through which adverse financial shocks tighten financial constraints and might render it beneficial to delay liquidations. However, the observed persistent delay cannot be explained by solely exogenous financial shocks. Accompanying the delayed process, a significant technological change in the corporate sector has been documented: the reliance on physical capital as an input of production has reduced since 1970s while intangible capital, including informational, organizational and knowledge capital, has been playing an increasingly important role. (See Corrado and Hulten (2010)). Moreover, the decreasing share of tangible capital has been related to the tightening financial constraints and a decline of corporate borrowing.²

By incorporating the technological change into the model of Cui (2017), this paper creates an endogenous tightening of the financial constraints and derives conditions under

¹See Eisfeldt and Rampini (2006), and Cui (2017).

²Falato, Kadyrzhanova, and Sim (2013), Brown, Fazzari, and Petersen (2009) and Gatchev, Spindt, and Tarhan (2009).

which the rising reliance on intangible capital indirectly provides incentives to delay liquidations. Specifically, the technological change is linked to the capital liquidation decision through their interactions with the financial constraints. In the model, the tangible capital is differentiated from the intangible capital assuming that only the former can be pledged as a collateral for borrowing. Both analytic and quantitative results show that, a drop in the tangible share that worsens unproductive firms' borrowing capacities has the potential of raising their inclination to postpone liquidation. The delay thus damps an efficient reallocation of capital to the more productive firms and impairs aggregate investment, output and Total Factor Productivity (TFP). In addition, in terms of the relationship between firm productivity and firm leverage, the model reveals that the financially constrained productive firms tend to have larger leverage ratios than their unproductive peers, a finding that is supported by micro-foundations and does not emerge if the two types of capital are not distinguished.

The model features a continuum of firms each run by an entrepreneur who receives idiosyncratic productivity shocks and pays a random liquidation cost.³ The liquidation cost creates incentives to delay the liquidation and thus an option of continuing to stay in business. The liquidation decision-making at an individual level is first investigated before an aggregation of the economy. While a productive entrepreneur who receives high productivity chooses to invest in both types of capital and finance his activity by borrowing against collateral, an unproductive entrepreneur, instead of selling all capital and saving in bonds, might choose to stay in business producing unproductively, for fear of the high liquidation cost incurred if he otherwise liquidated. As a result, each period the unproductive entrepreneur has to weigh the option value of liquidating against that of staying, giving rise to an endogenous threshold level of the liquidation cost.

The financial constraints which affects firm leverages are bridges between the technological change and the liquidation decision. The model has implications on firm leverages both over time and across firms. Over time, as the decreasing share of tangible capital lowers the collateral available for borrowing, the debt capacity of a firm as well as its beginning- and end-of-period leverages drop consequently. Across firms and given the same tangible share, the model predicts that the productive firms that are not financially constrained have a lower leverage ratio than the unproductive. However as the tangible share shrinks over time and the productive firms become financially constrained at a certain time, their leverage ratio exceeds that of the unproductive.⁴ This result arises from the assumption that the productive firms are able to exploit the borrowing benefit

³See Shi, Cao, et al. (2015), Cui and Radde (2016), Eisfeldt and Rampini (2006), and Kurlat (2013) for reasons that cause liquidation cost.

⁴The feature that productive firms have higher borrowing capacity is consistent with the micro-foundation where productive firms have more to lose once default. See Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Buera and Shin (2013) and Buera and Shin (2013) among other.

of tangible capital by flexibly deviating from its long-run share, while the unproductive can only produce using a fixed proportion of capital.

The relationship between the leverage and the liquidation decision is then examined. For the unproductive entrepreneur who has to choose between liquidating and staying, a lower *end-of-period leverage* targeted points the entrepreneur to stay in business which is now associated with a lighter future debt burden, as long as the current net worth is high enough for the firm to sustain. A lower *beginning-of-period leverage* also plays a role of delaying the liquidation if the entrepreneur is sufficiently impatient and values today more than tomorrow. Because the entrepreneur cares little about the future value generated by liquidating and saving, but he cherishes the current value of continuing to run a business which has the benefit of bearing a lower debt burden.

Intuitions above provide a simple explanation to the main findings of this paper: over time as the reliance on intangible capital increases, the lowering leverage makes it optimal for an unproductive entrepreneur to delay the liquidation and stay in business, as long as he is impatient and has sufficient net worth to sustain. In addition, the same rule applies across firms in a fixed time point: those unproductive entrepreneurs with relatively lower beginning-of-period leverages (but who target the same end-of-period leverage as the productive) are more likely to delay the liquidation.

I then aggregate the model to examine the steady-state response of the economy to a falling tangible share, as well as compare sensitivity of the economy to shocks before and after the change. As the long-run tangible share falls, the shrinking debt capacity lowers the investment and wage rate while raises the profit. The higher profit depresses the willingness to liquidate and impairs the efficiency. Following a reduction in the tangible share, the economy becomes less sensitive to a negative financial shock because of the less reliance on debt financing. Coming to the productivity shock, an off-setting effect of the liquidation mechanism moderates the responses of major variables as the tangible share falls.

In literature, Caggese and Perez-Orive (2017) also develop a mechanism through which the rising share of intangible capital has a negative effect on the capital reallocation and efficiency. They argue that the increasing share of the intangible reduces the borrowing capacity and cash holding of the corporate sector, leading firms to increasingly rely on retained earnings for capital purchase. However, a decrease in the interest rate impairs firms' ability to buy capital using retained earnings, because it raises the capital price and damps the accumulation of saving. To construct the link between the technological change and capital reallocation, their mechanism relies on the reduction in the interest rate, which is an exogenous factor.⁵ As opposed to their less direct mechanism, the

⁵The force that brings real interest rates lower in their model is a higher propensity to save in the household sector due to an increase in household longevity and a decrease in the rate of time preference

model in this paper builds a more direct link by shifting from the buy-side to the sell-side of capital stock. Thanks to the rich mechanism of liquidation decision-making, the technological change that lowers the borrowing capacity exerts direct effects on the option value of liquidating and staying.

The model in this paper is constructed based on Cui (2017), who introduces a channel to investigate the interactions between the financial constraints and the liquidation decision. Instead of modeling financial constraints to response to exogenous credit shocks as Cui (2017) does, I endogenize the tightening of the financial constraints by differentiating the tangible capital from the intangible and assuming that only the former can be collateralized for borrowing. As firms rely less on the tangible capital, their borrowing capacity shrinks and the financial constraints tighten due to a reduction of the collateral. In addition, Cui (2017) only considers the steady-state liquidation decision when the productive firms share the same leverage ratio with the unproductive. Differentiating types of capital and modeling the tangible capital as the exclusive collateral contribute to the result that the productive firms have a different leverage ratio than the unproductive. Moreover, Cui (2017) considers only the liquidation decision of those entrepreneurs who were unproductive previously and remain unproductive at present. This paper completes the analysis by considering in addition the decision of those who were productive previously and turns unproductive only lately.

The model predicts that the firms' leverage falls in response to a drop in the tangible share, which is consistent with the empirical findings of Bates, Kahle, and Stulz (2009) who relate the decrease in corporate borrowing to the rising reliance on intangible capital. Theoretical works on this topic include Bates et al. (2009), Falato et al. (2013) and Perotti et al. (2017), who develop models to examine how the rising intangible share shrinks the debt capacity and increases cash holdings. This work adds to this strand of literature by taking the financial change as an intermediary and investigating how it links the rising reliance on intangible capital to the delayed process of capital liquidation.

The rest is structured as follows. Section 1.2 summarizes stylized facts that support the assumptions and setups of the model. Section 1.3 introduces a model in which capital composition of the firm affects the liquidation decision through the financial constraints. Section 1.4 investigates effects of a decreasing tangible share on the liquidation decision and the aggregate economy both analytically and quantitatively. Section 1.5 concludes.

1.2 Stylized Facts

In this section, I summarize stylized facts that support assumptions and setups of the model.

1. The reallocation of capital in the U.S. is highly pro-cyclical, which is opposed to the efficiency requirement. The misallocation or the delayed liquidations of capital have been empirically linked to the relative importance of tangible and intangible capital.

The observed cyclical behavior of capital reallocation diverges from a seemingly logical view towards it. One aspect of an efficient usage of capital involves reallocating capital from the less productive entities to the more productive. Accordingly, recessions, during which period more firms turn unproductive, should see more liquidations and restructurings of capital stock. In contrast, after studying both the level and the turnover rates ⁶ of the reallocation using firm-level Compustat data from 1971 to 2000, Eisfeldt and Rampini (2006) document that the reallocation of existing productive capital is highly pro-cyclical across firms. Consistently, Cui (2017) also finds a pro-cyclical liquidation-to-expenditure ratio, which is defined as the ratio of aggregate capital liquidation to the total capital expenditure. Since capital liquidations that are supposed to take place during recessions are observed mostly in normal times, the process of capital liquidations is delayed.

The misallocation or delayed liquidations of capital have been linked empirically to the relative importance of tangible and intangible capital. Using productivity dispersion to measure misallocation efficiency, Caggese and Perez-Orive (2017) find that inefficiency rises in sectors with high average shares of intangible capital over the period from 1980 to 2015, while it remains roughly constant in tangible-intensive sectors over the same period.

2. The rising reliance on intangible capital has been related to increasing cash-holdings of the non-financial corporate sector in the U.S..

It is well documented that the non-financial corporations in the U.S. have shifted from a net borrowing position to a net saving position around 2000 and have steadily increased their cash-holdings over the first decades of this century ⁷. Accompanying this is a technological change in the corporate sector: the reliance on physical capital as an input of production has reduced gradually since 1970s while the intangible capital, including informational, organizational and knowledge capital, has been given an increasingly important role ⁸. A firm-level study by Falato et al. (2013) documents that the intangible capital as a share of the total capital went from around 0.2 in the 1970s to 0.5 in the 2000s.

Empirical evidence suggests that the decrease in corporate borrowing can be attributed to the rising reliance on intangible capital, which is difficult to be pledged as collateral for borrowing. Falato et al. (2013) find robust evidences that the intangible capital has

⁶Turnover rate of capital in Eisfeldt and Rampini (2006) is defined as reallocation normalized by the total market value of Compustat firms

⁷See Armenter and Hnatkovska (2017), Quadri (2017)

⁸See Brown et al. (2009), Corrado and Hulten (2010)

emerged as the most important determinant of cash holdings at firm-level. Brown et al. (2009) and Gatchev et al. (2009) document that most of the U.S. firms finance their R&D expenditures, as well as other non-physical inputs including marketing expenses and product development, out of retained earnings and equity issues.

To model the relative importance of tangible and intangible capital in this work, firms are assumed to aggregate the tangible and intangible capital using a CES function, in which a parameter σ measures the long-run share of tangible capital in production and the ratio $\frac{\sigma}{1-\sigma}$ represents the long-run target of the tangible-intangible composition. The rising importance of intangible capital is thus described by a reduction in σ .

3. The decreasing share of tangible capital increases cash-holdings of the corporate sector by undermining the borrowing capacities and tightening the financial constraints of entities.

The ineligibility of the intangible capital as a borrowing collateral has a root in its nature and manifests itself in reality. Williamson (1988), Shleifer and Vishny (1992) and Giglio and Severo (2012) address redeployability as an essential character of a good collateral. The redeployability requires an asset being costlessly seized and sold in the market for a price similar to its value in the first best use. Accordingly, as the intangible capital is fundamentally less available to other firms than its parent, its value in alternative uses is reduced by the lack of specific knowledge or competences. Thus, the intangible capital is rarely recognized as a kind of collateral for lending while the tangible capital such as plants, equipment and machines are generally accepted by financial intermediaries as collateral.

Empirical findings have linked the reliance on intangible to cash-holdings through the financial constraints. For example, Giglio and Severo (2012) find that a positive relationship between intangible capital and cash is especially strong for the financially constrained firms.

These stylized facts motivate the setup of the financial constraints where borrowing is bounded up at a fraction of the tangible capital, which is the only collateral for borrowing.

1.3 The Model

In this section, a model involving agents' optimization problem is specified. As the model is quite similar to the one in Cui (2017) at the aggregate level, the analysis in this section follows Cui (2017) closely. There are two types of agents in the economy, namely a representative household and a unit of continuous entrepreneurs. Both types of agents take as given the aggregate state variable X , which deserves further explanation at the

end of this section.

1.3.1 Representative Household's Problem

Taking the previously accumulated bond B_r as given and discounting the future value at a rate of $\beta^r > \beta$ (β is discount factor of the entrepreneurs), the representative household chooses the amount of consumption C_r , labor supply L_r and bonds B_r that will be left into the next period, subjecting to a resource constraint. The resource constraint says that: the consumption and new savings in form of bonds are financed by labor income and returns from previous savings. Wage rate $\omega = \omega(X)$ and interest rate $R = R(X)$ are functions of the aggregate state X and taken as given. The problem of the representative household is summarized by the following Bellman equation:

$$v^r(B_r; X) = \max_{C_r, L_r, B_r'} \{u^r(C_r, L_r) + \beta^r \mathbb{E}[v^r(B_r'; X') | X]\} \quad \text{s.t.}$$

$$C_r + B_r' = \omega L_r + R B_r. \quad (1.1)$$

Assuming the household has a GHH utility in which the consumption and labor supply are not additive separable, i.e.,

$$u(C_r, L_r) = \frac{\left(C_r - \frac{\mu L_r^{1+\nu}}{1+\nu}\right)^{1-\varepsilon} - 1}{1-\varepsilon},$$

where $\varepsilon = 1$ is set to allow the same log utility as the entrepreneurs'. The household's policy function is determined by

$$\omega = \mu L_r^\nu \quad (1.2)$$

and

$$\mathbb{E}_X \left[\frac{\beta^r u_c \left(C_r' - \frac{\mu (L_r')^{1+\nu}}{1+\nu}\right)}{u_c \left(C_r - \frac{\mu (L_r)^{1+\nu}}{1+\nu}\right)} R' \right] = 1. \quad (1.3)$$

1.3.2 Entrepreneurs' Problem

Technology

The firm of an entrepreneur i produces with three types of inputs: tangible capital k_{Ti} , intangible capital k_{Ni} and labor hours h_i . The tangible capital and intangible capital are combined to form the aggregate capital stock $k_i = k(k_{Ti}, k_{Ni})$, which has a function form of constant-return-to-scale (CRS). A technology, which is also constant-return-to-scale,

is used to produce goods for consumption:

$$y_i = (z_i k_i)^\alpha (A h_i)^{1-\alpha}, \text{ where } k_i = k(k_{Ti}, k_{Ni}).$$

A is an aggregate productivity shock. $\alpha \in (0, 1)$ is the share of capital. $z_i \in \{z_l, z_h\}$ with $z_h > z_l > 0$ is an idiosyncratic productivity shock that follows a Markov process. The associated transition probability is:

$$\begin{aligned} p^{hl} &= \text{Prob}(z_{it} = z_l | z_{it-1} = z_h) \\ p^{lh} &= \text{Prob}(z_{it} = z_h | z_{it-1} = z_l) \end{aligned}$$

where $0 < p^{hl} < 1$, $0 < p^{lh} < 1$ and $p^{hl} + p^{lh} < 1$.

The entrepreneur owns the capital and hire labor hours competitively from the labor market, taking the wage rate as given. The profit of entrepreneur i from production is then

$$\Pi(z_i, k_i; \omega) = \max_{h_i} (z_i k_i)^\alpha (A h_i)^{1-\alpha} - \omega h_i.$$

It can be shown that $\Pi(z_i, k_i; \omega) = z_i \pi(\omega) k_i$, i.e., the profit is a linear function of the aggregate capital stock k_i . The optimal labor hours hired by entrepreneur i and the aggregate profit rate are correspondingly

$$\begin{aligned} h_i^* &= \frac{z_i k_i}{A} \left[\frac{(1-\alpha)A}{\omega} \right]^{\frac{1}{\alpha}} \\ \pi &= \alpha \left(\frac{(1-\alpha)A}{\omega} \right)^{\frac{1-\alpha}{\alpha}}. \end{aligned} \tag{1.4}$$

It follows that $\Pi_i = z_i \pi(\omega) k_i = \alpha y_i$ and $\omega h_i = (1-\alpha) y_i$. The amount of resources distributed to the entrepreneur accounts for a fraction α of the total output produced, while the rest goes to the household.

Resale Constraints

The law of motion for capital stock is

$$\begin{aligned} k_{Ti,t+1} &= (1-\delta) k_{Ti,t} + I_{Ti,t} \\ k_{Ni,t+1} &= (1-\delta) k_{Ni,t} + I_{Ni,t}, \end{aligned}$$

where $I_{Ti,t}$ and $I_{Ni,t}$, satisfying $I_{Ti,t} \geq -(1-\delta) k_{Ti,t}$ and $I_{Ni,t} \geq -(1-\delta) k_{Ni,t}$, are entrepreneur i 's investment in tangible capital and intangible capital, respectively. Hence the total amount of capital (Here I differentiate the **total amount of capital stock**

$k_{Ti} + k_{Ni}$, which is the sum of the tangible capital and intangible capital, from the **aggregate capital** k_i , which is an aggregation of the tangible capital and intangible capital in the production function), evolves as a sum of the last-period depreciated total amount of capital stock and the new investment of capital, i.e.,

$$k_{Ti,t+1} + k_{Ni,t+1} = (1 - \delta) k_{Ti,t} + (1 - \delta) k_{Ni,t} + I_{Ti,t} + I_{Ni,t}.$$

When an entrepreneur decides to run a business (by holding capital and producing), the firm produces subject to the following resale constraint:

$$k_{Ti,t+1} + k_{Ni,t+1} \geq (1 - \delta) k_{Ti,t} + (1 - \delta) k_{Ni,t},$$

which indicates that $I_{Ti,t} + I_{Ni,t} \geq 0$. When the entrepreneur chooses to liquidate the firm and saves in bonds, he incurs $k_{Ti,t+1} = k_{Ni,t+1} = 0$.

It's noteworthy that the resale constraint imposes a restriction only on the total amount of capital stock ($k_{Ti} + k_{Ni}$) and leaves the individual tangible and intangible capital (k_{Ti} and k_{Ni}) unrestricted, which may give rise to an equilibrium in which the entrepreneur invests in only one type of capital. In the following part, I will explain further why the setup of two separate resale constraints, one for each type of capital, is not used. For the moment, presence of additional assumptions ensures that the possibility of the entrepreneur investing solely in one type of capital is ruled out. Specifically, assumptions will enter the model such that if a high productivity z_h is drawn, the entrepreneur always invests in both types of capital; if a low productivity z_l is drawn and the entrepreneur chooses to keep running a firm, he maintains the current amount of total capital so that $I_{Ti,t} + I_{Ni,t} = 0$, and meanwhile adjusts the ratio of tangible capital to intangible capital to a fixed value. In the latter case, non-positive values of $I_{Ti,t}$ or $I_{Ni,t}$ is allowed, meaning a partial liquidation is possible.

Preference

The entrepreneur obtains log-utility from consumption (or dividends) and incurs a utility cost ζ in a case when he chooses to liquidate the firm:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \log(c_t) - \mathbf{1}_{\{\text{liquidating}\}} \zeta \}, \quad 0 < \beta < 1.$$

ζ is an i.i.d. stochastic liquidation cost with a CDF $F(\cdot)$. In each period a firm receives liquidation cost randomly, which will impact the liquidation-decision process by decreasing the option value of liquidating. A high ζ represents a high liquidation cost and the

entrepreneur might find it optimal to stay for another period running the business rather than to liquidate and incur the liquidation cost. In addition, from the perspective of modeling, the utility-cost form of ζ has the merit of allowing a closed-form solution.

Financial Constraints

The entrepreneur can save and borrow in bonds at an interest rate R . $\beta^h > \beta$ is assumed, i.e., the household is more patient than the entrepreneurs, making the latter net borrowers. The bond market is incomplete in that an occasionally binding financial constraint is in place:

$$Rb' \geq -\theta(1 - \delta)k'_T.$$

Following Hart and Moore (1994) and Kiyotaki and Moore (1997), I assume that the debt is contracted to be risk-free and is collateralized by the capital assets instead of firm's output due to unverifiability of the latter. Amount that can be borrowed is bounded up at a maximum which accounts for a fraction $\theta \in [0, 1]$ of the collateral. In case of default, the lender should be able to recover θ fraction of the collateral. Deviating from Kiyotaki and Moore (1997), I differentiate tangible capital and intangible capital and assume that only the tangible capital is eligible to serve as collateral.

Along with the production function which takes both types of capital as inputs, the financial constraints imply that, the more the firm relies on the intangible capital, the less tangible capital is available to be pledged. And so that the borrowing constraints is more likely to bind. Among other studies, Falato et al. (2013) examines how rising intangible capital shrinks the debt capacity of firms and increases their cash holdings.

Recursive Problem of the Entrepreneur

Denote v as the entrepreneur's optimal value, given the individual state (k_T, k_N, b, z, ζ) and the aggregate state X . The recursive problem of the Entrepreneur is represented by the Bellman equation below:

$$v(k_T, k_N, b, z, \zeta; X) = \max_{\text{liquidate/stay}} \{v^0(k_T, k_N, b, z, \zeta; X), v^1(k_T, k_N, b, z; X)\},$$

where v^0 and v^1 satisfy:

$$v^0(k_T, k_N, b, z, \zeta; X) = \max_{b'} \{ \log(c) - \zeta + \beta \mathbb{E}[v(0, 0, b', z', \zeta'; X) | z, X] \} \quad \text{s.t.}$$

$$c + b' = z\pi k(k_T, k_N) + (1 - \delta)k_T + (1 - \delta)k_N + Rb \quad (1.5)$$

$$v^1(k_T, k_N, b, z; X) = \max_{k'_T, k'_N, b'} \log(c) + \beta \mathbb{E}[v(k'_T, k'_N, b', z', \zeta'; X) | z, X] \quad \text{s.t.}$$

$$c + k'_T + k'_N + b' = z\pi k(k_T, k_N) + (1 - \delta)k_T + (1 - \delta)k_N + Rb \quad (1.6)$$

$$Rb' \geq -\theta(1 - \delta)k'_T$$

$$k'_T + k'_N \geq (1 - \delta)k_T + (1 - \delta)k_N.$$

In each period, the optimization problem of an entrepreneur involves comparing two option values: v_0 and v_1 , the value of liquidating the firm and that of staying in business, respectively. By choosing to liquidate the firm, the entrepreneur bears a randomly assigned liquidation cost and sells both types of capital. Since no capital evolves into next period, i.e., $k'_T = k'_N = 0$, the entrepreneur chooses only the optimal level of bond holdings b' to achieve v_0 . (I follow the convention that negative value of b' represents borrowing) Resource constraint (1.5) says that consumption (or dividend) and purchase of new bonds are financed by profits from production, previous capital stock and returns from bonds. By staying in business and running a firm, the entrepreneur obtains v_1 by choosing the optimal holdings of next-period tangible, intangible capital and bonds. Resource constraint (1.6) indicates that the entrepreneur uses profits from production, previous capital stock and returns from bonds to finance consumption, new investment and purchase of new bonds. The optimization process takes the financial constraints and resale constraints into consideration. The former sets an upper bound to borrowing and the latter means that running a firm incurs no liquidation.

With the problems of the household and the entrepreneurs stated, I can now explain the aggregate state $X = (D, S)$. $D \equiv D(k_T, k_N, b, z)$ is the aggregate distribution of individual states. The updating rule of D is H , i.e., $D' = H(X)$. $S \equiv S(A, \theta)$ is the set of aggregate shocks whose law of motion G is exogenous and so independent of D , i.e., $S' = G(S)$. The law of motion of X can then be summarized as $X' = (D'(X), G(S))$.

1.3.3 Entrepreneur's Policy Function

Before moving on to characterize the entrepreneur's policy function, I shall first illustrate a property of the value function which enables a close-form decision rule.

Proposition 1

For $\forall m > 0$, the value function satisfies

$$v(mk_T, mk_N, mb, z, \zeta; X) = v(k_T, k_N, b, z, \zeta; X) + \frac{\log m}{1 - \beta}. \quad (1.7)$$

Proof. See Appendix A. ■

Equation (1.7) implies that as a firm scales up or down, the value of the firm to the entrepreneur changes by only a constant term, given the same individual and aggregate productivity. Define the time t beginning-of-period ratio of the tangible (and intangible) capital to total equity as below:

$$\lambda_T \equiv \frac{k_T}{k_T + k_N + b} \quad (1.8)$$

$$\lambda_N \equiv \frac{k_N}{k_T + k_N + b}. \quad (1.9)$$

Then, $\lambda_T + \lambda_N \equiv \frac{k_T + k_N}{k_T + k_N + b}$ represents the leverage ratio, because the more the firm borrows (the smaller b is), the larger $\lambda_T + \lambda_N$ is. Additionally, the leverage pair (λ_T, λ_N) are invariant as the firm scales up or down. Moreover, it can be proved that two firms that differ only in size and thus share the same beginning-of-period leverage pair (λ_T, λ_N) , will target the same end-of-period leverage pair (λ'_T, λ'_N) . (See Appendix) With these properties, it's convenient to re-scale the model and replace the state (k_T, k_N, b) with (λ_T, λ_N) which completely captures evolution of as well as relationships among k_T , k_N and b . To sum up, the property stated in equation (1.7) is useful in that it helps reduce the number of state variables by one and enables closed-form policy function below.

Policy Function

Proposition 2

There exists a shadow value of capital $q = q(\lambda_T, \lambda_N, z, \zeta; X)$. $q < 1$ if the entrepreneur is running a firm and subject to a binding resale constraint; $q = 1$ in two cases: the entrepreneur being subject to a slack resale constraint or being subject to no resale constraint (he is liquidating or has already liquidated the firm). The entrepreneur's net worth satisfies

$$n = z\pi k(k_T, k_N) + [(1 - \delta)k_T + (1 - \delta)k_N]q + Rb.$$

Then the policy functions for the consumption $c(k_T, k_N, b, z, \zeta; X)$, total amount of capital stock $k'_T(k_T, k_N, b, z, \zeta; X) + k'_N(k_T, k_N, b, z, \zeta; X)$, and bond holding $b'(k_T, k_N, b, z, \zeta; X)$ have the following closed-form:

$$\begin{aligned} c &= (1 - \beta)n \\ k'_T + k'_N &= \frac{\lambda'_T + \lambda'_N}{(q - 1)(\lambda'_T + \lambda'_N) + 1} \beta n \\ b' &= \frac{1 - \lambda'_T - \lambda'_N}{(q - 1)(\lambda'_T + \lambda'_N) + 1} \beta n, \end{aligned}$$

where $\lambda'_T(\lambda_T, \lambda_N, z, \zeta; X)$, $\lambda'_N(\lambda_T, \lambda_N, z, \zeta; X)$ and $q(\lambda_T, \lambda_N, z, \zeta; X)$ are jointly determined.

Proof. See Appendix ■

The shadow price of capital q is a function of μ , the multiplier associated with the resale constraint $k'_T + k'_N \geq (1 - \delta)(k_T + k_N)$. q reflects tightness of the resale constraint and how much the capital price is distorted. When the resale constraint is slack, $\mu = 0$ and $q = 1$; when it binds, $\mu > 0$ and $q < 1$. Moreover, under a binding resale constraint, the less resource (net worth) the entrepreneur has to sustain the current activities, the tighter the resale constraint is, so that the shadow price q is more distorted. The policy functions imply that $q(k'_T + k'_N) + b' = \beta n$. That is, a fraction $1 - \beta$ of the net worth is consumed by the entrepreneur and the remaining fraction β is saved in forms of capital investment and bonds holding. This rule holds true regardless of the binding state of the resale constraints. I leave the characterization of q , λ'_T and λ'_N into the next proposition.

1.3.4 Entrepreneur's Decision Rules

Each period an entrepreneur decides among three options: investing—running a firm and investing in capital, staying—running a firm but only staying with the current level of capital, and liquidating—selling all the capital. Based on the relative returns on capital and bonds, three possible scenarios might occur over time. First, the return on bonds is so high that the entrepreneur always liquidates the capital and invests in bonds once he has a firm. Second, the return on capital is so high that he always invests. Third, the entrepreneur liquidates only occasionally. I narrow things down to the more interesting third scenario in which entrepreneurs switch among options based on the individual productivity drawn each period. Specifically, overtime if z_h is drawn so that running a firm is profitable, the z_h -entrepreneur always chooses to invest. If z_l is drawn instead, the z_l -entrepreneur weighs the value of running an unproductive firm against that of liquidating it. In this subsection, assumptions that are necessary to reach the intended equilibrium are discussed. Then choices of the end-of-period leverage (λ'_T, λ'_N) are considered for the z_h - and z_l -entrepreneur separately. Finally, the decision rule of whether to liquidate or stay in business is investigated for the z_l -entrepreneur.

Assumptions

To generate an equilibrium in which the entrepreneur acts on the individual productivity shocks drawn, several restrictions are set on the parameters. First, I assume

$$z_l \pi \frac{\partial k}{\partial k_s} + 1 - \delta < R < z_h \pi \frac{\partial k}{\partial k_s} + 1 - \delta, \quad s = T, N,$$

or equivalently $z_l < \frac{R-(1-\delta)}{\pi \frac{\partial k}{\partial k_s}} < z_h$ holds, where $z_j \pi \frac{\partial k}{\partial k_s} + 1 - \delta$, $j \in \{h, l\}$ is the rate of return on capital, and R is the rate of return on bond-holdings. The right part requires the value of z_h to be large enough in order to generate a return on capital that is higher than that on bonds. Consequently, an entrepreneur drawing z_h always finds it optimal to run a firm by investing in capital rather than to liquidate and save in bonds. In contrast, the left part requires z_l to be small enough so that the return on capital is lower than that on bonds. If the liquidation cost is zero, an entrepreneur drawing z_l always liquidates to maximize his lifetime utility. However, existence of the positive liquidation cost gives rise to a third choice: staying with the current level of capital for another period if the liquidation cost is too high. Thus an endogenous threshold level of the liquidation cost is to be determined, a level below which the entrepreneur finds it optimal to liquidate. This is left to the next part.

The assumption above enables a distinction between two types of entrepreneurs, the z_h -entrepreneur and the z_l -entrepreneur. In the following, I discuss the determination of the shadow value q and the leverage ratios λ'_T , λ'_N for the z_h -entrepreneur and the z_l -entrepreneur separately. The z_l -entrepreneur's choice between staying and liquidating will also be analyzed.

Before proceeding, some notations are clarified here. The z_h - and z_l -entrepreneurs are modeled to make decisions separately and be independent of their previous productivity. Specifically, denote time t end-of-period pair of leverage as $(\lambda'_{Tj}, \lambda'_{Nj})$, where $j \in \{h, l\}$. Then $\lambda'_{Th} = \lambda'_T(\lambda_{Tj}, \lambda_{Nj}, z_h, \zeta; X)$, $\lambda'_{Nh} = \lambda'_N(\lambda_{Tj}, \lambda_{Nj}, z_h, \zeta; X)$, $\lambda'_{Tl} = \lambda'_T(z_l, \zeta; X)$ and $\lambda'_{Nl} = \lambda'_N(z_l, \zeta; X)$. The j in the subscript shows a z_j is drawn in time t . Then the time t beginning-of-period leverage pair, which also serves as the state variable of the model, is $(\lambda_{Tj}, \lambda_{Nj})$, indicating a z_j was drawn in time $t - 1$. Note that neither of λ'_{Tl} and λ'_{Nl} is dependent on $(\lambda_{Tj}, \lambda_{Nj})$, i.e. the z_l -entrepreneurs will be modeled to decide on the end-of-period leverage irrespective of the previous productivity drawn. More detail on this assumption will be covered later.

Similarly, k'_{Tj} and k'_{Nj} are the end-of-period tangible and intangible capital, given z_j is drawn in period t . Using the definition of the leverage ratios, rewrite the financial constraint as

$$\frac{k'_{Tj}}{k'_{Tj} + b'_j} = \frac{\lambda'_{Tj}}{1 - \lambda'_{Nj}} \leq \frac{1}{1 - \theta(1 - \delta)/R} = \lambda_\theta. \quad (1.10)$$

Note that the upper bound of borrowing is denoted as λ_θ for simplicity.

The Productive Entrepreneur

As has been discussed, a z_h -entrepreneur always invests in both types of capital, so that the resale constraint is slack and $q = 1$. Then I have the proposition below.

Proposition 3

The leverage ratios λ'_{Th} and λ'_{Nh} jointly satisfy

$$\mathbb{E}_{z,X} \left[\frac{z' \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Th}} + 1 - \delta - R'}{z' \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + [(1 - \delta) \lambda'_{Th} + (1 - \delta) \lambda'_{Nh}] q' + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] \geq 0 \quad (1.11)$$

and

$$\mathbb{E}_{z,X} \left[\frac{\frac{\lambda'_{Nh}}{1 - \lambda'_{2h}} \left(z' \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Th}} + 1 - \delta - R' \right) - \left(z' \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Nh}} + 1 - \delta - R' \right)}{z' \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + [(1 - \delta) \lambda'_{Th} + (1 - \delta) \lambda'_{Nh}] q' + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] = 0. \quad (1.12)$$

Proof. See Appendix ■

Condition (1.11) and (1.12) are essentially derived by combining euler equations of the capital and bonds. They correspond with

$$v_{k'_T} - v_{b'} = \frac{1}{1 - \lambda'_{Nh}} \mu_\lambda \quad (1.13)$$

$$v_{k'_N} - v_{b'} = \frac{\lambda'_{Th}}{(1 - \lambda'_{Nh})^2} \mu_\lambda, \quad (1.14)$$

where $v_{k'_T}$, $v_{k'_N}$ and $v_{b'}$ are the marginal value of the tangible capital, intangible capital and bond holdings, respectively.⁹ $\mu_\lambda > 0$ is the multiplier associated with financial constraint (1.10). Moving on to the z_h -entrepreneur's decision on $(\lambda'_{Th}, \lambda'_{Nh})$, if the financial constraint is slack, he chooses the tangible and intangible capital as if there's no restriction on borrowing. Since $\mu_\lambda = 0$, it follows from (1.13) and (1.14) that the entrepreneur optimizes by choosing the pair of leverage ratios to equate $v_{k'_T}$, $v_{k'_N}$ and $v_{b'}$. If the firm is financially constrained however, the entrepreneur borrows with a limit and adjusts the pair of leverage ratios to satisfy $\mu_\lambda > 0$ and the financial constraint $\frac{\lambda'_{Th}}{1 - \lambda'_{Nh}} = \lambda_\theta > 1$. Then it holds that $v_{b'} < v_{k'_T} < v_{k'_N}$. That is, the entrepreneur deviates from the leverage ratios of the slack case by reducing investment in both types of capital. In addition, under a binding financial constraint, the investment in the tangible capital benefits the firm by expanding its borrowing capacity, so the reduction in the tangible is less than that in the intangible, leading to a ratio of tangible to intangible capital that is higher than the one in the slack case.

⁹After normalizing and transforming the original problem into one where λ_{Th} and λ_{Nh} play as state variables, $v_{k'_T}$, $v_{k'_N}$ and $v_{b'}$ are evaluated at $k'_T = \lambda'_{Th}$, $k'_N = \lambda'_{Nh}$ and $b' = 1 - \lambda'_{Th} - \lambda'_{Nh}$ respectively.

The Unproductive Entrepreneur

As mentioned at the beginning of section 1.3.4, a liquidation cost stops the entrepreneur from liquidating the firm immediately after drawing z_l . Instead, he makes liquidation decisions by weighing the value of liquidating against that of staying. Assuming for the moment that the z_l -entrepreneur has already chosen to stay in business, then the decision rule of his leverage ratios is investigated first, before the threshold value of the liquidation cost is determined.

Consider again the financial constraint in the form of leverage ratios, $\frac{\lambda'_T}{1-\lambda'_N} \leq \lambda_\theta$. Let pair $(\lambda_T^c, \lambda_N^c)$ denote the leverage of a firm under a binding financial constraint, i.e., $\frac{\lambda_T^c}{1-\lambda_N^c} = \lambda_\theta$. Then it can be proved that under a given state, $\lambda'_T + \lambda'_N$ is maximized when the firm borrow to the maximum, and the corresponding maximum value is $\lambda_T^c + \lambda_N^c$. Specially for the z_l -entrepreneur, denote the maximum value as $\lambda_{Tl}^c + \lambda_{Nl}^c$. Then I have Proposition 4.

Proposition 4

Suppose the entrepreneur draws z_l and the state $(\lambda_{Tj}, \lambda_{Nj})$, $j = h, l$ satisfies

$$(\lambda_{Tl}^c + \lambda_{Nl}^c) \beta \left[z_l \pi k \left(\frac{\lambda_{Tj}}{\lambda_{Tj} + \lambda_{Nj}}, \frac{\lambda_{Nj}}{\lambda_{Tj} + \lambda_{Nj}} \right) + 1 - \delta + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) \right] < 1 - \delta, \quad (1.15)$$

then both his financial constraint and resale constraint bind if he chooses to stay in business.

Proof. See Appendix ■

The intuition behind Proposition 4 is that for an unproductive entrepreneur who chooses to keep running a firm, the net worth of the firm is so low that even if the entrepreneur borrows to the maximum, resources are still not sufficient to sustain the consumption and investment. If there's no liquidation cost, the entrepreneur would sell the capital for financial flexibility. However, the existence of the liquidation cost creates incentive to stay with the current capital for another period, which subjects the entrepreneur to a binding resale constraint.

In the type of equilibrium where both the financial constraint and resale constraint bind for a z_l -entrepreneur, I have

$$\frac{\lambda'_{Tl}}{1 - \lambda'_{Nl}} = \lambda_\theta, \quad (1.16)$$

and $k'_{Tl} + k'_{Nl} = (1 - \delta)(k_{Tj} + k_{Nj})$, $j \in \{h, l\}$. When the entrepreneur decides to keep running his firm with a low productivity, he sustains his consumption by borrowing to

the maximum amount allowed and refraining from increasing the total amount of capital stock. To pin down λ'_{Tl} and λ'_{Nl} , I assume additionally that the z_l -entrepreneur aggregates capital using **Leontief Function**, $k_l = k(k_{Tl}, k_{Nl}) = \min\left\{\frac{k_{Tl}}{\sigma}, \frac{k_{Nl}}{1-\sigma}\right\}$, in which the firm always use the two types of capital in a fixed proportion: $\frac{k_{Tl}}{\sigma} = \frac{k_{Nl}}{1-\sigma} \equiv k_l$. Therefore, if z_l is drawn and the entrepreneur chooses to stay, he adjusts the ratio of the tangible and intangible capital to satisfy $\frac{k'_{Tl}}{k'_{Nl}} = \frac{\sigma}{1-\sigma}$, while keeps the total amount of capital unchanged.¹⁰ Normalizing with the definition of the leverage ratios, I have

$$\frac{\lambda'_{Tl}}{\lambda'_{Nl}} = \frac{\sigma}{1-\sigma}. \quad (1.17)$$

This fixed relationship enables me to rewrite the financial constraint more compactly. Define $\lambda'_l = \lambda'_{Tl} + \lambda'_{Nl}$, then the constraint (1.16) can be transformed to

$$\lambda'_l \equiv \lambda'_{Tl} + \lambda'_{Nl} = \frac{1}{1 - \frac{\sigma\theta(1-\delta)}{R}} \equiv \lambda_\sigma. \quad (1.18)$$

Notice that $\lambda_\sigma = \frac{1}{1 - \frac{\sigma\theta(1-\delta)}{R}}$ is defined for simplicity. Thanks to the foregone relationship between λ_{Tl} and λ_{Nl} , λ_l can entirely replace the state pair $(\lambda_{Tl}, \lambda_{Nl})$ and serve as the state of the z_l -entrepreneur. After λ'_l is solved for, λ'_{Tl} and λ'_{Nl} can be immediately backed out using (1.17).

Instead of pinning down λ'_{Tl} and λ'_{Nl} by modeling the resale constraint as a restriction on *the total amount of capital* and meanwhile assuming Leontief aggregation function, an alternative is modeling resale constraint for *each individual type of capital separately*, i.e., $k'_T \geq (1-\delta)k_T$ and $k'_N \geq (1-\delta)k_N$. By doing so, when both resale constraints bind, I arrive at $k'_{Tl} = (1-\delta)k_{Tj}$ and $k'_{Nl} = (1-\delta)k_{Nj}$. λ'_{Tl} and λ'_{Nl} can be pinned down by $\frac{\lambda'_{Tl}}{\lambda'_{Nl}} = \frac{\lambda_{Tj}}{\lambda_{Nj}}$. However, this setup potentially causes unit root. Because suppose the entrepreneur was unproductive last period, then it holds that $\frac{\lambda'_{Tl}}{\lambda'_{Nl}} = \frac{\lambda_{Tl}}{\lambda_{Nl}}$, which means steady state values of λ_{Tl} and λ_{Nl} cannot be pinned down in this case. Assuming a Leontief function fixes the ratio $\frac{\lambda'_{Tl}}{\lambda'_{Nl}}$ at $\frac{\sigma}{1-\sigma}$, which resolves the unit root problem by breaking down the connection between $(\lambda'_{Tl}, \lambda'_{Nl})$ and $(\lambda_{Tl}, \lambda_{Nl})$. If the current unproductive entrepreneur was productive last period, then the last-period ratio $\frac{\lambda_{Tl}}{\lambda_{Nl}}$ might deviate from $\frac{\sigma}{1-\sigma}$, which means the entrepreneur needs to adjust each type of capital to meet the fixed ratio entering the new period. However, it turns out in the quantitative practice that, the adjustment required is in fact so trivial that it can be neglected.¹¹ If the current unproductive entrepreneur was otherwise also unproductive last period, then

¹⁰That means a one-to-one exchange between the tangible and intangible capital can take place, generating partial liquidations of either tangible or intangible capital ($I_T + I_N = 0$, and either $I_T \leq 0$ or $I_N \leq 0$ holds).

¹¹This is because the aggregation function of a z_h -entrepreneur is a CES kind so that $\frac{\lambda'_{Th}}{\lambda'_{Nh}}$ does not deviate much from $\frac{\sigma}{1-\sigma}$.

no adjustment is needed, as the fixed ratio has already been reached last period. To sum up, this is in effect an equilibrium that is very close to the result which is intended to reach by assuming $k'_T \geq (1 - \delta) k_T$ and $k'_N \geq (1 - \delta) k_N$ separately. Therefore, I assume the Leontief aggregation so that the leverage ratios can be pinned down by equation (1.16) and (1.17).

Option Values: Weighing Liquidating Against Staying

In this part, I consider the liquidation decision-making process of a z_l -entrepreneur. Specifically, how he weighs the option value of liquidating against that of staying and determines the threshold level of the liquidation cost.

Before proceeding, I first define variables that are exclusive to the current-period z_l -entrepreneur. These variables are intertemporal in that they are dependent on productivity of both today and yesterday. First $\zeta_{jl} = \zeta((\lambda_{Tj}, \lambda_{Nj}), z_l; X)$ is the threshold liquidation cost and $q_{jl} = q((\lambda_{Tj}, \lambda_{Nj}), z_l; X)$ is the shadow price of capital, where the beginning-of-period leverage pair $(\lambda_{Tj}, \lambda_{Nj})$ indicates a z_j was drawn previously, and z_l indicates a z_l is drawn currently. With the Leontief aggregation, $(\lambda_{Tl}, \lambda_{Nl})$ boils down to λ_l .

Further for ease of convenience, I use some foregone facts to re-denote variables so that both the beginning-of-period leverage and the targeted end-of-period leverage are explicit. Recall that as long as z_l is drawn, the entrepreneur who stays in business is financially constrained and targets $\lambda'_l = \lambda_\sigma$. Then replacing z_l by $\lambda'_l = \lambda_\sigma$ and ignoring the aggregate state X for simplicity at the moment, ζ_{jl} and q_{jl} can be rewritten as $\zeta_{jl} = \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$, and $q_{jl} = q((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$. To sum up, since $\lambda_l = (\lambda_{Tl}, \lambda_{Nl}) = \lambda_\sigma$, I have $\zeta_{hl} = \zeta((\lambda_{Th}, \lambda_{Nh}), \lambda_\sigma)$ and $\zeta_{ll} = \zeta(\lambda_\sigma, \lambda_\sigma)$. Then I have the following Proposition.

Proposition 5

1. For each $j \in (h, l)$, there exists a unique threshold $\zeta_{jl} = \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma; X)$, a liquidation cost below which the z_l -entrepreneur liquidates the entire firm. The shadow price is

$$q_{jl} = \begin{cases} 1 & \text{if } \zeta < \zeta_{jl}, \\ q_{jl}((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma; X) & \text{if } \zeta \geq \zeta_{jl}. \end{cases}$$

When the z_l -entrepreneur chooses to stay in business, $q_{jl} < 1$ satisfies

$$(1 - \delta)(\lambda_{Tj} + \lambda_{Nj}) \left[q_{jl} - 1 + \frac{1}{\lambda'_{Tl} + \lambda'_{Nl}} \right] = \beta n^*, \quad (1.19)$$

where $n^* = z_l \pi k (\lambda_{Tj}, \lambda_{Nj}) + (1 - \delta) (\lambda_{Tj} + \lambda_{Nj}) q_{jl} + R(1 - \lambda_{Tj} - \lambda_{Nj})$ is the net worth of staying which is normalized by dividing $k_{Tj} + k_{Nj} + b$.

2. Define further $\Delta_{jl} = \Delta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma; X)$, where

$$\Delta_{jl} = \log \left(\frac{z_l \pi k (\lambda_{Tj}, \lambda_{Nj}) + (1 - \delta) (\lambda_{Tj} + \lambda_{Nj}) + R(1 - \lambda_{Tj} - \lambda_{Nj})}{z_l \pi k (\lambda_{Tj}, \lambda_{Nj}) + (1 - \delta) (\lambda_{Tj} + \lambda_{Nj}) q_{jl} + R(1 - \lambda_{Tj} - \lambda_{Nj})} \right). \quad (1.20)$$

Then, for each $j \in \{h, l\}$, the threshold liquidation cost $\zeta_{jl} = \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma; X)$ satisfies the following forward-looking equation:

$$\frac{\Delta_{jl}}{1 - \beta} - \zeta_{jl} = \beta p^{lh} \frac{\log(\tilde{\lambda}' n'_h / R')}{1 - \beta} + \beta p^{ll} \frac{\log(\tilde{\lambda}' n'_l / R')}{1 - \beta} + \beta p^{ll} \left(-\zeta'_{ll} + \int_0^{\zeta'_{ll}} F(\zeta') d\zeta' \right), \quad (1.21)$$

where:

$$\begin{aligned} \tilde{\lambda}' &= \frac{\lambda'_{Tl} + \lambda'_{Nl}}{1 + (q_{jl} - 1)(\lambda'_{Tl} + \lambda'_{Nl})} \\ n'_h &= z_h \pi' + (1 - \delta) + R' \left(\frac{1}{\lambda'_{Tl} + \lambda'_{Nl}} - 1 \right) \\ n'_l &= z_l \pi' + (1 - \delta) + R' \left(\frac{1}{\lambda'_{Tl} + \lambda'_{Nl}} - 1 \right). \end{aligned}$$

Proof. See Appendix. ■

As has been mentioned, q_{jl} reflects the tightness of the resale constraint and the distortion of consumption. If the z_l -entrepreneur has decided to keep running the firm, he then subjects himself to the binding resale constraint and has to maintain the current level of capital with a low productivity. The decision of not selling capital distorts the consumption. The less net worth the entrepreneur has, the harder for him to maintain the current level of capital, so that more consumption is distorted, and the resale constraint turns tighter.

ζ_{jl} is the threshold level of the liquidation cost that equates the option value of staying and that of liquidating. As mentioned previously, the existence of the random liquidation cost ζ lowers the option value of liquidating and creates the incentive for the z_l -entrepreneur to stay for another period in business, hoping to draw a higher productivity tomorrow. If the liquidation cost is however small, the option value of liquidating is higher than that of staying, the z_l -entrepreneur chooses to sell the firm for more liquidity. ζ_{jl} is thus the threshold level that makes the entrepreneur indifferent between liquidating and staying.

Next consider Δ_{jl} , of which the denominator is the normalized current-period net worth

if the z_l -entrepreneur chooses to stay while the numerator is the normalized current-period net worth if he chooses to liquidate. Δ_{jl} is then the difference between net worth of liquidating and staying. Note that $\Delta_{jl} > 1$ holds, due to the distortion of shadow price if stay ($q_{jl} < 1$). With the definition of Δ_{jl} , the threshold ζ_{jl} is determined according to the proposition.

Left-hand side of equation (1.21) captures the gain of liquidating over staying for the entrepreneur, and right-hand side of it represents the option value of staying, reflecting a forward looking perspective. ζ_{jl} solves (1.21) by equating the gain of liquidating and the option value of staying, i.e., an z_l -entrepreneur drawing a liquidation cost $\zeta = \zeta_{jl}$ is indifferent between liquidating and staying. If however the liquidation cost drawn is higher than the threshold value, the entrepreneur who finds it too costly to liquidate and incur the cost will choose to stay for another period in business, waiting for a recovery of the productivity.

For a z_l -entrepreneur, $F(\zeta_{jl})$ is the probability of liquidating while $1 - F(\zeta_{jl})$ is the probability of staying. Then, equation (1.11) and (1.12) each, which determines the end-of-period leverage ($\lambda'_{Th}, \lambda'_{Nh}$) of the period t z_h -entrepreneur, can be factored into three terms: the entrepreneur remaining productive in $t + 1$, turning unproductive in $t + 1$ so liquidating, and turning unproductive but choosing to stay for another period. (See Appendix)

1.3.5 Aggregation and Recursive Equilibrium

Wealth Evolution

As is revealed in the policy function, regardless of the individual productivity, both types of entrepreneur save a fixed fraction β of their net worth. In addition, a fraction λ'_{Tj} of the total saving is assigned to the tangible capital and another fraction λ'_{Nj} is assigned to the intangible capital. As a result of these linearity, bonds and both types of capital stock move in proportion with the changing aggregate net worth, making it possible to track the wealth evolution. For $j \in \{h, l\}$, denote K_{Tj} and K_{Nj} as the aggregate tangible and intangible capital held by the z_h - and z_l -entrepreneurs. Then $K_j = K(K_{Tj}, K_{Nj})$ is an aggregation of the aggregate tangible and intangible capital. Denote B as the aggregate bond holdings of entrepreneurs who ran no firms at $t - 1$. The aggregate evolution of wealth is captured in the following equations:

$$K'_{Th} = \lambda'_{Th}\beta \sum_j \left[\frac{z_h\pi K_j}{K_{Tj} + K_{Nj}} + (1 - \delta) + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) \right] (K_{Tj} + K_{Nj}) p^{jh} + \lambda'_{Th}\beta R p^{lh} B \quad (1.22)$$

$$K'_{Nh} = \lambda'_{Nh}\beta \sum_j \left[\frac{z_h \pi K_j}{K_{Tj} + K_{Nj}} + (1 - \delta) + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) \right] (K_{Tj} + K_{Nj}) p^{jh} + \lambda'_{Nh}\beta R p^{lh} B \quad (1.23)$$

$$K'_{Tl} = \sigma (1 - \delta) \sum_j \left[1 - F(\tilde{\zeta}_{jl}) \right] p^{jl} (K_{Tj} + K_{Nj}) \quad (1.24)$$

$$K'_{Nl} = (1 - \sigma) (1 - \delta) \sum_j \left[1 - F(\tilde{\zeta}_{jl}) \right] p^{jl} (K_{Tj} + K_{Nj}) \quad (1.25)$$

$$B' = \beta \sum_j F(\tilde{\zeta}_{jl}) \left[\frac{z_l \pi K_j}{K_{Tj} + K_{Nj}} + (1 - \delta) + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) \right] (K_{Tj} + K_{Nj}) p^{jl} + \beta R p^{ll} B. \quad (1.26)$$

Equation (1.22) and (1.23) illustrate sources of end-of-period tangible and intangible capital held by the mass of productive entrepreneurs. A fraction $\lambda'_{Th}\beta$ of the net worth held by entrepreneurs who turns productive in t goes to end-of-period tangible capital and a fraction $\lambda'_{Nh}\beta$ goes to the intangible capital. Equation (1.24) and (1.25) say that for entrepreneurs who turn unproductive at t , the capital will be held by those who choose to stay in business. Among the aggregate beginning-of-period capital of those who stay, the tangible capital accounts for a fraction of σ , with the remainder going to the intangible. Equation (1.26) describes evolution of the aggregate bonds in the hands of the entrepreneurs who do not run firms. They come from the net worth of those unproductive entrepreneurs who liquidate in t and the net worth of those entrepreneurs who did not run firms in $t - 1$.

Recursive Equilibrium

$\mathcal{L} \subset \mathcal{R}_+^4$ is a compact set of all possible values of leverage ratios $(\lambda_{Th}, \lambda_{Nh}, \lambda_{Tl}, \lambda_{Nl})$. $\mathcal{K} \subset \mathcal{R}_+^6$ is a compact set of all possible $(B_H, B, K_{Th}, K_{Nh}, K_{Tl}, K_{Nl})$. Then $\mathcal{L} \times \mathcal{K}$ contains all possible values of endogenous states. $\mathcal{A} \subset \mathcal{R}_+^2$ is a compact set containing all possible exogenous states (A, θ) . Finally, $\mathcal{X} = \mathcal{L} \times \mathcal{K} \times \mathcal{A}$ is the compact set containing all possible state variables.

Recursive Competitive Equilibrium is a mapping from \mathcal{X} to \mathcal{X} that consists of the household's policy function $(C_H, L_H, B'_H) : \mathcal{X} \rightarrow \mathcal{R}_+^3$, leverage pair $(\lambda'_{Tj}, \lambda'_{Nj}) : z_j \times \mathcal{X} \rightarrow \mathcal{L}$ (for $j \in \{h, l\}$), z_l -entrepreneurs' decision rules $(q_{jl}, \Delta_{jl}, \zeta_{jl}) : z_l \times \mathcal{X} \rightarrow \mathcal{R}^3$, and pricing $(\omega, \pi, R') : \mathcal{X} \rightarrow \mathcal{R}_+^3$ that satisfy

1. the household's policy function (C_H, L_H, B'_H) solves (1.1), (1.2), and (1.3);

2. leverage ratios $(\lambda'_{Tl}, \lambda'_{Nl}, \lambda'_{Th}, \lambda'_{Nh})$ satisfy (1.16), (1.17), (1.11) and (1.12);
3. z_l -entrepreneurs' decision rules $(q_{jl}, \Delta_{jl}, \zeta_{jl})$ are determined by (1.19), (1.20) and (1.21);
4. wage rate ω solves (1.4), and R' and π are adjusted to clear the credit market and labor market

$$\sum_{j=h,l} \left[\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right] (K_{Tj} + K_{Nj}) + B + B_r = 0 \quad (1.27)$$

$$\left(\frac{\pi}{\alpha} \right)^{\frac{1}{1-\alpha}} \sum_j (p^{jh} z_h + p^{jl} z_l) K_j = AL_r. \quad (1.28)$$

Aggregate Variables

The output of an individual entrepreneur i is $y_i = \frac{\pi}{\alpha} z_i k_i$. Then aggregate output is calculated as the sum of output for z_h - and z_l -entrepreneurs:

$$Y = \frac{\pi}{\alpha} \left[\sum_j (p^{jh} z_h + p^{jl} z_l) K_j \right],$$

where $K_j = K(K_{Tj}, K_{Nj})$ for $j = h, l$. Based on the form of the individual production function $y_i = (z_i k_i)^\alpha (AL)^{1-\alpha}$, total factor productivity on aggregate level can be defined as

$$TFP \equiv \frac{Y}{\left(\sum_j K_j \right)^\alpha L^{1-\alpha}} = \frac{(\sum_i z_i k_i)^\alpha A^{1-\alpha} L^{1-\alpha}}{\left(\sum_j K_j \right)^\alpha L^{1-\alpha}} = A^{1-\alpha} \left[\frac{\sum_j (p^{jh} z_h + p^{jl} z_l) K_j}{\sum_j K_j} \right]^\alpha.$$

Then define $r_k \equiv \frac{K_h}{K_l}$ as the ratio of capital stock holding by z_h - and z_l -entrepreneurs. In addition, define $\bar{z}^h \equiv p^{hh} z_h + p^{hl} z_l$ as the average productivity for previously productive entrepreneurs, and $\bar{z}^l \equiv p^{lh} z_h + p^{ll} z_l$ the average productivity for previously unproductive entrepreneurs. The TFP can be rewritten as

$$TFP = A^{1-\alpha} \left(\frac{(p^{hh} z_h + p^{hl} z_l) r_k + p^{lh} z_h + p^{ll} z_l}{r_k + 1} \right)^\alpha = A^{1-\alpha} \left(\bar{z}^h - \frac{\bar{z}^h - \bar{z}^l}{r_k + 1} \right)^\alpha. \quad (1.29)$$

Notice that in (1.29), if $\bar{z}^h > \bar{z}^l$ holds, TFP is increasing in r_k . In effect, this form defines TFP as a measurement of efficiency. Allocating more capital stock to the productive firms leads to improvement in efficiency.

The aggregate liquidation of capital is the sum of aggregate liquidation of tangible capital (L_T) and that of intangible capital (L_N). In addition, each aggregate liquidation

term is the full liquidation (CL) plus the adjustment of capital (CA):.

$$L = L_T + L_N = (CL_T + CA_T) + (CL_N + CA_N).$$

For more explanation to CL and CA , see Appendix.

Aggregate capital expenditure is the aggregate value that productive entrepreneurs spend on purchasing capital, both old capital from liquidation and new capital invested:

$$CX_T = K'_{Th} - (1 - \delta) \sum_j p^{jh} K_{Tj}, \quad CX_N = K'_{Nh} - (1 - \delta) \sum_j p^{jh} K_{Nj}.$$

Aggregate capital investment, is the change in capital stock over one period. It's calculated as aggregate capital expenditure less aggregate capital liquidation:

$$I = I_T + I_N = (CX_T - L_T) + (CX_N - L_N) = CX - L.$$

The Liquidation-Expenditure Ratio (L/X) is defined as the ratio of aggregate capital liquidation to aggregate capital expenditure:

$$L/X = \frac{L}{L + I} = \frac{L}{CX}.$$

1.4 Model Results

In this section, I investigate the effects of a decreasing tangible share on the economy both analytically and quantitatively. Analytically, I present how the technological change affects the liquidation decision of the entrepreneurs. Quantitatively, I show changes in the aggregate variables and impulse responses to shocks before and after the technological change.

1.4.1 Calibration

Capital Aggregation Function of the productive entrepreneur

As has been discussed in the subsection of the z_l -entrepreneur's decision rule, I model the unproductive firm to aggregate capital with Leontief function, in which the firms always hold the tangible and intangible capital in a fixed proportion. As for the z_h -entrepreneur, I consider two types of aggregation functions, the regular Constant Elasticity Substitution (CES) production function and the Leontief production function as a special case of the former. Let me start with the Leontief function which is simpler and more intuitive.

With **Leontief Function** the productive firms share the same aggregation technology as the unproductive firm, i.e.,

$$\frac{k_{Th}}{\sigma} = \frac{k_{Nh}}{1-\sigma} \equiv k_h(k_{Th}, k_{Nh}).$$

Apply the definition of the leverage ratios, it follows that

$$\frac{\lambda_{Th}}{\sigma} = \frac{\lambda_{Nh}}{1-\sigma}. \quad (1.30)$$

The Leontief technology substantially simplifies the z_h -entrepreneur's decision process. All he needs is to produce with a fixed proportion of the tangible and intangible capital subject to the financial constraint. Recall the state pair of the z_h -entrepreneur is $(\lambda_{Th}, \lambda_{Nh})$. With the Leontief function, the ratio of λ_{Th} to λ_{Nh} is fixed at $\frac{\sigma}{1-\sigma}$. This relationship helps reduce the number of the state variables. That is, let $\lambda_h = \lambda_{Th} + \lambda_{Nh}$ so that the state pair $(\lambda_{Th}, \lambda_{Nh})$ can be entirely summarized by λ_h , the same as in the case of the z_l -entrepreneur where λ_l is enough to serve as the state. Again, given the fixed ratio between λ_{Th} and λ_{Nh} , the z_h -entrepreneurs' financial constraint $\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \leq \lambda_\theta$ can be rewritten as

$$\lambda'_h = \lambda'_{Th} + \lambda'_{Nh} \leq \frac{1}{1 - \frac{\theta\sigma(1-\delta)}{R}} = \lambda_\sigma, \quad (1.31)$$

which shares the same form as the z_l -entrepreneur's financial constraint.

To sum up, when the productive firms aggregate capital with the Leontief technology, the state $(\lambda_{Th}, \lambda_{Nh})$ can be summarized by λ_h and the z_h -entrepreneur chooses only the total leverage λ'_h . After λ'_h is obtained, λ'_{Th} and λ'_{Nh} can immediately be backed out using (1.30).

Constant Elasticity Substitution (CES) Function has the following form:

$$k_h(k_{Th}, k_{Nh}) = \left[\sigma \left(\frac{k_{Th}}{\sigma} \right)^{-\rho} + (1-\sigma) \left(\frac{k_{Nh}}{1-\sigma} \right)^{-\rho} \right]^{-\frac{1}{\rho}}.$$

The elasticity of substitution is $\frac{1}{1+\rho}$. When $\rho \rightarrow \infty$, the function boils down to the Leontief form, indicating zero substitution between the two production factors. The parameter σ can be thought of as a long-run share of the tangible capital in production, while the ratio $\frac{\sigma}{1-\sigma}$ is a technological constraint in capital choice towards which composition of the two types of capital target in the long run. The actual share of the tangible capital can deviate from the central tendency given by σ , especially when the tangible capital provides the benefit of expanding the debt capacity of a firm. As a result, the CES function of the productive entrepreneurs implies more flexibility of adjusting the capital composition compared to their unproductive peers who have to produce with a fixed

Table 1.1: Parameters

Description	Parameter	Value	Target
Household discount factor	β^h	0.98	Annual risk-free rate 2%
Household risk aversion	ε	1	Log-utility
Elasticity of labor supply	ν	1	Elasticity=1
Utility weight on leisure	μ	4.0931	Working hours 0.25
Capital depreciation rate	δ	0.0925	Investment/Output=0.185
Capital share of output	α	0.33	Capital share=1/3
Entrepreneur discount factor	β	0.9044	Capital/Output=2
Share of tangible capital	σ	0.5	Tangible=Intangible
Elasticity of substitution	ρ	-0.3	Standard value
Transition probability	p^{hh}	0.8450	Persistence=0.69, s.d.=0.18
Transition probability	p^{ll}	0.8450	$p^{ll} = p^{hh}$
Idiosyncratic productivity, high	z_h	2.1247	Persistence=0.69, s.d.=0.18
Idiosyncratic productivity, low	z_l	0.4707	$\log(z_h^\alpha) = -\log(z_l^\alpha)$
Financial constraint	θ	0.8319	Debt/Output=0.74
Upper bound of $F(\cdot)$	$\bar{\zeta}$	30.0454	L/X ratio=0.3
Persistence, financial shock	ρ_θ	0.9279	Cyclical behavior of debt
Persistence, productivity shock	ρ_A	0.2955	Cyclical behavior of output
Std.dev. financial shock	σ_θ	0.0581	Cyclical behavior of debt
Std.dev., productivity shock	σ_A	0.0134	Cyclical behavior of output

proportion of each type of capital.

Parameters

I follow the approach in Cui (2017) and calibrate the model parameter to match U.S. long-run aggregate statistics from 1971 to 2015 in annual frequencies.

As a convention, the capital share $\alpha = 0.33$ is set. Then the procedure developed in Tauchen (1986) is used to calibrate the two-point Markov process of the idiosyncratic productivity. Specifically, set the transition probability to satisfy $p^{hh} = p^{ll}$ and denote standard deviation of the shock as ξ , then the two levels of the idiosyncratic shock are

$$\log(z_h^\alpha) = \xi \quad \text{and} \quad \log(z_l^\alpha) = -\xi.$$

The moments in the data is taken from Ábrahám, White, et al. (2006) which fits the idiosyncratic productivity to an AR(1) process. The persistence is $p = 0.69$, while the standard deviation of the shock is $d = 0.18$. Matching the two-point Markov chain to moments in the data, ξ and p^{hh} can be solved as $\xi = \sqrt{\frac{d^2}{1-p^2}}$ and $p^{hh} = \frac{1+p}{2}$. Correspondingly, $z_h = 2.1247$ and $z_l = 0.4707$ can be solved for.

The household's discount factor β^h is calibrated to match an annual real risk-free rate

of 2%. The utility function of the household has the following form:

$$u^r(C_r, L_r) = \log \left(C_r - \frac{\mu L_r^{1+\nu}}{1+\nu} \right).$$

The elasticity of labor supply ν is typically set to be 1. $\mu = 4.0931$ is calibrated to match percent of total hours used on working $L_r = 0.25$.

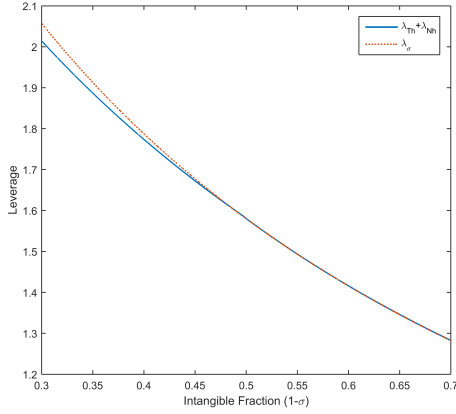
A uniform distribution is used for the distribution function of liquidation cost $F(\cdot)$ to keep the solution tractable. The lower bound is zero, while the upper bound is $\bar{\zeta}$. The remaining parameters, including the depreciation rate δ , the financial constraint θ , the entrepreneurs' discount factor β , and the upper bound of the distribution of liquidation cost $\bar{\zeta}$, are calibrated to match several U.S. long-run aggregate statistics from 1971 to 2015 in steady state.

The aggregate productivity shocks and financial shocks both follow AR(1) processes, with ρ_A and ρ_θ being the persistence and $\epsilon_t^A \sim N(0, \sigma_A^2)$ and $\epsilon_t^\theta \sim N(0, \sigma_\theta^2)$ being i.i.d random variables. The estimation result in Cui (2017) is used, which is estimated using cyclical components of output and debt observed.

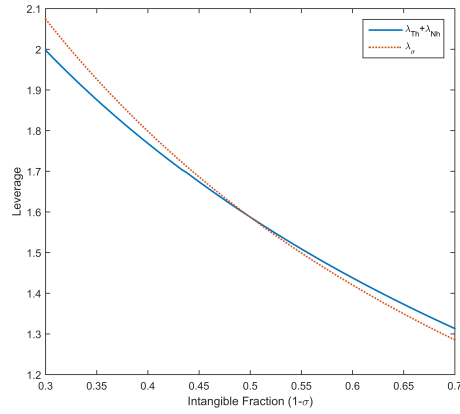
The calibration result is summarized in Table 1.1. With the parameters, the model is solved around the steady state using log-linearization. The long-run share of tangible capital σ is set to be 0.5 (corresponds to intangible share of 0.5 in 2000s) as benchmark. Assuming σ changes linearly, I vary it from 0.7 (corresponds to intangible share of 0.3 in 1980s) to 0.3 (a projected value corresponding to a intangible share of 0.7) following Falato et al. (2013) to see how entrepreneurs respond with their liquidation decision.

1.4.2 Analytic Results

In this subsection, effects of a decreasing share of tangible capital on the liquidation decisions of the z_l -entrepreneur is analyzed. The reduction is reflected by a decrease in σ , which then influences the liquidation decision through its impact on the choices of the leverage ratios λ'_{Tj} and λ'_{Nj} . As can be seen later, λ_{Tj} and λ_{Nj} enter the liquidation decision equation in forms of two combinations: one in the form of the capital aggregation $k \left(\frac{\lambda_{Tj}}{\lambda_{Tj} + \lambda_{Nj}}, \frac{\lambda_{Nj}}{\lambda_{Tj} + \lambda_{Nj}} \right)$, the other in the form of the total leverage $\lambda_{Tj} + \lambda_{Nj}$. In effect, the first form shows up only when the z_h -entrepreneur produces using CES aggregation and is financially constrained. Moreover, as σ changes, its influence on the capital aggregation function is limited and negligible. Hence, in the following, changes of the total leverage $\lambda_{Tj} + \lambda_{Nj}$ in response to the reduction of σ are first explored, followed by their effects on the liquidation decisions.



(a) Leontief function



(b) CES function

Figure 1.1: Changes of the leverages as the intangible share increases

Note: $\lambda_{Th} + \lambda_{Nh}$ is the total leverage of the z_h -entrepreneur, and λ_σ is the total leverage of the z_l -entrepreneur. As σ goes down, the financial constraint of the z_l -entrepreneur is always binding, and it holds that $\frac{\lambda_{Tl}}{\lambda_{Nl}} = \frac{\sigma}{1-\sigma}$. The financial constraint of the z_h -entrepreneur turns binding at the point where $\lambda_{Th} + \lambda_{Nh}$ intersects λ_l . To the left of the intersection, the financial constraint is slack and it holds that $\frac{\lambda_{Th}}{\lambda_{Nh}} = \frac{\sigma}{1-\sigma}$. To the right of the intersection, the financial constraint is binding and $\frac{\lambda_{Th}}{\lambda_{Nh}} = \frac{\sigma}{1-\sigma}$ hold if Leontief aggregation is used; $\frac{\lambda_{Th}}{\lambda_{Nh}} > \frac{\sigma}{1-\sigma}$ hold if CES aggregation is used.

The Effects of Tangible Share Reduction on Leverage Ratios

The key parameter σ as the long-run share of tangible capital impacts the decision rule of the leverages through both the capital aggregation functions and the financial constraints. In this part, I analyze how the steady-state total leverage changes as the tangible share falls, using different capital aggregation functions of the z_h -entrepreneur. As the financial constraints of both types of entrepreneurs are essential in the analysis, I repeat them below.

Financial constraints faced by the z_l -entrepreneur: $\lambda'_l = \lambda'_{Tl} + \lambda'_{Nl} = \lambda_\sigma$.

Financial constraints faced by the z_h -entrepreneur (Leontief): $\lambda'_h = \lambda'_{Th} + \lambda'_{Nh} \leq \lambda_\sigma$.

Financial constraints faced by the z_h -entrepreneur (CES): $\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \leq \lambda_\theta$, where $\lambda_\sigma = \frac{1}{1-\frac{\theta\sigma(1-\delta)}{R}}$ and $\lambda_\theta = \frac{1}{1-\frac{\theta(1-\delta)}{R}}$.

Figure 1.1 summarizes content of this part.

Given the same σ , compare $\lambda_{Th} + \lambda_{Nh}$ and λ_l

If the z_h -entrepreneur aggregates capital with Leontief function as the z_l -entrepreneur does, and if the z_h -entrepreneur borrows to the limit, it follows immediately that both entrepreneurs target the same total leverage, i.e., $\lambda'_{Th} + \lambda'_{Nh} = \lambda'_h = \lambda'_l$ (The right part of Figure 1.1 (a)). In addition, when $\lambda_h = \lambda_l$ holds, a special steady-state equilibrium

emerges. That is, all else equal, two entrepreneurs, one is previously productive while the other is previously unproductive, make the same liquidation decision if they both turn unproductive at present. This follows directly by noticing $\zeta(\lambda_h, \lambda_l) = \zeta(\lambda_l, \lambda_l)$. When the z_h -entrepreneur faces a slack financial constraint, I have $\lambda'_{Th} + \lambda'_{Nh} = \lambda'_l < \lambda'_l$ (The left part of Figure 1.1 (a)).

If the z_h -entrepreneur aggregates capital with CES technology, first consider the case in which the financial constraint is slack ($\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} < \lambda_\theta$). Recall that the z_h -entrepreneur chooses the end-of-period leverage ratios λ'_{Th} and λ'_{Nh} to equate the marginal value of tangible capital, intangible capital and bond-holdings. Given this rule, the CES technology implies an optimal tangible-to-intangible ratio of $\frac{\sigma}{1-\sigma}$, i.e., $\frac{\lambda'_{Th}}{\lambda'_{Nh}} = \frac{\sigma}{1-\sigma}$. Together with $\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} < \lambda_\theta$, it can be proved that $\lambda'_{Th} + \lambda'_{Nh} < \lambda'_l = \lambda_\sigma$ (See Appendix). Consider next the case in which the entrepreneurs is financial constrained ($\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} = \lambda_\theta$) and can not borrow as much as possible to finance the investment to an efficient level. They choose the leverage pair so that $\frac{\lambda'_{Th}}{\lambda'_{Nh}} > \frac{\sigma}{1-\sigma}$, reflecting the benefit of the tangible capital in expanding borrowing capacity. Together with $\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} = \lambda_\theta$, it can be proved that $\lambda'_{Th} + \lambda'_{Nh} > \lambda'_l = \lambda_\sigma$ (See Appendix).

To sum up, given a certain σ , with the Leontief technology, $\lambda_{Th} + \lambda_{Nh} \leq \lambda_l = \lambda_\sigma$ holds in the steady state. In contrast, the CES function enables the z_h -entrepreneur to deviate from the fixed ratio $\frac{\sigma}{1-\sigma}$, making it possible to reach a higher total leverage than the z_l -entrepreneur does, specifically, when the financial constraint binds.

Effects of a falling σ on $\lambda_{Th} + \lambda_{Nh}$ and λ_l

How do the steady-state total leverages $\lambda_{Th} + \lambda_{Nh}$ and λ_l change as the tangible fraction decreases? The answer is, as σ decreases, the financial constraints become tighter and both types of entrepreneurs target lower steady-state total leverages.

For the z_l -entrepreneur, since $\lambda'_l = \frac{1}{1 - \frac{\theta\sigma(1-\delta)}{R}}$ always holds, it's obvious that the reduction of the tangible capital share impairs the borrowing capacity and depresses the leverage.

Situation is more complex for the z_h -entrepreneur. If the borrowing constraint binds, the decrease in σ reduces the firm's reliance on tangible capital. Since less tangible capital can be pledged for borrowing, the borrowing constraint tightens which raises the multiplier μ given other things unchanged. Equation (1.13) and (1.14) shows that the entrepreneur has to reduce the investment in both tangible and intangible capital in response to a deteriorate financial condition. The resulting lower λ'_{Th} and λ'_{Nh} imply a smaller total leverage $\lambda'_{Th} + \lambda'_{Nh}$ as σ decreases. If the borrowing constraint is slack, though $\frac{\lambda'_{Th}}{\lambda'_{Nh}} = \frac{\sigma}{1-\sigma}$ always holds, more conditions are needed to determine how $\lambda'_{Th} + \lambda'_{Nh}$ changes. In my framework, it can be proved that $\lambda'_{Th} + \lambda'_{Nh}$ shrinks as σ decreases (See Appendix for the proof).

In a word, the reduction of the tangible share tightens the financial constraints of both entrepreneurs, forcing them to move to lower levels of total leverages in the steady state.

The Effects of the Decreased Leverages on the Liquidation Decisions

As I have proved that the lower tangible share is connected with lower leverages, the next step is to investigate the effects of the decreased leverages on the liquidation decisions. Since the liquidation threshold equation in steady state is essential to the analysis, I rewrite it here in a way that is convenient to work with:

$$\begin{aligned} & -\zeta_{jl} + \beta p^l \left[\zeta_{ll}(\lambda_\sigma, \lambda_\sigma) - \int_0^{\zeta_{ll}(\lambda_\sigma, \lambda_\sigma)} F(\zeta) d\zeta \right] \\ & = \frac{\beta}{1-\beta} \left[\sum_j p^{lj} \log \left(\frac{z_j \pi + 1 - \delta}{R} + \frac{1}{\lambda_\sigma} - 1 \right) - \log \left(q_{jl} + \frac{1}{\lambda_\sigma} - 1 \right) \right] - \frac{\Delta_{jl}}{1-\beta}, \end{aligned} \quad (1.32)$$

in which $q_{jl} = q((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$ is determined through

$$(1-\delta) \left[q_{jl} - 1 + \frac{1}{\lambda_l} \right] = \beta \left[z_l \pi k_{\lambda,j} + (1-\delta) q_{jl} + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) \right]. \quad (1.33)$$

$\Delta_{jl} = \Delta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$ is determined through

$$\Delta_{jl} = \log \left(\frac{z_l \pi k_{\lambda,j} + 1 - \delta + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right)}{z_l \pi k_{\lambda,j} + (1-\delta) q_{jl} + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right)} \right). \quad (1.34)$$

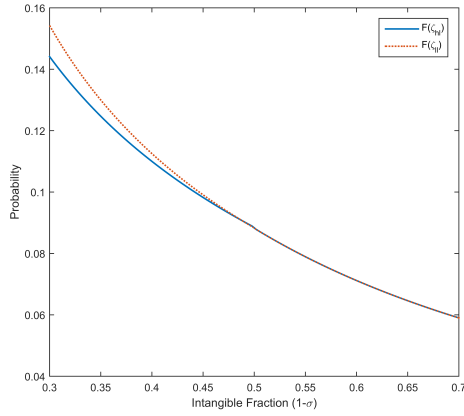
Note that $\zeta_{jl} = \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$.¹² The following proposition can be proved.

Proposition 6

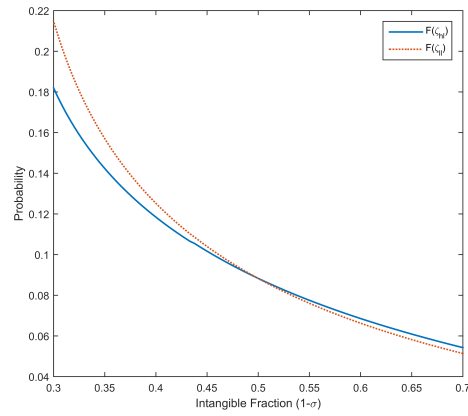
1. If the lower productivity z_l is sufficiently large and $\beta R > 1 - \delta$, $\frac{\partial \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} > 0$ holds for $j = h, l$. Specifically,
 - If $j = l$, $\frac{\partial \zeta(\lambda_\sigma, \lambda_\sigma)}{\partial \lambda_\sigma} > 0$
 - If $j = h$, $\frac{\partial \zeta((\lambda_{Th}, \lambda_{Nh}), \lambda_\sigma)}{\partial \lambda_\sigma} > 0$.
2. If β is sufficiently small, $\zeta((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma) \geq \zeta((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma)$ if $\lambda_{T1} + \lambda_{N1} \geq \lambda_{T2} + \lambda_{N2}$.

Proof. See Appendix. ■

¹²Since if $j = l$ the beginning-of-period leverage ratio is $(\lambda_{Tl}, \lambda_{Nl}) = \lambda_\sigma$, it holds that $\zeta_{ll} = \zeta(\lambda_\sigma, \lambda_\sigma)$. In addition, $k_{\lambda,j} = k\left(\frac{\lambda_{Tj}}{\lambda_{Tj} + \lambda_{Nj}}, \frac{\lambda_{Nj}}{\lambda_{Tj} + \lambda_{Nj}}\right)$. Specially, $k_{\lambda,j} = 1$ if Leontief aggregation is used.



(a) Leontief function



(b) CES function

Figure 1.2: Changes of the liquidation probabilities of the unproductive entrepreneurs

Note: $F(\zeta_{hl}) = F(\zeta((\lambda_{Th}, \lambda_{Nh}), \lambda_\sigma))$ and $F(\zeta_{ul}) = F(\zeta(\lambda_\sigma, \lambda_\sigma))$ are liquidation probabilities of the z_l -entrepreneur. As σ falls, financial constraint of the z_h -entrepreneur gets tighter. $F(\zeta_{hl})$ intersects $F(\zeta_{ul})$ at the point where the financial constraint turns binding.

Figure 1.2 shows changes of the liquidation probability as the tangible share decreases under the benchmark parameters which satisfy Proposition 6. The solid line is $F(\zeta_{hl}) = F(\zeta((\lambda_{Th}, \lambda_{Nh}), \lambda_\sigma))$, the liquidation probability of the current z_l -entrepreneur who was productive previously and thus starts the current period with a leverage $(\lambda_{Th}, \lambda_{Nh})$. The dash line is $F(\zeta_{ul}) = F(\zeta(\lambda_\sigma, \lambda_\sigma))$, the liquidation probability of the current z_l -entrepreneur who was unproductive last period and thus starts the current period with λ_σ . The liquidation probability $F(\zeta_{jl})$ is increasing in ζ_{jl} , the threshold cost under which the unproductive entrepreneur finds it optimal to liquidate. Since the threshold cost is so closely connected with the liquidation probability, I sometimes use them without distinctions for ease of convenience. Two observations can be obtained from Figure 1.2 as illustrations of Proposition 6.

Observation 1: The steady-state liquidation probability falls as the long-run share of tangible capital shrinks, regardless of the aggregation technology used by the z_h -entrepreneur, and no matter whether the current z_l -entrepreneur was productive or unproductive previously.

Observation 2: Given the same tangible share, entrepreneurs who start the period with a higher total leverage are more likely to liquidate than those who are lower leveraged at the beginning of the period (See Figure 1.1 and Figure 1.2 jointly).

In the rest of this subsection, the two observations are explained one by one.

Observation 1

Observation 1 describes the variation of the liquidation probability as σ decreases. It states that the steady-state liquidation probability $\zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$ falls as σ decreases, no matter whether the entrepreneur starts the period with the leverage pair $(\lambda_{Th}, \lambda_{Nh})$ (solid line) or $(\lambda_{Tl}, \lambda_{Nl}) = \lambda_l = \lambda_\sigma$ (dash line). Since the reduction of σ pulls down both $\lambda_{Tj} + \lambda_{Nj}$ and λ_σ , the goal is to prove $\zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$ falls as $\lambda_{Tj} + \lambda_{Nj}$ and λ_σ decrease.

Part 1 of Proposition 6 says that the liquidation probability falls as the *end-of-period* leverage decreases, given the beginning-of-period leverage pair unchanged. To understand Part 1, it's important to investigate how the fall in the end-of-period leverage affects the right-hand-side of equation (1.32), which captures the value of staying over that of liquidating. On one side, a lower level of the targeted leverage distorts the shadow price and points the entrepreneur to liquidating. Intuitively, the shadow price q_{jl} captures how hard it is for the z_l -entrepreneur to sustain the current level of total capital. The harder it is, the lower the shadow price is. A lower level of the targeted leverage means less future borrowing and makes it harder to sustain the total capital, pulling down q_{jl} . Therefore, a smaller λ_σ distorts the shadow price and raises the value of liquidating. On the other side, a lower level of targeted leverage raises the net worth left into the future for a z_l -entrepreneur who chooses to stay, because lower leverage means lower future repayment of debt. In short, two effects of the falling end-of-period leverage, the distortion effect on the shadow price and the enhancing effect on the future net worth, compete to affect the value of staying and liquidating.

I show that as long as z_l and R are sufficiently large, the distortion effect on q_{jl} is dominated by the enhancing effect on future net worth. This is intuitive, because a relatively rich entrepreneur (who has high z_l and R) concerns little about the distortion of the capital price (See the denominator of (1.34)). Thus, the equilibrium option value of staying goes up as λ_σ falls. After showing that the entrepreneur is more willing to stay as the targeted leverage falls, it is straightforward to prove that liquidation probability falls as a result.

Part 2 of Proposition 6 says that the liquidation probability falls as the *beginning-of-period* leverage decreases, given the same end-of-period leverage targeted. To gain some intuition on this, feed each of $\zeta_1 = \zeta((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma)$ and $\zeta_2 = \zeta((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma)$ back into (1.32) to get two equations and subtract one from the other, the resulted relation is obtained:

$$(1 - \beta)(\zeta_1 - \zeta_2) = \left[\beta \log \left(q((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma) + \frac{1}{\lambda_\sigma} - 1 \right) + \Delta((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma) \right] - \left[\beta \log \left(q((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma) + \frac{1}{\lambda_\sigma} - 1 \right) + \Delta((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma) \right]. \quad (1.35)$$

This equation shows that difference between the two liquidation probabilities (LHS) can be attributed to difference between two terms (RHS). The first term captures a *future* effect, which drives the entrepreneur towards liquidation by pulling down the net worth left into the future if staying. The second term is a *current* effect, which also points the entrepreneur to liquidation, by amplifying the current net worth gain of liquidating over that of staying.

To understand the impact of the beginning-of-period leverage $\lambda_T + \lambda_N$ on each term, first note that given the same end-of-period leverage λ_σ , $q((\lambda_T, \lambda_N), \lambda_\sigma)$ is decreasing in $\lambda_T + \lambda_N$, because a larger beginning-of-period leverage increases debt repayment, making it harder to sustain the current level of capital stock and thus distorting the capital price. Second, $\Delta((\lambda_T, \lambda_N), \lambda_\sigma)$ is increasing in $\lambda_T + \lambda_N$. Though a heavier debt burden impairs the value of both liquidating and staying, a larger $\lambda_T + \lambda_N$ amplifies the current net worth gain of liquidating over that of staying, by distorting the capital price if the entrepreneur chooses to stay. If β is sufficiently small, i.e., if the entrepreneur does not value future much, the current effect dominates the future effect, which means a larger beginning-of-period leverage leads to a stronger inclination for liquidating. Putting it from the opposite side, an impatient entrepreneur who starts the period with a lower level of total leverage bears a lighter debt burden and less distorted capital price, and is therefore more likely to stay in business than one who has a higher beginning-of-period leverage.

I summarize to end this part by reviewing the process of the liquidation decision making. Consider a previously productive entrepreneur who turns unproductive. First holding the end-of-period leverage unchanged, the entrepreneur becomes less willing to liquidate his firm as the beginning-of-period leverage falls (Part 2 of Proposition 6). Then as the end-of-period leverage falls, his willingness to liquidate is depressed further (Part 1(b) of Proposition 6). The entrepreneur thus ends up with a lower probability of liquidating. Consider next a previously unproductive entrepreneur who remains unproductive. An entrepreneur of such kind has the same beginning- and end-of-period leverages. As the leverages fall, he also turns less likely to liquidate (Part 1(a) of Proposition 6). In sum, regardless of the previous productivity, a current unproductive entrepreneur turns more likely to stay in business as the financial conditions deteriorates and the leverage goes down.

Observation 2

Observation 2 compares the liquidation probability across lines, i.e, how does the liquidation probability of those who were previously productive differ from those who were

previously unproductive. Since the only difference between the two is their beginning-of-period leverages, the proof to observation 2 is similar to the proof to part 2 of observation 1.

Specifically, Observation 2 states that given the same tangible fraction σ , if $\lambda_{Th} + \lambda_{Nh} \geq \lambda_\sigma$ holds, then $\zeta((\lambda_{Th}, \lambda_{Nh}), \lambda_\sigma) \geq \zeta(\lambda_\sigma, \lambda_\sigma)$; if $\lambda_{Th} + \lambda_{Nh} < \lambda_\sigma$ holds, then $\zeta((\lambda_{Th}, \lambda_{Nh}), \lambda_\sigma) < \zeta(\lambda_\sigma, \lambda_\sigma)$. In short, previously productive entrepreneurs who start the current period with a higher total leverage have a greater steady-state probability to liquidate their firms, and vice versa. This follows immediately from part 2 of Proposition 7 by replacing $(\lambda_{T1}, \lambda_{N1})$ and $(\lambda_{T2}, \lambda_{N2})$ with $(\lambda_{Th}, \lambda_{Nh})$ and λ_σ .

1.4.3 Quantitative Results

In this subsection, I vary the value of the long-run tangible share to investigate the change in the aggregate variables as well as responses of the economy to standard-deviation shocks.

Aggregate Variable

Figure 1.3 shows steady-state variations of some aggregate variables as the long-run intangible share rises from 0.3 (1980s) to 0.5 (2000s) to 0.7 (projected), keeping others unchanged.

It has been discussed earlier that the liquidation probability goes down as firms decrease their reliance on the tangible capital. This is because the shrinking debt capacity on one hand pulls down the interest rate (R) and on the other hand inhibits the productive firms from investing (I) and hiring labors, which depresses wage rate and raises profit π . Lower interest rate means it's less attractive to liquidate the firm and save while the higher profit increases net worth of the unproductive firms who stay, with both effects pushing up the option value of staying. As a result, aggregate liquidation (L) falls, accompanying a reduction of the investment (I). The reduction in L exceeds that in I which is reflected through the falling liquidation expenditure ratio, since $L/X = \frac{L}{L+I}$. As the liquidation process slows down, capital cannot be allocated efficiently to the more productive firms, resulting in a lower ratio of the productive to unproductive capital ($\frac{K_h}{K_l}$) which impairs the TFP.

Impulse Responses to Shocks

In this part, I compare the economy's sensitivity to shocks before and after a reduction in the tangible share (from $\sigma = 0.5$ to $\sigma = 0.4$). Figure 1.4 shows the impulse responses

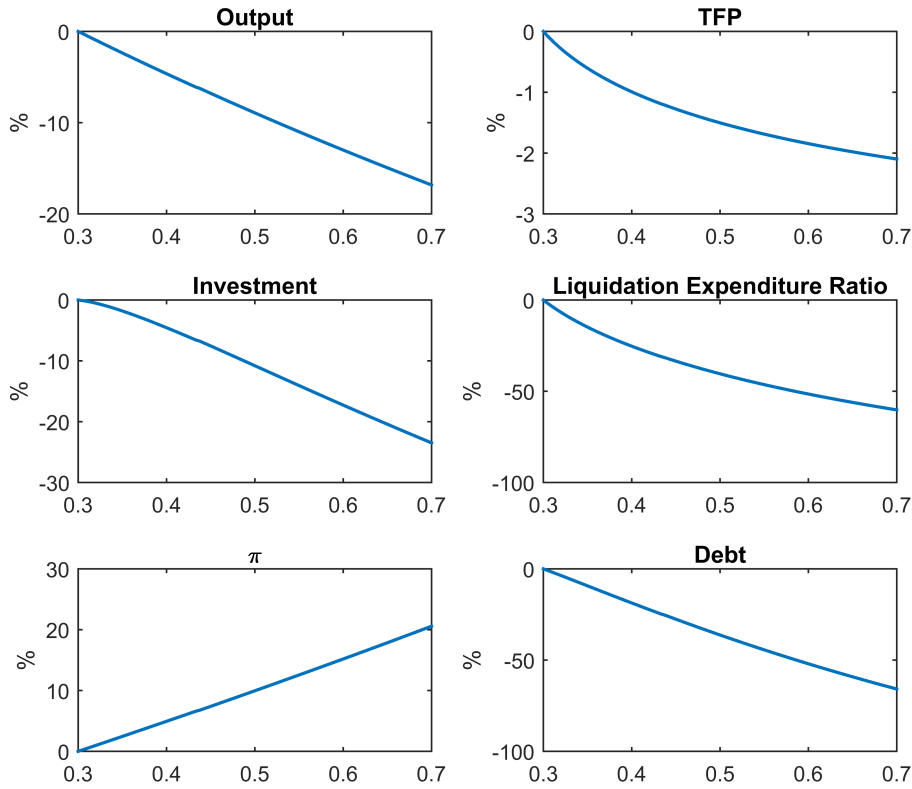


Figure 1.3: Variations of the aggregate variables

to a negative one standard-deviation aggregate productivity shock. Figure 1.5 shows responses to a negative one standard-deviation financial shock.

For the moment ignore the different sensitivities before and after the reduction in tangible share and focus only on the impulse response to shocks. The economy responds to productivity and financial shocks differently. In response to a productivity shock, the investment, consumption, output and TFP all drop. In effect however, the drops have been alleviated by an offsetting effect created by the liquidation mechanism. Specifically, the reduction in the profit rate (π) decreases the net worth and therefore the benefit of staying, motivating more unproductive firms to liquidate and transfer their capital to the productive firms. L-X ratio shoots up due to both the drop in the investment (I) and the increase in the liquidation (L). Besides, more liquidation raises savings of the unproductive entrepreneurs and thus more resources are available to lend to the productive. In a word, the liquidation mechanism stops the economy from plunging too much in response to a negative productivity shock. By contrast, in response to the negative financial shock, the liquidation mechanism reinforces the deterioration of the economy. As the financial condition worsens, the investment falls due to less financial resource available. Meanwhile, productive firms hire less labors, which depresses the

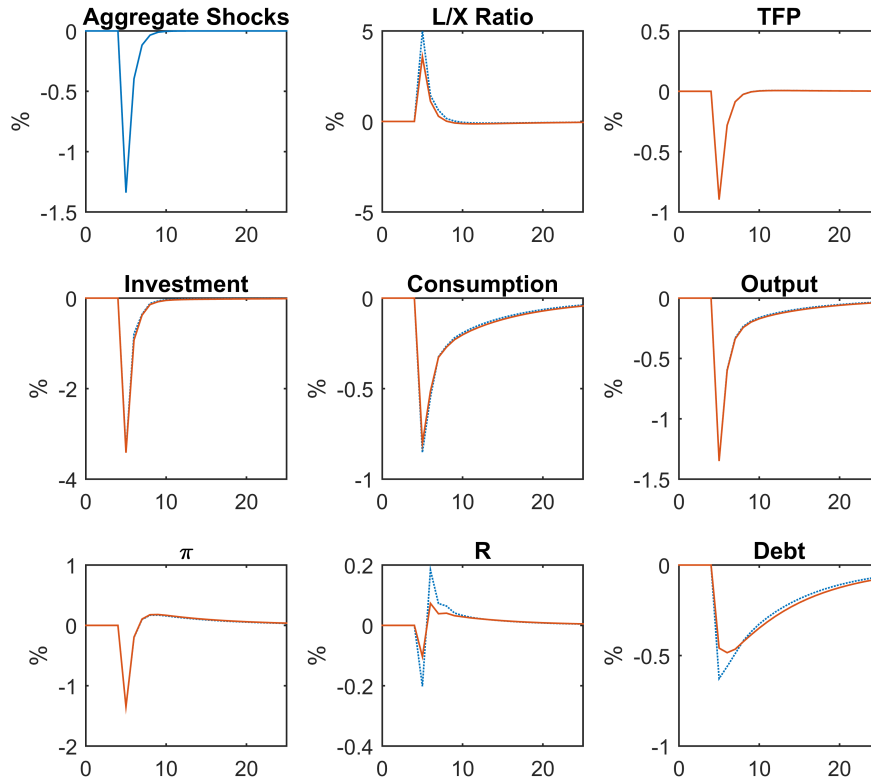


Figure 1.4: Impulse responses to a negative aggregate productivity shock

Note: The shock takes place in period five. All variations are plotted as percentage changes from the corresponding steady-state levels. Solid lines represent impulse responses to shocks before σ reduces, while dashed lines represent the situation after σ reduces.

wage rate and raises the profit. Unproductive firms thus find it more attractive to stay and keep producing, slowing down the process of transferring capital to the productive and further impairing the investment, output and TFP. The L-X ratio drops because the reduction in liquidation (L) exceeds that in investment (I).

Now divert to the economy's different sensitivities to shocks before and after the reduction in the tangible share. The size of the responses to a productivity shock and a financial shock are discussed separately. After the reliance on tangible capital reduces, the debt, interest rate, profit and L/X ratio show greater responses to the productivity shock while the investment, consumption, output and TFP display smaller responses. This is a reflection of the offsetting effects of the liquidation mechanism: the reduction in the profit rate promotes the capital reallocation and efficiency, which alleviates the deterioration of the aggregate economy. After the long-run share of tangible capital falls, the response of the economy becomes more moderate, simply because the economy now has less tangible capital as collateral to borrow and thus counts less on debt to finance economic activities.

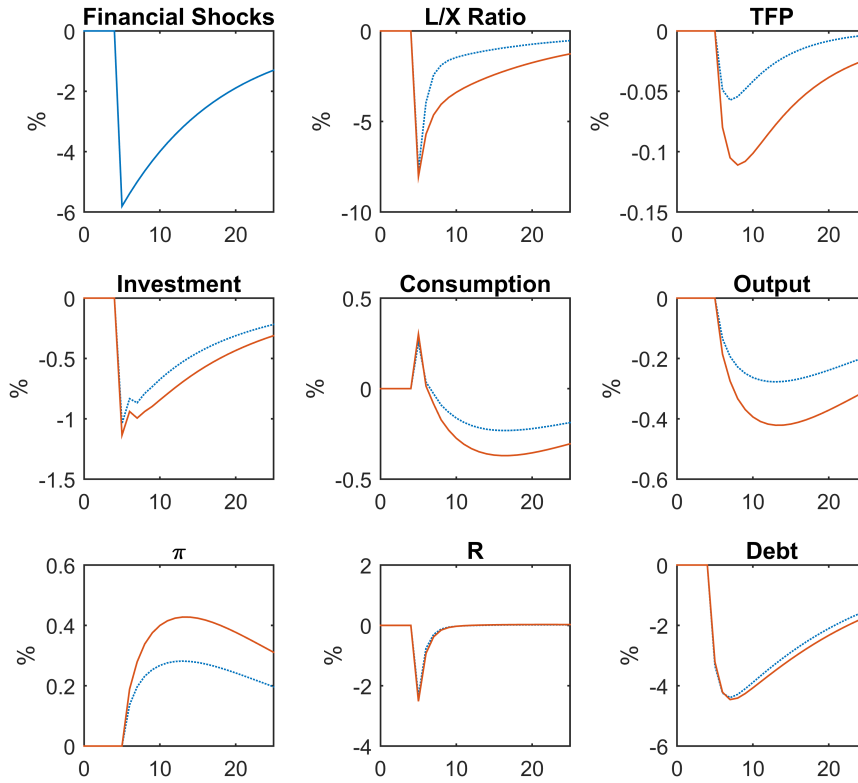


Figure 1.5: Impulse responses to a negative financial shock

Note: The shock takes place in period five. All variations are plotted as percentage changes from the corresponding steady-state levels. Solid lines represent impulse responses to shocks before σ reduces, while dashed lines represent the situation after σ reduces.

To sum up, following a reduction in the tangible share, the economy becomes less sensitive to a negative financial shock while an off-setting effect of the liquidation mechanism moderates the responses of major production variables to a productivity shock.

1.5 Conclusion

This paper builds a model in which the financial constraints link the technological change in the corporate sector to capital liquidation decisions. The model shows that the rising intangible capital share shrinks borrowing capacity, which can then delays the liquidation.

The aim of this work is not to examine the delay of capital liquidation over time, but rather to model a mechanism through which the share of intangible capital can play a role in affecting the liquidation decision. Specifically, it investigates how the liquidation decision changes in response to a fall in the tangible share, holding other factors invariant. However, in the real world and over time, there are potentially many factors that could

influence the liquidation process. For example, any factor that would cause a change in the firm profit or the interest rate, can effectively encourage (lower profit or higher interest rate) or delay (higher profit or lower interest rate) the liquidation, which is possibly an interesting topic for future studies.

Appendix

Proof to Proposition 1

Take the aggregate state X as given, define the operator \mathcal{T}

$$\begin{aligned} \mathcal{T}v(k_T, k_N, b, z, \zeta; X) &= \max \{v^0(k_T, k_N, b, z, \zeta; X), v^1(k_T, k_N, b, z; X)\} \\ v^0(k_T, k_N, b, z, \zeta; X) &= \max_{b'} \log [z\pi k(k_T, k_N) + (1 - \delta)(k_T + k_N) + Rb - b'] - \zeta \\ &\quad + \beta \mathbb{E}[v(0, 0, b', z', \zeta'; X) | z, X] \\ v^1(k_T, k_N, b, z, \zeta; X) &= \\ &\quad \max_{b'} \log [z\pi k(k_T, k_N) + (1 - \delta)(k_T + k_N) + Rb - k'_T - k'_N - b'] \\ &\quad + \beta \mathbb{E}[v(0, 0, b', z', \zeta'; X) | z, X] \\ &\quad \text{s.t. } Rb' \geq -\theta(1 - \delta)k'_T \\ &\quad k'_T + k'_N \geq (1 - \delta)k_T + (1 - \delta)k_N \end{aligned}$$

The value function is the fixed point of the contract mapping in some closed function space \mathcal{V}_1 (Stokey, Lucas, and Prescott, 1989). To prove that the value function has the property stated in equation (1.7), I prove in the following that for any arbitrary v which has the property, the contraction mapping \mathcal{T} preserves the same property. Therefore, the unique fixed point $v = v^*$ of \mathcal{T} satisfies equation (1.7).

Consider entrepreneur A with state (k_T, k_N, b, z, ζ) whose optimal policy is (k'_T, k'_N, b') and entrepreneur B with state $(mk_T, mk_N, mb, z, \zeta)$ and one possible policy for him is (mk'_T, mk'_N, mb') . Their value function satisfies equation (1.7). The states show that entrepreneur B is running a firm that is $m(m > 0)$ times that of A . As the optimal policy (k'_T, k'_N, b') satisfies all constraints for (k_T, k_N, b) , obviously policy (mk'_T, mk'_N, mb') also

satisfies the constraints for (mk_T, mk_N, mb) , making it a feasible policy. Then we have

$$\begin{aligned}
& \mathcal{T}v^1(mk_T, mk_N, mb, z; X) \\
& \geq \log [z\pi k(mk_T, mk_N) + (1 - \delta)mk_T + (1 - \delta)mk_N + Rmb - mk'_T - mk'_N - mb'] \\
& \quad + \beta \mathbb{E} [v(mk'_T, mk'_N, mb', z'; X) | z, X] \\
& = \log m [z\pi k(k_T, k_N) + (1 - \delta)k_T + (1 - \delta)k_N + Rb - k'_T - k'_N - b'] \\
& \quad + \beta \mathbb{E} [v(k'_T, k'_N, b', z'; X) | z, X] + \frac{\beta \log m}{1 - \beta} \\
& = \log [z\pi k(k_T, k_N) + (1 - \delta)k_T + (1 - \delta)k_N + Rb - k'_T - k'_N - b'] \\
& \quad + \beta \mathbb{E} [v(k'_T, k'_N, b', z'; X) | z, X] + \frac{m}{1 - \beta}
\end{aligned}$$

and

$$\begin{aligned}
& \mathcal{T}v^0(mk_T, mk_N, mb, z, \zeta; X) \\
& \geq \log [z\pi k(mk_T, mk_N) + (1 - \delta)mk_T + (1 - \delta)mk_N + Rmb - mb'] - \zeta \\
& \quad + \beta \mathbb{E} [v(0, 0, mb', z', \zeta'; X) | z, X] \\
& = \log m [z\pi k(k_T, k_N) + (1 - \delta)k_T + (1 - \delta)k_N + Rb - b'] - \zeta \\
& \quad + \beta \mathbb{E} [v(0, 0, b', z', \zeta'; X) | z, X] + \frac{\beta \log m}{1 - \beta} \\
& = \log [z\pi k(k_T, k_N) + (1 - \delta)k_T + (1 - \delta)k_N + Rb - b'] - \zeta \\
& \quad + \beta \mathbb{E} [v(0, 0, b', z', \zeta'; X) | z, X] + \frac{m}{1 - \beta}
\end{aligned}$$

That is,

$$\mathcal{T}v(mk_T, mk_N, mb, z, \zeta; X) \geq \mathcal{T}v(k_T, k_N, b, z, \zeta; X) + \frac{\log m}{1 - \beta}$$

Scale with $\frac{1}{m}$ and following the same procedure, we get

$$\mathcal{T}v(k_T, k_N, b, z, \zeta; X) \geq \mathcal{T}v(mk_T, mk_N, mb, z, \zeta; X) - \frac{\log m}{1 - \beta}$$

Combine the two equations, we have

$$\mathcal{T}v(mk_T, mk_N, mb, z, \zeta; X) = \mathcal{T}v(k_T, k_N, b, z, \zeta; X) + \frac{\log m}{1 - \beta}$$

Hence the contraction mapping \mathcal{T} preserves the same property of an arbitrary v . As v is the unique fixed point, v satisfies $v(mk_T, mk_N, mb, z, \zeta; X) = v(k_T, k_N, b, z, \zeta; X) + \frac{\log m}{1 - \beta}$.

Then suppose (k'_T, k'_N, b') is the optimal policy that results in $v(k_T, k_N, b)$, with (λ'_T, λ'_N) being the corresponding leverage pair. Then according to equation (1.7) and the definition of v , the end-of-period leverage pair (λ'_T, λ'_N) that yields $v(k_T, k_N, b)$ (optimal value that

can be reached) also yields $v(mk_T, mk_N, mb)$ keeping all else equal.

Deriving the Policy Function and Shadow Price (Proposition 2 and Part 1 of Proposition 5)

$v(mk_T, mk_N, mb, z; X) = v(k_T, k_N, b, z; X) + \frac{\log m}{1-\beta}$ can be applied.

Take out $k'_T + k'_N + b' = \frac{k'_T + k'_N}{\lambda'_T + \lambda'_N}$, we get

$$\begin{aligned} L = \max_{k'_T + k'_N, \lambda'_T, \lambda'_N} & \log \left(z\pi k(k_T, k_N) + (1-\delta)k_T + (1-\delta)k_N + Rb - \frac{k'_T + k'_N}{\lambda'_T + \lambda'_N} \right) \\ & + \beta \mathbb{E} [v(\lambda'_T, \lambda'_N, 1 - \lambda'_T - \lambda'_N, z', \zeta'; X) | z, X] + \frac{\beta \log \frac{k'_T + k'_N}{\lambda'_T + \lambda'_N}}{1 - \beta} \\ & + \mu [k'_T + k'_N - (1 - \delta)(k_T + k_N)] \end{aligned}$$

Notice that, to maximize, we need to optimize over aggregate level of capital $k'_T + k'_N$ instead of level of each capital separately. First order condition w.r.t $k'_T + k'_N$

$$\frac{-\frac{1}{\lambda'_T + \lambda'_N}}{z\pi k + (1-\delta)k_T + (1-\delta)k_N + Rb - \frac{k'_T + k'_N}{\lambda'_T + \lambda'_N}} + \frac{\beta}{(1-\beta)(k'_T + k'_N)} + \mu = 0$$

Define q as

$$\begin{aligned} & \frac{z\pi k + [(1-\delta)k_T + (1-\delta)k_N]q + Rb}{z\pi k + (1-\delta)k_T + (1-\delta)k_N + Rb} \\ & = \frac{1}{1 + (1-\beta)\mu(k'_T + k'_N)} \end{aligned}$$

when $\mu = 0$, $q = 1$; when $\mu > 0$, $q < 1$. Denote

$$n = z\pi k + [(1-\delta)k_T + (1-\delta)k_N]q + Rb$$

Then the following can be derived no matter resale constraint binds or not:

$$\begin{aligned} c &= (1-\beta)n \\ q(k'_T + k'_N) + b' &= \beta n \\ k'_T + k'_N &= \frac{\lambda'_T + \lambda'_N}{(q-1)(\lambda'_T + \lambda'_N) + 1} \beta n \\ b' &= \frac{1 - \lambda'_T - \lambda'_N}{(q-1)(\lambda'_T + \lambda'_N) + 1} \beta n \end{aligned}$$

When the resale constraint binds, we have

$$k'_T + k'_N + b' = \frac{k'_T + k'_N}{\lambda'_T + \lambda'_N} = \frac{(1 - \delta)(k_T + k_N)}{\lambda'_T + \lambda'_N} = \frac{\beta n}{(q - 1)(\lambda'_T + \lambda'_N) + 1} \quad (1.36)$$

Notice that $k'_T + k'_N = (1 - \delta)(k_T + k_N)$ has been applied. Next divide 1.36 by $k_T + k_N + b$, then the equation is transformed to

$$\frac{(1 - \delta)(\lambda_T + \lambda_N)}{\lambda'_T + \lambda'_N} = \frac{\beta n^*}{(q - 1)(\lambda'_T + \lambda'_N) + 1} \quad (1.37)$$

where $n^* = z\pi k(\lambda_T, \lambda_N) + [(1 - \delta)\lambda_T + (1 - \delta)\lambda_N]q + R(1 - \lambda_T - \lambda_N)$. The shadow price q is solved from this equation.

Now apply $k'_T + k'_N = \frac{\lambda'_T + \lambda'_N}{(q-1)(\lambda'_T + \lambda'_N) + 1} \beta n$ to the value function, we get

$$\begin{aligned} v^1(k_T, k_N, b, z; X) &= \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} + \frac{\log n}{1 - \beta} \\ &+ \beta \max_{\lambda'_T, \lambda'_N} \left\{ -\frac{\log[1 + (q - 1)(\lambda'_T + \lambda'_N)]}{1 - \beta} + v(\lambda'_T, \lambda'_N, 1 - \lambda'_T - \lambda'_N, z'; X') \right\} \end{aligned}$$

$v^1(k_T, k_N, b, z; X)$ has the following form

$$v^1(k_T, k_N, b, z; X) = J^1(\lambda_T, \lambda_N, z; X) + \frac{\log n}{1 - \beta},$$

where

$$\begin{aligned} J^1(\lambda_T, \lambda_N, z; X) &= \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} \\ &+ \beta \max_{\lambda'_T, \lambda'_N} \left\{ -\frac{\log[1 + (q - 1)(\lambda'_T + \lambda'_N)]}{1 - \beta} + v(\lambda'_T, \lambda'_N, 1 - \lambda'_T - \lambda'_N, z'; X') \right\}. \end{aligned}$$

$v^0(k_T, k_N, b, z; X)$ has the following form

$$v^0(k_T, k_N, b, z; X) = J^0(z; X) - \zeta + \frac{\log(z\pi k + (1 - \delta)k_T + (1 - \delta)k_N + Rb)}{1 - \beta},$$

where $J^0(z; X) = \log(1 - \beta) + \frac{\beta \log \beta}{1 - \beta} + \beta \mathbb{E}[v(0, 0, 1, z', \zeta'; X') | z, X]$

Derive FOCs (Proposition 3)

$$\begin{aligned} L &= \max_{\lambda'_T, \lambda'_N} \left\{ -\frac{\log[1 + (q - 1)(\lambda'_T + \lambda'_N)]}{1 - \beta} + v(\lambda'_T, \lambda'_N, 1 - \lambda'_T - \lambda'_N, z'; X') \right\} \\ &+ \mu_\lambda \left[\frac{1}{1 - \theta(1 - \delta)/R} - \frac{\lambda'_T}{1 - \lambda'_N} \right] \end{aligned}$$

FOCs:

$$v_{k'_T} - v_{b'} = \frac{q-1}{(1-\beta)[1+(q-1)(\lambda'_T + \lambda'_N)]} + \frac{1}{1-\lambda'_N} \mu_\lambda \quad (1.38)$$

$$v_{k'_N} - v_{b'} = \frac{\lambda'_T}{(1-\lambda'_N)^2} \mu_\lambda \quad (1.39)$$

Where

$$v_{k_T}(k_T, k_N, b, z, \zeta, X) = \frac{z\pi \frac{\partial k}{\partial k_T} + (1-\delta)q}{(1-\beta)n}$$

$$v_{k_N}(k_T, k_N, b, z, \zeta, X) = \frac{z\pi \frac{\partial k}{\partial k_N} + (1-\delta)q}{(1-\beta)n}$$

$$v_b(k_T, k_N, b, z, \zeta, X) = \frac{R}{(1-\beta)n}$$

Substitute out μ_λ , we get

$$v_{k'_N} - v_{b'} - \frac{\lambda'_T}{1-\lambda'_N} (v_{k'_T} - v_{b'}) = \frac{\lambda'_T}{1-\lambda'_N} \cdot \frac{q-1}{(1-\beta)[1+(q-1)(\lambda'_T + \lambda'_N)]} \quad (1.40)$$

If z_h is drawn in period t

Apply $q = 1$ and $\mu_\lambda \geq 0$ to 1.38, I have

$$v_{k'_T} - v_{b'} \geq 0$$

or, given z_h is drawn and apply the liquidation probability $F(\zeta'_{hl})$

$$\begin{aligned} & p^{hh} \mathbb{E}_X \left[\frac{z_h \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Th}} + 1 - \delta - R'}{z_h \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + (1-\delta)(\lambda'_{Th} + \lambda'_{Nh}) + R'(1-\lambda'_{Th} - \lambda'_{Nh})} \right] + \\ & p^{hl} \mathbb{E}_X F(\zeta'_{hl}) \left[\frac{z_l \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Th}} + 1 - \delta - R'}{z_l \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + (1-\delta)(\lambda'_{Th} + \lambda'_{Nh}) + R'(1-\lambda'_{Th} - \lambda'_{Nh})} \right] + \\ & p^{hl} \mathbb{E}_X [1 - F(\zeta'_{hl})] \left[\frac{z_l \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{lh}} + (1-\delta)q'_{hl} - R'}{z_l \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + (1-\delta)(\lambda'_{Th} + \lambda'_{Nh})q'_{hl} + R'(1-\lambda'_{Th} - \lambda'_{Nh})} \right] \\ & \geq 0 \end{aligned}$$

Apply $q = 1$ to 1.39, I have

$$\frac{\lambda'_T}{1-\lambda'_N} (v_{k'_T} - v_{b'}) = v_{k'_N} - v_{b'}$$

or

$$\begin{aligned}
& p^{hh} \mathbb{E}_X \left[\frac{\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \left(z_h \pi' \frac{\partial k'}{\partial \lambda'_{Th}} + 1 - \delta - R' \right) - \left(z_h \pi' \frac{\partial k'}{\partial \lambda'_{Nh}} + 1 - \delta - R' \right)}{z_h \pi' k (\lambda'_{Th}, \lambda'_{Nh}) + (1-\delta) (\lambda'_{Th} + \lambda'_{Nh}) + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] + \\
& p^{hl} \mathbb{E}_X F \left(\tilde{\zeta}_{hl} \right) \left[\frac{\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \left(z_l \pi' \frac{\partial k'}{\partial \lambda'_{Th}} + 1 - \delta - R' \right) - \left(z_l \pi' \frac{\partial k'}{\partial \lambda'_{Nh}} + 1 - \delta - R' \right)}{z_l \pi' k (\lambda'_{Th}, \lambda'_{Nh}) + (1-\delta) (\lambda'_{Th} + \lambda'_{Nh}) + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] + \\
& p^{hl} \mathbb{E}_X \left[1 - F \left(\tilde{\zeta}_{hl} \right) \right] \left[\frac{\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \left(z_l \pi' \frac{\partial k'}{\partial \lambda'_{Th}} + (1-\delta) q'_{hl} - R' \right) - z_l \pi' \frac{\partial k'}{\partial \lambda'_{Nh}} - (1-\delta) q'_{hl} - R'}{z_l \pi' k (\lambda'_{Th}, \lambda'_{Nh}) + (1-\delta) (\lambda'_{Th} + \lambda'_{Nh}) q'_{hl} + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] \\
& \hspace{25em} = 0
\end{aligned}$$

If z_l is drawn in period t

All constraints bind, so we have

$$\frac{\lambda'_{Tl}}{1 - \lambda'_{Nl}} = \frac{1}{1 - \theta (1 - \delta) / R}$$

With Leontief function, I have

$$\frac{\lambda'_{Tl}}{\lambda'_{Nl}} = \frac{\sigma}{1 - \sigma}$$

Combine the two conditions, I get

$$\lambda'_{Tl} + \lambda'_{Nl} = \lambda'_l = \frac{1}{1 - \sigma \theta (1 - \delta) / R}$$

Binding Financial and Resale Constraint (Proposition 4)

First, it is known from Proposition 2 that $q = 1$ when the resale constraint does not bind. And the amount that the firm saves $k'_T + k'_N + b'$ has been shown to be a fixed fraction β of the net worth $n = z\pi k + (1 - \delta) k_T + (1 - \delta) k_N + Rb$. Hence under a given state (k_T, k_N, b) , $\lambda'_T + \lambda'_N = \frac{k'_T + k'_N}{k'_T + k'_N + b'} = 1 - \frac{b'}{k'_T + k'_N + b'}$ is maximized when the the firm borrow to the maximum (when b' reaches its smallest possible value), and the corresponding maximum value is $\lambda_T^c + \lambda_N^c$ as is defined.

To prove Proposition 4, start with a z_l -entrepreneur who chooses to stay in business and thus subject himself to the resale constraint. As has been proved, $k'_T + k'_N + b' = \beta [z\pi k + (1 - \delta) k_T + (1 - \delta) k_N + Rb]$ holds for both types of entrepreneur. Recall that $\lambda'_T + \lambda'_N = \frac{k'_T + k'_N}{k'_T + k'_N + b'}$, substitute $k'_T + k'_N + b'$ out, I get

$$k'_{Tl} + k'_{Nl} = (\lambda'_{Tl} + \lambda'_{Nl}) \beta [z_l \pi k + (1 - \delta) k_{Tj} + (1 - \delta) k_{Nj} + Rb].$$

If the productivity is so low that even if the z_l -entrepreneur borrows to the limit and realizes the leverage pair $(\lambda_{Tl}^c, \lambda_{Nl}^c)$, I still have the right-hand-side satisfying

$$(\lambda_{Tl}^c + \lambda_{Nl}^c) \beta [z_l \pi k + (1 - \delta) k_{Tj} + (1 - \delta) k_{Nj} + Rb] < (1 - \delta) (k_{Tj} + k_{Nj}), \quad (1.41)$$

then $k'_{Tl} + k'_{Nl} < (1 - \delta) (k_{Tj} + k_{Nj})$ is implied, i.e. the resale constraint is violated. Since the resale constraint restricts the entrepreneur to keep the current total amount of capital, it must be the case that the resale constraint binds. Divide both sides of (1.41) by $(k_{Tj} + k_{Nj} + b) (\lambda_{Tj} + \lambda_{Nj})$ and apply the definition of leverage ratio, we arrive at (1.15).

Liquidation Decision (Proposition 5)

When z_h is drawn in t , the entrepreneur always invests, i.e. $q((\lambda_{Tj}, \lambda_{Nj}), z_h, \zeta; X) = 1$. If we look at the functional form of J in section 1.5, we can tell that the beginning-of-period leverage pair $(\lambda_{Tj}, \lambda_{Nj})$ does not affect value of J . Therefore we have

$$J(\lambda_{Tj}, \lambda_{Nj}, z_h, \zeta; X) = J(z_h; X) = J_h$$

When the entrepreneur draws z_l in t and decides to liquidate, $q((\lambda_{Tj}, \lambda_{Nj}), z_l, \zeta; X) = 1$, again the beginning-of-period leverage pair $(\lambda_{Tj}, \lambda_{Nj})$ does not affect the value of J and we have

$$J(\lambda_{Tj}, \lambda_{Nj}, z_l, \zeta; X) = J(z_l, \zeta; X) = J_l - \zeta$$

When the entrepreneur draws z_l in period t and decides to stay, $q((\lambda_{Tj}, \lambda_{Nj}), z_l, \zeta; X) = q_{jl} < 1$. Whether the firm turns unproductive from a productive or unproductive state affect J through affecting the shadow price of capital q_{jl} , and therefore we have J is dependent on $(\lambda_{Tj}, \lambda_{Nj})$

$$J(\lambda_{Tj}, \lambda_{Nj}, z_l, \zeta; X) = J(\lambda_{Tj}, \lambda_{Nj}, z_l; X) = J_s$$

Those z_l entrepreneurs compare the values of staying with capital and liquidating. They liquidate the entire firm when ζ is smaller than the endogenous liquidation threshold $\zeta_{jl} = \zeta_{jl}((\lambda_{Tj}, \lambda_{Nj}), z_l; X)$, which satisfies

$$J_s + \frac{\log n_s}{1 - \beta} = J_l + \frac{\log n_l}{1 - \beta} - \zeta_{jl} \quad (1.42)$$

$n_s = z_l \pi k + (1 - \delta) (k_T + k_N) q_{jl} + Rb$, and $n_l = z_l \pi k + (1 - \delta) (k_T + k_N) + Rb$.

The threshold level ζ_{jl} solves

$$\zeta_{jl} = J_l - J_s + \frac{\Delta_{jl}}{1 - \beta},$$

$\Delta_{jl} = \log\left(\frac{n_{\text{liquidate}}}{n_{\text{stay}}}\right)$, $n_{\text{stay}} = z_l \pi k(\lambda_{Tj}, \lambda_{Nj}) + (1 - \delta)(\lambda_{Tj} + \lambda_{Nj}) q_{jl} + R(1 - \lambda_{Tj} - \lambda_{Nj})$,
and $n_{\text{liquidate}} = z_l \pi k(\lambda_{Tj}, \lambda_{Nj}) + (1 - \delta)(\lambda_{Tj} + \lambda_{Nj}) + R(1 - \lambda_{Tj} - \lambda_{Nj})$.

Define:

$$\begin{aligned} \tilde{\lambda}' &= \frac{1}{1 + (q_{jl} - 1)(\lambda'_{Tl} + \lambda'_{Nl})} \\ n'_h &= z_h \pi' k(\lambda'_{Tl}, \lambda'_{Nl}) + (1 - \delta)\lambda'_{Tl} + (1 - \delta)\lambda'_{Nl} + R'(1 - \lambda'_{Tl} - \lambda'_{Nl}) \\ n'_l &= z_l \pi' k(\lambda'_{Tl}, \lambda'_{Nl}) + (1 - \delta)\lambda'_{Tl} + (1 - \delta)\lambda'_{Nl} + R'(1 - \lambda'_{Tl} - \lambda'_{Nl}) \\ L &= z_l \pi' k(\lambda'_{Tl}, \lambda'_{Nl}) + (1 - \delta)(\lambda'_{Tl} + \lambda'_{Nl}) q'_{ll} + R'(1 - \lambda'_{Tl} - \lambda'_{Nl}) \end{aligned}$$

Then, we have for an entrepreneur who draw z_l today and does not liquidate the whole firm (stay), the value function is equivalent to

$$\begin{aligned} J_s &= \log(1 - \beta) + \beta p^{lh} \mathbb{E}_X \left[J'_h + \frac{\log \beta (\tilde{\lambda}' n'_h)}{1 - \beta} \right] \\ &+ \beta p^{ll} \mathbb{E}_X \int_0^{\tilde{\zeta}'_l} \left[J'_l - \zeta' + \frac{\log \beta (\tilde{\lambda}' n'_l)}{1 - \beta} \right] dF(\zeta') \\ &+ \beta p^{ll} \mathbb{E}_X \int_{\tilde{\zeta}'_l}^{+\infty} \left[J'_s + \frac{\log \beta (\tilde{\lambda}' L)}{1 - \beta} \right] dF(\zeta') \end{aligned} \quad (1.43)$$

The right-hand side includes the utility from consumption $\log(1 - \beta)$ and three possible continuation values next period: value of drawing z_h and investing, value of drawing z_l and liquidating and value of drawing z_l and stay with capital.

For an entrepreneur who draw z_l today and liquidate the whole firm (liquidate), the value function is equivalent to

$$J_l = \log(1 - \beta) + \beta p^{lh} \mathbb{E}_X \left[J'_h + \frac{\log(\beta R')}{1 - \beta} \right] + \beta p^{ll} \mathbb{E}_X \left[J'_l + \frac{\log(\beta R')}{1 - \beta} \right] \quad (1.44)$$

where the right hand side includes the utility from consumption and two possible continuation values next period: value of drawing z_h and investing, and value of drawing z_l and remaining not running firms.

One similar recursion for $J_h(X)$ can be obtained. I leave it out here since it will be

canceled out anyway.

Finally, subtract (1.44) from (1.43) and obtain the following forward looking equation for ζ_l :

$$\frac{\Delta_l}{1-\beta} - \zeta = \beta p^{lh} \frac{\log(\tilde{\lambda}' n'_h / R')}{1-\beta} + \beta p^{ll} \frac{\log(\tilde{\lambda}' n'_l / R')}{1-\beta} + \beta p^{ll} \left(-\zeta' + \int_0^{\zeta'} F(\zeta') d\zeta' \right) \quad (1.45)$$

Left-hand side of (1.45) captures gain of liquidating over staying for the entrepreneur: a time t net worth gain if he chooses to liquidate over the net worth if he otherwise chooses to stay in business. The gain $\frac{\Delta_{jl}}{1-\beta}$ is resulted from the binding resale constraint which leads to a deterioration in capital price ($q_{jl} < 1$) if the entrepreneur chooses to stay rather than liquidate in t . Meanwhile, the gain is reduced by incurring the liquidation cost ζ_{jl} .

Right-hand side of (1.45) represents the option value of staying, reflecting a forward looking perspective. Standing on period t , the entrepreneurs who draw z_l have two choices: keeping producing with a low productivity (staying) and waiting for future z_h , or selling all capital and earning returns on bonds (liquidating). The first term captures the value of staying over liquidating for the period t z_l -entrepreneur if he turns productive (with probability p^{lh}) in period $t+1$, with $\log(\tilde{\lambda}' n'_h)$ being the normalized net worth under high productivity and R' being the return on bonds if the entrepreneurs otherwise liquidated in t . Similarly, the second term captures the value of staying over liquidating for the period t z_l -entrepreneur if he remains unproductive (with probability p^{ll}) in period $t+1$, with $\log(\tilde{\lambda}' n'_l)$ being the normalized net worth under low productivity. The last term captures the value of choosing to stay in period t and liquidate in period $t+1$ over the value of choosing to liquidate earlier in period t .

Aggregate Full Capital Liquidation and Capital Adjustment

With probability $F(\zeta_{jl})$, previously z_j -entrepreneur liquidate the whole firm if they are currently unproductive. The aggregate value of tangible and intangible capital that are fully liquidated are respectively

$$CL_T = (1-\delta) \sum_j F(\zeta_{jl}) p^{jl} K_{Tj} \quad CL_N = (1-\delta) \sum_j F(\zeta_{jl}) p^{jl} K_{Nj}.$$

With probability $1 - F(\zeta_{hl})$, the previously productive entrepreneurs who turns unproductive adjust the ratio of tangible and intangible capital to be $\frac{\sigma}{1-\sigma}$ by partially liquidate or purchase a certain type of capital. The last-period unproductive entrepreneurs that remain unproductive today need not adjust because they have finished adjusting earlier

when they just turned unproductive from productive. For entrepreneurs who stay, the aggregate adjustment of tangible and intangible capital are respectively

$$CA_T = (1 - \delta)(1 - F(\zeta_{hl}))p^{hl}K_{Th} - \sigma(1 - \delta)(1 - F(\zeta_{hl}))p^{hl}(K_{Th} + K_{Nh})$$

and

$$CA_N = (1 - \delta)(1 - F(\zeta_{hl}))p^{hl}K_{Nh} - (1 - \sigma)(1 - \delta)(1 - F(\zeta_{hl}))p^{hl}(K_{Th} + K_{Nh})$$

Positive value of CA_T (CA_N) indicates partial selling of capital while negative value indicates purchase of capital. Notice also that adjustments of tangible and intangible capital sum up to be 0, consistent with the unchanged total level of capital, which is $(1 - \delta)(1 - F(\zeta_{hl}))p^{hl}(K_{Th} + K_{Nh})$.

The Effects of a Decreasing Tangible Share on Leverages

First, compare $\lambda'_{Th} + \lambda'_{Nh}$ and λ'_σ for a given σ . Consider the case in which z_h -entrepreneur aggregates capital using CES technology

Part 1: Financial constraint is slack, so $\frac{\lambda'_{Th}}{\lambda'_{Nh}} = \frac{\sigma}{1 - \sigma}$ and $\frac{\lambda'_{Th}}{1 - \lambda'_{Nh}} < \frac{1}{1 - \frac{\theta(1 - \delta)}{R}}$ hold. Prove that $\lambda'_{Th} + \lambda'_{Nh} < \frac{1}{1 - \frac{\sigma\theta(1 - \delta)}{R}}$.

Proof:

Start with $\frac{\lambda'_{Th}}{\lambda'_{Nh}} = \frac{\sigma}{1 - \sigma}$, so $\lambda'_{Th} + \lambda'_{Nh} = \frac{\lambda'_{Th}}{\sigma}$ and $\lambda'_{Nh} = \frac{(1 - \sigma)\lambda'_{Th}}{\sigma}$ can be derived.

Plug λ'_{Nh} into $\frac{\lambda'_{Th}}{1 - \lambda'_{Nh}} < \frac{1}{1 - \frac{\theta(1 - \delta)}{R}}$ and rearrange, it follows that

$$\lambda'_{Th} < \frac{1}{\frac{1}{\sigma} - \frac{\theta(1 - \delta)}{R}}$$

Therefore,

$$\begin{aligned} \lambda'_{Th} + \lambda'_{Nh} &= \frac{\lambda'_{Th}}{\sigma} \\ &< \frac{1}{\frac{1}{\sigma} - \frac{\theta(1 - \delta)}{R}} \cdot \frac{1}{\sigma} \\ &= \frac{1}{1 - \frac{\sigma\theta(1 - \delta)}{R}} \end{aligned} \tag{1.46}$$

Part 2: Financial constraint binds, so $\frac{\lambda'_{Th}}{\lambda'_{Nh}} > \frac{\sigma}{1 - \sigma}$ and $\frac{\lambda'_{Th}}{1 - \lambda'_{Nh}} = \frac{1}{1 - \frac{\theta(1 - \delta)}{R}}$ holds. Prove that $\lambda'_{Th} + \lambda'_{Nh} > \frac{1}{1 - \frac{\sigma\theta(1 - \delta)}{R}}$.

Proof:

Start with $\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} = \frac{1}{1-\frac{\theta(1-\delta)}{R}}$, then it can be derived that $\lambda'_{Th} + \lambda'_{Nh} = 1 + \frac{\theta(1-\delta)}{R}\lambda'_{Th}$ and $\lambda'_{Nh} = 1 + \left[\frac{\theta(1-\delta)}{R} - 1 \right] \lambda'_{Th}$.

Plug λ'_{Nh} into $\frac{\lambda'_{Th}}{\lambda'_{Nh}} > \frac{\sigma}{1-\sigma}$, it follows that

$$\lambda'_{Th} > \frac{1}{\frac{1}{\sigma} - \frac{\theta(1-\delta)}{R}}$$

Therefore,

$$\begin{aligned} \lambda'_{Th} + \lambda'_{Nh} &= 1 + \frac{\theta(1-\delta)}{R}\lambda'_{Th} \\ &> 1 + \frac{\theta(1-\delta)}{R} \cdot \frac{1}{\frac{1}{\sigma} - \frac{\theta(1-\delta)}{R}} \\ &= \frac{1}{1 - \frac{\sigma\theta(1-\delta)}{R}} \end{aligned}$$

Next prove if z_h -entrepreneur faces a slack borrowing constraint, when σ falls, $\lambda'_{Th} + \lambda'_{Nh}$ also falls.

Proof:

If the borrowing constraint is slack, it holds true that $\frac{\lambda'_{Th}}{\lambda'_{Nh}} = \frac{\sigma}{1-\sigma}$ for both Leontief and CES aggregation. Because entrepreneurs who run firms are net borrowers ($b' < 0$), it holds that $\lambda'_{Th} + \lambda'_{Nh} > 1$. Together with $\frac{\lambda'_{Th}}{\lambda'_{Nh}} = \frac{\sigma}{1-\sigma}$, $\lambda'_{Th} > \sigma$ can be inferred. Therefore, as σ falls, for $\frac{\lambda'_{Th}}{\lambda'_{Nh}} = \frac{\sigma}{1-\sigma}$ to hold, one percent decrease in σ requires decrease that is larger than one percent in λ'_{Th} . Since $\lambda'_{Th} + \lambda'_{Nh} = \frac{\lambda'_{Th}}{\sigma}$ and the numerator falls faster than the denominator, it must be the case that $\lambda'_{Th} + \lambda'_{Nh}$ decreases as σ decreases.

Effects of a Falling Share of Tangible Capital on Liquidation Decision (Proposition 6)

Part 1: Derive conditions for $\frac{\partial \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} > 0$ to hold.

A. Prove $\frac{\partial H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} \leq 0$

Denote the right-hand-side of 1.32 as $H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$. To derive conditions for $\frac{\partial \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} > 0$ to hold, we first derive conditions for $\frac{\partial H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} < 0$ to hold.

$$H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma) = \beta \frac{\left[\sum_j p^{lj} \log \left(\frac{z_j^{\pi+1-\delta}}{R} + \frac{1}{\lambda_\sigma} - 1 \right) - \log \left(q_{jl} + \frac{1}{\lambda_\sigma} - 1 \right) \right]}{1 - \beta} - \frac{\Delta_{jl}}{1 - \beta} \quad (1.47)$$

Now use equation 1.19 to get

$$q_{jl} + \frac{1}{\lambda_\sigma} - 1 = \frac{\beta}{(1-\beta)(1-\delta)} \left[z_l \pi k_{\lambda,j} + 1 - \delta + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) - (1-\delta) \frac{1}{\lambda_\sigma} \right] \quad (1.48)$$

where $k_{\lambda,j} = k \left(\frac{\lambda_{Tj}}{\lambda_{Tj} + \lambda_{Nj}}, \frac{\lambda_{Nj}}{\lambda_{Tj} + \lambda_{Nj}} \right)$. With our specific production function, we have $k_{\lambda,h} < 1$ and $k_{\lambda,l} = 1$. Then plug 1.48 and the definition of Δ_{jl} 1.20 into 1.47, we get

$$\begin{aligned} (1-\beta) H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma) &= C \\ &- \beta \log(z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1) - (1-\delta) \lambda_\sigma^{-1}) \\ &+ \beta p^{lh} \log(z_h \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1)) + \beta p^{ll} \log(z_l \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1)) \\ &+ \log(z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1) - (1-\delta) \lambda_\sigma^{-1}) \\ &- \log(z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1)) \end{aligned} \quad (1.49)$$

C is a constant term. Next take derivative w.r.t λ_σ^{-1} , we get

$$\begin{aligned} (1-\beta) \frac{\partial H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma^{-1}} &= \frac{(\beta-1)(1-\delta)}{z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1) - (1-\delta) \lambda_\sigma^{-1}} \\ &+ \beta \sum_j \frac{p^{lj} R}{z_j \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1)} \end{aligned} \quad (1.50)$$

Apply $z_h > z_l$ to the second term of 1.50, we get

$$\begin{aligned} (1-\beta) \frac{\partial H^r}{\partial \lambda_\sigma^{-1}} &\geq \frac{\beta R}{z_h \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1)} \\ &+ \frac{(\beta-1)(1-\delta)}{z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1) - (1-\delta) \lambda_\sigma^{-1}} \end{aligned} \quad (1.51)$$

Case 1: $\lambda_{Tj} + \lambda_{Nj} \leq \lambda_\sigma$

Notice that in this case, we always have $\frac{\lambda_{Tj}}{\lambda_{Nj}} = \frac{\sigma}{1-\sigma}$, so that $k_{\lambda,j} = 1$ holds. Apply $\lambda_{Tj} + \lambda_{Nj} \leq \lambda_\sigma$, we get

$$\begin{aligned} (1-\beta) \frac{\partial H^r}{\partial \lambda_\sigma^{-1}} &\geq \frac{\beta R}{z_h \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1)} \\ &- \frac{(1-\beta)(1-\delta)}{z_l \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1) - (1-\delta) \lambda_\sigma^{-1}} \end{aligned}$$

Subtract one term and add one term, we get

$$\begin{aligned}
(1 - \beta) \frac{\partial H^r}{\partial \lambda_\sigma^{-1}} &\geq \frac{\beta R}{z_h \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1)} \\
&\quad - \frac{\beta [R - (1 - \delta)]}{z_l \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1}} \\
&\quad + \frac{\beta [R - (1 - \delta)] - (1 - \beta)(1 - \delta)}{z_l \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1}}
\end{aligned} \tag{1.52}$$

For the sum of the first two terms to be non-negative, the necessary and sufficient condition is

$$\frac{R\pi}{1 - \delta} (z_h - z_l) \leq z_h \pi + 1 - \delta - R \tag{1.53}$$

The third term is non-negative if and only if $\beta R > 1 - \delta$.

Case 2: $\lambda_{Tj} + \lambda_{Nj} > \lambda_\sigma$

Starting from 1.51, add one term and subtract the same term, I arrive at

$$\begin{aligned}
(1 - \beta) \frac{\partial H^r}{\partial \lambda_\sigma^{-1}} &\geq \frac{\beta R}{z_h \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1)} \\
&\quad - \frac{\beta [R - (1 - \delta)]}{z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1}} \\
&\quad + \frac{\beta [R - (1 - \delta)] - (1 - \beta)(1 - \delta)}{z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1}}
\end{aligned}$$

If compare the second term with the second term in 1.52, we see that $z_l \pi k_{\lambda,j} + 1 - \delta + R((\lambda_{Tj} + \lambda_{Nj})^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1} < z_l \pi + 1 - \delta + R(\lambda_\sigma^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1}$. Hence for the sum of the first two terms to be non-negative, z_l needs to be even larger than the value required in 1.53. Again, the third term is non-negative as long as $\beta R > 1 - \delta$.

Now we have proved that under certain conditions, $\frac{\partial H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma^{-1}} \geq 0$, then under the same conditions, $\frac{\partial H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} \leq 0$.

B. Prove $\frac{\partial \zeta(\lambda_\sigma, \lambda_\sigma)}{\partial \lambda_\sigma} > 0$

Define the left-hand-side of 1.32 as $H^l(\zeta_{jl}, \zeta(\lambda_\sigma, \lambda_\sigma))$, then

$$H^l(\zeta_{jl}, \zeta(\lambda_\sigma, \lambda_\sigma)) = -\zeta_{jl} + \beta p^{ll} \left[\zeta(\lambda_\sigma, \lambda_\sigma) - \int_0^{\zeta(\lambda_\sigma, \lambda_\sigma)} F(\zeta) d\zeta \right], \tag{1.54}$$

where $\zeta_{jl} = \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)$. When $j = l$, the beginning leverage is $(\lambda_{Tl}, \lambda_{Nl}) = \lambda_\sigma$, we have $\zeta_{ll} = \zeta(\lambda_\sigma, \lambda_\sigma)$. Then the left-hand-side is $H^l(\zeta_{ll}, \zeta_{ll}) = H^l(\zeta(\lambda_\sigma, \lambda_\sigma), \zeta(\lambda_\sigma, \lambda_\sigma))$.

Taking derivatives w.r.t $\zeta(\lambda_\sigma, \lambda_\sigma)$, I get

$$\frac{dH^l(\zeta(\lambda_\sigma, \lambda_\sigma), \zeta(\lambda_\sigma, \lambda_\sigma))}{d\zeta(\lambda_\sigma, \lambda_\sigma)} = -1 + \beta p^l (1 - F(\zeta(\lambda_\sigma, \lambda_\sigma))) < 0 \quad (1.55)$$

The inequality holds true because β , p^l and F all range between 0 and 1.

We have proved in part A that $\frac{\partial H^r((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} \leq 0$ (RHS is decreasing in λ_σ), because the equality holds true, it must be the case that $\frac{\partial H^l(\zeta_{jl}, \zeta(\lambda_\sigma, \lambda_\sigma))}{\partial \lambda_\sigma} \leq 0$ (LHS is decreasing in λ_σ). Specially for $j = l$, $\frac{\partial H^l(\zeta_{ll}, \zeta(\lambda_\sigma, \lambda_\sigma))}{\partial \lambda_\sigma} \leq 0$. Since we just proved that $\frac{dH^l(\zeta_{ll}, \zeta(\lambda_\sigma, \lambda_\sigma))}{d\zeta(\lambda_\sigma, \lambda_\sigma)} < 0$, we see that $\frac{\partial \zeta(\lambda_\sigma, \lambda_\sigma)}{\partial \lambda_\sigma} > 0$ must hold.

C. Prove $\frac{\partial \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} > 0$

Next I use $\frac{\partial \zeta(\lambda_\sigma, \lambda_\sigma)}{\partial \lambda_\sigma} > 0$ and $\frac{\partial H^l(\zeta_{jl}, \zeta(\lambda_\sigma, \lambda_\sigma))}{\partial \lambda_\sigma} \leq 0$ to prove $\frac{\partial \zeta((\lambda_{Tj}, \lambda_{Nj}), \lambda_\sigma)}{\partial \lambda_\sigma} > 0$. Now taking derivatives of $H^l(\zeta_{jl}, \zeta(\lambda_\sigma, \lambda_\sigma))$ w.r.t λ_σ , we get

$$\frac{\partial H^l(\zeta_{jl}, \zeta(\lambda_\sigma, \lambda_\sigma))}{\partial \lambda_\sigma} = -\frac{\partial \zeta_{jl}}{\partial \lambda_\sigma} + \beta P^l \frac{\partial \zeta(\lambda_\sigma, \lambda_\sigma)}{\partial \lambda_\sigma} [1 - F(\zeta(\lambda_\sigma, \lambda_\sigma))]$$

We know that $\frac{\partial \zeta(\lambda_\sigma, \lambda_\sigma)}{\partial \lambda_\sigma} > 0$ so that the second term on the right-hand-side is positive, therefore the first term must be negative for $\frac{\partial H^l(\zeta_{jl}, \zeta(\lambda_\sigma, \lambda_\sigma))}{\partial \lambda_\sigma} < 0$ to hold. We have proved that $\frac{\partial \zeta_{jl}}{\partial \lambda_\sigma} > 0$.

Part 2: Derive conditions under which $\zeta((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma) > \zeta((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma)$ if $\lambda_{T1} + \lambda_{N1} > \lambda_{T2} + \lambda_{N2}$.

Using $\zeta((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma) - \zeta((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma) = H^r((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma) - H^r((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma)$ and start from 1.49, we have

$$\begin{aligned} & (1 - \beta) [H^r((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma) - H^r((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma)] \\ &= (1 - \beta) \log \left(\frac{z_l \pi k_{\lambda, j} + 1 - \delta + R((\lambda_{T2} + \lambda_{N2})^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1}}{z_l \pi k_{\lambda, j} + 1 - \delta + R((\lambda_{T1} + \lambda_{N1})^{-1} - 1) - (1 - \delta) \lambda_\sigma^{-1}} \right) \\ & - \log \left(\frac{z_l \pi k_{\lambda, j} + 1 - \delta + R((\lambda_{T2} + \lambda_{N2})^{-1} - 1)}{z_l \pi k_{\lambda, j} + 1 - \delta + R((\lambda_{T1} + \lambda_{N1})^{-1} - 1)} \right) \\ &= (1 - \beta) \log \left(\frac{S_2 - (1 - \delta) \lambda_\sigma^{-1}}{S_1 - (1 - \delta) \lambda_\sigma^{-1}} \right) - \log \left(\frac{S_2}{S_1} \right) \end{aligned} \quad (1.56)$$

$$S_1 = z_l \pi k_{\lambda, j} + 1 - \delta + R((\lambda_{T1} + \lambda_{N1})^{-1} - 1), \quad S_2 = z_l \pi k_{\lambda, j} + 1 - \delta + R((\lambda_{T2} + \lambda_{N2})^{-1} - 1).$$

Since $\lambda_{T1} + \lambda_{N1} > \lambda_{T2} + \lambda_{N2}$, we have $S_1 < S_2$, and $\frac{S_2 - (1 - \delta) \lambda_\sigma^{-1}}{S_1 - (1 - \delta) \lambda_\sigma^{-1}} > \frac{S_2}{S_1} > 1$. For $H^r((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma) - H^r((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma) > 0$ to hold, using 1.56 we get β has to satisfy

the following condition:

$$\beta < 1 - \frac{\log\left(\frac{S_2}{S_1}\right)}{\log\left(\frac{S_2 - (1-\delta)\lambda_\sigma^{-1}}{S_1 - (1-\delta)\lambda_\sigma^{-1}}\right)} \quad (1.57)$$

That is, when β is sufficiently small, given the condition that $\lambda_{T1} + \lambda_{N1} > \lambda_{T2} + \lambda_{N2}$, we have $\zeta((\lambda_{T1}, \lambda_{N1}), \lambda_\sigma) > \zeta((\lambda_{T2}, \lambda_{N2}), \lambda_\sigma)$.

To sum up, when the entrepreneur is sufficiently impatient, the larger leverage he starts with at the beginning of the period, the more likely he would liquidate at that period.

A collection of equilibrium conditions

Households' decisions

$$\mu L_r^v = \omega \quad (1.58)$$

$$\mathbb{E}_X \left[\frac{\beta^r u' \left(C_r' - \frac{\mu(L_r')^{1+v}}{1+v} \right)}{u' \left(C_r - \frac{\mu(L_r)^{1+v}}{1+v} \right)} R' \right] = 1 \quad (1.59)$$

$$C_r + B_r' = \omega L_r + R B_r \quad (1.60)$$

Entrepreneurs portfolio choices and liquidation decisions

z_h -entrepreneur

$$\begin{aligned} & p^{hh} \mathbb{E}_X \left[\frac{z_h \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Th}} + 1 - \delta - R'}{z_h \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + (1 - \delta)(\lambda'_{Th} + \lambda'_{Nh}) + R'(1 - \lambda'_{Th} - \lambda'_{Nh})} \right] + \\ & p^{hl} \mathbb{E}_X F(\zeta'_{hl}) \left[\frac{z_l \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Th}} + 1 - \delta - R'}{z_l \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + (1 - \delta)(\lambda'_{Th} + \lambda'_{Nh}) + R'(1 - \lambda'_{Th} - \lambda'_{Nh})} \right] + \\ & p^{hl} \mathbb{E}_X [1 - F(\zeta'_{hl})] \left[\frac{z_l \pi' \frac{\partial k(\lambda'_{Th}, \lambda'_{Nh})}{\partial \lambda'_{Th}} + (1 - \delta) q'_{hl} - R'}{z_l \pi' k(\lambda'_{Th}, \lambda'_{Nh}) + (1 - \delta)(\lambda'_{Th} + \lambda'_{Nh}) q'_{hl} + R'(1 - \lambda'_{Th} - \lambda'_{Nh})} \right] \\ & \geq 0 \end{aligned} \quad (1.61)$$

$$\begin{aligned}
& p^{hh} \mathbb{E}_X \left[\frac{\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \left(z_h \pi' \frac{\partial k'}{\partial \lambda'_{Th}} + 1 - \delta - R' \right) + \left(z_h \pi' \frac{\partial k'}{\partial \lambda'_{Nh}} + 1 - \delta - R' \right)}{z_h \pi' k (\lambda'_{Th}, \lambda'_{Nh}) + (1 - \delta) (\lambda'_{Th} + \lambda'_{Nh}) + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] + \\
& p^{hl} \mathbb{E}_X F(\zeta'_{hl}) \left[\frac{\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \left(z_l \pi' \frac{\partial k'}{\partial \lambda'_{Th}} + 1 - \delta - R' \right) + \left(z_l \pi' \frac{\partial k'}{\partial \lambda'_{Nh}} + 1 - \delta - R' \right)}{z_l \pi' k (\lambda'_{Th}, \lambda'_{Nh}) + (1 - \delta) (\lambda'_{Th} + \lambda'_{Nh}) + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] + \\
& p^{hl} \mathbb{E}_X [1 - F(\zeta'_{hl})] \left[\frac{\frac{\lambda'_{Th}}{1-\lambda'_{Nh}} \left(z_l \pi' \frac{\partial k'}{\partial \lambda'_{Th}} + (1 - \delta) q'_{hl} - R' \right) + z_l \pi' \frac{\partial k'}{\partial \lambda'_{Nh}} + (1 - \delta) q'_{hl} - R'}{z_l \pi' k (\lambda'_{Th}, \lambda'_{Nh}) + (1 - \delta) (\lambda'_{Th} + \lambda'_{Nh}) q'_{hl} + R' (1 - \lambda'_{Th} - \lambda'_{Nh})} \right] \\
& \hspace{20em} = 0 \\
& \hspace{20em} (1.62)
\end{aligned}$$

z_l -entrepreneur

$$\frac{\lambda'_{Tl}}{1 - \lambda'_{Nl}} = \frac{1}{1 - \theta(1 - \delta)/R}$$

Replace λ'_{Tl} and λ'_{Nl} with $\lambda'_i = \lambda'_{Tl} + \lambda'_{Nl}$, I get

$$\lambda'_i = \frac{1}{1 - \theta(1 - \delta)\sigma/R} \quad (1.63)$$

λ'_{Tl} and λ'_{Nl} can be backed out by

$$\lambda'_{Tl} = \sigma \lambda'_i$$

$$\lambda'_{Nl} = (1 - \sigma) \lambda'_i$$

Entrepreneurs liquidation decisions

$$[(1 - \delta) \lambda_{Tj} + (1 - \delta) \lambda_{Nj}] \left(q_{jl} - 1 + \frac{1}{\lambda'_{Tl} + \lambda'_{Nl}} \right) = \beta n^*$$

where $n^* = z_l \pi k (\lambda_{Tj}, \lambda_{Nj}) + [(1 - \delta) \lambda_{Tj} + (1 - \delta) \lambda_{Nj}] q_{jl} + R(1 - \lambda_{Tj} - \lambda_{Nj})$.

If z_l was drawn last period:

$$(1 - \delta) \left(q_{ll} - 1 + \frac{1}{\lambda'_i} \right) = \beta n_i^* \quad (1.64)$$

where $n_i^* = z_l \pi + (1 - \delta) q_{ll} + R \left(\frac{1}{\lambda'_i} - 1 \right)$

If z_h was drawn last period:

$$[(1 - \delta) \lambda_{Th} + (1 - \delta) \lambda_{Nh}] \left(q_{hl} - 1 + \frac{1}{\lambda'_i} \right) = \beta n_h^* \quad (1.65)$$

where $n_h^* = z_l \pi k (\lambda_{Th}, \lambda_{Nh}) + [(1 - \delta) \lambda_{Th} + (1 - \delta) \lambda_{Nh}] q_{hl} + R(1 - \lambda_{Th} - \lambda_{Nh})$.

$$\Delta_{jl} = \log \left(\frac{n_{\text{liquidate}}^*}{n_{\text{stay}}^*} \right)$$

where $n_{\text{stay}}^* = z_l \pi k (\lambda_{Tj}, \lambda_{Nj}) + (1 - \delta) (\lambda_{Tj} + \lambda_{Nj}) q_{jl} + R (1 - \lambda_{Tj} - \lambda_{Nj})$, and $n_{\text{liquidate}}^* = z_l \pi k (\lambda_{Tj}, \lambda_{Nj}) + (1 - \delta) (\lambda_{Tj} + \lambda_{Nj}) + R (1 - \lambda_{Tj} - \lambda_{Nj})$.

If l was drawn last period:

$$\Delta_{ll} = \log \left(\frac{z_l \pi + 1 - \delta + R \left(\frac{1}{\lambda_l} - 1 \right)}{z_l \pi + (1 - \delta) q_{ll} + R \left(\frac{1}{\lambda_l} - 1 \right)} \right) \quad (1.66)$$

If h is drawn last period:

$$\Delta_{hl} = \log \left(\frac{z_l \pi k (\lambda_{Th}, \lambda_{Nh}) + (1 - \delta) (\lambda_{Th} + \lambda_{Nh}) + R (1 - \lambda_{Th} - \lambda_{Nh})}{z_l \pi k (\lambda_{Th}, \lambda_{Nh}) + (1 - \delta) (\lambda_{Th} + \lambda_{Nh}) q_{hl} + R (1 - \lambda_{Th} - \lambda_{Nh})} \right) \quad (1.67)$$

$$\frac{\Delta_{jl}}{1 - \beta} - \zeta_{jl} = \beta p^{lh} \frac{\log \left(\tilde{\lambda}' n'_h / R' \right)}{1 - \beta} + \beta p^{lu} \frac{\log \left(\tilde{\lambda}' n'_l / R' \right)}{1 - \beta} + \beta p^{lu} \left(-\zeta'_{lu} + \int_0^{\zeta'_{lu}} F(\zeta') d\zeta' \right) \quad (1.68)$$

where:

$$\begin{aligned} \tilde{\lambda}' &= \frac{(\lambda'_{Tl} + \lambda'_{Nl})}{1 + (q_{jl} - 1) (\lambda'_{Tl} + \lambda'_{Nl})} \\ n'_h &= z_h \pi' + 1 - \delta + R' \left(\frac{1}{\lambda'_l} - 1 \right) \\ n'_l &= z_l \pi' + 1 - \delta + R' \left(\frac{1}{\lambda'_l} - 1 \right) \end{aligned}$$

If l was drawn last period:

$$\tilde{\lambda}'_l = \frac{\lambda'_l}{1 + (q_{ll} - 1) \lambda'_l}$$

If h is drawn last period:

$$\tilde{\lambda}'_h = \frac{\lambda'_l}{1 + (q_{hl} - 1) \lambda'_l}$$

Wealth evolution conditions

$$\begin{aligned} K'_{Th} &= \lambda'_{Th} \beta \sum_j \left[z_h \pi K + (1 - \delta) (K_{Tj} + K_{Nj}) + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) (K_{Tj} + K_{Nj}) \right] p^{jh} \\ &\quad + \lambda'_{Th} \beta R p^{lh} B \end{aligned} \quad (1.69)$$

$$K'_{Nh} = \lambda'_{Nh} \beta \sum_j \left[z_h \pi K + (1 - \delta) (K_{Tj} + K_{Nj}) + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) (K_{Tj} + K_{Nj}) \right] p^{jh} + \lambda'_{Nh} \beta R p^{lh} B \quad (1.70)$$

$$K'_{Tl} = \sigma (1 - \delta) \sum_j \left[1 - F(\tilde{\zeta}_{jl}) \right] p^{jl} (K_{Tj} + K_{Nj}) \quad (1.71)$$

$$K'_{Nl} = (1 - \sigma) (1 - \delta) \sum_j \left[1 - F(\tilde{\zeta}_{jl}) \right] p^{jl} (K_{Tj} + K_{Nj}) \quad (1.72)$$

$$B' = \beta \sum_j F(\tilde{\zeta}_{jl}) \left[\frac{z_l \pi K}{K_{Tj} + K_{Nj}} + (1 - \delta) + R \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) \right] (K_{Tj} + K_{Nj}) p^{jl} + \beta R p^{ll} B \quad (1.73)$$

Market clearing conditions

$$\pi = \alpha \left(\frac{(1 - \alpha) A}{\omega} \right)^{\frac{1-\alpha}{\alpha}} \quad (1.74)$$

$$\sum_j \left(\frac{1}{\lambda_{Tj} + \lambda_{Nj}} - 1 \right) (K_{Tj} + K_{Nj}) + B + B_h = 0 \quad (1.75)$$

$$\left(\frac{\pi}{\alpha} \right)^{\frac{1}{1-\alpha}} \left[(p^{lh} z_h + p^{ll} z_l) K(K_{Tl}, K_{Nl}) + (p^{hh} z_h + p^{hl} z_l) K(K_{Th}, K_{Nh}) \right] = AL_h \quad (1.76)$$

where or $K(K_{Th}, K_{Nh}) = K_{Th}^\sigma K_{Nh}^{1-\sigma}$, and $K(K_{Tl}, K_{Nl}) = K_{Tl} + K_{Nl}$, $K_{Tl} = \sigma K$, $K_{Nl} = (1 - \sigma) K$

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Chapter 2

Revisiting Capital Controls in a Two-country Model with Financial Constraints

2.1 Introduction

Over the 20th century, there have been long-time debates on the desirability of capital controls, which are defined as policies imposed to restrict free movements of capital flows across country borders. In the wake of the global financial crisis, many emerging markets have tightened their capital accounts in response to large capital inflows, which intensifies the discussion of reassessing capital controls. Using a two-country business cycle model, this paper takes the capital-control policies as research objectives, examining their effects on the movements of Terms of Trade (TOT) and potentials to improve international risk sharing, as well as discussing the welfare implications.

To study the effects of capital controls, an occasionally binding financial constraint is incorporated into the classic two-country stochastic model developed by Backus, Kehoe, and Kydland (1994). Each country uses labor and capital to produce country-specific intermediate goods which are then used as input to produce final goods for consumption and investment. Since the country-specific intermediate goods are imperfectly substitutable, an international spot market exists for trading purpose. A financial market is in place and it's incomplete in that only a non-contingent bond is traded and household borrowing is subject to the borrowing constraint. The borrowing structure of the model is asymmetric, with one country which is more impatient always borrowing from the other country. Capital controls enter the economy as a restriction on the financial constraint that shrinks the external borrowing capacity. It can either be viewed as a control on capital inflows in the borrowing country or a control on capital outflows in the lending

country.

After the model is calibrated to parameters that are representative for both developed and developing countries, the transition process of the economy to a new equilibrium after the implementation of capital controls is first investigated. This exercise shows that after capital controls are put in place and the external borrowing capacity shrinks, the TOT moves in favor of the borrowing country, reflecting a relative scarcity of the domestically produced intermediate goods and implying a possible improvement of the welfare. A welfare study confirms that the capital-control policies can be welfare-improving if the borrowing country has a relatively high influence power on the TOT (meaning that traded goods are less easier to be substituted). However, as the influence power decreases, the borrowing country turns out to suffer a welfare loss from capital controls. For the lending country, no welfare improving parameterization is found, meaning that the capital-control policies cannot be Pareto-improving in this model.

Impulse response exercises show that capital controls play a certain role in improving international risk sharing between countries. This again stems from the effect of capital controls on the TOT. Since the TOT moves inversely to relative productivity of the countries, appreciation of TOT in the relatively less productive country works as an insurance against adverse shocks to the country. The implementation of capital controls gives rise to an equilibrium in which the TOT responses more intensely to a standardized shock, and the international risk sharing is improved in this sense.

The work in this paper is most closely related to Heathcote and Perri (2016), who also use the two-country model of Backus et al. (1994) as a workhorse. Instead of implementing restrictions on the financial constraints, capital controls enter their model as a tax or subsidy on saving that is proportional to the net foreign asset position. With a symmetric asset market, a country can have its own capital-control stance and take either positive or negative net asset positions, enabling an investigation on the so called "capital-control war", in which both countries can set their optimal tax rates, taking as given the policy choice of their partner. With this setup, they find parameterizations under which the implementation of capital controls can be Pareto improving by causing favorable changes in the TOT and international risk sharing. This paper share the same finding that capital control policies in the borrowing country can improve its welfare when trade in the international spot market is limited, because the TOT of the borrowing country, which serves as an insurance against drops in productivity, appreciates more than when goods can be traded more freely. However, the setup in this paper fails to find a parameterization under which the capital-control policy improves the global welfare. Other papers that study how capital controls benefit a country by causing favorable changes in international prices includes Costinot, Lorenzoni, and Werning (2014) and Edwards (1989).

Another strand of literature studies the desirability of capital controls in terms of externality. This class of literature employs a financial constraint which contains prices that can be affected by domestic borrowing and lending behaviors. (See Korinek (2018), Bianchi (2011), and Benigno, Chen, Otrok, Rebucci, and Young (2016)) The capital-control policy proves to be welfare-improving in these works because it helps resolve the problem of borrowing externality, which arises from the fact that individual agents fail to internalize the effects of their borrowing behavior on equilibrium prices and tend to borrow more than the efficient amount in a social planner's eyes. The study on the capital-control policy in this paper also rely on the usage of financial constraints, but the borrowing externality is not a concern. Financial constraints in this paper are where capital control policy enters the model, and the tightness of the constraints captures the intensity of the capital-control policies.

The rest of this paper is structured as follows. Section 2.2 introduces the framework in which an asymmetric borrowing structure is incorporated into a classic two-country business cycle model. Section 2.3 calibrates the model to representative parameters for both developed and developing countries. Section 2.4 describes and analyzes the result. Section 2.5 concludes.

2.2 The model

In this section, we analyze the effects of capital control in a standard international real business cycle model with incomplete financial market and endogenous credit constraints. We closely follow the framework of Heathcote and Perri (2002) in this section.

The world economy consists of two countries, $i \in \{1, 2\}$, each country is populated by an infinitely lived representative household and produces country specific goods. The economy experiences in each period one event $s_t \in S$, where S is the set of all possible events. Let $s^t = (s_0, s_1, s_2, \dots, s_t)$ denote the history of events before and including t , the probability of s^t is given by $\pi(s^t)$ at time 0.

Households Household consumes $c_i(s^t)$ out of final good and supplies labor $n_i(s^t)$ to intermediate-good-producing firms (i -firms). In each period t , utility function of the representative household is increasing in consumption and decreasing in labor supply.

$$U(c_i, n_i) = \frac{c_i^{1-\gamma}}{1-\gamma} - \frac{n_i^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}$$

where ϵ is the Frisch elasticity of labor and γ captures risk aversion and inter-temporal elasticity of substitution.

The representative household in each country is the owner of capital stock $k_i(s^t)$ and supplies it to the domestic i -firms together with labor. Both capital and labor are internationally immobile. In return, household receive intermediate good and have it traded on the frictionless international spot market, where the law of one price holds. After trading on the international spot market as well as the financial market, households sell their compositions of intermediate goods to the domestic final-good-producing firms (f -firms). The final good received from f -firms may be either consumed or invested. The law of motion for capital is given by

$$k_i(s^{t+1}) = (1 - \delta) k_i(s^t) + x_i(s^t), \quad (2.1)$$

where δ is the depreciation rate and $x_i(s^t)$ is the newly invested final good in country i after history s^t .

Intermediate-good-producing firms (i -firms) The i -firms are perfectly competitive and take the labor and capital supplied by households as inputs to produce country-specific intermediate goods, namely good a in country 1 and good b in country 2. The production function is Cobb-Douglas in capital and labor

$$F(z_i(s^t), k_i(s^t), n_i(s^t)) = e^{z_i(s^t)} k_i^\theta(s^t) n_i^{1-\theta}(s^t), \quad (2.2)$$

where $z_i(s^t)$ is an exogenous technology shock. The shocks $z(s^t) = [z_1(s^t), z_2(s^t)]$ follows a VAR (1) process which is given by

$$z(s^t) = Az(s^{t-1}) + \varepsilon_z(s^t) \quad (2.3)$$

where A is the 2×2 productivity transition matrix, and $\varepsilon(s^t)$ is a vector of 2×1 i.i.d random variables with a covariance matrix Σ . Let $w_i(s^t)$ and $r_i(s^t)$ denote the wage and rental rate on capital in units of the intermediate good produced in country i , the i -firm maximizes

$$\max_{k_i(s^t), n_i(s^t)} \{ F(z_i(s^t), k_i(s^t), n_i(s^t)) - w_i(s^t) n_i(s^t) - r_i(s^t) k_i(s^t) \}, \quad (2.4)$$

subject to $k_i(s^t), n_i(s^t) \geq 0$.

Final-good-producing firms (f -firms) The f -firms are perfectly competitive and produce final good using intermediate goods a and b with a constant returns to scale technology

$$G_i(a_i(s^t), b_i(s^t)) = \begin{cases} \left[\varpi a_i(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\varpi) b_i(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 1, \\ \left[(1-\varpi) a_i(s^t)^{\frac{\sigma-1}{\sigma}} + \varpi b_i(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, & i = 2, \end{cases} \quad (2.5)$$

where σ is the elasticity of substitution between intermediate good a and b . Good a and good b are perfect substitutes when σ approaches infinity and perfect complements when σ approaches zero. ϖ determines the degree to which domestic f -firms are home bias in using home produced intermediate good. The f -firm's static maximization problem in country i after history s^t is given by

$$\max_{a_i(s^t), b_i(s^t)} \{G_i(a_i(s^t), b_i(s^t)) - q_i^a(s^t) a_i(s^t) - q_i^b(s^t) b_i(s^t)\} \quad (2.6)$$

subject to $a_i(s^t), b_i(s^t) \geq 0$, where $q_i^a(s^t)$ and $q_i^b(s^t)$ are the prices of good a and good b in units of domestically produced final good in country i .

Households' problems In time 0, the representative household in each country maximizes

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta_i^t U(c_i(s^t), 1 - n_i(s^t)), \quad (2.7)$$

subject to the budget constraint (country 1)

$$\begin{aligned} & c_1(s^t) + x_1(s^t) + q_1^a(s^t) Q(s^t) B_1(s^t) \\ = & q_1^a(s^t) (r_1(s^t) k_1(s^t) + w_1(s^t) n_1(s^t)) + q_1^a(s^t) B_1(s^{t-1}), \end{aligned} \quad (2.8)$$

and an occasionally binding credit constraint (country 1)

$$-Q(s^t) q_1^a(s^t) B_1(s^t) \leq \kappa q_1^a(s^t) F(z_1(s^t), k_1(s^t), n_1(s^t)). \quad (2.9)$$

The budget constraint of country 2 is analogous. In the equations above, β_i^t is the discount factor. In our benchmark calibration, the discount factors for the two countries are different, with household in country 1 being more impatient and having a lower value of β . In our model setup, only a single non-contingent bond is traded in the international financial market. In country i after history s^t , the representative household buy bond of quantity $B_1(s^t)$ at a price $Q(s^t)$ (in units of good a) and is paid one unit of good a in period $t+1$, ignoring the state in $t+1$. The credit constraint requires that the value of the bond bought by the household cannot exceed a fraction κ of the domestic GDP.

In the non-stochastic steady state, because of the impatience, household in country 1 always borrows from country 2 to the maximum amount allowed by the credit constraint, rendering binding constraints.

To sum up, the household in each country chooses $c_i(s^t) \geq 0$ and $n_i(s^t) \in [0, 1]$ for all $t \geq 0$ to maximize equation (2.7) subject to the appropriate budget constraints and credit constraints given by equations (2.8) and (2.9), taking as given the productivity shocks, initial capital stocks and the initial distribution of bonds.

Market clearing conditions In equilibrium, the set of prices should be such that, for all $t \geq 0$, all markets clear when households solve their optimization problems taking these prices as given.

Intermediate goods market clearing requires

$$a_1(s^t) + a_2(s^t) = F(z_1(s^t), k_1(s^t), n_1(s^t)) \quad (2.10)$$

$$b_1(s^t) + b_2(s^t) = F(z_2(s^t), k_2(s^t), n_2(s^t)). \quad (2.11)$$

Final goods market clearing requires

$$c_i(s^t) + x_i(s^t) = G_i(a_i(s^t), b_i(s^t)), \quad i = 1, 2. \quad (2.12)$$

Bond market clearing requires

$$B_1(s^t) + B_2(s^t) = 0. \quad (2.13)$$

Equilibrium For all s^t and $t \geq 0$, an equilibrium is a set of price that clears all the markets when households solves their problems given these prices. The good market clears when conditions 2.10, 2.11 and 2.12 are satisfied. The asset market clears when condition 2.13 is satisfied.

Additional variables of interest Terms of trade

$$p(s^t) = \frac{q_i^a(s^t)}{q_i^b(s^t)} \quad i = 1, 2. \quad (2.14)$$

Table 2.1: Benchmark Parameters

Description	Parameter value
Quarterly discount factor of country 1	$\beta_1 = 0.98$
Quarterly discount factor of country 2	$\beta_2 = 0.99$
Risk aversion	$\gamma = 1$
Frisch elasticity of labor supply	$\epsilon = 1$
Capital share	$\theta = 0.36$
Quarterly depreciation rate	$\delta = 0.025$
Elasticity of substitution between intermediate goods	$\sigma = 1.5$
Degree of home bias in final goods production	$\omega = 0.65$
Productivity transition matrix	$A = \begin{bmatrix} 0.95 & 0.00 \\ 0.00 & 0.95 \end{bmatrix}$
Std. dev. of productivity shocks	$\sigma_{\varepsilon_1} = 0.02, \sigma_{\varepsilon_2} = 0.02$
Correlation of productivity shocks	$corr(\varepsilon_1, \varepsilon_2) = 0.3$
Parameter of financial constraints	$\kappa = 1.6$

Exchange rate

$$rx(s^t) = \frac{q_1^a(s^t)}{q_2^a(s^t)} = \frac{q_1^b(s^t)}{q_2^b(s^t)}. \quad (2.15)$$

2.3 Calibration

With an occasionally binding financial constraint in its structure, the model is solved using a dynare toolkit. This toolkit provided by Guerrieri and Iacoviello (2015) applies a first-order perturbation approach to solve dynamic models with occasionally binding constraints in a piece-wise fashion.

After being solved, the model is calibrated to a quarterly frequency. Table 2.1 shows the values calibrated for parameters. Most parameter values follow those in Heathcote and Perri (2016), which are standard values in literature and are representative for both developed and developing countries.

Discount factor of foreign household is 0.99, a standard value for quarterly model, matching quarterly interest rate of 0.01. Home household is modeled to be more impatient than foreign household in that the former has a lower discount factor of 0.98, which matches a quarterly interest rate of 0.02.

The parameter of risk aversion is set to be 1 so that utility is logarithmic in consumption. The Frisch elasticity of labor supply is also set to 1. Capital share θ and depreciation rate δ are both standard value used in literature.

Since the model has an asymmetric borrowing structure, the productivity processes

for the two countries are assumed to be symmetric to make variations comparable. The shock persistence ρ is set to 0.95, which is in the middle of estimates used in business cycle studies. Standard deviation and correlation of the shocks are respectively 0.02 and 0.3, which gives plausible result for an average of developed and developing countries.

ω which captures a country's degree of home bias is set to match an import share of 0.3, which is approximately the average for OECD economies in 2014. Heathcote and Perri (2016). The elasticity of substitution between inputs σ , which governs a country's power to affect TOT, is set to be 1.5 in the baseline model. In the experiment, I extend the value of σ to be lower (0.9) and higher (5) to check robustness of the result.

The capital control parameter κ is set to 1.6 in the baseline model, which matches average ratio of quarterly external debt to GDP in Argentina over the period 2003q3-2015q2 (Ratio of quarterly external debt to annual GDP is 0.4. Data source: International Financial Statistics (IFS)). I then lower the value of κ to 1.2 to study the impact of capital control on the economy.

2.4 Results

2.4.1 Transition of the Economy

Figure 2.1 shows transitions of main variables to their new steady-state values after the capital-control policy is implemented, or putting differently, response of the economy to a permanent decline of the capital-control parameter κ from 1.6 to 1.2. Different values of the substitution rate σ are considered, which reflects the influence power of a country on the TOT.

I first explain the transition to a new steady state under the benchmark value of $\sigma = 1.5$, which is represented by solid lines in the figure. For ease of convenience, the borrowing country is described as home country and the lending country is foreign country. Immediately after the shrink of borrowing capacity in the home country, main variables display one-time sharp deviations from their old steady-state values, a result arising from the fact that the home country has to pay the relatively large amount of previous debt using the now small borrowing capacity. After the permanent change of κ , the foreign country experiences a one-time increase in resources, leading to a rise in its investment (x_2) which then boosts its output (y_2). Meanwhile, the need for more final goods raises the demand for both types of intermediate goods (a_2 and b_2) as input, with the former pushing up the home output (y_1) and export (a_2). Coming to the home country, its investment (x_1) is cut down sharply due to limited resources available, along with drops in the demand of a_1 , and drops in b_1 driven by the relatively high

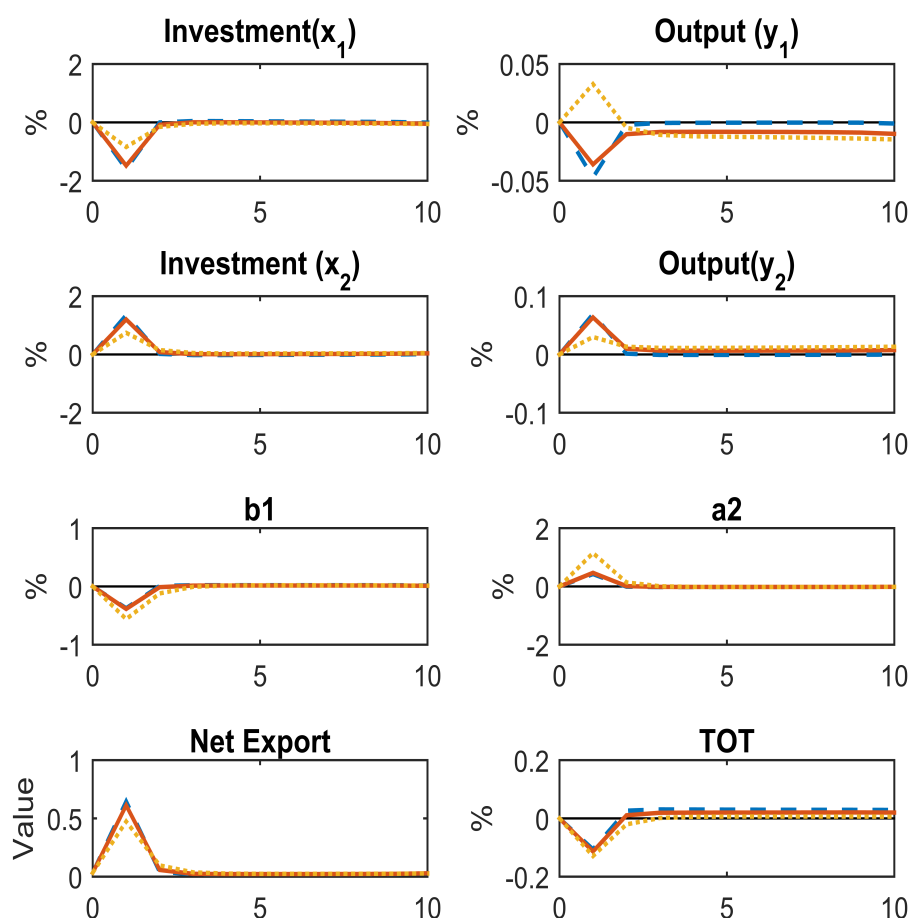


Figure 2.1: Transitions to new steady states after the capital-control policy is implemented

Transitions under various degrees of substitution elasticity $\sigma = 1.5$, $\sigma = 0.9$ and $\sigma = 5$ are represented by solid lines, dashed lines and dotted lines, respectively

complementary between them (relatively low σ). As home country is the main user of a_1 , the decline in it explains the depreciation in TOT immediately after the change.

As time elapses, the sharp deviations die out and variables gradually reach their new steady-state values. In relatively long-run, as the home capital investment (x_1) reaches a lower new steady-state value due to the limited amount of foreign finance after capital control has been imposed, the home output (y_1) goes down to a lower value finally. The foreign output (y_2) also falls back but stays at a higher new value, a result of the more abundant resources for domestic investment. The relative scarcer supply of the home produced input (a) pushes up its price, making both countries substitute it with the foreign produced input (b). Both countries incur larger trade deficits. The larger trade deficit in the home country arises from its heavier reliance on the imported good (b_1) and

reduced amount of export (a_2), while in the foreign country comes from the higher import price (qa_2). The TOT, whose appreciation shows a relatively scarcer supply of the home output, signals a welfare improvement to the home households, with the intuition being that lower import price benefits the home country who is a net importer.

To check the robustness, I vary the value of σ which characterizes the influence power of a country on the TOT and the corresponding transition paths are depicted in the figure. The dashed line in Figure 2.1 represents transitions of main variables under $\sigma = 0.9$, a relatively high degree of complementary between inputs. Given that inputs are hard to substitute one another, each country finds it difficult to divert to the relatively cheaper input and thus is forced to keep relying on the relatively more expensive input. Consequently, the higher complementary amplifies the return on the relatively scarce and expensive input. Therefore after the implementation of the capital-control policy, the home output drops immediately due to the initially low price of good a , but in the long-run, as the home country owns the scarcer input and has more power to enhance the TOT appreciation due to the high complementary, the TOT appreciates more in the home country and depreciates more in the foreign country than in the benchmark case.

The dotted line in Figure 2.1 displays the transitions under $\sigma = 5$, a relatively high degree of substitution between inputs. When it becomes more easier to substitute between inputs, the power of home bias comes into effect. The outcome is thus a result of the interaction between home bias and the high substitution. In the long-run, the home country flexibly increases the use of the more cheaper imported input (b_1) as well as the more expensive domestic input (a_1) due to the home bias, while the foreign country substitutes the more expensive imported input (a_2) with the cheaper domestic input (b_2) and the home bias reinforces the size of this substitution. In the short-run, the same logic applies, but the role of the two countries reverses. Home country substitutes the more expensive imported input (b_1) with the cheaper domestic input (a_1), while foreign country increases the use of both inputs (a_2 and b_2). As a result of the substitution, it's noticeable that the rise in the reliance on good a is so large that it even drives up home output (y_1). Although the TOT ought to appreciate because of the higher demand of input a , the relatively more supply of the latter ends up leading to the opposite. To sum up, when it's easier to substitute between inputs, both countries can flexibly use the relatively cheap input and thus the return on the relative scarcity is depressed by the high substitution possibility. As the home country, the owner of the scarcer input, now has less power to enhance the TOT movement, so the TOT appreciates the least in all three cases.

Let $V_i(z_1, z_2, k_1, k_2, B; \kappa)$ denote the expected lifetime utility for the two countries. Define the welfare gains from capital controls as the constant percentage increase in consumption in the economy that leaves the representative agent in country i indifferent

Table 2.2: Welfare Gain of the Capital-control Policy

w_i (%)	Elasticity of substitution (σ)				
	0.9	1.5	5	25	28
Country 1	0.2444	0.1300	-0.0176	-0.0553	-0.0553
Country 2	-0.5601	-0.4479	-0.2651	-0.1791	-0.1752

between $\kappa_1 = 1.6$ and $\kappa_2 = 1.2$. Since the utility function is separable and log in consumption, the welfare gain w_i can be expressed as

$$w_i = \exp(V_i(z_1, z_2, k_1, k_2, B; \kappa_2) - V_i(z_1, z_2, k_1, k_2, B; \kappa_1)) - 1$$

The welfare changes for the home country and the foreign country under various values of σ are summarized in Table 2.2, where country 1 is the borrowing country and country 2 is the lending country.

Table 2.2 shows that the welfare gain from the capital-control policy for the home country is decreasing in the substitution rate σ and turns negative at a relatively high value of σ , while the welfare gain for the foreign country is increasing in σ but stays negative even if I extend σ to the maximum value that the model can cope with.¹ Therefore, no Pareto-improving capital controls is found in this model.

2.4.2 Impulse Responses to Shocks

Impulse responses to a one s.d. positive productivity home shock are shown in Figure 2.2, with the dashed lines represent impulse responses before the capital-control policy is in place and the solid lines represent those after the implementation of the policy.

First focus on the impulse responses before the capital-control policy is implemented. A positive productivity shock to the home country (the borrowing country) gives rise to an increase in the domestic investment and labor supply, and a fall in the foreign investment and labor supply, reflecting the home households' desire to produce more with relatively higher productivity and the foreign households' willingness to sacrifice the domestic investment to lend more abroad. As a result, the home output increases a lot while the foreign output increases only slightly. The depreciation of the TOT shows a relative abundance of the home output. Meanwhile, the increase in the home output enlarges its borrowing capacity, so the home households can borrow more to invest and

¹when the inputs become extremely substitutable, both countries use only home produced input and it's unnecessary for the home country to borrow from abroad. But the model does not allow for financial autarky.

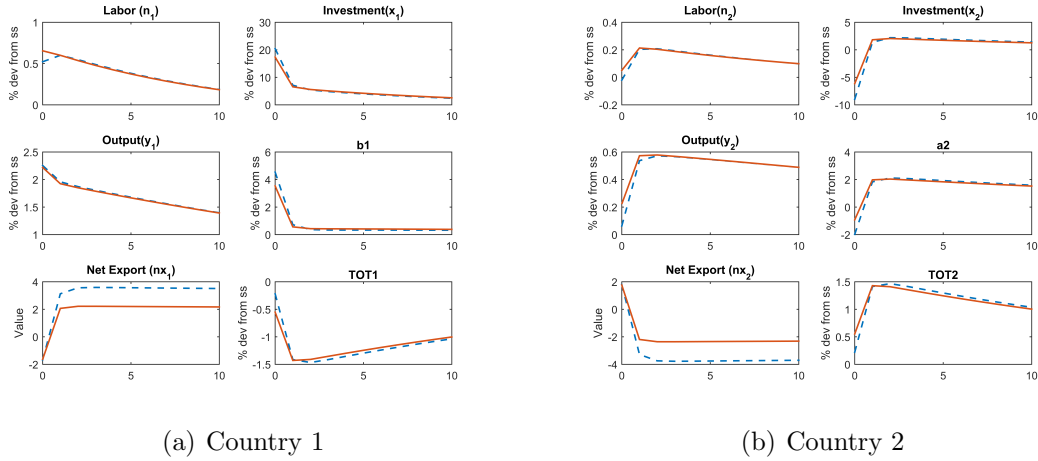


Figure 2.2: Impulse responses to a 1 s.d. positive productivity shock in Country 1

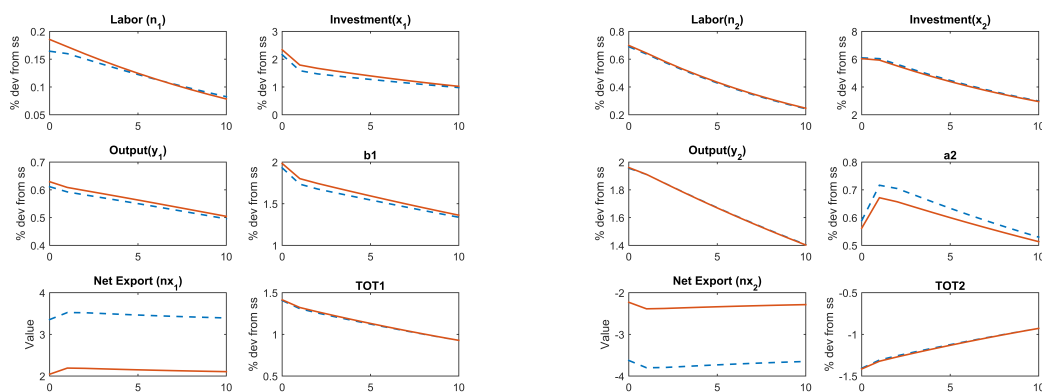
Note: The dashed lines represent impulse responses before the capital-control policy is put in place and the solid lines represent those after capital controls.

run a current account deficit. Under a relatively low rate of substitution, the home country fails to substitute the more expensive foreign produced input b_1 with the cheaper domestically produced input a_1 , which is consistent with the current account deficit. In response to the positive shock in the home country, the foreign country runs a current account surplus, because it imports less and exports more.

Now consider the impulse responses after the capital-control policy is imposed. In response to a positive productivity shock, the home country borrows abroad to increase the domestic investment and run a trade deficit by importing more. After the capital control, the borrowing capacity of the home households shrinks. Consequently investment increases less than before and the current account deficit narrows in response to the same shock, i.e. the home country has to import relatively less. Therefore the impulse responses see both a milder increase in the home import (b_1) and a milder decrease in the home export (a_2). As has been shown in Heathcote and Perri (2002), the current account deficit has an effect of mitigating deterioration of the TOT.² As a result, when the current account deficit narrows, the TOT is observed to deteriorate more after the shock. The worse TOT reduces the value of the output and accounts for the less sharp increase in the output.

The opposite is happening in the foreign country. After the capital-control policy is imposed, sacrifice of the foreign investment moderates in response to the shock in the home country because lending to the home country is reduced. The foreign current

² $\hat{p} = \phi \left[(\hat{y}_2 - \hat{y}_1) - \psi \frac{\hat{n}_x}{y} \right]$, where $\phi = \frac{1}{2s(\sigma-1)}$ and $\psi = \frac{2s-1}{1-s}$. Variables with hat represent deviations from the steady states. s denotes the steady state share of locally produced intermediate goods in final goods production and $s > 0.5$ when country are home bias. Under the benchmark parameters, $\phi > 0$ and $\psi > 0$. See Heathcote and Perri (2002).



(a) Country 1

(b) Country 2

Figure 2.3: Impulse responses to a 1 s.d. positive productivity shock in Country 2

Note: The Dashed lines represent impulse responses before the capital-control policy is put in place and the solid lines represent those after capital controls.

account surplus is smaller, consistent with the increase in investment. The TOT moves in favor of the foreign country by more than before, which increases the value of the foreign output by more than before.

As is emphasized by Cole and Obstfeld (1991), the TOT, as a reflection of relative scarcity of production, typically moves inversely to the relative productivity of countries, which thus improves the international risk sharing between countries and works as an insurance against adverse shocks. Accordingly, the implementation of capital controls which leads to a greater response of the TOT to shocks, plays a role in improving the international risk sharing.

Impulse responses to a one s.d. positive productivity foreign shock are shown in Figure 2.3. The responses are not simply a reverse of the responses to a home shock, a fact attributed to the asymmetric borrowing structure of the model. Specifically, if a positive one s.d. productivity shock strikes the foreign country (lending country), while the foreign investment and labor supply increase which push up the output as in the previous case, the investment and labor supply in the home country both increase rather than decrease. In addition, increase in the home output in response to the foreign shock is larger than the increase in the foreign output in response to a home shock. These results stem from the fact that as a borrowing country, the home does not sacrifice the domestic investment and output to lend abroad when the foreign country turns more productive. Also, the increase in the home investment to the foreign shock can be partly credited to the larger borrowing capacity following the increase in the home output. The TOT moves in favor of the home country, while the opposite movement takes place in the foreign country. The increase in the home investment accounts for the increase in the home import (b_1),

however the increase is not large enough to trigger a current account deficit, but only resulting in a more moderate increase in the current account surplus.

Except for changes that are associated with the asymmetric borrowing structure, other changes of the impulse responses to a foreign shock after capital control is just the reverse of those to a home shock, indicating again the role of capital controls to improve risk sharing across borders.

2.5 Conclusion

By incorporating financial constraints to a classical two-country business cycle model, this paper examines several dimensions of the capital-control policies: their effects on the Terms of Trade (TOT), potentials to improve international risk sharing, as well as the welfare implications. The practices show that the capital-control policies that restrict external borrowings can improve the welfare of the borrowing country by giving rise to a favorable movement of the TOT. But implementation of controls cannot be Pareto-improving in which case the welfare of the lending country is also a concern. In addition, the impulse response practices show that the implementation of capital controls give rise to an equilibrium in which TOT responses more intensely to a standardized shock, and the international risk sharing is improved in this sense.

Due to the simplicity of the model, this work fails to cover several features that might be important in analyzing the role of capital controls. For example, the implementation of capital controls in this model relies on the tightening of the financial constraints. The controls on capital inflows in the borrowing country and on capital outflows in the lending country are in effect two sides of the same practice, making it unfeasible to study the so called "capital-control war". In addition, this work overlooks the economic and social environment in which the capital-control policies are imposed, for example, exchange rate policies, the macroeconomic stance, the domestic financial system and the institutional quality. Those are all factors that might affect the impact of capital controls on the economy and thus deserve further investigations.

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Chapter 3

Capital Controls and Economic Growth: Threshold Effects of Country-specific Features

3.1 Introduction

3.1.1 A Brief Introduction

Over the 20th century, there have been long-time debates on the desirability of capital controls, which are defined as policies imposed to restrict free movements of capital flows across country borders. In the wake of the global financial crisis, many emerging markets have implemented capital-control policies to restrict capital floods from developed economies, which intensifies the discussion on reassessing capital controls. One of the main concerns about capital controls among others is their impacts on economic growth, though the results remain inconclusive in literature. To break down the ambiguity in the growth effects of capital controls, this paper brings into the two-dimensional relationship a third variable, which are country-specific social and economic features that might influence the interaction between the capital-control policies and economic growth. The underlying logic is that benefits of an open capital account can only be realized when some threshold conditions are satisfied, otherwise capital controls might benefit the growth by playing a role in sheltering domestic framework from foreign impacts. Using a recent threshold regression model, this paper investigates existence of certain threshold levels in the country-specific features, levels at which the effects of capital controls on economic growth reverse.

A recent methodology, Structural Threshold Regression (STR) model, is applied to

examine the role of country-specific features in affecting the interaction between capital controls and economic growth. The threshold regression model (or sample splitting model) is a nonlinear regression model which was first introduced by Hansen (2000). The model internally splits the sample and sorts data into regimes based on the value of a particular threshold variable. It enables an endogenous identification of the threshold level along with significance diagnoses with the help of a confidence interval constructed using the likelihood ratio. The slope coefficients under different regimes and the corresponding significance levels are also estimated. It is Kourtellos, Stengos, and Tan (2016) that develop the STR model to allow for endogeneity in both the threshold variables and the slope variables. This improvement is especially valuable for the type of growth model used in this work, which potentially suffers from serious endogeneity problems leading to inconsistent estimates.

With the STR model in hand, the goal of this paper is not to track how the effect of capital controls varies as the threshold variable changes in value, but rather to identify intrinsic threshold levels in the country-specific features and examine different effects of capital controls in regimes that the thresholds split into. This study makes use of the asymptotic theory and significance diagnoses of the STR methodology.

As a measurement of capital controls, the de jour data constructed by Fernández, Klein, Rebucci, Schindler, and Uribe (2016) based on IMF's Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER) is used. The dataset covers 100 countries over a period from 1995 to 2013 and provides information of capital flow restrictions on 32 transaction categories. This measurement well suits the research concern of this work in several aspects. As it captures the legal restrictions on capital account transactions, the de jour data enables a study that lays an emphasis on the policy of capital controls instead of the actual openness of capital account, which can be influenced by multiple factors besides the policy stances. In addition, the differentiation of the 32 categories permits a flexible study on various types of capital controls. Moreover, the 100 countries contained in the dataset disperse over a large span of social and economic development, providing considerable information to identify the threshold levels.

Four country-specific features are investigated as threshold variables that might affect the impact of capital controls on economic growth, namely performance of the domestic financial system (degrees of financial development), quality of the institutional framework (institutional quality), macroeconomic policies (size of government sector) and international trade (trade openness). For each of these thresholds, effects of various types of capital-control policies are examined, including aggregate capital controls, controls on capital inflow, debt securities, portfolio equity and bank credit, as well as controls on direct investment.

In the practice which takes the degrees of financial development as a threshold variable, this paper fails to identify a lower threshold level at which the effect of capital controls on growth turns from positive to negative, as opposed to the result of Prasad, Kose, and Taylor (2009). Consistent with the result of Prasad et al. (2009), a higher threshold level is identified, above which capital controls have a significant positive effect on growth and below which the effect of capital controls is obscure. To check the potential existence of a lower threshold level, an additional regression is conducted to the lower-regime only, and a sub-threshold is found where the effect of capital controls on growth turns from significantly positive to significantly negative. However, this sub-threshold is quite insignificant. Compared with the work of Prasad et al. (2009) who make use of a much earlier data set, the insignificance of the potential sub-threshold can be in part explained by the improvement of the financial systems in recent decades: countries all over the world have been climbing up on the ladder of the financial-development degree and some have moved out of the lowest financial-development region where capital controls could be beneficial to their growth, thus obscuring the sign of the lower-regime and making the effect insignificant.

Besides financial development, studies in this paper on other country-specific features give some meaningful results which support the view that the domestic framework can play a role in promoting economic efficiency and growth. However, the corresponding broad 95% confidence intervals in most of the studies signal large uncertainties around the thresholds, hence few threshold levels are considered significant. There are evidences that below a certain level of the institutional quality, capital controls have positive effects on the economic growth. Above the same level, controls on debt and equity might harm the economic growth potentially, which provides support to the view that high institutional quality plays a role in allocating resources efficiently. Thus a country with higher institutional quality can enjoy the benefit of an open capital account better compared to a country with poorer institutional quality. In the study on the macroeconomic policy, fiscal policy is measured by size of the government while monetary policy is measured by inflation variation. There are strong evidences that capital controls of various kinds are beneficial if the size of the government sector is too big to preserve efficiency. Moreover, significant threshold levels are identified in the measurement of government size when aggregate capital controls, controls on capital inflows and controls on debt are investigated. Also the significantly positive coefficient of capital controls support the view that capital controls can be beneficial in terms of economic growth in countries where the inflation rate fluctuates greatly. Studies on all types of capital controls show evidences that capital controls are less beneficial for countries that are more open to trade, providing support to the view that an open capital account could lead to misallocation of resources if the comparative advantages of the domestic sectors are protected.

The following part reviews the literature and sorts out ideas that motivate this work.

3.1.2 Motivations and Literature Review

One of the main concerns about capital controls among others is their impacts on the economic growth, however great ambiguity still exists in the literature. Standard theory believes that capital account liberalization promotes economic growth and efficiency by providing financing for high-return investments, enhancing international risk-sharing, and facilitating more efficient allocation of capital. Controls on the capital account, as opposed to an open accounts, play an adverse role in promoting growth because they impede free movements of capital flows. In contrast to the traditional wisdom, it has been argued in recent studies that capital controls foster economic efficiency and growth by stabilizing the real exchange rate and foreign financing, which allows for a more independent monetary policy to cool down over-investment and asset bubbles. This more tolerant perspective towards capital controls has motivated a number of countries to tighten their capital accounts after the global financial crisis. The IMF staff also accepted controls on certain types of inflows as a part of the toolkit (Ostry et al. (2010), Magud and Reinhart (2006)). Most of the papers focus on the influence of lifting capital controls, i.e. capital account liberalization. However, there are only weak or mixed evidences for a positive effect of capital account liberalization on growth. Comprehensive effect of the cost-benefit trade-off remains inconclusive. (see Kose, Prasad, Rogoff, and Wei (2006) for a detailed review).

To break down the ambiguity in the growth effects of capital controls, the concern has been diverted to cross-country disparities. A cross-group comparison reveals a negative relationship between income levels and the intensity of capital controls. Specifically, the average index of capital controls over a period from 1995 to 2013 in the low-income countries and middle-income countries are 0.63 and 0.48, respectively, while that of the high-income countries is a much lower 0.19. This observation raises the question of why the low-income countries tend to be more conservative in capital account liberalization, or further, are capital controls beneficial in terms of economic growth for them? Compared to the high-income countries, the low-income developing economies are often associated with weak domestic environments, for example, less developed financial systems, inadequate institutional frameworks and unstable macroeconomic policy stances. Can the disparities in the country-specific features add any clue to answer the question of growth effect of capital controls?

In fact, researchers have been trying to bring into the previous two-dimensional relationship a third variable, i.e. certain features that characterize domestic social and economic frameworks. They argue that the impact of capital controls on economic growth may be

nonlinear and conditioned on some country-specific features. The domestic framework, including policies and institutions, can play a role in affecting the growth effects of the capital-control policies. For example, in a host country with a developed financial system and good institutional quality, transparency and access to finance are guaranteed and investors' rights are well protected. In such an environment, capital can be efficiently channeled to profitable projects and therefore enhance the economic growth. In contrast, in a country with poor financial and institutional framework, an open capital account can expose the vulnerable domestic system to foreign impacts. Without adequate prudential supervisions in place, the resulted inefficient investments and asset bubbles can cause more challenging problems. Therefore, the benefits of an unfettered capital account can be obscured. (see Kraay (1998), Acemoglu, Johnson, Robinson, and Thaicharoen (2003), Blackburn and Forgues-Puccio (2010), Ju and Wei (2010), 2010) Based on these arguments, the existence of certain threshold levels, the levels in the country-specific features at which the effects of capital controls on growth reverse, are speculated. The underlying logic is that the benefits of an open capital account can only dominant and be realized when some threshold conditions are satisfied, otherwise capital controls might benefit the growth by sheltering domestic frameworks from foreign impact.

Following the literature, I narrow down the thresholds to be studied to five country-specific features. They capture, respectively, performance of the domestic financial systems (degrees of financial development), institutional frameworks (institutional quality), macroeconomic policies (size of government sector) and international trade (trade openness). A more developed financial system which is deep, efficient and adequately available to individuals is essential to allocate foreign finance to productive sectors. Institutions of high quality, which reflects good corporate and public governance as well as an effective legal framework, plays an important role in directing resources to high-return projects. A relatively open capital account is more likely to succeed if it is supported by good macro-policies that foster efficient allocations of capital. Capital account liberalization could lead to misallocation of resources if the international trade market is relatively less open so that sectors of comparative advantages are protected and the foreign capital has to flow into less competitive sectors.

In literature, most empirical studies focus on one particular country-specific feature and report some interesting results. Results reported by studies on multiple features vary a lot which can be partly attributed to different empirical methods, measurements of key characters and time horizons used.¹ Moreover, as is pointed out by Prasad et al. (2009), a majority of the studies use a simple linear interaction specification, leaving the

¹Examples of the former includes Hermes and Lensink (2003), Alfaro, Chanda, Kalemli-Ozcan, and Sayek (2004) and Alfaro et al. (2004) who study financial development; Edwards (2001)'s work on economic development; Klein (2005) on institutional quality. For examples of the latter, see Bekaert, Harvey, and Lundblad (2005), Arteta, Eichengreen, and Wyplosz (2001) and Kraay (1998)

statistical significance and confidence intervals of the threshold levels out of consideration. One exception is thus Prasad et al. (2009) who identify the potential threshold conditions with a quadratic interaction specification and derive confidence intervals of the financial openness coefficients for different levels of thresholds. Using a panel data of 84 countries covering the period of 1975-2004, they find clearly identifiable thresholds in financial depth and institutional quality, while evidence of threshold levels in other variables such as the trade openness and macro policies are not as strong. This paper shares the same research objective as Prasad et al. (2009), but differs from them in the research goals and the methodology being used. Instead of tracking how the effect of capital controls varies as the threshold variable changes in value, I seek to identify an intrinsic threshold in each country-specific feature and examine different effects of capital controls in regimes the threshold splits into. The studies make use of the asymptotic theory and significance diagnose in the framework of the STR methodology.

The rest of the paper is structured as follows. Section 3.2 presents sources of data and measurements in the empirical model. Section 3.3 briefly introduces the model and empirical strategy used to study threshold effects of capital controls. Section 3.4 shows the results and section 3.5 concludes.

3.2 Data and Measurements

3.2.1 Index of Capital Controls

There are broadly two distinct groups of indicators in measuring the degree of financial integration or financial openness in the literature: *de jure* measures and *de facto* measures. The *de jure* measures capture the legal restrictions or controls on capital account transactions, while the *de facto* measures exploit observable phenomena resulting from capital mobility, for example the stocks of foreign assets and liabilities used in Prasad et al. (2009). Quinn, Schindler, and Toyoda (2011) examine the *de jure* and *de facto* measures comprehensively, and assure that they capture though different but both useful and valid facet of international financial openness. Specifically, the *de facto* measures are often regarded as a reflection of how much an economy is actually integrated into the international capital markets. This group of measurement reflects influences of various factors on the capital account, not only the influences of legal restrictions as *de jure* measures do, but also those of many political and economic factors. As the *de facto* measures are only imperfectly related to government's policy stances, I use the *de jure* measures in this study to lay emphasis on the capital-control policies.

My study uses the *de jure* data introduced in Fernández et al. (2016). Like most

of the de jour measures, this dataset is based on IMF's Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER). As is documented in Quinn et al. (2011), Schindler (2009) KA index of capital controls is the most finely gradated measure stemming from AREAER. The KA index covers 91 countries over the period 1995 to 2005, and distinguishes restrictions on inflows and outflows over six asset categories. The dataset of Fernández et al. (2016), based on the methodology of Schindler (2009), extends the KA index to include four more asset categories, covering 100 countries over a longer period from 1995 to 2013.

The dataset provides information of restrictions on 32 transaction categories. Despite different directions of capital flows and asset categories, it also distinguishes various types of transactions according to the residency of the buyer or the seller, and whether the transaction represents a purchase or a sale or issuance. The narrative description in the AREAER has been used to determine whether or not there is restriction in a certain transaction category, with a 1 representing the presence of a restriction and a 0 representing no restriction. For various research objectives, it is possible to aggregate the data series correspondingly. The most basic aggregation is the indicators of controls on inflows and outflows for the ten asset categories. An overall index of capital controls is constructed as the average of the index of capital controls on inflows and outflows.

This new index of capital controls well suits our research concern in three aspects. First, the index is an aggregation of capital-control policies in many transaction categories. As the implementation of restrictions varies over time across categories, the intensity of controls is captured, which allows an investigation into the impact of capital controls on growth. Second, the dataset distinguishes between controls on capital inflows and capital outflows as well as controls on various asset categories. This feature of the dataset permits a study on influences of various types of capital controls. Different types of capital controls could differ in their impacts on growth. For example, it is generally believed the FDI and portfolio equity flows are more stable than debt flows. Therefore controls on debt can be more beneficial than controls on FDI, as the former reduces risk, while the latter impedes introduction of more advanced technology and concepts. Finally, the identification of threshold effects works efficiently if the dispersion of countries at different stages of development is large. The dataset contains in total 100 countries, with 42 high income countries, 26 upper middle income countries, 32 lower middle and low income countries. The complex constitution of the observations is large to provide enough dispersion in the country-specific features and allow for a fine search of the threshold effect.

In this study, for each potential threshold variable, five types of capital controls are to be considered, namely aggregate capital controls (ka in the data), controls on capital inflow (kai in the data), controls on debt securities, controls on equity and bank credit,

and controls on direct investment (*di* in the data). Controls on debt securities is an aggregation of controls on money market instrument (*mm* in the data set) and bond (*bo* in the data). Controls on equity and bank credit is an aggregation of controls on equity (*eq* in the data), financial credits (*fc* in the data) and collective investments (*ci* in the data).

3.2.2 Measurements of the Country-specific Features

The Degree of Financial Development

I deviate from the convention of approximating the degree of financial development with measures of financial depth: the ratio of private credit to GDP or stock market capitalization to GDP. Instead, a multidimensional dataset capturing the features of both the financial institutions (FI) and the financial markets (FM) is used. The new dataset is introduced by Svirydzienka (2016) and is originally developed in the context of the IMF Staff Discussion Note (). It is constructed for 183 countries on annual frequency between 1980 and 2013. In contrast to measuring the financial development with one single indicator of financial depth, the new index is defined as a combination of three characters for the FI and FM: depth (size and liquidity of markets), access (ability of individuals to access financial services), and efficiency (ability of institutions to provide financial services at low cost and with sustainable revenues, and the level of activity of capital markets). I make use of the financial development index from 1995 to 2013 to match the time horizon of the capital-control index.

The Institutional Quality

I use the World Bank Governance Indicators (WGI) to measure the institutional quality. This indicators capture the institutional quality in six aspects: voice and accountability; political instability and violence; government effectiveness; regulatory quality; rule of law; and control of corruption. The simple average of these six indexes is used as a proxy for the institutional quality. The aggregate index ranges approximately from -2.5 to 2.5, with higher values corresponding to better performances. This dataset containing 215 countries are available from 1996 onward.

The Fiscal Policy

The fiscal policy is measured with the ratio of government expenditure to GDP, which indicates the size of the government sector.

The Monetary Policy

The degree of variation of the consumer price inflation is used to capture the monetary policy stances. It is calculated as the standard deviation of annual inflation rates over the period considered.

The Trade Openness

I construct the measurement of the trade openness from the Penn World Table. The trade openness is the sum of exports and imports of goods and services divided by the GDP.

3.2.3 Control Variables in Growth Regressions

The set of control variables include variables that are typically included in the cross-country growth regressions. The initial income is measured by the logarithm of the real GDP per capita in 1990, a year prior to the start of the period being used to ensure the absence of endogeneity. The rate of investment to GDP and years of secondary school enrollment are both averaged over the period considered. The average growth rate of population is calculated as the logarithm difference over period divided by the length of the period.

In each regression corresponding to a certain threshold variable, the intended threshold variable along with all other potential threshold variables are taken as control variables to avoid potential bias resulted from omitting variables.

3.3 Methodology

In this section, I introduce the econometric model and empirical strategy used to study the threshold effects of capital controls. An additional subsection explains how the endogeneity is dealt with.

3.3.1 Structural Threshold Regression Model

The threshold model or sample splitting model is a nonlinear regression model which was first introduced by Hansen (2000)). The model internally splits the sample and sorts data into regimes based on the value of a particular threshold variable. The threshold level, as a dividing line between the regimes, is unknown and is thus endogenously estimated, and

its confidence interval, the slope coefficients under different regimes and the corresponding significant levels are estimated all together based on a solid theory. In its original work however, the model allows for neither endogeneity in the slope variables nor in the threshold variables. Caner and Hansen (2004) develop the statistic theory to deal with the endogeneity in the slope variables. However, if the endogeneity in the threshold variable is ignored, both Hansen (2000) and Caner and Hansen (2004) yield inconsistent estimates. (See Yu (2013)) It is Kourtellos et al. (2016) that develop the Structural Threshold Regression (STR) model to allow for endogeneity in both the threshold variables and the slope variables. Different from Hansen (2000) and Caner and Hansen (2004), Kourtellos et al. (2016) estimate the threshold parameter based on a two-stage concentrated least squares method that involves an inverse Mills ratio bias correction term in each regime.

The following STR model is considered:

$$y_i = \begin{cases} \beta'_{x1}x_i + u_i, & q_i \leq \gamma \\ \beta'_{x2}x_i + u_i, & q_i > \gamma \end{cases}$$

where y_i is the dependent variable in the two linear models, which are split by q_i , the threshold variable. x_i is a $p \times 1$ vector of the slope variables including an intercept and z_i is an $l \times 1$ ($l \geq p$) vector of instrument variables. u_i is the error term with $\mathbf{E}(u_i | \mathcal{F}_i) = 0$, where \mathcal{F}_{i-1} is a sigma field generated by $\{z_{i-j}, x_{i-1-j}, q_{i-1-j}, u_{i-1-j} : j \geq 0\}$. The parameters to be estimated include the threshold level, $\gamma \in \Gamma$, where Γ is a strict subset of the support of q_i and the slope coefficients $\beta_x = (\beta'_{x1}, \beta'_{x2})' \in \mathbf{R}^{2p}$.

The STR model can be written as follows to allow for endogeneity in both slope and threshold variables:

$$y_i = \beta'_{x1}g_{xi}I(q_i \leq \gamma) + \beta'_{x2}g_{xi}I(q_i > \gamma) + \kappa\Lambda_i(\gamma) + e_i^*,$$

where $g_{xi} = \mathbf{E}(x_i | \mathcal{F}_{i-1}) = \Pi'_x z_i$ is the conditional expectation from the reduced form $x_i = \Pi'_x z_i + v_{xi}$. κ is the inverse Mills ratio bias correction term. $e_i^* = \beta'_{x1}v_{xi}I(q_i \leq \gamma) + \beta'_{x2}v_{xi}I(q_i > \gamma) + \varepsilon_i$ with $\mathbf{E}(e_i^* | \mathcal{F}_{i-1}) = 0$.

The threshold parameter is estimated using a two-step concentrated least squares method and the slope parameters is estimated using a 2SLS or a GMM method. Specifically, the first step is to estimate the reduced form $q_i = \pi'_q z_i + v_{qi}$ and $x_i = \Pi'_x z_i + v_{xi}$ and construct the predicted inverse Mills ratio (IMR) term using the fitted values $\hat{q}_i = \hat{\pi}'_q z_i$ and $\hat{x}_i = \hat{g}_{xi} = \hat{\Pi}'_x z_i$. The IMR term is $\hat{\Lambda}_i(\gamma) = \hat{\lambda}_{1i}(\gamma)I(q_i \leq \gamma) + \hat{\lambda}_{2i}(\gamma)I(q_i > \gamma)$, where $\hat{\lambda}_{1i}(\gamma) = -\frac{\phi(\gamma - z'_i \hat{\pi}_q)}{\Phi(\gamma - z'_i \hat{\pi}_q)}$, and $\hat{\lambda}_{2i}(\gamma) = \frac{\phi(\gamma - z'_i \hat{\pi}_q)}{1 - \Phi(\gamma - z'_i \hat{\pi}_q)}$. $\phi(\cdot)$ and $\Phi(\cdot)$ are the normal pdf and cdf, respectively.

Stack the slope variables and the instrument variables respectively with the inverse

Mills ratio in $\hat{\mathcal{X}}_i(\gamma) = (x'_i(q_i \leq \gamma), x'_i(q_i > \gamma), \hat{\Lambda}_i(\gamma))'$, and $\hat{Z}_i(\gamma) = (z'_i(q_i \leq \gamma), z'_i(q_i > \gamma), \hat{\Lambda}_i(\gamma))'$.

The second step is to estimate the threshold γ by concentration using the predicted values obtained in the first step. For each γ , regress y_i on $\hat{\mathcal{X}}_i(\gamma)$ and the instruments $\hat{Z}_i(\gamma)$, obtaining the conditional 2SLS or GMM estimator $\hat{\theta}(\gamma) = (\hat{\beta}_{x1}(\gamma)', \hat{\beta}_{x2}(\gamma), \hat{\kappa}(\gamma))'$. The concentrated least square criterion is defined as:

$$S_n(\gamma) = \sum_{i=1}^n (y_i - \hat{\beta}_{x1}(\gamma)' \hat{g}_{xi} I(q_i \leq \gamma) - \hat{\beta}_{x2}(\gamma)' \hat{g}_{xi} I(q_i > \gamma) - \hat{\kappa}(\gamma) \hat{\Lambda}_i(\gamma))^2$$

Then γ is estimated by choosing the value which minimizes the CLS criterion

$$\hat{\gamma} = \arg \min_{\gamma} S_n(\gamma)$$

Once the threshold estimate $\hat{\gamma}$ is obtained, the slope parameters θ can be estimated by 2SLS or GMM. Let $\hat{\mathcal{X}}(\hat{\gamma})$ and $\hat{Z}(\hat{\gamma})$ be the matrices of stacked vectors $\hat{\mathcal{X}}_i(\hat{\gamma})$ and $\hat{Z}_i(\hat{\gamma})$. Let also Y be the stacked vector of y_i . Let $\hat{\mathcal{X}} = \hat{\mathcal{X}}(\hat{\gamma})$ and $\hat{Z} = \hat{Z}(\hat{\gamma})$ denote the matrices evaluated at $\hat{\gamma}$. Then the 2SLS estimator of $\theta = (\beta'_{x1}, \beta'_{x2}, \kappa)'$ is given by

$$\hat{\theta}_{2SLS} = (\hat{\mathcal{X}}' \hat{Z} (\hat{Z}' \hat{Z})^{-1} \hat{Z}' \hat{\mathcal{X}})^{-1} \hat{\mathcal{X}}' \hat{Z} (\hat{Z}' \hat{Z})^{-1} \hat{Z}' Y$$

The weight matrix $\hat{\Sigma}^*$ can be constructed using the 2SLS residual $\hat{e}_{i,2SLS} = y_i - \hat{\mathcal{X}}_i(\hat{\gamma})' \hat{\theta}_{2SLS}$. Then the GMM estimator is given by

$$\hat{\theta}_{GMM} = (\hat{\mathcal{X}}' \hat{Z} \hat{\Sigma}^{*-1} \hat{Z}' \hat{\mathcal{X}})^{-1} \hat{\mathcal{X}}' \hat{Z} \hat{\Sigma}^{*-1} \hat{Z}' Y$$

The corresponding estimated covariance matrix is $\hat{V}_{GMM} = (\hat{\mathcal{X}}' \hat{Z} \hat{\Sigma}^{*-1} \hat{Z}' \hat{\mathcal{X}})^{-1}$.

Consistency and asymptotic distribution of the threshold and slope parameters can be derived. See Kourtellos et al. (2016) for more details.

The likelihood ratio (LR) statistic is defined by $LR_n(\gamma) = n \frac{S_n(\gamma) - S_n(\hat{\gamma})}{S_n(\hat{\gamma})}$. Under certain assumptions, the asymptotic distribution of the LR test under $H_0 : \gamma = \gamma_0$ is given by $LR_n(\gamma_0) \rightarrow \eta^2 \psi$, where the distribution of ψ is $P(\psi \leq x) = (1 - e^{-x/2})(1 - e^{-\sqrt{\varphi}x/2})$. Hence, the asymptotic distribution of $LR_n(\gamma_0)$ is nonstandard and depends on two nuisance parameters η^2 and ψ .

The confidence intervals for the threshold parameter can be constructed by inverting the likelihood ratio test statistic LR_n . The assumption of constant threshold effect, conditional homoskedasticity and Gaussian errors gives proof that inferences based on

the inversion of the likelihood ratio test are asymptotically conservative. The confidence region is defined by $\hat{\gamma} = \{\gamma : LR_n(\gamma) \leq \hat{\eta}^2 c_\alpha\}$, where c_α is the $(1 - \alpha)100^{th}$ percentile distribution of ψ and can be computed by numerically solving the equation $(1 - e^{-x/2})(1 - e^{-\sqrt{\varphi}x/2}) = 1 - \alpha$ for known values of φ .

3.3.2 Empirical Strategy

To investigate the threshold effect of country-specific features on the interaction between capital controls and growth, the following STR model is used:

$$\Delta \ln GDP_{00-13,i} = \mu + \beta_1 CC_i I(q_i \leq \gamma) + \beta_2 CC_i I(q_i > \gamma) + x'_i \delta + u_i \quad (3.1)$$

The dependent variable is the average annual growth rate of real GDP per capita over the period 2000-2013, calculated as the logarithm difference over the period divided by the length of period. μ_i is the intercept. q_i is the threshold variable. I is an indicator function, which takes on values of 1 and 0 depending on the value of q_i , allowing slope coefficients to switch between regimes. There are two types of slope variables or control variables. I differentiate CC , the index of capital controls, from other control variables. β_1 is the slope coefficient of CC in regime 1, and β_2 is the slope coefficient of CC in regime 2. x_i , containing both endogenous and exogenous variables, is the vector of all control variables other than CC . The control variables, as has been mentioned in Section 3.2.3, include the initial income, the rate of investment to GDP, the average growth rate of population, the years of secondary school enrollment as well as all the potential thresholds variables.

To focus on the key variable of interest, I allow only the coefficient of CC to switch between regimes, while all other slope coefficients contained in δ are restricted to be independent of regimes. This design has no effect on the distribution theory. Restrictions implemented on the coefficients can be written as

$$R'\theta = c$$

where R is an $m \times n$ matrix of rank n , where m is the number of total coefficients, and n is the number of restrictions on coefficients. c is an n -dimensional vector of constants. The constrained GMM estimator is related to the unconstrained GMM estimator by

$$\hat{\theta}_{CGMM} = \hat{\theta}_{GMM} - \hat{V}_{GMM} R (R' \hat{V}_{GMM} R)^{-1} (R' \hat{\theta}_{GMM} - c)$$

and the constrained covariance matrix is

$$\hat{V}_{CGMM} = \hat{V}_{GMM} - \hat{V}_{GMM}R(R'\hat{V}_{GMM}R)^{-1}R'\hat{V}_{GMM}$$

The model can be extended to include multiple thresholds. I first consider the case of two potential thresholds and there is only weak evidence for this. So in the next section we only report the results of one threshold.

3.3.3 Endogeneity

Endogeneity of important control variables in the growth regression models always calls for concerns. For example, the initial income measured by the real GDP per capita at the beginning of the time period is endogenous by construction; restrictions on capital flows may be driven by movements in the GDP, which leads to concerns on loop of causality; the indicator of the financial development and economic growth may stem from common factors such as the institutional framework. In this section, I present how I deal with endogeneity in the empirical model.

Despite control variables that are typically included in the growth regression, i.e. the initial income, the investment ratio, the years of secondary school and the population growth rate, I control all the potential threshold variables in the regressions. Specifically, in each regression corresponding to a certain threshold variable, the intended threshold variable itself along with all other threshold variables to be investigated are included as control variables, as they can directly affect the economic growth besides their indirect effects as thresholds. An omission of these variables may lead to omitted-variable bias.

The initial income is measured with the logarithm of the real GDP per capita in 1990, a year prior to the start of the time period to avoid endogeneity problem.

To correct bias caused by other potential endogenous variables, I use lagged values of the corresponding variables as instruments. The lagged values are natural instruments, because on one hand they are correlated with the endogenous variable while on the other hand they are less likely to be correlated with current growth shocks. The time horizon of the dataset available is from 1995 to 2013. The observations from 2000 to 2013 of each country are averaged over time and used in my cross-country regressions, while observations prior to 2000 are taken as instruments.

An eligible instrument should meet two requirements simultaneously, namely the relevance and the validity. It must be correlated with the endogenous variables conditional on the other covariates, and at the same time uncorrelated with the error term. I follow the procedure below to pick the eligible instruments and identify the endogeneity. I first

include all lagged observations of the suspected endogenous variables as instruments and perform regressions using the two-step efficient generalized method of moments (GMM). Then, a series of tests are executed. The first set of test tests the relevance of the instruments as a whole. An LM test is performed to test the underidentification, under the null hypothesis of which the model is identified, meaning that the instruments are correlated with the endogenous regressors. A test of weak identification succeeds to diagnose whether the instruments are only weakly correlated, which gives rise to poorly performed estimators. All the regressions performed in our practice have passed the two tests above, indicating the instruments are relevant as a whole. The second set of test is the overidentifying restrictions test, specifically Hansen’s J test for efficient GMM estimator, which aims to diagnose if the instruments are jointly uncorrelated with the error term. In the third set of tests, I test the exogeneity of the individual regressor and the instruments. With the information provided by the third set of tests, the label of variables is adjusted: instruments that fail to pass the test are excluded, while previously suspected endogenous regressors that lack evidence of endogeneity are brought back as exogenous. After the adjustment, the first two sets of tests are implemented again to ensure the relevance and validity of the new set of instruments.

3.4 Results

This section presents results of the Structural Threshold Regression. For each threshold, I consider five different types of restrictions on the capital account, i.e. aggregate capital control (**Ka**), controls on capital inflow (**Kai**), controls on debt securities (**Debt**, including controls on money market instruments and bonds), controls on portfolio equity and bank credit (**Equity**, including controls on financial credits, equity and collective investment) and controls on direct investment (**Di**). The index of controls on debt securities is an aggregation of controls on two asset categories: money market instruments and bonds, while the index of controls on portfolio equity and bank credit is an aggregation of three categories: financial credits, equities, and collective investment. As the coefficients of the four regular controls in the growth model are all significant and the signs of which are as expected, I report only results concerning the identification of the threshold levels and the capital-control index which are of the main interests.

3.4.1 Financial Development

A more developed financial system which is deep, efficient and adequately available to individuals is essential to allocate the foreign finance to the productive sectors. Theoretically, Mendoza, Quadrini, and Rios-Rull (2007) show that in countries where financial

Table 3.1: The Financial Development as a Threshold Variable

CapCtrl	Ka	Kai	Debt	Equity	Di	Ka-Lower Regime
$\beta_1 (q \leq \gamma)$	-0.56 (0.39)	0.32 (0.46)	-0.34 (0.71)	-0.40 (0.39)	-0.01 (0.33)	0.36*** (0.13)
$\beta_2 (q > \gamma)$	2.23*** (0.49)	2.85*** (0.75)	1.90* (1.00)	1.46*** (0.48)	1.92*** (0.26)	-0.8** (0.38)
$\beta_1 - \beta_2$	-2.78*** (0.62)	-2.54*** (0.89)	-2.24* (1.20)	-1.85*** (0.60)	-1.93*** (0.38)	1.16*** (0.36)
Threshold (γ)	0.49	0.49	0.49	0.49	0.49	0.208
95% CI	0.490-0.493	0.24-0.60	0.19-0.60	0.49-0.52	0.3-0.6	0.15-0.34
# obs	92	92	92	92	92	58
# obs below γ	58	58	58	58	58	21
# obs in CI	2	38	49	3	25	58

globalization is not accompanied by the development of the domestic financial system, financial openness leads to an increase in the cost of borrowing which hurts the poor and results in a sizable welfare loss. In a different setup, Aoki, Benigno, and Kiyotaki (2010) show that when the domestic financial system is underdeveloped, the capital account liberalization is not necessarily beneficial because TFP stagnates in the long-run or employment decreases in the short-run. Accordingly, a threshold level in the degree of financial development is anticipated at which restrictions on the capital account turn from beneficial to harmful to the economic growth. However, I fail to recognize such a threshold.

The results (See Table 3.1) are quite consistent across different categories of capital controls, which are mainly resulted from the high correlations in the data, including the correlation of the aggregate capital controls and controls on inflows, and the correlation of controls on debt securities and on equities. 58 of the 92 observations are estimated to be below a threshold level of 0.49 in the degree of financial development. The widths of the 95% asymptotic confidence intervals of the thresholds vary a lot across different categories. When the aggregate restriction on capital is controlled, the confidence interval contains only two observations, indicating a little uncertainty around the threshold estimated. The confidence interval of the threshold identified is also narrow, with three observations contained, when restriction on equity flows is controlled. The broadest confidence interval occurs when restriction on debt flows is controlled, containing more than half of the total observations.

Above the threshold estimated, all five types of controls have significantly positive effects on growth, while no significant effect is recognized below this level. However, the difference between the two regimes is quite significant, which indicates that it is evidently

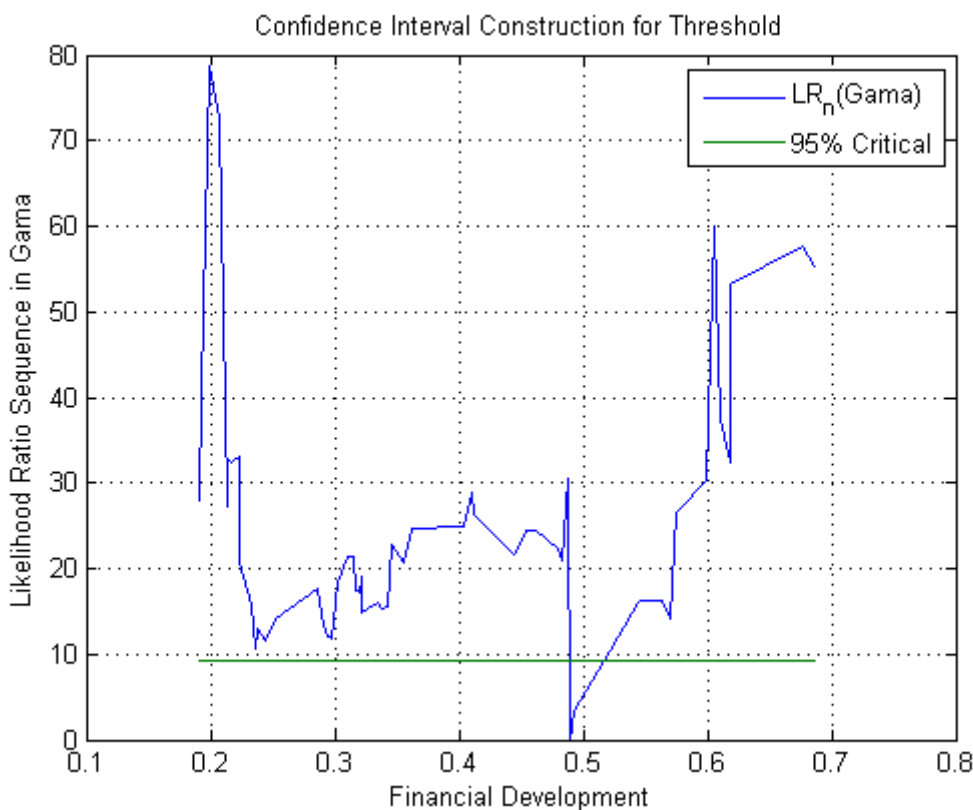


Figure 3.1: Concentrated likelihood ratio

more beneficial for countries with degrees of financial development above the threshold level to implement capital controls than for countries with lower degrees of financial development, a result that is quite the opposite to what has been expected.

In an effort to find more information to account for the insignificant coefficients of the capital control variables in the lower regime, I plot the concentrated likelihood ratio function in Figure 3.1. The threshold level estimated is a value of the threshold variable at which the likelihood ratio hits the zero axes. Beside the main dip created by the identified threshold, there is another major dip in the likelihood ratio to the right of it. I take out the 58 observations whose degree of financial development fall in the lower regime and implement another threshold regression separately. The result is reported in the last column (Ka-Lower Regime) of Table 3.1 where a threshold 0.208 in the lower regime is identified.² Below this threshold level, the aggregate capital control has a significantly positive effect on growth for the 21 countries with the most undeveloped financial systems in the data. In contrast, aggregate capital control in the 37 countries above this threshold has a significantly adverse effect on growth. The difference between these two sub-regimes is significant, whereas the confidence interval of the threshold is rather broad, spanning the whole range of γ in the lower regime. The opposite effects of these two sub-regimes

²Note that this is different from another endogenously estimated threshold in a model with two thresholds. I also implement the model of two thresholds and no supportive evidence is observed.

Table 3.2: Evolution of the Degree of Financial Development

Number of Countries	< 0.208	[0.208, 0.49]	> 0.49
1995-1999	34	31	27
2009-2013	16	37	39

offset each other and result in the insignificant capital-control coefficients of the lower regime.

My result is partly consistent with Prasad et al. (2009) who find that the impact of capital controls on growth takes an inverted U-shape as the threshold variable rises. Using the de facto data as a measure of the financial openness and the private credit to GDP ratio as a measure of the financial development degree, they report two thresholds in the financial development degree which effectively set the observations into three regimes: only in the mid-regime has the financial openness a positive marginal effect on growth. Both the lower- and upper-regimes see a negative effect of financial openness on growth. Like the second threshold in Prasad et al. (2009), the STR model also identifies a threshold level above which a relatively less open financial account is good for growth. However the STR model fail to identify a lower significant threshold level at which the effect of capital controls on growth turns from positive to negative.

The time horizon considered in Prasad et al. (2009) is from 1975 to 2004, the last five years of which overlap our first five years, but in general a period much prior to mine. During 1995 and 2013, the financial systems of most countries in the data kept evolving and improving. Suppose the potential threshold levels in the financial development degree do not change over time, then the evolution and improvement mean that many countries have moved out of the lower sub-regime (including countries whose degrees of financial development are below 0.208) in which capital controls have a positive growth effect. As these countries join the upper sub-regime (including countries whose degrees of financial development are between 0.208 and 0.49) in which capital controls have a negative effect, the previously (possibly) positive aggregate effect of capital controls in the entire lower regime (an aggregation of the lower sub-regime and the upper sub-regime) is thus obscured.

To see the evolution of the financial development, I take the two threshold levels given in the regression above (0.208 and 0.49) as nodes to examine how the degree of financial development evolves from 1995-1999 (the first five years of the data) to 2009-2013 (the last five year of the data). The result is shown in Table 3.2. The average degree of financial development of the 92 countries over the period 1995-1999 is compared to the one over the period 2009-2013. The result shows clear improvement in the degree of

Table 3.3: The Institutional Quality as a Threshold Variable

CapCtrl	Ka	Kai	Debt	Equity	Di
$\beta_1 (q \leq \gamma)$	1.02** (0.48)	1.57** (0.63)	1.68*** (0.47)	1.4*** (0.54)	0.73** (0.37)
$\beta_2 (q > \gamma)$	-0.78 (0.86)	0.27 (0.90)	-1.55* (0.88)	-1.36* (0.81)	0.27 (0.49)
$\beta_1 - \beta_2$	1.8* (0.93)	1.31 (1.03)	3.22*** (0.98)	2.76*** (0.93)	0.46 (0.59)
Threshold (γ)	-0.108	-0.028	-0.108	-0.108	-0.028
95% CI	-0.42-0.32	-0.61-0.51	-0.56-0.84	-0.61-0.51	-0.45-0.94
# obs	92	92	92	92	92
# obs below γ	40	43	40	40	43
# obs in the CI	23	36	46	36	41

financial development, with 18 countries moving out of the lowest regime and entering higher regimes. The improvement of the domestic financial system provides some clue to answer why I fail to recognize a lower threshold level in the threshold variable of financial development at which effect of capital controls on growth turn from positive to negative. If this speculation is correct, then a lower regime in which capital controls have a significantly negative effect on growth can be anticipated to form gradually. That is, more and more countries in the world economy are moving out of the region of low financial development degree where capital controls can be beneficial and joining the region where capital controls can be harmful.

3.4.2 Institutional Quality

Acemoglu et al. (2003) argue that an institutional environment is more important than geographic factors in explaining economic development, in the sense that better institutional framework encourages investment. Institutions of high quality, which reflect good corporate and public governance as well as an effective legal framework, can play important roles in directing resources to high-return projects. In a host country with sufficiently good institutional quality which contributes to allocating capital efficiently, a more open capital account is expected to better facilitate economic growth.

Table 3.3 shows that all five regressions give roughly the same threshold level in the index of institutional quality. However, the widths of the corresponding confidence intervals are rather broad and vary across different regressions. Roughly 40 of the total 92 observations are in the lower regime, and the coefficients on capital controls in this regime are significantly positive, indicating that capital controls in countries with lower

institutional quality are beneficial and associated with higher growth rate. The third and fourth regressions show that controls on debt and equities are harmful in countries with relatively high institutional quality, while the evidence for this in other regressions is not as strong. Three of the five regressions show that aggregate capital controls, controls on debt securities and on equities are significantly more beneficial in countries with relatively low institutional quality.

3.4.3 Macroeconomic Policies

A relatively open capital account is more likely to be successful if it is supported by good macro-policies that foster efficient allocations of capital. In this section, I report the role played by macro policies on the interaction between capital controls and growth. Specifically, I examine potential threshold levels in two variables: fiscal policy measured by the size of the government and monetary policy measured by the inflation variation.

The Fiscal Policy (the Size of the Government Sector)

The size of the government can impact the growth rate on two opposite directions. On one hand, a government of a large size can be associated with more inefficient operations, more rent-seeking activities, excess burdens of taxation, and policies that distort economic incentives and lower productivity, which reduce the economic efficiency and impede the growth. On the other hand, government can benefit growth through providing public good to correct externalities and through giving interventions to offset market failures. If the former effect dominates, a negative role of a big government in intermediating the growth effect of an open capital account is expected, which means above a certain level of the government size, restrictions on capital flows can be beneficial for growth.

Table 3.4 shows that the thresholds estimated vary across regressions. The widths of the corresponding confidence intervals also vary, with threshold levels in the first three regressions being exactly estimated, and the interval in the fourth regression spanning more than half of the observations. All five regressions give strong evidences that capital controls of various types are beneficial when the size of the government sector is big enough, and are significantly more beneficial than when the government sector is small. In the last three regressions, where the capital-control variables are less aggregated, capital controls associated with a small government have significantly adverse effects on growth. Compared to controls on the debt securities, controls on the equities and direct investment turn beneficial at a relatively higher threshold level, providing evidence that flows in forms of equities and direct investment are safer than flows in form of debts.

Table 3.4: The Size of the Government Sector as a Threshold Variable

CapCtrl	Ka	Kai	Debt	Equity	Di
$\beta_1 (q \leq \gamma)$	-0.57 (0.48)	-0.87 (0.61)	-0.79* (0.45)	-1.31** (0.58)	-0.65* (0.35)
$\beta_2 (q > \gamma)$	2.22*** (0.38)	2.21*** (0.58)	2.21*** (0.42)	2.20*** (0.50)	1.98*** (0.35)
$\beta_1 - \beta_2$	-2.79*** (0.55)	-3.08*** (0.71)	-3.00*** (0.51)	-3.51*** (0.64)	-2.63*** (0.44)
Threshold (γ)	0.146	0.146	0.149	0.157	0.15
95% CI	0.146-0.146	0.146-0.146	0.149-0.149	0.12-0.20	0.13-0.16
# obs	92	92	92	92	92
# obs below γ	36	36	38	44	39
# obs in the CI	1	1	1	56	16

Table 3.5: The Inflation Volatility as a Threshold Variable

CapCtrl	Ka	Kai	Debt	Equity	Di
$\beta_1 (q \leq \gamma)$	1.58*** (0.38)	1.91*** (0.45)	-0.46 (0.41)	-0.81** (0.40)	-0.20 (0.24)
$\beta_2 (q > \gamma)$	0.99** (0.44)	1.13** (0.51)	1.46*** (0.46)	1.45*** (0.48)	1.00*** (0.32)
$\beta_1 - \beta_2$	0.58 (0.44)	0.77 (0.47)	-1.92*** (0.92)	-2.27*** (0.48)	-1.20*** (0.27)
Threshold (γ)	0.89	0.89	1.59	1.59	0.746
95% CI	0.04-1.67	0.04-1.67	0.014-1.76	0.014-1.76	0.014-1.76
# obs	91	91	91	91	91
# obs below γ	45	45	72	72	39
# obs in the CI	60	60	62	62	62

Monetary Policy

A high volatility of inflation raises uncertainty in the price level and can lead to an unpleasant investment environment which reduces the growth rate. It increases the risk premium of long-term arrangements and gives rise to unanticipated redistribution of wealth. In this case, the high inflation volatility is expected to hinder the benefits of financial openness.

I follow Prasad et al. (2009) to measure the monetary policy with standard deviations of the inflation rate. The result is shown in Table 3.5. The threshold levels estimated vary across different categories, and their confidence intervals are all rather broad. The first two regressions identify significantly positive effects of controls on growth in both the lower regime and upper regime of inflation variance, while no significant difference

Table 3.6: The Trade Openness as a Threshold Variable

CapCtrl	Ka	Kai	Debt	Equity	Di
$\beta_1 (q \leq \gamma)$	0.87** (0.41)	1.08** (0.50)	1.12*** (0.40)	-0.02 (0.38)	0.81*** (0.30)
$\beta_2 (q > \gamma)$	-3.26*** (1.24)	-2.0 (1.3)	-3.3*** (1.0)	-4.16*** (1.29)	-0.32 (0.57)
$\beta_1 - \beta_2$	4.14*** (1.17)	3.08** (1.22)	4.43*** (0.95)	4.14*** (1.24)	1.13** (0.57)
Threshold (γ)	0.65	0.65	0.61	0.61	0.65
95% CI	0.27-0.78	0.27-1.04	0.61-0.65	0.29-0.65	0.29-0.82
# obs	92	92	92	92	92
# obs below γ	60	60	56	57	60
# obs in the CI	53	63	6	45	54

between the two regimes is recognized. The results of the last three regressions show negative effects of controls on growth if the variance of inflation is lower than the estimated threshold, though only significant when the control on equities is considered. There is evidence that capital controls in countries where the inflation rate is more volatile are more growth enhancing than in countries where the inflation rates are relatively more stable.

To sum up, there are evidences that capital controls are beneficial in terms of economic growth in countries where inflation rate is highly fluctuated, though evidence for the difference between the two regimes is not as strong. The estimated threshold level is however insignificant due to the broad confidence intervals.

3.4.4 Trade Openness

Capital account liberalization can lead to misallocation of resources if the sectors of comparative advantages are protected and the foreign capital has to flow into the less competitive sectors. (Eichengreen (2001), Brecher and Alejandro (1977)) Trade openness in this case can facilitate the efficient allocation of the foreign financing and promote growth. Based on this argument, a threshold level is expected in the measurement of trade openness below which capital controls are beneficial to avoid the misallocation of capital.

The five regressions in Table 3.6 estimate very similar threshold levels in the trade openness, below which roughly two thirds of the observations lie. The widths of the confidence intervals vary, minimized in the case where restrictions on the debt securities are controlled, with only 6 observations inside. Four of the five regressions show significantly

positive effects of capital controls on growth in countries that are less open to trade and three of the five regressions show significantly negative effects in countries that are more open to trade. All five regressions reveal strong evidence that capital controls are more beneficial in countries that are less open to trade.

3.5 Conclusion

Using a recent Structural Threshold Regression (STR) model, this paper investigates the existence of threshold levels in several country-specific features, the level at which the effect of capital controls on economic growth reverses. Though threshold levels estimated in most of the regressions have rather broad confidence intervals and are hard to identify, there are still some meaningful and interesting findings concerning the effects of capital controls on the economic growth at different levels of the domestic social and economic framework. Those evidences provide some support to views on the role played by the domestic framework in promoting economic efficiency and growth.

There are several interesting topics remaining for future research. In the section of the degree of financial development, it is mentioned that compared with the result of Prasad et al. (2009), practice in this paper fails to identify a lower threshold level at which the effect of capital controls turn from positive to negative. Rather, only a higher threshold level is identified, above which capital controls have significantly positive effect on growth. The improvement of the financial development degree over time is speculated to be the main reason for the inconsistency results. If the speculation holds correct, it could be expected that as more and more countries moving up and stepping across the threshold levels on the ladder of financial development, a threshold level at which the effect of capital controls turn from significant negative to positive can be identified. This can be taken as a research topic for future data.

This paper conducts a cross-sectional study on the threshold effect, because the STR methodology is a regression model for cross-sectional data. In the literature, there are practices which apply the threshold regression method to dynamic panel data (See Kremer, Bick, and Nautz (2013)). However on one hand, these practices have no support of a strong asymptotic theory; on the other hand, they allow for the endogeneity in the slope variables only, while the endogeneity in the threshold variables is not dealt with. That answers why the dynamic panel threshold methodology is not applied to my practice. A future research on the very topic of this work can be conducted when a sounded theory of the dynamic panel threshold regression is developed. This kind of study will have a power of taking advantage of the rich information in the panel data which provides more dispersion over time and is thus important to the identification of the threshold levels.

Table 3.7: Variables and Sources

Variables	Sources
Growth rate of PPP per capita GDP (log difference over period divided by period length)	Penn World Tables (version 9.0)
Real GDP per capita of 1990	Penn World Tables (version 9.0)
Average investment to GDP	Penn World Tables (version 9.0)
Years of secondary school enrollment over 25 years old	Barro and Lee (2016) database, updated to 2010
Average annual growth rate of population (log difference over period divided by period length)	Penn World Tables (version 9.0)
Degree of financial development	Measurement from Svirydzenka (2016)
Institutional quality	World Bank Governance Indicators (WGI)
Government expenditure	Penn World Tables (version 9.0)
Annual CPI inflation	International Financial Statistics (IFS)
Current price trade openness (sum of exports and imports of goods and services) to GDP	Penn World Tables (version 9.0)
Aggregate capital controls	Variable <i>ka</i> in Fernández, Klein, Rebucci, Schindler, and Uribe (2016)
Aggregate controls on capital inflow	Variable <i>kai</i> in Fernández, Klein, Rebucci, Schindler, and Uribe (2016)
Capital controls on debt securities	An aggregation of controls on money market instruments (<i>mm</i>) and bond (<i>bo</i>) in Fernández, Klein, Rebucci, Schindler, and Uribe (2016)
Capital controls on equity and bank credit	An aggregation of controls on financial credits (<i>fc</i>), equity (<i>eq</i>), and collective investments (<i>ci</i>) in Fernández, Klein, Rebucci, Schindler, and Uribe (2016)
Capital controls on direct investment	Variable <i>di</i> in Fernández, Klein, Rebucci, Schindler, and Uribe (2016)

Table 3.8: Countries in the Data Set of Capital Control by Income Groups

High (42)		Upper Middle (26)	Lower Middle/Low (32)
Australia	Saudi Arabia	Algeria	Bangladesh
Austria	Singapore	Angola	Bolivia
Bahrain	Slovenia	Argentina	Burkina Faso
Belgium	Spain	Brazil	Cote d'Ivoire
Brunei Darussalam	Sweden	Bulgaria	Egypt
Canada	Switzerland	China	El Salvador
Chile	U.A.E.	Colombia	Ethiopia
Cyprus	United Kindom	Costa Rica	Georgia
Czech Republic	United States	Dominican Republic	Ghana
Denmark	Uruguay	Ecuador	Guatemala
Finland		Hungary	India
France		Iran	Indonesia
Germany		Jamaica	Kenya
Greece		Kazakhstan	Kyrgyz Republic
Hong Kong		Lebanon	Moldova
Iceland		Malaysia	Morocco
Ireland		Mauritius	Myanmar
Israel		Mexico	Nicaragua
Italy		Panama	Nigeria
Japan		Peru	Pakistan
Korea		Romania	Paraguay
Kuwait		South Africa	Philippines
Latvia		Thailand	Sri Lanka
Malta		Tunisia	Swaziland
Netherlands		Turkey	Tanzania
New Zealand		Venezuela	Togo
Norway			Uganda
Oman			Ukraine
Poland			Uzbekistan
Portugal			Vietnam
Qatar			Yemen
Russia			Zambia

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