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Productive government spending, growth, and sequential voting

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Abstract

This paper considers an endogenous growth model with productive government spending in which overlapping generations of agents vote sequentially on policy. With sequential majority voting, there is a multiplicity of politico-economic equilibria originating from self-fulfilling policy expectations. Some of these equilibria are Pareto-inefficient and there are endogenous cycles. A constitutional rule providing partial commitment significantly shrinks the set of politico-economic equilibria, removing all inefficiencies and cycles. However, a likely outcome is that government size is too high relative to the growth-maximizing size.

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1. Introduction

A country's growth rate is influenced by public policy. Distortionary taxation lowers the private returns to savings and investment, whereas productive government expenditures raise productivity and enhance growth. These policy features are the reason of the inverted U-shaped relationship between government size and growth in Barro (1990). Also the empirical growth literature includes various policy measures in growth regressions.[1]

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[1] Using panel data for OECD countries over the period 1970–1995, Kneller et al. (1999) find a significant negative impact of distortionary taxation (income taxes) and a significant positive impact of productive government expenditure (including infrastructure, education, defense, health) on growth. Canning and Pedroni (1999) analyze the effects of infrastructure spending on growth and find an inverted U-shaped relationship with some countries on either side of the growth-maximizing level.

The perception that policy differences to a large extent explain growth differences has turned the interest of researchers to exploring the origins of policy differences. The political-economy literature describes collective choice mechanisms and explains how fundamentals (preferences and technologies) together with political institutions determine political outcomes. Alesina and Rodrik (1994) consider a growth model in the spirit of Barro (1990), focusing on how income inequality affects policy and growth. However, a critical feature of this model is that taxes are voted on in period zero only and then are constant over time. As has been pointed out by Krusell et al. (1997), these solutions are typically not time-consistent, even when the median voter does not change over time (as turns out to be the case in the model of Alesina and Rodrik). Further, even if policy commitment were possible, the optimal policy would not involve a constant tax rate.[2]

This paper considers a model in which endogenous growth is enhanced by productive government services and in which agents vote on policy sequentially. I employ an overlapping generations structure that allows explicit solutions of the dynamic policy game to be derived. Within the overlapping generations framework, there is a positive externality of productive government spending on the welfare of future generations that is not internalized, since there is no bequest motive. This raises the question whether the sequential voting procedure leads to an inefficiently low growth path due to a too low level of spending (and taxation). I show, however, that this need not be the case, especially so if the current policy maker can commit the policy maker in the subsequent period to a certain policy.

In the basic version of the model, government spending is financed by a proportional tax on labor income only.[3] Only the young generation supplies labor (and pays taxes), whereas both young and old generations benefit from higher productive spending through higher (gross) wages and a higher capital return. Population growth is positive and the young generation has the majority at each point in time and decides policy.

The first result is that there is a multiplicity of politico-economic equilibria depending on the coordination of self-fulfilling policy expectations. Current policy makers have to predict the effect of their policy decision on future policy outcomes, which feeds back into today's policy decision. It turns out that many policy expectations can be self-fulfilling, implying that a continuum of stationary equilibria (and many others, including endogenous cycles) emerges. In some of these equilibria, the level of spending is inefficiently low (i.e. they can be Pareto-dominated by another policy path of higher spending and taxation in all periods). This inefficiency is, however, not the result of the externality on future generations' welfare mentioned above, but comes from pessimistic self-fulfilling expectations of policy makers: a government declines to expand spending and taxation above an inefficiently low level, since it fears a spending cut in the future in response to such a policy today.

The question therefore arises whether a political mechanism can be designed that allows current policy makers to commit future policy makers to a certain policy in order to

[2] Similar objections apply to the growth model of Bertola (1993) and to the extension of Alesina and Rodrik's model by Fiaschi (1999).

[3] Most results are robust to taxation of capital income, provided that capital income is taxed at a lower rate than labor income, as appears to be the case in most developed countries.

avoid an inefficiently low level of public spending. Following an idea of Azariadis and Galasso (1996), a political mechanism that provides commitment is a "constitutional rule" under which (large) fiscal policy adjustments require not only a simple majority, but the approval by a qualified majority. When such a rule is implemented, the set of politico-economic equilibria is significantly reduced and all inefficient (Pareto-dominated) equilibria (which involve too low spending and taxation) disappear. Instead, however, the tax rate in the politico-economic equilibrium exceeds the growth-maximizing tax rate, provided that the interest elasticity of savings is not too large. Intuitively, if an incumbent government (elected by the majority of workers) can commit a future government to a certain policy, the young generation not only benefits from higher spending today (through a higher gross wage), but also from higher spending in the future (through a higher capital return). This induces the incumbent government to choose a policy level that exceeds the level maximizing net labor income, and that also exceeds the growth-maximizing policy whenever the interest elasticity of savings is not too high. Thus, the politico-economic equilibrium under the constitutional rule is likely to lead to a "too high" government size relative to the growth-maximal levels (i.e. which is at the downward-sloping part of Barro's curve). Nevertheless, these equilibria are not Pareto-inefficient, but currently living generations benefit at the expense of almost all future generations.

These results relate to Alesina and Rodrik (1994) who consider an infinitely lived agent model assuming that productive government expenditures are financed by a tax on capital income only. They find that majority voting (under commitment, at date zero) leads to a tax rate exceeding the growth-maximizing tax rate. The reason is that voters benefit from higher wages due to higher spending, but the higher wages are not reflected in a higher growth rate (which only depends on the net capital return). Fiaschi (1999) extends Alesina and Rodrik's model to two independent tax rates on capital and labor income. By a similar argument, he finds that the labor (capital) income tax rate in the politico-economic equilibrium is too low (high) relative to its growth-maximizing level. My contrasting result is due to the fact that, in a Diamond-type overlapping generations economy, savings come out of labor income, so that both labor and capital income taxes are detrimental to growth, unlike in infinitely lived agent economies.[4]

The analysis is based on Markovian (recursive) equilibria in which policy depends only on last period's policy choice. Even stronger indeterminacy results in dynamic policy games can easily be obtained when policy depends on the whole history of past policy choices. With such history dependence, a considerably larger set of equilibria can be supported by appropriate trigger strategies.[5] To avoid such indeterminacies, Krusell et al. (1997) and Krusell and Rios-Rull (1999) propose to restrict policy functions to be Markovian. In spite of such a restriction, I obtain a multiplicity of stationary policy functions (and consequently a continuum of stationary politico-economic equilibria). Similar indeterminacy results for Markovian policy functions have recently been obtained

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[4] See Uhlig and Yanagawa (1996) and Bertola (1996) for a discussion of the impact of capital and labor tax rates on growth in overlapping generation models.

[5] Formally, such folk theorems in games with overlapping generations of players have been proven by Kandori (1992) and Smith (1992). For applications in policy games, see Chari and Kehoe (1990) and Azariadis and Galasso (1996).

by Grossman and Helpman (1998) in a linear overlapping generations model with endogenous growth and intergenerational transfers. Sequential majority voting, therefore, does generally not characterize determinate intertemporal equilibria, unless further restrictions (such as a constitutional rule) are imposed.[6]

The paper is organized as follows. The next section introduces the model for exogenous policy. Sections 3 and 4 solve the model under sequential majority voting, establishing the indeterminacy result and the existence of inefficient equilibria and endogenous cycles. Section 5 introduces the constitutional rule. Section 6 provides concluding remarks and discusses extensions. Proofs not included in the text are contained in Appendix A.

2. The model

Consider a version of the Diamond (1965) overlapping generations model in discrete time $t=0,1,2,\ldots$. Each generation consists of a continuum of individuals living for two periods. Individuals supply one unit of labor in the first lifetime period, wish to consume in young and in old age, and transfer income between periods by holding shares of the capital stock which is the only asset. The population size of the young generation (i.e. the labor force) in period t is denoted as L_t. I assume positive population growth to ensure that at each point in time the young generation has the majority of votes.

The government provides productive government services that proportionately raise the effectiveness of labor and are financed by a proportional tax on labor income. This specification implies that the aggregate production technology exhibits increasing returns to scale, so that endogenous growth is possible, similar to the growth models of Barro (1990) and Barro and Sala-i Martin (1992). In this section, government policy is exogenous.

A continuum of firms produce a single homogenous consumption/investment good from private inputs of capital and labor via a constant returns, concave and differentiable production function $F(K_t,A_tL_t)$. K_t and L_t denote capital and labor input, and A_t is labor-augmenting technological progress. $f(k_t)=F(k_t,1)$ denotes the production function in intensive form which gives output per unit of effective labor as a function of capital per unit of effective labor $k_t=K_t/(A_tL_t)$. Factors receive their marginal product, so that the gross interest rate R_t and the wage w_t are

$$R_t = 1-\delta+f'(k_t),$$

$$w_t = A_t w(k_t). \tag{1}$$

δ is the depreciation rate, and $w(k)=f(k)-kf'(k)$ denotes the wage per unit of effective labor which is increasing in k.

[6] There are political-economy models (for instance, Saint-Paul and Verdier, 1993; Glomm and Ravikumar, 1992, 1995) in which different generations of agents vote sequentially, but in which agents need not be forward-looking because they are not concerned about the future. Consequently no indeterminacies arise.

The government provides productive government services G_t which are financed by a proportional tax on labor income:

$$G_t = \tau_t w_t L_t. \tag{2}$$

Capital income is not taxed, but Section 6 considers how this can be relaxed. Note, however, that in this economy, the tax on labor income is not distortionary, since labor is supplied inelastically (in fact, it is equivalent to a lump-sum tax on the young generation), whereas a tax on capital income lowers the return on capital and thereby distorts the savings/investment decision.

Government services per worker raise the effectiveness of labor A_t according to the linear technology

$$A_t = a\left(\frac{G_t}{L_t}\right). \tag{3}$$

One can think of the government providing public goods that are rival and not excludable, in the sense that the provision of the public goods raises the productivity of all workers, but that public goods are subject to congestion. As argued by Barro (1990) and Barro and Sala-i Martin (1992), most productive public services (such as expenditures on infrastructure and education) are characterized by these features. The assumption that government spending *per worker* determines the effectiveness of labor implies that there are no scale effects of labor supply. In particular, there exists a balanced growth path even when population is growing.[7] Note also that productive government expenditures increase the effectiveness of labor immediately and have no lasting effects. Although this assumption is not innocuous, it seems reasonable in an overlapping generations model with an interpretation of a period lasting 30 years. I would, however, refrain from interpreting government services as investment into accumulable factors of production such as human or physical capital.

Eqs. (1)–(3) imply that capital per unit of effective labor is uniquely determined by the tax rate:

$$w(k_t) = \frac{1}{a\tau_t}. \tag{4}$$

The wage per unit of effective labor $w(k)$ is strictly increasing in k, but need not converge to infinity when k tends to infinity. However, I assume that $\lim_{k\to\infty} w(k) \equiv \bar{w} > 1/a$. This assumption ensures that for all $1 \geq \tau_t > \underline{\tau} \equiv 1/(a\bar{w})$, there is a unique k_t, denoted as $k(\tau_t)$, satisfying Eq. (4). Obviously, k_t decreases in τ_t: higher government spending raises the effectiveness of labor and lowers capital per unit of effective labor.[8]

[7] If the effectiveness of labor was proportional to total government spending instead of spending per worker, positive population growth would lead to an explosive growth path, as in other AK-type growth models which are based on externalities.

[8] If government spending is too low ($0 < \tau_t \leq \underline{\tau}$), there is no solution k_t to Eq. (4), and in this case the effectiveness of labor must be zero: $A_t = 0$. Section 6 discusses how this assumption can be relaxed so that growth is possible even under laissez-faire.

For given policy τ_t (and so for given k_t), the aggregate production function has constant returns to capital:

$$F(K_t, A_t L_t) = A_t L_t f(k_t) = \frac{f(k_t)}{k_t} K_t. \tag{5}$$

The capital productivity $f(k_t)/k_t$ is decreasing in k_t, and thus increasing in τ_t. If policy is constant over time, capital productivity is constant, and the model reduces to a simple "AK type" growth model. The aggregate net labor income is also linear in capital:

$$w_t(1-\tau_t)L_t = \frac{w(k_t)}{k_t}(1-\tau_t)K_t = \Phi(\tau_t)(1-\tau_t)K_t, \tag{6}$$

where $\Phi(\tau) \equiv (1/a\tau k(\tau))$ is the gross wage per unit of capital. Φ is increasing in τ whenever $w(k)/k$ is decreasing in k,[9] and Φ equals zero at $\underline{\tau}$ since $\lim_{k\to\infty} w(k)/k = 0$. Hence, net labor income $\Phi(\tau)(1-\tau)$ equals zero at $\tau = \underline{\tau}$ and at $\tau = 1$, and is positive in between. I assume that net labor income is a unimodal function of the tax rate:

Assumption 1: There exists a unique tax rate τ^* maximizing net labor income $\Phi(\tau)(1-\tau)$. Net labor income is strictly increasing for $\tau \in (\underline{\tau}, \tau^*)$ and strictly decreasing for $\tau \in (\tau^*, 1)$.

It can be verified that Assumption 1 is satisfied for all CES production technologies. To save notation, express the gross interest rate also as a function of the tax rate:

$$R(\tau) \equiv 1 - \delta + f'(k(\tau)).$$

Evidently, the interest rate is increasing in the tax rate (which is tantamount to spending).

I assume that consumers' utility $u(c_t, c_{t+1})$ is a linear homogenous, quasi-concave and differentiable function of consumption in both lifetime periods. Thus, an individual born in period t with first-period income I_t who faces an interest rate R_{t+1} saves $s(R_{t+1})I_t$ where the savings rate $s(R_{t+1})$ is independent of income. The indirect utility of that individual can be written $v(R_{t+1})I_t$ where $v(R) \equiv u(1 - s(R), s(R)R)$. Note that savings need not be increasing in the interest rate (this is only the case when the substitution effect dominates the income effect), but that indirect utility is strictly increasing in the interest rate, i.e. $v' > 0$. With aggregate net labor income given by Eq. (6), capital accumulates according to

$$K_{t+1} = s(R(\tau_{t+1}))\Phi(\tau_t)(1-\tau_t)K_t. \tag{7}$$

Of course, the economy will only grow at a positive rate if s and/or Φ are big enough.[10] The utility of generation t is[11]

$$u_t = v(R(\tau_{t+1}))\Phi(\tau_t)(1-\tau_t)K_t. \tag{8}$$

[9] $w(k)/k$ is (globally) decreasing under the assumption that the elasticity of factor substitution is not too small; otherwise, it is decreasing only when k is large enough.

[10] Note that the old generation sells the capital stock $(1-\delta)K_t$ from the previous period to the young generation. It is implicitly embodied in the right-hand side of Eq. (7).

[11] More precisely, u_t/L_t is the utility of each individual belonging to generation t, but since all individuals of each generation are identical, this distinction is of no importance.

3. Sequential majority voting

I now assume that policy is decided sequentially by governments who are elected by a majority of votes. Assuming positive population growth, the (identical) young individuals have a majority of votes at each point in time. Thus, government at time t aims to maximize utility of generation t which is given by Eq. (8).

The modelling of sequential voting in this economy entails two complications. First, the policy decided by generation t affects the return on capital which is owned by generation $t-1$. Under the assumption of rational expectations, generation $t-1$ forecasts the capital return correctly when it decides its savings. In particular, savings in $t-1$ and thereby the capital stock in period t depend on the policy decision of period t. On the other hand, when generation t votes on τ_t, the capital stock is already in place, and policy deviations cannot alter it anymore. Therefore, voters in period t take the capital stock K_t as given. Since voters' utility can be expressed as $u_t = \Pi(\tau_{t+1}, \tau_t) K_t$ where

$$\Pi(\tau_+, \tau) \equiv v(R(\tau_+))\Phi(\tau)(1-\tau) \tag{9}$$

is the "stage payoff", generation t aims to maximize the stage payoff $\Pi(\tau_{t+1}, \tau_t)$ irrespective of the size of the (given) capital stock K_t. Hence, the economy is described by a dynamic game between an infinity of governments representing generations whose payoff functions are $\Pi(\tau_{t+1}, \tau_t)$. Once a solution of this game is determined, the capital stock evolves according to Eq. (7), and this growth path determines utility of all generations according to Eq. (8).

Second, when policy is decided, the government needs to predict rationally the effect of current policy on future policy decisions. In general, policy in period $t+1$ may be any function of the past policy history $(\tau_t, \tau_{t-1}, \ldots, \tau_0)$. Allowing for such history-dependent policy functions, the set of subgame perfect equilibria may be highly indeterminate, since many outcomes can be enforced by appropriate punishment strategies.[12] To plausibly limit the set of potential equilibria, I restrict attention to recursive equilibria in which policy may depend only on last period's policy decision. Hence, I consider Markovian policy functions of the form $\tau_{t+1} = \psi(\tau_t)$ and assume that the functional relationship ψ does not change over time.[13]

The problem of the government in period t is to maximize its objective function $\Pi(\tau_{t+1}, \tau_t)$ under the prediction that next period's government adjusts taxes according to the policy function $\tau_{t+1} = \psi(\tau_t)$. In a stationary situation, this prediction is self-fulfilling whenever government t finds it optimal to decide policy according to the same policy function ψ. That is, the optimal decision of government t is given by $\tau_t = \psi(\tau_{t-1})$ for any

[12] Such folk-theorem results in games with overlapping generations of players have been proven by Kandori (1992) and Smith (1992).

[13] This approach follows Krusell et al. (1997) and Krusell and Rios-Rull (1999) who assume that policy depends on last period's policy decision and on a set of minimal state variables. Such state variables (for example, the capital stock) do not need to enter here for the reasons indicated in the previous paragraph.

given past policy decision τ_{t-1}. Formally, a *stationary policy function* is a function ψ:[0,1] → [0,1] that satisfies

$$\psi(\tau_0) \in \underset{0 \leq \tau \leq 1}{\operatorname{argmax}}\ \Pi(\psi(\tau), \tau) \qquad \forall \tau_0 \in [0, 1].$$

A *politico-economic equilibrium* (PEE) is a sequence of tax rates $(\tau_t)_{t \geq 0}$ generated by a stationary policy function ψ, i.e. which satisfies $\tau_{t+1} = \psi(\tau_t)$ for all $t \geq 0$ and $\tau_0 \in \operatorname{argmax} \Pi(\psi(\tau),\tau)$. A PEE $(\tau_t)_{t \geq 0}$ is stationary if $\tau_{t+1} = \tau_t$ for all $t \geq 0$.

Since the payoff function does not explicitly depend on past policy, one may wonder why a policy function should depend on past policy at all. If every policy decision is independent of past policy decisions, the incumbent government need not take the reaction of the future government into account and would simply set a constant tax rate in each period. I would, however, argue that it is plausible to assume that policy should depend on the previous period's policy outcome. When policy in some period is decided by a majority of votes of the young generation, the old generation is still alive and its past policy decision may well affect the current policy outcome. Section 5 discusses explicitly how the policy outcome is partly determined by the old generation when it has the right to veto policy changes. The model may also be augmented by lagged effects of government spending on productivity. In this case, past policy decisions would explicitly enter the payoff function and they should enter policy functions as well. Whether or not multiplicity of equilibrium would be removed remains an open issue.

In what follows, I show that there exist many stationary policy functions that give rise to a large set of PEE. In particular, there is a continuum of stationary PEE, some of which are Pareto-dominated by other PEE, and there are cyclical solutions of any arbitrary order. It is obvious from the definition that any policy function which is nontrivial (i.e. which depends on the past by assigning different outcomes to different past policies) must be such that governments are indifferent between all possible policy outcomes. Policy functions will therefore be closely related to indifference curves of the payoff function Π. On the one hand, $\Pi(\cdot,\tau)$ is strictly increasing in next period's policy since young generations benefit from higher spending in the future which raises the return on capital. Assumption 1 implies, on the other hand, that $\Pi(\tau_+,\cdot)$ has a unique maximum at τ^* for any future policy level τ_+. Hence, in (τ,τ_+) space, the indifference curves of Π have a unique minimum at $\tau = \tau^*$, as shown in Fig. 1.

Since $\Pi(\tau_+, 0) = \Pi(\tau_+,1) = 0$ for any $\tau_+ \in [0,1]$, all indifference curves of positive payoff level leave the unit square $(\tau,\tau_+) \in [0,1]^2$ twice at the boundary $\tau_+ = 1$. For convenience, I assume that indifference curves cut the diagonal at most twice. This is an implication of the following assumption.

Assumption 2: The function $\Pi(\tau,\tau)$ has a unique maximum at $\hat{\tau} \in (0,1)$, is strictly increasing for $\tau \in (\underline{\tau},\hat{\tau})$ and strictly decreasing for $\tau \in (\hat{\tau},1)$.

Clearly, the tax rate maximizing the stationary payoff $\Pi(\tau,\tau)$ exceeds the tax rate maximizing payoff for given future policy $\Pi(\tau_+,\cdot)$, i.e. $\hat{\tau} > \tau^*$. The reason is that voters benefit from higher government spending in the next period which is financed by a tax on the next generation. Assumption 2 implies that indifference curves intersect the diagonal at most twice. Let $\pi_0 = \Pi(0,\tau^*)$ and $\hat{\pi} = \Pi(\hat{\tau},\hat{\tau})$. Then for any payoff level $\pi \in [\pi_0,\hat{\pi})$, the

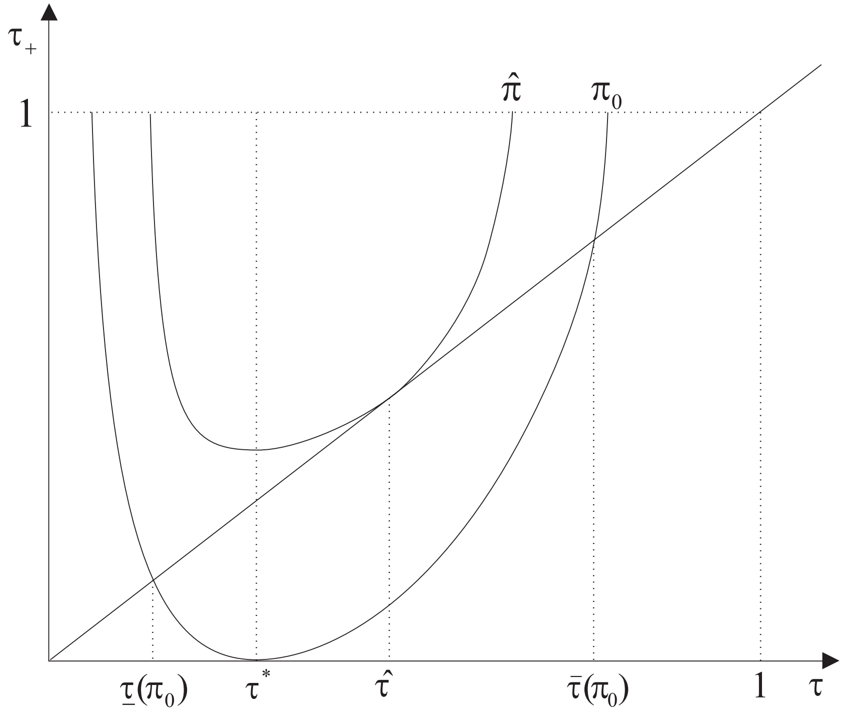

Fig. 1. Indifference curves of $\Pi(\tau_{+},\tau)$.

indifference curve at level π cuts the diagonal at two tax rates denoted as $\underline{\tau}(\pi) < \hat{\tau} < \bar{\tau}(\pi)$ (see Fig. 1).

For any $\pi \geq \pi_0$, denote by $\tau_{+} = G_{\pi}(\tau)$ the explicit solution of the indifference curve $\Pi(\tau_{+},\tau) = \pi$ whenever $\tau_{+} \leq 1$. Since policy makers must be indifferent between different policy outcomes, stationary policy functions coincide with an indifference curve (parameterized by the maximum payoff π) on the interval of past policy outcomes. That is, $\psi(\tau) = G_{\pi}(\tau)$ whenever τ was the policy outcome in the period before. I need to make sure that iterations of ψ also yield optimal policy outcomes in all future periods. This requires two restrictions. First, the policy attaining the payoff level π cannot exceed $\bar{\tau}(\pi)$ since otherwise the sequence of future policies $\psi^k(\tau)$ would yield an unbounded (and eventually infeasible) sequence of tax rates. Therefore, policy functions parameterized by the maximum payoff level π are given by

$$\psi(\tau) = \min(G_{\pi}(\tau), \bar{\tau}(\pi)). \tag{10}$$

Second, if τ^* was the policy outcome in the period before, the policy decision today, $\psi(\tau^*)$, and in the next period, $\psi^2(\tau^*)$, should not exceed the maximum feasible tax rate $\bar{\tau}(\pi)$ as well. This is guaranteed for the payoff level π as shown in Fig. 2, and it will also be true for all higher payoff levels $\pi \geq \tilde{\pi}$.

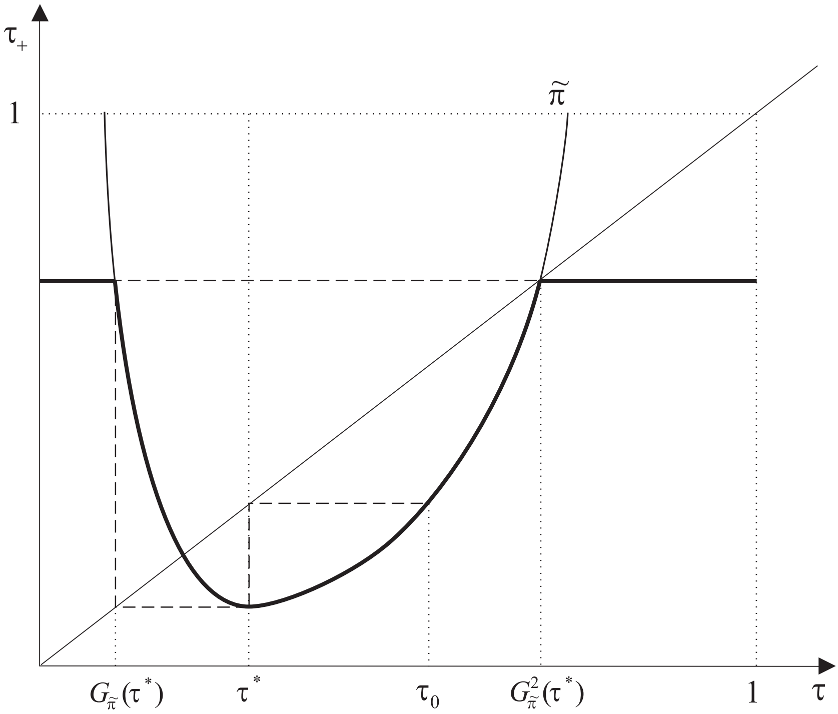

Fig. 2. The stationary policy function (10) of level $\tilde{\pi}$.

Hence, whenever $\pi\in[\tilde{\pi},\hat{\pi}]$, there exists a stationary policy function attaining the payoff level π which is given by Eq. (10). This is stated in the following proposition which is formally proven in Appendix A.

Proposition 1: *There exists* $\tilde{\pi}$ *and a continuum of stationary policy functions* ψ *as defined by* Eq. (10) *which are parameterized by* $\pi\in[\tilde{\pi},\hat{\pi}]$. *For any such* ψ, *every sequence* $(\tau_t)_{t\geq 0}$ *that satisfies* $\tau_{t+1}=\psi(\tau_t)$ *for all* t *and* $\tau_0\in\arg\max\,\Pi(\psi(\tau),\tau)$ *is a PEE. In particular, every* $\tau\in[\underline{\tau}(\tilde{\pi}),\bar{\tau}(\tilde{\pi})]$ *is a stationary PEE.*

The dynamics of policy and consequently the economy's growth path are highly indeterminate.[14] The policy path (and thus the growth path) depends on the expectations of current policy makers regarding future policy outcomes. There are many self-fulfilling expectations which are parameterized by the payoff parameter π. Pessimistic governments would achieve a lower stage payoff than optimistic ones. I will show in the next section how these different beliefs relate to growth and utility of all generations. I will also show that not all PEE converge to some long-run policy path but some may even fluctuate

[14] As mentioned before, a stronger indeterminacy result is easily obtained if time-dependent policy functions are considered. In fact, any policy sequence yielding payoffs of at least π in each period is a subgame perfect equilibrium. To see this, consider an arbitrary sequence $(\tau_t)_{t\geq 0}$ that satisfies $\Pi(\tau_{t+1},\tau_t)\geq\tilde{\pi}$, $t\geq 0$. Any deviation from this path (say in period t to some $\tau_t'\neq\tau_t$) can be credibly punished by successors $t+k$, $k\geq 1$, choosing $\tau'_{t+k}=G^k_{\tilde{\pi}}(\tau_t')$ which pays off $\tilde{\pi}$ to all generations $t+i$, $i\geq 0$.

forever. Thus, the economy's growth path depends very much on policy expectations and on policy history.

4. Growth, efficiency and cycles

This section shows how the PEE of Proposition 1 relate to growth and utility of all generations. Some PEE turn out to be inefficient in the sense that they are Pareto-dominated by other PEE. Subsequently, it will be shown that not all PEE converge to a stationary equilibrium, but that there exist permanently fluctuating PEE which may be cyclical of any arbitrary order or even irregular.

Note first that none of the PEE is first-best Pareto-efficient since the level of productive government spending is necessarily too low. Pareto efficiency requires that the net output $F(K_t,A_tL_t)-G_t$ is maximized in every period. Using (Eq. (3)), this implies the first-order condition $F_L(K_t,aG_t)a=1$ or, equivalently, $G_t=w_tL_t$. This condition corresponds to the natural efficiency condition ("Barro rule") that the output share of government spending equals the share it would get if public services were supplied competitively (see Barro, 1990). This share equals the wage share in this specification of the model.[15] Provided that only labor income is taxed, the efficiency condition requires that all labor income must be taxed away which implies zero growth and output in all future periods. A Pareto optimal path with positive growth can only be decentralized by appropriate lump-sum taxation of young *and* old generations. A proportional tax on income (or another tax on capital income) would be distortionary, however, and would not lead to Pareto optimality, though a shift from labor to capital taxes might increase the growth rate if the interest elasticity of savings is not too large (see Uhlig and Yanagawa, 1996).

On the other hand, it is worth mentioning that any growth path of the model (as in other models of endogenous growth) is dynamically efficient in the sense that it is not possible to raise consumption in all periods by just "eating" a fraction of the capital stock. However, as shown in a recent paper by Wigger (2001), the impossibility of dynamic inefficiency in models of endogenous growth does not preclude the possibility of Pareto-improving intergenerational transfers.

In the following, I rule out other policy instruments such as capital taxation or intergenerational transfers, and turn to the second-best analysis within the policy regime in which labor taxation is the only available policy instrument. I restrict attention to stationary PEE which have $\tau_t=\tau$ with $\tau\in[\underline{\tau}(\tilde{\pi}),\bar{\tau}(\tilde{\pi})]$ for all $t\geq 0$, and ask whether these equilibria can be Pareto-ranked. The utility of generation $t\geq 0$ is

$$\Pi(\tau,\tau)K_t=\Pi(\tau,\tau)(s(R(\tau))\Phi(\tau)(1-\tau))^tK_0.$$

Furthermore, there is an initial old generation (generation -1) holding the initial capital stock and whose utility (=consumption in period 0) is $R(\tau)K_0$. Clearly, generation -1

[15] Barro (1990) does not explicitly consider labor as a factor of production, but interprets instead the capital stock to include human capital, so that the optimal government size would be considerably smaller. Section 6 proposes an extension of the model with the same feature.

only benefits from higher taxes, generation 0 benefits from higher stage payoff $\Pi(\tau,\tau)$ which is maximal at $\hat{\tau}$, and all future generations benefit from a higher stage payoff *and* from a higher growth rate $s(R(\tau))\Phi(\tau)(1-\tau)$. Let τ_g denote the tax rate maximizing the growth rate.

It is clear that all tax rates below $\tau_P \equiv \min(\hat{\tau},\tau_g)$ are Pareto-dominated by the tax rate τ_P since all generations prefer the tax rate τ_P to any lower tax rate. All tax rates greater than or equal to τ_P cannot be Pareto-ranked, however. Generations far in the future benefit from tax rates close to τ_g whereas generation 0 benefits most from the tax rate $\hat{\tau}$. Finally, generation -1 benefits from tax rates greater than $\max(\tau_g,\hat{\tau})$. Whenever savings are increasing in the interest rate, the growth-maximizing tax τ_g exceeds the tax rate maximizing net labor income, τ^*. Thus, $s' \geq 0$ implies that $\tau_g \geq \tau^*$, and therefore $\tau_P \geq \tau^*$. In particular, all stationary PEE below τ^* are Pareto-dominated by other PEE which have higher public spending and higher growth.

Intuitively, this inefficiency result rests on pessimistic self-fulfilling policy expectations of voters. When the economy is stuck in an inefficient equilibrium $\tau < \tau^*$, voters believe that an expansion of public spending and taxation (to τ^*, say) would be followed by a cut in public spending in the next period. This belief turns out to be self-fulfilling since the next generation (who cuts public spending) trusts in an expansion of public spending in the subsequent period.

Thus, underlying this inefficiency are "cyclical" expectations regarding future policy. Such pessimistic expectations even give rise to volatile sequences of fiscal policy which do not converge to a stationary equilibrium. Indeed, a straightforward application of the theorem of Li and Yorke (1975) implies that there exist cyclical sequences of any arbitrary order, as well as irregular policy sequences. To see this, consider the policy function corresponding to the lowest possible payoff level $\tilde{\pi}$, and consider the pre-image of τ^* to the right of τ^*, denoted as τ_0 (see Fig. 2). Then the following condition is satisfied:

$$\psi^2(\tau_0) < \psi(\tau_0) = \tau^* < \tau_0 < \psi^3(\tau_0) = \bar{\tau}(\tilde{\pi}),$$

and, by the theorem of Li and Yorke, there exist infinitely many cycles of arbitrary periodicity, as well as an uncountable set of irregular (chaotic) solutions. Moreover, the actual policy path depends on the initial policy decision which is also indeterminate since there are many optimal policies.

The following proposition summarizes the findings of this section.

Proposition 2:

(a) If savings are non-decreasing in the interest rate, there are stationary PEE which are Pareto-dominated by other PEE. In these Pareto-dominated PEE, government spending and growth are too low.

(b) There exist cyclical PEE of any arbitrary order and an uncountable set of irregular PEE. The actual policy (and growth) path not only depends on the free parameter π representing policy expectations, but also on history (i.e. the inital policy decision).

5. Constitutional rules

The possibility of inefficient outcomes of the political decision process is based on the inability of the incumbent government to commit future governments to a particular fiscal policy. Pessimistic expectations of future policy outcomes lead the current government to choose an inefficiently low level of public spending, and such pessimistic beliefs can even lead to volatile sequences of fiscal policy.

Therefore, the question arises whether the current government can commit future fiscal policy makers to a certain policy in order to avoid an inefficiently low policy outcome. A possibility providing partial commitment of policy decisions is a constitutional system under which "large" fiscal policy adjustments require not only a simple majority of voters but the approval by a qualified majority. Following Azariadis and Galasso (1996), I introduce a "constitutional rule" which gives the majority of voters the right to propose fiscal policy, but which allows the minority to veto large fiscal policy changes.

More specifically, suppose that in every period the majority (i.e. the young generation) makes a fiscal policy proposal p_t. An ε-constitutional rule gives the minority the right to veto the policy proposal whenever it deviates from the previous period's policy decision τ_{t-1} by more than ε percent. Thus, a simple majority is sufficient to enforce small policy adjustments, whereas larger adjustments require the approval of a wider majority. If a policy proposal is rejected, the policy of the previous period is maintained. Hence, in each period, the political decision process follows two stages: in the first stage, the young generation proposes p_t, and in the second stage, the old generation has the right to choose

$$\tau_t \in \{\tau_{t-1}, p_t\} \quad \text{whenever} \quad p_t \notin [\tau_{t-1}(1-\varepsilon), \tau_{t-1}(1+\varepsilon)],$$

whereas $\tau_t = p_t$ whenever $p_t \in [\tau_{t-1}(1-\varepsilon), \tau_{t-1}(1+\varepsilon)]$.

Since old voters always prefer higher spending and taxes, they veto any tax proposal below $\tau_{t-1}(1-\varepsilon)$, whereas they accept (or have to accept) any other proposal. Anticipating this behavior, the majority predicts correctly that all proposals above $\tau_{t-1}(1-\varepsilon)$ (but only those) are enforceable. Hence, the constitutional rule implies that the young generation decides $\tau_t \geq \tau_{t-1}(1-\varepsilon)$ directly.

It is intuitively clear that this constitutional system provides partial commitment. Every generation knows that it needs not fear large spending cuts in the future and is therefore willing to expand public spending today. Provided that the constitution rule is strict enough (ε is low enough), this removes all inefficiencies and fluctuations.

Formally, a policy function $\psi:[0,1] \rightarrow [0,1]$ under the ε-constitutional rule satisfies

$$\psi(\tau_0) \in \underset{\tau_0(1-\varepsilon) \leq \tau \leq 1}{\operatorname{argmax}} \ \Pi(\psi(\tau), \tau) \qquad \forall \tau_0 \in [0, 1].$$

In particular, every policy function satisfies $\psi(\tau) \geq \tau(1-\varepsilon)$ for all $\tau \in [0,1]$. The lowest payoff level must be higher under the constitutional rule: since the successive government's policy is constrained by $\psi(\tau) \geq \tau(1-\varepsilon)$, a generation that starts from a low level of public spending (for example, from $\tau_0 = 0$) can guarantee a payoff of at least $\hat{\pi}_\varepsilon \equiv \Pi(\tau(1-\varepsilon), \tau)$. Hence only maximum payoff levels $\pi \geq \hat{\pi}_\varepsilon$ are achievable. Either the economy starts from a sufficiently low level of policy and is able to achieve the maximum payoff π immediately, or the inherited policy τ_0 is too high so that the incumbent government decides to cut policy to the lowest possible level $\tau_0(1-\varepsilon)$. All policy functions of Lemma 3 with $\pi \geq \hat{\pi}_\varepsilon$ remain policy functions under the constitutional rule if they are appropriately augmented at large tax rates. Specifically, for any $\pi \in [\hat{\pi}_\varepsilon, \hat{\pi}]$, there is a policy function under the ε-constitutional rule which is given by

$$\psi(\tau) = \begin{cases} \min(G_\pi(\tau), \bar{\tau}(\pi)) & , \quad \text{if } \tau(1-\varepsilon) \leq \bar{\tau}(\pi), \\ \tau(1-\varepsilon) & , \quad \text{if } \tau(1-\varepsilon) > \bar{\tau}(\pi). \end{cases}$$

Fig. 3 shows the policy function with the lowest possible payoff level $\hat{\pi}_\varepsilon$. Clearly, all stationary PEE must be in the interval $[\underline{\tau}(\hat{\pi}_\varepsilon), \bar{\tau}(\hat{\pi}_\varepsilon)]$. The constitutional rule does not completely eliminate indeterminacy, but as $\varepsilon \to 0$, the set of stationary PEE shrinks to the single stationary equilibrium at $\hat{\tau}$. Further, if ε is sufficiently small (i.e. if the constitutional rule is strict enough), there are no fluctuations and all PEE converge to some stationary PEE.

The resulting growth rate is generally below the maximum level that would be achieved at τ_g. Earlier generations benefit at the expense of future generations, either because of too high or too low policy levels. Whether spending and taxes are too high or too low relative to the growth-maximizing policy level depends on the interest elasticity of savings. As is shown in Lemma 4 of Appendix A, the growth-maximizing tax τ_g is smaller than the stationary policy under the strict constitutional rule $\hat{\tau}$ if and only if $s'(R)R/s(R) < s(R)$ at $R = R(\tau_g)$. Thus, whenever the interest elasticity of savings is lower than the savings rate, growth is too low due to excessive spending and taxation.[16] Intuitively, whenever the interest elasticity of savings is not too large, the growth-enhancing effect of future spending is lower than the positive effect on the utility of voters. Therefore, the constitutional rule enforces incumbent governments to decide a too high level of taxation and spending relative to the growth-maximizing level. This result contrasts to Barro (1990) who shows (in an infinite-horizon framework) that the utility maximizing policy coincides with the growth-maximizing policy only if the production function is Cobb–Douglas.[17] In the model of sequential policy making (and a strict constitutional rule), the utility maximizing policy of each generation corresponds to the growth-maximizing policy only when the interest elasticity of savings coincides with the savings rate. However, if the interest elasticity of savings is lower, the policy leads to excessively high spending and taxation relative to the policy path of maximum growth. On the other hand, when $\tau_g < \hat{\tau}$ and $\varepsilon > 0$ is sufficiently small, no stationary PEE is Pareto-dominated by other

[16] Clearly, if the intertemporal utility function is Cobb–Douglas, savings are independent of the interest rate and this condition is satisfied. Several empirical studies find negative or insignificantly small values of the interest elasticity of savings. See Uhlig and Yanagawa (1996) for a simple calibration exercise showing that a condition like ours may well be satisfied.

[17] For more general production functions, maximization of growth and utility yield different policy outcomes, depending on the elasticity of substitution between capital and government spending.

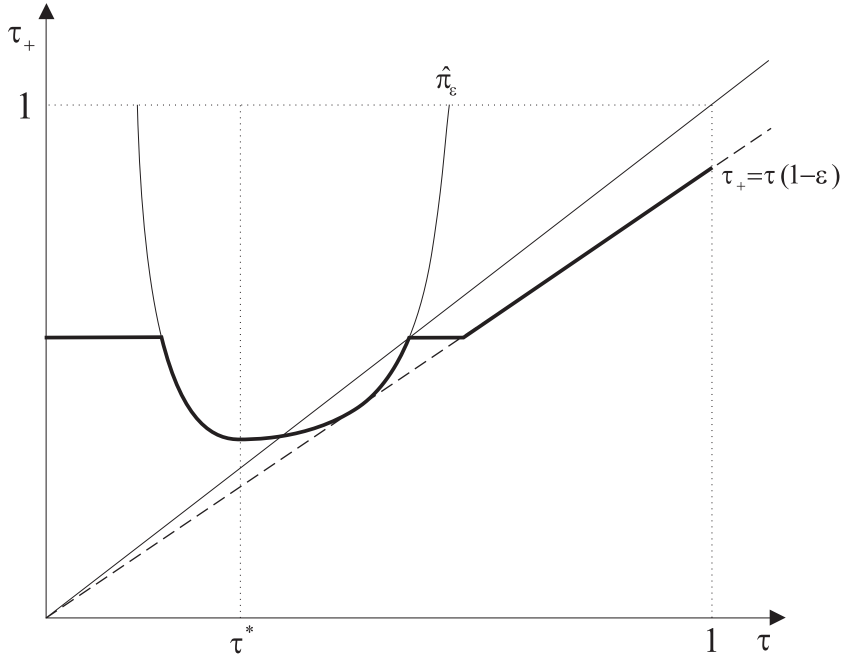

Fig. 3. A stationary policy function under the constitutional rule.

policy outcomes, but earlier generations benefit at the expense of future generations. Hence I can state the following:

Proposition 3: *If ε is sufficiently small, all PEE converge to a stationary equilibrium close to $\hat{\tau}$ If $s'(R)R/s(R) < s(R)$ is satisfied, $\hat{\tau} > \tau_g$. In this case, no stationary PEE (for ε sufficiently small) is Pareto-dominated by other feasible policy outcomes, but growth is too low due to excessive spending and taxation.*

It is valuable to consider the limit case of a strict constitutional rule $\varepsilon = 0$ in which every policy adjustment requires the approval of a qualified majority. Such a constitutional rule has been considered by Azariadis and Galasso (1996) to study the political economy of social security in a pure-exchange overlapping generations model. Similar to the economy in this paper, the old generation benefits from a larger size of the government and would veto any tax/transfer cut. On the other hand, the rule provides commitment to the young generation when it decides transfers in the beginning. Starting from a low transfer level, all PEE converge to the golden-rule level of transfers.

A similar outcome obtains in this economy for $\varepsilon = 0$: starting from low levels of taxation, all PEE converge to the stationary PEE at $\hat{\tau}$. On the other hand, once a higher tax rate is in place, the economy would stick to it since the old generation can veto any tax cut. Such an excessive policy might be the outcome after certain fundamental parameters have changed. If there is some flexibility, however, at least small tax cuts can be enforced and all such excessive equilibria disappear.

6. Conclusions and extensions

I have analyzed a growth model in which productive government spending is decided endogenously and sequentially by overlapping generations of voters. It turns out that there is a multiplicity of politico-economic equilibria that are due to self-fulfilling policy expectations. This multiplicity even emerges when policy functions are restricted to be Markovian. Some of these equilibria involve endogenous fluctuations, and other equilibria are characterized by an inefficiently low level of government size.

Political mechanisms providing (partial) commitment (such as a constitutional rule) can reduce the multiplicity largely and remove inefficiencies and fluctuations. Nevertheless, the growth rate is typically below the maximally achievable growth, so that earlier generations benefit at the expense of future generations. Furthermore, for low interest elasticities of savings (which are compatible with stylized facts), the government size exceeds its growth-maximizing level.

I discuss briefly whether these conclusions remain valid in variations of the model. Detailed derivations are available from the author on request. First, results are the same if there is a tax rate on capital income that is lower than the labor income tax rate. That is, if capital income is taxed at rate $\tau_{Kt} = \lambda\tau_t$ where $\lambda < 1$, the policy level maximizing net labor income is lower than the policy level maximizing net capital income, as in the model of Section 2. Again it follows that there is a continuum of stationary PEE, some of which are inefficient. A constitutional rule would eliminate the inefficiencies, but would lead to a policy above the growth-maximizing level. Second, the model can easily be augmented by a learning-by-doing externality as a second source of growth. This would imply that growth would be positive even under laissez-faire. If the impact of the knowledge-externality is not too strong relative to the impact of productive government spending, there exists again a unique tax rate τ^* maximizing net labor income, and the results would be the same.

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Appendix A

Proof of Proposition 1: Proposition 1 is demonstrated by deriving three lemmas. The first lemma is a straightforward reformulation of the definition of a stationary policy function. It states that every stationary policy function maps all past policies into optimal policy outcomes, and that payoffs at all optimal policy outcomes coincide. Hence, the graph of a stationary policy function coincides with an indifference curve of Π at all optimal policies.

Lemma 1: *A function ψ:[0,1] → [0,1] is a stationary policy function if and only if there exists a maximum payoff level π such that*

$$\psi(\tau) \le G_\pi(\tau) \quad \text{and} \quad \psi^2(\tau) = G_\pi(\psi(\tau)) \quad \forall \tau \in [0,1].$$

Proof: From the definition of a stationary policy function follows immediately that a function ψ is a stationary policy function if and only if there exists a maximum payoff level $\pi \in \mathbb{R}$ such that

$$\Pi(\psi(\tau),\tau) \le \pi \quad \text{and} \quad \Pi(\psi^2(\tau),\psi(\tau)) = \pi \quad \forall \tau \in [0,1].$$

These two conditions state that π is the maximum payoff level and that $\psi(\tau)$ is an optimal policy for any past policy τ. Using the explicit solution of the indifference curve, G_π, the claim follows. □

Lemma 1 implies that after one iteration of ψ, ψ and G_π coincide. From the shape of the indifference curves follows that no tax rate above the upper stationary tax rate $\bar{\tau}(\pi)$ can be a feasible policy outcome since this would require ever higher (and eventually infeasible) tax rates in the future.

Lemma 2: *If ψ is a stationary policy function with $\pi = \max \Pi(\psi(\tau),\tau)$, then $\psi(\tau) \le \bar{\tau}(\pi)$ for all $\tau \in [0,1]$.*

Proof: Suppose that ψ is a stationary policy function such that $\pi = \max \Pi(\psi(\tau),\tau)$ and that there exists a τ such that $\psi(\tau) = \tau_1 > \bar{\tau}(\pi)$. From Lemma 1 follows that $\psi(\tau_1) = G_\pi(\tau_1)$ and, by the same argument, all iterations of ψ and G_π coincide: $\psi^k(\tau_1) = G_\pi^k(\tau_1)$, $k > 1$. As Fig. 1 shows, such a policy path cannot be feasible since tax rates would eventually exceed unity. Ever higher (and eventually infeasible) tax rates are required to sustain the initial tax rate τ_1 as a policy outcome. Hence ψ cannot be a stationary policy function. □

Lemmas 1 and 2 state that the stationary policy function ψ with payoff level π is bounded above by G_π and by $\bar{\tau}(\pi)$. Since G_π attains its minimum at τ^* and since (from Lemma 1) $\psi(\tau^*) \le G_\pi(\tau^*)$, $G_\pi(\tau^*)$ is an upper bound on the minimum feasible policy outcome. Moreover, for $\psi(\tau^*)$ to be a feasible policy outcome, next period's policy $\psi^2(\tau^*)$ must fall short of the upper bound $\bar{\tau}(\pi)$. As will be shown in the following lemma, a necessary condition for this to be the case is that $G_\pi^2(\tau^*) \le \bar{\tau}(\pi)$. Moreover, this condition is also sufficient for the existence of a stationary policy function.

Lemma 3: *Let $\pi \in \mathbb{R}$. There exists a stationary policy function ψ such that $\pi = \max \Pi(\psi(\tau),\tau)$ if and only if $\pi \in [\pi_0, \hat{\pi}]$ and*

$$G_\pi^2(\tau^*) \le \bar{\tau}(\pi). \tag{11}$$

Whenever Eq. (11) is satisfied, a stationary policy function is given by Eq. (10).

Proof: Suppose first that ψ is a stationary policy function yielding payoff π. Then clearly $\pi \ge \pi_0$ since $\Pi(\psi(\tau^*),\tau^*) \ge \Pi(0,\tau^*) = \pi_0$. Moreover, $\pi \le \hat{\pi}$ since otherwise any sequence of policy outcomes would be strictly increasing and eventually become infeasible. Suppose that Eq. (11) is violated, $G_\pi(\tau^*) > \bar{\tau}(\pi)$. This implies that $G_\pi(\tau^*) < \tau^*$ (otherwise Eq. (11) would be fulfilled, see Fig. 1). Hence, since $\psi(\tau^*) \le G_\pi(\tau^*)$ (because of Lemma 1) and since G_π is decreasing on $[\psi(\tau^*), G_\pi(\tau^*)]$, it follows that $G_\pi(\psi(\tau^*)) \geq G_\pi^2(\tau^*)$. However, this contradicts $\psi \leq \bar{\tau}(\pi)$ since

$$\psi^2(\tau^*) = G_\pi(\psi(\tau^*)) \geq G_\pi^2(\tau^*) > \bar{\tau}(\pi).$$

Conversely, suppose that Eq. (11) is satisfied and consider the policy function (10). To show that ψ is a stationary policy function, I check that the conditions in Lemma 1 are satisfied. Evidently, the first inequality $\psi \leq G_\pi$ holds by definition of ψ. Take now some $\tau \in [0,1]$ and suppose first that $\psi(\tau) = \bar{\tau}(\pi) \leq G_\pi(\tau)$. Then, by definition of ψ, $\psi^2(\tau) = \bar{\tau}(\pi) = G_\pi(\psi(\tau))$. Hence, the second condition of Lemma 1 is satisfied. Suppose second that $\psi(\tau) = G_\pi(\tau) \leq \bar{\tau}(\pi)$. To show that the second condition of Lemma 1 is satisfied, it suffices to show that

$$G_\pi^2(\tau) \leq \bar{\tau}(\pi) \tag{12}$$

holds. Indeed, when Eq. (12) holds, the definition of ψ implies that $\psi(G_\pi(\tau)) = G_\pi^2(\tau)$ and therefore

$$\psi^2(\tau) = \psi(G_\pi(\tau)) = G_\pi^2(\tau) = G_\pi(\psi(\tau)).$$

To show Eq. (12), notice that $G_\pi(\tau) \in [G_\pi(\tau^*), \bar{\tau}(\pi)]$. Suppose first that $G_\pi(\tau) \in [\tau^*, \bar{\tau}(\pi)]$. Then $G_\pi^2(\tau) \leq G_\pi(\bar{\tau}(\pi)) = \bar{\tau}(\pi)$ since G_π is increasing on $[\tau^*, \bar{\tau}(\pi)]$. Suppose second that $G_\pi(\tau) \in [G_\pi(\tau^*), \tau^*]$. Then $G_\pi^2(\tau) \leq G_\pi^2(\tau^*) \leq \bar{\tau}(\pi)$ by Eq. (11) and since G_π is decreasing on $[G_\pi(\tau^*), \tau^*]$. This completes the proof. □

For the policy function (10), the set of optimal policies is the interval $I = \{\tau : G_\pi(\tau) \leq \bar{\tau}(\pi)\}$. Every tax rate within this interval yields the same payoff π. Condition (11) guarantees that the interval I remains invariant under the policy function: for every past policy outcome $\tau \in I$, the actual policy decision is optimal, i.e. $\psi(\tau) \in I$. Hence every government behaves according to policy function ψ whenever it expects successors to do the same.

Now I complete the proof of Proposition 1. Note first that condition (11) is satisfied whenever $G_\pi(\tau^*) \geq \tau^*$. Hence, Eq. (11) holds for any $\pi \in [\pi^*, \hat{\pi}]$ where $\pi^* \equiv \Pi(\tau^*, \tau^*)$. In particular, $G_{\pi^*}^2(\tau^*) = \tau^* < \bar{\tau}(\pi^*)$. On the other hand, Eq. (11) is not satisfied at $\pi_0 = \Pi(0, \tau^*)$ since $G_{\pi 0}(\tau^*) = 0$, and thus $G_{\pi 0}^2(\tau^*) = 1 > \bar{\tau}(\pi_0)$. Since both $\bar{\tau}(\cdot)$ and $G_\cdot^2(\tau^*)$ are continuous functions of π by the implicit function theorem, there exists a $\tilde{\pi} \in (\pi_0, \pi^*)$ such that Eq. (11) holds with equality at $\pi = \tilde{\pi}$ and is satisfied strictly for all $\pi \in [\tilde{\pi}, \pi^*]$ (see Fig. 2). Lemma 3 implies that for any $\pi \in [\tilde{\pi}, \hat{\pi}]$, there exists a stationary policy function as given by Eq. (10). Moreover, no payoffs outside this interval can be achieved by a stationary policy function. □

Lemma 4: *$\tau_g < \hat{\tau}$ if and only if $((s'(R)R)/s(R)) < s(R)$ at $R = R(\tau_g)$.*

Proof: Let $\Phi_n(\tau) \equiv \Phi(\tau)(1-\tau)$ denote net labor income. Since τ_g maximizes growth, the F.O.C.

$$s'(R(\tau_g))R'(\tau_g)\Phi_n(\tau_g) + s(R(\tau_g))\Phi_n'(\tau_g) = 0 \tag{13}$$

is satisfied. Assumption 2 implies that $\tau_g < \hat{\tau}$ if and only if $(d/d\tau)\Pi(\tau,\tau)>0$ at $\tau=\tau_g$ which means

$$v'(R(\tau_g))R'(\tau_g)\Phi_n(\tau_g) + v(R(\tau_g))\Phi_n'(\tau_g) > 0.$$

Using Eq. (13) and $R'>0$, $\Phi_n>0$, this condition is equivalent to

$$\frac{s'(R)R}{s(R)} < \frac{v'(R)R}{v(R)} \quad \text{at } R = R(\tau_g). \tag{14}$$

From the definition of v and the optimality condition $u_1 = u_2R$ follows that $v' R/v = u_2sR/u$. Furthermore, since u is linear homogenous and since $u_1 = u_2R$, it follows that $u = u_1(1-s) + u_2sR = u_1 = u_2R$, and thus $v'R/v = u_2sR/u = s$. Hence, Eq. (14) is equivalent to the condition of the lemma. □

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