Essays in Life-Cycle Finance:

Understanding Personal Investment and Consumption Choices

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This dissertation is dedicated to my mum Guoping and dad Shengwei.

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Executive Summary

This dissertation is a collection of three stand-alone research papers containing theoretical and empirical research on household portfolio choice and consumption decision in the life time. Household finance is a new interesting field in financial economics that has attracted a lot of attention among financial economists recently. Campbell even gave his presidential address titled “Household Finance” at the 2006 AFA annual meeting. Modeling household finance is different from traditional finance because household financial problems have many special features such as long but finite horizon, non-traded human capital, illiquid housing, borrowing constraints, and complex taxation. Also, household asset demands are important in determining asset prices. Therefore, household portfolio choice is a valuable topic for a doctoral dissertation.

Empirical analyses show that low-wealth households tend to be *non-participants* concerning risky assets and tend to hold a large portfolio share of risky asset if they participate at all (*gamblers*). So far, few theoretical models have been able to satisfactorily provide a joint understanding of these two observations. Chapter Two solves a novel life-cycle model in which social links to relatives, friends, and public welfare programs play the key role in explaining both types of behaviors. Social and family transfers induce low-wealth households to over-consume and depress the savings motive since social transfers are available only if personal savings are exhausted. The depressed investment demand further increases the fixed participation cost and thus, increases non-participation. On the other hand, if low-wealth households decide to hold any risky asset, they tend to hold rather large and risky investments because the down side risk is insured by help from relatives and public welfare.

Chapter Three numerically solves the optimal life-cycle portfolio choice when the model is calibrated to match the empirical retirement age distribution: people tend to retire at markedly higher rates around the firm's early retirement age and the age at which the full pension can be received. The model shows that financial incentives for keeping investors in labor force and low leisure preference for young investors endogenously restrict investors from retiring early. Thus, this chapter suggests a novel effect of the early retirement option on portfolio choice. As opposed to results from earlier models, the optimal portfolio share of stock does not increase monotonically prior to retirement. Wealthy investors might find it optimal to reduce the stock share in their early stage of life in order to decrease the possibility of having insufficient wealth for retirement when they are older. Thus, the model predicts either an increasing or a hump-shaped pattern for life-cycle stock holding, consistent with empirical observations.

Chapter Four presents new evidence contradicting the existence of the portfolio composition puzzles concerning household finance: portfolio risk is empirically increasing in age and wealth which is contradicting Merton's (1971) solution. The puzzles cause serious problems in assessing the classical theoretical models that have been developed to rationalize households' portfolio choices. This chapter investigates the 2005 Panel Study of Income Dynamics data and shows that, when the household portfolio includes real estate and private business and allows for leverage, the portfolio risk for young and low-wealth households is in general higher than old and rich households, which is consistent with the predictions of classical models.

Zusammenfassung

Diese Dissertation ist eine Sammlung von drei eigenständigen Arbeiten, welche theoretische und empirische Forschung über die Portfoliowahl von Haushalten und Konsumentscheidungen innerhalb der Lebenszeit eines Menschen beinhalten. Household Finance ist ein neues, interessantes Feld innerhalb der Finanzökonomie, welches kürzlich sehr viel Aufmerksamkeit unter Finanzökonomen erhalten hat. Campbell hielt sogar auf der jährlichen AFA Konferenz 2006 eine Rede mit dem Titel "Household Finance". Die Modellierung von Houshold Finance unterscheidet sich vom traditionellen Finanzwesen, weil die Probleme des Ersteren viele spezielle Eigenschaften aufweisen, wie den langfristigen aber endlichen Zeithorizont, nicht handelbares Humankapital, nicht liquide Unterkünfte, Beschränkungen im Kreditbereich und komplexe Besteuerungen. Zusätzlich sind Anlagenachfragen der Haushalte für die Bestimmung der Anlagepreise relevant. Daher ist die Portfoliowahl von Haushalten ein wertvolles Thema für eine Dissertation.

Empirische Analysen zeigen, dass Haushalte mit niedrigem Vermögen dazu tendieren, nicht an riskanten Anlageformen zu partizipieren und bei Teilnahme einen hohen Anteil an risikoreichen Anlagen zu wählen (Spielertypus). Bislang waren theoretische Modelle noch nicht in der Lage eine befriedigende Erklärung für ein gemeinsames Verständnis dieser zwei Beobachtungen zu geben. Kapitel zwei löst ein neues Life Cycle Modell, in welchem soziale Verbindungen zu Verwandten, Freunden und öffentlichen Wohlfahrtsprogrammen eine Schlüsselrolle zur Erklärung dieser beiden Verhaltensweisen darstellen. Sozial- und Familientransferzahlungen verleiten

Haushalte mit niedrigem Wohlstandsniveau zu übermäßigem Konsum und verwässern die Sparanreize. Die verringerte Investitionsnachfrage erhöht die fixen Kosten weiter und verringert daher die Teilnahme. Auf der anderen Seite entscheiden sich an riskanten Anlageformen teilnehmende Haushalte mit niedrigem Wohlstandsniveau tendenziell für höhere und risikoreichere Investitionen, weil ihr Risiko durch Hilfe von Verwandten und öffentliche Wohlfahrt quasi versichert ist.

Kapitel drei löst die optimale Wahl eines Lebenszyklus Portfolios und des Ruhestandsalters, wenn ein früher Ruhestand durch ein bestimmtes Intervall beschränkt ist, z.B. zwischen dem frühest möglichen Alter und dem obligatorischen Ruhestandsalter. Das Modell ist in der Lage mit der empirischen Verteilung des Ruhestandsalter übereinzustimmen: (1) Investoren tendieren mit erhöhter Wahrscheinlichkeit zu einem Ruhestand in der Nähe des frühest geeigneten Alters und dem Ruhestandsalter, ab welchem sie volle Pensionsansprüche erhalten; (2) ein nicht zu vernachlässigender Anteil der Investoren begibt sich zwischen diesen beiden Spitzen in den Ruhestand. Im Gegensatz zu den Ergebnissen früherer Modelle steigt der optimale Portfolioanteil von Aktien nicht monoton vor dem Ruhestand. Vermögende Investoren dürften es optimal finden den Aktienanteil zu reduzieren, wenn ein früherer Ruhestand nicht geeignet erscheint. Daher sagt das Modell in Übereinstimmung mit den Beobachtungen entweder ein steigendes oder hügelförmiges Muster für das Halten von Aktien in der Lebenszeit vorher.

Kapitel vier präsentiert neue Beweise, welche das Rätsel der Zusammenstellung des Portfolios im Bereich der Haushaltsfinanzen widerlegen. Die empirisch beobachteten Zusammenstellungen der Portfolios werfen Probleme bei Berücksichtigung klassischer theoretischer Modelle auf, welche zur Rationalisierung der Portfoliowahl von Haushalten entwickelt wurden. Dieses Kapitel untersucht die Daten der „Panel Study of Income Dynamics" von 2005 und zeigt, dass bei Berücksichtigung von Wohneigentum, privaten Unternehmen und Fremdkapitalaufnahme im Portfolio eines Haushaltes, das Risiko der Portfolios von jungen und kaum vermögenden Haushalten generell höher ist, als bei alten und reichen Haushalten, was mit den Vorhersagen klassischer Modelle übereinstimmt.

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Chapter One

Introduction

The dissertation draws upon the modern science of finance to address several important issues in household finance, that is, the investment and consumption choices over the lifetime. The new science of finance has had a profound impact on the practice of institutional risk management. Sophisticated enterprise-wide risk-management systems are widely employed today by financial service firms and a growing number of non-financial firms. By comparison, applications of this new science to the important life-cycle issues faced by households have been very limited. Online financial planning 'tools' and 'optimizers' lag far behind the best theory. Although the household portfolio and consumption issues, such as housing, labor income uncertainty, etc, have appeared actively in the recent literature, enduring puzzles to bridge the theoretic models and the real world are still calling for doctoral dissertations and research papers: many households do not hold any risky asset (non-participation puzzle); many households do not hold a complete set of assets (portfolio incompleteness puzzle); richer households have an increasing portfolio share of stocks in wealth (increasing portfolio puzzle), etc.

This dissertation aims to contribute to the literature by combining the new science of finance such as option pricing and the advanced quantitative methodologies with the life-cycle model. The essays are circulating around the several core questions. How do people change their risk-taking corresponding to different levels of wealth and age facing different risks in their lifetime? How does the borrowing constraint affect people's decisions? How is the household-based analysis related to the whole financial market? How can the empirical findings be interpreted based on the consumption-based life-cycle models? In particular, the dissertation is interested in the consequences of choices with features of options and non-linear constraints. Three essays with different focuses, thus, are proposed to shed a light on these questions.

Chapter Two solves a novel life-cycle model in which social links to relatives, friends, and public welfare programs play the key role in explaining investment behavior for low-wealth households. The literature is puzzled by the seemingly contradictory investment decisions observed among low-wealth households. First, they are more likely to be *non-participants* in risky asset markets than wealthy households. In various data sets, wealth is shown to have a strong positive effect on public equity participation and private business ownership (Haliassos and Bertaut 1995, Campbell 2006, and Calvet et al. 2006). Second, low-wealth households tend to hold rather large and risky investments if they participate at all (*gamblers*), although the portfolio share of risky assets is generally thought to be monotonically increasing in wealth. Campbell (2006) reports that wealth influences the portfolio share of risky assets in a quadratic pattern. In the lower part of the wealth distribution, shares of risky assets decrease with wealth. Similar results can also be found in Vissing-Jorgensen (2002) and Guiso et al. (2002).

I show that the two investment decisions of low-wealth households can be jointly understood with a simple modification of the standard life-cycle model. The central ingredients are social links to relatives, friends, and public welfare programs, which are added to the standard model in the form of an implicit put option. As households' net worth decreases, the put option becomes more likely to be in the money. The household has two alternative ways of benefiting from the put: either by increasing current consumption so that there would not be enough financial wealth to justify the entry cost of the risky assets; or by increasing the share of risky investments because the downside risk is insured by the put.

I also perform an empirical analysis in this chapter that identifies the effects of family and social transfers on participation and portfolio shares. The data strongly supports the prediction that family and social transfers explain both the "*non-participant*" and "*gambler*" choices for low-wealth households.

Chapter Three numerically solves the optimal life-cycle portfolio choice when the model is calibrated to match the empirical retirement age distribution: people tend to retire at markedly higher rates around the firm's early retirement age and the age at which the full pension can be received. Saving for early retirement has been one of the primary motivations for individuals to invest in the stock market. Realizing the option feature of early retirement, the literature has concluded that such flexibility leads to a higher stock proportion of wealth for investors (see Sundaresan and Zapatero 1997, Lachance 2003, Choi and Shim 2006, and Farhi and Panageas 2007). However, investors typically do not retire until age 62. Indeed, the distribution of retirement age implied by these models becomes implausible because in practice very few people retire before they get into their 60s. Consequently, the question arises: Will the optimal portfolio choice change for a model that generates a realistic retirement age distribution?

The key result of this chapter that differs from earlier literature is that investors might find it optimal to reduce their stock holding even though wealth is increasing. The reason is the following. When investors are not old enough, their leisure preference is low and financial penalty in retirement benefit for early retirement is high. Consequently, the critical wealth level is extremely high and these investors are virtually restricted from exercising the option. However, as investors get older, the critical wealth gets lower. If rich investors still hold a risky portfolio, their wealth might end up below the critical wealth level as they get older and lose the chance to exercise the early retirement option. Therefore, rich investors might find it optimal to reduce the portfolio risk when they are young.

As opposed to other empirical studies, Chapter Four presents new evidence showing that a household's portfolio choice is consistent with the policy functions derived from the standard life-cycle model. The standard life-cycle model predicts that household portfolio risk is decreasing in age and wealth. Nevertheless, previous empirical evidence is at odds with this prediction. The following contradictions are regarded as "*the composition puzzles*" of the household portfolio: (i) the risky asset share is either slightly increasing or constant for increasing wealth; (ii) risky asset holdings have typically been low at low ages, and then either increasing or hump-shaped over the life cycle; (iii) young and low wealth households are constrained to borrow and do not tend to hold stocks (see Guiso et al. (2002 and 2003), Campbell (2006), Ameriks and Zeldes (2004), and Poterba and Samwick (2001)). These contradictions raise serious concerns about the validity of the standard life-cycle model that is widely used in the financial planning literature.

This chapter shows that, the household's portfolio risk, measured as portfolio standard deviation, is generally decreasing in age and wealth. Two forces drive this result. First, households can borrow through real estate mortgage and other debt to increase portfolio risk. Second, real estate and private business are regarded as two additional risky assets and are included in the computation of portfolio risk.

I estimate the portfolio risk surface over a bivariate age-wealth space using OLS regressions. Quadratic and multiplicative terms are used to capture the non-linearity of household portfolio risk and the interactive effects between age and wealth. The result shows that when leverage is considered in portfolio risk, the estimated portfolio risk surface is downward sloping in age and wealth. This pattern is present even when the regressions are controlled for a variety of demographic variables such as family units, marriage, employment, education, and house ownership. I reproduce the results of earlier empirical papers on the composition puzzles by considering different definitions of the household portfolio and I show that failure to allow for non-stock risky assets and leverage gives rise to the composition puzzles.

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Chapter Two

Non-Participant or Gambler: Investment Decisions of Low-wealth Households

Empirical analyses show that low-wealth households tend to be *non-participants* concerning risky assets and tend to hold a large portfolio share of risky asset if they participate at all (*gamblers*). So far, few theoretical models have been able to satisfactorily provide a joint understanding of these two observations. This chapter solves a novel life-cycle model in which social links to relatives, friends, and public welfare programs play the key role in explaining both types of behaviors. Social and family transfers induce low-wealth households to over-consume and depress the savings motive since social transfers are available only if personal savings are exhausted. The depressed investment demand further increases the fixed participation cost and thus, increases non-participation. On the other hand, if low-wealth households decide to hold any risky asset, they tend to hold rather large and risky investments because the down side risk is insured by help from relatives and public welfare.

2.1 Introduction

> Welfare benefits are means tested, i.e., they are paid only to those with incomes (and assets) below some level. (…) Over the past four decades, the spending on means tested programs (except Medicaid) has remained relatively constant (rising from 1.0 percent of GDP to 1.3 percent of GDP) …. (Martin Feldstein, the Presidential address of AEA, 2005)

The above quote highlights the importance of understanding the investment decisions of low-wealth/income households to the investment of billions of dollars in welfare programs. However, the literature is puzzled by the seemingly contradictory investment decisions observed among low-wealth households. First, they are more likely to be *non-participants* in risky asset markets than wealthy households. In various data sets, wealth is shown to have a strong positive effect on public equity participation and private business ownership (Haliassos and Bertaut 1995, Campbell 2006, and Calvet et al. 2006). Second, low-wealth households tend to hold rather large and risky investments if they participate at all (*gamblers*), although the portfolio share of risky assets is generally thought to be monotonically increasing in wealth. Campbell (2006) reports that wealth influences the portfolio share of risky assets in a quadratic pattern. In the lower part of the wealth distribution, shares of risky assets decrease with wealth. Similar results can also be found in Vissing-Jorgensen (2002) and Guiso et al. (2002).

As of yet, only a few explanations for the non-participation puzzle have been proposed based on traditional rational models. Vissing-Jorgensen (2002) suggests that half of the non-participation can be optimal in the presence of modest transaction costs. Linnainmaa (2005) shows that the combination of learning about investment opportunities and short-sale constraints can generate the observed non-participation. Liu and Zhou (2006) explain the non-participation by the existence of a large wealth shock and the lack of insurance. Despite an outpouring of research in household portfolios recently, the gambling behavior for low-wealth households has received almost no attention. More importantly, we lack a successful theory that provides a joint explanation of why low-wealth households either cannot tolerate any risk at all or take extremely high risk.

I show that the two investment decisions of low-wealth households can be jointly understood with a simple modification of the standard life-cycle model. The central ingredients are social links to relatives, friends, and public welfare programs, which are added to the standard model in the form of an implicit put option. As households' net worth decreases, the put option becomes more likely to be in the money. The household has two alternative ways of benefiting from the put: either by increasing current consumption so that there would not be enough financial wealth to justify the entry cost of the risky assets; or by increasing the share of risky investments because the downside risk is insured by the put.

I solve numerically for the optimal portfolio and savings decisions for a finitely-lived investor facing mortality risk, short-sale constraints, and receiving labor income with disastrous labor income risk.[1] Cocco et al. (2005) study a similar life cycle model of consumption and portfolio choice with uninsurable labor income risk. Modeling disastrous income risk can be dated back to precautionary saving theory (Carroll 1992, 1997).

Despite these similar features, this chapter differs from the previous studies in several important dimensions. First, family and social transfers are modeled as a put option dependent upon wealth and income. The idea comes from the social insurance literature in which Social Security can be regarded as a put option because it is implicitly designed to guarantee a minimal consumption level for participants (Constantinides et al. 2005). I broaden this concept to include monetary help from relatives and friends and explicitly model it in an asset allocation decision. The guarantee of a minimal wealth level differs from the unemployment insurance considered in Michaelides (2003). The unemployment insurance only depends on the event of unemployment, while the transfer option considered in this chapter also depends on wealth. Transfers are provided only in the case that the household has experienced the income shock and has limited wealth.

Second, households are allowed to borrow for leveraging the portfolio, but they cannot borrow for consumption. This extension from the standard life-cycle model essentially allows the portfolio share of risky assets to be larger than one, while restricting the net worth to be positive. I do this for two reasons. First, in practice, households do borrow while holding risky assets.[2] Allowing for leverage will show how dramatically the implicit put option of family help and social transfers increase the riskiness of the household portfolio. Second, the portfolio implications of fully relaxing the borrowing constraint are not the focuses of this chapter since they have been

[1] In this chapter, the disastrous labor income risk is modeled as receiving zero income with some probability.

[2] According to the 2005 Panel Study of Income Dynamics, about 54% of total households have outstanding mortgages, credit card charges, student loans, medical or legal bills, or loans from relatives. About 49% of total households have both debt and risky assets.

examined in Cocco et al. (2005). Therefore, allowing for portfolio leverage and keeping the net worth positive seems to be an effective way of investigating the impact of the social welfare while keeping the present model in line with the standard models in the literature (See, for instance, Merton 1969, Carroll 1992, 1997, and Cocco et al. 2005).

Finally, I perform an empirical analysis that identifies the effects of family and social transfers on participation and portfolio shares. The analysis is carried out in two steps. In the first step, I estimate a probit model for the dummy of receiving monetary help from relatives and friends or social welfare programs. Based on the estimated model, I further compute the predicted score for each household, which represents the propensity to receive transfers. In the second step, the predicted score is used as a regressor in the probit regression for participation and the OLS regression for portfolio shares in three asset categories, namely stocks, non-home real estate and private business, and primary residence and vehicles. The staged estimation is used to identify transfer effects since transfer effects do not depend on whether the household receives transfers but instead on the likelihood of receiving transfers.

The data strongly supports the prediction that family and social transfers explain both the "*non-participant*" and "*gambler*" choices for low-wealth households. An increase in the predicted score from the 20th percentile to the 80th percentile decreases the probability of being a stockholder from 15.9 to 11.8 percent, increases the portfolio share of stocks from 15.9 to 20.5 percent, and increases the portfolio share in non-home real estate and private business from 29.3 to 35.3 percent for the participants. After the transfer effects are captured, the coefficient of wealth turns positive, implying an increasing wealth effect on portfolio share in risky assets, consistent with common intuition. Most regressors in the regression of the portfolio share of primary residence and vehicles have different signs from the other two asset classes. This is related to the dual purposes (consumption and investment) of owning a primary house and vehicles.[3] Households purchase a primary house and vehicles not only as an investment vehicle but also for deriving utility from consuming them.

The chapter is organized as follows. Section 2.2 describes the economic model. Section 2.3 discusses households' optimal consumption and portfolio choices. Section 2.4 presents empirical evidence. Section 2.5 concludes.

2.2 The economic model

[3] In fact, the consumption motive involved in purchasing a house and vehicles has been carefully considered in Cocco (2005), Yao and Zhang (2005), and Van Hemert (2007).

A. Preferences

The benchmark discrete-time model follows Merton (1969) in a multi-period optimization set up. The household has a time-separable power utility:

$$U_1 = E_1 \sum_{t=1}^{T} \delta^{t-1} (\prod_{j=0}^{t-2} p_j) \left(p_{t-1} \frac{C_t^{1-\gamma}}{1-\gamma} \right). \tag{2.1}$$

The household lives for T periods. $C_t > 0$ is the level of date t consumption. $\gamma > 0$ is the coefficient of relative risk aversion. $\delta < 1$ is the time preference parameter. p_t denotes the probability that the household is alive at $t+1$, conditional on being alive at date t.

The preferences in (2.1) are the same as the ones used in Cocco et al. (2005) except that the present model abstracts from modeling the bequest motive. Households with a desire to bequeath wealth would be expected to save more. This could affect the optimal portfolio choice more on the dimension of age than on the dimension of wealth since young and old households face quite different mortality risk. Due to the fact that this chapter focuses on the wealth effect rather than the life-cycle effect for the portfolio choice, the absence of the bequest motive will have limited effects on the results. Also, Cocco et al. (2005) conclude that, overall, the effects of the bequest motive are not very large with a moderate intensity of the bequest motive.

Power utility in (2.1) implies that the consumer's willingness to substitute consumption over time is the reciprocal of the coefficient of relative risk aversion. Yet it is unclear that these two concepts should be linked so tightly. Epstein and Zin (1989) propose a recursive formulation of intertemporal utility which disentangles risk aversion and intertemporal substitution. Cocco et al. (2005) show that increasing the elasticity of intertemporal substitution from 0.1 (corresponding to the risk aversion parameter of 10) to 0.2 or 0.5 makes the investor more willing to substitute intertemporally. Consequently, the investor saves less and increases the share of risky assets. Rather than focus on the intertemporal life-cylce portfolio choice, this chapter aims to explain the relationship between wealth and portfolio choice. Therefore, the benchmark model adopts the classical power utility.

Alternative preferences also include habit formation: utility depends on the consumption of a reference group or the individual's own past consumption (see for example Constantinides 1990). Polkovnichenko (2007) and Gomes and Michaelides

(2003) introduce habit formation preferences in a life-cycle model of consumption and portfolio choice. In contrast to the risky portfolio and non-saving behavior observed for the low-wealth households, they conclude that habit formation preferences lead to more savings and more conservative portfolios.

B. Investments

The household can invest in a riskfree asset and a risky asset. The riskfree asset has a constant gross real return of R_f and the corresponding log return is r. The risky asset has a gross real return R_t. It follows a geometric Brownian motion and the log return has constant mean μ and volatility σ. For a given proportion allocated to risky assets α_t, the log returns of the portfolio are normally distributed over each discrete time step of length Δt (the time between $t-1$ and t) with mean $\mu_{\alpha,\Delta t} = \left[\alpha\mu + (1-\alpha)r - \frac{1}{2}\alpha^2\sigma^2\right]\Delta t$ and volatility $\sigma_{\alpha,\Delta t} = \alpha\sigma\sqrt{\Delta t}$.

The household is allowed to borrow for leveraging the portfolio, but they cannot borrow for consumption. In addition, the household faces the short-sale constraint. Therefore,

$$\alpha_t \geq 0. \tag{2.2}$$

The short constraint (2.2) ensures that the household's allocation to the risky asset is non-negative at all dates. The household, however, can borrow through mortgage loans or credit cards. According to the 2005 Panel Study of Income Dynamics, the mean of mortgage loans across all households is \$53,171 and the mean across credit card loans and student loans is only \$8,513. Since the amount of mortgage loans is much higher than that of credit card loans, mortgage loans are considered the primary way to borrow. For mortgage loans, houses are usually required as collateral. As a result, home equity (difference between the house value and mortgage) is generally positive unless the house value has dropped dramatically. Therefore, it is reasonable to restrict the net worth to be positive. In this case, the borrowing constraint is partially relaxed so that the gambling behavior induced by social welfare can be shown while the main features of the model are kept in line with the literature. The portfolio implications of fully relaxing the borrowing constraint can be found in Cocco et al. (2005).

C. The labor income process

If markets are complete so that labor income can be capitalized and its risk insured, labor income can be regarded deterministic as in Merton (1971). Carroll (1992, 1997) and Cocco et al. (2005) argue that labor income risk is uninsurable due to the market incompleteness. Consequently, the labor income process is described by a random walk model in which households face a temporary shock and a permanent shock. In addition to these shocks, Carroll (1997) also considers a disastrous labor income shock. In particular, labor income is modeled as being zero with some probability.

Modeling risky labor income by a random walk process with temporary and permanent shocks can increase the credibility of the model. The cost, however, is to add another state variable to the optimization. This has barely been a problem for the literature since the indirect utility is homogeneous with respect to current labor income so that current labor income can be normalized to one. Thus, labor income can be eliminated from the state space. In the present model, however, the indirect utility is not homogeneous with respect to current labor income because the insured level of wealth by social welfare is exogenously determined and not dependent on the individual's current labor income. This implies that modeling risky labor income by some normally distributed shocks will add one dimension to the state space of the present model.

Considering the tradeoff between tractability and realistically modeling, this chapter adopts a reduced form of modeling risky labor income. Each period, the household faces the possibility of a disastrous labor income shock. In particular, the stochastic process underlying labor income is characterized only by two states: the employed state which receives a deterministic income and the disastrous state which receives zero income. Such modeling of risky labor income assumes that the main source of labor income risk stems from the disastrous income shock. This assumption is supported by the results in Cocco et al. (2005) showing that labor income with normally distributed shocks behaves very similar to deterministic income while a small probability (0.5%) of a zero labor income draw dramatically lowers the optimal equity share. Therefore, the simplified modeling for risky labor income is believed to have minor effects on the main results of this chapter.

The transition probabilities between the two states are modeled to be different and conditional on the income state in the previous period.[4] The employed household

[4] An example of equal transition probabilities can be seen in Carroll (1997).

receives the disastrous shock with probability λ_e, while the distressed household with probability λ_d. Formally, the labor income process is given by:

$$Y_t = \begin{cases} \bar{Y}_t & \text{with prob.} \quad \begin{matrix} 1-\lambda_e & \text{if } Y_{t-1} = \bar{Y}_{t-1} \\ 1-\lambda_d & \text{if } Y_{t-1} = 0 \end{matrix} \\ 0 & \text{with prob.} \quad \begin{matrix} \lambda_e & \text{if } Y_{t-1} = \bar{Y}_{t-1} \\ \lambda_d & \text{if } Y_{t-1} = 0 \end{matrix} \end{cases}, \tag{2.3}$$

where $\bar{Y}_t$ is the pre-determined labor income received at t.

D. Family and social transfers

Monetary aid from relatives, friends, and social welfare programs is modeled as a put option, insuring that the household has a minimum wealth *MinW*. Constantinides et al. (2005) admit that social insurance implicitly guarantees a minimum consumption level for its participants and price this put option explicitly. Rather than pricing, this chapter aims to investigate the portfolio implications in the presence of such put option. In particular, this chapter associates the exercise of the put to the labor income states as well as wealth, given the fact that social welfare programs are usually tailored for low-income or unemployed households. Households qualified for family and social transfers need to be in the disastrous state ($Y_t = 0$) and to have limited resources ($W_t <$ *MinW*).[5] Formally, the wealth held in period t after being adjusted by the put is:

$$W_t' = W_t + \max(0, MinW - W_t) I(Y_t = 0) \tag{2.4}$$

where I is the indicator function and *max* is a function which takes the larger value between 0 and *MinW* minus W_t.

One may object that the eligibility of social welfare does not necessarily require the qualified household to be fully unemployed with zero income. It will be shown later that a less extreme income shock received in the disastrous state does not affect the main results. This is not surprising. Labor income is part of total wealth. Family and social transfers insure the household to have a minimum total wealth. Therefore, the amount of disastrous income shock will only affect the amount of the transfer. But the wealth level

5 See Feldstein (2005) for a detailed discussion of welfare programs.

after being adjusted by the put is always equal to the minimum wealth level regardless of how severe the shock is.

It should be noted that the present modeling of social insurance combines two strands of literature. One strand of literature considers social insurance as a "benefits guarantee" or put purely conditional on wealth (see Feldstein et al. 2001, Smetters 2001, and Pennacchi 1999). The other strand focuses on unemployment insurance which essentially provides a positive lower bound for labor income (see Michaelides 2003). The portfolio implications in the case of unemployment insurance have been investigated by Cocco et al. (2005). However, the portfolio implications when social insurance is considered as a put are still not well studied. Thus, this chapter contributes to the literature by exploring the portfolio implications in the presence of social insurance.

E. The optimization problem

In period t, the household starts the period with wealth W_t. If the household is employed, then labor income Y_t is realized. If the household receives a disastrous income shock, then the wealth is adjusted by the family and social transfers depending on the value of W_t. Following Cocco et al. (2005), I denote cash-on-hand in period t by

$$X_t = W_t' + Y_t \ , \tag{2.5}$$

conditional on the employment status. This is understood as the wealth that includes labor income and possible family and social transfers received in period t. In the numerical solutions, $\log(X_t)$ is discretized to take values on a grid. Then the household makes decisions for consumption and asset allocation in a portfolio composed of a risky asset and a riskless asset. The intertemporal budget constraint is then given by:

$$W_{t+1} = R_{t+1}^p (W_t' + Y_t - C_t) , \tag{2.6}$$

where R_{t+1}^p is the portfolio return held from period t to period $t+1$:

$$R_{t+1}^{p} = \alpha_t R_{t+1} + (1-\alpha_t) R_f \,. \tag{2.7}$$

R_{t+1} is the gross real return of the risky asset and R_f is the gross real return of the riskless asset. The problem the household faces is that of maximizing (2.1) subject to constraints (2.2) through (2.7): the labor income process， the investment process，and the implicit put option. The control variables of the problem are $\{C_t, \alpha_t\}_{t=1}^{T}$. The state variables are $\{t, X_t, Y_t\}_{t=1}^{T}$. The previous literature often normalizes the variables with respect to current permanent labor income in order to reduce the dimensionality of the state space, e.g., consumption choice is expressed as the ratio of consumption to current permanent labor income.[6] This normalization does not work in the presence of a fixed strike price of an option that is independent of individuals' labor income, i.e., the minimum wealth provided by family help and public welfare aims to provide basic living necessities but not to compensate the loss of labor income. As is explained in Section 2.C, the state variable labor income (Y_t) is characterized by two states rather than by some more complicated distribution.

The Bellman equation for this problem is given by:

$$J_t(X_t, Y_t) = \underset{C_t>0, \alpha_t \geq 0}{Max} \left[\frac{C_t^{1-\gamma}}{1-\gamma} + \delta p_t E_t[J_{t+1}(X_{t+1}, Y_{t+1})]\right], \text{ for } t \leq T$$

$$\text{where } W_{t+1} = (X_t - C_t)(\alpha_t R_{t+1} + (1-\alpha_t)\bar{R}_f),$$

$$X_{t+1} = W_{t+1} + \max(0, MinW - W_{t+1}) I(Y_{t+1} = 0).$$

The problem cannot be solved analytically due to the discontinuous option payoff. The policy functions are derived numerically by using backward induction. In the last period the investor consumes all available wealth so that the value function corresponds to the indirect utility function. This value function can be iteratively substituted in the Bellman equation, yielding solutions for the previous periods. In order to coincide with the Geometric Brownian Motion of the risky returns, log cash-on-hand values are discretized in a grid structure. The grid has equal time increments as well as equal steps in $\log(X_t)$. From each grid point, I allow for a multinomial forward move to a relatively large number of subsequent grid points (e.g., 41) at the next time step. The

[6] See details of such normalization in Cocco et al. (2005).

associated probabilities are calculated by using the discrete normal distribution with a specified value for the control variables, i.e., the stock proportion α. In order to have the indirect utility by the end of the next period lie on the grid points even with a choice of riskless portfolio ($\alpha = 0$), the grid structure is assumed to grow at the riskfree rate. Thus, the choices of α vary from 0 to 2 in steps of 0.01. Appendix 2.A shows that the approximated probabilities do not depend on the level of $\log(X)$ or time and they are solely functions of α. Such property saves time when running the numerical algorithm because the probabilities need to be computed only once for all grid points.

In most cases, $\log(X_{t+1})$ including labor income and possible social welfare will not land on grid points at the next time step since labor income and social welfare are not proportional to $\Delta(\log X)$. In order to obtain the corresponding indirect utilities for the $\log(X_{t+1})$ not landing on grid points, spline estimation is used. Given the relatively large number of grid points, the estimation error is limited.[7] Such interpolation has also been used in other recent papers in the literature (e.g., Cocco et al. 2005).[8]

F. Parameterization

Estimation of the labor income process

The PSID family data from 1970 to 1992 is used to estimate the deterministic labor income as a function of age and other characteristics. One important distinction from previous empirical work is the definition of labor income. I decompose all reported income into two categories: labor income and wealth-dependent transfers. First, labor income includes wage, bonus and commission, and unemployment compensation. Second, wealth-dependent transfers include supplemental social income, help from relatives and friends, and other welfare transfers. Since this chapter explicitly models transfers as a put option, transfer income should be separated from labor income. Another advantage is that we can explain what happens to the 0.5% households who do

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[7] For more details about this methodology see Appendix 2.A and Hodder and Jackwerth (2007).

[8] Alternatively, one can avoid interpolation by fixing the $\log(X_{t+1})$ values equal to the grid points at the next step. Since these values have been adjusted by labor income and social welfare, we need to deduct them to obtain the financial wealth at the next time step. Dividing this financial wealth by the current log cash-on-hand value $\log(X_t)$, we obtain the corresponding return. We further compute the associated probability for this return. Such algorithm increases the time of computing because the associated probability for each grid point is different and needs to be computed case by case. A more severe problem with this algorithm is the quality of discretization of the normal distribution. Since the forward nodes are not evenly spaced and are mostly located in the left side of the normal distribution due to the deduction of labor income, the forward nodes cannot well approximate the normal distribution. Therefore, spline estimation is adopted in this chapter.

not report any income. From the perspective of this chapter, these people are unemployed and do not qualify for welfare and help since they have enough personal savings.[9] All nominal values are converted into real terms by using the Consumer Price Index with 1992 as the base year.

To control for education the sample is split into three groups: the observations without high school and college education, a second group with high school education but without a college degree, and lastly college graduates. The following cross-sectional time series regression is estimated:

$$\log(Y_{it}) = f(t, Z_{it}) + \varepsilon_{it}. \tag{2.8}$$

For each education group I assume that the deterministic function $\bar{Y}_t = f(t, Z_{it})$ is additively separable in t and Z_{it}. The variable t denotes age. The vector Z_{it} includes other personal characteristics such as marital status and family size. I fit a third-order polynomial to the age dummies to obtain the profiles. Table 2.1 and Figure 2.1 report the results for the three education groups. The coefficients of the age dummies are clearly significant and the labor income is shown to have a hump shape in age, in line with intuition and stylized facts (See Cocco et al. 2005 and Gourinchas and Parker 2002). Retirement income is modeled as a constant fraction of the labor income in the last working-year. This fraction, termed as the replacement rate, is calibrated as the ratio of the average of labor income for retirees in a given education group to the average of labor income in the last working-year prior to retirement. The results are shown in the bottom row of Table 2.1. The R-squared coefficients tend to be low for non-educated and high school graduates and on average are 8% lower than the model estimated in Cocco et al. (2005).

The transition probabilities λ_e and λ_d relate to the income requirement for applying for public welfare. In the United States, the Department of Health and Human Services issues the poverty guidelines each year in the *Federal Register* for administrative purposes –determining, for instance, financial eligibility for certain federal welfare programs[10]. I use three levels, i.e., 100%, 125%, and 150%, as the income threshold for determining whether the household is in the disastrous state and

[9] Cocco et al. (2005) attribute this phenomenon to measurement error.

[10] For example, the Food Stamp Program sets the income eligibility level at the 125% poverty guidelines in 2007.

hence eligible for welfare and help. Using the most recent PSID family data in the years 1999, 2001, 2003, and 2005, I compute the proportion of households that have changed their income states within a certain period for using as a proxy of transition probability. Since questions related to income and wealth in the PSID data are retrospective[11] (for instance, those asked in 1999 refer to the 1998 calendar year), I apply the poverty guideline in the previous year to the current sample year. Table 2.2 reports historical frequencies of receiving the income shock from 1999 to 2005 conditional on the income state in the previous period, i.e., whether the family income is above or below the income thresholds.

Table 2.1: Labor income process

The table shows the results of a cross-sectional estimation with fixed effects. The estimation is based on the PSID family data from 1970 to 1992. The log real income is regressed on family size, marital status (dummy), and age dummies. The sample excludes households with female heads, retirees, non-respondents, students, and housewives. T-statistics are in parentheses. ***Significant at 1%; **significant at 5%; *significant at 10%.

Independent Variable Log Real Income	Non-educated Coefficient	High school coefficient	College Coefficient
Family size	0.0148**	0.0081*	0.0316***
	(2.09)	(1.83)	(4.77)
Marital status	0.0409	0.0670***	0.0770***
	(1.32)	(4.25)	(3.41)
Age	0.1223***	0.1993***	0.3561***
	(4.62)	(14.19)	(14.12)
$Age^2/10$	-0.0253***	-0.0409***	-0.0687***
	(-3.94)	(-11.55)	(-11.29)
$Age^3/100$	0.0015***	0.0026***	0.0043***
	(2.93)	(9.12)	(9.12)
Constant	8.1697***	7.1358***	4.5987***
	(23.95)	(41.04)	(13.97)
No. of observations	9445	26388	11875
No. of groups	1138	2898	1199
Average obs per group	8.3	9.1	9.9
R-square within	0.0315	0.0401	0.1425
F-stat	53.92	195.98	354.63
Replacement rate	0.8513	0.6612	0.9350

The table shows several interesting facts. First, much poverty is transitory. Only 30% of poor households stay poor in the next period. This observation is also noticed by Barrett and Swallow (2006) and Baulch and Hoddinott (2000). They find that people

[11] Surveys are mostly conducted in Spring of each year, and therefore income and wealth data are for the previous year.

commonly suffer – or even choose – short-term income losses that push them below an inherently arbitrary poverty line for a relatively brief period of time. Then they recover without explicit external assistance. Second, the probability of an income shock conditional on receiving the shock in the previous period is not significantly higher than the probability conditional on not receiving the shock. This fact supports the implicit assumptions of equal probabilities applied in the previous literature (See Carroll 1992, 1997 and Cocco et al. 2005). Third, the two year probability (on the main diagonals in Table 2.2) does not differ from the four year and six year probabilities. Such time-stability suggests that the model is robust to the choice of decision frequency. Since the frequency of the PSID data (2 years) is not in accordance to the time steps of the benchmark model (5 years), I use the average of four year and six year transition probabilities as the proxy for using in the calibration. Consequently, λ_e and λ_d for the 100%, 125%, and 150% of the poverty guideline are 0.28 and 0.27, 0.32 and 0.31, and 0.36 and 0.35. Table 2.2 also shows that the transition probabilities increase as the thresholds increase. The benchmark model is calibrated to the 125% of the poverty guideline. The other two levels (100% and 150%) are also calibrated for robustness. The results do not show qualitative differences.

Other parameters

The discount factor δ is set to be 0.96, and the coefficient of relative risk aversion γ is 10. This is the upper bound for risk aversion considered reasonable by Mehra and Prescott (1985). Lower values are also considered as robustness checks. The mean equity premium is 4% for the benchmark case. The riskfree rate is 2% and the volatility of the risky asset is 0.157. These parameters are taken from Cocco et al. (2005) so that the results of this chapter are comparable to the literature.

The minimum wealth that the social welfare insures is $15,000 per year in real terms. This is about the average of the 125% poverty guideline for a one-person family ($12,763) and a two-people family ($17,113). The set of parameters is displayed in Table 2.3.

I consider a household who starts working at age 25, retires at age 65, and dies with certainty at age 75 if this has not happened earlier. The mortality probabilities for different ages are taken from United States Life Tables (Anderson 1998).

2.3 Optimal consumption and portfolio choices

Figure 2.1: Fitted labor income process

This figure shows the labor income profiles of three education groups across age. The profiles are generated based on the regression results in Table 2.1. The household is assumed to be married and to have two children.

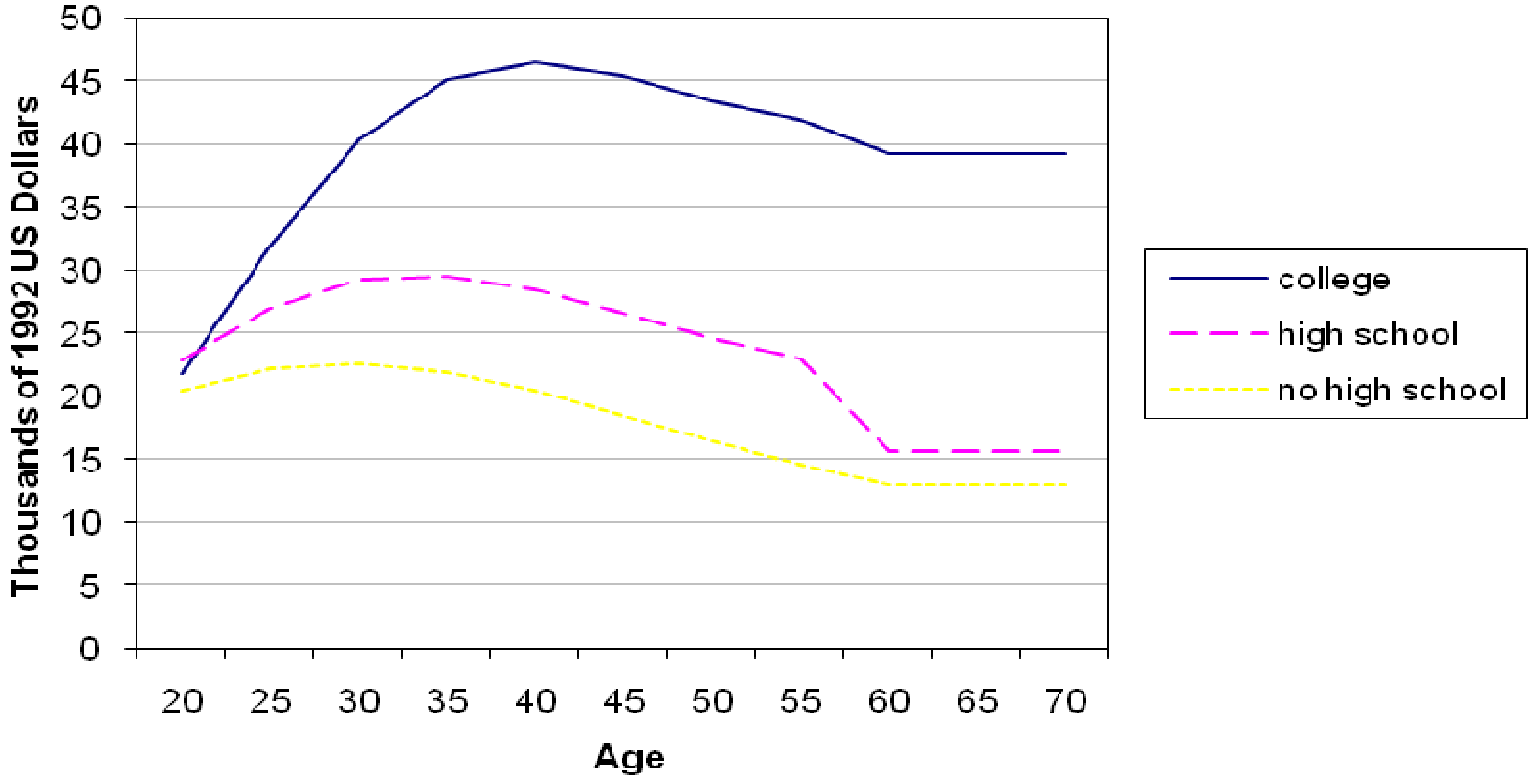

Table 2.2: Historical frequencies of transition between income states

The table reports historical conditional frequencies of receiving an income shock within a certain period. The left column denotes the starting year of the period and the upper row denotes the end year of the period. The income shock is defined as having income less than a particular threshold. The thresholds are set to be 100%, 125%, and 150% of the poverty guideline issued by the Department of Health and Human Services. There are two frequencies for each period. The left is conditional on receiving normal income in the previous period and the right is conditional on receiving an income shock in the previous period.

(a) The threshold is 100% of the poverty guideline.

	2001		2003		2005	
1999	0.237	0.241	0.278	0.257	0.274	0.275
2001	-	-	0.263	0.263	0.278	0.257
2003	-	-	-	-	0.277	0.274

(b) The threshold is 125% of the poverty guideline.

	2001		2003		2005	
1999	0.281	0.279	0.316	0.294	0.314	0.317
2001	-	-	0.316	0.296	0.322	0.314
2003	-	-	-	-	0.315	0.317

(c) The threshold is 150% of the poverty guideline.

	2001		2003		2005	
1999	0.318	0.321	0.371	0.327	0.360	0.356
2001	-	-	0.359	0.335	0.363	0.355
2003	-	-	-	-	0.354	0.359

Table 2.3: Benchmark parameters

The table lists the parameters used to calibrate the benchmark model. Investors with cash-on-hand less than the minimum wealth and being unemployed are qualified to receive social welfare which amounts to the difference between the minimum wealth and the cash-on-hand.

Time preference parameter	δ	0.96	Interest rate	r	0.02
Minimum wealth	$MinW$	15000	Mean	μ	0.06
Risk aversion coefficient	γ	10	Volatility	σ	0.157

In this section, I solve the model numerically and characterize the optimal choices for the reference household considered to be a 35 year-old married male with two children and a high school diploma. In order to highlight the novel effects of help and welfare on investment decisions, the results are compared amongst three models: the benchmark model of the chapter, the model in Cocco et al. (2005) with deterministic labor income (the CGM model)[12], and the benchmark model without labor income.

[12] The original CGM model (2005) is calibrated to the risky labor income instead of deterministic labor income. However, it has been found that the portfolio and consumption choices are quite similar in both

A. Consumption decisions

All the three models exhibit a decreasing trend on the consumption-wealth ratio (see Figure 2.2), deviating from the Merton's (1969) solution. Merton (1969) shows that the optimal consumption-wealth ratio is constant in the absence of labor income. The CGM model (the dashed line) shows that including labor income will increase the consumption propensity for the poor and cause a decreasing consumption-wealth ratio in wealth. However, the put option has the same effect. This can be seen from the benchmark model without labor income (the dotted line). Since labor income is absent in this case, the decreasing pattern of the consumption-wealth ratio should be solely attributed to the put option. This is not surprising since social welfare and family help can be regarded as conditional labor income. Resulting from both of the income and option effects, the benchmark model has a decreasing consumption-wealth ratio as well.

Despite the common decreasing pattern of the consumption-wealth ratio amongst the three models, the speed of decrease is quite different depending on the wealth level. In the model with the option effect only, rich households have a constant consumption-wealth ratio while poor households increase their consumption propensity dramatically. In the CGM model with deterministic labor income, however, the decreasing pattern of the consumption-wealth ratio can be observed for both rich and poor households. This is due to the fact that the put option for rich households is far out of the money and their decisions are barely affected by the presence of the option. So they have a constant consumption-wealth ratio. Low-wealth households, however, are induced to consume more than they would consume without the option. Such over-consumption has two consequences in addition to higher consumption in the current period. First, if the wealth level is above the minimum wealth (the option is out of the money), over-consumption can decrease wealth and increase the likelihood of exercising the option in future periods. Second, if the wealth level is below the minimum wealth (the option is in the money), over-consumption can lead to higher transfer income in future periods. In this case, the household has little incentive to work and thus more incentive to completely live on social welfare or family help.

Extraordinary over-consumption induced by the welfare option has an important impact on the non-participation puzzle in the literature. So far, fixed transaction cost theory has been widely acknowledged as an aid in resolving the puzzle (see Vissing-Jorgensen 2002). Households saving less are more likely not to invest in stocks

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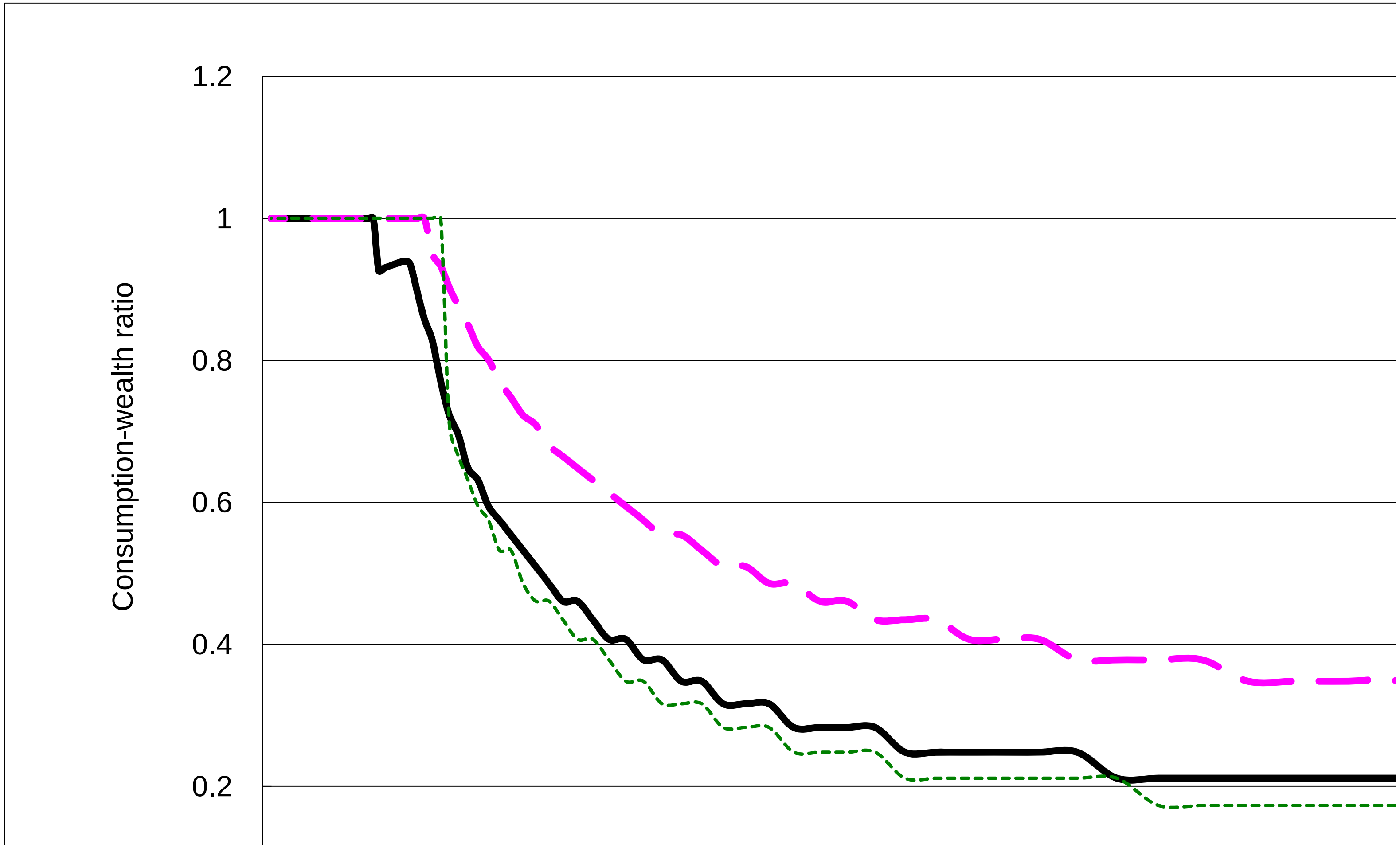
cases.

Figure 2.2: Consumption-wealth ratio for the reference household

The figure illustrates the optimal consumption-wealth ratio as a function of total wealth (cash-on-hand) for the reference household.



because the small amount of investment does not pay off for the relatively large fixed transaction cost. The help and welfare option significantly decreases the saving motive for low-wealth households and thus makes their risky investment unattractive, thereby lowering participation in risky assets.

B. Portfolio choices

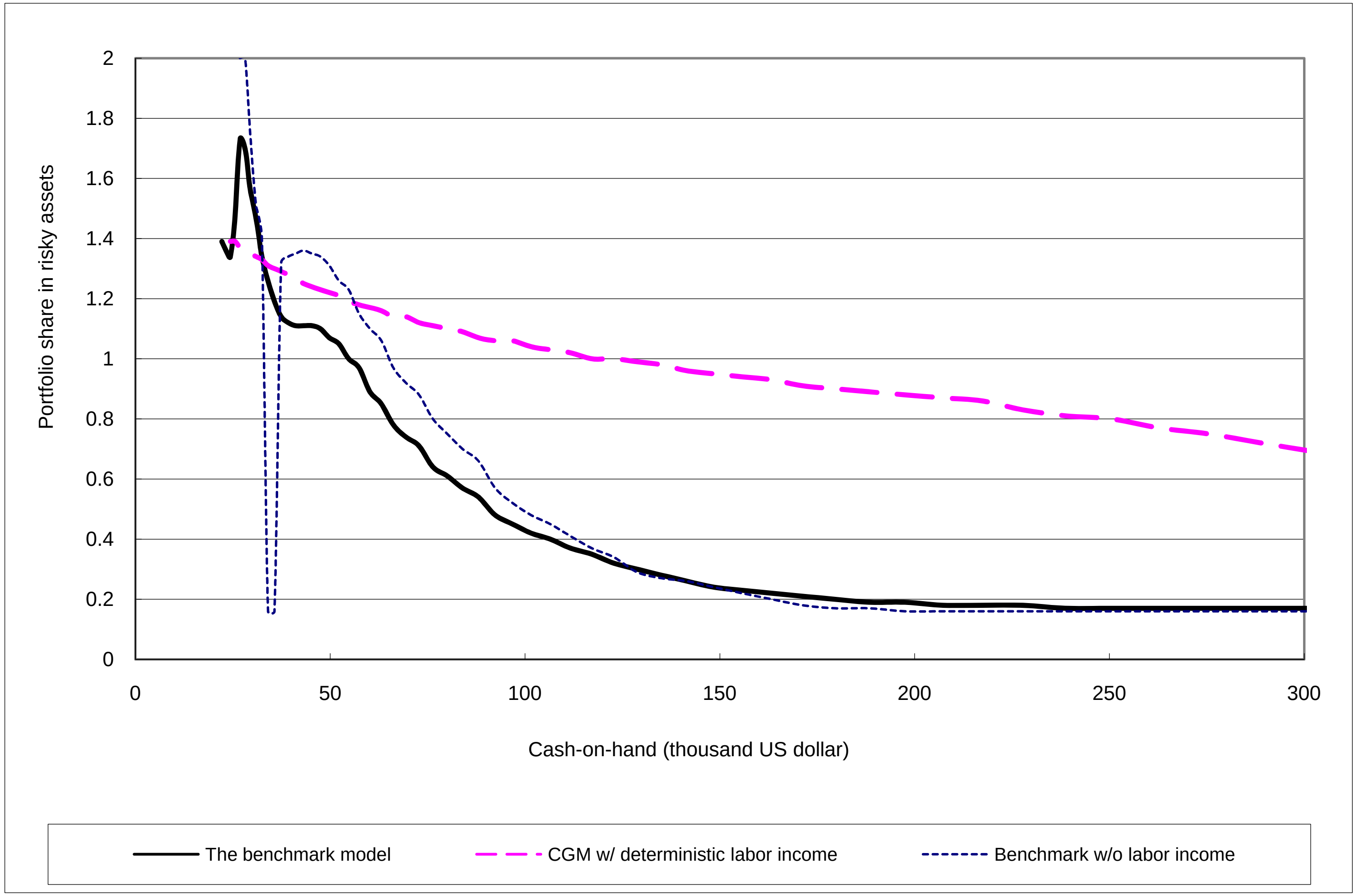
Option effects and labor income effects on household portfolio choices are presented in Figure 2.3. Labor income effects are illustrated by the CGM model with the deterministic labor income (the dashed line) that is calibrated to a 35 year-old married male with two children and a high school diploma. As is analytically solved in Merton (1971), portfolio share of the risky asset is decreasing in wealth. This is because human capital, behaving much like a riskless asset, takes a higher fraction of total wealth for low-wealth households than rich households. In order to balance the risk taking in the portfolio, low-wealth households should invest more in risky assets. (See, e.g., Campbell et al. 2001, Cocco et al. 2005, Davis and Willen 2000, Haliassos and Michaelides 2003, Jagannathan and Kocherlakota 1996, and Viceira 2001).

In general, option effects are also characterized by deceasing portfolio share of the risky asset in wealth. Such gambling behavior, however, can only be observed among low-wealth households (cash-on-hand below $150,000), while rich households choose according to Merton's (1969) constant portfolio solved in the frictionless and complete market without labor income. This is illustrated by the benchmark model without labor income (see the dotted lines). Intuitively, if wealth is not far above the minimum wealth, the downside risk of investment is reduced by the possible compensation from social welfare and family help. Therefore, low-wealth households are induced to increase their portfolio share of the risky asset in order to exploit the equity premium given the relatively low risk.

The constant risk taking for rich households not only can distinguish option effects from labor income effects, but also can serve to rationalize the quadratic pattern of portfolio share of the risky asset in the empirical data. Though option effects do not explain the increasing portfolio share of the risky asset for rich households, they do not impose a counterintuitive decreasing portfolio investment as labor income effects do. It means that the introduction of the welfare option will lead to heterogeneous risk-taking patterns between low-wealth and rich households. This is an important advantage regarding the calibration of portfolio choices for rich households. In order to adjust the decreasing pattern derived from classical models for rich households to the empirically

Figure 2.3: Portfolio share of risky assets for the reference household

The figure shows the optimal portfolio shares of the risky asset as a function of wealth (cash-on-hand) for the reference household.



increasing pattern, we need to assume a rather steeply decreasing risk aversion for rich households. However, with the benchmark model, we only need to change the flat pattern to an increasing pattern. Thus, a modestly decreasing risk aversion for rich households together with the welfare option is sufficient to explain the observed quadratic pattern of the risk-taking in the household portfolio.

We also notice that portfolio share of the risky asset with option effects does not decrease monotonically. Graphically, there are peaks of portfolio share for households with $35000 cash-on-hand. The households would rather reduce their risk taking than gamble further because the welfare option is now deep in the money.[13] These households find that they will benefit from the welfare option for most of the possible portfolio choices. Therefore the marginal value from the welfare option does not compensate the marginal risk.

The benchmark model (the solid line) behaves quite similar to the model with the option effects only, indicating that option effects dominate the labor income effects. This is because the zero income risk in the benchmark model implies that labor income is not a safe asset and households should hold fewer stocks when they are young. Cocco et al. (2005) confirms that even a 0.5% probability of a zero labor income draw dramatically lowers the optimal stock share.

Figure 2.4 plots the optimal portfolio choices for a married male with two children when he is 35, 45, and 60 years. The policy functions are shifted downwards as the age increases, indicating that young investors should hold more aggressive portfolios than the old. This is because young households have relative longer time to the maturity of the welfare option (death) than the old. In this case, young households can take more risk.

In summary, low-wealth households spend all wealth in consumption when wealth is below a critical value which depends on the probability of income shock, the minimum wealth level insured by family and social transfers, and other standard parameters. Above the critical value, they hold large portfolio shares in risky assets. This portfolio share is decreasing in wealth. Rich households, however, choose the same consumption and portfolio policies as given by the Merton (1969) solution.

C. Sensitivity analysis

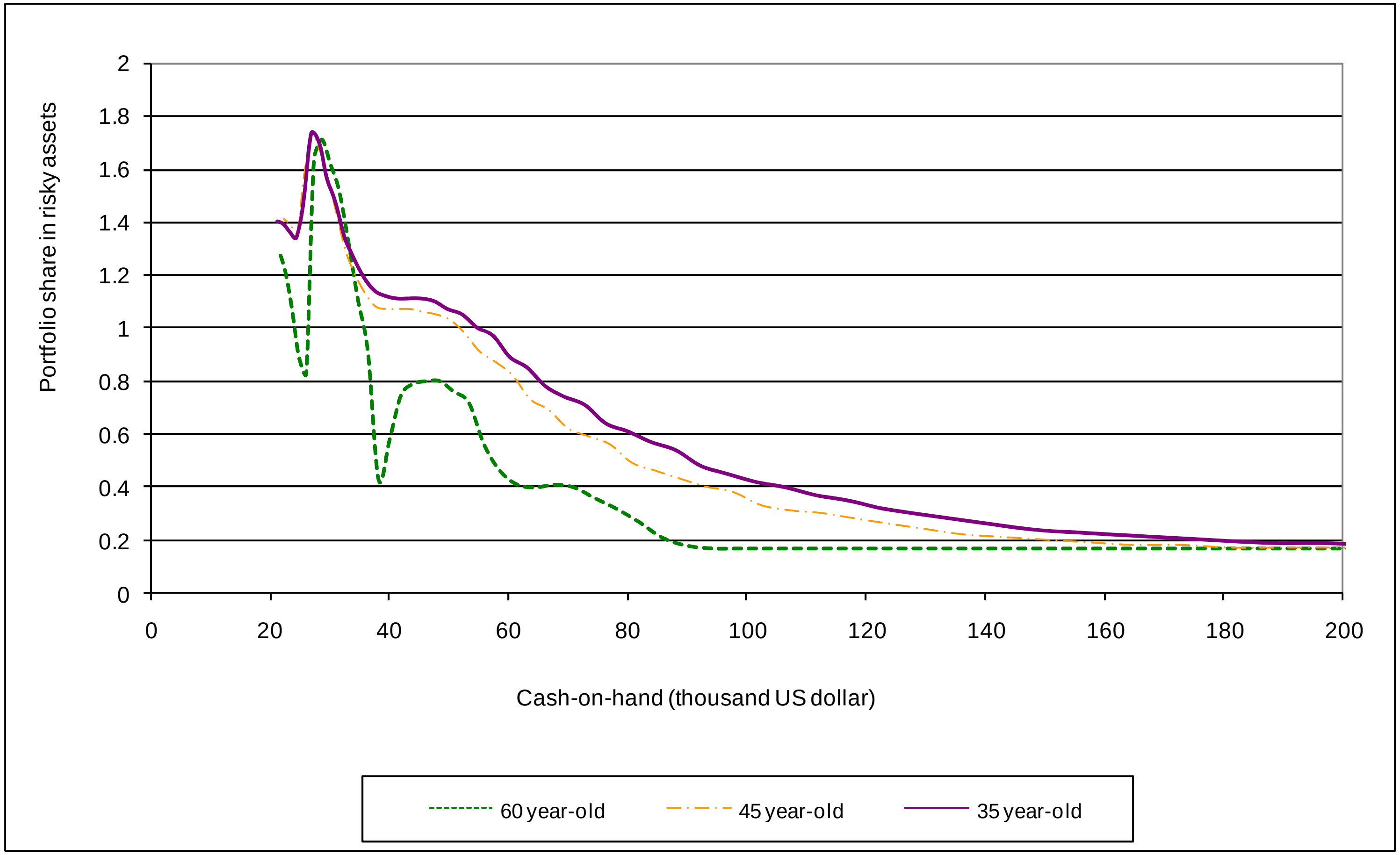
Risk aversion

The effect of decreasing risk aversion is presented in Figure 2.5. Lowering risk

[13] Similar peak effects due to the option can also be found in Hodder and Jackwerth (2007).

Figure 2.4: Life-cycle portfolio choice for the reference household

The figure shows the optimal portfolio shares of the risky asset as a function of wealth (cash-on-hand) for the reference household at the age of 35, 45, and 60.



aversion (reducing the risk aversion coefficient to 5) affects not only the portfolio share directly due to higher risk tolerance (see Figure 2.5(a)), but also increases consumption propensity and decreases savings (see Figure 2.5(b)). The increased consumption propensity will leave less wealth to the future. Therefore, the gambling behavior for more risk-tolerant households (gamma = 5) is observed at the wealth level of $30,000 while the gambling for more risk-averse households (gamma = 10) is observed at the wealth level of $26,000. As a result, the portfolio choice is shifted to the right for low-wealth households and shifted upwards for rich households. Interestingly, risk-tolerant households will gamble less heavily than risk-averse households when they face the same option (the peak portfolio share of risky assets for risk-tolerant households is lower than risk-averse households). This is because the incentive effect from the put option is relatively smaller for risk-tolerant households than risk-averse households. Despite these differences, the general shape of the portfolio choice function does not change when the risk aversion is decreased.

Equity premium

With respect to raising the equity premium, Figure 2.6 presents the effects on portfolio and consumption choices. The equity premium is increased from 4% to 5.75%.[14] Figure 2.6(b) shows that the optimal consumption choice is barely affected. Figure 2.6(a) shows that raising equity premium has heterogeneous effects on the optimal portfolio choice depending on the wealth level. For rich households, high equity premium increases the demand for risky assets. For low-wealth households, raising equity premium decreases the portfolio share of risky assets. This is because when equity premium increases, the probability for the investment to drop below the strike price is lower. The incentive to gamble is then smaller. The general pattern of portfolio and consumption choice remains the same.

The insured minimum wealth

The level of the insured minimum wealth is a critical decision for policy makers of Social Insurance. It is interesting to investigate the impact of changing the level of minimum wealth on households' portfolio choice and consumption. Figure 2.7 presents the effects of increasing the minimum wealth from $15,000 to $20,000 on portfolio and consumption choices. The consumption-wealth ratio is increased due to the higher insured minimum wealth. Consequently, there are more low-wealth households that

[14] This number is found in Cocco et al. (2005).

Figure 2.5(a): Different levels of risk aversion – portfolio choice

The figure shows the optimal portfolio shares of the risky asset as a function of wealth (cash-on-hand) for the reference household when the risk aversion coefficient is 5 and 10.

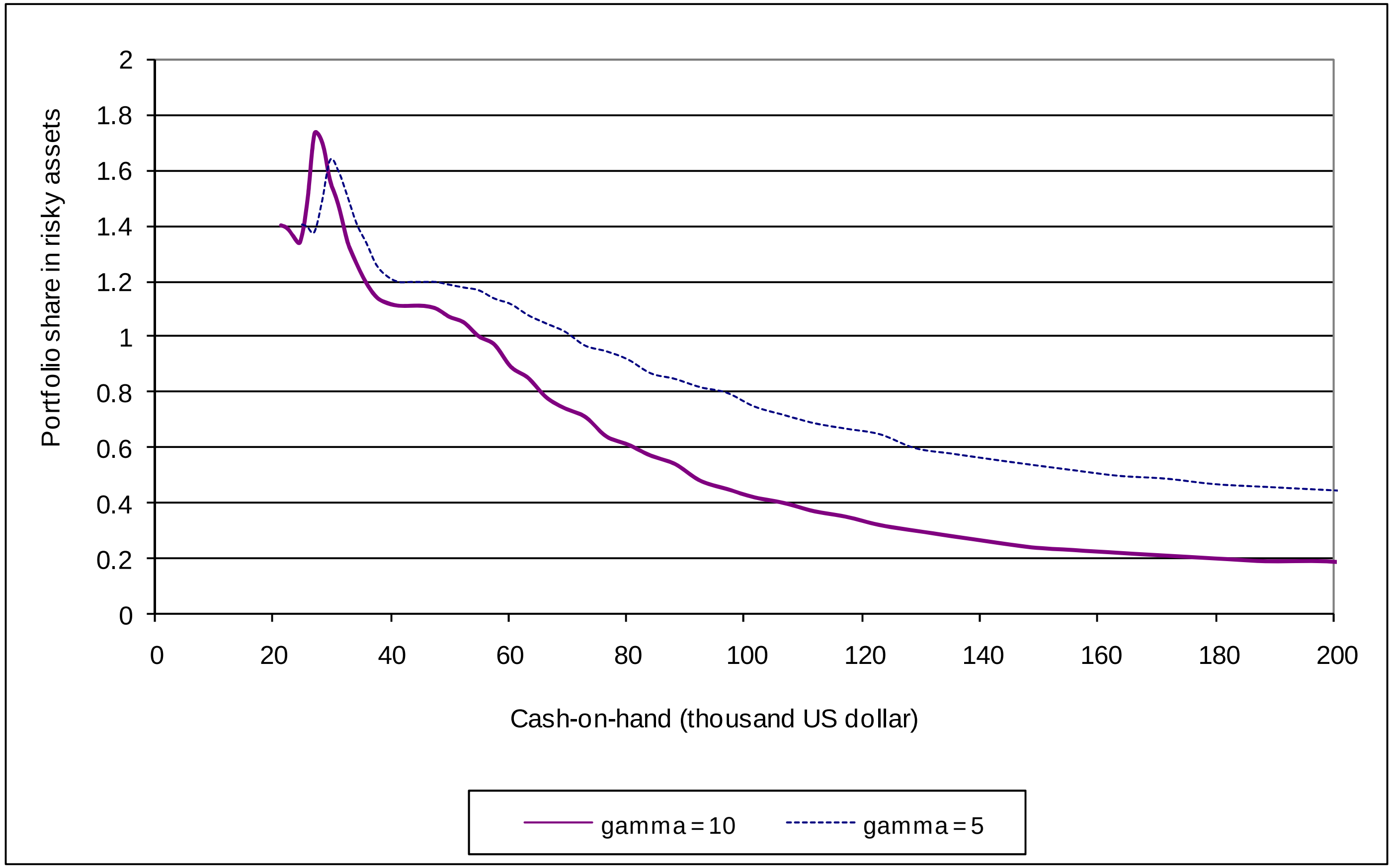


Figure 2.5(b): Different levels of risk aversion – consumption-wealth ratio

The figure shows the optimal consumption-wealth ratio as a function of wealth (cash-on-hand) for the reference household when the risk aversion coefficient is 5 and 10.

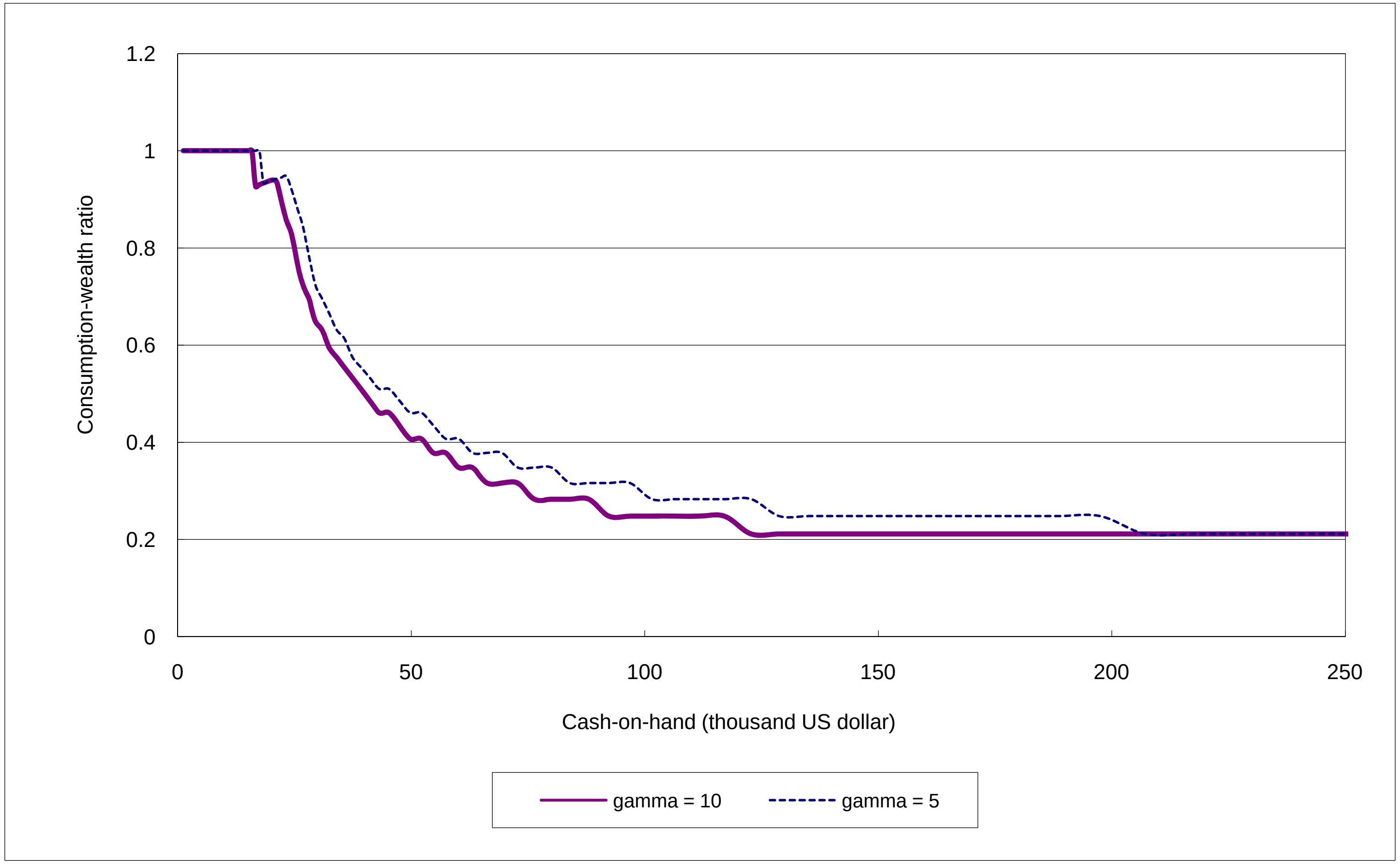


Figure 2.6(a): Different levels of risk premium – portfolio choice

The figure shows the optimal portfolio shares of the risky asset as a function of wealth (cash-on-hand) for the reference household when the risk premium is 4% and 5.75%.

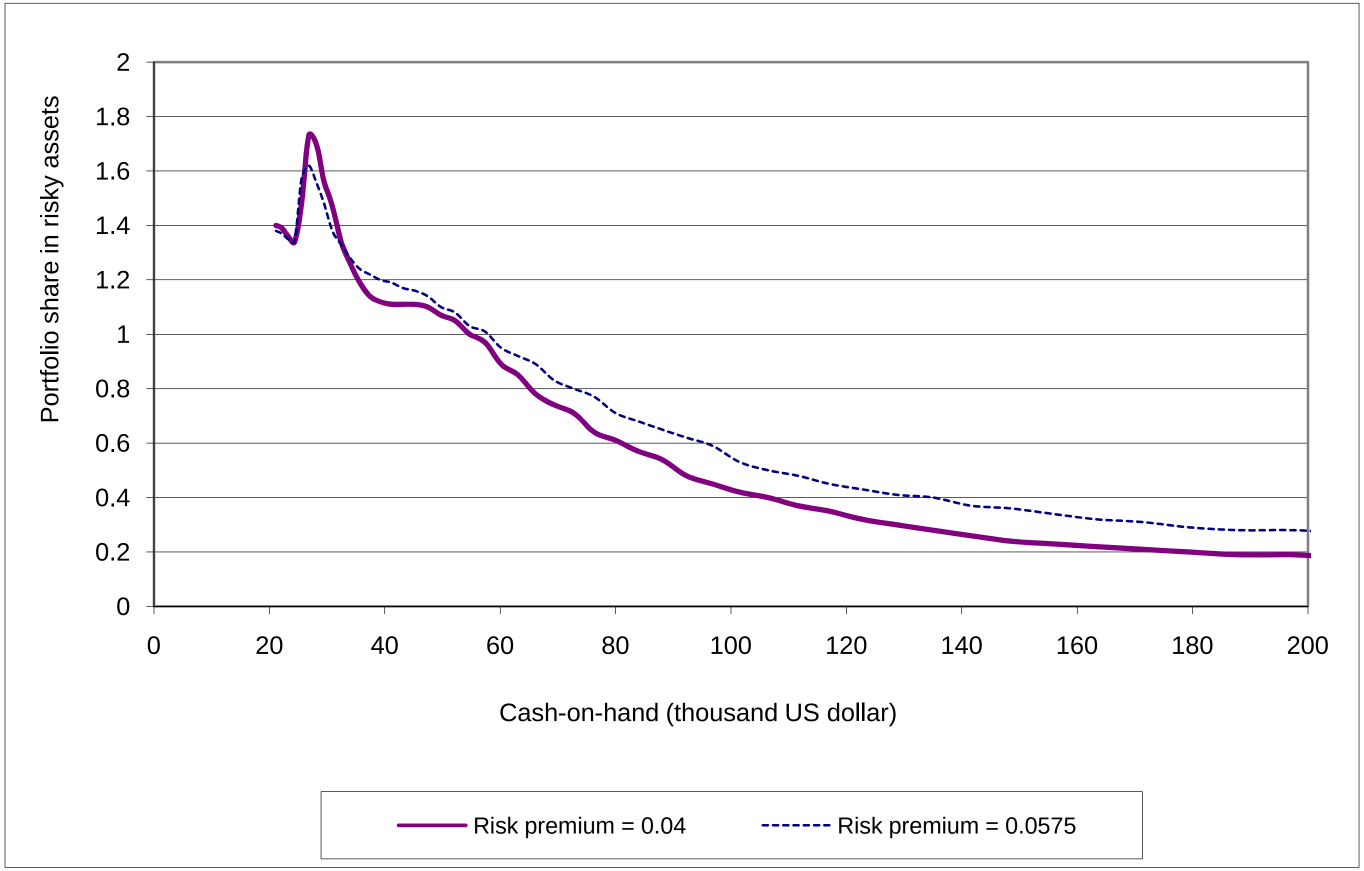


Figure 2.6(b): Different levels of risk premium – consumption-wealth ratio

The figure shows the optimal consumption-wealth ratio as a function of wealth (cash-on-hand) for the reference household when the risk premium is 4% and 5.75%.

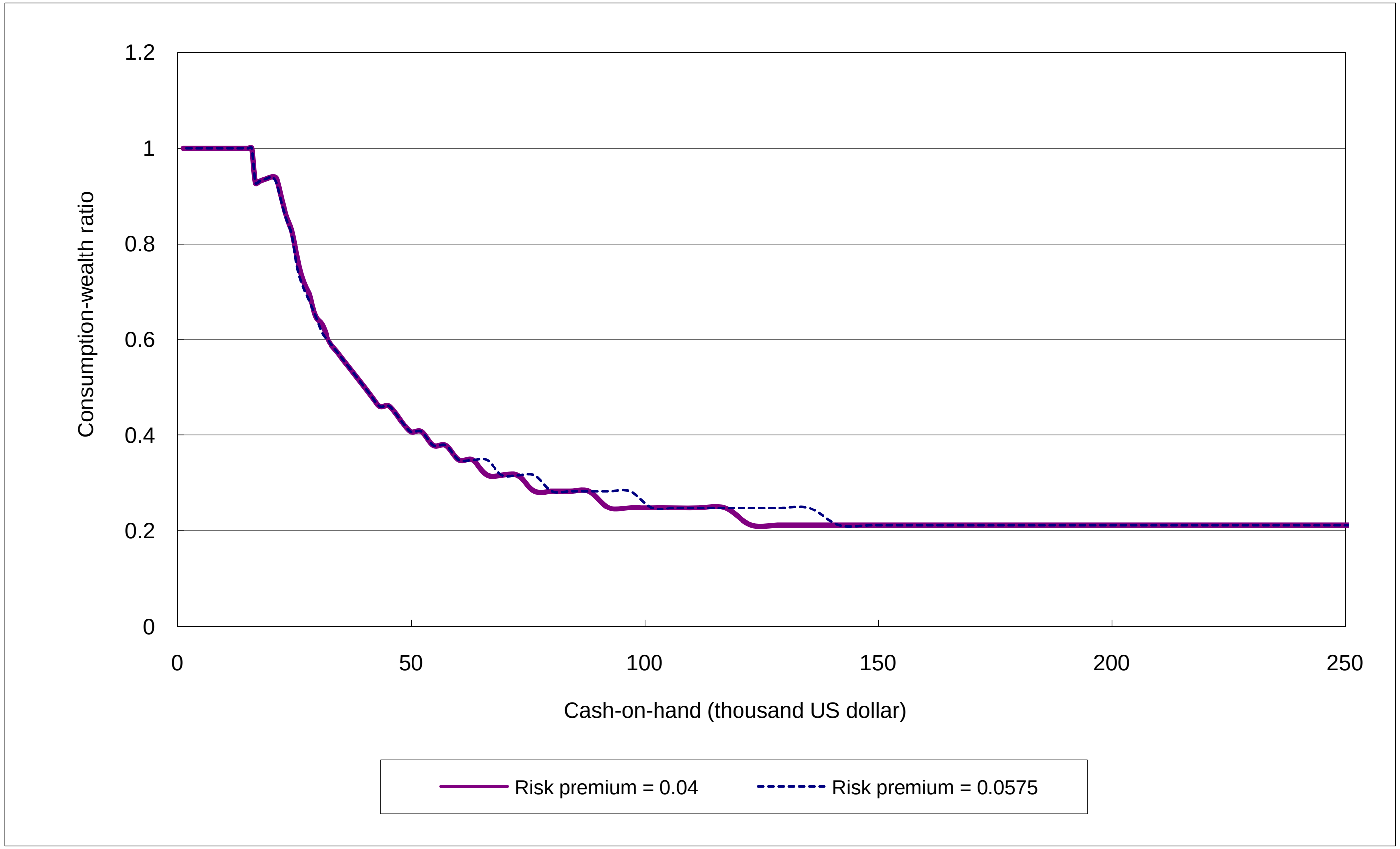


Figure 2.7(a): Different levels of minimum wealth – portfolio choice

The figure shows the optimal portfolio shares of the risky asset as a function of wealth (cash-on-hand) for the reference household when the minimum wealth is 15,000 and 20,000.

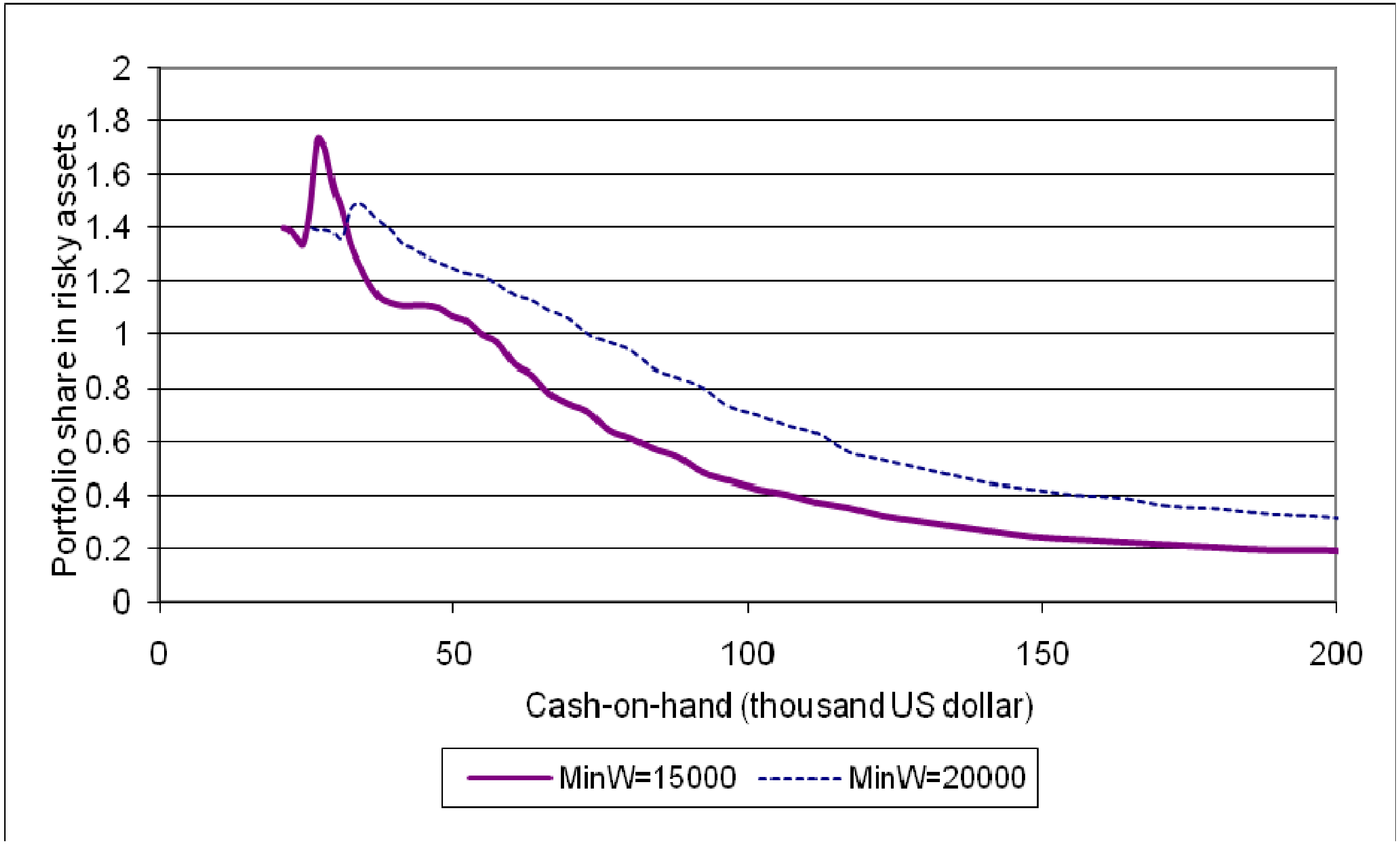
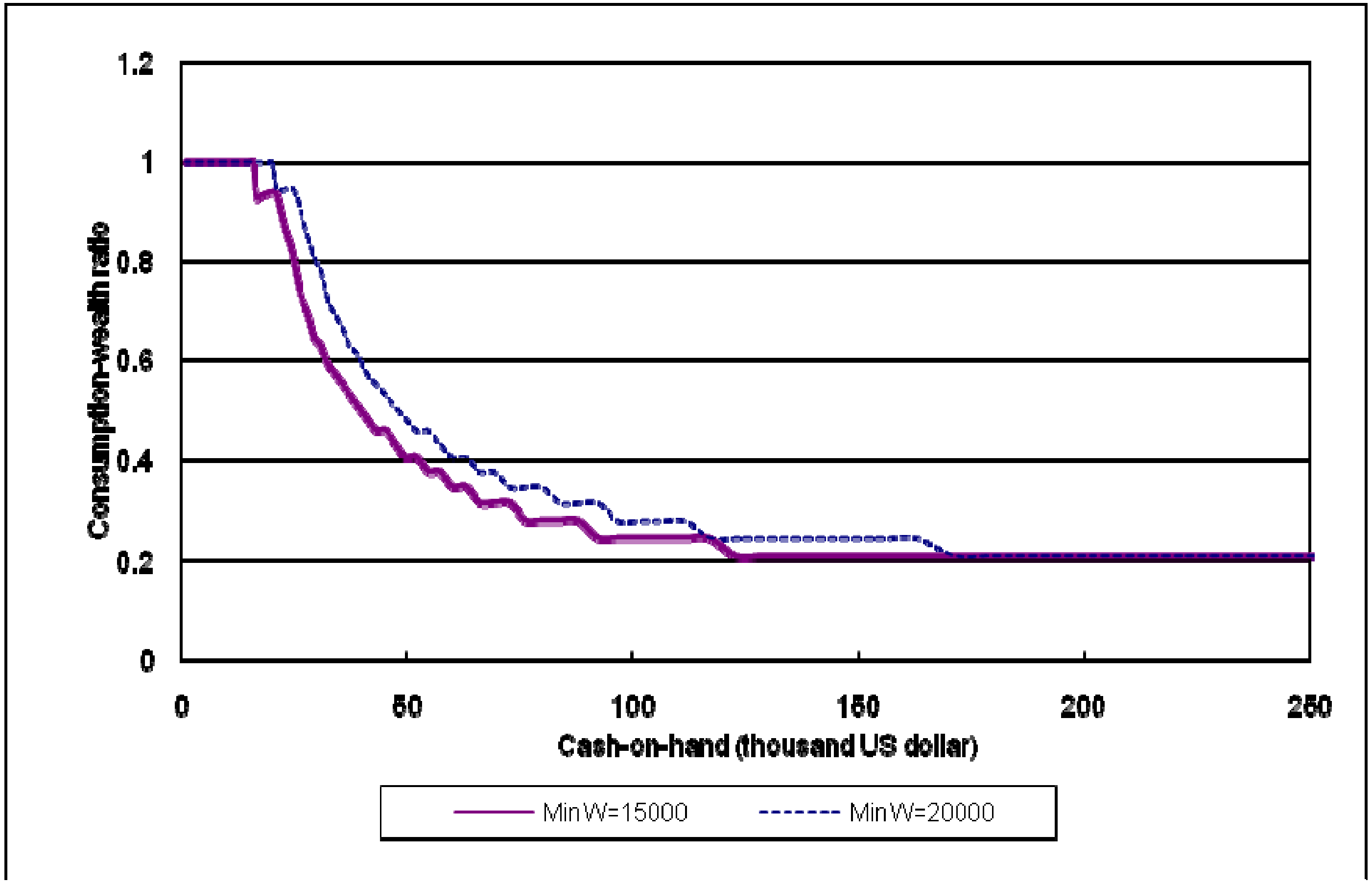


Figure 2.7(b): Different levels of minimum wealth – consumption-wealth ratio

The figure shows the optimal consumption-wealth ratio as a function of wealth (cash-on-hand) for the reference household when the minimum wealth is 15,000 and 20,000.



consume everything. The peak of portfolio share of risky assets is also moved to richer households. Graphically, the portfolio share of risky assets is shifted to the right. However, the main result of the benchmark model (social and family transfers increase the propensity to hold the risky asset and portfolio share of risky assets) is not changed.

2.4 Empirical evidence

In this section, I present the empirical evidence for the effects of social and family transfers on the investment decisions of low-wealth households. Although in recent years abundant research in empirical household portfolio has been carried out due to the increasing accessibility of high-quality datasets, effects of social and family transfers have not been investigated. The challenge is to measure the effects of social and family transfers appropriately. It is important to understand from the analysis in the previous section that the effects of transfers rely on the likelihood of receiving transfers in the future rather than whether the household has received transfers or not in the past.

The following empirical work answers two questions related to the transfer effects. First, does the likelihood to receive transfers decrease households' propensity to participate in the risky asset? Secondly, does the likelihood to receive transfers increase the portfolio share of the risky asset?

The empirical analysis relies on the most recent wave of the PSID family data for 2005. The PSID data provides detailed information on households' income including welfare transfers and family help, which serves well the purposes of this chapter. Also, it reports holdings of individual assets, which enables us to carry out conditional analysis for different assets. Following Bertaut and Starr-McCluer (2002), risky assets are classified into three main categories based on risk and liquidity. First, *equity* includes directly held stock and retirement accounts. Second, *non-financial assets* include investment real estate and private business. Third, *other non-financial assets* include primary residence and vehicles. Riskfree assets include all bank accounts and bonds. Debt includes mortgage loans, real estate loans, and credit card loans. Net worth is computed as the sum of all assets less all debts. Table 2.4 presents summary statistics for different assets as well as other household characteristics. Asset share is the ratio of the asset to net worth. A household is viewed as a *participant* in the particular risky asset if the corresponding asset share is positive. For instance, the equity share of a *participant in equity* is positive. If the variable does not refer to a particular asset, then a *participant* is defined as a household holding a positive share in at least one risky asset.

Table 2.4 presents weak evidence that risky asset participants tend to have higher

Table 2.4: Summery statistics

The table reports the summary statistics of the main assets and household characteristics in the 2005 PSID survey. The number of observations of households is 5376. There are 1725 equity participants, 956 nonfinancial assets (investment real estate and private business) participants, 4662 other nonfinancial assets (primary residence and vehicles) participants, and 4730 participants in any risky asset.

	All households			Participants			Nonparticipants		
	Mean	Median	Std Dev	Mean	Median	Std Dev	Mean	Median	Std Dev
Equity ($)	52,924	0	506,525	164,938	38,000	883,985	-	-	-
Nonfinancial assets ($)	55.528	0	633,182	312,259	75,000	1,475,208	-	-	-
Other nonfinancial ($)	139,731	74,000	211,997	161,132	104,000	219,951	-	-	-
Riskfree ($)	24,579	2,000	141,714	32,755	5,000	162,781	-	-	-
Debt ($)	61,685	14,000	100,407	87,962	58,000	109,842	-	-	-
Net worth ($)	211,077	36,500	975,141	240,262	55,700	1,036,180	-	-	-
Net worth per unit ($)	97,457	14,500	447,454	110,973	22,099	475,411	-	-	-
Labor income ($)	53,521	34,898	136,834	58,955	40,000	144,892	13,733	7,000	17,478
Age	44.23	43.00	15.63	44.63	44.00	15.26	41.26	39.00	17.84
Family units	2.64	2.00	1.43	2.70	2.00	1.42	2.25	2.00	1.49
High-school dummy	0.73	1.00	0.44	0.76	1.00	0.43	0.53	1.00	0.50
College dummy	0.27	0.00	0.44	0.29	0.00	0.46	0.07	0.00	0.26
Employment dummy	0.78	1.00	0.41	0.82	1.00	0.39	0.54	1.00	0.50
House owner dummy	0.58	1.00	0.49	0.66	1.00	0.47	0.00	0.00	0.04

labor income, to receive higher education, and to be house owners. The former two elements are traditionally known to be associated with participation in risky assets (Calvet et al. 2007). The last element shows that housing and vehicles are important risky assets in a household portfolio. Housing and vehicle ownership increases risky asset participation significantly. However, housing and vehicles differ from other financial assets in that they serve a dual purpose (Yao and Zhang 2005). It is both a durable consumption good from which the owner derives utility and also an investment tool. Therefore, transfer effects are analyzed separately for these assets in this chapter.

A. Identification of the likelihood of receiving transfers

The numerical solution in the previous section demonstrates that transfer effects depend on the probability of income shock, household wealth level, and other demographic characteristics that affect the eligibility to receive social transfers and family help. All of these variables imply a certain likelihood of receiving transfers. This paper adopts a probit model to estimate this likelihood of receiving transfers for each household. The dependent variable is a binary variable that records whether the household has received any social transfers and/or monetary aid from relatives and friends in 2005. The sample has 5376 observations. About 38.2% of households received transfers in 2005. The independent variables include total family wealth and its quadratic term, employment dummy, retirement dummy, family labor income, education dummies, and family units. Table 2.5 shows the results of the probit model.

Wealth has a negative effect on the probability of receiving transfers. Most social welfare programs exclusively support households with limited resources. Similar to wealth, labor income negatively impacts the likelihood of receiving transfers. This is not only because some welfare programs are targeted at low-income households, but also because low income jobs are characterized by high unemployment risk. In contrast, retirees receive stable pension income and face a limited probability of income shock. Finally, education increases households' skills needed to attain a stable job. This holds true, however, only for college education.

After the likelihood to receive transfers is identified, a linear prediction based on the estimates of coefficients can be computed for each household. I assume that such likelihood is stable in the short run, which means in the future the household will receive transfers with this estimated likelihood. Then I can investigate the relationship between portfolio share of assets and this likelihood.

Table 2.5: The probit model for the likelihood of receiving transfers

The table reports the probit estimation for the likelihood of receiving social welfare and help from relatives and friends. The dependent variable is a binary variable that records whether the household has received any social transfers and monetary help from relatives and friends in 2005. The sample includes 5376 observations. ***Significant at 1%; **significant at 5%; *significant at 10%.

Independent variable	coefficient	t-stat
Total wealth/1000000	-0.209***	-4.52
Total wealth/1000000 squared	0.006***	4.61
Employment dummy	-0.178***	-3.14
Retirement dummy	-0.423***	-5.43
Labor income/1000000	-0.279*	-1.78
College diploma dummy	-0.064	-1.50
High school diploma dummy	0.016	0.39
Family units	-0.044***	-3.5
Constant	0.052	0.81

B. Transfer effects on the participation decision

Table 2.6 reports probit regressions of participation decisions in different asset classes on the likelihood of receiving transfers, on financial characteristics, and on demographic characteristics using the 2005 PSID data. The sample is smaller than in Tables 2.4 and 2.5 since households whose net worth lies in the top or bottom one percent of observations are dropped. This step is taken in order to diminish the effect of outliers.

There is clear evidence of a negative effect of the likelihood of receiving transfers on participation decisions in every asset class. These effects are economically important as well as statistically significant. We first look at the participation decision in stock, the asset most extensively investigated in the literature. An increase in the likelihood of receiving transfers from the 20^{th} percentile to the 80^{th} percentile decreases the probability of being a stockholder from 15.9 to 11.8 percent, when holding all other variables in the regression constant at their means. The negative marginal effects on the participation in real estate investment and private business and primary residence and vehicles are even larger. The negative effects are consistent with the theoretical

Table 2.6: Transfer effects on participation decisions

The table reports probit regressions of participation decisions in different asset classes on the likelihood of receiving transfers, financial, and demographic household characteristics using the 2005 PSID data. For each regression, the linear coefficient, t-statistics, and marginal effect of each predicting variable are reported. The marginal effect is assessed by computing the impact on the participation decision of increasing a continuous regressor from the 20th percentile to the 80th percentile, or of setting a dummy variable equal to one, while holding the other variables in the regression constant at their means. The sample size and proportion of participation are reported below the regressions.

	Participant in equity			Participant in real estate investment and private business			Participate in primary residence and vehicles		
	Estimate	t-stat	Change	Estimate	t-stat	Change	Estimate	t-stat	Change
Score	-0.855***	-4.13	-4.14%	-1.367***	-6.67	-4.66%	-1.959***	-8.89	-8.57%
Net worth per unit/1000000 ($)	7.033***	20.54	11.14%	4.455***	16.37	5.43%	6.819***	9.00	19.28%
$(\text{Net worth per unit}/1000000)^2$	-4.113***	-13.85	-	-2.416***	-11.24	-	-4.075***	-9.55	-
Age	0.002	1.48	1.45%	-0.005**	-3.02	-2.16%	-0.004**	-2.42	-2.23%
College diploma dummy	0.542***	10.83	13.57%	-0.053	-1.00	-0.81%	0.017	0.22	0.34%
High school diploma dummy	0.296***	5.57	6.04%	0.067	1.24	1.02%	0.239***	4.45	5.05%
Labor income/1000000	4.749***	10.69	7.23%	-0.118	-0.78	-1.33%	13.431***	12.06	22.74%
Constant	-1.900***	-20.66	-	-1.524***	-16.90	-	0.059	0.64	-
Sample size	5268			5268			5268		
Participation rate	32.09%			17.78%			86.72%		

prediction that social welfare and family help depress households' investment motive and thus increase the probability of being non-participants in risky assets for low-wealth households.

Education has been found to positively affect participation in stock market in the literature (Vissing-Jorgensen 2002, Campbell 2006, and Calvet et al. 2007). In Table 2.6, education enters the equity participation regression with the same positive signs. Education effects on private business participation are insignificant as also found in Campbell (2006). This might be due to the fact that entrepreneurship is not a major focus of education. Ageeffects are very weak and ambiguous. Wealth is found to have positive effects on participation for all assets, consistent with a fixed cost of participation. Labor income has important positive effects for the participation in equity and primary residence but it has an insignificant impact on participation in private business. An intuitive interpretation is that entrepreneurs live mostly on business dividends and not on labor income.

C. Transfer effects on the portfolio choice

It is known that the wealthy invest differently than do poorer households (Tracy et al. 1999, Heaton and Lucas 2000, Carroll 2002). In order to focus on the behavior of a representative household, especially of low-wealth households, the quantile regression of the median is adopted instead of the OLS regression. Just as OLS regressions estimate the mean, median regressions estimate the median of the dependent variable expressed as functions of observed covariates. For household data, medians are more representative than means. Given the extreme skewness of the wealth distribution, median regression renders more robust estimates than mean regression.

Table 2.7 reports median regressions of portfolio shares of different asset classes on the likelihood of receiving transfers, financial variables, and demographic characteristics. The sample is even smaller than that used in Table 2.6 since now non-participating households and households with zero or negative net worth are also dropped.

As for portfolio choices on equity and private business, the likelihood of receiving transfers has positive effects on the corresponding portfolio share. An increase in the likelihood of receiving transfers from the 20^{th} percentile to the 80^{th} percentile increases the conditional portfolio share of equity by 4.57 percent and the share of private business and real estate investments by 6 percent. The 5 percent change is relatively large considering that the median portfolio share of equity is merely 23 percent. These

Table 2.7: Transfer effects on portfolio choices

The table reports median regressions of portfolio shares of different asset classes conditional on participation on the likelihood of receiving transfers, financial, and demographic household characteristics using the 2005 PSID data. For each regression, the linear coefficient, t-statistics, and marginal effect of each predicting variable are reported. The marginal effect is assessed by computing the impact on the portfolio share of increasing a continuous regressor from the 20th percentile to the 80th percentile, or of setting a dummy variable equal to one, while holding the other variables in the regression constant at their means. The sample size and the median portfolio share of the asset for the participants are reported below the regressions.

	Portfolio share of equity			Portfolio share of real estate investment and private business			Portfolio share of primary residence and vehicles		
	Estimate	t-stat	Change	Estimate	t-stat	Change	Estimate	t-stat	Change
Score	0.208***	2.60	4.57%	0.272***	2.78	6.00%	-0.288**	-2.45	-6.34%
Net worth per unit/1000000 ($)	0.246**	2.56	2.57%	0.519***	4.65	5.44%	-2.982***	-17.53	-31.21%
$(\text{Net worth per unit}/1000000)^2$	-0.084	-1.09	-	-0.113	-1.52	-	1.750***	11.89	-
Age	0.003***	3.91	7.29%	-0.005***	-4.81	-12.44%	-0.004***	-4.15	-10.91%
College diploma dummy	0.087***	4.69	8.69%	-0.051**	-2.00	-5.08%	0.126***	4.14	12.62%
High school diploma dummy	-0.008	-0.33	-0.82%	-0.088***	-2.99	-8.76%	0.119***	3.90	11.91%
Labor income/1000000	0.115***	2.93	0.84%	-0.202	-1.51	-1.48%	0.744***	7.55	5.43%
Constant	0.096**	2.39	-	0.671***	12.49	-	1.252***	23.68	-
Sample size	1615			890			4083		
Median portfolio share	0.23			0.32			1.00		

positive effects suggest that the existence of social welfare and family help induce households to gamble as they turn poor.

Further evidence in support of the existence of transfer effects is that after controlling for transfer effects portfolio shares of equity and private business are increasing in wealth. This means that the decreasing pattern for the share of risky asset is due to the transfer effect. Once the decreasing component is explained by the transfer effect, the quadratic pattern of risky portfolio share estimated in Campbell (2006) is replaced by a monotonically increasing function estimated in this chapter, which is consistent with intuition in that wealthy households hold more risky portfolios than the poor households.

Age has a positive effect on equity share but negative effects on the shares of private business and primary residence. The negative effects are consistent with the rules of thumb suggested by financial planners: as people age, they should shift investments away from stocks and towards bonds (Jagannathan and Kocherlakota 1996). Table 2.6 shows that as people age, they shift investments away from private business, real estate investments, primary residence, and vehicles. Another interpretation is that people purchase their primary residence with the help of a mortgage when they are young and net worth is low. As a result, portfolio share of primary residence tends to be high. As they age, they pay back the mortgage loan gradually and the net worth increases. The portfolio share of primary residence thus decreases. The positive effect on equity share is due to the increased holding of stocks through retirement accounts as people age.

Labor income has weak effects on portfolio share of equity and private business. The previous section shows that in the Merton (1971) model, labor income can explain the decreasing portfolio share of the risky asset with wealth (see Figure 2.3) and that labor income could potentially be the cause of the gambling behavior of low-wealth households. The evidence here, however, does not support this theory.

Most regressors in the regression of the portfolio share of primary residence and vehicles have different signs than those in the other two asset classes. This is related to the dual purposes (consumption and investment) of owning a primary house and vehicles. First, social welfare and family help are not expected to cover mortgage debt. If the net worth becomes low due to a sharp decrease in housing prices and the house owner incurs an income shock, the house can only be foreclosed or liquidated because welfare programs only support basic living expenses. The loss of primary housing consumption and liquidation cost might be larger than the insurance benefit of social welfare. In this case, the likelihood of receiving transfers would not boost the share of the primary housing. Second, due to the inelastic demand of owing a house, low-wealth households have to purchase the house with rather large mortgage loans. With a same house, low-wealth households have a higher share of primary housing than the rich. This is why wealth has a negative sign and quadratic pattern. Third, the present value of future labor income determines the amount of mortgage loan that the household can afford. A high mortgage loan leads to a high portfolio share for primary housing. Therefore, labor income has a positive effect.

D. Robustness

An alternative way to identify the transfer effect is to impose an assumption: those who receive transfers in the current period will be more likely to receive transfers in the future. In this case, we can use the dummy of receiving transfers as an explanatory variable to test the empirical relationship between transfers and household portfolio. Based on the theoretical results in Section 2.3, we would expect that the sign of the dummy of receiving transfers is negative in the regression of participation and positive in the regression of portfolio risk.

Another robustness check is related to the truncation for the sample. In previous regressions, the richest and poorest households are excluded from the sample in order to eliminate the effect of outliers. However, since median regressions are more robust to outliers than OLS regressions, we should not expect that the results will be qualitatively changed.

The results are presented in Table 2.8 and 2.9. The signs of coefficient for the welfare dummy are consistent to the theoretical predictions of this chapter. In general, welfare is found to have negative effects on households' decisions to participate in asset markets and positive effects on portfolio shares of risky assets. However, most estimates are statistically insignificant. This suggests that whether the household has received transfers in the current period has less explanatory power than the likelihood of receiving transfers in the future for explaining households' investment decisions.

2.5 Conclusion

In this chapter, I solve a life-cycle model of consumption and portfolio choice in the presence of social welfare and family help. The model is proposed as an explanation of two empirical puzzles simultaneously observed among low-wealth households. They either choose to be *non-participants* in risky asset markets or *gamblers* by taking on a

Table 2.8: One-stage estimation on participation decisions

The table reports probit regressions of participation decisions in different asset classes on the likelihood of receiving transfers, financial, and demographic household characteristics using the 2005 PSID data. For each regression, the linear coefficient, t-statistics, and marginal effect of each predicting variable are reported. The marginal effect is assessed by computing the impact on the participation decision of increasing a continuous regressor from the 20th percentile to the 80th percentile, or of setting a dummy variable equal to one, while holding the other variables in the regression constant at their means. The sample size and proportion of participation are reported below the regressions.

	Participant in equity			Participant in real estate investment and private business			Participate in primary residence and vehicles		
	Estimate	t-stat	Change	Estimate	t-stat	Change	Estimate	t-stat	Change
Family units	-0.111***	-6.16	-11.74%	0.002	0.11	0.14%	0.201***	5.93	2.82%
Age	0.000	0.07	0.13%	-0.006***	-3.17	-3.83%	-0.011***	-4.43	-1.19%
Labor income/1000000	2.200***	4.88	5.59%	-0.147	-0.79	-0.25%	0.150	0.862	0.05%
Welfare dummy	-0.064	-1.2	-2.22%	-0.083	-1.49	-1.92%	-0.057	-0.64	-0.24%
College diploma dummy	0.518***	10.09	19.06%	-0.060	-1.13	-1.40%	-0.073	-0.7	-0.32%
High school diploma dummy	0.252***	4.31	8.61%	-0.041	-0.67	-0.97%	-0.107	-1.14	-0.42%
Ln(Net worth)	0.432***	23.82	50.97%	0.415***	22.58	35.56%	0.297***	16.01	4.99%
Constant	-5.325***	-28.44	-	-5.190***	-26.09	-	-1.041***	-5.42	-

Table 2.9: One-stage estimation on portfolio decisions

The table reports median regressions of portfolio shares of different asset classes conditional on participation on the likelihood of receiving transfers, financial, and demographic household characteristics using the 2005 PSID data. For each regression, the linear coefficient, t-statistics, and marginal effect of each predicting variable are reported. The marginal effect is assessed by computing the impact on the portfolio share of increasing a continuous regressor from the 20^{th} percentile to the 80^{th} percentile, or of setting a dummy variable equal to one, while holding the other variables in the regression constant at their means. The sample size and the median portfolio share of the asset for the participants are reported below the regressions.

	Portfolio share of equity			Portfolio share of real estate investment and private business			Portfolio share of primary residence and vehicles		
	Estimate	t-stat	Change	Estimate	t-stat	Change	Estimate	t-stat	Change
Family units	-0.022***	-3.05	-6.50%	-0.048***	-3.72	-14.25%	0.109***	9.99	32.73%
Age	0.004***	5.53	9.91%	-0.005***	-3.99	-14.79%	-0.008***	-7.34	-22.28%
Labor income/1000000	0.049	1.61	0.35%	-0.155	-1.49	-1.13%	0.001	0.18	0.10%
Welfare dummy	0.034*	1.75	3.40%	0.025	0.63	2.48%	-0.031	-0.92	-3.13%
College diploma dummy	0.091***	5.31	9.08%	-0.070**	-2.00	-7.05%	-0.002	-0.04	12.62%
High school diploma dummy	0.011	0.44	1.07%	-0.069	-1.59	-6.91%	0.065*	1.79	6.53%
Ln(Net worth)	-0.022***	-3.42	-8.01%	-0.042***	3.36	15.01%	-0.072***	-8.25	-25.70%
Constant	0.317***	4.29	-	0.292**	1.99	-	1.921***	20.8	-

large share of risky assets. Both of these two behaviors are linked to the existence of social welfare and family help, modeled as a put option. Welfare and help depress the investment motive for the low-wealth households. The decrease in investment increases the marginal cost of the fixed transaction cost of entering risky asset markets. Thus, welfare and aid decrease the propensity to invest in risky assets. On the other hand, the welfare option induces low-wealth households to increase their portfolio share of risky assets. With realistic parameters, the model fits the empirically observed decreasing portfolio share of risky assets with wealth.

Empirical evidence supporting the effects of social welfare and family help on participation decisions and portfolio choices predicted by the theoretical model is presented in this chapter. A proxy for measuring the transfer effects, the likelihood of receiving transfers, is shown to have negative effects on participation and positive effects on portfolio shares in equity and private business. The weak labor income effects show that the traditional Merton model cannot fully explain the gambling behavior of low-wealth households. I also find that gambling behavior with respect to the primary residence cannot be explained by the transfer option, emphasizing the dual purposes of owning a primary residence and vehicles.

The chapter suggests that some non-linear household portfolio choices can be explained by non-linear return structures or real options within the framework of rational models. The non-linear features of some social welfare programs open a novel dimension for financial planners to advise portfolio choices.

Appendix 2.A: Numerical Solution

The basic structure of the model uses the methodology described in Hodder and Jackwerth (2007). Following their notations, I use grids of cash-on-hand values X and time t, with $\Delta(\log X)$ constant as well as time steps Δt of equal length. I use 120 log value steps with the upper boundary equal to 300,000 and lower boundary equal to 1,000. Hence, $\Delta(\log X)$ is 0.047532. In order to have the indirect utility at the next period lie on the grid points even with a choice of a riskless portfolio ($\alpha = 0$), the grid structure is assumed to grow at the riskfree rate and so are the upper and lower boundaries. Time to maturity is 50 years. The number of time steps is 10. Hence, Δt is 5 years. The frequency of rebalancing is set to be low because households are reluctant to change their portfolio due to the transaction cost and inertia (see Brunnermeier and Nagel 2006). The choices of α vary from 0 to 2 in steps of 0.01.

To calculate the probabilities of moving from one cash-on-hand value at time t to all possible values that can be reached at $t + \Delta t$, the range of 41 grid points with index i equal to -20, …, 0, …, 20 are used. The probabilities for those possible moves depend on the choice of α (stock proportion). For a given α, I calculate the probabilities based on the normal density times a normalization constant so that the computed probabilities sum to one:

$$p_{i,\alpha,\Delta t} = \frac{\dfrac{1}{\sqrt{2\pi}\sigma_{\alpha,\Delta t}} \exp\left[-\dfrac{1}{2}\left(\dfrac{r\Delta t + i\Delta\log(X) - \mu_{\alpha,\Delta t}}{\sigma_{\alpha,\Delta t}}\right)^2\right]}{\sum_{j=-20}^{20} \dfrac{1}{\sqrt{2\pi}\sigma_{\alpha,\Delta t}} \exp\left[-\dfrac{1}{2}\left(\dfrac{r\Delta t + j\Delta\log(X) - \mu_{\alpha,\Delta t}}{\sigma_{\alpha,\Delta t}}\right)^2\right]} \tag{2.A1}$$

where $\mu_{\alpha,\Delta t} = \left[\alpha\mu + (1-\alpha)r - \frac{1}{2}\alpha^2\sigma^2\right]\Delta t$ and $\sigma_{\alpha,\Delta t} = \alpha\sigma\sqrt{\Delta t}$. (2.A1) shows that the approximated probabilities do not depend on the level of $\log(X)$ or time and they are solely functions of α.

For each period, the state space is constructed by 121 grids for cash-on-hand and two states of unemployment and employment. In addition, two special indirect utilities are required for the state of zero cash-on-hand in order to allow zero saving to be admissible. For unemployed households, the grid points below the minimum wealth are adjusted to be the same as the minimum wealth. For the employed households, the cash-on-hand is adjusted by labor income. This adjustment will generally cause the

$\log(X_{t+1})$ not to land on grid points at the next time step. In order to obtain the corresponding indirect utilities for the $\log(X_{t+1})$ not landing on grid points, spline estimation is used. Given the relatively large number of grid points, the estimation error is limited. Such interpolation has also been used in other recent papers in the literature (e.g., Cocco et al. 2005). Alternatively, one can avoid interpolation by fixing the $\log(X_{t+1})$ values equal to the grid points at the next step. Since these $\log(X_{t+1})$ values have been adjusted by labor income and social welfare, we need to deduct them to obtain the financial wealth at the next time step. Dividing this financial wealth by the current log cash-on-hand value $\log(X_t)$, we obtain the corresponding return. We further compute the associated probability for this return. Such algorithm increases the time of computing because the associated probability for each grid point is different and needs to be computed case by case. A more severe problem with this algorithm is the quality of discretization of the normal distribution. Since the forward nodes are not evenly spaced and are mostly located in the left side of the normal distribution due to the deduction of labor income, the forward nodes cannot well approximate the normal distribution. Therefore, spline estimation is adopted in this chapter. Given the choices of consumption and the proportion of risky assets, the expected utility of being unemployed or employed in the next period can be separately computed. Then the expected utility associated with the given choices can be computed by multiplying the corresponding probabilities of employment and unemployment conditional on the current status of employment. The optimal choices are the results of maximizing over the expected utilities of all admissible choices.

One distinction of this model from previous studies which also assume zero income risk is to allow the household to consume all his cash-on-hand because in the next period his cash-on-hand will either be adjusted to the minimum wealth in case of unemployment or be adjusted by the labor income in case of employment. Both cases will prevent the household from having nothing to eat. However, in other studies (e.g., Carroll 1997) the choice of zero savings is implicitly excluded as an optimal choice since in case of receiving zero income in the next period the power utility will be minus infinity.

When implementing the backward sweep through the grid, one difficulty is in dealing with behavior at the boundaries. The terminal step is trivial in that I calculate the terminal utility using the terminal wealth. To calculate the indirect utilities of the grid points close to the upper and lower boundaries, I use buffers of cash-on-hand values above the upper boundary and below the lower boundary. The lower buffer for the unemployment case is trivial since the indirect utilities associated with grid points which are below the minimum wealth are all equal to the indirect utility for the grid point equal to the minimum wealth. For the indirect utilities associated to the upper buffer grid points, I use an approximation based on the indirect utilities on the upper boundary $J_{t,ub}$, which can be decomposed into two parts according to the Bellman equation:

$$J_{t,ub} = \frac{C_{t,ub}^{\;1-\gamma}}{1-\gamma} + \delta p_t E_{t,ub}[J_{t+1}(X_{t+1}, Y_{t+1})]. \tag{2.A2}$$

The first term comes from the current consumption and the second term comes from the expected future utility. The closed form solution in Merton (1971) shows that the optimal consumption is linear in wealth in the absence of labor income. Assuming that the optimal consumption policies (consumption-wealth ratio) for the upper buffer grid points are the same as for the upper boundary, we can compute the first part for the upper buffers with index *ub-i* equal to *ub-20, …, ub-1,* by

$$\frac{(C_{t,ub} X_{t,ub-i} / X_{t,ub})^{1-\gamma}}{1-\gamma}. \tag{2.A3}$$

Assuming further that the expected future utilities for the buffer grid points are the same as the one on the boundary, we can approximate the indirect utilities for the upper buffer as the following:

$$J_{t,ub-i} = \frac{(C_{t,ub} X_{t,ub-i} / X_{t,ub})^{1-\gamma}}{1-\gamma} + \delta p_t E_{t,ub}[J_{t+1}(X_{t+1}, Y_{t+1})]. \tag{2.A4}$$

This approach is potentially suboptimal; however, the distortion results in ripples only some 20-50 steps below the upper boundary, affecting mainly the early time steps.

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Chapter Three

Life-Cycle Portfolio Choice with a Realistic Retirement Age Distribution

This chapter numerically solves the optimal life-cycle portfolio choice when the model is calibrated to match the empirical retirement age distribution: people tend to retire at markedly higher rates around the firm's early retirement age and the age at which the full pension can be received. The model shows that financial incentives for keeping investors in labor force and low leisure preference for young investors endogenously restrict investors from retiring early. Thus, this chapter suggests a novel effect of the early retirement option on portfolio choice. As opposed to results from earlier models, the optimal portfolio share of stock does not increase monotonically prior to retirement. Wealthy investors might find it optimal to reduce the stock share in their early stage of life in order to decrease the possibility of having insufficient wealth for retirement when they are older. Thus, the model predicts either an increasing or a hump-shaped pattern for life-cycle stock holding, consistent with empirical observations.

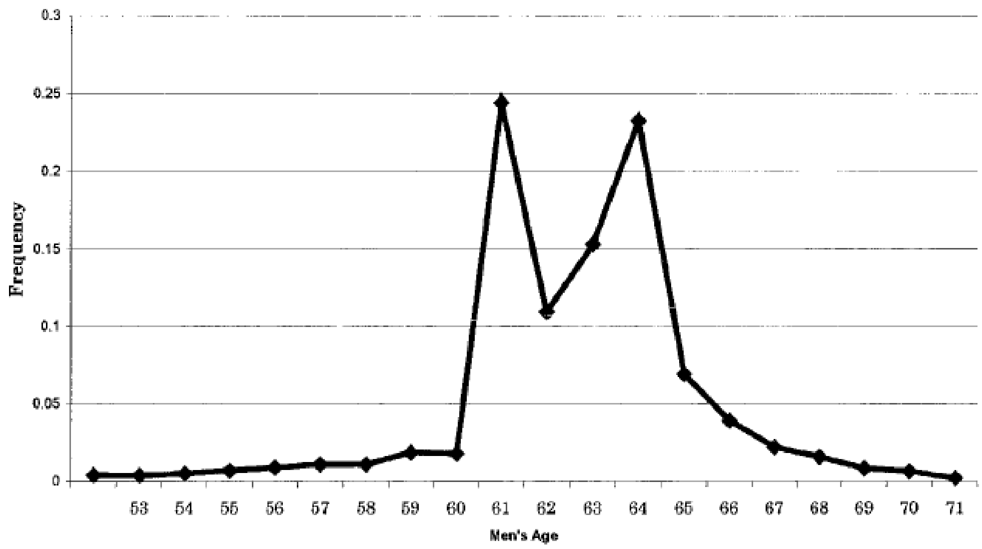
3.1 Introduction

> The first baby boomers are turning 62 this year. (…) Almost a third of the 2.9 million boomers born in 1946 plan to apply for benefits this year according to a recent MetLife Mature Market Institute survey (U.S. News & World Report, January 29, 2008).

Saving for early retirement has been one of the primary motivations for individuals to invest in the stock market. Realizing the option feature of early retirement, the literature has concluded that such flexibility leads to a higher stock proportion of wealth for investors (see Sundaresan and Zapatero 1997, Lachance 2003, Choi and Shim 2006, and Farhi and Panageas 2007). As is quoted above, however, investors typically do not retire until age 62. Indeed, the distribution of retirement age implied by these models becomes implausible[15] because in practice very few people retire before they get into their 60s. Consequently, the question arises: Will the optimal portfolio choice change for a model that generates a realistic retirement age distribution?

Figure 3.1: Retirement age distributions for HRS men, 1992. Source: Lumsdaine and Mitchell 1999.

This figure depicts the empirical retirement age distributions for male respondents in the 1992 Health and Retirement Study.



This chapter aims to answer this question by solving the life-cycle portfolio choice in an early retirement model that is able to match the empirical retirement age distribution as plotted in Figure 3.1. Financial incentives for keeping investors in labor

15 See the comment of Farhi and Panageas (2007) on the absence of a retirement deadline.

force (i.e., investors are not eligible for Social Security's benefits until age 62) and heterogeneous leisure and time preferences have been used in the literature[16] to explain the two spikes at ages 62 and 65. Therefore, this chapter models financial incentives and leisure preference as functions of retirement age. Thus, the tradeoff between pension wealth and leisure depends on time. Such time-dependence allows us: (1) to explain the high empirical correlation between retirement and some particular ages and (2) to explore the implications for optimal life-cycle portfolio choices. The contribution of this chapter is to derive the optimal portfolio choice in the presence of a realistic retirement age distribution that is endogenously generated.

I solve numerically for the optimal portfolio and retirement age decisions using a model that is calibrated to the empirical retirement age distributions. I consider an investor who seeks to maximize his expected utility derived from retirement wealth and leisure associated with early retirement. The investor faces borrowing and short-sale constraints. The investor can invest his retirement wealth in two assets: a riskless bond and a stock. The investor adjusts the asset allocation of the retirement plan once a year.

The main results of the chapter differ from the previous literature in the following aspects:

First, the literature has concluded that the investor will enter retirement when a critical wealth threshold is reached. This chapter relates the critical wealth level to retirement benefits and leisure preference. Since retirement benefits and leisure preference are functions of retirement age, the critical wealth level also depends on the retirement age. The critical wealth level is low when the investor has a high leisure preference and needs to give up small amount of retirement benefits for retiring early. Since young investors typically have a low leisure preference and need to give up a large amount of retirement benefits, they do not retire early.

Second, since young investors are endogenously restricted from exercising the early retirement option due to their low leisure preference and large loss of retirement benefits, they will decrease the portfolio risk when they are rich enough. For rich and young investors, holding a large stock share increases the probability that wealth will fall below the critical wealth in later periods when the critical wealth level drops dramatically. Therefore, the precautionary investment strategy will be optimal for rich investors. Less wealthy investors, however, will keep increasing the stock share to "gamble" on early retirement since their wealth is still below the critical wealth level. These two behaviors combine to create a hump shape for the policy function of stock

[16] See Gustman and Steinmeier (2002).

share in age and wealth. This is a novel effect of early retirement option on the portfolio choice. In the previous literature, the early retirement option can only increase the portfolio risk.

Third, with the increasing leisure preference and realistic financial incentives based on the current regulations of pension plans, this chapter endogenously generates the retirement age decisions that closely match the empirical retirement age distribution in Figure 3.1. Especially, investors are seen to enter retirement at markedly higher rates at the early retirement age and the age at which the full pension can be received. It turns out that a slight change of the speed of increasing leisure preference at ages 62 and 65 can capture the two spikes in the empirical retirement age distribution that have puzzled the literature.

Fourth, the model provides another rational explanation for the empirical life-cycle portfolio choice. Young investors, who typically have accumulated limited wealth, should hold a low proportion in stock. As young investors get older and their wealth approaches the critical wealth, they increase the stock proportion in the portfolio. Middle-aged investors will either decrease the stock holding if the critical wealth is reached or increase the stock holding if not.

The literatures on retirement and portfolio choice converge in the model adopted in this chapter. Consider first the literature on retirement decisions. A partial listing would include Gustman and Steinmeier (2002), Stock and Wise (1990), Rust (1994), Laezar (1986), Rust and Phelan (1997), and Diamond and Hausman (1984). Most of these models are structural estimations. They aim to explain the retirement age decisions by financial incentives and heterogeneous time and leisure preference. But the impact of retirement decisions on the portfolio choice is missing in this literature.

The second thread is the literature on the life-cycle portfolio choice. In that literature, Ameriks and Zeldes (2004), Faig and Shum (2002), Heaton and Lucas (2000), and Poterba and Samwick (2001) find that empirical risky asset holdings have typically been low at young ages, and then either increasing or hump-shaped over the life cycle. These empirical results have been at odds with predictions of theoretical models such as Merton (1971), Kocherlakota and Jagannathan (1996), and Cocco et al. (2005), which suggest that stockholdings should be decreasing in age. In this literature, retirement decisions are either missing or modeled without an option framework.

Finally, few papers combine these two lines of research to analyze the impact of retirement decisions on the life-cycle portfolio choice. Some results of this chapter share some similarities with this literature including Bodie et al. (1992), Lachance (2003), Choi and Shim (2006), and Farhi and Panageas (2007). Despite the similarities, this chapter differs from the literature in the following aspects. First, the early retirement is endogenously restricted for young investors in this chapter while Lachance (2003) and Choi and Shim (2006) model early retirement as a free boundary problem, i.e., the early retirement option can be exercised anytime and alive infinitely. Farhi and Panageas (2007) only consider an exogenous deadline. Second, financial incentives or penalties related to early retirement are explicitly modeled in an analytical life-cycle model in this chapter. Third, leisure can only be adjusted discontinuously in this chapter while Bodie et al. (1992) assumes that labor supply can be adjusted in a continuous fashion. Since in many jobs workers either work full time or they are retired, the discontinuous function of leisure is more realistic. More importantly, this leads to a stronger option effect.

The structure of the chapter is as follows: Section 3.2 contains the model setup, assumptions, and a description of the numerical methodology and parameter choices. Section 3.3 presents the policy functions. Section 3.4 calibrates the leisure preference to the empirical retirement age decisions and presents the optimal life-cycle portfolio choice with the optimal choice of leisure preference. Section 3.5 concludes.

3.2 The model

This section presents the settings and assumptions of the benchmark model. Then the numerical methodology used to solve the model is briefly introduced together with the parameter choices.

A. Preferences

An investor considering his retirement plan seeks to maximize his expected utility at the terminal date of T by choosing an optimal portfolio to manage his retirement wealth and an optimal age of retirement. The utility function has the form

$$U(W_T, \tau) = \frac{W_T^{1-\gamma}}{1-\gamma} + L_\tau \tag{3.1}$$

where W_T is the total retirement wealth at the age of T, τ is the retirement age, and γ is the relative risk aversion parameter. L_τ measures the lump sum utility gain from retirement. The model is finite and the investor dies for sure at the age of T. The investor starts the retirement planning at the age of T_0.

This chapter abstracts from modeling intermediate consumption and labor income by assuming that the investor participates in a pension plan that requires saving part of the labor income for retirement. The remaining labor income covers consumption and personal savings. Personal savings, like housing or a private business, are part of the investor's wealth. This chapter, however, is interested in the portfolio choice for the retirement wealth that is only composed of defined benefit and defined contribution pension plans. Therefore, only the terminal retirement wealth enters into the utility function. Also, such modeling enables us to identify the effect of an early retirement option on portfolio choice while disregarding labor income effects.[17] The option effect can be clearly seen by comparing this chapter's results with Merton's (1969) standard solution. As the utility does not depend on intermediate consumption, the time preference and mortality risk are irrelevant for the optimal solution and they disappear from the utility function.

The preferences specified in (3.1) use an additive utility function over wealth and leisure. It is straightforward to allow for the more usual multiplicative Cobb-Douglas utility function over leisure and wealth. The above additive form is adopted because it is commonly used in finance literature and it allows for risk aversion coefficient γ to be greater than 1.

The additive functional form is also used in recent theoretical papers, e.g. Lachance (2003) and Liu and Neis (2004) and empirical studies, e.g. French (2001). These additive function forms have the same feature as the function (3.1): the function is additive at the utility level. Alternatively, one can plug the sum of leisure and wealth into the power utility (e.g., $\frac{(W+L)^{1-\gamma}}{1-\gamma}$). In this case, the function is additive at the variable level and a leisure gain is equivalent to an increase in wealth.

Another concern arises with the utility form of leisure. The preferences (3.1) assume that utility is linear in leisure, which implies that investors are risk neutral with respect to leisure and marginal utility from leisure is constant. There are two criticisms for the linear utility in leisure. One is that investors might not be risk neutral regarding leisure. The other is that marginal utility from leisure should be diminishing. In the context of this chapter, these concerns seem to be irrelevant since leisure is a deterministic binary variable in this chapter. Therefore, this chapter adopts a simple linear function for leisure as the benchmark model to investigate the relationship

[17] As is shown in Cocco et al. (2005), the portfolio share of stock is decreasing in age and wealth in the presence of (risky) labor income.

between early retirement option and portfolio choice. The simple linear function also decreases the number of parameters for calibrating the model to the empirical data. The alternative preferences that allow for risk aversion and diminishing marginal utility in leisure can be any concave functions (e.g., a power utility for leisure).

The above alternative preferences differ in many aspects, but as long as leisure can only be adjusted in a discrete fashion, i.e., the investor can work either full time or retire, the option effect of early retirement on the portfolio choice should be present. Later, a robustness check with these alternative preferences will be presented.

B. Leisure

The tradeoff between the utility gain from early retirement and the reduction of retirement benefit plays a critical role in the model. The utility gain L_τ is modeled to be linear in the time to the investor's death ($T-\tau$). It means that the earlier the investor enters retirement, the more leisure he will obtain. Therefore,

$$L_\tau = \delta \frac{T-\tau}{T-T_0} \tag{3.2}$$

where δ is a constant. From (3.2), one can see immediately that the leisure gain from early retirement is zero when retiring at T and reaches the maximum of δ when retiring at T_0.

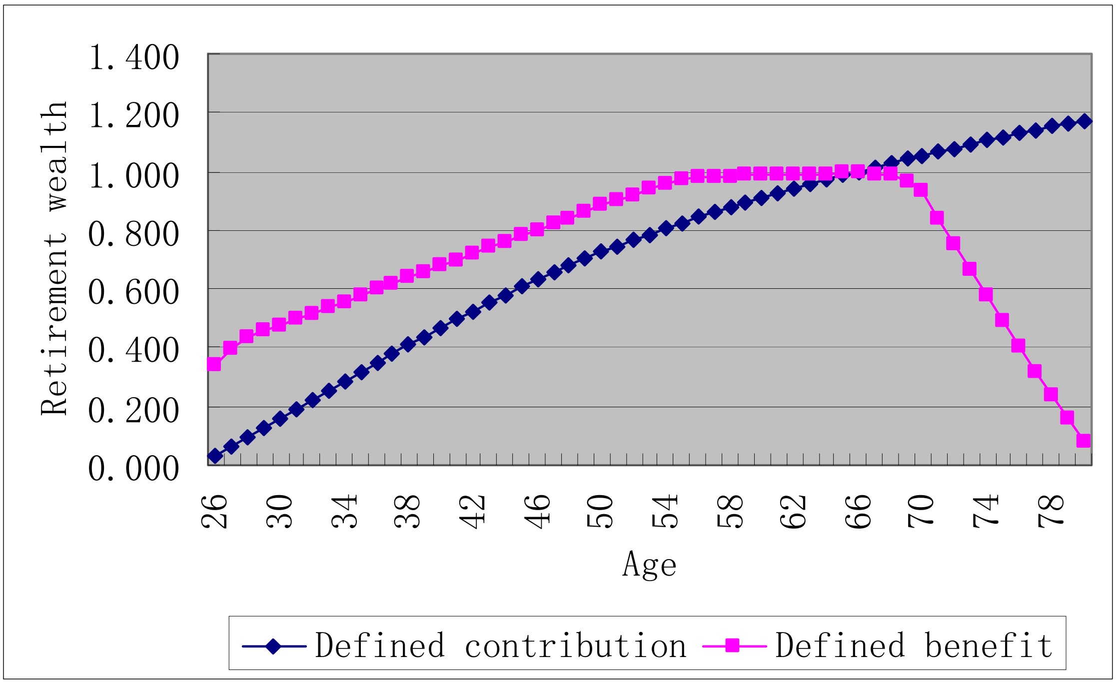
δ measures the value of leisure relative to retirement wealth and is assumed to be constant in the benchmark model. This assumption is questionable. For example, Gustman and Steinmeier (2002) assume that the preference of leisure is increasing as the investor ages. Young investors find leisure less attractive than old investors due to several reasons. First, the cost to work is higher for old investors than young investors. Second, marginal utility from leisure is diminishing, thus, not working for an additional year is less valuable for young investors than old investors. The cost for updating the constant δ to a more complicated function is the increase in the number of parameters. Therefore, the constant preference of leisure is a reasonable assumption to start the analysis. Later, we will extend the model to adopt a nonlinear structure for the preference of leisure.

C. Retirement wealth

The retirement wealth depends on when the investor chooses to retire. Typical defined benefit pension plans such as Social Security offer financial incentives to keep the investor in the labor force before the Normal Retirement Age. For instance, the benefit at age 62 is cut to 70% of the full benefit which would be available at the Normal Retirement Age of 67 (See Munnell 2006). However, early retirement implies that the investor will receive pension income for more years provided the life expectancy is fixed. The retirement wealth also comes from defined contribution pension plans. The investor contributes to the cumulated pension benefit before retirement. After retirement, the investor stops the contribution and the pension benefit reaches the maximum. In this case, early retirement implies less retirement wealth from defined contribution pension plans. Figure 3.2 illustrates the complicated relationship between the retirement wealth and the retirement age. The retirement wealth is denoted as the sum of the annual retirement income up to the age of 80. The income is assumed to be reinvested at the riskfree rate. The retirement wealth for the investor who retires at 66[18] is normalized to 1. The life-time labor income profile for this investor is assumed to follow the estimated results for high school graduates in Cocco et al. (2005).

Figure 3.2: Retirement wealth and retirement age

This figure depicts the relationship between the retirement wealth and retirement age. The retirement wealth is denoted as the sum of the annual retirement income up to the age of 80. The income is assumed to be reinvested at the riskfree rate. The retirement wealth for the investor who retires at 66 is normalized to 1.



[18] 66 is the normal retirement age for individuals born after 1943 in the US.

Figure 3.2 shows that the retirement wealth from the defined contribution plan is increasing as the investor postpones the time of retirement. Since the defined contribution plan is similar to the personal saving, the investor will accumulate more wealth if he works longer. The retirement wealth from the defined benefit plan, however, is concave in retirement age. This is the result of a few institutional rules related to Social Security benefits (e.g., individuals can only receive Social Security benefits from 62; individuals can receive a bonus for Social Security benefits if they delay retirement; and such bonus no longer applies for retiring after 70). In summary, the investor faces a significant loss in retirement wealth if he retires early. This observation gives rise to the research question of this chapter: if the investor cannot choose to retire early in his life due to the substantial loss of retirement wealth, how would such constraint change the life cycle portfolio choice for the investor?

To model such complicated relationship between the retirement wealth and the retirement age, the investor is assumed to have an initial wealth W_0 that is equal to the present value of the retirement wealth for an investor retiring at the age of 66. If the investor retires at a different age of *T*, the retirement wealth will be adjusted by the difference between the retirement wealth at *T* and 66 based on Figure 3.2. Such adjustment can either be a loss or a plus to the retirement wealth, depending on the retirement age. Therefore, the utility in (3.1) should be restated as the following:

$$U(W_T, \tau) = \frac{(W_T + \beta B_\tau + (1-\beta)C_\tau)^{1-\gamma}}{1-\gamma} + L_\tau . \tag{3.3}$$

where B_τ is the adjustment for the retirement wealth from defined benefit plans and C_τ from defined contribution plans. β controls the relative weight of the defined benefit pension wealth in the total retirement wealth.

The previous research shows that there exists a critical wealth threshold such that the investor will not retire until wealth has exceeded this threshold.[19] Since this chapter explicitly models the retirement wealth as a function of the retirement age, the critical wealth threshold is not only endogenously determined by investor preferences, but also depends on time.

D. Financial assets

[19] See, for instance, Farhi and Panageas (2007), Lachance (2003), and Choi and Shim (2006).

The investor has an initial retirement wealth W_0 that is equal to the present value of all the future retirement benefits for the investor retiring at the age of 66. W_0 is assumed to be exogenously determined by the life-time labor income, the contribution rate, and the retirement income based on the pension plans. The investor can borrow against future retirement benefits and determine asset allocation. The borrowing constraint has been extensively discussed in the literature[20]. The main implication for the life-cycle portfolio choice is that young investors are borrowing constrained so that they invest a high proportion of wealth in stocks. This chapter allows the investor to borrow against future retirement benefits because this chapter focuses on the effect of the option to retire early on the portfolio choice. In order to identify the early retirement effects, it is useful to compare this chapter's optimal portfolio choice with the benchmark model (Merton 1969) that is characterized by a constant portfolio choice. Thus, any deviation from this constant solution will be the effect of the early retirement option. Other papers such as Farhi and Panageas (2007) also relax the borrowing constraint.

The investor can invest in a bond and a stock portfolio (from now on, simply stock). The bond has a constant gross real return of R_f and the corresponding log return is r. The stock has a gross real return R_t. It follows a geometric Brownian motion and the log return has constant mean μ and volatility σ. For a given proportion allocated to stock, α_t, the log returns of the retirement wealth are normally distributed for each discrete time step of length Δt (the time length between t and $t+1$) with mean $\mu_{\alpha_t,\Delta t} = \left[\alpha_t\mu + (1-\alpha_t)r - \frac{1}{2}\alpha_t^2\sigma^2\right]\Delta t$ and volatility $\sigma_{\alpha_t,\Delta t} = \alpha_t\sigma\sqrt{\Delta t}$. The investor is assumed to face the short-sale constraint through the following inequality:

$$0 \le \alpha_t . \tag{3.4}$$

E. The optimization problem

In each period t, the investor starts the period with retirement wealth W_t and determines asset allocation of their retirement wealth and decides whether or not to retire. After retirement, the retirement wealth is assumed to be reinvested at the riskfree

[20] See, for example, Cocco et al. (2005).

rate until T. The intertemporal budget constraint is then given by:

$$W_{t+1} = R^p_{t+1} W_t , \tag{3.5}$$

where R^p_{t+1} is the portfolio return held from period t to period $t+1$:

$$R^p_{t+1} = \alpha_t R_{t+1} + (1-\alpha_t)\bar{R}_f . \tag{3.6}$$

The problem the investor faces is to maximize (3.3) subject to constraints (3.2), (3.4), (3.5), and (3.6). The control variables of the problem are $\{\alpha_t, \tau\}_{t=1}^{T}$. The state variables are $\{t, W_t\}_{t=1}^{T}$. The Bellman equation for this problem is given by:

$$J_{W,T} = U_{W,T} ,$$
$$J_{X,t} = Max[E_t J_{X,t+1}] .$$

Since the early retirement option can only be exercised at a set number of times, this problem has a feature of the Bermudan option, and cannot be solved analytically. I derive the policy functions for portfolio share of stock and the optimal retirement age numerically by using backward induction and a grid structure for approximating the expected utilities.[21]

First, I establish a grid structure. The log wealth values are sampled on a grid. That grid has equal time increments as well as equal steps with respect to $\ln(W)$. I choose the grid spacing and riskfree rate such that a strategy of being fully invested in the bond (α_t=0) will always end up on a grid point. From each grid point, I allow a multinomial forward move to a relatively large number of subsequent grid points (e.g., 101) at the next time step. Potential forward moves are structured such that they land on grid points, and the associated probabilities are calculated by using the discrete normal distribution conditional on the portfolio share of stock (α_t).

Second, backward induction is carried out. In the last period, the value function corresponds to the indirect utility function. For each terminal retirement wealth at T, I calculate the associated utility, and then step backwards in time to $T-1$. At each

[21] See more details about this methodology in the Appendix 3.A and Hodder and Jackwerth (2007).

possible wealth level, the expected utilities for each admissible choice of α_t are calculated assuming the investor continues to work until the next period. I also calculate the utilities associated with the investor's decision to retire. If the investor chooses to exercise the retirement option, the current wealth will be invested at the riskfree rate until T and the terminal wealth will be adjusted by the financial incentives (or penalties) of pension plans. The utility is computed as the sum of the power utility with respect to the terminal wealth W_T and the leisure gain from retirement.

The highest of those expected utilities from keeping the option alive and the utility of exercising the option is chosen as the optimal indirect utility for that wealth level which we denote as $J_{W,T-1}$. I record the optimal indirect utilities and the associated optimal share of stock as well as the retirement decision for each wealth level. This procedure is then repeated stepping backward in time for all time steps.

F. Choosing parameters

I consider an investor who starts working at age 26 and dies with certainty at age 80. The investor makes the portfolio choice once a year. Therefore there are 54 time steps. The normal retirement age is 66[22]. The coefficient of relative risk aversion γ is 10. This is the upper bound for risk aversion considered reasonable by Mehra and Prescott (1985). Lower values are also considered in the robustness check. There are a total of 800 log steps between the lower and upper boundaries. The mean equity premium is 4% for the benchmark case. The riskfree rate is set to be 1.7% so that 5 log wealth steps are equal to $r\Delta t$. The volatility of the risky asset is 0.157. These parameters are mostly taken from Cocco et al. (2005) so that the results of this chapter are comparable to the literature on life-cycle portfolio choice.

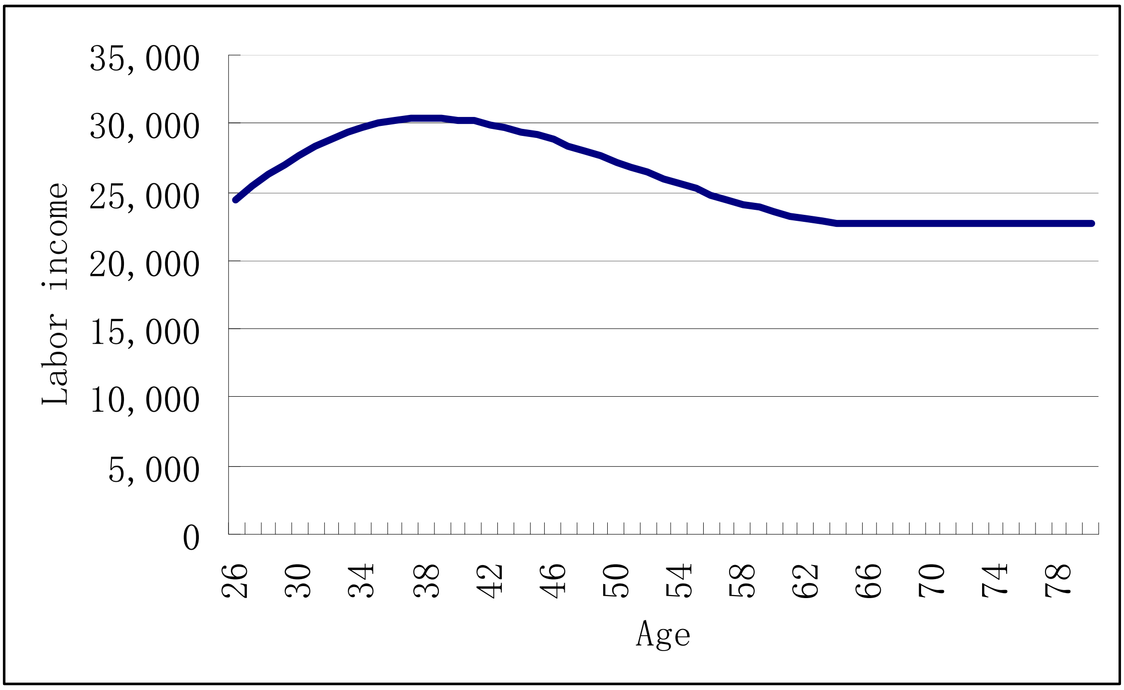
The retirement wealth from defined contribution pension plans is calibrated to the life-time labor income profile for a high school graduate. In Cocco et al. (2005), labor income is estimated as a function of the polynomial of age and personal characteristics. Figure 3.3 presents the life-time labor income for a married investor with a high school diploma and two children. The replacement ratio for the retirement income is 0.68212 which is estimated in Cocco et al. (2005). The retirement wealth from defined benefit pension plans is computed using the benefit calculators provided by the official website of the US Social Security Administration. The retirement wealth depends on the

[22] The normal retirement age is the age at which retirement benefits are equal to the "primary insurance amount", i.e., retirement benefits are not reduced. For an investor born during the period from 1943 to 1954, the normal retirement age is 66 in the US.

retirement age (see Figure 3.2) and is composed of pension income from both defined benefit and defined contribution plans. Assuming the contribution rate is 15% of labor income, the retirement wealth for retiring at the normal retirement age (66) can be computed for both defined contribution and benefit plans. Consequently, the relative weight of defined benefit β is obtained. In the benchmark case, the calibrated value for β is 0.48. The total retirement wealth for retiring at age 66 is normalized to 1. The upper boundary is 8 and the lower boundary is 0.5. Therefore, each log wealth step is equal to [log(8/0.5)]/800. The initial wealth is twice of the present value of the total retirement wealth for retiring at age 66 so that all the simulated wealth paths will be above the lower boundary. Table 3.1 summarizes the parameter choices.

Figure 3.3: Life-time labor income

This figure presents the life-time labor income for a married investor with a high school diploma and two children. The replacement ratio after retirement is 0.68212. This figure is calibrated based on the results from Cocco et al. (2005).



The only free parameter for calibrating the model to the observed retirement age distribution is the leisure preference δ. After some tests, three values that represent a high, medium, or low level of leisure preference are chosen to generate a sensible retirement age distribution that is as close as to the observed retirement age distributions for male respondents in the Health and Retirement Study as depicted in Figure 3.1. In particular, the calibrated retirement age distribution is calibrated such that the two peaks at age 62 (when early retirement is allowed) and age 65 (when the unreduced pension can be received) are also present.

Table 3.1: Parameter choices

Starting age of the retirement planning	T_0	26
The age at which the investor dies for sure	T	80
Number of time steps	n	54
Number of grid points for log wealth		800
Log wealth step		(log(8/0.5))/800 ≈ 0.003466
Future nodes for the Normal approximation		1+2×50 = 101
Risk aversion parameter	γ	10
Riskfree return	r	0.017
Mean	μ	0.057
Volatility	σ	0.157
Weight of defined benefit	β	0.48
Initial retirement wealth	W_0	0.784571
Leisure preference	δ_1	0.001
	δ_2	0.0001
	δ_3	0.00005

3.3 The benchmark case

This section first presents the policy functions of portfolio choice and retirement decision that solve the benchmark model in the last section. The policy functions are displayed over the bivariate state space of log wealth and time. Before looking at the policy functions, it is useful to recall Merton's (1969) solution for complete-markets, ignoring labor income. Merton showed that the optimal fraction of wealth invested in the risky asset is constant, independent of wealth and age, and depends only on risk aversion and the moments of the asset's excess return:

$$\alpha = \frac{\mu - r}{\gamma\sigma^2}. \tag{3.8}$$

With the parameters in Table 3.1, this implies that the investor should keep about 16% share of his portfolio in stock at any point in the state space. Any deviation from this optimal share can be considered to be a result of introducing the early retirement option.

Next, the empirical predictions of the model for the life cycle portfolio choice are discussed and compared with some stylized evidence. In a third step, using the policy functions derived from the benchmark model, portfolio choices and retirement decisions of 10000 investors are simulated over the life-cycle. This step generates a simulated retirement age distribution and compares this with the empirical distribution. Finally, this section performs a sensitivity analysis with respect to the alternative utility forms and parameter choices.

A. Retirement decisions

Figure 3.4 presents the policy functions for the optimal portfolio choice and retirement decision.[23] The portfolio share of stock is set to 0 if the investor chooses to retire because the investor is assumed to hold a riskless portfolio after retirement. These areas are labeled "retirement" in the figures and lie on the floors characterized by high wealth and old age. The result shows that the early retirement option generates a critical wealth level similar to previous studies. Whenever the investor's wealth exceeds the critical wealth, he retires. Contrary to what we expected, the critical wealth is increasing in age, which means that young investors need even less wealth than old investors to prefer retirement despite the fact that the retirement wealth for young retired investors is reduced much more than for old retired investors. Theoretically, this is because the utility gain from leisure outweighs the loss in retirement wealth. However, Figure 3.1 shows that the number of young investors who report retirement is very low. Such inconsistency implies that the leisure preference for young investors is relatively too high compared with old investors.

Despite the poor calibration for young investors, the model successfully predicts the retirement behavior for investors older than 70. The empirical retirement age distribution in Figure 3.1 shows that almost no investors will delay retirement after 70. The calibrated retirement decisions in Figure 3.4 predict exactly the same. The main reason is that the bonus for delaying retirement no longer applies after 70. In other words, there are no more financial incentives for giving up leisure. The successful prediction that no investors will retire after 70 is an important feature of the present model that differs from the literature. The literature typically uses free boundary models (i.e., the early retirement option can be exercised anytime and alive infinitely) to solve

[23] The benchmark model is solved for three different values of leisure preference. Since the policy functions behave qualitatively quite similarly, only the policy functions with leisure preference equal to 0.0001 are displayed.

the optimal retirement decisions. A serious critique for these models is that they cannot generate a sensible retirement age distribution. For example, Farhi and Panageas (2007) notice that in the infinite horizon case, there is no notion of "life" cycle, since time plays no explicit role in the solution. Poor investors will continue working even after 80 years old as long as their wealth does not exceed the critical wealth. To overcome this drawback, exogenous mandatory retirement is introduced to the model. However, mandatory retirement is against the US law. The advantage of the present model is that it can generate an endogenous retirement deadline at the age of 70. The retirement behavior depicted by Figure 3.4 also indicates the dramatic impact from the financial penalty and incentives embedded in pension plans.

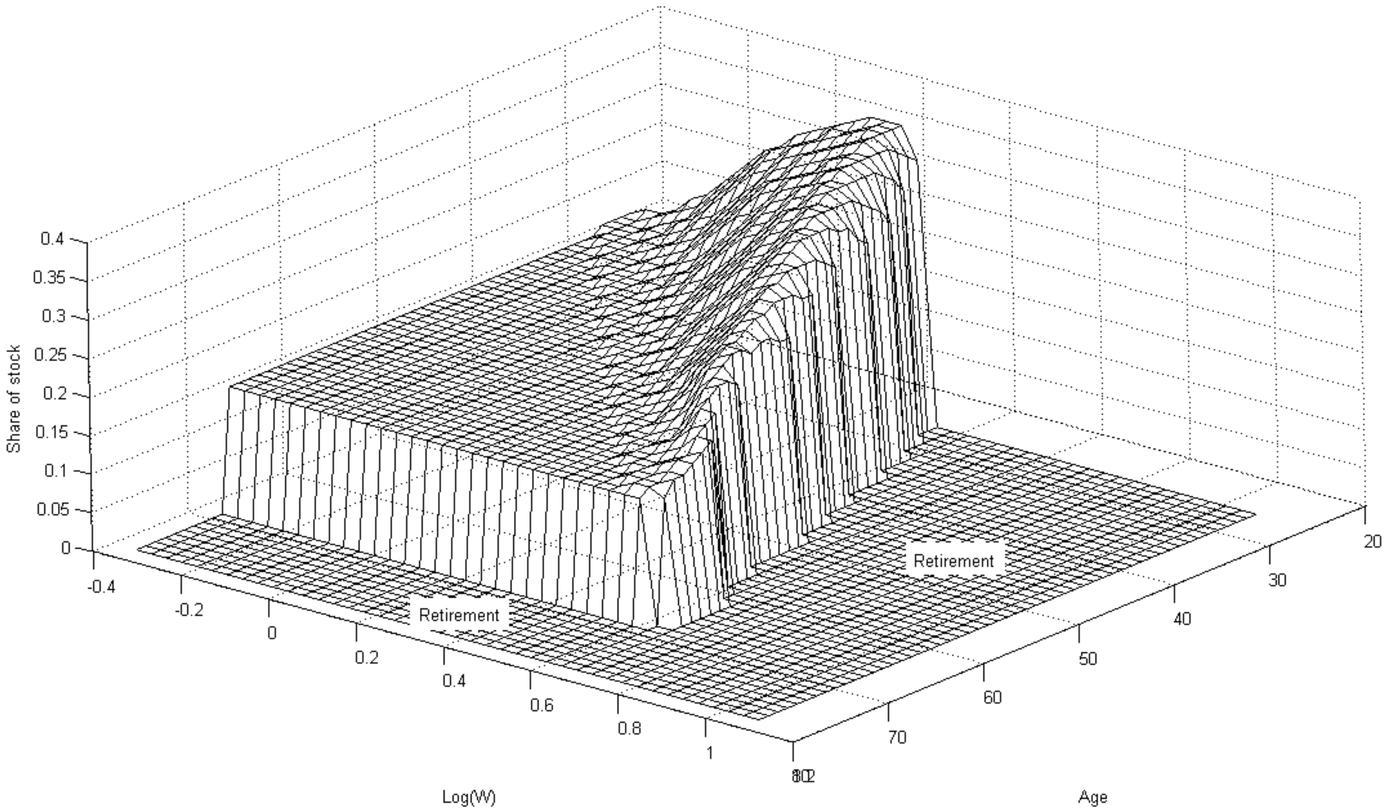
B. Portfolio choices

The portfolio share surface in Figure 3.4 is characterized by an "option ridge" in the middle of the surface. This ridge reflects the most intuitive effect of the early retirement option on portfolio choice. As previous studies have concluded, prior to retirement an investor invests a higher proportion of his wealth in risky assets. Based on the option pricing theory, the option is more valuable when the price of the underlying asset is higher and the time to maturity is longer. Correspondingly, Figure 3.4 confirms that the gambling effect is stronger when the investor's wealth is closer to the critical wealth and when the investor is younger. Old and poor investors hold 16% share in stock as predicted by Merton's solution in (3.8). They are not affected by the early retirement option, because their wealth is far below the critical wealth. The early retirement option is almost worthless since it is deep out of the money and is about to expire.

An interesting and novel effect found in Figure 3.4 is that the portfolio share of stock is not monotonically increasing before wealth exceeds the critical wealth. The investor reduces the portfolio risk when wealth is very close to the critical wealth. Such precautionary behavior is related to the restriction of trading frequency and the level of the critical wealth. The investor cannot continuously adjust the portfolio share of stock due to transaction cost. Holding a very risky portfolio for the current period, the investor increases the both probabilities of gaining and losing in the next period. If the investor is unlucky, the wealth level in the next period might be below the critical wealth and the investor cannot exercise the early retirement option. Instead, if the investor holds a relatively more conservative portfolio, the chance to exercise the option in the next period will be higher. But such behavior depends on the critical wealth level in the next period. If the critical wealth level will be much higher than the current period,

Figure 3.4: Policy functions for the benchmark case

This figure presents the policy functions for portfolio share of stock and retirement decision for the benchmark model with leisure preference equal to 0.0001. After retirement, the investor is assumed to invest at the risk-free rate.



the investor has to gamble further on the early retirement option. In Figure 3.4, we have seen that the critical wealth is slightly increasing in age. Therefore, the precautionary portfolio choice is not that pronounced.

In Figure 3.5, we illustrate this precautionary portfolio choice in a clearer way by imposing a time restriction on the early retirement option. The same benchmark model is applied here except that the investor cannot retire before 62, the earliest age that investors can claim for retirement benefits from social security. In Figure 3.5, the portfolio surface is hump-shaped. Young and rich investors hold a conservative portfolio as suggested by Merton's solution in (3.8). Such behavior is due to the time constraint for exercising the early retirement option. These young and rich investors would retire early in their lives if they were allowed to do so. Now that they are not allowed to exercise the option, holding a risky portfolio cannot be an optimal investment strategy for them. The high risk increases the likelihood of negative returns so that the wealth might end up below the critical wealth in the future when retirement becomes eligible. Therefore, rich and young investors tend to decrease the portfolio risk ensuring their ability to exercise the option. As a result, there also exists a critical wealth threshold before 62. Before this critical wealth is reached, investors will increase the stock share. After this critical wealth is reached, investors will decrease the stock share.

C. The simulated retirement age distributions

Figure 3.6 presents and compares the simulated retirement age distributions with different leisure preference parameters. There are three interesting observations in this figure. First, Figure 3.6 shows that the increase of leisure preference will shift the distribution to the left. Intuitively, if investors in general prefer leisure rather than wealth, they are inclined to retire early. Second, although low leisure preference induces investors to retire later, no investors will retire after 70. Consequently, about 25% of investors retire at age of 69. Third, none of the simulated distribution matches the observed distribution in Figure 3.1. Especially, the two spikes at age of 62 and 65 are not present in Figure 3.6. After comparing Figure 3.1 and 6, we can conclude that, leisure preference for investors before 60 is over-estimated and for investors after 60 is under-estimated. This evidence suggests that we need to deviate from the benchmark case in modeling leisure preference L_τ.

D. Empirical predictions and evidence

Although this chapter investigates the optimal choice of portfolio with respect to

Figure 3.5: Policy functions for the time-constrained benchmark case

This figure presents the policy functions for portfolio share of stock and retirement decision for the benchmark case with a time constraint for the retirement option. The investor cannot exercise the option before age of 62. After retirement, the investor is assumed to invest at the risk-free rate.

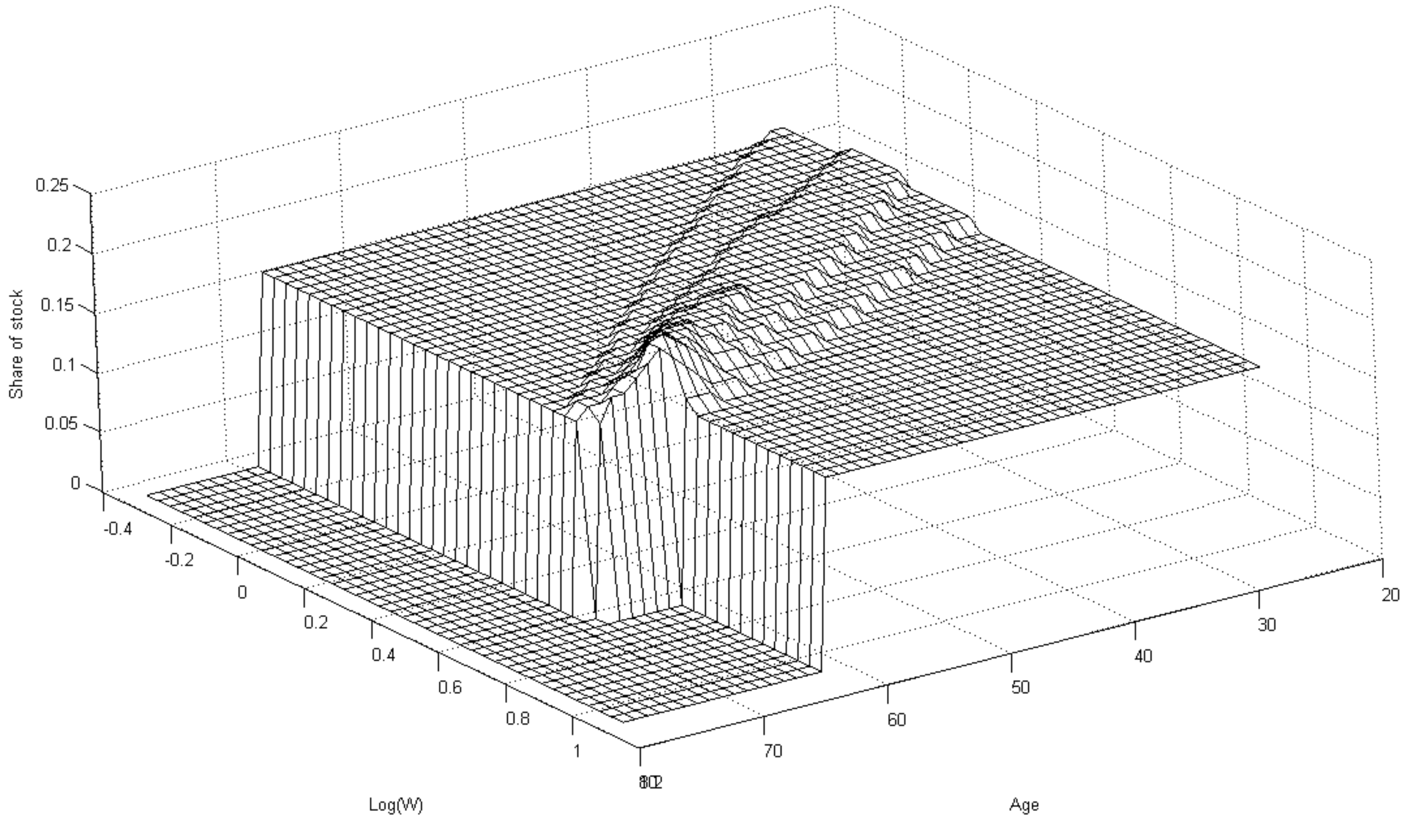
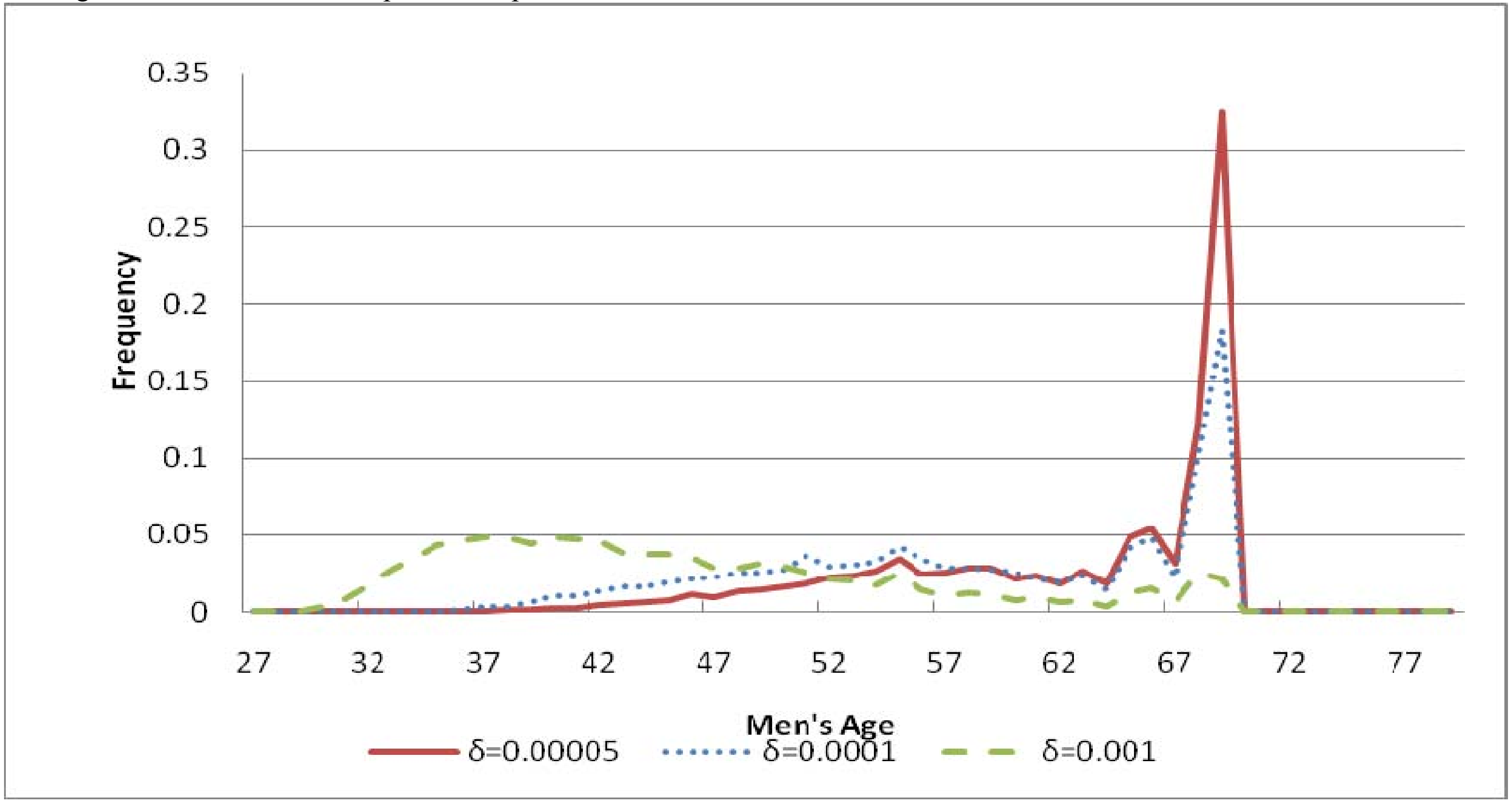


Figure 3.6: Simulated retirement age distributions for the benchmark case

This figure plots the simulated retirement age distributions for 10000 investors based on the policy functions of the benchmark case. The distributions are generated for three leisure preference parameters.



retirement wealth, since retirement savings seems to be the primary motivation for investing in the stock market, the findings of the model might also shed light on life-cycle household portfolio choice.

The model produces the following empirical predictions for the optimal portfolio choice over the life cycle. First, young investors, who typically have accumulated limited wealth, should hold a low proportion of stock. Second, as young investors get older and wealth approaches the critical wealth, they increase the stock proportion in the portfolio. Finally, middle-aged investors will decrease stock holdings if they are wealthy enough. The last prediction comes from the precautionary investment behavior which is a novel finding of this chapter.

While the studies by Ameriks and Zeldes (2004), Bertaut and Haliassos (1997), Heaton and Lucas (2000), and Poterba and Samwick (2001) document that it is hard to obtain unambiguous and clear-cut results concerning the life-cycle profile for the fraction of wealth invested in stock, there exist stylized facts to support the predictions of the benchmark case. Indeed, several studies report that risky asset holdings are typically low at young ages, and then are either increasing or hump-shaped over the life cycle (see for example, Ameriks and Zeldes (2004), Faig and Shum (2002), Heaton and Lucas (2000), and Poterba and Samwick (2001)). Combining the first and second predictions, we can explain why the stock share increases in age. The third prediction is especially important in order to rationalize the hump shape because it provides a novel argument: some middle-aged investors are weary of investing in risky assets because they have acquired enough wealth for retiring in their 60s and they are afraid of losing the chance to retire by then if they hold a risky portfolio.

Ever since Merton's (1969, 1971) theoretical prediction that young investors hold more stock than old investors, the literature has been puzzled by the empirical evidence of low stock holdings for young investors. This puzzle has inspired a strand of papers in the literature. For instance, the recent work by Benzoni et al. (2007) argues that the cointegration between stock and labor markets induces young investors to hold low proportions of their wealth in stock. The predictions of the benchmark case contribute to the explanation of the puzzle in two aspects. First, they provide a new explanation for the low stock holdings of young investors. Second, these predictions serve to explain why in later life stages investors either keep increasing stock holdings or choose a more conservative portfolio.

E. Sensitivity analysis

Concavity for leisure

The benchmark case assumes that the utility is linear in leisure. We want to check whether a concave utility for leisure will change the results of the benchmark case. In Figure 3.7, the utility has the following form:

$$U(W_T,\tau)=\frac{(W_T+\beta B_\tau+(1-\beta)C_\tau)^{1-\gamma}}{1-\gamma}+\frac{L_\tau^{1-\theta}}{1-\theta}. \quad (3.9)$$

θ is the risk aversion parameter for leisure and takes a value of 3. Leisure preference has the same form as in (3.2) but δ is equal to 500 in order to generate a sensible retirement age distribution.

The policy functions of retirement decision and portfolio choice in Figure 3.7 share several common features as in the benchmark case. First, investors will reduce their portfolio risk when their wealth is close to the critical wealth. Such precautionary investment behavior is even more pronounced due to the concave leisure utility. Concavity implies a declining marginal utility. It means that the marginal utility from retiring early is larger for old investors (when leisure gain is small) than for young investors (when leisure gain is large). Therefore, we see that in Figure 3.7 the critical wealth level is almost constant in age. As is explained in the previous text, investors will be conservative if their wealth is close to or has exceeded the critical wealth in the future. The more their wealth has exceeded the critical wealth, the more conservative they will be. Second, no investors will retire after 70. Third, the surface of the portfolio choice is upward sloping in wealth. It means that in general the early retirement option increases the portfolio share of stock prior to retirement.

Non-separate wealth and leisure

A concave utility for leisure can also be modeled in the following form:

$$U(W_T,\tau)=\frac{(W_T+\beta B_\tau+(1-\beta)C_\tau+L_\tau)^{1-\gamma}}{1-\gamma} \quad (3.10)$$

In this case, leisure is treated as a complete substitution for wealth. Again, all the parameters have the same values as in the benchmark case except δ is reset to be 1. Figure 3.8 gives us a different picture of optimal portfolio choice and retirement decisions. First, the portfolio share of stock is slightly decreasing in wealth, contradicting the results of the benchmark case and the literature as well. Second, it is low wealth investors rather than rich investors that are induced to retire early by the early retirement option. All these odd results are related to the utility form used in (3.10). In the previous literature and the benchmark case of this chapter, the critical wealth exists because wealth and leisure are separate. Rich investors find working for more wealth is less attractive than leisure due to the declining marginal utility of wealth. With wealth and leisure added together as in (3.10), however, working for more wealth is as valuable as leisure because the marginal utility is the same. For example, the investor is considering whether he should work for one day to earn $100 or to take a day off. When he is poor, he thinks $100 is more valuable than a day's off so he chooses to work. Should he change his mind when he turns rich because $100 means less for him now? The answer is negative because a day's leisure is also valued less than when he is poor. Moreover, after retirement the wealth is assumed to be invested at the risk-free rate which is less than the expected return of a risky portfolio. Therefore, rich investors will be more reluctant to retire because they have more wealth for investment than poor investors. To conclude, the separate utility of wealth and leisure is the necessary condition for the result that early retirement option increases the stock proportion in the portfolio.

Risk aversion and equity premium

The policy functions derived from the benchmark case are also affected by other parameters. Risk aversion and equity premium are the critical parameters that measure how attractive stock is. In the benchmark case, we choose the upper bound for risk aversion and the lower bound for equity premium. We need to check whether lower risk aversion or a higher equity premium will change the portfolio choice and retirement behavior in Figure 3.4.

The effect of decreasing risk aversion is presented in Figure 3.9 and the effect of increasing equity premium is presented in Figure 3.10. In general, lowering risk aversion (reducing the risk aversion coefficient to 5) and increasing equity premium (from 4% to 5.75%)[24] increases the portfolio share of stock. However, the surface of the stock share is still upward sloping and the precautionary behavior is still present before wealth exceeds the critical wealth. Therefore, lowering risk aversion and increasing equity premium just lift the surface of portfolio choice up but the general pattern of portfolio choice and retirement decisions does not change.

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24 This number is found in Cocco et al. (2005).

Figure 3.7: Policy functions for additive power utility

This figure presents the policy functions for portfolio share of stock and retirement decision for the model with the utility form specified in (3.9).

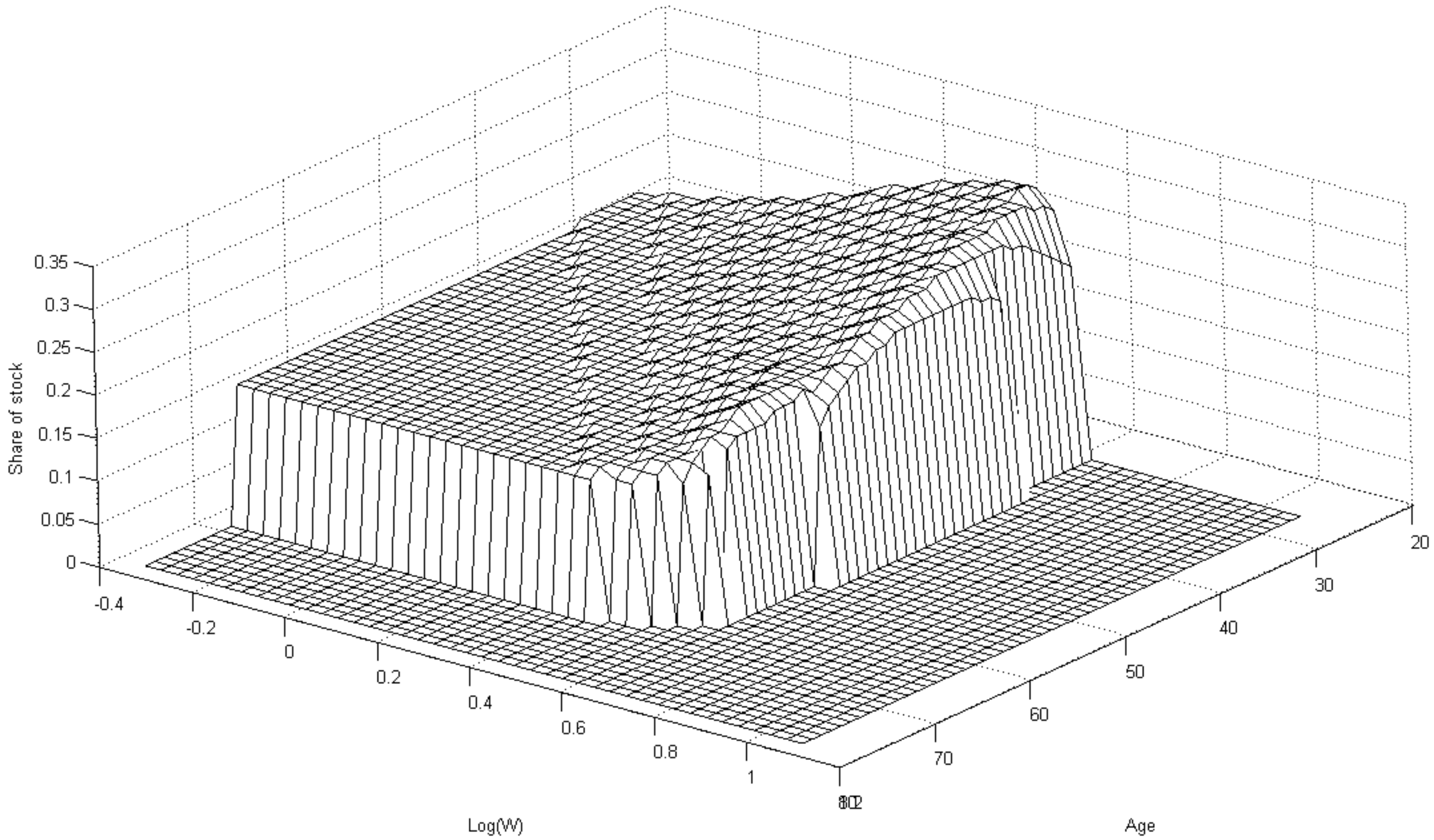


Figure 3.8: Policy functions for additive wealth and leisure

This figure presents the policy functions for portfolio share of stock and retirement decision for the model with the utility form specified in (3.10).

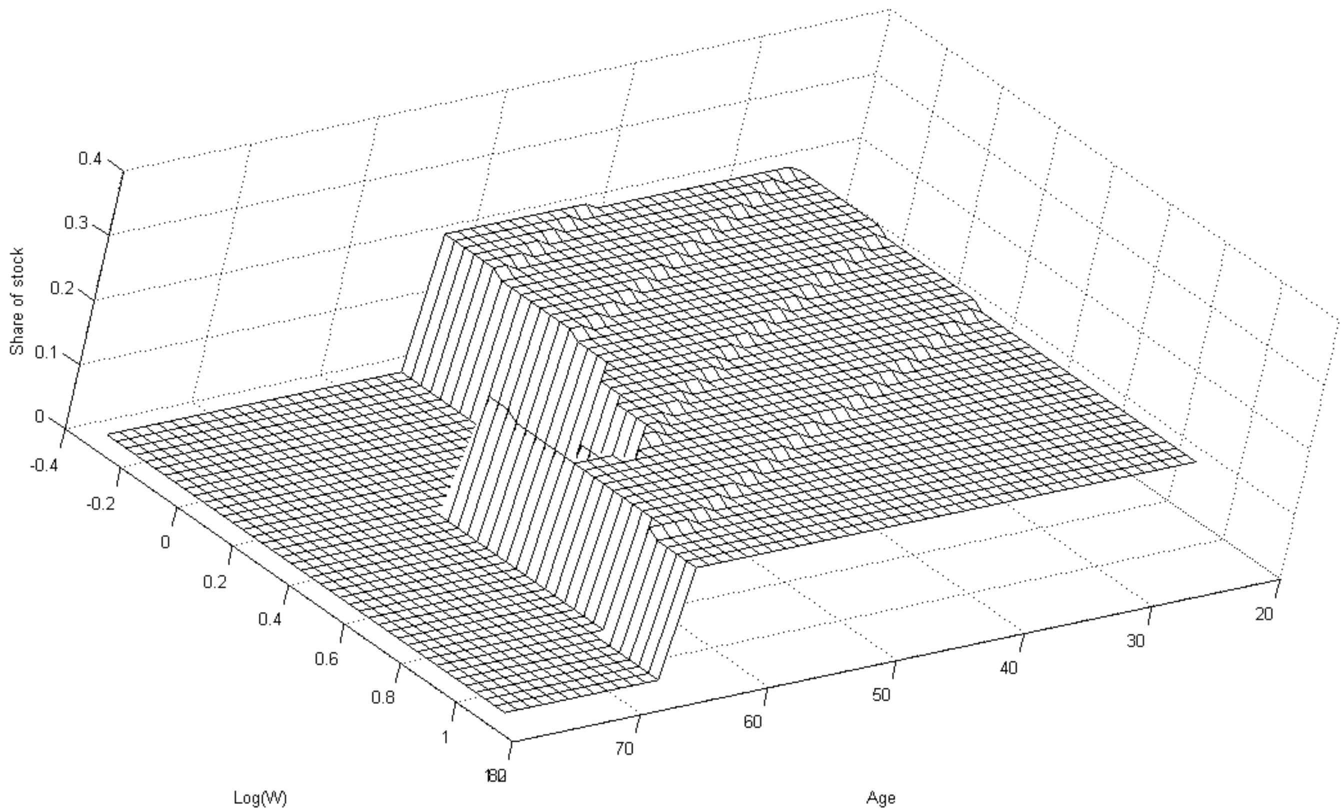


Figure 3.9: Policy functions for low risk aversion

This figure presents the policy functions for portfolio share of stock and retirement decision for the benchmark case with the risk aversion parameter equal to 5.

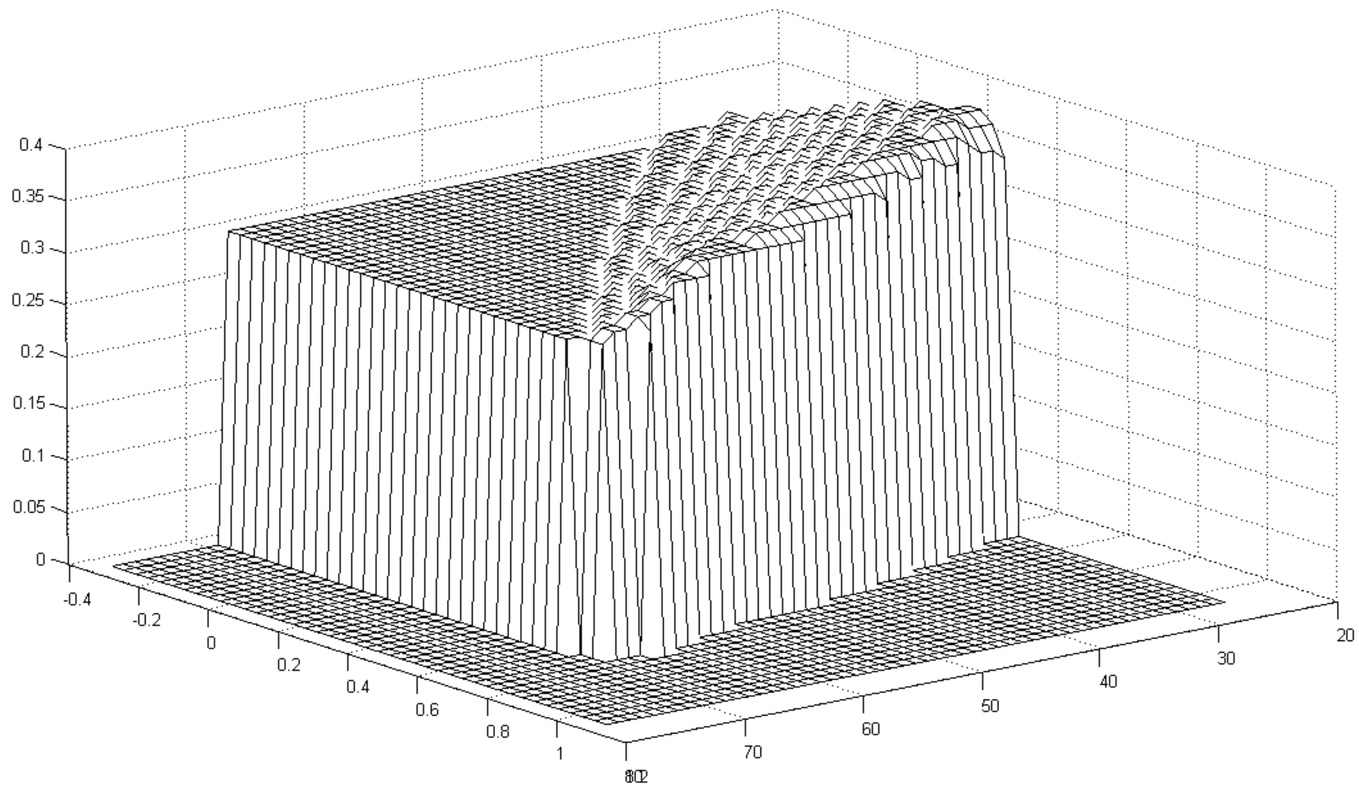
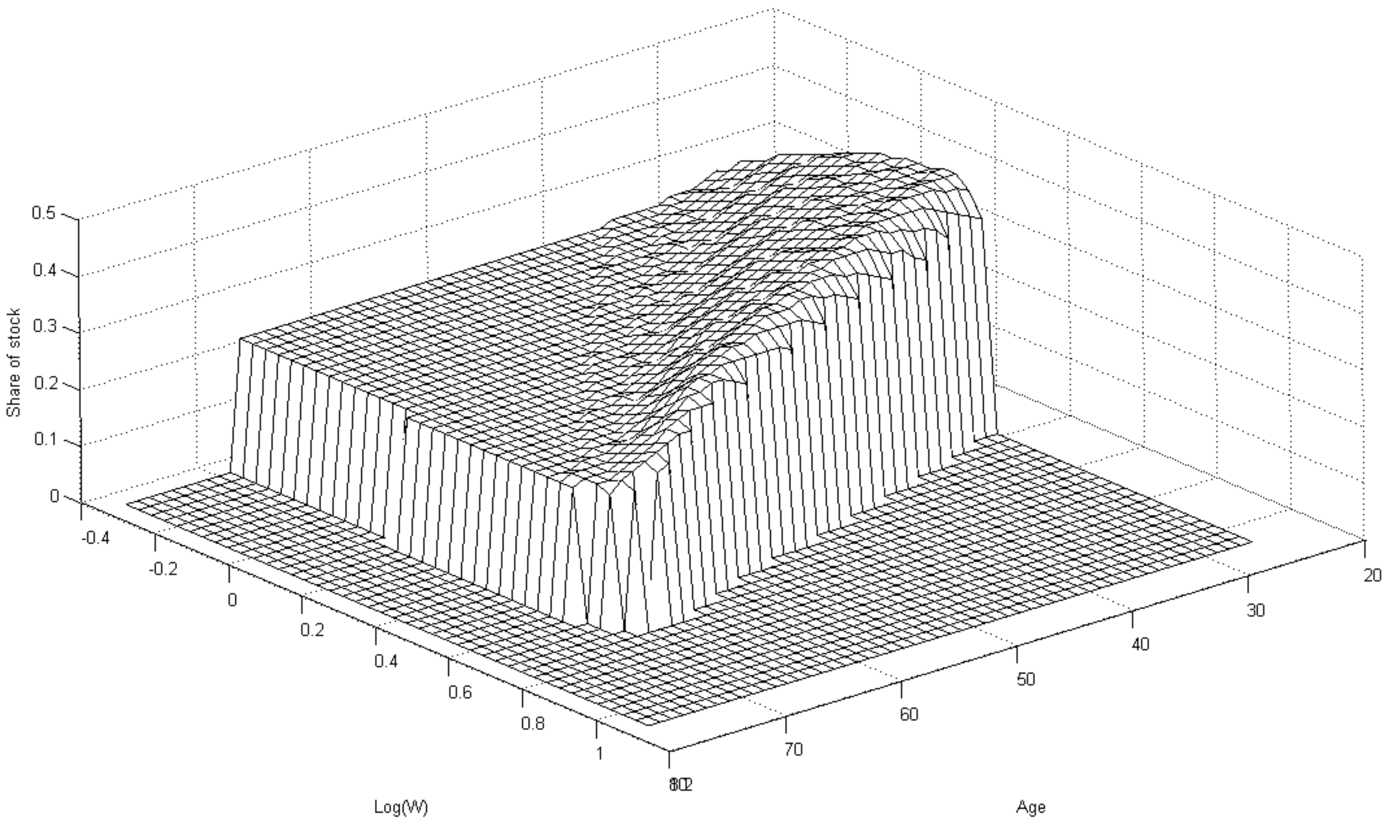


Figure 3.10: Policy functions for high equity premium

This figure presents the policy functions for portfolio share of stock and retirement decision for the benchmark case with the equity premium equal to 5.75%.



3.4 Calibrating leisure preference

In the last section, the policy functions of portfolio choice and retirement decisions are derived for the benchmark case. However, the simulated retirement age distributions in Figure 3.6 cannot match the observed distribution in Figure 3.1 exactly. It has been mentioned that leisure preference is a free parameter for calibrating the model to the observed retirement age distribution. Therefore, this section aims to calibrate an appropriate function of leisure preference based on the observed retirement age distribution.

A. The functions of leisure preference

In the benchmark case, the leisure preference δ is assumed to be constant across time. In order to match the observed and complicated retirement behavior (e.g., the spikes at age of 62 and 65), the leisure preference should be modeled as a function of time $\delta(\tau)$.

Gustman and Steinmeier (2002) assume that the leisure preference is increasing as the investor ages. The simplest way to model this is the linear function:

$$\delta(\tau) = \delta_1 \tau . \tag{3.11}$$

If we add a constant to the linear function, then we need to calibrate two parameters:

$$\delta(\tau) = \delta_0 + \delta_1 \tau . \tag{3.12}$$

Gustman and Steinmeier (2002) adopt an exponential function:

$$\delta(\tau) = \exp(\delta_0 + \delta_1 \tau) . \tag{3.13}$$

Last but not least, Coile and Gruber (2000) find the dummy for age 62 is strong and significant to explain the retirement decision. Therefore, we can use mixed linear functions to postulate the two spikes in the observed distribution. It means that there are structural changes for the slope of the increasing leisure preference at age of 62 and 65. For example,

$$\delta(\tau) = \begin{cases} \delta_1 \tau, & \tau \le 61 \\ \delta_2 \tau, & 62 \le \tau \le 65 \\ \delta_3 \tau, & 66 \le \tau \end{cases}. \tag{3.14}$$

The parameters of the above functions need to be calibrated to the observed retirement age distribution and then we can compare the performance of different functions. The parameters are calibrated in order to minimize the sum of squared error (the difference between the simulated frequencies and the observed frequencies).

Table 3.2: Performance of leisure parameters

This table reports the calibrated parameters for minimizing the sum of the squared error between the simulated retirement age distribution and the observed retirement age distribution.

Function	Parameters	SSE
$\delta(\tau) = \delta$	δ = 0.0002	0.11668
$\delta(\tau) = \delta_1 \tau$	δ_1 = 0.0000048	0.093073
$\delta(\tau) = \delta_0 + \delta_1 \tau$	δ_0 = -0.000001, δ_1 = 0.000004	0.091558
$\delta(\tau) = \exp(\delta_0 + \delta_1 \tau)$	δ_0 = 21.5793, δ_1 = -10	0.12841
$\delta(\tau) = \begin{cases} \delta_1 \tau, & \tau \le 61 \\ \delta_2 \tau, & 62 \le \tau \le 65 \\ \delta_3 \tau, & 66 \le \tau \end{cases}$	δ_1 = 0.0000046 δ_2 = 0.0000047 δ_3 = 0.0000044	0.046869

B. Calibration

Table 3.2 presents the calibrated parameters and the corresponding sum of squared errors. As expected, the linearly increasing leisure preference performs better than the benchmark case in which the leisure preference is constant. Adding a constant to the linear function only improves the performance slightly. Considering that increasing the number of parameters also increases the calibration error, the linear function without a constant seems to be an economic model for the leisure preference. The exponential function for modeling leisure preference cannot match the empirical distribution. The sum of squared error is low simply because the simulated probabilities for all the ages are zero. The mixed linear functions have the best performance based on the smallest sum of squared error. It confirms the structural change for the age of 62 as documented by Coile and Gruber (2000). The literature, however, has not provided a convincing story for such structural change. Gustman and Steinmeier (2002) propose that a bimodal distribution of time preference can explain the peaks in retirement both at ages 62 and at

65. This chapter, however, shows that a very small change of the speed at which the leisure preference increases (from 0.0000046 to 0.0000047) is sufficient to generate the peak at age 62.

Figure 3.11 further depicts the simulated retirement age distributions for different modeling of leisure preference. We can see that the mixed linear function of leisure preference is able to match the two empirical spikes in retirement at ages 62 and 66. However, the simulated retirement age distribution is still not perfect as it generates another spike at age 70. Of course, we can keep changing the functional form of the leisure preference until the empirical retirement age distribution is fitted perfectly. But this is beyond the scope of the present chapter. The main objective of this chapter is to derive the optimal life-cycle portfolio choice when the endogenous retirement decisions are realistically calibrated to match the empirical retirement age distribution, especially the two spikes at ages 62 and 66. This objective is analyzed in Figure 3.12.

The portfolio choice in Figure 3.12 is very similar to the model with exogenous time constraints in Figure 3.5. The portfolio surface is generally hump-shaped. This confirms the argument in the motivation of the chapter. If the investor is restricted from exercising the early retirement option in his early stage of the life cycle due to the low leisure preference or financial incentive, then the rich investor will decrease the portfolio risk in order to increase the chance of exercising the option later. The investor will behave as they are exogenously time-constrained which is supported by the empirical retirement data.

3.5 Conclusion

This chapter investigates the impact of early retirement option on the optimal life-cycle portfolio choice in a model that can endogenously generate a realistic retirement age distribution. The key result that differs from earlier literature is that investors might find it optimal to reduce their stock holding even though wealth is increasing. The reason is the following. When investors are not old enough, their leisure preference is low and financial penalty in retirement benefit for early retirement is high. Consequently, the critical wealth level is extremely high and these investors are virtually restricted from exercising the option. However, as investors get older, the critical wealth gets lower. If rich investors still hold a risky portfolio, their wealth might end up below the critical wealth level as they get older and lose the chance to exercise the early retirement option. Therefore, rich investors might find it optimal to reduce the portfolio risk when they are young.

Figure 3.11: Simulated retirement age distributions

This figure presents the simulated retirement age distributions for the constant, linear, and mixed linear leisure preference. The empirical retirement age distribution from Figure 1 is modified for the change of normal retirement age adopted in the calibration.

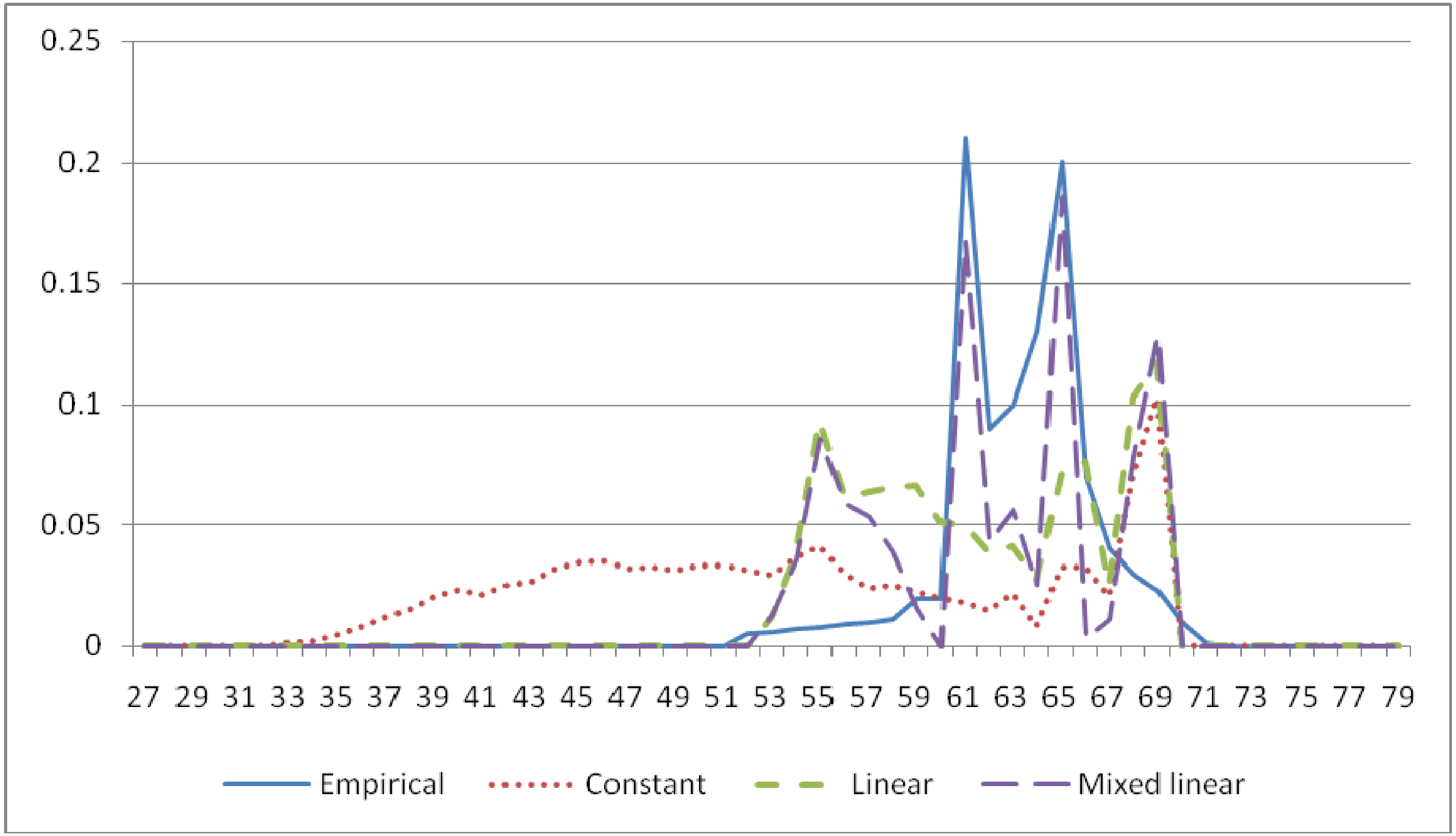
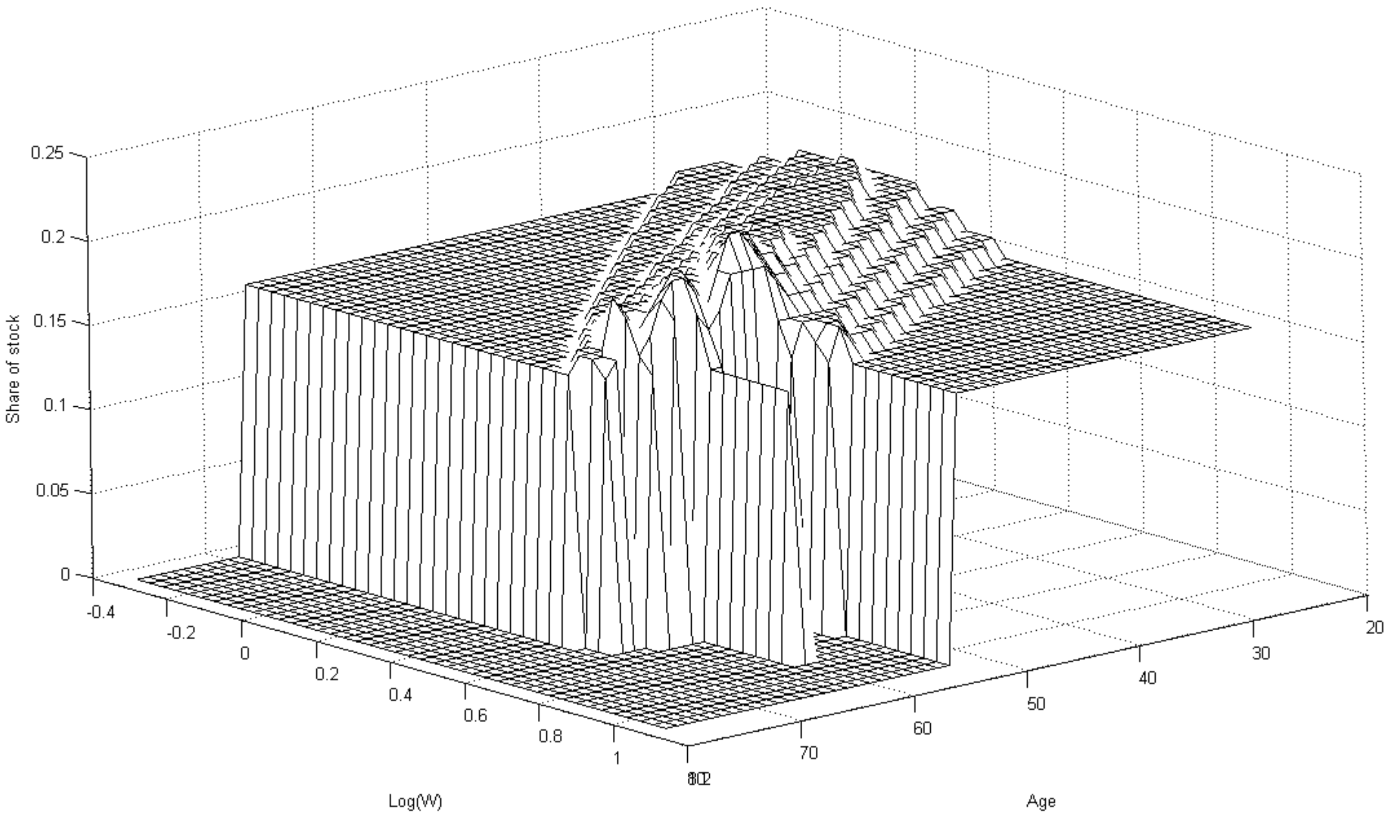


Figure 3.12: Policy functions with optimal leisure preference

This figure presents the policy functions of the stock share and retirement decisions for the benchmark model with the optimal leisure preference that is calibrated to the empirical retirement age distribution.



Hence, this model predicts that young investors, who typically have accumulated limited wealth, should hold a low proportion of wealth in stock. As young investors get older and the wealth gets closer to the critical wealth, they increase the stock proportion in their portfolio. Middle-aged investors either decrease stock holdings if their wealth has exceeded the critical wealth or increase stock holdings further if not. These predictions can explain why empirically there exist both increasing and decreasing patterns for the stock holdings of middle-aged investors.

This chapter suggests that institutional rules for retirement and financial incentives in the pension plans can change the demand for stock and labor force participation dramatically. This has serious implications for analyzing Social Security policy and retirement pension plans. In order to avoid the funding crisis in the next several decades for Social Security, proposals have been taken to increase the Social Security normal retirement age and early retirement age, both to encourage individuals to stay in labor force. On the other hand, the growing popularity of 401(k) retirement plans gives individuals a great amount of freedom to choose an asset allocation for retirement. Individuals need more financial advice in choosing an optimal portfolio. Typically, the analysis of Social Security policies and giving financial advices for individual investors are separate. This chapter shows that Social Security policies can not only affect investors' retirement decisions but also portfolio choice. Therefore, financial advice for individuals should take Social Security policies into account.

An extension to this model could involve the inclusion of intermediate consumption into the utility function. Thus, the retirement planner would not only care about the retirement wealth which he lives on after retirement but also the quality of life before retirement. Another extension could be the inclusion of labor income. Risky labor income results in unpredictable cash flows to the retirement wealth. This background risk might reduce the demand for stock holdings.

Appendix 3.A: Numerical procedure

The numerical procedure follows Hodder and Jackwerth (2007). The fundamental construct is a grid of log wealth values $\ln(W)$ and time t with $\Delta \ln(W)$ and Δt constant. The initial log wealth value is on the grid, and I choose the grid spacing such that 5 $\Delta \ln(W)$ is equal to $r\Delta t$. This is done to have the wealth process lie on the grid even in the case for α equal to 0.

To approximate the normal distribution, I use 101 grid points with index i equal to -50, …, 0, …, 50. The probabilities for those possible moves depend on the choice of α (stock proportion). For a given α, I calculate the probabilities based on the normal density times a normalization constant so that the computed probabilities sum to one:

$$p_{i,\alpha,\Delta t} = \frac{\frac{1}{\sqrt{2\pi}\sigma_{\alpha,\Delta t}} \exp\left[-\frac{1}{2}\left(\frac{i\Delta \ln(W) - \mu_{\alpha,\Delta t}}{\sigma_{\alpha,\Delta t}}\right)^2\right]}{\sum_{j=-50}^{50} \frac{1}{\sqrt{2\pi}\sigma_{\alpha,\Delta t}} \exp\left[-\frac{1}{2}\left(\frac{j\Delta \ln(W) - \mu_{\alpha,\Delta t}}{\sigma_{\alpha,\Delta t}}\right)^2\right]} \tag{3.A1}$$

where $\mu_{\alpha,\Delta t} = \left[\alpha\mu + (1-\alpha)r - \frac{1}{2}\alpha^2\sigma^2\right]\Delta t$ and $\sigma_{\alpha,\Delta t} = \alpha\sigma\sqrt{\Delta t}$. The choices of α vary from 0 to 1 in steps of 0.01. (3.A1) shows that the approximated probabilities do not depend on the level of $\log(W)$ or time and they are solely functions of α.

I now calculate the expected indirect utilities and initialize the indirect utilities at the terminal date J_T to the utility of the investor $U_{W,T}$ where his retirement wealth fits precisely on the grid. I start stepping backwards in time from the terminal date T in steps of Δt. At each wealth level, the expected indirect utilities are calculated for all choices of α using the probabilities defined in (3.A1). I also calculate the utilities associated with the investor choosing to enter retirement. If the investor chooses to exercise the retirement option, the current wealth will be invested at the riskfree rate until T and the terminal wealth W_T will be adjusted by the financial incentives (or penalties) of pension plans. The utility is computed as the sum of the power utility with respect to the terminal wealth and the leisure gain from retirement. The highest of those expected utilities from keeping the option alive and the utility of exercising the option is chosen as the optimal indirect utility for the corresponding wealth level which we denote as $J_{W,T-\Delta t}$. I record the corresponding α as the optimal portfolio choice and the corresponding time as the optimal retirement age if entering retirement has the higher utility than staying in work. If the investor chooses to retire, α is set to be 0 since we assume that after retirement the wealth will be fully invested in bonds. This procedure steps back to T_0, at which the investor starts the retirement planning.

When implementing the backward sweep through the grid, we have to deal with behavior at the boundaries. The terminal step is trivial in that we calculate the terminal utility from the terminal wealth. For the grids close to the upper boundary and lower boundary, I keep a buffer of wealth levels above and below the boundary so that the expected indirect utility can be calculated by looking up values from such points. The upper buffer values can be calculated directly based on the wealth level since the optimal decision is known to be to retire. I compute the indirect utilities for the lower buffer grids as the buffer utilities in the previous time step multiplying the ratio of the current indirect utility on the boundary to the indirect utility on the boundary of the previous time step. I do not assume that these values are correct except for the upper buffers, but they all work very well. By expanding the grid structure, the distortion ripples do not affect the grid points we are interested in.

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Chapter Four

Revisiting the Composition Puzzles of the Household Portfolio: New Evidence

This chapter presents new evidence contradicting the existence of the portfolio composition puzzles concerning household finance: portfolio risk is empirically increasing in age and wealth which is contradicting Merton's (1971) solution. The puzzles cause serious problems in assessing the classical theoretical models that have been developed to rationalize households' portfolio choices. This chapter investigates the 2005 Panel Study of Income Dynamics data and shows that, when the household portfolio includes real estate and private business and allows for leverage, the portfolio risk for young and low-wealth households is in general higher than old and rich households, which is consistent with the predictions of classical models.

4.1 Introduction

This chapter presents new empirical evidence contradicting the existence of the composition puzzles concerning household portfolios. In order to illustrate these puzzles, I begin with Merton's (1971) solution for a standard life-cycle model in the presence of human capital. This type of classical models assumes an investor has a certain or uncertain life span. Income is exogenously determined. Each year, the investor decides how much to consume and how to allocate savings among various asset classes subject to several constraints, namely budget, borrowing, and short sale constraints. The investor's objective is to maximize expected discounted life time utility. Denoting the present value of future labor income by $PV(FY)$ and the share of the risky asset by α, Merton's result can be rewritten as:

$$\alpha = \frac{\mu}{\gamma\sigma^2}\left(1 + \frac{PV(FY)}{W}\right). \tag{4.1}$$

Thus, the share of the risky asset equals the multiplication of two parts. The first part is a constant that is equal to the ratio of the mean of the risky asset's excess return μ to the multiple of the variance σ^2 and the risk aversion parameter γ. The second part is related to the ratio of present value of future labor income $PV(FY)$ to wealth W. Equation (4.1) generates three predictions for portfolio composition: (i) holding human capital constant, the risky asset share is decreasing in wealth; (ii) holding wealth constant, young households with larger human capital choose a higher risky asset share than aged households with less human capital; (iii) low wealth and young households might even borrow the riskless asset to invest more than 100% of their available financial wealth in the risky asset.

Nevertheless, previous empirical evidence is at odds with the predictions from equation (1). The following contradictions are regarded as "*the composition puzzles*"[25] of the household portfolio: (i) the risky asset share is either slightly increasing or constant for increasing wealth;[26] (ii) risky asset holdings have typically been low at low ages, and then either increasing or hump-shaped over the life cycle;[27] (iii) young and

[25] See Guiso, L., M. Haliassos, and T. Jappelli (2002).

[26] Campbell (2006) finds the wealth effect has a quadratic pattern with a minimum share at $70,000. Guiso et al. (2003) find that the cross-sectional relationship between the level of the risky asset share and the level of wealth is essentially flat.

[27] See e.g., Ameriks and Zeldes (2004) and Poterba and Samwick (2001).

low wealth households are constrained to borrow and do not tend to hold stocks.[28] These contradictions raise serious concerns about the validity of the standard life-cycle model that is widely used in the financial planning literature.

In an attempt to reconcile theory and observation, the literature has so far mainly focused on adapting theoretical models to observations. For instance, Cocco et al. (2005) consider the labor income process to be stochastic. However, they conclude that labor income with normally distributed shocks generates policy functions very close to those generated by riskless income. Although some models that impose particular assumptions on preferences or the labor income process have managed to explain parts of the composition puzzles,[29] the literature has surprisingly ignored the potential resolution of the puzzles through careful investigation of the data.

As opposed to other empirical studies, this chapter presents evidence showing that a household's portfolio choice is consistent with the policy functions derived from the standard life-cycle model. The household's portfolio risk, measured as portfolio standard deviation, is generally decreasing in age and wealth. Two forces drive this result. First, households can borrow through real estate mortgage and other debt to increase portfolio risk. Second, real estate and private business are regarded as two additional risky assets and are included in the computation of portfolio risk.

I estimate the portfolio risk surface over a bivariate age-wealth space using OLS regressions. Quadratic and multiplicative terms are used to capture the non-linearity of household portfolio risk and the interactive effects between age and wealth. The result shows that when leverage is considered in portfolio risk, the estimated portfolio risk surface is downward sloping in age and wealth. This pattern is present even when the regressions are controlled for a variety of demographic variables such as family units, marriage, employment, education, and house ownership. I reproduce the results of earlier empirical papers on the composition puzzles by considering different definitions of the household portfolio and I show that failure to allow for non-stock risky assets and leverage gives rise to the composition puzzles.

Nonparametric regression is applied to insure that the nonlinearity in the data is well captured by the OLS regressions. Also, alternative definitions for the wealth variable and alternative estimates of the variance-covariance matrix do not change the main results of the chapter.

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[28] See Haliassos and Michaelides (2002).

[29] For instance, Polkovnichenko (2007) solves a life-cycle model with additive and endogenous habit formation preferences for an investor who has stochastic uninsurable labor income. Benzoni et al. (2007) investigate portfolio choice when labor income and dividends are cointegrated.

The rest of the chapter is organized as follows. Section 4.2 presents a literature review. Section 4.3 describes the econometric methodologies and models. Section 4.4 introduces the data. Section 4.5 presents the main results. Section 4.6 provides robustness analyses. Section 4.7 concludes.

4.2 Literature

This chapter's distinct contribution is to include real estate and private business in the household portfolio, to allow borrowings through mortgages, and to estimate the portfolio risk function over the age-wealth space. The estimated risk function does not support the existence of the composition puzzles.

In seeking to explain the composition puzzles, both of the theoretical and empirical literatures have realized the importance of real estate and private business. They argue that young and low-wealth investors are observed to have low stockholdings because their wealth is dominated by housing wealth or private business.

Some earlier theoretical papers have analyzed the impact of real estate and private business on portfolio choice. Flavin and Yamashita (2002) use a mean-variance efficiency framework to examine the household's optimal portfolio choice when owner-occupied housing is included in the list of available assets. Cocco (2005) presents the solution of a dynamic life-cycle model in the presence of stochastic labor income and housing. Yao and Zhang (2005) examine the optimal dynamic portfolio decisions for investors who acquire housing services from either renting or owning a house. All these theoretical models suggest that housing ownership crowds out stock holdings. Young households, who typically have large holdings of real estate relative to their net worth, are highly leveraged and therefore forced to reduce stock holdings.

On the other hand, the empirical literature has studied the relationship between the stockholding and the ownership of real estate and private business. Before we go through the literature, it is important to note that the relative importance of stockholdings is sensitive to how the portfolio is defined.[30] The literature has adopted three main definitions of the portfolio: (i) *liquid assets* are the sum of stocks, cash and bonds; (ii) *financial assets* are liquid assets plus house value, vehicles, other real estate, and private business; (iii) *total assets* are financial assets plus human capital. *Liquid net worth*, *financial net worth*, and *total net worth* are the corresponding assets minus various types of debt including mortgages and consumer loans.

Flavin and Yamashita (2002) tabulate the mean ratio of different risky assets to

[30] See Heaton and Lucas (2000).

financial net worth by age groups using the 1989 PSID wealth data. They report that the share of stockholdings is increasing and the share of housing wealth is decreasing in age. This supports the theoretical prediction that real estate crowds out the stockholding in early stages of life-cycle. Cocco (2005) uses cross-sectional Tobit regressions to analyze the determinants of stockholdings relative to liquid assets, financial assets, and total assets. He also finds that real estate and private business holdings crowd out the relative share of stockholdings.

Other empirical studies, however, find ambiguous results. Yao and Zhang (2005) find that the share of real estate to financial net worth and owning a business crowd out the share of stock to financial net worth. But the crowding-out effect is insignificant for the share of stock to liquid assets. The crowding-out effect of owning a private business is reported in Heaton and Lucas (2000). However, Yao and Zhang (2005) find that owning a business increases the stock proportion for renters because of the diversification benefits. Brunnermeier and Nagel (2006) treat housing and private business wealth as background wealth and include them in the regression equation as additional control variables. Such treatment ignores the extent to which households leverage their portfolio by taking out mortgages. With the same housing wealth, households which choose a high ratio of mortgage debt to house value will take more risk than a low debt ratio.

In summary, the empirical literature has shown that owning real estate and private business can serve as a substitute for holding stock. Such theory and evidence can resolve the composition puzzles only if the stock is the only risky asset in the household portfolio. However, based on equation (4.1), the risky asset refers to a portfolio of risky assets, including real estate and private business. So the research question of interest for the present chapter is whether households optimally choose their portfolio risk as predicted by equation (4.1) when real estate, private business, and debt are included in the portfolio.

4.3 Modeling

From equation (4.1) we can see that the demand for the risky asset is a function of certain state variables, including wealth, age, and other demographic characteristics. The empirical literature uses the portfolio share of stock or risky assets to measure the demand.[31] This is true in the classical theoretical models in which a portfolio is composed of a riskless asset and a risky asset. Given the accessibility of extensive

[31] See e.g., Campbell (2006).

household-level surveys offering detailed information on portfolio composition, we now understand that a household portfolio is composed of different risky assets.

Portfolio share might not be appropriate for measuring the demand for risky assets due to the following two concerns. First, the risks associated with stocks, real estate, and private business are different. Investing all of the wealth in stocks is of course less risky than investing the same amount of money in private business although the share of the risky asset is one for both portfolios. Second, the correlation between different risky assets will change the total risk of the portfolio even though the relative portfolio shares of assets stay constant. For example, assume investor A invests one fourth of wealth in bonds, stocks, real estate, and private business respectively. The portfolio share for all the risky assets is 75%. Assume investor B invests exactly 75% of wealth in stocks and 25% in bonds. In this case, one cannot conclude that investor A and B's portfolios have the same risk. Even if in a special case in which the three risky assets have the same variance, the total risks of the two portfolios are different due to the correlation between assets. In most cases, the three risky assets are not perfectly correlated. Therefore, investor A's portfolio is more diversified than investor B's portfolio and has a lower total risk.

Therefore, in a framework with multiple risky assets, the portfolio share of the single risky asset α_t in equation (4.1) cannot measure the total demand of risky assets for the investor. Nor can the sum of each risky asset's portfolio share be an appropriate measure. To measure the total demand for multiple risky assets, this chapter proposes to adopt the standard deviation of the household portfolio:

$$\sigma_{it} = \sqrt{\alpha_{it}^T \Omega_t \alpha_{it}} \tag{4.2}$$

where σ_{it} is the standard deviation of the household portfolio for household i at time t, α_{it} is the vector of asset weights (or portfolio share), and Ω_t is the variance-covariance matrix of risky assets. The household portfolio includes riskless asset (cash and bonds), stocks, private business, and real estate. It is important to note that, if the household portfolio only has one single risky asset, then σ_{it} is just a multiple of α_t in (1) with the multiple being the volatility of the single risky asset. Thus, we can formulate the demand function for risk in a framework with multiple risky assets:

$$\sigma_{it} = f(W_{it}, age_{it}, Z_{it}) + \varepsilon_{it} \quad (4.3)$$

where W_{it} is the amount of financial net worth, age_{it} is the age of the head of the household, and Z_{it} is a vector of additional explanatory variables.

A. Accounting for nonlinear effects

Most recent theoretical models of life-cycle portfolio choice[32], in which stochastic income, liquidity constraints, and transaction costs are incorporated, do not render analytic closed-form solutions for the asset demand. The simulated policy functions are usually non-linear in age and wealth.[33] Despite the unknown functional form and probable non-linearity in the demand function (4.2), the empirical literature has extensively used standard ordinary least squares regressions to estimate the demand for risky assets. Some works include a quadratic term to allow for some curvature.[34] In all these cases, however, the researcher assumes a particular underlying functional form of the data. The empirical portfolio risk might be more complicated than is taken into account by the quadratic term.

This chapter uses both of the parametric and nonparametric regressions to capture the non-linearity of household portfolio risk. First, OLS regression with level and quadratic terms for wealth and age is applied. Second, as a robustness check for more complicated non-linearities, the method of *local linear regression* introduced in Fan and Gijbels (1996) is adopted. Formally, we want to estimate the following equation

$$\sigma_{it} = m(\ln W_{it}, age_{it}) + \varepsilon_{it} \quad (4.4)$$

where m is the nonparametric estimator over the age-wealth space. With this method, the relationship between the portfolio risk, age, and wealth is given flexibility to characterize its own shape.

Local linear regression is preferred to *kernel regression* because local linear regression estimates offer several improvements, including improved performance at the boundaries and a reduction in bias. This property is especially important when using

32 See Cocco et al. (2005) and Cocco (2005).

33 For instance, Cocco et al. (2005) calibrate a life-cycle model with uninsurable labor income and a borrowing constraint and find that the risky asset share plotted vs. age exhibits a hump shape.

34 See e.g. Vissing-Jorgensen (2002) and Campbell (2006).

micro data for households in which the observations are bounded in age and wealth. For instance, Brunnermeier and Nagel (2006) exclude observations for a household if the head of household is retired or, liquid net worth or financial net worth is less than $10,000 in order to eliminate the effects of outliers. Such sample selection imposes boundaries in age (65) and in wealth ($10,000). In this case, kernel estimators are biased due to the missing data on the other side of the boundary while local linear regression can mitigate this problem to a large extent.[35]

B. Accounting for interactive effects

One of the major concerns with respect to earlier empirical works is the ambiguity of the estimated age and wealth effects on the portfolio share of stocks. Guiso et al. (2003) find that the cross-sectional relationship between the level of the risky asset share and the level of wealth is essentially constant, while Campbell (2006) finds the wealth effect follows a quadratic pattern with a minimum share of $70,000. Regarding the life-cycle effect, Ameriks and Zeldes (2004) estimate a hump-shaped age effect on the equity share. Campbell (2006) finds that the age effect is rather weak, i.e., the estimated coefficient of age is small and insignificant. Note however, that the above results are all obtained without considering the effects of real estate and private business.

This chapter suggests that the interactive effects between age and wealth explain the inconsistent results as of yet to be found in the literature. The OLS regression includes both of quadratics and covariate of wealth and age as the following:

$$\sigma_{it} = \beta_0 + \beta_1 \ln W_{it} + \beta_2 (\ln W_{it})^2 + \beta_3 Age_{it} + \beta_4 (Age_{it})^2 + \beta_5 (\ln W_{it}) Age_{it} + \delta Z_{it} + \varepsilon_{it}, \quad (4.5)$$

where Z_{it} is a vector of additional explanatory variables and δ is a vector of coefficients. The intuition for the interactive effects is the following. Financial assets dominate human capital for rich households. So the hump-shaped life-cycle portfolio risk might not characterize the risk taking behavior of rich households. Depending on the choices of sample weights and selection, different results for the age effect might be estimated when using linear models without the cross-product of age and wealth. Also, the risk taking behavior of elderly households might be different towards a change in the wealth level. These concerns are documented in Carroll (2002) and Hurd (2002).

[35] For more details about the differences between local linear regression and kernel regression, a survey by DiNardo and Tobias (2001) is a nice reference.

Alternatively, in seeking to effectively capture the interactive effect between age and wealth, the nonparametric regression is carried out over a bivariate age-wealth space. We cannot, however, further increase the number of space dimensions due to the curse of dimensionality – the rate at which the nonparametric estimator converges becomes too low. Fortunately, age and wealth are the two variables of greatest interest in the literature. Therefore, equation (4.4) is estimated to detect the interactive effect.

C. Accounting for leverage effects

In reality, the household is restrained from borrowing against the future income, so the household cannot lever the portfolio as predicted by the theoretical models. This is one of the previously mentioned composition puzzles. However, if real estate is included in the household portfolio, then the household can borrow against the house value. Meanwhile, the household can still hold a large amount of investment in risky assets such as residential housing and stocks. Such possibility of leverage can potentially significantly affect the portfolio risk function specified in (4.3). In order to investigate the leverage effect, the standard deviation of the portfolio σ_{it} is computed by equation (4.2). In particular, the weights are defined as the ratios of the asset value to financial net worth (the sum of all the assets minus all the debts), which reflects the leverage effect.

4.4 Data

In this work the Panel Study of Income Dynamics (PSID) is used. Although this dataset studies and tracks households over time, this chapter mainly focuses on the cross-sectional variation of household portfolio risk. As a result, the most recent wave for the year 2005 is analyzed. Ameriks and Zeldes (2004) state that even with perfect data, it is difficult to disentangle empirically the effects that age has on portfolio shares (age effects) from effects related to a person's date of birth (cohort effects) or effects related to the date of observation (time effects). There is no way to separately identify time, age, and cohort effects without imposing further assumptions. This chapter studies the 2005 data alone so that time effects can be excluded. The assumption that is imposed on this chapter is the absence of cohort effects. That is, portfolio choice is independent of when the investor is born.

Variable definitions – I define three broad categories of risky assets. Stocks include any shares of stock in publicly held corporations, mutual funds, investment trusts, or indirect holdings through private annuities or Individual Retirement Accounts. Real estate includes the owner-occupied house and other real estate holdings. Private business includes investments in a farm or business. The riskless asset is defined as the sum of checking or savings account balances, money market funds, certificates of deposit, government savings bonds, bond funds, and treasury bills. Debt is the sum of mortgage loans, credit card charges, student loans, and loans from relatives. The risk and return for vehicles are difficult to measure, and the value of vehicles depreciates relatively quickly; they are thus excluded from the household portfolio. Liquid assets are defined as the sum of holdings of stocks and riskless asset. Financial assets are defined as the sum of liquid assets, private business, and real estate. Financial net worth is defined as the financial assets minus debt.

Variance-covariance matrix – In order to compute the portfolio risk, we need to estimate the variance-covariance matrix for the three risky assets. The House Price Index (HPI) issued by the Office of Federal Housing Enterprise Oversight is used to measure the risk of real estate investments. The HPI is a quarterly, weighted, repeat-sales index, and a broad measure of the movement of single-family house prices. Data on the S&P 500 is used to measure stock returns. Data on returns for private business is difficult to obtain. The smallest companies by capitalization listed in the US equity market might be a decent proxy for private business, although many private businesses are not publicly listed. The Russell Microcap Index represents the 2000th to 4000th security in the US equity market by capitalization. The median market capitalization is 207 million. The quarterly data starts from the 2nd quarter in 2000 and ends with the 3rd quarter in 2007. The log returns are all adjusted by the inflation rate. The variance-covariance of the three risky assets is presented in Table 4.1.

Table 4.1: The estimated variance-covariance matrix

The returns of stocks, real estate, and private business are measured by the S&P 500, the HPI, and the Russell Microcap Index respectively.

	Stocks	Real estate	Private business
Stock	0.0287		
Real estate	0.0007	0.0007	
Private business	0.0338	0.0007	0.0562

The drawback of the estimated variance-covariance matrix in Table 4.1 is that the business represented by the Russell Microcap Index might still be too big for private business. An alternative is to use the historical data of the PSID Family Income Files to generate the time series of annual growth rates of housing value and business income.

The variance-covariance matrix is computed based on these time series and a stock index. Palia et al. (2007) compute the standard deviations of real estate, private business, and stock, and the correlations of these assets for each household using the 1976-1997 PSID data. The average of the variances and covariances over all households is shown in Table 4.2. The estimated variance-covariance matrices in Table 4.1 and 4.2 are quite different. In Section 4.6, a robustness check for changing the input of variance-covariance matrix is performed.

Table 4.2: The variance-covariance matrix estimated by Palia et al. (2007)

The variances of real estate and private business and the covariances of stocks, real estates, and private business are taken from Table II of Palia et al. (2007). They are computed based on the annual growth rates of housing value and business income for the households in the 1976-1997 PSID data and the annual gross return of CRSP value-weighted market index. However, the variance of stock is computed using the S&P 500 stock index since Palia et al. (2007) do not report the variance of stock.

	Stocks	Real estate	Private business
Stocks	0.028696		
Real estates	0.036328	0.105625	
Private business	-0.11736	0.34256	0.253009

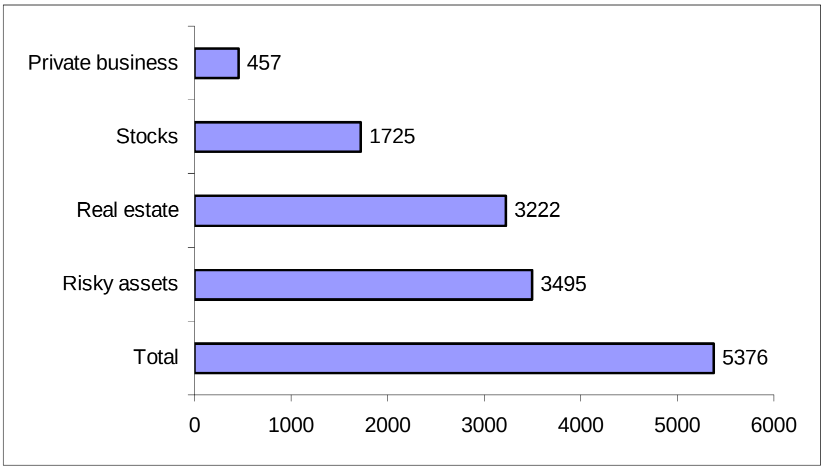
Sample selection – An important feature of household portfolio choices is that a large fraction of households does not own risky assets. This leads to the classic sample selection problem. Fortunately, the sample selection bias of the present model is much smaller than in other studies in which all non-stockholders are excluded, because the present model only excludes households who do not own any type of risky asset. Figure 4.1 shows the numbers of observations categorized by assets. Among the three categories of risky assets, real estate is the most popular. About 60% of households own real estates, compared with 32% of households for stocks and 9% for private business. If we measure the portfolio risk by the share of stocks in the portfolio, we would exclude 68% of all observations. In this chapter, however, the proportion of excluded data is reduced to 35%. In order to ensure positive asset weights, I also exclude households with negative or zero financial net worth.

Summary statistics – Table 4.3 presents summary statistics. The portfolio risk is computed based on equation (4.2) and the variance-covariance matrix for risky asset owners. The top panel shows the cross-sectional statistics for all households. The bottom panel shows statistics for households which at least own one main category of risky asset.

As the table shows, the risky asset owners are generally older and richer than non-participants in risky assets, which is consistent with the findings of previous studies.[36] However, the risky asset owners also have higher debt than other households. Leverage can be an important reason for households to own risky assets.

Figure 4.1: Number of observations by assets

This figure reports the frequencies for risky assets held. Households who own real estate, stocks, or a private business are regarded as risky asset owners.



The distribution of net worth has strong positive skewness. Taking logs eliminates much of this skewness. In order to reduce the impact of outliers, I further exclude observations corresponding to the smallest and largest 2 percentile portfolio risk. Therefore, the minimum portfolio risk is 1.6% and the maximum is 57.8%, among 3115 observations.

4.5 Results

A. Replicating the puzzles

Empirical studies in the literature provide evidence that is at odds with the theoretical predictions. Campbell (2006) finds that the portfolio share of stock (or risky assets) has a U-shape pattern in wealth. Guiso et al. (2003) find that the cross-sectional relationship between the level of the risky asset share and the level of wealth is essentially flat. Ameriks and Zeldes (2004) and Poterba and Samwick (2001) find that portfolio share of stock (or risky assets) is typically low at low ages, and then either increasing or hump-shaped over the life-cycle.

Can we find similar patterns if we run regressions for the same dependent variables as used in the literature? This analysis depends on how the portfolio is defined. Portfolio share of stock is the most widely used dependent variable in the literature. In this case,

[36] See e.g., Vissing-Jorgensen (2002).

Table 4.3: Summary statistics

The table reports the summary statistics of the main assets and household characteristics for the 2005 PSID survey. Risky asset owners refer to households who owned at least one risky asset including stocks, real estate, or a private business.

Variable	Mean	10th pct.	median	90th pct.
	All households			
Bonds and bank accounts ($)	24,579	0	2,000	45,000
Stocks ($)	52,923	0	0	106,000
Real estate ($)	156,293	0	70,000	400,000
Private business ($)	26,739	0	0	0
Debts ($)	61,685	0	14,000	182,500
Net worth ($)	198,849	-6,950	25,200	472,000
Age	44	25	43	66
Sex of head dummy	0.72	0	1	1
Marriage dummy	0.51	0	1	1
Number of children	0.83	0	0	2
House ownership dummy	0.58	0	1	1
Employment dummy	0.78	0	1	1
High school diploma dummy	0.73	0	1	1
College diploma dummy	0.27	0	0	1
Family units	2.6	1	2	5
	Risky asset owners			
Bonds and bank accounts ($)	38,250	0	6,000	77,000
Stocks ($)	87,554	0	300	200,000
Real estate ($)	253,422	22,000	158,000	520,000
Private business ($)	44,298	0	0	8,000
Debts ($)	90,537	0	62,000	222,500
Net worth ($)	332,987	12,810	111,000	725,500
Age	48	30	47	69
Sex of head dummy	0.82	0	1	1
Marriage dummy	0.67	0	1	1
Number of children	0.78	0	0	2
House ownership dummy	0.91	1	1	1
Employment dummy	0.81	0	1	1
High school diploma dummy	0.79	0	1	1
College diploma dummy	0.35	0	0	1
Family units	2.8	1	2	5
Portfolio risk	0.23	0.03	0.07	0.19

Table 4.4: Risk, age, and wealth for holding stocks

Each column presents estimates from an OLS regression. The dependent variable is the portfolio risk, measured by the standard deviation of the portfolio. The portfolio is composed of bonds and stocks. Liquid assets are defined as the sum of bonds and stocks. T-statistics are in parentheses. ***Significant at 1%; **significant at 5%; *significant at 10%.

	Portfolio risk		
	(1)	(2)	(3)
Constant	0.052798*	0.042109	0.048772
	(1.86)	(1.42)	(1.63)
Age	0.001902***	0.002306***	0.002287***
	(5.55)	(4.86)	(4.56)
Age^2	-0.000016***	-0.000014***	-0.000015***
	(-5.17)	(-3.85)	(-3.78)
Ln(Liquid assets)	0.001364	0.001411	0.001322
	(0.27)	(0.28)	(0.26)
$(Ln(Liquid\ assets))^2$	0.000115	0.000247	0.000245
	(0.51)	(0.98)	(0.97)
Age × Ln(liquied assets)		-0.000058	-0.00005
		(-1.23)	(-1.06)
Family units			-0.00097
			(-0.61)
Sex of head			-0.0054**
			(-2.36)
Number of children			0.000316
			(0.17)
Marital status			-0.00036
			(-0.12)
House ownership			0.004054
			(1.45)
Employment			-0.00058
			(-0.2)
High school diploma			0.000383
			(0.16)
College diploma			0.000425
			(0.24)
Adjusted R^2	0.0726	0.0729	0.0846
No. of observations	1546	1546	1546

the portfolio is only composed of the riskless asset and stock. The portfolio risk computed based on formula (4.2) is just a multiple of the portfolio share of stock. Therefore, these two dependent variables are effectively the same. Table 4.4 presents the estimates from the OLS regression for the specification (4.5) when the portfolio is defined to include only the riskless asset and stock. Three model specifications are estimated: (1) level and quadratic terms of wealth and age; (2) plus interactive term; (3) plus control variables.

Column (1) displays the relationship between the stockholding, wealth and age. Consistent to Ameriks and Zeldes (2004), age effects are statistically significant. The hump-shape is also present. Consistent to Guiso et al. (2003), wealth effects are rather weak. The interactive effect in column (2) is not significant. Figure 4.2 presents the economic relationship between portfolio risk, wealth, and age, using the estimated coefficients in column (2). Portfolio risk is low for young investors and hump-shaped over the life-cycle. Portfolio risk is slightly increasing in wealth. All these facts are contradictory to the theoretical predictions that portfolio risk should be decreasing in age and wealth. After controlling for additional explanatory variables in column (3), the signs and statistical significance of the coefficients are not changed.

Table 4.5 presents the estimates from the OLS regression for the specification (4.5) when the portfolio is defined to include riskless asset, stock, real estate, and private business. This analysis corresponds to the literature that the household bears risks not only from stocks, but also from real estate and private business (See e.g., Cocco 2005 and Heaton and Lucas 2000). These background risks will crowd out the portfolio share of stock. It is not clear as of yet, however, whether the total risk of the household portfolio including background risks will behave as the theory predicts.

In column (1), only wealth effects are statistically significant. Consistent with Campbell (2006), wealth has a quadratic effect on the portfolio risk. Different from Table 4.4, the hump-shaped age effect is not identified. Instead, the portfolio risk is U-shaped in age. However, this effect is rather weak and statistically insignificant. In column (2), the interactive effect is significant, which is also different from Table 4.4. Figure 4.3 presents the economic relationship between portfolio risk, age, and wealth when the household is augmented with real estate and private business. The age effects are dominated by the wealth effects. The portfolio risk is increasing in wealth. Such pattern, however, is against the theoretical prediction that poor households should hold more risky portfolios than rich households. The composition puzzles are still present even though background risks are taken into account. With control variables in column

Figure 4.2: The risk surface estimated by OLS for holding stocks

This figure plots the risk surface over the age-wealth space as estimated in the second column of Table 4.3.

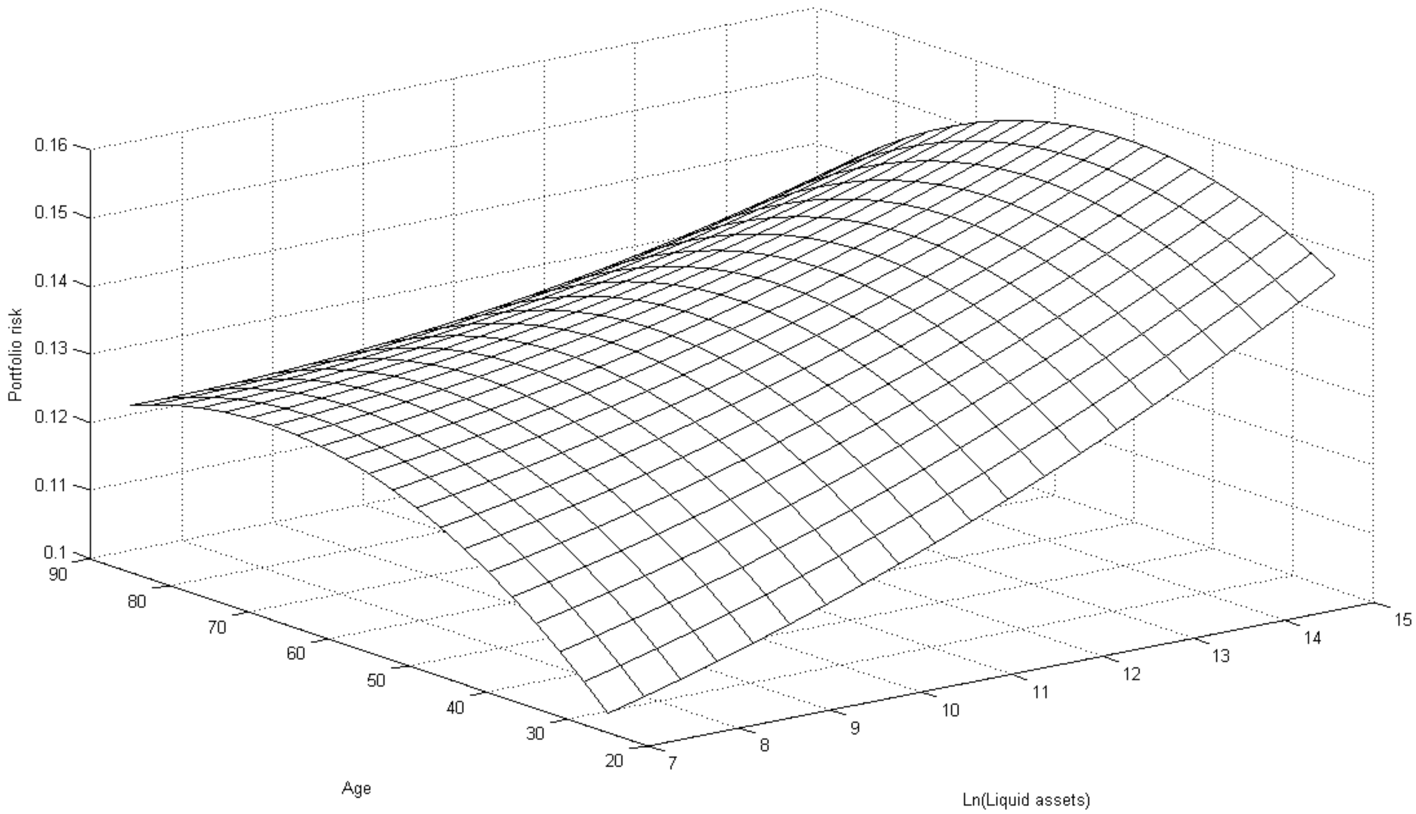


Figure 4.3: The unleveraged risk surface estimated by OLS for holding risky assets

This figure plots the risk surface over the age-wealth space as estimated in the second column of Table 4.4.

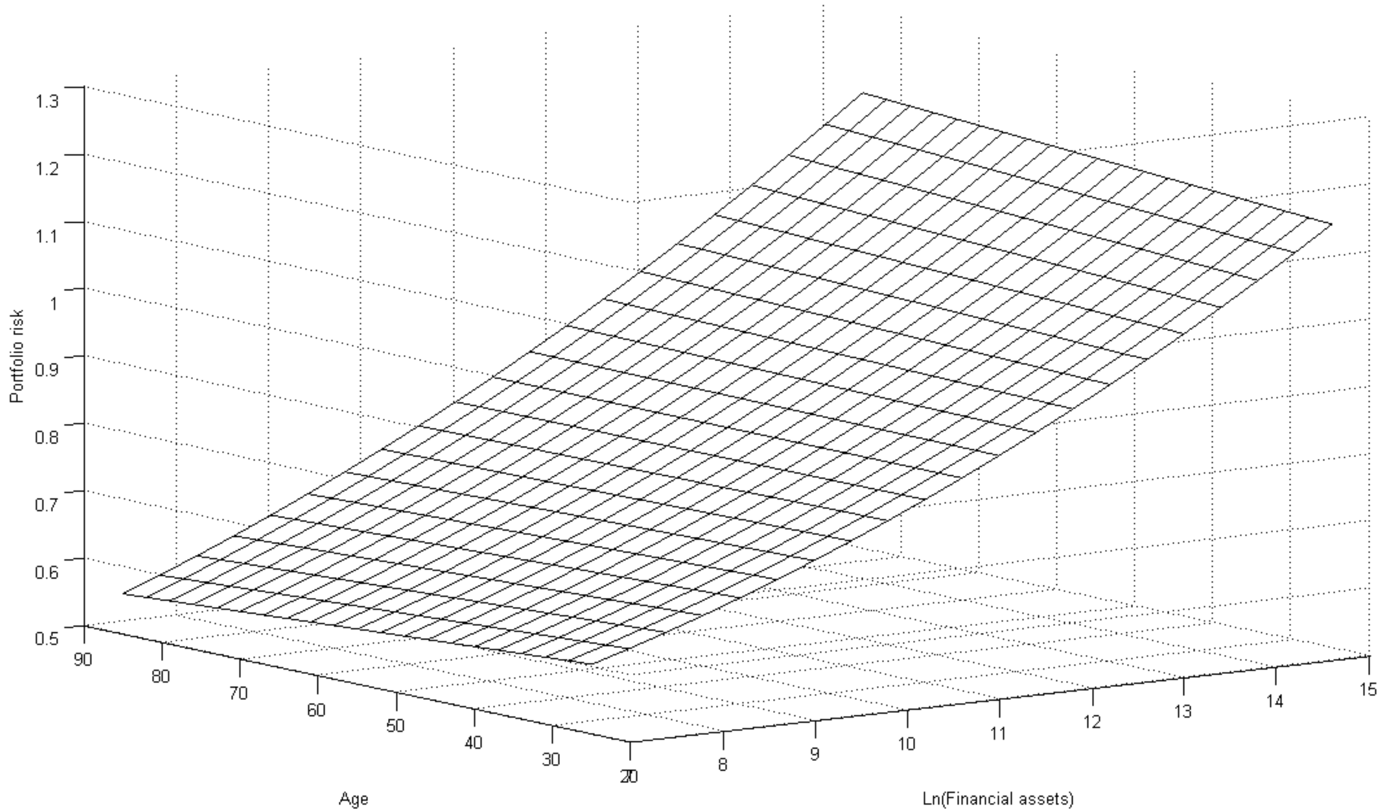


Table 4.5: Unleveraged risk, age, and wealth for holding all risky assets

Each column presents estimates from an OLS regression. The dependent variable is the portfolio risk, measured by the standard deviation of the portfolio. The portfolio is composed of bonds, stocks, private business, and real estate. Financial assets refer to the sum of bonds, stocks, private business, and real estate. T-statistics are in parentheses. ***Significant at 1%; **significant at 5%; *significant at 10%.

	Portfolio risk		
	(1)	(2)	(3)
Constant	0.479643***	0.523657***	0.342841***
	(19.96)	(20.94)	(14.75)
Age	-0.000053	-0.00204***	-0.00033
	(-0.24)	(-5.16)	(-0.91)
Age^2	0.000003	0.00000	-0.000003*
	(1.51)	(0.17)	(-1.65)
Ln(Financial assets)	-0.08002***	-0.007899***	-0.05061***
	(-19.96)	(-19.79)	(-13.69)
$(Ln(Financial\ assets))^2$	0.003558***	0.003124***	0.0023***
	(20.85)	(16.93)	(13.71)
Age × Ln(Financial assets)		0.000188***	0.000081***
		(5.98)	(2.84)
Family units			-0.00188**
			(-2.32)
Sex of head			0.001801
			(1.04)
Number of children			0.001042
			(1.08)
Marital status			-0.00042
			(-0.27)
House ownership			-0.05653***
			(-26.79)
Employment			0.00079
			(0.49)
High school diploma			0.00474***
			(3.8)
College diploma			0.000473
			(0.42)
Adjusted R^2	0.1572	0.1665	0.3391
No. of observations	3115	3115	3115

Table 4.6: Leveraged risk, age, and wealth for holding all risky assets

Each column presents estimates from an OLS regression. The dependent variable is the portfolio risk, measured by the standard deviation of the portfolio. The portfolio is composed of bonds, stocks, private business, real estates, and all types of debts. Financial net worth refers to the sum of bonds, stocks, private business, and real estate minus all the debts. T-statistics are in parentheses. ***Significant at 1%; **significant at 5%; *significant at 10%.

	Portfolio risk		
	(1)	(2)	(3)
Constant	0.727082***	0.873784***	0.83635***
	(20.92)	(23.18)	(22.06)
Age	-0.00216***	-0.00728***	-0.00715***
	(-4.37)	(-9.93)	(-9.59)
Age^2	0.000013***	-0.000006	-0.000000
	(2.73)	(-1.28)	(-0.04)
Ln(Financial net worth)	-0.088***	-0.09156***	-0.0887***
	(-14.97)	(-15.76)	(-15.19)
$(Ln(Financial\ net\ worth))^2$	0.00332***	0.00217***	0.001989***
	(12.83)	(7.64)	(7.02)
Age × Ln(Financial net worth)		0.000613***	0.000587***
		(9.34)	(8.99)
Family units			-0.00298
			(-1.48)
Sex of head			0.00999**
			(2.33)
Number of children			0.005501**
			(2.29)
Marital status			0.007888**
			(2.04)
House ownership			-0.01143**
			(-2.45)
Employment			0.015136***
			(3.8)
High school diploma			0.007031**
			(2.27)
College diploma			0.013287***
			(4.88)
Adjusted R^2	0.1774	0.1996	0.2196
No. of observations	3115	3115	3115

(3), the statistical significance and signs of age variables are slightly different. The dominant wealth effects, however, preserve their signs of coefficients and statistical significance.

B. Leverage effects

If the household portfolio is further broadened to include debt, the leverage effect will kick in. Table 4.6 presents the estimates from the OLS regression for the specification (4.5) when the portfolio is defined to include all riskless assets, risky assets and debt. This analysis has not been fully studied by the empirical literature.

In column (1), all the variables are statistically significant, strongly suggesting the presence of nonlinear pattern between portfolio risk, age, and wealth. Both wealth and age effects are characterized by a U-shape. The interactive effect in column (2) is also statistically significant. The important economic relationship between portfolio risk, age, and wealth is plotted in Figure 4.4. Two observations from Figure 4.4 are interesting. First, we find that portfolio risk is decreasing both in age and wealth. Such evidence contradicts the existence of the composition puzzles that have been documented in the literature. The leverage effect is the driving force for the result in Figure 4.4. Since young and poor households typically need to borrow through mortgage or credit card loans, their risky investments in real estate, stock, and private business are leveraged. Consequently, they have larger risky portfolios than old and rich households. Second, we find nonlinear and interactive effects between age and wealth. For instance, for young households, portfolio risk is decreasing in wealth; while for old households, portfolio risk is slightly increasing. On the other hand, for poor households, portfolio risk is decreasing in age; while for rich households, portfolio risk is flat or increasing in age. Such pronounced nonlinearity might be the reason that the previous empirical studies yield ambiguous wealth and age effects. Finally, the estimates of wealth and age variables are robust when controlling for the demographic variables.

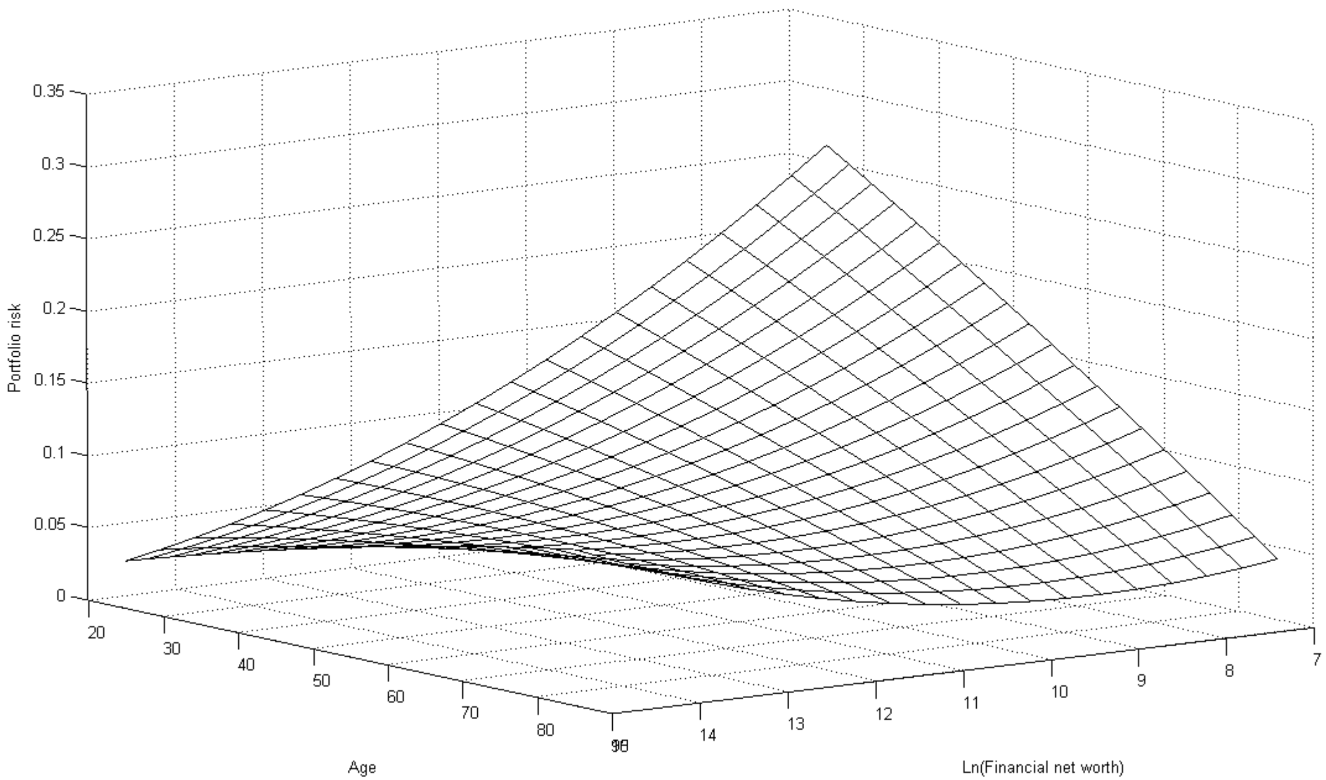
4.6 Robustness check

A. Nonparametric estimation

Given the fact that the function of portfolio risk is highly nonlinear over the age-wealth space, we might be concerned about whether the particular underlying functional form of specification (4.5) has been correctly specified to capture the nonlinearity. Nonparametric regression does not impose an assumption on the functional form and gives the data flexibility to characterize its own shape. Therefore,

Figure 4.4: The leveraged risk surface estimated by OLS for holding risky assets

This figure plots the risk surface over the age-wealth space as estimated in the second column of Table 4.5.



nonparametric estimation is used to check whether the benchmark model has been well specified to reflect the nonlinearity in the data.

Figure 4.5 presents a graph of the function $\hat{m}$ as is specified in equation (4.4). The bandwidth is selected to be 0.7. The selected bandwidth is larger than Silverman's (1986) rule of thumb, which in this case is 0.21. The choice of a large bandwidth is for focusing on the general pattern of the surface of portfolio risk.

Figure 4.5 confirms the estimates from the OLS regressions in Table 4.6. The north corner of the surface reflects the portfolio choices for young and low-wealth households. Their family net worth ranges from $1,097 (log wealth equals 7) to $22,026 (log wealth equals 10). Their age is between 25 and 40. This group corresponds to 7% of the observations in the sample. The standard deviation of their portfolio is estimated to be around 40%. The south corner of the surface presents the portfolio choices for the old and rich households. The net worth of this group of the population is between $442,410 and $3,269,000 and age lies between 60 and 80, which corresponds to 6% of the observations in the sample. The estimated standard deviation of portfolios for this group is on average approximately 28%. Having compared the portfolio risk function in these two corners, we can see that the portfolio standard deviation of a young and low-wealth household is about 12% higher than an old and rich household. These observations do not support the existence of the composition puzzles.

In this context, there are strong non-linear patterns and interactive effects between age and wealth. In the wealth dimension, wealth exhibits quadratic dependence on the portfolio risk. The age effects are also quite different depending on the wealth level. The portfolio risk surface is characterized by a hump shape along the age dimension for households with net worth less than $22,000. The richer households maintain or slightly decrease their risk tolerance as they age. Such heterogeneity across wealth can explain the ambiguous empirical findings for the age effect in the literature.[37]

The most obvious limitation for nonparametric regression is that it is difficult to include many variables due to the curse of dimensionality. The analyses so far are based on family data and we cannot control for the specific composition of individual families as we do in the OLS regression. Does the number of family units change the portfolio risk surface significantly? To answer this question, the whole sample is accordingly divided into 4 sub-samples based on the number of family units. In Figure 4.6, I estimate the portfolio risk surface for the sub-samples with one, two, three, and more

[37] For instance, Ameriks and Zeldes (2004) report the hump shape, while Campbell (2006) finds that the age effect is rather weak.

Figure 4.5: Non-parametric estimation of portfolio risk

This figure presents the portfolio risk surface with the band width choice of 0.7. The sample is as same as the one in the OLS regressions.

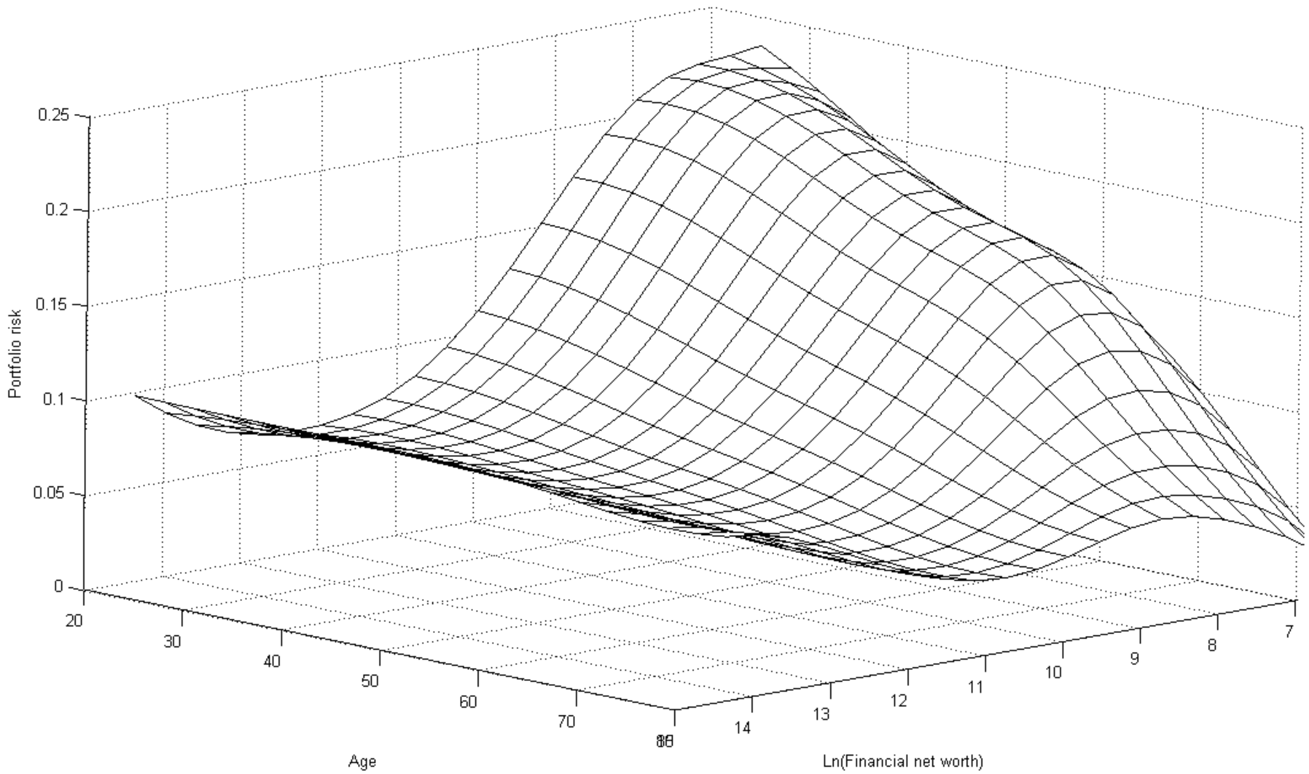
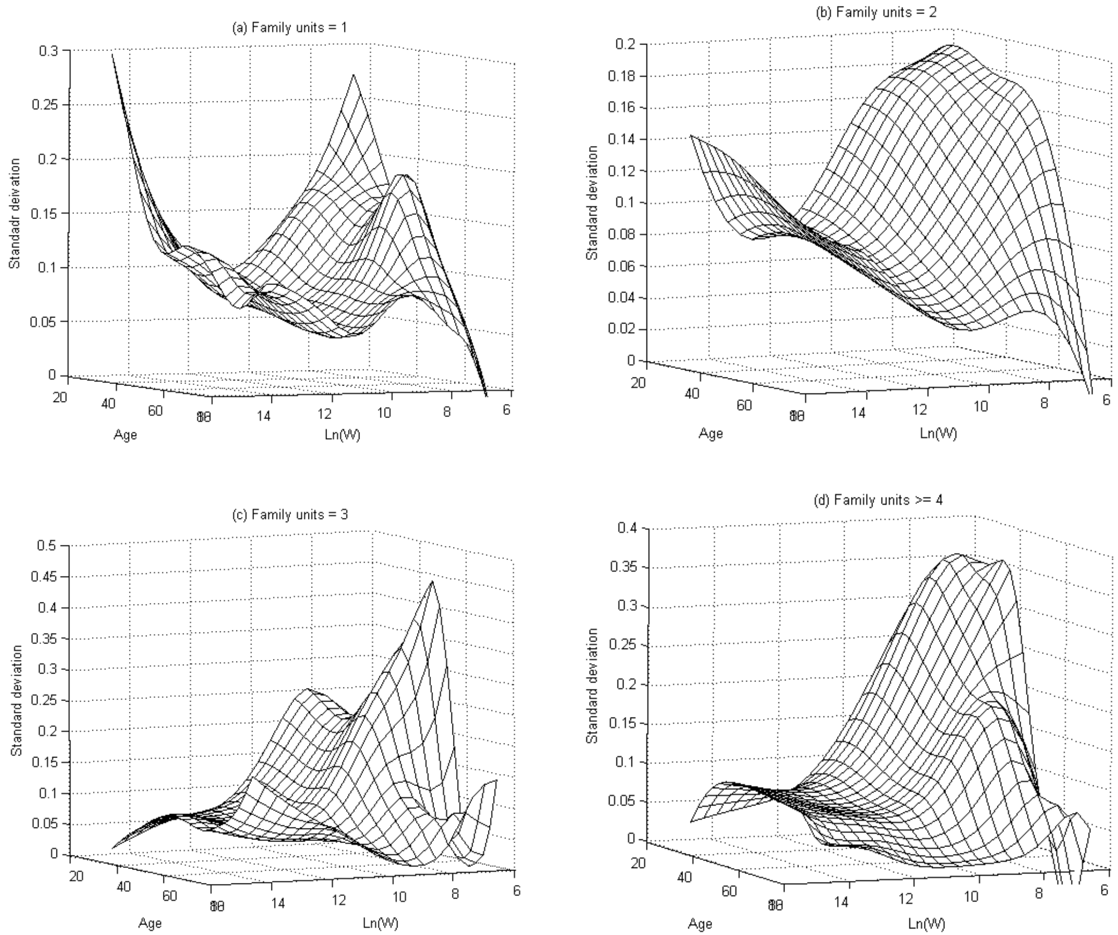


Figure 4.6: The portfolio risk and family units

This panel of figures presents the portfolio risk surfaces for sub-samples with different numbers of family units.



than four family units. The data yields 536 observations of one-unit family, 1067 observations of two-unit families, 617 observations of three-unit families, and 900 observations of four-unit families.

On average, the portfolio risk at the north corner is higher than at the south corner, which confirms the robustness of the empirical fact that the portfolio risk is decreasing in age and wealth. One might argue that the west corners in Figure 4.6(a) and 4.6(b) are abnormally characterized by high risk. We also see such a pattern in Figure 4.2, but we cannot find this pattern in Figure 4.6(c) and 4.6(d) in which presumably the head of household is old and has children. This could imply that the west corners in Figure 4.6(a) and 4.6(b) represent the children of rich families who start their family business alone. They have stable and sufficient future income due to inheritance. Therefore they can easily leverage their consumption and investments in real estate or private business and lever the overall portfolio risk.

Marriage is another important control variable since the data is on the household level and the single and married are given the same weight. Marriage is also associated with a large consumption demand, for e.g. a bigger house. Do married couples choose portfolio risk in a different manner as compared to single persons? To answer this question, I select two sub-samples from the data based on marital status: married and single (including widowed, divorced, and separated). The married sub-sample has 2189 observations and the single sub-sample has 1012 observations.

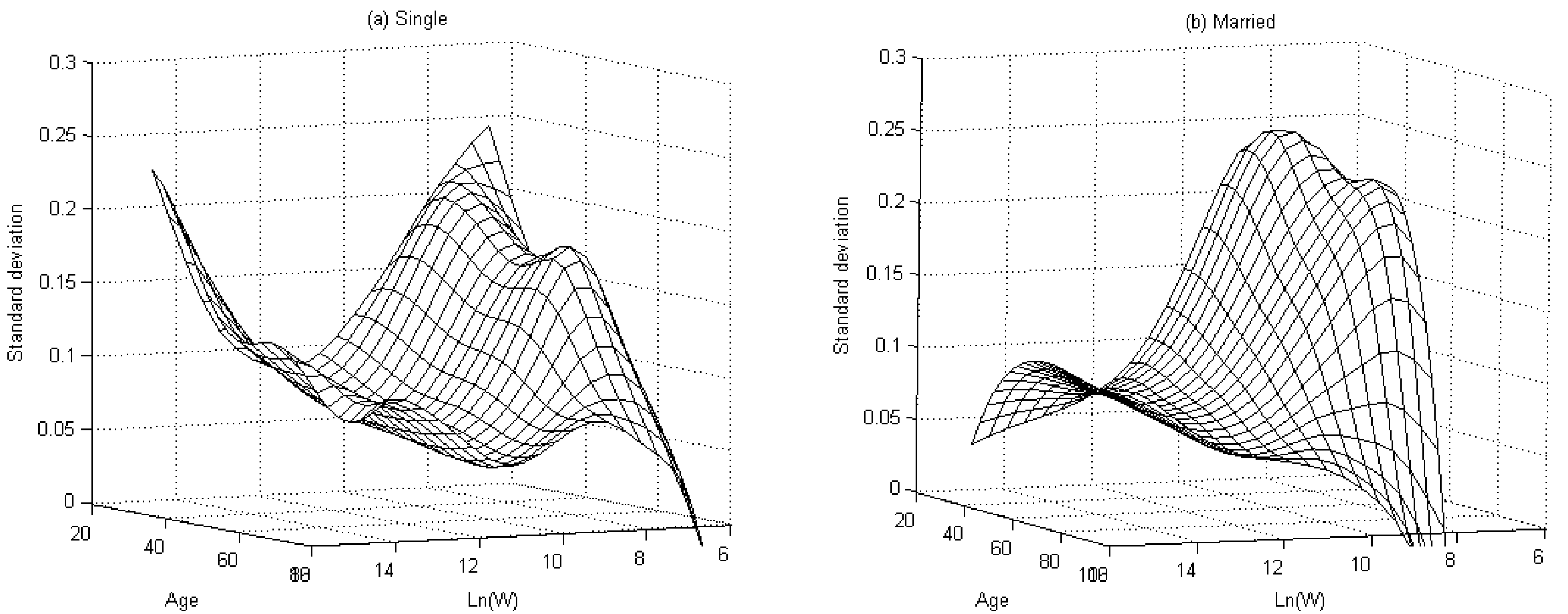
The result is presented in Figure 4.7. In general, married couples tend to invest in the same manner as single persons. Single young rich households are more risk tolerant than married ones. As mentioned before, it could be the case that young, rich people tend to be single, or there could simply be some outliers which happen to be young, single, and rich who have very risky portfolios. Both cases do not affect the generally downward sloping surface in age and wealth.

B. Alternative definitions of wealth

As Heaton and Lucas (2000) note, the relative importance of stockholdings is sensitive to how the portfolio is defined. In the OLS regressions for the benchmark model, the definition of wealth variable is linked to how the portfolio is defined. For instance, if the portfolio is defined to include only the riskless asset and stock, then wealth is defined to be the sum of riskless asset and stock. The broadest definition of portfolio includes all assets and debt. In this case, wealth is financial net worth. The question is: Are the estimates from the OLS regressions robust to the definition of

Figure 4.7: The portfolio risk and marital status

This panel of figures presents the portfolio risk surfaces for sub-samples with different marital status.



wealth?

In Table 4.7, we are interested in how the risk from holding stocks varies with age and wealth. In the benchmark model, wealth is the sum of liquid assets: riskless asset and stock, because that is how the portfolio is defined. In Table 4.7, wealth is defined to be financial net worth, which is the sum of all assets minus debt. Will stockholdings varies in a different way with liquid assets and net worth of the family? Comparing Table 4.4 and 4.7, we find that the signs and statistical significance for most coefficients do not change. The signs for wealth are different in the two tables. However, the estimates are statistically insignificant in both cases. The main effects, age effects, are robust to the alternative definition of wealth.

Similarly, Table 4.5 and 4.8 both study how the unleveraged portfolio risk changes with age and wealth. The portfolio is defined to include all the assets. Wealth refers to the sum of all the assets in Table 4.5, while in Table 4.8, wealth is net worth. Despite the different definitions for the wealth variables, Table 4.5 and 4.8 generate quite similar estimates. Signs and statistical significance of coefficients are the same.

This robustness check confirms that the driving force for the composition puzzles is whether the portfolio includes all risky assets and allows leverage, but not how family wealth is defined.

C. Alternative variance-covariance matrix

In the previous text, the variance-covariance matrix used in the benchmark model has been criticized and an alternative variance-covariance matrix is proposed. It should be noted that portfolio risk is a function of asset weights and the variance-covariance matrix. Asset weights are still computed as the way in the benchmark case. Also, the same criterion is used to filter outliers: observations corresponding to the smallest and largest 2 percentile portfolio risk are excluded. Therefore, the sample has the same size as the benchmark model with 3115 observations.

Table 4.9 presents the estimates from the OLS regression. In column (1), portfolio risk is generally decreasing in age and wealth. The quadratic terms of age and wealth are statistically insignificant. Column (2) shows that the interactive effect is statistically significant. With control variables in column (3), most of the estimates preserve their signs and statistical significance. Figure 4.8 presents the economic relationship between portfolio risk, age, and wealth. We find that young and low-wealth households have more risky household portfolios than old and rich households, suggesting the absence of composition puzzles.

Table 4.7: Risk for holding stocks with alternative definition of wealth

Each column presents estimates from an OLS regression. The dependent variable is the portfolio risk, measured by the standard deviation of the portfolio. The portfolio is composed of bonds and stocks. Financial net worth refers to the sum of bonds, stocks, private business, and real estate minus all the debts. T-statistics are in parentheses. ***Significant at 1%; **significant at 5%; *significant at 10%.

	Portfolio risk		
	(1)	(2)	(3)
Constant	0.081003***	0.078028***	0.086729***
	(2.75)	(2.61)	(2.79)
Age	0.002199***	0.002442***	0.002586***
	(6.13)	(4.36)	(4.42)
Age^2	-0.000018***	-0.000017***	-0.000019***
	(-5.52)	(-4.63)	(-4.71)
Ln(Financial net worth)	-0.00307	-0.00358	-0.00462
	(-0.61)	(-0.7)	(-0.87)
$(Ln(Financial\ net\ worth))^2$	0.000193	0.000272	0.000312
	(0.9)	(1.07)	(1.19)
Age × Ln(Financial net worth)		-0.000028	-0.000028
		(-0.56)	(-0.55)
Family units			-0.00139
			(-0.86)
Sex of head			-0.0054*
			(-1.65)
Number of children			0.000429
			(0.23)
Marital status			-0.00013
			(-0.04)
House ownership			0.003906
			(1.28)
Employment			-0.00187
			(-0.64)
High school diploma			0.001039
			(0.42)
College diploma			0.002768
			(1.57)
Adjusted R^2	0.0494	0.049	0.0524
No. of observations	1546	1546	1546

Table 4.8: Unleveraged risk for holding all risky assets with alternative definition of wealth

Each column presents estimates from an OLS regression. The dependent variable is the portfolio risk, measured by the standard deviation of the portfolio. The portfolio is composed of bonds, stocks, private business, and real estate. Financial net worth refers to the sum of bonds, stocks, private business, and real estate minus all the debts. T-statistics are in parentheses. ***Significant at 1%; **significant at 5%; *significant at 10%.

	Portfolio risk		
	(1)	(2)	(3)
Constant	0.246702***	0.264523***	0.234837***
	(15.07)	(15.24)	(15.2)
Age	-0.0004*	-0.00112***	-0.00028
	(-1.83)	(-3.49)	(-0.96)
Age^2	0.000005**	0.000002	-0.000003
	(2.44)	(0.89)	(-1.41)
Ln(Financial net worth)	-0.04199***	-0.04199***	-0.03288***
	(-14.83)	(-14.85)	(-13.16)
$(Ln(Financial\ net\ worth))^2$	0.002125***	0.001934***	0.001644***
	(16.8)	(13.72)	(1.19)
Age × Ln(Financial net worth)		0.00009***	0.000066***
		(3.06)	(2.56)
Family units			-0.00186**
			(-2.32)
Sex of head			0.001179
			(0.69)
Number of children			0.001191
			(1.25)
Marital status			-0.00038
			(-0.25)
House ownership			-0.05624***
			(29.2)
Employment			0.001843
			(1.16)
High school diploma			0.004041***
			(3.28)
College diploma			0.000572
			(0.53)
Adjusted R^2	0.1558	0.1581	0.3561
No. of observations	3115	3115	3115

Figure 4.8: Portfolio risk with alternative variance-covariance matrix

This figure plots the risk surface over the age-wealth space as estimated in the second column of Table 4.9.

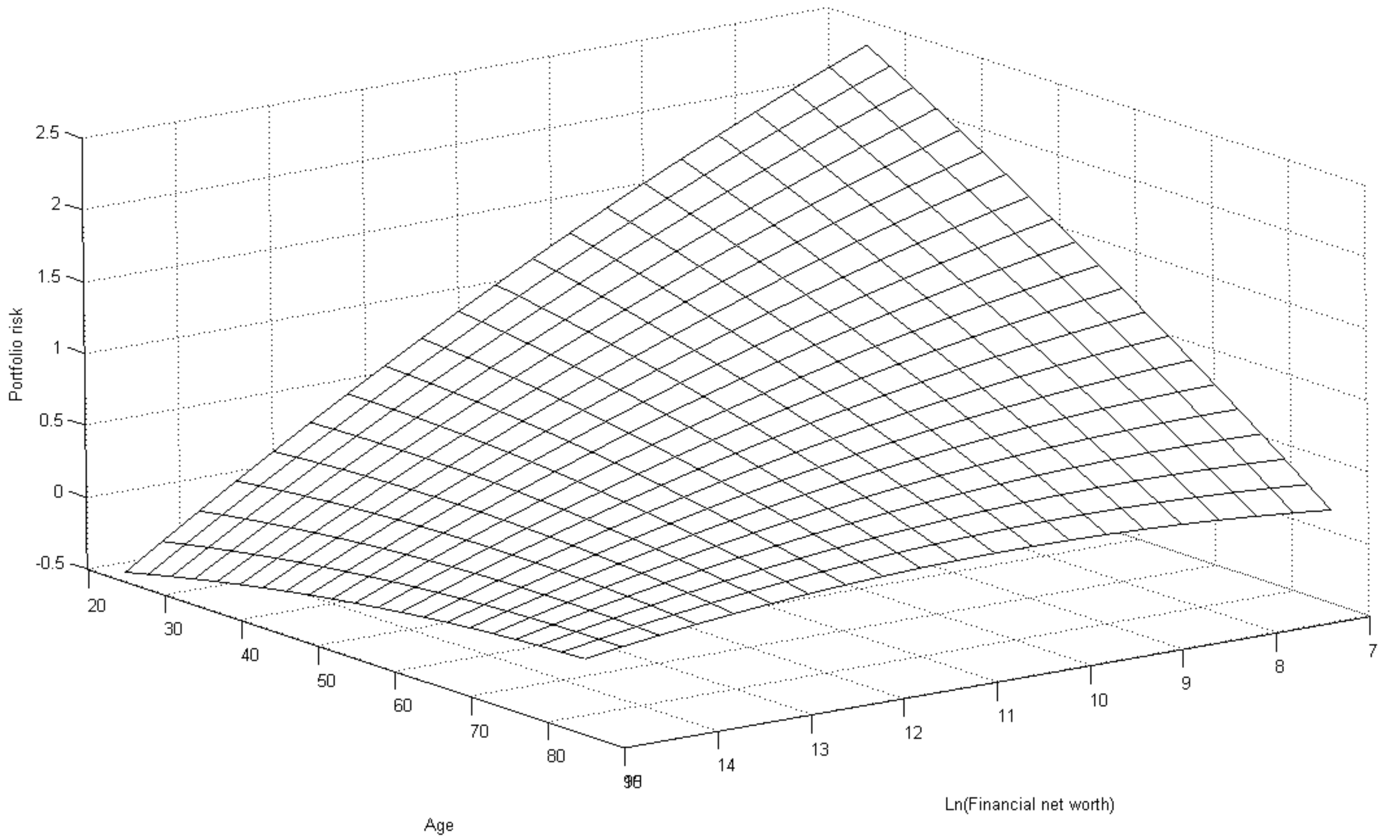


Table 4.9: Leveraged risk for holding all risky assets with alternative variance-covariance matrix

Each column presents estimates from an OLS regression. The dependent variable is the portfolio risk, measured by the standard deviation of the portfolio and computed with alternative variance-covariance matrix in Table 4.2. The portfolio is composed of bonds, stocks, private business, real estate, and debt. Financial net worth refers to the sum of bonds, stocks, private business, and real estate minus all the debts. T-statistics are in parentheses. ***Significant at 1%; **significant at 5%; *significant at 10%.

	Portfolio risk		
	(1)	(2)	(3)
Constant	3.930353***	5.331025***	5.973774***
	(11.36)	(14)	(16.48)
Age	-0.01797***	-0.06504***	-0.07684***
	(-3.62)	(-8.71)	(-10.7)
Age^2	0.000073	-0.000093*	0.0000495
	(1.56)	(-1.85)	(0.99)
Ln(Financial net worth)	-0.21271***	-0.25336***	-0.46725***
	(-3.66)	(-4.39)	(-8.48)
$(\text{Ln(Financial net worth)})^2$	-0.00037	-0.01046***	-0.00354
	(-0.15)	(-3.72)	(-1.33)
Age × Ln(Financial net worth)		0.005536***	0.005592***
		(8.36)	(8.97)
Family units			-0.00858
			(-0.45)
Sex of head			0.022804
			(0.58)
Number of children			0.042938*
			(1.87)
Marital status			0.123459***
			(3.32)
House ownership			0.869927***
			(18.43)
Employment			0.112121***
			(2.97)
High school diploma			0.020768
			(0.7)
College diploma			0.119664***
			(4.58)
Adjusted R^2	0.2714	0.2872	0.3766
No. of observations	3115	3115	3115

The present analysis suggests that the main result of this chapter is robust to alternative variance-covariance matrices. Since portfolio risk is a function of asset weights and variance-covariance matrix, we can conclude that asset weights are of the first order importance to derive the results of this chapter. In this sense, this chapter does not deviate too much from the literature in which asset weights are treated as the dependent variables.

4.7 Conclusion

I present new evidence which contradicts the existence of portfolio composition puzzles. When the household portfolio allows for non-stock assets and leverage, young and low-wealth households are shown to hold more risky portfolios than old and rich households, which is consistent with the predictions of standard financial models. For a long time, the literature has been struggling to modify the classical finance models and/or to create new theories to resolve the composition puzzles. This chapter suggests that the classical models might in fact very well describe how the households adjust the total risk of household portfolio to the change of age and wealth. Asset allocation advice from financial planners and other professionals is also validated by the data: as individuals age, they should reduce their portfolio risk. Future research should focus on the relative proportions of different classes of assets in the household portfolio.

This chapter also suggests that leverage risk is an important source of household portfolio risk. Given the fact that the growing popularity of 401(k) retirement plans gives individuals a great amount of freedom to choose an asset allocation, individuals need to understand leverage ratio, partially determined by their housing demand and consumption level, should affect their asset allocation in the retirement plans.

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Erklärung

Ich versichere hiermit, dass ich die vorliegende Arbeit mit dem Thema

Essays in Life-Cycle Finance:
Understanding Personal Investment and Consumption Choices

ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus anderen Quellen direkt oder indirekt übernommenen Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Weitere Personen, insbesondere Promotionsberater, waren an der inhaltlich materiellen Erstellung dieser Arbeit nicht beteiligt. Die Arbeit wurde bisher weder im In- noch im Ausland in gleicher oder ähnlicher Form einer anderen Prüfungsbehörde vorgelegt.

Konstanz, den 30/11/2008 ____________________

(Fangyi Jin)