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Two-Fraction Electron System on a Thin Helium Film

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A systematic theoretical investigation of microwave absorption of 2–dimensional electron systems above a thin helium film in the presence of a cyclotron resonance magnetic field is presented. To explain the measured data, a two-fraction structure of the electron system is introduced. One component corresponds to the free electron motion, the second one takes into account electron localization near the potential minimum caused by the roughness of the substrate. Within this model the general dependence of microwave absorption becomes understandable. The details of the observed cyclotron resonance line-shift are discussed.

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1. INTRODUCTION

Two-dimensional (2D) electron systems, localized on the surface of liquid helium, have been studied widely by cyclotron resonance (CR) absorption techniques. Both the shift of the position of the CR absorption line and the change of the linewidth provide information about the fundamental transport parameters of the electron system.

Existing cyclotron resonance data [1,2] for the electron system on a thin helium film show interesting features. First of all the free electron CR–line is permanently presented in the general magnetic field dependence of microwave absorption. There is neither any "dimple" nor roughness corrections to its position. The small CR–line shift comes just from the very thin helium film thickness, d, and has an unexpected film thickness dependence. The absence of a conventional dimple CR–line shift appears quite strange, especially in consideration of the direct observation of the dimple effect in transport measurements [3].

Secondly, the CR–line width has an unexpected asymmetry (see Fig. 1). This asymmetry grows as the film thickness decreases, and finally (for $d \leq 10^{-5}$ cm) the asymmetry contribution in the CR–linewidth looks as a dominating effect in the general CR–line structure (see Fig. 2).

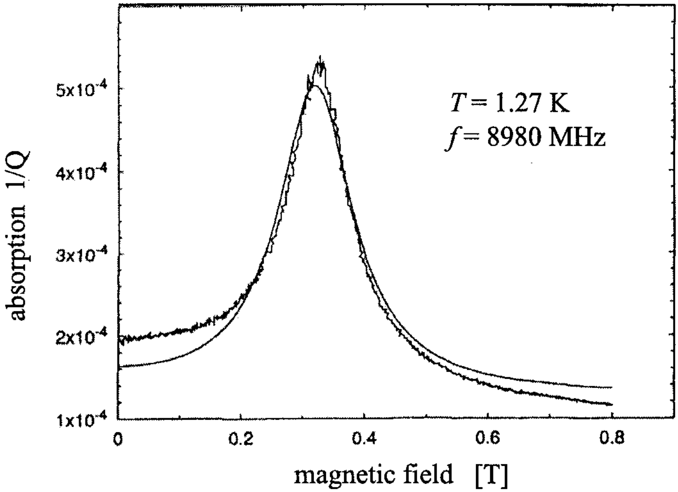

Fig. 1. Shown are CR–data from Ref. [1] for 2D–electrons on a thin helium film covering a hostaphan surface. The solid line corresponds to the Drude absorption with the fitting of the maximum and the half-width of the experimental curve. It can be noted how the asymmetry develops: the left side of the experimental data is above the Drude prediction, whereas the right side is below. The electron density is 2×10^8 cm^{-2}.

We explain the indicated CR–line features for electrons on thin helium films within the so-called two–fraction scenario in the 2D electron distribution. One of these components, n_e, corresponds to the free electron motion along the helium surface. The second one, n_l, is represented by possible electron localization in dimples of the liquid helium film or in potential wells due to roughness of the solid substrate. It is evident that

$$n_e + n_l = n_s \tag{1}$$

where n_s is the total 2D electron density, which has to be fixed during the microwave measurements. However, the relationship between these fractions is flexible. Therefore, the problem is reduced to the definition of n_l and n_e and to the construction of the effective conductivity which is also dependent on n_l and n_e.

2. BASIC DEFINITIONS

Roughness: The substrates of Refs. [1] and [2] are optically smooth. Nevertheless they could be rough at a level of 10^{-5} cm. And if the helium film thickness has the same scale, then the influence of substrate roughness becomes important.

A rough substrate with the profile $\delta(x)$ is characterized by some amplitude distribution (e.g. Gaussian) as

$$D(\delta) = \frac{1}{(2\pi\Delta^2)^{1/2}} \exp\left(-\delta^2/2\Delta^2\right) \tag{2}$$

where $\Delta^2 =< \delta^2 >$ is the mean-square amplitude deviation in vertical direction. We need also an auto-correlation function for $\delta(x)$, such as

$$< \delta(x)\delta(x-x') > \; = \; \Delta^2 \exp\left(-x'^2/2\zeta^2\right), \tag{3}$$

here ζ is the correlation length in horizontal direction. Using Eq. (3) and the second derivative of the substrate profile, δ'', we can write the correlation function

$$< \delta''\delta'' > \; = \; \beta^2 \; \equiv \; 3\Delta^2/\zeta^4 \tag{4}$$

and the corresponding δ'' distribution as

$$D(\delta'') = \frac{1}{(2\pi\beta^2)^{1/2}} \exp\left[-(\delta'')^2/(2\beta^2)\right] . \tag{5}$$

The helium film, defined as $d(x) = d + \xi(x)$, will round the substrate profile $\delta(x)$. Via such a screening of the roughness we can introduce both the active tops (with the positive ξ'' top derivative) and the passive tops (here $\xi'' < 0$). The coupling electron energy, V_a, near some active tops has the following form

$$V_a \geq -\Lambda/d_a, \quad \Lambda = \frac{e^2(\epsilon_d - 1)}{4(\epsilon_d + 1)}, \quad f/d_a^3 \simeq \rho g h + \alpha < \delta''\delta'' >^{1/2} \tag{6}$$

where ϵ_d is the dielectric constant of the solid substrate, α the liquid helium surface tension, f the van der Waals constant, and h the substrate level with respect to the bulk helium surface. For the validity of Eq. (6) we assume that the local derivatives ξ'' above the active tops are less than the corresponding derivatives δ'', so that

$$< \xi''\xi'' > \quad \leq \quad < \delta''\delta'' > \; .$$

Definition (6) shows that the thickness of the helium film at the local tops is formed both by h (typical for a flat helium film), and an additional Laplace pressure, which compresses a helium film at the active tops.

The next important characteristics is the density of the active tops, n_a. We define, that

$$n_a^{-1/2} \int\limits_{\delta_c}^{+\infty} D(\delta) d\delta = (\zeta^2)^{1/2} \; , \quad \zeta^2 \exp{(\delta_c^2/2\Delta^2)} \simeq n_c^{-1} \tag{7}$$

$$n_c^{-1} + (R - \Delta)^2 = R^2 \; , \quad \alpha/R \simeq \rho g h \; , \quad d \ll h \; . \tag{8}$$

Here Δ is from Eq. (2), ζ is from Eq. (3), and R is the so-called screening radius. If h is small enough and therefore R big with respect to Δ (i.e., $R^2 \gg \Delta^2$), then n_c from Eq. (8) becomes negligible, because

$$n_c^{-1} \simeq 2R\Delta.$$

Under these conditions δ_c from Eq. (7) goes to infinity and the corresponding density $n_a \to 0$. Increasing h leads to a decrease of δ_c. Simultaneously we have a growing in both n_c and n_a.

It is natural also to introduce the "top" eigen–frequencies, ω_a. Using the coupling electron energy in a form

$$V_a \simeq -\Lambda/d_a(x) \; , \quad d_a(x) \simeq d_r(0)(1 + \delta'' x^2/2(d_a(0)) \tag{9}$$

and the expansion of this energy near the minimum of the tops, we get

$$V_a(x) \simeq -\Lambda/d_a(0) + k_a x^2/2 \; , \quad k_a = \Lambda\delta''/d_a(0)^2 \; , \quad \omega^2 = k_a/m \; . \tag{10}$$

For the realization of Eq. (9) and (10) we assume that the surface of the helium film near the tops is practically flat (i.e., $R^2 \gg \Delta^2$), and so the local distance $d(x)$ between the electron and the position of the top is only sensitive to δ''.

It is necessary to note that the characteristics of our helium film on rough substrates does not overlap with the description in Refs. [4] and [5]. The formalism in Ref. [4] is correct, if

$$[\xi(x) + \delta(x)] \quad \ll \quad < d >$$

In our case, however, we use the statistic $\delta(x)$ distribution. So in this situation, as it follows from Eq. (2) and (5), the deviations in the $\delta(x)$ distribution can be arbitrarily big. And therefore it is necessary to use the definitions (2) to (10).

Chemical Potential: For the definition of the chemical potential, μ, we follow the tradition in semiconductors [6]. So the question arises regarding condition (1), with the components n_l and n_e versus the chemical potential,

$$n_l + n_e = n_s, \quad n_l = \frac{n_a}{\exp\left[(V_a - \mu)/T\right] + 1}, \quad V_a < 0 \tag{11}$$

$$n_e = \frac{n_e^o}{\exp\left(-\mu/T\right) + 1},$$

$$n_e^o \to \begin{cases} = mT/(2\pi\hbar^2), & T \gg \hbar\omega_c \\ = 1/(\pi l_H^2), & T \ll \hbar\omega_c\ , \end{cases}$$

where l_H is the magnetic length. Thus Eq. (11) is the definition of μ, as function of V_a, n_a, T, n_s, and H. So, in some limiting cases we get for the chemical potential:

$$\mu \to \begin{cases} = -T\ln\left[(n_i - n_s)/n_s\right]\ , & n_a \to 0 \\ = V_a - T\ln\left[(n_a - n_s)/n_s\right]\ , & n_a > n_s\ . \end{cases} \tag{12}$$

Absorption Properties: Let us now consider the absorption properties. Under the conditions of Ref. [1] the classical absorption picture is fulfilled. Using, in addition, the Drude approximation we have the following free electron absorption, $Q_e^{-1}(\omega, \omega_c)$,

$$Q_e^{-1}(\omega, \omega_c) = \frac{e^2\tau E_{\|}^2}{m}\, \frac{n_e(1 + \omega^2\tau^2 + \omega_c^2\tau^2)}{(1 - \omega^2\tau^2 + \omega_c^2\tau^2)^2 + 4\omega^2\tau^2}\ . \tag{13}$$

Here τ is the elastic time relaxation, $E_{\|}$ is the effective *ac* electric field along the helium surface, and n_e is the free electron density from Eq. (11).

The corresponding absorption $Q_l^{-1}(\omega, \omega_c)$ due to *local states* is

$$Q_l^{-1}(\omega, \omega_c) = \frac{e^2\tau E_{\|}^2}{m} n_l \int_0^{\Gamma} D(\gamma) f(\omega, \omega_c, \omega_a)\, d\gamma \tag{14}$$

$$\text{where} \quad f(z, x, t) = \frac{(z - t)^2 - z - zx - 2(z - z)^2}{\left[(z - t)^2 - z - zx\right]^2 + 4z(z - t)^2}$$

$$\text{with} \qquad z = \omega^2\tau^2, \quad x = \omega_c^2\tau^2, \quad t = \omega_a^2\tau^2,$$

$$\gamma = \tfrac{md^2}{\Lambda\tau^2}\omega_a^2\tau^2, \quad \Gamma = m\omega^2 d^2/\Lambda.$$

Here n_l is from Eq. (11), the function $\omega(\gamma)$ from Eq. (10), and $D(\gamma)$ from Eq. (5). The integral limits are written using the information with respect to the small probability for eigen–modes $\omega_a > \omega$ excitation.

The total absorption

$$Q^{-1} = Q_e^{-1} + Q_l^{-1} \tag{15}$$

contains five external parameters: d, n_s, τ, Δ, and ζ. It is reasonable to assume that d and n_s are well defined independently. The scale of τ can be estimated, using the known mobility calculations for 2D electrons on helium. As a result the above calculations have two fitting parameters, Δ and ζ, which have to be extracted from the experimental data. A convenient way for this purpose is the fitting of the ratio between the absorption amplitudes at ω_c and the amplitude, when $H = 0$. The second fitting parameter could be extracted from the shift of the CR–peak position.

3. DISCUSSION

Starting with the discussion of the general definitions, (13) to (15), we need a numerical treatment. Such an activity will be developed elsewhere. Here we propose just some analytic conclusions (mainly with respect to the existence of single electron dimples).

First of all, the existence of the peak absorption in the total Q expression, Eq. (15), is possible due to the contribution of Q_e^{-1}, Eq. (13), in these losses. The position of such a peak, when ω is fixed, follows from the condition

$$\omega_c^2\tau^2 = \omega^2\tau^2 - 1/[4(\omega\tau)^2] \quad . \tag{16}$$

As the local absorption Q_l^{-1}, Eq. (14), has a monotonic structure as function of the magnetic field in the actual interval of $0 < \omega_c\tau \leq 1$ (see inset in Fig. 2) Eq. (16) shows that a "relaxation" shift of the CR–line is developed in the right direction. So decreasing of $\omega\tau$ shifts the CR–line towards smaller magnetic fields, see Fig. 2.

To give numbers, we get for peak "2" (in Fig. 2)

$$\omega^2\tau^2 \simeq 0.69 \quad . \tag{17}$$

In the case of peak "3"

$$\omega^2\tau^2 \simeq 0.59 \quad . \tag{18}$$

As one can see from Fig. 2 the quality in both resonances, (17) and (18), is quite bad. Nevertheless the position of these peaks corresponds to the free

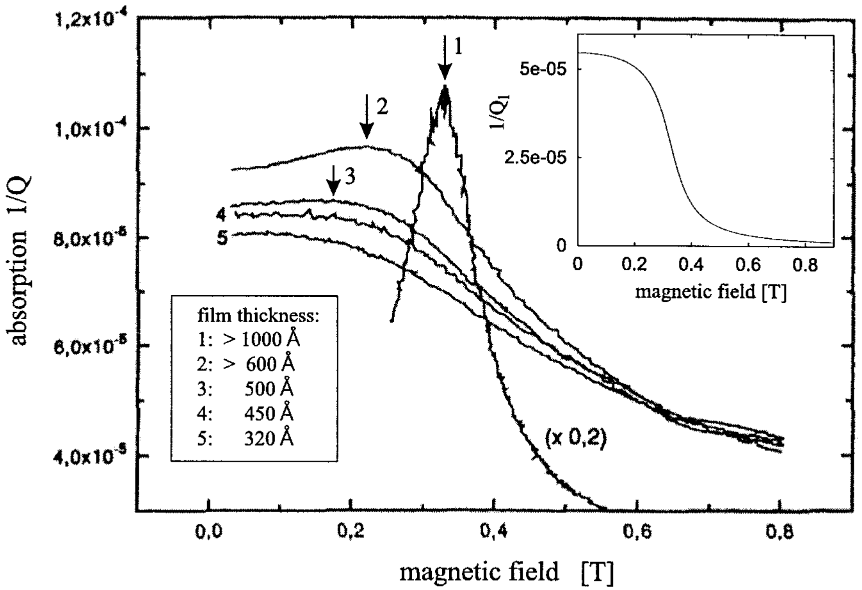

Fig. 2. Shown is the thickness dependence of the CR–absorption, from Ref. [1]. Line "1" is the same as in Fig. 1. The scale of this line, however, is reduced by a factor 5 to allow comparison with lines "2" to "5". The inset shows the behaviour of Q_l^{-1} as defined in Eq. (14).

electron motion with strong friction. If, e.g., one uses the dimple formalism for calculating peaks "2" and "3" (in this case $\Delta\omega_c \sim d^{-4}$, Ref. [7]), we get

$$\frac{\Delta\omega_3}{\Delta\omega_2} = \frac{d_2^4}{d_3^4} = 2.1 \quad . \tag{19}$$

However from the experimental data in Fig. 2, we get

$$\frac{\Delta\omega_3}{\Delta\omega_2} = \frac{\omega_1 - \omega_3}{\omega_1 - \omega_2} = 1.5 \quad . \tag{20}$$

The numbers in (19) and (20) are not consistent. Therefore the shifts $\Delta\omega_c$ from Fig. 2 can not be explained using the "dimple" language.

4. CONCLUSIONS

We have used a two-fraction model for the interpretation of existing CR–absorption data in a 2D electron system on thin helium films. This model helps to understand the reasons for the conservation of the free electron CR–line position simultaneously with the dramatic decrease of the CR–amplitude versus the decreasing helium film thickness. Besides the monotonic part in Eq. (14) (see also Fig. 2) the unusual asymmetric behaviour of the observed CR–absorption is explained.

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