Three Essays on Social Networks in Economics

Dissertation

zur Erlangung des akademischen Grades eines
Doktors der Wirtschaftswissenschaften (Dr.rer.pol.)

vorgelegt von

Livia Shkoza

an der

Universität Konstanz

Sektion Politik – Recht – Wirtschaft
Fachbereich Wirtschaftswissenschaften

Konstanz, 2022

Tag der mündlichen Prüfung: 29. Januar 2023

1. Referent: Prof. Dr. Winfried Pohlmeier
2. Referent: Prof. Dr. Derya Uysal
3. Referent: Prof. Dr. Guido Schwerdt

To my parents, Sonila and Arben Shkoza.
I am very proud I am their daughter.

Acknowledgement

My thesis is on the importance of social interactions, and my experience in the last few years is a true testament to how relationships have the power to change our lives. I have been lucky enough to meet some of the greatest people in my life in Konstanz, and I would like to dedicate a few words to them.

I feel blessed for having had the chance to first hear about Econometrics from Prof. Dr. Winfried Pohlmeier. Just like parents teach their children, he also teaches by setting an example. I learned to be independent, not only in research but also in life. I learned from him the value of a positive attitude, trusting the process, and having different passions in life. When I first heard the term *Doktorvater*, I thought it was so meaningful, and Prof. Pohlmeier gives it true value. I am immensely grateful that he is my mentor.

I would like to thank my second supervisor, Prof. Dr. Derya Uysal, who has offered me helpful support and feedback over the years. I am very thankful not only for our professional conversations, but also for the time I spent with her and her family. I would like to thank my third supervisor, Prof. Dr. Guido Schwerdt, for his support. I enjoyed his course in my first year of the Ph.D. and our later encounters.

My sincere thanks go to Prof. Dr. Roxana Halbleib. She and her family have made me feel so welcome and supported on many occasions. Our conversations hold a special place in my heart. It has been a true pleasure to know and spend some time with Prof. Dr. Lyudmila Grigoryeva. I would like to thank her for her time and advice. I would also like to thank Verena Kretz for her patience and support in all administrative issues, but not only.

I was incredibly lucky to have great colleagues, who turned into dear friends and supporters. I met Dr. Jana Mareckova as my tutor in Advanced Econometrics, and without her help and support I would not be here. The same goes for Dr. Anastasia Simmet, whose support has been constant. Dr. Ekaterina Kazak was always there for advice and help.

The philosophical and practical discussions with Dr. Phillip Heiler, about Econometrics and everything else, remain one of the best memories from my Ph.D. time. Christian Mücher is a good friend and colleague. I am very happy I shared the office with him. I would like to thank Dr. Gerhard Fechteler, who has been an amazing friend and colleague. Working with him has been a true pleasure and a great learning opportunity. Many thanks go to Dr. Maurizio Daniele for always being there for me.

While it has been a constant emotional challenge to be away from my family, it also made me realize how lucky I am to have them. My mother, the biggest force I know, is my most loyal companion, especially during the Ph.D. Her pride is the ultimate driving motivation for everything I do. My brother is the person that can make me smile anytime. My love for him is infinite. My father passed away a long time ago, but I never lost him. For most of my life, I have wanted to honor his memory, and this Ph.D. matches the depth of my love and longing for him.

I would like to thank my dear friends, Diona Gjermeni and Nertila Poci Saglam, who have supported me unconditionally. I met Diona at the beginning of my Ph.D., and she quickly became a very important person in my life. Thanks to her, I met Nertila, who, together with her amazing family, has made me feel at home.

Livia Shkoza Konstanz, November 2022

Table of Contents

List of Tables

List of Figures

Summary

As human beings we are wired to connect with one another. The quality of a person's life crucially depends on the quality of her relationships. Research findings support the hypothesis that social connections have a high positive correlation with life longevity and recovery from an illness. In contrast, the absence of social connections, loneliness, increases the risk of early disease and death, more than smoking. The impact of social connections on mental health is so profound, that it exceeds the effect of exercise and ideal body weight on health (Doty, 2016).

From experience we can all attest to the fact that relationships determine the paths we take, the jobs we choose or how we vote. Because social interactions affect many aspects of our lives, it is crucial to investigate their impact on our behaviour, the types of network structures that are more likely to emerge and why individuals choose their peers. Economics is considered primarily to be the study of markets, where individuals interact through an anonymous process of price formation. Because of this view, for a long time social interactions have not been an important concern for economists. However, in a broader view, at the core of economics is the allocation of resources, and individuals' response to incentives. From this perspective, social interactions become fundamental, because they are shaped by incentives, and subsequently affect the allocation of resources. As Manski (2000) recommends, economists focusing both on policy, and on theory should be concerned about empirical research on social interactions. The impact of nonmarket interactions on schooling outcomes, employment patterns, crime rates and residential segregation has been a question for years and sound empirical analysis is crucial to inform policy. From the theoretical perspective, research should focus on the relevant and prevalent classes of social interactions.

In fact, most empirical studies on the impact of social interactions document correlation among peers' outcomes. However, this does not prove that the individual is affected by

the peers. This correlation could be due to homophily, the tendency to choose similar individuals as peers. This type of correlation is summarized by the correlated effects. It is important to note that correlated effects are not social effects. Identifying the causal effect of social interactions is challenging, due to first, the correlated effects, and second, due to the reflection problem. Because the individual affects his peers, and his peers also determine his outcome, it is impossible to distinguish between endogenous (the impact of the peers' behaviour) and exogenous peer effects (the influence of peers' characteristics) (Manski, 1993).

The growing interest in networks has inspired the developments in the study of peer effects. Contrary to previous assumptions, individuals have their separate peer groups, within a partition of the population, such as a classroom, resulting in a social network. Seeing peer effects in this light, Bramoullé et al. (2009) showed that given the network information, it is possible to solve the reflection problem, and separately identify endogenous and exogenous peer effects. They consider the correlated effects as network fixed effects, and use local or global differencing (between or within transformation). The reflection problem is essentially an endogeneity problem, and instruments are derived based on network intransitivity. Intransitive triads are those where individual i is linked to j, and j is linked to k, but i and k are not linked. So, the characteristic of k impacts the outcome of j, which impacts the outcome of i. But, the characteristic of k is not directly impacting the outcome of i, and can therefore be used as a valid instrument.

The literature has since then grown rapidly, in different directions. Relying on social networks theory, the primary focus at the theoretical level is the simultaneity between outcomes and network formation (Jackson, 2010). Empirical researchers have investigated the impact of social networks on several outcomes, such as welfare participation, criminal activity, obesity and education (see Bertrand et al., 2000; Calvó-Armengol et al., 2009; Patacchini and Zenou, 2008; Trogdon et al., 2008). At the heart of the econometric literature is identification. One important strand of the literature is inspired by the spatial econometric literature (see Bramoullé et al., 2009; Lee et al., 2010), and relies on distribution-free methods, such as IV or GMM, to estimate the peer effects.

This dissertation is concerned with the identification and estimation of social network peer effects in educational settings. The first chapter contributes to the literature on heterogeneous peer effects by proposing an Instrumental Variable - Minimum Distance (IV-MD) estimator, which accounts for classroom characteristics in estimating peer effects. The second chapter incorporates the social network approach in the estimation of stress contagion in school. The third chapter focuses on isolated individuals, and explores two matters: the right definition of an isolated individual, and the consequences of removing them. In the three chapters we rely on the linear model of peer effects, which captures in a compact way the impact of the peers on the individual. It includes the local-aggregate and local-average endogenous peer effects, i.e., the impact of the aggregate and of the average behaviour of the peers. The former effect corresponds to the hypothesis that the utility of the individual is increasing in the peers' efforts. The latter effect comprises the assumption that the individual tries to conform to the norm of the group, the norm being represented by the average peers' outcome.

The goal of the first project is to include heterogeneity in the estimation of peer effects. A traditional strategy in the literature is to estimate homogeneous coefficients across all distinct networks. However, as we find in our data, the networks tend to have very different structures. To overcome this limitation, we propose a novel IV-MD estimation strategy, which is easy to implement, and effective in accounting for network heterogeneity. We augment the composite model proposed by Liu et al. (2014), which includes the local-aggregate and local-average peer effects hypotheses. Suppose we have L networks. Instead of grouping all L networks in a block diagonal structure (which is the case when homogeneous peer effects are estimated), we estimate the composite model by IV for each network separately, and obtain L sets of first stage parameters. In a second stage, we suppose that the first stage peer effects parameters are explained by a set of network specific observable factors - size and fraction of girls. The structural parameters measure the effect of the network factors on the peer effects parameters. The Minimum Distance estimator used in the second stage minimizes the weighted quadratic difference between the first stage parameters and the structural parameters. In our study the impact of network factors has a structural interpretation, contrary to the vast empirical studies that are based on reduced form approaches. Moreover, we can easily test the different specifications against each other using the MD test statistics. Our Monte Carlo simulations reveal reasonable coverage probabilities, even when the networks are dense, and identification gets weak as a consequence. The numerical insights support the inclusion of the local-aggregate component.

The empirical application is based on a unique dataset of 85 school classes of secondary schools in Germany. Due to limited network data availability, most studies on peer effects rely on the Add Health dataset, where students were asked to nominate up to five male and five female friends, within the school. Contrary to this, in our data the students were asked to nominate their peers within the classroom, without limiting their number. Using the rich Gymnasiasten-Studie data, we are able to construct the networks of social interactions, compute the average grades, extract the results of a standard psychometric Intelligence Structure Test, and many individual characteristics. In our empirical application we find that network size and gender composition are significant determinants of peer effects, while the homogeneous model produces insignificant estimates. In addition, we find that both hypotheses are relevant in determining the education outcome of the individual: the aggregate outcome of the peers and the norm of the group. Our results show that class size has a negative impact on peer influence, while the sign of gender composition is different for the two hypotheses.

The goal of the second project is to incorporate the network framework into the study of stress in school. As a major determinant of school outcomes, stress is not only determined by individual characteristics, such as health or family background, but also by the stress experience of the peers. Stress in school is widely researched, and it has been linked to many factors, such as time pressure or examinations. However, the role of the peers in experiencing stress has been overall neglected. While the role of peer influence in academic achievement is recognized and studied, the evidence on stress transmission is scarce. In fact, studies show that negative emotions are more contagious than positive emotions, and in this sense the study of stress contagion becomes more meaningful.

Previous studies on stress contagion rely on multilevel regression models, which do not account for the impact of the peers. To overcome this, we investigate stress contagion in the context of a structural network approach. We argue that the seating plan plays a crucial role, because it determines the network of close peers. To model the interdependencies among those sitting next to each other we borrow from the spatial, e.g. Anselin (2003), and the network peer effects literature, e.g. Bramoullé et al. (2009). Using a panel dataset on students' stress levels, we identify and estimate a parameter of stress contagion, and inspect different factors that may aggravate the stress experience. We model the dependent variable - the stress level of individual i at time t - as a linear function of the sum of the stress levels of i's spatial peers, the individual characteristics, and the exogenous characteristics of the peers. The novelty of our model rests in the time varying endogenous peer effects parameter, which depends on the contextual factors measuring the situation in the classroom. We estimate the model by Maximum Likelihood.

Our empirical results are based on a unique dataset exploited first by Kärner et al. (2017). The data stems from two classrooms, with different seating arrangements, which we use to construct the networks. Classroom A presents a traditional seating arrangement, following a teacher-centered organization style, while in classroom B the students sit in round tables, working more in groups and receiving less teacher instruction. The data on stress is self-reported by the students, who were asked at fixed time points to report their stress and coping levels. Finally, video recordings enable us to construct context factors reflecting the overall situation in the classroom. We consider two dependent variables:

pressure to succeed and *time pressure*. As exogenous individual characteristics we use the average of the two variables: *understanding* and *self-confidence*. Finally, as context factors we include: *teacher instruction*, *teacher time pressure*, *complexity*, *cooperative work* and *individual work*.

We find significant peer effects across all specifications, confirming the hypothesis that stress transmits across peers, i.e., the more stressed the spatial peers are, the more stressed is the individual. Our results show that better *understanding* and higher *self-confidence* imply a less stressful experience. Counterintuitively, we find that when the peers cope better, the individual is more stressed. However a plausible explanation for this results is social comparison, i.e., when the individual compares himself to the peers, to assess that the peers are coping better than him, resulting in individual stress. Finally, we find that accounting for different context factors is crucial, as it provides important insights. *Cooperative work* has a positive impact on stress contagion only in classroom B. The role of the teacher is crucial, as it has a negative effect in all our specifications. The reducing effect of *teacher instruction* on stress contagion is more prevalent in classroom B, which is sensible, as students in class B spend more time working independently, and a teacher intervention releases some of that responsibility and stress.

The third project focuses on isolated individual in social networks. Two matters, that are not investigated yet in the literature, are considered: the definition of an isolated individual and the consequences of removing them. An isolated person claims to have no peers, i.e., his outdegree is 0. As a result, this person is not subject to endogeneity, as he is not influenced by any peers. However, this person might receive links, as he might be a peer to some connected individuals. In this case his indegree is not 0. The presence of isolated individuals is not harmful to the identification strategy, on the contrary, there is evidence they might be helpful to identify the social multiplier (Boucher and Fortin, 2015). The literature so far has not reached a final conclusion on the best way to treat these individuals. In fact, most empirical studies in the literature remove isolated nodes from the dataset.

To shed some light on this matter, we consider two definitions of isolated individuals and different network structures. We define a *completely isolated* individual as someone with outdegree and indegree equal to 0, and a *partially isolated* one as someone with outdegree 0. A *completely isolated* person does not impact the structure of the network, but his equation serves as an identifying restriction, which can be used to build a moment function. Removing these observations results in less observations and less moment functions, which in the GMM framework translates into an asymptotically inefficient estimator. A *partially isolated* individual might receive some links. If we remove them, not only are the observations lost, but also the regressors of the individuals who send the links change. As a consequence, the IV estimator is biased. We show analytically how the IV estimator of the local-aggregate model is inefficient and inconsistent, when *completely isolated* and *partially isolated* individuals are removed, respectively. The results can be easily extended to the local-average specification.

In our Monte Carlo simulations we inspect the behaviour of the estimates for two different networks: Erdös Rényi and Small World networks. The latter type has attributes more similar to observed networks, namely the small diameter (the shortest path between the two most distant nodes). The numerical insights of our simulations suggest that identifying those with outdegree 0 as isolated individuals, and removing them, is the most harmful strategy, in terms of MSE and bias. The estimates are biased in all scenarios, and their variances are larger. Removing the *completely isolated* generally has negligible negative consequences, especially if the networks are small.

References

Anselin, L. (2003). "Spatial Econometrics". In: *A Companion to Theoretical Econometrics*. Ed. by Baltagi, B. H. John Wiley & Sons, Ltd. Chap. 14, pp. 310–330.

Bertrand, M., E. F. P. Luttmer, and S. Mullainathan (2000). "Network Effects and Welfare Cultures". *The Quarterly Journal of Economics* 115 (3), pp. 1019–1055.

Boucher, V. and B. Fortin (Feb. 2015). *Some Challenges in the Empirics of the Effects of Networks*. IZA Discussion Papers 8896. Institute of Labor Economics (IZA).

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). "Identification of peer effects through social networks". *Journal of Econometrics* 150 (1), pp. 41–55.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009). "Peer effects and social networks in education". *The Review of Economic Studies* 76 (4), pp. 1239–1267.

Doty, J. (2016). *Into the Magic Shop: A Neurosurgeon's Quest to Discover the Mysteries of the Brain and the Secrets of the Heart*. Penguin Publishing Group.

Jackson, M. O. (2010). *Social and economic networks*. Princeton University Press.

Kärner, T., D. Sembill, C. Aßmann, E. Friederichs, and C. H. Carstensen (2017). "Analysis of person-situation interactions in educational settings via cross-classified multilevel longitudinal modeling: illustrated with the example of students' stress experience". *Frontline Learning Research* 5 (1), pp. 16–42.

Lee, L.-f., X. Liu, and X. Lin (2010). "Specification and estimation of social interaction models with network structures". *The Econometrics Journal* 13 (2), pp. 145–176.

Liu, X., E. Patacchini, and Y. Zenou (2014). "Endogenous peer effects: Local aggregate or local average?" *Journal of Economic Behavior and Organization* 103, pp. 39–59.

Manski, C. F. (1993). "Identification of Endogenous Social Effects: The Reflection Problem". *The Review of Economic Studies* 60 (3), pp. 531–542.

– (2000). "Economic Analysis of Social Interactions". *Journal of Economic Perspectives* 14 (3), pp. 115–136.

Patacchini, E. and Y. Zenou (2008). "The strength of weak ties in crime". *European Economic Review* 52 (2), pp. 209–236.

Trogdon, J., J. Nonnemaker, and J. Pais (2008). "Peer effects in adolescent overweight". *Journal of Health Economics* 27 (5), pp. 1388–1399.

Zusammenfassung

Als Menschen sind wir darauf ausgerichtet, miteinander in Verbindung zu treten. Die Qualität des Lebens eines Menschen hängt entscheidend von der Qualität seiner Beziehungen ab. Forschungsergebnisse stützen die Hypothese, dass soziale Beziehungen eine hohe positive Korrelation mit der Lebenserwartung und der Genesung von Krankheiten haben. Demgegenüber erhöht das Fehlen sozialer Beziehungen, die Einsamkeit, das Risiko einer frühen Erkrankung und eines frühen Todes, stärker als Rauchen. Die Auswirkung sozialer Beziehungen auf die psychische Gesundheit ist so tiefgreifend, dass sie sogar die Wirkung von Bewegung und Körpergewicht auf die Gesundheit übertrifft (Doty, 2016).

Aus eigener Erfahrung können wir alle bestätigen, dass Beziehungen die Wege bestimmen, die wir einschlagen, die Berufe, die wir wählen oder wie wir wählen. Da sich soziale Interaktionen auf viele Aspekte unseres Lebens auswirken, ist es von entscheidender Bedeutung zu untersuchen, wie sie sich auf unser Verhalten auswirken, welche Arten von Netzwerkstrukturen sich am ehesten herausbilden und wie Individuen ihre Peers auswählen. Die Wirtschaftswissenschaften gelten in erster Linie als die Lehre der Märkte, auf denen Individuen durch einen Prozess der Preisbildung interagieren. Aufgrund dieser Sichtweise waren soziale Interaktionen lange Zeit kein wichtiges Thema für Wirtschaftswissenschaftler. Bei genauerer Betrachtung steht jedoch die Zuweisung von Ressourcen und die Reaktion des Einzelnen auf Anreize im Mittelpunkt der Wirtschaftswissenschaften. Aus dieser Perspektive sind soziale Interaktionen von grundlegender Bedeutung, da sie durch Anreize beeinflusst werden und einer der Faktoren sind, die die Ressourcenallokation bestimmen. Manski (2000) empfiehlt Wirtschaftswissenschaftlern, die sich sowohl mit Politik als auch mit Theorie beschäftigen, sich mit der empirischen Forschung zu sozialen Interaktionen befassen. Die Auswirkungen nicht-marktbezogener Interaktionen auf Schulbildung, Beschäftigungsmuster, Kriminalitätsraten und Wohnsegregation sind seit Jahren ein Thema und eine fundierte empirische Analyse ist für die politische Entscheidungsfindung von entscheidender Bedeutung. Aus theoretischer Sicht sollte sich die

Forschung auf die relevanten und vorherrschenden Klassen von sozialen Interaktionen konzentrieren.

Tatsächlich belegen die meisten empirischen Studien über die Auswirkungen sozialer Interaktionen eine Korrelation zwischen den Ergebnissen der Peers. Dies beweist jedoch nicht, dass der Einzelne durch die Peers beeinflusst wird. Diese Korrelation könnte auf Homophilie zurückzuführen sein, d. h. auf die Tendenz, ähnliche Personen als Peers zu wählen. Diese Art der Korrelation wird unter dem Begriff korrelierte Effekte zusammengefasst. Es ist wichtig zu beachten, dass korrelierte Effekte keine sozialen Effekte sind. Die Ermittlung der kausalen Wirkung sozialer Interaktionen ist eine Herausforderung, zum einen aufgrund der korrelierten Effekte und zum anderen aufgrund des Reflexionsproblems. Da das Individuum seine Peers beeinflusst und diese auch sein Ergebnis bestimmen, ist es unmöglich, zwischen endogenen (den Auswirkungen des Verhaltens der Freunde) und exogenen Peer-Effekten (dem Einfluss der Eigenschaften der Peers) zu unterscheiden (Manski, 1993).

Das gesteigerte Bewusstsein für die Relevanz sozialer Netzwerke führte in den letzten Jahren zu einem erhöhten Forschungsinteresse. Im Gegensatz zu früheren Annahmen haben Individuen ihre eigenen Peer-Gruppen innerhalb einer Teilmenge der Bevölkerung, wie z. B. einem Klassenzimmer, was zu einem sozialen Netzwerk führt. Unter diesem Gesichtspunkt zeigten Bramoullé u. a. (2009), dass es angesichts der Netzwerkinformationen möglich ist, das Reflexionsproblem zu lösen und endogene und exogene Peer-Effekte getrennt zu identifizieren. Sie betrachten die korrelierten Effekte als feste Netzwerkeffekte und verwenden eine lokale oder globale Differenzierung. Das Reflexionsproblem ist im Wesentlichen ein Endogenitätsproblem und die Instrumente werden auf der Grundlage der Intransitivität des Netzwerks abgeleitet. Intransitive Triaden sind solche, bei denen die Individuum i mit j und j mit k verbunden sind, nicht aber i mit k. Das Merkmal von k wirkt sich also auf das Ergebnis von j aus, das sich wiederum auf das Ergebnis von i auswirkt. Das Merkmal von k wirkt sich jedoch nicht direkt auf das Ergebnis von i aus und kann daher als gültiges Instrument verwendet werden.

Die Literatur entwickelte sich seither rasant in verschiedene Richtungen. Der Schwerpunkt der theoretischen Forschung liegt auf der gemeinsame Modellierung von Ergebnissen und Netzwerkbildung (Jackson, 2010). Empirische Forschung untersuchte hauptsächlich die Auswirkungen sozialer Netzwerke auf verschiedene Ergebnisse, wie z. B. auf die Teilnahme an Sozialhilfe, kriminellen Aktivitäten, Fettleibigkeit und Bildung (siehe Bertrand u. a., 2000; Calvó-Armengol u. a., 2009; Patacchini und Zenou, 2008; Trogdon u. a., 2008). Im Mittelpunkt der ökonometrischen Literatur steht die Identifizierung der Peer-Effekte. Ein wichtiger Teil der Literatur ist von der räumlichen ökonometrischen Literatur inspiriert (siehe Bramoullé u. a., 2009; Lee u. a., 2010) und stützt sich auf verteilungsfreie Methoden, wie IV oder GMM, um die Peer-Effekte zu schätzen.

Die vorliegende Dissertation befasst sich mit der Identifikation und Schätzung von Peer-Effekten in sozialen Netzwerken im Bildungsbereich. Das erste Kapitel leistet einen Beitrag zur Literatur über heterogene Peer-Effekte, indem es einen Instrumentalvariablen-Minimum-Distanz (IV-MD)-Schätzer vorschlägt, der Klassenzimmereigenschaften bei der Schätzung von Peer-Effekten berücksichtigt. Im zweiten Kapitel wird der Ansatz des sozialen Netzwerks auf die Schätzung der Stressansteckung unter Schulkindern angewendet. Das dritte Kapitel konzentriert sich auf isolierte Individuen und untersucht zwei Fragen: was ist die richtige Definition eines isolierten Individuums und was sind die Folgen des Ausschlusses dieser Individuen von der Analyse. In den drei Kapiteln stützen wir uns auf ein lineares Modell der Peer-Effekte, das auf kompakte Weise die Auswirkungen der Peers auf das Individuum erfasst. Es umfasst die lokal-aggregierten und lokal-durchschnittlichen endogenen Peer-Effekte, d. h. die Auswirkungen des aggregierten und des durchschnittlichen Verhaltens der Peers. Der erste Effekt entspricht der Hypothese, dass der Nutzen des Individuums mit den Bemühungen der Peers zunimmt. Der zweite Effekt umfasst die Annahme, dass der Einzelne versucht, sich der Norm der Gruppe anzupassen, wobei die Norm durch das durchschnittliche Ergebnis der Peers repräsentiert wird.

Ziel des ersten Projekts ist die Berücksichtigung von Heterogenität bei der Schätzung von Peer-Effekten. Eine traditionelle Strategie in der Literatur ist die Schätzung homogener Koeffizienten für alle unterschiedlichen Netzwerke. Wie wir jedoch in unseren Daten feststellen, weisen die Netzwerke, in unserem Fall Klassenzimmer, in der Regel sehr unterschiedliche Strukturen auf. Um diese Einschränkung zu überwinden, schlagen wir eine neuartige IV-MD-Schätzstrategie vor, die einfach zu implementieren ist und die Heterogenität der Netzwerke effektiv berücksichtigt. Wir erweitern das von Liu u. a. (2014) vorgeschlagene zusammengesetzte Modell, das die Hypothesen der lokal-aggregierten und lokal-durchschnittlichen Peer-Effekte beinhaltet. Angenommen wir haben L Netzwerke. Anstatt alle L Netzwerke gemeinsam zu einer Blockdiagonalstruktur zu gruppieren (was der Fall ist, wenn homogene Peer-Effekte geschätzt werden), schätzen wir das zusammengesetzte Modell durch IV für jedes Netzwerk separat und erhalten L Sätze von Parametern der ersten Stufe. In einem zweiten Schritt nehmen wir an, dass die Parameter der ersten Stufe für die Peer-Effekte durch eine Reihe von netzwerkspezifischen beobachtbaren Faktoren - Größe und Anteil der Mädchen - erklärt werden. Die Strukturparameter messen den Effekt der Netzwerkfaktoren auf die Peer-Effekt-Parameter. Der in der zweiten Stufe verwendete Minimum-Distanz-Schätzer minimiert die gewichtete quadratische Differenz zwischen den Parametern der ersten Stufe und den strukturellen Parametern. In unserer Studie wird der Einfluss der Netzwerkfaktoren strukturell interpretiert, im Gegensatz zu den zahlreichen empirischen Studien, die auf reduzierten Formansätzen beruhen. Außerdem können wir die verschiedenen Spezifikationen mit Hilfe der MD-Teststatistiken leicht gegeneinander testen. Unsere Monte-Carlo-Simulationen zeigen angemessene Abdeckungswahrscheinlichkeiten, selbst wenn die Netzwerke dicht sind und die Identifizierung infolgedessen schwach wird. Die numerischen Erkenntnisse unterstützen die Einbeziehung der lokalen Aggregatkomponente.

Die empirische Anwendung basiert auf einem einzigartigen Datensatz von 85 Schulklassen von Sekundarschulen in Deutschland. Aufgrund der begrenzten Verfügbarkeit von Netzwerkdaten stützen sich die meisten Studien zu Peer-Effekten auf den Add Health-Datensatz, bei dem die Schüler gebeten wurden, bis zu fünf männliche und fünf weibliche Freunde innerhalb der Schule zu benennen. Im Gegensatz dazu wurden die Schülerinnen und Schüler in unseren Daten gebeten, ihre Peers innerhalb des Klassenzimmers zu benennen, ohne deren Anzahl zu begrenzen. Anhand der umfangreichen Daten der Gymnasiasten-Studie können wir die Netzwerke sozialer Interaktionen konstruieren, die Durchschnittsnoten berechnen, die Ergebnisse eines standardisierten psychometrischen Intelligenz-Struktur-Tests und viele individuelle Merkmale extrahieren. In unserer empirischen Anwendung stellen wir fest, dass die Größe des Netzwerks und die Zusammensetzung des Geschlechts signifikante Determinanten der Peer-Effekte sind, während das homogene Modell insignifikante Schätzungen liefert. Darüber hinaus stellen wir fest, dass sowohl das Gesamtergebnis, als auch die Norm der Gruppe für die Bestimmung des Bildungsergebnisses des Einzelnen relevant sind: Unsere Ergebnisse zeigen, dass die Klassengröße eine negative Auswirkung auf den Einfluss der Peers hat, während das Vorzeichen der Geschlechterzusammensetzung für die beiden Hypothesen unterschiedlich ist.

Ziel des zweiten Projekts ist es, den Netzwerkrahmen in die Untersuchung von Stress in der Schule einzubeziehen. Als wichtige Determinante für schulische Leistungen wird Stress nicht nur durch individuelle Merkmale wie Gesundheit oder familiären Hintergrund bestimmt, sondern auch durch die Stresserfahrung der Peers. Stress in der Schule ist weithin erforscht und wurde mit vielen Faktoren wie Zeitdruck oder Prüfungen in Verbindung gebracht. Die Rolle der Peers beim Erleben von Stress wurde jedoch meist nicht betrachtet. Während die Rolle des Einflusses von der Peers auf die schulischen Leistungen anerkannt und untersucht ist, gibt es nur wenige Belege für die Übertragung von Stress. Tatsächlich zeigen Studien, dass negative Emotionen ansteckender sind als positive Emotionen. In diesem Sinne ist die Untersuchung der Stressansteckung von großer Bedeutung.

Bisherige Studien zur Stressansteckung stützen sich auf mehrstufige Regressionsmodelle, bei denen die Auswirkungen der Peers nicht berücksichtigt werden. Um diese Lücke in der Literatur zu schließen, untersuchen wir die Stressansteckung im Rahmen eines strukturellen Netzwerkansatzes. Wir argumentieren, dass der Sitzplan eine entscheidende Rolle spielt, da er das Netzwerk enger Peers bestimmt. Um die Interdependenzen zwischen den Sitznachbarn zu modellieren, verwenden wir Ergebnisse aus der Literatur zu räumliche Ökonometrie, z.B. Anselin (2003), und der Netzwerk-Peer-Effekte-Literatur, z.B. Bramoullé u. a. (2009). Anhand eines Paneldatensatzes über das Stressniveau von Studenten ermitteln und schätzen wir einen Parameter für die Stressansteckung und untersuchen verschiedene Faktoren, welche die Stresserfahrung verschlimmern können. Wir modellieren die abhängige Variable - das Stressniveau des Individuums i zum Zeitpunkt t - als eine lineare Funktion der Summe der Stressniveaus der räumlichen Peers von i, der individuellen, und der exogenen Merkmale der Peers. Die Neuheit unseres Modells liegt in dem zeitlich variierenden endogenen Peer-Effekt-Parameter, der von den Kontextfaktoren abhängt, welche die Situation im Klassenzimmer messen. Wir schätzen das Modell mittels Maximum Likelihood.

Unsere empirischen Ergebnisse beruhen auf einem einzigartigen Datensatz, der zuerst von Kärner u. a. (2017) genutzt wurde. Die Daten stammen aus zwei Klassenzimmern mit unterschiedlichen Sitzordnungen, die wir für die Konstruktion der Netzwerke verwenden. Klassenzimmer A weist eine traditionelle Sitzordnung auf, die einem lehrerzentrierten Organisationsstil folgt, während in Klassenzimmer B die Schüler an runden Tischen sitzen, mehr in Gruppen arbeiten, und weniger Anweisungen des Lehrers erhalten. Die Daten zum Stress beruhen auf Selbstauskünften der Schüler, die zu bestimmten Zeitpunkten zu ihrem Stress- und Bewältigungsniveau befragt wurden. Schließlich ermöglichen uns die Videoaufzeichnungen, Kontextfaktoren zu konstruieren, die die Gesamtsituation im Klassenzimmer widerspiegeln. Wir betrachten zwei abhängige Variablen: *Erfolgsdruck* und *Zeitdruck*. Als exogene individuelle Merkmale verwenden wir den Durchschnitt der Variablen *Verständnis* und *Selbstvertrauen*. Schließlich schließen wir als Kontextfaktoren: *Lehrerinstruktion*, *Lehrerzeitdruck*, *Komplexität*, *kooperatives Arbeiten* und *individuelles Arbeiten* ein.

Wir finden signifikante Peer-Effekte für alle Spezifikationen, was die Hypothese bestätigt, dass sich Stress über Peers überträgt, d.h. je gestresster die räumlichen Peers sind, desto gestresster ist der Einzelne. Unsere Ergebnisse zeigen, dass ein besseres Verständnis und ein höheres Selbstvertrauen eine weniger stressige Erfahrung bedeuten. Kontraintuitiv stellen wir fest, dass der Einzelne mehr gestresst ist, wenn die Peers besser zurechtkommen. Eine plausible Erklärung für dieses Ergebnis ist der soziale Vergleich. D. h. wenn eine Person sich mit den Peers vergleicht und feststellt, dass die Peers besser zurechtkommen als sie, kann dies zu individuellem Stress führen. Schließlich stellen wir fest, dass die Berücksichtigung verschiedener Kontextfaktoren von entscheidender Bedeutung ist, da sie wichtige Erkenntnisse liefert. Kooperative Arbeit wirkt sich nur im Klassenraum B positiv auf die Stressansteckung aus. Die Rolle der Lehrkraft ist von entscheidender Bedeutung, da sie in allen unseren Spezifikationen einen negativen Effekt hat. Der reduzierende Effekt des Lehrerunterrichts auf die Stressansteckung ist in Klasse B stärker ausgeprägt, was intuitiv ist, da die Schüler in Klasse B mehr Zeit damit verbringen, selbstständig zu arbeiten, und ein Eingriff des Lehrers einen Teil dieser Verantwortung und des Stresses freisetzt.

Das dritte Projekt konzentriert sich auf isolierte Individuen in sozialen Netzwerken. Es werden zwei Aspekte betrachtet, die in der Literatur noch nicht untersucht wurden: die Definition eines isolierten Individuums und die Folgen des Entfernens einer solchen Person. Eine isolierte Person behauptet, keine Peers zu haben, d. h. ihr Outdegree ist 0. Folglich unterliegt diese Person nicht der Endogenität, da sie nicht von Peers beeinflusst wird. Allerdings könnte diese Person Links erhalten, da sie ein Peer für andere verbundene Personen sein könnte. In diesem Fall ist der Indegree nicht 0. Das Vorhandensein isolierter Personen ist für die Identifikationsstrategie nicht schädlich, im Gegenteil, es gibt Hinweise darauf, dass sie bei der Identifikation des sozialen Multiplikators (Boucher und Fortin, 2015) hilfreich sein könnten. In der Literatur ist man bisher noch nicht zu einem endgültigen Schluss gekommen, wie diese Personen am besten zu behandeln sind. In der Tat entfernen die meisten empirischen Studien in der Literatur isolierte Personen aus dem Datensatz.

Um etwas Licht in diese Angelegenheit zu bringen, betrachten wir zwei Definitionen von isolierten Personen und unterschiedliche Netzwerkstrukturen. Wir definieren eine *vollständig isolierte* Person als jemanden mit Outdegree und Indegree gleich 0 und eine *teilweise isolierte* Person als jemanden mit Outdegree 0. Eine *vollständig isolierte* Person hat keinen Einfluss auf die Struktur des Netzwerks, aber ihre Gleichung dient als identifizierende Einschränkung, die zur Erstellung einer Momentfunktion verwendet werden kann. Das Entfernen dieser Beobachtungen führt zu weniger Beobachtungen und weniger Momentfunktionen, was im GMM-Rahmen zu einem asymptotisch ineffizienten Schätzer führt. Wenn wir *teilweise isolierte* Individuen entfernen, gehen nicht nur die Beobachtungen verloren, es ändern sich auch die Regressoren der Personen, die die Links senden. Dies hat zur Folge, dass der IV-Schätzer verzerrt ist. Wir zeigen analytisch, wie der IV-Schätzer des lokalen Aggregatmodells ineffizient und inkonsistent ist, wenn *vollständig isolierte* bzw. *teilweise isolierte* Individuen entfernt werden. Die Ergebnisse lassen sich leicht auf die Spezifikation des lokalen Durchschnitts ausweiten.

In unseren Monte-Carlo-Simulationen untersuchen wir das Verhalten der Schätzungen für zwei verschiedene Netzwerke: Erdös Rényi und Small World Netzwerke. Der letztere Typ weist Eigenschaften auf, die empirisch beobachteten Netzwerken ähnlicher sind, nämlich einen kleinen Durchmesser (der kürzeste Weg zwischen den beiden am weitesten entfernten Knotenpunkten). Die numerischen Erkenntnisse unserer Simulationen legen nahe, dass die Entfernung *teilweise isolierte* Individuen die schädlichste Strategie in Bezug auf den MSE und die Verzerrung ist. Die Schätzungen sind in allen Szenarien verzerrt und ihre Varianzen sind größer. Das Entfernen der vollständig isolierten Individuen hat im Allgemeinen vernachlässigbare negative Auswirkungen, insbesondere wenn die Netzwerke klein sind.

Literatur

Anselin, L. (2003). “Spatial Econometrics”. In: *A Companion to Theoretical Econometrics*. Ed. by Baltagi, B. H. John Wiley & Sons, Ltd. Chap. 14, pp. 310–330.

Bertrand, M., E. F. P. Luttmer, and S. Mullainathan (2000). “Network Effects and Welfare Cultures”. *The Quarterly Journal of Economics* 115 (3), pp. 1019–1055.

Boucher, V. and B. Fortin (Feb. 2015). *Some Challenges in the Empirics of the Effects of Networks*. IZA Discussion Papers 8896. Institute of Labor Economics (IZA).

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). “Identification of peer effects through social networks”. *Journal of Econometrics* 150 (1), pp. 41–55.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009). “Peer effects and social networks in education”. *The Review of Economic Studies* 76 (4), pp. 1239–1267.

Doty, J. (2016). *Into the Magic Shop: A Neurosurgeon’s Quest to Discover the Mysteries of the Brain and the Secrets of the Heart*. Penguin Publishing Group.

Jackson, M. O. (2010). *Social and economic networks*. Princeton University Press.

Kärner, T., D. Sembill, C. Aßmann, E. Friederichs, and C. H. Carstensen (2017). “Analysis of person-situation interactions in educational settings via cross-classified multilevel longitudinal modeling: illustrated with the example of students’ stress experience”. *Frontline Learning Research* 5 (1), pp. 16–42.

Lee, L.-f., X. Liu, and X. Lin (2010). “Specification and estimation of social interaction models with network structures”. *The Econometrics Journal* 13 (2), pp. 145–176.

Liu, X., E. Patacchini, and Y. Zenou (2014). “Endogenous peer effects: Local aggregate or local average?” *Journal of Economic Behavior and Organization* 103, pp. 39–59.

Manski, C. F. (1993). “Identification of Endogenous Social Effects: The Reflection Problem”. *The Review of Economic Studies* 60 (3), pp. 531–542.

– (2000). “Economic Analysis of Social Interactions”. *Journal of Economic Perspectives* 14 (3), pp. 115–136.

Patacchini, E. and Y. Zenou (2008). “The strength of weak ties in crime”. *European Economic Review* 52 (2), pp. 209–236.

Trogdon, J., J. Nonnemaker, and J. Pais (2008). “Peer effects in adolescent overweight”. *Journal of Health Economics* 27 (5), pp. 1388–1399.

Chapter 1

Peer effects heterogeneity and social networks in education

Abstract

This study focuses on the role of heterogeneity in network peer effects by accounting for network-specific factors and different driving mechanisms of peer behavior. We propose a novel Instrumental Variable–Minimum Distance (IV-MD) estimation approach and apply this approach to estimate peer effects on school achievement exploiting the network structure of friendships within classrooms. We use a unique network dataset from German upper secondary schools. We show that accounting for heterogeneity is not only crucial from a statistical perspective, but also yields new structural insights into how class size and gender composition affect school achievement through peer behavior.

1.1. Introduction

In the social sciences, it is an uncontested hypothesis that social interactions shape an individual's behavior and the behavior of groups of individuals as a whole. A popular framework of representing these interactions are peer effects models, which have a straightforward interpretation but also suffer from their stylized nature. This especially holds true in education where the identification and estimation of peer effects is a crucial issue. Closely related to the identification issue is the question to what extend the strength of peer effects is driven by observable network specific factors. For instance, in the context of individual educational attainment obvious candidates for such factors are class size and gender composition, two factors frequently considered to be major determinants of individual performance at school.

This paper takes a closer look on the role of heterogeneity within network peer effects models by augmenting the linear-in-means peer effects model in various dimensions. In particular, we focus on heterogeneous peer effects by accounting for network-specific factors - class size and gender composition - and allowing for different driving mechanisms of peer behavior. Since there is no suitable tool to estimate and test for those dimensions of heterogeneity in peer effects, we propose a novel and easy-to-implement Instrumental Variable-Minimum Distance (IV-MD) approach. Using our proposed approach, we provide empirical evidence on the existence and importance of peer effects heterogeneity in education based on a previously unexploited dataset for German secondary schools containing unique information on friendship networks within classes. We show that neglecting heterogeneity due to network specific features leads to insignificant and potentially misleading findings.

In his seminal work, Manski (1993) explains the dependence of an individual's behavior on the behavior of others in a socially interactive environment via three possible effects: (i) endogenous peer effects (the individual is influenced by the peers' behavior), (ii) exogenous peer effects (the individual is affected by the peers' characteristics), and (iii) correlated effects (individuals' outcomes are similar due to similar environments or common unobserved shocks). Separate identification of these effects is far from trivial and several strategies, primarily driven by data availability issues, have been proposed (see Bramoullé et al., 2020, for a comprehensive survey on the existing literature). Bramoullé et al. (2009) show that exogenous information on the second (and higher) order peers can serve as valid instruments to identify the endogenous peer effect, when network data is observed. This idea of exploiting the network structure by using exogenous variation in the covariates of the second-order peers ('peers of the peers') as valid instruments is similar to the IV/GMM estimation approaches for spatial lag models (e.g. Kelejian and Prucha, 1998, 1999).

The composite peer effects specification proposed by Liu et al. (2014) is an extension of the pioneering Bramoullé et al. (2009) linear-in-means model. We rely on this specification for the first stage of our IV-MD estimation approach because of its desirable features. First, the network-specific peer effects can be given a structural interpretation, as the model relies on a microeconomic foundation, i.e., utility-maximizing behavior in a Nash equilibrium (similar to Blume et al., 2015; Calvó-Armengol et al., 2009, among others). Secondly, the model allows for two channels through which peer effects operate. One hypothesis is that individuals align their behavior with the norm of their peer group represented by the mean behavior of the peers, because deviating from the norm may inflict utility losses. This idea is reflected in the composite model by the local-average part, which uses the mean behavior of the peer group as a predictor for the individual's behavior (for example, Boucher et al., 2014). An alternative hypothesis is that an individual's return to their own effort increases with the aggregate quality of the peers. This is reflected in the local-aggregate part of the composite model. Thus, the composite model allows the data to decide in which way and how strongly peer behavior affects an individual's behavior. In this study, we incorporate heterogeneity by modelling the two peer effects parameters, the local-average and the local-aggregate coefficients of the composite model, as functions of network specific features.

Irrespective of the identification strategies applied, the implicit assumption made in most of the peer effect studies is that the endogenous peer effects are homogeneous across networks. Several recent studies investigate other dimensions of heterogeneity in social networks. Using a laboratory experiment on individual performance Beugnot et al. (2019) study the role of gender heterogeneity in social networks based on the model by Arduini et al. (2020). From a technical point of view the approach of Arduini et al. (2020) is similar to the composite model as both approaches allow for different peer effects resulting from the presence of several networks. However, in the study by Beugnot et al. (2019) the focus of interest is to detect differences in peer effects between males and females, while our study focuses on the question whether networks operate differently depending on the gender composition.

The empirical evidence on heterogeneous peer effects in education exploiting network information is rather limited. A notable exception is Calvó-Armengol et al. (2009), who investigate the relationship between peer effects and the network topology. They only provide graphical evidence that the strength of the network effect varies with certain structural network measures, such as density, asymmetry, and redundancy. Contrary to

Calvó-Armengol et al. (2009) we study the link between network characteristics and the endogenous peer effect in a structural way within a regression approach. Another study modelling peer effects heterogeneity in education is Masten (2018). In this study the endogenous peer effect is pair-specific, purely random and, contrary to our approach, not driven by observable factors.

The empirical evidence on the role of peer effects in education is rather scarce due to the limited availability of appropriate network data. To the best of our knowledge, most of the existing research papers on network peer effects in education use the National Longitudinal Study of Adolescent to Adult Health (Add Health) (see Boucher et al., 2014; Bramoullé et al., 2009). In our paper, we study heterogeneous peer effects on school grades using unique network data from 85 school classes of secondary schools in Germany. Besides using network data which has not been used before, our study also contributes to the empirical literature on the determinants of school performance by providing deeper insights on how certain network features may affect individual performance via peer behaviour. In particular, our study provides a better understanding of how gender composition and class size affect an individual's school grades by enhancing peer behavior.

One strand of the literature on gender effects in school performance concentrates on the difference of outcomes for girls in single-sex and coeducational classes (see for a review Mael et al., 2005; Morse, 1999). The results based on observational studies are somewhat mixed: some studies provide evidence for positive effects of single-sex schools, whereas others suggest no difference. The other strand of the literature identifies the gender peer effect using exogenous variation in gender due to experimental or quasi-experimental research design. Hoxby (2002) and Lavy and Schlosser (2011) find that the proportion of female students has positive effects on students' cognitive achievements. They do not find a differential effect on boys and girls. However, none of these studies explicitly takes into account the classrooms' network structure. The common point in these studies is that gender or gender ratio enters the reduced form equations as a regressor. In contrast to this approach, in our structural approach, the gender ratio affects the outcome of academic success through the endogenous peer effect. This indirect effect has a clear structural interpretation in the sense that observed differences in academic success between classes with different gender compositions have their roots in different collaborative patterns captured by the peer effects.

Our study also contributes to the long-lasting debate on the effect of class size on academic success. The empirical evidence on this issue is by no means unambiguous. For example, Hanushek (1996) and Hoxby (2000) find no effect of class size reduction on achievement.

The results of Dobbelsteen et al. (2002) suggest that students in smaller classes do not have better academic performance (and even sometimes worse) than students in larger classes. On the other hand, Angrist and Lavy (1999) and Krueger (1999), and more recent studies, Fredriksson et al. (2012) and Heinesen (2010), report a substantial positive effect of reducing class size on academic achievement. Similar to the studies on gender effects, the vast majority of the empirical studies concentrate on direct effects of class size on school success within reduced form settings. Our study shows that peer effects decrease with class size.

Finally, instrumental variable strategies are an elegant way to identify endogenous peer effects without imposing strong distributional assumptions. However, identification via higher order peers' characteristics can be subject to the weak instrumental variable problem. To investigate this concern in the context of our approach, we conduct a Monte Carlo experiment and show that inference based on our IV-MD estimator is robust against the potential flaws of weak instruments. Additionally, we check for the consequences of potential over-parametrization and misspecification of the peer effects specification considered.

The outline of this paper is as follows. In Section 1.2 we introduce the composite network model and elaborate on its identification conditions. In that section, we also introduce the new IV-MD approach for the modelling and estimation of heterogeneous peer effects. In Section 1.3, we describe our network data and discuss further implementation issues. Section 1.4 contains the major empirical findings, while Section 1.5 summarizes the Monte Carlo simulation design and findings. Section 1.6 concludes and gives an outlook for future research.

1.2. The Network Model and Estimation

As a starting point we rely on the composite peer effects model proposed by Liu et al. (2014). We assume there is a finite set of N agents, partitioned into L independent networks, and write n_l for the number of agents in the lth network ($l = 1, \ldots, L$). The social connections for network l are indicated in the adjacency matrix $A_l = [a_{ij,l}]$, where $a_{ij,l} = 1$ if agent i in network l is connected with agent j, and $a_{ij,l} = 0$ otherwise. The diagonal elements $a_{ii,l}$ are set to zero. The reference group of agent i in network l is the set of their peers, and the size of the reference group is the (out)degree $a_{i,l} = \sum_{j=1}^{n_l} a_{ij,l}$. Write $G_l = [g_{ij,l}]$ for the row-normalized adjacency matrix of network l, with elements $g_{ij,l} = a_{ij,l}/a_{i,l}$, where by construction $0 \leq g_{ij,l} \leq 1$ and $\sum_{j=1}^{n_l} g_{ij,l} = 1$.

The econometric specification for the composite peer effects model is

$$y_{i,l} = \beta_{1l} \sum_{j=1}^{n_l} a_{ij,l} y_{j,l} + \beta_{2l} \sum_{j=1}^{n_l} g_{ij,l} y_{j,l} + \sum_{j=1}^{n_l} g_{ij,l} x'_{j,l} \gamma_l + x'_{i,l} \delta_l + \eta_l + \epsilon_{i,l}, \quad E[\epsilon_l | x_l, A_l, \alpha_l] = 0 \tag{1.1}$$

for $i = 1, \dots, n_l$ and $l = 1, \dots, L$. Note that the parameters of our model are heterogeneous, i.e., network-specific. The coefficients β_{1l} and β_{2l} capture the local-aggregate and the local-average endogenous peer effects, respectively. The contextual peer effects are captured by γ_l. Finally, the correlated effect is given by η_l. Liu et al. (2014) show that the econometric model (1.1) is derived from a utility maximizing network game with a unique Nash equilibrium.

Under homogeneity of the peer effects, i.e., $\beta_{1l} = \beta_1$, $\beta_{2l} = \beta_2$, $\gamma_l = \gamma$, $\delta_l = \delta$ for $l = 1, \dots, L$, our model reduces to the model of Liu et al. (2014). The composite model nests two specifications, the local-aggregate ($\beta_{2l} = 0$) and the local-average model ($\beta_{1l} = 0$).

Provided that the network adjacency matrix A_l is exogenous conditional on the control variables x_l and the network fixed effects η_l, identification can be achieved for the homogeneous local-average and local-aggregate network models, as well as the composite model under intransitivity and variation in network measures (see Liu et al., 2014). Formally, the identification of the local-average model requires linear independence of I_l, G_l, G_l^2 and G_l^3 and of I, A, G and AG for the local-aggregate model when there are different outdegrees.

In matrix notation, the general econometric model (1.1) takes the form

$$Y_l = \beta_{1l} A_l Y_l + \beta_{2l} G_l Y_l + G_l X_l \gamma_l + X_l \delta_l + \eta_l \iota_{n_l} + \epsilon_l, \qquad l = 1, \dots, L\,, \tag{1.2}$$

where $Y_l = (y_{1,l}, \dots, y_{n_l,l})'$, $X_l = (x_{1,l}, \dots, x_{n_l,l})'$, while $\epsilon_l = (\epsilon_{1,l}, \dots, \epsilon_{n_l,l})'$ and ι_{n_l} is an $n_l \times 1$ vector of ones. In a quasi-panel data fashion, we can partial out the network-specific effects by a within transformation by multiplying (1.2) by $J_l = I_{n_l} - \frac{1}{n_l} \iota_{n_l} \iota'_{n_l}$ from left. Because $J_l \iota_{n_l} = 0$, the transformed model is

$$J_l Y_l = \beta_{1l} J_l A_l Y_l + \beta_{2l} J_l G_l Y_l + J_l X_l \delta_l + J_l G_l X_l \gamma_l + J_l \epsilon_l, \qquad l = 1, \dots, L. \tag{1.3}$$

For simplicity of exposition, we rewrite the differenced model as follows:

$$\tilde{Y}_l = W_l \pi_l + \tilde{\epsilon}_l, \qquad l = 1, \dots, L, \tag{1.4}$$

where $\tilde{Y}_l = J_l Y_l$ is the transformed vector of dependent variables, $W_l = J_l \begin{bmatrix} A_l Y_l & G_l Y_l & X_l & G_l X_l \end{bmatrix}$ is the regressor matrix of dimension $n_l \times k_w$ with $k_w = 2(1 + k_x)$ and $\pi_l = \left(\beta_{1l}, \beta_{2l}, \delta_l', \gamma_l'\right)'$ is the parameter vector of dimension $k_w \times 1$.

In what follows, we assume that the parameters β_{1l} and β_{2l} for network-specific peer effects can be explained by a set of network-specific observable factors:[1]

$$\beta_{jl} = m_l' \beta_j\,, \qquad j = 1, 2, \tag{1.5}$$

where m_l is a $k_m \times 1$ vector of network-specific characteristics including an intercept. The model can be easily extended to accommodate for heterogeneity in the remaining parameters, δ and γ, but for the sake of easy interpretation we assume that they are the same across networks, i.e., $\gamma_l = \gamma$ and $\delta_l = \delta$. The restriction between the first stage reduced form parameter vector π_l and the structural form parameter vector $\theta = \left(\beta_1', \beta_2', \gamma', \delta'\right)'$ is given by

$$\pi_l(\theta) = M_l \theta\,, \tag{1.6}$$

with

$$M_l = \begin{bmatrix} m_l' & 0 & 0 & 0 \\ 0 & m_l' & 0 & 0 \\ 0 & 0 & I_{k_x} & 0 \\ 0 & 0 & 0 & I_{k_x} \end{bmatrix}_{2(1+k_x) \times 2(k_m + k_x)}.$$

Stacking the restrictions between the L reduced form parameter vectors π_l and the structural form parameter θ given by (1.6) into a hyper-system yields

$$\pi(\theta) = M\theta\,, \tag{1.7}$$

with $\pi(\theta) = (\pi_1(\theta)', \pi_2(\theta)', \ldots, \pi_L(\theta)')'$ and $M = (M_1', M_2', \ldots, M_L')'$.

Estimation

We propose estimating θ by Minimum Distance (MD) employing two estimation stages: First, the reduced form parameters π_l are estimated via Instrumental Variables (IV) for each l and stacked together in $\hat{\pi}$. In the second stage, the structural form parameter

[1] This assumption can be generalized by assuming that β_{jl} can be replaced by a linear predictor representation along the lines of Chamberlain (1984) for panel data models with correlated random effects.

θ is obtained by minimizing the distance between $\hat{\pi}$ and $\pi(\theta)$ based on the (estimated) asymptotically optimal weighting matrix.

Since the networks are assumed to be independent, a systems regression approach does not yield any gains in efficiency over a simple single equation estimation approach for the first estimation stage. As instruments we use the exogenous variables, their counterparts for the peers, aggregate characteristics of the peers, and the covariates of the second order peers as over-identifying instruments, i.e., $Z_l = J_l \begin{bmatrix} X_l & G_l X_l & A_l X_l & G_l^2 X_l \end{bmatrix}$. It is intuitive that our instruments are valid under intransitivity, since the characteristics of the second order friends have only an effect on the outcome through their effect on the outcomes of the first order friends. Thus, the instrumental variable estimator for each network is

$$\hat{\pi}_l = \left[W_l' Z_l \left(Z_l' Z_l \right)^{-1} Z_l' W_l \right]^{-1} W_l' Z_l \left(Z_l' Z_l \right)^{-1} Z_l' Y_l, \qquad l = 1, \ldots, L. \tag{1.8}$$

If heteroskedasticity is assumed, then for each network the asymptotic distribution of $\hat{\pi}_l$ is

$$\sqrt{n_l} \left(\hat{\pi}_l - \pi_l \right) \xrightarrow{d} \mathcal{N} \left(0, \Omega_l \right)$$

where $\Omega_l = \Delta_l V_l \Delta_l'$ with

$$\Delta_l = \left(\mathrm{E} \left[W_l' Z_l \right]' \mathrm{E} \left[Z_l' Z_l \right]^{-1} \mathrm{E} \left[W_l' Z_l \right] \right)^{-1} \mathrm{E} \left[W_l' Z_l \right]' \mathrm{E} \left[Z_l' Z_l \right]^{-1},$$
$$V_l = \mathrm{E} \left[Z_l' \tilde{\epsilon}_l \tilde{\epsilon}_l' Z_l \right].$$

For the homoskedastic case with $\mathrm{E} \left[\tilde{\epsilon}_l \tilde{\epsilon}_l' \middle| Z_l \right] = \sigma_l^2 I_{n_l}$ and $V_l = \sigma_l^2 \, \mathrm{E} \left[Z_l' Z_l \right]$, the asymptotic variance of $\hat{\pi}_l$ reduces to

$$\Omega_l = \sigma_l^2 \left(\mathrm{E} \left[W_l' Z_l \right]' \mathrm{E} \left[Z_l' Z_l \right]^{-1} \mathrm{E} \left[W_l' Z_l \right] \right)^{-1}.$$

The variance of $\hat{\pi}_l$ can be estimated using the sample counterpart of the asymptotic variance where expectations are replaced with sample means and the unknown population parameters with their estimates. For homoskedastic errors, the estimator for the variance covariance matrix is

$$\widehat{V} \left(\hat{\pi}_l \right) = \hat{\sigma}_l^2 \left(W_l' Z_l' (Z_l' Z_l)^{-1} W_l' Z_l \right)^{-1},$$

where $\hat{\sigma}_l^2$ is a consistent estimator of σ_l^2. For example, $\hat{\sigma}_l^2 = \frac{1}{n_l - k_w} \hat{\tilde{\epsilon}}_l' \hat{\tilde{\epsilon}}_l$ is a consistent estimator for σ_l^2, where $\hat{\tilde{\epsilon}}_l$ is the residual of the differenced model in Equation 1.4, i.e., $\hat{\tilde{\epsilon}}_{i,l} = \tilde{Y}_l - W_l \hat{\pi}_l$. If the error term is assumed to be heteroskedastic, $\mathrm{E} \left[Z_l' \tilde{\epsilon}_l \tilde{\epsilon}_l' Z_l \right]$ can be estimated by $Z_l' D Z_l / n_l$, where D is an $n_l \times n_l$ diagonal matrix with entries $\widehat{\tilde{\epsilon}}^2_{i,l}$. In that case, the variance can be estimated as follows:

$$\widehat{V}\left(\hat{\pi}_l\right) = \left[W_l' Z_l (Z_l' Z_l)^{-1} Z_l' W_l\right]^{-1} \left[W_l' Z_l (Z_l' Z_l)^{-1} Z_l' D Z_l (Z_l' Z_l)^{-1} Z_l' W_l\right] \left[W_l' Z_l (Z_l' Z_l)^{-1} Z_l' W_l\right]^{-1}.$$

For the stacked vector of reduced form parameters π, we get $\sqrt{N}\left(\hat{\pi} - \pi\right) \xrightarrow{d} \mathcal{N}\big(0, \Omega\big)$. Because of the independence of the networks, Ω is a block-diagonal matrix with diagonal elements equal to Ω_l, i.e., $\Omega = diag[\Omega_l]$.

In the second step, we estimate the structural form parameter by minimizing the weighted quadratic distance between the estimated reduced form parameter vector $\hat{\pi}(\theta) = \left(\hat{\pi}_1', \hat{\pi}_2', \ldots, \hat{\pi}_L'\right)'$ and $M\theta$ with respect to the structural parameter vector θ. Using the efficient weighting matrix, the inverse of any consistent estimator of Ω, the minimization problem can be written formally as follows:

$$\hat{\theta}_{MD} \equiv \arg\min_{\theta} \left[\hat{\pi} - M\theta\right]' \hat{\Omega}^{-1} \left[\hat{\pi} - M\theta\right]. \tag{1.9}$$

Given that the first stage is identified and the restriction between π and θ is linear, the MD estimation is equivalent to a generalized least squares regression of $\hat{\pi}$ on M:

$$\hat{\theta}_{MD} = \left(M' \hat{\Omega}^{-1} M\right)^{-1} M' \hat{\Omega}^{-1} \hat{\pi},$$

with $\sqrt{N}\left(\hat{\theta}_{MD} - \theta\right) \xrightarrow{d} \mathcal{N}\big(0, (M' \Omega^{-1} M)^{-1}\big)$. Since the optimal weighting matrix is block diagonal in our case, the minimum distance estimator can be simply computed as $\hat{\theta}_{MD} = \left(\sum_{l=1}^{L} M_l' \hat{\Omega}_l^{-1} M_l\right)^{-1} \left(\sum_{l=1}^{L} M_l' \hat{\Omega}_l^{-1} \hat{\pi}_l\right)$ even for a large number of networks and a large number of covariates. The standard errors can be estimated based on an asymptotic variance formula by replacing the unknown Ω with its consistent estimator.

1.3. Data

Our empirical study is based on the data of the *Gymnasiasten-Studie* (CAESR, 2007), a longitudinal survey of 3,385 10th grade students attending upper secondary school (*Gymnasium*) in the German federal state North Rhine-Westphalia (NRW) in the school year 1969/1970. Our data cotains 121 classes from 68 randomly selected upper secondary schools. The initial survey of the students provides information on their previous school grades as well as individual characteristics such as gender and age. Besides this initial survey, a standard psychometric Intelligence Structure Test (IST) was administered in the classroom during the data collection period. About ten years after the original survey, the students' grades were collected from the school archives. Central to our study is the network information collected in the Sociometric Test of the *Gymnasiasten Studie*. In order to construct the adjacency matrices A_l and G_l for each class, we use information about every student's assessment of whom he or she liked in the class based on the question:[2]

> "In every class there are fellow students who one likes more than others in the class. Some others one finds pretty unpleasant, and that is quite normal. Kindly first list the students who you personally like a lot."

The vast majority of the empirical papers studying network peer effects in education use the National Longitudinal Study of Adolescent to Adult Health (ADD Health) data. Our unique network data differs from the Add Health data in several ways. One difference is that the students in our data nominate their friends within the classroom, while in Add health data nominations are made within the schools. More importantly, in the Add Health survey, the respondents were asked to name up to ten (five female and five male) best friends. This might raise a truncation problem that does not occur with our dataset. Indeed, Griffith (2021) shows that peer effects with censored data are under-estimated.

We constructed our dataset by merging information from three different sources: student surveys, administrative data from school archives, and the sociometric test. We dropped an observation when any of the variables used in the empirical model were missing. Similarly, we dropped isolated individuals, i.e., those who did not name anyone as a friend. This leaves a sample of 2,385 students and 101 classes. It is worth noting that the majority of our missing observations are due to non-compliance of some schools in providing the archive data. After cleaning the data, we excluded classes with fewer than

[2] The original question in German is "*In jeder Klasse gibt es Mitschüler, die man sympathisch findet und die man mehr als andere in der Klasse gut leiden kann. Einige findet man sicher recht unsympathisch, und das ist auch ganz normal so. Würden sie nun zunächst einmal die Schüler nennen, die Sie persönlich gut leiden können.*"

18 students, because the first-stage estimates of a small network suffer from low degrees of freedom.[3] Excluding small classes leads to 2,165 students in 85 classes. Table 1.1 contains the summary statistics of the variables used in our empirical analysis and some network statistics. In the left panel of the table we present the variables before applying the class size restriction. One can see that the sample means are not substantially affected by the class size restrictions.

Table 1.1: Summary Statistics

	Entire Sample		Estimation Sample	
	Mean	Std. Dev.	Mean	Std. Dev.
Outcome Variables				
GPA[a]	3.20	0.48	3.19	0.48
German	3.46	0.76	3.45	0.76
Math	3.51	0.96	3.51	0.95
Individual characteristics				
IQ	40.20	9.11	40.03	9.14
Previous GPA	3.19	0.49	3.19	0.49
Age	15.38	0.87	15.37	0.87
Network measures				
Class size	27.82	6.01	29.19	4.92
Relative Class size[b]	0.71	0.15	0.75	0.13
Sample Class size[c]	23.61	5.98	25.47	4.23
Female share	0.45	0.43	0.49	0.43
Sample female share[c]	0.45	0.43	0.49	0.44
Density	0.24	0.06	0.22	0.04
Clustering	0.03	0.02	0.02	0.01
N	2,385		2,165	
L	101		85	

Note: Own calculations. We exclude classes with fewer than 18 students in the estimation sample. a: Better grades are represented by lower values. b: relative class size denotes the class size divided by the total number of observations. c: Sample class size refers to the number of remaining students in the classroom after dropping the individuals with missing information either in the survey data, administrative data, or in the sociometric test. The same holds for the female share measure.

We measure the academic performance using the average of the final grades (GPA) for all compulsory and elective courses at the end of the school year 1969/70. We use the

[3]We estimated the model using several thresholds, but in general the results did not change qualitatively.

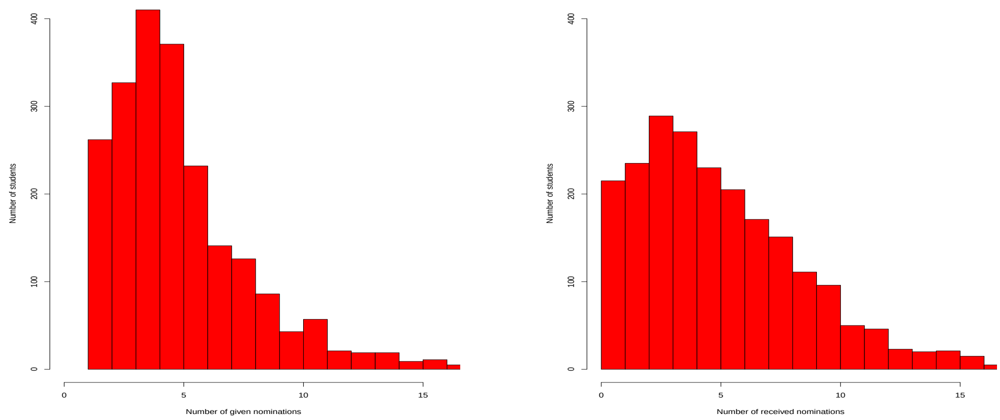
administrative data collected from the school archives to construct the GPA. At the time of the survey the choice of different courses within a class was very limited, i.e., all students of the same class had to take the same courses. Selection of certain specializations (e.g., languages, mathematics and sciences, humanistic secondary school) took place with the choice of the specific secondary school. Therefore, the GPA within a given network is based on mostly the same subjects. The grades are measured in terms of the German grading system: with 1 ("very good") being the best grade and 6 ("insufficient") as the worst grade. Besides the overall GPA, we will take a closer look at the scores in Mathematics and German in order to detect potential differences in peer behavior across subjects.

The individual heterogeneity in our model is captured by the student's IQ score, the overall GPA from the previous school year, and the student's age. The IQ is constructed from the correctly solved questions of the IST. In order to account for network heterogeneity in the peer effects of the local-average and local-aggregate models, we allow the two peer effect parameters β_{1l} and β_{2l} in Equation (1.2) to depend on class-specific factors. As such factors we use the relative class size, i.e., the size of the network relative to the total number of observations, and the fraction of girls in the class. As we already discussed in detail in Section 1.1, the literature on the effects of the class size and gender (ratio) on school outcomes is very rich. In general, the main consideration is the direct causal link from class size to the outcome. It seems, however, reasonable to look for a potential indirect link through heterogeneous peer effects. In fact, Lin (2014) estimates the peer effects for large, i.e., larger than the median, and small classes separately, using the Add Health dataset, and finds that the peer effects are considerably different for the two groups. She also conducted similar analyses of various network attributes, including the gender ratio. Surprisingly, she does not find a significant difference between the peer effects of the two subsamples by gender proportion. However, experimental evidence about gender diversity and performance shows that team collaboration is greatly improved by the presence of women in the group (see, for example, Bear and Woolley, 2011, and references therein).

The network density is defined as the ratio of all connections in a network to the number of potential connections. Thus, the denser a network is, the closer to unity is the density. In our sample, the density of the networks varies between 0.14 and 0.37, with 45 classes having a lower density than the mean. Clustering, on the other hand, measures all transitive triads relative to the total number of triads. It is a measure of the probability that two of i's peers nominate each other. In our dataset this measure varies between 0.004 and 0.08, suggesting that we can rely on peers of peers for identification.

To obtain a better understanding of the network structure and its potential role for peer effects, we take a look at the summary statistics of the friends' nominations. The average number of friends a student named (outdegree) is 5.48, which indicates that the students take the selection of friends seriously. Figure 1.1 depicts the distribution of friends. The distribution of outdegrees and indegrees indicates that the networks are sparse which is essential for the identification of network-specific endogenous peer effects. Most of the students name around five people that they like, and very few name more than ten peers.

Figure 1.1: Distribution of naming friends

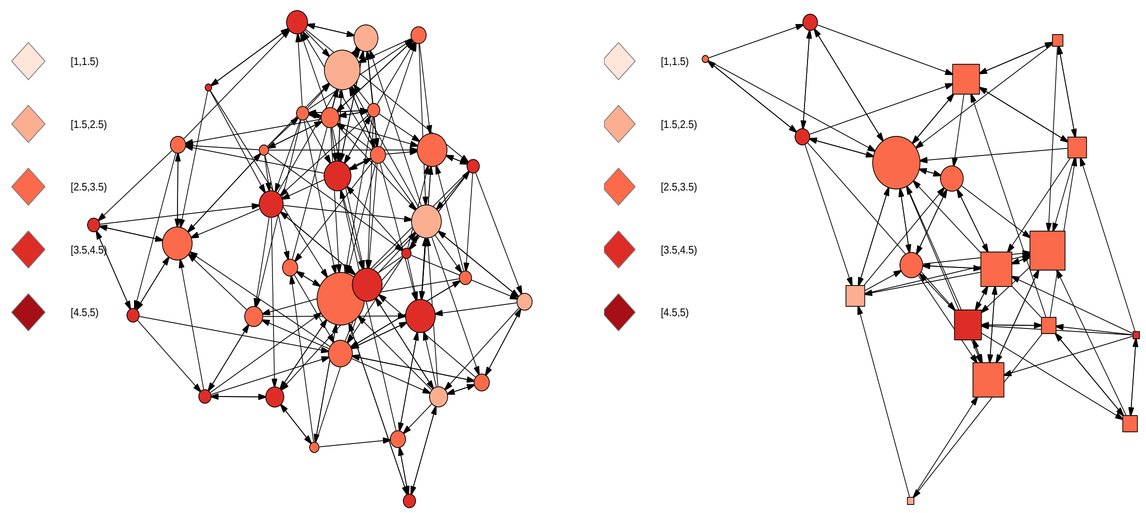


Histogram of the names given as friends (outdegrees) and individuals named as friends (indegrees). The median (mean) for the outdegrees and as well as for the indegrees is 5 (5.8). Source: *NRW Gymnasiasten-Studie*.

Our motivation for considering potential heterogeneity in peer effects results from the observation that the network structures and characteristics vary substantially across networks. We claim that the differences in networks might affect how peer effects operate. With the help of some network graphs, we illustrate the variation of the school classes (networks) in terms of individual performance, class size, gender, and network structure. The size of a node is proportional to the outdegree, its color indicates the GPA score (lighter colors represent better performance), and the shape of the node indicates the gender. Since plotting all networks together for visual inspection would give too small a picture to be detected, we concentrate on four classes.

First, we plot the largest and smallest classrooms in Figure 1.2. Second, Figure 1.3 depicts the two networks with the highest and the lowest densities in the sample. Without the intention of stressing the following argument too much, a comparison of the largest with the smallest network in Figure 1.2 illustrates that the performance of students might depend on their degree of connectivity and the class size. For both networks, the better performing students are slightly more central (being named as friends more often), while particularly in the larger network, the less well-connected students are also associated with lower performance.

Figure 1.2: The largest and the smallest classroom networks

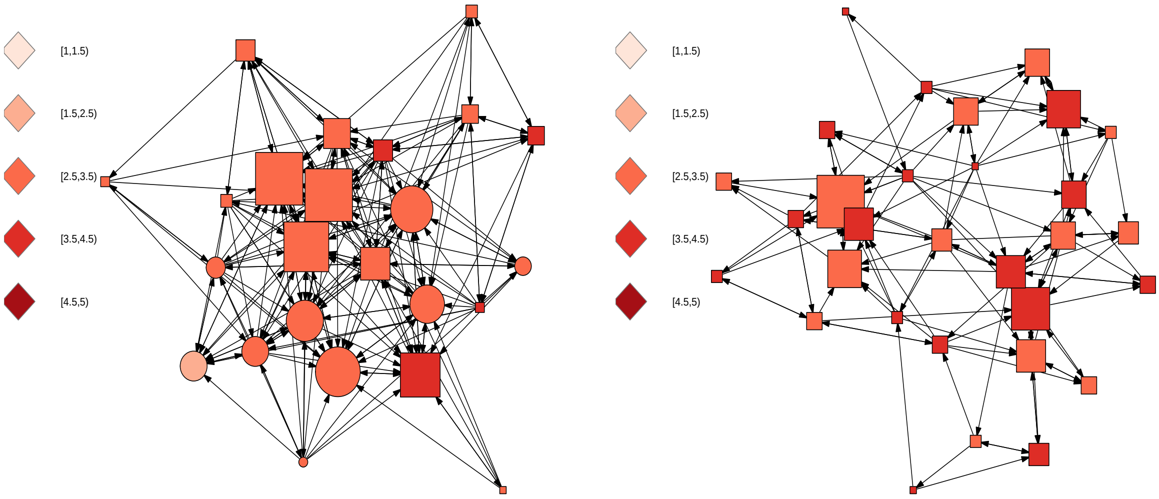


Note: The size of a node is proportional to its outdegree, its color indicates the GPA score (lighter colors representing better performance), and the shape of the node indicates the gender, i.e., circles represent female students and squares represent male students. Left: Largest classroom network, $n = 35$, density $= 0.15$, clustering $= 0.007$, girls class. Right: smallest classroom, $n = 18$, density $= 0.26$, clustering $= 0.03$, female ratio $= 0.33$. Source: *NRW Gynasiasten-Studie*.

Figure 1.3 provides further exploratory evidence that the network structure varies across classrooms. The least dense network on the right reveals two major clusters, while the dense network on the left is centered around a single cluster of students. In the densest network we find several very popular students, who have average grades. The least dense classroom corresponds to a boys class, where there are no good students and the most central ones are again average students. Comparing the network graphs for different subjects we hardly find any differences. The four classroom networks depicted appear very similar if the color of the nodes is based not on the GPA scores but on the grades in Math and German.[4]

[4]The corresponding network graphs for the two other outcome variables can be obtained from the authors on request.

Figure 1.3: Classroom networks with the highest and the lowest density



Note: The size of a node is proportional to its outdegree, its color indicates the GPA score (lighter colors representing better performances), and the shape of the node indicates the gender, i.e., circles represent female students and squares represent male students. Left: Densest classroom network, $n = 24$, density = 0.37, clustering = 0.07, female ratio = 0.37. Right: least dense classroom network, $n = 30$, density = 0.14, clustering = 0.004, boys class. Source: *NRW Gynasiasten-Studie*.

1.4. Empirical Results

The primary specification in our study is the heterogeneous composite model given by (1.2), which includes the local-aggregate and the local-average peer effects. Although most of the empirical studies focus on peer effects as a result of norm behavior, and therefore favor the local-average model, ex-ante, both hypotheses on how peers affect individual educational achievement are reasonable. In fact, the two effects may complement or even counteract each other. As mentioned above, our main outcome variable of interest is the GPA. However, since peer effects may operate differently depending on the subject, we also study the peer effects for Math and German (see Tables 1.4 and 1.5 in the Appendix). As predetermined or exogenous explanatory variables we use the GPA of the previous year, IQ, and age, as well as their counterparts for the student's peer group. As network-specific characteristics we use the relative class size [5] and the fraction of girls.

Before applying our IV-MD estimator, we make sure that the identification conditions, i.e., the linear independence of the relevant matrices, stated in Section 1.2 is satisfied for each classroom. Table 1.2 summarizes the estimation results for the composite, local-aggregate, and local-average models with heterogeneous peer effects based on the IV-MD approach with globally differenced variables. In order to ease the interpretation of the estimation results, we centered the network-specific characteristics around their means, so that the two intercept terms in (1.5) reflect the aggregate and the average peer effects for a class with mean characteristics.

First and most importantly, our estimation results reveal that taking into account heterogeneity in peer effects turns out to be absolutely crucial. We find clear evidence that both peer effects significantly differ by class size and gender composition. Several studies focusing on homogeneous peer effects find insufficient statistical evidence for the peer effects (e.g. Boucher et al., 2014; Liu et al., 2014). Similar to these studies, we also find for our sample insignificant estimates when peer effects are assumed to homogeneous (see Table 1.6 in the Appendix).

Second, the way peers affect a student's performance also matters, as both mechanisms, the local-aggregate and the local-average one, have a positive impact on a student's educational attainment in a representative classroom with average relative size and gender composition. The comparison of MD statistics in the last row of Table 1.2 shows that the heterogeneous local models are rejected in favor of the heterogeneous composite model.

In the composite model both intercepts turn out to be positive at the 1% significance level. This means that if the peers perform better individually or on average, then so does

[5] We use the relative class size, so that asymptotically this variable does not vanish.

the individual. It is important to note that the coefficients of the two local models are not directly comparable in magnitude. The peer effects due to local-aggregate behavior are proportional to the number of peers (outdegree of student), i.e., the larger the student's peer group, the stronger that student's performance is affected by their peers. Therefore, we compute the impact of a one unit change in the GPA of the peers on the individuals's GPA, for both models. For a class with average characteristics the local-aggregate effect would exceed the local-average effect if the student has more than 52 peers. Noting that the median outdegree in our sample is 5 (see Figure 1.1) we can conclude that the local-average peer effect generally dominates the local-aggregate peer effect.

Table 1.2: IV-MD Estimation Results for GPA

	Heterogeneous Peer Effects Model		
	Composite	Local-aggregate	Local-average
Local-aggregate peer effect			
Intercept	0.0021***	0.0012***	
	(0.0003)	(0.0003)	
Relative Class Size	-0.0094***	-0.0109***	
	(0.0025)	(0.0025)	
Female Share	-0.0027***	-0.0010	
	(0.0008)	(0.0008)	
Local-average peer effect			
Intercept	0.1099***		0.1526***
	(0.0262)		(0.0293)
Relative Class Size	-0.3768***		-0.3252***
	(0.1096)		(0.1229)
Female Share	0.1386***		0.1432***
	(0.0326)		(0.0353)
Own characteristics			
IQ	-0.0033***	-0.0031***	-0.0027***
	(0.0004)	(0.0004)	(0.0004)e
Previous GPA	0.7344***	0.7401***	0.7317***
	(0.0071)	(0.0071)	(0.0076)
Age	-0.0049	-0.0036	-0.0023
	(0.0038)	(0.0039)	(0.0043)
Peers' characteristics			
IQ	0.0002	-0.0003	0.0001
	(0.0008)	(0.0008)	(0.0008)
Previous GPA	-0.0249	0.084***	-0.0465*
	(0.0246)	(0.0144)	(0.0274)
Age	-0.0203***	-0.0148*	-0.023**
	(0.0076)	(0.0081)	(0.0089)
MD statistics (d.f.)	3071.27 (668)	3122.10 (671)	3137.59 (671)

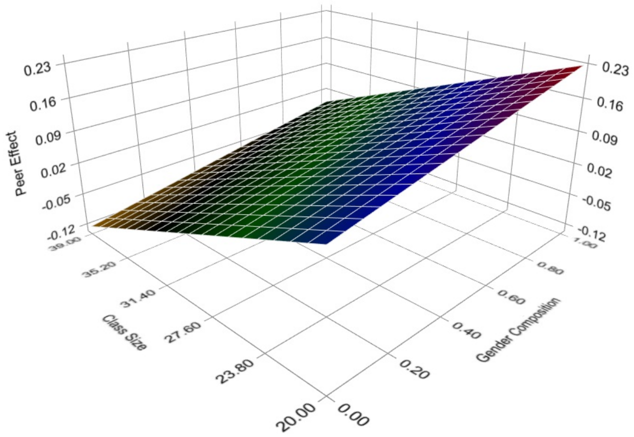
Estimates of the three model variants obtained by IV-MD estimation. The first column corresponds to the composite model, the second column corresponds to the local-aggregate model and the third column corresponds to the local-average model estimation results. Robust standard errors are reported in parentheses. First stage errors are assumed to be heteroskedastic, $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$, N=2165, L=85.

Interestingly, the coefficients on the female share variable in local-aggregate and local-average part operate in different directions. The gender effect is large and positive for the local-average part but small and negative for the local-aggregate part. Interpreting the local-average effect as a proxy for norm behavior, we conclude that our estimates indicate that in girls-only classes, norm behavior is much more present than in boys-only classes. In order to illustrate the sizes of the effects of the two components, consider two classes of average relative size, one being a girls-only class while the other class is a boys-only class. In this case, the overall peer effect of a student with 5 friends in girls-only class is 0.1841 ($= 5 \times (0.0021 - 0.0027 \times (1 - 0.49)) + (0.1099 + 0.1386 \times (1 - 0.49))$) compared to 0.0590 for a student with the same number of friends in a boys-only class.

The effect of class size is negative and significant for both peer effect mechanisms, meaning that for larger classrooms, the enhancing contribution of peer behavior diminishes. Assuming an average female share, the peer effect of the smallest class size is 0.2223, while for the largest class size the overall peer effect is 0.0142. It is important to emphasize that this class size effect is novel in the literature, as it operates through peer behavior. It operates in addition to a potential direct effect of class size on a student's performance, which is traditionally considered in studies on the determinants of educational achievement. Because we take global differencing, the effect of class size on peer behavior is a second channel for the impact of class size on educational achievement. Unlike the conventional direct effect of class size obtained from reduced form specifications, the effect of class size through peer behavior indicates the role of social interactions in a classroom, which then partly determines the individual performance. Therefore this approach also offers a specific explanation of why certain classes have certain performances.

Figure 1.4 depicts the size of the combined peer effect over different class sizes and gender compositions. For larger classes with a low fraction of female students, the peer effect is negative. Classes with a mean female composition have a negative overall peer effect if the class size is larger than 34. All in all, the combined peer effect ranges from -0.12 to 0.23.

Figure 1.4: Peer Effects by Class Size and Gender Composition



Surface of peer effect by gender composition and class size for an outdegree of 5 based on the parameter estimates of the composite model given in Table 1.2. The peer effect denotes the change in a student's GPA score due to a one unit change in the GPA of all peers assuming a median outdegree of 5.

The coefficients on own IQ and own previous GPA have the expected signs. Not very surprisingly, the GPA of the previous year is a very good predictor of current performance. Students with a higher IQ also perform better. Our results do not suggest a significant impact of age. For the exogenous peer effects, we observe that having smarter or less smart peers does not have an impact on the individual outcome. Neither does the previous GPA of the peers. The results show that having older peers helps to have better grades.

Columns 2 and 3 in Table 1.2 summarize the results from the heterogeneous local models. The impact of the female share in the local-aggregate model is no longer significant, but has the same sign as in the composite model and is similar in magnitude. In the local-aggregate model, we see that having peers with better grades has a negative influence on the individual outcome. Other estimates are similar to those for the composite model.

The IV-MD estimates of the heterogeneous local and composite models with grades in Math and German as dependent variables are given in Tables 1.4 and 1.5 in the Appendix. For these two dependent variables we also find heterogeneity in peer effects in terms of the class characteristics and the transmission mechanism. With a few exceptions, these findings for the two subjects are consistent with the findings for the overall GPA score. However, a notable exception is the significant and negative coefficient on female share for the local-average effect for Math, which indicates that the role of gender composition in educational outcomes has to be discussed in the light of the field of study. Similarly, the positive coefficient of class size for Math and German suggests that the role of class size on peer behavior also depends on the field of study.

Our IV-MD approach for the heterogeneous composite model allows us to test against a number of nested specifications using the Minimum χ^2-statistics defined as the difference between the MD-statistics of the nested model and the unrestricted alternative with degrees of freedom equal to the number of restrictions. A comparison of the MD-statistics presented in the last row of Table 1.6 for the nested homogeneous specifications with their counterparts for the heterogeneous specifications reveal that for all three outcome variables, the H_0 of homogeneous peer effects has to be rejected against the heterogeneous peer effects models. These findings hold for the heterogeneous composite model as well as for the heterogeneous local models.

1.5. Monte Carlo Evidence

In what follows we try to tackle two crucial questions related to the quality of peer effects estimation under heterogeneity by means of Monte Carlo simulations. The first question is how the network density affects the estimation performance. To explore this question we consider three different network designs. In two designs all of the independent networks have the same density. We generate networks such that they have a density value of 0.14 and 0.37, respectively, as these two figures correspond to the lowest and highest densities in our empirical data. For the third design, we generate the data such that 20% of the networks have low density (0.14) and the rest have high density (0.37). It is documented in the literature that, for the local-average models, the instrument strength of the second order friends' characteristics decreases with increasing network density (see Bramoullé et al., 2009; Startz and Wood-Doughty, 2017). Intuitively, the reason is that with increasing network density more individuals become connected as first-order friends and the number of second order friends decreases. However, note that our dataset contains networks of different densities. We suppose that our IV-MD approach incorporating the information of high and low density networks generates reliable estimates even if some of the networks are dense. Our Monte Carlo findings support this presumption.

The second question is related to the consequences of over-parametrization and misspecification. Since both, local-aggregate and local-average specifications, can theoretically be justified in the education context, we argue that using the composite model as the one which nests both types of peer effects is the superior strategy. In particular, we raise the question to what extent the composite model can serve as a robustification strategy when one of the local models holds true. This, however, is only a reasonable strategy for the applied researcher, if over-parametrization does not lead to serious efficiency losses and the inference remains valid. To study this question, we simulate network data from different data generating processes (DGP) based on the composite (*DGP-COMP*), the local-aggregate (*DGP-AGG*) and the local-average (*DGP-AVG*) model, such that (i) both types of peer effects affect the outcome, (ii) only the local-aggregate part affects the outcome, and (iii) only the local-average part affects the outcome. We then estimate the peer effects for each of the network DGPs.

We simulate the network data such that major features of the simulated data match the features of our data and take parameter estimates from our empirical study as the true parameter values. We start by randomly drawing the class size and gender share for 85 classes as they are the network specific factors that are considered to affect the peer effects in our empirical study. We draw class sizes from a uniform distribution from 20 to 40. Gender shares are drawn such that the ratio of female, male and mixed classrooms matches the ratio of classrooms in the empirical application and the share in the mixed classrooms is uniformly distributed. For each classroom, we then compute β_{1l} and β_{2l} according to Eq. (1.5), using the estimated parameters from our empirical application as the true parameters. For each simulation, we generate a random asymmetric network with the probability of any link equal to the chosen density. To keep the DGPs rather simple, we use only one exogenous characteristic, which is generated from a normal distribution with mean equal to 40 and standard deviation equal to 9, to match the IQ variable's features. Lastly, the outcome variable is generated according to three different DGPs. Each DGP corresponds to the reduced form equation of one of the three models, i.e., the composite model (*DGP-COMP*), local-aggregate model ($\beta_2 = 0$, *DGP-AGG*), and local-average model ($\beta_1 = 0$, *DGP-AVG*):

$$\textit{DGP-COMP} \qquad Y_l = (I_{n_l} - \beta_{1l}A_l - \beta_{2l}G_l)^{-1}(\alpha\iota + X_l\delta_1 + G_lX_l\gamma_1 + \varepsilon_l) \tag{1.10}$$

$$\textit{DGP-AGG} \qquad Y_l = (I_{n_l} - \beta_{1l}A_l)^{-1}(\alpha\iota + X_l\delta_1 + G_lX_l\gamma_1 + \varepsilon_l) \tag{1.11}$$

$$\textit{DGP-AVG} \qquad Y_l = (I_{n_l} - \beta_{2l}G_l)^{-1}(\alpha\iota + X_l\delta_1 + G_lX_l\gamma_1 + \varepsilon_l). \tag{1.12}$$

The error term ε is drawn from a normal distribution with mean 0 and variance 0.1, and the parameters δ_1 and γ_1 are chosen to be equal to the estimated parameters of the variable IQ.

Our simulations are based on $R = 1000$ Monte Carlo samples. For each parameter we report the mean bias (MBias), the mean squared error (MSE), the relative standard errors (RelSE) and the empirical coverage probability (COVP). The simulation results based on the data generated from the composite model are summarized in Table 1.3. Further results on the consequences of over-parametrization when the data results from the more parsimonious local-aggregate model or from the local-average model are reported in the Appendix in Tables 1.7 and 1.8, respectively. The boxplots in Figure 1.5 and 1.6 provide further insights into the distributional properties of the Monte-Carlo estimates of the peer effects.

Estimates of the Composite Model (DGP-COMP)

Table 1.3 summarizes the simulation results when the data is generated using the composite network model. For the correctly specified composite model (figures in the left block of the table), the parameter estimates of the local-aggregate part and the parameter estimate on non-peer effect variable *IQ* are well-determined and quite robust with respect to the underlying network density. Generally, the MSEs of the local-aggregate parameters decrease with increasing network density. The MSE reduction amounts to more than

50 percent for the high density network compared to the low density network. An even stronger but opposite effect can be observed for the MSEs of the local-average component. Here the MSEs of the local-average parameters more than quadruple for a network with density 0.37 compared to the low density network with a density of 0.14, which reflects the aforementioned relationship between instrument strength and network density. The boxplots in Figure 1.5 nicely depict the improvements for the local-aggregate parameters with increasing network density, while the boxplots of Figure 1.6 show the deterioration of the estimates of the local-average part when the network density increases. The observed relationship between network density and MSE of the peer effects components also holds for the misspecified models (Table 1.3 middle and right panel), the correctly specified sparse models and the over-parametrized specifications reported in Tables 1.7 and 1.8 in the Appendix.

Table 1.3: Monte Carlo Results for the Composite model (*DGP-COMP*)

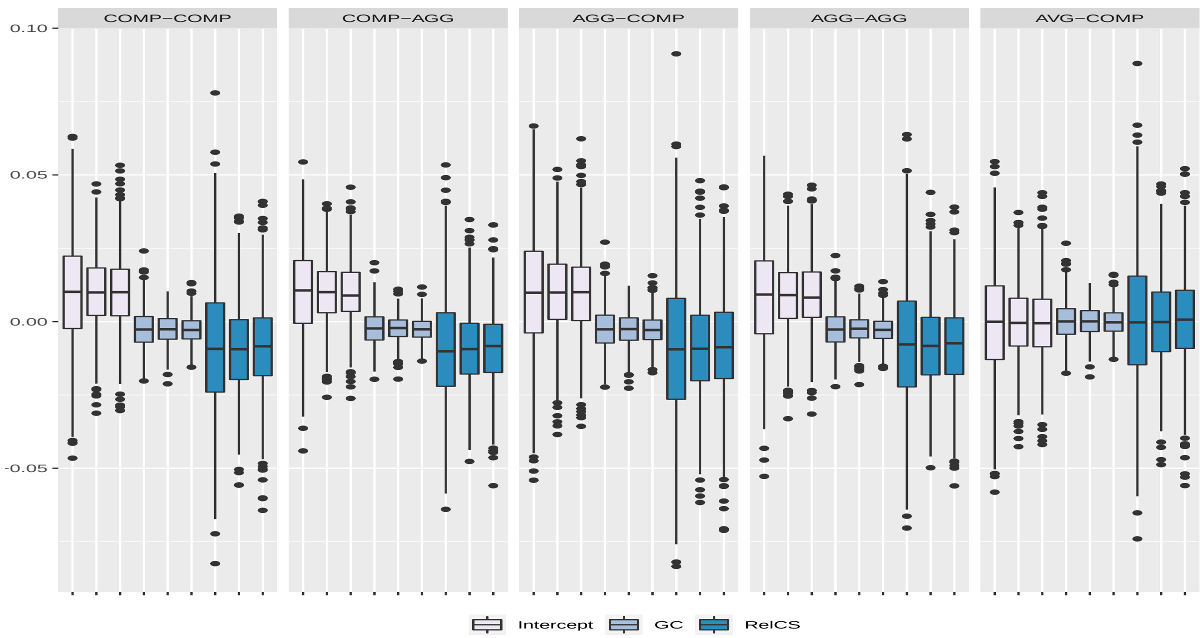
	True Model		Estimated Model												
	Composite		Composite				Local-aggregate				Local-average				
			MBias	MSE	RelSE	COVP	MBias	MSE	RelSE	COVP	MBias	MSE	RelSE	COVP	
Density = 0.14	*Local-aggregate peer effect*														
	Intercept	*0.0105*	*-0.0004*	*0.32*	*0.96*	*0.94*	-0.0002	0.24	0.86	0.90					
	RelCS	*-0.0094*	*0.0001*	*0.49*	*0.95*	*0.93*	-0.0004	0.35	0.86	0.91					
	GC	*-0.0027*	*0.0002*	*0.04*	*1.02*	*0.95*	0.0005	0.03	0.89	0.92					
	Local-average peer effect														
	Intercept	*0.3246*	*-0.0898*	*234.42*	*1.07*	*0.96*					-0.0437	251.81	1.09	0.96	
	RelCS	*-0.3768*	*0.0108*	*381.25*	*1.09*	*0.97*					-0.0342	419.09	1.11	0.96	
	GC	*0.1386*	*-0.0121*	*45.74*	*1.03*	*0.95*					-0.0122	51.80	1.03	0.95	
	IQ	*-0.0033*	*-0.0000*	*0.00*	*1.02*	*0.96*	-0.0000	0.00	0.91	0.93	-0.0000	0.00	1.02	0.96	
	IQ	*0.0002*	*-0.0004*	*0.00*	*0.89*	*0.91*	-0.0004	0.00	0.91	0.91	-0.0004	0.00	0.90	0.92	
Density = 0.14+0.37	*Local-aggregate peer effect*														
	Intercept	*0.0105*	*-0.0005*	*0.15*	*1.01*	*0.95*	-0.0007	0.11	0.89	0.92					
	RelCS	*-0.0094*	*0.0003*	*0.22*	*1.01*	*0.95*	0.0002	0.17	0.89	0.92					
	GC	*-0.0027*	*0.0001*	*0.02*	*1.00*	*0.95*	0.0003	0.02	0.90	0.92					
	Local-average peer effect														
	Intercept	*0.3246*	*-0.2417*	*764.75*	*1.00*	*0.94*					-0.1710	800.72	1.03	0.95	
	RelCS	*-0.3768*	*0.0031*	*1205.44*	*1.02*	*0.95*					-0.0468	1341.23	1.04	0.95	
	GC	*0.1386*	*-0.0084*	*134.37*	*0.99*	*0.94*					-0.0183	148.99	1.01	0.95	
	IQ	*-0.0033*	*-0.0000*	*0.00*	*1.04*	*0.96*	-0.0000	0.00	0.94	0.93	-0.0000	0.00	1.05	0.96	
	IQ	*0.0002*	*-0.0007*	*0.01*	*0.87*	*0.91*	-0.0003	0.01	0.89	0.92	-0.0007	0.01	0.88	0.91	
Density = 0.37	*Local-aggregate peer effect*														
	Intercept	*0.0105*	*-0.0006*	*0.15*	*0.97*	*0.94*	-0.0006	0.10	0.90	0.93					
	RelCS	*-0.0094*	*0.0007*	*0.23*	*0.96*	*0.94*	0.0003	0.15	0.90	0.93					
	GC	*-0.0027*	*-0.0001*	*0.02*	*1.03*	*0.97*	0.0000	0.02	0.90	0.92					
	Local-average peer effect														
	Intercept	*0.3246*	*-0.3713*	*1150.08*	*1.06*	*0.95*					-0.3035	1203.75	1.09	0.97	
	RelCS	*-0.3768*	*-0.0269*	*1789.81*	*1.06*	*0.96*					-0.0499	1994.82	1.08	0.97	
	GC	*0.1386*	*-0.0046*	*196.87*	*1.05*	*0.96*					-0.0165	216.18	1.08	0.96	
	IQ	*-0.0033*	*-0.0000*	*0.00*	*0.99*	*0.95*	-0.0000	0.00	0.90	0.92	-0.0000	0.00	0.98	0.95	
	IQ	*0.0002*	*-0.0013*	*0.02*	*0.88*	*0.90*	-0.0004	0.01	0.89	0.92	-0.0013	0.02	0.88	0.90	

Note: Monte Carlo simualtion results based on 1000 replications. The true DGP is the composite model. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error. RelSE: Relative standard errors, COVP: Empirical Coverage probability for 95% confidence intervall. Numbers in italics indicate estimation results based on the correctly specified model.

For both components the relative standard errors are generally close to one, indicating that the asymptotic standard errors approximate the standard errors in finite samples closely. Moreover, for the composite model the empirical coverage probabilities are close to the nominal value of 0.95. Since the empirical coverage probabilities are computed using the critical values under standard normality, we conclude that conventional asymptotic inference is sufficiently accurate. Moreover, the empirical coverage probabilities prove to be robust with respect to the network densities. Even for the high network density design, where the instrument strength for the local-average is relatively weak, the standard errors and the empirical coverage probabilities are well determined.

The middle and the right panel of Table 1.3 provide insights into the consequences of misspecification. Estimation of the local aggregate-model (middle panel) while omitting the local-average components has no major consequences for the bias of the parameter estimates. The omission of the hard to estimate local-average component yields MSE reductions for the included parameters (see the box-plots COMP-COMP and COMP-AGG in Figure 1.5 for comparison). However, the inference gets slightly distorted. The relative standard errors and the coverage probabilities are too small and produce too many rejections of the null hypothesis. The situation for the misspecified local-average model (right panel of Table1.3) is somewhat different. Here we find no efficiency gains by omitting the local-aggregate component (see also the box-plots COMP-COMP and COMP-AVG in Figure 1.6).

Figure 1.5: Boxplots of parameter estimates of the local-aggregate part

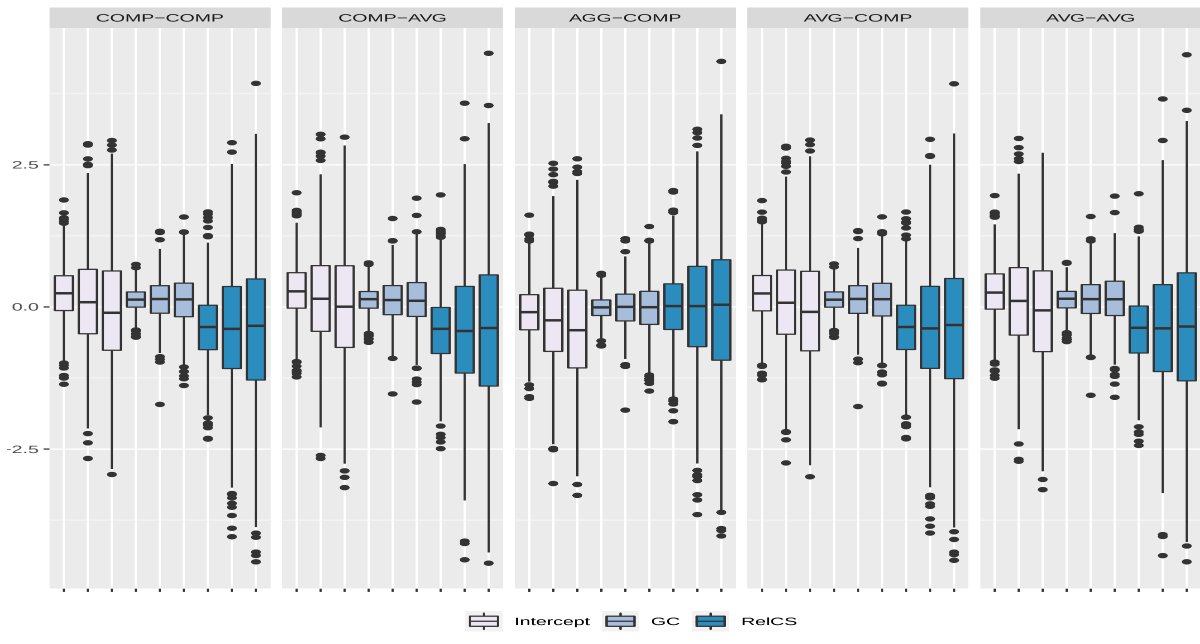


Note: Boxplots of parameter estimates of the local-aggregate part from 1000 simulations. Each subplot corresponds to a different combination of the true data generation process and estimated model. The first acronym in the title of the subplot refers to the underlying true data model and the second one refers to the estimation model used, e.g. COMP-AGG indicates that the true data generating process is based on a composite model but a local-aggregate model is estimated (i.e., AVG part omitted). For each of the subplot, we report the parameter estimates of the local-aggregate part for the three density designs (0.14, 0.14 and 0.37, 0.37) from left to right.

Over-parametrization

Local-aggregate and local-average peer effects are very similar in nature as they both depend on the same underlying adjacency matrix so that they are strongly correlated by construction. We therefore study the effects of over-parametrization by comparing the estimates of the correctly specified local-aggregate model (Table 1.7) and the correctly specified local-average (Table 1.8) with the estimates of the over-parametrized composite model. Using the correct and parsimonious local-aggregate specification leads to the expected MSE reductions (see Table 1.7). However, the inclusion of the local-aggregate component in the local-average model proves to be advantageous as we find a small MSE reduction of around 10 percent when estimating the local-average component by the richer composite model (see Table 1.8). This means that the moments of the local-aggregate parameters are also informative for the estimation of the local-average parameters. As the coverage probabilities of the redundant parameters are not distorted in any of the over-parameterized composite models it appears to be a reasonable strategy to include the local-aggregate component even when the true model is the local-average one and the local-average component when the true model is the local-aggregate one. We also see hardly any difference in box plots AGG-COMP vs. AGG-AGG in Figure 1.6 and AVG-COMP vs. AVG-AVG in Figure 1.5 indicating that the over-parametrization does not harm the part which is correctly included. The inclusion of the irrelevant part leads mostly to insignificant coefficient estimates as seen in AGG-COMP in Figure 1.6 and AVG-COMP in Figure 1.5.

Figure 1.6: Boxplots of the parameter estimates of the local-average part



Note: Boxplots of parameter estimates of the local-average part from 1000 simulations. Each subplot corresponds to a different combination of the true data generation process and estimated model. The first acronym in the title of the subplot refers to the underlying true data model and the second one refers to the estimation model used, e.g. COMP-AVG indicates that the true data generating process is based on a composite model but a local-average model is estimated (i.e. AGG part omitted). For each of the subplot , we report the parameter estimates of the local-average part for the three density designs (0.14, 0.14 and 0.37, 0.37) from left to right.

1.6. Conclusions

This paper contributes to the growing literature on the empirical analysis of social networks. In particular, we focus on the role of heterogeneity in network peer effects by accounting for network-specific factors and different driving mechanisms of peer behavior. For our empirical study of the role of network peer effects on educational attainment, we use a unique network dataset of 85 school classes of secondary schools in Germany, which allows us to exploit exogenous variation in second degree friends to identify the endogenous peer effects.

The novel IV-MD approach applied proves to be a valuable tool to study parametrically rich network environments. Our Monte-Carlo simulations reveal reasonable coverage probabilities even in the presence of dense networks. The inclusion of a local aggregate component proves to be a valuable specification strategy when the estimates of the local-average components suffer from weak identification in the presence of dense networks.

As network-specific factors, we find that the size of the network (i.e., of the school class) and gender composition are important determinants of the peer effects, while conventional model specifications with homogeneous peer effects turn out to be too crude and lead to insignificant findings. In addition to the network-specific factors, heterogeneity in terms of the underlying behavioral assumptions matters. In particular, we have shown that a student's educational attainment is affected by both the pure size of their peer group, as reflected by the local-aggregate model, and the norm behavior captured by the local-average model.

Our study contributes to the voluminous empirical literature on the determinants of educational attainment. We show that an increase in the class size alone reduces peer behavior and may even lead to negative peer effects in very large classes. Unlike the vast majority of empirical studies in this field, which are largely based on reduced form approaches, our approach gives rise to a structural interpretation of why class size and gender composition matter and why these factors differ across subjects. For instance, our study sheds light on peer behavior as a specific channel through which class size affects educational attainment.

We regard our study as a promising starting point for more realistic modeling of heterogeneous network behavior and for a deeper understanding of how networks operate. Future work should be devoted to more elaborate specifications of network heterogeneity (e.g., nonlinear or nonparametric peer effects) as well as to the analysis of the relationship between network structures (e.g., properties of the adjacency matrices) and the identification of network peer effects.

1.7. Appendix

Table 1.4: IV-MD Estimation Results: German

	Heterogeneous Peer Effects Model		
	Composite	Local-aggregate	Local-average
Local-aggregate peer effect			
Intercept	0.0024***	0.0016**	
	(0.0007)	(0.0007)	
Relative Class Size	0.0049	0.0053	
	(0.0055)	(0.0056)	
Female Share	-0.005***	-0.0029*	
	(0.0016)	(0.0016)	
Local-average peer effect			
Intercept	0.0338		0.1442***
	(0.0267)		(0.0311)
Relative Class Size	0.8538***		0.576***
	(0.1805)		(0.2017)
Female Share	0.2476***		0.2099***
	(0.0525)		(0.0580)
Own characteristics			
IQ	-0.0046***	-0.0058***	-0.0053***
	(0.0009)	(0.0009)	(0.0010)
Previous GPA	0.8238***	0.8722***	0.8458***
	(0.0160)	(0.0165)	(0.0172)
Age	-0.0508***	-0.0496***	-0.0653***
	(0.0088)	(0.0092)	(0.0097)
Peers' characteristics			
IQ	0.0025	-0.0013	0.0059***
	(0.0017)	(0.0018)	(0.0018)
Previous GPA	0.0107	0.0191	0.0202
	(0.0390)	(0.0329)	(0.0440)
Age	0.0023	-0.0315*	0.0343**
	(0.0165)	(0.0181)	(0.0172)
MD statistics (d.f.)	2750.33 (668)	2796.71 (671)	2774.81 (671)

Estimates of the three model variants obtained by IV-MD estimation. The first column corresponds to the composite model, the second column corresponds to the local-aggregate model and the third column corresponds to the local-average model estimation results. The IV-MD estimates of the two submodels are based on first stage IV-estimates of the submodels. Robust standard errors are reported in parentheses. First stage errors are assumed to be heteroskedastic, $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$, N=2165, L=85.

Table 1.5: IV-MD Estimation Results: Math

	Heterogeneous Peer Effects Model		
	Composite	Local-aggregate	Local-average
Local-aggregate peer effect			
Intercept	0.0004	-0.0002	
	(0.0007)	(0.0008)	
Relative Class Size	0.0107*	0.0037	
	(0.0064)	(0.0066)	
Female Share	-0.0047***	-0.0036*	
	(0.0018)	(0.0019)	
Local-average peer effect			
Intercept	-0.0146		-0.0001
	(0.0285)		(0.0338)
Relative Class Size	1.2058***		0.4928**
	(0.1946)		(0.2335)
Female Share	-0.1942***		-0.1405**
	(0.0551)		(0.0614)
Own characteristics			
IQ	-0.0162***	-0.0172***	-0.0171***
	(0.0011)	(0.0012)	(0.0012)
Previous GPA	0.8370***	0.8178***	0.8678***
	(0.0197)	(0.0207)	(0.0216)
Age	0.0695***	0.0615***	0.0626***
	(0.0106)	(0.0111)	(0.0122)
Peers' characteristics			
IQ	0.0185***	0.0143***	0.0109***
	(0.0020)	(0.0022)	(0.0024)
Previous GPA	-0.0938**	0.025	-0.0929**
	(0.0416)	(0.0386)	(0.0460)
Age	0.0036	0.0583***	0.0577**
	(0.0216)	(0.0212)	(0.0237)
MD statistics (d.f.)	3078.52 (668)	3134.75 (671)	3087.13 (671)

Estimates of the three model variants obtained by IV-MD estimation. The first column corresponds to the composite model, the second column corresponds to the local-aggregate model and the third column corresponds to the local-average model estimation results. The IV-MD estimates of the two submodels are based on first stage IV-estimates of the submodels. Robust standard errors are reported in parentheses. First stage errors are assumed to be heteroskedastic, $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$, N=2165, L=85.

Table 1.6: Homogeneous Model Estimates

	GPA			German			Math		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Aggregate Peer Effect	0.0007	0.0008		0.0030**	0.0030**		0.0014	0.0012	
	(0.0006)	(0.0006)		(0.0012)	(0.0012)		(0.0015)	(0.0014)	
Average Peer Effect	0.8025		0.4181	-0.0977		-0.0277	0.2464		0.1863
	(0.6213)		(0.6822)	(0.6231)		(0.6381)	(0.6165)		(0.6191)
Own characteristics									
IQ	-0.0032***	-0.0032***	-0.0032***	-0.0061***	-0.0062***	-0.0063***	-0.0197***	-0.0195***	-0.0197***
	(0.0008)	(0.0008)	(0.0008)	(0.0016)	(0.0016)	(0.0016)	(0.0022)	(0.0021)	(0.0022)
Previous GPA	0.7295***	0.7351***	0.7322***	0.7971***	0.7966***	0.7969***	0.8093***	0.8059***	0.8083***
	(0.0164)	(0.0150)	(0.0159)	(0.0309)	(0.0308)	(0.0310)	(0.0377)	(0.0371)	(0.0376)
Age	-0.0095	-0.0159**	-0.0130	-0.0616***	-0.0608***	-0.0629***	0.0297	0.0318	0.0294
	(0.0102)	(0.0079)	(0.0102)	(0.0178)	(0.0166)	(0.0178)	(0.0219)	(0.0213)	(0.0219)
Peers' characteristics									
IQ	0.0034	0.0014	0.0024	0.0029	0.0032	0.0029	0.0125	0.0078*	0.0112
	(0.0024)	(0.0016)	(0.0024)	(0.0039)	(0.0035)	(0.0040)	(0.0128)	(0.0044)	(0.0128)
Previous GPA	-0.5522	0.0607**	-0.2548	0.1640	0.0766	0.1191	-0.2161	-0.0112	-0.1582
	(0.4798)	(0.0297)	(0.5247)	(0.5613)	(0.0613)	(0.5756)	(0.5190)	(0.0730)	(0.5181)
Age	-0.0122	-0.0268	-0.0192	-0.0567	-0.0516	-0.0535	0.0203	0.0276	0.0223
	(0.0217)	(0.0174)	(0.0219)	(0.0497)	(0.0340)	(0.0506)	(0.0472)	(0.0436)	(0.0471)
MD statistics	3135.11	3165.16	3166.51	2804.18	2805.10	2818.05	3147.42	3151.12	3148.02
d.f.	672	673	673	672	673	673	672	673	673

IV estimates for the three models. For each outcome variable, the first column presents the results for the composite model, the second column presents the results for the local-aggregate model, and the third column presents the results for the local-average model estimation results. Robust standard errors are in parentheses. First stage errors are assumed to be heteroskedastic. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$, N=2165, L=85.

Table 1.7: Monte Carlo Results for the Aggregate model (DGP-AGG)

	True Model		Estimated Model							
	Local-aggregate		Composite				Local-aggregate			
			MBias	MSE	RelSE	COVP	MBias	MSE	RelSE	COVP
Density = 0.14	*Local-aggregate peer effect*									
	Intercept	0.0105	-0.0005	0.42	0.96	0.94	*-0.0018*	*0.31*	*0.87*	*0.91*
	RelCS	-0.0094	0.0002	0.63	0.95	0.94	*0.0015*	*0.45*	*0.87*	*0.91*
	GC	-0.0027	0.0002	0.05	1.02	0.96	*0.0001*	*0.04*	*0.90*	*0.92*
	Local-average peer effect									
	Intercept	0	-0.0898	230.58	1.07	0.96				
	RelCS	0	0.0102	374.12	1.10	0.96				
	GC	0	-0.0149	45.59	1.03	0.95				
	IQ	-0.0033	-0.0000	0.00	1.03	0.96	*-0.0000*	*0.00*	*0.91*	*0.94*
	IQ	0.0002	-0.0004	0.00	0.89	0.91	*-0.0001*	*0.00*	*0.91*	*0.92*
Density = 0.14+0.37	*Local-aggregate peer effect*									
	Intercept	0.0105	-0.0006	0.20	1.01	0.95	*-0.0016*	*0.15*	*0.90*	*0.91*
	RelCS	-0.0094	0.0005	0.29	1.01	0.95	*0.0014*	*0.22*	*0.90*	*0.92*
	GC	-0.0027	0.0001	0.03	1.00	0.96	*0.0001*	*0.02*	*0.90*	*0.93*
	Local-average peer effect									
	Intercept	0	-0.2338	745.45	1.00	0.95				
	RelCS	0	-0.0053	1181.96	1.02	0.95				
	GC	0	-0.0092	129.97	1.00	0.95				
	IQ	-0.0033	-0.0000	0.00	1.04	0.96	*-0.0000*	*0.00*	*0.94*	*0.93*
	IQ	0.0002	-0.0007	0.01	0.87	0.91	*0.0001*	*0.01*	*0.89*	*0.92*
Density = 0.37	*Local-aggregate peer effect*									
	Intercept	0.0105	-0.0007	0.20	0.97	0.94	*-0.0014*	*0.13*	*0.91*	*0.93*
	RelCS	-0.0094	0.0008	0.29	0.96	0.94	*0.0013*	*0.19*	*0.91*	*0.93*
	GC	-0.0027	-0.0001	0.02	1.04	0.96	*-0.0002*	*0.02*	*0.90*	*0.92*
	Local-average peer effect									
	Intercept	0	-0.3591	1117.56	1.06	0.95				
	RelCS	0	-0.0413	1760.83	1.06	0.96				
	GC	0	-0.0024	192.01	1.05	0.96				
	IQ	-0.0033	-0.0000	0.00	0.99	0.95	*-0.0000*	*0.00*	*0.91*	*0.92*
	IQ	0.0002	-0.0013	0.02	0.88	0.90	*0.0000*	*0.01*	*0.88*	*0.91*

Note: Monte Carlo simualtion results based on 1000 replications. The true DGP is the local-aggregate model. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error. RelSE: Relative standard errors, COVP: Empirical Coverage probability for 95% confidence intervall. Numbers in italics indicate estimation results based on the correctly specified model.

Table 1.8: Monte Carlo Result for the Local-average model (*DPG-AVG*)

	True Model		Estimated Model							
	Local-average		Composite				Local-average			
			MBias	MSE	RelSE	COVP	MBias	MSE	RelSE	COVP
Density = 0.14	*Local-aggregate peer effect*									
	Intercept	0	-0.0003	0.33	0.96	0.94				
	RelCS	0	0.0000	0.50	0.95	0.94				
	GC	0	0.0002	0.04	1.01	0.95				
	Local-average peer effect									
	Intercept	0.3246	-0.0906	235.87	1.07	0.95	*-0.0659*	*254.38*	*1.09*	*0.96*
	RelCS	-0.3768	0.0117	382.42	1.09	0.96	*-0.0165*	*417.93*	*1.11*	*0.96*
	GC	0.1386	-0.0118	45.86	1.03	0.95	*-0.0048*	*51.80*	*1.03*	*0.95*
	IQ	-0.0033	-0.0000	0.00	1.02	0.95	*-0.0000*	*0.00*	*1.02*	*0.96*
	IQ	0.0002	-0.0004	0.00	0.89	0.91	*-0.0004*	*0.00*	*0.90*	*0.92*
Density = 0.14+0.37	*Local-aggregate peer effect*									
	Intercept	0	-0.0004	0.16	1.01	0.95				
	RelCS	0	0.0003	0.23	1.01	0.95				
	GC	0	0.0001	0.02	1.00	0.95				
	Local-average peer effect									
	Intercept	0.3246	-0.2420	765.19	1.00	0.94	*-0.2189*	*820.29*	*1.03*	*0.95*
	RelCS	-0.3768	0.0033	1206.81	1.02	0.95	*-0.0089*	*1336.35*	*1.04*	*0.96*
	GC	0.1386	-0.0083	134.43	0.99	0.94	*-0.0023*	*149.07*	*1.01*	*0.95*
	IQ	-0.0033	-0.0000	0.00	1.04	0.96	*-0.0000*	*0.00*	*1.05*	*0.96*
	IQ	0.0002	-0.0007	0.01	0.87	0.91	*-0.0007*	*0.01*	*0.88*	*0.91*
Density = 0.37	*Local-aggregate peer effect*									
	Intercept	0	-0.0004	0.16	0.97	0.94				
	RelCS	0	0.0006	0.24	0.95	0.94				
	GC	0	-0.0001	0.02	1.03	0.96				
	Local-average peer effect									
	Intercept	0.3246	-0.3710	1144.82	1.06	0.95	*-0.3687*	*1227.54*	*1.10*	*0.96*
	RelCS	-0.3768	-0.0283	1780.81	1.06	0.96	*-0.0009*	*1957.62*	*1.09*	*0.97*
	GC	0.1386	-0.0038	196.66	1.05	0.96	*0.0077*	*215.12*	*1.08*	*0.96*
	IQ	-0.0033	-0.0000	0.00	0.98	0.95	*-0.0000*	*0.00*	*0.98*	*0.95*
	IQ	0.0002	-0.0013	0.02	0.88	0.90	*-0.0013*	*0.02*	*0.88*	*0.89*

Note: Monte Carlo simualtion results based on 1000 replications. The true DGP is the local-average model. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error. RelSE: Relative standard errors, COVP: Empirical Coverage probability for 95% confidence intervall. Numbers in italics indicate estimation results based on the correctly specified model.

References

Angrist, J. and V. Lavy (1999). “Using Maimonides’ rule to estimate the effect of class size on scholastic achievement”. *Quarterly Journal of Economics* 114 (2), pp. 533–575.

Arduini, T., E. Patacchini, and E. Rainone (2020). “Treatment Effects With Heterogeneous Externalities”. *Journal of Business & Economic Statistics* (4), pp. 826–838.

Bear, J. B. and A. W. Woolley (2011). “The role of gender in team collaboration and performance”. *Interdisciplinary Science Reviews* 36 (2), pp. 146–153.

Beugnot, J., B. Fortin, G. Lacroix, and M. C. Villeval (2019). “Gender and peer effects on performance in social networks”. *European Economic Review* 113, pp. 207–224.

Blume, L. E., W. A. Brock, S. N. Durlauf, and R. Jayaraman (2015). “Linear social interactions models”. *Journal of Political Economy* 123 (2), pp. 444–496.

Boucher, V., Y. Bramoullé, H. Djebbari, and B. Fortin (Jan. 2014). “Do peers affect student achievement? Evidence from Canada using group size variation”. *Journal of Applied Econometrics* 29 (1), pp. 91–109.

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). “Identification of peer effects through social networks”. *Journal of Econometrics* 150 (1), pp. 41–55.

– (2020). “Peer Effects in Networks: A Survey”. *Annual Review of Economics* 12 (1), pp. 603–629.

CAESR (2007). *Dataset Gymnasiastenstudie*. Central Archive for Empirical Social Research, Cologne, Germany.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009). “Peer effects and social networks in education”. *The Review of Economic Studies* 76 (4), pp. 1239–1267.

Chamberlain, G. (1984). “Panel data”. In: *Handbook of Econometrics*. Ed. by Griliches, Z. and Intriligator, M. D. Elsevier, pp. 1247–1318.

Dobbelsteen, S., J. Levin, and H. Oosterbeek (2002). “The causal effect of class size on scholastic achievement: Distinguishing the pure class size effect from the effect of changes in class composition”. *Oxford Bulletin of Economics and Statistics* 64 (1), pp. 17–38.

Fredriksson, P., B. Öckert, and H. Oosterbeek (Nov. 2012). “ Long-term effects of class size”. *The Quarterly Journal of Economics* 128 (1), pp. 249–285.

Griffith, A. (2021). “Name Your Friends, but Only Five? The Importance of Censoring in Peer Effects Estimates using Social Network Data”. *Journal of Labor Economics* 0 (ja), null.

Hanushek, E. A. (1996). “Measuring investment in education”. *The Journal of Economic Perspectives* 10 (4), pp. 9–30.

Heinesen, E. (2010). “Estimating class-size effects using within-school variation in subject-specific classes”. *The Economic Journal* 120 (545), pp. 737–760.

Hoxby, C. M. (2000). “The effects of class size on student achievement: New evidence from population variation”. *The Quarterly Journal of Economics* 115 (4), pp. 1239–1285.

– (2002). “The power of peers: How does the makeup of a classroom influence achievement”. *Education Next* Summer 2 (2.2), pp. 57–63.

Kelejian, H. H. and I. R. Prucha (1998). “A generalized spatial two-stage least squares procedure for estimating a spatial autoregressive model with autoregressive disturbances”. *The Journal of Real Estate Finance and Economics* 17 (1), pp. 99–121.

Kelejian, H. H. and I. R. Prucha (1999). "A Generalized Moments Estimator for the Autoregressive Parameter in a Spatial Model". *International Economic Review* 40 (2), pp. 509–533.

Krueger, A. B. (May 1999). "Experimental estimates of education production functions". *The Quarterly Journal of Economics* 114 (2), pp. 497–532.

Lavy, V. and A. Schlosser (2011). "Mechanisms and impacts of gender peer effects at school". *American Economic Journal: Applied Economics* 3 (2), pp. 1–33.

Lin, X. (2014). "Network attributes and peer effects". *Economics Bulletin* 34 (3), pp. 2060–2079.

Liu, X., E. Patacchini, and Y. Zenou (2014). "Endogenous peer effects: Local aggregate or local average?" *Journal of Economic Behavior and Organization* 103, pp. 39–59.

Mael, F., A. Alonso, D. Gibson, K. Rogers, and M. Smith (2005). *Single-sex versus coeducational schooling: A systematic review. Doc# 2005-01.* Tech. rep.

Manski, C. F. (1993). "Identification of Endogenous Social Effects: The Reflection Problem". *The Review of Economic Studies* 60 (3), pp. 531–542.

Masten, M. A. (2018). "Random Coefficients on Endogenous Variables in Simultaneous Equations Models". *The Review of Economic Studies* (2), pp. 1193–1250.

Morse, S. (1999). "Separated by sex: A critical look at single-sex education for girls". *Journal of Chemical Education* 76, p. 615.

Startz, R. and A. Wood-Doughty (2017). "Improved estimation of peer effects using network data". Dissertation. Department of Economics, University of California, Santa Barbara.

Chapter 2

A Network Model of Stress Contagion in the Classroom: Do Learning Arrangements Matter?

Abstract

This paper examines the role of learning arrangements on stress contagion among students in the classroom. Based on a formal network model, we investigate the spatial contagion of stress in the classroom as a function of seating arrangement and didactic-methodological context factors (e.g., teacher instruction, cooperative work, complexity of the learning content). Using a novel panel data set for two different classes with different seating plans, we quantify stress contagion among students. Our empirical results show that contextual factors play a crucial role in stress contagion and that it varies depending on the learning arrangement.

2.1. Introduction

Stress is a major source of individual performance differences among students in the classroom. Its extent is not only determined by individual factors such as health, mental predisposition, family background inter alia, but also by the contagion of stress by classmates. Such emotional contagions are likely to create a multiplier effect in the sense that the direct effect of an exogenous stress triggering event is bolstered by an endogenous stress effect via the stress levels of classmates. Yet, the mechanisms of stress contagion in the classroom are not yet well understood and hard to assess empirically.

Stress in school is an important topic as stress may cause, for instance, mental health (Auerbach et al., 2018) or learning problems (Vogel and Schwabe, 2016). Kouzma and Kennedy (2004) found that main sources of students' stress are school-related, as for instance, examinations, high workload, time pressure, and peer-related issues such as stress contagion. So far, existing research on stress in school has largely neglected the role of stress contagion, especially with regard to peer-to-peer emotional crossover. Yet, there is a number of studies examining peer influence, which can in part be linked to the concept of emotional contagion among students (Burgess et al., 2018). Research suggests that peers have a large impact on each other's emotions, moods and thus experiences during class. While peer influence usually implies the pressure to conform to a specific behavior, social and emotional contagion among peers is a concept reflecting on more general, often implicit peer processes (Dishion and Dodge, 2005). Classmates do have a critical role when it comes to academic achievement and performance in school (see Fortuin et al., 2016; Gašević et al., 2013; King, 2020; King and Datu, 2017; King and Mendoza, 2020). Concerning this matter, many studies explore concepts with a positive connotation such as motivation, goal achievement and positive school behavior, leaving aside that also stress can crossover from one student to another.

Stress contagion describes stressors that are transferred from one individual, through their role within a group, to others. A shared threat in the classroom can trigger school stress and, when transmitted, causes a range of reactions and adaptations in other members, i.e. classmates (Wethington, 2000). Bakker and Schaufeli (2000) found that the prevalence of perceived burnout among teachers is most strongly related to individual teachers' stress when they often communicated with each other. Therefore, it can be assumed that the likelihood of stress crossover processes in class become intensified, especially when classmates talk to each other frequently, for example due to tasks that must be done in groups. Difficulties experienced by the individual may result in shared and even amplified stress by others. Thus, stress contagion among peers is related to learning, school performance and general well-being, and consequently, it is critical to establish a better understanding of the underlying mechanisms.

Previous research on stress contagion has focused primarily on multilevel regression models (see Becker et al., 2014; Frenzel et al., 2018; Oberle and Schonert-Reichl, 2016; Warwas and Helm, 2017) leaving aside the fact that sources of similar behavior cannot be separated. Therefore, we propose a structural network approach of stress contagion in the classroom with which different sources of individual stress behavior generated by the peers can be disentangled, i.e., by exploiting network information we are able to identify endogenous from exogenous peer effects of stress contagion.

In particular, we take a closer look on the spatial aspects of stress contagion determined by classroom seating arrangements. We argue that the spatial organization of the classroom may play a significant role on how a student's stress level stress spills over to the local peers. For this we "borrow" from the statistical approaches considering complex interdependencies between observational units as they have been popularized in the spatial econometric literature, e.g. Anselin (2003) and Elhorst (2014), as well as in the literature on network peer effects, e.g. Bramoullé et al. (2009, 2020). Based on panel data on students' stress levels we propose a novel approach which allows us to identify and estimate a parameter of stress contagion. In particular, our model is able to shed light on the role of context specific stress factors that aggravate stress contagion.

Our empirical results are based on a unique data set exploited first by Kärner et al. (2017). The data contain information on two school classrooms, whose seating charts we use to construct the networks consisting of spatial peers. Moreover, the students were asked to report their stress levels and how they were coping with the situation. Furthermore, video recordings allow to construct context variables, which reflect the overall situation in the classroom at each time.

The paper is organized as follows. In Section 2.2 we define stress contagion and describe conditions of contagion processes from a psychological perspective. Section 2.3 introduces our network econometric approach and discusses how it can be used to characterize context related stress contagion among students. In Section 2.4 we describe the data underlying our empirical study and provide some summary statistics, while in Section 2.5 we present our empirical findings. Section 2.6 concludes and contains a discussion on potential future work.

2.2. Stress Contagion

2.2.1. Definition and Conditions of Stress Contagion

In general emotional contagion is defined by Hatfield et al. (1994, p. 5) as "the tendency to automatically mimic and synchronize facial expressions, vocalizations, postures, and movements with those of another person and, consequently, to converge emotionally." Westman and Vinokur (1998) discuss three potential mechanisms through which emotional and even stress contagion can occur: (i) both interaction partners are exposed to the same stressor because they share the social environment, (ii) direct transfer of stress from one person to the other person due to an empathic reaction on behalf of the receiver, and (iii) indirect transfer via behavioral interaction (Härtel and Page, 2009). These three different mechanisms find their formal counterparts in the econometric literature on peer effects estimation, which distinguishes between different sources of similar behavior among individuals. Here (i) refers to the exogenous or contextual peer effects and unobserved correlated effects, (ii) is related to the endogenous peer effect and (iii) to social multiplier effect as a result of all interactions (see Manski, 1993, and Section 2.3 for a more formal definition within a peer effects network model).

Rather than focusing on specific emotions such as joy or anger, we take a closer look at the crossover of an emotional or psychological state, namely stress. Stress, subjectively experienced by a person, may then result from subjective appraisal of a stressful or demanding situation and the situational coping within the current interpersonal situation (Lazarus and Folkman, 1984). Bakker et al. (2009) describe specific conditions under which processes of emotional contagion are most likely. According to the authors, among other things, the frequency of interactions and the spatial and personal proximity or distance influence the probability of stress contagion. In the light of the literature, it can be assumed that corresponding peer effects are more likely among students when they sit next to each other in the classroom and work on tasks together. With regard to our peer-effects model, conditions for stress contagion in class will be systematized in (1) peer-related factors (peers as a stressor, peers as a resource) and (2) context-related factors.

Peer-related factors: peers as a stressor. Peers can act as a source of stress. Collaborative work in the classroom requires time for coordination, communication, and mutual feedback. Insufficient group work is often associated with lower group effectiveness, disengagement of individual students, and negative performance evaluations by fellow group members (Arvey and Murphy, 1998; Druskat and Wolff, 1999). Due to processes of stress contagion (Westman and Vinokur, 1998), experienced time pressure, uncertainty, or pressure to succeed may spread across the team or group. Existing research shows, for instance, that stress may be caused by group assessments because of different effort and mark expectations among group members, and a lack of individual control over the outcome quality of joint group work (Pitt et al., 2018). Hon and Chan (2013) distinguish between two types of work related stress. They found that team task conflict was positively associated with challenge-related stress and that team relationship conflict was positively associated with hindrance-related stress. Also, learning environments that challenge individual students too much and therefore create social comparison with skilled peers constitute a stressful experience with unfavorable emotions such as shame or anxiety, possibly also determining the stress levels of several students involved in the group (Pekrun, 2006). It was also found that social comparison with similar others can moderate stress contagion effects in the sense that emotional exhaustion is more pronounced in individuals who perceived that their colleagues were also burned-out (Bakker and Schaufeli, 2000).

Peer-related factors: peers as a resource. Peers can act as a resource for coping with stress. Working in a group may be less stressful, as the presence of other students represents potential sources of informational and emotional support, which have positive effects on psychological adjustment because collaboration fosters knowledge exchange and the distribution of cognitive effort among group members (Kirschner et al., 2009; Sweller et al., 2011). Furthermore, a positive climate within teams/groups buffers the effects of contagion processes by providing group members with emotional support during stressful periods (Cohen and Wills, 1985). In a randomized experimental field study on effects of the work setting on stress and emotions during experiments in biology classes Minkley et al. (2017) find that conducting experiments alone carries the risk of self-attributed failure signified by elevated stress. In contrast, conducting an experiment in a group is less stressful, as the peers constitute a source of social support. Accordingly, Kärner et al. (2021) show that support from socially similarly situated individuals, i.e., peers are perceived as being particularly effective and helpful.

Context-related factors. In addition to peer-related factors for stress contagion, there are context-related factors because interaction partners share the same social environment and are exposed to the same stressful condition (Westman and Vinokur, 1998). In a classroom setting, all students share the same "educational treatment". In general, lessons can be described by the proportions of teacher instruction, individual work and partner or group work of students that are observable. In this regard, processes of stress contagion may be more likely in educational situations where students work together in teams or groups compared to situations where they just listen to the teacher and do not collaborate with each other. Furthermore, the complexity of learning contents plays a role in the extent to which lessons are generally experienced as stressful, as well as the stress of the teacher, who, for example, influences the pace of teaching through their own time pressure (Kärner et al., 2017).

2.2.2. Previous approaches in researching stress contagion

Processes of stress and emotional contagion have been investigated in working contexts (see Bakker et al., 2009), couple relationships (see Song et al., 2011) or mother-infant relationships (see Waters et al., 2014). Crossover processes are also a topic in educational sciences as, for instance, emotional contagion between principals and teachers (see Westman and Etzion, 1999) and between teachers and students is investigated (see Frenzel et al., 2018, 2009; Warwas and Helm, 2017). For teacher-student relationship, for instance, crossover of own experiences (Bakker, 2005) and enjoyment (Frenzel et al., 2018) or associations between teachers' burnout and students' motivation (Shen et al., 2015) were studied.

Popular empirical approaches to study emotional and stress contagion based on experimental data are, the investigation of contagion processes via systematic variation of facial expression conditions (e.g. Hatfield et al., 2014), the analysis of dyadic relationships via actor-partner interdependence model (see Cook and Kenny, 2005) or the use of physiological measures and methods of behavioral observation (e.g. Donker et al., 2018). Empirical studies on contagion effects within dyadic relationships using observational data rely on linear regression approaches, structural equation modeling, and multilevel modeling (Cook and Kenny, 2005). Becker et al. (2014) for instance, investigate the relationship between teachers' emotions (anger, anxiety, enjoyment), their instructional behavior, and students' emotions in class. The authors use an experience sampling approach and model intra-individual variability via multilevel regression analysis (repeated measures are nested within persons who are nested within classes). Oberle and Schonert-Reichl (2016) investigate relationships between teachers' burnout levels and students' hypothalamic-pituitary-adrenal functioning by measuring students' salivary cortisol levels by means of multilevel regression model.

The studies quoted above are more or less reduced form approaches in the sense that they do not explicitly account for the network aspects of stress contagion in the classroom. However, observed similarities in stress outcomes among students can be a consequence of different mechanisms, all leading to the same or similar reduced form settings. Therefore, empirical studies based on reduced form set-ups leave room for wide range of interpretations (see Manski, 2000). In particular, reduced form settings are silent on how stress is transmitted between students via peer behavior and the role context factors play in this transmission process. In contrast, our structural, network based approach allows us to disentangle various factors explaining similarities in stress behavior among students. In particular, we can identify stress contagion due to endogenous peer behavior and its multiplier effect.

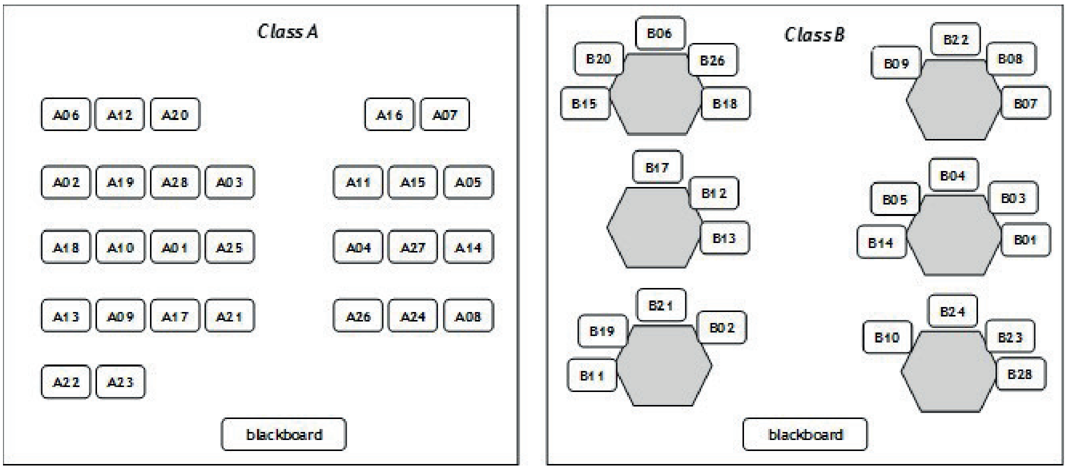
2.3. Measuring Stress Contagion in a Network - An Econometric Approach

In the following we treat the classroom as a network where seating neighbors are treated as spatial peers in the sense that the seating neighbors may effect each other through their own stress levels. Thus, we hypothesize that the specific seating plan of the classroom implies a specific network structure and therefore determines the way and extend individual stress spills over to other classmates. In our econometric model the specific seating plan of a classroom with n students is algebraically represented by the adjacency matrix $A = [A_{ij}]_{n \times n}$, which summarizes social interactions. If students i and j are direct seating neighbors (spatial peers) A_{ij} is 1, and 0 indicates absence of a direct neighborhood. By definition, the diagonal elements of A are 0, reflecting the fact that students do not interact with themselves. In the following we code the seating plan as an undirected network. In this case A is symmetric such that stress contagion between seating neighbors always operates in both directions.

In our empirical study, we consider two classrooms with different seating plans as given in Figure 2.1. Students of Class A study in a classroom with a teacher centered, block-wise seating plan, while students of Class B study in a learner centered teaching arrangement with learning clusters.

Based on the two seating plans we construct the corresponding adjacency matrices for the classroom networks. For instance, taking a look at classroom A in Figure 2.1, we consider as spatial peers the students sitting next to each person. So, the peers of $A06$ are $A12$ and $A20$, meaning that the elements of the adjacency matrix A, $A_{6,12}$, $A_{6,20}$, $A_{12,6}$, $A_{20,6}$ are 1. For Classroom B we consider as spatial peers the students sitting in the same table.

Figure 2.1: Seating plans of Class A and Class B



In our econometric model the dependent variable y_{it} denotes the stress level student i reports at time t. We model the individual stress level as a linear function of the sum of the stress levels of the i-th student's spatial peers ("linear in sums model"):

$$y_{it} = \mu_i + \delta_t \sum_{j=1}^{n} A_{ij} y_{jt} + \beta_x x_{it} + \delta_x \sum_{j=1}^{n} G_{ij} x_{jt} + \varepsilon_{it}, \qquad (i = 1, \ldots, n,\ t = 1, \ldots, T). \tag{2.1}$$

Student i-th stress level is also driven by other exogenous factors x_{it} and by the exogenous factors of the spatial peers.

According to the seating plans, the classrooms are made of groups, and everyone interacts with everyone within the group (except for themselves). What is necessary for identification in this case is to have different group sizes (see Bramoullé et al., 2009, p.45). Indeed, for both classrooms, there is variation in the number of students sharing a table.

Our model allows to distinguish three different mechanisms (Manski, 1993, 2000) which may generate similarities in the stress behavior of students: the endogenous peer effect, the exogenous (or contextual) peer effect and correlated effects. Most interesting in the context of stress contagion is the endogenous peer effect represented by the parameter δ_t. Its existence generates stress contagion in the sense that the stress level of student i varies with the stress levels of his peers. For our model we assume complementary behavior, i.e. the stress level of student i depends on the aggregate of the stress levels of his spatial peers such that more spatial peers generate more stress. In the network peer effects literature such a model specification is called a local aggregate model, see e.g. Calvó-Armengol et al. (2009) for an analysis of peer effects for individual school performance within a friendship network. An alternative specification is the local average model, which has an interesting game-theoretical interpretation in terms of norm or conformity behavior. In such a set-up the utility of a student decreases with the deviation of his behavior from the social norm represented by the average of the peer group. Here we refrain from following this idea because we do not regard a student's stress level as a control variable within a model of utility maximizing behavior.

Note that the presence of an endogenous peer effect leads to stress contagion depending on the specific form of the network and the size of δ_t. Let $Y_t = (y_{1t}, y_{2t}, \ldots, y_{nt})'$ be the vector of stress levels of the students at time t and let c_0 and $c_0 + \Delta_c$ the intervention levels without and with an additional intervention of intensity Δ_c, then the change in the stress levels due to the increased intervention Δ_c is given by

$$\Delta Y_t = Y_t(c_0 + \Delta_c) - Y_t(c_0) = \Delta_c \cdot v(A, \delta_t), \tag{2.2}$$

where $v(A, \delta_t)$ denotes the Katz-Bonacich centrality vector with δ_t determining the decaying influence of higher order neighbors. Although we assume above that the intervention level is raised for all students by the same amount its impact on the stress level generally differs depending on the centrality of the student. Moreover, even if only a fraction of the students is assigned to an intervention every student is affected by the intervention though the the network. Identification of students with a high centrality may therefore be helpful to optimize a teacher's activities to reduce the overall stress level in class.

Finally, by construction the regressor $\sum_{j=1}^{n} A_{ij} y_{jt}$, capturing endogenous stress contagion implies an endogeneity problem as the error term ε_{it} is correlated with this regressor. Thus an ordinary least squares regression yields biased and inconsistent parameter estimates.

A novel element of our model is to allow for a time varying endogenous contagion effect. We model δ_t as a time varying parameter depending on the contextual factor Z_t.

$$\delta_t = \delta_0 + \delta Z_t \tag{2.3}$$

For an easier interpretation we scale the context variable by min-max scaling as $Z_t = \big(\tilde{Z}_t - \min\{\tilde{Z}_t\}/\big(\max\{\tilde{Z}_t\} - \min\{\tilde{Z}_t\}\big)$, where $\tilde{Z}_t$ is the unscaled variable. Thus $\delta_t = \delta_0$ is the size of the endogenous contagion effect when the corresponding context variable takes it lowest value, while $\delta_t = \delta_0 + \delta$ gives the endogenous peer effect at the maximum level of Z_t.

For the exogenous peer effect in (2.1) we use a local average representation, where the adjacency matrix $G = [G_{ij}]_{n \times n}$ is the row normalized version of A. Thus, the term

$\sum_{j=1}^{n} G_{ij} x_{jt}$ picks up changes in the stress level of student i due to changes in the contextual factors of his spatial peers.

Finally similar stress behavior of student i and his peers may also be present due to similar unobservable exogenous factors captured by the individual effect μ_i and/or the error term. Contrary to the endogenous contagion effect, the exogenous peer effect and the correlated effects do not generate social multiplication.

2.4. Data and the Characteristics of the two Classrooms

Our empirical findings are based on a data set first exploited by Kärner et al. (2017). Within a short-term longitudinal study, 53 students (two classes; 18 males, 35 females; mean age = 19.53 ± 4.76 years) from a public German vocational training school were investigated during nine school lessons on the subject "economic business processes". The study was approved by the Bavarian Ministry of Education and Cultural Affairs and was conducted in accordance with the Declaration of Helsinki. Participants were of full age, and if they were not, the parents of the underage participant signed declarations of consent prior to participating.

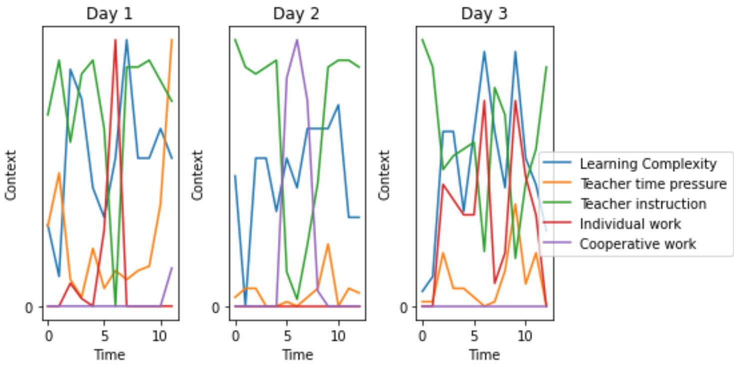
The students' stress experience and situational coping as well as the time pressure experienced by the teachers were sampled at 10-minute intervals using mobile handheld-computers (Palm Tungsten E2®). In general, the continuous-state-sampling method used offers a high degree of ecological validity and differs from other experience-sampling methods due to the high frequency of its measurements and the equidistance of its measurement points (see Csikszentmihalyi and Larson, 1987). There was a maximum number of 38 measurement points per person in the current study, distributed over three days. Participants had the opportunity to rate their actual experience on a continuous rating scale from 0 (= "I fully disagree") to 100 (= "I fully agree"). Students' situational coping was measured using two items: "I can cope with actual demands" (students' *self-confidence*) and "I understand the subject matter" (students' *understanding*). The students' stress experience was measured via two items: "I am under time pressure" (students' *time pressure*) and "I'm under pressure to succeed" (students' *pressure to succeed*). Furthermore, the teachers were asked every 10 minutes about their own experience of stress in terms of time pressure using the item "I am under time pressure" (*teacher time pressure*). A total of 1932 measurements were processed for each of the four variables of student experience. In addition, the lessons were recorded for subsequent video-based analysis of classroom characteristics (e.g. group work).

Context-related factors in the classrooms were assessed via video-based time-sampling analysis using the classification scheme proposed by Seidel et al. (2001). Time intervals of 15 sec. each were coded. Afterwards, the single coded 15-sec. intervals were aggregated to 10-min. intervals via sum scores, synchronizing the context conditions and the person-related data (students' stress experience and situational coping). To assess the reliability of the codings, one third of the videos were coded by two independent coders, finding a satisfactory Cohen's kappa of 0.73. The following instruction-related variables were coded: *teacher instruction* (teacher explains and lectures to the class), *individual work* of students (learners work individually and independently of the teacher on tasks or problems) and *cooperative work* of students (learners work in teams or groups independently of the teacher on tasks or problems). In addition, to assess the *complexity* of the learning content during the lesson, time intervals of 1 minute each were coded and we used a four-point Likert scale based on Bloom's taxonomy to assess the complexity of the learning content and the tasks to be worked on (increasing levels of difficulty: 0 = "apply already known content", 1 = "analyze contexts", 2 = "synthesize complex ideas", 3 = "evaluate complex problems"; between-coder-correlation = 0.82).

Figure 2.2 and 2.3 plot for each day and class the five different the (scaled) context factors (see also the summary statistics of the unscaled variables are in Table 2.6 of Appendix 2.7). With respect to learning *complexity*, we find that there is similar variation in the two classrooms, also reflected in the average values of these variables. The plots reveal significant differences between the two seating arrangements for the variable *teacher's time pressure*. In classroom A there is very little time pressure that the teacher experiences, while the teacher in class B is constantly under time pressure, especially in the middle of the day.

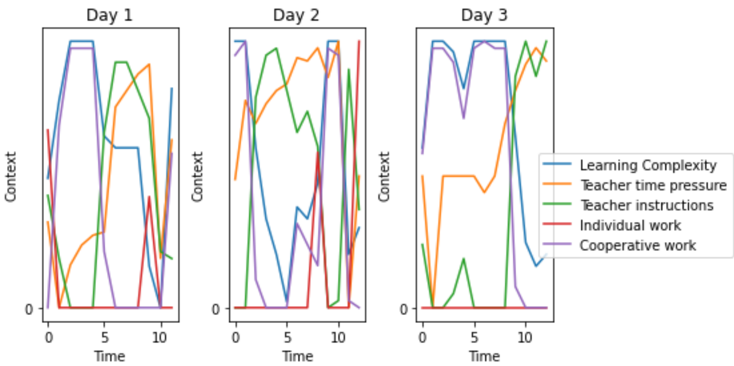
Note, that there is a remarkable difference in the level of *teacher instruction*. The teacher in class A is continuously instructing, even if at a low level, whereas in class B there are longer periods of no instructions. Finally, the level of *individual* and *cooperative work* is significantly different for the two classes. While students in class B are continuously collaborating, for students of A there are a few moments of group within a day.

Figure 2.2: Context variables of class A



The context variables are normalized between 0 and 1.

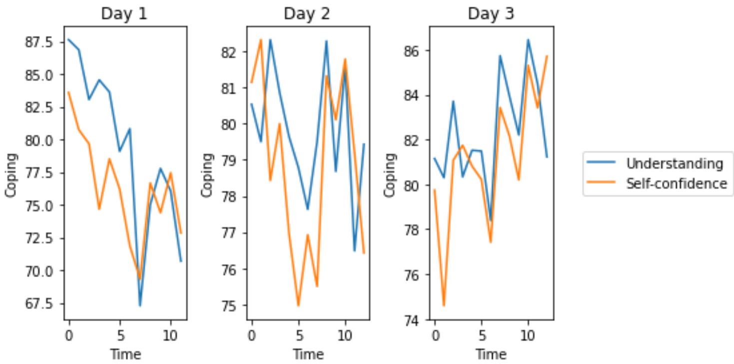
Figure 2.3: Context variables of class B



The context variables are normalized between 0 and 1.

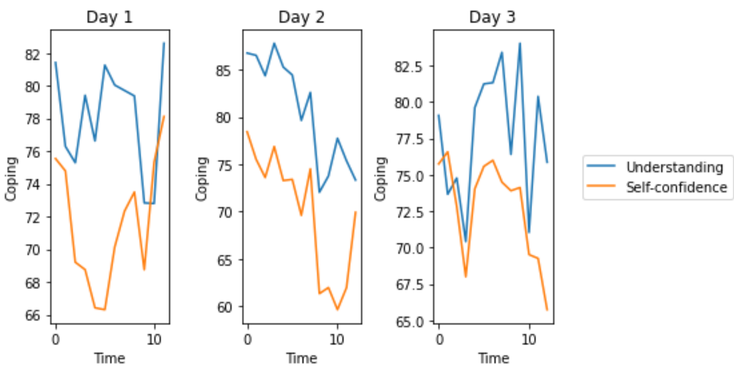
Figures 2.4 and 2.5 plot the coping variables averaged across individuals, while the corresponding plots for the stress variables can be found in Figure 2.6 and 2.7, respectively (see also the summary statistics in Table 2.7 of the Appendix). Comparing Figure 2.4 and 2.5, we clearly see the high correlation between *understanding* and *self-confidence*. Furthermore, students in class A are coping slightly better on average than the students in class B.

Figure 2.4: Personal coping variables in class A



The graphs contain the average coping variables across individuals.

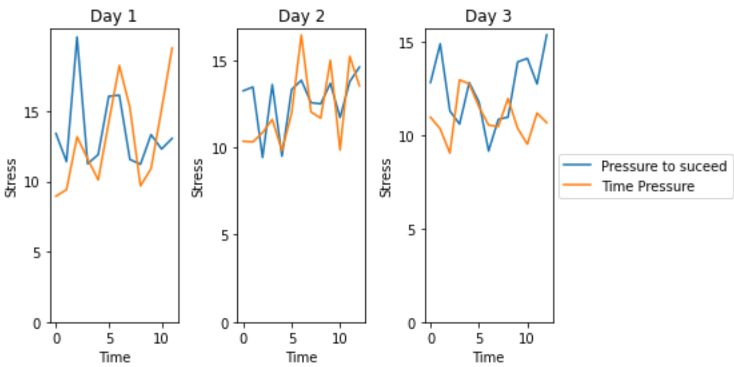
Figure 2.5: Personal coping variables in class B



The graphs contain the average coping variables across individuals.

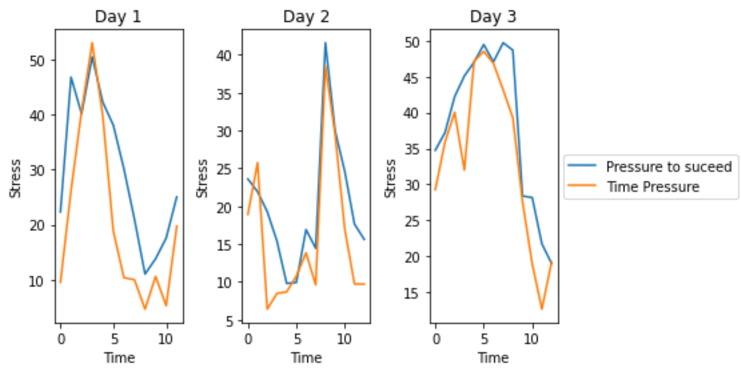
Figure 2.6 and 2.7 clearly reveal that the students in class B are on average more stressed. Furthermore, we find that *pressure to succeed* is generally higher than *time pressure*.

Figure 2.6: Stress variables in class A



The graphs contain the average stress variables across individuals.

Figure 2.7: Stress variables in class A



The graphs contain the average stress variables across individuals.

Our descriptive data analysis reveals that the two classes differ not only in their seating plans, but also regarding the context factors and thus the general teaching arrangements. Therefore from a pedagogical perspective the differences in the attributed roles of teachers and students and the associated different realizations of teaching approaches are most notable.

2.5. Empirical Findings

We estimated the spatial peer effects model by maximum likelihood for both class rooms with context specific contagion as given by equations (2.1) and (2.3).[1] As the literature shows, not only peer- and context-related factors affect stress contagion, but also person-related characteristics (e.g. empathy, gender Bakker et al., 2009)) generally can have an influence. The individual effect μ_i accounts for these individual factors. The panel structure of our data allows us to eliminate μ_i by within-transformation. We use as dependent variables *pressure to succeed* and *time pressure* as two alternative but related concepts of stress measurement. The different context factors *teacher instruction*, *complexity*, *teacher time pressure* and *individual work* and *cooperative work* are potentially driving the strength of the stress contagion. We present separate model estimates for each of the context factors.[2]

For all our estimated specifications we find clear evidence that stress contagion effects are present for both classrooms. With the exception of one case (dependent variable *time pressure* and context factor *cooperative work*, see Table 2.4) stress contagion is

[1]The Python code is available from the authors on request. Estimation details are given in Appendix 2.7.

[2]In a later version of the paper, we will report on estimates where we will use factors obtained principal components analysis (PCA) as context factors.

more pronounced for the learner centred classroom B compared to the teacher centred classroom A. This finding is congruent with the inherent didactic objective of a learner centred arrangement to promote interaction between students.

Considering the contagion parameter evaluated at the sample mean $\delta_t(\bar{Z})$ we find that the difference in stress contagion in classroom B compared to classroom A is stronger for the stress measurement *pressure to succeed* than for the measurement *time pressure*. Moreover, regardless of the specific context factor used, stress contagion in classroom B is significantly effected by the context factor, while it has no significant effect for the classroom A.

Table 2.1 contains the estimation results for the context factor *teacher instruction*. Considering stress measure *pressure to succeed*, we find that the direct spatial peer effect evaluated at the mean level of the context variable teacher instruction is 0.076 for classroom B compared to 0.043 for classroom A. Thus students in the learner centred classroom B are affected almost twice as much by the stress of their spatial peers. On the other hand, considering stress measure *time pressure* in more detail (columns 3 and 4 of Table 2.1), the difference in stress contagion for the two classrooms is only marginal.

Note, that for classroom B the role of the teacher is more decisive on how strongly stress spills over. The role of the teacher is crucial for mitigating stress spill-overs in classroom B, while the effect of *teacher instruction* on the endogenous peer effect parameter of classroom A turns out to be comparatively small and not significantly determined. The role of *teacher instruction* for the mitigation of stress contagion becomes most evident if one considers the extreme case of the lowest level of teacher instruction ($Z_t = 0$), i.e. $\delta_t = \delta_0$. For this case the difference between the two classrooms is most pronounced.

Table 2.1: MLE results. Context Variable: *Teacher Instruction*

	Dependent Variable:			
	Pressure to Succeed		*Time Pressure*	
	Class A	Class B	Class A	Class B
Endogenous peer effect				
Intercept	0.052***	0.126***	0.080***	0.151***
	(0.020)	(0.009)	(0.018)	(0.008)
Teacher Instruction	-0.017	-0.091***	-0.035*	-0.150***
	(0.021)	(0.015)	(0.021)	(0.018)
$\delta_t(\bar{Z})$	0.043***	0.076***	0.060***	0.067***
	(0.014)	(0.011)	(0.013)	(0.011)
Own characteristic				
Coping	-0.126***	-0.146***	-0.290***	-0.268***
	(0.026)	(0.050)	(0.027)	(0.051)
Exogenous peer effect				
Coping	0.211***	0.244***	0.347***	0.270***
	(0.029)	(0.061)	(0.030)	(0.062)
σ^2	77.957***	382.822***	79.191***	386.233***
	(3.387)	(17.792)	(3.445)	(17.963)
log-lik $\times 10^{-3}$	3.829	4.191	3.840	4.201

Class A denotes the teacher oriented classroom (n = 25). Class B denotes the students oriented classroom (n = 25), T = 38 for both class rooms. Standard errors in brackets. * denotes significance at 10%, ** at 5% and *** at 1 % significance level. Standard errors were computed by the inverse of the Hessian.

Continuing with the role of the teacher, we consider the time pressure that the teacher experiences as a determining factor of stress contagion among peers. Table 2.2 contains the estimation results for the context factor *teacher time pressure*. This context factor is only relevant in the student oriented classroom. The contagion parameter evaluated at the mean *teacher time pressure* is larger in class B. In the student oriented classrooms when the teacher is under stress, the endogenous peer effect is smaller. The teacher is under time pressure during instructions phase, meaning that during this time the students are not actively challenged, and therefore spreading less stress.

Table 2.2: MLE results. Context Variable: *Teacher Time Pressure (TP)*

	Dependent Variable:			
	Pressure to Succeed		*Time Pressure*	
	Class A	Class B	Class A	Class B
Endogenous peer effect				
Intercept	0.039***	0.143***	0.054***	0.174***
	(0.014)	(0.012)	(0.014)	(0.012)
Teacher TP	0.012	-0.073***	0.021	-0.100***
	(0.039)	(0.018)	(0.032)	(0.021)
$\delta_t(\bar{Z})$	0.044***	0.117***	0.061***	0.138***
	(0.017)	(0.009)	(0.014)	(0.008)
Own characteristic				
Coping	-0.126***	-0.153***	-0.291***	-0.270***
	(0.026)	(0.051)	(0.027)	(0.052)
Exogenous peer effect				
Coping	0.210***	0.229***	0.346***	0.256***
	(0.030)	(0.062)	(0.030)	(0.063)
σ^2	77.995***	390.162***	79.371***	401.161***
	(3.388)	(18.193)	(3.454)	(18.800)
log-lik $\times 10^{-3}$	3.830	4.203	3.841	4.225

Class A denotes the teacher oriented classroom ($n = 25$). Class B denotes the students oriented classroom ($n = 25$), T = 38 for both class rooms. Standard errors in brackets. * denotes significance at 10%, ** at 5% and *** at 1 % significance level. Standard errors were computed by the inverse of the Hessian.

Table 2.3 summarizes the estimation results for the context factor *complexity*, which appears to be strong driving force for stress contagion in class B, while it does not play a significant role in class A. Most interestingly, *complexity* is mitigating stress contagion in terms of *time pressure*, i.e. students are less affected by the time pressure of their peers, if the course material is complex. This stands in contrast to the effect of *complexity* in the transmission of the pressure to succeed. The opposite signs on *complexity* indicate, that the two endogenous stress variables capture different phenomena.

Table 2.3: MLE results. Context Variable: *Complexity*

	Dependent Variable:			
	Pressure to Succeed		*Time Pressure*	
	Class A	Class B	Class A	Class B
Endogenous peer effect				
Intercept	0.037*	-0.004	0.047**	0.593***
	(0.019)	(0.019)	(0.021)	(0.060)
Complexity	0.008	0.133***	0.020	-0.396***
	(0.027)	(0.019)	(0.030)	(0.043)
$\delta_t(\bar{Z})$	0.041***	0.073***	0.059***	0.363***
	(0.014)	(0.011)	(0.013)	(0.03)
Own characteristic				
Coping	-0.126***	-0.148***	-0.291***	-0.320***
	(0.026)	(0.050)	(0.027)	(0.060)
Exogenous peer effect				
Coping	0.210***	0.241***	0.347***	0.325***
	(0.029)	(0.061)	(0.030)	(0.072)
σ^2	77.994***	378.905***	79.346***	528.104***
	(3.388)	(17.580)	(3.453)	(34.361)
log-lik $\times 10^{-3}$	3.830	4.185	3.841	4.356

Class A denotes the teacher oriented classroom (n = 25). Class B denotes the students oriented classroom (n = 25), T = 38 for both class rooms. Standard errors in brackets. * denotes significance at 10%, ** at 5% and *** at 1 % significance level. Standard errors were computed by the inverse of the Hessian.

Finally, we inspect how stress contagion behaves during periods of *cooperative* or *individual work*. Phases of student group work (Table 2.4) in class B are associated with a significant increase in stress contagion between the peers sitting close to each other, as they work cooperatively and independently on complex tasks and content, and have to account for their work results to the teacher and their peers. As there is more communication and exchange, there is a more prominent channel to transmit stress.

Table 2.4: MLE results. Context Variable: *Cooperative Work*

	Dependent Variable:			
	Pressure to Succeed		*Time Pressure*	
	Class A	Class B	Class A	Class B
Endogenous peer effect				
Intercept	0.041***	0.047***	0.056***	0.024*
	(0.014)	(0.013)	(0.013)	(0.015)
Cooperative work	-0.001	0.085***	0.020	0.133***
	(0.027)	(0.013)	(0.025)	(0.016)
$\delta_t(\bar{Z})$	0.041***	0.069***	0.061***	0.059***
	(0.015)	(0.011)	(0.013)	(0.012)
Own characteristic				
Coping	-0.126***	-0.146***	-0.291***	-0.266***
	(0.026)	(0.050)	(0.027)	(0.051)
Exogenous peer effect				
Coping	0.210***	0.246***	0.348***	0.268***
	(0.029)	(0.061)	(0.030)	(0.062)
σ^2	78.005***	381.609***	79.340***	385.653***
	(3.389)	(17.724)	(3.452)	(17.925)
log-lik $\times 10^{-3}$	3.830	4.189	3.841	4.200

Class A denotes the teacher oriented classroom ($n = 25$). Class B denotes the students oriented classroom ($n = 25$), T = 38 for both class rooms. Standard errors in brackets. * denotes significance at 10%, ** at 5% and *** at 1 % significance level. Standard errors were computed by the inverse of the Hessian.

Table 2.5 summarizes the estimation results when the determinant of the peer effect parameter is *individual work*. Individual work has a significant positive effect only on the stress contagion parameter of class A (dependent variable *time pressure*). This is due to the fact that when the students become active out of their passivity and have to work on their own, this becomes noticeable in the stress contagion. However, the stress contagion parameter evaluated at the average context factor is highly significant and always larger in class B, implying that the spillovers are larger in the student oriented sitting arrangements.

Table 2.5: MLE results. Context Variable: *Individual Work*

	Dependent Variable:			
	Pressure to Succeed		*Time Pressure*	
	Class A	Class B	Class A	Class B
Endogenous peer effect				
Intercept	0.037***	0.108***	0.050***	0.126***
	(0.014)	(0.009)	(0.014)	(0.008)
Individual work	0.024	-0.000	0.042*	-0.024
	(0.026)	(0.033)	(0.025)	(0.036)
$\delta_t(\bar{Z})$	0.039***	0.108***	0.054***	0.124***
	(0.014)	(0.009)	(0.013)	(0.008)
Own characteristic				
Coping	-0.126***	-0.150***	-0.290***	-0.269***
	(0.026)	(0.051)	(0.027)	(0.052)
Exogenous peer effect				
Coping	0.211***	0.243***	0.346***	0.271***
	(0.029)	(0.062)	(0.030)	(0.064)
σ^2	77.943***	394.780***	79.193***	408.431***
	(3.386)	(18.454)	(3.446)	(19.199)
log-lik $\times 10^{-3}$	3.829	4.210	3.840	4.236

Class A denotes the teacher oriented classroom ($n = 25$). Class B denotes the students oriented classroom ($n = 25$), T = 38 for both class rooms. Standard errors in brackets. * denotes significance at 10%, ** at 5% and *** at 1 % significance level. Standard errors were computed by the inverse of the Hessian.

With regard to coping, it can be seen that on an individual level it contributes significantly to a reduction in the stress experience in all models, for both stress parameters considered and in both classes (Tables 2.1 to 2.5; coping as an own characteristic). The negative sign is in line with the intuition that a better *understanding* and higher *self-confidence* contribute to a less stressful reaction. Furthermore, it can be seen in all models that the coping skills of the peers contribute to a significant increase in the individual stress experience (Tables 2.1 to 2.5; coping as an exogenous peer effect). The positive exogenous peer effect seems counter-intuitive at first but can be plausibly explained by the assumption of social comparison processes (cf. Pekrun, 2006). Social comparison can lead to stress itself when a student, after comparing himself with his peers, gets the impression from verbal or behavioural expressions of peers that they understand the subject matter better or are better able to cope with class-related demands than oneself.

2.6. Conclusions

The aim of this study is to investigate the extent to which spatial peers and contextual factors contribute to stress contagion processes among students in the classroom. The data underlying our analyses included information on two school classes, whose seating plans we used to construct networks consisting of spatial peers. We processed self-report data from the students on their experience of stress and coping, self-report data from the teachers on their experience of time pressure, and data based on video observation to map the context factors characterizing the two learning arrangements.

At this point it is already obvious that the design of the study has both strengths and limitations. The naturalistic setting and the high frequency of data collection (every 10 minutes) offer a high degree of ecological validity, which is additionally ensured by the situational nature of the data collection. However, on the other hand, the naturalistic setting carries the risk of confounding variables that may bias the results. As described above, we can at least ensure that no person-related factors (e.g., empathy, gender) confound our results regarding stress contagion because we conducted within transformation. However, we cannot exclude the possibility that there were context-specific confounding variables that influenced the results. Therefore, a combination of naturalistic setting and experimental laboratory conditions seems to be useful for further research. For example, specific class-related stressors could be identified in naturalistic learning contexts in order to validate them in later laboratory experiments on stress contagion processes.

As the descriptive data show, the two classes investigated differ noticeably in their particular learning arrangements: the teacher in class A tends to follow a teacher-centered teaching style, which, compared to class B, is characterized by a proportionally higher share of *teacher instruction* and *individual work* as well as a low share of *cooperative work*. In class B, the situation is quite the opposite. Furthermore, the *complexity* of the subject matter is somewhat higher in class B compared to class A and the teacher in class B experiences much more time pressure compared to the teacher in class A. The learners in class B experience on average more stress in terms of *pressure to succeed* and *time pressure* and slightly lower coping experiences in terms of *understanding* of subject matter and *self-confidence* in dealing with classroom demands compared to the learners in class A.

Previous studies on stress contagion processes mainly used ordinary (multiple) regression analysis, structural equation modeling or multilevel modeling as statistical approaches. Only exceptions used network-based approaches and to our knowledge no previous study investigated stress contagion in students in real classroom settings using a network-based approach. Therefore, in our study we used a peer effects network model, which methodologically goes beyond previous approaches to the analysis of contagion processes.

In all models we found significant peer effects for both stress indicators, i.e., the higher the stress experience of the immediate peers in the classroom, the higher the individual stress experience of the students. In all specification we find that the better the own coping experience, the lower the individual stress experience. Our results suggest that the better the immediate peers in the classroom are coping, the higher (and not lower) the stress experience of the students. However, this initially counterintuitive effect can be explained by social comparison processes as described in section 2.5. From a pedagogical perspective, it seems relevant in this case to think about interventions that support peers to function as resources rather than stressors. This could be achieved, for example, by changing the accentuation of the reference norms, so that external evaluations (e.g. performance) refer less to the social reference norm and more to the individual or criterion-related reference norm.

With regard to the context factors under consideration, a heterogeneous picture emerges depending on the class. The assumption that more *cooperative work* leads to an increase in stress contagion is valid in class B, but not in class A, where the relevant effects are not significant. In class A, however, the effect of *individual work* on stress contagion with regard to *time pressure* is significantly positive, which is consistent with the expectations. The effect of *teacher instruction* is negative and for the most part significant in both classes for both stress indicators. However, it should be noted that the teacher's interventions lead to substantial effects in class B in particular. This can be explained by the fact that the learners in class B have to work much more independently and on their own responsibility on more complex tasks during the course of the study, and that the teacher's intervention contributes to the learners' reduction of stress. With regard to the *complexity* of the learning content, we find a significant positive effect on stress contagion only for class B for the variable *pressure to succeed*. For *time pressure*, there is a significant negative effect in class B, which can be explained by the fact that the learners in class B are given sufficient time to work independently on their tasks. What remains is the negative effect of *teacher time pressure*, only found in class B for the two stress variables and can be explained by the fact that in phases of teacher instruction it is the teacher who is under time pressure.

As our analysis show, the picture consolidates that in the study of contagion processes the consideration of context factors is indispensable for the understanding of corresponding processes and also when, from a pedagogical perspective, the question arises as to how lessons can be designed to be as low in stress as possible while at the same time offering a high learning potential.

2.7. Appendix

Let $\tilde{x}_t$ contain the exogenous individual characteristics x_t and the peers' characteristics Gx_t and $\beta = [\beta_1, \quad \beta_2]'$. The log-likelihood of the model reads

$$\ln L = -\frac{NT}{2} ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T} e_t{}'e_t + \sum_{t=1}^{T} ln|I_N - \delta_t G|, \tag{2.4}$$

with $e_t = y_t - \delta_t G y_t - \tilde{x}_t \beta$.
We treat μ as fixed effects, and estimate

$$\ln L = -\frac{NT}{2} ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T} e_t^{*\prime} e_t^* + \sum_{t=1}^{T} ln|I_N - \delta_t G|, \tag{2.5}$$

where $e_t^* = y_t^* - \delta_t G y_t^* - \tilde{x}_t^* \beta$. The $*$ denotes

$$y_t^* = y_t - \frac{1}{T}\sum_{t=1}^{T} y_t,$$

$$\delta_t G y_t^* = \delta_t G y_t - \frac{1}{T}\sum_{t=1}^{T} \delta_t G y_t$$

and

$$\tilde{x}_t^* = \tilde{x}_t - \frac{1}{T}\sum_{t=1}^{T} \tilde{x}_t.$$

Taking the derivative with respect to σ and replacing the estimator $\hat{\sigma}^2 = \frac{1}{NT}\sum_{t=1}^{T} e_t^*{\prime} e_t^*$, we get the concentrated log-likelihood

$$\ln L = C - \frac{-NT}{2} \ln\left(\sum_{t=1}^{T} e_t^{*\prime} e_t^*\right) + \sum_{t=1}^{T} ln|I_N - \delta_t G|. \tag{2.6}$$

The data was collected in 3 days, and as a result we compute the log-likelihood for each day. To estimate the standard errors, we estimate a numerical Hessian, and use the inverse of the negative Hessian as an estimator for the variance-covariance matrix of the ML estimator.

Table 2.6: Summary statistics

	Mean		Std.Dev	
	A	B	A	B
Complexity	0.97	2.09	0.24	0.84
Teacher time pressure	7.84	61.14	11.13	21.92
Teacher instruction	6.97	4.01	2.79	3.72
Individual work	1.29	0.21	2.23	0.65
Cooperative work	0.65	4.32	2.08	4.14

Summary statistics of the context factors

Table 2.7: Summary statistics

	Mean		Std.Dev	
	A	B	A	B
Pressure to succeed	12.86	29.42	2.06	13.05
Time pressure	12.03	23.59	2.50	14.37
Understanding	80.54	78.91	4.00	4.63
Self-confidence	78.86	71.45	3.67	4.80

Summary statistics of the stress and coping variables

References

Anselin, L. (2003). "Spatial Econometrics". In: *A Companion to Theoretical Econometrics*. Ed. by Baltagi, B. H. John Wiley & Sons, Ltd. Chap. 14, pp. 310–330.

Arvey, R. D. and K. R. Murphy (1998). "Performance evaluation in work settings". *Annual Review of Psychology* 49 (1), pp. 141–168.

Auerbach, R. P., P. Mortier, R. Bruffaerts, J. Alonso, C. Benjet, P. Cuijpers, K. Demyttenaere, D. D. Ebert, J. G. Green, P. Hasking, E. Murray, M. K. Nock, S. Pinder-Amaker, N. A. Sampson, D. J. Stein, G. Vilagut, A. M. Zaslavsky, and R. C. Kessler (2018). *WHO World Mental Health Surveys International College Student Project: Prevalence and distribution of mental disorders*. Press Release.

Bakker, A. B. (2005). "Flow among music teachers and their students: The crossover of peak experiences". *Journal of Vocational Behavior* 66 (1), pp. 26–44.

Bakker, A. B., M. Westman, and I. J. H. van Emmerik (2009). "Advancements in crossover theory". *Journal of Managerial Psychology* 24 (3), pp. 206–219.

Bakker, A. B. and W. B. Schaufeli (2000). "Burnout Contagion Processes Among Teachers1". *Journal of Applied Social Psychology* 30 (11), pp. 2289–2308.

Becker, E. S., T. Goetz, V. Morger, and J. Ranellucci (2014). "The importance of teachers' emotions and instructional behavior for their students' emotions - An experience sampling analysis". *Teaching and Teacher Education* 43, pp. 15–26.

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). "Identification of peer effects through social networks". *Journal of Econometrics* 150 (1), pp. 41–55.

– (2020). "Peer Effects in Networks: A Survey". *Annual Review of Economics* 12 (1), pp. 603–629.

Burgess, L. G., P. M. Riddell, A. Fancourt, and K. Murayama (2018). "The Influence of Social Contagion Within Education: A Motivational Perspective". *Mind, Brain, and Education* 12 (4), pp. 164–174.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009). "Peer effects and social networks in education". *The Review of Economic Studies* 76 (4), pp. 1239–1267.

Cohen, S. and T. A. Wills (1985). "Stress, Social Support, and the Buffering Hypothesis". *Psychological Bulletin* 98 (2), pp. 310–357.

Cook, W. L. and D. A. Kenny (2005). "The actor-partner interdependence model: A model of bidirectional effects in developmental studies". *International Journal of Behavioral Development* 29 (2), pp. 101–109.

Csikszentmihalyi, M. and R. Larson (1987). "Validity and Reliability of the Experience-Sampling Method". *Journal of Nervous and Mental Disease* 175 (9), pp. 526–536.

Dishion, T. and K. Dodge (July 2005). "Peer Contagion in Interventions for Children and Adolescents: Moving Towards an Understanding of the Ecology and Dynamics of Change". *Journal of abnormal child psychology* 33, pp. 395–400.

Donker, M. H., T. van Gog, and M. T. Mainhard (2018). "A Quantitative Exploration of Two Teachers with Contrasting Emotions: Intra-Individual Process Analyses of Physiology and Interpersonal Behavior". *Frontline Learning Research* 6 (3), pp. 162–185.

Druskat, V. U. and S. B. Wolff (1999). "Effects and timing of developmental peer appraisals in self-managing work groups". *Journal of Applied Psychology* 84 (1), pp. 58–74.

Elhorst, J. P. (2014). *Spatial econometrics : from cross-sectional data to spatial panels.* Springer briefs in regional science. Heidelberg [u.a.]: Springer.

Fortuin, J., M. van Geel, and P. Vedder (2016). "Peers and academic achievement: A longitudinal study on selection and socialization effects of in-class friends". *The Journal of Educational Research* 109 (1), pp. 1–6.

Frenzel, A. C., B. Becker-Kurz, R. Pekrun, T. Goetz, and O. Ludtke (2018). "Emotion Transmission in the Classroom Revisited: A Reciprocal Effects Model of Teacher and Student Enjoyment". *Journal of Educational Psychology* 110 (5), pp. 628–639.

Frenzel, A. C., T. Goetz, O. Ludtke, R. Pekrun, and R. E. Sutton (2009). "Emotional Transmission in the Classroom: Exploring the Relationship Between Teacher and Student Enjoyment". *Journal of Educational Psychology* 101 (3), pp. 705–716.

Gašević, D., A. Zouaq, and R. Janzen (2013). ""Choose Your Classmates, Your GPA Is at Stake!": The Association of Cross-Class Social Ties and Academic Performance". *American Behavioral Scientist* 57 (10), pp. 1460–1479.

Härtel, C. E. J. and K. M. Page (2009). "Discrete emotional crossover in the workplace: the role of affect intensity". *Journal of Managerial Psychology* 24 (3), pp. 237–253.

Hatfield, E., J. Cacioppo, and R. Rapson (1994). *Emotional Contagion.* New York, NY: Cambridge University Press.

Hatfield, E., L. Bensman, P. D. Thornton, and R. L. Rapson (2014). "New Perspectives on Emotional Contagion: A Review of Classic and Recent Research on Facial Mimicry and Contagion". *Interpersona: An International Journal on Personal Relationships* 8 (2), p. 21.

Hon, A. H. Y. and W. W. Chan (2013). "The Effects of Group Conflict and Work Stress on Employee Performance". *Cornell Hospitality Quarterly* 54 (2), pp. 174–184.

Kärner, T., D. Sembill, C. Aßmann, E. Friederichs, and C. H. Carstensen (2017). "Analysis of person-situation interactions in educational settings via cross-classified multilevel longitudinal modeling: illustrated with the example of students' stress experience". *Frontline Learning Research* 5 (1), pp. 16–42.

Kärner, T., J. K. Weiß, and K. Heinrichs (2021). "A social perspective on resilience: social support and dyadic coping in teacher training". *Empirical Research in Vocational Education and Training* 13 (24).

King, R. B. (2020). "Mindsets are contagious: The social contagion of implicit theories of intelligence among classmates". *British Journal of Educational Psychology* 90 (2), pp. 349–363.

King, R. B. and J. A. Datu (2017). "Happy classes make happy students: Classmates' well-being predicts individual student well-being". *Journal of school psychology* 65, 116—128.

King, R. B. and N. B. Mendoza (2020). "Achievement goal contagion: mastery and performance goals spread among classmates". *Social Psychology of Education* 23, pp. 795–814.

Kirschner, F., F. Paas, and P. A. Kirschner (2009). "A Cognitive Load Approach to Collaborative Learning: United Brains for Complex Tasks". *Educational Psychology Review* 21 (1), pp. 31–42.

Kouzma, N. M. and G. A. Kennedy (2004). "Self-reported sources of stress in senior high school students". *Psychological Reports* 94 (1), pp. 314–316.

Lazarus, R. S. and S. Folkman (1984). *Stress, Appraisal, and Coping.* New York: Springer.

Manski, C. F. (1993). “Identification of Endogenous Social Effects: The Reflection Problem”. *The Review of Economic Studies* 60 (3), pp. 531–542.

– (2000). “Economic Analysis of Social Interactions”. *Journal of Economic Perspectives* 14 (3), pp. 115–136.

Minkley, N., T. Ringeisen, L. B. Josek, and T. Kärner (2017). “Stress and emotions during experiments in biology classes: Does the work setting matter?” *Contemporary Educational Psychology* 49, pp. 238–249.

Oberle, E. and K. A. Schonert-Reichl (2016). “Stress contagion in the classroom? The link between classroom teacher burnout and morning cortisol in elementary school students”. *Social Science & Medicine* 159, pp. 30–37.

Pekrun, R. (Dec. 2006). “The Control-Value Theory of Achievement Emotions: Assumptions, Corollaries, and Implications for Educational Research and Practice”. *Educational Psychology Review* 18, pp. 315–341.

Pitt, A., F. Oprescu, G. Tapia, and M. Gray (2018). “An exploratory study of students’ weekly stress levels and sources of stress during the semester”. *Active Learning in Higher Education* 19 (1), pp. 61–75.

Seidel, T., I. M. Dalehefte, and L. Meyer (2001). “Videoanalysen – Beobachtungsschemata zur Erfassung von „Sicht-Strukturen“ im Physikunterricht”. In: *Erhebungs- und Auswertungsverfahren des DFG-Projekts „Lehr-Lern-Prozesse im Physikunterricht – eine Videostudie*. Ed. by Prenzel, M., Duit, R., Euler, M., Lehrke, M., and Seidel, T. Kiel: IPN-Materialien, 41–58.

Shen, B., N. McCaughtry, J. Martin, A. Garn, N. Kulik, and M. Fahlman (2015). “The relationship between teacher burnout and student motivation”. *British Journal of Educational Psychology* 85 (4), pp. 519–532.

Song, Z. L., M. D. Foo, M. A. Uy, and S. H. Sun (2011). “Unraveling the Daily Stress Crossover Between Unemployed Individuals and Their Employed Spouses”. *Journal of Applied Psychology* 96 (1), pp. 151–168.

Sweller, J., P. L. Ayres, and S. Kalyuga (2011). *Cognitive load theory. Explorations in the learning sciences, instructional systems and performance technologies.* New York: Springer.

Vogel, S. and L. Schwabe (2016). “Learning and memory under stress: implications for the classroom”. *Npj Science Of Learning* 1, p. 16011.

Warwas, J. and C. Helm (2017). “Enjoying working and learning in vocational education: a multilevel investigation of emotional crossover and contextual moderators”. *Empirical Research in Vocational Education and Training* 9 (1), p. 11.

Waters, S. F., T. V. West, and W. B. Mendes (2014). “Stress Contagion Physiological Covariation Between Mothers and Infants”. *Psychological Science* 25 (4), pp. 934–942.

Westman, M. and D. Etzion (1999). “The crossover of strain from school principals to teachers and vice versa”. *Journal of Occupational Health Psychology* 4 (3), pp. 269–278.

Westman, M. and A. D. Vinokur (1998). “Unraveling the relationship of distress levels within couples: Common stressors, empathic reactions, or crossover via social interaction?” *Human Relations* 51 (2), pp. 137–156.

Wethington, E. (July 2000). “Contagion of stress”. *Advances in Group Processes* 17, pp. 229–253.

Chapter 3

Isolated individuals in social networks. How does their treatment impact the peer effects estimates?

Abstract

The focus of this study is the treatment of isolated individuals in social networks when estimating peer effects. It is common practice to remove the isolated individuals and so far, there has not been a proper investigation on the consequences of their removal. We formally distinguish between two definitions of isolated nodes and systematically analyse the effect of their removal on the IV estimates. This study disentangles the effects of the two dimensions of the removal: the loss in observations and the change in regressors. In a linear model of peer effects we show that the IV estimator is inefficient but consistent when removing completely isolated individuals, and inconsistent when removing partially isolated individuals. Our Monte Carlo study results provide numerical insights into the effects of the different modelling strategies on the parameter estimates in the local-aggregate and local-average linear models, and suggest that removing partially isolated individuals is the most harmful strategy.

3.1. Introduction

Being social creatures, interactions and relationships hold a special place in our lives. The people we are related to, whether family members, school classmates, work colleagues, friends, acquaintances, or teachers, shape our lives by influencing our behavior and decisions. In the recent years the interest in modelling and estimating the effects of social interactions using the social network approach has increased. At the theoretical level the main concern is the impact that social networks have on outcomes and how the outcomes in return affect network formation (see Jackson, 2011). At the empirical level the main focus is the impact of social interactions on outcomes, with applications on welfare participation (Bertrand et al., 2000), coordination and cooperation (Cassar, 2007), criminal activity (Patacchini and Zenou, 2008), obesity (Trogdon et al., 2008), and education (Calvó-Armengol et al., 2009). At the econometric level the focus is the identification of peer effects in models of social networks. One strand of literature is inspired by the spatial literature (Lee et al., 2010), and linear models of peer effects are estimated using non-experimental data.

In the econometric literature one of the standard specifications is the local-average model, which describes the dependence of individual behavior on the average behavior of the peer group. The local-aggregate specification models the behavior of the individual as a function of the aggregate behavior of the peers. It is a standard assumption that the network does not contain isolated individuals. It is believed that removing these individuals from the data does not have any consequences, because, by construction, they do not affect the structure of the network. However, Boucher and Fortin (2015) show that the presence of isolated individuals provides identification conditions for the social multiplier and the source of social interactions. A social multiplier arises when a common shock has a direct effect on the impacted individuals and an indirect effect through social effects. The logic is that isolated and non-isolated individuals have different best-response functions, providing a set of two equations. With these two equations it is possible to identify all parameters, which in turn is sufficient to test for the presence of a social multiplier. Based on this result, removing these observations is problematic.

The information on the social interactions is recorded in the adjacency matrix. The adjacency matrix of an undirected network is symmetric, as the links are reciprocal. If the links are not reciprocal, then the network is directed, and the adjacency matrix is not symmetric. The indegree denotes the number of links an individual receives and the outdegree the number of links a person sends. In an undirected network the indegree is equal to the outdegree, and an isolated person is a complete outlier, he does neither receive nor send any links. In this case, removing this person only implies the loss in observations. However, in a directed network a person who sends no links might receive links. Therefore, there are two possible definitions of an isolated person: outdegree zero (no sent links) or outdegree and indegree 0 (also no received links). The answer to this question determines the quality of the peer effects estimates. We will show in this paper that the effect of a removal differs for the two definitions.

Bramoullé et al. (2009) assume that the network contains no isolated individuals. We assume they refer to individuals with outdegree equal to 0. Liu et al. (2014) report that they remove isolated individuals from the estimation sample, without stating their definition of an isolated individual. Cohen-Cole et al. (2017) remove the isolated individuals, considering an individual isolated if he neither sends nor receives nominations. Liu et al. (2016) also consider undirected networks where no agent is isolated for ease of presentation. Most of these studies rely on the Add Health dataset. Dieye and Fortin (2017) also rely on this dataset to estimate peer effects in obesity. They report that isolated individuals make up 23% of their sample. They consider an individual to be isolated if they claim to have friends. So, their definition is based on the sent nominations.

The purpose of this study is to analyse the effect of removing isolated individuals and to show, whether it affects the properties of the peer effects estimates in a linear model. We investigate this question by inspecting the two definitions of isolated individuals and different network structures. We define a *completely isolated* individual as someone with outdegree and indegree equal to zero, and a *partially isolated* one as someone with outdegree zero. A *completely isolated* person does not impact the structure of the network, but his equation serves as an identifying restriction, which can be used to build a moment function. Removing these observations results in fewer observations and fewer moment functions, which in the GMM framework translates into an asymptotically less efficient IV estimator of the peer effects parameters. This holds if the parameters of the isolated and connected individuals are the same. Additionally, because an isolated individual does not have peers, his equation does not suffer from endogeneity and no instruments are required. A *partially isolated* individual might receive links. Consider a network of three individuals. Individual i sends no links, but individual j sends a link to i and k. When individual i is completely removed from the network, the peer variables of j, which were initially $f(y_i, y_k)$ and $f(x_i, x_k)$, now become $f(y_k)$ and $f(x_k)$. In this case, not only is the observation of i lost, but also the regressors of j change. As a consequence, the IV estimator is biased. We show analytically how the IV estimator of the local-aggregate model is inefficient and inconsistent, when *completely isolated* and *partially isolated* individuals are removed, respectively. The results can be easily extended to the local-average specification.

In our Monte Carlo simulations we inspect the behavior of the estimates for two different networks. In graph theory random networks are extensively used to study how the assumptions on link formation impact the structure of the network. Erdös Rényi graphs (Erdös and Rényi, 1959) are commonly used, and their primary feature is that the probability of link formation is constant and independent. As a result, the degree distribution follows the Poisson distribution. Because these graphs might be too simplistic, we also consider Small World networks (Watts and Strogatz, 1998). They have attributes more similar to observed networks, e.g. the small diameter (the shortest path between the two most distant nodes). The numerical insights of our simulations suggest that identifying those with outdegree zero as isolated individuals, and removing them, is the most harmful strategy. The estimates are biased in all scenarios, and their variances are larger. Removing the *completely isolated* generally has negligible negative consequences.

This paper is organized as follows. Section 3.2 introduces the linear model and derives the properties of the IV estimator when isolated individuals are removed. Section 3.3 briefly introduces the Erdös Rényi and Small World graphs. In Section 3.4 we describe the Monte Carlo simulation and summarize the findings. Section 3.5 concludes and gives an outlook on future research questions related to isolated individuals.

3.2. The Framework

Consider a network of n individuals, where each individual i has the peer group P_i of size n_i. The network information is saved in the adjacency matrix $A \in \mathbb{R}^{n \times n}$, with elements $A_{ij} = 1$ if i claims to be connected to j. The diagonal elements A_{ii} are zero, as it is assumed that no individual is a peer to himself. Recall that in case of an undirected network A is symmetric. Let us define two cases of isolated individuals: *completely isolated* (CI) and *partially isolated* (PI). The *completely isolated* individual is the individual that is completely detached from the network, he neither receives nor sends any links. The *partially isolated* individual sends no links but might receive some. The applicability of these definitions depends on the type of network. Therefore, in an undirected network an isolated individual is *completely isolated*. A directed network on the other hand can have *completely isolated* and *partially isolated* individuals. When a *completely isolated* individual is removed, denote this case by *CI-CR*, then we deal with only a loss in observations. We refer by *PI-CR* to the case when the *partially isolated* individual is completely removed.

3.2.1. Completely isolated individuals

Let us consider the local-aggregate peer effects model without exogenous peer effects for the sake of simplicity. The results can be easily extended to include exogenous peer effects and to the local-average model. We first consider the network with *completely isolated* individuals. Let the dummy variable C_i take the values

$$C_i = \mathbb{1}\left[\sum_{j=1}^{n} A_{ij} \neq 0\right] \mathbb{1}\left[\sum_{i=1}^{n} A_{ji} \neq 0\right]. \tag{3.1}$$

So, $C_i = 1$ if i is connected. The general model is

$$Y_i = \theta_1 C_i \sum_{j=1}^{n} A_{ij} Y_j + X_i' \theta_2 + \varepsilon_i, \tag{3.2}$$

where $\mathrm{E}[\varepsilon | X, A] = 0$. We assume that the network is stochastic and exogenous conditional on X. The endogenous peer effects are captured by θ_1. The endogeneity of $\sum_{j=1}^{n} A_{ij} Y_j$ is generally addressed by instrumental variables. The parameter vector of interest is $\theta = (\theta_1, \theta_2')'$. The model for connected individuals is defined

$$Y_i = \theta_1 \sum_{j=1}^{n} A_{ij} Y_j + X_i' \theta_2 + \varepsilon_i, \quad \text{for } i \text{ with } C_i = 1. \tag{3.3}$$

Note that for individuals with no peers (3.2) takes the form

$$Y_i = X_i' \theta_2 + \varepsilon_i, \quad \text{for } i \text{ with } C_i = 0. \tag{3.4}$$

Equation (3.4) serves to build an additional moment restriction to identify θ_2. Therefore, removing this observation also translates into less moment functions, which results in an inefficient estimator. Additionally, equation (3.4) does not suffer from endogeneity.

Denote the number of connected individuals by $n_1 = \sum_{i=1}^{n} C_i$ and the number of isolated individuals by $n_0 = \sum_{i=1}^{n} (1 - C_i)$, such that $n = n_1 + n_0$. The $k \times 1$ vector of individual characteristics X_i is observed for all the individuals in the sample. Denote by Z_{0i} the $q_0 \times 1$ vector of excluded instruments, with $q_0 \geq k$.

The $(2k + q_0) \times 1$ vector of moment functions for model (3.2) is given by:

$$\psi(Y_i, X_i, Z_{0i}, \theta) = \begin{pmatrix} \psi_1(Y_i, X_i, Z_{0i}, \theta) \\ \psi_0(Y_i, X_i, \theta_2) \end{pmatrix} \in \mathbb{R}^{(2k+q_0)\times 1},$$

where ψ_1 is the $q_1 \times 1$ moment function for the connected individuals

$$\psi_1(Y_i, X_i, Z_{0i}, \theta) = C_i Z_i \varepsilon_i \in \mathbb{R}^{q_1 \times 1} \quad i = 1, .., n_1,$$

with $Z_i = [X_i', Z_{0i}']'$ and $q_1 = k + q_0$. ψ_0 is the $k \times 1$ vector of moment functions for isolated individuals

$$\psi_0(Y_i, X_i, \theta_2) = (1 - C_i) X_i \varepsilon_i \in \mathbb{R}^{k \times 1} \quad i = 1, ..., n_0.$$

Note that

$$\mathrm{E}[\psi_1(Y_i, X_i, Z_{0i}, \theta)] = P[C_i = 1] \ \mathrm{E}[Z_i \varepsilon_i] = 0,$$

and

$$\mathrm{E}[\psi_0(Y_i, X_i, \theta_2)] = P[C_i = 0] \ \mathrm{E}[X_i \varepsilon_i] = 0.$$

Now define the estimator $\hat{\theta}_{All}$ of θ that is based on $\psi = (\psi_1', \psi_0')'$

$$\hat{\theta}_{All} = \arg\min_{\theta} \left[\frac{1}{n} \sum_{i=1}^{n} \psi(Y_i, X_i, Z_{0i}, \theta) \right]' W_n^1 \left[\frac{1}{n} \sum_{i=1}^{n} \psi(Y_i, X_i, Z_{0i}, \theta) \right], \tag{3.5}$$

with limiting distribution

$$\sqrt{n} \left(\hat{\theta}_{All} - \theta \right) \xrightarrow{d} N(0, (\underbrace{A_0' V_0^{-1} A_0}_{\Sigma_1})^{-1}), \tag{3.6}$$

where $A_0 = E[\frac{\partial \psi(Y_i, X_i, Z_{0i}, \theta)}{\partial \theta'}]$, $V_0 = V[\psi(Y_i, X_i, Z_{0i}, \theta)]$ and W_n^1 is the optimal weighting matrix.

Similarly, the estimator $\hat{\theta}_{CI-CR}$ that relies only on connected individuals and is based on the moment function ψ_1 is given by

$$\hat{\theta}_{CI-CR} = \arg\min_{\theta} \left[\frac{1}{n} \sum_{i=1}^{n} \psi_1(Y_i, X_i, Z_{0i}, \theta) \right]' W_n^2 \left[\frac{1}{n} \sum_{i=1}^{n} \psi_1(Y_i, X_i, Z_{0i}, \theta) \right].$$

Denote the selection matrix $S = [I_{q_1 \times q_1}, 0_{q_0 \times q_1}]$. The asymptotic distribution of $\hat{\theta}_{CI-CR}$ is given by

$$\sqrt{n} \left(\hat{\theta}_{CI-CR} - \theta \right) \xrightarrow{d} N(0, (\underbrace{A_0' S' (S V_0 S')^{-1} S A_0}_{\Sigma_2})^{-1}),$$

where $\psi_1(Y_i, X_i, Z_{0i}, \theta) = S\psi(Y_i, X_i, Z_{0i}, \theta)$. To compare the estimators we follow Pohlmeier (2022). In order to show that $\Sigma_2 - \Sigma_1 \geq 0$, we can equivalently show that $\Sigma_1^{-1} - \Sigma_2^{-1} \geq 0$, i.e.,

$$A_0' V_0^{-1} A_0 - A_0' S'(SV_0S')^{-1} SA_0 \geq 0. \tag{3.7}$$

Pohlmeier (2022) shows that the difference between these matrices is an idempotent quadratic form, which is by construction positive semidefinite. This shows that the asymptotic variance-covariance matrix of the GMM estimator based on q moment functions is never larger than the estimator based on $q_1 < q$ moment functions. To summarize, removing the *completely isolated* nodes from the sample results in an asymptotically inefficient estimator.

3.2.2. Partially isolated individuals

When the *partially isolated* individuals are completely removed from the network the regressors and the instruments change as well. First, let us define for a directed network a dummy variable that indicates whether someone is in the network

$$D_i = \mathbb{1}\left[\sum_{j=1}^{n} A_{ij} \neq 0\right]. \tag{3.8}$$

The number of connected persons is $n_1 = \sum_{i=1}^{n} D_i$ and the number of isolated individuals is $n_0 = \sum_{i=1}^{n}(1 - D_i)$. Consider the local-aggregate model with one regressor:

$$Y_i = \theta_1 D_i \sum_{j=1}^{n} A_{ij} Y_j + X_i\theta_2 + \varepsilon_i. \tag{3.9}$$

The instrument is $Z_{0i} = D_i \sum_{j=1}^{n} A_{ij} X_j$ (Liu et al., 2014). After removing individuals with outdegree zero ($D_i = 0$), the model can be written as

$$Y_i = \theta_1 \sum_{j=1}^{n} D_j A_{ij} Y_j + X_i\theta_2 + \varepsilon_i, \quad \text{for } i \text{ with } D_i = 1, \tag{3.10}$$

where $D_j = 1$ denotes if the peer j is part of the network. Consequently, if we remove the *partially isolated* individuals completely, the instruments will become

$$\tilde{Z}_{0i} = D_i \sum_{j=1}^{n} A_{ij} D_j X_j, \tag{3.11}$$

and the moment function

$$\tilde{\psi}_1(Y_i, X_i, \tilde{Z}_{0i}, \theta) = D_i \tilde{Z}_{0i} \varepsilon_i. \tag{3.12}$$

The estimator $\hat{\theta}_{PI-CR}$ of θ in this case can be defined as

$$\hat{\theta}_{PI-CR} = \arg\min_{\theta} \left[\frac{1}{n} \sum_{i=1}^{n} \tilde{\psi}_1(Y_i, X_i, \tilde{Z}_{0i}, \theta) \right]' W_n^3 \left[\frac{1}{n} \sum_{i=1}^{n} \tilde{\psi}_1(Y_i, X_i, \tilde{Z}_{0i}, \theta) \right],$$

where W_n^3 is the optimal weighting matrix. In the following we show that $\hat{\theta}_{PI-CR}$ is inconsistent.

Let D be an $n_1 \times n$ matrix, which is obtained from an identity matrix by removing n_0 rows corresponding to isolated individuals. The $n_1 \times n_1$ transformed adjacency matrix that only includes connected individuals is obtained as DAD'. The model in matrix notation is given by

$$\begin{aligned} DY &= \theta_1 DAD'DY + DX\theta_2 + D\varepsilon \qquad (3.13) \\ &= \theta_1 \tilde{Y}^p + \tilde{X}\theta_2 + \tilde{\varepsilon}, \end{aligned}$$

where $\tilde{Y}^p$ is the $n_1 \times 1$ vector of peers' outcomes with elements $D_i \sum_{j=1}^{n} D_j A_{ij} Y_j$. The excluded instruments matrix $\tilde{Z}_0$ with elements $D_i \sum_{j=1}^{n} A_{ij} D_j X_j$ is

$$\begin{aligned} \tilde{Z}_0 &= DAD'DX \qquad (3.14) \\ &= \tilde{X}^p, \end{aligned}$$

Define $\tilde{W}$ and $\tilde{Z}$ the $n_1 \times 2$ regressor and instrument matrices[1], respectively

$$\tilde{W} = \begin{bmatrix} \tilde{Y}^p & \tilde{X} \end{bmatrix}, \quad \tilde{Z} = \begin{bmatrix} \tilde{X} & \tilde{X}^p \end{bmatrix}.$$

Recall that, under homoskedasticity, the GMM estimator is identical to the IV estimator. Therefore, under homoskedasticity and since the model is exactly identified, the estimator $\hat{\theta}_{PI-CR}$ can be written as

$$\begin{aligned} \hat{\theta}_{PI-CR} &= (\tilde{Z}'\tilde{W})^{-1}\tilde{Z}'\tilde{Y} \qquad (3.15) \\ &= (\tilde{Z}'\tilde{W})^{-1}\tilde{Z}'D(W\theta + \varepsilon) \qquad (3.16) \\ &= (\tilde{Z}'\tilde{W})^{-1}\tilde{Z}'DW\theta + (\tilde{Z}'\tilde{W})^{-1}\tilde{Z}'D\varepsilon, \end{aligned}$$

[1]We assume the intercept has been partialled out.

where $W = [Y^p \quad X]$ and Y^p is the original $n \times 1$ vector of peers' outcomes with elements $\sum_{j=1}^n A_{ij} Y_j$. Let us consider the probability limit of this estimator.

$$\begin{aligned}
\operatorname*{plim}_{n\to\infty} \hat{\theta}_{PI-CR} &= \operatorname*{plim}_{n\to\infty}(\frac{1}{n}\tilde{Z}'\tilde{W})^{-1} \operatorname*{plim}_{n_1\to\infty}(\frac{1}{n}\tilde{Z}'DW)\theta + \operatorname*{plim}_{n\to\infty}(\frac{1}{n}\tilde{Z}'\tilde{W})^{-1} \operatorname*{plim}_{n\to\infty}(\frac{1}{n}\tilde{Z}'D\varepsilon) \\
&= \operatorname*{plim}_{n\to\infty}(\frac{1}{n}\tilde{Z}'\tilde{W})^{-1} \operatorname*{plim}_{n\to\infty}\frac{1}{n}\tilde{Z}'DW\theta,
\end{aligned}$$

where $\operatorname{plim}_{n\to\infty}(\frac{1}{n}\tilde{Z}'D\varepsilon) = \mathrm{E}[D_i \tilde{Z}_i \varepsilon_i] = 0$.

Let us take a look at the two elements: $\tilde{Z}'\tilde{W}$ and $\tilde{Z}'DW$.

$$\tilde{Z}'\tilde{W} = \begin{bmatrix} \sum_{i=1}^n D_i X_i \tilde{Y}_i^p & \sum_{i=1}^n D_i X_i^2 \\ \sum_{i=1}^n D_i \tilde{X}_i^p \tilde{Y}_i^p & \sum_{i=1}^n D_i X_i \tilde{X}_i^p \end{bmatrix}$$

and

$$\tilde{Z}'DW = \begin{bmatrix} \sum_{i=1}^n D_i X_i Y_i^p & \sum_{i=1}^n D_i X_i^2 \\ \sum_{i=1}^n D_i \tilde{X}_i^p Y_i^p & \sum_{i=1}^n D_i X_i \tilde{X}_i^p \end{bmatrix},$$

with probability limits

$$\operatorname*{plim}_{n\to\infty} \frac{1}{n}(\tilde{Z}'\tilde{W}) = \begin{bmatrix} \tilde{M}_1 & \tilde{M}_2 \\ \tilde{M}_3 & \tilde{M}_4 \end{bmatrix} \tag{3.17}$$

and

$$\operatorname*{plim}_{n\to\infty} \frac{1}{n}(\tilde{Z}D'W) = \begin{bmatrix} M_1 & \tilde{M}_2 \\ M_3 & \tilde{M}_4 \end{bmatrix}. \tag{3.18}$$

The derivation of $\tilde{M}_1$, $\tilde{M}_2$ $\tilde{M}_3$, $\tilde{M}_4$, M_1 and M_3 can be found in the Appendix. Inserting equations (3.17) and (3.18) to the probability limit of $\hat{\theta}_{PI-CR}$ we get

$$\begin{aligned}
\operatorname{plim} \hat{\theta}_{PI-CR} &= \begin{bmatrix} \tilde{M}_1 & \tilde{M}_2 \\ \tilde{M}_3 & \tilde{M}_4 \end{bmatrix}^{-1} \begin{bmatrix} M_1 & \tilde{M}_2 \\ M_3 & \tilde{M}_4 \end{bmatrix} \theta \qquad (3.19) \\
&= \frac{1}{\tilde{M}_1\tilde{M}_4 - \tilde{M}_2\tilde{M}_3} \begin{bmatrix} \tilde{M}_4 & -\tilde{M}_2 \\ -\tilde{M}_3 & \tilde{M}_1 \end{bmatrix} \begin{bmatrix} M_1 & \tilde{M}_2 \\ M_3 & \tilde{M}_4 \end{bmatrix} \theta. \qquad (3.20) \\
&= \begin{bmatrix} \frac{M_1\tilde{M}_4 - \tilde{M}_2 M_3}{\tilde{M}_1\tilde{M}_4 - \tilde{M}_2\tilde{M}_3}\theta_1 \\ \frac{\tilde{M}_1 M_3 - M_1\tilde{M}_3}{\tilde{M}_1\tilde{M}_4 - \tilde{M}_2\tilde{M}_3}\theta_1 + \theta_2 \end{bmatrix}.
\end{aligned}$$

For $\hat{\theta}_{PI-CR}$ to be consistent, it has to hold that

$$M_1 = \tilde{M}_1,$$

and

$$M_3 = \tilde{M}_3.$$

The asymptotic bias depends on the differences between M_1 and $\tilde{M}_1$, and M_3 and $\tilde{M}_3$, which are driven by the difference between $\mathrm{E}[Y_i^p|X]$ and $\mathrm{E}[\tilde{Y}_i^p|X]$. Recall that Y^p is the original vector of peers' outcomes, and $\tilde{Y}^p$ is the transformed vector of peers' outcomes, which is affected by the removal of the *partially isolated* individuals. Consequently, the asymptotic bias is determined by the change in the regressors that is caused by the removal of *partially isolated* individuals.

3.3. Random Networks

Random network models are popular in graph theory to investigate how different assumptions on link formation lead to different network structure properties. In this section we give a brief introduction to two types of random graphs: Erdös Rényi and Small World graphs. We rely on these types of graphs in the simulation study to generate the networks.

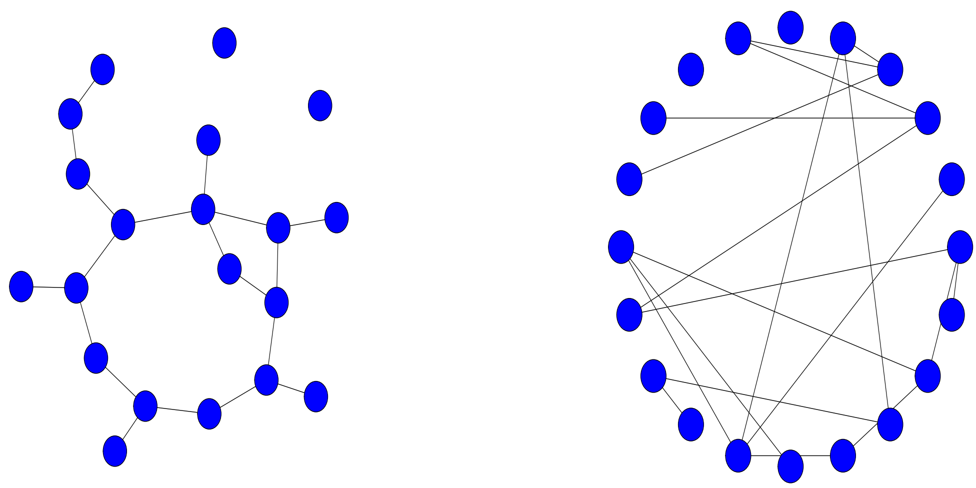
The Poisson random graph model is the most basic one. It was independently proposed by Solomonoff and Rapoport (1951), Gilbert (1959) and Erdös and Rényi (1959, 1960, 1961). Consider a set of nodes $N = \{1, \ldots, n\}$, where the link between any 2 nodes i and j is formed with probability p. Furthermore, the formation of links is independent. The probability of observing a network with m links is

$$p^m(1-p)^{\frac{n(n-1)}{2}-m}.$$

The degree of each node is approximated by a binomial distribution, and for large n and small p this binomial expression is approximated by a Poisson distribution. Because the degree distribution is approximated by a Poisson distribution, the class of random graphs for which each link is formed independently with equal probability is referred to as the class of Poisson random networks. When p is very small, below $\frac{1}{n}$, most nodes are isolated. When p grows beyond $\log(n)$, the isolated nodes disappear (Jackson, 2011).

One important feature of observed networks is the combination of relatively small diameters and high levels of clustering. The diameter of a network is the shortest distance between the two most distant nodes in the network. The most distant nodes are those with the longest shortest path, where the shortest path between two nodes denotes the minimal number of steps from one to the other. Clustering refers to the tendency of nodes to link based on similar characteristics and therefore forming subgroups within the network. Because basic random networks do not capture these characteristics, Watts and Strogatz (1998) constructed the Small World network model, which incorporates the small diameter. In this model, the nodes are initially linked according to a highly clustered lattice, which does not have a small diameter, i.e., a circle of nodes, each connected to neighbors that are k or less steps further. In the next step, some nodes are disconnected and reconnected uniformly at random. With probability π, a link is removed and replaced between two nodes that were not connected. For $\pi = 0$ the network is a lattice, and for $\pi = 1$ the network is a Poisson random network. Even the smallest π ensures a smaller distance between any two nodes compared to the original lattice (Jackson, 2011).

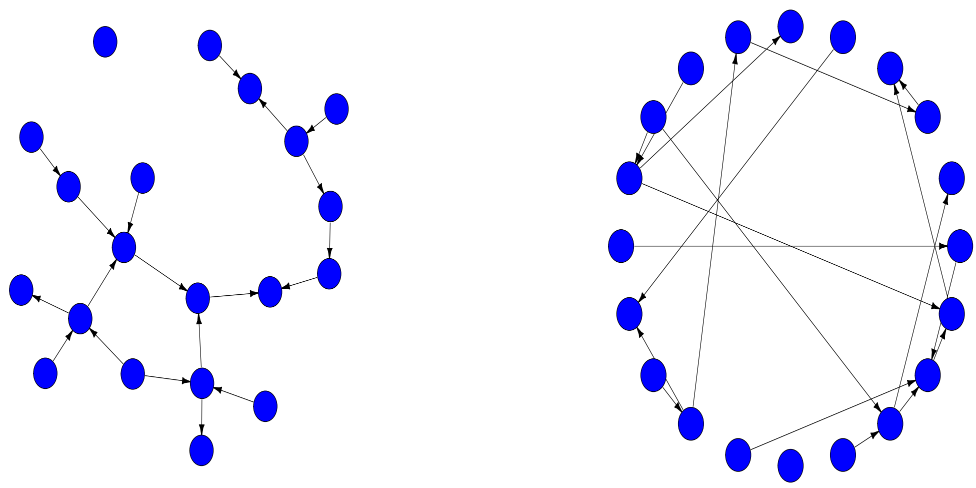
Figure 3.1: Erdös Rényi and Small World Graphs



On the left side there is a Erdös Rényi network, and on the right side a Small World network. The networks have 20 nodes and density 0.05.

Figure 3.1 depicts two simulated networks, one relying on the Erdös Rényi, the other on the Small World process. There are two completely isolated nodes in both networks. While the number of isolated nodes is the same, we find significant differences in the degree distributions. In the Erdös Rényi there are 12 nodes that are connected to 2 other nodes, and the rest is only connected to one other node. In the Small World graph we find that at least 4 nodes are connected to 3 other nodes, and one node is connected to four others.

Figure 3.2: Erdös Rényi and Small World Graphs



On the left side there is a Erdös Rényi network, and on the right side a Small World network. The networks have 20 nodes and density 0.05.

In Figure 3.2 we depict two directed networks. Both networks contain only one completely isolated node. The maximal number of received links in the Erdös Rényi graph is 3, and the nodes send at most 2 links. Most of the nodes do receive at least one link, while in the Small World graph it is more common for nodes to receive no links. The nodes send and receive at most 2 links. This small illustration alone reveals how two networks with the same number of nodes and of links, and therefore same density, can reveal different social structures. In general, the type of network formation process has a significant impact on the expected number of isolated individuals.

3.4. Monte Carlo Experiments

To examine the properties of the estimators in finite samples we run Monte Carlo simulations. We generate data based on the local-aggregate (abbreviated by AGG) model as in (3.21) and the local-average (abbreviated by AVG) model as in (3.22)

$$Y = (I - \beta_{agg} A)^{-1}(\alpha\iota + X\gamma_1 + GX\gamma_2 + \varepsilon), \tag{3.21}$$

and

$$Y = (I - \beta_{avg} G)^{-1}(\alpha\iota + X\gamma_1 + GX\gamma_2 + \varepsilon), \tag{3.22}$$

where G is the row-normalized adjacency matrix. We follow Liu et al. (2014) and use the aggregate peers' characteristics and the average second order peers' characteristics as instruments for the local-aggregate and local-average models, respectively.

We vary the density, the size and the type of the networks. We follow Bramoullé et al. (2009) and Startz and Wood-Doughty (2017) for the coefficients and the variables. In Table 3.1 we give on overview of the parameters of the simulations. The tables with the simulation results are in the Appendix. Table 3.2 gives an overview of the scenarios considered.

Table 3.1: Data generating process

Coefficients	
α	0.7683
β_{agg}	0.3666
β_{avg}	0.4666
γ_1	0.0834
γ_2	0.1507
Variables	
X	$\log\mathcal{N}(0,3)$
ε	$\mathcal{N}(0,0.1)$
Networks	
Size (N)	50 and 70
Density (d)	0.025, 0.05 and 0.1
Type	Erdös Rényi (ER) and Small World (SW)
Simulations	
Number of replication (R)	5000
Measures	
Mbias	$\frac{1}{R}\sum_{r=1}^{R}(\hat{\theta}_{(r)}-\theta)$
MSE	$\frac{1}{R}\sum_{i=1}^{R}(\theta-\hat{\theta}_{(r)})^2$

The table summarizes the parameters of the Monte Carlo experiments

Table 3.2: Overview of the simulation scenarios

d = 0.025	
Table 3.3	Local-aggregate specification
Table 3.4	Local-average specification
Table 3.9	Number of isolated individuals
d = 0.05	
Table 3.5	Local-aggregate specification
Table 3.6	Local-average specification
Table 3.10	Number of isolated individuals
d = 0.1	
Table 3.7	Local-aggregate specification
Table 3.8	Local-average specification
Table 3.11	Number of isolated individuals

The table summarizes the information in the tables of the Appendix.

Table 3.3 summarizes the simulations results for the local-aggregate model with networks of density 0.025. Before interpreting the results, let us inspect Table 3.9, where we clearly see that SW networks tend to contain significantly less isolated individuals, especially *completely isolated* ones. Starting with networks of size 50, we find that the $\hat{\theta}_{CI-CR}$ generally has a slightly larger bias than $\hat{\theta}_{All}$, but smaller than $\hat{\theta}_{PI-CR}$. This suggests that removing everyone with outdegree zero has a dominant negative impact on the accuracy of the estimates. While the estimates of γ_1 are almost not affected by the removal of the *completely isolated* individuals with respect to MSE, the same does not hold for the peer effects parameters, β and γ_2. We clearly see the deterioration in MSE from the loss in observations. However, $\hat{\theta}_{PI-CR}$ has the largest MSE, suggesting again that removing observations and changing regressors has the strongest negative effect on the properties of the estimator.

For the larger networks, a decrease in the number of isolated individuals does not change the pattern for ER graphs. While the bias and MSE of the estimators decrease with increasing network size, the bias and MSE of $\hat{\theta}_{PI-CR}$ remain the largest, suggesting that removing everyone with outdegree zero is harmful. This pattern is not very stable for the SW networks, where $\hat{\theta}_{PI-CR}$ does not always have the largest MSE .

In Table 3.4 the simulation results for the local-average model are summarized. Overall we find the same behavior of the estimators in case of $N = 50$. One important difference is in the MSEs of the coefficients for ER graphs, where the negative impact of removing everyone with outdegree 0 is notably higher. This could be due to the difference in instruments: in the local-aggregate we use the first order peers' and in the local-average we rely on the second order peers' characteristics. This pattern stays the same also for larger networks, including SW networks. Overall, the results for the local-average model are more stable compared to the local-aggregate model. They all lead to the conclusion that removing observations and changing regressors biases the estimator and increases its variance. Removing only *completely isolated* individuals has very little impact on the estimator.

Tables 3.5 and 3.6 summarize the simulation results for networks of density 0.05. The patterns we find here are the same as those for networks of density 0.025. Finally, for networks of density 0.1 we report the results in Tables 3.7 and 3.8. Although there are less isolated individuals (see Table 3.11), the consequences of removing *partially isolated* individuals are still evident. In most cases $\hat{\theta}_{PI-CR}$ has the largest MSE and bias. Most importantly, while the MSEs of $\hat{\theta}_{All}$, $\hat{\theta}_{PI-CR}$ and $\hat{\theta}_{CI-CR}$ tend to equate for networks of size 70, the bias of $\hat{\theta}_{PI-CR}$ is still prominent.

3.5. Conclusions

This paper contributes to the growing literature on the relevance of social interactions. In particular, we focus on isolated individuals. It is a common assumption in the literature that isolated individuals are absent. The common practice in empirical applications is to simply remove the observations without peers (e.g. Bramoullé et al., 2009). The general belief is that a removal has no impact on the properties of the peer effects estimator.

We consider a linear peer effects model, where the network is assumed to be exogenous. We consider two types of isolated individuals. *Completely isolated* are those who neither send nor receive links. Removing them results in less observations and less moment restrictions that can be used to build identifying moment functions. Based on the GMM framework, we show that this case leads to an inefficient estimator of peer effects. *Partially isolated* individuals are those who send no links but might receive some, i.e., anyone with outdegree zero. Removing these from the network, not only reduces the number of observations, but also changes the regressors of those who send the links. We show that, as a consequence, the estimator is inconsistent.

To demonstrate the behavior of the IV estimator when the isolated nodes are removed, we run several Monte Carlo experiments. We start with simulating Erdös Rényi random graphs, as one of the most basic types of graphs. The main property of the Erdös Rényi graph is that the probability of forming a link is constant and independent. This results in a degree distribution that follows the Poisson distribution. Moreover, to display characteristics of real networks, such as high clustering and small diameters, we also consider Small World graphs. We consider two standard specifications of peer effects models: the local-aggregate and the local-average models. Finally, we consider the size and the density of the network potential determinants of the impact of removing isolated individuals and simulate networks of sizes 50 and 70, and densities 0.025, 0.05 and 0.1

The simulations show that the impact of the loss in observations and change in regressors depends on all the network characteristics discussed above. In larger and denser networks, the number of isolated individuals decreases, and therefore, also the estimates are less affected. We find that the negative consequences remain more prominent in the local-average model. The patterns we find are similar for both types of networks. There are two major tendencies: removing *completely isolated* individuals has a very low negative impact on the estimates, while removing *partially isolated* individuals has serious negative consequences on the bias and MSE of the estimates.

This paper demonstrates that an applied researcher working with observed networks should always consider two aspects related to isolated individuals. First, the definition of an isolated individual is important. Defining someone as isolated only based on the outdegree leads to a biased and inconsistent peer effects estimator, for different network sizes, densities and structures. The more reasonable definition is to define as isolated only those who are completely detached from the network, i.e., individuals that neither receive nor send any links. This is a suitable definition because first, observed networks rarely contain them, and second, removing them has negligible consequences on the estimates. Second, the model specification plays an important role. The estimator of the local-aggregate model is generally less sensitive to the removal of isolated individuals.

Future work should consider modelling peer effects in a way that the information on these individuals is exploited instead of deleted. It could be beneficial to estimate the links of those who appear to be isolated, based on the links of those with similar characteristics, for instance. This would result in a peer effects model that incorporates network formation.

3.6. Appendix

Probability limit of the IV estimator

The following derivations are based on the assumption that the network remains sparse with $n_1 \to \infty$.

$$\frac{1}{n}\sum_{i=1}^{n} D_i X_i \tilde{Y}_i^p \xrightarrow{p} \mathrm{E}\left[D_i X_i \tilde{Y}_i^p\right],$$

where

$$\begin{aligned}\mathrm{E}\left[D_i X_i \tilde{Y}_i^p\right] &= \mathrm{E}[D_i X_i \mathrm{E}[\tilde{Y}_i^p|X]] \\ &= P[D_i = 1]\mathrm{E}[X_i \mathrm{E}[\tilde{Y}_i^p|X]] \\ &= \tilde{M}_1.\end{aligned}$$

$$\frac{1}{n}\sum_{i=1}^{n} D_i X_i^2 \xrightarrow{p} \mathrm{E}\left[D_i X_i^2\right],$$

with

$$\begin{aligned}\mathrm{E}\left[D_i X_i^2\right] &= \mathrm{P}\left[D_i = 1\right]\mathrm{E}[X_i^2] \\ &= \tilde{M}_2.\end{aligned}$$

$$\frac{1}{n}\sum_{i=1}^{n} D_i \tilde{X}_i^p \tilde{Y}_i^p \xrightarrow{p} \mathrm{E}\left[D_i \tilde{X}_i^p \tilde{Y}_i^p\right],$$

where

$$\begin{aligned}\mathrm{E}\left[D_i \tilde{X}_i^p \tilde{Y}_i^p\right] &= \mathrm{E}\left[D_i \tilde{X}_i^p \mathrm{E}[\tilde{Y}_i^p|X]\right] \\ &= \mathrm{P}\left[D_i = 1\right]\mathrm{E}\left[\tilde{X}_i^p[\tilde{Y}_i^p|X]\right] \\ &= \tilde{M}_3.\end{aligned}$$

$$\frac{1}{n}\sum_{i=1}^{n} D_i X_i \tilde{X}_i^p \xrightarrow{p} \mathrm{E}[D_i X_i \tilde{X}_i^p],$$

with

$$\begin{aligned}
\mathrm{E}\left[D_i X_i \tilde{X}_i^p\right] &= \mathrm{P}\left[D_i = 1\right] \mathrm{E}\left[X_i \tilde{X}_i^p\right] \\
&= \tilde{M}_4.
\end{aligned}$$

$$\frac{1}{n}\sum_{i=1}^{n} D_i X_i Y_i^p \xrightarrow{p} \mathrm{E}\left[D_i X_i Y_i^p\right],$$

with

$$\begin{aligned}
\mathrm{E}\left[D_i X_i Y_i^p\right] &= \mathrm{E}\left[D_i X_i \mathrm{E}[Y_i^p|X]\right] \\
&= \mathrm{P}\left[D_i = 1\right] \mathrm{E}\left[X_i \mathrm{E}\left[Y_i^p|X\right]\right] \\
&= M_1.
\end{aligned}$$

$$\frac{1}{n}\sum_{i=1}^{n} D_i \tilde{X}_i^p Y_i^p \xrightarrow{p} \mathrm{E}\left[D_i \tilde{X}_i^p Y_i^p\right],$$

with

$$\begin{aligned}
\mathrm{E}\left[D_i \tilde{X}_i^p Y_i^p\right] &= \mathrm{E}[D_i \tilde{X}_i^p \mathrm{E}[Y_i^p|X]] \\
&= \mathrm{P}[D_i = 1]\mathrm{E}[\tilde{X}_i^p \mathrm{E}[Y_i^p|X]] \\
&= M_3.
\end{aligned}$$

Monte Carlo Simulations

Table 3.3: Simulation results for the IV estimation of the local-aggregate model

	True	MBias			MSE		
		$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$
		N = 50					
		Erdös Rényi graphs					
β	0.3666	0.0003	-0.0088	-0.0018	0.0147	0.9734	0.0709
γ_1	0.0834	0.0001	-0.0003	-0.0001	0.0001	0.0032	0.0001
γ_2	0.1507	-0.0008	-0.0582	-0.0006	0.0043	0.2835	0.0049
		Small World graphs					
β	0.3666	0.0018	0.0004	-0.0024	0.0140	1.6243	0.1006
γ_1	0.0834	0.0001	0.0012	0.0001	0.0001	0.0083	0.0001
γ_2	0.1507	0.0001	-0.0377	0.0007	0.0011	0.1113	0.0044
		N = 70					
		Erdös Rényi graphs					
β	0.3666	0.0000	0.0018	-0.0000	0.0002	0.0047	0.0003
γ_1	0.0834	-0.0001	0.0005	-0.0001	0.0000	0.0007	0.0000
γ_2	0.1507	-0.0002	-0.0271	-0.0002	0.0001	0.0034	0.0001
		Small World graphs					
β	0.3666	-0.0018	0.0016	-0.0018	0.0118	0.0014	0.0118
γ_1	0.0834	-0.0001	-0.0001	-0.0001	0.0001	0.0002	0.0001
γ_2	0.1507	0.0013	-0.0092	0.0012	0.0048	0.0008	0.0048

Note: Monte Carlo simulation results based on 5000 replications. The DGP is the local-aggregate model. The density of the networks is 0.025. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error.

Table 3.4: Simulation results for the IV estimation of the local-average model

	True	MBias			MSE		
		$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$
		N = 50					
		Erdös Rényi graphs					
β	0.4666	0.0009	-0.5086	0.0008	0.0034	442.2416	0.0039
γ_1	0.0834	0.0002	0.0036	0.0001	0.0001	0.2867	0.0001
γ_2	0.1507	-0.0004	-0.0003	-0.0004	0.0003	4.3593	0.0003
		Small World graphs					
β	0.4666	0.0015	-0.1844	0.0014	0.0024	1.6595	0.0026
γ_1	0.0834	0.0002	-0.0055	0.0002	0.0001	0.0891	0.0001
γ_2	0.1507	0.0000	-0.0091	0.0000	0.0002	0.1069	0.0002
		N = 70					
		Erdös Rényi graphs					
β	0.4666	0.0005	-0.0666	0.0006	0.0019	7.1327	0.0021
γ_1	0.0834	-0.0000	0.0017	-0.0001	0.0000	0.0211	0.0000
γ_2	0.1507	-0.0002	-0.0333	-0.0002	0.0001	0.4225	0.0001
		Small World graphs					
β	0.4666	-0.0000	-0.0555	-0.0001	0.0024	0.4439	0.0025
γ_1	0.0834	-0.0000	0.0011	-0.0000	0.0000	0.0128	0.0000
γ_2	0.1507	0.0002	-0.0034	0.0002	0.0001	0.0355	0.0001

Note: Monte Carlo simulation results based on 5000 replications. The DGP is the local-average model. The density of the networks is 0.025. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error.

Table 3.5: Simulation results for the IV estimation of the local-aggregate model

		MBias			MSE		
	True	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$
		N = 50					
		Erdös Rényi graphs					
β	0.3666	0.0001	0.0004	0.0001	0.0001	0.0002	0.0001
γ_1	0.0834	-0.0000	0.0001	-0.0000	0.0001	0.0006	0.0001
γ_2	0.1507	-0.0002	-0.0160	-0.0002	0.0002	0.0014	0.0002
		Small World graphs					
β	0.3666	-0.0001	0.0024	-0.0001	0.0004	0.0129	0.0004
γ_1	0.0834	-0.0001	-0.0000	-0.0001	0.0001	0.0003	0.0001
γ_2	0.1507	-0.0003	-0.0109	-0.0003	0.0002	0.0018	0.0002
		N = 70					
		Erdös Rényi graphs					
β	0.3666	-0.0010	-0.0001	-0.0010	0.0183	0.0079	0.0183
γ_1	0.0834	0.0001	-0.0001	0.0001	0.0002	0.0001	0.0002
γ_2	0.1507	0.0004	-0.0049	0.0004	0.0028	0.0007	0.0028
		Small World graphs					
β	0.3666	-0.0009	-0.0010	-0.0009	0.0038	0.0030	0.0038
γ_1	0.0834	-0.0001	-0.0003	-0.0001	0.0001	0.0001	0.0001
γ_2	0.1507	0.0003	-0.0013	0.0004	0.0005	0.0006	0.0005

Note: Monte Carlo simulation results based on 5000 replications. The DGP is the local-aggregate model. The density of the networks is 0.05. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error.

Table 3.6: Simulation results for the IV estimation of the local-average model

		MBias			MSE		
	True	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$
		N = 50					
		Erdös Rényi graphs					
β	0.4666	0.0017	-1.1461	0.0017	0.0050	5800.6705	0.0053
γ_1	0.0834	-0.0000	0.0084	-0.0000	0.0001	0.3781	0.0001
γ_2	0.1507	-0.0004	0.0878	-0.0004	0.0003	52.2159	0.0003
		Small World graphs					
β	0.4666	-0.0006	-0.0472	-0.0007	0.0044	0.3869	0.0046
γ_1	0.0834	-0.0000	0.0007	-0.0000	0.0001	0.0008	0.0001
γ_2	0.1507	-0.0000	-0.0077	-0.0000	0.0002	0.0023	0.0002
		N = 70					
		Erdös Rényi graphs					
β	0.4666	-0.0003	-0.0363	-0.0002	0.0057	0.2759	0.0058
γ_1	0.0834	-0.0000	-0.0002	-0.0000	0.0000	0.0001	0.0000
γ_2	0.1507	0.0000	-0.0035	0.0000	0.0002	0.0010	0.0002
		Small World graphs					
β	0.4666	-0.0002	-0.0071	-0.0002	0.0088	0.0575	0.0088
γ_1	0.0834	0.0000	0.0001	0.0000	0.0000	0.0002	0.0000
γ_2	0.1507	0.0003	-0.0009	0.0003	0.0002	0.0005	0.0002

Note: Monte Carlo simulation results based on 5000 replications. The DGP is the local-average model. The density of the networks is 0.05. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error.

Table 3.7: Simulation results for the IV estimation of the local-aggregate model

		MBias			MSE		
	True	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$	$\hat{\theta}_{All}$	$\hat{\theta}_{PI-CR}$	$\hat{\theta}_{CI-CR}$
		N = 50					
		Erdös Rényi graphs					
β	0.3666	0.0005	-0.0008	0.0005	0.0145	0.0093	0.0145
γ_1	0.0834	0.0000	0.0009	0.0000	0.0002	0.0017	0.0002
γ_2	0.1507	-0.0006	0.0005	-0.0006	0.0044	0.0128	0.0044
		Small World graphs					
β	0.3666	-0.0038	-0.0004	-0.0038	0.0782	0.0187	0.0782
γ_1	0.0834	0.0002	0.0003	0.0002	0.0001	0.0002	0.0001
γ_2	0.1507	0.0010	-0.0017	0.0010	0.0053	0.0035	0.0053
		N = 70					
		Erdös Rényi graphs					
β	0.3666	-0.0125	0.0088	-0.0125	0.7182	3.2110	0.7182
γ_1	0.0834	0.0000	0.0031	0.0000	0.0027	0.0474	0.0027
γ_2	0.1507	0.0099	-0.0091	0.0099	0.3469	2.1767	0.3469
		Small World graphs					
β	0.3666	-0.0005	0.0003	-0.0005	0.0933	0.0920	0.0933
γ_1	0.0834	-0.0005	-0.0001	-0.0005	0.0009	0.0009	0.0009
γ_2	0.1507	0.0001	-0.0012	0.0001	0.0518	0.0520	0.0518

Note: Monte Carlo simulation results based on 5000 replications. The DGP is the local-aggregate model. The density of the networks is 0.1. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error.

Table 3.8: Simulation results for the IV estimation of the local-average model

	True	MBias $\hat{\theta}_{All}$	MBias $\hat{\theta}_{PI-CR}$	MBias $\hat{\theta}_{CI-CR}$	MSE $\hat{\theta}_{All}$	MSE $\hat{\theta}_{PI-CR}$	MSE $\hat{\theta}_{CI-CR}$
		N = 50					
		Erdös Rényi graphs					
β	0.4666	-0.0003	-0.0006	-0.0004	0.0384	0.1392	0.0384
γ_1	0.0834	0.0002	0.0001	0.0002	0.0001	0.0001	0.0001
γ_2	0.1507	-0.0002	-0.0009	-0.0002	0.0006	0.0007	0.0006
		Small World graphs					
β	0.4666	0.0059	-0.0051	0.0059	0.0862	0.1603	0.0862
γ_1	0.0834	0.0001	0.0002	0.0001	0.0001	0.0002	0.0001
γ_2	0.1507	-0.0003	-0.0016	-0.0003	0.0010	0.0013	0.0010
		N = 70					
		Erdös Rényi graphs					
β	0.4666	-0.0009	-0.0009	-0.0009	0.0671	0.0685	0.0671
γ_1	0.0834	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
γ_2	0.1507	-0.0001	-0.0002	-0.0001	0.0006	0.0006	0.0006
		Small World graphs					
β	0.4666	0.0006	-0.0029	0.0006	0.0407	0.0477	0.0407
γ_1	0.0834	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
γ_2	0.1507	0.0001	-0.0003	0.0001	0.0005	0.0005	0.0005

Note: Monte Carlo simualtion results based on 5000 replications. The DGP is the local-average model. The density of the networks is 0.1. MBias: Mean of the bias over all Monte Carlo replications. MSE: Mean squared error.

Table 3.9: Number of isolated individuals

	Erdos Reny PI	Erdos Reny CI	Small World PI	Small World CI
$N = 50$	25	14	20	9
$N = 70$	24	10	12	3

The maximum number of partially isolated and completely isolated individuals across simulations. Density of the networks is 0.025.

Table 3.10: Number of isolated individuals

	Erdos Reny		Small World	
	PI	CI	PI	CI
$N = 50$	12	5	9	3
$N = 70$	8	2	4	1

The maximum number of partially isolated and completely isolated individuals across simulations. Density of the networks is 0.05

Table 3.11: Number of isolated individuals

	Erdos Reny		Small World	
	PI	CI	PI	CI
$N = 50$	4	1	3	1
$N = 70$	2	0	3	0

The maximum number of partially isolated and completely isolated individuals across simulations. Density of the networks is 0.1

References

Bertrand, M., E. F. P. Luttmer, and S. Mullainathan (2000). “Network Effects and Welfare Cultures”. *The Quarterly Journal of Economics* 115 (3), pp. 1019–1055.

Boucher, V. and B. Fortin (Feb. 2015). *Some Challenges in the Empirics of the Effects of Networks*. IZA Discussion Papers 8896. Institute of Labor Economics (IZA).

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). “Identification of peer effects through social networks”. *Journal of Econometrics* 150 (1), pp. 41–55.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009). “Peer effects and social networks in education”. *The Review of Economic Studies* 76 (4), pp. 1239–1267.

Cassar, A. (2007). “Coordination and cooperation in local, random and small world networks: Experimental evidence”. *Games and Economic Behavior* 58 (2), pp. 209–230.

Cohen-Cole, E., X. Liu, and Y. Zenou (May 2017). “Multivariate choices and identification of social interactions”. *Journal of Applied Econometrics* 33.

Dieye, R. and B. Fortin (Jan. 2017). *Gender Peer Effects Heterogeneity in Obesity*. CIRANO Working Papers 2017s-03. CIRANO.

Erdös, P. and A. Rényi (1959). “On Random Graphs I”. *Publicationes Mathematicae Debrecen* 6, p. 290.

– (1960). “On the evolution of random graphs”. *Publ. Math. Inst. Hungary. Acad. Sci.* 5, pp. 17–61.

– (1961). “On the strength of connectedness of a random graph”. *Acta Math. Acad. Sci. Hung.* 12, pp. 261–267.

Gilbert, E. N. (1959). “Random Graphs”. *The Annals of Mathematical Statistics* 30 (4), pp. 1141 –1144.

Jackson, M. O. (2011). “Chapter 12 - An Overview of Social Networks and Economic Applications*”. In: ed. by Benhabib, J., Bisin, A., and Jackson, M. O. Vol. 1. Handbook of Social Economics. North-Holland, pp. 511–585.

Lee, L.-f., X. Liu, and X. Lin (2010). “Specification and estimation of social interaction models with network structures”. *The Econometrics Journal* 13 (2), pp. 145–176.

Liu, X., E. Patacchini, and Y. Zenou (2014). “Endogenous peer effects: Local aggregate or local average?” *Journal of Economic Behavior and Organization* 103, pp. 39–59.

Liu, X., E. Patacchini, and E. Rainone (2016). “Peer effects in bedtime decisions among adolescents: a social network model with sampled data”. *Econometrics Journal* 20 (3), pp. 103–125.

Patacchini, E. and Y. Zenou (2008). “The strength of weak ties in crime”. *European Economic Review* 52 (2), pp. 209–236.

Pohlmeier, W. (2022). *Advanced Econometrics*. Lecture notes, Department of Economics, University of Konstanz.

Solomonoff, R. and A. Rapoport (1951). *The bulletin of mathematical biophysics* 13, pp. 107–117.

Startz, R. and A. Wood-Doughty (2017). “Improved estimation of peer effects using network data”. Dissertation. Department of Economics, University of California, Santa Barbara.

Trogdon, J., J. Nonnemaker, and J. Pais (2008). “Peer effects in adolescent overweight”. *Journal of Health Economics* 27 (5), pp. 1388–1399.

Watts, D. J. and S. H. Strogatz (1998). “Collective dynamics of ‘small-world’ networks”. *Nature* 393, pp. 440 –442.

Complete References

Angrist, J. and V. Lavy (1999). “Using Maimonides’ rule to estimate the effect of class size on scholastic achievement”. *Quarterly Journal of Economics* 114 (2), pp. 533–575.

Anselin, L. (2003). “Spatial Econometrics”. In: *A Companion to Theoretical Econometrics*. Ed. by Baltagi, B. H. John Wiley & Sons, Ltd. Chap. 14, pp. 310–330.

Arduini, T., E. Patacchini, and E. Rainone (2020). “Treatment Effects With Heterogeneous Externalities”. *Journal of Business & Economic Statistics* (4), pp. 826–838.

Arvey, R. D. and K. R. Murphy (1998). “Performance evaluation in work settings”. *Annual Review of Psychology* 49 (1), pp. 141–168.

Auerbach, R. P., P. Mortier, R. Bruffaerts, J. Alonso, C. Benjet, P. Cuijpers, K. Demyttenaere, D. D. Ebert, J. G. Green, P. Hasking, E. Murray, M. K. Nock, S. Pinder-Amaker, N. A. Sampson, D. J. Stein, G. Vilagut, A. M. Zaslavsky, and R. C. Kessler (2018). *WHO World Mental Health Surveys International College Student Project: Prevalence and distribution of mental disorders.* Press Release.

Bakker, A. B. (2005). “Flow among music teachers and their students: The crossover of peak experiences”. *Journal of Vocational Behavior* 66 (1), pp. 26–44.

Bakker, A. B., M. Westman, and I. J. H. van Emmerik (2009). “Advancements in crossover theory”. *Journal of Managerial Psychology* 24 (3), pp. 206–219.

Bakker, A. B. and W. B. Schaufeli (2000). “Burnout Contagion Processes Among Teachers1”. *Journal of Applied Social Psychology* 30 (11), pp. 2289–2308.

Bear, J. B. and A. W. Woolley (2011). “The role of gender in team collaboration and performance”. *Interdisciplinary Science Reviews* 36 (2), pp. 146–153.

Becker, E. S., T. Goetz, V. Morger, and J. Ranellucci (2014). “The importance of teachers’ emotions and instructional behavior for their students’ emotions - An experience sampling analysis”. *Teaching and Teacher Education* 43, pp. 15–26.

Bertrand, M., E. F. P. Luttmer, and S. Mullainathan (2000). “Network Effects and Welfare Cultures”. *The Quarterly Journal of Economics* 115 (3), pp. 1019–1055.

Beugnot, J., B. Fortin, G. Lacroix, and M. C. Villeval (2019). “Gender and peer effects on performance in social networks”. *European Economic Review* 113, pp. 207–224.

Blume, L. E., W. A. Brock, S. N. Durlauf, and R. Jayaraman (2015). “Linear social interactions models”. *Journal of Political Economy* 123 (2), pp. 444–496.

Boucher, V., Y. Bramoullé, H. Djebbari, and B. Fortin (Jan. 2014). “Do peers affect student achievement? Evidence from Canada using group size variation”. *Journal of Applied Econometrics* 29 (1), pp. 91–109.

Boucher, V. and B. Fortin (Feb. 2015). *Some Challenges in the Empirics of the Effects of Networks.* IZA Discussion Papers 8896. Institute of Labor Economics (IZA).

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). "Identification of peer effects through social networks". *Journal of Econometrics* 150 (1), pp. 41–55.

– (2020). "Peer Effects in Networks: A Survey". *Annual Review of Economics* 12 (1), pp. 603–629.

Burgess, L. G., P. M. Riddell, A. Fancourt, and K. Murayama (2018). "The Influence of Social Contagion Within Education: A Motivational Perspective". *Mind, Brain, and Education* 12 (4), pp. 164–174.

CAESR (2007). *Dataset Gymnasiastenstudie.* Central Archive for Empirical Social Research, Cologne, Germany.

Calvó-Armengol, A., E. Patacchini, and Y. Zenou (2009). "Peer effects and social networks in education". *The Review of Economic Studies* 76 (4), pp. 1239–1267.

Cassar, A. (2007). "Coordination and cooperation in local, random and small world networks: Experimental evidence". *Games and Economic Behavior* 58 (2), pp. 209–230.

Chamberlain, G. (1984). "Panel data". In: *Handbook of Econometrics.* Ed. by Griliches, Z. and Intriligator, M. D. Elsevier, pp. 1247–1318.

Cohen, S. and T. A. Wills (1985). "Stress, Social Support, and the Buffering Hypothesis". *Psychological Bulletin* 98 (2), pp. 310–357.

Cohen-Cole, E., X. Liu, and Y. Zenou (May 2017). "Multivariate choices and identification of social interactions". *Journal of Applied Econometrics* 33.

Cook, W. L. and D. A. Kenny (2005). "The actor-partner interdependence model: A model of bidirectional effects in developmental studies". *International Journal of Behavioral Development* 29 (2), pp. 101–109.

Csikszentmihalyi, M. and R. Larson (1987). "Validity and Reliability of the Experience-Sampling Method". *Journal of Nervous and Mental Disease* 175 (9), pp. 526–536.

Dieye, R. and B. Fortin (Jan. 2017). *Gender Peer Effects Heterogeneity in Obesity.* CIRANO Working Papers 2017s-03. CIRANO.

Dishion, T. and K. Dodge (July 2005). "Peer Contagion in Interventions for Children and Adolescents: Moving Towards an Understanding of the Ecology and Dynamics of Change". *Journal of abnormal child psychology* 33, pp. 395–400.

Dobbelsteen, S., J. Levin, and H. Oosterbeek (2002). "The causal effect of class size on scholastic achievement: Distinguishing the pure class size effect from the effect of changes in class composition". *Oxford Bulletin of Economics and Statistics* 64 (1), pp. 17–38.

Donker, M. H., T. van Gog, and M. T. Mainhard (2018). "A Quantitative Exploration of Two Teachers with Contrasting Emotions: Intra-Individual Process Analyses of Physiology and Interpersonal Behavior". *Frontline Learning Research* 6 (3), pp. 162–185.

Doty, J. (2016). *Into the Magic Shop: A Neurosurgeon's Quest to Discover the Mysteries of the Brain and the Secrets of the Heart.* Penguin Publishing Group.

Druskat, V. U. and S. B. Wolff (1999). "Effects and timing of developmental peer appraisals in self-managing work groups". *Journal of Applied Psychology* 84 (1), pp. 58–74.

Elhorst, J. P. (2014). *Spatial econometrics : from cross-sectional data to spatial panels.* Springer briefs in regional science. Heidelberg [u.a.]: Springer.

Erdös, P. and A. Rényi (1959). "On Random Graphs I". *Publicationes Mathematicae Debrecen* 6, p. 290.

Erdös, P. and A. Rényi (1960). “On the evolution of random graphs”. *Publ. Math. Inst. Hungary. Acad. Sci.* 5, pp. 17–61.

– (1961). “On the strength of connectedness of a random graph”. *Acta Math. Acad. Sci. Hung.* 12, pp. 261–267.

Fortuin, J., M. van Geel, and P. Vedder (2016). “Peers and academic achievement: A longitudinal study on selection and socialization effects of in-class friends”. *The Journal of Educational Research* 109 (1), pp. 1–6.

Fredriksson, P., B. Öckert, and H. Oosterbeek (Nov. 2012). “ Long-term effects of class size”. *The Quarterly Journal of Economics* 128 (1), pp. 249–285.

Frenzel, A. C., B. Becker-Kurz, R. Pekrun, T. Goetz, and O. Ludtke (2018). “Emotion Transmission in the Classroom Revisited: A Reciprocal Effects Model of Teacher and Student Enjoyment”. *Journal of Educational Psychology* 110 (5), pp. 628–639.

Frenzel, A. C., T. Goetz, O. Ludtke, R. Pekrun, and R. E. Sutton (2009). “Emotional Transmission in the Classroom: Exploring the Relationship Between Teacher and Student Enjoyment”. *Journal of Educational Psychology* 101 (3), pp. 705–716.

Gašević, D., A. Zouaq, and R. Janzen (2013). ““Choose Your Classmates, Your GPA Is at Stake!”: The Association of Cross-Class Social Ties and Academic Performance”. *American Behavioral Scientist* 57 (10), pp. 1460–1479.

Gilbert, E. N. (1959). “Random Graphs”. *The Annals of Mathematical Statistics* 30 (4), pp. 1141 –1144.

Griffith, A. (2021). “Name Your Friends, but Only Five? The Importance of Censoring in Peer Effects Estimates using Social Network Data”. *Journal of Labor Economics* 0 (ja), null.

Hanushek, E. A. (1996). “Measuring investment in education”. *The Journal of Economic Perspectives* 10 (4), pp. 9–30.

Härtel, C. E. J. and K. M. Page (2009). “Discrete emotional crossover in the workplace: the role of affect intensity”. *Journal of Managerial Psychology* 24 (3), pp. 237–253.

Hatfield, E., J. Cacioppo, and R. Rapson (1994). *Emotional Contagion*. New York, NY: Cambridge University Press.

Hatfield, E., L. Bensman, P. D. Thornton, and R. L. Rapson (2014). “New Perspectives on Emotional Contagion: A Review of Classic and Recent Research on Facial Mimicry and Contagion”. *Interpersona: An International Journal on Personal Relationships* 8 (2), p. 21.

Heinesen, E. (2010). “Estimating class-size effects using within-school variation in subject-specific classes”. *The Economic Journal* 120 (545), pp. 737–760.

Hon, A. H. Y. and W. W. Chan (2013). “The Effects of Group Conflict and Work Stress on Employee Performance”. *Cornell Hospitality Quarterly* 54 (2), pp. 174–184.

Hoxby, C. M. (2000). “The effects of class size on student achievement: New evidence from population variation”. *The Quarterly Journal of Economics* 115 (4), pp. 1239–1285.

– (2002). “The power of peers: How does the makeup of a classroom influence achievement”. *Education Next* Summer 2 (2.2), pp. 57–63.

Jackson, M. O. (2010). *Social and economic networks*. Princeton University Press.

– (2011). “Chapter 12 - An Overview of Social Networks and Economic Applications*”. In: ed. by Benhabib, J., Bisin, A., and Jackson, M. O. Vol. 1. Handbook of Social Economics. North-Holland, pp. 511–585.

Kärner, T., D. Sembill, C. Aßmann, E. Friederichs, and C. H. Carstensen (2017). "Analysis of person-situation interactions in educational settings via cross-classified multilevel longitudinal modeling: illustrated with the example of students' stress experience". *Frontline Learning Research* 5 (1), pp. 16–42.

Kärner, T., J. K. Weiß, and K. Heinrichs (2021). "A social perspective on resilience: social support and dyadic coping in teacher training". *Empirical Research in Vocational Education and Training* 13 (24).

Kelejian, H. H. and I. R. Prucha (1998). "A generalized spatial two-stage least squares procedure for estimating a spatial autoregressive model with autoregressive disturbances". *The Journal of Real Estate Finance and Economics* 17 (1), pp. 99–121.

– (1999). "A Generalized Moments Estimator for the Autoregressive Parameter in a Spatial Model". *International Economic Review* 40 (2), pp. 509–533.

King, R. B. (2020). "Mindsets are contagious: The social contagion of implicit theories of intelligence among classmates". *British Journal of Educational Psychology* 90 (2), pp. 349–363.

King, R. B. and J. A. Datu (2017). "Happy classes make happy students: Classmates' well-being predicts individual student well-being". *Journal of school psychology* 65, 116—128.

King, R. B. and N. B. Mendoza (2020). "Achievement goal contagion: mastery and performance goals spread among classmates". *Social Psychology of Education* 23, pp. 795–814.

Kirschner, F., F. Paas, and P. A. Kirschner (2009). "A Cognitive Load Approach to Collaborative Learning: United Brains for Complex Tasks". *Educational Psychology Review* 21 (1), pp. 31–42.

Kouzma, N. M. and G. A. Kennedy (2004). "Self-reported sources of stress in senior high school students". *Psychological Reports* 94 (1), pp. 314–316.

Krueger, A. B. (May 1999). "Experimental estimates of education production functions". *The Quarterly Journal of Economics* 114 (2), pp. 497–532.

Lavy, V. and A. Schlosser (2011). "Mechanisms and impacts of gender peer effects at school". *American Economic Journal: Applied Economics* 3 (2), pp. 1–33.

Lazarus, R. S. and S. Folkman (1984). *Stress, Appraisal, and Coping.* New York: Springer.

Lee, L.-f., X. Liu, and X. Lin (2010). "Specification and estimation of social interaction models with network structures". *The Econometrics Journal* 13 (2), pp. 145–176.

Lin, X. (2014). "Network attributes and peer effects". *Economics Bulletin* 34 (3), pp. 2060–2079.

Liu, X., E. Patacchini, and Y. Zenou (2014). "Endogenous peer effects: Local aggregate or local average?" *Journal of Economic Behavior and Organization* 103, pp. 39–59.

Liu, X., E. Patacchini, and E. Rainone (2016). "Peer effects in bedtime decisions among adolescents: a social network model with sampled data". *Econometrics Journal* 20 (3), pp. 103–125.

Mael, F., A. Alonso, D. Gibson, K. Rogers, and M. Smith (2005). *Single-sex versus coeducational schooling: A systematic review. Doc# 2005-01.* Tech. rep.

Manski, C. F. (1993). "Identification of Endogenous Social Effects: The Reflection Problem". *The Review of Economic Studies* 60 (3), pp. 531–542.

– (2000). "Economic Analysis of Social Interactions". *Journal of Economic Perspectives* 14 (3), pp. 115–136.

Masten, M. A. (2018). "Random Coefficients on Endogenous Variables in Simultaneous Equations Models". *The Review of Economic Studies* (2), pp. 1193–1250.

Minkley, N., T. Ringeisen, L. B. Josek, and T. Kärner (2017). "Stress and emotions during experiments in biology classes: Does the work setting matter?" *Contemporary Educational Psychology* 49, pp. 238–249.

Morse, S. (1999). "Separated by sex: A critical look at single-sex education for girls". *Journal of Chemical Education* 76, p. 615.

Oberle, E. and K. A. Schonert-Reichl (2016). "Stress contagion in the classroom? The link between classroom teacher burnout and morning cortisol in elementary school students". *Social Science & Medicine* 159, pp. 30–37.

Patacchini, E. and Y. Zenou (2008). "The strength of weak ties in crime". *European Economic Review* 52 (2), pp. 209–236.

Pekrun, R. (Dec. 2006). "The Control-Value Theory of Achievement Emotions: Assumptions, Corollaries, and Implications for Educational Research and Practice". *Educational Psychology Review* 18, pp. 315–341.

Pitt, A., F. Oprescu, G. Tapia, and M. Gray (2018). "An exploratory study of students' weekly stress levels and sources of stress during the semester". *Active Learning in Higher Education* 19 (1), pp. 61–75.

Pohlmeier, W. (2022). *Advanced Econometrics.* Lecture notes, Department of Economics, University of Konstanz.

Seidel, T., I. M. Dalehefte, and L. Meyer (2001). "Videoanalysen – Beobachtungsschemata zur Erfassung von „Sicht-Strukturen" im Physikunterricht". In: *Erhebungs- und Auswertungsverfahren des DFG-Projekts „Lehr-Lern-Prozesse im Physikunterricht – eine Videostudie.* Ed. by Prenzel, M., Duit, R., Euler, M., Lehrke, M., and Seidel, T. Kiel: IPN-Materialien, 41–58.

Shen, B., N. McCaughtry, J. Martin, A. Garn, N. Kulik, and M. Fahlman (2015). "The relationship between teacher burnout and student motivation". *British Journal of Educational Psychology* 85 (4), pp. 519–532.

Solomonoff, R. and A. Rapoport (1951). *The bulletin of mathematical biophysics* 13, pp. 107–117.

Song, Z. L., M. D. Foo, M. A. Uy, and S. H. Sun (2011). "Unraveling the Daily Stress Crossover Between Unemployed Individuals and Their Employed Spouses". *Journal of Applied Psychology* 96 (1), pp. 151–168.

Startz, R. and A. Wood-Doughty (2017). "Improved estimation of peer effects using network data". Dissertation. Department of Economics, University of California, Santa Barbara.

Sweller, J., P. L. Ayres, and S. Kalyuga (2011). *Cognitive load theory. Explorations in the learning sciences, instructional systems and performance technologies.* New York: Springer.

Trogdon, J., J. Nonnemaker, and J. Pais (2008). "Peer effects in adolescent overweight". *Journal of Health Economics* 27 (5), pp. 1388–1399.

Vogel, S. and L. Schwabe (2016). "Learning and memory under stress: implications for the classroom". *Npj Science Of Learning* 1, p. 16011.

Warwas, J. and C. Helm (2017). "Enjoying working and learning in vocational education: a multilevel investigation of emotional crossover and contextual moderators". *Empirical Research in Vocational Education and Training* 9 (1), p. 11.

Waters, S. F., T. V. West, and W. B. Mendes (2014). "Stress Contagion Physiological Covariation Between Mothers and Infants". *Psychological Science* 25 (4), pp. 934–942.

Watts, D. J. and S. H. Strogatz (1998). “Collective dynamics of ‘small-world’ networks”. *Nature* 393, pp. 440 –442.

Westman, M. and D. Etzion (1999). “The crossover of strain from school principals to teachers and vice versa”. *Journal of Occupational Health Psychology* 4 (3), pp. 269–278.

Westman, M. and A. D. Vinokur (1998). “Unraveling the relationship of distress levels within couples: Common stressors, empathic reactions, or crossover via social interaction?” *Human Relations* 51 (2), pp. 137–156.

Wethington, E. (July 2000). “Contagion of stress”. *Advances in Group Processes* 17, pp. 229–253.

Eigenabgrenzung

Das erste Kapitel, *Peer effects heterogeneity and social networks in education*, ist in Zusammenarbeit mit Herrn Prof. Dr. Winfried Pohlmeier (Universität Konstanz) und Prof. Dr. Derya Uysal (Universität München) entstanden. Meine individuelle Leistung bei der Erstellung dieses Kapitels ist 33%.

Das dritte Kapitel, *A Network Model of Stress Contagion in the Classroom: Do Learning Arrangements Matter?*, ist in Zusammenarbeit mit Herrn Prof. Dr. Winfried Pohlmeier (Universität Konstanz) und Prof. Dr. Tobias Kärner (Universität Hohenheim) entstanden. Meine individuelle Leistung bei der Erstellung dieses Kapitels ist 33%.

Das dritte Kapitel, *Isolated individuals in social networks. How does their treatment impact the peer effects estimates?*, habe ich ohne Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel erstellt.