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The multiple-volunteers principle ⋆

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ABSTRACT

We present a class of simple transfer-free rules for assigning an unpleasant task among a group of agents: agents decide simultaneously whether or not to volunteer; if the number of volunteers exceeds a threshold number, the task is assigned to a volunteer; otherwise, the task is assigned to a non-volunteer. In particular, the rule may ask for multiple volunteers although one agent is sufficient to perform the task. In a setting in which agents care about who performs the task, any multiple-volunteers rule yields a strict interim Pareto improvement over random task assignment. Some volunteers rule is utilitarian optimal across all transfer-free binary mechanisms, and a rule with a large threshold reaches the first-best approximately if the group is large. Similar results hold for the problem of assigning a pleasant task. In that case, the task is assigned to a volunteer if and only if there are sufficiently few volunteers.

1. Introduction

Imagine a group of people from which one must be selected as the performer of a task. Examples include the selection of a person to stand first in line in a dangerous (e.g. military battle) situation, the selection of the salesperson who deals with an unpleasant customer, or the selection of the chairperson of a department at a university. The performance of the task is similar to the provision of a public good, with the two special characteristics that this particular public good *must* be provided and that monetary transfers are not feasible.

The optimal task assignment depends on the agents' preferences, which are private information. If each person was solely interested in minimizing the probability of being personally selected for the task, then due to incentive constraints and the lack of remuneration, no screening of any private information would be possible, whatever the prevailing rule is. Fortunately, in many situations, people are also genuinely interested in *who* performs the task. This can be because some people would perform the task at a higher quality than others or because people are at least slightly altruistic and care about others' costs of being selected. Formally, we assume that a general positive affine transformation of the performing agent's (privately known) payoff determines the other agents' payoffs. Perhaps surprisingly, this payoff and information structure has rarely been considered in the literature.

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The rule that first comes to mind as a potential improvement over random assignment of the task is the *any-volunteers rule*: all agents are asked simultaneously about who would like to "volunteer"; if at least one agent volunteers, the task is assigned randomly among the volunteering agents; if no agent volunteers, the task is assigned randomly among all agents. The any-volunteers rule can lead to expected welfare gains relative to a random assignment because agents can self-select into volunteers and non-volunteers according to their private information. But, and this is the starting point of our paper, other equally simple rules can yield even higher welfare.

For example, a *two-volunteers rule* may be used which stipulates that the task is assigned among the non-volunteers unless at least two volunteers come forward. As we show, if performing the task is very costly, or if agents' altruism is very small, then in the unique equilibrium of the any-volunteers rule no volunteers will come forward so that the task will be assigned randomly among the agents. In contrast, the two-volunteers rule always has an equilibrium in which all types of the agents (volunteers as well as non-volunteers) are strictly better off than with random assignment. This guaranteed-improvement property generalizes to rules with thresholds greater than two. The basic intuition is that by committing to select a non-volunteering agent for the task if the threshold is not reached, the designer effectively creates a cost of non-volunteering that always keeps up some volunteering activity. We suggest the term *multiple-volunteers principle* for the heuristic of requiring multiple volunteers although one agent would be sufficient to perform a task.

Throughout the paper, we focus on the class of *anonymous binary mechanisms* without transfers: agents choose simultaneously from just *two* available actions. The binary-action assumption is restrictive in our setting, where the agents' private information is continuously distributed. As the examples of the any-volunteers rule, the random-assignment rule, and the multiple-volunteers rules make clear, the assumption still allows for a lot of flexibility in designing mechanisms. The strongest argument in favor of anonymous binary mechanisms is their simplicity. The rules of a binary mechanism are easily explained to players. Moreover, our solution concept of symmetric Bayes-Nash equilibrium is particularly plausible in such mechanisms: with just two alternatives, a player can quickly find a best response, whether she applies trial and error or introspection.[1] Simple mechanisms are particularly appealing if the task assignment must be completed quickly. Although a restriction to only two actions may sometimes be too strong, it is a natural starting point, and binary mechanisms can be surprisingly powerful.

We show that the utilitarian optimum in the class of anonymous binary mechanisms is an *i-volunteers rule* with some threshold i. Such an i-volunteers rule assigns the task randomly to one of the volunteers if there are more than i volunteers and to one of the non-volunteers if there are fewer than i. The any-volunteers rule and the multiple-volunteers rules are i-volunteers rules, but the random-assignment rule is not.

We also consider the limit case of a large group of agents. Any sufficiently large fixed threshold i for the number of required volunteers leads, in the limit, to some efficiency gains relative to random assignment and to some efficiency loss relative to the first best. But if the threshold is adapted optimally as the group size increases, the first-best can be approximated arbitrarily closely in a large group, meaning that it becomes almost certain that the task will be assigned to an individual with maximum type. In particular, in a large population it is optimal to set a large threshold. There is no substantial efficiency loss from restricting attention to binary mechanisms without monetary transfers. Proving our results requires novel methods, since the equilibrium conditions for binary-action games, though one-dimensional, are nonlinear, and we have to compare the equilibrium welfare across all such games.

Literature

Volunteering as a strategic game has mainly been studied in private value settings or under complete information (both of which are limits of our model). In the simplest version of the volunteers' game, the task is a public good that is provided if and only if at least one volunteer comes forward, and every volunteer pays the (homogeneous) cost of providing the public good. Olson (2009, first edition: 1965) conjectured that the equilibrium probability of a volunteer coming forward decreases in the group size. Olson's conjecture was proved by Diekmann (1985) assuming complete information. Nöldeke and Peña (2020) generalize this result for volunteers' games where multiple contributors may be required to produce the public good.[2]

In an incomplete-information setting with private values, Bergstrom and Leo (2015) characterize the type distributions for which Olson's conjecture holds. We obtain, in essence, a conclusion opposite to the Olsen conjecture: Volunteering can be organized more efficiently in a large group than in a small group.

The volunteer game has also been studied in a *coordinated* version, meaning that one of the volunteers is randomly selected to provide the public good (this rule is similar to the any-volunteers rule with the crucial difference that the public good is not provided if no volunteer comes forward). Weesie and Franzen (1998) prove Olson's conjecture for this game, assuming complete information.[3]

In the settings with private values or complete information discussed above, incentives for volunteering arise from the threat that the public good is not provided *at all* rather than, as in our setting, from the threat that the public good is provided by the *wrong*

[1] More generally, the literature on choice overload (e.g., Jacob et al. (2024)) has established that the number of available options is a crucial factor determining whether agents can be expected to behave rationally.

[2] De Jaegher (2020) shows that there are cases in which (i) multiple, say k, contributors are needed to produce the public good, (ii) in equilibrium no contributor will come forward, and (iii) a positive equilibrium level of contributing can be restored by committing to produce the public good only if at least $k + 1$ contributors come forward. This result can be seen as an instance of the multiple-volunteers principle.

[3] Makris (2009), still maintaining complete information, characterizes the symmetric equilibria in the coordinated volunteers game where multiple contributors may be required to produce the public good. Requiring multiple contributors implies that there exists an additional, unattractive, equilibrium in which nobody contributes. Makris explores the conditions under which this equilibrium vanishes.

agent. Our model could be extended to allow for the possibility that the public good is not provided at all, but given our preference model, the wrong-agent threat is already a powerful incentive device.

Our preference model, in which a general affine function of the performing agent's type determines the other agents' payoffs, first appears in Li et al. (2016). They restrict attention to settings with two agents, but—in contrast to us—allow preferences to be ex-ante asymmetric across agents. The focus is on cheap talk rather than on mechanism design, where the receiver's preferences correspond to our planner's objective. It is shown that in the receiver-preferred equilibrium, the precision of both agents' communication is increasing in each agent's preference alignment with the planner.

An analysis of transfer-free mechanisms without the restriction to binary mechanisms can be found in Bhaskar and Sadler (2019). There, the agents' preferences for the allocation of a resource are partially aligned because of positive externalities. The setting is a special case of the pleasant-task variation of our preference model, with the difference that in their setting, the task does not have to be allocated. Bhaskar and Sadler (2019) identify an interesting class of mechanisms called *bin-mechanisms*, which partition the type space into intervals ("bins") and assign the task to the types in the highest reported bin. They also define non-simple ("better") bin mechanisms and show that one such mechanism is welfare maximizing among all transfer-free rules if the distribution of types is uniform.

Closely related to the multiple-volunteers principle as a design element is the idea of a conditional volunteering strategy by an individual agent. Schelling (2006) casts the idea of "volunteering if 20 others do likewise" (p. 95). In the context of public good provision, with private values or complete information, Reischmann and Oechssler (2018) and Oechssler et al. (2022) allow agents to send binding messages of the form "I am willing to contribute x units to the public good if in total y units are contributed". They show theoretically and experimentally that such conditional-contribution mechanisms are powerful tools for improving welfare. Earlier contributions have proposed so-called provision point mechanisms as simple rules that can lead to increased public good provision (Palfrey and Rosenthal, 1984; Bagnoli and Lipman, 1989). In these mechanisms, the cost of the public good acts as a threshold, as the public good is provided only if the sum of contributions equals or exceeds its cost.

An important strand of literature that focuses on two actions in settings with (often) continuously distributed types is the literature on voting over two alternatives, where each agent can only vote for either alternative (abstention may constitute a third action). This literature presents many positive results for large population limits, concerning either full information aggregation (e.g., Feddersen and Pesendorfer (1997), Krishna and Morgan (2011)) or first-best utilitarian welfare (e.g., Ledyard and Palfrey (2002), Krishna and Morgan (2015)). These results are different from our limit result. Voting limit results typically rely on the law of large numbers, while our result relies on bounds for tail probabilities of the Poisson distributions. This is because in the voting models, the planner's (or pivotal voter's) optimal choice in a large population relies on some *average* of the individuals' private information, while in our case the identity of the agent with the *maximum* type is of interest.

Between our assumption that the public good must be provided now and the opposite assumption that it can be avoided lies the possibility that the provision can be delayed. The possibility of delay naturally leads to a war-of-attrition game in which each agent waits, or engages in some other costly search process, until someone agrees to provide the service. In a private values setting with heterogeneous costs, such a volunteering game has been analyzed by Bliss and Nalebuff (1984). In equilibrium, the agent with the lowest cost of providing the service is the first to volunteer, but substantial waiting costs may have to be incurred before a volunteer is found.[4]

2. Model

A task of public interest must be assigned among a group of agents $1, \dots, n$, where $n \geq 2$. An allocation is a probability distribution $q = (q_1, \dots, q_n)$, where q_j is the probability that agent j is assigned the task. Note that $\sum_{j=1}^{n} q_j = 1$, that is, we assume that the task has to be done by one of the agents.

2.1. Preferences and information

Each agent $j \in \{1, \dots, n\}$ is privately informed about her type $t_j \in [t_L, t_H]$. Agents are symmetric from an ex-ante point of view. Each agent's type is independently distributed; the distribution function F is strictly increasing and continuously differentiable with density $F' = f$. We denote the expected type by $\bar{t} = \int_{t_L}^{t_H} t \mathrm{d}F(t)$.

If agent j performs the task, then her utility is t_j and the utility of each other agent is $\alpha t_j + \beta$, where $\alpha \in \mathbb{R}_{>0}$ and $\beta \in \mathbb{R}$ are agent-independent parameters of the utility function. Hence, if the allocation is $q = (q_1, \dots, q_n)$ and types are $(t_1, \dots, t_n)$, then agent j's expected utility is given by $q_j t_j + \sum_{k \neq j} q_k(\alpha t_k + \beta)$. Note that cases with $\alpha = 0$ (i.e., private values) are excluded.[5]

We assume that agents of all types prefer someone of their own type providing the service over doing it themselves:

Assumption 1.

$$t < \alpha t + \beta \text{ for all } t \in [t_L, t_H]. \tag{1}$$

[4] See also Bilodeau and Slivinski (1996) for a related model with complete information. See Klemperer and Bulow (1999) for a general approach to war-of-attrition games, and see LaCasse et al. (2002) and Sahuguet (2006) for more special extensions.

[5] In the private-values case ($\alpha = 0$), only the uniformly random allocation of the task is incentive compatible under Assumption 1. Our setting includes arbitrarily close approximations of the private values case.

This means that finding a volunteer for the task is indeed difficult, or put differently, that the task is unpleasant.[6] Different applications of the model correspond to specific values and interpretations of the parameters α and β.

Heterogeneous ability. The model captures situations in which the opportunity cost of performing the task is the same for all agents, but each agent is privately informed about her ability to perform the task. An agent's ability type t then equals the benefit created for each agent in the group if the agent is selected, so that in this application $\alpha = 1$. All agents prefer to select a more capable agent, but the selected agent also has to bear an opportunity cost of β. Assumption 1 means that the opportunity cost of providing the task is positive, $\beta > 0$. Other values for α are obtained by allowing an agent's cost to depend on her ability.

Altruism and heterogeneous cost. Another interpretation of the model is that the agents differ in the cost of providing the service, have identical abilities, and are at least somewhat altruistic in the sense that they feel bad if someone else has to incur a cost for their benefit. In this case, we may think of the ability or benefit as being normalized to 0 and the provision cost being $-t > 0$ for an agent of type t, leading to parameters $\beta = 0$ and $t_H < 0$. The parameter $\alpha > 0$ measures the degree of altruism. Assumption 1 means that the utility function places higher weight on own payoff than on others' payoffs, $\alpha < 1$.

2.2. Anonymous binary mechanisms and volunteering

A social planner (or the group of agents themselves) commits to the rules of a mechanism to allocate the task among the agents. We are interested in symmetric equilibria of anonymous mechanisms that allow only two messages (or actions).[7] There can be a third option to reject the mechanism. We assume that if any agent rejects the mechanism, each agent receives an outside payoff that is equal to the payoff from (uniformly-)random assignment, by which we mean that the task is always assigned with equal probability to any agent. It could for example be the case that in the event that not all agents agree to participate in the mechanism, someone from the group is picked at random and forced to perform the task by a third party. Since we will show (in Section 3.3) that in any equilibrium of any mechanism each type of agent is at least as well off as in the uniformly-random assignment, we can as well take the agents' participation in the mechanism for granted.

An *anonymous binary mechanism* is characterized by two properties. The first is that the agents simultaneously choose between two actions, denoted by "Y" and "N". The second is that the allocation depends only on the number of Y-players (rather than on their identities), such that each Y-player is chosen with equal probability and each N-player is chosen with equal probability. Such a mechanism is hence characterized by a list

$$\rho = (p_1, \dots, p_{n-1}),$$

where for all $j = 1, \dots, n-1$, the number $p_j \in [0, 1]$ denotes the probability that the task is assigned to a randomly selected Y-player if there are j Y-players; with probability $1 - p_j$, it is assigned to a randomly selected N-player if there are j Y-players.

For example, the *uniform-assignment rule* is given by $p_j = j/n$ for all j; this mechanism induces the uniformly-random allocation $q = (\frac{1}{n}, \dots, \frac{1}{n})$ independently of the agents' actions. The *any-volunteers rule* is defined by $p_j = 1$ for all $j = 1, \dots, n-1$. If the number of Y-players is 0 or n, any mechanism allocates the task randomly among all agents. We use the definitions $p_0 = 0$ and $p_n = 1$ for the probability that the task is assigned to one of the Y-players in these cases.

Commitment to the mechanism implies that once the task is allocated to an agent by the mechanism, performing the task is not voluntary for that agent. We define *volunteering* in our setting as taking an action in the mechanism that in expectation leads to being selected for the task with a probability weakly higher than $1/n$. A non-volunteering agent is not protected against having to do the task, but will have to do it with a probability less than $1/n$.

2.3. Equilibria

Given any mechanism $\rho = (p_1, \dots, p_{n-1})$, a symmetric strategy profile is characterized by a function σ that determines the strategy for each agent, where $\sigma(t)$ denotes the probability that type $t \in [t_L, t_H]$ plays Y. Given any symmetric strategy profile σ, the ex-ante probability that a given agent plays Y is denoted by

$$y(\sigma) = \int \sigma(t) \mathrm{d}F(t).$$

The resulting expected type of a Y-player is denoted by

$$t_Y(\sigma) = \frac{1}{y(\sigma)} \int \sigma(t) t \mathrm{d}F(t) \quad \text{if } y(\sigma) > 0,$$

and of an N-player by

$$t_N(\sigma) = \frac{1}{1 - y(\sigma)} \int (1 - \sigma(t)) t \mathrm{d}F(t) \quad \text{if } y(\sigma) < 1.$$

[6] The opposite case of a pleasant task is discussed in Section 7.4.
[7] This assumption is discussed in Sections 7.1 and 7.2.

The expected benefit that accrues to every other agent if the task is assigned to an a-player is denoted by

$$u_a(\sigma) = \alpha t_a(\sigma) + \beta \quad \text{for } a = Y, N.$$

The (interim) expected utility $U_a^\rho(\sigma, t)$ of any type-t agent who takes action a and anticipates that the other agents will use the strategy σ is

$$U_Y^\rho(\sigma, t) = \sum_{j=0}^{n-1} B_{y(\sigma)}^{n-1}(j)\left(p_{j+1}\frac{j u_Y(\sigma) + t}{j+1} + (1 - p_{j+1})u_N(\sigma)\right), \tag{2}$$

$$U_N^\rho(\sigma, t) = \sum_{j=0}^{n-1} B_{y(\sigma)}^{n-1}(j)\left(p_j u_Y(\sigma) + (1 - p_j)\frac{(n-j-1)u_N(\sigma) + t}{n-j}\right). \tag{3}$$

Using the binomial distribution, $B_y^{n-1}(j) = \binom{n-1}{j}(1-y)^{n-1-j}y^j$ denotes the probability that, from the point of view of the given agent, j of the $n-1$ other agents choose Y, given that each of them chooses Y with probability y. The function σ is a symmetric equilibrium in a mechanism ρ if the following implications hold for all types t:

$$\text{if } \sigma(t) > 0 \text{ then } \mathrm{U}_\mathrm{Y}^\rho(\sigma, t) - U_N^\rho(\sigma, t) \geq 0,$$
$$\text{if } \sigma(t) < 1 \text{ then } \mathrm{U}_\mathrm{Y}^\rho(\sigma, t) - U_N^\rho(\sigma, t) \leq 0.$$

2.4. Goals: Guaranteed improvement, ex-ante utility maximization

The paper has two interdependent goals. First, we would like to identify mechanisms that can guarantee an improvement over the benchmark of the uniform-assignment rule. We say that a mechanism has the *guaranteed-improvement property* if, for all type distributions F and all values of the preference parameters α and β, there exists an equilibrium such that all types of agents obtain a strictly higher (interim) expected utility than in the uniform-assignment rule.

Second, we consider a planner who across all mechanism-equilibrium combinations (ρ, σ) maximizes each player's ex-ante expected utility

$$\int \max\{U_Y^\rho(\sigma, t), U_N^\rho(\sigma, t)\}\mathrm{d}F(t).$$

Note the different natures of the two goals. The guaranteed-improvement property refers to an interim Pareto-comparison –no welfare function is maximized there. Also, the guaranteed-improvement property has a robustness flavor in the sense that the mechanism is fixed across all parameter constellations, whereas for the second goal, the planner takes a parameter constellation as given and maximizes across mechanisms. The two goals are related because the proof of our characterization of welfare-maximizing mechanisms relies on already knowing that some mechanism improves upon the uniform-assignment rule.

3. Preliminary results

In 3.1, we introduce four auxiliary functions that are central to simplifying our analysis. In 3.2, we introduce the convention that the message Y is the volunteering action. In 3.3, we show that we can restrict attention to equilibria in partition strategies, and we explain why we can take the participation of the agents for granted. Proofs are relegated to the Appendix.

3.1. Auxiliary functions

The agents' expected utilities can be expressed in terms of four functions, h_Y, h_N, q_Y and q_N. Taking the point of view of a given agent who has chosen an action (Y or N), the functions q_Y and q_N describe the probability of personally getting assigned the task, and the functions h_Y and h_N describe the probability that someone in the group of Y-playing agents gets assigned the task.

Let $y \in [0, 1]$ denote every other (i.e., not the given) agent's probability of playing Y. Given any mechanism $\rho = (p_1, \ldots, p_{n-1})$, the probability that anyone of the Y-playing agents is selected for the task, conditional on the event that the given agent chooses action $a = Y, N$, is denoted $h_a^\rho(y)$, i.e.,

$$h_Y^\rho(y) = \sum_{j=0}^{n-1} B_y^{n-1}(j)p_{j+1} \tag{4}$$

and

$$h_N^\rho(y) = \sum_{j=0}^{n-1} B_y^{n-1}(j)p_j. \tag{5}$$

The probability that the given agent is selected for the task if she chooses action $a = Y, N$ is denoted $q_a^\rho(y)$, i.e.,

$$q_Y^\rho(y) = \sum_{j=0}^{n-1} B_y^{n-1}(j)\frac{p_{j+1}}{j+1} \tag{6}$$

and

$$q_N^\rho(y) = \sum_{j=0}^{n-1} B_y^{n-1}(j)\frac{1-p_j}{n-j}. \tag{7}$$

We drop the upper index ρ whenever the mechanism is clear. As an example, consider these functions for the case of the uniform-assignment rule. Here, $q_Y(y) = q_N(y) = 1/n$ and, from the point of view of a given agent, the expected number of other Y-players is equal to $y(n-1)$. By playing Y, the agent adds in herself. Thus,

$$h_Y(y) = \frac{1+y(n-1)}{n} \text{ and } h_N(y) = \frac{y(n-1)}{n} \text{ in the uniform-assignment rule.} \tag{8}$$

Some algebraic relations between the auxiliary functions hold independently of the underlying mechanism and are particularly useful. First, the ex-ante probability that any given agent is selected can be written as

$$yq_Y(y) + (1-y)q_N(y) = \frac{1}{n} \quad \text{for all } y \in [0,1]. \tag{9}$$

Hence, q_N can be expressed in terms of q_Y. Second, a simple formula is available for expressing q_Y in terms of h_Y and h_N. To see it, note that the probability that the task is assigned to a Y-player can be expressed in the form $yh_Y + (1-y)h_N$. Alternatively, the same probability can be expressed in the form nyq_Y because every Y-playing agent is selected with the same probability. Thus,

$$nyq_Y(y) = yh_Y(y) + (1-y)h_N(y) \quad \text{for all } y \in [0,1]. \tag{10}$$

Third, for the derivatives of q_Y and q_N, we have the formulas

$$q_Y'(y) = \frac{1}{y}(h_Y(y) - h_N(y) - q_Y(y)) \quad \text{and} \tag{11}$$

$$q_N'(y) = \frac{1}{1-y}(h_N(y) - h_Y(y) + q_N(y)) \quad \text{for all } y \in (0,1). \tag{12}$$

The proof of these formulas, which relies on well-known properties of Bernstein polynomials, is relegated to the Appendix.

3.2. Interpreting action Y as volunteering

For any mechanism, we can construct an equivalent mechanism by switching the labels of the actions Y and N. Formally, any mechanism $\rho' = (p_1, \dots, p_{n-1})$ is equivalent to the mechanism $\rho = (1-p_{n-1}, \dots, 1-p_1)$ in the following sense: for any action profile in $\{N, Y\}^n$ that is played in mechanism ρ', if the action of every agent is changed, the new action profile induces the same allocation in ρ as was induced by the original action profile in ρ'. As an immediate consequence, a strategy σ is a symmetric equilibrium in ρ if and only if $1-\sigma$ is a symmetric equilibrium in ρ'.

By switching to an equivalent mechanism if necessary, we can restrict attention to mechanism-equilibrium combinations (ρ, σ) such that playing Y leads to a weakly higher probability of getting assigned the task than playing N. We will maintain this restriction throughout the paper. Formally (cf. (9)),

$$q_N^\rho(y(\sigma)) \le \frac{1}{n} \le q_Y^\rho(y(\sigma)). \tag{13}$$

Accordingly, we call the action Y "volunteering" and the action N "non-volunteering".

For illustration, consider the mechanism $\rho' = (0, \dots, 0)$. In this mechanism, the task is assigned to the non-volunteers unless all n agents volunteer; we call this the *all-volunteers* rule. By switching the labels of the actions Y and N, we obtain the equivalent mechanism $\rho = (1, \dots, 1)$; i.e., the any-volunteers rule. It is straightforward to show that

$$q_N^\rho(y) = \frac{(1-y)^{n-1}}{n} \le \frac{1}{n} \le \frac{1-(1-y)^n}{ny} = q_Y^\rho(y) \quad \text{for all } y \in (0,1]. \tag{14}$$

Thus, in any equilibrium in the any-volunteers rule the message Y can be interpreted as volunteering. The opposite is the case in the all-volunteers rule because $q_N^{\rho'}(1-y) = q_Y^\rho(y)$ and $q_Y^{\rho'}(1-y) = q_N^\rho(y)$.

3.3. Partition strategies and participation

Using the auxiliary functions, the agent's expected utility from playing Y or, resp., N, in any mechanism ρ can be conveniently written as

$$U_Y^\rho(\sigma, t) = q_Y^\rho \cdot t + (h_Y^\rho - q_Y^\rho)u_Y + (1 - h_Y^\rho)u_N, \tag{15}$$

$$U_N^\rho(\sigma, t) = q_N^\rho \cdot t + h_N^\rho u_Y + (1 - h_N^\rho - q_N^\rho)u_N, \tag{16}$$

where we have omitted the argument $y(\sigma)$ for the functions on the right-hand side. The first term in these expressions captures the payoff that arises from the event that the agent is selected herself, the second term captures the payoff in the event that the task is performed by a Y-player (other than the agent herself), and the third term captures the payoff in the event that the task is performed by an N-player (again, not including the agent herself).

Throughout our analysis, it will be crucial to distinguish mechanism-equilibrium combinations in which an agent's probability to be assigned the task is independent of her action (i.e. $q_N = q_Y$) from combinations where it is not (i.e. $q_N < q_Y$). To understand why, consider the expected payoff gain from volunteering for an agent of type t who expects all others to play according to some strategy σ,

$$U_Y(\sigma,t) - U_N(\sigma,t) = (q_Y - q_N)(t - u_y) + (h_Y - h_N - q_N)(u_Y - u_N). \tag{17}$$

This difference is independent of the agent's type t if $q_N = q_Y$, and it is strictly increasing in the type if $q_N < q_Y$. In the latter case, it follows that a player's best response to any strategy, and hence any equilibrium strategy σ, is a *partition strategy*, that is, there exists a cut-off type $\hat{t} \in [t_L, t_H]$ such that $\sigma(t) = 0$ for all $t < \hat{t}$ and $\sigma(t) = 1$ for all $t > \hat{t}$. If instead $q_N = q_Y$, the equilibrium does not have to be a partition strategy, but any such mechanism-equilibrium combination is payoff-equivalent to the random-assignment rule and thus can also be implemented with a partition strategy.

Lemma 1. *In any mechanism-equilibrium combination* (ρ,σ) *with* $q_N^\rho(y(\sigma)) = q_Y^\rho(y(\sigma))$ *or* $y(\sigma) = 0$ *or* $y(\sigma) = 1$*, all types of agents obtain the same expected utility as in the uniform-assignment rule.*

To deal with the case $q_N < q_Y$, we introduce more notation concerning partition strategies. We identify any partition strategy σ with the Y-playing probability $y(\sigma)$. Given any partition strategy $y \in [0,1]$ and defining the cut-off type $\hat{t}(y) = F^{-1}(1-y)$, we can calculate the expected type of a volunteer and a non-volunteer, respectively, as

$$t_Y(y) = E[t|t \geq \hat{t}(y)] \ \text{ and } \ t_N(y) = E[t|t \leq \hat{t}(y)].$$

We define the continuous extensions $t_Y(0) = t_H$ and $t_N(1) = t_L$. It holds that $t_Y(y) > t_N(y)$. Since $\alpha > 0$, volunteers create a higher expected benefit than non-volunteers,

$$u_Y(y) = \alpha t_Y(y) + \beta > \alpha t_N(y) + \beta = u_N(y). \tag{18}$$

Since in equilibrium any volunteering type has a larger than average type, a revealed-preference argument yields a clearcut payoff comparison with the uniform-assignment rule.

Lemma 2. *In any mechanism-equilibrium combination* (ρ,σ) *with* $q_N^\rho(y(\sigma)) < q_Y^\rho(y(\sigma))$ *and* $0 < y(\sigma) < 1$*, all types of agents obtain a strictly higher expected utility than in the uniform-assignment rule.*

As for the agents' participation decisions, we assume that the uniform-assignment rule is used as a default rule if at least one agent rejects the mechanism. We also assume that each agent decides about accepting or rejecting the mechanism given her own type. Thus, taken together, Lemma 1 and Lemma 2 justify the agents' participation in any mechanism.

For later use, note that an interior partition strategy $y \in (0,1)$ is an equilibrium in a mechanism ρ if and only if the cut-off type $\hat{t}(y)$ is indifferent between the two actions, that is,

$$\Delta^\rho(y) = 0 \text{ where } \Delta(y) = U_Y^\rho(y,\hat{t}(y)) - U_N^\rho(y,\hat{t}(y)). \tag{19}$$

4. Improvements over the uniform-assignment rule

In this section, we introduce a particular class of anonymous binary mechanisms, i-volunteers rules, that are central to the paper. We show that most, but not all, i-volunteers rules satisfy the guaranteed-improvement property. The existence of mechanisms satisfying the guaranteed-improvement property also serves as a preparatory step for Section 5, where we show that the solution to the planner's problem always involves an i-volunteers rule.

Definition 1. For any $i = 1,\ldots,n-1$, a mechanism $(p_1,\ldots,p_{n-1})$ is called an i-volunteers rule with threshold i if $p_i > 0$, $p_j = 1$ for all $j > i$, and $p_j = 0$ for all $j < i$.

In an i-volunteers rule, each agent anticipates that volunteering puts her in a lottery box together with the other volunteers if there are more than i volunteers in total. If the required number of i volunteers is not reached (or, put differently, the number of non-volunteers exceeds $n-i$), then all volunteers are released from the task and the non-volunteers are put into a lottery box. We call such an i-volunteers rule *pure* if $p_i = 1$. If $p_i < 1$, the required number of volunteers is i with probability p_i and $i+1$ with probability $1-p_i$, which is determined after agents have made their choices.

In an i-volunteers rule, playing Y can be interpreted as an act of conditional volunteering: volunteering if enough others do likewise. Volunteers know that they will only be assigned the task if there are at least $i-1$ other volunteers. If the number of other volunteers equals $i-1$, then the agent is pivotal, i.e., her own action choice has an impact on whether the task is assigned via a lottery among the volunteers or via a lottery among the non-volunteers. A lower threshold hence increases the probability that a volunteer is assigned the task, but this does not mean that the threshold should always be set as low as possible: a higher threshold may increase the probability that an agent is pivotal for the selection of a volunteer and can therefore increase the volunteering rate.

With this definition, the any-volunteers rule is a pure 1-volunteer rule, i.e., it has threshold 1 and $p_1 = 1$. The uniform-assignment rule is an i-volunteers rule only if the group consists of two agents, with threshold $i = 1$ and $p_1 = 0.5$. In larger groups, the question arises whether an i-volunteers rule can always guarantee an improvement over random assignment. This question will be addressed next.

Remark 1. The any-volunteers rule does not have the guaranteed-improvement property: an equilibrium outcome different from the uniform-assignment rule exists if and only if $\alpha\bar{t} + \beta < t_H$.

The intuition behind this result is as follows. The inequality $\alpha\bar{t} + \beta < t_H$ means that any agent with a sufficiently high type prefers to do the task herself rather than letting a different agent with an average type do it. In the any-volunteers rule, this is exactly the trade-off faced by a high-type agent who believes that no other agent will contribute: switching her action from non-volunteering to volunteering raises the probability that she herself gets assigned the task by $1 - 1/n$ and reduces, by the same amount, the probability that a non-volunteer other than herself (i.e., an agent of average type) is selected. Thus, volunteering is optimal if and only if the inequality in the remark holds. For a formal proof of Remark 1, we refer the reader to Theorem 5 in Section 7.3 which includes Remark 1 as a special case.

The result that the any-volunteers rule does not have the guaranteed-improvement property provides a counterpoint to Theorem 1 below, which shows that the property holds for rules with larger thresholds. We call an i-volunteers rule a *multiple-volunteers* rule if it is sufficiently different from the any-volunteers rule (and its mirror rule, the all-volunteers rule).

Definition 2. An i-volunteers rule $(p_1, \dots, p_{n-1})$ is called a multiple-volunteers rule if $p_1 < 1/n$ and $p_{n-1} > 1 - 1/n$.

This condition is satisfied for any i-volunteers rule with $2 \le i \le n-2$. A multiple-volunteers rule exists if and only if $n > 2$.

Theorem 1. *Any multiple-volunteers rule has the guaranteed-improvement property.*

The fact that the guaranteed-improvement property holds for multiple-volunteers rules, but not for the any-volunteers rule, supports the heuristic that multiple volunteers should be asked to come forward even when a single volunteer is enough for the task; we call this heuristic the *multiple-volunteers principle*.

Note the strength of the conclusion of Theorem 1: the strict interim-improvement over the uniform-assignment rule holds at all preference parameters α and β that satisfy the assumptions of our model, including values of α that are arbitrarily close to 0 and arbitrarily large β. In particular, in the heterogenous-ability interpretation of the model, an improvement can be achieved even if the opportunity cost of providing the task is much larger than its individual benefit.

The conclusion of Theorem 1 is strikingly different from the situation in settings of pure private values. There each agent only cares about her personal probability of getting assigned the task. In such settings, no type-specific incentives can be provided without monetary transfers, so that the uniformly-random allocation obtains always.

To prove Theorem 1, we fix a multiple-volunteers rule $\rho = (p_1, \dots, p_{n-1})$ and drop the argument ρ from all functions. The proof proceeds by showing that the maximal element $\hat{y}$ of the set $\{y \in [0,1] \mid \Delta(y) = 0\}$, which is an equilibrium because $\Delta(\hat{y}) = 0$, satisfies

$$0 < \hat{y} < 1 \quad \text{and} \quad q_Y(\hat{y}) > \frac{1}{n}. \tag{20}$$

From this, the theorem is immediate because Lemma 2 can be applied.

Note first that for any multiple-volunteers rule, the all-volunteering strategy $y = 1$ is never an equilibrium. If an agent who expects all other agents to volunteer also volunteers, then she expects the task to be assigned to a volunteer with probability $h_Y(1) = 1$ and to herself with probability $q_Y(1) = 1/n$. If instead the agent does not volunteer, she expects the task to be assigned to a volunteer with probability $h_N(1) = p_{n-1}$ and to herself with probability $q_N(1) = 1 - p_{n-1}$. Plugging these values into (19) yields that

$$\Delta(1) = (\frac{1}{n} - 1 + p_{n-1})(t_L - \alpha\bar{t} - \beta).$$

The first factor on the right-hand side is > 0 in any multiple-volunteers rule, while the second is negative by Assumption 1. Thus,

$$\Delta(1) < 0, \tag{21}$$

contradicting equilibrium.

To show (20), we have to study the functions q_Y and Δ. The shape of Δ depends crucially on $h_Y - h_N$, which captures the impact of an agent's switch from non-volunteering to volunteering on the probability (computed from the switching agent's point of view) that the task gets assigned to a volunteer. Fig. 1 illustrates these expressions as functions of the volunteering rate.

In two technical lemmas, which are proved in the Appendix, we establish qualitative properties of the relevant functions that are illustrated in Fig. 1.[8] First, the specific form of a multiple-volunteers rule implies that $h_Y - h_N$ is quasi-concave, i.e., first strictly increasing and then strictly decreasing.

Lemma 3. *For any multiple-volunteers rule, there exists $y^{*m} \in (0,1)$ such that, for all $y \in (0,1)$, $(h_Y - h_N)'(y) > 0$ if $y < y^{*m}$, and $(h_Y - h_N)'(y) < 0$ if $y > y^{*m}$.*

Recall from (11) that the function q_Y is increasing where it is below the function $h_Y - h_N$ and decreasing where it is above that function. The intersection points of q_Y and $h_Y - h_N$ are the same as the critical points of q_Y. Our second technical result is that there exists only a single such intersection point, which must be a maximum of q_y.

Lemma 4. *For any multiple-volunteers rule, there exists $y^{m1} \in (0,1)$ such that, for all $y \in (0,1)$, $h_Y(y) - h_N(y) > q_Y(y)$ if $y < y^{m1}$ and $h_Y(y) - h_N(y) < q_Y(y)$ if $y > y^{m1}$.*

[8] The functions can take more complex shapes than in the example of Fig. 1. In particular, the function Δ can have more zeros. Moreover, while in Fig. 1 it is true that $\hat{y} < y^{m1}$, this inequality can be reversed, such as when $\beta = 0.1$ and all other parameters are the same as in the figure.

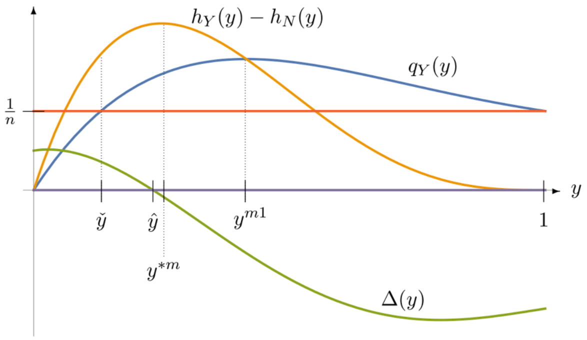

Fig. 1. The diagram illustrates the quasi-concavity of q_Y and $h_Y - h_N$ as well as existence of a partition equilibrium strategy $\hat{y}$ with $\Delta(\hat{y}) = 0$ and $q_Y(\hat{y}) > 1/n$ for an example with $n = 5$ agents and the pure 2-volunteers rule. The type of each agent is uniformly distributed on $[-1, 0]$ and the preference parameters are $\alpha = \beta = 1$.

This single-crossing property implies that q_Y is quasi-concave, increasing up to the point $y^{m1} \in (0, 1)$, then decreasing. Intuitively, in a multiple-volunteers rule, by volunteering an agent reduces her personal selection probability if the others' volunteering rate is sufficiently low, and otherwise increases it.

The crucial point of comparison is the agent's ex-ante selection probability $1/n$, marked as a horizontal line in Fig. 1. At $y = 0$, i.e., if nobody else volunteers, a volunteering agent expects to be selected with probability p_1. Thus, $q_Y(0) < 1/n$ for any multiple-volunteers rule, that is, at the point 0 the function q_Y is below the $1/n$-line. On the other hand, at $y = 1$, if everybody else volunteers, then volunteering herself puts the agent in a lottery box with everybody else, that is, $q_Y(1) = 1/n$. Quasiconcavity of q_Y then implies the existence of a unique volunteering rate $\check{y}$ where the function q_Y crosses the $1/n$-line from below.

At $\check{y}$, because an agent's action has no impact on her own selection probability, the benefit from volunteering is equal to

$$\Delta(\check{y}) = (h_Y(\check{y}) - h_N(\check{y}) - \frac{1}{n})(u_Y(\check{y}) - u_N(\check{y})). \tag{22}$$

Note that at the point $\check{y}$, q_Y is increasing, which means $h_Y(\check{y}) - h_N(\check{y}) > 1/n$, establishing that $\Delta(\check{y}) > 0$. Hence, it must be true that $q_Y(\hat{y}) > 1/n$ at the last point $\hat{y}$ where the function Δ crosses the y-axis. Note that such a point exists because of the intermediate value theorem and (21). This shows (20) and completes the proof of Theorem 1.

5. Solving the planner's problem

Now we turn to the planner's problem of solving for the ex-ante optimal anonymous binary mechanism. Before presenting our main characterization result in 5.2, we solve in 5.1 the planner's problem without equilibrium constraints.

5.1. *The first-best benchmark*

Consider a planner who is restricted to anonymous binary mechanisms, but can prescribe any strategy, to be used by all players. The planner seeks a mechanism-strategy combination that maximizes each agent's ex-ante expected utility. Given any mechanism ρ and any strategy σ, a volunteer is selected with probability $ny(\sigma)q_Y^\rho(y(\sigma))$ and a non-volunteer with probability $n(1 - y(\sigma))q_N^\rho(y(\sigma))$. Thus, the expected type of the selected agent is

$$W^\rho(\sigma) = ny(\sigma)q_Y^\rho(y(\sigma))t_Y(\sigma) + n(1 - y(\sigma))q_N^\rho(y(\sigma))t_N(\sigma). \tag{23}$$

Each agent is selected with probability $1/n$. This implies the ex-ante expected utility

$$(\frac{1}{n} + (1 - \frac{1}{n})\alpha)W^\rho(\sigma) + (1 - \frac{1}{n})\beta.$$

Since $\alpha > 0$, the planner's objective boils down to maximizing the welfare objective $W^\rho(\sigma)$. In terms of our two leading interpretations, this means that the planner maximizes the expected ability or minimizes the expected cost of the selected agent.

$$\textit{Binary-first-best problem:} \quad \max_{p_1 \ldots, p_{n-1}, \sigma} W^{p_1 \ldots, p_{n-1}}(\sigma)$$

$$\text{s.t.} \quad 0 \le p_j \le 1 \qquad (j = 1, \ldots, n-1),$$
$$0 \le \sigma(t) \le 1 \qquad (t \in [t_L, t_H]).$$

In the theorem below we characterize the binary first-best and show that it cannot be reached in equilibrium, due to the free-riding problem that is present in our model.

Theorem 2. *The solution to the binary-first-best problem involves the any-volunteers rule* $(1,\dots 1)$ *and a partition strategy* y^{**} *with* $(W^{(1,\dots,1)})'(y^{**}) = 0$.[9] *The binary first-best cannot be obtained as an equilibrium of any anonymous binary mechanism.*

The details of the proof are in the Appendix. The first part is straighforward because a partition strategy is the best way to split the type space, and one volunteer is enough for the task. To see the second part, that the binary first-best cannot be attained in equilibrium, we consider any mechanism ρ together with any partition strategy y and establish a formula for the welfare effect of marginally increasing the volunteering rate y:

$$\frac{\alpha}{n}(W^{\rho})'(y) = \Delta^{\rho}(y) + (q^{\rho}_Y(y) - q^{\rho}_N(y))(\alpha\hat{t}(y) + \beta - \hat{t}(y)). \tag{24}$$

This formula—which we will also use as a tool towards characterizing the binary-second-best in Theorem 3 below—captures how the misalignment between the planner's welfare goal and an agent's equilibrium condition depends on the strength of the marginal type's free-riding incentive. Now take ρ to be the any-volunteers rule and evaluate (24) at a binary first-best partition strategy $y = y^{**}$, which satisfies $(W^{\rho})'(y^{**}) = 0$ and $q^{\rho}_Y(y^{**}) - q^{\rho}_N(y^{**}) > 0$ as computed in (14). One then sees that the equilibrium condition $\Delta(y^{**}) = 0$ could only be satisfied if there was no free-riding incentive, $\alpha\hat{t}(y^{**}) + \beta = \hat{t}(y^{**})$. Hence, social welfare and individual incentives would only be aligned if $\beta = 0$, which in the heterogeneous-ability interpretation means zero volunteering cost, and $\alpha = 1$, which in the altruism interpretation means that the other agents fully internalize the cost imposed on the selected agent. This case is, however, ruled out by Assumption 1. This completes the proof of Theorem 2.

An *entirely* unrestricted planner would always assign the task to an agent with the highest type among all agents in the group. Given that a continuum of types exists, this first-best solution can obviously not be reached exactly in the binary first best. But, as we will show later, in a large group it can be approximated arbitrarily closely even in the binary *second* best.

5.2. *The planner's second-best problem*

We now consider a planner who maximizes each agent's ex-ante expected utility across all symmetric equilibria of anonymous binary mechanisms.

Given any mechanism ρ and partition strategy $y \in [0,1]$, the expected type of the selected agent (cf. (23)) can be written as

$$W^{\rho}(y) = nyq^{\rho}_Y(y)t_Y(y) + n(1-y)q^{\rho}_N(y)t_N(y). \tag{25}$$

This leads to the

Binary-second-best problem:

$$\max_{\rho=(p_1\dots,p_{n-1}),y} W^{\rho}(y)$$

$$\begin{aligned} \text{s.t.}\quad & 0 \le p_j \le 1 \qquad (j = 1,\dots,n-1),\\ & 0 \le y \le 1,\\ & \Delta^{\rho}(y) = 0,\\ & q^{\rho}_N(y) \le q^{\rho}_Y(y). \end{aligned}$$

Relative to the binary-first-best problem, the new constraints are the restriction to partition strategies y, the equilibrium constraint $\Delta(y) = 0$, and the constraint $q_N(y) \le q_Y(y)$. The new constraints are justified by the results in Section 3.3. Here is the main result in this section.[10]

Theorem 3. *Let* $n \ge 3$. *Any solution to the binary-second-best problem involves an* i*-volunteers rule for some threshold* $i \in \{1,\dots n-1\}$.

The result focuses on groups of a least three agents; with $n = 2$ agents, the any-volunteers rule is always optimal.[11]

This result reveals the structure of the optimal mechanism. It does not characterize the optimal value of the threshold i; this value will depend on the exogenous model parameters. However, we can say a bit more. First, the results in Section 4 imply that one prominent i-volunteers rule—the any-volunteers rules—will often be sub-optimal. Second, the results in Section 6 imply that i-volunteers rules with arbitrarily large thresholds i will be optimal as the group size n becomes large.

We prove Theorem 3 in two steps. The first step is to identify a relaxation which expresses that the planner could not gain if she was allowed to give the cut-off type a strict incentive to volunteer. Formally, any solution to the binary-second-best problem also solves the relaxed problem in which the constraint $\Delta^{\rho}(y) = 0$ is replaced by the inequality $\Delta^{\rho}(y) \ge 0$. This follows from the free-riding incentives in our model as expressed in (24).

[9] For example, if there are only two agents, the first-order condition simplifies to $\hat{t}(y^{**}) = \bar{t}$, that is, in the optimum the marginally volunteering type is the expected type.

[10] The result extends to settings in which each agent's type space is binary. In such a setting, by the revelation principle, the binary-action restriction entails no loss of generality, implying that an i-volunteers rule is utilitarian optimal among all transfer-free mechanisms.

[11] Let $n = 2$. Given any mechanism $\rho = (p_1)$, it is straightforward to verify that $\Delta^{\rho}(y) = (p_1 - 1/2)(\hat{t}(y) - (\alpha\bar{t} + \beta))$. Since $t_L - (\alpha\bar{t} + \beta) < 0$ because of Assumption 1, a partition equilibrium y with $0 < y < 1$ exists if and only if $t_H > \alpha\bar{t} + \beta$. If this condition holds, the any-volunteers rule is optimal with $\hat{t}(y) = \alpha\bar{t} + \beta$. The any-volunteers rule yields a strict improvement over random assignment in this case, but compared to the first-best cut-off $\bar{t}$, fewer agent types volunteer. If $t_H < \alpha\bar{t} + \beta$, then only a uniformly-random allocation is possible. In this case, any rule is optimal.

As for the second step, because some improvement over the uniformly-random assignment is possible by Theorem 1, the constraint $q^{\rho}_N(y) \leq q^{\rho}_Y(y)$ is not binding at the optimum. Fixing the optimal volunteering rate y^* in the relaxed binary-second-best problem, we consider the Lagrangian first-order conditions with respect to the rule-defining probabilities $p_1, \dots, p_{n-1}$. Using the non-negativity of the Lagrangian multiplier of the constraint $\Delta^{\rho}(y) \geq 0$ together with the stochastic independence of the agents' types, the optimality of an i-volunteers rule follows. The details are in the Appendix.

6. Volunteering in a large group

In this section, we consider a large group of agents and show that the first-best welfare level can be approximated arbitrarily closely.[12]

Theorem 4. *As the group size n converges to infinity, the binary-second-best welfare level converges to the first-best limit welfare level t_H.*

This result is important because it shows that binary mechanisms, although being very simple with just two possible actions for each agent, are sufficient to approximate the first best in a large group. The reason a binary mechanism *may be* good enough is that the information to be extracted from the agents is binary as well: each agent is essentially asked whether or not her type is close to the highest feasible type. The non-obvious feature is that there exists an i-volunteers rule and an equilibrium in which it becomes almost certain that at least i volunteers will come forward if the population is sufficiently large.

We do not use the any-volunteers rule in this construction. The any-volunteers rule implements the binary first best (Theorem 2). But once we take the equilibrium conditions into account, the any-volunteers rule is in many cases not better than a random assignment no matter how large the population of agents (Remark 1). Instead, our construction involves i-volunteers rules with large thresholds i.

To prove Theorem 4 (for details see the Appendix), it is sufficient to focus on pure i-volunteers rules. As an intermediate step, we focus on the large-population limit behavior in any i-volunteers rule with a given i above some lower bound (Lemma 5). Any such i-volunteers rule sustains non-vanishing (per-capita) welfare improvements over the uniform-assignment rule in the large-population limit, although none of these rules becomes fully efficient in the limit. This leaves open the possibility that the planner can approximate the first best in a large population by increasing the threshold i beyond any bound. In the proof of Theorem 4, we then confirm this approximation, using the Chernoff bounds for tail probabilities of Poisson random variables. The conclusion that in a large population it is optimal to require that many volunteers come forward provides additional support for the *multiple-volunteers principle*.

Lemma 5 refers to an i-volunteers rule where the threshold i exceeds some lower bound (27) that depends on the preference parameters α and β and the type distribution F. For any such rule and appropriate equilibrium sequence, we compute the large-population limit of the expected number of volunteers (28) and the limit welfare (29). The limit welfare is strictly larger than $\bar{t}$, the welfare from the uniform-assignment rule. In other words, the welfare improvement from using an i-volunteers rule does not vanish in the limit.

A central role is played by the Poisson distribution. For any $z > 0$, let

$$\text{Pois}(z)(j) = e^{-z}\frac{z^j}{j!}$$

denote the probability of the realization $j = 0, 1, \dots$ according to the Poisson distribution with mean z. The corresponding hazard-rate function is

$$h^{\text{Pois}(z)}(j) = \frac{\text{Pois}(z)(j)}{\sum_{k=j}^{\infty}\text{Pois}(z)(k)} = \frac{1}{j!\sum_{k=j}^{\infty}\frac{z^{k-j}}{k!}}. \tag{26}$$

Lemma 5. *Consider a pure i-volunteers rule ρ^i with threshold*

$$i > \frac{\alpha t_H + \beta - t_H}{\alpha(t_H - \bar{t})}. \tag{27}$$

For any group size $n > i$, there exists a partition equilibrium y_n such that as $n \to \infty$, the expected number of volunteers

$$ny_n \to z^*, \quad \text{where } h^{\text{Pois}(z^*)}(i) = \frac{\alpha t_H + \beta - t_H}{i\alpha(t_H - \bar{t})}. \tag{28}$$

For the welfare evaluated at rule ρ^i and equilibrium y_n,

$$W^{\rho^i}(y_n) \to \bar{t} + (t_H - \bar{t})\sum_{j=i}^{\infty}\text{Pois}(z^*)(j). \tag{29}$$

The lower bound (27) on i guarantees that the right-hand side in (28) is less than 1, so that the Eq. (28) has a solution z^*. Moreover, z^* is uniquely determined by (28) because the hazard-rate $h^{\text{Pois}(z)}(j)$ is strictly decreasing in the mean z.

The intuition behind Lemma 5 is as follows (for proof details see the Appendix). First, recall that the Poisson distribution is the limit of binomial distributions as the number of draws grows large and the expected number of successes stabilizes. Thus, if the

[12] Note that, in a large group, each agent's ex-ante expected utility is essentially given by $\alpha W + \beta$, where W is the welfare defined in (23). Thus, the limit welfare statement apply equally in terms of ex-ante expected utility.

expected number of volunteers converges to some z^*, then the number of volunteers in a large population approximately follows a Poisson distribution with mean z^*. In the following we will explain the equation (28) from which z^* can be computed. In a large group, each agent will volunteer with a small probability, that is, the marginally volunteering type is approximately equal to t_H. Her decision to volunteer is pivotal for assignment to a volunteer if exactly $i-1$ other agents volunteer, which happens with probability

$$\mathrm{Pois}(z^*)(i-1) = \frac{i}{z^*}\mathrm{Pois}(z^*)(i).$$

In this event, the marginal agent's decision to volunteer implies that the task will be assigned to a top type rather than to an average type, yielding an additional benefit of approximately $\alpha(t_H - \bar{t})$.

Any given agent's payoff, however, is also affected by the probability that she is selected herself for the task if she volunteers. In a large population, this probability is approximately equal to

$$\sum_{j\geq i-1} \mathrm{Pois}(z^*)(j)\frac{1}{j+1} = \frac{1}{z^*}\sum_{j\geq i}\mathrm{Pois}(z^*)(j). \tag{30}$$

In this event, the agent's payoff will be determined by her own type rather than by the benefit provided by someone else, contributing a payoff loss of approximately $\alpha t_H + \beta - t_H$ for the marginal type. Overall, the marginal type's payoff gain from volunteering is approximately equal to

$$\frac{i}{z^*}\mathrm{Pois}(z^*)(i)\alpha(t_H - \bar{t}) - \frac{1}{z^*}\sum_{j\geq i}\mathrm{Pois}(z^*)(j)(\alpha t_H + \beta - t_H). \tag{31}$$

This is equal to zero because the marginal type is indifferent between volunteering and not volunteering, yielding (28).[13]

7. Discussion

7.1. Asymmetric equilibria

We focus on symmetric equilibria because of their simplicity and because coordination on an asymmetric equilibrium is difficult in a symmetric environment. In an asymmetric equilibrium, the agent's cutoff type $\hat{t}_j$ can depend on the agent's identity j. Thus, the conditions for equilibria with interior cutoffs boil down to a system of n equations rather than one equation as in our analysis. There can also exist equilibria with cutoffs at the boundary so that some agents always contribute and others never do. For a class of examples, consider the pure k-volunteers rule with $2 \leq k \leq (n+1)/2$. Suppose that $\alpha\bar{t} + \beta \geq t_H$ so that even the highest-type agent prefers to let the task be done by another agent of average type. Then there exists an equilibrium in which the agents $j = 1, \ldots, k-1$ use a partition strategy with cutoff type $\hat{t}_j = t_L$ so that they volunteer for sure, while the remaining agents $j = k, \ldots, n$ use a partition strategy with cutoff type $\hat{t}_j = t_H$ so that they never volunteer. Note that no agent $j \leq k-1$ will ever be selected for the task. If she deviates and plays N, then she must sometimes do it herself without improving the type selection if somebody else does it. Any agent $j \geq k$ will be selected for the task with probability $1/(n-k+1)$. If she deviates and plays Y, then she does it herself with probability $1/k$, without improving the type selection if somebody else does it. Because $1/(n-k+1) < 1/k$, she does not want to deviate. This class of asymmetric equilibria leads to no improvement over the random-assignment rule. On the other hand, in a symmetric equilibrium an improvement over the random assignment is possible by Theorem 1.

The example suggests that asymmetric equilibria tend to be less efficient than symmetric equilibria. Replacing an agent-independent intermediate cutoff type $\hat{t}$ with small cutoff types for some agents and large cutoff types for others means that the task allocation will eventually depend more on an agent's identity and depend less on her type, thus reducing overall efficiency.

7.2. Restrictions of the mechanism

In this section, we discuss the restrictions of the class of mechanisms that we consider. First, our results also hold in settings in which each agent's type space is binary. In such a setting, by the revelation principle, the restriction to symmetric equilibria of anonymous binary mechanisms entails no loss of generality, implying that an i-volunteers rule is utilitarian optimal among all transfer-free mechanisms.[14] From this perspective, our results simply show that the mechanisms that are optimal among all mechanisms in settings with a binary type space continue to be optimal in the restricted set of binary mechanisms when the type space is continuous.

[13] In the case $i = 1$, we can make a connection to the literature on of public-good provision with complete information. Bergstrom and Leo (2015) define the coordinated volunteer's dilemma as the game in which, similar to the any-volunteers rule, the task is performed by a randomly selected volunteer if and only if at least one volunteer comes forward; if nobody volunteers, then the task is not performed at all. The task has a commonly known provision cost c and a commonly known benefit b to each agent; they consider symmetric equilibria in mixed strategies. As shown by Bergstrom and Leo (2015), in their setting formulas analogous to those in Lemma 5 for the heterogenous-ability case (i.e., $\alpha = 1$ and $\beta = c$) hold if $t_H - \bar{t}$ is replaced by b. This is intuitive because, in our model, the social benefit of the volunteering decision of the marginal type is that the task is done at highest quality, t_H, rather than average quality, $\bar{t}$. The lower bound for the threshold i in Lemma 5 is then satisfied for the any-volunteers rule ($i = 1$) if $1 > c/b$, implying that a sequence of equilibria with a strictly positive limit for the expected number of volunteers exists if $c < b$.

[14] Any mechanism that is not anonymous can be transformed into a symmetric game by uniformly randomizing the roles among the players before the game starts.

Our focus on binary mechanisms is motivated by their simplicity. In situations where a volunteer is needed and payments are not used, simplicity is often a very important property of a mechanism. We have shown that binary mechanisms, in spite of their simplicity, are quite powerful. Efficiency could be increased with more than two messages, but the absence of transfers would still preclude a first-best allocation.[15]

An entirely different approach to reduce the complexity of the agents' strategic considerations would be a restriction to dominant-strategy or ex-post incentive-compatible mechanisms with arbitrary action spaces. But this only leaves the possibility of random assignment.

Remark 2. The only ex-post incentive-compatible (EPIC) task-assignment rule is the one that induces the uniformly-random allocation for all type profiles.

The proof of this result is relegated to the Appendix.[16]

To better understand the limits of the restriction to binary messages, we next consider the question whether something can be gained if we maintain the restriction to the two messages Y and N per agent for simplicity reasons, but allow for sequential moves. In a sequential mechanism, the agents can condition their actions on previous actions, which may allow them to communicate more information about their types. Such a rule may be symmetrized by randomizing the order in which the agents move. We show with a simple example that the welfare from the optimal sequential binary mechanism can be higher than the binary-second-best welfare from Section 5.2. Let $n = 2$, F uniform on $[0, 1]$, and $\alpha = 1$. We consider a sequential version of the any-volunteers rule, in which the two agents sequentially play Y or N and the task is assigned to the second mover if and only if the second mover plays Y. As we show in the Appendix, if β is sufficiently small, then there exists an equilibrium in which the first mover chooses Y if her type is above the cutoff $1/2 + 4\beta$. If the second mover observes Y, she is optimistic that the first mover would do a good job and she only chooses Y herself if her type is larger than the cutoff $3/4 + 3\beta$. If the second mover observes N, she chooses Y if her type is larger than the cutoff $1/4 + 3\beta$. Note that all three cutoff types are different, reflecting the asymmetric roles of the first and second mover and the information revealed by the first mover. One can verify that the resulting welfare is higher than in the any-volunteers rule with simultaneous moves if β is sufficiently close to zero.

For a given i-volunteers rule, it can also happen that the sequentiality of moves reduces welfare relative to simultaneous moves. To see this, let $n = 3$ and consider the sequential version of the pure 2-volunteers rule. In the Appendix we show that in any perfect-Bayesian partition equilibrium the outcome will be that the first mover will play N while the two followers will play Y, independently of the type profile, if β is sufficiently large. Thus, the random assignment obtains. This contrasts with the simultaneous-move version of the pure 2-volunteers rule where an improvement is possible for any β by Theorem 1. In summary, the introduction of sequential moves implicitly opens new communication channels, which can improve the allocation, but can also exacerbate the free-riding problem.

The use of monetary transfers is ruled out in our setting since the very essence of volunteering is that there is no adequate remuneration. The underlying reasons are manifold, including social customs, income effects, and legal restrictions. If both arbitrary monetary transfers and messages were admitted, then the efficient allocation could be obtained by an auction-like mechanism that asks all agents for their types, assigns the task to the highest type and makes a payment to the selected type equal to $\beta - (1-\alpha)\hat{t}_{(n-1)}$, where $\hat{t}_{(n-1)}$ denotes the second-highest reported type.

7.3. Rules with random default

The rules that we identified as the welfare-optimal binary mechanisms are not the only binary mechanisms in our setting. Other binary mechanisms are less efficient, but might have other desirable properties. In this section, we consider the possibility of assigning the task randomly if the required number of volunteers is not reached instead of assigning it to the non-volunteers.

Definition 3. For any $i = 1, \ldots, n-1$, a mechanism $(p_1, \ldots, p_{n-1})$ is called an i-volunteers rule with random default and threshold i if $p_j = 1$ for all $j > i$, $p_i > i/n$, and $p_j = j/n$ for all $j < i$.

That is, an i-volunteers rule with random default is similar to a regular i-volunteers rule except that the assignment falls back to uniformly random if not enough volunteers show up. Intuitively, such a mechanism guarantees that non-volunteers are never worse off when participating in the mechanism than with random assignment of the task, no matter how many volunteers actually come forward.

We have assumed that each agent decides about participating in a given mechanism after learning her own type, but before learning anything else. An alternative assumption would be that during the execution of the mechanism each agent observes the actual number of volunteers before the assignment decision is made, and she can trigger the reversion to the random-assignment rule

[15] Bhaskar and Sadler (2019) provide a partial characterization of the utilitarian optimal transfer-free mechanism with an unrestricted message space in a setting that is closely related to a special case of the pleasant-task variation of our model. For a uniform type distribution, Bhaskar and Sadler (2019) show the optimality of partitioning the type space into intervals and allocating the task to an agent with a type in the highest reported bin, where allocation to the highest type within that interval is more likely with lower reports of the other agents; we conjecture a similar structure will play a role in settings with an unpleasant task.

[16] Feng et al. (2023) derive a similar conclusion in a class of allocation problems without monetary transfers, showing that only constant rules are EPIC. However, their technical assumptions exclude our setting.

at this point. It can be shown that under the resulting stronger participation constraints, an i-volunteers rule with random default is second-best optimal; see Goldlücke and Tröger (2023).

The rules with random default satisfy this added robustness requirement, but their lack of threat to assign the task specifically to the non-volunteers also introduces a weakness.

Theorem 5. *No i-volunteers rule with random default has the guaranteed-improvement property: an equilibrium outcome different from the uniform-assignment rule exists if and only if $\alpha t_H + \beta - t_H < i\alpha(t_H - \bar{t})$.*

In the heterogeneous-ability interpretation of our model, this inequality implies that volunteering breaks down if the opportunity cost of volunteering β is too high. In the altruism interpretation, it means that volunteering breaks down if the level of altruism α is too small. Theorem 5 generalizes Remark 1, which corresponds to the case $i = 1$. The proof is relegated to the Appendix.

7.4. Assigning a pleasant task

So far we have maintained Assumption 1. Here we revert it to consider cases with excessive volunteering incentives.[17] Instead of volunteering for an unpleasant task, agents who choose the message Y propose themselves for a promotion, a prize for some achievement, or a coveted project assignment. Agents then prefer to be selected over another agent with the same merit, but they also care about the merit of the agent who is selected.[18]

To model the problem of assigning a pleasant task, we keep the condition that the performer's utility is a positive affine transformation of the other agents' utilities, but we assume that agents of all types prefer doing the task themselves rather than letting someone else of their own type do the task. The latter condition means that getting people to refrain from volunteering can be difficult. Thus, we maintain the assumption $\alpha > 0$ but change Assumption 1 to

Assumption 2.

$$t > \alpha t + \beta \text{ for all } t \in [t_L, t_H]. \tag{32}$$

In the heterogeneous-ability case, the new assumption holds if the opportunity cost of volunteering is strictly negative, $\beta < 0$. In the altruism case, the new assumption is valid if the types are strictly positive, $t_L > 0$, which means that agents differ in the *benefit* they receive from the task. The optimal anonymous binary mechanism now has to reward an agent's decision to refrain from volunteering, by assigning the task to a volunteer with higher probability. A mechanism $(p_1, \dots, p_{n-1})$ is called a *maximum-i-volunteers rule* if there exists a threshold i $(1 \le i \le n-1)$ such that $p_j = 1$ for all $1 \le j < i$ and $p_j = 0$ for all $n - 1 \ge j > i$. With a maximum-volunteers rule, the task is awarded to the one of the volunteers if there are sufficiently *few* volunteers.

The main results can be adapted to the case of a pleasant task. In particular, one can show that if Assumption 2 holds, the binary-second-best mechanism is a maximum-i-volunteers rule for some threshold $i \in \{1, \dots, n-1\}$. Moreover, maximum-i-volunteers rules are sufficient to approximate the first-best as the group size increases.

8. Conclusion

We have shown that simple binary mechanisms can be powerful tools for assigning a task among a group of agents. Multiple-volunteers rules, which assign the task to a volunteer if and only if multiple volunteers come forward, are particularly useful. We expect this multiple-volunteers principle to extend to settings in which multiple tasks must be assigned, in the sense that one should typically require a number of volunteers that is larger than the number of tasks. While we think that binary mechanisms are a natural starting point due to their practicality, the kind of preferences and private information that we have modeled clearly deserve attention beyond binary mechanisms, including environments with monetary transfers.

CRediT authorship contribution statement

Susanne Goldlücke: Writing – review & editing, Writing – original draft, Validation, Methodology, Formal analysis, Conceptualization; **Thomas Tröger:** Writing – review & editing, Writing – original draft, Validation, Methodology, Funding acquisition, Formal analysis, Conceptualization.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

None.

[17] See the working paper version (Goldlücke and Tröger, 2023) for formal results.

[18] Another example is the setting of Li et al. (2016), in which two agents would like their own project to be chosen, but also have a common interest in the quality of the chosen project. The setting of Bhaskar and Sadler (2019) also fits in with $\beta = 0$, except that they also allow for a non-provision of the public good.

Appendix A. Proofs

Proof of formulas (11) and (12). Using the definition (6),

$$yq_Y(y) = \sum_{j=0}^{n-1} \frac{(n-1)!y^{j+1}(1-y)^{n-(j+1)}}{(j+1)!(n-(j+1))!} p_{j+1} = \frac{1}{n}\sum_{j=0}^{n-1} B_y^n(j+1)p_{j+1}.$$

Taking derivatives on both sides yields

$$yq_Y'(y) + q_Y(y) = \frac{1}{n}\sum_{j=0}^{n-1} \frac{d}{dy} B_y^n(j+1)p_{j+1}.$$

Thus, using the following standard identity about the derivative of a Bernstein polynomial,

$$\frac{d}{dy}B_y^n(j) = n\Big(B_y^{n-1}(j-1) - B_y^{n-1}(j)\Big) \text{ for } j = 1,\dots,n-1,$$
$$\frac{d}{dy}B_y^n(j) = nB_y^{n-1}(j-1) \text{ for } j = n$$

we obtain

$$\begin{aligned} yq_Y'(y) + q_Y(y) &= \sum_{j=0}^{n-1} B_y^{n-1}(j)p_{j+1} - \sum_{j=0}^{n-2} B_y^{n-1}(j+1)p_{j+1} \\ &= \sum_{j=0}^{n-1} B_y^{n-1}(j)p_{j+1} - \sum_{j=0}^{n-1} B_y^{n-1}(j)p_j. \end{aligned}$$

Now (11) follows from the definitions (4) and (5). Using (9), the derivative of q_Y can be used to calculate (12). □

Proof. (Proof of Lemma 1) The conclusion is obvious if $y(\sigma) = 0$ or $y(\sigma) = 1$ because then only one action is used in equilibrium. Suppose that $0 < y(\sigma) < 1$ and $q_N = q_Y$.[19] In this case, (9) implies $q_N = 1/n = q_Y$.

In cases with $u_Y = u_N$, the law of iterated expectations implies $yu_Y(y) + (1-y)u_N(y) = \alpha\bar{t} + \beta$, with $\bar{t}$ the expected type. Hence, $u_Y = u_N = \alpha\bar{t} + \beta$. Thus, both payoff expressions (15) and (16) are equal to $(1-\frac{1}{n})(\alpha\bar{t}+\beta) + \frac{1}{n}t$, which is the payoff from the uniform-assignment rule.

The remaining possibility is that $u_Y \neq u_N$. From (17), it is sufficient to show that the four auxiliary functions take the same values as when the uniform-assignment rule is used. We already have this for the functions q_N and q_Y.

Taking the difference of (15) and (16),

$$U_Y(\sigma,t) - U_N(\sigma,t) = (h_Y - h_N - \frac{1}{n})(u_Y - u_N),$$

which is independent of t. The utility difference must be equal to 0 because otherwise one action is strictly optimal for all types, implying $y(\sigma) = 0$ or $y(\sigma) = 1$. We conclude that $h_Y - h_N = 1/n$. On the other hand, (10) implies that $y(\sigma) = y(\sigma)h_Y + (1-y(\sigma))h_N$. Solving the system of these two equations leads to the random-assignment formulas (8). □

Proof. (Proof of Lemma 2) Using (9), $q_N < 1/n < q_Y$. Thus, $nyq_Y > y$ and $n(1-y)q_N < (1-y)$. This together with (18) implies

$$nyq_Y(y)u_Y(y) + n(1-y)q_N(y)u_N(y) > yu_Y(y) + (1-y)u_N(y) = \alpha\bar{t} + \beta, \tag{A.1}$$

where the last equality relies on the law of iterated expectations. Using (15) and (16), we see that, for any type t,

$$\begin{aligned} &yU_Y(y,t) + (1-y)U_N(y,t) \\ &= \big(y(h_Y - q_Y) + (1-y)h_N\big)u_Y + \big(y(1-h_Y) + (1-y)(1-h_N-q_N)\big)u_N + \frac{1}{n}t. \end{aligned}$$

Recall from (10) that

$$nyq_Y = yh_Y + (1-y)h_N.$$

Using (9), we get a similar formula for q_N,

$$n(1-y)q_N = y(1-h_Y) + (1-y)(1-h_N).$$

Therefore, it follows that

$$\begin{aligned} yU_Y(y,t) + (1-y)U_N(y,t) &= (n-1)(yq_Yu_Y + (1-y)q_Nu_N) + \frac{1}{n}t \\ &\overset{(A.1)}{>} (1-\frac{1}{n})(\alpha\bar{t}+\beta) + \frac{1}{n}t, \end{aligned}$$

which is the agent's payoff from the uniform-assignment rule. Therefore, also the equilibrium payoff $\max\{U_Y(y,t), U_N(y,t)\}$ is strictly larger than the payoff from the uniform-assignment rule. □

[19] We drop again the argument $y(\sigma)$ from the auxiliary functions.

Proof. (Proof of Lemma 3) Fix a multiple-volunteers rule with threshold i. Plugging in the definitions,

$$h_Y(y) - h_N(y) = B_y^{n-1}(i)(1-p_i) + B_y^{n-1}(i-1)p_i. \tag{A.2}$$

Since there are no multiple-volunteers rules for $n = 2$, we assume that $n \geq 3$. Suppose that $i = 1$. Using (A.2), it is straightforward to verify that

$$(h_Y - h_N)'(y) = (1-y)^{n-3}(n-1)l_1(y),$$

where we use the auxiliary function

$$l_1(y) = 1 - 2p_1 - y(n - 1 - np_1),$$

which is linear in y. Note that $l_1(1) = (2-n)(1-p_1) < 0$. Since in a multiple-volunteers rule, $p_1 < 1/n$, it also holds that $l_1(0) = 1 - 2p_1 > 0$ and that $l_1(y)$ is strictly decreasing in y, implying the claim of the lemma. The case $i = n-1$ is treated analogously to the case $i = 1$.

Now suppose that $1 < i < n-1$. Using (A.2), it is straightforward to verify that

$$(h_Y - h_N)'(y) = \frac{(n-1)!y^{i-2}(1-y)^{n-2-i}}{(i)!(n-i)!}l_i(y),$$

where we use the auxiliary function

$$\begin{aligned} l_i(y) = \ & y(1-y) - 2p_i)i(n-i) \\ & - y^2(1-p_i)(n-i)(n-i-1) \ + \ (1-y)^2 p_i i(i-1). \end{aligned}$$

Note that $l_i(0) = (i-1)ip_i > 0$ if $p_i > 0$, and $l'(0) = i(n-i) > 0$ if $p_i = 0$, implying that

$$l(y) > 0 \quad \text{for all } y > 0 \text{ that are sufficiently close to } 0.$$

Similarly, $l_i(1) = -(n-i)(n-i-1)(1-p_i) < 0$ if $p_i < 1$, and $l'(1) = i(n-i) > 0$ if $p_i = 1$, implying that

$$l(y) < 0 \quad \text{for all } y < 1 \text{ that are sufficiently close to } 1.$$

Thus, by the intermediate value theorem, $l_i(y^{*m}) = 0$ for some $y^{*m} \in (0,1)$. Moreover, y^{*m} is unique because l_i is quadratic in y. Hence, for all $y \in (0,1)$,

$$l_i(y) > 0 \text{ if } y < y^{*m}, \text{ and } l_i(y) < 0 if\, y > y^{*m},$$

which proves the lemma. □

Proof. (Proof of Lemma 4)

From (11), the intersection points of q_Y and $h_Y - h_N$ are the critical points of q_Y. We can also use this formula to calculate the second derivative for all $y \in (0,1)$:

$$q_Y''(y) = \frac{(h_Y(y) - h_N(y))' - 2q_Y'(y)}{y}. \tag{A.3}$$

Thus, at any critical point y^* of q_Y, the number $q_Y''(y^*)$ has the same sign as $(h_Y(y^*) - h_N(y^*))'$. Hence, using Lemma 3, any critical point $y^* \in (0, y^{*m})$ must be a local minimum, and any critical point $y \in (y^{*m}, 1)$ must be a local maximum.

It remains to show that for any multiple-volunteers rule,

$$h_Y(y) - h_N(y) - q_Y(y) > 0 \text{ for all } y > 0 \text{ sufficiently close to } 0, \tag{A.4}$$

$$\text{and} \quad h_Y(y) - h_N(y) - q_Y(y) < 0 \text{ for all } y < 1 \text{ sufficiently close to } 1. \tag{A.5}$$

Once we have shown (A.4), which means that q_Y is increasing close to zero, it becomes clear that q_Y cannot have a critical point in $(0, y^{*m})$ and must be increasing on the whole interval. It then follows from (A.4) that somewhere in the interval $[y^{*m}, 1)$, the function q_Y has a maximum at which it intersects $h_Y - h_N$.

To show (A.4) and (A.5), let $(p_1, \ldots, p_{n-1})$ be any multiple-volunteers rule with threshold i. At $y = 1$, if everybody else volunteers, a volunteering agent expects to be selected with probability $q_Y(1) = 1/n$. Using (A.2), we see that $h_Y(1) - h_N(1) = 1 - p_{n-1}$. Using the assumption $p_{n-1} > 1 - \frac{1}{n}$,

$$h_Y(1) - h_N(1) - q_Y(1) = 1 - p_{n-1} - \frac{1}{n} < 0,$$

implying (A.5). Using

$$q_Y(y) = \sum_{j=i}^{n-1} B_y^{n-1}(j)\frac{1}{j+1} + B_y^{n-1}(i-1)\frac{p_i}{i}.$$

and (A.2), we see that

$$h_Y(y) - h_N(y) - q_Y(y) = \binom{n-1}{i-1} y^{i-1} p_i (1 - \frac{1}{i}) + \mathcal{O}(y^i),$$

Thus, (A.4) holds if $i \geq 2$. If $i = 1$, then

$$h_Y(y) - h_N(y) = p_1 + (n-1)y(1-p_1) - (n-1)yp_1 + \mathcal{O}(y^2),$$
$$q_Y(y) = p_1 + (n-1)y\frac{1}{2} - (n-1)yp_1 + \mathcal{O}(y^2).$$

Thus, (A.4) follows from $1 - p_1 > 1 - 1/n > 1/2$. □

Proof. (Proof of Theorem 2.) To solve the binary-first-best problem, fix any mechanism-strategy combination $(\rho, \sigma) = (p_1, \ldots, p_{n-1}, \sigma)$. By switching to an equivalent mechanism if needed, we can assume that $q_Y^\rho(y(\sigma)) \geq q_N^\rho(y(\sigma))$.

Since $y(\sigma)t_Y(\sigma) + (1 - y(\sigma))t_N(\sigma) = \bar{t}$, we can write the objective in terms of t_Y:

$$W^\rho(\sigma) = (q_Y^\rho(y(\sigma)) - q_N^\rho(y(\sigma)))ny(\sigma)t_Y(\sigma) + nq_N^\rho(y(\sigma))\bar{t}.$$

We show first that the binary first-best is attained by a partition strategy. For a given strategy σ, we define the partition strategy $\overline{y} = y(\sigma)$ and show that

$$W^\rho(\overline{y}) \geq W^\rho(\sigma). \tag{A.6}$$

Given our definition of the partition strategy, to show (A.6), it remains to show that $t_Y(\overline{y}) \geq t_Y(\sigma)$. To see this inequality, note that for all $t \geq \hat{t}(\overline{y})$,

$$\int_{\hat{t}(\overline{y})}^{t} \mathrm{d}F(\tau) = \bar{y} - \int_{t}^{t_H} \mathrm{d}F(\tau) \leq \bar{y} - \int_{t}^{t_H} \sigma(\tau)\mathrm{d}F(\tau) = \int_{t_L}^{t} \sigma(\tau)\mathrm{d}F(\tau).$$

Using integration by parts,

$$\begin{aligned}
\overline{y}t_Y(\overline{y}) &= \int_{\hat{t}(\overline{y})}^{t_H} t\mathrm{d}F(t) \\
&= -\int_{\hat{t}(\overline{y})}^{t_H} \int_{\hat{t}(\overline{y})}^{t} \mathrm{d}F(\tau)\mathrm{d}t + t_H \int_{\hat{t}(\overline{y})}^{t_H} \mathrm{d}F(\tau) \\
&\geq -\int_{t_L}^{t_H} \int_{t_L}^{t} \sigma(\tau)\mathrm{d}F(\tau)\, \mathrm{d}t + t_H \int_{t_L}^{t_H} \sigma(\tau)\mathrm{d}F(\tau) \\
&= \int_{t_L}^{t_H} t\sigma(t)\mathrm{d}F(t) \;=\; \overline{y}t_Y(\sigma).
\end{aligned}$$

As a next step, we fix an arbitrary partition strategy $y \in [0,1]$ and solve for the optimal mechanism ρ given y. In the last step, we maximize over y. Using (9), we can write the objective W in terms of $q_Y^\rho(y)$, as

$$W^\rho(y) = nyq_Y^\rho(y)(t_Y(y) - t_N(y)) + t_N(y). \tag{A.7}$$

Therefore, the optimal mechanism ρ maximizes $q_Y^\rho(y)$. Thus, from (6) the any-volunteers rule is the unique optimal mechanism if $0 < y < 1$.

Any binary-first-best y^{**} satisfies $0 < y^{**} < 1$ because otherwise the uniformly-random allocation would obtain. Thus, the binary-first-best partition y^{**} is found by solving the first-order condition $(W^\rho)'(y^{**}) = 0$.

Next we prove (24). We will drop the argument ρ from all functions. Using (25) and applying the product differentiation rule, we find

$$\begin{aligned}
\frac{1}{n}W'(y) &= \frac{d}{dy}\big(yq_Y(y)t_Y(y) + (1-y)q_N(y)t_N(y)\big) \\
&= q_Y t_Y - q_N t_N + yq_Y' t_Y + (1-y)q_N' t_N + yq_Y t_Y' + (1-y)q_N t_N',
\end{aligned}$$

where we have dropped the argument y from all functions after the last equal sign. After computing

$$t_Y'(y) = \frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{1}{y}\int_{\hat{t}(y)}^{t_H} t\mathrm{d}F(t)\right) = \frac{\hat{t}(y) - t_Y(y)}{y}$$

and

$$t_N'(y) = \frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{1}{1-y}\int_{t_L}^{\hat{t}(y)} t\mathrm{d}F(t)\right) = \frac{t_N(y) - \hat{t}(y)}{1-y},$$

and plugging in the expressions from (11) and (12), we find

$$\begin{aligned}
\frac{\alpha}{n}W'(y) &= \alpha t_Y(h_Y - h_N - q_Y) + \alpha t_N(h_N - h_Y + q_N) + \alpha\hat{t}(q_Y - q_N) \\
&= u_Y(h_Y - h_N - q_Y) + u_N(h_N - h_Y + q_N) + (\alpha\hat{t} + \beta)(q_Y - q_N).
\end{aligned}$$

Combining this with the formula

$$\Delta(y) = (h_Y - h_N - q_N)(u_Y - u_N) + (q_Y - q_N)(\hat{t} - u_Y)$$

from (17) and (19), formula (24) follows.

The remaining arguments are given in the text below Theorem 2. □

Proof. (Proof of Theorem 3) Consider any solution $(\rho^*, y^*) = ((p_1^* \dots, p_{n-1}^*), y^*)$. From Theorem 1 it follows that the value reached at the solution improves upon the uniform-assignment rule. Thus, $0 < y^* < 1$ and $q_N^{\rho^*}(y^*) < q_Y^{\rho^*}(y^*)$ by Lemma 1.

Step 1. We show that (ρ^*, y^*) also solves the relaxed binary-first-best problem in which the constraint $\Delta^\rho(y) = 0$ is replaced by the inequality $\Delta^\rho(y) \geq 0$.

Suppose otherwise that the relaxed problem has a solution (ρ, y) such that $\Delta(y) > 0$. Applying (24), we see that

$$(W^\rho)'(y) > (q_Y^\rho(y) - q_N^\rho(y))((\alpha - 1)\hat{t}(y) + \beta).$$

The right-hand side is ≥ 0 because of Assumption 1. This is a contradiction to optimality because none of the constraints on y is binding.

Step 2. Fixing y^*, the remaining relaxed binary-first-best problem is a linear maximization problem over $\rho = (p_1, \dots, p_{n-1})$. Hence the Lagrange conditions are necessary and sufficient, without any qualification. Let $\lambda \geq 0$ denote the Lagrangian multiplier for the constraint $\Delta^\rho(y^*) \geq 0$. Due to $q_N^\rho(y^*) < q_Y^\rho(y^*)$, the Lagrangian multiplier for the constraint $q_N^\rho(y^*) \leq q_Y^\rho(y^*)$ equals 0. Thus, using the Lagrangian $L(p_1, \dots, p_{n-1}) = W^\rho(y^*) + \lambda\Delta^\rho(y^*)$, for all $j = 1, \dots, n-1$,

$$\begin{aligned} &\text{if } \frac{\partial L}{\partial p_j} > 0, \text{ then } p_j^* = 1, \\ &\text{if } \frac{\partial L}{\partial p_j} < 0, \text{ then } p_j^* = 0. \end{aligned} \tag{A.8}$$

Using (25),

$$\frac{\partial W^\rho}{\partial p_j} = B^n(j)\big(t_Y - t_N\big) = \frac{B^{n-1}(j)}{n-j} n(1-y^*)\big(t_Y - t_N\big),$$

where we have dropped the argument y^* from all functions. Similarly, using the definition (19), which is

$$\Delta(y) = (h_Y - h_N - q_N)(u_Y - u_N) + (q_Y - q_N)(\hat{t} - u_Y),$$

$$\frac{\partial \Delta^\rho}{\partial p_j} = \frac{B^{n-1}(j)}{n-j}\left(\left(\frac{1-y^*}{y^*} - 1\right)\hat{t} + \left(j\frac{1-y^*}{y^*} - (n-j)\right)(u_Y - u_N) - \frac{1-y^*}{y^*}u_Y + u_N\right).$$

Thus,

$$\frac{\partial L}{\partial p_j} = \underbrace{\frac{B^{n-1}(j)}{n-j}}_{>0}\left(j\lambda \underbrace{\left(\frac{1-y^*}{y^*} + 1\right)(u_Y - u_N)}_{>0} + [\text{ terms independent of } j]\right). \tag{A.9}$$

Consider the case that $\lambda > 0$.

From (14) and the paragraph below it, the all-volunteers rule $\rho' = (0, \dots, 0)$ satisfies $q_Y^{\rho'}(y') < q_N^{\rho'}(y')$ for all $y' \in [0, 1)$. Thus, recalling the constraints of the binary-second-best problem,

$$(p_1^* \dots, p_{n-1}^*) \neq \rho'.$$

Let i be the smallest integer such that $p_i^* > 0$. Then $\partial L/\partial p_i \geq 0$ by (A.8), and (A.9) implies that $\partial L/\partial p_j > 0$ for all $j > i$, implying $p_j^* = 1$ by (A.8). Thus, $(p_1^* \dots, p_{n-1}^*)$ is an i-volunteers rule.

It remains to consider the case $\lambda = 0$. Then $\partial L/\partial p_j = \partial W^\rho/\partial p_j > 0$ for all j, implying $p_j^* = 1$ for all j, that is, ρ^* is the any-volunteers rule. □

Proof. (Proof of Lemma 5)

Before presenting the proof, we provide a roadmap. *Step 0* recalls the Poisson approximation of the binomial distribution. Using the lower bound (27) on the volunteering threshold i, *Step 1* shows that there exists a sequence of partition strategies along which the expected number of volunteers does not vanish and the associated cut-off types strictly prefer volunteering. In *Step 2* we lower the marginal type until that type becomes indifferent; this yields a sequence of equilibria with non-vanishing expected numbers of volunteers. In *Step 3* we show that this sequence is bounded, which implies existence of a limit point. *Step 4* is to show that any such limit point z^* satisfies the equation (28) that is stated in the theorem. Since the left-hand-side of that equation is strictly decreasing, the limit point is unique. *Step 5* establishes the formula (29) for the limit welfare.

Step 0. Consider a sequence of numbers $(x_n)_n$, $x_n \in [0, 1]$ such that $nx_n \to z$ for some number $z > 0$. Then

$$\lim_n B_{x_n}^{n-1}(j) = e^{-z}\frac{z^j}{j!} \quad \text{for } j = 0, 1, \dots \tag{A.10}$$

and

$$\lim_n \sum_{j=i-1}^{n-1} B_{x_n}^{n-1}(j)\frac{1}{j+1} = e^{-z}\sum_{j=i-1}^{\infty}\frac{z^j}{(j+1)!}. \tag{A.11}$$

Formula (A.10) is the well-known Poisson limit theorem. To see (A.11), define $B^{n-1}_{x_n}(j) = 0$ for all $j > n-1$ and note that

$$\sum_{j=i-1}^{\infty} e^{-z}\frac{z^j}{(j+1)!} \overset{(A.10)}{=} \sum_{j=i-1}^{\infty} \lim_n B^{n-1}_{x_n}(j)\frac{1}{j+1}$$

$$= \lim_n \sum_{j=i-1}^{\infty} B^{n-1}_{x_n}(j)\frac{1}{j+1} = \lim_n \sum_{j=i-1}^{n-1} B^{n-1}_{x_n}(j)\frac{1}{j+1}.$$

In the rest of the proof, we will use the functions h_Y, h_N, q_Y, q_N, and Δ as applied to the given pure i-volunteers rule.

Step 1. Fix a number $\underline{z}$ such that

$$0 < \underline{z} < \ln\left(\frac{\alpha(t_H - \bar{t})i}{\alpha t_H + \beta - t_H}\right), \tag{A.12}$$

which is possible because the lower bound (27) guarantees that the argument of the function $\ln(\dots)$ is larger than 1. For all n, define $\underline{y}_n = \underline{z}/n$. We show that $\Delta(\underline{y}_n) > 0$ for all sufficiently large n.

Because $\underline{y}_n \to 0$, only the highest type volunteers in the large-population limit. Thus, as $n \to \infty$,

$$\hat{t}(\underline{y}_n) \to t_H, t_Y \to t_H, \text{ and } t_N \to \bar{t},$$

which implies

$$u_Y - u_N \to \alpha(t_H - \bar{t}) \text{ and } \hat{t} - u_Y \to t_H - (\alpha t_H + \beta). \tag{A.13}$$

Using the definitions (4) and (5), we find

$$h_Y(y) - h_N(y) = B^{n-1}_y(i-1), \tag{A.14}$$

where applying (A.10) with $j = i-1$ yields

$$h_Y(\underline{y}_n) - h_N(\underline{y}_n) \to e^{-\underline{z}}\frac{\underline{z}^{i-1}}{(i-1)!} \quad \text{as} \quad n \to \infty. \tag{A.15}$$

Moreover, the definition (6) yields

$$q_Y(y) = \sum_{j=i-1}^{n-1} B^{n-1}_y(j)\frac{1}{j+1}, \tag{A.16}$$

such that applying (A.11) yields

$$q_Y(\underline{y}_n) \to e^{-\underline{z}} \sum_{j=i-1}^{\infty} \frac{\underline{z}^j}{(j+1)!}. \tag{A.17}$$

Finally, using (7),

$$q_N(\underline{y}_n) = \sum_{j=0}^{i-1} B^{n-1}_y(j)\frac{1}{n-j} \le \frac{1}{n-i} \to 0 \quad \text{as} \quad n \to \infty. \tag{A.18}$$

Recall the function Δ from (19),

$$\Delta(y) = (h_Y - h_N - q_N)(u_Y - u_N) + (q_Y - q_N)(\hat{t} - u_Y).$$

We use the limits (A.13), (A.15), (A.17) and (A.18) to calculate

$$\lim_n \Delta(\underline{y}_n) = e^{-\underline{z}}\frac{\underline{z}^{i-1}}{(i-1)!}\alpha\underbrace{(t_H - \bar{t})}_{>0} + e^{-\underline{z}}\sum_{j=i-1}^{\infty}\frac{\underline{z}^j}{(j+1)!}\underbrace{(t_H - (\alpha t_H + \beta))}_{<0 \text{ by Assumption 1}}. \tag{A.19}$$

Note that

$$\sum_{j=i-1}^{\infty}\frac{\underline{z}^j}{(j+1)!} \le \sum_{j=i-1}^{\infty}\frac{\underline{z}^{i-1}\underline{z}^{j-i+1}}{i!(j-i+1)!} = \frac{\underline{z}^{i-1}}{i!}e^{\underline{z}}.$$

Thus, (A.19) implies

$$\lim_n \Delta(\underline{y}_n) \ge \frac{\underline{z}^{i-1}}{(i-1)!}\left(e^{-\underline{z}}\alpha(t_H - \bar{t}) - \frac{\alpha t_H + \beta - t_H}{i}\right) > 0,$$

where the last inequality follows from (A.12). This completes *Step 1*.

Step 2. The arguments leading to (21), which extend to the case $i = 1$, show that $\Delta(1) < 0$. Since for all sufficiently large n, *Step 1* has shown that $\Delta(\underline{y}_n) > 0$, the intermediate-value theorem implies the existence of $y_n \ge \underline{y}_n$ such that $\Delta(y_n) = 0$. By construction, $ny_n > \underline{z}$. This implies that $\liminf_n ny_n > 0$. This completes *Step 2*.

In the following, let $z_n = ny_n$ be any sequence with $\Delta(y_n) = 0$ and $\liminf_n z_n > 0$.

Step 3. Using a proof by contradiction, we show in the following that the sequence $(z_n)_n$ is bounded. Suppose otherwise, that is, that the expected number of volunteers z_n tends to infinity.

Given that only i volunteers are needed, it should then be the case that the probability that the task is assigned to a non-volunteer falls to zero along the sequence. We will show in the following that this probability tends to 0 even if it is first multiplied by z_n.

We first show that

$$B^n_{y_n}(j) \to 0 \quad \text{for all } j = 0, 1, \ldots. \tag{A.20}$$

To see this, note that due to elementary properties of the binomial distribution,

$$B^n_{y_n}(j) \le n^j y_n^j (1 - y_n)^{n-j} = (ny_n)^j (1 - y_n)^{n-j},$$

which implies

$$\ln\left(B^n_{y_n}(j)\right) \le j \ln(ny_n) + (n - j)\ln(1 - y_n).$$

Hence, using that $\ln(1 - y_n) \le -y_n$ and $y_n \le 1$,

$$\ln\left(B^n_{y_n}(j)\right) \le j \ln(ny_n) - (n - j)y_n \le j \ln(z_n) - z_n + j \to -\infty.$$

which implies (A.20).

Since by elementary properties of the binomial distribution,

$$ny_n B^{n-1}_{y_n}(j) = (j + 1)B^n_{y_n}(j + 1), \tag{A.21}$$

we can already deduce from (A.20) that

$$z_n B^{n-1}_{y_n}(j) \to 0 \quad \text{for all } j = 0, 1, \ldots, \tag{A.22}$$

which, using (A.14), means

$$\lim_{n\to\infty} z_n(h_Y(y_n) - h_N(y_n)) = 0. \tag{A.23}$$

From (A.22), and because $y_n \le 1$, it also follows that

$$\lim_{n\to\infty} \sum_{j=0}^{i-1} B^{n-1}_{y_n}(j) \frac{z_n}{z_n - jy_n} = 0,$$

which is equivalent to

$$\lim_{n\to\infty} nq_N(y_n) = 0. \tag{A.24}$$

Multiplying the equilibrium condition $\Delta(y_n) = 0$ with z_n and rewriting it using (9), we get

$$0 = ny_n(h_Y(y_n) - h_N(y_n) - q_N(y_n))(u_Y(y_n) - u_N(y_n)) + (1 - nq_N(y_n))(\hat{t}(y_n) - u_Y(y_n)).$$

Now we consider the limit $n \to \infty$ of this expression. By (A.23) and (A.24) the first term vanishes. Thus, by (A.24), it follows that

$$\lim_n \hat{t}(y_n) - u_Y(y_n) = 0.$$

However, using Assumption 1,

$$u_Y(y_n) - \hat{t}(y_n) = \alpha t_Y(y_n) + \beta - \hat{t}_n \ge \alpha\hat{t}(y_n) + \beta - \hat{t}_n \ge \min_{t\in[t_L,t_H]} \alpha t + \beta - t > 0,$$

which contradicts the zero limit derived above. This completes *Step 3*.

Step 4. Since the sequence z_n is bounded, there exists a convergent subsequence. Let z_{n_k} be such a subsequence, and consider any limit point z^* of z_{n_k}. We will show in the following that

$$h^{\text{Pois}(z^*)}(i) = \frac{\alpha t_H + \beta - t_H}{i\alpha(t_H - \bar{t})}.$$

A computation analogous to that leading to (A.19) in *Step 1* implies

$$\lim_k \Delta(y_{n_k}) = e^{-z^*} \frac{(z^*)^{i-1}}{(i-1)!}\alpha(t_H - \bar{t}) + e^{-z^*} \sum_{j=i-1}^{\infty} \frac{(z^*)^j}{(j+1)!}(t_H - (\alpha t_H + \beta)).$$

Applying the equilibrium condition $\Delta(y_{n_k}) = 0$,

$$0 = e^{-z^*} \frac{(z^*)^{i-1}}{(i-1)!}\alpha(t_H - \bar{t}) + e^{-z^*} \sum_{j=i-1}^{\infty} \frac{(z^*)^j}{(j+1)!}(t_H - (\alpha t_H + \beta)).$$

After multiplying by z^*/i and switching to the variable $j' = j + 1$ in the sum,

$$0 = e^{-z^*} \frac{(z^*)^i}{i!} \alpha(t_H - \bar{t}) + \sum_{j'=i}^{\infty} e^{-z^*} \frac{(z^*)^{j'}}{j'!} \frac{t_H - (\alpha t_H + \beta)}{i}.$$

Thus,

$$0 = \text{Pois}(z^*)(i)\alpha(t_H - \bar{t}) - \sum_{j'=i}^{\infty} \text{Pois}(z^*)(j') \frac{\alpha t_H + \beta - t_H}{i}.$$

This implies the claimed formula (28).

From (26) one sees that $h^{\text{Pois}(z)}(i)$ is strictly decreasing in z. Thus, the limit point z^* is unique, showing that the sequence $(z_n)_n$ converges to z^*.

Computations analogous to those leading to (A.17) and (A.18) now show that

$$\lim_n q_Y(y_n) > 0 \quad \text{and} \quad \lim_n q_N(y_n) = 0.$$

Thus, $q_Y(y_n) > q_N(y_n)$ for all sufficiently large n. Together with *Step 2* this shows that all such y_n are indeed partition equilibria. (For small n, if $q_Y(y_n) < q_N(y_n)$, then the strategy y_n must be redefined as an arbitrary partition equilibrium.) This completes *Step 4*.

Step 5. The probability that the task is assigned to a volunteer in the equilibrium with the partition strategy y_n is

$$\sum_{j=i}^{n} B_{y_n}^n(j) \overset{(A.21)}{=} \sum_{j=i}^{n} \frac{z_n}{j} B_{y_n}^{n-1}(j-1)$$

Thus, using *Step 0*,

$$\lim_n \sum_{j=i}^{n} B_{y_n}^n(j) = z^* \sum_{j=i}^{\infty} \frac{(z^*)^{j-1}}{j!} e^{-z^*} = \sum_{j=i}^{\infty} \frac{(z^*)^j}{j!} e^{-z^*}$$

Welfare is equal to the equilibrium value of the expected type of the selected agent,

$$W(y_n) = t_N(y_n) + \sum_{j=i}^{n} B_{y_n}^n(j)(t_Y(y_n) - t_N(y_n)),$$

such that

$$\lim_{n\to\infty} W(y_n) = \bar{t} + \sum_{j=i}^{\infty} \frac{(z^*)^j}{j!} e^{-z^*}(t_H - \bar{t}).$$

This completes the proof of the theorem. □

Proof. (Proof of Theorem 4) For any group size n, let W_n^* denote the binary-second-best optimal welfare level. For any $i < n$, let $W^{\rho^i}(y_n)$ denote the welfare in the equilibrium y_n in the i-volunteers rule ρ^i. Then certainly

$$\lim_{n\to\infty} W_n^* \geq \lim_{i\to\infty} \lim_{n\to\infty} W^{\rho^i}(y_n).$$

We will show in the following that the right-hand side equals the first-best limit welfare t_H, which implies the same for the left-hand side.

Using the shortcut $\kappa = (\alpha t_H + \beta - t_H)/(\alpha(t_H - \bar{t}))$, equation (28) can also be written as

$$i\text{Pois}(z^*)(i) = \kappa \sum_{j=i}^{\infty} \text{Pois}(z^*)(j)$$

or, using the definition of $\text{Pois}(z^*)(i)$,

$$i \frac{e^{-z^*}(z^*)^i}{i!} = \kappa \sum_{j=i}^{\infty} \text{Pois}(z^*)(j). \tag{A.25}$$

We will use the following (Chernoff) bounds for tail probabilities as applied to a Poisson random variable with mean z:

$$\sum_{j=l}^{\infty} \text{Pois}(z)(l) \leq \frac{e^{-z}(ez)^l}{l^l} \quad \text{for all } l \geq z, \tag{A.26}$$

$$\sum_{j=0}^{l} \text{Pois}(z)(j) \leq \frac{e^{-z}(ez)^l}{l^l} \quad \text{for all } \ l \leq z. \tag{A.27}$$

To prove (A.26), let X denote a Poisson distributed random variable with mean z. Then

$$E[\left(\frac{l}{z}\right)^X] = \sum_{k=0}^{\infty} \left(\frac{l}{z}\right)^k \frac{z^k e^{-z}}{k!} = \sum_{k=0}^{\infty} \frac{l^k}{k!} e^{-z} = e^{l-z}.$$

Thus, (A.26) follows from Markov's inequality:

$$\Pr[X \geq l] = \Pr\left[\left(\frac{l}{z}\right)^X \geq \left(\frac{l}{z}\right)^l\right] \leq \frac{E[\left(\frac{l}{z}\right)^X]}{\left(\frac{l}{z}\right)^l} = e^{l-z}\left(\frac{z}{l}\right)^l.$$

The proof of (A.27) is analogous.

We begin by showing that

$$z^* \geq i \text{ for all sufficiently large } i. \tag{A.28}$$

Suppose that $z^* < i$. Applying (A.26),

$$\sum_{j=i}^{\infty} \text{Pois}(z^*)(j) \leq \frac{e^{-z^*}(ez^*)^i}{i^i}.$$

Using (A.25), we obtain

$$i\frac{e^{-z^*}(z^*)^i}{i!} \leq \kappa\frac{e^{-z^*}(ez^*)^i}{i^i}.$$

After cancelling terms,

$$\frac{i^{i+1}}{(i)!e^i} \leq \kappa.$$

By Stirling's formula, the left-hand side tends to infinity as $i \to \infty$, yielding a contradiction. This shows (A.28).

In particular, $\sqrt{i-1}/z^* \to 0$ as $i \to \infty$. Because the right-hand side of (A.25) is bounded by κ, it also follows that

$$\frac{e^{-z^*}(z^*)^{i-1}\sqrt{i-1}}{(i-1)!} \to 0.$$

By Stirling's formula,

$$\frac{e^{i-1-z^*}(z^*)^{i-1}}{(i-1)^{(i-1)}} \to 0.$$

Thus, using (A.27) with $l = i - 1$,

$$1 - \sum_{j=i}^{\infty} \text{Pois}(z^*)(j) \leq \frac{e^{i-1-z^*}(z^*)^{i-1}}{(i-1)^{(i-1)}} \to 0,$$

implying $\lim_{i\to\infty} \sum_{j=i}^{\infty} \text{Pois}(z^*)(j) = 1$. Using (29), this implies

$$\lim_{i\to\infty} \lim_{n\to\infty} W^{\rho^i}(y_n) = t_H.$$

□

Proof. (Proof of Remark 2) Consider any anonymous EPIC task-assignment rule $(Q_1, \ldots, Q_n)$, where $Q_i(\tau)$ denotes the probability that the task is assigned to agent i at type profile $\tau = (t_1, \ldots, t_n)$, with $\sum_{j=1}^n Q_j(\tau) = 1$. That the rule is EPIC means that, for all i, $t_1, \ldots, t_n$, and t_i',

$$t_i Q_i(t_i, t_{-i}) + \sum_{j\neq i}(\alpha t_j + \beta)Q_j(t_i, t_{-i}) \geq t_i Q_i(t_i', t_{-i}) + \sum_{j\neq i}(\alpha t_j + \beta)Q_j(t_i', t_{-i}).$$

By standard revealed-preference arguments, this implies that Q_i is weakly increasing in t_i.

Anonymity means that $Q_{\pi(i)}(t_{\pi(1)}, \ldots, t_{\pi(n)}) = Q_i(t_1, \ldots, t_n)$ for any permutation π of the set $\{1, \ldots, n\}$, any agent i, and any type profile $(t_1, \ldots, t_n)$. In particular, for any i and j, using the permutation that switches i and j, one sees that for all type profiles $\tau = (t_1, \ldots, t_n)$,

$$Q_i(\tau) = Q_j(\tau) \quad \text{if } t_i = t_j. \tag{A.29}$$

Our goal is to show that

$$Q_i(\tau) = \frac{1}{n} \quad \text{for all } i. \tag{A.30}$$

We prove this by induction. For all $k = 1, \ldots, n$, consider the following statement A_k: For all type profiles τ such that the largest $n - k + 1$ components are identical, we have $Q_i(\tau) = 1/n$ for all $i = 1, \ldots, n$.

The statement A_1 is true by (A.29). We will show $A_k \Rightarrow A_{k+1}$ for all $k < n$.

Assume A_k. To prove A_{k+1}, consider any type profile $\tau = (t_1, \ldots, t_n)$ such that the largest $n - k$ components are identical. Without loss of generality, we consider an ordered type profile, $t_1 \leq \cdots \leq t_n$. Thus, there exists a type t such that $t_{k+1} = \cdots = t_n = t$. From A_k,

$$Q_i(t_1, \ldots, t_{j-1}, t, t_{j+1}, \ldots, t_n) = \frac{1}{n} \quad \text{for all } i \text{ and all } j \leq k. \tag{A.31}$$

It remains to consider type profiles t such that

$$t_j < t \quad \text{for all } j \le k. \tag{A.32}$$

Because Q_j is weakly increasing in agent j's type,

$$Q_j(t_1, \ldots, t_n) \le \frac{1}{n} \quad \text{for all } j \le k. \tag{A.33}$$

EPIC implies that agent k with type t does not gain from reporting type t_k instead. This together with (A.31) implies that

$$t\frac{1}{n} + \sum_{j<k}(\alpha t_j + \beta)\frac{1}{n} + \sum_{j>k}(\alpha t + \beta)\frac{1}{n} \ge tQ_k(\tau) + \sum_{j<k}(\alpha t_j + \beta)Q_j(\tau) + \sum_{j>k}(\alpha t + \beta)Q_j(\tau).$$

Subtracting $\alpha t + \beta = \sum_{j=1}^{n} Q_j(\tau)(\alpha t + \beta)$ on both sides yields

$$(t - \alpha t - \beta)\frac{1}{n} + \sum_{j<k}\alpha(t_j - t)\frac{1}{n} \ge (t - \alpha t - \beta)Q_k(\tau) + \sum_{j<k}\alpha(t_j - t)Q_j(\tau),$$

which is equivalent to

$$\underbrace{(t - \alpha t - \beta)}_{<0}\left(\frac{1}{n} - Q_k(\tau)\right) + \sum_{j<k}\underbrace{\alpha(t_j - t)}_{<0}\left(\frac{1}{n} - Q_j(\tau)\right) \ge 0,$$

where the underbraced inequalities follow from Assumption 1 and (A.32). The above inequality together with (A.33) implies $Q_j(\tau) = \frac{1}{n}$ for all $j \le k$. Thus, $\sum_{j>k} Q_j(\tau) = 1 - k/n$. From (A.29), $Q_{k+1}(\tau) = \cdots = Q_n(\tau)$, implying $Q_j(\tau) = 1/n$ for all $j > k$. This completes the proof of (A.30). □

Proof. (Proof of Theorem 5)

For any i-volunteers rule with random default it holds that $i + 1 \le n$, which implies $(n - i)/n \le (n - i)/(i + 1)$, or

$$1 - \frac{i}{n} \le \frac{n - i}{i + 1}. \tag{A.34}$$

Hence,

$$1 - p_i < \frac{n - i}{i + 1}. \tag{A.35}$$

Using the definitions (6) and (7), for any y,

$$q_Y(y) - q_N(y) =$$

$$B_y^{n-1}(i-1)\underbrace{\left(\frac{p_i}{i} - \frac{1}{n}\right)}_{>0} + B_y^{n-1}(i)\underbrace{\left(\frac{1}{i+1} - \frac{1-p_i}{n-i}\right)}_{>0 \text{ by (A.35)}} + \sum_{j=i+1}^{n-1} B_y^{n-1}(j)\underbrace{\left(\frac{1}{j+1} - 0\right)}_{>0}. \tag{A.36}$$

If $y > 0$, then at least one of the binomial probabilities above is strictly positive, implying $q_Y(y) > q_N(y)$. This shows that for all $y \in (0, 1]$, any equilibrium is a partition equilibrium.

To get a better understanding of equilibrium, we need an explicit expression for $\Delta(y)$ (defined in 19). Using (4), (5), and (6),

$$\begin{aligned} h_Y - h_N - q_Y &= \sum_{j=0}^{i-2} B_y^{n-1}(j)\frac{1}{n} + B_y^{n-1}(i-1)(p_i - \frac{i-1}{n}) + B_y^{n-1}(i)(1 - p_i) \\ &\quad - \sum_{j=0}^{i-2} B_y^{n-1}(j)\frac{1}{n} - B_y^{n-1}(i-1)\frac{p_i}{i} - B_y^{n-1}(i)\frac{1}{i+1} - \sum_{i+1}^{n-1} B_y^{n-1}(j)\frac{1}{j+1} \\ &= B_y^{n-1}(i-1)\cdot(i-1)\left(\frac{p_i}{i} - \frac{1}{n}\right) + B_y^{n-1}(i)\left(\frac{i}{i+1} - p_i\right) \\ &\quad - \sum_{i+1}^{n-1} B_y^{n-1}(j)\frac{1}{j+1}. \end{aligned}$$

Similarly, using (4), (5), and (7),

$$h_Y - h_N - q_N = B_y^{n-1}(i-1)\cdot i\left(\frac{p_i}{i} - \frac{1}{n}\right) + B_y^{n-1}(i)\cdot(1 - p_i)\left(1 - \frac{1}{n-i}\right).$$

The "if-and-only-if" inequality in the statement of the theorem can be written as

$$t_H + (i-1)(\alpha t_H + \beta) > i(\alpha\bar{t} + \beta). \tag{A.37}$$

Let us first consider cases in which this holds.

Consider $y \approx 0$. Then $B_y^{n-1}(i-1)$ is much larger then $B_y^{n-1}(j)$ for all $j \ge i$, implying

$$\frac{q_Y - q_N}{B_y^{n-1}(i-1)} \approx \frac{p_i}{i} - \frac{1}{n},$$

$$\frac{h_Y - h_N - q_Y}{B_y^{n-1}(i-1)} \approx (i-1)\Big(\frac{p_i}{i} - \frac{1}{n}\Big),$$
$$\frac{h_Y - h_N - q_N}{B_y^{n-1}(i-1)} \approx i\Big(\frac{p_i}{i} - \frac{1}{n}\Big).$$

Thus,

$$\frac{\Delta(y)}{B_y^{n-1}(i-1)} \approx \Big(\frac{p_i}{i} - \frac{1}{n}\Big)\big(\hat{t} + (i-1)u_Y - iu_N\big).$$

From $y \approx 0$ it follows that $\hat{t} \approx t_H$, $u_Y \approx \alpha t_H + \beta$, and $u_N \approx \alpha\bar{t} + \beta$. Thus, in the case considered, $\hat{t} + (i-1)u_Y - iu_N > 0$ if y is close to 0, implying that $\Delta(y) > 0$ if y is close to 0. Since $\Delta(1) < 0$ as in (21), by continuity of Δ, there exists y such that $\Delta(y) = 0$, yielding the desired equilibrium.

Now consider cases in which the opposite inequality of (A.37) holds. Since $\Delta(1) < 0$, it is sufficient to show that $\Delta(y) < 0$ for all $0 < y < 1$. Moreover, because $\Delta(y)$ is linear in p_i, it is sufficient to consider the extreme cases $p_i = 1$ and $p_i \approx i/n$.

In fact, it is sufficient to consider the case $p_i = 1$ for any i. The case $p_i \approx i/n$ leads to the rule $(p_1, \dots, p_{n-1}) \approx (1/n, \dots, i/n, 1, \dots, 1)$, which is already covered by the first case, with an incremented definition of the threshold i. The only case not covered by this then is the case $i = n-1$, $p_i \approx i/n$; this leads to the uniform-assignment rule, where $\Delta(y) = 0$ for all y; by linearity of $\Delta(y)$ in p_{n-1}, we conclude that $\Delta(y) < 0$ for all other volunteers rules with random default.

Assume $p_i = 1$. Note that, using Assumption 1, $\hat{t} < \alpha\hat{t} + \beta < u_Y$. Thus, if after plugging the expressions for $q_Y - q_N$ and $h_Y - h_N - q_Y$ that were obtained above into (19) we drop the sums $\sum_{i+1}^{n-1}$, then we obtain a strict upper bound for $\Delta(y)$, that is,

$$\Delta(y) < B_y^{n-1}(i-1) \cdot\left(\Big(\frac{1}{i} - \frac{1}{n}\Big)\big(t_Y + (i-1)u_Y - iu_N\big) + \frac{y}{1-y}\frac{n-i}{i}\frac{1}{i+1}(t_Y - u_Y)\right).$$

where we have also used the inequality $\hat{t} < t_Y$, have plugged in the above expression for $h_Y - h_N - q_N$, and have used that

$$B_y^{n-1}(i) = B_y^{n-1}(i-1)\frac{y}{1-y}\frac{n-i}{i}.$$

Now using (A.34) we obtain

$$\frac{\Delta(y)}{B_y^{n-1}(i-1)\cdot(\frac{1}{i} - \frac{1}{n})} < t_Y + (i-1)u_Y - iu_N + \frac{y}{1-y}(t_Y - u_Y).$$

Because the opposite of (A.37) holds, $t_Y - u_Y \le i\alpha(\bar{t} - t_Y)$. Thus,

$$\frac{\Delta(y)}{B_y^{n-1}(i-1)\cdot(\frac{1}{i} - \frac{1}{n})} < i\alpha(\bar{t} - t_N) + \frac{y}{1-y}i\alpha(\bar{t} - t_Y)).$$

Hence,

$$\frac{\Delta(y)(1-y)}{B_y^{n-1}(i-1)\cdot(\frac{1}{i} - \frac{1}{n})i\alpha} < (1-y)(\bar{t} - t_N) + y(\bar{t} - t_Y) = 0,$$

where the equation follows from the law of iterated expectations. Thus, $\Delta(y) < 0$. □

Examples in Section 7.2. To see that the welfare from the optimal sequential binary mechanism can be higher than the binary-second-best welfare from Section 5.2, we consider the example with $n = 2$, F uniform on $[0,1]$, and $\alpha = 1$. Recall from Footnote 11 that the any-volunteers rule is optimal in our main model. We consider the case that $\beta > 0$ is small, in particular $\beta < 0.5$.[20] In this case, the any-volunteers rule leads to the partition equilibrium with cut-off $\hat{t} = \frac{1}{2} + \beta$. For $\beta = 0$, the binary first best is obtained, which yields a surplus of $S^{**} = \frac{5}{8}$.

Consider now the sequential version of the any-volunteers rule. A partition equilibrium is characterized by the first-mover cutoff type $\hat{t}_1$, the second-mover cutoff type $\hat{t}_{2,N}$ that applies after she observes the first-mover action N, and the corresponding second-mover cutoff type $\hat{t}_{2,Y}$ if she observes Y. The claim is that if β is sufficiently small, then there exists an equilibrium with $\hat{t}_1 = 1/2 + 4\beta$, $\hat{t}_{2,N} = 1/4 + 3\beta$, and $\hat{t}_{2,Y} = 3/4 + 3\beta$.

To verify this claim, note that for the second mover it is a best response to choose Y if her own type is larger than the expected utility from letting the first mover perform the task. In the case of the uniform distribution, this condition leads to the cutoff types $\hat{t}_2(N) = \hat{t}_1/2 + \beta$ and $\hat{t}_2(Y) = (\hat{t}_1 + 1)/2 + \beta$. For $\hat{t}_1 = 1/2 + 4\beta$, we obtain the values above. The first mover, who is assigned the task unless the second mover volunteers, plays Y if

$$(1 - \hat{t}_{2,Y})\Big(\frac{\hat{t}_{2,Y} + 1}{2} + \beta - t_1\Big) \ge (1 - \hat{t}_{2,N})\Big(\frac{\hat{t}_{2,N} + 1}{2} + \beta - t_1\Big).$$

[20] If $\beta \ge 0.5$, the any-volunteers rule leads to random assignment, as does any transfer-free mechanisms.

For $\hat{t}_{2,N} = 1/4 + 3\beta$ and $\hat{t}_{2,Y} = 3/4 + 3\beta$ we obtain

$$(\frac{1}{4} - 3\beta)(\frac{7}{8} + \frac{5}{2}\beta - t_1) \geq (\frac{3}{4} - 3\beta)(\frac{5}{8} + \frac{5}{2}\beta - t_1),$$

which leads to the cutoff value as defined above.

One can then verify that the resulting welfare is higher than in the (symmetric) equilibrium of the any-volunteers rule with simultaneous moves if β is small enough. In particular, in the limit $\beta = 0$, the resulting welfare is equal to $\frac{21}{32} > \frac{5}{8}$.

Consider now the second example of the pure 2-volunteers rule with $n = 3$ agents. Suppose the agents decide sequentially, first Agent 1, then Agent 2, then Agent 3.

Assume $\beta > (1 + 2\alpha)t^H - 3\alpha t^L$. This assumption implies that each agent prefers being assigned the task with probability 0 to being assigned the task with probability $1/3$, independently of what happens if the task is not assigned to the agent. This is because the payoff from being assigned the task with probability $1/3$ is bounded above by $(1/3)t^H + (2/3)(\alpha t^H + \beta)$, and the payoff from being assigned the task with probability 0 is bounded below by $\alpha t^L + \beta$.

A perfect Bayesian equilibrium is called a *partition equilibrium* if at each history where an agent moves (i) she uses a partition strategy and (ii) the other agents' belief concerning the expected type of the agent is strictly smaller if she plays N than if she plays Y. (Note that (ii) has bite off the equilibrium path.)

We will show that in any partition equilibrium the final history will be NYY independently of the type profile, so that the allocation will be uniformly random.

Because with simultaneous moves there is an equilibrium that is strictly better than uniformly random, we arrive at the conclusion that sequential moves can be harmful.

We will use backward induction. At the history YY, Agent 3 knows that she will be assigned the task with prob $1/3$ when playing Y and with prob 0 when playing N. Given the assumption on β, she plays N.

At the history YN, Agent 3 knows that she will get the task with probability $1/2$, no matter whether she plays Y or N. With the remaining probability $1/2$, by playing N she brings the task to N-player 2 and by playing Y she brings the task to Y-player 1. Given the partition form of the equilibrium, bringing the task to Y-player 1 is better, so Agent 3 will play Y.

In summary, at history Y, Agent 2 knows that by playing Y she induces Agent 3 to play N, so the history will be YYN and Agent 2 will share the task equally with Y-player 1. But if Agent 2 plays N, then she induces Agent 3 to play Y so that she will avoid the task altogether. Thus, given the assumption about β, Agent 2 will play N.

We conclude that Agent 1 if she plays Y induces the history YNY so she gets assigned the task with probability $1/2$.

Next, we will show that if Agent 1 plays N, then she induces the history NYY.

At history NY, Agent 3 will play Y because she gets the task with probability $1/2$ also if she plays N, but given the partition form of the equilibrium, she'd rather share the task with a Y-player than an N-player.

At history NN, player 3 will play Y to avoid the task, given the assumption about β. So, at history N, Agent 2 anticipates that by playing N she induces the history NNY, whereas by playing Y she induces the history NYY. With either action, she gets assigned the task with probability $1/2$. Given the partition form of the equilibrium, she'd rather share the task with a Y-player than an N-player. Thus, Agent 2 plays Y at history N.

We conclude that Agent 1 by playing N induces the history NYY. Thus, Agent 1 will play N to avoid the task. The final history will by NYY independently of the type profile, with the result that the allocation will be uniformly random.

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