Essays on Firm Dynamics and Labor Market Flows

Dissertation

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Data Sources

Chapter 1 and Chapter 2 use the weakly anonymous Establishment History Panel 1975-2010. Data access was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and/or remote data access. All results have been cleared for disclosure.

Chapter 3 uses administrative firm data for Germany (*Amtliche Firmendaten für Deutschland*, AFiD), which are provided by the Research Data Centers of the Federal and State Statistical Offices. All results have been cleared for disclosure.

Author Contribution

Chapter 1 and Chapter 2 are my own work developed under the guidance of Leo Kaas as well as Nawid Siassi and Carlos Carrillo-Tudela. Chapter 3 was developed jointly with Leo Kaas. Leo Kaas provided much stirring. The model as well as the calibration was done by him. I did the data work necessary for calibrating and motivating the paper. I sourced and evaluated the literature for relevance, in particular in the early stages of the project. The chapter in its current form is a refined iteration of the idea that was started in the first quarter of 2014, where in the beginning I was involved in the model part as well. However, we eventually specialized. Therefore, my overall contribution may be assessed at 40 - 50%.

Summary

Firm dynamics – firm entry, growth, and exit – are central to the creation and destruction of jobs as well as the flow of workers between employment and unemployment. The economic gains such as wage and productivity growth brought about by the dynamics of firms and the labor market flows are realized behind vast heterogeneity and changing conditions in the product market as well as the input markets, in particular, the labor market. For instance, labor differs in skill and experience; jobs differ by the skill requirements; and firms differ by productivity and product demand. Required, therefore, is a reasonably efficient matching process not only between workers and firms but also between firms and customers.

My dissertation aims at providing new empirical and theoretical insights on firm dynamics and labor market flows. It analyses German firm or establishment data with detailed information on product prices, quantities sold, and employment histories. It also develops and simulates models of firm dynamics with richer labor and product market features, taking into account recent advances made in the literature. The models facilitate explaining the documented empirical distributions and the interaction of firm dynamics and labor and product markets as comprehensively as possible.

The dissertation consists of three independent research essays organized in three chapters. It is a cumulative thesis. Chapter 1 and Chapter 2 are a study on the employer-level drivers of labor market flows with a focus on occupational heterogeneity and misallocation. Chapter 2 seeks to explain the empirical results documented in Chapter 1 with a calibrated general equilibrium quantitative model. It studies the joint interaction of labor market flows, misallocation, and occupational labor heterogeneity. Chapter 3 is a study on firm dynamics and the labor market. It distinguishes the supply and demand drivers of the dynamics of firms, prices, output, and employment.

Chapter 1 is titled "Occupational Labor Market Flows". This chapter investigates the cross-sectional patterns of the rates of occupational job and worker flows. Disregarding the occupational margin of job flows, implies a job to worker flow ratio of 34.8%, which is smaller than the 46.1% with occupation-based gross job flows. Thus, the latter is an improved measure of job flows and captures better the importance of job flows for worker flows. The chapter finds that one in four reallocated occupational jobs results in no net job changes, consistent with simultaneous job creation and destruction at the level of the establishment. These findings may be explained by imperfect substitutability of occupational labor in production, differences in the costs of occupational job turnover, occupation-specific labor productivity heterogeneity, or (labor) misallocation. Furthermore, while labor market flow rates in specific occupations, and for all occupations as a whole, have not trended at all, their relative shares have trended linearly over time, consistent with job polarization. This observation allows to treat the aggregate occupational employment trends separately from the cross-sectional dynamics, isolating the importance of cross-sectional heterogeneity.

Chapter 2 is titled "Labor Market Flows and Misallocation with Occupational Heterogeneity". The starting point of this chapter is the fact that where firms operate with more

than one occupation, then an occupation classifies a workplace into subunits with similar skill requirements. Then, these subunits expand and shrink at different rates. When occupational job reallocation is accounted for, then gross job reallocation over and above net job creation is considerably large. This *firm-level residual job reallocation* leads to more worker reallocation per job reallocation than was previously documented. The chapter discusses empirical evidence for the phenomenon, explains it as a result of labor quality heterogeneity, misallocation of factor inputs such as labor, and quantifies their roles in a search model of firm dynamics. Due to computational complexity, the chapter ends with preliminary quantitative results.

Chapter 3 is titled "Firm Dynamics with Frictional Product and Labor Markets". It is jointly written with Leo Kaas. The chapter analyses the joint dynamics of prices, productivity and employment across firms. It develops a dynamic equilibrium model of heterogeneous firms who compete for workers and customers in frictional labor and product markets. Using panel data on prices and output for German manufacturing firms, the model is calibrated to evaluate the quantitative contributions of productivity and demand for the labor market. Product market frictions decisively dampen the firms' employment adjustments in response to productivity shocks. Further, the chapter analyses the impact of shocks to the first and second moments of idiosyncratic risk on macroeconomic outcomes. An increase in demand uncertainty induces declines in output and employment together with rising cross-sectional dispersion of price and output growth which are typical features of recessions in our data.

Zusammenfassung

Unternehmensdynamiken - Markteintritt, Wachstum und Marktaustritt - sind von grundlegender Bedeutung für das Entstehen und die Verluste von Arbeitsplätzen sowie für den Wechsel zwischen Beschäftigung und Arbeitslosigkeit. Die durch Unternehmens- und Arbeitsmarktdynamiken entstehenden ökonomischen Zugewinne wie Einkommen und Produktivitätswachstum entstehen vor dem Hintergrund einer immensen Heterogenität und sich ändernden Voraussetzungen auf Produkt- und vorgelagerten Märkten. Dies gilt insbesondere für den Arbeitsmarkt. So unterscheiden sich Arbeitskräfte hinsichtlich ihrer Qualifikationen und Erfahrungen; Arbeitsplätze unterscheiden sich in Bezug auf ihre Qualifikationsanforderungen; Unternehmen unterscheiden sich in Hinblick auf ihre Produktivität und Nachfrage. Daher ist ein einigermaßen effizienter Matching-Prozess nicht nur zwischen Arbeitnehmern und Unternehmen sondern auch zwischen Unternehmen und Kunden nötig.

Diese Dissertation liefert neue empirische und theoretische Erkenntnisse in Hinblick auf Unternehmensdynamiken und Arbeitsmarktsströme zu geben. Hierzu werden deutsche Unternehmens- oder Niederlassungsdaten analysiert, die detaillierte Informationen zu Produktpreisen, und der Anzahl der Verkaufseinheiten und Beschäftigungshistorien umfassen. Außerdem werden Modelle für Unternehmensdynamiken mit umfangreichen Arbeits- und Produktmarkteigenschaften entwickelt und simuliert. Diese Modelle erlauben eine möglichst umfassende Erklärung der bekannten empirischen Verteilungen und den Interaktionen zwischen Unternehmensdynamiken sowie Arbeits- und Produktmärkten.

Die vorliegende Dissertation umfasst drei unabhängige Forschungspapiere, die in drei Kapitel aufgeteilt sind. In diesem Sinne ist es eine kumulative Arbeit. Kapitel 1 und 2 betrachten die einflussfaktoren der Arbeitsmarktströme mit einem Fokus auf Beschäftigungsheterogenität und Fehlallokation. Kapitel 2 versucht mittels eines kalibrierten, quantitativen allgemeinen Gleichgewichtsmodells die empirischen Befunde aus Kapitel 1 zu erklären. Es werden die gemeinsamen Interaktionen von Arbeitsmarktströmen, Fehlallokationen und Beschäftigungsheterogenität untersucht. Kapitel 3 ist eine Studie zu Unternehmensdynamiken und dem Arbeitsmarkt. Es werden dabei die angebots- und nachfrageseitigen Determinanten der Unternehmens-, Preis-, Output- und Beschäftigungsdynamiken unterschieden.

Kapitel 1 trägt den Titel "Occupational Labor Market Flows" [Beschäftigungsbezogene Arbeitsmarkströme]. Es werden die im Querschnitt zu beobachtenden Muster der Beschäftigungs- und Arbeitnehmerströme betrachtet. Werden die tätigkeitsbezogenen Aspekte der Arbeitsplatzströme vernachlässigt, liegt das Verhältnis von Job- zu Arbeitnehmerströmen bei 34,8%. Werden diese Aspekte jedoch berücksichtigt, ergibt sich ein Verhältnis von 46,1%. Demnach ist letzteres ein besseres Maß für die Beschäftigungsströme und ist besser in der Lage Beschäftigungs- und Arbeitnehmerströme zu beschreiben. In diesem Kapitel zeigt sich, dass ein Viertel aller umverteilten Arbeitsplätze in keinen Netto-Arbeitsplatzverlust resultiert - also Arbeitsplätze auf Ebene der Niederlassung simultan geschaffen und verloren werden. Dieser Befund könnte auf eine unvollkommene Substituierbarkeit von Berufen in der Produktion, Unterschiede hinsichtlich der Kosten der Arbeitsplatzfluktuation, berufsbezogene Produktivitätsheterogenität oder Arbeitsplatzfehlallokation hindeuten. Während die Rate der Arbeitsmarktströme in spezifischen Berufen sowie sämtlichen Berufen sich nicht verändert haben, hat ihr Anteil über die Zeit zugenommen. Diese Beobachtung erlaubt es, aggregierte Beschäftigungstrends unabhängig von den Querschnittsdynamiken zu behandeln und so die Bedeutung der Querschnittsheterogenität zu isolieren.

Kapitel 2 trägt den Titel "Labor Market Flows and Misallocation with Occupational Heterogeneity" [Arbeitsmarktströme und Fehlallokationen mit beruflicher Heterogenität]. Das Kapitel nimmt seinen Ausgangspunkt darin, dass in Unternehmen, die mehr als einen Beruf umfassen, jede berufliche Tätigkeit eine Untereinheit mit vergleichbaren Qualifikationsanforderungen darstellt. In diesem Fall wachsen und schrumpfen diese Untereinheiten mit unterschiedlichen Geschwindigkeiten. Wenn die Reallokationen von berufsbezogenen Jobs berücksichtigt wird, sind die über Netto-Arbeitsplatzschaffungen hinausgehenden Bruttoverschiebungen von Arbeitsplätzen sehr umfangreich. Diese residuale Arbeitsplatzverschiebung auf Ebene der Unternehmen resultiert in mehr Arbeitnehmerverschiebungen pro Arbeitsplatzverschiebung als zuvor dokumentiert. In diesem Kapitel werden empirische Belege für dieses Phänomen diskutiert und als Resultat von Arbeitsqualitätsheterogenität und Fehlallokation der Faktoreinsätze wie Arbeit erklärt. Die Rolle dieser Variablen wird im Rahmen eines Suchmodells von Unternehmensdynamiken quantifiziert. Aufgrund der hohen Komplexität der notwendigen Berechnungen, schließt dieses Kapitel mit vorläufigen quantitativen Ergebnissen.

Kapitel 3 trägt den Titel "Firm Dynamics with Frictional Product and Labor Markets" [Unternehmensdynamiken mit friktionalen Produkt- und Arbeitsmärkten]. Es wurde gemeinsam mit Leo Kaas verfasst. Im Kapitel werden Preis-, Produktivitäts- und Beschäftigungsdynamiken über Unternehmen hinweg gemeinsam analysiert. Es wird ein dynamisches Gleichgewichtsmodell auf Basis von heterogenen Unternehmen entwickelt, die um Arbeitnehmer und Kunden in friktionalen Arbeits- und Produktmärkten konkurrieren. Das Model wird mittels Paneldaten für Preise und Output für deutsche Unternehmen im produzierenden Gewerbe kalibriert um die Beiträge von Produktivität und Nachfrage im Arbeitsmarkt zu quantifizieren. Produktmarktfriktionen reduzieren in bedeutendem Umfang die Beschäftigungsanpassungen der Unternehmen als Reaktion auf Produktivitätsschocks. Außerdem wird in diesem Kapitel analysiert wie sich Schocks auf den ersten und zweiten Moment des idiosynkratischen Risikos makroökonomisch auswirken. Ein Anstieg der Nachfrageunsicherheit führt zu einer Verringerung des Outputs und der Beschäftigung bei gleichzeitig wachsender Querschnittsdispersion von Preis- und Outputwachstum. Dies sind typische Eigenschaften von Rezessionen in unseren Daten.

CHAPTER 1

Occupational Labor Market Flows

1.1. Introduction

Labor market flows refer to worker flows, in the form of hires and separations, and to job flows, measured in terms of employment changes. Through labor market flows, workers migrate between employment and unemployment while firms reshuffle their skill composition in addition to expanding or contracting.

Research on labor market flows associates job flows to net job changes but abstracts away from studying the flow of jobs and workers by narrow skill groups.[2] Yet, as firms expand or contract, they likely destroy jobs of certain skill types and, simultaneously, create jobs of other skill types. To the extent that simultaneous job creation and destruction at the level of the firm is significant, abstracting away from the skill composition of jobs, potentially limits a better understanding of the functioning of the labor market.

My contribution to the literature is to address this gap by studying occupational labor market flows in the cross-section by focusing on the patterns at the establishment level. Since a job comprises a particular skill set required to perform the tasks relevant for the given job, an occupation is more interesting as a characteristic of the job than other arbitrary groupings of workers such as by age.

To proceed, I consider that an occupation represents a set of skills required to perform the tasks of the job at hand. In particular, I adopt the divide that workers in a given occupation constitute a micro-economic unit. Each one of these units is affected by aggregate, establishment-specific, or unit-specific factors. Then, by definition, when the unit shrinks, jobs are destroyed. Jobs are created when it expands.

Where establishments operate with more than one occupation, zero-growth establishments may also create and destroy jobs. By contrast, if job flows are based on net employment changes, then only positively (negatively) growing establishments create (destroy) jobs.[3] Consider an establishment that grows by zero workers which it achieves by expanding employment of managers by two and shrinking employment of engineers by two. The establishment reallocates zero jobs on net but reallocates a total of four jobs. Thus, pooling engineering and managerial jobs together, implies that jobs are created (destroyed) only where the establishment expands (shrinks) in employment.

In fact, the limitation of net employment changes for measuring job flows is well-known in the literature. For instance, Burgess, Lane, and Stevens (2000) articulated:

[2] The primary reason is the lack of more disaggregated data. Relevant references include but are not limited to, e.g., Davis and Haltiwanger (1992), Anderson and Meyer (1994), Hamermesh, Hassink, and Ours (1996), Davis and Haltiwanger (1999), Abowd, Corbel, and Kramarz (1999), Burgess, Lane, and Stevens (2000), Davis, Faberman, and Haltiwanger (2006), Bachmann et al. (2017), and Bellmann, Gerner, and Upward (2017).

[3] Minimalistically speaking, a job creation is thought of as the act of filling a new vacant position. Since a replacement hire is never considered as job creation, a positive job change is the correct definition of a job creation.

> Our definition of job flows is the standard one: the net change in employment at the employing unit. But another, equally valid, view of a job links it to a task, a particular set of skills. Thus when a firm reconfigures its skill mix keeping the total number of jobs the same, replacing jobs of one type (one task, one skill type) with jobs of another type, under this view there would be both job creation and job destruction.

Because labor market flows are measured at the employer level, such as the establishment, it is possible to better understand their employer-level drivers, which is why it is important to obtain as accurate measurements of job flows as possible.

The study of occupational labor market flows necessitates a consideration of not only establishment factors common to all occupations but also factors specific to a particular occupation. In particular, since the 1980s, labor markets in the US and European countries have witnessed a change in the structure of occupational employment whereby there has been a decline in employment of routine-intensive medium-skill occupations accompanied by a rise in employment of non-routine low- and high-skill occupations. According to the literature, the polarization of labor markets is affecting the structure of employment in specific occupations *within* industries and the overall employment structure *between* industries.[4] In my data and as documented in the job polarization literature, occupational employment has trended since 1976 but job and worker flow rates in specific occupations and for all occupations combined have not trended at all. This allows me to abstract away from the aggregate trends and to focus on the cross-sectional dynamics of occupational job and worker flows, which elevate establishment heterogeneity.

The data used is called the Establishment History Panel (BHP) of the German Institute for Employment Research (IAB), which has historical records of stocks and flows of workers for one-digit occupations since the year 1975. The sample I use covers the period 1975 – 2010. It is sufficient to paint a long run picture of the labor market flows. Recent related studies use the same data, e.g., Bellmann, Gerner, and Upward (2017, in part) and Bachmann et al. (2017, at quarterly frequency). The same data is also used as the employer-side of the matched employer-employee data used in influential studies such as Dustmann, Ludsteck, and Schönberg (2009) and Card, Heining, and Kline (2013). Although the data in the last three studies comes from the same population, my panel is constructed differently from theirs.

I report that employment of manual occupations, characteristic of routine-intensive medium-skill occupations, declined in the late 1970s through the 2000s. At the same time, employment of all other occupations, characteristic of non-routine low- and high-skill occupations, rose during the same time. By contrast, occupational worker and job flow rates did not trend at all. Therefore, the trending occupation-specific employment shares can be thought of as aggregate effects around which the occupational flow rates have own dynamics.

With occupation-based job flow rates, observed net job creation rate is significantly different from the sum of occupation-based job creation and destruction rates. I find that, on average, approximately 25% of all occupation-based job flows do not lead to net job creation, implying simultaneous job creation and destruction at the level of the establishment. Further, ignoring the occupational margin of job reallocation, implies a job to external worker flow

[4] See evidence by Autor, Katz, and Kearney (2006), and Autor and Dorn (2013) for the United States; Spitz-Oener (2006) for Germany; Goos and Manning (2007) for the United Kingdom; and Goos, Manning, and Salomons (2009), Goos, Manning, and Salomons (2014), and Michaels, Natraj, and Reenen (2014) for European countries.

ratio of approximately 36%, which is smaller than the approximately 47% with occupation-based job flows.[5] Hence, the extent of worker churn, defined as worker flows over and above job flows, is overestimated when occupational job flows are not taken into account.

These patterns characterize the entire distribution of establishment growths. When it comes to the variability of occupational job and worker flows in the cross-section, I find that all labor market flows are highly dispersed, (mostly) positively skewed, and have heavy tails. This implies that, over a period of one year, most establishments neither hire or separate with workers nor create or destroy corresponding jobs, be it in specific occupations or in all occupations as a whole. Moreover, the kurtosis values are large since where establishments hire or separate with workers, then they do so at much larger rates than would be suggested by their concentrations; and analogously for job creation and destruction.

My data lacks both wage and productivity information to be able to make causal predictions. However, search and matching theory, see, e.g., Pissarides (2000) and Cahuc, Marque, and Wasmer (2008), predict that total and occupation-specific labor productivity changes drive job surplus values above or below zero so that some jobs are created and other are destroyed. Furthermore, Caliendo, Monte, and Rossi-Hansberg (2015), and Caliendo, Mion, et al. (2015) document a strong correlation between firm productivity and internal occupational organization, Caliendo, Monte, and Rossi-Hansberg (2015) find that the probability of adding or dropping a layer, defined by wage-ranked occupations, is correlated with value added while Caliendo, Mion, et al. (2015) estimate considerable productivity gains to occupational reorganization.

My research belongs to the empirical literature on labor market flows, in particular, as contributed by Davis and Haltiwanger (1992), Burgess, Lane, and Stevens (2000), Davis, Haltiwanger, and Schuh (1996), Abowd, Corbel, and Kramarz (1999), Davis, Faberman, and Haltiwanger (2006), and Davis, Faberman, and Haltiwanger (2012). Other references include Anderson and Meyer (1994), Hamermesh, Hassink, and Ours (1996), Albaek and Sorensen (1998), Bellmann, Gerner, and Upward (2017) and Bachmann et al. (2017). Except for Abowd, Corbel, and Kramarz (1999) who study occupational hires and separation rates as well as Bauer and Bender (2004) with two-years of data, the data in all of the remaining contributions does not allow a disaggregation by occupation. Coincidentally, my research stands between the polarization literature already cited above and the well-known empirical literature on labor market flows.

Section 1.2 presents the concepts of labor market flows formally. Section 1.3 discusses the data. Section 1.4 presents the empirical results. Section 1.5 concludes.

1.2. Empirical Labor Market Flow Concepts

This section presents concepts that are used to study labor market flows, a generic term that refers to worker flows – hires and separations, and to job flows – job creation and destruction. In this section, workers in a given occupation can be viewed as constituting a micro-economic unit that interacts with other units within and across establishments, in the sense that jobs get reallocated between units and workers flow between units. When the unit shrinks, jobs are destroyed. Jobs are created otherwise. Within an establishment, workers may reallocate across occupations via common mechanisms of promotions and demotions.

The section applies definitions of labor market flow rates used in Burgess, Lane, and Stevens (2000), applied at the occupational level. See also, e.g., Davis and Haltiwanger

[5] In specific occupations, I find that occupational worker flows rates are two to three times the corresponding magnitudes of job flow rates.

(1992), Davis, Haltiwanger, and Schuh (1996), Abowd, Corbel, and Kramarz (1999), and Davis, Faberman, and Haltiwanger (2006).

The unit of observation is the establishment. Because a given firm might own more than one business enterprises or establishments, the establishment is the lowest appropriate unit for studying labor markets for, otherwise, establishment job flows would be found to be lower simply because of aggregating to the firm level. At the same time, hires and separations might be over-estimated simply because of the presence of intra-firm worker migrations. From the perspective of the establishment, however, they are establishment-to-establishment worker flows. Furthermore, much of the related literature uses establishment data.

1.2.1. Definitions. Denote by L_{iot} the total number of jobs in occupation o held by establishment i in period t. Let JF_{iot} define occupation-o job flows. Occupational job flows measure the period-to-period changes in the number of occupational jobs held by an establishment. That is $JF_{iot} = L_{iot} - L_{io,t-1}$. New hires, denoted H_{iot}, are the number of workers who in period t flow into jobs in occupation o at establishment i from other establishments, unemployment, or out of the labor force. Job separations, denoted S_{iot}, refer to the number of workers who flow out of occupation-o jobs at establishment i in period t to other establishments, unemployment, or out of the labor force. Separations result from lay-offs or quits. Hires and separations are jointly referred to as worker flows. I often write type-o job or worker flows to refer to job or worker flows corresponding to occupation o.

As a percentage of establishment occupational employment, worker and job flows are related as follows

$$JFR_{iot} = \frac{H_{iot} - S_{iot} + X_{iot}}{\overline{L}_{iot}}, \tag{1.1}$$

where $\overline{L}_{iot} = (L_{iot} + L_{io,t-1})/2$ is average occupational employment and X_{iot} is the net flow of workers from other occupations into occupation o within the establishment. Internal job flows sum to zero, i.e., $\sum_o X_{iot} = 0$. Dividing by between-period average occupational employment averages out the influence of temporary shocks that may lead to temporary fluctuations in occupational employment over the periods as the establishment regresses towards a long-run level. The rates of hires, separations, and net internal worker flows for occupation o are denoted $HR_{iot} = H_{iot}/\overline{L}_{iot}$, $SR_{iot} = S_{iot}/\overline{L}_{iot}$, and $XR_{iot} = X_{iot}/\overline{L}_{iot}$, respectively.

It follows from (1.1) that, job flow rates are defined over the interval $[-2, 2]$ and encompasses new, continuing, and exiting establishments. New establishments have $JFR_{iot} = 2$ while exiting establishments have $JFR_{iot} = -2$ because the former have zero separations and the latter have zero hires. The same holds for establishment's first cohort of type-o hires or last cohort of type-o workers, respectively.[6]

When JFR_{iot} is positive, (net) occupational job creation is said to occur. Conversely, when JFR_{iot} is negative, (net) occupational job destruction is said to occur. Formally, occupational job creation and destruction rates, denoted JCR_{iot} and JDR_{iot}, respectively, are defined as

$$JCR_{iot} = JFR_{iot} \text{ if } JFR_{iot} \geq 0 \ , \tag{1.2a}$$

$$JDR_{iot} = |JFR_{iot}| \text{ if } JFR_{iot} \leq 0 \ . \tag{1.2b}$$

[6] In addition to being bounded between -2 and 2 and defined for continuing, new, and exiting establishments, net occupation job creation rates are symmetric about zero and are less susceptible to the so-called regression-to-the-mean bias whereby establishment growth and size are misleadingly negatively related–see Davis, Haltiwanger, and Schuh (1996).

It follows that $JFR_{iot} = JCR_{iot} - JDR_{iot} = HR_{iot} - SR_{iot} + XR_{iot}$. The sum of job creation and destruction gives rise to the total amount of job reallocation.

1.2.1.1. *Aggregation.* Since an establishment may create jobs in one occupation and simultaneously destroy other jobs in another, total establishment job flow rates are weighted sums of occupation-specific job flow rates. Define *occupation-based* job creation and destruction as

$$JCR_{it} = \sum_o \frac{\overline{L}_{iot}}{\overline{L}_{it}} JCR_{iot}\,, \tag{1.3a}$$

$$JDR_{it} = \sum_o \frac{\overline{L}_{iot}}{\overline{L}_{it}} JDR_{iot}\,, \tag{1.3b}$$

where $\overline{L}_{it} = \sum_o \overline{L}_{iot}$. By contrast, if the establishment operates only one occupation or where labor is assumed to be homogeneous, there can not be simultaneous job creation and destruction.

Aggregate time-series can be generated by summing up weighted establishment-level rates:

$$YR_t = \sum_i \frac{\overline{L}_{it}}{\sum_i \overline{L}_{it}} YR_{it}\,, \tag{1.4}$$

where $YR_{it} \in \left\{HR_{it}, SR_{it}, XR_{it}, JFR_{it}, JCR_{it}, JDR_{it}\right\}$. Furthermore,

$$YR_{ot} = \sum_i \frac{\overline{L}_{iot}}{\sum_i \overline{L}_{iot}} YR_{iot}\,. \tag{1.5}$$

where $YR_{iot} \in \left\{HR_{iot}, SR_{iot}, XR_{iot}, JFR_{iot}, JCR_{iot}, JDR_{iot}\right\}$.

1.2.1.2. *Link with Job Polarization.* In the context of job polarization, reviewed by Acemoglu and Autor (2011)), the literature has identified trends in the occupational employment shares $\sum_i \overline{L}_{iot} / \sum_i \overline{L}_{it}$. These trends are such that the employment of medium-skilled occupations has declined relative to low- and high-skilled occupations. In such a context, the aggregate flow rate YR_t may be reformulated as

$$YR_t = \sum_o \frac{\sum_i \overline{L}_{iot}}{\sum_i \overline{L}_{it}} YR_{ot} \equiv \sum_o \widetilde{YR}_{ot}. \tag{1.6}$$

It follows that the aggregate dynamics of $\widetilde{YR}_{ot}$ are due to type-o employment share, which in the above enters as a weight, and type-o worker or job flow rate. My research focuses on the component YR_{ot}.

1.2.1.3. *Worker Churning Flows.* In the theoretical consideration such as by Mortensen and Pissarides (1994), job creation (destruction) is synonymous with hiring (separation) because firms operate with one worker. By contrast, recent studies attempt to account for the fact that establishments hire and separate with workers at the same time so that not all worker flows may be accounted for by job flows. See, e.g., Kaas and Kircher (2015) and Schaal (2017) for quantitative-theoretical formalizations and Burgess, Lane, and Stevens (2000), Davis, Faberman, and Haltiwanger (2006), and Davis, Faberman, and Haltiwanger (2013) for empirical facts.

To determine the role of job flows for the flow of workers, Burgess, Lane, and Stevens (2000) calculate churning flows, defined as worker flows over and above job flows:

$$\widetilde{CFR}_{it} = HR_{it} + SR_{it} - \left|JFR_{it}\right|. \tag{1.7}$$

Observe that where the total number of occupations is one or workers are homogeneous, then $|JFR_{it}| = JCR_{it} + JDR_{it}$. Dividing throughout by $HR_{it} + SR_{it}$, I can calculate the amount of worker flows that is due to job flows.

Burgess, Lane, and Stevens (2000) thought that churning flow rates so defined elevate the role of churning and understate the role of job flows for the reallocation of workers. Taking occupational job flows into account, I measure churning flow rate as

$$CFR_{it} = HR_{it} + SR_{it} - (JCR_{it} + JDR_{it}), \tag{1.8}$$

where, without loss of generality, internal worker flows are ignored. The fundamental difference is that job reallocation is here occupation-based while only net job creation at the establishment level can be calculated in Burgess, Lane, and Stevens (2000).

1.2.1.4. *Excess Job Reallocation.* Recall once again that $JFR_{it} = \sum_o \overline{L}_{iot} JFR_{iot} / \overline{L}_{it}$. If all occupational labor is grown at the same rate, then $JFR_{it} = JFR_{iot}$. If on the other hand, imperfect substitutability of occupation labor in production (due to, e.g., differences in labor supply, limits to occupational labor mobility between firms, and so on), or occupation-specific labor productivity shocks imply that the rates of occupational job creation and destruction are not the same, then $JFR_{it} \neq JFR_{iot}$.

Define the excess job reallocation rate XJR_{it} as the difference between gross establishment job reallocation and net job flows:[7]

$$XJR_{it} = (JCR_{it} + JDR_{it}) - |JFR_{it}|. \tag{1.9}$$

If $XJR_{it} = 0$, no simultaneous job creation and job destruction occurs, which amounts to the case of one-occupation establishments. If $XJR_{it} = (JCR_{it} + JDR_{it}) > 0$, then job creation in some occupations is accompanied with an equal amount of job destruction in others, a situation in which there is a cancelling out as establishments expand one occupation and equally contract another.

A significant average value of $XJR_{it}/(JCR_{it} + JDR_{it}) \in (0, 1)$, if found, (i) would reveal the extent to which occupational jobs are simultaneously created and destroyed (at the level of the establishment), and (ii) it could reflect the amount of establishment-level idiosyncrasy in the reallocation of occupational jobs, complementarity of occupational labor in production, or the differences in the costs of occupational job turnover.

Yet, it might be that the net expansion and contraction of establishments' total employment in an industry far dominate the establishment occupational employment growths so that excess job reallocation is insignificant. Consider establishments belonging to sector k, where, more generally, k can be any set of establishments in a particular grouping of establishments. The sectoral excess job reallocation is

$$\sum_{i \in k} z_{it} XJR_{it} = \sum_{i \in k} z_{it} JRR_{it} - \sum_{i \in k} z_{it} |JFR_{it}|, \tag{1.10}$$

where $JRR_{it} = JCR_{it} + JDR_{it}$ and $z_{it} = \overline{L}_{it} / \sum_{i \in k} \overline{L}_{it}$. The the sector's average job reallocation rates over and above the sector's net job creation rate is

$$\underbrace{\sum_{i \in k} z_{it} JRR_{it} - |\sum_{i \in k} z_{it} JFR_{it}|}_{\equiv \widehat{XJR}_{k,t}} = \sum_{i \in k} z_{it} XJR_{it} + \sum_{i \in k} z_{it} |JFR_{it}| - |\sum_{i \in k} z_{it} JFR_{it}|, \tag{1.11}$$

[7] This residual job reallocation is here referred to as (establishment-level) excess job reallocation to be the establishment-level counterpart to the terminology Davis and Haltiwanger (1992) use, but at the industry level.

The left-hand side is the average excess job reallocation rate in sector k, i.e., the sector's weighted sum of job creation and job destruction rates minus the sector's net job creation rate. The first term on the right-hand side is the establishment-level component of excess job reallocation while the second term is the between-establishments component. The between-establishment component is the average rate of net job changes occurring between establishments belonging to group k over and above the group's net job creation rate. I write $\widehat{XJR}$ to denote the left-hand side to distinguish it from XJR.

1.3. Data

The data comes from the Establishment History Panel (BHP) of the German Institute for Employment Research (IAB). It covers a 50% random sample of all establishments with operations in Germany. See Gruhl, Schmucker, and Seth (2012) for the official documentation.

I have access to the pure random sample, the alternative being a stratified random sample based on employment size. The period covered is 1975 to 2010 for West Germany and 1992 to 2010 for East Germany, with a reference date of 30th June of each year.

The establishments in the data employ full-time workers, including apprentices, covered by social security (about 70% of total employment (Fuchs and Weyh (2010)) as filed by employers to the social security system. Since 1999, marginal or unpaid employees whose average daily wage is zero, e.g., employees in maternity protection, periods of sickness longer than 42 days or sabbaticals have been included. Excluded are the self-employed and civil servants.

The BHP comprises a representative cross sectional data of German establishments. I combine the cross sections into a longitudinal panel of establishment employment histories, each after merging with the following Extension Files: (i) Worker Flows, with occupational hires and separations, and (ii) Entry and Exit, with records on establishment entry and exit. These Extension Files are provided separately and on a project basis.

An establishment is uniquely identified by an artificial ID based on the industry and region of operation, meaning that a multi-branch establishment may have one ID if the constituent branches and the parent establishment are in the same industry and administrative district (or *Kreis*) (Hethey and Schmieder 2010). Establishments that appear (exit) before (after) the reference date are recorded in the following year.

Whereas there is no information on ownership structure or legal form, establishment employment can be analysed based on industry, federal state, age or size. Since the classification of industries changes over time, I work with the time-consistent classification provided with the BHP, namely the 1993 Classification of Economic Activities (up to 3-digit codes).

To avoid counting as new or exit instances such as of mergers or acquisitions where establishments continue operating under a different identity, I consider establishment entry and exit only where it is unambiguously clear. Section 1.A.1 further elaborates.

There are about 330 different 3-digit occupation codes based on the KldB75 Classification of Occupations that German employers use when filing to the social security system information about their employees. According to the official documentation, occupations are not tied to educational qualification nor job position within the establishment.[8].

Based on the level of the job requirement and the sector in which the job is performed, the BHP coalesces the occupations into Blossfeld classification of occupations yielding 12 1-digit occupations ensuring time-consistency–see Blossfeld (1987). In a given occupation, the

[8]However, in Germany there is a strong correlation between the level of education attained and the occupation of an employee–see Dustmann (2004)

data makes no distinction of employer-to-employer transitions, those hired from or separated to non-employment, or those that switch occupations internally. Quits or layoffs are never reported. Internal worker flows may result from promotions, demotions, coding errors, or erroneous classification of a worker's occupation.[9]

The panel is then cleaned for its twelve occupations and the labor flow definitions in Section 1.2 applied. The occupations reported are (i) agricultural occupations, (ii) unskilled manual occupations, (iii) skilled manual occupations, (iv) unskilled services occupations, (v) skilled services occupations, (vi) unskilled occupations in administration and commerce, (vii) skilled occupations in administration and commerce, (viii) technicians, (ix) semiprofessionals, (x) engineers, (xi) professionals, and (xii) managers. Throughout the analysis, agricultural occupations are ruled out. To organize the occupational groups, I refer to occupations (ii)–(iii) as *manual occupations*, (iv)–(v) as *services occupations*, (vi)–(vii) as *commercial and administrative occupations*, and (viii)–(xii) as *professional occupations*.

From correspondence with a member of the IAB, between 1992 and 2000 there was noticeable above-average decreases and increases in the number of notifications in about 14 districts. The reasons given are notification problems at one or more establishments in the districts. For brevity these districts are not mentioned here. As will be seen later, they may help explain some sharp changes in the time series for some occupations such as professionals. There are also spikes in the year 1999 for aforementioned reasons. These spikes are so systematic that they could not be minimized with trimming the data based on rehires, temporary outflows or marginal part-time workers.

Excluding East Germany prior to 1993, the merged panel has a total of 34,576,439 establishment-year observations. I drop these sectors: agriculture, hunting, forestry and fishing; private households with employed persons; extraterritorial activities; and missing sectors. I remove establishment-year observations with only agricultural occupations and make sure that total employment growth equals the difference between hires and separations.

I ignore hires and separations at establishments where net internal worker flows is unequal zero, or at establishments with entry, or exit types other than small, large, and medium or small death and atomized death.[10] I also ignore occupation-specific hires and separations if the occupation group is absent, equivalently with zero occupational employment in two consecutive years. For occupational labor market flows, I use 27,128,908 establishment-year observations from 1976 to 2010. Finally, I drop establishments operating in the former East Germany in order to simplify the analysis and benchmark my study with existing studies most of which focus on West Germany. This reduces the sample size to about 23,772,268.[11]

1.4. Results

1.4.1. Basic Cross-Sectional Features. The final 1976-2010 West German sample has 24,801,331 establishment-year observations of which 23,772,268 have non-missing worker and job flow rates due to cleaning such as where certain establishment entry and exit types are excluded. The distribution of establishment age in years is 10.43 (mean), 8.63 (sd), and 8.17 (median).

[9]By computing internal net worker flows as residuals after pooling the data to particular levels of aggregation, I certainly limit the amount of errors assuming that instances of systematic coding errors or erroneous internal worker flows is likely minimal.

[10]Section 1.A.1 discusses these entry and exit types in more details.

[11]This quoted number of observations just refers to the data with valid labor market flow rates.

Table 1.1. Cross-Sectional Distribution of Occupational Employment

			Occ. Employment			Intensity		
	N	Share	Mean	Sd	p50	Mean	Sd	p50
All occupations	23772268		15.05	128.94	4			
Manual occ.	9360746	0.327	4.92	66.26	0	0.246	0.367	0.0
Unskilled manual	3935702	0.157	2.36	43.64	0	0.070	0.208	0.0
Skilled manual	7620874	0.170	2.56	26.99	0	0.176	0.319	0.0
Services occ.	11472260	0.200	3.00	22.72	0	0.280	0.392	0.0
Unskilled services	9071586	0.151	2.27	20.28	0	0.171	0.313	0.0
Skilled services	4167999	0.049	0.73	6.21	0	0.109	0.284	0.0
Comm. and admin. occ.	14910672	0.301	4.52	34.60	1	0.355	0.398	16.67
Unskilled comm.	7134930	0.100	1.51	12.34	0	0.140	0.295	0.0
Skilled comm.	11563200	0.200	3.01	27.32	0	0.215	0.330	0.0
Professional occ.	6862539	0.168	2.53	40.68	0	0.115	0.258	0.0
Semi-professionals	1518888	0.056	0.84	15.24	0	0.032	0.153	0.0
Technicians	2634005	0.050	0.75	14.03	0	0.031	0.133	0.0
Professionals	1046278	0.014	0.21	9.58	0	0.009	0.065	0.0
Engineers	1174899	0.025	0.37	14.70	0	0.011	0.076	0.0
Managers	2925999	0.024	0.36	6.42	0	0.033	0.139	0.0

Notes: "occ." stands for occupations; "Comm. and admin. occ." stands for commercial and administrative occupations; "Unskilled/Skilled comm." stands for unskilled/skilled commercial and administrative occupations. The column N contains numbers of non-missing observations used in job and worker flow data, which defer from those used in the remainder of the columns of the table because of cleaning such as when certain establishment entry and exit types are excluded. Number of observations used are 24801331, for Occ. Employment columns, and 24777702, for Intensity columns. Based on the pooled sample covering the years 1976 - 2010. Establishments such as exiting ones with zero workers have undefined occupational employment intensity and thus make the small difference in the numbers of observations.

Table 1.1 shows more descriptive statistics about the data. It focuses on the distribution of occupation employment as well as occupational employment intensity defined as occupational employment share (of total establishment employment) per establishment (in column "Intensity", i.e., L_{iot}/L_{it}).

The table shows that the median establishment has zero occupation-specific employment and that most establishments are small with a median of 4 workers. About 33% of all workers are employed in manual occupations, 30% in commercial and administrative occupations, 20% in services occupations, and 17% in professional occupations. Occupational employment intensity is below 30%, except for commercial and administrative occupations where nevertheless is below half. It follows, therefore, that the establishments employ multiple occupations.

In both manual and commercial and administrative occupations, occupational employment shares and intensities are increasing with skill but are decreasing with skill in services occupations, and between semi-professionals and professionals, and between technicians and

Table 1.2. Establishment and Employment Shares by Sector (%)

	Employment Share	Establishment Share
Mining and quarrying	0.89	0.21
Manufacturing	31.08	13.48
Electricity, gas and water supply	1.03	0.25
Construction	7.45	11.01
Wholesale and retail trade	16.22	24.28
Hotels and restaurants	2.93	7.16
Transport, storage and communication	5.27	4.67
Financial intermediation	3.90	2.67
Real estate, renting and business activities	9.24	15.10
Public administration and defence; compulsory social security	5.97	1.69
Education	2.70	2.47
Health and social work	9.17	9.42
Other community, social and personal service activities	4.14	7.59

Table 1.3. Establishment Shares by Size

	0–	20–	50–	250–	500–
Employment Share (%)	27.90	13.22	25.09	10.11	23.68
Establishment Share(%)	89.19	6.37	3.71	0.44	0.29

Table 1.4. Establishment Shares by Age (Years)

	0–	3–	6–	11–	16–
Employment Share (%)	11.59	13.33	19.29	16.13	39.66
Establishment Share (%)	21.98	18.03	21.65	14.30	24.04

Table 1.5. Establishment Shares by Growth (%)

	[-200, -20)	[-20, -5)	[-5, 5]	(5, 20]	(20, 200]
Employment Share	6.84	15.84	44.28	19.94	13.09
Establishment Share	15.82	7.68	44.76	8.87	22.87

engineers. Lastly, occupational employment is highly dispersed but occupational employment intensity is relatively less dispersed. However, both are positively skewed.

Table 1.2 reports that a little over one-half of establishments are in the wholesale and retail trade, real estate, or manufacturing but most workers (31%) are employed in the manufacturing sector, followed by wholesale and retail trade (16%), and by health and social work (9%). Subsection 1.A.2 in the Appendix concludes that establishments in sectors with the highest employment shares are not necessarily the largest by average employment. Furthermore, sectors certainly have a main occupation they employ.

The distribution of establishments by employment size, age groups, and groups of employment growth rates are reported in Table 1.3, Table 1.4, and Table 1.5. Subsection 1.A.2 describes the distributions by occupation and by establishment groups.

Table 1.3 reports that nearly 90% of establishments have fewer than 20 workers. Yet, around 70% of all employment is found at establishments with at least 20 workers.

The establishment shares by age, reported in Table 1.4, are fairly uniform, ranging from 22% at younger than 3 years old establishments to 24% at establishments aged 16 years or older. According to the table, employment share is in general increasing with the age of the establishment except between age-groups 6 to 10 years and 11 to 15 years.

Table 1.5 shows the establishment and employment shares by shrinking, stable and expanding establishments. It shows that stable-growth establishments within [-5%, 5%] are the most frequent and employ most workers. Medium-growth establishments in the growth categories of [-20%,-5%) and (5%,20%] are only 15% but together employ nearly 36% of all workers. Fastest growing establishments, among which are new and exiting, are small but more numerous than medium-growth establishments.

1.4.2. Labor Market Flows and Job Polarization. Job polarization refers to the decline in the employment share of medium-skilled routine-intensive occupations relative to low- and high-skilled non-routine-intensive occupations during the last three to four decades. Job polarization is studied by classifying occupations based on the routine-intensity of the tasks performed in the occupations. According to Acemoglu and Autor (2011), routine tasks involve a series of steps which may be well executed by a computer while non-routine tasks are characterized by flexibility, creativity, problem-solving, and adaptability.

To arrive at “routine” vs “non-routine” classification, the occupations are ranked by their routine task intensity (RTI) index – see, e.g., Autor and Dorn (2013) and Goos, Manning, and Salomons (2014). However, as in Jaimovich and Siu (2020), my occupations can be given the same classification without directly calculating the RTIs. Table 1.18 in the Appendix shows how such a mapping is achieved. Accordingly, jobs in manual occupations represent *routine manual* jobs; jobs in services occupations represent *non-routine manual* jobs; jobs in commercial and administrative occupations represent *routine cognitive* jobs; and jobs in professional occupations, i.e., technicians, semi-professionals, engineers, professionals and managers, are *non-routine cognitive* jobs.

1.4.2.1. *Trends in Occupational Employment.* Table 1.6 reports the mean and changes in occupational employment shares during the late 1970s through the 1980s, and during the 1990s and 2000s. The division of the years takes advantage of the history of the German labor market, in particular the re-unification in 1990 and the inclusion of marginal part-time workers into the social security system. Shown in the table in the columns ”Mean” are the average occupational employment shares during the period. The percentage changes in occupational employment shares shown in the Δ columns refer to the share at the end of the period relative to the base year of the period.

The table shows that employment of manual occupations, characteristic of routine-intensive medium-skill occupations, declined in the late 1970s through the 2000s. At the same time, employment of all other occupations, characteristic of non-routine low- and high-skill occupations, rose during the same time. Note that employment of services occupations fell between 1976–1989 before rising throughout the 1990s and 2000s. In Subsection 1.A.3, the trends are presented visually year by year.

1.4.2.2. *Time Series of Occupational Labor Market Flows.* Bellmann, Gerner, and Upward (2017) finds that establishments in Germany primarily use the hiring margin to adjust

Table 1.6. Levels and Changes in Occupational Employment Shares (%)

	1976 – 1989			1990 – 1998			1999 – 2010		
	Mean	Share in 1976	Δ	Mean	Share in 1990	Δ	Mean	Share in 1999	Δ
Manual occupations	39.6	42.5	-5.4	34.1	36.9	-5.2	25.8	28.4	-4.6
Unskilled manual	19.5	21.9	-4.1	15.8	17.7	-3.3	12.3	13.3	-1.9
Skilled manual	20.2	20.6	-1.2	18.3	19.2	-1.9	13.5	15.1	-2.6
Services occ.	17.0	17.2	-0.3	17.4	17.1	0.2	24.0	22.5	2.5
Unskilled services	12.9	13.5	-1.1	12.6	12.6	-0.4	18.5	17.3	2.0
Skilled services	4.2	3.7	0.8	4.9	4.5	0.6	5.5	5.2	0.6
Comm. and admin. occ.	28.3	27.1	2.2	30.3	29.2	1.9	31.5	30.9	0.6
Unskilled comm.	10.0	9.8	0.0	9.5	9.8	-0.5	10.4	10.3	0.1
Skilled comm.	18.3	17.3	2.1	20.7	19.4	2.5	21.1	20.5	0.5
Professional occ.	14.6	12.8	3.4	17.8	16.4	3.1	18.2	17.8	1.4
Semi-professionals	4.0	3.0	1.8	5.7	4.9	1.9	6.9	6.4	1.3
Technicians	5.3	5.0	0.4	5.6	5.5	0.0	4.3	4.7	-0.6
Professionals	1.0	0.8	0.4	1.2	1.2	0.4	1.8	1.7	0.4
Engineers	2.1	1.8	0.6	2.7	2.5	0.5	2.6	2.6	0.1
Managers	2.2	2.1	0.2	2.5	2.3	0.4	2.5	2.4	0.2

Notes: See Table 1.1 for number of observations.

their employment.[12] If trends in occupational employment shares are affecting the cross-sectional rates of labor market flows, then we can start by analyzing the time-series of occupational hires rates. The time series of occupational hires and separation rates are shown in Figure 1.1.

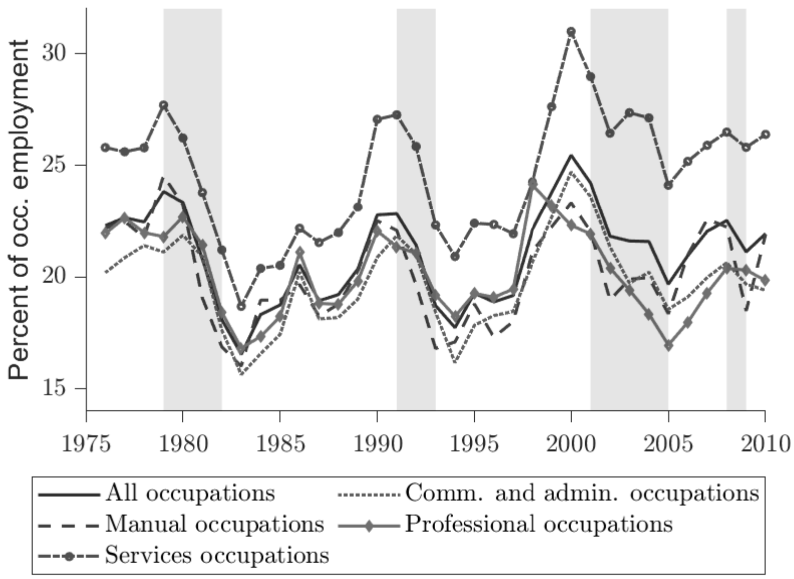
The hires rate for all occupations peaks as the economy enters a recession, sharply declines during it and reaches a trough after it, and the cycle repeats.[13] The figure shows that across occupations the pattern is similar. The figure shows that the separation rates are rising in a recession and falling in non-recessionary years.

That trends in the occupational composition of aggregate employment do not seem to matter for the cross-sectional rates of labor market flows is also observed in the time series of the occupational job creation and destruction rates as shown in Figure 1.2. The figures display more cyclical patterns than would be suggested by linearly trending patterns typical in the job polarization analyses.

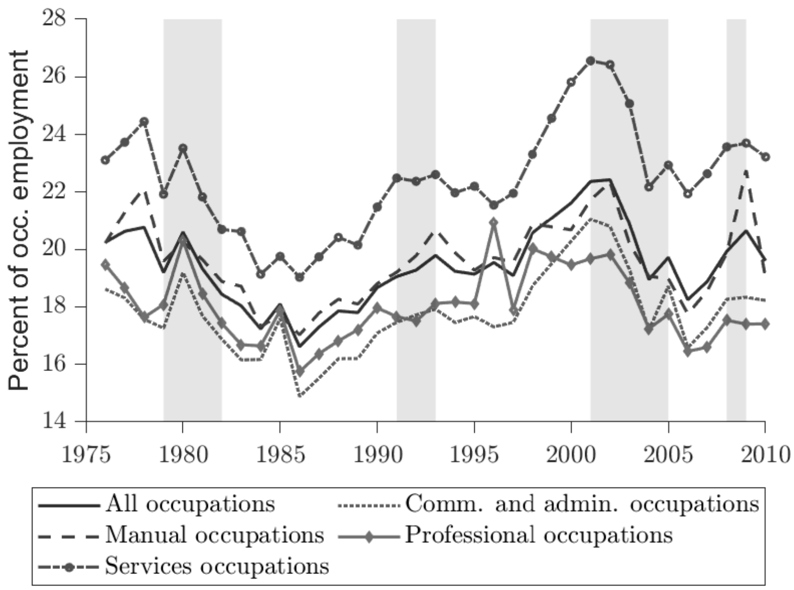
In the above, the most important fact to remember is that the job polarization phenomenon is precise in that patterns for medium-skilled routine-intensive occupations relative to low- and high-skilled non-routine-intensive occupations are noticeably different. Such a pattern is not observed for the occupational labor market flow rates in my data.

[12]Institutional features of the German labor market, namely employee dismissal policies that make it relatively more costly to adjust employment downwards explain the plausibility of this finding.

[13]Observe that no detrending is performed because it's the trends that the figure seeks to uncover and not cyclical patterns.



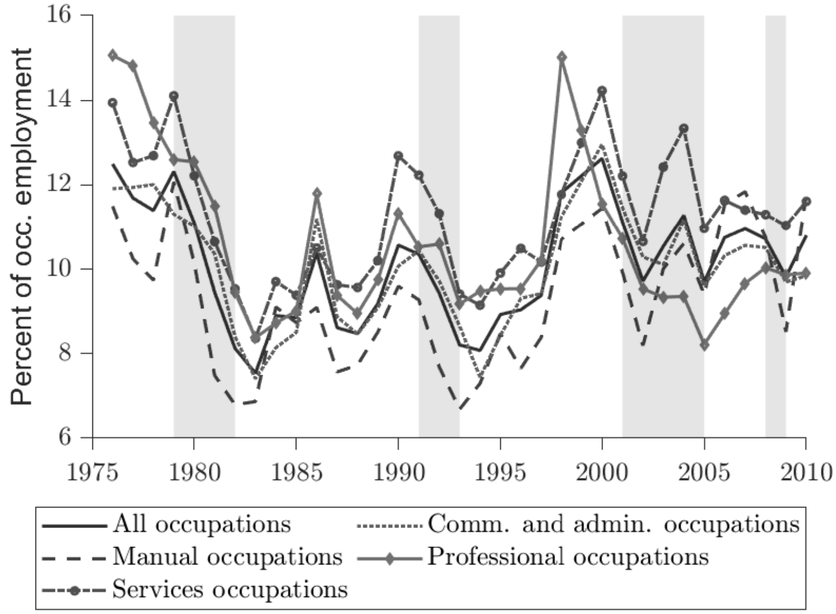
(A) Hires rate



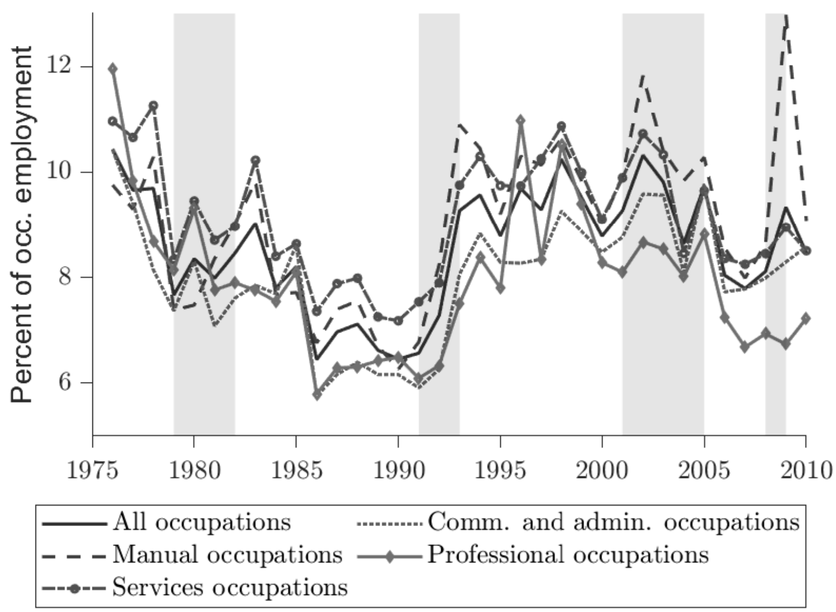
(B) Separation Rate

FIGURE 1.1. Occupational Worker Flow Rates

Notes: Original series where value for year 1999 is a linear interpolant, eliminating distorting spikes. The gray bars are recessionary years obtained from the 2009/2010 Annual Report of the German Government's Council of Economic Experts (*Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung*, page 260). See Table 1.1 for notations.



(A) Job Creation Rate



(B) Job Destruction Rate

Figure 1.2. Occupation-Based Job Flow Rates

Notes: See Figure 1.1.

1.4.3. Labor Market Flows in the Cross Section. In this section, I discuss the cross-sectional distribution of occupational labor market flow rates. Establishment occupational labor market flow rates are weighted by the corresponding average occupational employment between any two years so that the mean rates equal the aggregate rates. Other statistics are weighted by the same weights but are cross-sectional, i.e., they do not coincide with the aggregate statistics. Where occupations are pooled together, the weight is the total average employment of the constituting occupations. For formal derivations see Subsection 1.2.1.1 and its parent section. Furthermore, for reasons described in Section 1.3, the data point for the year 1999 is throughout omitted or, in graphical presentations, linearly interpolated .

Each of the weighted statistics is calculated separately for each year. The simple averages across the years are the ones that I report so that they can be interpreted as the long-run statistics, and where temporary recessions and booms have been averaged out.

To further characterize the variability of occupational labor market flows in the cross-sections, the following quantile-based statistics are used (cf: Bartelsman, Dobbelaere, and Peters (2015)):

$$dispersion = \frac{q_{.75} - q_{.25}}{q_{.75} + q_{.25}}, \tag{1.12a}$$

$$skewness = \frac{(q_{.75} - q_{.50}) - (q_{.50} - q_{.25})}{q_{.75} - q_{.25}}, \tag{1.12b}$$

$$kurtosis = \frac{q_{.90} - q_{.10}}{q_{.75} - q_{.25}}. \tag{1.12c}$$

The dispersion is the ratio of the width of the distribution to its location. The skewness compares the difference between the upper quartile and the median and the median and the lower quartile relative to the width of the distribution. It measures the symmetry of the distribution where zero skewness implies symmetric distribution. A negative skewness implies that the distribution has a longer left tail but its mass is concentrated on the right; and oppositely for positive skewness. The kurtosis, estimated as the difference between the $90th$ and $10th$ percentiles of the distribution relative to its width, measures how heavy the tails of the distribution are. If the kurtosis is high, the dispersion results from extreme but infrequent realizations. If the kurtosis is low, the dispersion results from frequent and moderate deviations.

1.4.3.1. *Occupational Worker Flow Rates.* Table 1.7 presents the cross-sectional statistics for occupational worker flow rates. Subsection 1.A.4 in the Appendix discusses job-to-job worker flows, rehires or separations, and part-time worker flows.

Focusing first on the left panel which has the mean rates of worker flows, the table says that for all occupations in any given year nearly one in five workers will be new hires but nearly the same number will not be working at the same establishment. Whereas the mean rate is large, the median is significantly smaller. The median hires and separation rates imply that 50% of establishments will hire or separate with up to 30% of their work force annually. While the mean occupation-specific hires and separate rates are just around the aggregate mean rate, differences exist where it is reported the hires rates range from 15.05% for technicians to 29.50% for professionals. The pattern and magnitude are roughly the same for separation rates.

Hires rates are increasing in skill and between technicians and engineers and between semi-professionals and professionals, except in commercial and administrative occupations where they are decreasing in skill. Separation rates, are however decreasing in skill for the said occupations, except in commercial and administrative occupations where they are again

TABLE 1.7. Occupational Worker Flow Rates

	Mean			Sd		Median		Dispersion		Skewness		Kurtosis	
	HR	SR	XR	HR	SR	HR	SR	HR	SR	HR	SR	HR	SR
All occupations	0.209	0.194	0.000	0.244	0.208	0.147	0.142	0.522	0.464	0.228	0.241	2.333	2.417
Manual occ.	0.201	0.196	-0.003	0.265	0.243	0.125	0.129	0.613	0.521	0.279	0.295	2.392	2.537
Unskilled manual	0.198	0.198	-0.002	0.277	0.255	0.116	0.128	0.662	0.505	0.270	0.285	2.387	2.533
Skilled manual	0.204	0.194	-0.003	0.279	0.256	0.125	0.123	0.699	0.646	0.233	0.257	2.269	2.294
Services occ.	0.245	0.224	-0.000	0.317	0.270	0.162	0.159	0.789	0.701	0.135	0.143	1.964	2.052
Unskilled services	0.243	0.225	-0.000	0.326	0.284	0.152	0.153	0.829	0.701	0.173	0.184	1.961	2.061
Skilled services	0.250	0.219	-0.000	0.335	0.278	0.174	0.158	1.000	1.000	-0.02	-0.030	1.702	1.641
Comm. and admin. occ.	0.197	0.177	0.001	0.272	0.231	0.128	0.123	0.614	0.578	0.197	0.195	2.363	2.365
Unskilled comm.	0.227	0.208	-0.001	0.326	0.275	0.143	0.149	0.954	0.916	0.028	-0.071	1.835	1.784
Skilled comm.	0.182	0.162	0.001	0.272	0.238	0.115	0.106	0.728	0.728	0.098	0.075	2.270	2.252
Professional occ.	0.201	0.180	0.005	0.271	0.243	0.139	0.127	0.675	0.617	0.087	0.093	2.116	2.237
Semi-professionals	0.251	0.214	0.002	0.250	0.217	0.195	0.167	0.385	0.374	0.248	0.241	2.542	2.635
Technicians	0.150	0.148	0.008	0.283	0.267	0.064	0.074	0.956	0.913	0.258	0.128	2.244	2.210
Professionals	0.295	0.255	-0.000	0.364	0.331	0.221	0.186	0.798	0.836	-0.06	-0.025	1.749	1.686
Engineers	0.171	0.151	0.002	0.301	0.281	0.084	0.072	0.946	0.960	0.191	0.161	2.076	2.072
Managers	0.174	0.159	0.012	0.364	0.327	0.013	0.022	1.000	1.000	0.890	0.800	2.527	2.294

Notes: See Table 1.1 for notations.

decreasing. The mean rates for managers are in general below the aggregate rates and are among the lowest.

The column XR of the table reports the annual mean net flow rates within establishments. Net internal worker flows ought to be compared with the net external worker flows, i.e., the difference between hires and separations. For instance, in professional occupations, net internal worker flows is $0.005 \times 100/(0.201 - 0.180) = 23.81\%$ relative to the net external worker flows.

According to the table, manual and services occupations migrate to other occupations within the establishment annually. At the same time, on average, workers in the other occupations tend to migrate within to professional and commercial and administrative occupations. In manual occupations, high-skilled workers have larger net internal outflows, but this difference might not be significant; as is not in commercial and administrative occupations. Led by managers and technicians, net internal worker flows are the largest in the professional occupations.

The standard deviation (sd) columns of the table show that many establishments on average have lower or higher than the average hires and separation rate. However, where the worker flow rates are positive, then, depending on the occupation, they can be up to two to three times the mean rate.

The large dispersion and skewness figures indicate that the widths of the distribution of the hires and separation rates are far away from the locations of the distribution so much that much of the flow rates are positively skewed. The only exceptions are skilled services occupations, professionals, unskilled commercial and administrative occupations where the skewness has a small negative value.

Since worker flow rates are concentrated at lower percentiles, the high kurtosis values say that where establishments hire or separate with workers, then they do so at much larger rates than would be suggested by their concentrations.

TABLE 1.8. Distribution of Occupation-Based Job Flow Rates

	Mean		Sd		Median		Dispersion		Skewness		Kurtosis	
	JCR	JDR	JCR	JDR	JCR	JDR	JCR	JDR	JCR	JDR	JCR	JDR
All occupations	0.101	0.085	0.222	0.185	0.034	0.028	0.963	0.983	0.378	0.384	2.396	2.362
Manual occ.	0.093	0.090	0.240	0.224	0.008	0.015	1.000	1.000	0.835	0.710	2.865	2.587
Unskilled manual	0.091	0.093	0.257	0.246	0.001	0.010	1.000	1.000	0.976	0.846	3.863	2.742
Skilled manual	0.094	0.088	0.258	0.236	0.000	0.000	1.000	1.000	1.000	1.000	3.255	3.120
Services occ.	0.112	0.091	0.291	0.240	0.001	0.000	1.000	1.000	0.991	0.995	3.240	3.213
Unskilled services	0.109	0.092	0.300	0.257	0.000	0.000	1.000	1.000	1.000	1.000	3.748	3.520
Skilled services	0.121	0.089	0.315	0.246	0.000	0.000	1.000	1.000	1.000	1.000	3.536	5.777
Comm. and admin. occ.	0.100	0.080	0.260	0.216	0.008	0.003	1.000	1.000	0.824	0.919	2.891	3.102
Unskilled comm.	0.114	0.096	0.318	0.267	0.000	0.000	1.000	1.000	1.000	1.000	4.260	3.483
Skilled comm.	0.093	0.072	0.265	0.229	0.001	0.000	1.000	1.000	0.986	1.000	3.371	5.406
Professional occ.	0.105	0.079	0.264	0.234	0.022	0.004	1.000	1.000	0.519	0.892	2.702	3.456
Semi-professionals	0.093	0.055	0.234	0.193	0.018	0.000	1.000	1.000	0.617	0.995	2.552	9.597
Technicians	0.096	0.086	0.283	0.265	0.000	0.000	1.000	1.000	0.995	1.000	3.935	4.276
Professionals	0.142	0.103	0.360	0.324	0.009	0.001	1.000	1.000	0.858	0.995	3.573	12.592
Engineers	0.109	0.087	0.299	0.280	0.004	0.000	1.000	1.000	0.929	1.000	3.231	5.913
Managers	0.136	0.110	0.380	0.334	0.000	0.000	1.000	1.000	1.000	1.000	8.651	9.145

Notes: See Table 1.1 for notations.

1.4.3.2. *Occupational Job Flow Rates.* The cross-sectional distribution of occupational job flows are shown in Table 1.8, according to which an average of 10% of all jobs at establishments are created and 8.5% are destroyed every year. The table shows that the average job flow rates vary across occupations, ranging from 9.10% in unskilled manual occupations to 14.20% among professionals but judging by the overall mean rate, mean occupational job flow rates do not deviate very far from the aggregate rate (all occupations pooled together).

In manual and services occupations, job creation rates are increasing with skill but job destruction rates are decreasing with skill. Both job creation and destruction rates are decreasing in skills in commercial and administrative occupations, and between technicians and engineers, and between semi-professionals and professionals. Job flow rates for managers are above the aggregate mean rate and are among the largest.

The occupational job flow rates are dispersed, positively skewed and have a large kurtosis. As judged by the low median job flow rates, most establishments do not create or destroy jobs every year. Hence, the dispersion values, and the skewness values are of the same magnitude by job creation or job destruction across occupations. Additionally, the kurtosis are so large since the statistics include fastest growing or shrinking establishments as well as new and exiting establishments whose job flow rates approach the maximum of 2.

Compared to worker flow rates in Table 1.7, worker flows are on average much greater than job flows. On average, they are two to three times job flow rates in specific occupations and all occupations put together. The dispersion and skewness columns show further that occupation-specific job flows are in general more dispersed, more skewed, and have larger kurtosis.

1.4.3.3. *Conventional Job Flow Rates.* In Table 1.9, conventionally defined job flow rates, which disregard occupational employment growth dynamics by basing on the net growth rate of the total employment size of establishments are presented. The table also reports the

Table 1.9. Distribution of Conventional Job Flow Rates

	Mean (%)		Sd ([0,1])		Median (%)		Dispersion ([0,1])		Skewness ([0,1])		Kurtosis ([1, ∞])	
	JCR	JDR	JCR	JDR	JCR	JDR	JCR	JDR	JCR	JDR	JCR	JDR
All occupations	0.078	0.062	0.216	0.178	0.002	0.001	1.000	1.000	0.951	0.984	3.043	3.373
Manual occupations	0.085	0.082	0.238	0.222	0.001	0.004	1.000	1.000	0.987	0.926	3.421	2.889
Services occupations	0.107	0.086	0.291	0.239	0.000	0.000	1.000	1.000	1.000	1.000	3.668	3.731
Comm. and admin. occ.	0.089	0.069	0.256	0.211	0.000	0.000	1.000	1.000	0.990	1.000	3.410	4.190
Professional occ.	0.095	0.069	0.261	0.231	0.004	0.000	1.000	1.000	0.902	1.000	3.020	5.812

Notes: See Table 1.1 for notations.

Table 1.10. Net Job Creation and Excess Job Reallocation

	Mean (%)		Sd ([0,1])		Median (%)		Dispersion ([0,1])		Skewness ([0,1])		Kurtosis ([1, ∞])	
	JFR	XJR	JFR	XJR	JFR	XJR	JFR	XJR	JFR	XJR	JFR	XJR
All occupations	0.015	0.046	0.297	0.114	0.001	0.008	2.844	1.000	0.098	0.637	3.021	2.582
Manual occupations	0.003	0.016	0.346	0.075	-0.004	0.000	-3.055	∞	0.009	∞	2.931	∞
Services occupations	0.021	0.010	0.400	0.063	0.000	0.000	0.604	∞	0.099	∞	3.488	∞
Comm. and admin. occ.	0.020	0.022	0.350	0.104	0.000	0.000	1.775	∞	0.178	∞	3.486	∞
Professional occupations	0.026	0.020	0.367	0.084	0.018	0.000	3.944	1.000	0.326	1.000	3.600	7.421

Notes: See Table 1.1 for notations.

flow rates in the broader occupations if they were conventionally defined, disregarding the constituting occupations.

The table shows that conventional job creation and destruction rates are on average 0.5% to 2.3% lower than occupation-based job flow rates. While the medians are even lower, their kurtosis is larger. Given that the means are lower, yet the standard deviations and dispersions are of similar magnitudes, conventional job flow rates are more variable relative to the occupation-base job flow rates.

1.4.3.4. *Excess Job Reallocation Rate.* That where establishments operate with more than one occupation simultaneously create and destroy jobs so that ignoring occupational job flows results in lower job flow rates is not too surprising. But how can the extent of simultaneous job creation and destruction at the level of the establishment level be appreciated?

Consider Table 1.10. Since excess job reallocation measures the amount of job reallocation over and above net job creation, it is interesting to find that the mean excess job reallocation rate exceeds net job creation rate by a factor of three; and indeed, across all occupations except services occupations are mean excess job reallocation rates larger.

Interpreted differently, an average excess job reallocation rate of 0.046 implies that $0.046 \times 100/(0.101+0.085) = 24.7\%$ of all occupational job reallocation do not lead to net employment expansion or contraction (the denominator is the sum of job creation and destruction rate taken from Table 1.8). Further, excess job reallocation is less dispersed and is in general positively skewed with a large kurtosis.

In Table 1.11, I report the mean occupational job flow rates, in particular, in order to show the significance of establishment excess job reallocation as compared to the *between-establishment excess job reallocation.* Recall that the latter is simply the amount of net job

TABLE 1.11. Occupational Job Flow Rates by Sector

	JFR	XJR	JCR	JDR	$\widehat{XJR}$	$\frac{XJR}{\widehat{XJR}}$
Mining and quarrying	-0.013	0.041	0.063	0.077	0.126	0.325
Manufacturing	-0.001	0.038	0.077	0.078	0.155	0.247
Electricity, gas and water supply	-0.003	0.031	0.056	0.059	0.112	0.273
Construction	0.012	0.049	0.116	0.104	0.207	0.236
Wholesale and retail trade	0.028	0.060	0.125	0.097	0.194	0.308
Hotels and restaurants	0.075	0.075	0.198	0.124	0.247	0.304
Transport, storage and communication	0.025	0.041	0.118	0.094	0.187	0.221
Financial intermediation	0.012	0.016	0.064	0.052	0.104	0.156
Real estate, renting and business activities	0.056	0.059	0.164	0.108	0.215	0.274
Public administration and defence	0.005	0.037	0.066	0.061	0.122	0.302
Education	0.028	0.041	0.106	0.079	0.158	0.259
Health and social work	0.041	0.045	0.104	0.062	0.124	0.363
Other service activities	0.036	0.048	0.125	0.089	0.177	0.268

Notes: See Table 1.2 for fuller sectoral names.

creation of establishments belonging to the sector over and above the sector's net job creation.[14] It tells the story of the amount of establishment heterogeneity within a sector. By contrast, establishment-level excess job reallocation tells the story of the amount of occupational heterogeneity within an establishment.

As in Table 1.10, net job creation rate in any sector is away from zero.[15] The annual net job creation rate for the top three sectors in the table, led by the manufacturing sector, is negative but it is positive in all other sectors. Despite the differences across sectors in the sign and level of the net job creation rates, all sectors create and destroy more jobs than it can be explained by the respective industry's overall growth. Moreover, within a sector, establishment job creation and destruction rates are significantly away from the net job creation rate. Two metrics are used to measure the significance of each: the latter with the establishment excess job reallocation (column XJR) and the former with the between-establishment excess job reallocation (column $\widehat{XJR}$).

Throughout, $\widehat{XJR}$ is at least three times XJR. That implies establishment overall expansion or contraction is more important for the reallocation of occupational jobs. Nevertheless, the last column reports that the contribution of establishment excess job reallocation, i.e., establishment-level simultaneous job creation and destruction, ranges from 15.6% in the financial intermediation sector to 36.3% in the health and social work sector. Note further that these shares are in general around a quarter to one-third.

In Figure 1.3, I show that the reported establishment-level excess job reallocation rates are truly characteristic of the cross-section as they do not exhibit any dominant trends over time. In Figure 1.4, it is reported that excess job reallocation is in general present across the whole distribution of employment growth.[16]

[14]This is the component which Davis and Haltiwanger (1999) studied.

[15]In general, they are skewed so that their growth rates are not symmetric around zero.

[16]Due to data restrictions, to ensure sufficient anonymity and timely delivery of the results, occupations are pooled into two broad groups generalized to low-skilled and high-skilled occupations.

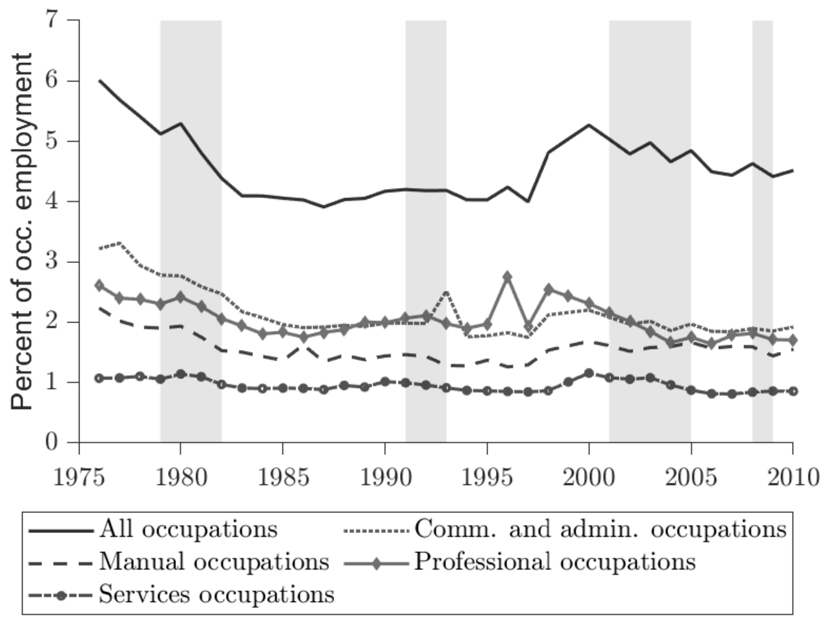


FIGURE 1.3. Excess Job Reallocation Rates

Notes: See Figure 1.1.

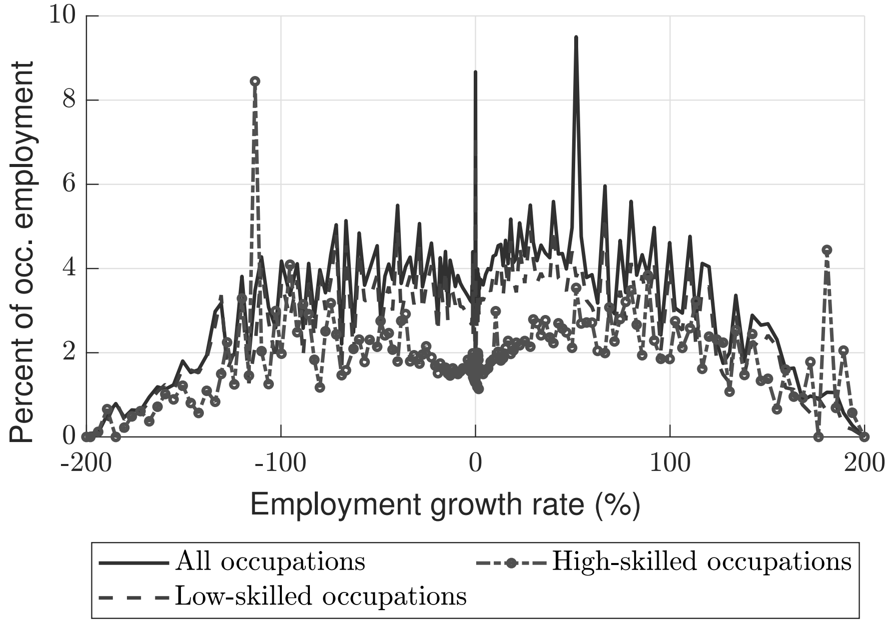
1.4.4. Relation Between Job Flows and Worker Flows. I have so far described the distributions of the occupation job and worker flow rates. But, how are they related? This is the subject of churning flows.

Since on average more workers are reallocated relative to jobs, the amount of worker churning, i.e., worker flows over and above job flows, can be calculated. However, my data does not have information on internal worker flows. Therefore, I calculate the ratio of job flows to (external) worker flows.

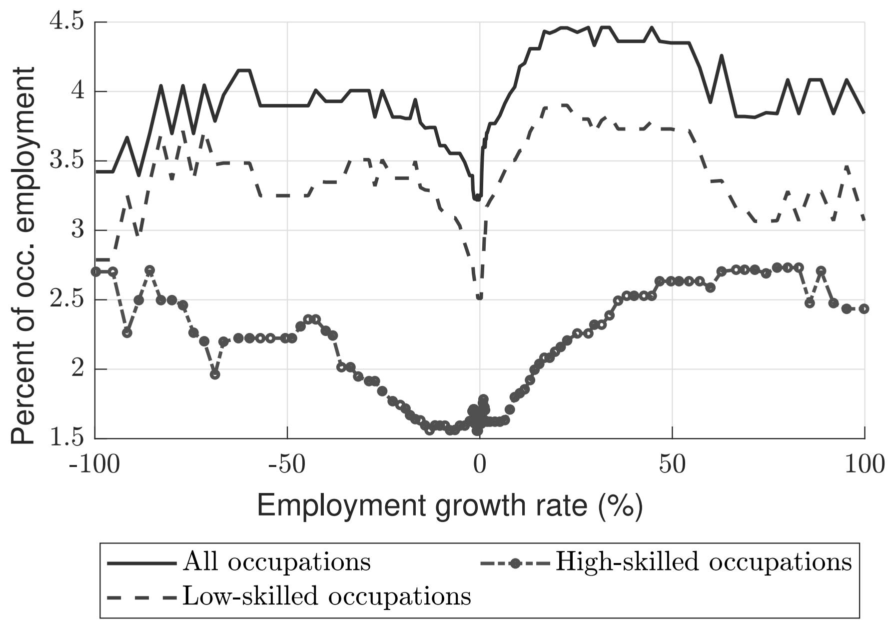
The results are reported in Table 1.12. The table shows that job flows matter for the reallocation of external workers not only in specific occupations but also for all workers as a whole. The ratios are not perfect but are comparable across occupations. In particular, the occupation-based ratios range from 43.4% in services occupations to 48.30% in professional occupations occupations. In narrower, occupations the lowest ratio is around 31.7% for semi-professionals but the highest is 74.3% for managers. The ratios with conventional job flows are lower overall and in specific occupational groups.

In Figure 1.5, the relation between worker flows and net job creation is visualized as an alternative way of relating worker and job flows.[17] The figure not only shows the relationship between hires and separation rates but also the origins of worker churning. Precisely, the figure shows that establishments with large as well as those with fairly stable percentage employment changes simultaneously hire and separate with workers. Therefore, net job creation can not explain all of the worker flows observed in the cross section. Furthermore,

[17]Unfortunately, due to data restrictions, it was not possible to obtain the graphs for more occupations in time while ensuring sufficient anonymity. The graphs are based on an older sample where both West and East Germany were considered as one.



(A) Original



(B) Smoothed

FIGURE 1.4. Excess Job Reallocation Rates and Employment Growth

Notes: "High-skilled occupations" are all professional occupations while "low-skilled occupations" are the remaining occupations. The plots are mean vs mean where the x-axis is the employment-wighted mean of the employment growth rate for all workers at establishments falling in a certain employment growth rate interval and the the y-axis is the employment-wighted mean excess job reallocation for growth interval. The size of the intervals is the smaller the smaller the absolute employment growth rate.

TABLE 1.12. Job to Worker Flow Ratio

	$\frac{JRR}{HR+SR}$ (Occupation-Based)	$\frac{\widetilde{JRR}}{HR+SR}$ (Conventional)
All occupations	0.461	0.348
Manual occupations	0.461	0.422
Unskilled manual	0.463	
Skilled manual	0.459	
Services occupations	0.434	0.413
Unskilled services	0.430	
Skilled services	0.449	
Comm. and admin. occupations	0.480	0.422
Unskilled comm. and admin.	0.482	
Skilled comm. and admin.	0.479	
Professional occupations	0.483	0.430
Semi-professionals	0.317	
Technicians	0.608	
Professionals	0.433	
Engineers	0.609	
Managers	0.743	

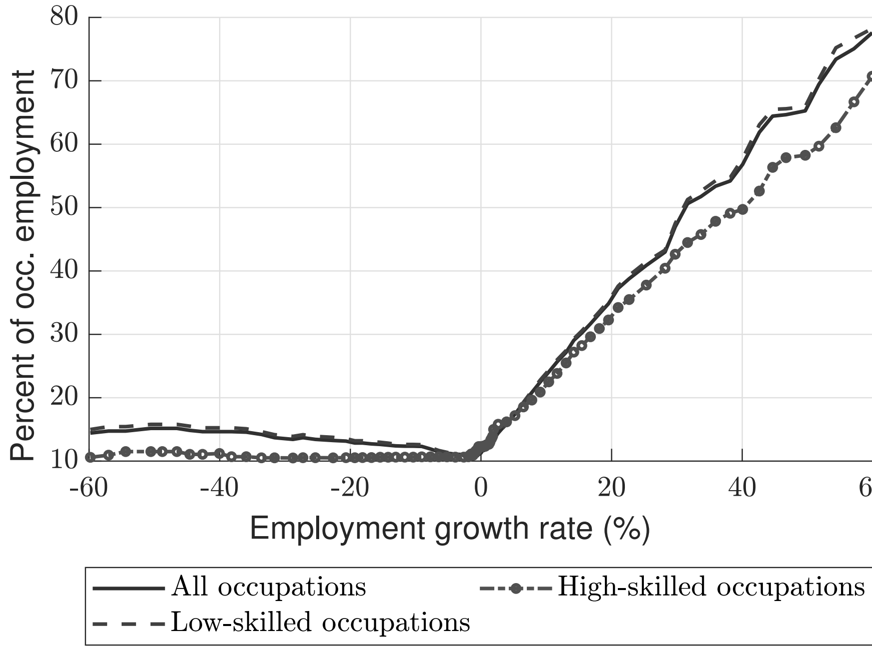
the figures show that the relationship is non-linear with the plot for the hiring rates the mirror image of that of separation rates. In other words, to grow, establishments hire more than they separate, and to shrink establishments must separate with more workers than they match. Consequently, fast growing (shrinking) establishments hire (separate with workers) more than one-for-one with the growth rate. Throughout, the mean rates for high-skilled occupations are smaller than the low-skilled occupations, and are below the 45 degree line because the x-axis is total employment growth rate.

1.5. Conclusion

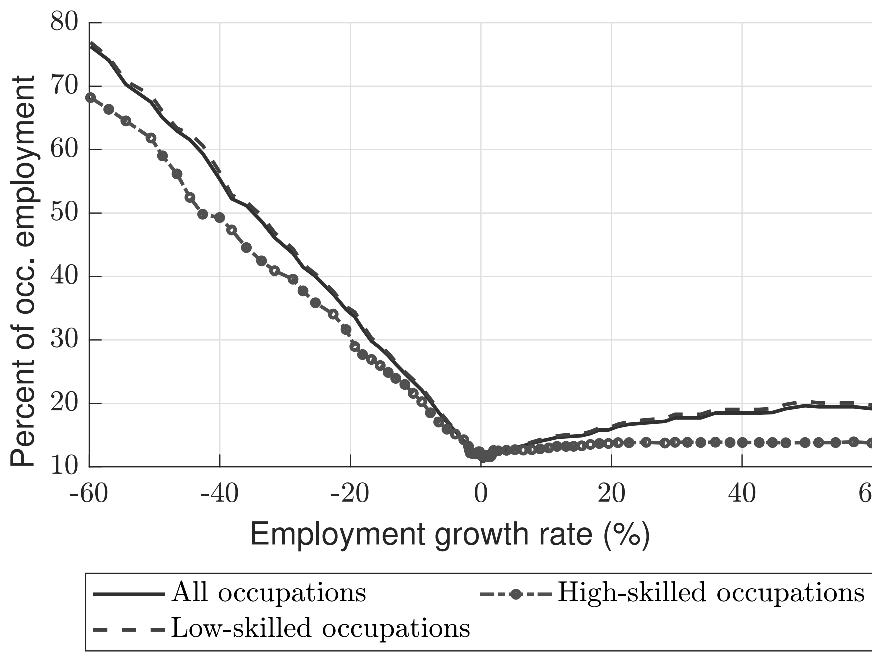
The literature has long known the limitations of associating one-for-one the job flows and net employment changes. In particular, such definition potentially implied that job flow rates were underestimated and their importance for the flow of workers understated. However, such limitation was so far simply mentioned in the literature but never was it studied. My research makes strides in this regard.

Different from the conventional definitions, establishment level total job creation and destruction are in this study occupation-based where corresponding occupation-specific rates are appropriately weighted and aggregated up. By doing so, I find that indeed the occupation-based job flow rates are significantly larger. Extending existing decompositions, I show that occupation-based job flow rates explain more worker flows than could conventionally be done.

My research also shows that the findings are robust and representative across the whole distribution of establishment growth. Furthermore, the cross-sectional patterns of occupational job and worker flows are different from the recent job polarization phenomenon.



(A) Hires Rate



(B) Separation Rate

FIGURE 1.5. Occupational Worker Flow Rates and Employment Growth

Notes: Data used cover both West and East Germany. See further notes under Figure 1.4.

Since these patterns are characteristic of the employer or the firm, to be able to explain them using quantitative-theoretic models of the labor market, alternative approaches that account for labor and job heterogeneity as well as heterogeneity in occupational labor productivity would have to be used. Recent considerations are Cahuc, Marque, and Wasmer (2008) and Bagger, Christensen, and Mortensen (2014). Both would have to be reconciled with single worker-type approaches of, e.g., Kaas and Kircher (2015) and Schaal (2017).

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1.A. Data

In this section, further data treatment and additional results are discussed.

1.A.1. Additional Information About the Data. When an establishment changes ownership or legal form, its ID changes and all workers are recorded as separations or hires as the case may be. However, separations and hires of this kind are erroneous because the establishment continues and workers have not gone through a job search process to secure their positions.

In the BHP, new and exiting establishments are each classified into seven entry and exit types.[18] A new (exiting) establishment could be a genuine foundation (exit) or a spin-off (takeover), among other types. The classification of new establishments (and analogously for exiting ones) is based on a comparison of the number of worker inflows and outflows at the establishment in question and at the originating establishment(s). For example, "medium and large entry" class of new establishments has at most 30% of all its new hires having come from the same establishment, whereas the "atomized death" class of exiting establishments has at most 30% of all workers finding employment at the same establishment in the following year. It is atomized because upon exit employees are randomly scattered across establishments or into non-employment. Each new establishment in the "small entry" class has at most three new hires, two of whom were not employed at the same establishment in the previous year. The converse holds for "small death" exit type, i.e., no two employees end up in the same establishment in the following year.

The above two types of entry and exit are unambiguously correctly classified and have well-known features, i.e., entry rates are strongly procyclical and have decreasing employment growth as they age while the exit rates are strongly countercyclical (Hethey and Schmieder 2010). Hires and separations for these two types of entry and exit are the only ones considered in this study. Flows at other types of entry or exit are deleted. The two classes of entry (exit) together account for a rounded figure of 50% of total employment-weighted share of all new

[18]Thanks to Hethey and Schmieder (2010), establishment entry and exit are carefully coded based on information on the cluster of worker flows by comparing June-to-June number and characteristics of employees together with a comparison of the inflow and outflow of workers at or from the receiving or originating establishments. This reduces errors emanating from basing establishment entry and death entirely on dates of appearance in or disappearance from the cross-sections.

(exiting) establishments based on first (last) date of appearance. Moreover, Hethey and Schmieder (2010) point out the contribution of establishment entry or exit in overall job reallocation otherwise defined, i.e., not based on worker inflows and outflows, is substantially overstated.

New or exiting establishments whose types are not specified by the method of Hethey and Schmieder (2010) together with those outside the classes of “small entry”, “medium and large entry”, “small death”, and “atomized death” are never included in the analysis in the years they first or last appear. I pull the two entry and exit types into one category of new and exiting establishments, respectively.

1.A.2. Further Features of Occupational Employment. In this section, additional basic sample statistics are reported.

Compared with Table 1.2, Table 1.13 reports that sectors with the highest employment shares do not necessarily employ more workers on average. The largest establishments by total employment are in the mining sector and utilities which are nearly ten times the size of the smallest establishments in hotels and restaurants.

While there is large variation across sectors by the average occupational employment size, sectors are certainly more specialized by the occupations they employ. In particular, Table 1.14 reports that the highest magnitudes of occupational employment intensity range from 44.67% in commercial and adminstration occupations in the real estate sector to 90.94% in the financial sector for the same occupations. That is the table shows that for each sector there is a main occupation employed with a share as low as 44.67% to as high as 90.94%.

Table 1.15 reports that occupational employment intensity is increasing with establishment size in manual and professional occupations but it is decreasing in services and commercial and administrative occupations. However, some differences exist among the constituting occupations.

Table 1.16 shows that the older the establishment the larger is the size. However, occupational employment intensity is at best uniformly distributed across establishment age groups.

According to Table 1.17, stable-growth establishments attain the average employment size of 15 workers, have the highest employment intensity in services and commercial and administration occupations but medium-growth establishments employ more workers on average and have the highest employment intensity in manual and professional occupations.

1.A.3. Trends in Occupational Employment. Following Jaimovich and Siu (2020), Blossfeld occupations can be given the same interpretation used in the job polarization literature. By means of Table 1.18, I describe how Blossfeld occupations can be classified in terms of the routine intensity. This classification is widely used in the job polarization literature–see Acemoglu and Autor (2011) for a review of the literature.

In Figure 1.6 and Figure 1.7 occupational employment growth over times is presented.

The figures show that there is job polarization in the BHP data where trends in occupational employment shares point to declines in the routine-intensive manual occupations relative to non-routine intensive services occupations and professionals, and relative to routine cognitive jobs characteristic of commercial and administrative occupations. As I argue in the text, what is driving these trends certainly does not seem to have much influence on the occupation-specific rates of worker and job flows.

Causal evidence for Germany by Dauth et al. (2017) suggests that robots, which are interpreted as one instrument by which routine-intensive jobs are being replaced, are affecting

TABLE 1.13. Mean Occupational Employment By Sector ([0,∞])

	Mining and quarrying	Manufacturing	Electricity, gas, and water supply	Construction	Wholesale and retail trade	Hotels and restaurants	Transport, storage, and communication	Financial intermediation	Real estate, renting, and business activities	Public admin., and defence; compulsory social security	Education	Health and social work	Other community, social, and personal service activities
All occupations	63.32	34.69	62.39	10.18	10.05	6.16	16.96	22.00	9.21	53.18	16.48	14.65	8.20
Manual occupations	44.36	20.58	28.36	7.93	1.79	1.91	2.20	0.18	1.25	6.01	0.95	0.77	0.56
Unskilled manual	33.53	11.87	6.50	2.75	0.56	0.08	0.68	0.03	0.68	3.24	0.23	0.13	0.26
Skilled manual	10.83	8.71	21.87	5.17	1.23	1.83	1.53	0.15	0.57	2.77	0.72	0.64	0.30
Services occupations	6.81	2.62	7.16	0.44	1.64	3.55	8.28	1.02	2.52	10.99	2.48	6.15	3.91
Unskilled services	5.86	2.44	6.34	0.42	1.46	3.43	7.67	0.96	2.31	9.69	2.18	1.83	1.87
Skilled services	0.95	0.18	0.82	0.02	0.17	0.12	0.60	0.06	0.21	1.30	0.30	4.32	2.04
Comm. and admin. occupations	4.92	6.36	14.70	1.18	5.85	0.57	5.49	20.25	3.31	24.21	2.41	1.20	1.92
Unskilled comm. and admin.	0.58	1.92	2.74	0.20	3.37	0.35	1.70	1.36	0.60	5.32	0.51	0.33	0.52
Skilled comm. and admin.	4.33	4.44	11.96	0.98	2.48	0.21	3.80	18.89	2.72	18.89	1.90	0.88	1.40
Professionals	7.15	5.10	11.96	0.62	0.72	0.12	0.97	0.54	2.10	10.24	10.51	6.46	1.70
Semi-professionals	0.03	0.16	0.40	0.00	0.01	0.05	0.02	0.03	0.06	4.74	6.80	5.13	0.91
Technicians	4.95	2.87	6.45	0.32	0.26	0.01	0.39	0.04	0.65	2.73	0.65	0.46	0.16
Professionals	0.09	0.09	0.29	0.01	0.08	0.01	0.02	0.07	0.13	0.62	2.20	0.75	0.23
Engineers	0.98	1.27	3.65	0.19	0.07	0.00	0.28	0.07	0.54	1.59	0.67	0.04	0.10
Managers	1.09	0.71	1.17	0.10	0.30	0.05	0.26	0.33	0.71	0.56	0.19	0.08	0.31

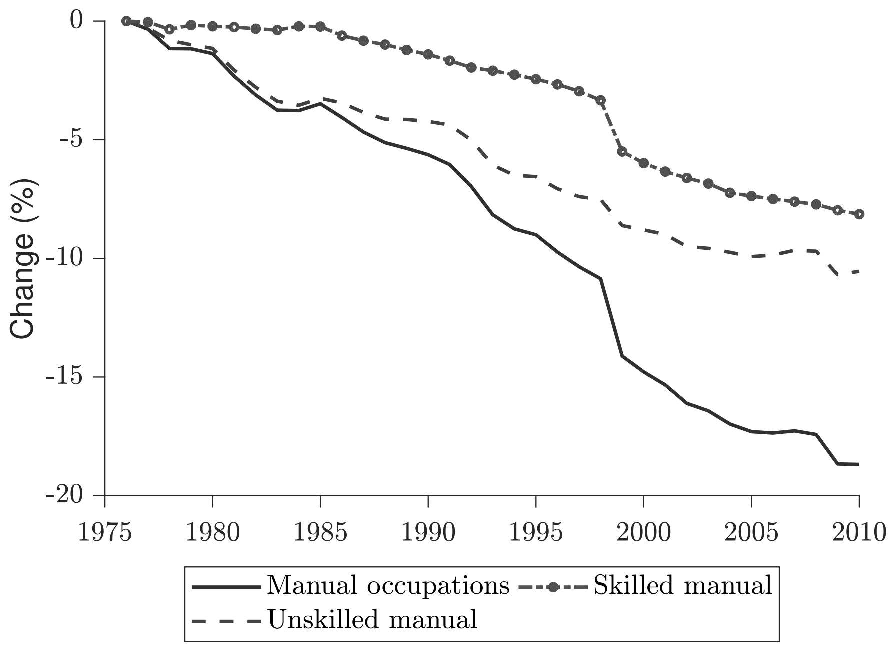
Notes: See Table 1.1 and Table 1.11 for number of observations.

workers in exposed occupations through the entry margin, namely by ever lower entry of younger workers into such occupations.

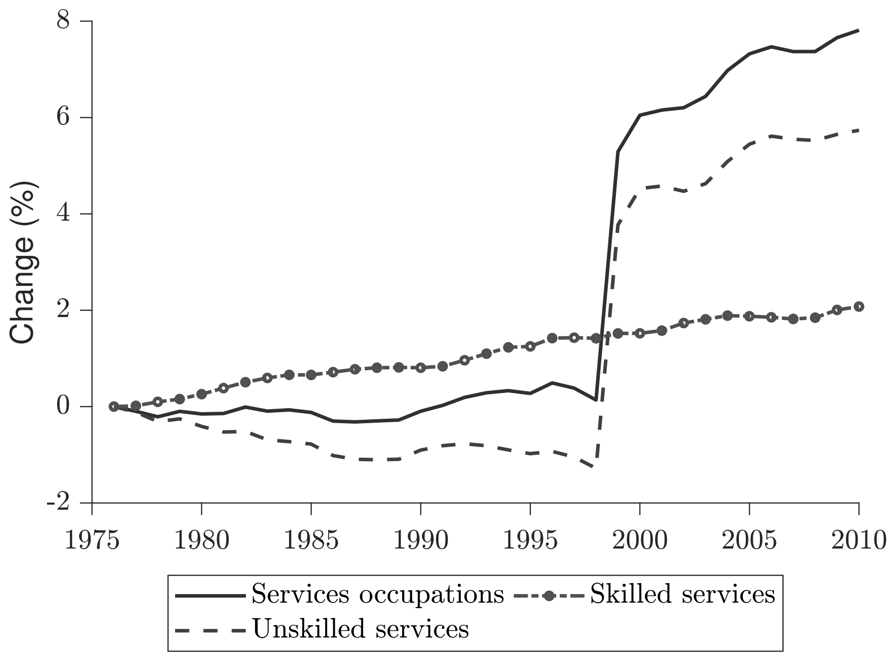
Observe that inclusion of marginal part-time workers in the social security reporting system since 1999, as well as the Hartz labor market reforms of the early 2000s that might have led to an expansion of some of the unskilled jobs in the 2000s, see Carrillo-Tudela, Launov, and Robin (2018), help explain why in the figures, unskilled services and unskilled commercial and administrative occupations suddenly jump and start to trend upwards.

1.A.4. Job-to-Job Worker Flows and Other Flows. I have also computed the rates of hires and separations for workers who move between establishments, marginal part-time workers, rehires and temporary separations. Unfortunately, it is not possible to separate these by occupation. These results are reported in Table 1.19 as percentage shares of average total employment.

From Table 1.19 and as compared to Table 1.7, the proportion of the average job to job worker flows, marginal part-time workers and rehires/temporary separations are about 46-47%, 30-33%, and 8.88-9.83% of all worker flows, respectively. Furthermore, they are dispersed and positively skewed with a high kurtosis.



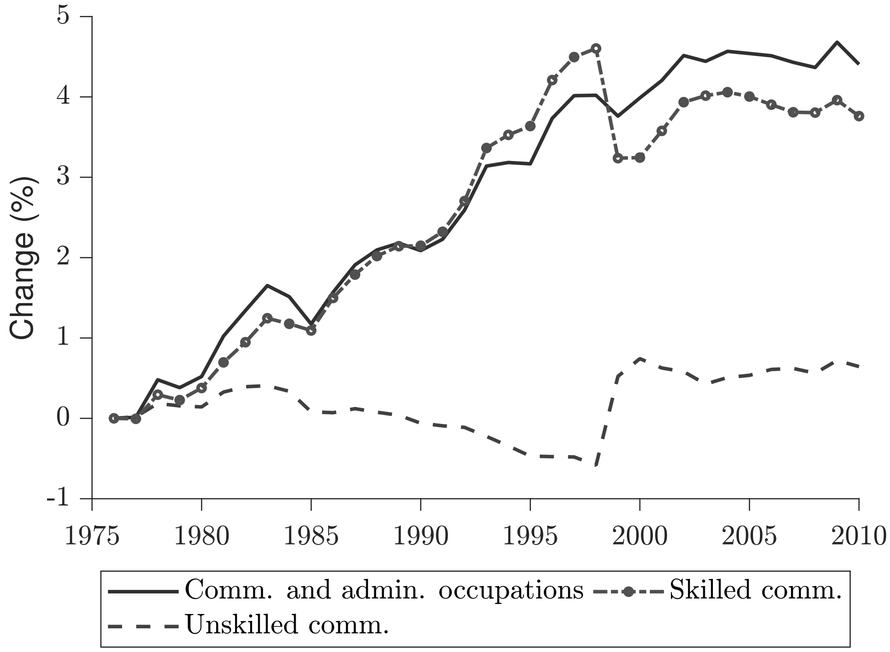
(a) Manual Occupations



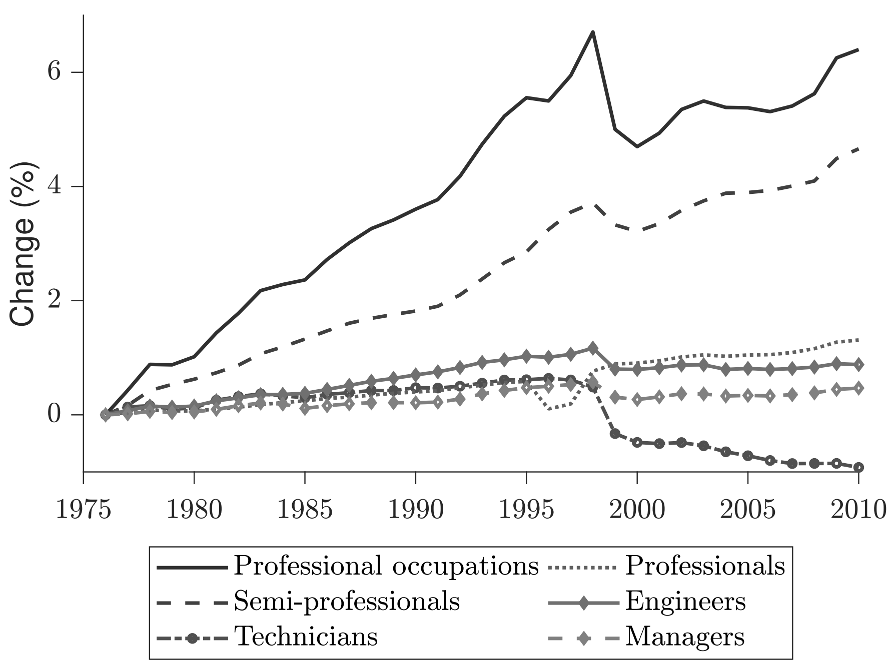
(b) Services Occupations

Figure 1.6. Trends in Employment of Manual and Services Occupations

Notes: See Table 1.6.



(A) Commercial and Administrative Occupations



(B) Professional and Managerial Occupations

FIGURE 1.7. Trends in Employment of Commercial and Administrative Occupations, and Professional and Managerial Occupations

Notes: See Table 1.6.

TABLE 1.14. Occupational Employment Intensity By Sector (%)

	Mining and qua-rrying	Manu-fact-uring	Electri-city, gas, and water supply	Constr-uction	Whole-sale and retail trade	Hotels and resta-urants	Trans-port, storage, and comm-unication	Fina-ncial inter-medi-ation	Real estate, renting, and business activities	Public admin., and defence; compul-sory social security	Edu-cation	Health and social work	Other comm-unity, social, and personal service activities
Manual occupations	53.88	57.71	57.53	77.54	16.84	29.76	4.64	0.52	5.72	12.66	1.96	1.51	3.85
Unskilled manual	43.04	21.23	20.21	18.15	3.96	1.40	2.06	0.32	2.76	10.33	0.69	0.51	2.26
Skilled manual	10.84	36.48	37.31	59.39	12.88	28.36	2.58	0.20	2.97	2.33	1.27	1.00	1.59
Services occupations	21.33	6.03	9.80	2.95	14.12	57.18	59.24	6.03	27.54	20.70	14.78	73.61	59.63
Unskilled services	20.94	5.33	8.86	2.64	11.77	55.43	57.52	5.44	22.45	18.65	13.05	9.92	18.45
Skilled services	0.39	0.70	0.94	0.30	2.35	1.76	1.73	0.59	5.09	2.04	1.73	63.69	41.19
Comm. and admin. occupations	16.04	26.77	17.83	16.09	60.61	11.35	32.12	90.94	44.67	48.71	19.84	9.47	21.59
Unskilled comm. and admin.	2.22	12.58	2.89	3.48	35.03	7.48	8.02	9.06	8.26	10.12	4.36	2.70	6.60
Skilled comm. and admin.	13.82	14.19	14.94	12.62	25.58	3.88	24.09	81.88	36.40	38.59	15.47	6.77	14.98
Professionals	8.38	9.37	14.34	3.32	7.75	1.53	3.89	2.46	21.73	15.38	63.04	15.27	14.05
Semi-professionals	0.07	0.43	0.46	0.03	0.15	0.53	0.11	0.09	0.49	5.66	57.22	10.20	7.13
Technicians	3.96	5.67	6.88	1.47	2.57	0.11	1.19	0.17	6.87	3.55	0.80	2.37	0.92
Professionals	0.11	0.14	0.20	0.04	1.09	0.06	0.07	0.20	1.10	0.89	3.57	2.11	1.59
Engineers	1.03	0.93	2.99	0.60	0.44	0.04	0.37	0.17	4.25	1.92	0.31	0.07	0.52
Managers	3.20	2.20	3.81	1.17	3.50	0.80	2.16	1.82	9.03	3.36	1.13	0.51	3.90

Notes: See Table 1.1 and Table 1.11 for number of observations or notations.

TABLE 1.15. Occupational Employment by Establishment Size

	0–		20–		50–		250–		500–	
	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)
All occupations	4.71		31.23		101.73		344.96		1232.92	
Manual occupations	1.31	23.55	10.36	33.19	33.32	32.69	117.64	34.06	460.75	35.69
Unskilled manual	0.37	6.01	4.28	13.34	17.97	17.16	68.31	19.76	271.50	20.63
Skilled manual	0.94	17.54	6.08	19.85	15.34	15.54	49.33	14.30	189.24	15.06
Services occupations	1.26	28.99	6.42	20.46	19.57	19.38	58.83	17.10	168.24	14.79
Unskilled services	0.78	28.99	5.53	20.46	16.53	19.38	48.40	17.10	135.95	14.79
Skilled services	0.48	11.86	0.89	2.86	3.04	3.03	10.43	3.01	32.30	2.91
Comm. and admin. occupations	1.54	35.97	9.92	31.91	32.27	31.88	100.87	29.40	303.22	26.69
Unskilled comm. and admin.	0.63	14.41	3.59	11.56	10.24	10.36	27.91	8.16	76.02	6.92
Skilled comm. and admin.	0.91	21.56	6.33	20.35	22.03	21.52	72.96	21.24	227.20	19.77
Professionals	0.58	11.11	4.36	13.89	16.04	15.52	66.15	19.00	294.90	22.32
Semi-professionals	0.18	2.92	1.70	5.34	6.17	6.11	25.36	7.24	81.52	7.65
Technicians	0.15	2.92	1.18	3.74	4.77	4.51	19.98	5.77	93.80	6.85
Professionals	0.05	0.89	0.24	0.77	0.92	0.86	4.80	1.35	32.84	2.20
Engineers	0.05	0.98	0.46	1.45	2.07	1.92	9.35	2.68	61.69	3.76
Managers	0.15	3.40	0.78	2.58	2.11	2.12	6.65	1.96	25.05	1.86

TABLE 1.16. Occupational Employment by Establishment Age

	0–		3–		6–		11–		16–	
	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)
All occupations	7.94		11.13		13.40		16.97		24.82	
Manual occupations	2.73	23.14	3.89	23.87	4.59	24.44	5.81	25.61	7.45	25.97
Unskilled manual	1.30	6.81	1.87	6.91	2.17	6.79	2.79	6.90	3.61	7.49
Skilled manual	1.43	16.33	2.01	16.96	2.42	17.65	3.02	18.71	3.84	18.48
Services occupations	1.73	28.45	2.32	28.88	2.73	28.81	3.31	27.41	4.74	26.52
Unskilled services	1.31	18.53	1.73	18.06	2.03	17.25	2.46	15.16	3.65	16.08
Skilled services	0.43	9.92	0.58	10.82	0.70	11.56	0.85	12.25	1.08	10.44
Comm. and admin. occupations	2.35	36.69	3.27	35.91	3.97	35.22	5.01	34.93	7.65	34.70
Unskilled comm. and admin.	2.35	15.25	3.27	14.72	3.97	14.05	5.01	13.51	7.65	12.68
Skilled comm. and admin.	1.41	21.44	2.04	21.19	2.54	21.17	3.31	21.42	5.44	22.02
Professionals	1.09	11.45	1.61	10.97	2.04	11.13	2.76	11.63	4.85	12.29
Semi-professionals	0.27	2.57	0.44	2.74	0.59	2.96	0.85	3.32	1.88	4.37
Technicians	0.37	3.06	0.54	3.04	0.67	3.10	0.88	3.20	1.24	2.93
Professionals	0.07	0.74	0.11	0.78	0.15	0.86	0.21	0.97	0.47	1.09
Engineers	0.16	1.15	0.24	1.06	0.31	1.04	0.42	1.04	0.70	1.01
Managers	0.22	3.92	0.28	3.35	0.33	3.17	0.40	3.09	0.57	2.89

Table 1.17. Occupational Employment by Employment Growth

	[-200, -20)		[-20, -5)		[-5, 5]		(5, 20]		(20, 200]	
	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)	Mean ([0,∞])	Intensity (%)
All occupations	6.51		31.02		14.89		33.83		8.61	
Manual occupations	2.16	27.16	11.53	31.60	4.72	20.85	11.23	30.57	2.55	25.45
Unskilled manual	0.90	7.27	5.80	10.48	2.34	5.72	5.41	9.81	1.07	7.04
Skilled manual	1.26	19.89	5.73	21.12	2.38	15.13	5.82	20.76	1.48	18.41
Services occupations	1.48	27.84	5.50	23.27	2.82	29.57	6.57	23.77	2.19	28.23
Unskilled services	1.06	16.32	4.44	15.68	2.09	17.96	5.17	15.99	1.59	16.83
Skilled services	0.42	11.52	1.06	7.59	0.73	11.61	1.40	7.78	0.60	11.40
Comm. and admin. occupations	2.01	34.00	9.23	31.93	4.48	38.38	10.00	31.04	2.62	33.82
Unskilled comm. and admin.	0.77	14.06	3.29	12.80	1.38	14.84	3.24	11.82	1.02	13.68
Skilled comm. and admin.	1.25	19.94	5.94	19.13	3.11	23.54	6.76	19.22	1.60	20.14
Professionals	0.83	10.54	4.62	12.69	2.79	10.87	5.87	14.11	1.21	12.08
Semi-professionals	0.20	2.49	1.23	4.07	0.97	2.97	2.17	5.18	0.38	3.21
Technicians	0.28	3.16	1.63	3.57	0.81	2.74	1.58	3.55	0.33	3.23
Professionals	0.05	0.77	0.26	0.96	0.25	0.83	0.50	1.15	0.10	0.97
Engineers	0.12	1.04	0.76	1.17	0.42	0.92	0.80	1.27	0.16	1.23
Managers	0.18	3.08	0.74	2.92	0.34	3.42	0.82	2.96	0.24	3.44

Table 1.18. Classifying Blossfeld Occupations by Routine Intensity

	Classification by Jaimovich and Siu (2020)	
Occupations by Blossfeld (1987)	Routine-Intensity Category	Examples (cf: Jaimovich and Siu (2020))
Manual occupations	Routine manual	“blue collar” jobs, such as machine operators and tenders, mechanics, dressmakers, fabricators and assemblers, and meat processing workers.
Services occupations	Non-routine manual	service jobs, including janitors, gardeners, manicurists, bartenders, home care aides, and personal care workers.
Commercial and administrative occupations	Routine cognitive	occupations in sales, and office and administrative support; examples include secretaries, bank tellers, retail salespeople, travel agents, mail clerks, and data entry keyers
Professional occupations	Non-routine cognitive	managerial, professional and technical workers, such as physicians, public relations managers, financial analysts, computer programmers, and economists

Table 1.19. Other Margins of Worker Flow Rates

	Mean		Sd		Median		Dispersion		Skewness		Kurtosis	
	HR	SR	HR	SR	HR	SR	HR	SR	HR	SR	HR	SR
On-the-Job Flows	0.097	0.092	0.171	0.156	0.055	0.050	0.763	0.757	0.193	0.257	2.222	2.293
Part-time	0.070	0.058	0.166	0.135	0.004	0.003	1.000	1.000	0.894	0.906	3.126	3.229
Rehires or Temp. Sep.	0.021	0.017	0.054	0.049	0.009	0.006	1.000	1.000	0.156	0.292	1.950	2.011

Notes: "Rehires or Temp. Sep." stands for rehires or temporary separations. Numbers of observations for part-time hires and separations are 9954239, and 9277528, respectively. First observation of part-time hires occurs in year 1999 and that of part-time separations occurs in year 2000. The number of observations for the remainder of the variables is each 23772268.

CHAPTER 2

Labor Market Flows and Misallocation with Occupational Heterogeneity

2.1. Introduction

A key feature of a labor market is its ability to reallocate jobs between firms and to match workers of different skill sets to jobs of conformable skill requirements. However, sub-optimal firm-worker matches lead to substantial differences in labor productivity between firms and to worker churn, the amount of worker flows over and above job flows, as firms and workers re-evaluate their matches.[1]

My contribution to the literature is a quantitative model for studying the equilibrium interaction between labor market flows, misallocation, and occupational labor heterogeneity. These aspects of the labor market have so far been studied separately. For instance, Fujita and Nakajima (2016), Kaas and Kircher (2015) and Schaal 2017 model the interaction between labor market flows and firm dynamics; Bagger, Christensen, and Mortensen (2014) is an empirical contribution on occupational labor heterogeneity and misallocation; and Burgess, Lane, and Stevens (2000), Davis, Faberman, and Haltiwanger (2006), and Davis, Faberman, and Haltiwanger (2013) are empirical contributions on labor market flows.

By definition, a factor of production is misallocated if factor-specific marginal productivity differs across firms such as due to labor quality differences. Labor market flows include job and worker flows. Job flows are measured at the level of the employer such as the establishment as employment changes. Worker flows are employer-level hires and separations. Their magnitudes, determinants, and relationship are of great interest empirically and theoretically.[2] Furthermore, an occupation as a collection of jobs with similar skill requirements classifies a workplace into subunits with similar characteristics and which are affected by the same factors.

I first examine a lesser known aspect of the labor market flows, namely simultaneous job creation and destruction at the establishment level using establishment employment history data for Germany where it is possible to measure worker and job flows in broad 1-digit occupations.[3] I find that occupation-specific job flow rates differ at a given establishment so much that even establishments with zero employment growth create jobs in some occupations, and simultaneously destroy jobs in other occupations.

[1]Relevant references on labor productivity include Iranzo, Schivardi, and Tosetti (2008), Bagger, Christensen, and Mortensen (2014), and Bartelsman, Dobbelaere, and Peters (2015). See, e.g., Burgess, Lane, and Stevens (2000) and Davis, Faberman, and Haltiwanger (2006) on work churning flows.

[2]See empirical studies, e.g., Davis and Haltiwanger (1992), Anderson and Meyer (1994), Davis and Haltiwanger (1999), Abowd, Corbel, and Kramarz (1999), Burgess, Lane, and Stevens (2000), Davis, Faberman, and Haltiwanger (2006). See models by, e.g., Mortensen and Pissarides (1994), Hopenhayn and Rogerson (1993), and Fujita and Nakajima (2016).

[3]In Kimasa (2020), I discuss in detail the cross-sectional patterns of simultaneous occupational job and worker flows. Building on the same data, I extract a sub-sample.

The magnitude of occupation-based job reallocation over and above net job creation are not only considerably large but are well distributed across the entire distribution of the establishment growth rates. The average magnitude of this *establishment-level excess job reallocation* relative to net job creation is about 24%.[4] Furthermore, occupation-based average job reallocation rate is 5% larger than is conventionally measured, which disregard the occupational aspect of job flows. I find that 12% more worker flows can be explained by the improved measure of job flows than by conventional definitions of job flows. By implication, the extent of worker churn is smaller than what conventional measures suggest.

Absent disaggregated data, the existing empirical literature focuses on net job creation thus connecting one-for-one net employment growth and job reallocation–see Burgess, Lane, and Stevens (2000, p. 479).[5] Consequently, changes in the overall employment size of establishments and changes in the distribution of employment opportunities within and between establishment for jobs of different skill requirements are indistinguishable.

If differences in occupational labor productivity, imperfect substitutability of occupational labor, or differences in the costs of occupational job turnover mean occupational labor is reallocated at different rates and hence more jobs are reallocated than the firm expands or contracts, then labor and job heterogeneity are key for a better understanding of the dynamics of firms and worker and job flows. By proposing an equilibrium model that delivers firm-level simultaneous job creation and destruction, my research attempts to fill the gap.

My model is a Diamond-Mortensen-Pissarides (DMP) framework with firm dynamics and directed labor market search, extending the model of Kaas and Kircher (2015) by adding occupational heterogeneity and job upgrading. Firms in my model operate with two types of imperfectly substitutable labor, namely high-skilled and low-skilled occupational labor that differ in productivity, matching rates, and reallocation costs. Correlated occupational labor productivity affect not only the flow of jobs of own type but also the total number of jobs firms reallocate. Employed low-skilled workers may be upgraded to become high-skilled at a general conversion cost.[6] Occupational mobility in unemployment and search on the job are both abstracted from.

Following Bagger, Christensen, and Mortensen (2014), I can decompose labor productivity into three components: *labor quality heterogeneity*, *occupational labor misallocation*, and *total factor misallocation*. The first component refers to the differences across firms in the average quality of labor. The second component captures any managerial inefficiencies at distributing a given stock of labor across occupations, leading to a suboptimal level of quality-adjusted labor. The last component refers to the differences in the marginal productivity of quality-adjusted labor. It may be due to a misallocation of capital, labor, or other factors of production.

The equilibrium in my model with within-firm job upgrading is not socially efficient. This result contrasts Kaas and Kircher (2015) without job upgrading and the efficient result of

[4] In line with previous terminology used but at the industry level, in particular in Davis and Haltiwanger (1999), this residual job reallocation is referred to as the *establishment-level excess job reallocation*. It is an objective of this research to explain this phenomenon.

[5] See, e.g., Davis and Haltiwanger (1992), Anderson and Meyer (1994), Davis and Haltiwanger (1999), Abowd, Corbel, and Kramarz (1999), Davis, Faberman, and Haltiwanger (2006), for more empirical contributions, and, e.g., Mortensen and Pissarides (1994), Hopenhayn and Rogerson (1993), and Fujita and Nakajima (2016) for theoretical contributions.

[6] Thus, internally acquired skills are not firm specific.

Moen and Rosén (2004) who have job upgrading.[7] With commitment on contracting as in these papers, joint surplus optimization does not fully internalize all externalities from search because upgraded workers, who keep their skills upon an upgrade, search in the same markets as high-skilled workers when unemployed. This search behavior cannot be dynamically contracted away. Therefore, a surplus remains that must be shared between the firm and its workers for as long as the employment relationship continues.

I calibrate the model in order to jointly match the cross-sectional patterns of labor productivity as well as occupational job and worker flows. For explaining the role of misallocation and occupational heterogeneity for the dynamics of firms and labor market flows it is key to capture the shapes of the distributions of the key variables. The model does well in capturing the shapes of the distributions of labor market flow rates, labor productivity, and skill-intensity. However, it over- and under-estimates the first-order moments. The simulation results are preliminary and should be considered illustrative alone.

The calibrated model results show that labor-productivity differences are 100.06% due to overall misallocation and -0.06% due to occupational labor quality heterogeneity. Occupational labor misallocation by itself contributes about 34.25% while the contribution of aggregate misallocation, due to differences in quality-adjusted marginal labor productivity is 65.81%. Where the model has matched the shape of the distributions better, the contribution of labor quality heterogeneity has been positive but typically below 5%. Therefore, it is better to put the contribution of occupational labor misallocation at 25-35%, and that due to quality-adjusted marginal labor productivity above 50%.

In the context of known results, Bagger, Christensen, and Mortensen (2014) document that occupational labor misallocation within firms explain about a quarter of the cross-sectional variance of log labor productivity in Swedish manufacturing. The contribution of aggregate factor misallocation is 65% and the remainder 9% due to labor quality heterogeneity. Their findings are robust across sectors.

My model is related to a growing literature that features heterogeneous firms in the labor market without search frictions, e.g., Hopenhayn and Rogerson (1993) or with search frictions, e.g., Smith (1999), Cahuc, Marque, and Wasmer (2008), Elsby and Michaels (2013), Hawkins (2013), Acemoglu and Hawkins (2014), Schaal (2017), Kaas and Kircher (2015), and Fujita and Nakajima (2016). Only Cahuc, Marque, and Wasmer (2008) on this list have a model of firm dynamics with heterogeneous labor. However, their paper has no quantitative analysis.

The closest models to mine are Kaas and Kircher (2015) and Schaal (2017). The key differences between my model and Kaas and Kircher (2015) have been elaborated above. In all essential aspects, Kaas and Kircher (2015) and Schaal (2017) are similar, except that Schaal (2017) focuses on productivity volatility shocks and that the unique efficient search equilibrium is in each delivered differently. While they both rely on the free entry of firms, there is just one free-entry condition in Kaas and Kircher (2015) but in Schaal (2017) there are as many as there are unique firms, i.e., one free-entry condition for each submarket of the labor market and vacancy creation costs are equalized across submarkets.

By studying occupation-specific labor market flows, my research builds on a well-known empirical literature without occupational labor such as Davis and Haltiwanger (1992), Burgess, Lane, and Stevens (2000), Davis, Haltiwanger, and Schuh (1996), Davis and Haltiwanger (1999), and Davis, Faberman, and Haltiwanger (2006). Others include Anderson and Meyer

[7] Shi (2016) models on-the-job training where equilibrium is socially optimal. However, a job in Shi (2016) is a form of capital which can be improved continuously via investment in training. The notion of skill types is, however, absent.

(1994), Hamermesh, Hassink, and Ours (1996), Albaek and Sorensen (1998), and Bellmann, Gerner, and Upward (2017).

My research is also related to an empirical research that relates labor quality heterogeneity and firm performance such as Bagger, Christensen, and Mortensen (2014), Iranzo, Schivardi, and Tosetti (2008), Caliendo, Monte, and Rossi-Hansberg (2015), Caliendo, Mion, et al. (2015), and Bartelsman, Dobbelaere, and Peters (2015). The organization of occupational labor in firms is undertaken by Caliendo, Monte, and Rossi-Hansberg (2015) and Caliendo, Mion, et al. (2015). Caliendo, Monte, and Rossi-Hansberg (2015) find that firms reorganize the occupations they manage when adapting to exogenous shocks. Caliendo, Mion, et al. (2015) estimate considerable productivity gains to firm reorganization.

In Section 2.2, key empirical labor market flow concepts are introduced. Section 2.3 describes the data and presents the empirical results. Section 2.4 introduces the model. Section 2.7 explores the model quantitatively. Section 2.8 concludes.

2.2. Labor Market Flow Concepts

The definitions of labor market flows applied are derived from Burgess, Lane, and Stevens (2000). See also Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1996), Abowd, Corbel, and Kramarz (1999), and Davis, Faberman, and Haltiwanger (2006).

Consider an establishment i in period $t = 1, ..., T$. The establishment employees L_{iot} workers in occupation $o = 1, ..., O$. Let H_{iot} and S_{iot} be occupation-o newly hired workers and separations, respectively. They are external flows of workers who transition between employment and non-employment states, and between establishments. Let X_{iot} be the net internal worker flows into occupation o from other occupations within the same establishment and observe that internal net worker flows sum to zero, i.e., $\sum_o X_{iot} = 0$. The rate of occupational job flows, JFR_{iot}, is the average change in the number of occupation-o jobs held by an establishment:

$$JFR_{iot} = HR_{iot} - SR_{iot} + XR_{iot} \,,$$

where $HR_{iot} = \frac{H_{iot}}{\overline{L}_{iot}}$ is hires rate, $SR_{iot} = \frac{S_{iot}}{\overline{L}_{iot}}$ is separation rate, $XR_{iot} = \frac{X_{iot}}{\overline{L}_{iot}}$ is net internal worker flow rate, and $\overline{L}_{iot} = 0.5 \times (L_{iot} + L_{io,t-1})$ is average employment of occupation o.

The rates of occupational job creation (JCR_{iot}) and destruction (JDR_{iot}) are defined as

$$JCR_{iot} = JFR_{iot} \text{ if } JFR_{iot} \geq 0 \;\;,$$

and

$$JDR_{iot} = |JFR_{iot}| \text{ if } JFR_{iot} \leq 0 \;\;.$$

That is, a job is created where occupational employment change is positive, it is destroyed otherwise.[8]

Let YR_{iot} be an element of $\{HR_{iot}, SR_{iot}, XR_{iot}, JFR_{iot}, JCR_{iot}, JDR_{iot}\}$. Then YR_{iot} can be aggregated to various classifications such as the establishment or industry. Establishment-level rates are

$$YR_{it} = \sum_o \frac{\overline{L}_{iot}}{\overline{L}_{it}} YR_{iot} \,,$$

[8]Job creation and destruction are so defined to separate them from worker flows in particular so as to not count every hire as a job creation since, e.g., a replacement hire just replaces an existing position.

where $\overline{L}_{it} = \sum_o \overline{L}_{iot}$. Aggregate rates are

$$YR_{ot} = \sum_i \frac{\overline{L}_{iot}}{\sum_i \overline{L}_{iot}} YR_{iot}\,,$$

and

$$YR_t = \sum_i \frac{\overline{L}_{it}}{\sum_i \overline{L}_{it}} YR_{it}\,.$$

The establishment-level job flow rates are here *occupation-based* because they account for all jobs reallocated by the establishment for workers in different occupations.

The above definitions of occupational job creation and destruction extend the standard definition of job creation and destruction. Conventionally defined job creation and destruction, on the other hand, disregard occupational job reallocation. They are defined as

$$\widetilde{JCR}_{it} = JFR_{it} \text{ if } JFR_{it} \geq 0 \;\;,$$

and

$$\widetilde{JDR}_{it} = |JFR_{it}| \text{ if } JFR_{it} \leq 0\,,$$

where $JFR_{it} = HR_{it} - SR_{it}$. HR_{it} and SR_{it} are hires and separation rates, which equal occupation-based hires and job separation rates.

It is easy to see that, with non-disaggregated data, where job creation is strictly positive, job destruction is zero. By contrast, where occupational jobs are created and destroyed at different rates, then at the level of an establishment there can be simultaneous job creation and destruction. The difference

$$(JCR_{it} + JDR_{it}) - |JFR_{it}| \geq 0$$

is what I label as *establishment-level excess job reallocation*, i.e., it is job reallocation over and above net job creation. In this sense, job reallocation rates may be underestimated under conventional definitions–see Burgess, Lane, and Stevens (2000, p. 479).

The literature is also interested in the role played by job flows for the reallocation of workers. This can be done better with *occupation–based* job flows since they are an improved measure of the amount of job reallocation in the labor market. Let the fraction of (external) worker flows due to occupation-based job reallocation be

$$WFR_{it} = \frac{JCR_{it} + JDR_{it}}{HR_{it} + SR_{it}},$$

where the internal margin of worker flows is ignored since the data does not report them separately.

Now contrast this way of relating job and worker flows, to "churning" flows introduced by Burgess, Lane, and Stevens (2000) who decompose worker flows into a component due to job reallocation and a residual, which they label churning flow rate (CFR). They employ this definition to non-disaggregate data as

$$HR_{it} + SR_{it} = |JFR_{it}| + CFR_{it}.$$

Churning flows are worker flows over and above job flows. Burgess, Lane, and Stevens (2000) note that this measure elevates the role of churning and understates the role of job flows for the reallocation of workers. In Burgess, Lane, and Stevens (2000),

$$|JFR_{it}| = \widetilde{JCR}_{it} + \widetilde{JDR}_{it}$$

since workers are homogeneous.

2.3. Data

2.3.1. The Establishment History Panel. The data is a panel of German establishments called the Establishment History Panel 1975 – 2010 (BHP) of the Institute for Employment Research (IAB). The BHP 1975 – 2010 covers a 50 percent random sample of all establishments with operations in Germany over 1975 to 2010 for West Germany and 1992 to 2010 for East Germany as of 30th June of a given year.

For this sample of establishments, the data reports stocks and flows of workers in broad 12 1-digit occupations. I pool these occupations into two occupational classes generalising to low-skilled occupations (occupation-l) and to high-skilled occupations (occupation-h), with agricultural occupations excluded from the analysis.[9] Occupation-l occupations are (i) unskilled manual occupations, (ii) skilled manual occupations, (iii) unskilled services occupations, (iv) skilled services occupations, (v) unskilled occupations in administration and commerce, and (vi) skilled occupations in administration and commerce. Occupation-h occupations are (i) technicians, (ii) semiprofessionals, (iii) engineers, (iv) professionals, and (v) managers.

I consider the homogeneous period 2004-2007 for West Germany, i.e., the years post Hartz reforms so as to rule out the Great Recession but more importantly to limit the influence of the long-run changes in occupational dynamics, partly due to changing nature of occupations as may be affected by technological as well as structural change–see in Goos, Manning, and Salomons (2014) and the papers they cite.[10] The reason I focus on West Germany is that most pre-existing studies on Germany focus on West Germany. This way I can relate to them and can borrow from them certain data moments necessary for calibrating the model. Dropped as well are the following sectors: (i) agriculture, hunting, forestry and fishing, (ii) private households with employed persons, (iii) Extraterritorial activities, and (iv) Missing sectors. Lastly, I drop all establishments without any of the two types of occupations. Additional data details and cleaning can be found in Kimasa (2020).

2.3.2. Results.

2.3.2.1. *Descriptive Statistics.* The 2004-2007 unbalanced sub-panel has 2,570,827 establishment year observations. The basic descriptive statistics of the establishments are reported in Table 2.1, where it can be concluded that the establishments are small and employment is positively skewed. The mean employment of low-skilled occupations is nearly five times as large as that of high-skilled occupations.

Much of the labor market flows relate with employment growths. Table 2.2 reports the distribution of establishments and employment in the data by groups of employment growth rates. The table shows the relative importance of stable-growth establishments which puts in perspective their relevance for the occupational job flow rates to be reported.

The mean total or occupation-specific employment sizes displayed in the table reveal that the largest class of establishments are medium-growth establishments, $JFR \in [-20\%, -5\%)$, and $JFR \in (5\%, 20\%]$, followed by the stable-growth establishments with growth rates $JFR \in [-5\%, 5\%]$. The columns on establishment share show that stable-growth establishments are the most frequent even when weighted by their occupational employment size (last three columns). They are followed by medium-growth establishments. The high-growth

[9] For each of the two broad occupations that I focus on, job reallocation rates are appropriate weighted sums of the constituting occupational rates. With only two occupations to highlight the mechanism, the model can only speak to occupation-based job reallocation with two occupations.

[10] In Kimasa (2020), I report that it is possible to separate the job polarization phenomenon from the cross-sectional dynamics.

TABLE 2.1. Descriptive Statistics

	Occupational Employment		
	All	Low-Skilled	High-Skilled
mean	12.536	10.329	2.138
max	10032	10021	7196
sd	59.083	45.008	24.101
p50	4	3	0

TABLE 2.2. Basic Data Statistics by Employment Growth Groups

	Mean Employment (#)			Establishment Share (%)				(%)
	All Occ.	Low -Skilled Occ.	High -Skilled Occ.	Establ. Share	All Occ. Empl. Share	Low-Skilled Occ. Empl. Share	High-Skilled Occ. Empl. Share	High-Skilled Occ. Empl. Intensity
[-200, -20)	5.516	4.793	0.692	14.30	6.29	6.6	4.63	12.55
[-20, -5)	24.265	20.028	4.112	8.31	16.08	16.1	15.98	16.95
[-5, 5]	11.289	9.024	2.197	49.33	44.42	43.1	50.68	19.46
(5, 20]	27.630	22.918	4.585	9.03	19.91	20.0	19.37	16.60
(20, 200]	8.756	7.657	1.049	19.03	13.29	14.1	9.34	11.98
Total	12.536	10.329	2.138	100.00	100.0	100.00	100.00	17.06

Notes: "Empl." refers to Employment. "Occ." refers to Occupations(al). "Establ." refers to Establishment. "High-Skilled Occ. Empl. Intensity " is the share of employment of high-skilled occupations where the denominator is the total employment of the group. "#" stands for head count; one minus it gives the low-skilled intensity.
Source: Own calculations using data and methodology as described in Section 2.3.

establishments, $JFR \in [-200\%, -20\%)$, and $(20\%, 200\%]$, are the smallest but are the second most frequent, by establishment count.

The table also reports in the last column the intensity of high-skilled occupational employment growth where the denominator is the group's total employment. Start with the last row of the column where the aggregate intensity, equivalently, the proportion of high-skilled occupational employment is reported to be about 17.06%. Across employment growth groups, high-skilled employment intensity is the higher the more stable the establishments grow.

2.3.2.2. *Occupational Labor Market Flow Rates.* Table 2.3 reports the magnitudes of the rates of job and worker flows for all occupations and between low-skilled and high-skilled occupations. As can be seen, the average net employment growth rate is positive. Associated with this net growth are large magnitudes of both job and worker flows. On average, worker flow rates are larger than job flow rates, implying that much of the reallocation of workers does not contribute directly to the net expansion and contraction of establishments. The table also reports that both worker and job flows are decreasing in skill, but the ratio of job to worker flows hardly varies across skill groups. The last two columns of the table report the amount of occupational switching within establishments, showing that low-skilled occupations

Table 2.3. Rates of Job and Worker Flows (%)

	JFR	HR	SR	JCR	JDR	WFR	XR	$\frac{XR}{JFR}$
All Occ.	1.95	20.90	18.95	10.47	8.52	47.67	0.00	0.00
Low-Skilled Occ.	2.07	21.47	19.35	10.76	8.68	47.62	-0.05	-2.18
High-Skilled Occ.	1.36	18.08	16.93	8.94	7.58	47.20	0.21	15.79

Notes: Reported as weighted averages, the rates correspond to the aggregate rates. Recall that occupational employment growth can be computed as $JFR_{iot} = HR_{iot} - SR_{iot} + XR_{iot}$, $o \in \{l, h\}$. I first calculate the weighted averages JFR_{ot}, JFR_{ot} and JFR_{ot} in each year. Then, the yearly weighted means are averaged over 2005 to 2007. Finally, XR_o is calculated as the residual $XR_o = JFR_o - HR_o + SR_o$. I make sure that XR_{iot} sum to zero.
Source: Own calculations using data and methodology as described in Section 2.3.

Table 2.4. Occupation-Based vs Conventional Job Flows (%)

	Occupation-Based			Conventional		
	JCR	JDR	WFR	JCR	JDR	WFR
All Occ.	10.47	8.52	47.67	8.17	6.23	36.14
Low-Skilled Occ.	10.76	8.68	47.62	8.77	6.70	37.89
High-Skilled Occ.	8.94	7.58	47.20	8.08	6.72	42.28

Notes: See Table 2.3.

upgrade to become high-skilled, which is an important margin for the employment growth of high-skilled occupations accounting for 15.79%.

Table 2.4 compares occupation-based job flow rates and those conventionally defined, which disregard occupational employment growth dynamics by basing on the net growth rate of the total employment size of establishments. Accounting for occupational job reallocation leads to larger magnitudes of job reallocation rates as well as a larger job to worker flow ratio. At around 19% (= 10.45% + 8.52%), my occupation-based average job reallocation rate is around 5% higher than is conventionally measured with non-disaggregated data. Furthermore, occupation-based job reallocation rates explain around 12% more of external worker flow rates, the sum of the rates of hires and separations (from other establishments or non-employment).

As an indication of the extent to which a given establishment in my data simultaneously creates and destroys jobs, I find that about 24% of reallocated occupational jobs do not contribute to observed net employment change. In other words, *establishment-level excess job reallocation* is nearly a quarter. In Figure 2.1, it is shown that excess job reallocation is prevalent across the whole distribution of the employment growth.

2.3.2.3. *Implications.* Consistent with the existing empirical literature, the preceding evidence has shown that job flows are large in magnitude and matter for worker flows. However, calculating job flows at the level of the establishment as the total of occupation-specific job flows has led to significantly larger magnitudes. Since changes in the number of occupational jobs reflect the factors affecting their growth, they should influence the inflow and outflow of workers in the corresponding jobs. Indeed, occupation-based job flows explain more worker flows than under conventional estimates.

Consider now previous contributions such as Abowd, Corbel, and Kramarz (1999), Burgess, Lane, and Stevens (2000), and Davis, Faberman, and Haltiwanger (2006), who document that establishments with large as well as those with fairly stable percentage employment changes simultaneously hire and separate with workers. The authors argue that the large magnitudes

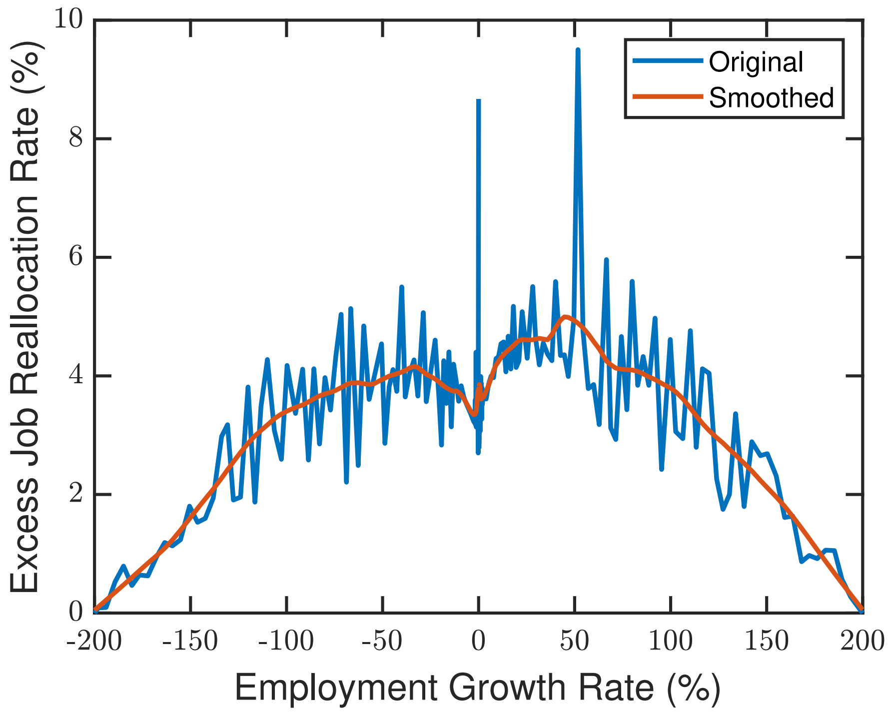


FIGURE 2.1. Excess Job Reallocation by Employment Growth

Notes: To generate this graph, the employment growth is discretized into bins whose lengths decrease as zero employment growth is approached. For each bin, computed are employment-weighted means of employment growth rates and the excess job reallocation rates. The figure shows these means. See Table 2.3 data source.

of worker flow rates observed at stable-growth establishments are a result of worker quits and replacement hires, which are due to establishment-worker match heterogeneity (see, in particular, Burgess, Lane, and Stevens (2000)). To the extent that jobs for workers of different skills, such as defined by occupations, are created and destroyed at different rates, worker quits and replacement hires cannot be the only mechanism explaining the large worker churning flows this previous body of literature identify.

I would like to quantitatively explain the determinants of the occupational labor market flows expanding on existing research. The search and matching theory with or without firm dynamics, see e.g., Pissarides (2000), Kaas and Kircher (2015), and Fujita and Nakajima (2016), generate labor market flows through labor productivity changes in response to which firms expand or contract their employment. In these models, labor is homogeneous and there is zero excess job flows. Cahuc, Marque, and Wasmer (2008) can theoretically generate differences in occupational job flow rates and hence excess job reallocation with occupation-specific labor turnover and imperfectly substitutable occupational labor. Thus, what determine which occupational jobs are created or destroyed include differences in the costs of occupational labor turnover, complementarity in occupational labor, and occupation-specific labor productivity. Underlying this interaction is the efficiency of the labor market at matching workers and occupational jobs and the efficiency of firms at distributing a given stock of matched workers across teams or occupations. Section 2.4 proposes an equilibrium model with firm dynamics, search frictions, and occupational labor quality differences that is rich enough to address labor misallocation and labor market flows.

2.4. Model

This section describes a frictional labor market within the Diamond-Mortensen-Pissarides (DMP) framework. To this framework, I introduce missallocation and occupational heterogeneity thus extending Kaas and Kircher (2015).

2.4.1. Environment. The model is set in an infinity time-horizon with discrete time periods denoted by t. There is an endogenous measure of firms and a unity measure of workers. Firms and workers are risk-neutral, and discount the future with the common factor $\beta \in (0,1)$. Workers' consumption equals to their incomes. Firms maximize discounted profit values and employ workers in low-skilled or high-skilled occupations. Firms may upgrade low-skilled workers to become high-skilled. There is no skill downgrade. The aggregate share of workers employed in low-skilled and high-skilled occupations is endogenous. Unemployed workers sorting into the occupations is exogenously given. Workers in a given occupation are identical. Denote occupations by $o \in \{l,h\}$ where l stands for low-skilled and h for high-skilled.

2.4.1.1. *Technology and Productivity.* Firms employ a continuum of workers of any type. The productivity of a firm is governed by discrete stochastic productivity states $z = (x, \alpha) \in \{(x_j, \alpha_k) : x_j, \alpha_k \in \mathbb{R}_+, j = 1, \ldots, n, k = 1, \ldots, m, n, m \in \mathbb{N}\}$. z is a draw from the finite set Z. The productivity level x is common to all workers in Hicks-neutrality sense. The productivity level α is the average quality or productivity of occupation-h labor relative to occupation-l labor.[11]

The productivity state z is idiosyncratic to the firm, i.e., it is independent and identically distributed across firms. z follows a first-order stationary Markov chain $(z_t : t \geq 0)$ with transition probability $\pi_{zz_+} = \text{Prob}(z_{t+1}|z_t)$. Denote by $\pi^0(z)$ the initial probability distribution of the productivity states.

The nature of the stochastic productivity of firms implies that the employment-size distribution of firms is going to be a consequence of the history of realized productivity states. Therefore, individual firms can be identified by the history of the stochastic shock processes. Let a firm's shock history be z^a where $a = 0, 1, \ldots$ is the age of the firm.

Let n_l and n_h be the number of occupation-l and high-skilled workers at a firm with productivity state z. The output of the firm is $y = xf(\alpha, n_l, n_h)$ with $f(\alpha, 0, 0) = 0$, where f exhibits decreasing returns to scale, increasing in all of its arguments, is concave in (n_l, n_h), and satisfies the Inada condition $\lim_{n_o \to \infty} \frac{\partial f}{\partial n_o} = 0$ for all i.

2.4.1.2. *Wages.* Occupation-o unemployed workers enjoy an income equal to b_o from home production. High-skilled workers are more productive when unemployed so that $b_h \geq b_l$. Employed workers supply one unit of labor in any given period and can earn occupation-specific compensation $w_o(z^a)$. Upgraded workers earn $w_p(z^a)$. Low-skilled workers incur $k_p \geq 0$ if upgraded (cf. Moen and Rosén (2004)).

2.4.1.3. *Entry and Exit.* There is a large pool of inactive homogeneous firms. Some inactive firms enter the economy every period if entry is sufficiently profitable. Entrants draw their productivity from the distribution $\pi^0(z)$. They employ workers in at least one of the occupations. To operate with occupation-o, entrants pay an entry cost equal to $k_o \geq 0$.

Firms exit the economy if operating in it is not profitable enough. All firms pay a fixed operating cost $k_f \geq 0$. Since, e.g., Hopenhayn (1992), such cost ensures endogenous exit.

[11] In the real world, relative occupational labor productivity α is ultimately determined by the distribution of worker ability in the economy. Then, the level of α for any given firm depends on the labor market allocation of ability. Thus, α in the model is interpreted as the *ex-post* average relative quality of labor firms realize.

The exit rate is $\delta \in [\underline{\delta}, 1]$ where $\underline{\delta} \in [0, 1)$. It is 1 in the exit event and $\underline{\delta}$ otherwise. Thus, firm discount factor is $\beta(1-\delta)$.

2.4.1.4. *Separations and Job Upgrading.* Firms may lay-off all or part of their workforce. They optimally choose occupation-specific separation rate $s_o \in [\underline{s}_o, 1]$. The lower bound $\underline{s}_o \geq 0$ is an exogenous common rate at which occupation-o workers flow into unemployment. Firms may also upgrade workers in low-skilled occupation to become high-skilled after paying some convex adaptation or conversion cost. To upgrade any low-skilled workers, the firm must also be posting occupation-h vacancies, which implicitly implies that a certain fraction of occupation-h vacancies are allocated for internal worker upgrading. Skills acquired through job upgrading are general in the sense that they are perfectly transferable across firms.[12] Upgraded workers become high-skilled forever and are separated with at the rate $s_h(z^a)$.

Let $s_p \geq 0$ be the rate at which low-skilled jobs are upgraded. The cost of upgrading $h_p = s_p(1-s_l)n_l$ low-skilled workers is $c_p(h_p)$. The conversion cost function is twice differentiable and strictly increasing and convex with $c_p(0) = 0 = c_p'(0)$. The superscript denotes first-order derivative.

2.4.1.5. *Search and Matching.* Firms with vacant employment opportunities search for unemployed workers. Unemployed workers search for vacancies. Employed workers do not search. Search in the labor market is directed. That is firms and workers meet in submarkets that differ in the matching rates and the values of posted contracts. Contracts specify the terms of association. Workers observe all posted contracts. Both firms and workers can visit only one submarket, conditional on the occupation. Low-skilled and high-skilled workers search in occupation-specific submarkets.[13]

The cost of $v_o \geq 0$ vacancies is $c_o(v_o)$ with $c_o(0) = 0 = c_o'(0)$ and $c_o'' \geq 0$. The double superscript denotes second-order derivative. Let q_o be the occupation-specific number of workers per vacancy. It is the "queue" of occupation-o workers searching for occupation-o vacancies. The matching rate, $m(q_o)$, determines the number of matches per posted vacancies in occupation-o submarkets. It satisfies $m(0) = 0$, $m' \geq 0$, and $m'' \leq 0$. The job finding probability is $m(q_o)/q_o$, which is decreasing in the occupation-specific queue length.

2.4.1.6. *Contracts.* Firms post employment contracts in the occupation-o submarkets in order to attract searching workers. The contracts can be formulated as

$$c_{h,a} = \{w_h(z^a), s_h(z^k), \delta(z^k)\}_{k>a}\,, \tag{2.1a}$$

$$c_{l,a} = \{w_l(z^a), s_l(z^k), \delta(z^k), s_p(z^k), w_p(z^k), s_h(z^k)\}_{k>a}\,, \tag{2.1b}$$

$$c_{p,a} = \{w_p(z^a), s_h(z^k), \delta(z^k)\}_{k>a}\,. \tag{2.1c}$$

Occupation-h contract specifies the wage for high-skilled workers and future retention probability $(1 - s_h(z^k))(1 - \delta(z^k))$. By contrast, the low-skilled contract accounts for the fact that these workers may be upgraded to the wage w_p and may be separated with at the rate s_h. Firms fully commit to the state-contingent contracts. The workers are paid the same cohort-dependent occupation-specific wages for the duration of the employment. Separation rates are the same for all workers of the same occupation.[14]

[12] See Subsection 2.4.2.5 for further discussion.

[13] The segmentation of the labor market into occupation-specific submarkets may be due to production technology whereby workers with different characteristics perform different tasks or if firms can advertise both the human-capital requirements of the job and the wages (cf: Moen and Rosén (2004)). This segmentation result is the subject of Inderst (2005), Inderst (2005), and Menzio and Shi (2010).

[14] The assumption of cohort-independent separation rates is assumed so as to simplify the theoretical analysis. It is also the computationally feasible case.

2.4.1.7. *Timing.* Events in the model occur in the following sequence. At the very beginning of a period, productivity states are revealed and new firms enter. Next, firms produce, post contracts, and decide on lay-offs, job upgrading, and exit. Lastly, unemployed workers are matched with vacant jobs, upgraded low-skilled workers are converted to high-skilled occupations, separated workers flow into unemployment, some firms exit, and compensations are made. The sequence repeats in the next period.

2.4.2. Surplus Values. In this section, I describe the dynamic equations for the values of unemployment and employment as well as the value of firms, all of which may be expressed as surplus values.

2.4.2.1. *Worker's Surplus Values.* Unemployed workers who observe all posted contracts seek to maximize the expected value from job search. They are rewarded with contracts that make them indifferent between all job search opportunities.

Let $\rho_o(c_{o,a}, q_o(z^a)$ denote the occupation-o expected surplus value for the worker from employment in contract $c_{o,a}$ when visiting a submarket with a queue length $q_o(z^a)$. Let U_o and $E_o(z, w_o)$ be the value of unemployment and employment in occupation o, respectively. The optimal contract and search behavior for occupation-o workers satisfy:

$$\rho_o(c_{o,a}, q_o(z^a)) = \beta(1-\delta)\frac{m(q_o(z^a))}{q_o(z^a)}\mathbb{E}_{z^a}\left[E_o(z^{a+}, c_{o,a+}) - U_o\right], \tag{2.2a}$$

$$\rho_o^*(c_{o,a}, q_o(z^a)) = \max_{(c_{o,a}, q_o(z^a)_{a\geq 0}} \rho_o(c_{o,a}, q_o(z^a)), \tag{2.2b}$$

In the above, $\mathbb{E}_{z^a}$ is the conditional expectation operator, conditioned on the productivity history z^a, and z_{a+} is next period's productivity history.

While (2.2a) is a relationship between contract $c_{o,a}$ and the queue length $q_o(z^a)$, (2.2b) is an indifference condition for the worker which each active submarket fulfills, in the sense that submarkets with a lower value have zero queue lengths and zero contract postings. According to (2.2a), higher-wage contracts have longer queues, and lower-job finding probabilities. The surplus value for unemployed workers is time-invariant in the stationary economy.

The dependency of $\rho_o^*(c_{o,a}, q_o(z^a))$ on firm characteristics can be suppressed since the maximum occupation-o expected surplus value is the equilibrium surplus value. Hence, let $\rho_o = \rho_o^*(c_{o,a}, q_o(z^a))$. Then, the value of occupation-o unemployment satisfies

$$U_o = b_o + \rho_o + \beta U_o. \tag{2.3}$$

The value of unemployment for occupation-o unemployment workers equals the flow value b_o in the current period plus an optional expected surplus value ρ_o associated with job search plus the present value of future unemployment. It is useful to define the flow value from unemployment:

$$(1-\beta)U_o = b_o + \rho_o. \tag{2.4}$$

The employment values take the following recursive forms

$$E_l(z, c_{l,\tau}) = w_l(z^\tau) - (1 - s_l(z^a))s_p(z^a)k_p$$
$$+ \beta\delta(z^a)U_l + \beta(1 - \delta(z^a))s_l(z^a)U_l$$
$$+ \beta(1 - \delta(z^a))(1 - s_l(z^a))s_p(z^a)\mathbb{E}_{z^a}E_h(z^{a+}, c_{p,\tau+})$$
$$+ \beta(1 - \delta(z^a))(1 - s_l(z^a))(1 - s_p(z^a))\mathbb{E}_{z^a}E_l(z^{a+}, c_{l,\tau+}) \,, \tag{2.5a}$$
$$E_h(z^a, c_{h,\tau}) = w_h(z^\tau) + \beta\delta(z^a)U_h$$
$$+ \beta(1 - \delta(z^a))\mathbb{E}_{z^a}\big[s_h(z^a)U_h + (1 - s_h(z^a))E_h(z^{a+}, c_{h,\tau+})\big] \,, \tag{2.5b}$$
$$E_h(z^a, c_{p,\tau}) = w_p(z^\tau) + \beta\delta(z^a)U_h$$
$$+ \beta(1 - \delta(z^a))\mathbb{E}_{z^a}\big[s_h(z^a)U_h + (1 - s_h(z^a))E_h(z^{a+}, c_{p,\tau+})\big] \,. \tag{2.5c}$$

for cohort $\tau \leq a$ of workers with $w_o(z^{\tau+}) = w_o(z^\tau)\,, o \in \{l, h\}$ and $w_p(z^{\tau+}) = w_p(z^\tau)$ because the occupational wages are flat but are cohort specific. The value of employment in the current period equals the occupation-specific wage. Upgraded workers additionally pay the upgrading cost k_p. The value of employment next period depends on presence or absence of worker or firm exit. For high-skilled workers, next period's value equals the unemployment value if either the worker is laid off or the firm exits the economy. The same wage is earned in the subsequent period if retained. Analogously for lows-skilled employed workers. Different from high-skilled workers is that low-skilled workers may be upgraded to the value E_h with associated wage denoted w_p.

2.4.2.2. *Optimal Job Upgrading Contract.* For externally hired workers, the set of equations (2.2) characterizes the incentive compatible contracts. For upgraded workers, the optimal contract must ensure that upgraded workers have no incentive to quit the next period in order to search for high-skilled jobs at a potentially higher wage. This condition is

$$\rho_h \leq \beta(1 - \delta(z^a))(1 - s_l(z^a))s_p(z^a)\mathbb{E}_{z^a}\big[E_h(z^{a+}, c_{p,a+}) - U_h\big]. \tag{2.6}$$

It says that the expected surplus value from employment as an upgraded worker is at least greater than the occupation-h optional value of search. Condition (2.6) must hold with equality since the firm maximizes its profit when it pays as little wage compensation as is optimally possible.

An optimal upgrading contract must also be at least better than the worker's outside option, which is the unemployment value as low skilled:

$$\beta(1 - \delta(z^a))(1 - s_l(z^a))s_p(z^a)\mathbb{E}_{z^a}\big[E_h(z^{a+}, c_{p,a+}) - U_l\big] \geq 0. \tag{2.7}$$

Plugging (2.6) at equality into (2.7) yields the combined condition

$$\rho_h + \beta(1 - \delta(z^a))(1 - s_l(z^a))s_p(z^a)(U_h - U_l) \geq 0\,. \tag{2.8}$$

2.4.2.3. *Joint Surplus Value.* In addition to the surplus maximization problem (2.2) for unemployed workers, a second key result in a directed-search framework is that firms and workers optimize their joint surplus. I now want to show that this result holds in an environment with occupational heterogeneity.

Let n_p be the stock of upgraded workers at an arbitrary firm in history z^a. The stock of high-skilled workers is the sum of upgraded workers, n_p, and external hires, $\tilde{n}_h$, i.e., $n_h = \tilde{n}_h + n_p$. Let $(\tilde{n}_{l,\tau}, \tilde{n}_{h,\tau})$ be retained cohort-τ externally hired workers and $n_{p,\tau}$ be cohort-τ retained upgraded workers. Let $F(z, \ell, c)$ be the present value of a firm of age a whose exogenous state is the productivity state z, and its endogenous state comprises of the stock of retained workers, $\ell = (n_l, \tilde{n}_h, n_p)$, as well as the set of all contracts for the different cohorts,

$c_a = \{(c_{l,\tau}, c_{h,\tau}, c_{p,\tau}) : \tau < a\}$, of workers employed by the firm. Let G be the joint surplus value of the firm and its workers. The following proposition summarizes the relationship between F and G:

PROPOSITION 2.1. The surplus value between the firm and its workers satisfies

$$G(z^a, \ell) = F(z^a, \ell, c_a) + \sum_{\substack{\tau<a \\ o\in\{l,h\}}} \tilde{n}_{o,\tau}[E_o(z^a, c_{o,\tau}) - U_o] + \sum_{\tau<a} n_{p,\tau}[E_h(z^a, c_{p,\tau}) - U_h], \tag{2.9}$$

where $G(z^a, \ell)$ is defined recursively as

$$G(z^a, \ell) = \max_{\substack{v_l, v_h, q_l, q_h, \\ s_l, s_h, s_p, \delta}} xf(\alpha, n_l, n_h) - k_f - \sum_{o\in\{l,h\}} \big[c_o(v_o) + b_o n_o + \rho_o(n_o + q_o v_o)\big] - c_p(h_p)$$

$$- k_p h_p + \beta(1-\delta)(U_h - U_l)h_p + \beta(1-\delta)\mathbb{E}_{z^a} G(z^{a+}, \ell_+) \tag{2.10a}$$

$$\text{s.t.} \quad \delta \in [\underline{\delta}, 1],\ q_o,\ v_o \geq 0,\ s_o \in [\underline{s}_o, 1],\ s_p \in [0,1]\,,\ \text{and}\,, \tag{2.10b}$$

$$0 \leq \rho_h n_l + \beta(1-\delta)(U_h - U_l)h_p\,, \tag{2.10c}$$

$$n_h = \tilde{n}_h + n_p\,, \tag{2.10d}$$

$$h_p = s_p(1 - s_l)n_l\,, \tag{2.10e}$$

$$\ell = (n_l,\, \tilde{n}_h, n_p)\,,\ \ell_+ = (n_{l,+}, \tilde{n}_{h,+}, n_{p,+})\,, \tag{2.10f}$$

$$n_{l,+} = (1 - s_l)n_l - h_p + m(q_l)v_l\,, \tag{2.10g}$$

$$\tilde{n}_{h,+} = (1 - s_h)\tilde{n}_h + m(q_h)v_h\,, \tag{2.10h}$$

$$n_{p,+} = (1 - s_h)n_p + h_p\,, \tag{2.10i}$$

$$s_p = 0,\ \text{if}\ v_h = 0\,, \tag{2.10j}$$

Proposition 2.1 implies that the firm optimizes its value if it optimizes the joint surplus with its workers, and vice versa. Its proof (see 2.A.1) relies on the facts that worker preferences are linear and firms are pre-committed to previous contracts so that the terms in the summation operators in (2.9) are all predetermined. This independence from the contracts implies that the job surplus value takes the recursive form (2.10), which is independent of the occupation-specific employment contracts. Absent contracts in (2.10) reduces the dimensionality of the firm's problem greatly.

The joint surplus value equation (2.10) comprises of the flow output, net of the operating costs, net of the costs of recruitment, net of the opportunity costs of employment, net of the utility costs from job search, net of the costs of job upgrading (see second line of equation (2.10a)). The costs of upgrading workers, comprises of the conversion costs and the flow cost the workers incur from a job upgrade as well as the cost the firm incurs in the design of an incentive-compatible job upgrading contract (see (2.10c)). Optimization of (2.10) is subject to the bound constraints (2.10b), the evolution of labor next period (2.10g), (2.10h) and (2.10i), and optimal upgrading contract constraint (2.10c).

2.4.2.4. *Optimal Policy.* By writing $n_h = \tilde{n}_h + n_p$, and setting $k_p = 0$ for simplicity, the endogenous state space can be collapsed to (n_l, n_h). Then, the first-order optimality conditions with respect to q_o, and v_o are, respectively,

$$0 = -\rho_o v_o + \beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{i,+}}m'(q_o)v_o\,, \tag{2.11a}$$

$$0 = -c_o'(v_o) - \rho_o q_o + \beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{i,+}}m(q_o)\,. \tag{2.11b}$$

Combining them yields a positive relationship between matching rates and vacancies:

$$c_o'(v_o) = \rho_o\Big[\frac{m(q_o)}{m'(q_o)} - q_o\Big]\,. \tag{2.12}$$

Due to convex vacancy filling costs and concave matching function, equation (2.12) says that firms that post more occupation-o vacancies attract more occupation-o workers per vacancy and have higher matching rates.

Let $\lambda \geq 0$ be the multiplier on constraint (2.10c). The first order optimality conditions with respect to s_h, s_l, and s_p are, respectively,

$$\beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{h,+}} \leq 0\,, \tag{2.13a}$$

$$\begin{aligned}\Big[c_p'(h_p) - \beta(1-\delta)(1+\lambda)(U_h-U_l) - \beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{h,+}}\Big]\frac{\partial h_p}{\partial s_l}&\\ +\beta(1-\delta)(1-s_p)n_l\frac{\partial G(z^{a+},\ell_+)}{\partial n_{l,+}} &\leq 0\,,\end{aligned} \tag{2.13b}$$

$$\begin{aligned}\Big[c_p'(h_p) - \beta(1-\delta)(1+\lambda)(U_h-U_l) - \beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{h,+}}\Big]\frac{\partial h_p}{\partial s_p}&\\ +\beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{l,+}}\frac{\partial h_p}{\partial s_p} &\leq 0\,,\end{aligned} \tag{2.13c}$$

where the inequality is binding for $s_h < 1$, $s_l < 1$, and $s_p < 1$, respectively.

The optimal occupation-h lay-offs satisfy condition (2.13a). This condition is different from the optimal q_h, determining the optimal rate of matching with high-skilled workers. According to condition (2.11a), optimal $q_h > 0$ satisfies

$$\beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{h,+}} > 0\,,$$

saying that to hire high-skilled workers, the expected continuation value must be sufficiently positive. Therefore, the firm does not both lay-off and hire high-skilled workers.

Consider conditions (2.13b) and (2.13c). Using $\frac{\partial h_p}{\partial s_l} = -s_p n_l$, $\frac{\partial h_p}{\partial s_p} = (1-s_l)n_l$ and combining conditions (2.13b) and (2.13c), the optimal occupation-l lay-off and upgrading rates satisfy

$$\beta(1-\delta)\mathbb{E}_{z^a}\frac{\partial G(z^{a+},\ell_+)}{\partial n_{l,+}} \leq 0\,.$$

Thus, similar to occupation-h optimal recruitment and lay-off policy, the firm does not both lay-off and hire low-skilled workers.

The preceding results are in fact an extension of those in Kaas and Kircher (2015) to an environment with occupational heterogeneity. That the firm hires when doing so is sufficiently profitable and it lays-off workers when it not longer is profitable to hold onto the existing workers implies that there exists an open set of inactive firms, i.e., firms with zero vacancy posting, zero matching rates, and zero lay-offs similar to Elsby and Michaels (2013) and Kaas and Kircher (2015).

Conditions (2.11a) and (2.13c) imply the following condition for optimal job upgrading and queue lengths:

$$c_p'(h_p) \leq \beta(1-\delta)(1+\lambda)(U_h - U_l) + \frac{\rho_h}{m'(q_h)} - \frac{\rho_l}{m'(q_l)}\,, \tag{2.14}$$

which is binding for $s_p < 1$. The optimal job upgrading rate trades-off occupation-l and occupation-h matching rates. Other things constant, job upgrading rate is increasing in the occupation-h queue length; it is decreasing in the occupation-l queue length. The firm has an additional instrument, namely a higher job upgrading rate when increasing its stock of high-skilled workers.

The optimal occupation-l lay-off, $s_l < 1$, and job upgrading rate s_p for a firm hiring occupation-h workers can be derived using (2.11a) and (2.13c) as

$$c_p'(h_p) \leq \beta(1-\delta)(1+\lambda)(U_h - U_l) + \frac{\rho_h}{m'(q_h)}\,, \tag{2.15}$$

with equality for $s_p < 1$. Keeping other things constant, q_h and s_p are positively related but s_l and q_h are negatively related. To expand employment of occupation-h the firm uses both margins of job upgrading and external hires and in doing so it prefers not to lay-off additional occupation-l workers. The binding case implies

$$c_p''(h_p)\frac{dh_p}{dq_h} = -\frac{\rho_h}{m'(q_h)^2}m''(q_h) \geq 0\,. \tag{2.16}$$

It follows that $\frac{ds_p}{ds_l} \geq 0$, given n_l. Therefore, optimal occupation-l lay-off and job upgrading are positively related. To separate with low-skilled workers, the firm prefers to upgrade some of them.

2.4.2.5. *Occupational Mobility and Occupation-Specific Skills.* In the model, there is no occupational mobility for the unemployed and skills are considered to be specific to occupations but occupational skills are general across firms. Below I rationalize these choices.

Kambourov and Manovskii (2009b) and Kambourov and Manovskii (2009a) document that returns to occupational tenure are substantial, which point to occupation-specificity of skills (or human capital more generally). On the contrary, occupational mobility on the job is large, e.g., Groes, Kircher, and Manovskii (2015) and Carrillo-Tudela, Hobijn, et al. (2016), which implies that occupational skills are partly general. The model, however, abstracts from on-the-job search, and therefore occupational mobility for employed workers is beyond its scope.

Carrillo-Tudela and Visschers (2014) find that occupational switching among the unemployed is sizable accounting for about half of all completed unemployment spells. Since the model is stationary, i.e., there are no aggregate shocks, and worker productivity changes are derived from firm productivity changes, it would be hard to rationalize occupational switching for the unemployed. In this sense, the model takes as given the sorting of the unemployeed workers into the occupations. Further, it would seem implausible to suppose that separated workers, those in upgraded jobs, return to their previous occupation-l unemployment state.

2.4.3. Equilibrium. This section discusses the properties of the stationary equilibrium for the model.

Let $\mu(z^a)$ be the count of existing firms of age $a > 0$. Let $\mu_o^0 \geq 0$ be the count of new firms in occupation-o labor markets. The number $\mu(z^a)$ of firms z^a evolves as

$$\mu(z^{a+1}) = (1 - \delta(z^a))\pi_{zz_+}\mu(z^a)\,, \tag{2.17}$$

with $\mu(z^0) = \sum_o \pi^0(z)\mu_o^0$.

New firms enter as long as the expected net value from entry is at most zero:

$$\sum_z \pi^0(z)[F_o(z, \ell_0, c_0) - k_o] \leq 0,\ o \in \{l, h\}\,, \tag{2.18}$$

where the subscript in $F_o(z, \ell_0, c_0)$ denotes new firms in occupation-o labor markets with state $(z, \ell_0, c_0) = (z, 0, 0, 0, \{\})$, and $q_o, v_o > 0$. It is the solution to the original $F(z, \ell_0, c_0)$ with positive vacancy positing in the occupation-o labor market.

The distribution of all workers satisfies an aggregate resources constraint stating that the total labor force is equal to unity. Accordingly, the distribution of occupation-o employed workers denoted $n_o(z^a)$ at firms z^a and unemployed workers $q_o(z^a)v_o(z^a)$ who are searching for occupation-o job at the same firms satisfies:

$$\sum_{\substack{a \geq 0, z^a,\\ o \in \{l,h\}}} \mu(z^a)[n_o(z^a) + q_o(z^a)v_o(z^a)] = 1\,. \tag{2.19}$$

Note that at any firm the net internal worker flows is zero.

Definition 2.2. A stationary competitive search equilibrium with labor quality heterogeneity, job upgrading, and missallocation is a list

$$\{\rho_o, \mu_o^0, \delta(z^a), s_o(z^a), s_p(z^a), q_o(z^a), v_o(z^a), w_o(z^a), w_p(z^a)\}$$

for $a \geq 0, o \in \{l, h\}$ such that (i) firms and workers optimize their joint surplus (2.9) according to (2.10a), (ii) job upgrading is optimal, satisfying (2.6), and (2.7), (iii) the aggregate resources constraint (2.19) is fulfilled, (iv) the free entry condition (2.18) holds for all $o \in \{l, h\}$, and (v) the number of firms z^a evolves according to (2.17).

Proposition 2.3. A stationary competitive search equilibrium exists. It is unique if firm entry is strictly positive, i.e., $\mu_o^0 > 0$ for all $o \in \{l, h\}$.

See Subsection 2.A.2 for proof. That strictly positive entry of firms yields a unique equilibrium is one variant of many similar conditions for uniqueness in the search literature such as the ones I have cited.

The stationary competitive equilibrium is solved for iteratively. The optimal search values (ρ_l, ρ_h) are the elements of the fixed point of the vector-valued root-finding problem (2.18) such that (2.19) is satisfied, and where the optimal recruitment policies, in particular $(q_o(z^a), v_o(z^a), s_p(z^a))_{a\geq 0}$, solve (2.10). For any vector of worker surplus values $(\tilde{\rho}_l, \tilde{\rho}_h)$, the number of new firms $\tilde{\mu}_0 \leq \tilde{\mu}_l^0 + \tilde{\mu}_h^0$ can be solved for as a residual of the resources constraint (2.19) where $\tilde{\mu}_o^0$ is count of new firms in occupation-o submarkets. New firms enter until condition (2.18) holds with equality, from which the entry costs $\tilde{k}_o$ are deduced. This is repeated until $\tilde{k}_o = k_o$. If $\mu_0 > 0$, a unique fixed point for (ρ_l, ρ_h) exists because of continuity of (2.18), boundedness of (ρ_l, ρ_h), and Brouwer's fixed-point theorem. The search values are bounded because, if ρ_o is too large, value from entry is too low and no new firms enter in occupation-o markets. When ρ_o is too small, no workers search. Uniqueness of (ρ_l, ρ_h) implies uniqueness of a search equilibrium.

2.5. Social Optimality

Equilibrium with within-firm job upgrading is in general not socially optimal, as a second-best allocation, which contrasts the efficient result of Moen and Rosén (2004). The key difference is that job upgrading here occurs within the firm, whereas in Moen and Rosén (2004) there exists a training firm and, separately, a poaching firm. The training firm trains untrained workers which may be poached by the poaching firm via on-the-job search. Under conditions of commitment on contracting as here, Moen and Rosén (2004) can show that market allocations and second-best planner's allocations are equivalent. Under their arrangement, different types of workers and firms search in different markets, and joint surplus optimization fully internalizes all externalities from search. On the other hand, where jobs can be upgraded internally within the firm, upgraded workers when unemployed search in the same markets as high-skilled workers. This search behavior cannot be dynamically contracted. Therefore, a surplus remains that must be shared between the firm and the worker for as long as the employment relationship continues. Without job upgrading and exogenously fixed shares of occupation-l and high-skilled workers, social optimality would be restored. See Inderst (2005) or Menzio and Shi (2010) how precisely that is done.

2.5.1. The Planning Problem. Consider a social planner who is constrained by search frictions in the labor market. The planner takes as given the mass $\mu(z^a)$ of existing firms of age $a > 0$ with idiosyncratic history z^a and solves

$$\max_{\substack{\mu_l^0,\mu_h^0,\delta,\\ v_l,v_h,q_l,q_h,\\ s_l,s_h,s_p}} \sum_{t=0}^{\infty} \beta^t \Big\{ -\sum_{o\in\{l,h\}} \mu_o^0 k_o + \sum_{a\geq 0,z^a} \mu(z^a) \Big[x f(\alpha, n_l(z^a), n_h(z^a)) - k_f$$

$$- \sum_{o\in\{l,h\}} b_o n_o(z^a) - \sum_{o\in\{l,h\}} c_o(v_o(z^a)) - c_p(h_p(z^a)) - k_p h_p(z^a) \Big] \Big\} \tag{2.20a}$$

s.t. (2.17), (2.19), and,

$$\delta \in [\underline{\delta}, 1],\ \mu_o^0 \geq 0,\ q_o,\ v_o \geq 0,\ s_o \in [\underline{s}_o, 1],\ s_p \in [0,1]\,, \tag{2.20b}$$

$$n_h(z^a) = \tilde{n}_h(z^a) + n_p(z^a)\,, \tag{2.20c}$$

$$h_p(z^a) = s_p(z^a)(1 - s_l(z^a))n_l(z^a)\,, \tag{2.20d}$$

$$n_l(z^{a+1}) = (1 - s_l(z^a))n_l(z^a) - h_p(z^a) + m(q_l(z^a))v_l(z^a)\,, \tag{2.20e}$$

$$\tilde{n}_h(z^{a+1}) = (1 - s_h(z^a))\tilde{n}_h(z^a) + m(q_h(z^a))v_h(z^a)\,, \tag{2.20f}$$

$$n_p(z^{a+1}) = (1 - s_h(z^a))n_p(z^a) + h_p(z^a)\,, \tag{2.20g}$$

$$s_p(z^a) = 0 \text{ if } v_h(z^a) = 0\,, \tag{2.20h}$$

The planner decides on entry of new firms, the exit probability of incumbent firms, the amount of external recruitment (through vacancy postings and queue lengths), job separation rates, and the rate of converting low-skilled workers in order to maximize aggregate output net of the operating cost, net of the opportunity cost of employment, net of the vacancy creation costs, net of the job upgrading costs (incurred by both the firm and the worker), and net of the entry costs, subject to the evolution of firms (2.17), the aggregate resources constraint (2.19), and the evolution of occupation-specific labor of firms in any history z^a. I now show that the social planner wants to maximize the social value of individual firms z^a.

2.5.2. The Social Value of the Firm. The sequential planning problem (2.20) includes the contribution due to new firms of age $a = 0$ and a contribution due to incumbent firms of age $a > 0$. These contributions are completely separable in t, a, and, z^a. Due to (2.17), the effective discount factor is therefore $\beta(1-\delta(z^a))$. For given multiplier $\beta^t\rho \geq 0$ on the aggregate resources constraint (2.19), the Lagrangian function associated with the sequential planning problem optimizes the contribution of the individual firms indexed by their states $(z^a, n_l(z^a), \tilde{n}_h(z^a), n_p(z^a))$ for all $a \geq 0$. By the principal of optimality, the Lagrangian for the sequential planning problem of any such firm admits recursive functional equation representation.[15] Further, the resulting recursive functional equation representation implies that the social planner maximizes the social values of each firm defined as

$$\begin{aligned} G_a(z^a, \ell(z^a)) = \max_{\substack{v_l, v_h, q_l, q_h,\\ s_l, s_h, s_p, \delta}} & xf(\alpha, n_l(z^a), n_h(z^a)) - k_f - \sum_{o\in\{l,h\}} \left[c_o(v_o(z^a)) + b_o n_o(z^a)\right] \\ & - \rho \sum_{o\in\{l,h\}} \left[n_o(z^a) + q_o(z^a)v_o(z^a)\right] - c_p(h_p(z^a)) - k_p h_p(z^a) \\ & + \beta(1-\delta(z^a))\mathbb{E}_z G_{a+1}(z^{a+1}, \ell(z^{a+1}))\,, \end{aligned} \tag{2.21}$$

subject to $\ell(z^a) = (n_l(z^a), \tilde{n}_h(z^a), n_p(z^a))$, and, (2.20b) – (2.20h) for all $a \geq 0$. Summing (2.21) across a and z^a, yields the Bellman equation for the sequential planning problem (2.20). The sequential planning problem admits a functional equation representation as the total of the contribution of all firms in the economy and where each problem can be solved separately. It depends on a only through z^a.

The first-order condition of the planning problem with respect to $\mu_o^0 \geq 0$, for $o \in \{l, h\}$, yields

$$\sum_z \pi^0(z)[G_{0,o}(0,0,z) - k_o] \leq 0, \qquad (= 0 \text{ if } \mu_o^0 > 0)\,, \tag{2.22}$$

where $G_{0,o}(z, 0, 0, 0)$ denotes the social value of new firms that are active in occupation-o search markets. It is evaluated as $G_0(z, 0, 0, 0)$ where $q_o, v_o > 0$, and coincides with the decentralized optimal firm entry under the preceding conditions.

2.5.3. Efficiency. There are two notable differences between the social value of the firm, problem (2.21), and the joint surplus value of the firm and its workers, problem (2.10). First, the multiplier ρ and the optional values from search (ρ_o, ρ_h) and the term $\beta(1-\delta)(U_h - U_l)h_p$ in (2.10). Second is the constraint (2.10c) in (2.10). The last two are absent in (2.21) because the social planner has no information on the terms of the optimal contract for upgraded workers. Because the distribution of worker types is endogenous, only one aggregate resources constraint (2.19) exists. Were the aggregate stock of occupation-specific worker types exogenously fixed, there would arise two aggregate resources constraints, one for occupation-l labor, another one for occupation-h labor. Consequently two multipliers, denoted ρ_l^* and ρ_h^*, would be needed. Efficiency would be restored with $\rho_l^* = \rho_l$ and $\rho_h^* = \rho_h$ similar to other papers in the directed-search literature (see, e.g., Inderst (2005), Delacroix and Shi (2006), or Menzio and Shi (2010)). Forcing $\rho_l = \rho = \rho_h$ with job upgrading would not restore optimality unless $U_h \geq U_l$. Thus, within firm job upgrading renders the equilibrium inneficient.

Finally, Shi (2016) has a model with on-the-job training where social optimality achieves. A job in Shi (2016) model is a form of capital and training is an investment through which the

[15] See Kaas and Kircher (2015) for a derivation.

job is improved continuously. While in my model, firms invest resources in order to convert low-skilled jobs to high-skilled jobs, the notion of skill types is in Shi (2016) absent.

2.6. Occupational Heterogeneity and Missallocation

Occupation-l and occupation-h labor may combine in production through, e.g., the CES function:

$$xf(\alpha, n_l, n_h) = x[\alpha n_h^{\frac{\psi-1}{\psi}} + (1-\alpha)n_l^{\frac{\psi-1}{\psi}}]^{\frac{\psi}{\psi-1}\nu}, \tag{2.23}$$

where $\psi > 0$ is the elasticity of substitution between the labor inputs, and $\nu \in (0,1)$ is the degree of returns to scale in production.[16]

Let $n = n_l + n_h$ be total employment, $\tilde{n} = \alpha n_h + (1-\alpha)n_l$ be total labor quality, and n^* be quality adjusted labor input such that $y = xn^{*\nu}$. Each one of these is calculated at the level of the firm.

Following Bagger, Christensen, and Mortensen (2014) and borrowing their terminologies, I can decompose labor productivity into three components: *labor quality heterogeneity*, *occupational labor misallocation*, and *total factor misallocation.* That is labor productivity is

$$\underbrace{\frac{y}{n}}_{\substack{\text{labor}\\ \text{productivity}}} \propto \underbrace{\frac{\tilde{n}}{n}}_{\substack{\text{labor}\\ \text{quality heterogeneity}}} \cdot \underbrace{\frac{n^*}{\tilde{n}}}_{\substack{\text{occupational}\\ \text{labor misallocation}}} \cdot \underbrace{\nu\frac{y}{n^*}}_{\substack{\text{aggregate}\\ \text{factor misallocation}}}, \tag{2.25}$$

where $\propto$ is the proportionality symbol.

The first component on the right-hand side of (2.25) captures differences across firms in the average quality of labor. The second component captures any managerial inefficiencies at distributing a given stock of talent across occupations or teams they manage, which leads to a suboptimal level of quality-adjusted labor. The last component captures differences in the marginal productivity of quality-adjusted labor and may be due to capital, labor, or other factors of production being allocated in a manner that total factor productivity x can not justify. For example, if frictions such as information asymmetry in the labor market lead to sub-optimal worker-firm matches. Aggregate factor misallocation depends on total factor productivity x through y.

Log labor productivity is accounted for by labor quality heterogeneity and missallocation:

$$\underbrace{\log(\frac{y}{n})}_{\text{Log RLP}} \propto \underbrace{\log(\frac{\tilde{n}}{n})}_{\text{Log Labor Quality}} + \underbrace{\log(\frac{n^*}{\tilde{n}})}_{\text{Log Labor Input}} + \underbrace{\log(\nu\frac{y}{n^*})}_{\text{Log MLP}}, \tag{2.26}$$

where *RLP* is labor productivity, i.e., output per worker; *Labor Quality* is α-weighted average labor; *Labor Input* is an aggregator of the occupation-specific labor, which is the effective production labor input; and *MLP* is marginal labor productivity with respect to Labor Input.

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[16] As $\psi \to 1$, (2.23) reduces to the Cobb-Douglas form:

$$xf(\alpha, n_l, n_h) = x[n_h^{\alpha} n_l^{(1-\alpha)}]^{\nu}. \tag{2.24}$$

2.7. Quantitative Analysis

In this section, I evaluate the model quantitatively using German firm data by method of moments with equal weights. Part of the data moments are based on the BHP establishment data as discussed in the text. Part of it is borrowed from existing works, where special attention is paid at matching firm distribution of revenue labor productivity, employment, and, job and worker flows. Moments for revenue labor productivity as well as skill intensity come from actual firm data as studied by Bartelsman, Dobbelaere, and Peters (2015). Data on worker and job flows are based on the BHP data.[17]

The following definitions are introduced: *Skill intensity* is firm-level share of occupation-h employment. *Occupation-h employment share* is share of occupation-h employment at a level different from the firm. Where not specified, the variable or statistics refers to the total of occupation-l and occupation-h occupations.

The following quantile-based additional statistics further characterize the variability of firms (cf: Bartelsman, Dobbelaere, and Peters (2015)):

$$dispersion = \frac{q_{.75} - q_{.25}}{q_{.75} + q_{.25}}, \tag{2.27a}$$

$$skewness = \frac{(q_{.75} - q_{.50}) - (q_{.50} - q_{.25})}{q_{.75} - q_{.25}}, \tag{2.27b}$$

$$kurtosis = \frac{q_{.90} - q_{.10}}{q_{.75} - q_{.25}}. \tag{2.27c}$$

The dispersion measures the width of the distribution relative to its location. The skewness measures the symmetry of the distribution by comparing the difference between the upper quartile and the median and the median and the lower quartile relative to the width of the distribution. A skewness value equal to zero implies the distribution is symmetric. A negative value implies that the distribution has a longer left tail but its mass is concentrated on the right. The kurtosis is a measure of the heaviness of the tails. It is calculated as the difference between the $90th$ and $10th$ percentiles of the distribution relative to its width. A high kurtosis indicates that firm dispersion results from extreme but infrequent realizations. A low kurtosis implies frequent and moderate deviations.

2.7.1. Calibration.

2.7.2. Functional Forms and Productivity Shocks. The output function is given by the CES function (2.23). The parameters of the matching functions are governed by those of the aggregate matching function. In particular, I follow Kaas and Kircher (2015) in assuming a CES function given by

$$m(q_o) = (1 + kq_o^{-r})^{\frac{-1}{r}},$$

where I normalize k to unity.[18]

[17]It is implicitly assumed that establishment-level worker and job flows can be appropriately aggregated to the firm level, which means using establishment statistics is appropriate. Such aggregation ensures that the magnitude and dynamics of labor market flows are more accurately measured.

[18]It is straitforward to consider $k \neq 1$ or more flexible alternatives such as $m(q_o) = m_{i,0}(1 + k_o q_o^{-r})^{\frac{-1}{r}}$ with $m_{i,0} > 0$ while keeping the matching elasticity independent of the occupation. However, that introduces additional parameters to an already long list of parameters to calibrate endogenously.

The cost functions take the following familiar convex forms:

$$c_o(v_o) = \frac{c_{0,i}}{1+\gamma_o} v_o^{1+\gamma_o}, \quad c_{0,i} > 0, \gamma_o \geq 0\,,$$

$$c_p(s_p(1-s_l)n_l) = \frac{c_{0,p}}{1+\gamma_p}(s_p(1-s_l)n_l)^{1+\gamma_p}, \quad c_{0,p} > 0, \gamma_p \geq 0\,.$$

As surveyed by Bartelsman and Doms (2000) and Syverson (2011), the literature on productivity documents that firm productivity differences are jointly due to permanent and temporary differences. Motivated by this fact, let $\alpha = \alpha_0 \cdot (1+\tilde{\alpha})$ where α_0 and $\tilde{\alpha}$ are, respectively, permanent and transitory components with $\tilde{\alpha} \in [-1, 1]$.[19] Analogously, let $x = x_0 \cdot (1+\tilde{x})$.[20]

The permanent components of productivity (x_0, α_0) will be drawn once and for all by new firms from distributions with variance $\sigma^2_{x,0}$ and $\sigma^2_{\alpha,0}$, respectively. Thus type-x and type-α productivity may be correlated only through the transitory components.

The transitory $\tilde{x}$ and $\tilde{\alpha}$ follow the VAR(1) process:

$$\begin{bmatrix} \mathrm{erf}^{-1}(\tilde{x}_{t+1}) \\ \mathrm{erf}^{-1}(\tilde{\alpha}_{t+1}) \end{bmatrix} = \begin{bmatrix} \varphi_x & 0 \\ 0 & \varphi_\alpha \end{bmatrix} \begin{bmatrix} \mathrm{erf}^{-1}(\tilde{x}_t) \\ \mathrm{erf}^{-1}(\tilde{\alpha}_t) \end{bmatrix} + \begin{bmatrix} \upsilon_{x,t+1} \\ \upsilon_{\alpha,t+1} \end{bmatrix}, \tag{2.28}$$

with jointly normally distributed innovations. The function erf is the Gaussian error function whose range is $[-1, 1]$. The variance and covariance for the innovations are $\sigma^2_x, \sigma^2_\alpha$, and $\sigma_{\alpha x}$, respectively. In (2.28), the function erf is used to ensure that the productivity states are non-negative and to allow for good scaling of the stochastic processes.

In discretizing the VAR(1) process, the univariate normal distribution will provide the weights for selecting the points so that more likely points are selected. By contrast, Tauchen (1986) points are uniformly distribution. Similar technique is employed for the permanent common productivity x.

The values of the parameters are summarized in Table 2.5 and Table 2.6 and details of their selection described as follows.

2.7.3. Parameter Choices. The period in the model is set to a year. The discount factor $\beta = 0.96$ so that the annual interest rate is about 4%. The degree of returns to scale in production $\nu = 0.70$ corresponds to a labor-income share of about 70%. Considering that there is no consensus on an estimate for the aggregate elasticity of substitution ψ, I set $\psi = 1.6$.[21]

To calibrate r, I match an elasticity of the job finding rate with respect to the vacancy to unemployment ratio of about 0.346, an aggregate job finding probability of about 5% and a vacancy to unemployment ratio of about 9% all of which come from Kohlbrecher, Merkl, and Nordmeier (2016) for all Germany. These targets yield $r = 1.8056$.

The lay-off rate, the rate of employer-initiated worker separations, in the cross-section is 4.0% due to Bellmann, Gerner, and Upward (2017). In the BHP data, job-to-job separation

[19] Set up so that $\alpha = \alpha_0 \cdot (1 \pm |\tilde{\alpha}|)$. It follows that α is an epsilon above or below α_0. It is implemented as $\alpha = \max(0, \min(1, \alpha))$ in order to ensure bounds are satisfied.

[20] It would be relatively more standard to consider $x = x_0\tilde{x}$. However, for certain parameter choices, the realized x might take large and unrepresentative values, which might lead to poor scaling of the highly-nonlinear optimization problem, wasting valuable computational resources.

[21] Most US studies place the aggregate elasticity of substitution ψ to be greater than 1.0. The often-used value is 1.40 (cf. Katz and Murphy (1992)), which Spitz-Oener (2006) adopt for Germany. Acemoglu and Autor (2011) obtain an estimate ranging from 1.60 to 1.80 for the US. The paper by Dustmann, Ludsteck, and Schönberg (2009) suggests a much larger value than 1.80. However, they use wage date of full-time males of West Germany. Therefore, the choice of 1.60 is reasonable.

TABLE 2.5. Summary of Parameter Choices in the Benchmark Calibration

	Value	Description/Target
Exogenous:		
β	0.96	4% annual interest rate
ν	0.70	70% labor-income share
ψ	1.60	Elasticity of substitution between the labor types
k	1.00	Normalized matching function scale parameter
r	1.8056	Job finding rate elasticity wrt. to $1/q$
$\gamma_l = \gamma_h$	0.48	Elasticity of the average hiring cost wrt. to hires
γ_p	1.0	Common value, e.g., Shi (2016)
$\underline{\delta}$	0.00	Exogenous exit rate
k_f	0.00	Fixed operating cost
$\underline{s}_l$	5.33	Occupation-l worker quit rate (%)
$\underline{s}_h$	4.66	Occupation-h worker quit rate (%)
$\sigma_{x,0}$	0.663	Std. deviation of Log RLP
φ_x	0.064	Average growth rate of Log RLP
Endogenous:		
$\sigma_{\alpha,0}$	0.287201	Std. deviation of share of occupation-h employment
φ_α	-0.002695	Type-h employment share where growth is stable
σ_x	0.0289898	Std. deviation of total employment growth rate
σ_α	0.031085	Std. deviation of occupation-h employment growth rate
$\sigma_{\alpha x}$	-0.006113	$Cov(g, g_h)$
ρ_l	0.001348	Occupation-l establishment entry rate
ρ_h	0.001049	Occupation-h establishment entry rate
b_l	4.15212e-05	Skill premium
b_h	0.001598	Occupation-h separation rate
c_l	0.004636	Ratio between occupation-l and occupation-h matching rates
c_h	0.006165	Occupation-h hires rates, respectively
c_p	0.079368	Internal worker flow rate for high-skilled jobs
δ	0.053153	Median Log RLP of 0 (normalization)

Notes: Employment growths, separations and hires are in the cross-section measured as rates as defined in Section 2.3.
Source: Unless otherwise stated, moments for the empirical counterparts on productivity and skill intensity come from Bartelsman, Dobbelaere, and Peters (2015) whereas the rest are own calculations using data and methodology as described in Section 2.3.

TABLE 2.6. Permanent Components of Productivity (x_0, α_0)

x_0	0.3785	0.5922	0.7850	1.0000	1.2739	1.6886	2.6421
α_0	0.0359	0.1088	0.2082	0.3299	0.4714	0.6311	0.8076

rate is 9.73%. The lay-off to quit ratio is 43.38% with total separation rate given in Table 2.3. Due to data restrictions, I consider that the lay-off to quit ratio and the share of job-to-job separations to each be the same across occupations. Then, I obtain $\underline{s}_l = 5.33\%$ and $\underline{s}_h = 4.66\%$ also leveraging the results in Table 2.3.

The elasticity parameter $\gamma_l = 0.48$, which is the elasticity of recruitment costs with respect to the number of newly hired workers with an apprenticeship training according to

Muehlemann and Pfeifer (2016, Table A4).[22] I set $\gamma_h = \gamma_l$ so that differences between labor types in the hiring costs are in levels and not in elasticity. The elasticity $\gamma_p = 1.0$ as in Shi (2016), which is arbitrary.

The unemployment income value b_h is estimated to match the aggregate occupation-h lay-off rate of 8.24%. Here I subtract the share of job-to-job separation rate to 51.33%. This share is for all workers as a whole. Using Table 2.3 I back out the high-skilled lay-off rate. This choice is motivated by Elsby and Michaels (2013) and Schaal (2017) who target the employment to unemployment flow rate. As there is no on-the-job search in my model, all separations imply unemployment inflows.

I calibrate b_l to match a skill premium–log wage differential–of about 0.45 due to Dustmann, Ludsteck, and Schönberg (2009).[23] Absent transitory shocks, the relation between wage and the unemployment benefit was shown by Kaas and Kircher (2015) to be:

$$w = b + \rho + \frac{1 - \beta(1-s)(1-\delta)}{\beta(1-\delta)} \frac{q}{m(q)} \rho$$

where occupational heterogeneity was abstracted away. Observe that $m(q)/q$ is the job finding rate. This relation would be derived for occupation-o, $o \in \{l, h\}$, workers without internal job upgrading. In that case b_l is derived so that $\log(w_h) - \log(w_l) = 0.45 \rightarrow w_h = 1.568 w_l$. I calibrate b_l this way. Further, the monthly job finding rates for occupation-l and high-skilled workers are estimated in Hertweck and Sigrist (2015) to be 5.2% and 13.8%, respectively.[24] Then, b_l follows with internally calibrated ρ_l, ρ_h, b_h and δ.[25]

The scale parameter c_p targets an average share of occupation-h net internal worker flows of 0.21% reported in Table 2.3. The scale parameter c_h is calibrated to match occupation-h average hires rate of 8.88%. This is 49.1% of all occupation-h hires rate in Table 2.3 since 50.1% of all total hires in my data are job-to-job.

The relation $c_l = \frac{m(q_l)}{m(q_h)} c_h = 0.752 c_h$ is assumed in order to match the average occupation-l matching rate relative to occupation-h matching rate. I first observe that aggregate matching rate is an employment-weighted sum of occupation-specific rates, i.e., $m(q) = \omega_h \times m(q_h) + (1 - \omega_h) \times m(q_l)$. Then, $\frac{m(q_l)}{m(q_h)} = \omega_h \frac{m(q_l)}{q_l} \frac{1}{m(q)/q_l - \omega_l m(q_l)/q_l} = 0.561$. According to Hertweck and Sigrist (2015), the monthly job finding rate for high-skilled workers is 13.8% (or 100% annual). Then $1/q \approxeq 1/q_l$ at the annual frequency. Using the (monthly) aggregate and occupation-l job finding rates of 6.2% and 5.2% (cf: Hertweck and Sigrist (2015)) yields $c_l = 0.752 c_h$ given $\omega_h = 0.17$ taken from Table 2.2.

The parameters of the productivity processes are estimated to match data moments for log RLP, occupation-h employment shares, and total and occupation-h occupational employment growth. Both Bartelsman, Dobbelaere, and Peters (2015) and Bayer, Mecikovsky, and Meier

[22] Dustmann (2004) finds that between 64–67% percent of secondary school students go into apprenticeships. They should represent well-above half of all low-skilled workers defined by the occupations in the present study.

[23] The polarization literature, see review by Acemoglu and Autor (2011), documents a rising skill premium. The value 0.45 is here considered to be characteristic of the 2000s. The premium is based on a sample of males; it would be bigger for females. According to Dustmann (2004), prior to 2000 the premium is around 0.30 for males and 0.40 for females.

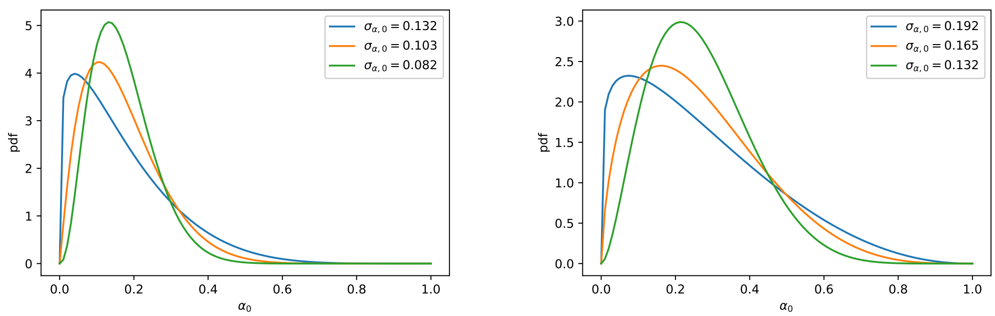
[24] The figures are multiplied by 12 to convert them to the annual frequency. They are capped at 100%.

[25] While this reduces the number of parameters to estimate, for some $(\rho_l, \rho_h, b_h, \delta)$, b_l might be negative, in which case a value close to zero is set.

TABLE 2.7. Empirical Log RLP vs Implied Log-Normally Distributed RLP

	dispersion	*skewness*	*kurtosis*
Empirical	0.094	-0.067	2.117
Implied	0.118	0.0	1.900
Scaled	0.094	0.0	1.900

Notes: See Table 2.5.



(A) $\mu_{\alpha_0} = 0.17$ (B) $\mu_{\alpha_0} = 0.27$

FIGURE 2.2. Example Distribution of α_0
Notes: The assumed parametric distribution of α_0 has its parameters set to deliver the shown expected values and variances.

(2018) find that permanent labor productivity contribute about 90% of the variance of labor productivity.[26] Therefore, $\sigma_{x,0}$ can be directly set to $0.90^{0.5}$ of the std. deviation of Log RLP.

First, I compare the empirical distribution of Log RLP and that which would arise were it log-normally distributed. I use as the expected value, the median Log RLP. Table 2.7 summarizes the results where it is shown that the empirical Log RLP is less dispersed, left skewed, and has heavier tails.

Second, I apply a scaling factor of 0.7935 to the std. deviation so that the dispersions become equal. Finally, $\sigma_{x,0} = 0.90^{0.5} \times 0.7935 \times 0.881 = 0.663$ where 0.881 is taken from Bartelsman, Dobbelaere, and Peters (2015).[27] It is attempted to match neither the skewness nor the kurtosis. However, these statistics could be captured by the transitory shocks.

To calibrate $\sigma_{\alpha,0}$, I first note that employment of occupation-h is zero at most establishments such that it is positively skewed with the median establishment employing zero high-skilled workers (see Table 2.1). Second, labor input $n^{*\nu}$ can be thought of as the expected output due to production by occupation-l and high-skilled workers where the probability weights are $(1-\alpha)$ and α, respectively. As a result, α can be interpreted as the parameter of a Bernoulli trial of occupation-h employment by firms. This parameter has an

[26] In Bartelsman, Dobbelaere, and Peters (2015), RLP is defined as real revenue per worker and high-skilled employment share is the share of employment of workers with at least a college or university degree.

[27] Bartelsman, Dobbelaere, and Peters (2015) study firms with at least 10 workers. Employment size in the simulation is normalized to some arbitrary maximum value. To hit the target of 10 workers, the empirical maximum number of workers employed by any firms is set to 50,000. Then 10 workers are normalized to a different range set in the model simulation.

Table 2.8. Covariances Between Employment Growth Rates

$Cov(g, g_l)$	$Cov(g, g_h)$	$Cov(g_l, g_h)$
0.230	0.151	0.037

Notes: Consider total, occupation-l , and occupation-h employment growth rates g , g_l , and g_h , respectively, and establishment j and period t. These covariances are calculated as approximations to, e.g., $\mathbb{E}\mathbb{E}_t\mathbb{E}_{j,t}(g_{jt} - \mathbb{E}_j g) \cdot (g_{jht} - \mathbb{E}_j g_h)$, where $\mathbb{E}_k$ is the conditional expectations operator conditioned on k. See Section 2.3 for data description and definitions.

empirical counterpart, which is the share of the count of establishments or firms that employ high-skilled workers.

Based on BHP data, the share of establishments with strictly positive occupation-h employment is 26.98%. Occupation-h employment share is 17%. Considering α_0 is defined over $[0, 1]$, I approximate it with the beta distribution, which is flexible enough to be left or right skewed or symmetric. Then, $\sigma_{\alpha,0}$ is estimated so that the corresponding beta distribution has an expected value of 17% and a std. deviation that matches the empirical std. deviation of skill intensity of 0.220 (cf: Bartelsman, Dobbelaere, and Peters (2015)).

As Figure 2.2 demonstrates, an expected value of 17% appears to characterize a positively skewed α_0 better than a mean of 26.98% in that the right tails are longer. Further, that firms should be larger in employment size than establishments, 17% might be more accurate since it is the employment-weighted version of the higher 26.98%.

From the BHP data, the standard deviation of total and type-h employment growth rates are 0.306 and 0.346, respectively. I estimate σ_x and σ_α to match them whereas $\sigma_{\alpha x}$ targets the average of within-establishment covariances between total and occupation-h employment growth rates.[28]

To calibrate the persistent parameter φ_x, I note that quality-adjusted labor productivity can be represented as

$$\log(\frac{y}{\tilde{n}}) = \log(x_o) + \log(1 + \tilde{x}) + \theta(;\tilde{n}) \, ,$$

where the function $\theta(;\tilde{n})$ takes $\tilde{n}$ as one of its inputs. Following the literature on misallocation, e.g., Hsieh and Klenow (2009), which argues that quality differences can be accounted for my considering quality-adjusted labor productivity, proxied by revenue per wage bill, the term $\theta(;\tilde{n})$ play a negligible role for the variance of the left-hand side. The first-order autocorrelation coefficient, denoted $\varrho(\cdot)$, can be derived as

$$\underbrace{\varrho(\log(\frac{y}{\tilde{n}}))}_{1-0.036=0.964} = \underbrace{\frac{\sigma^2_{\alpha,0}}{\sigma^2(\log(\frac{y}{\tilde{n}}))}}_{\approxeq 0.90} + \underbrace{\frac{\sigma^2(\log(1+\tilde{x}))}{\sigma^2(\log(\frac{y}{\tilde{n}}))}}_{=\varphi_x} + \frac{\sigma^2(\theta(;\tilde{n}))}{\sigma^2(\log(\frac{y}{\tilde{n}}))} \, ,$$

where $\sigma^2(.)$ is the variance of the variable in the parenthesis. The left-hand side targets the average growth of Log RLP of 0.036 due to Bartelsman, Dobbelaere, and Peters (2015). The fraction of the variance of labor productivity of 0.90 due to permanent differences is due to Bayer, Mecikovsky, and Meier (2018), and, more generally, Bartelsman, Dobbelaere, and Peters (2015) and Syverson (2011). Therefore, it follows that $\varphi_x \approxeq 0.964 - 0.90 = 0.064$.

Burgess, Lane, and Stevens (2000) among others find more worker churn at stable-growth establishments than at fast growing ones, which they attribute to a greater skill re-shuffling or

[28]Appealing to the law of large numbers, such covariance in the model is approximated by the simple between-firm covariance.

match reevaluation at such establishments. As a result, I estimate φ_α so that establishments with stable growth defined as $g \in [-5\%, 5\%]$ have a 0.195 share of type-h employment (cf: Table 2.2).

The dynamics of firm selection, and hence average firm size and productivity, are linked with the entry and fixed operating costs. The search values target an establishment entry rate of 4.8% for West Germany based on Fuchs and Weyh (2010).[29] ρ_l and ρ_h target a 90% and 10% share of the entry rate based on own calculations using the BHP data, respectively. Targeting ρ_o implies that k_o will be a residual of the free-entry condition (2.18). I set $k_f = 0 = \underline{\delta}$. I estimate δ to target a normalized 0.0 median of Log RLP.[30] Thus, firm exit events follow a Bernoulli process with parameter $\underline{\delta} + \delta$.

2.7.4. Quantitative Results. In this section, I discuss the results of the quantitative exercise. Where applicable, the model predicted moments are compared with respective data moments. Matching both the first-order moments and higher-order moments is challenging because of time constraints and computation complexity. Section 2.B in the Appendix discusses how the model is simulated and the challenges faced.

Consider Table 2.9, Table 2.10, Table 2.11, and Table 2.12. At the time of writing, simulation of the model was still ongoing. It is therefore not suprising that the results in the tables should be considered preliminary as they are far from the convergent solution.

To be able to make plausible predictions about the contributions of misallocation and occupational heterogeneity for the dynamics of firms and labor market flows, the quantitative results I report should match acceptably well the qualitative features of the cross-sectional distribution of labor productivity, and the labor market flows in terms of standard deviations, and percentile-based measures of dispersion, skewness and kurtosis. However, as the simulation moves between candidate points, it misses some of these targets, highlighting the high-dimensional nature of the objective function.

Across the various data moments targeted, the model produces first-order moments of the labor market flow rates as well as the skill intensity with the same signs. They are, however, lower or higher than their empirical counteparties. The mean growth rate of Log RLP, and the covariance between total employment growth and occupation-h employment growth have opposite signs.[31]

Standard deviation measures of firm dispersion are in the model lower than in the data. However, quantile based-dispersion and skewness of skill intensity are captured well. The model predicts well the skewness, and kurtosis of the skill-intensity but the dispersion of Log RLP has the oposite sign. The simulated dispersion of Log RLP might suffer from the fact

[29] In Fuchs and Weyh (2010), this is the share of job creation by new establishments. It equals employment-weighted establishment entry rate. Since firm entry rates are not available, I say empirical firm entry rate is around this value.

[30] The empirial median is 5.015 due to Bartelsman, Dobbelaere, and Peters (2015). Therefore, I work with a shifted empirical Log RLP distribution. This makes all target moments to be defined on a much more comparable set of values.

[31] In general, I have been able in some of my simulations to get the correct signs of the covariance between total employment growth and occupation-h employment growth (and its level close enough). I also can get lower mean growth rate of Log RLP, in absolute terms. However, the distributions of Log RLP into the components of misallocation and labor quality heterogeity have not so far been sensitive to both. I can also get large std. deviations of occupation-h employment growth without affecting the conclusions I make. The point is that, from the optimizing algorithm's point of view, the results I report achieve the best balance at this stage of the simulation.

that the median RLP in the model is normalized to zero. By contrast, the model gets nearly 90% of the empirical std. deviation of Log RLP.

The model suggests in Table 2.13 that misallocation is the main driver of the cross-section differences in log labor productivity. Together, misallocation accounts for 100.06%. Occupational labor misallocation by itself contributes about 34.25%. The contribution of aggregate misallocation, due to differences in quality-adjusted marginal labor productivity is 65.81%. Differences attributed to labor quality heterogeneity amount to -0.06%. The lower contribution of labor quality differences reflects the lower std. deviation of skill intensity the model predicts.

2.8. Conclusions

I have examined one aspect of the labor market, namely simultaneous job creation and destruction at the firm level. At the level of the firm, differences in the rates of occupational job and worker reallocation point to heterogeneity over and above the amount of heterogeneity that is due to net expansion or contraction of the firm. I have proposed that factor misallocation, in particular labor misallocation, as well as quality differences partly can explain the significant differences between the reallocation rates of occupational job and worker flows. A main task has been to deliver a model with labor market flows as well as labor productivity differences that is reach enough enough to capture occupational heterogeneity and misallocation.

I have considered two forms of misallocation. The first is driven by differences in the quality-adjusted marginal labor productivity, and the second is the occupational labor misallocation emanating from managerial inefficiencies in allocating a given stock of workers across occupations within the firm. I have preliminarily reported that aggregate misallocation, the first type of misallocation, far dominates, followed by occupational misallocation, and followed by differences in labor quality. It is interesting to note that differences in the average labor quality may contribute negatively to the the differences in the labor productivity.

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TABLE 2.9. Comparison of Model and Data Moments

	Model	Data
Targeted		
Std. deviation of Skill Intensity	0.0525	0.22
Occupation-h empl. share (stable firms (%))	37.49	19.50
Std. dev of employment growth	0.1326	0.306
Std. dev of occupation-h employment growth	0.0636	0.346
$Cov(g, g_h)$	-0.0011	0.151
Occupation-l exit rate (%)	5.496	4.32
Occupation-h exit rate (%)	5.496	0.48
Occupation-h separation rate (%)	4.664	8.24
Occupation-h hires rate (%)	7.193	8.88
Occupation-h internal hires rate (%)	3.122	0.21
Median Log RLP	0.0502	0.0
Untargeted		
Hires rate	10.77	10.43
Separation rate	7.19	9.22
Occupation-l hires rate (%)	12.86	10.54
Occupation-l separation rate (%)	5.33	9.42
Log RLP growth (%)	-33.16	3.60
Occupation-l internal hires rate	-1.82	-0.05
Job creation rate (%)	5.86	8.60
Occupation-l job creation rate (%)	5.91	8.77
Occupation-h job creation rate (%)	5.80	8.08
Job destruction rate (%)	0.03	6.66
Occupation-l job destruction rate (%)	0.04	6.70
Occupation-h job destruction rate (%)	0.01	6.72
Employment growth rate (%)	4.01	1.95
Occupation-l employment growth rate (%)	5.86	2.07
Occupation-h employment growth rate (%)	5.79	1.36
Std. dev of occupation-l employment growth	0.053	0.323
Std. dev of Log RLP	0.781	0.881
Std. dev of Log MLP	0.600	
Std. dev of Log Labor Input	0.417	
Std. dev of Log Labor Quality	0.105	

Notes: Occupation-o internal hires rate is share of internal hires rate in occupation-o employment. Occupation-specific job creation and job destruction are defined conventionally (compare with Table 2.4). Total job creation and destruction, i.e., for both occupations, is occupation-based where the number of occupations is two–low-skilled and high-skilled occupations each as a whole.

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TABLE 2.10. Quantile-Based Firm Dispersion

	Model	Data
Log RLP	8.52	0.094
Skill Intensity	0.451	0.667
Log MLP	-0.965	
Log Labor Input	0.407	
Log Labor Quality	-0.085	

TABLE 2.11. Quantile-Based Firm Skewness

	Model	Data
Log RLP	0.025	-0.067
Skill Intensity	0.456	0.392
Log MLP	0.052	
Log Labor Input	0.387	
Log Labor Quality	-0.674	

TABLE 2.12. Quantile-Based Firm Kurtosis

	Model	Data
Log RLP	1.843	2.117
Skill Intensity	3.204	2.250
Log MLP	1.705	
Log Labor Input	2.365	
Log Labor Quality	3.858	

Notes: See Table 2.5.

TABLE 2.13. Distribution of Labor Productivity

	Level	% Share in Var(Log RLP)
Cov(Log RLP, Log MLP)	0.402	65.81
Cov(Log RLP, Log Labor Input)	0.209	34.25
Cov(Log RLP, Log Labor Quality)	-0.000	-0.06

Notes: See Table 2.5.

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2.A. Proofs

2.A.1. Proof of Proposition 2.1. The optimization problem of the firm in state (z, ℓ, c) is the following Bellman equation:

$$F(z^a, \ell, c_a) = \max_{\substack{v_l, v_h, q_l, q_h,\\ s_l, s_h, s_p, \delta}} xf(\alpha, n_l, n_h) - k_f - \sum_{\substack{\tau<a\\ o\in\{l,h\}}} \tilde{n}_{o,\tau} w_o(z^\tau) - \sum_{\tau<a} n_{p,\tau} w_p(z^\tau)$$

$$- \sum_{o\in\{l,h\}} c_o(v_o) - c_p(h_p) + \beta(1-\delta)\mathbb{E}_z F(z^{a+}, \ell_+, c_{a+}) \tag{2.29a}$$

$$\text{s.t.} \quad \delta \in [\underline{\delta}, 1],\ q_o,\ v_o \geq 0,\ s_o \in [\underline{s}_o, 1],\ o \in \{l, h\},\ s_p \in [0, 1]\,, \tag{2.29b}$$

$$\tilde{n}_o = \sum_{\tau<a} \tilde{n}_{o,\tau},\ n_p = \sum_{\tau<a} n_{p,\tau},\ \tilde{n}_{l,\tau} = n_{l,\tau}\,, \tag{2.29c}$$

$$n_h = \tilde{n}_h + n_p,\ h_p = s_p(1-s_l)n_l\,, \tag{2.29d}$$

$$\ell = (n_l,\, \tilde{n}_h, n_p)\,, \tag{2.29e}$$

$$\ell_+ = (n_{l,+}, \tilde{n}_{h,+}, n_{p,+})\,, \tag{2.29f}$$

$$c_+ = \{(c_{l,\tau}, c_{h,\tau}) : \tau \leq a\} \tag{2.29g}$$

$$n_{l,+} = (1-s_l)n_l - h_p + m(q_l)v_l\,, \tag{2.29h}$$

$$\tilde{n}_{h,+} = (1-s_h)\tilde{n}_h + m(q_h)v_h\,, \tag{2.29i}$$

$$n_{p,+} = (1-s_h)n_p + h_p\,, \tag{2.29j}$$

$$\text{(2.2) if } q_o > 0,\ o \in \{l, h\} \text{ and }\,, \tag{2.29k}$$

$$\text{(2.6), and, (2.7) if } s_p > 0 \tag{2.29l}$$

$$s_p \geq 0 \text{ if } v_h > 0\,, \tag{2.29m}$$

$$\text{(2.8)} \tag{2.29n}$$

Using (2.5) and (2.3), occupation-specific wages are

$$w_l(z^\tau) = b_l + \rho_l + (1 - s_l(z^a))s_p(z^a)k_p + [E_l(z, c_{l,\tau}) - U_l]$$

$$- \beta(1-\delta(z^a))(1-s_l(z^a))(1-s_p(z^a))\mathbb{E}_{z^a}[E_l(z^{a+}, c_{l,\tau+}) - U_l]$$

$$- \beta(1-\delta(z^a))(1-s_l(z^a))s_p(z^a)\mathbb{E}_{z^a}[E_h(z_+, w_p) - U_l], \tag{2.30a}$$

$$w_h(z^\tau) = b_h + \rho_h + [E_h(z^a, c_{h,\tau}) - U_h]$$

$$- \beta(1-\delta)(1-s_h)\mathbb{E}_{z^a}[E_h(z^{a+}, c_{h,\tau+})) - U_h] \tag{2.30b}$$

$$w_p(z^\tau) = b_h + \rho_h + [E_h(z^a, c_{p,\tau}) - U_h]$$

$$- \beta(1-\delta)(1-s_h)\mathbb{E}_{z^a}[E_h(z^{a+}, c_{p,\tau+}) - U_h]\,, \tag{2.30c}$$

$\tau \leq a$.

Add to and subtract $\beta(1-\delta)(1-s_l)s_p n_l U_h$ from (2.30a) to obtain:

$$
\begin{aligned}
w_l(z^\tau) = {} & b_l + \rho_l + (1-s_l(z^a))s_p(z^a)k_p + [E_l(z, c_{l,\tau}) - U_l] \\
& - \beta(1-\delta(z^a))(1-s_l(z^a))(1-s_p(z^a))\mathbb{E}_{z^a}[E_l(z^{a+}, c_{l,\tau+}) - U_l] \\
& - \beta(1-\delta(z^a))(1-s_l(z^a))s_p(z^a)(U_h - U_l) \\
& - \beta(1-\delta(z^a))(1-s_l(z^a))s_p(z^a)\mathbb{E}_{z^a}[E_h(z^{a+}, c_{p,\tau+}) - U_h]\,.
\end{aligned}
\tag{2.31}
$$

The right-most value in (2.31) is the surplus value for low-skilled workers upgraded in this period. Upon conversion to become high-skilled, their outside option becomes U_h.

Equation (2.2a) implies

$$
\rho_o q_o(z^a)v_o(z^a) = \beta(1-\delta(z^a))m(q_o(z^a))v_o(z^a)\mathbb{E}_{z^a}[E_o(z^{a+}, c_{o,a})) - U_o]\,,
\tag{2.32}
$$

where the right-hand side is the surplus value for occupation-o new external hires.

Substitute the final wage equations into (2.29). From the result, subtract the left-hand side term in (2.32), and add to it its right-hand side equivalent term. This yields

$$
\begin{aligned}
F(z^a,\ell,c) = {} & \max_{\substack{v_l,v_h,q_l,q_h,\\ s_l,s_h,s_p,\delta}} xf(\alpha, n_l, n_h) - k_f - \sum_{o\in\{l,h\}} c_o(v_o) - c_p(h_p) \\
& + \beta(1-\delta)\mathbb{E}_z F(z^{a+}, \ell_+, c_{a+}) \\
& - \sum_{o\in\{l,h\}} [b_o n_o + \rho_o(n_o + q_o v_o)] - k_p h_p + \beta(1-\delta)(U_h - U_l)h_p \\
& - \sum_{\substack{\tau<a\\ o\in\{l,h\}}} \tilde{n}_{o,\tau}[E_o(z^a, c_{o,\tau}) - U_o] - \sum_{\tau<a} n_{p,\tau}[E_h(z^a, c_{p,\tau}) - U_h] \\
& + \beta(1-\delta)\Big\{\sum_{\tau<a}(1-s_l)(1-s_p)n_{l,\tau}\mathbb{E}_{z^a}[E_l(z^{a+}, c_{l,\tau+}) - U_l] \\
& + m(q_l)v_l\mathbb{E}_{z^a}[E_l(z^{a+}, c_{l,a}) - U_l]\Big\} \\
& + \beta(1-\delta)\Big\{\sum_{\tau<a}(1-s_h)\tilde{n}_{h,\tau}\mathbb{E}_{z^a}[E_h(z^{a+}, c_{h,\tau+})) - U_h] \\
& + m(q_h)v_h\mathbb{E}_{z^a}[E_h(z^{a+}, c_{h,a})) - U_h]\Big\} \\
& + \beta(1-\delta)\Big\{\sum_{\tau<a}(1-s_h)n_{p,\tau}\mathbb{E}_{z^a}[E_h(z^{a+}, c_{p,\tau+}) - U_h] \\
& + (1-s_l)s_p n_l\mathbb{E}_{z^a}[E_h(z^{a+}, c_{p,a}) - U_h]\Big\}\,,
\end{aligned}
$$

In the above, the continuation surplus values for occupation-l and occupation-h workers (last three lines) include surpluses for all retained workers and for new hires and upgraded workers.

The equivalence to (2.9), implies that the relation

$$
F(z^a, \ell, c) = G(z^a, \ell) - \sum_{\substack{\tau<a\\ o\in\{l,h\}}} \tilde{n}_{o,\tau}[E_o(z^a, c_{o,\tau}) - U_o] - \sum_{\tau<a} n_{p,\tau}[E_h(z^a, c_{p,\tau}) - U_h]\,,
\tag{2.34}
$$

has all the surplus terms cancel each other for all periods. That the surplus values in the current period cancel out is quickest to see. Future or continuation surpluses also cancel out after time shifting. ∎

2.A.2. Proof of Proposition 2.3. • *Existence.* To prove existence, dynamic programming arguments can be applied for any arbitrary firm that solves the equivalent problem of firms (2.10) for given bounded search values (ρ_l, ρ_h). Write $G(z, n_l, \tilde{n}_h, n_p; \rho_l, \rho_h) \equiv G(z, n_l, \tilde{n}_h, n_p)$ to emphasize that G is taken for some arbitrary finite search values (ρ_l, ρ_h). Problem (2.10) can then be defined over the finite set $Z \times [0, n_l^\infty] \times [0, \tilde{n}_h^\infty] \times [0, n_p^\infty]$ where the superscript ∞ on the bound means bound takes some large finite value. To do this two properties of the model are needed. First, it is the Inada condition assumed on f, namely $\lim_{n_o \to \infty} \frac{\partial f}{\partial n_o} = 0$ for all $o \in \{l, h\}$. Second, it is that for too large values of labor the firm finds it optimal to lay-off some of its workers. Problem (2.10) is therefore a continuous bounded discounted dynamic programming problem. Since discounted dynamic programming problems satisfy Blackwell's sufficient conditions for a contraction, it will be the case that an operator T such that $G = TG$, where TG is equal to the right hand side of (2.10), operates on bounded continuous functions of $(z, n_l, \tilde{n}_h, n_p; \rho_l, \rho_h)$ to produce other bounded continuous functions of $(z, n_l, \tilde{n}_h, n_p; \rho_l, \rho_h)$. Being a contraction means that a unique fixed-point solution G^* exists which is reached after an infinity application of T starting with any bounded and continuous G, i.e., $G^* = \lim_{k \to \infty} T^{k+1} G$, $k = 0, 1, \ldots$. Since G is continuous on the closed set $Z \times [0, n_l^\infty] \times [0, \tilde{n}_h^\infty] \times [0, n_p^\infty]$ a maximum exists, given finite (ρ_l, ρ_h). By uniqueness of G^*, an optimal policy $(\delta, q_o, v_o, s_o, s_p), o \in \{l, h\}$ that attains a maximum in (2.10) exists.

So far solution for $(\delta, q_o, v_o, s_o, s_p), o \in \{l, h\}$ can be obtained, indeed without knowledge about the wages. To solve for the wages, first consider occupation-h wages according to (2.30b) rearranged as

$$E_h(z^a, c_{h,a}) - U_h = w_h(z^a) - b_h - \rho_h + \beta(1 - \delta(z^a))(1 - s_h(z^a))\mathbb{E}_{z^a}[E_h(z^{a+}, c_{h,a+})) - U_h], \tag{2.35}$$

which is a Bellman equation taking (2.2a) as a constraint.

I can formulate the other wage equations as solutions to the problem of solving for (w_l, w_p) such that

$$\begin{aligned} E_l(z, c_{l,a}) - U_l &= w_l(z^a) - b_l - \rho_l - (1 - s_l(z^a))s_p(z^a)k_p \\ &\quad + \beta(1 - \delta(z^a))(1 - s_l(z^a))s_p(z^a)(U_h - U_l) \\ &\quad + \beta(1 - \delta(z^a))(1 - s_l(z^a))(1 - s_p(z^a))\mathbb{E}_{z^a}[E_l(z^{a+}, c_{l,a+}) - U_l] \\ &\quad + \beta(1 - \delta(z^a))(1 - s_l(z^a))s_p(z^a)\mathbb{E}_{z^a}[E_h(z^{a+}, c_{p,a+}) - U_h], \\ E_h(z^a, c_{p,a}) - U_h &= w_p(z^a) - b_h - \rho_h + \beta(1 - \delta(z^a))(1 - s_h(z^a))[E_h(z^{a+}, c_{p,a+}) - U_h], \end{aligned}$$

subject to (2.2a). The unemployment values are pinned down by equation (2.4). All these are Bellman equations which can be solved for by fixed-point algorithms such as by value iteration. ■

• *Uniqueness.* An equilibrium must jointly satisfy the free entry condition (2.18), the resources constraint (2.19), as well as the joint surplus objective (2.10). In the case of strictly positive entry, condition (2.18) holds with equality and therefore, a nested fixed point problem arises comprising of (2.18), (2.19), and (2.10). It can be solved iteratively, starting with an arbitrary guess for (ρ_l, ρ_h) and an arbitrary guess for the distribution of new firms with count (μ_l^0, μ_h^0) in each of the occupation-i labor markets. Then, the optimal search values (ρ_l, ρ_h), and the optimal masses of new firms (μ_l^0, μ_h^0) solve this fixed point problem. By contrast, a fixed point problem is absent in case of $\mu_o^0 = 0$ for some $o \in \{l, h\}$. Therefore, strictly positive firm entry implies uniqueness of (ρ_l, ρ_h). Lastly, Subsection 2.A.2 showed existence of an equilibrium for an arbitrary bounded search values (ρ_l, ρ_h), and that an optimal stationary policy exists, for any arbitrary firm. Convexity of the vacancy creation costs as well as the job upgrading costs, imply that optimal recruitment, upgrading, and layoff policies are unique. Therefore, uniqueness of (ρ_l, ρ_h) also implies uniqueness of the stationary equilibrium. ■

2.B. Model Simulation

The model is simulated in a similar manner to Kaas and Kircher (2015). In particular, I follow a fixed number of entrants through their lifecycles. Then, the stationary distribution of firms comprises the initial distribution of new firms and those that survival to reach a certain age. The starting number of new firms is set to 1×10^5 and the maximum age is 60 years. Permanent productivity levels for new firms are distributed according to the probability vectors of the discretized permanent productivity points. Temporary productivity shocks are initially uniformly distributed.

The simulation problem can be described as that of nested fixed-point problem. The outer fixed-point optimizes over the endogenous set of parameters (see Table 2.5). The inner fixed-point problem is the Bellman equation describing the joint surplus maximization problem (2.10a). This problem is solved by value function iteration, where at each iteration optimal recruitment and separation policies are solved. There are five variables to solve for: $(q_l, q_h, s_l, s_h, s_p)$. As was discussed in the text, (v_l, v_h) are functions of one or more of (q_l, q_h) . The problem is solved by means of constrained directed search algorithms. Theoretical foundations of these algorithms are found in Torczon (1997) and Lewis and Torczon (1999). The particular algorithm used is an adaptation of Hooke and Jeeves (1961) based on the implementation of Mark G. Johnson (see http://www.netlib.org/opt/hooke.c). Precisely, since his implementation assumes the optimization problem is unconstrained, I first adjust to bounds where a trial point falls beyond the bounds. Second, where an adjusted-to-bounds trial point violates the implied analytical results, the point is rejected by the extreme barrier method by setting the objective function value to minus infinity.

The state space is discretized based on a trial grid size and maximum non-binding employment level. Throughout, there is a trade-off between computational speed and accuracy or uncertainty of the results. The reported quantitative results are based on the following number of grid points: $7 \times 7 \times 7 \times 7$ for the permanent and temporary productivity states, and 35×35 for occupation-l and occupation-h employment, and a maximum occupation-specific employment level of 100. The grid size is not equidistant. To obtain relatively more accurate results, the grid size is the narrower the lower the employment level. In certain experimental results, permanent productivity grid sizes were much lower, e.g., $3 \times 3 \times 3 \times 3$, while the grid size for employment was typically set to 40–70. And the maximum occupation-specific employment ever tested is 10,000, but was found to be too large, led to a poorly scaled optimization problem, and lower grid sizes yielded somewhat volatile results.

There are 11 endogenous parameters in the benchmark calibration. To obtain a good initial parameter guess, a random sample of points is generated. The one that yields the lowest moment-matching objective function value becomes the initial parameter guess. Afterwards, search for optimal parameters is resumed using the Hooke and Jeeves algorithm. Sampling is done by using the latin hypercube sampling. Random sampling around a trial point is set off where it is judged no improvement is expected to be made. The sampling improves the guessed parameters.

The model qualifies as a large scale problem. I simultaneously simulate it on my personal computer as well as on the the Scientific Compute Cluster of the University of Konstanz

(SCCKN) or the High Performance Computing (HPC) cluster (bwUniCluster 2.0) for the German Federal state of Baden-Wuerttemberg where the University of Konstanz is one of the shareholders. While my personal computer has a much faster clock rate, it has a much lower number of cores. In SCCKN, I run one to three jobs with 100 - 300 cores per job for several weeks unless interrupted, e.g., for technical reasons. By contrast, access to bwUniCluster 2.0 for sufficient number of cores is limited to a maximum of 3 days. The simulation easily takes more than a month to converge, depending on initial guess point. For these reasons, I utilize the bwUniCluster and the SCCKN for continually sampling around a trial point. With bwUniCluster, it is possible to run multiple teams of processes, where each team has 100 - 300 cores. This results in more than a few thousands of cores. However, in May 2020, the bwUniCluster was a victim of a severe cyber attack and access to it was not possible until mid August, 2020 with new rules of access. As of this writing, I do not have access to the bwUniCluster.

To deliver the results I report, the model is implemented using the efficient C++ programming language. I program for distributed-memory computation. Many computational routines have been implemented from scratch. In fact, initial usage of Matlab was disappointing, which necessitated switching to C++.

CHAPTER 3

Firm Dynamics with Frictional Product and Labor Markets

3.1. Introduction

Firm heterogeneity matters for the labor market and for business-cycle dynamics. For instance, firms which differ in size, age or productivity create and destroy jobs at different rates and they respond to aggregate shocks in different ways, see e.g. Davis, Faberman, and Haltiwanger (2006), Haltiwanger, Jarmin, and Miranda (2013) and Moscarini and Postel-Vinay (2012). This motivates a large literature on the role of firms in macroeconomics, much of which builds on the seminal contributions of Hopenhayn (1992) and Hopenhayn and Rogerson (1993), sometimes augmented by richer labor market features.[1] In such models of firm dynamics, firms are hit by idiosyncratic transitory shocks to their revenue productivity which induces them to create or destroy jobs as they grow or contract over time. These shocks may reflect the price or the quantity components of revenue and hence could be induced by supply-side or demand-side disturbances. This seems a reasonable theoretical shortcut, given that most datasets have no separate information on firm-level prices and output. Yet one might expect that supply and demand affect the dynamics of firms in different ways and hence have distinct implications both for the cross-section as well as for aggregate dynamics.

Recent empirical findings highlight a prominent role of firm-specific demand for firm growth. Using U.S. data on narrowly defined industries that permit a distinction between price and quantity, Foster, Haltiwanger, and Syverson (2008) examine the separate contributions of demand and productivity for firm performance, finding that demand variation is the dominant driver of firm growth and firm survival.[2] Hottman, Redding, and Weinstein (2016) use price and sales information from scanner data to infer the sources of firm heterogeneity on the basis of a structural model of monopolistic competition. They show that demand differences and demand variation (as reflected in time-varying "firm appeal" and "product appeal" parameters) are a more important source of cross-sectional variation of sales and firm growth than are markups or cost heterogeneity.[3]

This paper aims to understand the respective roles of demand and supply for firm dynamics and the labor market. To this end, we develop an equilibrium model with heterogeneous firms producing differentiated products and frictions in product and labor markets. We

[1] Search and matching frictions in the labor market have been introduced into the Hopenhayn-Rogerson model framework by, e.g., Acemoglu and Hawkins (2014), Cooper, Haltiwanger, and Willis (2007), Elsby and Michaels (2013), Fujita and Nakajima (2016), Kaas and Kircher (2015) and Schaal (2017).

[2] The quinquennial manufacturing census data they use does not permit them to study the dynamics of firms over time. While there are no significant productivity differences across firms of different ages, younger firms charge lower prices than incumbents which suggests that these firms attempt to build a customer base (relationship capital). This idea motivates Foster, Haltiwanger, and Syverson (2016) to build a structural econometric model of firm dynamics in which product demand stochastically adjusts slowly as firms actively expend resources to build a customer base.

[3] Argente, Lee, and Moreira (2018) use similar data and show that most products are rather short-lived while firm appeal (i.e. demand) and product scope are the dominant factors of firm growth.

calibrate the model to match features of price and output adjustments of German manufacturing firms during the period 1995–2014. In particular, exogenous, persistent processes of firm-specific demand and productivity are calibrated to match the within-firm dynamics of prices and physical labor productivity. The quantitative model serves three purposes. First, we demonstrate how product market frictions matter for replicating quantitatively reasonable employment dynamics. Second, we quantify the importance of supply and demand for firm dynamics. Third, we use the model to explore the effects of aggregate first- or second-moment shocks to either demand or productivity and relate these findings to the business-cycle features in our data.

In Section 3.2, we build an equilibrium model in which heterogeneous firms compete for workers in a frictional labor market and simultaneously compete for retail buyers in a frictional product market. Demand (customer tastes) and physical productivity are firm-specific state variables, and idiosyncratic shocks to either of these variables have distinct implications for the price, output and employment adjustments of a firm. Search frictions in labor and product markets imply that firms need to adjust their workforce as well as their customer base slowly over time. We show that due to product market frictions, firm-specific demand and productivity shocks take a differential impact on firms' labor market adjustments. In particular, the extent of matching frictions in the product market crucially matters for a firm's employment policy in response to productivity shocks which is generally weaker when firms are more demand constrained.[4]

In Section 3.3, we calibrate this model to account for the joint dynamics of output and prices at the nine-digit product level in an administrative panel of manufacturing firms in Germany, which also contains information on employment and working hours. We use the product-level information on quantity and sales values to construct a firm-level price index and to measure quantity labor productivity. Idiosyncratic demand and productivity shocks are calibrated to match the firm-level dynamics of prices and quantity productivity in our data. Various parameters about product and labor market search frictions are calibrated to match recruitment and sales expenditures and worker and customer turnover rates. Importantly, the elasticity of the product market matching function determines how tightly firms are demand constrained and hence how quickly they adjust employment in response to productivity shocks. Indeed, we show that our model *without product market frictions* would generate too volatile employment dynamics when firms face empirically plausible shocks to demand and productivity. It would also generate a too strong co-movement between employment and productivity.

We then examine the separate contributions of demand and productivity shocks for firm dynamics. They obviously play rather distinct roles for the output and price adjustments of firms. Indeed, our model generates the empirical negative co-movement between firm-level prices and output, an untargeted moment, which is ultimately driven by productivity shocks. Both demand and productivity shocks are quantitatively important for the employment adjustments of firms for which productivity plays a slightly more prominent role. In the absence of productivity shocks (demand shocks), job destruction at continuing firms would fall by 55 percent (42 percent). Over a third of unemployment is induced by either one of these two forces.

[4]Search frictions in the product market are one way to think about costly customer acquisition. Another approach would consider information dissemination and customer awareness as in Dinlersoz and Yorukoglu (2012) or Perla (2019).

Lastly, we examine the impact of different aggregate shocks on the economy. We compare declines in average productivity or demand, on the one hand, as well as increases in the uncertainty of productivity or demand, on the other hand. Shocks to the first moments of either productivity or demand do not generate quantitatively large responses of employment and they do not induce counter-cyclical dispersions of firm growth that we document in our data. The lack of labor market amplification is a result of our calibration strategy which sets the flow income value during unemployment to match a reasonable replacement rate of unemployment benefits, thereby generating a relatively large surplus value of a job. With a different small surplus calibration, the labor market response of first-moment shocks to productivity or demand would be larger, and both these shocks would also induce (small) counter-cyclical responses of price and output growth dispersion.

We further feed into our model shocks to the second moment of demand or productivity. An increase of demand uncertainty induces sizable declines in output and employment, together with a rise of price and output growth dispersion. Higher productivity uncertainty, on the other hand, cannot generate the co-movement of aggregate output and employment. Likewise, productivity uncertainty shocks do not generate recessionary responses when product markets are frictionless. Based on these findings, we conclude that higher demand uncertainty is a plausible feature of recessionary episodes.

Related to our work are several recent contributions that introduce product market search frictions into macroeconomic models. Generally, search in product markets is meant to capture the observation that firms spend substantial time and resources for sales and marketing activities in order to attract customers.[5] In the presence of these frictions, Bai, Rios-Rull, and Storesletten (2019) and Michaillat and Saez (2015) argue that aggregate demand shocks play a more prominent role than aggregate technology shocks. Kaplan and Menzio (2016), Petrosky-Nadeau and Wasmer (2015) and Den Haan (2013) combine frictions in product and labor markets, introducing new mechanisms for business-cycle dynamics. Albrecht, Postel-Vinay, and Vroman (2013), Paciello, Pozzi, and Trachter (2019) and Shi (2016) examine price variability and sales policies in equilibrium models of product market search in which the customer base is a state variable. Unlike our paper, none of these contributions addresses firm heterogeneity and the role of firm-specific demand.

Firms in our model employ multiple workers and accumulate a customer base with multiple customers. In these respects, our model closely relates to Gourio and Rudanko (2014), who study customer acquisition as a costly and time-consuming process, as well as Kaas and Kircher (2015), who describe the hiring process under convex labor adjustment costs. Both papers use competitive search as in Moen (1997) so that firms use lower product prices to attract more customers or higher wages to attract more workers. Our paper combines these ingredients to develop a unified framework in which demand and productivity variation matters differentially for the dynamics of firms.[6] Different from other competitive-search environments, equilibrium in our model is not socially efficient which is due to the presence of monopolistically competitive firms. Nonetheless, we show that optimal firm policies can be characterized in a fairly tractable way as solutions to a joint-surplus maximization problem.

Our finding that uncertainty shocks help to generate plausible aggregate dynamics is in accordance with a recent literature on the role of uncertainty in macroeconomics. For instance,

[5] In the U.S., marketing expenditures are as high as 7.7% of GDP (Arkolakis 2010).

[6] Relatedly, Roldan and Gilbukh (2020) introduce idiosyncratic productivity risk in a model similar to Gourio and Rudanko (2014) which they calibrate in order to match features of price and sales dispersion from scanner data, to study the response of markups to aggregate shocks, among others. Unlike us, they do not consider firm-specific demand shocks or labor market features.

Bloom (2009), Bloom et al. (2018) and Bachmann and Bayer (2014) argue that time-varying uncertainty improves the fit of macroeconomic models with heterogeneous firms. Schaal (2017) considers a heterogeneous-firm model with labor market search, showing that uncertainty shocks help to understand the volatility of aggregate unemployment. In these articles, uncertainty shocks are introduced as increases in the volatility of idiosyncratic (revenue) productivity. Our contribution is that we are able to distinguish between demand uncertainty and (physical) productivity uncertainty as separate influence factors for the overall uncertainty at the firm level. We show that demand uncertainty is a more relevant feature of the business cycle than productivity (or cost) uncertainty.[7]

Our work further relates to an empirical literature which investigates the dispersion and dynamics of firm-level prices and productivity. While Abbott (1991) and Foster, Haltiwanger, and Syverson (2008) document dispersion of producer prices in specific industries, Carlsson and Skans (2012) and Carlsson, Messina, and Skans (2020) use Swedish firm-level data for the manufacturing sector, finding that unit labor costs are transmitted less than one-to-one to output prices, and that much of the variation in output prices remains unexplained by productivity shocks. Furthermore, they find that employment responds negligibly to productivity shocks, while permanent demand shocks are the main driving force of employment adjustment. Pozzi and Schivardi (2016) use data for Italian manufacturing firms to identify demand and TFP shocks as separate factors driving firm growth on the basis of a frictionless model. They find that the firms' responses to these shocks are limited which suggests the importance of adjustment frictions.

3.2. The Model

In this section we build a canonical model that describes the dynamics of firms in frictional product and labor markets. The model includes a representative household, retailers and firms. The household buys differentiated goods in a competitive market from retailers and supplies labor in a frictional labor market to firms. Retailers buy goods in a frictional product market from firms. All retailers and firms are owned by the representative household.

In the product market, firms compete for retailers ("buyers") via costly sales activities and by offering discounts on their products, which helps to accumulate a customer base. In the labor market, firms build up a workforce by spending resources on recruitment and by offering long-term contracts to new hires. Firms adjust their customer base and their employment stock in response to idiosyncratic demand and productivity shocks. Search in both markets is competitive: workers and buyers direct their search towards particular wage or price offers, trading off higher matching rates against lower match values.

We describe a stationary equilibrium in which search values of buyers and workers are constant over time, while individual firms' employment and output grow or shrink, depending on their idiosyncratic productivity and demand states. We then establish a theorem which describes a tractable equilibrium characterization by a joint surplus maximization problem.

3.2.1. The Environment. Representative household. The infinitely-lived household consumes differentiated goods produced by firms as well as a separate numeraire good. Utility of the household is

$$\sum_t \beta^t \left[e_t + u(C_t)\right] \quad , \quad \text{where } \; C_t = \left(\int \left[y_t(f) c_t(f)\right]^\theta d\mu_t(f) \right)^{1/\theta} \quad ,$$

[7]Different from this paper, Basu and Bundick (2017) and Leduc and Liu (2016) introduce demand uncertainty as time-varying volatility of the household's discount factor in New-Keynesian DSGE models.

e_t is consumption of the numeraire good, u is a concave utility function and β is the household's discount factor. C_t is a consumption aggregator which integrates over the measure μ_t of active firms f whose output the household consumes in $c_t(f)$ units. $y_t(f)$ are idiosyncratic (firm-specific and time-varying) preference parameters which reflect, for instance, firm- or product-specific tastes or brand values.[8] Parameter $\theta \in (0, 1)$ defines the substitution elasticity $(1 - \theta)^{-1} > 1$ between different products. In a stationary equilibrium, $C_t = C$ is a constant. All prices, wages and costs defined below are expressed in units of the numeraire good.

Workers. There is a constant stock $\bar{L}$ of workers who are members of the household. A worker can be either employed at a firm or unemployed. An unemployed worker earns b units of the numeraire good. Only unemployed workers search for jobs.

Retailers. There is a (potentially unlimited) number of retailers which buy consumption goods from firms and sell them to households at competitive retail prices. Retailers can be either attached to the customer base of a firm or unattached. In the former case, the retailer can buy the output good of this firm up to a fixed (unit) quantity. Selling (possibly smaller) quantities of this good to the household is costless; hence the profit of an attached retailer is simply the difference between the retail price and the purchase price. Alternatively, retailers are unattached in which case they may search for purchases from firms. Search involves payment of $\kappa > 0$ units of the numeraire good; once matched to a firm, the retailer can buy up to one unit of the good produced by the firm per period until a separation occurs.[9]

Production firms. A firm with L workers produces xL units of its unique output good.[10] x is the firm's idiosyncratic productivity. Shocks to x stand for any type of supply-side events such as technology changes or (unmodeled) price changes of factor inputs besides labor. If the firm has B retail buyers, it sells B units of output in the current period, given the unit purchase constraint. Because the good is non-storable, the firm is naturally constrained by $B \leq xL$ in any given period. If that inequality is strict, the firm wastes some of its output.[11]

Retail market. After a firm sells B units of its output to retailers, the latter sell these goods to the household at competitive retail prices. This implies that consumption of the firm's output is $c = B$, while the competitive retail price equals the household's marginal rate of substitution between the firm's output good and the numeraire good which is $u'(C)C^{1-\theta}y^{\theta}B^{\theta-1}$ where y is the firm-specific demand state. To simplify notation, write $M \equiv u'(C)C^{1-\theta}$ (a constant in steady state) which implies that the retail price of a firm's product with demand state y and customer base of size B is equal to $p^r = My^{\theta}B^{\theta-1}$.

Shocks. Both idiosyncratic states x and y follow a Markov process on a finite state space. We write $z = (x, y) \in Z$ and denote $\pi(z_+|z)$ the transition probability from z to z_+. For a firm of age a, we write $z^a = (z_0, \ldots, z_a)$ for the shock history from the entry period (firm age zero) up to the current period (firm age a). $\pi^a(z^a)$ denotes the unconditional probability of that history event.

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[8] These are the counterparts of the *firm and product appeal* parameters of Hottman, Redding, and Weinstein (2016).

[9] With this simple linear technology, retail firms can also be interpreted as collections of multiple purchase connections to production firms where each connection allows the purchase of a fixed (one unit) quantity of the firm's output. Any expansion of purchase connections requires the payment of the fixed search cost.

[10] In an earlier version of this paper, we assume that firms produce perfect substitutes ($\theta = 1$) while revenue is concave due to decreasing returns in production. It is straightforward to combine both features so that firm size is constrained both by curvature in demand and in production.

[11] It is a straightforward extension to introduce inventories at the computational cost of adding another state variable to the firm's problem.

Recruitment and sales activities. For recruitment and sales, the firm spends $r(R)$ and $s(S)$ units of the numeraire good, respectively, where R and S measure recruitment and sales effort. Both cost functions are increasing and convex.

Labor market search. Search in the labor market is competitive. Recruiting firms offer long-term contracts to new hires. They are matched with unemployed workers in submarkets that differ by the offered contract values. In a given submarket, a firm hires $m(\lambda) \leq \lambda$ workers per unit of recruitment effort, where λ measures unemployed workers per unit of recruitment effort in the submarket, and m is a strictly increasing and concave function. Hence, $m(\lambda)/\lambda$ is the probability that an unemployed worker finds a job in this submarket, a decreasing function of λ. An employment contract specifies wage payments and separation probabilities contingent on realizations of firm-specific shocks. We write $\mathcal{C}^a = (w^a(z^k), \delta_w^a(z^{k+1}))_{k \geq a}$ for the employment contract of a worker who is hired by a firm of age a. $w^a(z^k)$ is the wage that the worker earns when the firm has age $k \geq a$, conditional on the shock history z^k and conditional on staying employed at this firm. $\delta_w^a(z^{k+1})$ is the probability to separate from the firm in event history z^{k+1} with $k+1 > a$.

Product market search. Search in the product market is also competitive, but here firms cannot commit to long-term contracts.[12] Instead, firms that aim to expand the customer base offer discount prices p^d to new buyers. In all subsequent periods, attached retailers continue purchasing at this firm, but anticipate that the firm charges the reservation price (=retail price) p^r which makes the retailer exactly indifferent between buying and not buying. Unattached retailers and selling firms are matched in submarkets which differ by the buyers' match values. Per unit of sales effort, the firm attracts $q(\varphi) \leq \varphi$ new buyers, where φ is the measure of unattached buyers per unit of sales effort in the submarket, and q is an increasing and concave function. An unattached buyer searching for purchases is successful with probability $q(\varphi)/\varphi$, which is a decreasing function of φ.

Entry, separations and exit. New firms can enter the economy at cost $K > 0$ with zero workforce and zero customer base. They draw an initial productivity and demand state $z_0 = (x_0, y_0)$ from probability distribution π^0. Any existing firm, depending on its supply and demand shocks, separates from workers according to the contractual commitments. Separated workers can search for jobs in the same period. The firm may also decide not to serve some of its attached retailers who then leave the firm's customer base. Workers quit the job into unemployment with exogenous probability $\bar{\delta}_w$, and retailers leave the customer base of a firm with exogenous probability $\bar{\delta}_b$. This implies that the actual customer churn rate is bounded below by $\delta_b \geq \bar{\delta}_b$. Likewise, the contractual state-contingent worker separation rates are bounded below by $\delta_w \geq \bar{\delta}_w$. At the end of the period, any firm exits with probability δ in which case all its workers enter the unemployment pool and all its buyers become unattached.

Timing. The timing within a period is as follows. First, firm-specific demand and productivity shocks are realized. Second, some workers and buyers separate from firms. Third, firms search for new hires and buyers. Fourth, production takes place, workers are paid and goods are sold. Fifth, firms exit with probability δ.

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[12]The assumption that firms offer long-term contracts to workers though not to retailers is intended to reflect realistic features of worker-firm and customer-firm relationships. Although long-term contracts with customers are common in some industries, they tend to be rather short. For German manufacturing firms, Stahl (2010) finds that although 50% of sales are undertaken in written contracts, the average contract duration is just nine months. With our annual calibration, the absence of price commitment seems a plausible assumption. In Section 3.A in the Appendix, we describe an alternative pricing assumption with long-term contracts and no price discrimination between first-time buyers and repeat buyers.

3.2.2. Competitive Search Equilibrium. We describe a stationary equilibrium in which search values of workers and buyers, as well as the distributions of workers and buyers across firm types, are constant over time. Any firm's policy only depends on the idiosyncratic shock history z^a where a is the firm's age. Hence we identify the different firm types with z^a.

3.2.2.1. *Workers.* Let U denote the value of an unemployed worker and let $W(\mathcal{C}^a, z^k)$ denote the value of an employed worker in contract $\mathcal{C}^a$ at firm z^k with $k \geq a$. These values represent the marginal contribution of the worker to the representative household's utility. Unemployed workers observe the set of contracts $\mathcal{C}^a$ at firm types z^a and the corresponding market tightness λ in the submarkets in which value-equivalent contracts are offered. An unemployed worker's Bellman equation is

$$U = \max_{W(\mathcal{C}^a, z^a), \lambda} \frac{m(\lambda)}{\lambda} W(\mathcal{C}^a, z^a) + \Big(1 - \frac{m(\lambda)}{\lambda}\Big)[b + \beta U] \ , \tag{3.1}$$

where maximization is over all submarkets $(W(\mathcal{C}^a, z^a), \lambda)$. With probability $m(\lambda)/\lambda$, the worker finds employment in which case the continuation value is $W(\mathcal{C}^a, z^a)$. Otherwise the worker earns unemployment income b and remains unemployed to the next period.

The employment value $W(\mathcal{C}^a, z^k)$ satisfies the Bellman equation

$$W(\mathcal{C}^a, z^k) = w^a(z^k) + \beta(1-\delta)\mathbb{E}_{z^k} W'(\mathcal{C}^a, z^{k+1}) + \beta\delta U \ . \tag{3.2}$$

This worker earns the contractual wage $w^a(z^k)$ in the current period. At the end of the period, the firm exits with probability δ in which case the worker becomes unemployed. Otherwise the worker stays employed to the next period which yields continuation value $W'(\mathcal{C}^a, z^{k+1})$ where the prime indicates the employment value before the firm separates from workers, i.e.

$$W'(\mathcal{C}^a, z^{k+1}) = [1 - \delta^a_w(z^{k+1})] W(\mathcal{C}^a, z^{k+1}) + \delta^a_w(z^{k+1}) U \ . \tag{3.3}$$

With contractual separation probability $\delta^a_w(z^{k+1})$, the worker leaves the firm and can search for employment in the same period (continuation utility U). Otherwise the worker stays employed with continuation utility $W(\mathcal{C}^a, z^{k+1})$.

It is convenient to define the option value of search in submarket (W, λ) by

$$\bar{\rho}(W, \lambda) \equiv \frac{m(\lambda)}{\lambda}\Big(W - b - \beta U\Big) \ .$$

Then, the flow utility value of unemployment satisfies

$$(1-\beta)U = b + \rho \ , \tag{3.4}$$

where $\rho \equiv \max \bar{\rho}(W(\mathcal{C}^a, z^a), \lambda)$ is the maximal search value over all submarkets. It follows that any contract that attracts unemployed workers (i.e., $\lambda > 0$) yields the same search value ρ.

3.2.2.2. *Buyers.* Unattached retailers can search for purchases at cost $\kappa > 0$. They observe discount prices p^d offered by firms of different types (y, B), anticipating that this firm's output can be sold at retail price $p^r = My^\theta B^{\theta-1}$. A newly matched retailer makes profit $p^r - p^d$ in the first period and anticipates that the firm charges reservation (retail) prices in all subsequent periods. Therefore, the buyer's continuation value beyond the matching period is zero. Let φ denote buyer-to-sales-effort ratio in a generic submarket with matching probability $q(\varphi)/\varphi$. The expected gain from searching must equal the search cost:

$$\kappa = \max_{(p^d, p^r, \varphi)} \frac{q(\varphi)}{\varphi}\Big[p^r - p^d\Big] \ ,$$

where maximization is over all submarkets (p^d, p^r, φ). Any discount price that attracts new buyers (i.e., $\varphi > 0$) yields the same search value κ. It follows that discount prices are linked to market tightness φ via

$$p^d = p^r - \frac{\kappa\varphi}{q(\varphi)} \ . \tag{3.5}$$

3.2.2.3. *Firms.* A firm of type z^a takes as given the workers hired in earlier periods, L^τ, $\tau = 0, \dots, a-1$, together with their respective contracts $\mathcal{C}^\tau$.[13] It also takes as given the existing stock of the customer base B_-. Hence the firm's state vector is $\sigma = [(L^\tau, \mathcal{C}^\tau)_{\tau=0}^{a-1}, B_-, z^a]$. Let $J_a(\sigma)$ denote the value of the firm at the beginning of the period. The firm chooses recruitment policy $(\lambda, R, \mathcal{C}^a)$ and sales policy $(\delta_b, \varphi, S, p^d, p^r)$ to solve the problem

$$J_a(\sigma) = \max_{(\lambda,R,\mathcal{C}^a),(\delta_b,\varphi,S,p^d,p^r)} \left\{ p^r B_-(1-\delta_b) + p^d q(\varphi) S - W - r(R) - s(S) + \beta(1-\delta)\mathbb{E}J_{a+1}(\sigma_+) \right\} \tag{3.6}$$

subject to

$$\sigma_+ = [(L^{\tau+}, \mathcal{C}^\tau)_{\tau=0}^{a}, B, z^{a+1}] \ , \ \mathcal{C}^a = (w^a(z^k), \delta_w^a(z^{k+1}))_{k \geq a} \ , \ \delta_w^a(.) \geq \bar{\delta}_w \ , \tag{3.7}$$

$$L^{\tau+} = (1 - \delta_w^\tau(z^a))L^\tau \ , \ \tau = 0, \dots, a-1 \ , \ L^{a+} = m(\lambda)R \ , \tag{3.8}$$

$$W = \sum_{\tau=0}^{a} w^\tau(z^a) L^{\tau+} \ , \tag{3.9}$$

$$B = B_-(1 - \delta_b) + q(\varphi)S \ , \ \delta_b \geq \bar{\delta}_b \ , \tag{3.10}$$

$$B \leq xL \ , \ L = \sum_{\tau=0}^{a} L^{\tau+} \ , \tag{3.11}$$

$$\rho = \bar{\rho}(W(\mathcal{C}^a, z^a), \lambda) \ \text{ if } \ \lambda > 0 \ , \tag{3.12}$$

$$p^d = M y_a^\theta B^{\theta-1} - \frac{\kappa\varphi}{q(\varphi)} \ \text{ if } \ \varphi > 0 \ , \ p^r = M y_a^\theta B^{\theta-1} \ . \tag{3.13}$$

The firm's problem 3.6 is to maximize revenue from sales to attached and new buyers minus expenditures for wages, sales and recruitment costs, plus the expected continuation profit. The firm is committed to separation rates $\delta_w^\tau(z^a)$ for workers hired in previous periods $\tau < a$. For workers hired in this period, the firm commits to future separation rates, $\delta_w^a(z^{k+1}) \geq \bar{\delta}_w$, $k \geq a$. Together with wages $w^a(.)$, they define the contract $\mathcal{C}^a$ offered to new hires. Equations 3.8 say how employment in different worker cohorts evolves over time. Equation 3.9 states the wage bill of the firm. 3.10 says how the firm's customer base evolves. Because the firm is not committed in the product market, it decides buyer separation rates $\delta_b \geq \bar{\delta}_b$ (if required) freely.[14] Condition 3.11 says that the firm cannot sell more than what it produces with its current workforce L. Regarding wage contracts offered to new hires $\mathcal{C}^a$, as well as discount price offers p^d to new buyers, the firm respects the search incentives of workers and buyers, as expressed by constraints 3.12 and 3.13. That is, to attract more workers per recruitment effort (higher λ), the firm needs to offer a more attractive employment contract. Likewise, to attract more buyers per sales effort (higher φ), the firm needs to offer a lower discount

[13]Without loss of generality, all workers hired by a firm of a given type are hired in the same contract, which is an optimal policy of the firm (see Kaas and Kircher (2015) for a formal argument).

[14]Customer separations can only be optimal in response to adverse productivity shocks when the cost of employing (or hiring) $L = (B_-(1 - \bar{\delta}_b))/x$ workers exceeds the revenue $p^r B_-(1 - \bar{\delta}_b)$ plus the discounted continuation value of starting the next period with $B_-(1 - \bar{\delta}_b)$ customers.

price. The last equation in 3.13 says that the firm optimally charges the reservation price p^r on attached buyers.

3.2.2.4. *Equilibrium.* We can express all firm policy functions defined above to depend on the firm's history z^a, ignoring the dependence on pre-committed contracts and worker cohorts. This is feasible because such firm state variables evolve endogenously as functions of the firm's past shocks and policies. Hence, all firm policies (in stationary equilibrium) are functions of the idiosyncratic state history. For a firm of type z^a, write $\lambda(z^a)$ and $R(z^a)$ for the recruitment policy, $\varphi(z^a)$ and $S(z^a)$ for the sales policy, and so on.[15] Further define

$$L(z^a) = \sum_{\tau=0}^{a} L^\tau(z^a) \ , \tag{3.14}$$

$$B(z^a) = B(z^{a-1})[1-\delta_b(z^a)] + q(\varphi(z^a))S(z^a) \ , \tag{3.15}$$

for the stocks of workers and buyers in firm history z^a, where $L^\tau(z^a) = L^\tau(z^{a-1})[1-\delta_w^\tau(z^a)]$ if $a > \tau$, $L^a(z^a) = m(\lambda(z^a))R(z^a)$, and $B(z^{-1}) = 0$. Further, there are

$$N(z^a) = N_0(1-\delta)^a \pi^a(z^a) \tag{3.16}$$

firms of type z^a when N_0 is the mass of entrant firms in any period. We are now ready to define the stationary equilibrium.

Definition: *A stationary competitive search equilibrium is a list of value functions U, W, W', J_a, firm policies λ, R, φ, S, δ_b, $\mathcal{C}^a = (w^a(.), \delta_w^a(.))$, $(L^\tau)_{\tau=0}^a$, L, B, p^d, p^r which are all functions of the firm type z^a, entrant firms N_0, aggregate consumption C with $M = u'(C)C^{1-\theta}$, and a search value ρ such that:*

(a): Workers' value functions U, W, W' and the search value ρ describe optimal search behavior, equations 3.1–3.4.

(b): Retailers search optimally, equation 3.5, and aggregate consumption is given by

$$C = \left[\sum_{z^a} N(z^a)(y_a B(z^a))^\theta\right]^{1/\theta} \ . \tag{3.17}$$

(c): Firms' value functions J_a and policy functions solve problem 3.6–3.13, and $L(.)$, $B(.)$ and $N(.)$ evolve according to 3.14, 3.15 and 3.16.

(d): Firm entry is optimal. That is, $N_0 > 0$ and

$$K = \sum_{z^0} \pi^0(z^0) J_0(0, z^0) \ . \tag{3.18}$$

(e): Aggregate resource feasibility:

$$\bar{L} = \sum_{z^a} N(z^a)\Big\{L(z^a) + [\lambda(z^a) - m(\lambda(z^a))]R(z^a)\Big\} \ . \tag{3.19}$$

Aggregate resource feasibility (e) requires that any worker either belongs to the workforce $L(z^a)$ at one of $N(z^a)$ firms of type z^a or that the worker is unsuccessfully searching for a job in one of the submarkets where firms of type z^a search: precisely, $\lambda(z^a)R(z^a)$ workers are searching for employment per firm of type z^a, and share $1 - m(\lambda(z^a))/\lambda(z^a)$ of these workers are not successful and hence remain unemployed. In Section 3.A in the Appendix, we show how the aggregate resource constraint for the numeraire good can be derived using the budget constraint of the representative household and profits of firms.

[15]With abuse of notation, these functions are not indexed by the firm's age.

3.2.3. Characterization. The competitive search equilibrium permits a tractable solution which builds on the maximization of the joint surplus of a firm together with all workers and retail buyers linked to the firm. This problem ignores the impact of the firm's production on the representative household's consumer surplus. Therefore, as is well-known in environments with monopolistically competitive firms, equilibrium is not socially efficient.[16]

Write $G(L_-, B_-, z)$ for the surplus of a firm with productivity and demand state $z = (x, y)$ that begins a period with L_- workers and B_- buyers. This surplus includes the value that the firm generates for its owners as well as to all workers and retail buyers that are linked to the firm. It satisfies the recursion

$$
\begin{aligned}
G(L_-, B_-, z) = \max_{(\lambda,R,\delta_w),(\varphi,S,\delta_b)} \Big\{ & M(yB)^\theta - (b+\rho)L - r(R) - s(S) \\
& - \rho(\lambda - m(\lambda))R - \kappa\varphi S + \beta(1-\delta)\mathbb{E}_z G(L, B, z_+) \Big\} ,
\end{aligned}
\tag{3.20}
$$

subject to

$$
\begin{aligned}
& L = L_-(1-\delta_w) + m(\lambda)R \ , \ B = B_-(1-\delta_b) + q(\varphi)S \ , \\
& B \le xL \ , \ \delta_w \ge \bar{\delta}_w \ , \ \delta_b \ge \bar{\delta}_b \ .
\end{aligned}
$$

The joint surplus of a firm is the firm's revenue $p^r B = M(yB)^\theta$, minus the opportunity costs of employment, $(b+\rho)L$, recruitment and sales costs, $r(.)$ and $s(.)$, opportunity costs of job search for unemployed workers who unsuccessfully try to find employment at this firm, $\rho(\lambda - m(\lambda))R$, and search costs of new retailers aiming to join the customer pool of this firm, $\kappa\varphi S$. These search costs are incurred by $q(\varphi)S$ new buyers attracted by the firm, but also by $(\varphi - q(\varphi))S$ unsuccessful retailers who remain unmatched at the end of the period.

Proposition 3.1. Suppose that G solves the recursive joint surplus maximization problem 3.20 and that

$$
K = \sum_z \pi^0(z) G(0, 0, z)
\tag{3.21}
$$

holds. Further, let $N(z^a)$, $L(z^a)$ and $B(z^a)$ be defined recursively for given entry measure N_0 and iteration over the policy functions of problem 3.20, and suppose that the aggregate resource constraint 3.19 holds and that $M = u'(C)C^{1-\theta}$ with aggregate consumption given by 3.17. Then there exists a stationary competitive search equilibrium with identical firm policy functions as those described by 3.20, aggregate consumption C and workers' search value ρ.

The characterization by a joint surplus maximization problem greatly facilitates the computation of an equilibrium. Given a guess for ρ (search value) and M (marginal utility), solutions of 3.20 can be calculated by standard recursive methods. Updates for ρ and M are then obtained by using the entry condition 3.21 and the consumption aggregator 3.17.[17] All wages and prices can then be calculated using the respective search values of workers and retailers; see Section 3.A in the Appendix for details.

[16] Social optimality only obtains in the limiting case of perfect substitutes ($\theta \to 1$). We prove this result in a previous version of this paper where we set $\theta = 1$ and allow for decreasing returns in production.

[17] Specifically, if the value of an entrant is smaller (larger) than the entry cost, ρ must be decreased (increased). With the measure of entrant firms N_0 backed out from 3.16 and 3.19, aggregate consumption can be calculated from 3.17 which yields an update for M.

Proposition 1 extends welfare results for competitive search economies (cf. Moen (1997)) to an environment with monopolistically competitive firms and two-sided market frictions.[18] Kaas and Kircher (2015) prove similar results for multi-worker firms in an environment without product market frictions (and without monopolistic competition). Different from these well-known results, the competitive search equilibrium in our model is not efficient. Instead, it maximizes the joint surplus of firms, workers and retail buyers, without taking the impact on consumer surplus into account.

3.2.4. Product and Labor Market Interactions. To illustrate the role of market frictions for the impact of productivity and demand shocks, consider a one-period version of this model. Firms enter at cost K after which they draw productivity and demand states (x, y) and choose recruitment and sales policies to attract customers and workers. Using the characterization of Theorem 3.1, each firm maximizes the joint surplus,

$$M[yq(\varphi)S]^{\theta} - bm(\lambda)R - \rho\lambda R - \kappa\varphi S - r(R) - s(S) \ ,$$

subject to the constraint that all produced goods are sold, $xm(\lambda)R = q(\varphi)S$. As in the dynamic problem descibed above, joint surplus is the firm's revenue minus opportunity costs of workers and retailers, recruitment and sales costs. The four optimality conditions of this problem can be expressed as follows:

$$r'(R) = \rho\Big[\frac{m(\lambda)}{m'(\lambda)} - \lambda\Big] \ , \tag{3.22}$$

$$s'(S) = \kappa\Big[\frac{q(\varphi)}{q'(\varphi)} - \varphi\Big] \ , \tag{3.23}$$

$$M\theta(xy)^{\theta}[m(\lambda)R]^{\theta-1} - b = \frac{\rho}{m'(\lambda)} + \frac{x\kappa}{q'(\varphi)} \ , \tag{3.24}$$

$$xm(\lambda)R = q(\varphi)S \ . \tag{3.25}$$

Condition 3.22 says that across firms (which differ by x and y) recruitment effort and matching rates are positively related: firms hire more by spending more on recruitment (higher R) and by offering higher wages, thus attracting more workers (higher λ, cf. Kaas and Kircher (2015). Condition 3.23 expresses a similar relation in the product market: firms that spend more on sales also have lower discount prices (higher φ, cf. Gourio and Rudanko (2014).[19]

Condition 3.24 determines the size of firms and is unique to this model with two-sided frictions. The left-hand side expresses the marginal joint worker-firm surplus, i.e. marginal revenue net of the opportunity cost of work. The right-hand side is the marginal cost of employing one more worker *and* selling the output that this worker produces. It includes both the marginal cost of hiring the worker, $\rho/m'(\lambda)$, and the marginal cost of attracting x new customers which is $x\kappa/q'(\varphi)$. Both marginal costs critically depend on matching frictions. For instance, if the elasticity of q' is large (in absolute value), marginal customer acquisition costs rise steeply with the magnitude of customer expansion as summarized by the buyer-to-sales-effort ratio φ. This is the case when product market congestion externalities on the side of sellers are larger. Likewise, if the elasticity of m' is larger, marginal hiring costs rise steeply with the total amount of hires. Taken together, the two cost terms in 3.24 reflect how tightly the firm is constrained in both markets. In technical terms, this condition defines a

[18] In random search models, efficiency in two-sided search markets is far from trivial; see Petrosky-Nadeau, Wasmer, and Weil (2020) on the derivation of a double Hosios condition in a model with random search in product and labor markets.

[19] Both conditions are exactly the same in the dynamic model.

downward-sloping relation between the hiring indicator λ (note again that recruitment effort R increases in λ via 3.22) and the sales indicator φ (which is positively linked to sales effort S via 3.23), see Figure 3.1. Intuitively, if the firm hires and hence produces more, marginal revenue falls (the price declines in the supply of the firm's good) and marginal hiring costs increase. In response, the firm cuts sales expenditures and sets a smaller discount on the producer price.

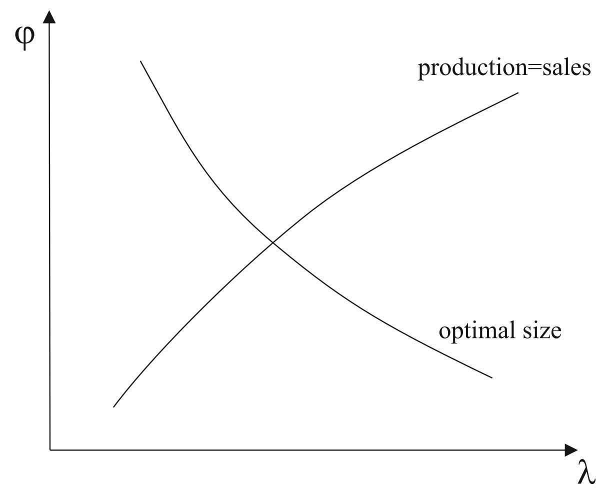


FIGURE 3.1. Optimal hiring and sales policies

Condition 3.25 is the requirement that production equals sales. After inverting 3.22 and 3.23 for R and S, this condition defines an increasing relation between the hiring indicator λ and the sales indicator φ (see Figure 3.1). Intuitively, if the firm hires and produces more, it requires more sales expenditures (and lower discount prices) to sell the additional output. Jointly, the two conditions 3.24 and 3.25 determine how optimal hiring and sales policies depend on firm characteristics, in particular on the productivity and demand states (x, y).[20]

The presence of product market frictions crucially matters for the firm's response to productivity and demand shocks. With frictionless product markets, buyers' and sellers' search costs are zero, $\kappa = 0$ and $s(.) = 0$, and the firm's optimal size is determined by its hiring policy (λ, R) which is the solution of 3.22 and 3.24 (without the last cost term). In this case, a one-percent increase of productivity x has *exactly the same impact on employment* as a one-percent increase of demand y. Likewise, the firm's revenue (price multiplied with output) responds identically to changes in x and y. The same is true in the dynamic model with frictionless product markets: a shock to x has the same impact on employment and revenue as a shock to y in the same proportion would have.[21]

Matters are very different when product market frictions come into play. An increase of demand (parameter y) induces the firm to hire more and to acquire more customers: the downward-sloping curve in Figure 3.1 shifts outwards so that λ and φ increase. Hence, the firm's output, employment and wage increase. Retail and producer prices are larger,

[20] Three further conditions are needed to determine the equilibrium in this one-period model: free entry of firms and the labor market resource constraint determine the search value ρ and the number of firms, and aggregate consumption determines $M = u'(C)C^{1-\theta}$. These conditions are irrelevant for the cross-sectional variation of firm policies which is the focus of the discussion in this subsection.

[21] This statement trivially follows from problem 3.20 when $\kappa = 0$ and $s(.) = 0$.

while the discount on the producer price increases. Conversely, an increase of productivity (parameter x) unambiguously induces the firm to acquire more customers: φ increases since both curves in Figure 3.1 shift up. Hence output increases while retail and producer prices fall, in combination with a greater discount. However, the labor market response is generally ambiguous and depends on how tight the demand constraint binds. If the product market search externality is strong enough, the firm may even choose to *decrease* employment in response to a positive productivity change. This happens if the elasticity of q' is relatively large (i.e., the elasticity of q is small) so that the downward-sloping curve in Figure 3.1 is relatively flat.

Generally, the presence of product market frictions dampens the employment and output response to productivity shocks. We make use of this observation in the next section where we calibrate the product market matching function elasticity in line with the firms' employment variability that we measure in our firm-level data.

3.3. Quantitative Analysis

In this section we calibrate the model to match statistics of price and productivity dynamics of German manufacturing firms. We do this to achieve three goals: First, we demonstrate the importance of product market frictions for the employment adjustments of firms. Second, we examine the quantitative role of demand and productivity for firm dynamics. Third, we use the model to explore the impact of aggregate shocks on macroeconomic outcomes which we compared with the cyclical features in our data. Before we do so, we describe our data and the calibration strategy.

3.3.1. Data and Measurement. This section describes the data and how we use them to construct firm-level measures of prices and quantity labor productivity. Further details and descriptive statistics are contained in Section 3.B in the Appendix. We use administrative firm data for Germany (*Amtliche Firmendaten für Deutschland*, AFiD), which are provided by the Research Data Centers of the Federal and State Statistical Offices.[22] We work with the panel *Industriebetriebe* (manufacturing establishments) and the module *Produkte* for the years 1995–2014. The former is an annual panel which builds on monthly, quarterly and annual census statistics covering all establishments in manufacturing, mining and quarrying with 20 or more employees. These data contain annual information on employment, revenue and wage bill, while working hours are available for a subsample of establishments. The module *Produkte* builds on quarterly production statistics and has recordings on quantities and revenues for nine-digit products for these establishments. We merge the two panels in order to construct a matched establishment-product panel. We deflate nominal variables to 2010 euros using the GDP deflator.

We drop establishments that do not operate throughout a given year or that report to employ fewer than 20 employees. We further drop products that are measured in different units across establishments (less than one percent). To reduce measurement problems arising from quality differences between firms and over time, we only consider those products which are measured in physical units (weight, length, area, or volume), whereas we remove all products which are measured in other units such as "items" or "pairs".[23] We further remove

[22]For more information and how to access the data, see Malchin and Voshage (2009) and the website of the Research Data Centers http://www.forschungsdatenzentrum.de.

[23]The underlying hypothesis is that products measured in physical units have a lower degree of processing, so that quality differences are less important. To give examples, our sample includes products "1720 32 144: Fabric of synthetic fibers (with more than 85% synthetic) for curtains (measured in m^2)" and "2112 30 200:

all products which are produced by less than six establishments in order to be able to compute a meaningful average price for each product.

We only consider those establishment-year observations where the valid products are representative in the sense that they contribute at least 50 percent of the total revenue of the establishment similar to (Foster, Haltiwanger, and Syverson 2008). The final sample includes 350,129 establishment-year observations. In Section 3.B in the Appendix, we present the distributions of establishments, employment and revenue by size class and by industry. Over a third of employment and revenue is concentrated in the largest five percent of establishments with more than 500 workers, while a third of establishments employ less than 50 workers.[24] Our data include single-establishment firms as well as establishments belonging to multi-establishment firms. Over three quarters of establishments and about fifty percent of employment are in single-establishment firms. The statistics that we use in the following analysis do not change much when we restrict the panel to single-establishment firms. We thus refer to "firms" in the rest of this paper, keeping in mind that our statistics are calculated at the establishment level.

Measuring Price and Productivity Dynamics. Firms in our model are hit by exogenous shocks to (physical) productivity x and demand y, both of which follow Markov processes. We parameterize them as AR(1) processes

$$\log(x_{it}) = \rho^x \log(x_{i,t-1}) + \sigma^x \varepsilon^x_{it} \ ,$$
$$\log(y_{it}) = \rho^y \log(y_{i,t-1}) + \sigma^y \varepsilon^y_{it} \ ,$$

where ε^x_{it} and ε^y_{it} are standard normally distributed. To inform the parameters ρ^x and σ^x for productivity dynamics, we obtain statistics for the firm dynamics of quantity labor productivity (output per unit of labor) which is identical to parameter x in our model. The demand state y cannot be directly observed in the data (or backed out from observables), but we can make use of the close connection between prices and firm-specific demand y via equation 3.13.[25] Therefore, we utilize information about price dynamics to estimate parameters ρ^y and σ^y via indirect inference, jointly with other internally estimated parameters as described below.

To construct measures of a firm-specific price index and quantity labor productivity, we proceed as follows. Let R_{ijt} and Q_{ijt} be the revenue and quantity values of product j in firm i and year t.[26] We define firm i's price of product j by $P_{ijt} = R_{ijt}/Q_{ijt}$ from which we obtain a quantity-weighted average price of good j in year t,

$$\bar{P}_{jt} \equiv \frac{\sum_i P_{ijt} Q_{ijt}}{\sum_i Q_{ijt}} \ .$$

Cigarette paper, not in the form of booklets, husks, or rolls less than 5 cm broad (measured in t)", whereas it does not include "1740 24 300: Sleeping bags (measured in 'items')" and "2513 60 550: Gloves made of vulcanized rubber for housework usage (measured in 'pairs')" (numeric codes based on product classification 2002).

[24]The average establishment in our sample is quite large: the mean (median) size is about 150 (70) employees, which is due to the fact that our sample is truncated below at 20 employees and that it covers the manufacturing sector.

[25]The firm's price in the model is a quantity-weighted average of prices charged on new and repeat buyers (see Section 3.A in the Appendix).

[26]To take into account that these products may represent less than 100 percent of the revenue of a firm, we follow Foster, Haltiwanger, and Syverson (2008) and scale these numbers up by the same proportionality factor to make sure that the sample revenue $\sum_j R_{ijt}$ equals the total revenue of firm i in year t. This adjustment is a valid modification of the data presuming that the goods in our sample are sufficiently representative for the set of all goods that this firm produces.

Recall that this summation is over at least six firms for any of the products that we consider. We use average product prices to measure a firm's quantity labor productivity. Whereas *revenue labor productivity* (RLP) is obtained by dividing a firm's actual revenue by labor hours, *quantity labor productivity* (QLP) divides the firm's output value at average market prices by labor hours:

$$\mathrm{RLP}_{it} \equiv \frac{\sum_j Q_{ijt} P_{ijt}}{H_{it}} \quad \text{and} \quad \mathrm{QLP}_{it} \equiv \frac{\sum_j Q_{ijt} \bar{P}_{jt}}{H_{it}} \ ,$$

where H_{it} are working hours. Revenue labor productivity is the product of quantity labor productivity and the firm's *price index* $\widetilde{P}_{it}$:

$$\mathrm{RLP}_{it} = \mathrm{QLP}_{it} \cdot \widetilde{P}_{it} \quad \text{where} \quad \widetilde{P}_i \equiv \frac{\sum_j P_{ijt} Q_{ijt}}{\sum \bar{P}_{jt} Q_{ijt}}. \tag{3.26}$$

This price index expresses the firm's actual revenue relative to the hypothetical revenue had the firm sold its products at the (quantity-weighted) average market prices.[27]

Since we are interested in within-firm dynamics of prices and productivity, we regress $\log \mathrm{QLP}_{it}$ and $\log \widetilde{P}_{it}$ on firm and time fixed effects and denote the residual terms by q_{it} and p_{it}, respectively. As noted above, q_{it} is the direct analogue of the exogenous variable $\log x_{it}$ in the model, so that we calibrate the autocorrelation and standard deviation parameters $\rho^x = 0.629$ and $\sigma^x = 0.266$ directly so as to replicate the autocorrelation (0.629) and standard deviation (0.342) of its empirical counterpart q_{it}.

Prices, on the other hand, are an endogenous model outcome and depend on all firm's exogenous and endogenous state variables. Yet, their statistical properties closely follow those of the firm's demand state y_{it}. Therefore, we estimate parameters ρ^y and σ^y to match the autocorrelation (0.644) and standard deviation (0.260) of the empirical price measure p_{it}.

A possible concern about our measurement of prices and productivity is the role of outliers and measurement error. To deal with the former, we remove all observations where the firm's price index or quantity labor productivity are below the 2nd or above the 98th percentiles of their respective distributions. Our restriction to products measured in physical units reduces the impact that quality differences take on prices. Nonetheless, if quantities are measured with error, our data would generate a spurious negative correlation between prices and quantities. To deal with this concern, we introduce measurement error into our model, re-estimate the parameters and show that our main results are similar to those of the benchmark model without measurement error (see Section 3.C in the Appendix for the results).

3.3.2. Parameterization. We calibrate the model at annual frequency. The first set of parameters is calibrated directly, while the remaining parameters are jointly estimated to match suitable data targets. We set the discount factor to $\beta = 0.96$ to reflect a four-percent interest rate. The firm exit rate is $\delta = 0.02$, corresponding to the annual exit rate of German firms with 20 or more workers (see Fackler, Schnabel, and Wagner (2013)).

The exogenous worker separation rate is set to $\bar{\delta}_w = 0.02$ so that the total separation rate in the stationary equilibrium is around 7 percent.[28] The exogenous customer separation rate is set to $\bar{\delta}_b = 0.43$; this number corresponds to the finding of Stahl (2010) that repeat customers account for 57% of the annual revenue in German manufacturing firms.

[27]This index is analogous to the construction of a household-level price index in Kaplan and Menzio (2015).

[28]These targets are based on Fuchs and Weyh (2010) who measure plant-level job creation and destruction rates from the IAB Establishment History Panel for the period 2000–2006.

We directly set the CES parameter $\theta = 0.8$ in line with markup estimates for German manufacturing sector of around 25 percent (cf. Deutsche Bundesbank (2017)). The utility function has constant elasticity, $u(C) = \frac{u_0}{1-\sigma} C^{1-\sigma}$ with $\sigma \geq 0$. We set $\sigma = 2/3$ so that the elasticity of industry demand corresponds to the mean estimate for U.S. manufacturing industries of Chang, Hornstein, and Sarte (2009). The marginal valuation of a good in the model equals $yu'(C)$ in units of the numeraire good. As the unit of measurement is arbitrary, we normalize the average value of $M = u'(C)C^{1-\theta}$ to unity by setting the mean value of the demand shock to $\log(\bar{y}) = 0$ and adjusting the scale parameter u_0 accordingly.

For recruitment and sales costs we adopt the cubic specifications $r(R) = r_0 R^3$ and $s(S) = s_0 S^3$. Convex adjustment costs give rise to sluggish adjustment of employment and customers, together with variation in wage offers and discount prices. Matching functions in the labor market and in the product market are Cobb-Douglas: $m(\lambda) = m_0 \lambda^{\mu}$ and $q(\varphi) = q_0 \varphi^{\gamma}$.[29] The labor market matching function elasticity is directly set to a standard value of $\mu = 0.5$ (e.g. Petrongolo and Pissarides (2001)). All further parameters in these cost and matching functions, namely $(r_0, s_0, m_0, q_0, \gamma)$ are internally calibrated, together with the two parameters for the AR(1) process of firm specific demand (ρ^y, σ^y), the entry cost parameter K, the product market search cost κ and unemployment income b.

These ten parameters are jointly estimated to match the following ten targets: (1) The scale parameters r_0 and s_0 are identified to match plausible shares of spending on recruitment and sales; specifically we target recruitment (sales) expenditures to be one (two) percent of GDP as in Christiano, Eichenbaum, and Trabandt (2016) (Arseneau and Chugh (2007)). (2) The scale parameter m_0 of the labor market matching function is set to match a stationary unemployment rate of 8.5 percent which is the data average of the OECD harmonized unemployment rate over the period 1995–2014; (3) For the two parameters q_0 and γ of the product market matching function, we make use of the insight that the elasticity γ is crucial for the firms' responsiveness of employment adjustments.[30] Therefore we target the standard deviation of log employment growth of 12.5 percent. The scale parameter q_0 is set such that the average matching probability of a buyer is 50 percent so that every second search attempt of a retail buyer is successful.[31] (4) The entry cost K, through its impact on the endogenous search value ρ determines average firm size. We target that the median firm in our data employs 70 workers.[32] (5) The retailer search cost parameter κ, via its impact on the difference between retail and producer prices, determines profits in the retail sector. We target that the gross operating surplus (retail profits) is 5% of total sales.[33] (6) The unemployment income parameter b is set to match 60 percent of the average wage, reflecting the unemployment

[29] For both matching functions, we make sure that matching rates of workers and shoppers ($m(\lambda)/\lambda$ and $q(\varphi)/\varphi$ resp.) do not exceed one; that is we set $m(\lambda) = \min(\lambda, m_0 \lambda^{0.5})$ and $q(\varphi) = \min(\varphi, q_0 \varphi^{0.5})$.

[30] See the discussion in subsection 2.4 for a formal argument. In our calibrated model, we verify this relationship numerically; see the results in Table 3.2 discussed below.

[31] Since no separate information on individual transactions and matching processes between retailers and producers is available, this choice is rather arbitrary. Therefore, we conduct a robustness analysis with respect to this parameter. These results and further sensitivity experiments with respect to the customer separation rate $\bar{\delta}_b$ and the sales expenditure target are presented in Section 3.C in the Appendix.

[32] Note that average productivity is normalized given that the mean of $\log x$ is equal to zero. Parameter K cannot be identified independently of the average values of firm productivity x because firm-level value functions are linearly homogeneous in the vector $(x, b^{1/\theta}, r_0^{1/\theta}, s_0^{1/(\theta-3)}, \rho^{1/\theta}, \kappa^{1/(\theta-1)}, B, S, K)$ (see problem 3.20, together with the assumed functional forms), so that all firm-level policies are independent of scaling transformations.

[33] See Table 45341-001 (gross operating surplus in retail excluding cars) at Statistisches Bundesamt (www.destatis.de).

replacement rate in Germany (cf. Krebs and Scheffel (2013)). (7) The standard deviation (0.260) and annual autocorrelation (0.644) of the empirical firm-specific price index (after controlling for firm fixed effects) identifies the AR(1) parameters ρ^y and σ^y. We estimate these ten parameters via a simulated method of moments procedure where we minimize the unweighted squared percentage distance between the empirical and the model-implied moments. All our parameter choices and targets are summarized in Table 3.1.

TABLE 3.1. Parameter Values and Calibration Targets

Directly calibrated parameters		
Parameter	Value	Explanation/Target
β	0.96	Annual interest rate 4%
δ	0.02	Firm exit rate (Fackler, Schnabel, and Wagner (2013))
$\bar{\delta}_w$	0.02	Worker separation rate 7%
$\bar{\delta}_b$	0.43	Customer retention rate 57%
θ	0.8	Markup 25%
σ	0.666	Price elasticity of industry demand -1.5
ρ^x	0.629	Autocorrelation of log QLP
σ^x	0.266	Standard deviation of log QLP

Internally calibrated parameters			
Parameter	Value	Target	Model
r_0	$3.16 \cdot 10^{-5}$	Recruitment costs (1% of output)	1.27%
s_0	$1.95 \cdot 10^{-3}$	Sales costs (2% of output)	2.19%
m_0	0.271	Unemployment rate (8.5%)	9.1%
q_0	2.182	Customer matching rate (50%)	50.3%
γ	0.362	Std. dev. employment growth (0.125)	0.129
b	0.186	Unemployment income (60% of av. wage)	60.7%
κ	0.0196	GOS in retail (5% of sales)	4.97%
K	135.98	Average firm size (70)	71.3
ρ^y	0.703	Autocorrelation of log price (0.644)	0.575
σ^y	0.170	Standard deviation of log price (0.260)	0.237

3.3.3. Implications of Product Market Frictions. Product market frictions are an essential feature of our model to generate plausible dynamics of employment, triggered by productivity and demand shocks which are calibrated in line with the empirical features of firm-level productivity and price changes. To see this, we compare various statistics of firm dynamics of our model with those of an alternative version of our model in which product market frictions are absent. In this alternative model, there are no search costs in product markets, i.e. we set $\kappa = s_0 = 0$, so that retail prices and producer prices coincide. To keep the comparison with the benchmark model as close as possible, we leave the parameters governing productivity and demand shocks unchanged and only re-calibrate three model parameters to match average firm size, the unemployment rate and the unemployment replacement rate.[34] As an alternative comparison, we also recalibrate the model without product market frictions from an agnostic standpoint, assuming that separate information on prices and quantities is not available so that we feed our model only with standard productivity shocks (i.e. an AR(1) process for $\log(x)$ while y is constant).[35]

[34]This requires setting the entry cost to $K = 158.7$, the matching function scale $m_0 = 0.601$ and unemployment income $b = 0.237$.

[35]For this model, we estimate $K = 109.7$, $b = 0.206$, $m_0 = 0.270$, $r_0 = 3.96 \cdot 10^{-5}$, $\sigma^x = 0.178$ and $\rho^x = 0.740$ to match firm size (70), replacement rate (60%), unemployment rate (8.5%), recruitment cost (1%

Table 3.2 compares statistical features of firm dynamics in the data and in our model with and without product market frictions (PMF). We look at the cross-sectional standard deviations and correlations of firm-level growth rates of prices ($\hat{p}$), physical productivity ($\hat{q}$), employment ($\hat{e}$) and output ($\hat{q}$). The first two rows of the table show that the benchmark model roughly replicates the standard deviations of price and productivity growth.

TABLE 3.2. Firm Dynamics With and Without Product Market Frictions

	Data	PMF (Benchmark)	PMF ($\gamma = 0.2$)	PMF ($\gamma = 0.6$)	No PMF (both shocks)	No PMS (x shocks)
$\sigma(\hat{p})$	0.210	0.199	0.192	0.211	0.184	0.060
$\sigma(\hat{q})$	0.281	0.334	0.336	0.334	0.303	0.224
$\sigma(\hat{e})$	0.126	0.129	0.111	0.159	0.451	0.119
$\sigma(\hat{y})$	0.289	0.393	0.379	0.415	0.668	0.298
$\rho(\hat{q}, \hat{e})$	-0.122	0.298	0.256	0.334	0.553	0.457
$\rho(\hat{y}, \hat{e})$	0.227	0.583	0.518	0.652	0.926	0.743
$\rho(\hat{p}, \hat{q})$	-0.644	-0.537	-0.471	-0.584	-0.670	-0.935
$\rho(\hat{p}, \hat{e})$	0.003	0.064	0.099	0.011	-0.252	-0.743
$\rho(\hat{p}, \hat{y})$	-0.638	-0.436	-0.388	-0.466	-0.474	-1.000
JC rate (%)	2.9	3.8	3.2	4.4	14.8	3.2
JD rate (%)	3.0	2.4	1.9	2.9	10.4	1.8

Note: $\hat{p}$, $\hat{q}$, $\hat{e}$ and $\hat{y}$ denote annual log growth rates of prices, physical productivity, employment and output, $\sigma(.)$ and $\rho(.)$ are cross-sectional standard deviations and correlation coefficients. Data statistics are based on the firm-specific price index and quantity labor productivity as defined in subsection 3.3.1 (Source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.)

Regarding the cross-sectional dispersion of employment growth, the model *with* product market frictions matches the data counterpart. This is implied by our calibration which identifies the product market matching function elasticity (parameter γ) by this data target. In fact, this elasticity determines how tightly firms are demand constrained and hence how quickly they are able to adjust employment in response to shocks. To demonstrate the role of this parameter, columns three and four show the model outcomes if parameter γ is either set to a lower ($\gamma = 0.2$) or a higher value ($\gamma = 0.6$), while all other parameters are re-estimated based on the same data targets. It can be seen that the standard deviation of employment growth and job flow rates are either too low (for $\gamma = 0.2$) or too high (for $\gamma = 0.6$) compared to the benchmark model and the data. Hence, and in line with our discussion in subsection 2.4, firms are more demand constrained when the product market search externality on the side of sellers is large (γ is low) so that employment is less responsive to productivity shocks.

In contrast, the model *without* product market frictions in which firms face the same shock processes for productivity and demand (column five) generates too large dispersion of employment growth. In this model, adjustment costs only arise from labor market frictions, whose underlying parameters are pinned down by labor market targets (recruitment costs, separation and matching rates). Product market frictions introduce additional adjustment costs which enables our model to replicate plausible firm-level variability of productivity, prices *and* employment.

of output), standard dev. of employment growth (12.5%) and autocorrelation of revenue labor productivity (0.611).

If separate price and output information was not available, a researcher might want to use the model without product market frictions with only one type of shock (here, as in most standard models, an AR(1) process for logged productivity x) in order to match the standard deviation of employment growth. The last column of Table 3.2 shows the outcome of this exercise (see footnote 35 for parameter values and calibration targets). Different from the calibration in column five, this model is forced to match the empirical standard deviation of employment growth. In turn, the implied volatilities of prices and quantity labor productivity are then far too low. We return to this model when we discuss the impact of aggregate shocks in subsection 3.5.

The dampening of employment adjustments arising from product market frictions can also be seen in the bottom two rows of the table which reports job creation and destruction rates.[36] While the benchmark model somewhat overpredicts (underpredicts) the magnitude of job creation (job destruction) for continuing firms, both job flow rates much too high in the model without product market frictions and both types of shocks (column five).

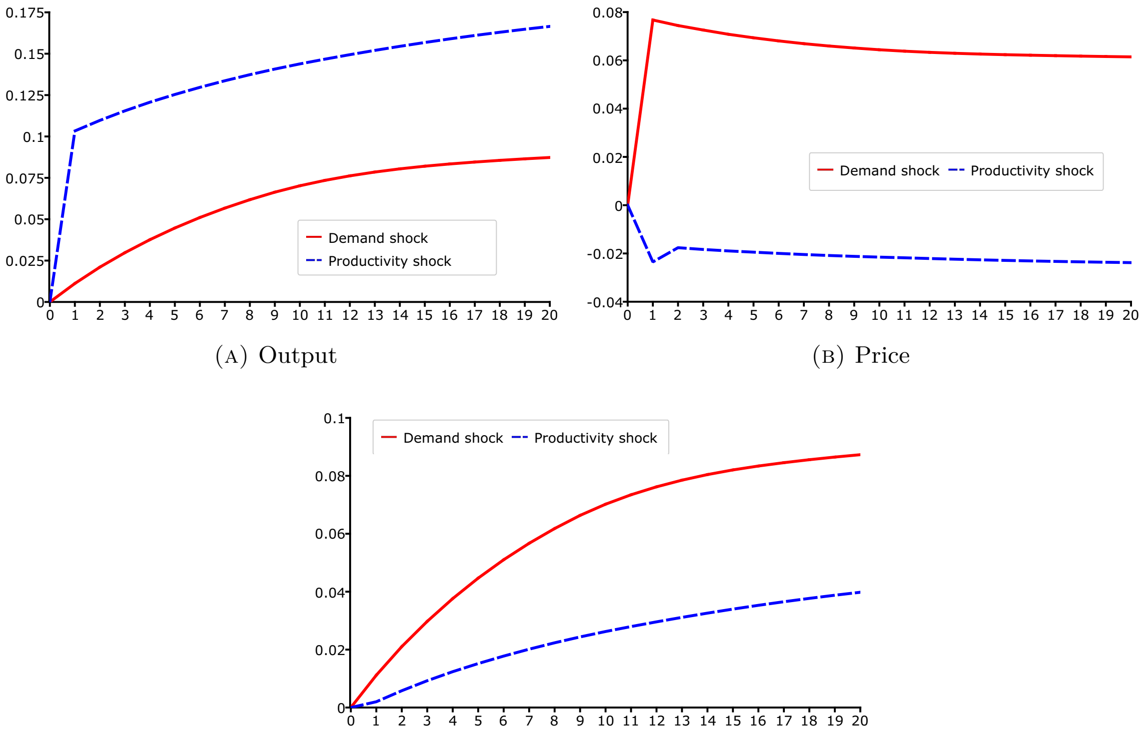
Further important differences can be observed regarding the co-movement of employment, output and quantity labor productivity. In the data, these cross-sectional correlations are rather modest (the one between productivity and employment is even negative), whereas the model without product market frictions produces too strong positive correlations between employment, output and quantity labor productivity. These co-movements are much weaker once product market frictions come into play. When it is costly to adjust the customer base, firms are more cautious to change employment in response to productivity changes.

3.3.4. The Role of Productivity and Demand for Firm Dynamics. To illustrate the separate roles of productivity and demand for firm dynamics, Figure 3.2 shows the responses of output, price and employment to a permanent 10-percent increase in either demand or productivity in period one. Higher demand (an increase of the taste parameter y) allows the firm to increase its price on impact for both its existing customers and for new customers. Over time, the firm expands employment and production so that it reduces the output price to sell the additional output to more customers. A positive productivity shock allows the firm to produce more output on impact. To sell the additional output, the firm cuts the discount price to attract new customers, and it also lowers the price for existing customers whose marginal valuation of the good decreases. Over time, from period two onward, the more productive firm starts to hire more workers and to attract more customers: output increases and the price declines further.

Figure 3.2 (c) shows that a demand shock induces a stronger adjustment of employment than a productivity shock of the same relative size. This is in contrast to the model without product market frictions, where an increase of x or y of the same size induces exactly the same response of employment. However, productivity shocks are more volatile in our calibrated model: the standard deviation of innovations to x (0.266) exceeds the one for innovations to y (0.170). Hence, it is a priori unclear how much these two forces matter for the firms' labor market adjustments.

To assess the separate roles of demand and productivity for firm dynamics, we report in Table 3.3 selected statistics for the benchmark model if either demand shocks or productivity shocks are absent. Both types of shocks contribute significantly to employment adjustments.

[36]Both in the data and in the model, we measure job creation and job destruction rates in the usual way: the job creation (destruction) rate is $jc_{it} = \frac{2\max(E_{it}-E_{i,t-1},0)}{E_{it}+E_{i,t-1}}$ ($jd_{it} = \frac{2\max(E_{i,t-1}-E_{i,t},0)}{E_{it}+E_{i,t-1}}$) where E_{it} ($E_{i,t-1}$) is year-t ($t-1$) employment at firm i. As in the data, the model sample is based on firms with 20 or more workers in both periods (hence it covers continuing firms only).



(A) Output

(B) Price

(C) Employment

FIGURE 3.2. Output and price responses to firm-level shocks.

Note: Output, price and employment response of a firm with average demand and productivity at its steady-state size in period zero which experiences a one-time permanent increase in productivity (blue, dashed) or demand (red, solid) by 10 percent in period one.

In the absence of demand shocks ("Only x shocks"), the model generates somewhat higher employment volatility, compared to the scenario where productivity shocks are absent. Also, productivity shocks account for a slightly larger share of job flows than demand shocks: Job destruction at continuing firms would fall by more than 50 (40) percent if productivity (demand) shocks were absent. The unemployment rate would fall by more than three percentage points in the absence of either one of the two forces.

Productivity shocks are considerably more relevant for output dynamics, whereas demand shocks contribute more to price volatility. The fact that our model reproduces the negative empirical co-movement between firm-level prices and output (an untargeted moment), and a negative co-movement between prices and productivity, is obviously induced by the presence of productivity shocks. The zero correlation between price and employment growth, both in the data and in the benchmark model, is explained by the offsetting forces of both shocks. If only one of the two shocks was active, price and employment growth would either be positively or negatively correlated, and price and output growth would either be perfectly negatively or positively correlated. And if productivity shocks were absent, employment and output growth would be perfectly correlated. Taking all these findings together, Table 3.3

TABLE 3.3. Firm Dynamics: Productivity and Demand Shocks

	Data	Both shocks	Only x shocks	Only y shocks
$\sigma(\hat{p})$	0.210	0.199	0.106	0.167
$\sigma(\hat{q})$	0.281	0.334	0.336	0.000
$\sigma(\hat{e})$	0.126	0.129	0.098	0.078
$\sigma(\hat{y})$	0.289	0.393	0.385	0.078
$\rho(\hat{q},\hat{e})$	-0.122	0.298	0.390	0.000
$\rho(\hat{y},\hat{e})$	0.227	0.583	0.595	1.000
$\rho(\hat{p},\hat{q})$	-0.644	-0.537	-0.969	0.000
$\rho(\hat{p},\hat{e})$	0.003	0.064	-0.465	0.327
$\rho(\hat{p},\hat{y})$	-0.638	-0.436	-0.964	0.327
JC rate (%)	2.9	3.8	2.7	2.5
JD rate (%)	3.0	2.4	1.4	1.1

Note: See the notes of Table 3.2 for explanations.

highlights that both productivity and demand shocks, and their interplay with product market frictions as implied by Table 3.2, are necessary features to capture the joint dynamics of prices, productivity and employment across firms.

3.3.5. Aggregate Dynamics. How does the model economy respond to aggregate shocks to the first or second moment of either productivity or demand risk? We are interested in the cyclical features of macroeconomic aggregates (i.e. output, employment and prices) as well as cross-sectional dynamics, in particular the dispersions of price and output growth across firms. To this end, we first look at the cyclicality of firm dispersion measures in our data. Then we analyze the impulse responses of different types of shocks in the model.

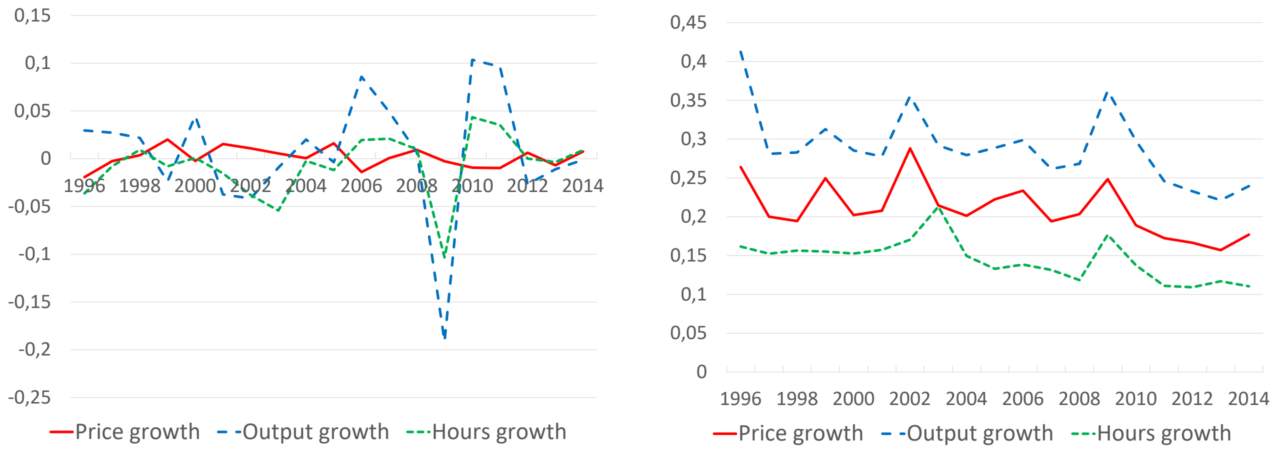
The literature documents counter-cyclical firm dispersion, based on the cross-sectional standard deviation of firms' revenue growth and other dispersion measures (e.g. Bloom et al. 2018). Our data allow us not only to confirm these findings for the manufacturing sector in Germany but also to document the separate cyclicalities of price and output growth dispersion.[37] We find that both standard deviations of output growth and price growth are counter-cyclical. Since log revenue growth is the sum of log price growth and log output growth, both price and output growth dispersion contribute to the counter-cyclicality of revenue growth dispersion.

Figure 3.3 shows time series of the cross-sectional means and standard deviations of price growth, output growth and hours growth. Germany had two recessions in the sample period (2002/03 and 2009). In both recessions, unsurprisingly, the means of output and hours growth go down.[38] Moreover, during the 2002/03 recession and the subsequent recovery, output growth leads hours growth. As shown in the right panel of Figure 3.3, all three dispersion measures go up in both recessions, albeit by different magnitudes, and again hours growth dispersion is lagging behind in the 2002/03 recession. Over the reported 19-year period, the

[37]See also Bachmann and Bayer (2014) who document countercyclical dispersion of firm-level growth rates of employment, value added and factor productivity for Germany during the period 1973–1998. Berger and Vavra (2018) find countercyclical price growth dispersion in U.S. data.

[38]We use hours instead of employment here because the labor market reforms in Germany during the 2000s (Hartz I–IV) have decisively altered the employment dynamics in Germany. In particular, aggregate employment barely fell during the Great Recession (cf. Burda and Hunt 2011).

means of the output and hours growth are pro-cyclical, while standard deviations of all three series are counter-cyclical.[39]



(A) Means (B) Standard deviations

FIGURE 3.3. Means and standard deviations of price, output and hours growth rates (1996–2014).

Data source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.

To see how macroeconomic variables and firm-level dispersion measures react to aggregate shocks, we compare the impulse responses of our model economy to four types of aggregate events: (i) a decrease in either mean productivity $\bar{x}$ or mean demand $\bar{y}$ by five percent; (ii) an increase in the standard deviation of shocks to firm productivity x or firm demand y by twenty percent (productivity or demand uncertainty shock).[40] In all four experiments, we let the initial shock decline with annual autocorrelation 0.7 and consider the adjustment path back to the original steady state.

Despite the characterization of the stationary equilibrium by a joint surplus maximization problem, the aggregate dynamics has no block-recursive solution, in contrast to other competitive-search models. The reason is that marginal utility of aggregate consumption responds to changes in the distribution of firms which in turn feeds back into the firms' problem. Therefore we need to loop over the transition path of aggregate consumption C_t together with workers' search values ρ_t to solve for the transition path.[41]

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[39] The correlations with (linearly) detrended value added in manufacturing are $(0.035, 0.743, 0.789)$ for the means of (price, output, hours) growth and $(-0.372, -0.514, -0.504)$ for the standard deviations of (price, output, hours) growth.

[40] Because AR(1) processes for the idiosyncratic states are expressed in the logs of x and y, we rescale the levels so that the means of x and y stay the same when the standard deviation of shocks increases.

[41] Our model in Section 2 is described in steady state without aggregate risk. To incorporate the latter, we are assuming here that the firms' wage contracts are contingent on the *aggregate* state of the economy. In response to aggregate events, contractual wages and separation rates adjust, which ensures that the response of the competitive-search equilibrium to these shocks is identical to the solution of simplified joint-surplus maximization problem that we consider.

Figure 3.4 shows the economy's response to negative *mean* productivity and demand shocks. The negative productivity shock generates a five percent decline of output on impact while employment shows a tiny increase on impact (about 0.1 percent) which is a result of a substantial drop of the workers' search value ρ_t in response to the shock which reduces wages and which makes it (slightly) more attractive for *incumbent* firms to hire. Potential entrants, however, find it less profitable to enter so that the number of firms falls over time, resulting in a long-term decline in aggregate employment, which is however relatively small (about -0.25 percent). Panel (c) shows that firms pass on the higher labor costs to customers: prices increase on impact by almost four percent.

The response to a negative demand shock is rather different. Again, entry falls and wages drop in response to the reduction in firm revenue and the declines of output and employment by 0.3 (0.25) are rather modest. Firms cut prices to accommodate customers' lower valuations of their products.

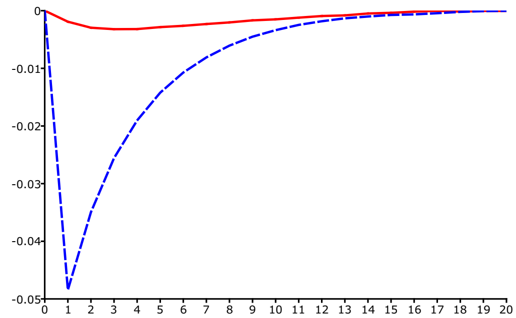
The bottom two graphs in Figure 3.4 show the responses of price and output growth dispersion to the two shocks. Negative productivity and demand shocks raise the dispersion of output growth and reduce the dispersion of price growth, with larger responses for declining productivity. Intuitively, both shocks reduce the level of aggregate production, while the magnitude of idiosyncratic uncertainty and adjustment costs stay the same. As growth rates are expressed in percentage terms, the dispersion of output growth rises while the dispersion of price growth falls. In both cases, however, the magnitudes are rather small.

We conclude that aggregate shocks to the first moments of productivity or demand do not generate the countercyclical dispersion of price growth that we observe in the data and they only lead to a small increase of output growth dispersion. Moreover, both shocks only generate modest declines of economic activity. The latter finding is a consequence of our calibration strategy which sets the flow income during unemployment (parameter b) equal to a plausible value of unemployment benefits in relation to wages. As a result, the surplus value of a job is relatively large which in turn implies that movements in productivity (likewise, in aggregate demand) cannot have large effects on the labor market (see, e.g., Ljungqvist and Sargent 2017). In contrast, if we calibrate b to a much higher value, for instance reflecting the value of leisure, our model can produce considerably more amplification (cf. Hagedorn and Manovskii 2008). What is more, and different from our benchmark calibration, negative first-moment shocks to productivity or demand can induce (small) increases of price and output growth dispersion. See Section 3.C in the Appendix for details.

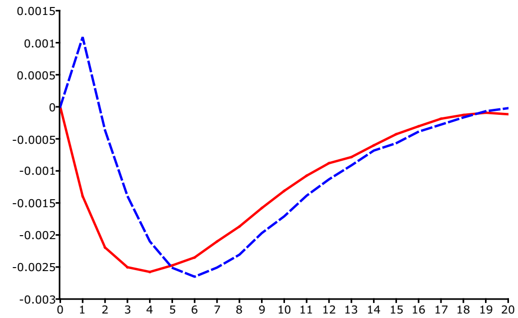
Quite different is the reaction of our model economy to uncertainty shocks, as we illustrate in Figure 3.5. In particular, the demand uncertainty shock generates declines in output and employment which are more sizable than the shocks to the first-moment of demand considered above. Furthermore, the two reported measures of firm dispersion rise in response to greater demand uncertainty (panels (e) and (f)).

An increase in productivity uncertainty, in contrast, generates a positive response of output (while employment drops). On the other hand, given that firms' output growth is strongly driven by productivity shocks (cf. Figure 3.2), greater productivity uncertainty triggers a large increase in the standard deviation of output growth (panel (e)).

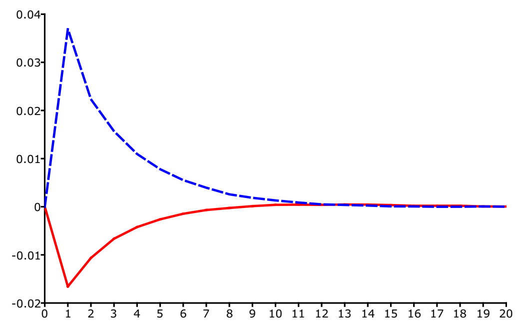
Based on these findings we conclude that higher demand uncertainty is a plausible feature of recessions: it can induce declines in output and employment together with increasing dispersion of price and output growth. On the other hand, aggregate shocks to productivity, either to the first or to the second moment, do not deliver meaningful impulse responses in our model.



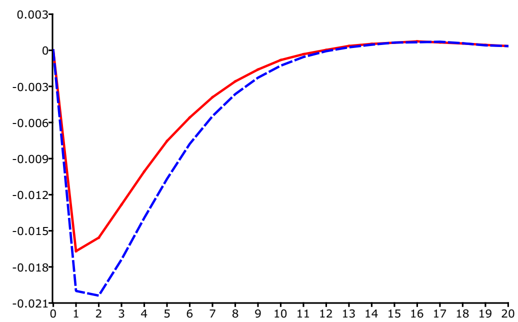
(A) Output



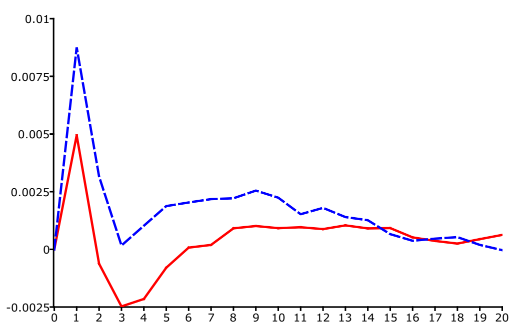
(B) Employment



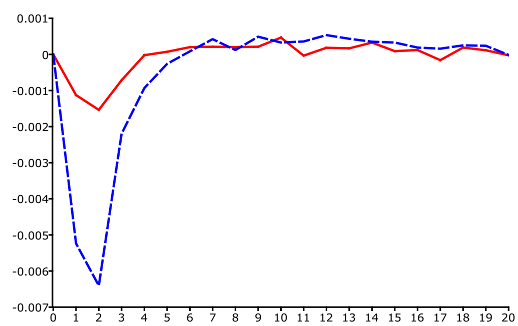
(C) Price



(D) Firms



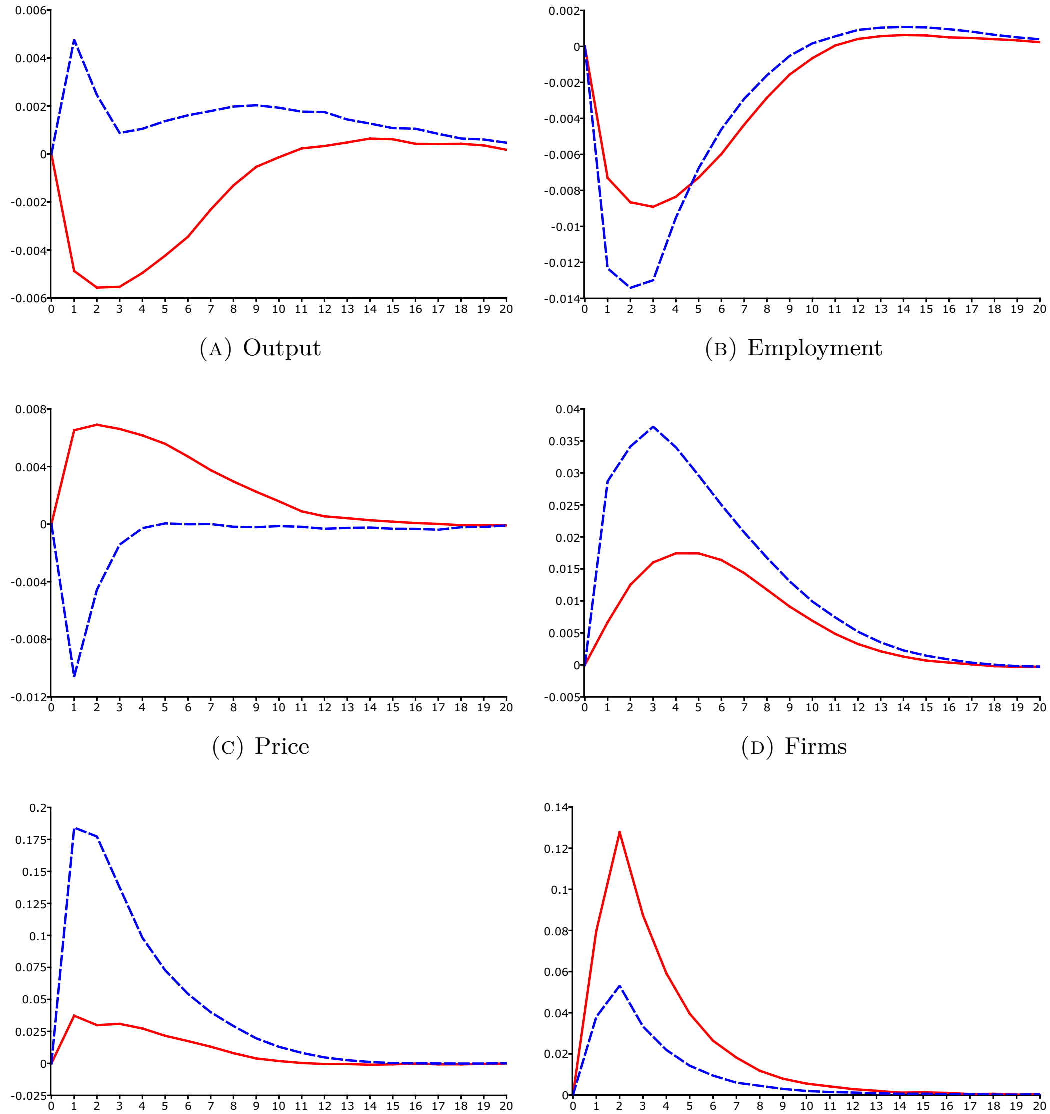
(E) Standard deviation of output growth



(F) Standard deviation of price growth

FIGURE 3.4. Responses to a five percent decrease of aggregate productivity (dashed, blue) and aggregate demand (solid, red).

By way of comparison, the model without product market frictions which is fed with productivity shocks only (see subsection 3.3 and the last column of Table 3.2) does not generate a recessionary reaction to an increase in uncertainty. This is shown by the short-dashed (red) impulse responses in Figure 3.6: in response to the 20 percent impact increase of productivity uncertainty (triggering isomorphic increases in output and price growth), output increases modestly while employment declines. On the other hand, a negative shock to aggregate productivity induces a sluggish and mild reduction of employment, but it does



(A) Output (B) Employment

(C) Price (D) Firms

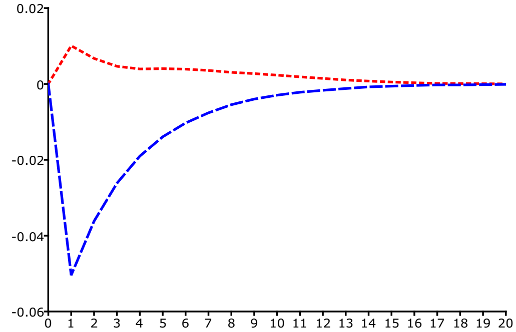
(E) Standard deviation of output growth (F) Standard deviation of price growth

FIGURE 3.5. Responses to an increase of aggregate productivity uncertainty (dashed, blue) and aggregate demand uncertainty (solid, red).

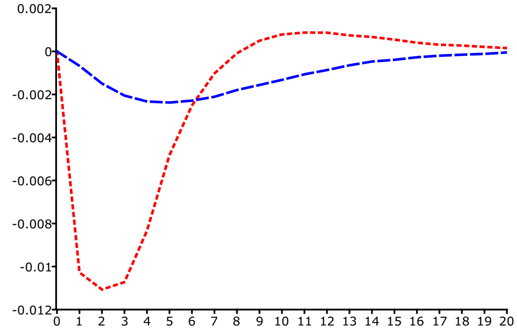
not increase the standard deviations of output or price growth (see the dashed, blue impulse responses).

3.4. Conclusions

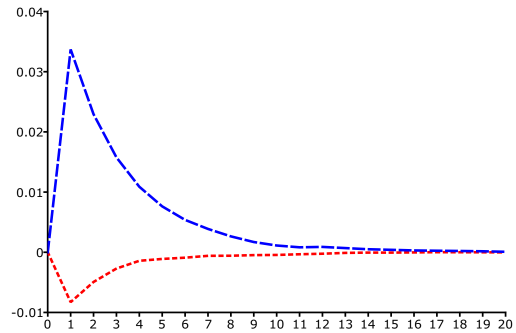
We introduce a model of heterogeneous firms which produce differentiated products and operate in frictional product and labor markets with convex sales and recruitment costs. Search frictions in the product market imply that firms are demand constrained, and hence must expend resources to spur demand. Likewise, frictions in the labor market make firms'



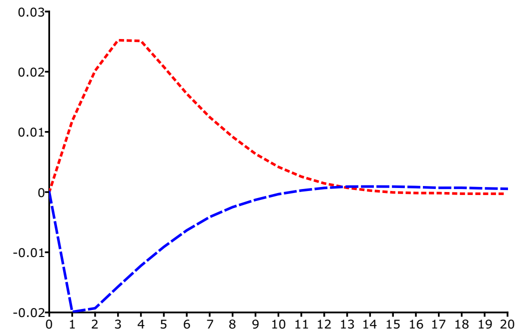
(A) Output



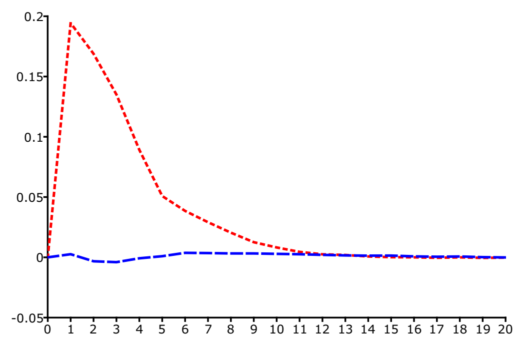
(B) Employment



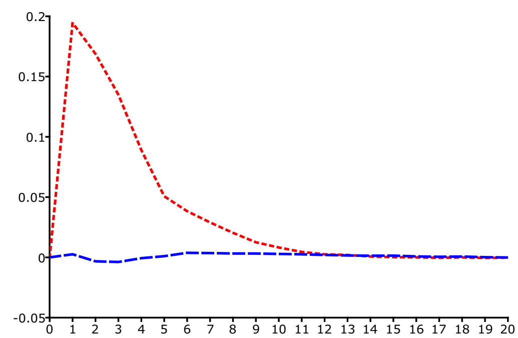
(C) Price



(D) Firms



(E) Standard deviation of output growth



(F) Standard deviation of price growth

FIGURE 3.6. Responses to a five percent decrease of aggregate productivity (dashed, blue) and aggregate uncertainty (short dashes, red) in the model without product market frictions.

adjustment to shocks sluggish, with consequences for the cross-sectional dynamics as well as for the aggregate economy.

We distinguish between firm-level productivity and demand shocks which affect the firms' output and pricing policies in different ways. The extent of product market frictions crucially determines how quickly firms are able to adjust employment in response to the shocks. The parameters of the shock processes are calibrated in order to match features of price and productivity dynamics of a panel of German manufacturing firms. We show that both demand and productivity shocks are necessary to describe the joint dynamics of prices, output and employment in our data.

By means of impulse response analyses, we highlight the importance of demand uncertainty for the business cycle. Considered in isolation, declines in the mean levels of either aggregate demand or aggregate productivity cannot generate plausible recessions in the model economy with counter-cyclical movements of firm growth dispersion as observed in our data. By contrast, demand uncertainty shocks can induce declines of output and employment together with rising output and price growth dispersion.

In sum, our work shows how product market conditions interact with labor market conditions to generate empirically plausible firm dynamics in a fairly tractable model framework. Due to our assumption of a representative household, some important product-labor market linkages that operate through the household sector, such as the different shopping behavior of unemployed workers (Krueger and Mueller 2010; Kaplan and Menzio 2016), are absent from our model. Other interesting features absent from this model are direct customer turnover ("search on the shop") or an intensive demand margin. Introducing such features might have important implications for aggregate dynamics and should be an interesting avenue for further research.

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3.A. Proofs and Derivations

3.A.1. Proof of Proposition 1. Consider (ρ, G, N_0, C) where G solves the recursive joint surplus maximization problem 3.20 together with 3.21. Further, aggregate consumption is 3.17 and the aggregate resource constraint 3.19 is satisfied when $L(z^a)$, $B(z^a)$ etc. are defined by the policy functions of problem 3.20. The proof proceeds in three steps. First, we construct candidate employment contracts and firm policies that resemble the joint surplus maximal solution. Second, we show that the extended policies maximize the joint surplus of a firm with commitment to previous contracts. Third, we show that the candidate policies also solve the private firm profit maximization problem and hence correspond to a competitive search equilibrium.

First, define candidate equilibrium contracts $\mathcal{C}^{a*} = (w^{a*}(z^k), \delta_w^{a*}(z^{k+1}))_{k \geq a}$ with separation rates $\delta_w^{a*}(z^k) \equiv \delta_w(z^k)$ from the policy functions of problem 3.20 (hence, separations are independent of the tenure in the firm). Candidate equilibrium wages $w(z^k)$ can be defined in different ways: for instance all workers may be paid flat wages over time, or all workers within the firm earn the same (equal treatment); see the corresponding equations at the end of this Appendix. We use equal-treatment wages for the remainder of this proof and hence specify candidate equilibrium wage contracts as $w^{a*}(z^k) = w(z^k)$, where $w(z^k)$ is defined as in 3.30 below. As in Section 3.2.2.3, define the generic state vector of the firm as $\sigma = [(L^\tau, \mathcal{C}^\tau)_{\tau=0}^{a-1}, B_-, z^a]$.

Second, let $G_a(\sigma)$ denote the *joint surplus* value of a firm, assuming that the firm takes as given previous worker cohorts L^τ and the precommitted separation rates as specified in contracts $\mathcal{C}^\tau$, $\tau < a$. For the contracts $(\mathcal{C}^{\tau*})_{\tau=0}^{a-1}$ in the candidate equilibrium (and the corresponding worker cohorts $L^{\tau*}$) write σ^* for the firm's state vector. We show that these contracts, together with the other joint-surplus maximizing firm policies, indeed solve the recursive joint-surplus maximization problem *with commitment*. The recursive problem to maximize the joint surplus (for firm owners, workers and buyers) is

$$G_a(\sigma) = \max_{(\lambda, R, \mathcal{C}^a), (\varphi, S, \delta_b)} \Big\{ M(y_a B)^\theta - (b+\rho)L - r(R) - s(S)$$

$$- \rho(\lambda - m(\lambda))R - \kappa\varphi S + \beta(1-\delta)\mathbb{E}G_{a+1}(\sigma_+) \Big\} , \tag{3.27}$$

subject to 3.7, 3.8, 3.10 and 3.11. Wage commitments in contracts $\mathcal{C}^\tau$ are obviously irrelevant for that problem. The same policies that solve problem 3.20, and in particular contracts $\mathcal{C}^{a*}$ for all $a \geq 0$, also solve problem 3.27. The only difference between these two problems is that the firm is precommitted to separation rates for existing workers in the latter but not in the former problem. But since policies for the latter problem are time consistent, both problems have the same solutions.

Third, it remains to show that these policies not only solve problem 3.27 but that they also maximize the *private profit value* of the firm, as specified in the recursive problem 3.6–3.13 with the same worker search value ρ. Substitution of 3.13 shows that

$$M(y_a B)^\theta - \kappa\varphi S = p^r B_-(1-\delta_b) + p^d q(\varphi) S \ .$$

Hence, the left-hand side of that term in problem 3.27 can be replaced by the right-hand side together with constraint 3.13. Further, we can write the labor costs

$$(b+\rho)L+\rho(\lambda-m(\lambda))R=(b+\rho)L_0+[b+\rho\frac{\lambda}{m(\lambda)}]m(\lambda)R\ , \tag{3.28}$$

with $L_0=\sum_{\tau=0}^{a-1}L^{\tau+}$ denoting employment of workers in previous cohorts. Given the pre-committed contracts $\mathcal{C}^{\tau*}$, $\tau<a$, the first term can be written

$$\begin{aligned}
(b+\rho)L_0 &= \sum_{\tau=0}^{a-1}[1-\delta_w^{\tau*}(z^a)]L^\tau\cdot(b+\rho)\\
&= \sum_{\tau=0}^{a-1}[1-\delta_w^{\tau*}(z^a)]L^\tau\Big[w^{\tau*}(z^a)-[W(\mathcal{C}^{\tau*},z^a)-U]+\beta(1-\delta)\mathbb{E}[W'(\mathcal{C}^{\tau*},z^{a+1})-U]\Big]\\
&= -\sum_{\tau=0}^{a-1}L^\tau[W'(\mathcal{C}^{\tau*},z^a)-U]+\sum_{\tau=0}^{a-1}L^{\tau+}w^{\tau*}(z^a)+\beta(1-\delta)\mathbb{E}\sum_{\tau=0}^{a-1}L^{\tau+}[W'(\mathcal{C}^{\tau*},z^{a+1})-U]\ .
\end{aligned}$$

For any contract $\mathcal{C}^a=(w^a(z^k),\delta_w^a(z^{k+1}))_{k\geq a}$ offered to new hires $m(\lambda)R=L^{a+}$, the second term in 3.28 can be written

$$\begin{aligned}
[b+\rho\frac{\lambda}{m(\lambda)}]m(\lambda)R &= [W(\mathcal{C}^a,z^a)-\beta U]m(\lambda)R\\
&= w^a(z^a)L^{a+}+\beta(1-\delta)\mathbb{E}[W'(\mathcal{C}^a,z^{a+1})-U]L^{a+}\ .
\end{aligned}$$

Substituting these expressions into 3.27 at $\sigma=\sigma^*$ shows

$$\begin{aligned}
G_a(\sigma^*)=\max_{(\lambda,R,\mathcal{C}^a),(\varphi,S,p,p^R,\delta_b)} &\Bigg\{p^rB_-(1-\delta_b)+p^dq(\varphi)S-W\\
&+\sum_{\tau=0}^{a-1}L^\tau[W'(\mathcal{C}^{\tau*},z^a)-U]-r(R)-s(S)\\
&+\beta(1-\delta)\mathbb{E}\Big\{G_{a+1}(\sigma_+^*)-\sum_{\tau=0}^{a-1}L^{\tau+}[W'(\mathcal{C}^{\tau*},z^{a+1})-U]-L^{a+}[W'(\mathcal{C}^a,z^{a+1})-U]\Big\}\Bigg\}\ ,
\end{aligned}\tag{3.29}$$

where maximization is subject to 3.8–3.13 with $\sigma_+^*=[(L^\tau,\mathcal{C}^{\tau*})_{\tau=0}^{a-1},(L^{a+},\mathcal{C}^a),B,z^{a+1}]$. In this maximization problem, the term $\sum_{\tau=0}^{a-1}L^\tau[W'(\mathcal{C}^{\tau*},z^a)-U]$ is predetermined and thus not subject to the maximization. Therefore, we can define the private firm value

$$J_a(\sigma)\equiv G_a(\sigma)-\sum_{\tau=0}^{a-1}L^\tau[W'(\mathcal{C}^\tau,z^a)-U]\ ,$$

i.e. the difference between the joint surplus value of firm z^a and the surplus value of the firm's workforce at the beginning of the period. Then problem 3.29 (at given state vector σ^*) is equivalent to problem 3.6. In particular, the firm policies λ, R, φ, S, p^d and p^r and $\mathcal{C}^{a*}$ that solve 3.29 also solve 3.6. Moreover, because of $G(0,0,z)=J_0(0,z)$, the entry condition 3.21 implies the equilibrium entry condition 3.18. Since the resource constraint is satisfied, the solution characterized by joint surplus maximization corresponds to a stationary competitive search equilibrium. □

3.A.2. Resource Constraint of the Numeraire Good. We can verify that the aggregate resource constraint for the numeraire good is satisfied in a stationary equilibrium. The budget constraint of the representative household in any period is

$$\sum_{z^a} N(z^a)p^r(z^a)B(z^a)+e = \sum_{z^a} N(z^a)\Big[\pi(z^a)+\sum_{\tau\leq a} L^\tau(z^a)w^\tau(z^a)\Big]+b\Big[\bar{L}-\sum_{z^a} N(z^a)L(z^a)\Big]-KN_0 \ .$$

The left-hand side expresses the household's consumption expenditures for the different goods and for the numeraire e. The right-hand side gives the household's income which includes wage and profit income at all firm types z^a plus income from unemployment minus expenditures for the entry costs of firms. Profit income of firm z^a is

$$\pi(z^a) = \ p^r(z^a)B(z^{a-1})[1-\delta_b(z^a)]+p^d(z^a)q(\varphi(z^a))S(z^a)-\sum_{\tau\leq a} L^\tau(z^a)w^\tau(z^a)-r(R(z^a))-s(S(z^a)) \ .$$

Using equation 3.5 and rearranging shows that the household's consumption of the numeraire good[42] is identical to unemployment income net of the costs for recruitment, sales, firm entry, and retailers' search costs, all of which are paid in the numeraire good:

$$e = b\Big[\bar{L}-\sum_{z^a} N(z^a)L(z^a)\Big]-\sum_{z^a} N(z^a)[r(R(z^a))+s(S(z^a))]-KN_0-\kappa\sum_{z^a} N(z^a)\varphi(z^a)S(z^a) \ .$$

3.A.3. Prices and Wages. The firms' output and employment policies follow directly from the solution of the joint surplus maximization problem 3.20. Hence they are independent of our specific assumptions on price or wage commitments. Now we show how prices and wages can be calculated based on the assumptions that (i) firms offer long-term flat wage contracts and (ii) firms offer discount prices to new buyers and cannot commit to long-term price contracts. We also discuss alternatives to both arrangements.

3.A.3.1. *Prices.* Given the solutions of firm-specific buyer stocks B and product market tightness φ (all conditional on firm type z^a), retail and discount prices are the solutions of 3.13. Then, the revenue of firm z^a is

$$Re(z^a) \equiv p^r(z^a)B(z^{a-1})(1-\delta_b(z^a)) + p^d(z^a)q(\varphi(z^a))S(z^a) = M[y_a B(z^a)]^\theta - \kappa\varphi(z^a)S(z^a) \ .$$

The firm's (average) price is $P(z^a) \equiv Re(z^a)/B(z^a)$ because $B(z^a)$ is the quantity of output units sold.

Other decentralizations without price discrimination (albeit with commitment) are also conceivable. Suppose for example that each firm charges the same price $p(z^a)$ for all its customers who expect that they will separate from firms with identical probability $\delta_b(z^a)$. Then optimal buyer search requires that

$$\kappa = \frac{q(\varphi(z^a))}{\varphi(z^a)}Q(z^a) \ ,$$

where the value of a buyer matched to a firm of type z^a satisfies the Bellman equation

$$Q(z^a) = My_a^\theta B(z^a)^{1-\theta} - p(z^a) + \beta(1-\delta)\mathbb{E}_{z^a}\Big([1-\delta_b(z^{a+1})]Q(z^{a+1})\Big) \ .$$

Given firm policies $\varphi(z^a)$ and $\delta_b(z^a)$, these two equations can be directly solved for non-discriminatory prices $p(z^a)$ and for the firms' revenue $Re(z^a) = p(z^a)B(z^a)$.[43]

[42]If $e < 0$, we assume that the household produces $-e$ units of the numeraire good which, together with unemployment income and net of shopping costs is identical to the firms' expenditures on entry, recruitment and sales.

[43]The model statistics on price dynamics that we report in Table 3.2 change very little under this different pricing assumption. Details are available upon request.

3.A.3.2. *Wages.* We show how to obtain wage schedules in the competitive search equilibrium. We distinguish between two cases: (i) All workers are paid the same wage within a firm (equal treatment); (ii) all workers are paid flat wages over time. In both cases, as in the joint-surplus maximization problem specified in the text, separation rates for all workers in a firm are assumed to be identical: $\delta_w^\tau(z^a) = \delta_w(z^a)$ where a is the age of the firm and τ is the worker cohort (the age of the firm when the worker was hired). Further, firms are able to commit to wage contracts.[44]

Equal Treatment. First consider an arrangement in which every firm pays the same wage to all its workers, i.e. $w^\tau(z^a) = w(z^a)$ for all $\tau \leq a$. In this case, worker values W and W' do not depend on the particular contract $\mathcal{C}^a$ and can therefore be written $W(z^a)$ and $W'(z^a)$, so that 3.1–3.4 become

$$\begin{aligned}
U &= \frac{m(\lambda(z^a))}{\lambda(z^a)} W(z^a) + \Big(1 - \frac{m(\lambda(z^a))}{\lambda(z^a)}\Big)[b + \beta U] \ , \\
W(z^a) &= w(z^a) + \beta(1-\delta)\mathbb{E}_{z^a} W'(z^{a+1}) + \beta\delta U \ , \\
W'(z^a) &= [1 - \delta_w(z^a)] W(z^a) + \delta_w(z^a) U \ , \\
U &= b + \rho + \beta U \ .
\end{aligned}$$

These equations can be solved for the worker surplus

$$W(z^a) - U = \rho\Big[\frac{\lambda(z^a) - m(\lambda(z^a))}{m(\lambda(z^a))}\Big] \equiv S^w(z^a) \ ,$$

and for wages:

$$w(z^a) = b + \rho + S^w(z^a) - \beta(1-\delta)\mathbb{E}_{z^a}\Big([1 - \delta_w(z^{a+1})] S^w(z^{a+1})\Big) \ . \tag{3.30}$$

Flat Wages. Consider now the case where every worker is paid a flat wage over time: w_τ is the constant wage of a worker hired in a firm at age τ, and $W(w_\tau, z^a)$ denotes the worker's value in this firm at history z^a, for $a \geq \tau$. We have the Bellman equations

$$\begin{aligned}
W(w_\tau, z^a) &= w_\tau + \beta(1-\delta)\mathbb{E}_{z^a} W'(w_\tau, z^{a+1}) + \beta\delta U \ , \\
W'(w_\tau, z^a) &= [1 - \delta_w(z^a)] W(w_\tau, z^a) + \delta_w(z^a) U \ , \\
U &= b + \rho + \beta U \ ,
\end{aligned}$$

from which we obtain

$$W(w_\tau, z^a) - U = w_\tau - b - \rho + \beta(1-\delta)\mathbb{E}_{z^a}(1 - \delta_w(z^{a+1}))\Big[W(w_\tau, z^{a+1}) - U\Big] \ .$$

Hence, $W(w_\tau, z^a) - U = A(z^a)(w_\tau - b - \rho)$ where $A(z^a)$ satisfies

$$A(z^a) = 1 + \beta(1-\delta)\mathbb{E}_{z^a}(1 - \delta_w(z^{a+1})) A(z^{a+1}) \ .$$

To solve for wages, note that for any wage offer w_τ in a firm of type z^τ,

$$\begin{aligned}
\rho = \frac{m(\lambda(z^\tau))}{\lambda(z^\tau)}\Big[W(w_\tau, z^\tau) - b - \beta U\Big] &= \frac{m(\lambda(z^\tau))}{\lambda(z^\tau)}\Big[W(w_\tau, z^\tau) - U + \rho\Big] \\
&= \frac{m(\lambda(z^\tau))}{\lambda(z^\tau)}\Big[A(z^\tau)(w_\tau - b - \rho) + \rho\Big] \ .
\end{aligned}$$

[44]Rudanko 2020 considers a model in which multi-worker firms apply an equal-treatment wage policy in the absence of commitment. The competitive-search equilibrium in this case is not efficient and it gives rise to endogenous wage rigidity.

This yields flat wages offered to new hires in a firm of type z^τ:

$$w_\tau = b + \rho + \frac{\rho}{A(z^\tau)} \frac{\lambda(z^\tau) - m(\lambda(z^\tau))}{m(\lambda(z^\tau))} \ .$$

3.B. Data

In this appendix we provide further details about the data.

Over the period 1995–2014 covered by the data, reporting on the number of hours changed and more than one classification standard of industries and products is used. Regarding the changes in reporting of hours worked, the first change is that all surveyed firms reported hours worked for the years up to 2006, but in subsequent years only firms with at least 50 workers reported working hours. The second change is that hours worked for white-collar workers were not included in the survey until 2002, whereas from 2003 onwards hours worked by all workers are reported. We deal with the second issue by imputing working hours of blue-collar workers on the other employees.

Between 2001 and 2002, the product classification standard changes from GP 95 (*Güterverzeichnis für Produktionsstatistiken* 95) to GP 2002; between 2008 and 2009, the standard changes from GP 2002 to GP 2009; and between years 2011 and 2012, the standard changes from GP 2009 to GP 2009 Version12. We first harmonize the product codes for the state of Mecklenburg-Vorpommern where GP 2002 in the year 2001 was used instead of GP 95 by applying the standard GP 95. Second, we remove all products that split or merge between classifications. We keep all products with the same measurement units, e.g., kilogram, meter etc, if they neither split nor merge between standards. We exploit available conversion codes from one standard to another when harmonizing the standards.[45]

The standards used for classifying industries are WZ 93 (*Klassifikation der Wirtschaftszweige*) from 1995 to 2002, WZ 2003 from 2003 to 2008, and WZ 2008 from 2009 to 2014. We convert all standards at the two-digit level to the WZ 2003 classification. The titles or descriptions of the standards WZ 93 and WZ 2003 are identical, allowing for a perfect conversion. When bringing WZ 2008 to WZ 2003, titles for four industries in WZ 2008 have no reasonable counterpart in WZ 2003. Likewise, five industries from WZ 2008 cannot be matched to WZ 2008. These industries are then left as they are. Further, we pool some two industries together and as a result of cleaning some industries are dropped.

Establishments in the data produce 2.46 products on average, with about 10 percent producing only one product and 55 percent producing more than five products. Table 3.4 presents the distribution of establishments, employment and revenue by size class. Over 40 (36) percent of revenue (employment) are concentrated in the largest five percent of establishments with more than 500 workers, while a third of establishments employ less than 50 (though 20 or more) workers.

Table 3.5 shows the percentage distribution of establishments, employment and revenue across two-digit industries. Observe that there are considerable size differences, with the largest establishments in the production of motor vehicles and the smallest establishments in recycling.

[45]See https://www.klassifikationsserver.de for these conversion codes. For the mapping of product classification GP 95 into GP 2002, the relevant document was downloaded from the internet, which can be shared upon request.

TABLE 3.4. Establishment Distribution by Employment Size (in %)

	Establishments	Employment	Revenue
20 − 49	33.65	6.78	4.81
50 − 249	53.27	37.78	34.69
250 − 499	8.28	18.76	20.08
500+	4.8	36.68	40.42

Source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.

TABLE 3.5. Establishment Distribution by Industry

	Establishment Share (%)	Employment Share (%)	Revenue Share (%)
Extraction of crude petroleum and natural gas	0.05	0.04	0.05
Basic metals	4.97	9.14	12.20
Chemicals and chemical products	5.60	11.45	15.89
Coke, refined petroleum products and nuclear fuel	0.17	0.18	0.36
Electrical machinery and apparatus n.e.c.	1.38	2.38	2.28
Fabricated metal products, except machinery and equipment	21.27	15.93	11.75
Food products and beverages	25.32	19.54	20.84
Furniture; manufacturing n.e.c.	0.63	0.74	0.73
Machinery and equipment n.e.c.	4.83	7.91	6.31
Medical, precision and optical instruments, watches and clocks	0.17	0.24	0.20
Motor vehicles, trailers and semi-trailers	1.05	4.82	4.30
Office machinery and computers	0.02	0.02	0.01
Other non-metallic mineral products	9.62	6.10	4.96
Other transport equipment	0.10	0.12	0.10
Pulp, paper and paper products	4.38	4.65	5.65
Radio, television and communication equipment and apparatus	0.04	0.07	0.06
Rubber and plastic products	11.62	10.77	8.97
Textiles	2.50	2.13	1.75
Tobacco products	0.03	0.04	0.04
Wearing apparel; dressing and dyeing of fur	0.09	0.07	0.05
Wood and of products of wood and cork, except furniture	3.10	2.10	2.26
Coal and lignite; extraction of peat	0.12	0.34	0.28
Other mining and quarrying	1.77	0.66	0.48
Publishing, printing and reproduction of recorded media	0.48	0.29	0.23
Recycling	0.47	0.27	0.29
Repair and installation of machinery and equipment	0.16	0.31	0.18

Data source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.

3.C. Extensions and Robustness

3.C.1. Alternative Product Market Parameters. Here we explore the consequences of re-calibrating several model parameters which are important for the customer dynamics in our model. First, in our benchmark calibration, we set $\bar{\delta}_b = 0.43$ so that only 57% of retail customers remain with the same producer from one year to the next. Here we show what happens if we set this parameter to the much lower value $\bar{\delta}_b = 0.1$. Second, parameter $q_0 = 2.18$ is calibrated such that a searching customer is matched with probability 1/2, an arbitrary number given the lack of direct evidence on matching processes between manufacturing and retail firms. Thus we compare the consequences of setting q_0 either to 50 percent (1.09) or 200 percent (4.36) of the benchmark level. Third, we target expenditures on sales to 4% of output, instead of 2% as in the benchmark calibration.

In all these experiments, we re-estimate all internally calibrated parameters. The model with lower customer separation rate requires a higher calibrated sales cost scale parameter ($s_0 = 0.0103$, about five times larger than the benchmark) to match the sales expenditure target. Likewise, the two alternative calibrations of parameter q_0 require mostly adjustments of parameter s_0 so that sales costs are roughly 2 percent of aggregate output, while all other re-calibrated parameters are similar to the benchmark model. The model with $q_0 = 4.36$ ($q_0 = 1.09$) has a customer matching rate of 81 (20) percent. Finally, the calibration with higher sales expenditures requires a much lower value of parameter γ (matching function elasticity). This induces firms to resort more to sales expenditures rather than discount prices in order to attract new customers.

Importantly, none of these alternative calibrations has a decisive impact on the dynamics of firm-level growth rates. This is shown in Table 3.6 which demonstrates that the model statistics of these alternative calibrations are rather similar to those of the benchmark model. The model with lower customer turnover ($\bar{\delta}_b = 10\%$) has however slightly lower rates of job creation and job destruction: firms prefer to adjust their employment a bit less if customer attrition is lower.

TABLE 3.6. The Impact of Alternative Product Market Parameters

	Benchmark	$\bar{\delta}_b = 10\%$	$q_0 = 4.36$	$q_0 = 1.09$	Sales cost 4% of output
$\sigma(\hat{p})$	0.199	0.197	0.188	0.201	0.186
$\sigma(\hat{q})$	0.334	0.336	0.336	0.336	0.336
$\sigma(\hat{e})$	0.129	0.127	0.120	0.126	0.127
$\sigma(\hat{y})$	0.393	0.385	0.392	0.393	0.383
$\rho(\hat{q}, \hat{e})$	0.298	0.224	0.326	0.312	0.206
$\rho(\hat{y}, \hat{e})$	0.583	0.526	0.585	0.585	0.512
$\rho(\hat{p}, \hat{q})$	-0.536	-0.567	-0.524	-0.537	-0.567
$\rho(\hat{p}, \hat{e})$	0.064	-0.044	0.024	0.024	0.118
$\rho(\hat{p}, \hat{y})$	-0.436	-0.510	-0.442	-0.451	-0.458
JC rate (%)	3.8	3.1	3.4	3.5	3.9
JD rate (%)	2.4	1.7	2.1	2.2	2.6

Note: See the notes of Table 3.2 for explanations.

3.C.2. Model with Measurement Error. If product quantities are measured with error in our data, a spurious negative correlation between product prices (measured as sales value divided by quantity) and output quantity would arise. Importantly, it would also lead to biased estimates of the autocorrelations and standard deviations of prices and quantity labor productivity which are key for our estimation of demand and productivity shocks. To deal with this concern, we introduce measurement error in output quantities into our model and then re-estimate our model again, based on model-generated statistics of quantity labor productivity and prices.

Specifically, suppose that true quantity labor productivity ($x =$QLP, quantity per unit of labor) follows the AR(1) process

$$\log(x_{it}) = \rho^x \log(x_{i,t-1}) + \sigma^x \varepsilon^x_{it} \ ,$$

with standard normally distributed ε_{it}, but that quantities (or labor hours) are measured with error, so that we measure in the data

$$\log(x_{it}^D) = \log(x_{it}) + m_{it} \ ,$$

where measurement error m_{it} is assumed to be normally distributed with mean zero and variance ν^2 and independently across firms and over time. Then

$$\text{cov}(\log x_{it}^D, \log x_{i,t-1}^D) = \text{cov}(\log x_{it}, \log x_{i,t-1}) = \rho^x \frac{(\sigma^x)^2}{1-(\rho^x)^2} \ ,$$

$$\text{var}(\log x_{it}^D) = \text{var}(\log x_{it}) + \nu^2 = \frac{(\sigma^x)^2}{1-(\rho^x)^2} + \nu^2 \ .$$

These two equations allow us to back out the AR(1) parameters ρ^x and σ^x, given our data estimates of the variance and autocorrelation of x_{it}^D and a given magnitude of measurement error ν.

For different values of ν, we can then proceed as follows: We re-calibrate parameters (ρ^x, σ^x) as described above and we set all directly calibrated parameters to the same values as before. All further, jointly estimated parameters, including the AR(1) parameters for demand shocks ρ^y and σ^y, are re-estimated based on the simulated model to which we add measurement error (of magnitude ν) in quantities. Compared to the benchmark model, the re-calibrated autocorrelation parameters ρ^x and ρ^y are larger, whereas the standard deviation parameters σ^x and σ^y are smaller.[46] The other re-estimated model parameters are rather similar.

In Table 3.7 we present the same statistics as shown in Table 3.2 when we introduce measurement error of magnitude $\nu = 5\%$ and $\nu = 10\%$ to those of the benchmark model. Most statistics are similar, except two of them: the negative correlations between price growth $\hat{p}$ and either productivity growth $\hat{q}$ or output growth $\hat{y}$ become more pronounced and even move closer to their data counterparts.

Table 3.7. The Impact of Measurement Error

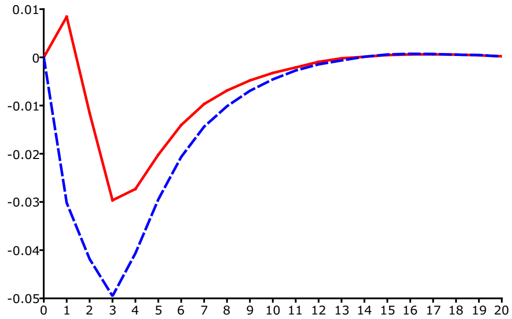
	Data	Benchmark	$\nu = 5\%$	$\nu = 10\%$
$\sigma(\hat{p})$	0.210	0.199	0.189	0.214
$\sigma(\hat{q})$	0.281	0.334	0.333	0.330
$\sigma(\hat{e})$	0.126	0.129	0.136	0.131
$\sigma(\hat{y})$	0.289	0.393	0.398	0.389
$\rho(\hat{q}, \hat{e})$	-0.122	0.298	0.323	0.296
$\rho(\hat{y}, \hat{e})$	0.227	0.583	0.612	0.588
$\rho(\hat{p}, \hat{q})$	-0.644	-0.537	-0.612	-0.693
$\rho(\hat{p}, \hat{e})$	0.003	0.064	-0.025	-0.038
$\rho(\hat{p}, \hat{y})$	-0.638	-0.436	-0.520	-0.600
JC rate (%)	2.9	3.8	3.9	3.7
JD rate (%)	3.0	2.4	2.5	2.3

Note: See the notes of Table 3.2 for explanations.

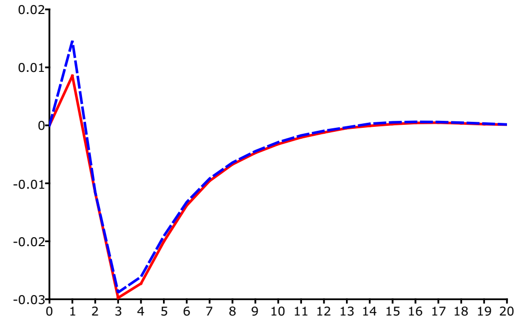
[46] With measurement error $\nu = 10\%$, we obtain $\rho^x = 0.688$, $\sigma^x = 0.238$, $\rho^y = 0.747$ and $\sigma^y = 0.128$.

3.C.3. Small Surplus Calibration. We consider an alternative calibration of our model in which we set the flow income from unemployment (parameter b) to a much higher value so that the surplus of a job is smaller and aggregate shocks can potentially generate a larger labor market response. To this end, we set b equal to 93 percent of the average wage and we recalibrate parameters K (entry cost), r_0 (recruitment cost scale), s_0 (sales cost scale) and m_0 (matching function scale) to match the same calibration targets as in the benchmark model: firm size, sales and recruitment costs (shares of output) and the unemployment rate. Parameters governing idiosyncratic productivity and demand processes remain the same.

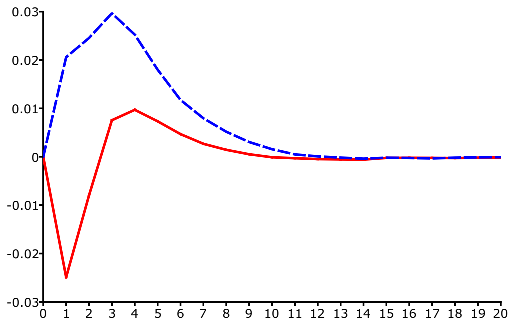
Figure 3.7 shows impulse responses to negative five-percent declines of aggregate productivity and aggregate demand parameters. In contrast to Figure 3.4, these first-moment shocks generate sizable declines of output and employment. Interestingly, both of these shocks also induce an increase of price and output growth dispersion, as shown in the bottom graphs of Figure 3.7, although they are relatively small in magnitude in comparison to the data (panel (b) of Figure 3.3).



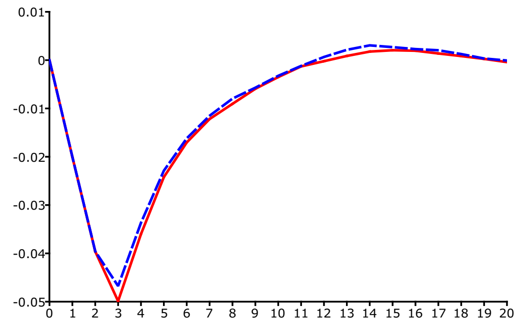
(A) Output



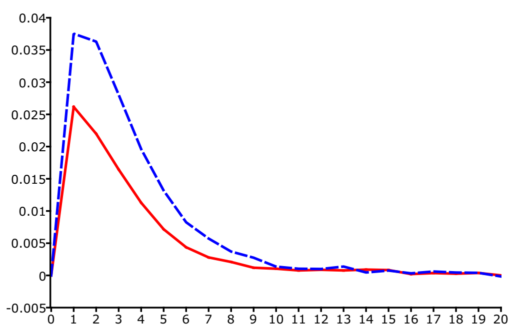
(B) Employment



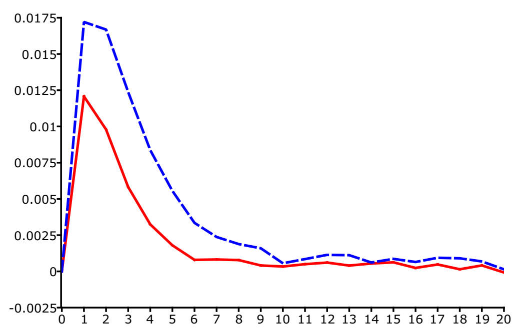
(C) Price



(D) Firms



(E) Standard deviation of output growth



(F) Standard deviation of price growth

Figure 3.7. Responses to a five percent decrease of aggregate productivity (dashed, blue) and aggregate demand (solid, red) in the model with small job surplus.