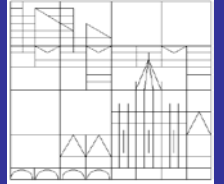




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Business Cycle: A Quantile Structural Vector
Autoregressive Approach***

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ASYMMETRIC EFFECTS OF UNCERTAINTY OVER THE BUSINESS CYCLE: A QUANTILE STRUCTURAL VECTOR AUTOREGRESSIVE APPROACH*

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Abstract

This paper proposes to relate conditional quantiles of stationary macroeconomic time series to the different phases of the business cycle. Based on this idea, I introduce a Bayesian Quantile Structural Vector Autoregressive framework for the analysis of the effects of uncertainty on the US real economy. For this purpose, I define a novel representation of the multivariate Laplace distribution that allows for the joint treatment of multiple equation regression quantiles. I find significant evidence for asymmetric effects of uncertainty over the US business cycle. The strongest negative effects are revealed during recession periods. During boom phases uncertainty shocks improve the soundness of the economy. Moreover, the phase of the financial sector matters when the real economy is at recession but not if the economy is at boom. When the financial system is in a bad state, an uncertainty shock leads to a deeper recession than in times when the financial system is in a good state.

Keywords: Uncertainty · Economic Cycles · Quantile SVAR · Multivariate Laplace

JEL-Codes: C32 · E44 · G01

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1 INTRODUCTION

In the wake of the Great Depression, Keynes (1936) notes that business cycles are asymmetric and Fisher (1933) remarks that cycles may be milder depending on the state of the different macrovariables. The past Global Financial Crisis suggests equivalent conclusions. It provides firm evidence for strong asymmetries – or nonlinearities – in the correlation of macroeconomic and financial time series that seem to depend on the state of each variable in the system. This is especially evident in the case of uncertainty. The severity of the last recession is argued to be partially explained by the high level of uncertainty (e.g., Stock and Watson (2012)). That is, the state of uncertainty has been a decisive factor for the intensity of the last recession. In general, Reinhart and Rogoff (2009), Claessens et al. (2012), or Brunnermeier and Sannikov (2013) suggest that the state of the financial sector is crucial for the response of an economy to a shock. Even though, many studies point out the important negative consequences of uncertainty for the economy (see e.g., Bloom (2009), Alexopoulos and Cohen (2009), Bachmann et al. (2013), Bloom et al. (2012), Leduc and Liu (2012), Gilchrist et al. (2013), and Carrière-Swallow and Céspedes (2013)), they do not investigate the nonlinear effects related to cycles. This however is crucial, e.g., for designing policies. Policies target economies at specific phases of their cycles and should thus take these asymmetries into account.

In light of these observations, the present study aims at contributing to the literature by finding evidence for the following three questions: (1) How asymmetric are the effects of uncertainty on the US real economy over the business cycle? (2) How decisive is the phase of the financial sector for the impact of an uncertainty shock on the US real economy? (3) How does the inference over the cycle compare to the effects at the *average state* of the economy?

For the analysis of asymmetries, I propose to relate conditional quantiles of stationary macroeconomic time series to the phases of their cycle. For instance, in the case of (conditional) GDP growth, lower quantiles, i.e., lower growth rates, can rather be attributed to recessions, whereas upper quantiles, i.e., higher growth rates, can be attributed to booms. To fix ideas, consider a univariate regression in which GDP growth is explained by uncertainty, lagged by one period.¹ This relationship can be estimated at different quantiles (τ) using the methodology provided by Koenker and Bassett (1978) or at the mean via ordinary least squares (OLS). Figure 1 depicts such regression lines (at various quantiles and at the mean) and the data points of US GDP growth versus lagged uncertainty. Further, filled circles represent recession periods.² It can be clearly seen that the lower quantile regression lines, i.e., for $\tau = \{0.1, 0.2\}$ describe the dependencies during recession periods, while upper quantile regression lines $\tau = \{0.8, 0.9\}$ portray the effect of uncertainty during non-recession, i.e. growth periods. Moreover, the graph yields evidence for nonlinearities in the data as slopes for the lower

¹For details on the definition of variables and their transformations, please see Appendix C.

²These are as defined by the National Bureau of Economic Research.

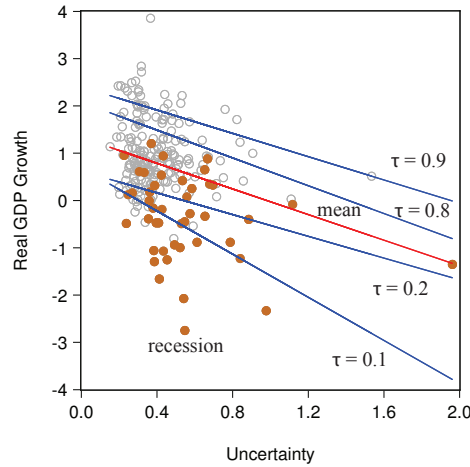


Figure 1: Scatter Plot with Regression Lines at the **Mean** and Different **Quantiles**

Note: The scatter plot shows the data points of quarterly US real GDP growth and US uncertainty lagged by one quarter for the time period 1954Q1 to 2012Q4. For details on the definition of variables see Appendix C. Filled circles depict recession periods as defined by the National Bureau of Economic Research. Regression lines are as indicated for different quantiles and for the mean using ordinary least squares.

quantiles differ from each other and the mean. In contrast, in the case of the upper quantiles the regression lines portray marginal effects similar to the mean. Thus, I argue that conditional quantiles capture the asymmetric dependencies over the cycle.

A study of the nonlinear dependencies within an economy requires a structural multiple equation framework, i.e., it is vital to model the interaction effects of a system of variables. Hence, I develop a framework that allows for such an analysis. It is called *Quantile Structural Vector Autoregressive* (QSVAR) model. For the formulation and estimation of the QSVAR, I propose a novel representation of the multivariate Laplace distribution that permits the joint treatment of multiple equation regression quantiles. In this, I build on the work by Koenker and Machado (1999) and Yu and Moyeed (2001) who show that the univariate Laplace distribution may be used to estimate regression quantiles. That is, in some sense, the Laplace distribution behaves to the quantile loss function as the Gaussian distribution behaves to the squared loss function.³ Yu and Moyeed (2001) suggest Bayesian methods for estimation. This path is taken here as well.

From a methodological perspective, this paper is closely related to Cecchetti and Li (2008). The authors are the first to present a quantile vector autoregressive framework that is estimated via general methods moments. They analyze different effects of asset price booms and busts on the economy in two dimensional systems. The present methodology has the advantage that it can easily accommodate higher dimensional models. Through its Bayesian orientation inference is carried out directly within the sampler, while the latter avenue runs into difficulties because of nontrivial matrix differentiations.

³The expression *in some sense* is clarified in Section 2.

In addition, as the present methodology analyzes interdependencies over the business cycle, it is also related to Markov switching (MS)-VAR models that were first introduced by Hamilton (1989). Markov switching models assume that the economy is governed by regimes that can be related to booms or busts. The current approach differs in two important aspects. First, each variable can be defined to be at some specific state of the cycle for the analysis, i.e., the quantiles may differ across equations. The MS approach estimates the entire system at one or the other regime. Second, the present methodology estimates the system over a fine grid of the cycle, whereas the MS tool is only feasible for a small number of regimes.

I find that the effects of uncertainty shocks on the US real economy are highly asymmetric over the business cycle. The effects are significantly different from the results for an economy at the *average* state at the lower and upper quantiles.

Results reveal the strongest negative effects on the economy during recession periods. During this phase of the cycle, findings are in line with the usual interpretation of uncertainty shocks. Uncertainty is argued to induce firms to pause investments and to defer consumer spending, leading to a decrease in aggregate demand.⁴ Prices fall and the monetary base shrinks, while the monetary authority lowers interest rates. During the deepest recessions uncertainty shocks contribute by 45% to the fluctuations in real GDP growth.⁵ The same figure for the *average* state of the economy amounts to 10%.

Strikingly, findings indicate that the economy may experience positive effects in response to an uncertainty shock, namely during the highest boom periods. At this phase a shock to uncertainty causes positive growth in GDP, inflation, money supply, and the federal funds rate. I argue, in light of the evidence, that it represents a sensible finding. This study measures uncertainty by volatility on the stock market. Thus, the measure represents changes in the prices of shares. During strong boom phases uncertainty shocks are found to lead to a rise in these prices. Firms and consumers, hence, interpret unexpected rises in volatility as an improvement of the economic outcome. On this ground I name these unexpected changes exuberance shocks. Further, I interpret this finding as evidence for speculative bubbles. An exuberance shock is highly important for fluctuations in inflation and money supply growth, which are key ingredients for the development and final burst of speculative bubbles. At last, the state of the financial sector is decisive for the effect of an uncertainty shock if the real economy is in recession. If the financial sector is in a bad state the resulting recession is found to be stronger. In contrast, the state of the financial system is not important for an economy at boom in the short run.

The paper is structured as follows. Section 2 provides the reader with the basics for understanding quantile regressions and presents the multivariate Laplace distribution for multiple equation quantile

⁴See e.g. Romer (1990) for a discussion on the effects on consumer spending. For the impact on firms see, e.g., Bernanke (1983)

⁵The lowest quantile treated in this paper is the 10% quantile, which is here referred to as the deepest recession.

regression. Section 3 introduces the QSVAR and outlines the sampler for its estimation. Section 4 depicts the empirical setup and presents the results. Section 5 concludes.

2 QUANTILE REGRESSION AND THE LAPLACE DISTRIBUTION

First, I give a short introduction to single equation quantile regression and highlight the link to the univariate Laplace distribution.⁶ In the following, I derive the multivariate generalization of the Laplace distribution that allows for the joint treatment of multiple equation quantile regressions.

2.1 Introduction to Single Equation Quantile Regression

The following exposition introduces quantile regression using an autoregressive process with one lag (AR(1)), i.e., $y_t = \phi_\tau y_{t-1} + v_t$ for $t = 1, \dots, T$, where T denotes the sample size, y_t the endogenous variable, ϕ_τ the autoregressive coefficient at quantile τ ($0 \leq \tau \leq 1$), and v_t the error term.⁷ To obtain the estimated marginal effect on the dependent variable at quantile τ , i.e., $\hat{\phi}_\tau$, one needs to solve – given τ – the minimization problem

$$\operatorname{argmin}_{\phi_\tau} \sum_{t=1}^T \rho_\tau(y_t - \phi_\tau y_{t-1}), \quad \text{where } \rho_\tau(v_t) = \begin{cases} v_t \cdot \tau & , \text{ if } v_t \geq 0 \\ v_t \cdot (\tau - 1) & , \text{ if } v_t < 0. \end{cases}$$

The loss function in the quantile framework, $\rho_\tau(v_t)$, leads to $\Pr(v_t < 0 | y_{t-1}) = \tau$. Or put differently, $Q_\tau(v_t | y_{t-1}) = 0$ where $Q_\tau(\cdot)$ yields the τ 'th quantile of the respective variable.⁸ This result entails that $Q_\tau(y_t | y_{t-1}) = \phi_\tau y_{t-1}$.

It has already been noted that the Laplace distribution behaves to the quantile loss function in some way similar as the Gaussian distribution to the squared loss function of OLS. The Laplace density for the above example may be written as

$$f_\tau(v_t) = \tau(1 - \tau) \exp\{-\rho_\tau(v_t)\}.\tag{1}$$

The Gaussian versus OLS case is similar in the sense that maximizing the product of the densities, i.e.,

$$\operatorname{argmax}_{\phi_\tau} \prod_{t=1}^T f_\tau(y_t - \phi_\tau y_{t-1})$$

yields a consistent estimate $\hat{\phi}_\tau$. This result for the univariate Laplace distribution is an important finding since it enables the use of Bayesian methods for quantile regressions, as Bayesian methods

⁶For a thorough introduction to quantile regression please refer to Koenker (2005).

⁷Quantile Autoregression in the frequentist context is discussed by Koenker and Xiao (2006).

⁸This is a similar condition as $E(v_t | y_{t-1}) = 0$ in the OLS framework.

⁹Yu and Zhang (2006) provide a detailed discussion of the characteristics of the univariate Laplace density used for quantile regression.

require a well defined likelihood function.¹⁰ In this context Sriram et al. (2013) show that the use of this density – even under an improper prior – leads to a consistent posterior distribution.

2.2 The Multivariate Laplace Distribution for Quantile Regression

This paper develops a multivariate Laplace distribution that can be used for multiple equation quantile regressions. In order to derive the formulation, it is best to begin with the general characteristic function of a univariate Laplace and derive the restrictions for univariate quantile regressions. This result can then be generalized to the multivariate setting.

The Laplace density of equation (1) is a special case of the general Laplace distribution. The characteristic function of this general density is given by

$$\Psi_{v_t}(s) = \frac{1}{1 + \frac{1}{2}\sigma^2 s^2 - ims},$$

where $m \in \mathbb{R}$, $\sigma \geq 0$, i is the imaginary unit, and s an arbitrary real number. m reflects the skewness parameter and σ the scale parameter. I denote the distribution as $\mathcal{L}(m, \sigma)$. For further details please see Kotz et al. (2001).¹¹

If one assumes that

$$m = \frac{1 - 2\tau}{\tau(1 - \tau)} \quad \text{and} \quad \sigma^2 = \frac{2}{\tau(1 - \tau)},$$

the distribution turns out to be the one presented in equation (1). For the proof please see Appendix A. Let this restricted distribution be denoted by $\mathcal{L}(m_\tau, \sigma_\tau)$.

To extend the above result to the multivariate setting note that, following Kotz et al. (2001), the characteristic function of a general multivariate Laplace is defined as

$$\Psi_{\mathbf{v}_t}(\mathbf{s}) = \frac{1}{1 + \frac{1}{2}\mathbf{s}'\boldsymbol{\Sigma}\mathbf{s} - i\mathbf{m}'\mathbf{s}},$$

where $\mathbf{v}_t \in \mathbb{R}^d$, $\mathbf{m} \in \mathbb{R}^d$, $\boldsymbol{\Sigma}$ is a $(d \times d)$ nonnegative definite symmetric matrix, and \mathbf{s} is a $(d \times 1)$ vector of arbitrary real numbers. In addition, d denotes the number of random variables. Let the general multivariate Laplace distribution be denoted by $\mathcal{L}_d(\mathbf{m}, \boldsymbol{\Sigma})$.

Kotz et al. (2001, p. 247) note that each component of \mathbf{v}_t , i.e., v_{jt} ($j = \{1, \dots, d\}$), admits a univariate representation. This implies that the same restrictions, which are required in the univariate framework,

¹⁰This finding spurred research on quantile regression in the Bayesian context. For instance, Alhamzawi and Yu (2013) discuss conjugate priors and variable selection. Priors are also discussed in Li et al. (2010) for the use of regularization as, e.g., via lasso. Benoit and van den Poel (2012) offer a model for quantile regression in the case of a dichotomous response variable. Geraci and Bottai (2007), Liu and Bottai (2009), Geraci and Bottai (2013), Luo et al. (2012), Reich et al. (2010), and Kobayashi and Kozumi (2012) present an approach for panel data. Chen and Gerlach (2009) present an approach that accounts for heteroskedasticity that is, e.g., present in financial data.

¹¹In general, i.e., in the univariate as well as in the multivariate setting Kotz et al. (2001) distinguish between a symmetric and an asymmetric distribution. For quantile regression the asymmetric distribution is of interest, as the symmetric distribution only covers the case of least absolute deviation regression. The symmetric is nested in the asymmetric distribution and given in case $m = 0$.

also apply for each component v_{jt} . Additionally, it entails that each univariate Laplace may be defined for a different quantile. Thus, the elements of \mathbf{m} and the diagonal elements of Σ have to fulfill the following criteria

$$m_j = \frac{1 - 2\tau_j}{\tau_j(1 - \tau_j)} \quad \text{and} \quad \sigma_{jj}^2 = \frac{2}{\tau_j(1 - \tau_j)}.$$

Let this restricted multivariate Laplace distribution be denoted by $\mathcal{L}_d(\mathbf{m}_\tau, \Sigma_\tau)$, where $\tau = (\tau_1, \dots, \tau_d)'$ is a $(d \times 1)$ vector of possibly different quantiles.

While the diagonal elements of Σ_τ are restricted, the off-diagonal elements of Σ_τ are not restricted. These control the covariances between the univariate asymmetric Laplace distributions.¹² The covariances can be decomposed into the product of the unrestricted correlations and the restricted variances, i.e., $\rho_{lk}\sigma_{\tau_l}\sigma_{\tau_k}$, where $l, k \in \{1, \dots, d\}$ and $\sigma_{\tau_j} = \sqrt{\frac{2}{\tau_j(1-\tau_j)}}$. In this way, Σ_τ may be decomposed to yield

$$\Sigma_\tau = \mathbf{S}_\tau \mathbf{R} \mathbf{S}_\tau', \quad (2)$$

where \mathbf{R} denotes the correlation matrix with ones on the diagonal and ρ_{lk} as off diagonal elements and $\mathbf{S}_\tau = \text{diag}(\sigma_{\tau_1}, \dots, \sigma_{\tau_d})$. This representation is used in the sampler proposed in this paper.

Finally, note that the quantile restrictions lead to a Laplace distribution with a variance that is, besides the correlation structure in \mathbf{R} , completely defined through τ .¹³ However, one might wish to relax this restriction on the variance. To this end, let \mathbf{B} denote a scaling parameter, that is defined by $\mathbf{B} = \text{diag}(b_1, \dots, b_d)$. Following Kotz et al. (2001, p. 254) it holds that $\mathbf{B}\mathbf{v}_t \sim \mathcal{L}_d(\mathbf{B}\mathbf{m}_\tau, \mathbf{B}\Sigma_\tau\mathbf{B}')$.

3 QUANTILE STRUCTURAL VECTOR AUTOREGRESSIONS

This paper provides a methodology for the estimation of reduced form quantile VARs. The structural model is then recovered in a second step by decomposing the covariance matrix of the error term. The reduced form VAR is given as

$$\mathbf{y}_t = \boldsymbol{\nu}_\tau + \sum_{i=1}^p \mathbf{A}_{\tau,i} \mathbf{y}_{t-i} + \mathbf{u}_t, \quad \text{for } t = 1, \dots, T, \quad (3)$$

where $\mathbf{y}_t = (y_{1t}, y_{2t}, \dots, y_{dt})'$ is a $(d \times 1)$ vector of endogenous variables, $\boldsymbol{\nu}_\tau$ is a $(d \times 1)$ vector of intercepts at quantiles $\tau = (\tau_1, \dots, \tau_d)'$, $\mathbf{A}_{\tau,i}$ for $i = 1, \dots, p$ denotes the matrix of lagged coefficients of size $(d \times d)$ also at quantiles $\tau = (\tau_1, \dots, \tau_d)'$, and $\mathbf{u}_t = (u_{1t}, u_{2t}, \dots, u_{dt})'$ is a $(d \times 1)$ vector of error terms.

To obtain the estimated coefficient matrices $\hat{\mathbf{A}}_{\tau,i}$ and $\hat{\boldsymbol{\nu}}_\tau$ I propose to assume

$$\mathbf{u}_t \sim \mathcal{L}_d(\mathbf{B}\mathbf{m}_\tau, \mathbf{B}\Sigma_\tau\mathbf{B}'),$$

¹²The bivariate case is discussed in Appendix A.2. This example shows explicitly how the correlation between several univariate asymmetric Laplace distributions is defined.

¹³The variance of the multivariate Laplace with quantile restrictions is given by $\mathbf{m}_\tau \mathbf{m}_\tau' + \Sigma_\tau$.

and develop in the following a sampler for its estimation.

In order to recover the structural model the covariance matrix of the error is required. Even though the assumption of a multivariate Laplace delivers consistent posteriors for $\mathbf{A}_{\tau,i}$ and ν_{τ} the methodology does not directly provide an estimate for the covariance matrix of the error. However, similar to Li et al. (2012) and Cecchetti and Li (2008), the covariance matrix can be defined in the following way:

$$\mathbf{\Omega}_{\tau} = (\omega_{jk}) = \left(\frac{\mathbb{E}[(\rho_{\tau_j}(u_{jt}))(\rho_{\tau_k}(u_{kt}))]}{\mathbb{E}[f_{u_{jt}}(0)]\mathbb{E}[f_{u_{kt}}(0)]} \right), \text{ where } j, k \in \{1, \dots, d\}. \quad (4)$$

$f_{u_{jt}}(0)$ denotes the pdf of u_{jt} evaluated at 0. For the structural model the estimated covariance matrix $\hat{\mathbf{\Omega}}_{\tau}$ may then be decomposed, e.g., by the Cholesky decomposition. For details on the estimation of $\mathbf{\Omega}_{\tau}$ please refer to Appendix B.1.

3.1 Estimation Setup

In this section I first describe a location scale mixture representation of the multivariate Laplace that is exploited in the Bayesian approach of this paper. It builds on the contributions by Tsionias (2003) and Kozumi and Kobayashi (2011) that are established in the context of the univariate Laplace distribution.

The mixture representation of the multivariate Laplace permits to cast the estimation problem into a maximization problem that involves the normal distribution. Thus, common results for estimation may be applied. More specifically, Kotz et al. (2001, p. 246) note that the multivariate Laplace can be represented in the following way:

$$\mathbf{u}_t = \mathbf{B}\mathbf{m}_{\tau}w_t + \sqrt{w_t}\mathbf{B}\mathbf{\Sigma}_{\tau}^{1/2}\mathbf{z}_t,$$

where w_t represents a standard exponential random variable ($w_t \sim \mathcal{E}(1)$) and \mathbf{z}_t is a d -dimensional random vector that is standard multivariate normal ($\mathbf{z}_t \sim \mathcal{N}_d(\mathbf{0}, \mathbf{I}_d)$, where \mathbf{I}_d is the identity matrix of dimension d). $\mathbf{\Sigma}_{\tau}^{1/2}$ represents the square root matrix $\mathbf{\Sigma}_{\tau}$, that yields $(\mathbf{\Sigma}_{\tau}^{1/2})(\mathbf{\Sigma}_{\tau}^{1/2})' = \mathbf{\Sigma}_{\tau}$.

This result allows to rewrite equation (3) to yield

$$\mathbf{y}_t = \nu_{\tau} + \sum_{i=1}^p \mathbf{A}_{\tau,i}\mathbf{y}_{t-i} + \mathbf{B}\mathbf{m}_{\tau}w_t + \sqrt{w_t}\mathbf{B}\mathbf{\Sigma}_{\tau}^{1/2}\mathbf{z}_t.$$

It follows that the conditional distribution of \mathbf{y}_t given \mathbf{A}_{τ} , $\mathbf{\Sigma}_{\tau}$, \mathbf{B} , w_t , and \mathcal{F}_{t-1} is normal, where \mathbf{A}_{τ} denotes the matrix of coefficients $(\nu_{\tau}, \mathbf{A}_{\tau,1}, \dots, \mathbf{A}_{\tau,p})'$ of dimension $((1 + dp) \times d)$, and \mathcal{F}_{t-1} is the information set that includes all relevant past values of \mathbf{y}_t . The first two conditional moments of \mathbf{y}_t are given by:

$$\mathbb{E}[\mathbf{y}_t | \mathbf{A}_{\tau}, \mathbf{\Sigma}_{\tau}, \mathbf{B}, w_t, \mathcal{F}_{t-1}] = \nu_{\tau} + \sum_{i=1}^p \mathbf{A}_{\tau,i}\mathbf{y}_{t-i} + \mathbf{B}\mathbf{m}_{\tau}w_t = \boldsymbol{\mu}_{\tau,t}$$

$$\mathbb{V}[\mathbf{y}_t | \mathbf{A}_{\tau}, \mathbf{\Sigma}_{\tau}, \mathbf{B}, w_t, \mathcal{F}_{t-1}] = w_t\mathbf{B}\mathbf{\Sigma}_{\tau}\mathbf{B}' = w_t\mathbf{\Sigma}_{\tau*},$$

where $\Sigma_{\tau^*} = \mathbf{B}\Sigma_{\tau}\mathbf{B}'$. Thus, it holds that

$$\mathbf{y}_t|\mathbf{A}_{\tau}, \Sigma_{\tau}, \mathbf{B}, w_t, \mathcal{F}_{t-1} \sim \mathcal{N}_d(\boldsymbol{\mu}_{\tau,t}, w_t\Sigma_{\tau^*}). \quad (5)$$

For the complete likelihood function please refer to Appendix B.2.

3.2 Bayesian Inference

This section introduces the conditional posterior distributions of $\boldsymbol{\alpha}_{\tau}$, Σ_{τ} , w_t , and \mathbf{B} , which are used in the proposed Metropolis-within-Gibbs sampler. $\boldsymbol{\alpha}_{\tau}$ denotes the column vector $\text{vec}(\mathbf{A}_{\tau})$ of size $(d(dp+1) \times 1)$. To ease the exposition I first cast the VAR model in compact form.

$$\mathbf{y} = (\mathbf{I}_d \otimes \mathbf{X})\boldsymbol{\alpha}_{\tau} + (\mathbf{B}\mathbf{m}_{\tau} \otimes \mathbf{I}_T)\mathbf{w} + \left(\mathbf{B}\Sigma_{\tau}^{1/2} \otimes \mathbf{W}^{1/2}\right)\mathbf{z},$$

where $\mathbf{y} = \text{vec}(\mathbf{y}_1, \dots, \mathbf{y}_T)'$ is a $(Td \times 1)$ vector of observations, $\mathbf{X} = (\mathbf{x}'_1, \dots, \mathbf{x}'_T)'$ is a $(T \times (dp+1))$ matrix, where $\mathbf{x}_t = (1, \mathbf{y}'_{t-1}, \dots, \mathbf{y}'_{t-p})$ represents a $(1 \times (dp+1))$ vector, $\mathbf{w} = (w_1, \dots, w_T)'$ is a $(T \times 1)$ vector and $\mathbf{W} = \text{diag}(\mathbf{w})$ reflects a $(T \times T)$ diagonal matrix. Thus, $\mathbf{W}^{1/2} = \text{diag}(\sqrt{w_1}, \dots, \sqrt{w_T})$. $\mathbf{z} = \text{vec}(\mathbf{z}_1, \dots, \mathbf{z}_T)$ denotes a $(Td \times 1)$ vector of standard multivariate normal random variables. Subsequently, the posteriors are introduced.

3.2.1 Conditional Posteriors of $\boldsymbol{\alpha}_{\tau}$ and Σ_{τ}

The prior is assumed to be of an independent Normal-inverse Wishart (\mathcal{IW}) type:¹⁴

$$\boldsymbol{\alpha} \sim \mathcal{N}(\underline{\boldsymbol{\alpha}}, \underline{\mathbf{V}}) \quad \text{and} \quad \Sigma \sim \mathcal{IW}(\underline{\Sigma}, \underline{\nu}).$$

Prior times likelihood yields the standard posterior probability density functions:¹⁵

$$\boldsymbol{\alpha}_{\tau}|\mathbf{y}, \Sigma_{\tau}, \mathbf{B}, \mathbf{w} \sim \mathcal{N}(\bar{\boldsymbol{\alpha}}_{\tau}, \bar{\mathbf{V}}_{\tau}) \quad \text{and} \quad \Sigma_{\tau}|\mathbf{y}, \boldsymbol{\alpha}_{\tau}, \mathbf{B}, \mathbf{w} \sim \mathcal{IW}(\bar{\Sigma}_{\tau}, \bar{\nu}),$$

where

$$\begin{aligned} \bar{\mathbf{V}}_{\tau} &= [\underline{\mathbf{V}} + ((\mathbf{B}\Sigma_{\tau}\mathbf{B}')^{-1} \otimes (\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}))]^{-1} \\ \bar{\boldsymbol{\alpha}}_{\tau} &= \bar{\mathbf{V}}_{\tau}[\underline{\mathbf{V}}^{-1}\underline{\boldsymbol{\alpha}} + ((\mathbf{B}\Sigma_{\tau}\mathbf{B}')^{-1} \otimes \mathbf{X}'\mathbf{W}^{-1})(\mathbf{y} - (\mathbf{B}\mathbf{m}_{\tau} \otimes \mathbf{I}_T)\mathbf{w})] \end{aligned}$$

and

$$\begin{aligned} \bar{\nu} &= \underline{\nu} + T \\ \bar{\Sigma}_{\tau} &= \underline{\Sigma} + (\mathbf{B}')^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\tau} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\tau})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\tau} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\tau})')(\mathbf{B})^{-1}. \end{aligned}$$

¹⁴All prior distributions are assumed to be independent of the remaining parameters. For instance, I assume for the prior of $\boldsymbol{\alpha}$ that $f(\boldsymbol{\alpha}|\Sigma, \mathbf{B}, w_t) = f(\boldsymbol{\alpha})$. As indicated, priors do not necessarily depend on the chosen quantiles τ .

¹⁵The decomposition of $(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\tau} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\tau})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\tau} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\tau})')$ into $(\mathbf{Y} - \mathbf{X}\hat{\mathbf{A}}_{\tau} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\tau})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\hat{\mathbf{A}}_{\tau} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\tau})')$ and $(\mathbf{A}_{\tau} - \hat{\mathbf{A}}_{\tau})\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}(\mathbf{A}_{\tau} - \hat{\mathbf{A}}_{\tau})$ also holds in this context.

3.2.2 Conditional Probability Density Function of the Latent Variable w_t

The conditional probability density of w_t is proportional to

$$f(w_t|\mathbf{y}_t, \mathbf{A}_\tau, \boldsymbol{\Sigma}_\tau, \mathbf{B}, \mathcal{F}_{t-1}) \propto w_t^{-d/2} \exp\left(-\frac{1}{2}(a_{\tau,t}w_t^{-1} + b_\tau w_t)\right),$$

with $a_{\tau,t} = (\mathbf{y}_t - \boldsymbol{\nu}_\tau - \sum_{i=1}^p \mathbf{A}_{\tau,i} \mathbf{y}_{t-i})' (\mathbf{B} \boldsymbol{\Sigma}_\tau \mathbf{B}')^{-1} (\mathbf{y}_t - \boldsymbol{\nu}_\tau - \sum_{i=1}^p \mathbf{A}_{\tau,i} \mathbf{y}_{t-i})$ and $b_\tau = 2 + \mathbf{m}'_\tau \boldsymbol{\Sigma}_\tau^{-1} \mathbf{m}_\tau$. This implies that w_t , conditional on the latter parameters, is proportional to a generalized inverse Gaussian with the following parameters:¹⁶

$$w_t|\mathbf{y}_t, \boldsymbol{\Sigma}_\tau, \mathbf{B}, \mathbf{A}_\tau, \mathcal{F}_{t-1} \sim \mathcal{GIG}(-d/2 + 1, a_{\tau,t}, b_\tau).$$

For details on the derivation please see Appendix B.3.

3.2.3 Conditional Posterior of \mathbf{B}

I assume a noninformative prior for \mathbf{B} , i.e. let

$$f(\mathbf{B}) = \text{const.}$$

The conditional posterior of \mathbf{B} then follows the likelihood of a $\mathcal{L}_d(\mathbf{B} \mathbf{m}_\tau, \boldsymbol{\Sigma}_{\tau*})$. Following Kotz et al. (2001), it is given by:

$$f(\mathbf{B}|\mathbf{y}, \boldsymbol{\alpha}_\tau, \boldsymbol{\Sigma}_\tau) \propto \prod_{t=1}^T \frac{2 \exp\left((\mathbf{y}_t - \mathbf{A}'_\tau \mathbf{x}'_t)' \boldsymbol{\Sigma}_{\tau*}^{-1} \mathbf{B} \mathbf{m}_\tau\right)}{(2\pi)^{d/2} |\boldsymbol{\Sigma}_{\tau*}|^{1/2}} \left(\frac{(\mathbf{y}_t - \mathbf{A}'_\tau \mathbf{x}'_t)' \boldsymbol{\Sigma}_{\tau*}^{-1} (\mathbf{y}_t - \mathbf{A}'_\tau \mathbf{x}'_t)}{2 + \mathbf{m}'_\tau \boldsymbol{\Sigma}_\tau^{-1} \mathbf{m}_\tau} \right)^{(-d/2+1)} K_{(-d/2+1)} \left(\sqrt{(2 + \mathbf{m}'_\tau \boldsymbol{\Sigma}_\tau^{-1} \mathbf{m}_\tau) ((\mathbf{y}_t - \mathbf{A}'_\tau \mathbf{x}'_t)' \boldsymbol{\Sigma}_{\tau*}^{-1} (\mathbf{y}_t - \mathbf{A}'_\tau \mathbf{x}'_t))} \right),$$

where $K_{(-d/2+1)}(\cdot)$ reflects the modified Bessel function of the second kind of order $-d/2 + 1$.

3.2.4 Metropolis-within-Gibbs Sampler

The sampling of the $\boldsymbol{\alpha}_\tau$ coefficients and w_t is a straightforward task using a Gibbs sampler. However, the draw of the correlations, i.e., the off-diagonal elements of \mathbf{R} that are represented in $\boldsymbol{\Sigma}_\tau$, and the scaling factors in \mathbf{B} cannot be considered as a similarly straightforward task.

I propose to use the conditional posterior of $\boldsymbol{\Sigma}_\tau$ and standardize each draw to infer on the correlations between the variables.¹⁷ Note that equation (2) yields that each draw of $\boldsymbol{\Sigma}_\tau$ can be rearranged as

$$\mathbf{R} = \mathbf{S}_\tau^{-1} \boldsymbol{\Sigma}_\tau \mathbf{S}_\tau^{-1}$$

¹⁶There are several algorithms available for the generation of random numbers from a generalized inverse Gaussian. I apply the one proposed by Devroye (2012) as it is computationally fast.

¹⁷The other option would be to use a Metropolis-Hasting algorithm and sample the off diagonal elements of $\boldsymbol{\Sigma}_\tau$. A Gibbs sampler, however, is preferred as every draw is accepted, thus convergence is faster. Simulation studies have shown that both options provide consistent estimates.

in order to conclude on the correlations. Thereby, one achieves that the diagonal elements of Σ_τ remain unchanged. This is important because quantile restrictions on the Laplace distribution have to remain fixed to obtain a consistent posterior for α_τ . Having drawn the new correlation matrix \mathbf{R} the covariance matrix Σ_τ can be updated using equation (2) again.

In the case of \mathbf{B} , the posterior probability density function is rather complicated as the matrix appears both in the mean and the variance of the conditional distribution of \mathbf{y}_t (e.g., equation (5)). Thus, for the draw of \mathbf{B} I propose to use a random walk Metropolis-Hasting (MH) algorithm that just requires that the conditional posterior probability density function can be evaluated (see Chib and Greenberg (1995)). In contrast to the Gibbs sampler, not every draw is accepted using the MH algorithm. At each draw an acceptance probability is calculated and compared to a random draw of a uniform random variable to decide on its acceptance. If not accepted, the previous draw is taken as the new draw. The acceptance probability is derived as in the following. Given a new draw of \mathbf{B} , called \mathbf{B}^* , and the last draw $\mathbf{B}^{(j-1)}$, where $j \in \{1, \dots, N\}$, it is

$$\alpha_{\text{MH},\mathbf{B}}(\mathbf{B}^{(j-1)}, \mathbf{B}^*) = \min \left[\frac{f(\mathbf{B}^* | \mathbf{y}, \alpha_\tau^{(j)}, \Sigma_\tau^{(j)}, \mathbf{w}^{(j)})}{f(\mathbf{B}^{(j-1)} | \mathbf{y}, \alpha_\tau^{(j)}, \Sigma_\tau^{(j)}, \mathbf{w}^{(j)})}, 1 \right].^{18}$$

I calibrate the acceptance probability to be between 0.2 and 0.5. For a good introduction to the MH algorithm see Koop (2006).

In the following, the algorithm is depicted for the case when draws of the scaling parameters are carried out jointly. This, of course, can be broken down into separate steps to ease the calibration of the acceptance rate. Furthermore, a random walk MH algorithm may be carried out using any symmetric distribution in the innovation part. This paper assumes a normal distribution.¹⁹

¹⁸In the depiction of the acceptance probabilities the draws of the other variables are also used as conditioning variables. Variables at draw (j) or $(j-1)$ are chosen in line with the algorithm presented in this section, however, they may of course vary according to the ordering in the sampler used.

¹⁹In practice, I draw the elements of \mathbf{B} separately. The scaling parameter c_d , i.e. for each draw is adjusted automatically in order to satisfy the acceptance ratio mentioned of 0.2 and 0.5.

Algorithm: Bayesian Quantile VAR

1. Define prior distribution for α_τ and Σ_τ and set starting values $\alpha_\tau^0, \Sigma_\tau^0$ and \mathbf{B}^0 . Set variance of the random walk innovation used in the MH step, c .
2. Repeat for $j = 1, 2, \dots, N$
 - (a) Gibbs Step 1:
For $t = 1, \dots, T$:
Draw $w_t^{(j)} | \mathbf{y}_t, \alpha_\tau^{(j-1)}, \Sigma_\tau^{(j-1)}, \mathbf{B}^{(j-1)}$
 - (b) Gibbs Step 2:
Draw $\alpha_\tau^{(j)} | \mathbf{y}, \Sigma_\tau^{(j-1)}, \mathbf{B}^{(j-1)}, \mathbf{w}^{(j)}$
 - (c) Gibbs Step 3:
 - i. Draw $\Sigma_\tau^{(j)} | \mathbf{y}, \alpha_\tau^{(j)}, \mathbf{B}^{(j-1)}, \mathbf{w}^{(j)}$
 - ii. Calculate $\mathbf{R}^{(j)} = \mathbf{S}_\tau^{-1} \Sigma_\tau^{(j)} \mathbf{S}_\tau^{-1}$
 - iii. Set $\Sigma_\tau^{(j)} = \mathbf{S}_\tau \mathbf{R}^{(j)} \mathbf{S}_\tau$
 - (d) MH Step 1:
 - i. Draw $\mathbf{v}_{**} \sim \mathcal{N}(\mathbf{0}, c\mathbf{I}_d)$
 - ii. Calculate $(b_1^*, \dots, b_d^*)' = (b_1^{(j-1)}, \dots, b_d^{(j-1)})' + \mathbf{v}_{**}$
 - iii. Evaluate $\alpha_{\text{MH}, \mathbf{B}}$
 - iv. Draw $u_{**} \sim \mathcal{U}(0, 1)$
 - v. If $u_{**} \leq \alpha_{\text{MH}, \mathbf{B}}$
set $(b_1^{(j)}, \dots, b_d^{(j)})' = (b_1^*, \dots, b_d^*)'$
else
set $(b_1^{(j)}, \dots, b_d^{(j)})' = (b_1^{(j-1)}, \dots, b_d^{(j-1)})'$

4 NONLINEAR EFFECTS OF UNCERTAINTY SHOCKS ON THE US REAL ECONOMY OVER THE CYCLE(S)

First, I detail the data, its transformations, the model setup, and the analysis as well as the identification of the structural shock. Second, I present the results of the analysis that assumes one cycle for the economy. Third, I discuss the findings on the asymmetries over the financial cycle. In each section, first the impulse response analysis and then the forecast error variance decomposition exercise are presented. A robustness analysis concludes this section.

4.1 *Empirical Issues*

Data and Transformations

To investigate the nonlinear effects of uncertainty shocks across quantiles this study includes variables that are commonly used in the analysis of financial market shocks.²⁰ That is, I formulate a small model of the US economy that also includes uncertainty alongside the standard variables as real economic activity, inflation, interest rates, and real money supply. The data set is of quarterly frequency and spans from 1954Q2 to 2012Q4.

Economic activity is measured by real GDP growth (Δq), inflation by growth in CPI (Δp), interest rates by changes in the effective federal funds rate (Δi), and real money supply by growth in real M2 (Δm). Uncertainty is measured by stock market volatility (u) and calculated as the sum of absolute returns of the Dow Jones Industrial index over each quarter.²¹ All variables, their transformations, and their times series plots are presented in Appendix C. In general, only stationary transformations of the series are included, as the derivation of quantiles for trending variables as, for instance, real GDP, is not sensible. It would imply an ordering over time and not over the different phases of the business cycle. Further, stock market volatility is converted such that lower quantiles can be attributed to recession periods and upper quantiles to booms. That is, u is multiplied by -1 to yield u^* . This entails that high levels of volatility which rather correspond to recession periods are in the lower quantiles of u^* . The other variables do not require any transformation as they are already aligned with the quantiles of GDP growth. This means that lower GDP growth tends to correspond to lower

²⁰See for instance Helbling et al. (2011), Meeks (2012), Hubrich and Tetlow (2012), or Fink and Schüller (2013).

²¹Studies differ strongly in the variable that is used to measure uncertainty. Apart from the measure applied in this study, Alexopoulos and Cohen (2009) construct an index on the basis of new paper articles to measure uncertainty. Bloom (2009) constructs a dichotomous index on the basis of stock market volatility that turns to one when, in response to an economic or political event, volatility rises above a certain threshold. Events are, e.g., the 9/11 terrorist attacks or the recent credit crunch during the Global Financial Crisis. Similarly, Carrière-Swallow and Céspedes (2013) measure only volatility above a certain threshold. The remaining time periods are assumed to be zero. Leduc and Liu (2012), Bachmann et al. (2013), and Bloom et al. (2012) exploit surveys on either consumers or businesses. Gilchrist et al. (2013) employ high-frequency firm-level stock market data.

inflation, lower interest rates, and lower money supply growth.²² In this manner, the model is set up in a way such that all the dynamics at one specific quantile reflect a similar phase of the cycle.²³

Model Setup, Estimation, and Structural Analysis

Throughout the study, I consider a non-informative prior, so that the data is allowed to drive the estimation of the parameters. The priors are

$$\boldsymbol{\alpha} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{d(pd+1)} \cdot 10) \quad \text{and} \quad \boldsymbol{\Sigma} \sim \mathcal{IW}(d, \mathbf{I}_d).$$

Due to the desire to use non-informative prior information, I am required to specify the model as parsimoniously as possible. Thus, I choose a lag length of 1.

The Metropolis-within-Gibbs sampler is set up with 15000 draws where 5000 are discarded as burn-in-draws. The convergence of the parameters is assured using trace plots.

In the structural analysis I normalize the size of the uncertainty shock across quantiles to ensure comparability. The magnitude of the shock is assumed to be one standard deviation of the volatility variable u^* . Putting this into context, the highest volatility spike during the last financial crisis was 7.5 standard deviations away from its historical average. The study considers a positive shock to risk, i.e., a rise in stock market volatility.²⁴ It is important to note that throughout the analysis I assume the economy to remain in the same state or quantile after the shock occurred. This represents an unrealistic assumption as a shock on a system would most likely cause an economy to change its state. Nonetheless, there is no theory that would indicate the path of this change. At last, the forecast error variance decomposition exercise uses the un-normalized decomposed covariance matrix as presented in equation (4). Its decomposition is discussed in the following.

Identification and Discussion of the Structural Shock

To identify the structural shock, first, the reduced form VAR is estimated. Second, the obtained covariance matrix is decomposed using a Cholesky decomposition, which implies a recursive structure for the shocks. The ordering of the Wold causal chain for the standard variables follows the one of monetary policy VARs (see, e.g., Leeper et al. (1996) or Christiano et al. (1999)). This entails that

²²The objective of aligning equations with the business cycle could also be achieved differently. The model is flexible enough to specify the quantile for each equation differently. Thus, instead of multiplying a series by -1 and estimating it, e.g., at $\tau = 0.2$, one could also omit the transformation and estimate the specific equation at $\tau = 0.8$. The approach followed in this paper, however, is preferred, as the presentation of the results is more intuitive. Otherwise, the reader would have to keep in mind to read the graphs for one variable at τ and for another at $1 - \tau$, to understand the effects of each model that has been estimated.

²³Of course, this methodology can accommodate any combination that the researcher requires as, e.g., medium GDP growth during periods of high inflation.

²⁴This entails that graphs of the impulse response analyses have been re-transformed, because the variable u^* has been used in the estimation process. Here a rise signifies a decrease in volatility.

the production sector is arranged first (economic activity, prices), then the policy sector is specified (interest rate, money). At last, I introduce the financial sector through stock market volatility. This order entails that the financial variable may be affected contemporaneously by shocks in the production and policy sector. However, an uncertainty shock does not affect the production or policy sector in the quarter of the shock. This is in line with the identification used in Gilchrist et al. (2013), who also research on the importance of uncertainty.²⁵

The identification of financial shocks in general is no trivial issue. Due to the fact that financial variables are sensitive to new information and this sensitivity is not constant it is difficult to pin down what exactly is driving the structural error of the variable. Some studies in this context, e.g., Meeks (2012), Helbling et al. (2011), Eickmeier et al. (2011), use sign restrictions. However, there are no theoretical concepts that would explain how the real economy reacts to an uncertainty shock at different quantiles.

In the SVAR study by Alexopoulos and Cohen (2009) the authors use a Cholesky decomposition as well. However, the authors permit that the uncertainty shock instantaneously affects the real economy.²⁶ In juxtaposition, I argue that the exact opposite identification is sensible. Uncertainty is a measure related to expected future economic developments. Thus, it is important that the uncertainty shock is not related to past or contemporaneous movements in the production or policy sector. I argue that expectations are the decisive factor driving uncertainty and should, thus, be accounted for in an analysis of the latter.

4.2 *Asymmetries over the Business Cycle*

This section assumes that the economy is driven by one cycle. Thus, variables in this part of the analysis are estimated at the same conditional quantile, i.e., $\tau = (\tau, \dots, \tau)'$.

4.2.1 *Impulse Response Analysis*

This exercise reveals two important findings: (1) Responses of the US real economy to an uncertainty shock are highly asymmetric over the business cycle; especially at the extremes of the cycle. There are strong negative effects on the economy during recessions, but also weak positive effects during booms. Moving from recession to boom the negative effects decrease and then turn positive and increase. (2) Responses to an uncertainty shock over the cycle are significantly different from the responses of the Gaussian model for most quantiles. The mean model predicts effects that are within the set of effects of all estimated models at the various quantiles; mostly being around the average

²⁵In addition to real variables, however, the authors include other financial variables, as e.g. credit spreads. These are allowed to be contemporaneously affected by an uncertainty shock.

²⁶Bloom (2009), Carrière-Swallow and Céspedes (2013), Leduc and Liu (2012) also use a Cholesky decomposition and order their uncertainty variable such that it may influence the real economy instantaneously. However, as indicated, their measure is different to the one employed in this paper. Thus, a different argument applies.

effect of the outcomes over the quantiles. In the following, I provide a description of the results. Subsequently, an economic interpretation is provided.

These results are portrayed in Figure 2. It shows the responses of each variable in the system to an uncertainty shock of one standard deviation that leads to a rise in volatility. Due to the transformation of u^* it is important to keep in mind that lower quantiles of this variables correspond to periods of high volatility. The left panel depicts the complete impulse responses for quantiles $\tau = 0.1$ (black) and 0.9 (gray).²⁷ The right panel portrays responses over the quantiles 2 quarters after the shock. These are depicted by bold (blue) lines. Further, it shows the effects that a standard Gaussian model would predict; thin (red) lines. The dotted lines refer to the 68% probability bands. Please refer to Appendix D, for the complete impulse responses in comparison to the mean for specific quantiles ($\tau = \{0.1, 0.2, 0.5, 0.8, 0.9\}$).

In the case of output growth (Δq), a shock at the lowest considered quantile leads to a strong negative decline of about 0.57 percentage points (p.p.) after half a year, while it causes a positive deviation of 0.13 p.p. at the highest quantile, i.e., the lowest state of volatility. The right panel, shows that the turning point, i.e., from a negative response to a positive one lies in the upper range of the quantiles. Thus, only at the highest quantiles output growth rises in response to an uncertainty shock.²⁸ The effect at the mean is measured to be -0.21 p.p after two quarters. It is, thus, about three times smaller than at the lowest estimated quantile. Further, probability bands of responses for the quantiles do not cross with the mean off the center of the distribution indicating significant differences.

Inflation (Δp) responds in a similar fashion except that the responses at the upper tail of the distribution are stronger than at the lower one. Prices fall at the lower tail by 0.17 p.p., while there is a rise of about 0.33 p.p. at the upper tail after 2 quarters. The mean response is by far smaller, 0.06 p.p. The probability bands of the models at the quantiles and the mean do not cross at lower parts, above the center, and the upper parts of the distribution.

There is also evidence for asymmetry in the response of interest rate changes (Δi). They are lowered by about 0.43 basis point (b.p.) at quarter 2 at the lowest quantile and raised by about 0.15 b.p. at the other extreme of the distribution. The mean response only amounts to a change of -0.13 b.p. As with inflation the probability bands of the models at the quantiles and the mean do not cross at the lower and upper parts of the distribution.

Using the quantile approach, results indicate that money growth (Δm) only responds significantly to an uncertainty shock at the lowest and highest phases of the cycle (except for $\tau = 0.45$). The maximum decrease after half a year is at $\tau = 0.1$ by 0.27 p.p. The maximum increase after 2 quarters is at $\tau = 0.9$ with 0.27 p.p. as well. The effect at the mean is slightly positive and amounts to 0.04

²⁷As quantiles are set mutually for all variables in this exercise, I report only a scalar quantile, i.e., for ease of exposition I do not report the vector of quantiles as $\tau = (\tau, \dots, \tau)'$.

²⁸Also at $\tau = 0.85$, there is a positive deviation in response to an uncertainty shock.

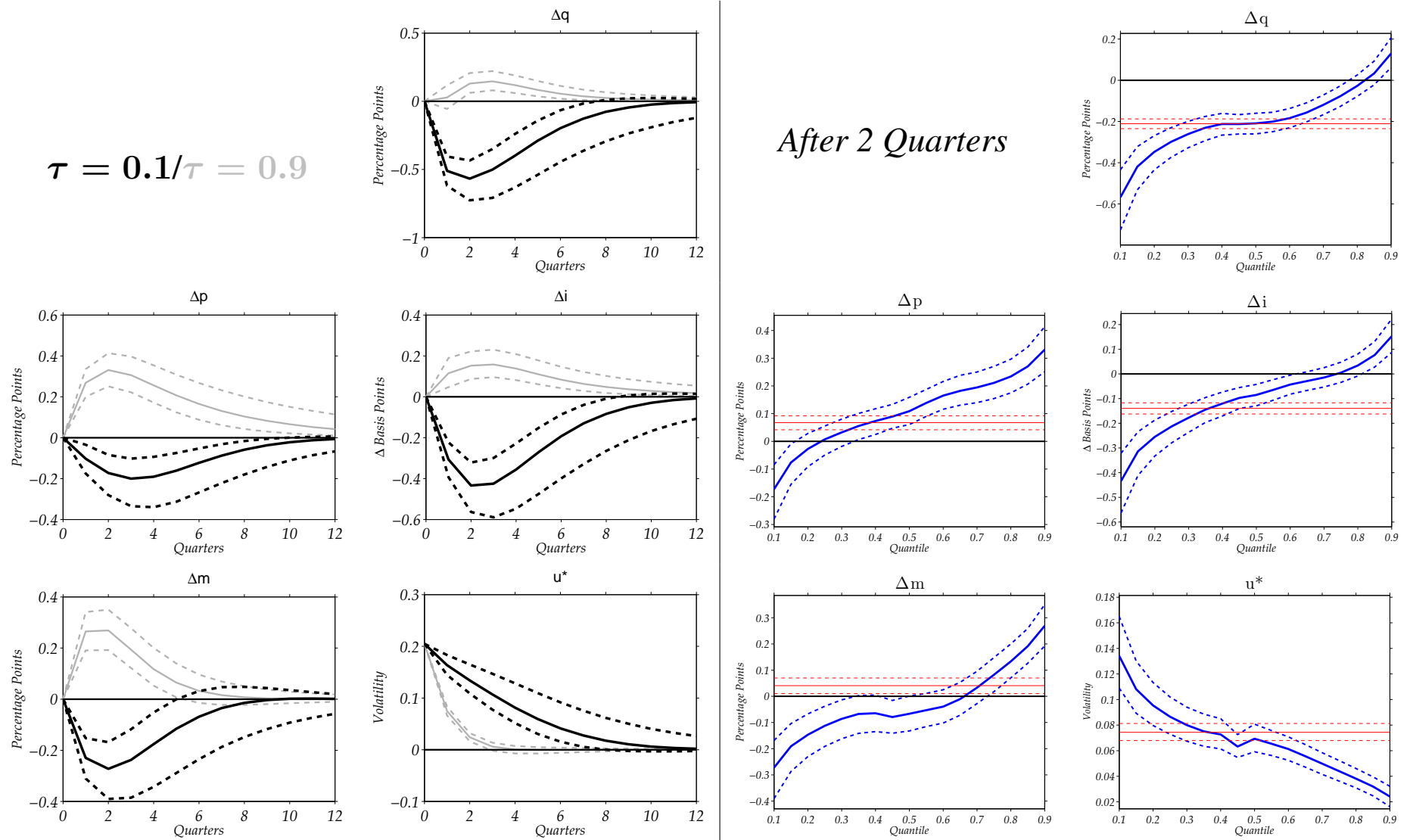


Figure 2: Response of the US Economy to an Uncertainty Shock: Left Panel: Responses at the lowest ($\tau = 0.1$) and highest ($\tau = 0.9$) quantile. Right panel: Responses After 2 Quarters Across Quantiles Compared to the Gaussian Model

Notes: The left panel depicts the complete impulse responses for quantiles $\tau = 0.1$ (black) and 0.9 (gray). The right panel portrays responses over the quantiles 2 quarters after the shock. It depicts impulse responses across quantiles (blue & thick lines) and at the mean (red & thin lines). The impulse of the uncertainty shock is normalized to one standard deviation of u^* . Solid lines refer to the median impulse response obtained at each quantile. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in the interest rate; Δm growth in money supply; u^* volatility. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

p.p.

The response of stock market volatility (u^*) is positive as assumed. Most strikingly, the persistence of the shock varies. At the lower parts of the distribution it is more persistent than at the mean, which can be explained by the phenomenon of volatility clustering during high states of volatility. Starting from $\tau = 0.65$ the shock is significantly less persistent than at the mean.

The findings during recession periods are in line with theoretical considerations. A shock to uncertainty leads firms to pause investments and hiring decisions; consumers postpone consumption. This has a negative effect on economic activity. Decreasing demand leads to downward pressure on prices and the stock of money in the economy shrinks. The monetary authority reduces interest rates in order to stimulate investments.

The decrease in negative effects of uncertainty when the economy passes from the worst recession towards periods of higher growth is also in line with theoretical considerations. On the one hand, the level of uncertainty decreases in this setup and, on the other hand, the economy itself is more sound and, thus, more immune to uncertainty.

At last, the positive effects during the highest boom periods raise some questions as they contradict the common understanding of the effects of uncertainty. Nonetheless, I argue that this finding is sensible. In light of the evidence, this shock should rather be interpreted as an exuberance shock. Investors during this phase of the cycle have highest expectations about future growth. Thus a shock to changes in the returns on the stock market is not perceived as a bad signal. In turn, output growth remains rising, inflation pressure grows, and money supply rises, while the monetary authority attempts to cool down the economy by increasing the interest rate. As discussed previously, volatility is driven by investors' expectations about the future development of the economy. Thus, this result can be supported by the latter argument. Further, this can be argued to be an empirical description of the phenomenon of speculative bubbles. At some point, even in response to a signal usually interpreted as negative, the economy remains on its growth path, which, at a certain moment, cannot be sustained by the economic fundamentals.

4.2.2 Forecast Error Variance Decomposition

Two important results emerge: (1) Uncertainty shocks are decisive for the US real economy. They explain up to 45% of fluctuations in GDP growth over 2 to 3 years. In general, the highest contribution of an uncertainty shock can be found during recession periods. During high phases of the economic cycle uncertainty shocks are only important for inflation and money supply growth. (2) A Gaussian model reports effects that are by far smaller. For instance, in the case of GDP growth and at $\tau = 0.1$, the contribution is more than 4 times higher (45% vs. 10%). These results support the findings of the impulse response analysis and the theoretical considerations. Uncertainty shocks in their common

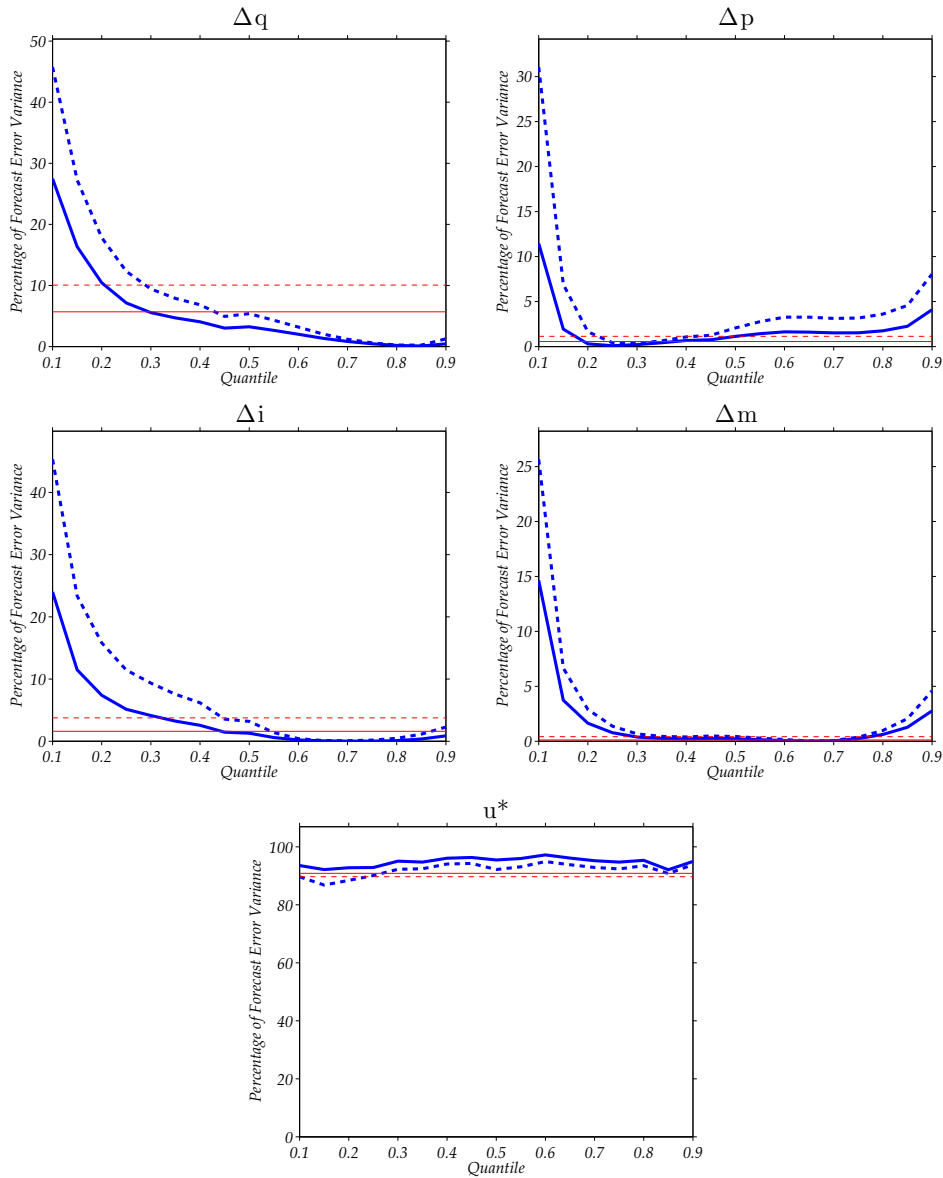


Figure 3: Contribution of Uncertainty Shocks to Fluctuations in US Economic Variables: Across Quantiles and at the Mean

Notes: The graph depicts quantile plots, where the x -axis represents the quantiles at which the specific model has been estimated and the y -axis the percentage of variance of the indicated variable explained by uncertainty shocks. The blue and thick lines represent the estimates at the quantiles. The red and thin lines represent the estimates using the Gaussian model. A solid line refers to the average forecast error variance between 1 and 4 quarters (1st year). A dashed line refers to the average from 5 to 12 quarters (2nd and 3rd year). Δq denotes GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* the transformed variable of stock market volatility. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

interpretation, i.e., in line with Bloom (2009), should have the strongest importance during recession periods where the economy is prone to instability. Their minor importance during boom phases does not contradict the hypothesis of the exuberance shock. It can be even argued to be underlined as during the highest boom phases there is an increase in the importance of the shock for fluctuations in

inflation and money growth (above mean) – both representing essential variables in a build-up of a speculative bubble, a situation where we find high prices as well as high liquidity.

The results are depicted in Figure 3. It shows the quantile plots of the importance of uncertainty shocks for fluctuations in each variable of the system as measured by forecast error variance decompositions. The bold (blue) lines represent the importance of the financial shock across quantiles, while the thin (red) lines refer to the importance at the mean. The two lines in each set (bold and thin) represent the figure for different horizons. The solid line refers to the average over one year and the dashed lines the average over two to three years.

The maximum importance of an uncertainty shock for changes in real GDP (Δq) is reported to be 34% (1 year) and 45% (2-3 years) for the two horizons at quantile $\tau = 0.1$, while the estimate at the mean amounts only to 8% and 10%. Furthermore, the minimum importance is given around the highest quantiles. Thus, the importance of an uncertainty shock for GDP growth is estimated to rise for recession periods: the deeper the recession, the higher is the contribution of uncertainty.

In the case of inflation (Δp), uncertainty plays an important role at both tails of the distribution. However, the highest magnitudes are revealed during the deepest recession ($\tau = 0.1$) with 14% and 31% for the two horizons. From the center of the distribution to the upper quantiles the importance remains higher than the one estimated by the Gaussian model, which is about 1% at both horizons. The highest importance during booms is 5 and 8% at quantile $\tau = 0.9$.

Furthermore, there is evidence that the contribution of uncertainty shocks to changes in interest rates (Δi) has a similar structure than in the case of GDP growth; almost no importance during boom phases and an increasing importance at the lower the quantile. The maximum importance is found at the lowest quantile, which is 30% and 45%. The mean figures are 2% and 4%.

The contribution of uncertainty shocks to fluctuations in money supply growth (Δm), on the other hand, has a similar structure as found for inflation. In both tails the contributions are important, however, they are strongest during recession periods. The strongest effects are found at the lower tail with 18% and 26% for the two horizons. The mean figures amount only to 0.2% and 0.4%. The importance at the highest quantile is 3 and 5%.

At last, the importance of an uncertainty shock for stock market volatility (u^*) is fairly stable across the distribution and remains at most parts higher than the one obtained using the Gaussian model. The importance is very high. Thus, other shocks of the system are not important for explaining fluctuations in stock market volatility.

4.3 Asymmetries over the Financial Cycle: Economy at Bust and Boom

This section analyzes the interaction of the cycle of the real economy and the financial cycle. The cycle of the real economy is fixed in this setup. More specifically, I analyze the asymmetric effects

of the financial cycle for an economy in a recession ($\tau = 0.2$) and during a boom ($\tau = 0.8$). Putting it differently the vector τ is assumed to be $(0.2, 0.2, 0.2, 0.2, \tau_5)'$ during the recession phase and $(0.8, 0.8, 0.8, 0.8, \tau_5)'$ during the boom phase. τ_5 is then allowed to vary to represent the financial cycle. I do not present the results for the strongest recessions and booms ($\tau = 0.1, 0.9$) to emphasize that asymmetries of the financial cycle are not only important at the most extreme tails of the distribution of the economy.

4.3.1 Impulse Response Analysis

Two results stand out: (1) For an economy in recession the state of the financial system (good or bad) is important. There is a stronger recession when the financial system is also at a bad state. E.g., the response of GDP growth to an uncertainty shock is more than twice as persistent if the financial system is at a bad state. (2) For an economy at boom the state of the financial system does not seem to be crucial in the short run. In the long run, the exuberance shock is more pronounced if the financial system is at a bad state. If the financial system is at a bad state the behavior of the economy is explosive. This could be argued to provide further evidence for speculative bubbles. It leads to the case that the developments in the real economy cannot be sustained by fundamentals. A burst of a speculative bubble, thus, becomes more likely. However, results in the long run have to be taken with caution. According to the findings it can be assumed, for instance in the last case, that the economy remains at a boom, while the financial sector is at the worst considered state. This would not remain. The cycles would converge, i.e., each sector would follow a path of phases until the financial and business cycle reflect the same state of the economy.

Figure 4 illustrates these impulse responses. The left panel depicts the responses to an uncertainty shock for an economy in a recession period $\tau = 0.2$ and the right panel shows the responses for the boom period $\tau = 0.8$. Each graph has two different sets of impulse responses. The gray ones refer to responses where the financial system is at a good state (low volatility, $\tau = 0.9$). The black ones show the case where the financial system is at a bad state (high volatility, $\tau = 0.1$). It, thus, compares the effects of an uncertainty shock in situations when the financial system is in its best and its worst state. *Economy at Bust:* The effect of a shock on GDP growth dies out twice as fast (1 1/2 year vs. more than 3 years) if the financial system is in a good state. Similar results hold for changes in interest rates. The reaction of the monetary authority is stronger and more persistent than if the financial system is in a good state. Responses of inflation and money supply do not differ significantly. Here, responses during a bad state of the financial system are reported to have higher variance. While the median response is different, the probability bands mostly overlap. The response of volatility indicates a highly persistent shock during bad states and a less persistent shock during good states.

Economy at Boom: Responses in the short run are reported to be very similar, i.e., the state of the

financial system does not seem to matter. However, in the long run shocks that occur in a bad financial state lead to an explosive behavior of the responses in the case of inflation, interest rate changes, money supply growth, and volatility. Thus it can be argued that the general notion of an exuberance shock does not depend on the state of the financial system. However, the magnitude and persistence seems to be important. A system with explosive behavior will end up faster in a burst of a speculative bubble than a system that still returns to an equilibrium.²⁹

4.3.2 Forecast Error Variance Decomposition

Two results stand out: (1) Uncertainty shocks have higher importance during a bad state of the financial cycle (2) Uncertainty is most important for GDP growth and changes in interest rates during recession, while it explains the highest fraction for inflation and money supply growth during booms. This again supports the hypothesis of the exuberance shock, as the variables involved in the build-up of speculative bubbles are most affected by uncertainty.

Figure 5 portrays the forecast error variance decompositions where stock market volatility is estimated over different phases of its cycle. The remaining variables of the US economy remain either at a recession period $\tau = 0.2$ or boom phase 0.8. As in the previous exercise the left panel shows the former and the right panel the latter setup. Again estimates are compared against the Gaussian model (thin lines). In addition, the percentage contribution is reported for the two horizons: average over one year (solid lines), and average over two and three years (dashed lines).

As noted, in general, i.e. across variables and states of the economy, a financial system in a bad state has a higher contribution for fluctuations in the real economy than a financial system in a good state. A financial shock in a bad state has higher importance for GDP growth and interest rates during recession periods than if the economy is in a good state. For GDP the difference is about 50% to about 4.5% (even smaller than at the mean) and for interest rates it is about 46% to 15% for the long horizon. In the case of inflation and money supply it is exactly the opposite. For an economy at boom an uncertainty shock in a bad state has stronger effects than the same shock for an economy at bust. The difference is about 45% to 7.5% for inflation and 28% to 12% for money supply growth over the longer time horizon. In the case of volatility the analysis depicts again a fairly stable contribution. Just in the case of a boom economy and a bad financial system other shocks in the system are found to have some importance for fluctuations in stock market volatility.

²⁹An analysis where $\tau = (0.9, 0.9, 0.9, 0.9, 0.1)'$, that is where the best and worst states of the economy and the financial system are estimated together, yields a similar result. The only difference is that in this setup the entire economy, i.e., also GDP growth, explodes in the positive direction.

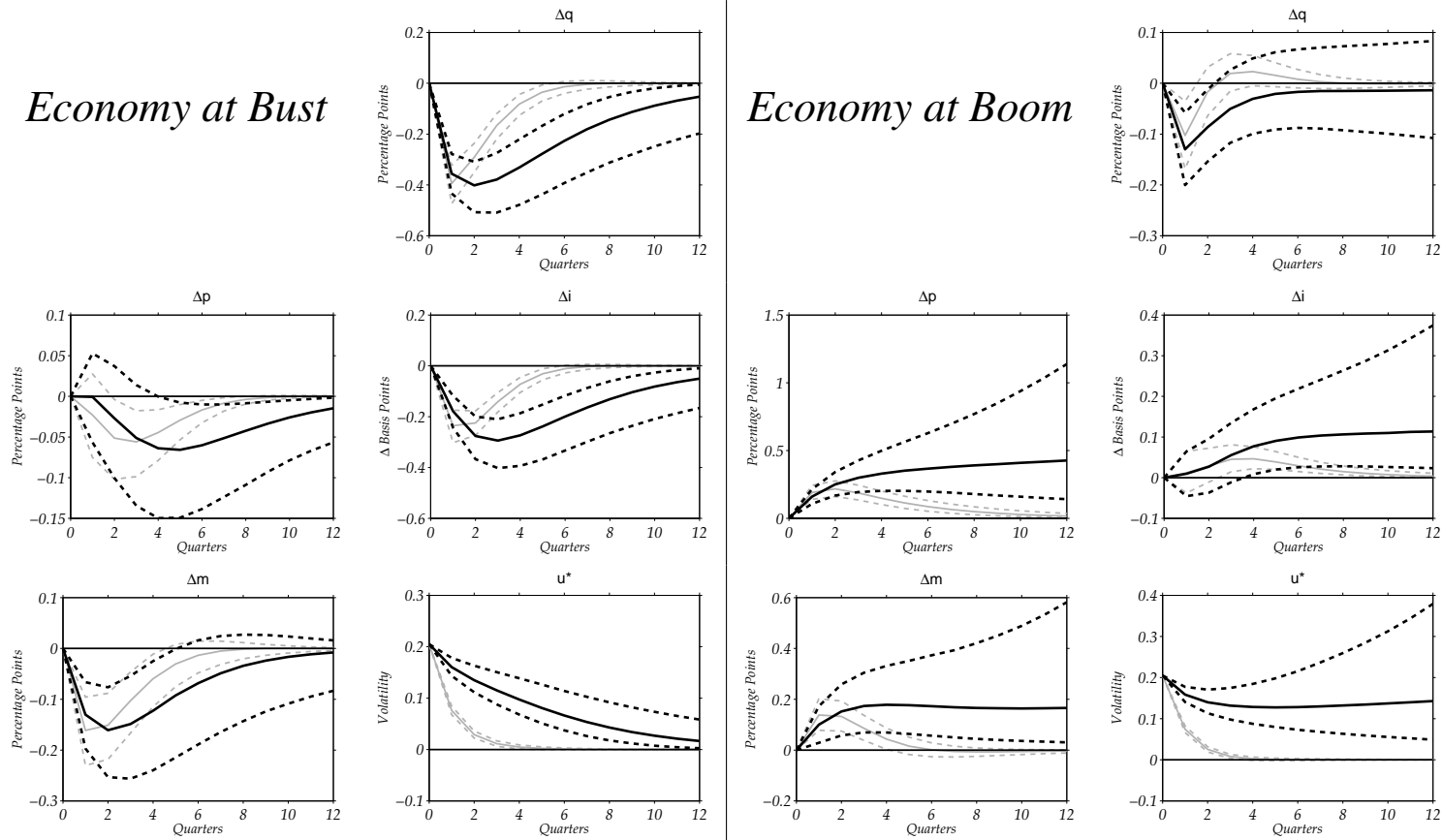
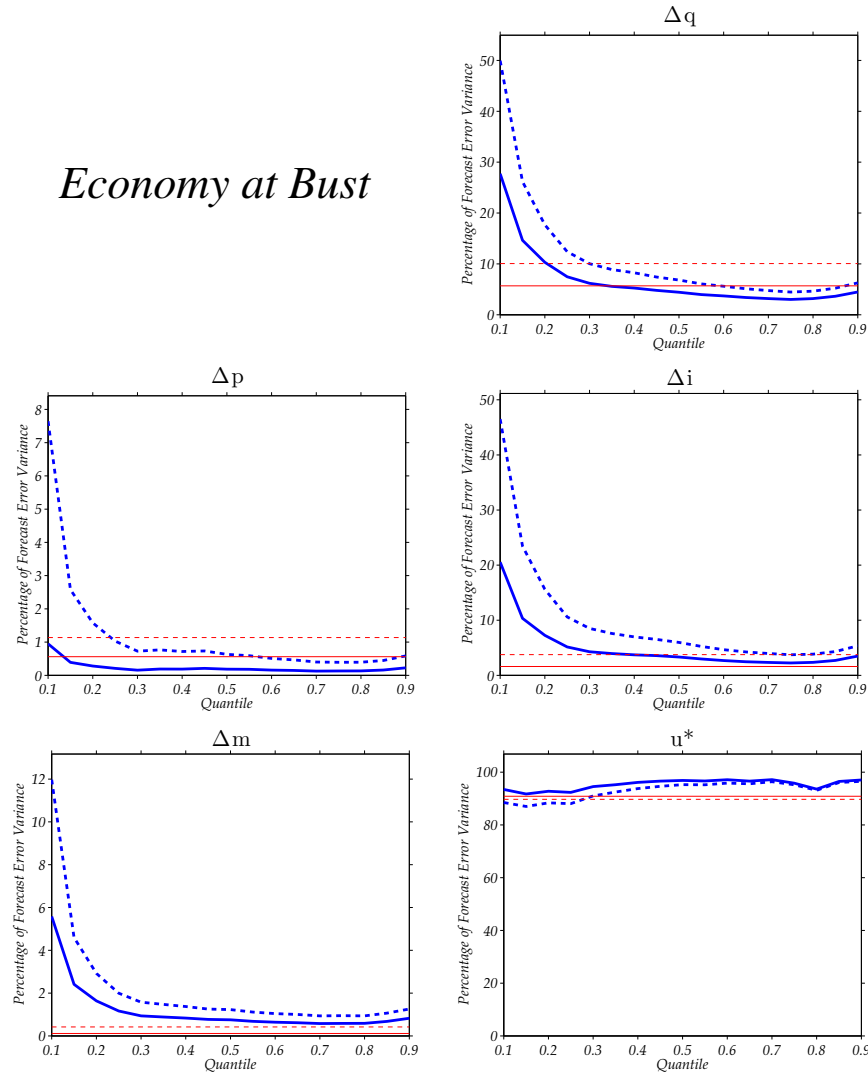


Figure 4: Response of the US Economy to an Uncertainty Shock During a Bust and Boom: Shock at $\tau_5 = 0.1$ and at $\tau_5 = 0.9$

Notes: The graph depicts the responses to an uncertainty shock. The left panel depicts an economy at $\tau = 0.2$, i.e., a recession and the right panel an economy at $\tau = 0.8$, i.e., a boom phase. The black lines describe the responses to a financial shock at $\tau_5 = 0.1$ and the gray lines portray the responses to a financial shock at $\tau_5 = 0.9$. The dashed lines correspond to posterior 68% probability bands. Δq denotes GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* the transformed variable of stock market volatility.

Economy at Bust



Economy at Boom

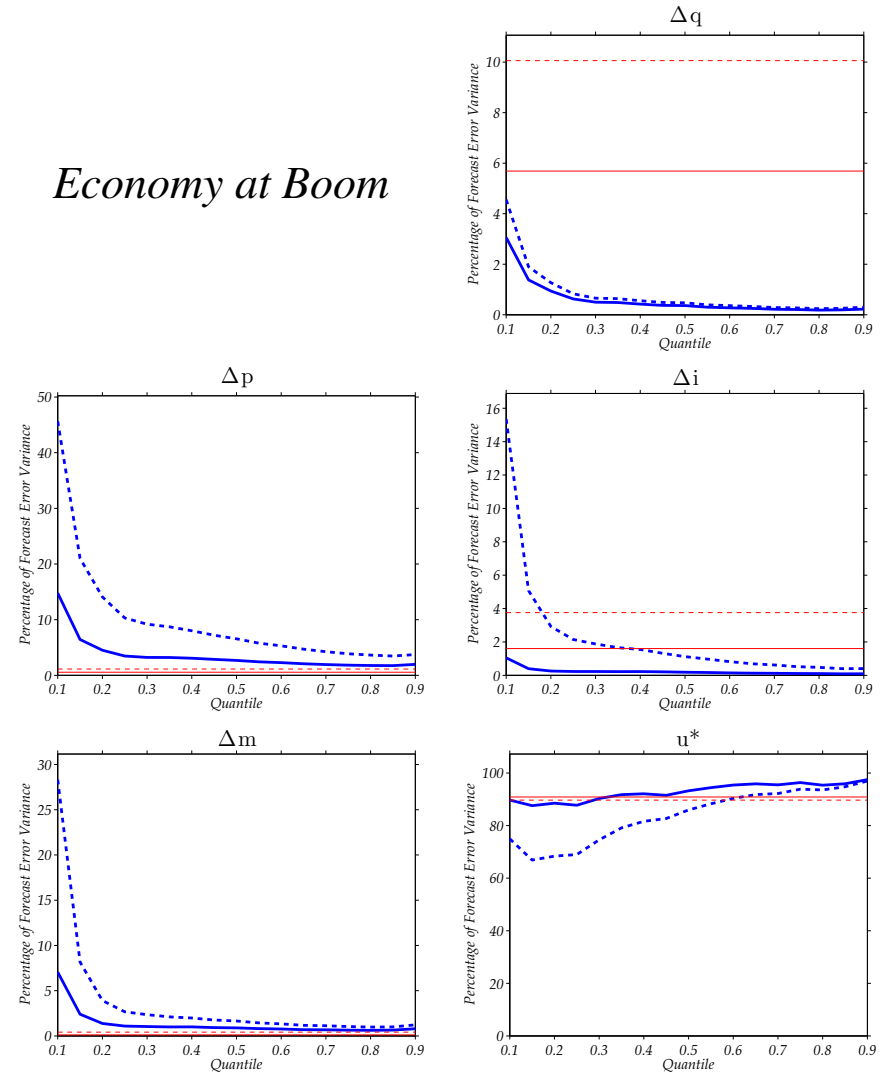


Figure 5: Contribution of Uncertainty Shocks to Fluctuations in US Economy at Bust ($\tau = 0.2$) and at Boom ($\tau = 0.8$)

Notes: The graph depicts quantile plots, where the x -axis represents the quantiles of the financial cycle τ_5 at which the specific model has been estimated and the y -axis the percentage of variance of the indicated variable explained by uncertainty shocks. The blue and thick lines represent the estimates at the quantiles. The red and thin lines represent the estimates using the Gaussian model. A solid line refers to the average forecast error variance between 1 and 4 quarters (1st year). A dashed line refers to the average from 5 to 12 quarters (2nd and 3rd year). Δq denotes GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* the transformed variable of stock market volatility. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

4.4 Robustness Analysis

In the following two kinds of robustness checks are conducted.³⁰ First, I include stock returns in the model of the US economy and, second, I analyze the effects of uncertainty on the German real economy as it shares similar characteristics with the US economy.

4.4.1 Including Stock Returns

In light of the evidence that an economy may experience a positive development in response to an uncertainty or exuberance shock, it is of interest how the stock market develops in response to such a shock. If it was an exuberance shock, one should see a rise in stock returns.

Results are depicted in Appendix E.1 and the exact definition of the variable that measures stock returns is shown in Appendix C.³¹ I introduce the variable stock returns into the model such that it may be affected by the uncertainty shock contemporaneously.

Most importantly, the results support my interpretation of the exuberance shock. An economy in its highest phase, i.e., $\tau = 0.9$, experiences a rise in the returns on the stock market in response to a shock. After one quarter the returns rise by about 1.9 p.p. In contrast, in a recession the stock market falls by about 1.3 p.p. With respect to the importance of an uncertainty shock for fluctuations in stock returns, findings indicate a higher contribution during recessions – where the highest contribution is at $\tau = 0.15$ with about 21% for both horizons.

4.4.2 German Economy

This section aims at finding further evidence for the asymmetric effects of uncertainty shocks. If it was a general phenomenon one should find equivalent results for countries that share similar characteristics. Especially with regard to the exuberance shock a robustness check should be most insightful as it contradicts common knowledge on uncertainty. For this analysis I consider the German economy. I model the German economy using the equivalent variables as in the US case. Stock market volatility is constructed using the DAX. Please see Appendix C for details on the data. Due to the shorter availability of the data the starting date of the analysis is now 1970Q2 (1954Q2 in the case of the US). Nonetheless, the other parts of the analysis as for instance lag length and priors remain unchanged to the previous analysis. Results are depicted in Appendix E.2.

³⁰As a third robustness check I consider a shock to monetary policy, i.e., an unexpected rise in the changes of interest rates, in order to see whether the findings about the uncertainty shock are model or data driven. At first sight, e.g., in the case of the high importance of an uncertainty shock during recession periods for all variables, the results might suggest that this represents an artefact of the model. However, the pattern of a monetary policy shock is different to that of an uncertainty shock. This outrules the possibility that the outcomes are model driven. Further details are available upon request from the author.

³¹I present the results for the setup that assumes one cycle for the economy. On the one hand, this is sufficient to provide insights into the meaning of the shock during a boom phase and, on the other, I avoid making assumptions on how the business and financial cycle converge. Phases of different macrovariables will not stay *orthogonal* to each other for a prolonged period of time.

Assuming *one cycle for the economy*, the impulse response analysis reveals a strong negative development of the German economy in response to an uncertainty shock. Further, this analysis also reveals a positive effect on the economic outcome during the highest boom phases. Again, responses for the tails of the distribution are shown to be significantly different from the Gaussian estimates. The forecast error variance decomposition analysis reveals a similar pattern for recession periods – even though stronger than for the US in the case of inflation, interest rates, and money – however, all contributions of uncertainty shocks are close to zero for boom periods. Considering the effect of an uncertainty shock along its *financial cycle* there is an equivalent pattern for an economy at bust. Only in case of GDP growth and interest rate changes, the response to an uncertainty shocks ebbs off faster than in the case of the US. For Germany the state of the financial system also seems to be crucial for developments of inflation; contrary to the case of the US in both scenarios (good and bad state of financial system). For a shock during boom phases, the state of the financial system does not seem to have an effect, as anyways the responses are insignificant. Only at the highest state of the economy, I obtain the same results as in the US, that the state of the financial system only matters for the size of the positive deviation.³² The variance decompositions, again reveal a similar pattern, just that the importance of an uncertainty shock is in general higher for an economy at bust than for an economy at boom (i.e., also for inflation and money supply, which is different compared to the US). For a booming economy the importance is below the mean importance, except for money supply, for most quantiles.

Overall, I conclude that the previous results are supported. Asymmetries over the business cycle and the financial cycle are an important characteristic of the German economy. There is also evidence for an exuberance shock. Of course, slight differences in the results to the US emerge. However, apart from obvious differences between the countries as e.g. institutions, in this setup differences can possibly be attributed to the sample period.

5 CONCLUDING REMARKS

This paper sheds light on the asymmetric effects of uncertainty on the US real economy. To provide evidence I argue that conditional quantiles may be related to the different phases of the business cycle. Building on this, I construct a novel methodology that allows for an analysis of one or more cycles - the quantile structural vector autoregressive model. For its estimation, I introduce a new representation of the multivariate Laplace distribution and propose a sampler for the estimation of the parameters.

The analysis provides evidence for the existence of strong asymmetries over the business and financial cycles. The asymmetries appear in many parts of the distribution, mostly in the tails of the

³²Results are not shown but can be obtained from the author on request

distribution, different from estimates provided by a Gaussian model. I find evidence that the commonly known uncertainty shock has two different effects on the outcome of the economy. During recession periods these are highly negative which is in line with theoretical considerations. However, the effects are also found to be positive during the highest boom phases. I interpret this shock as an exuberance shock as the stock market is affected positively as well. An exuberance shock entails a rise in equity prices. This can be argued to constitute empirical evidence for the phenomenon of speculative bubbles, as unexpected changes on the stock market leads to positive growth. This is supported by the result that prices and money are strongly determined by uncertainty shocks during boom phases, which are key ingredients in the build-up of speculative bubbles. At last, the state of the financial sector is decisive for the effect of an uncertainty shock if the real economy is in recession. If the financial sector is in a bad state the resulting recession is found to be stronger. In contrast, the state of the financial system is not important for an economy at boom in the short run.

The present study underlines the important changes in the dependencies over the cycles. Future research should, thus, provide further evidence for asymmetries over the cycle considering different shocks to the economy. Moreover, the framework should deliver important insights into cross-country spill-overs; especially considering the international differences in the phases of the cycles.

With regard to the methodology, there are also open questions left for future research. The use of different priors, maybe varying across quantiles, might be considered. Furthermore, it still remains unclear whether the approach to derive the structural model is best. Possibly one can estimate the contemporaneous relations directly from the multivariate Laplace distribution. Additionally, the methodology could be extended to deal with non-standard innovations, e.g. for the analysis of higher frequency financial data.

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A APPENDIX FOR LAPLACE DISTRIBUTION AND QUANTILE RESTRICTIONS

A.1 Quantile Restrictions for the Univariate Laplace Density

Assume v_t is Laplace distributed, i.e.,

$$f_\tau(v_t) = \tau(1 - \tau) \exp\{-\rho_\tau(v_t)\}, \text{ where } \rho_\tau(v_t) = \begin{cases} v_t \cdot \tau & , \text{ if } v_t \geq 0 \\ v_t \cdot (\tau - 1) & , \text{ if } v_t < 0. \end{cases}$$

Then the following has to hold for the characteristic function:

$$\begin{aligned} \Psi_{v_t}(s) &= E[\exp(isv_t)] \\ &= \int_{-\infty}^{\infty} \exp(isv_t) f_\tau(v_t) dv_t \\ &= \int_{-\infty}^0 \tau(1 - \tau) \exp(isv_t + (1 - \tau)v_t) dv_t + \int_0^{\infty} \tau(1 - \tau) \exp(isv_t - \tau v_t) dv_t \\ &= \tau(1 - \tau) \left(\frac{1}{is + (1 - \tau)} + \frac{1}{p - is} \right) \\ &= \frac{1}{1 + \frac{1}{\tau(1-\tau)}s^2 - i\frac{1-2\tau}{\tau(1-\tau)}s} \end{aligned}$$

The characteristic function as defined in Kotz et al. (2001) is

$$\Psi_{v_t}(s) = \frac{1}{1 + \frac{1}{2}\sigma^2 s^2 - ims}.$$

Thus the restrictions required are

$$m = \frac{1 - 2\tau}{\tau(1 - \tau)} \quad \text{and} \quad \sigma^2 = \frac{2}{\tau(1 - \tau)}.$$

A.2 Bivariate Laplace Distribution with Quantile Restrictions

Following Kotz et al. (2001), the bivariate characteristic function may be written as follows:

$$\Psi(s_1, s_2) = \frac{1}{1 + \frac{\sigma_{\tau_1}^2 s_1^2}{2} + \rho_{12} \sigma_{\tau_1} \sigma_{\tau_2} s_1 s_2 + \frac{\sigma_{\tau_2}^2 s_2^2}{2} - im_{\tau_1} s_1 - im_{\tau_2} s_2},$$

where the five parameters have to satisfy $m_{\tau_1} \in \mathbb{R}, m_{\tau_2} \in \mathbb{R}, \sigma_{\tau_1} \geq 0, \sigma_{\tau_2} \geq 0, \rho_{12} \in [-1, 1]$, and ρ_{12} denotes the correlation coefficient.

As can be seen, the bivariate Laplace can be thought of as two univariate Laplace distributions that are linked through ρ_{12} , the correlation coefficient.³³

³³However, as Kotz et al. (2001) note, even in the symmetric case, i.e., $m_{\tau_1} = 0$ and $m_{\tau_2} = 0$, when the random variables are uncorrelated, i.e., $\sigma_{\tau_1} \sigma_{\tau_2} \rho_{12} = 0$, they are not independent.

B APPENDIX FOR QUANTILE STRUCTURAL VECTOR AUTOREGRESSION AND BAYESIAN INFERENCE

B.1 Estimation of Covariance Matrix

The following discusses briefly how the covariance matrix

$$\boldsymbol{\Omega}_\tau = (\omega_{jk}) = \left(\frac{\mathbb{E}[(\rho_{\tau_j}(u_{jt}))(\rho_{\tau_k}(u_{kt}))]}{\mathbb{E}[f_{u_{jt}}(0)]\mathbb{E}[f_{u_{kt}}(0)]} \right), \text{ where } j, k \in \{1, \dots, d\},$$

may be calculated. Given the parameter estimates $\hat{\mathbf{A}}_{\tau,i}$ and $\hat{\boldsymbol{\nu}}_\tau$ one obtains $\hat{\mathbf{u}}_t$ in the standard way, i.e., $\hat{\mathbf{u}}_t = \mathbf{y}_t - \hat{\boldsymbol{\nu}}_\tau - \sum_{i=1}^p \hat{\mathbf{A}}_{\tau,i} \mathbf{y}_{t-i}$.

Following, the terms (i) $\mathbb{E}[(\rho_{\tau_j}(u_{jt}))(\rho_{\tau_k}(u_{kt}))]$ and (ii) $\mathbb{E}[f_{u_{jt}}(0)]$ can be replaced with their sample estimates. (i) is estimated by

$$\frac{1}{T} \sum_{t=1}^T (\tau_j - I(u_{jt} < 0))(\tau_k - I(u_{kt} < 0)).$$

(ii) refers to the probability density of u_{jt} evaluated at the point zero. It is obtained using a kernel density estimator on the residuals \hat{u}_{jt} using a Gaussian kernel, again, evaluated at zero.

B.2 The Conditional Likelihood Function of \mathbf{y}

Additionally to the definitions in Section 3, let $\boldsymbol{\mu}_\tau = \text{vec}((\boldsymbol{\mu}_{\tau,1}, \dots, \boldsymbol{\mu}_{\tau,T})') = (\mathbf{I}_d \otimes \mathbf{X})\boldsymbol{\alpha}_\tau + (\mathbf{B}\mathbf{m}_\tau \otimes \mathbf{I}_T)\mathbf{w}$. The complete likelihood of \mathbf{y} , i.e., for all observations $1, \dots, T$, may be written as

$$f(\mathbf{y}|\boldsymbol{\alpha}_\tau, \boldsymbol{\Sigma}_\tau, \mathbf{w}, \mathbf{B}) = \frac{1}{(2\pi)^{dT/2}} |\boldsymbol{\Sigma}_{\tau*} \otimes \mathbf{W}|^{-1/2} \exp \left[-\frac{1}{2} (\mathbf{y} - \boldsymbol{\mu}_\tau)' (\boldsymbol{\Sigma}_{\tau*} \otimes \mathbf{W})^{-1} (\mathbf{y} - \boldsymbol{\mu}_\tau) \right].$$

B.3 Derivation of the Conditional Posterior of w_t

Let $w \sim \mathcal{E}(1)$ and $\mathbf{y} \sim \mathcal{L}_d(\mathbf{m}, \boldsymbol{\Sigma})$. In order to show that the kernel of the $f(w|\mathbf{y})$ is proportional to that of a generalized inverse Gaussian distribution recall that the conditional density is obtained through

$$f(w|\mathbf{y}) = \frac{f(\mathbf{y}|w)f(w)}{f(\mathbf{y})}.$$

It has been shown that $f(\mathbf{y}|w)$ has a multivariate normal pdf, i.e.,

$$f(\mathbf{y}|w) = (2\pi)^{-d/2} |w\boldsymbol{\Sigma}|^{-1/2} \exp \left(-\frac{1}{2} (\mathbf{y} - \mathbf{m}w)' (w\boldsymbol{\Sigma})^{-1} (\mathbf{y} - \mathbf{m}w) \right)$$

Furthermore, $f(w) = \exp(-w)$. Neglecting $f(\mathbf{y})$ and the invariant terms of $f(\mathbf{y}|w)$,

$$\begin{aligned} f(w|\mathbf{y}) &\propto w^{-d/2} \exp\left(-\frac{1}{2}(\mathbf{y} - \mathbf{m}w)'(w\boldsymbol{\Sigma})^{-1}(\mathbf{y} - \mathbf{m}w) - w\right) \\ &= w^{-d/2} \exp\left(-\frac{1}{2}\left(\frac{\mathbf{y}'\boldsymbol{\Sigma}\mathbf{y}}{w} - \mathbf{y}'\boldsymbol{\Sigma}\mathbf{m} - \mathbf{m}'\boldsymbol{\Sigma}\mathbf{y} + w\mathbf{m}'\boldsymbol{\Sigma}\mathbf{m}\right) - w\right) \\ &\propto w^{-d/2} \exp\left(-\frac{1}{2}\left((\mathbf{y}'\boldsymbol{\Sigma}\mathbf{y})w^{-1} + (2 + \mathbf{m}'\boldsymbol{\Sigma}\mathbf{m})w\right)\right). \end{aligned}$$

The probability density function of a generalized inverse Gaussian denoted by $\mathcal{GIG}(\lambda, \chi, \psi)$, with $\lambda = -(d/2) + 1$, is given by

$$f(x|\lambda, \chi, \psi) = \frac{(\psi/\chi)^{\lambda/2}}{2K_{\lambda}(\sqrt{\chi\psi})} x^{\lambda-1} \exp\left\{-\frac{1}{2}(\chi x^{-1} + \psi x)\right\},$$

where $K_{\lambda}(\cdot)$ reflects the modified Bessel function of the second kind.

Hence,

$$f(w|\mathbf{y}) \propto \mathcal{GIG}(-d/2 + 1, \mathbf{y}'\boldsymbol{\Sigma}\mathbf{y}, 2 + \mathbf{m}'\boldsymbol{\Sigma}\mathbf{m}).$$

C DATA

Table 1: Data Sources, Descriptions, and Transformations

Variable	Symbol	Base Variable	Transformation	QC	Freq.	Conversion	Code
Real Economic Activity Growth	Δq	Real GDP	Log differences	-	Q	-	USGDP...D
Inflation	Δp	CPI All Items	Log differences	-	M	End of period	USCONPRCE
Change in Interest Rates	Δi	Federal Funds	differences	-	M	Average	USFEDFUN
Real Money Supply Growth	Δm	Money Supply M2	Constant prices	-	M	End of period	USMS2...B
Stock Market Volatility	u^*	Dow Jones Industrials	Log Differences Absolute value of log differences	-1	D	Sum of observations	DJINDUS(PI)
GDP Deflator		Chain-Type Price Index	-	-	Q	-	USGDP..CE
Robustness: <i>Stock Return</i>							
Stock Returns	r	Dow Jones Industrials	Log Differences	-	D	End of Quarter	DJINDUS(PI)
Robustness: <i>Germany</i>							
Real Economic Activity Growth	Δq	Real GDP	Constant prices Log Differences	-	Q	-	BDGDP...B
Inflation	Δp	CPI All Items	Log differences	-	M	End of period	BDCONPRCF
Interest Rate	Δi	Discount/Short Term EUR Repo	-	-	M	Average	BDPRATE.
Real Money Supply Growth	Δm	Money Supply M2	Constant prices Log Differences	-	M	End of period	BDM2....B
Stock Market Volatility	u^*	DAX 30	Log Differences Absolute value of log differences	-1	D	Sum of observations	DAXINDX(PI)
GDP Deflator		GDP Deflator	-	-	Q	-	BDQNA057E

Notes: *QC* denotes quantile conversion, i.e., how transformed variables are converted to align the quantiles with the business cycle. *D, M, Q* refers to daily, monthly, quarterly frequency respectively. 'Real Money' is obtained by dividing by 'GDP deflator'. Data codes from Datastream (Thomson Financial) are provided for each variable used in the analysis. All data are seasonally adjusted when necessary. In the robustness analysis GDP and Money Supply are converted into real variables using the GDP deflator indicated.

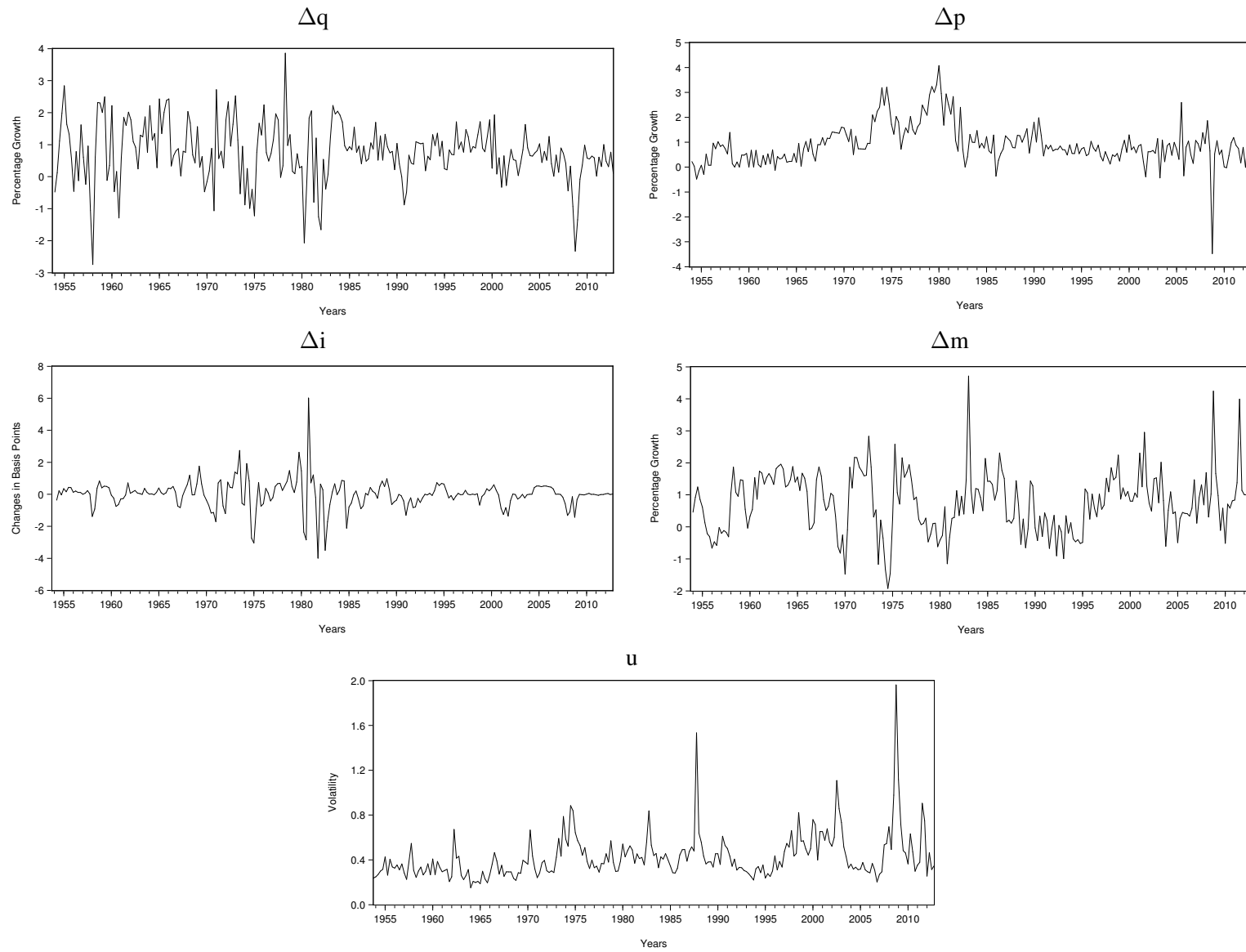


Figure 6: Time Series Graphs of US Economy

Notes: Δq denotes GDP growth, Δp inflation, Δi changes in interest rates, Δm growth in money supply, and u represents the untransformed measure of stock market volatility. The main analysis is conducted using $u^* = -u$. For further details on the variables see Table 1.

D COMPLETE IMPULSE RESPONSES

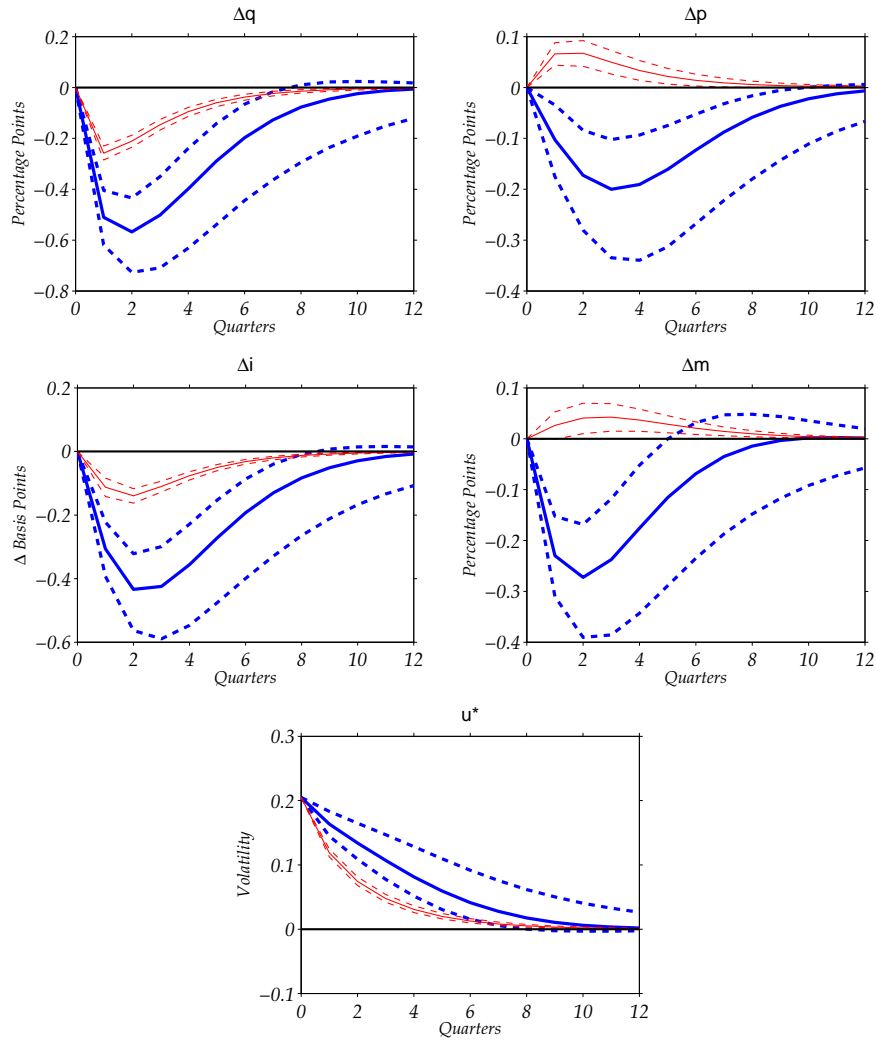


Figure 7: Response to Uncertainty Shock at $\tau = 0.1$

Notes: The graph depicts impulse responses at the quantile (τ) indicated (blue & thick lines) and at the mean (red & thin lines). Solid lines refer to the median impulse response. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* stock market volatility. The x -axis measures quarters, where the shock occurs in quarter zero.

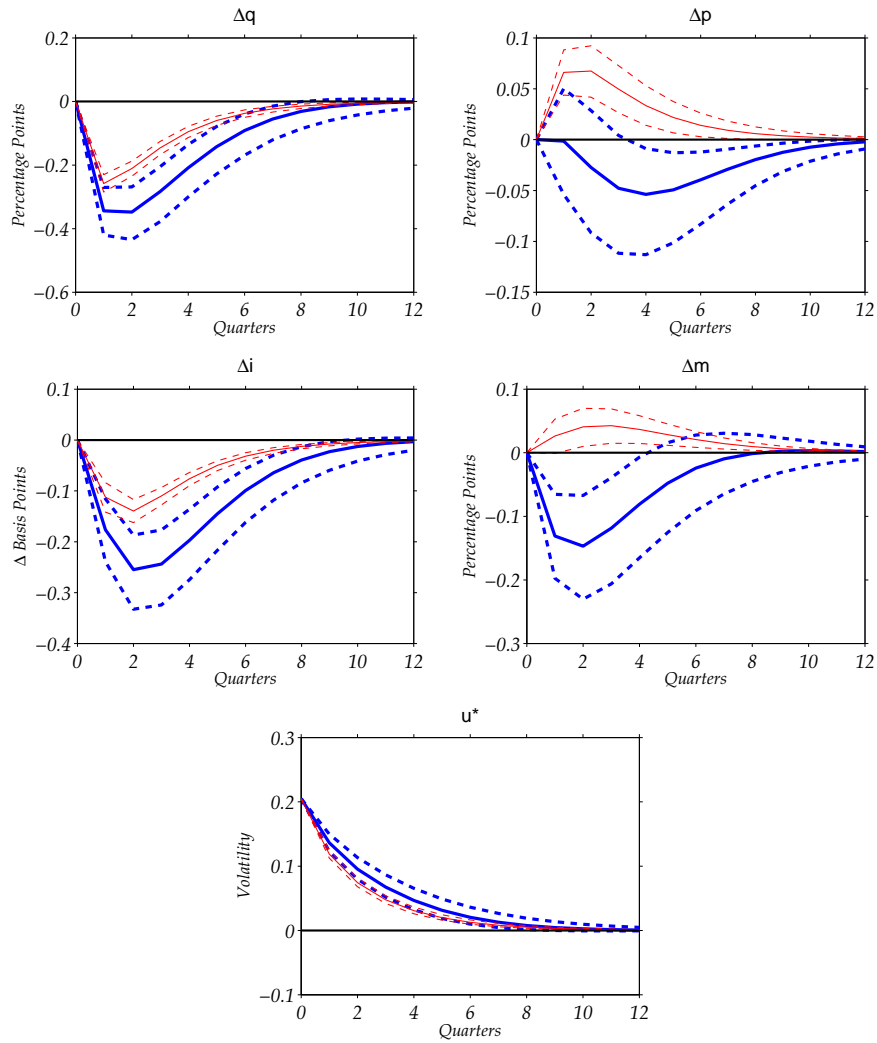


Figure 8: Response to Uncertainty Shock at $\tau = 0.2$

Notes: The graph depicts impulse responses at the quantile (τ) indicated (blue & thick lines) and at the mean (red & thin lines). Solid lines refer to the median impulse response. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* stock market volatility. The x -axis measures quarters, where the shock occurs in quarter zero.

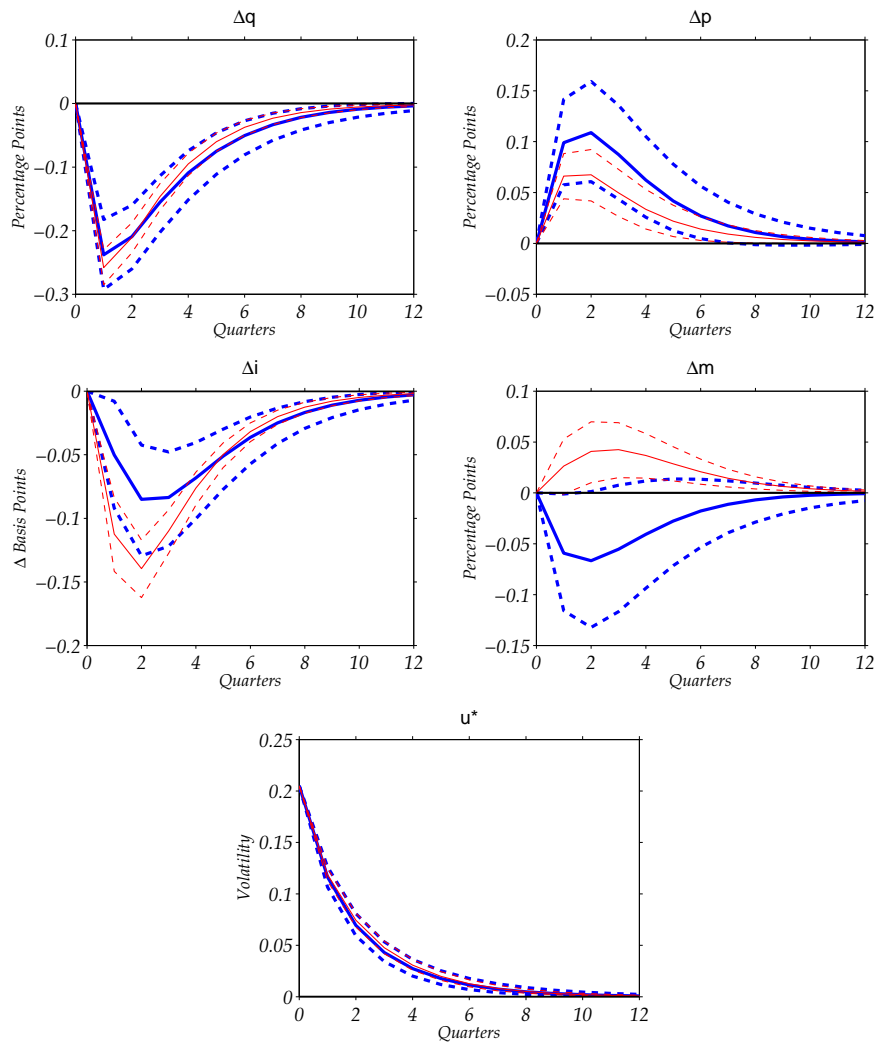


Figure 9: Response to Uncertainty Shock at $\tau = 0.5$

Notes: The graph depicts impulse responses at the quantile (τ) indicated (blue & thick lines) and at the mean (red & thin lines). Solid lines refer to the median impulse response. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* stock market volatility. The x -axis measures quarters, where the shock occurs in quarter zero.

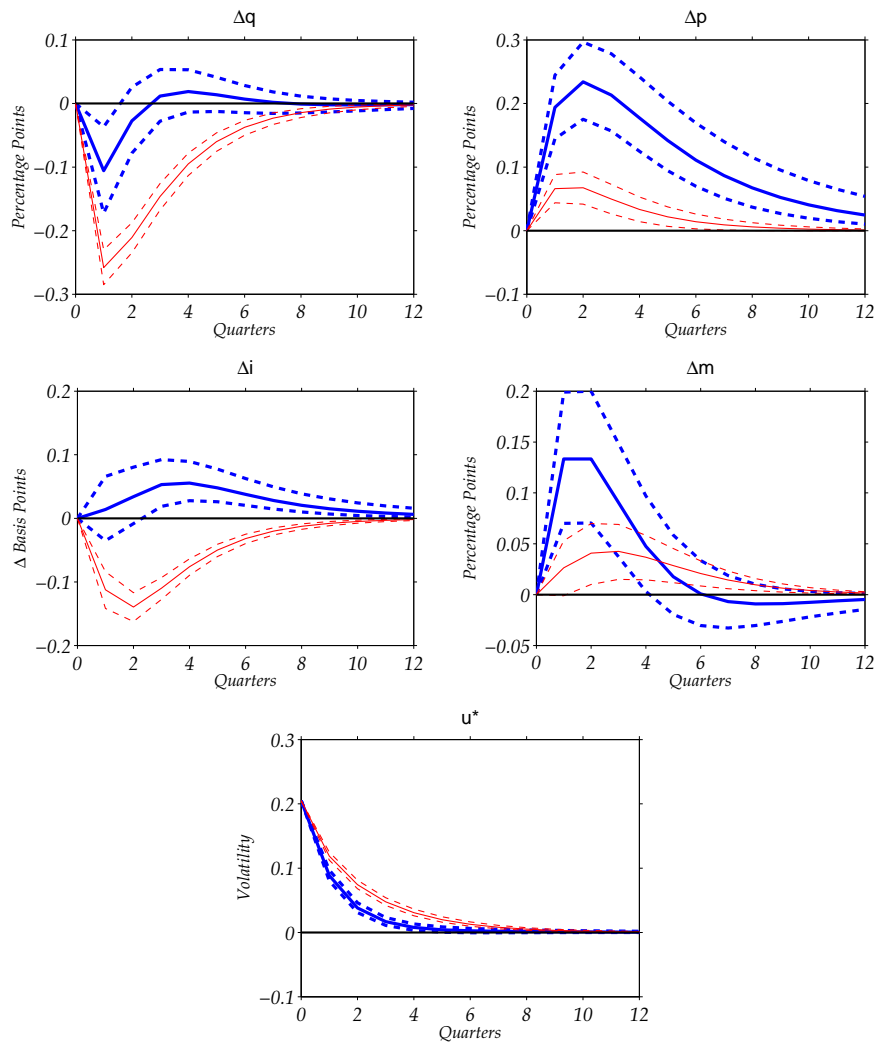


Figure 10: Response to Uncertainty Shock at $\tau = 0.8$

Notes: The graph depicts impulse responses at the quantile (τ) indicated (blue & thick lines) and at the mean (red & thin lines). Solid lines refer to the median impulse response. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* stock market volatility. The x -axis measures quarters, where the shock occurs in quarter zero.

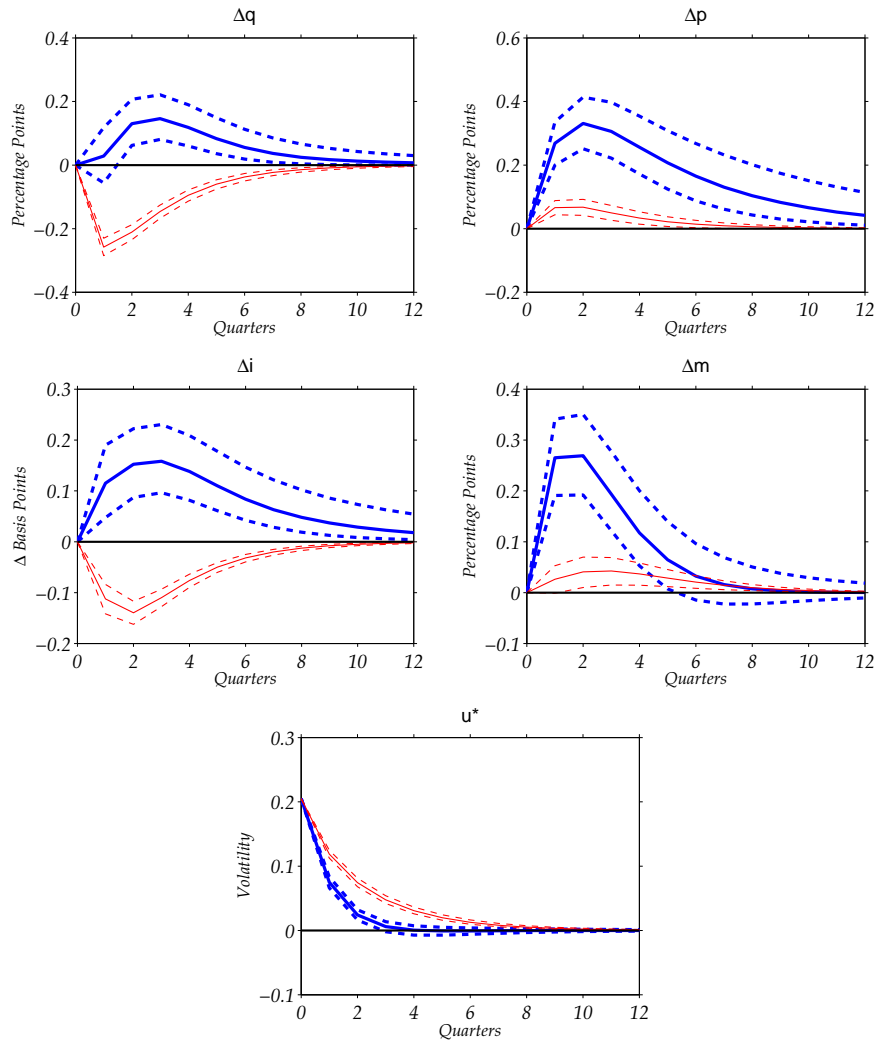


Figure 11: Response to Uncertainty Shock at $\tau = 0.9$

Notes: The graph depicts impulse responses at the quantile (τ) indicated (blue & thick lines) and at the mean (red & thin lines). Solid lines refer to the median impulse response. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* stock market volatility. The x -axis measures quarters, where the shock occurs in quarter zero.

E ROBUSTNESS ANALYSIS

E.1 Stock Returns

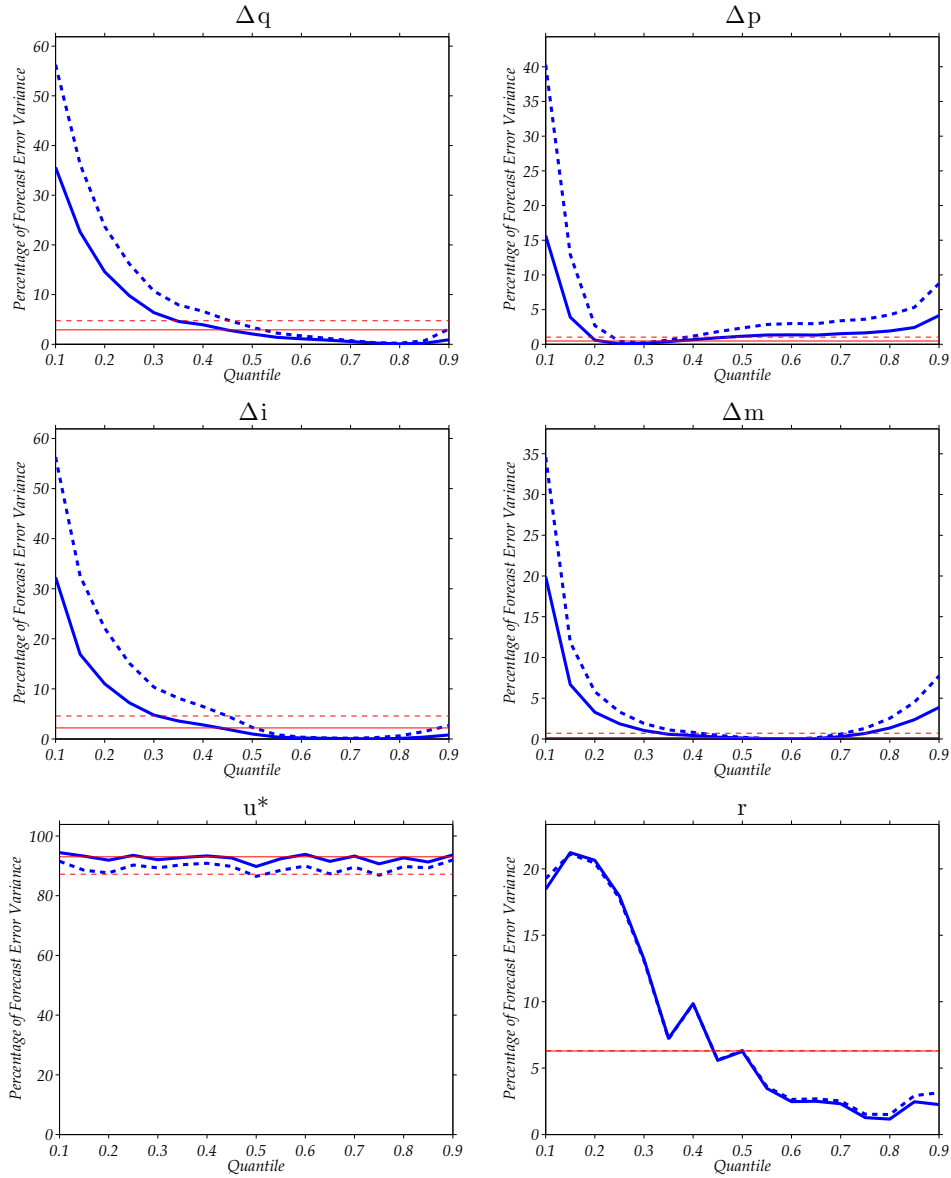


Figure 12: Contribution of Uncertainty Shocks to Fluctuations in US Economic Variables Including Stock Returns: Across Quantiles and at the Mean

Notes: The graph depicts quantile plots, where the x -axis represents the quantiles at which the specific model has been estimated and the y -axis the percentage of variance of the indicated variable explained by uncertainty shocks. The blue and thick lines represent the estimates at the quantiles. The red and thin lines represent the estimates using the Gaussian model. A solid line refers to the average forecast error variance between 1 and 4 quarters (1st year). A dashed line refers to the average from 5 to 12 quarters (2nd and 3rd year). Δq denotes GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* the transformed variable of stock market volatility; r the stock return. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

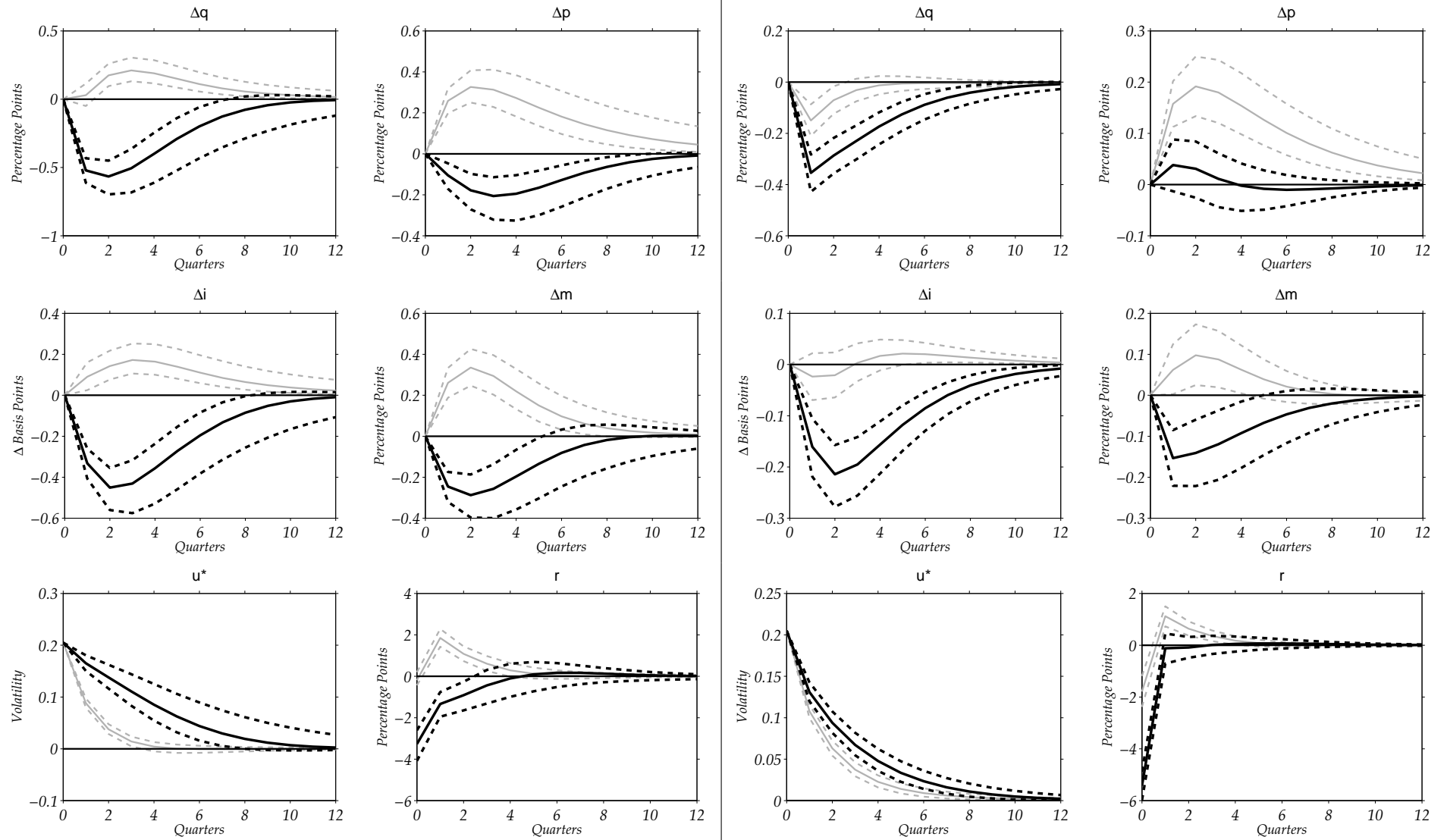


Figure 13: Response of the US Economy Including Stock Returns to an Uncertainty Shock: Left Panel: Responses at $\tau = 0.1$ (black) and $\tau = 0.9$ (gray). Right panel: Responses at $\tau = 0.3$ (black) and $\tau = 0.7$ (gray)

Notes: The left panel depicts the complete impulse responses for quantiles $\tau = 0.1$ (black) and 0.9 (gray). The right panel portrays the complete impulse responses for quantiles $\tau = 0.3$ (black) and 0.7 (gray). The impulse to uncertainty is normalized to one standard deviation of u^* . Solid lines refer to the median impulse response obtained at each quantile. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in the interest rate; Δm growth in money supply; u^* volatility; r stock returns. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

E.2 Germany

E.2.1 Asymmetries over the Business Cycle

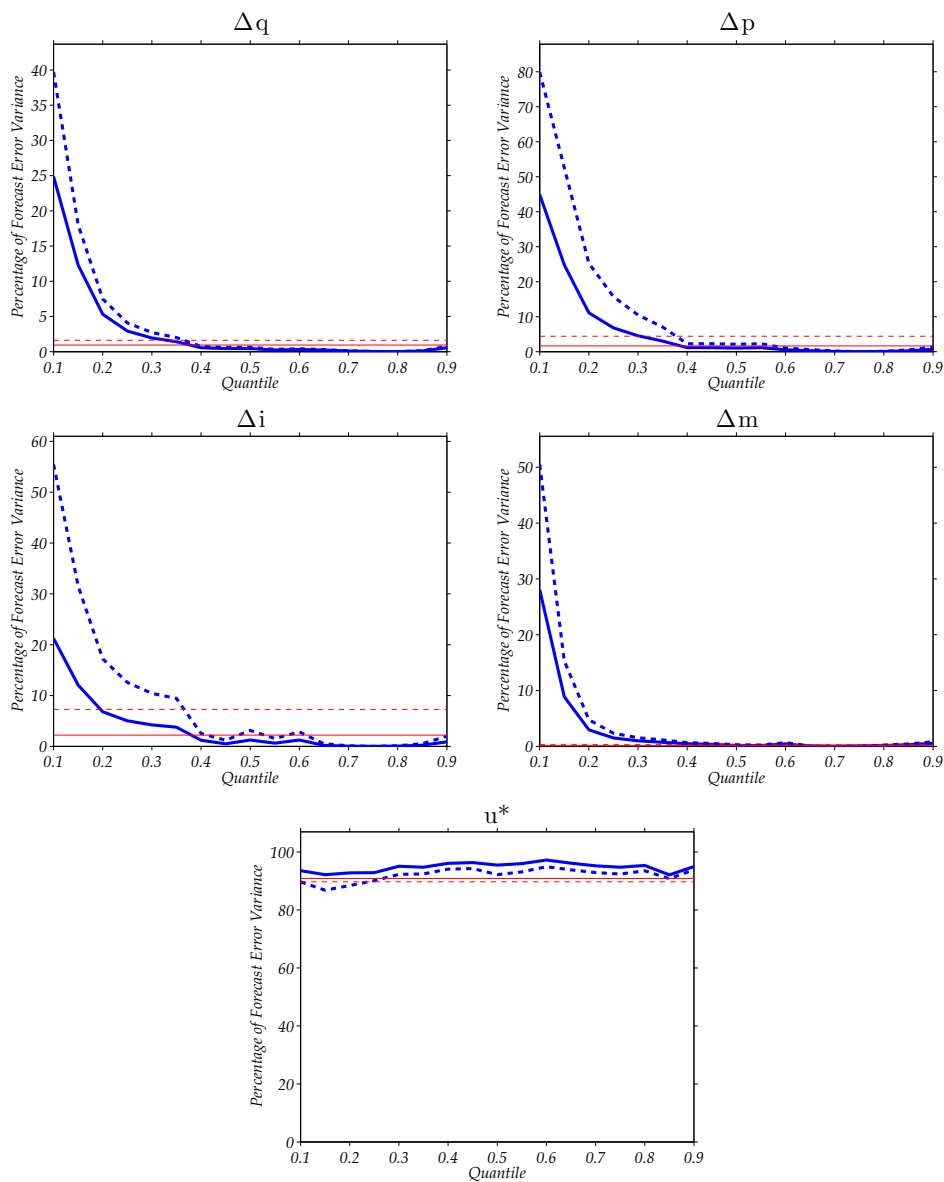


Figure 14: Contribution of Uncertainty Shocks to Fluctuations in German Economic Variables: Across Quantiles and at the Mean

Notes: The graph depicts quantile plots, where the x -axis represents the quantiles at which the specific model has been estimated and the y -axis the percentage of variance of the indicated variable explained by uncertainty shocks. The blue and thick lines represent the estimates at the quantiles. The red and thin lines represent the estimates using the Gaussian model. A solid line refers to the average forecast error variance between 1 and 4 quarters (1st year). A dashed line refers to the average from 5 to 12 quarters (2nd and 3rd year). Δq denotes GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* the transformed variable of stock market volatility. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

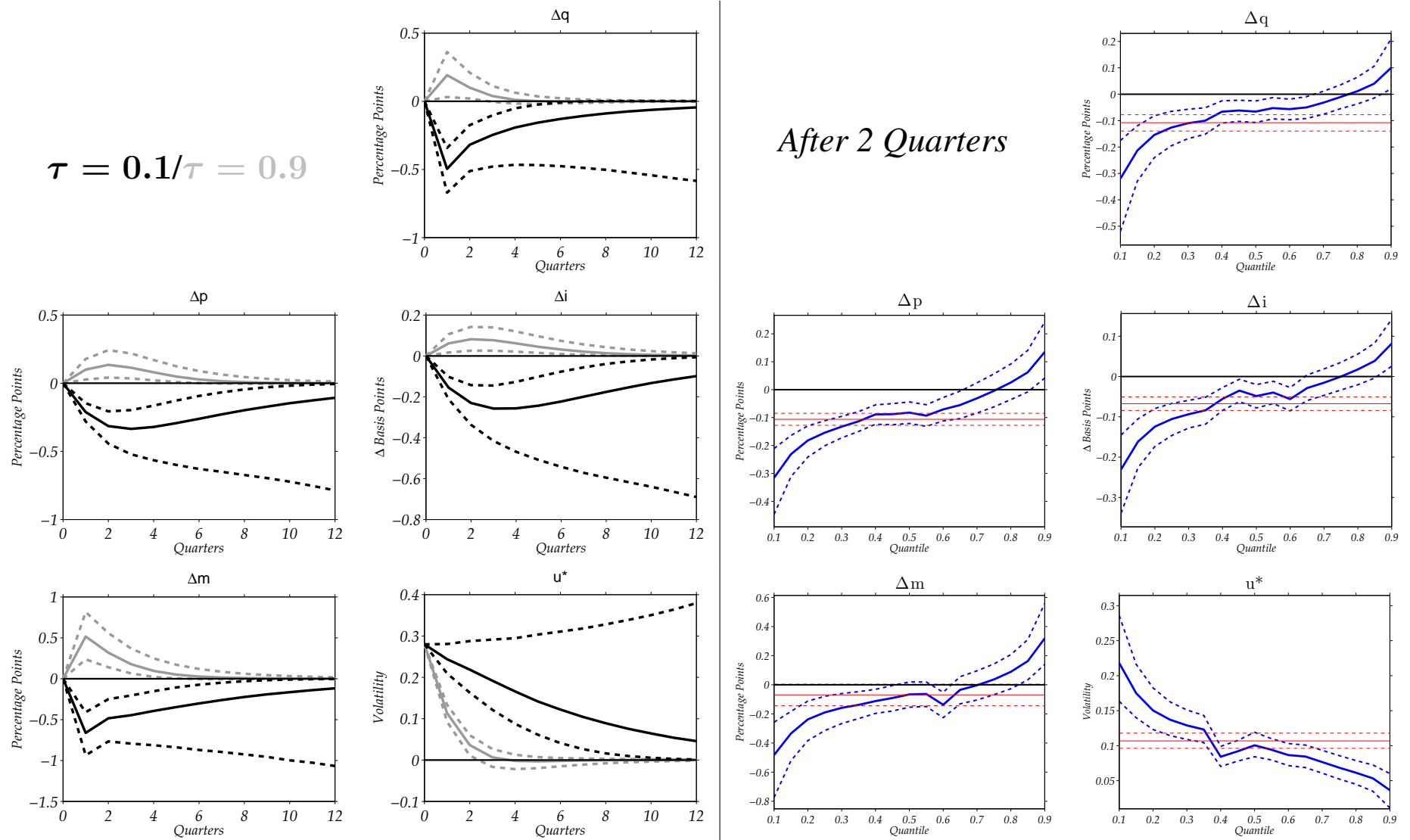


Figure 15: Response of the German Economy to an Uncertainty Shock: Left Panel: Responses at the lowest ($\tau = 0.1$) and highest ($\tau = 0.9$) quantile. Right panel: Responses After 2 Quarters Across Quantiles Compared to the Gaussian Model

Notes: The left panel depicts the complete impulse responses for quantiles $\tau = 0.1$ (black) and 0.9 (gray). The right panel portrays responses over the quantiles 2 quarters after the shock. It depicts impulse responses across quantiles (blue & thick lines) and at the mean (red & thin lines). The impulse to uncertainty is normalized to one standard deviation of u^* . Solid lines refer to the median impulse response obtained at each quantile. The dashed lines correspond to posterior 68% probability bands. Δq denotes real GDP growth; Δp inflation; Δi changes in the interest rate; Δm growth in money supply; u^* volatility. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

E.2.2 Asymmetries over the Financial Cycle: Economy at Bust and Boom

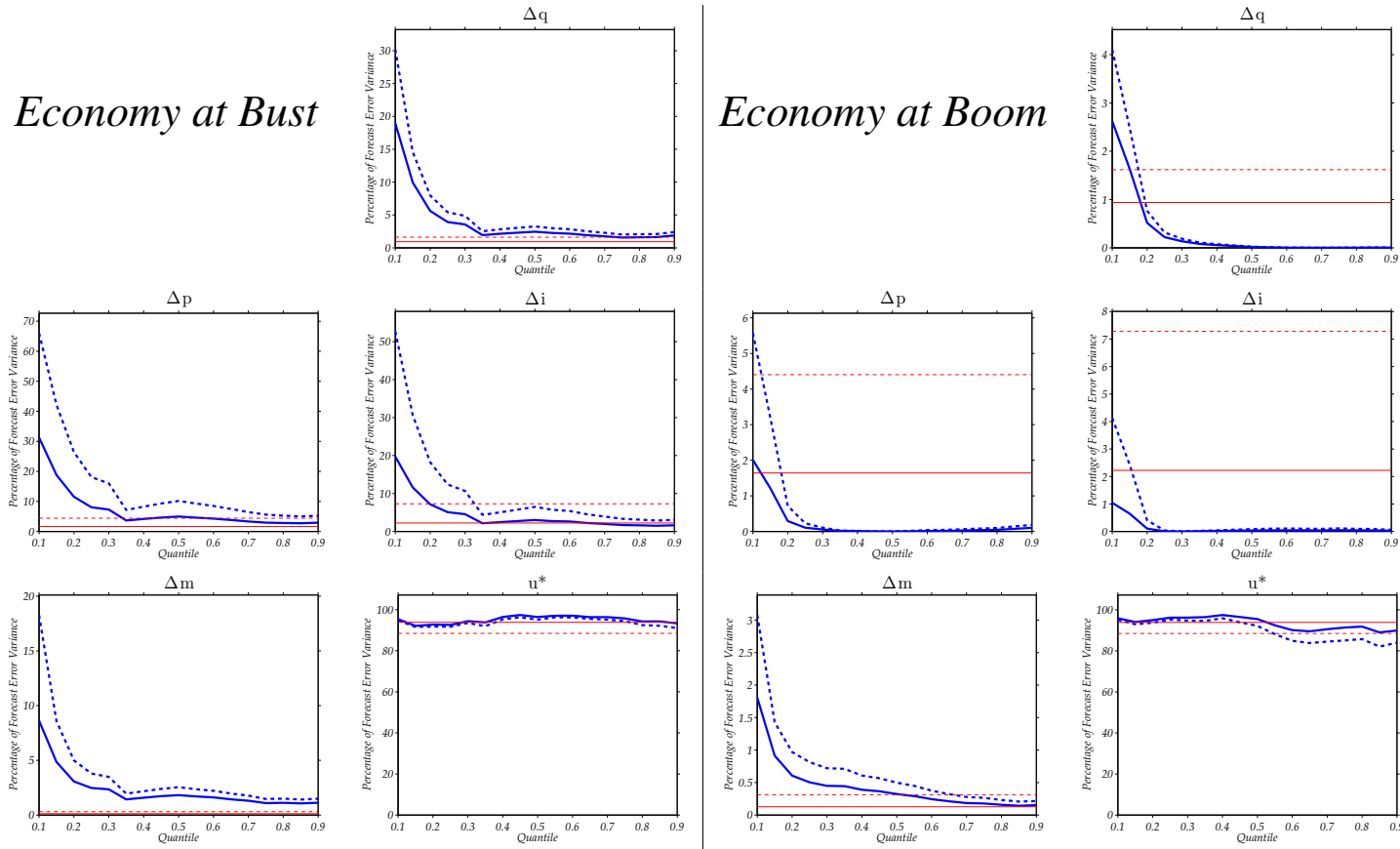


Figure 16: Contribution of Uncertainty Shocks to Fluctuations in the German Economy at Bust ($\tau = 0.2$) and at Boom ($\tau = 0.8$)

Notes: The graph depicts quantile plots, where the x -axis represents the quantiles of the financial cycle τ_5 at which the specific model has been estimated and the y -axis the percentage of variance of the indicated variable explained by uncertainty shocks. The blue and thick lines represent the estimates at the quantiles. The red and thin lines represent the estimates using the Gaussian model. A solid line refers to the average forecast error variance between 1 and 4 quarters (1st year). A dashed line refers to the average from 5 to 12 quarters (2nd and 3rd year). Δq denotes GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* the transformed variable of stock market volatility. Figures are calculated for $\tau = \{0.1, 0.15, \dots, 0.85, 0.9\}$.

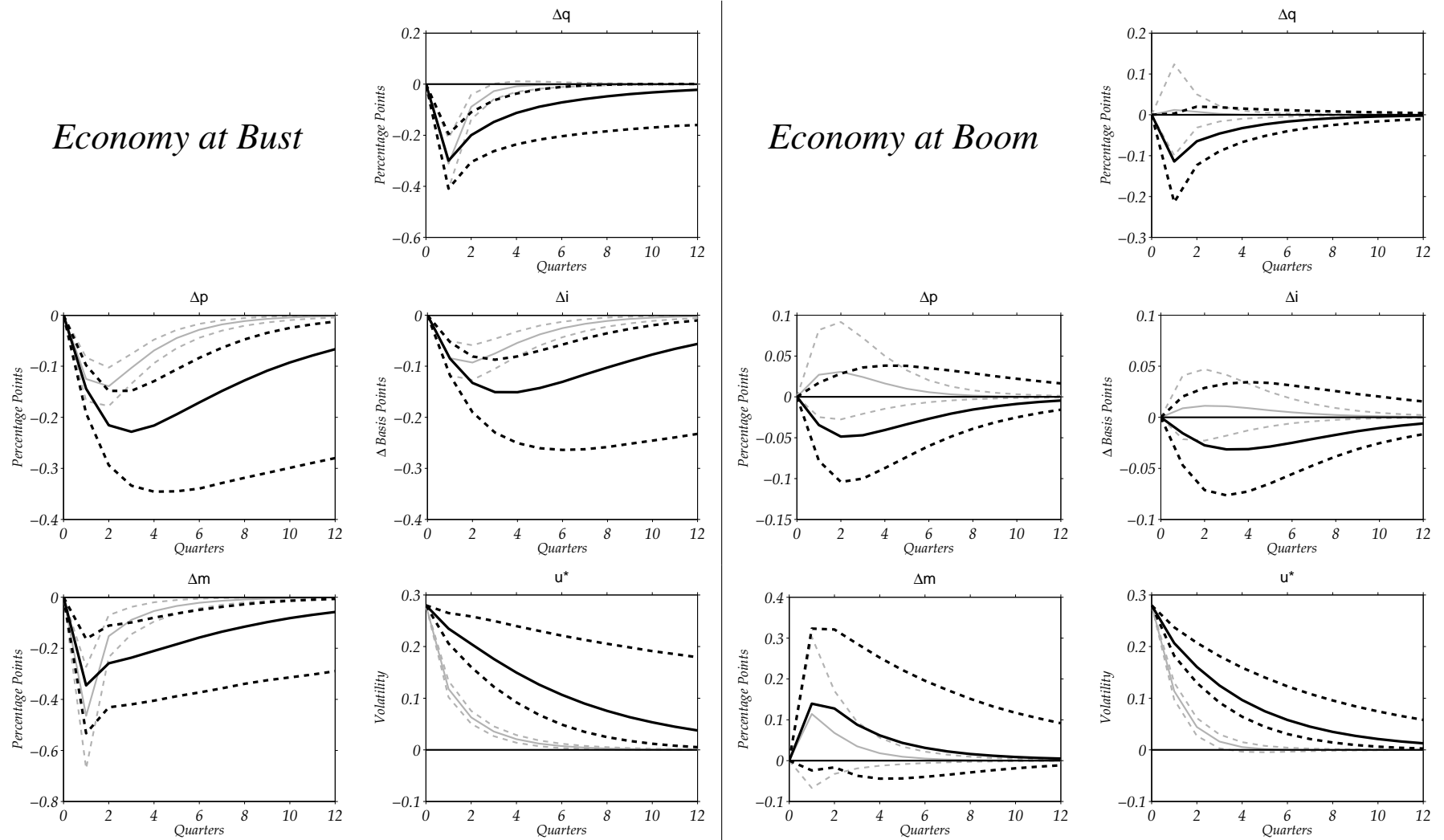


Figure 17: Response of the German Economy to an Uncertainty Shock During Bust and Boom: Shock at $\tau_5 = 0.1$ and at $\tau_5 = 0.9$

Notes: The graph depicts the responses to an uncertainty shock. The left panel depicts an economy at $\tau = 0.2$, i.e., a recession and the right panel an economy at $\tau = 0.8$, i.e., a boom phase. The black lines describe the responses to a financial shock at $\tau_5 = 0.1$ and the gray lines portray the responses to a financial shock at $\tau_5 = 0.9$. The dashed lines correspond to posterior 68% probability bands. Δq denotes GDP growth; Δp inflation; Δi changes in interest rates; Δm growth in money supply; u^* the transformed variable of stock market volatility.