A Datta-Das transistor with enhanced spin control

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We consider a two-channel spin transistor with weak spin-orbit induced interband coupling. We show that the coherent transfer of carriers between the coupled channels gives rise to an *additional* spin rotation. We calculate the corresponding spin-resolved current in a Datta-Das geometry and show that a weak interband mixing leads to enhanced spin control.

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The pioneering spin-transistor proposal of Datta and Das [1] best exemplifies the relevance of electrical control of magnetic degrees of freedom as a means of spin modulating charge flow. In this device [2], a spin-polarized current [3, 4] injected from the source is spin modulated on its way to the drain via the Rashba spin-orbit [5] (s-o) interaction, Fig. 1(a). The spin transistor operation relies on gate controlling [6] the strength α of the Rashba interaction which has the form $H_R = i\alpha\sigma_y\partial/\partial x$ in a strictly 1D channel [5]. Upon crossing the Rashba-active region of length L, a spin-up incoming electron emerges in the spin-rotated state

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \cos(\theta_R/2) \\ -\sin(\theta_R/2) \end{pmatrix}, \tag{1}$$

where $\theta_R = 2m^*\alpha L/\hbar^2 \equiv 2k_R L$ is the rotation angle and m^* is the electron effective mass [1]. The corresponding spin-resolved conductance is found to be $G_{\uparrow,\downarrow} = e^2(1 \pm \cos\theta_R)/h$.

Here we extend the above picture by considering a geometry with two weakly-coupled Rashba bands in the quasi-one-dimensional channel, Fig. 1(b). We treat the degenerate k states near the band crossings perturbatively in analogy to the nearly-free electron model [7]. This approach allows for a simple analytical description of the problem. We calculate the spin-resolved current by extending the usual procedure of Datta and Das [1] to account for weakly coupled bands. Our main finding is an *additional* spin rotation for injected electrons with energies near the band crossing [see shaded region around ε_F in Fig. 2]. As we derive later on, an incoming spin up electron in channel a emerges from the Rashba region in the rotated state

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \frac{1}{2}\begin{pmatrix} \cos(\theta_d/2)e^{-ik_R L} + e^{ik_R L} \\ -i\cos(\theta_d/2)e^{-ik_R L} + ie^{ik_R L} \\ -i\sin(\theta_d/2)e^{-ik_R L} \\ \sin(\theta_d/2)e^{-ik_R L} \end{pmatrix}, \tag{2}$$

where $\theta_d = \theta_R d/k_c$ is the additional spin rotation angle, d the interband matrix element and k_c the wave vector at the band crossing, Fig. 2. From (2) we can find the new spin-resolved conductance

$$G_{\uparrow,\downarrow} = \frac{e^2}{h}\begin{pmatrix} 1 + \cos(\theta_d/2)\cos\theta_R \\ 1 - \cos(\theta_d/2)\cos\theta_R \end{pmatrix}. \tag{3}$$

We now proceed to derive Eqs (2) and (3).

Model. We consider a quasi-one-dimensional wire of length L with two bands a and b described by $\varepsilon_{n,\sigma_z}(k) = \hbar^2k^2/2m^* + \epsilon_n$, $n = a, b$ and eigenfunctions $\varphi_{k,n,\sigma}(x, y) = e^{ikx}\phi_n(y)|\sigma\rangle/\sqrt{L}$, $\sigma = \uparrow, \downarrow$

where the $\phi_n(y)$'s denote the transverse confinement wave functions. In the presence of the Rashba s-o interaction, we can derive a Hamiltonian for the system in the basis of the uncoupled wave functions $\{\varphi_{k,n,\sigma_z}(x,y)\}$. This reads,

$$H_R = \begin{bmatrix} \varepsilon_+^a(k) & 0 & 0 & -\alpha d \\ 0 & \varepsilon_-^a(k) & \alpha d & 0 \\ 0 & \alpha d & \varepsilon_+^b(k) & 0 \\ -\alpha d & 0 & 0 & \varepsilon_-^b(k) \end{bmatrix}, \tag{4}$$

where $d \equiv \langle \phi_a(y)|\partial/\partial y|\phi_b(y)\rangle$, $\varepsilon_s^n(k) = \hbar^2 (k - sk_R)^2/2m^* + \epsilon_n - \epsilon_R$, $\epsilon_R \equiv \hbar^2 k_R^2/2m^*$, ($s = \pm$, $n = a, b$) and we have considered $|\sigma\rangle$ to be the eigenbasis of σ_y. For $d = 0$ the Hamiltonian in (4) is diagonal and yields uncoupled Rashba dispersions $\varepsilon_s^n(k)$ (thin lines in Fig. 2); the corresponding wave functions are $\varphi_{k,n,s}(x,y)$ (here $|\sigma\rangle \to |s = \pm\rangle = [|\uparrow\rangle \pm i|\downarrow\rangle]/\sqrt{2}$). Note that for $d = 0$ the bands cross for some values of k. For instance, for $k > 0$ a crossing occurs at $k_c = (\epsilon_b - \epsilon_a)/2\alpha$. For non-zero interband coupling $d \neq 0$ [8], we can diagonalize H_R exactly (see Mireles and Kirczenow in Ref. [8]) to find the new dispersions (thick lines in Fig. 2).

Bands near k_c. Since we are interested in transport with injection energies near the crossing, we follow below a simpler perturbative approach [7] to determine the energy dispersions and wave functions near k_c. Near the crossing we can solve the reduced Hamiltonian

$$\tilde{H}_R = \begin{bmatrix} \varepsilon_-^a(k) & \alpha d \\ \alpha d & \varepsilon_+^b(k) \end{bmatrix}, \tag{5}$$

which to lowest order yields

$$\varepsilon_\pm^{\text{approx}}(k) = \frac{\hbar^2 k^2}{2m^*} + \frac{1}{2}\epsilon_b + \frac{1}{2}\epsilon_a \pm \alpha d. \tag{6}$$

As shown in the inset of Fig. 2, Eq. (6) describes very well the anti-crossing of the bands near k_c. The corresponding *zero-order* eigenstates are

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left[|-\rangle_a \pm |+\rangle_b\right] = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ -i \end{pmatrix}_a \pm \begin{pmatrix} 1 \\ i \end{pmatrix}_b\right], \tag{7}$$

where the sub-indices indicate the respective channel. The analytical form in (6) allows us to determine the wave vectors k_{c1} and k_{c2} in Fig. 2 straightforwardly: we assume $k_{c1} = k_c - \Delta/2$ and $k_{c2} = k_c + \Delta/2$ and solve $\varepsilon_+^{\text{approx}}(k_{c1}) = \varepsilon_-^{\text{approx}}(k_{c2})$ (assumed $\sim \varepsilon_F$) to find

$$\Delta = \frac{2m^*\alpha d}{\hbar^2 k_c} = 2\frac{k_R}{k_c} d. \tag{8}$$

Note that to the lowest order used here the horizontal splitting Δ is constant and symmetric about k_c.

Boundary conditions. We now consider a spin-up electron entering the Rashba-active region of length L in the wire. Following the usual approach, we expand this incoming state in terms of the coupled Rashba states in the wire. We consider only the states k_{c1}, k_{c2}, and k_2 in the expansion

$$|\Psi\rangle = \frac{1}{2}|\psi_+\rangle e^{ik_{c1}x} + \frac{1}{2}|\psi_-\rangle e^{ik_{c2}x} + \frac{1}{\sqrt{2}}|+\rangle_a e^{ik_2 x}. \tag{9}$$

The above *ansatz* satisfies the boundary conditions for both the wave function and (to leading order) its derivative $x = 0$. More explicitly, the velocity operator condition [10] at $x = 0$ for an electron with $k = k_F$ yields

$$\begin{pmatrix} k_F \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} k_c + k_2 \\ -i\left(k_c - k_2 - 2k_R\right) \\ -\Delta/2 \\ -i\Delta/2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} k_c + k_2 \\ 0 \\ -\Delta/2 \\ -i\Delta/2 \end{pmatrix}, \tag{10}$$

where we used $k_2 - k_c = 2k_R$ (still valid to leading order). The 'four-vector' notation in (10) concisely specifies the spin states in channels a (upper half) and b (lower half). Note that Eq. (10) is satisfied provided that $\Delta \ll 4k_F$. This inequality is satisfied in our system for realistic parameters.

Underlying the *ansatz* in (9) is the assumption of unity transmission through the Rashba region. Here we have in mind the particular spin-transistor geometry sketched in Fig. 1(a): a gate-controlled Rashba-active region of extension L *smaller* than the total length L_0 of the wire. In this configuration, there are only small band offsets (which we neglect) of the order of $\epsilon_R \ll \varepsilon_F$ at the entrance ($x = 0$) and exit ($x = L$) of the Rashba region. Hence transmission is indeed very close to unity, see Ref. [9]. The boundary conditions at $x = L$ are also satisfied.

Generalized spin-rotated state. From Eq. (9) we find that a spin-up electron entering the Rashba

region at $x = 0$ emerges from it at $x = L$ in the spin-rotated state

$$\Psi_{\uparrow,L} = \frac{1}{4}\left[\begin{pmatrix} e^{-iL\Delta/2} \\ -ie^{-iL\Delta/2} \\ e^{-iL\Delta/2} \\ ie^{-iL\Delta/2} \end{pmatrix} + \begin{pmatrix} e^{iL\Delta/2} \\ -ie^{iL\Delta/2} \\ -e^{iL\Delta/2} \\ -ie^{iL\Delta/2} \end{pmatrix}\right] e^{ik_cL} + \frac{1}{2}\begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} e^{ik_2L}$$

$$= \frac{1}{2}e^{i(k_c+k_R)L}\begin{pmatrix} \cos(\theta_d/2)e^{-ik_RL} + e^{ik_RL} \\ -i\cos(\theta_d/2)e^{-ik_RL} + ie^{ik_RL} \\ -i\sin(\theta_d/2)e^{-ik_RL} \\ \sin(\theta_d/2)e^{-ik_RL} \end{pmatrix}, \tag{11}$$

which is essentially Eq. (2). Observe that in absence of interband coupling (i.e., $\theta_d = 0$) Eq. (11) reduces to the Datta-Das state in (1). An expression similar to (11) holds for the case of an incoming spin-down electron.

Spin-resolved current. For $x \geqslant L$ we have

$$\Psi_{\uparrow}(x \geqslant L, y) = \frac{1}{2}\begin{pmatrix} e^{-i\theta_R/2}\cos(\theta_d/2) + e^{i\theta_R/2} \\ -ie^{-i\theta_R/2}\cos(\theta_d/2) + ie^{i\theta_R/2} \end{pmatrix} e^{i(k_c+k_R)x}\phi_a(y) +$$

$$\frac{1}{2}\begin{pmatrix} -ie^{i\theta_R/2}\sin(\theta_d/2) \\ e^{i\theta_R/2}\sin(\theta_d/2) \end{pmatrix} e^{i(k_c-k_R)x}\phi_b(y), \tag{12}$$

which describes planes waves in the *uncoupled* channels a and b arising for an incoming spin-up electron in channel a. The total current follows straightforwardly (Landauer-Büttiker) from Eq. (12)

$$I_{\uparrow,\downarrow} = \frac{e}{h}eV[1 \pm \cos(\theta_d/2)\cos\theta_R]. \tag{13}$$

where $eV \ll \varepsilon_F$ is the applied bias between the source and drain. The spin-dependent conductance in (3) follows immediately from (13). Equation (13) clearly shows the additional modulation θ_d of the spin-resolved current due to s-o induced interband coupling. Figure 3 illustrates the angular dependence of $G_{\downarrow}$ as a function of θ_R and θ_d. The s-o mixing angle θ_d enhances the possibilities for spin control in the Datta-Das transistor.

Realistic parameters. For concreteness, let us consider infinite transverse confinement (width w). In this case, $\epsilon_b - \epsilon_a = 3\hbar^2\pi^2/2mw^2$ and the interband coupling constant $d = 8/3w$. We choose $\epsilon_b - \epsilon_a = 16\epsilon_R$, which implies (i) $\alpha = (\sqrt{3}\pi/4)\hbar^2/mw = 3.45 \times 10^{-11}$ eVm (and $\epsilon_R \sim 0.39$ meV) for $m = 0.05m_0$ and $w = 60$ nm, (ii) $\varepsilon(k_c) = 24\epsilon_R$ [ε_F should be tuned to $\sim \varepsilon(k_c)$], and

(iii) $k_c = 8\epsilon_R/\alpha$. Assuming $L = 69$ nm [Rashba region length, Fig. 1(a)], we find $\theta_R = \pi$ and $\theta_d = \theta_R d/k_c = \pi/2$, since $d/k_c \sim 0.5$. This is a conservative estimate. In principle, θ_d can be varied independently of θ_R via lateral gates which alter w. Note also that $\Delta/4k_F \sim 0.05$ [validity of Eq. (10)] for the above parameters. Finally, we note that the most relevant spin-flip mechanism (Dyakonov-Perel) should be suppressed in quasi-one-dimensional systems such as ours [11]. In addition, thermal effects are irrelevant in the experimentally feasible linear regime [12] we consider here

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Figures

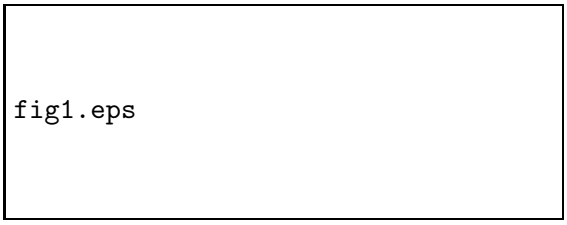
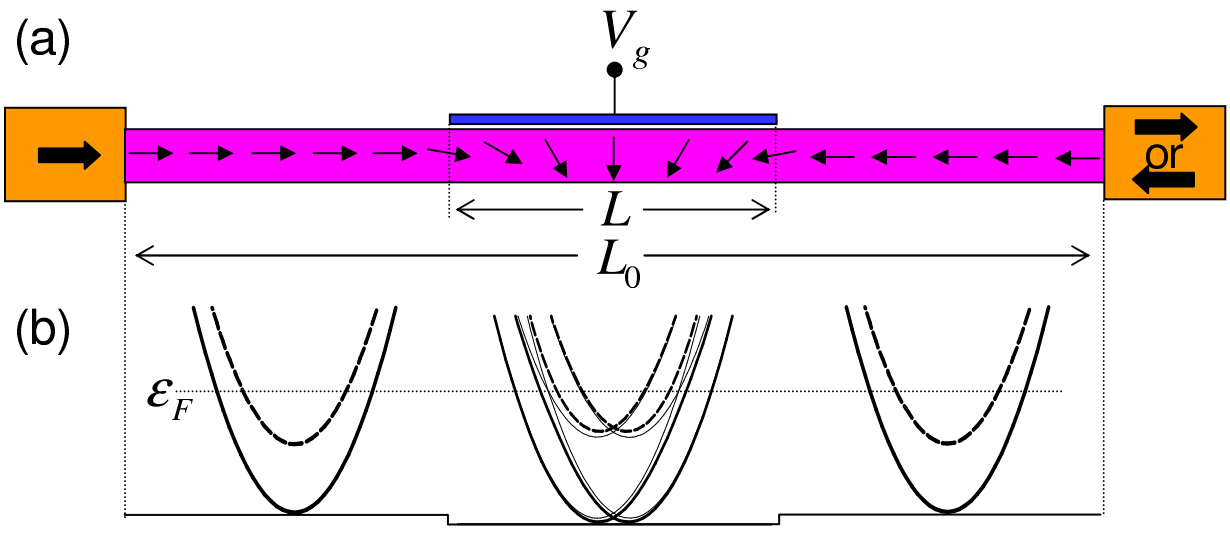
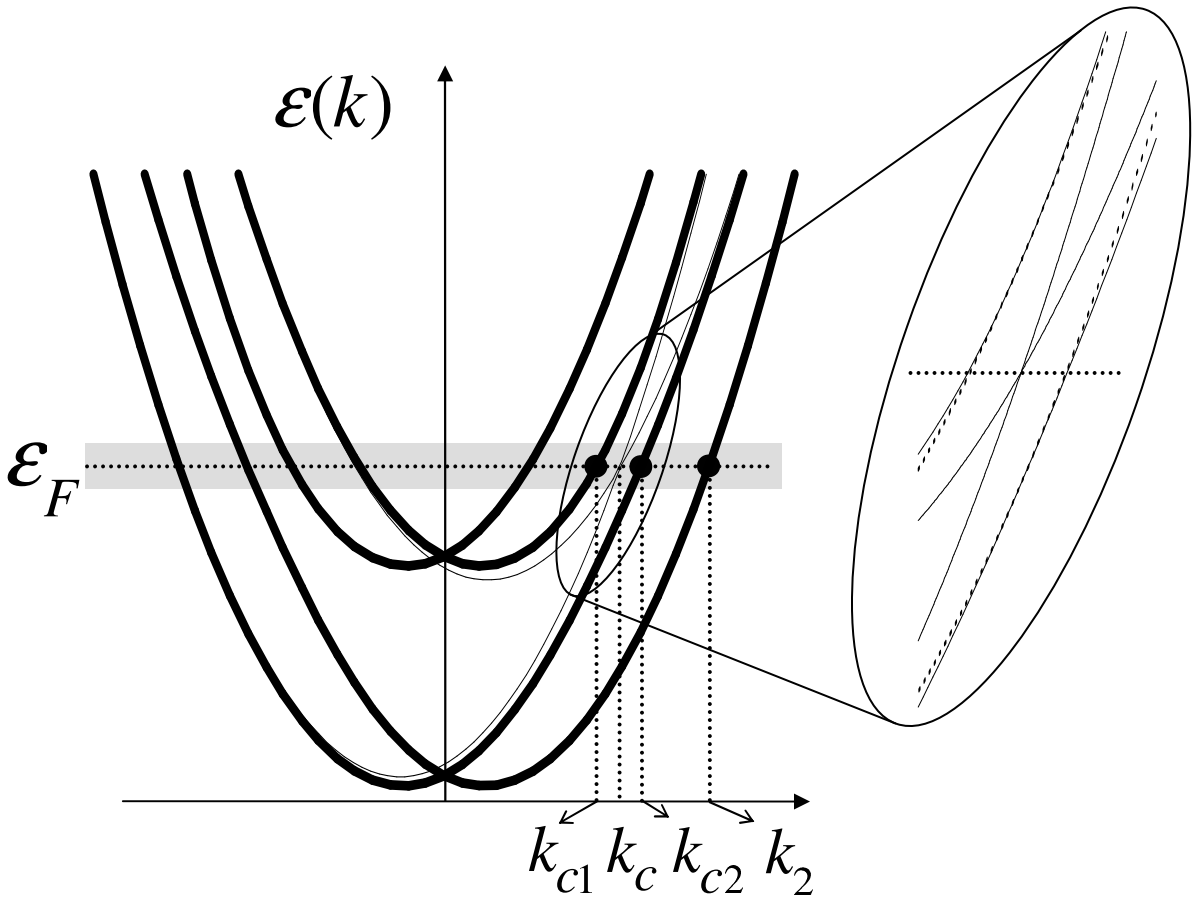


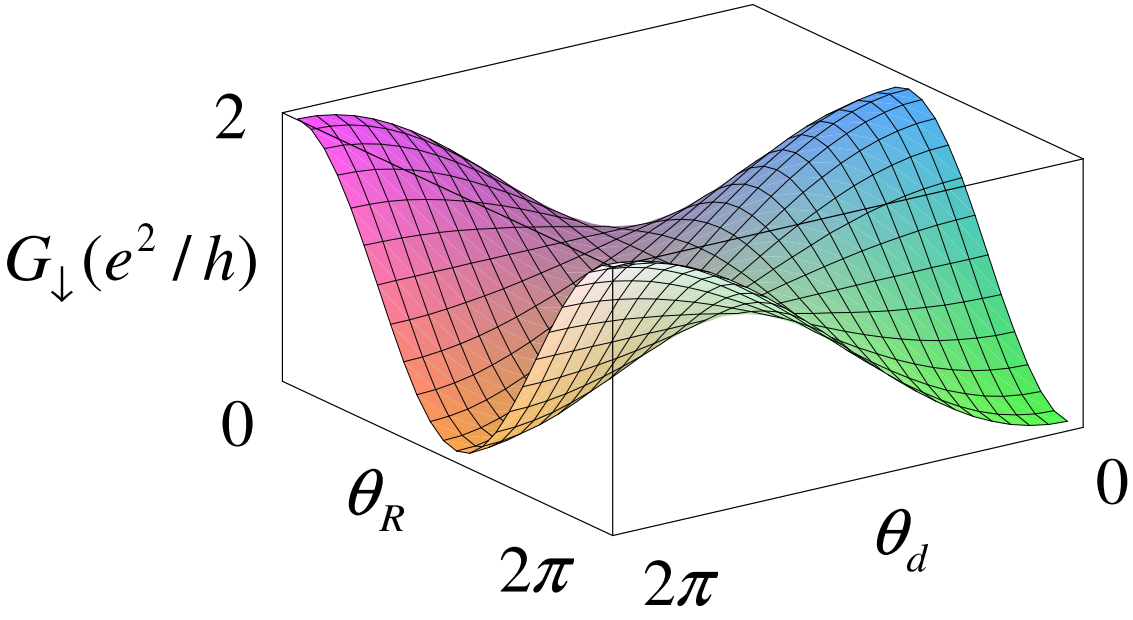
FIG. 1: Spin transistor geometry with a two-band channel. (a) The length L of the Rashba region is smaller than the total length L_0 of the wire. (b) Sketch of energy dispersions in the s-o active region with and without interband coupling (Rashba bands) and away from it (parabolic bands). Note the small band offsets between adjacent regions in the wire.

FIG. 2: Band structure in the presence of spin-orbit coupling. In absence of interband mixing the Rashba dispersions are uncoupled (thin solid lines) and cross at, e.g., k_c. For non-zero interband coupling the bands anti cross (thick solid lines). The inset shows a blowup of the dispersion region near the crossing: the approximate solution [dotted lines, perturbative approach, Eq. (6)] describes well the energy dispersions near k_c.

FIG. 3: Angular dependence of the spin-down conductance. The additional modulation θ_d due to s-o interband mixing and θ_R can be varied independently.


(a)
V_g
or
L
L_0
(b)
ε_F


$\varepsilon(k)$
ε_F
k_{c1} k_c k_{c2} k_2


2
$G_{\downarrow}(e^2/h)$
0
θ_R
2π
2π
θ_d
0