

# Nicht-Linearitäten in der Phillips-Kurve?: Empirische Belege für einige Europäische Länder

Wissenschaftliche Arbeit  
zur Erlangung des Grades eines Diplom-Volkswirtes  
im Fachbereich Wirtschaftswissenschaften  
der Universität Konstanz

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Bearbeitungszeit: 8. Dezember 2004– 8. Februar 2005

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Konstanz, den 07.02.2005

*To my parents, Verena, and Erling*

**Abstract:**

This dissertation studies the question of nonlinearities in the Phillips curve relationship in France, Germany and Italy. The implications from the theoretical models are that the mechanisms that make the Phillips curve nonlinear can work through different channels. Therefore, this thesis not just asks whether the Phillips curve is nonlinear but also why it is nonlinear. The results from logistic smooth transition regressions provide evidence for different mechanisms in Germany, compared to France and Italy. However, the results for France and Italy are rather mixed which might be due to the structural change during the mid 1980s.

Special thanks to Julian Callow for giving the idea for the topic, Laurence Boone, Nick Matthews, and James Nixon for all their helpful comments that created the starting point for this thesis. Special thanks also to Stefan Bergheim for his ideas and support during my studies. Furthermore many thanks to Andrea Salfner, Nick Netzer, and Michael Rauber for their comments and corrections, and Bernhard Obenhuber for essential software that made this dissertation possible.

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# 1. Introduction

## 1.1 Motivation

In a seminal paper published in 1958, A.W. Phillips studied the relationship between the rate of unemployment and the rate of change of wages in the United Kingdom between 1861 and 1957. He found that there existed an inverse relationship between the rate of unemployment and the rate of increase in money wages. The higher the rate of unemployment, the lower the rate of wage inflation. This relationship came to be known as the “Phillips curve”. Today, economists use the term Phillips curve to denote the link between inflation and the level of economic activity (see paragraph 1.2). This relationship is one of the main transmission mechanisms of monetary policy. An important element in the monetary policy strategy of the European Central Bank (ECB) is that price stability has to be maintained over the medium term. Therefore, the slope of the Phillips curve is of great importance, as it determines the output loss from bringing inflation down. If this trade-off between economic activity and inflation was linear and stable, there would be no need for avoiding excess demand pressure. In this case, the costs of wringing inflation out of the economy, once generated by high excess demand, are the same as the gains in terms of output that initially generated inflation. Avoiding large swings in the cycle might be desirable for other reasons, but a linear Phillips curve does not justify that. Therefore, the reaction of monetary policy to an increase in excess demand should be the same independent of the initial state of the business cycle or the current level of inflation. These points are not consistent with the stylized facts of business cycles, which suggest an asymmetric relationship between output and inflation<sup>1</sup>. Already Phillips (1958) discovered that the relationship between unemployment and wage inflation was nonlinear. He wrote

*"When the demand for labour is high and there are very few unemployed we should expect employers to bid wage rates up quite rapidly (...). On the other hand it appears that workers are reluctant to offer their services at less than the prevailing rates when the demand for labour is low and unemployment is high so that wage rates fall only very slowly. The relation between unemployment and the rate of change of wages is therefore likely to be highly nonlinear"*

*A.W. Phillips (1958), p. 283*

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<sup>1</sup> See De Long and Summers (1988) for evidence on asymmetries in business cycles.

The implications of such a convex Phillips curve for macroeconomic policy give the motivation for stabilising the output around its potential. Monetary policy makers have to be more aggressive in fighting inflation during strong excess demand, as bringing inflation back to initial level is more costly than the benefits from the initial increase. Therefore, monetary policy has to be more forward looking and it is more important to be aware of the current state of the business cycle. Another possibility is that the slope of the Phillips curve is higher during periods of higher average inflation. Therefore, decreasing inflation during periods of low average inflation is more costly than during high average rates of inflation. This would also imply that it is important to hold inflation at moderate levels to avoid the need for a costly decrease of the inflation rate. Furthermore, a stabilizing policy is easier to carry out at low inflation rates, as an increase in excess demand has initially low effects on inflation and therefore leaves time to react. When thinking about the shape of the Phillips curve, some economists have proposed that it could also be concave<sup>2</sup>. This would imply that an overheating of the economy is less inflationary than a recession and therefore be an argument against aggressive monetary policy in periods of high excess demand. In this case, a temporary rise in inflation can permanently lower unemployment and increase output.

Consequently, different shapes of the Phillips curve have considerably different implications for monetary policy. Therefore, it is important for monetary policymakers to precisely measure the slope of the curve. Furthermore, it is just as important to determine the underlying macroeconomic conditions that cause an increase or decrease in the slope. Hence, the usual linear econometric technique to estimate the shape of the Phillips curve is a generalization that neglects information that is of central importance for the strategy of monetary policy.

## 1.2 A General Representation of the Phillips Curve

The original Phillips curve relationship can be written as

$$g_w = -\beta(u - u^*) \quad (1)$$

where  $\beta > 0$  measures the responsiveness of wages to unemployment,  $g_w$  is the rate of wage inflation,  $u$  is the unemployment rate and  $u^*$  is the natural rate of unemployment,

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<sup>2</sup> See Stiglitz (1997) and Eisner (1997).

and  $u-u^*$  is the unemployment gap. However, this simple long run relationship between the change in wage inflation and unemployment is not as straightforward as it seems. When workers and firms bargain over wages, they are concerned about the real value of wages. If they expect the general price level to increase, they will include their expectations of inflation into wage contracts. Therefore, unemployment depends not on the level of inflation, but rather on the excess of inflation over that which was expected<sup>3</sup>. Therefore, the original wage-inflation Phillips curve from equation (1) can be rewritten as an expectation augmented wage-inflation Phillips curve

$$(g_w - \pi^e) = -\beta(u - u^*) \quad (2)$$

where  $\pi^e$  is the level of expected price inflation. Because the prices a company charges are closely connected to the wages it pays, economists use Phillips curves to relate general price inflation (as opposed to wage inflation) to unemployment rates. Under the assumption of a constant real wage, actual inflation  $\pi$  equals wage inflation and hence (2) can be rewritten as the expectations augmented Phillips curve

$$(\pi - \pi^e) = -\beta(u - u^*). \quad (3)$$

The link between output and unemployment is being explained by Okun's Law. Using Okun's law, the price inflation Phillips curve can be transformed into the output-inflation Phillips curve<sup>4</sup>

$$(\pi - \pi^e) = \beta(y - \bar{y}) \quad (4)$$

where  $y$  is output and  $\bar{y}$  potential output and the difference between the two is the output gap.  $\beta$  is the trade-off parameter, which is the essential parameter representing the transmission channel for monetary policy. Therefore, estimating  $\beta$  as accurately as possible is of great importance for monetary policy decision makers.

### 1.3 Outline

The next chapter gives an overview of the theoretical literature that forms the foundation for a nonlinear Phillips curve. The theoretical background is especially important as different models imply different shapes of the Phillips curve and therefore have different implications for monetary policy. This is an important issue, which the

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<sup>3</sup> See Friedman (1968).

<sup>4</sup> Blinder (1997) studies the relationship for the US and notes that Okun's law "closes the loop between real output growth and changes in unemployment with stunning reliability" (p.241).

majority of empirical studies on nonlinearities in the Phillips curve do not consider. These issues are discussed in the review over the empirical literature in chapter three. The empirical part of this study begins in chapter four with analysis of the largest European economies<sup>5</sup> France, Germany and Italy. In order to detect a change in the relationship, it is important to include periods of both high and low inflation. Over the course of the last couple of years, inflation in France, Germany, and Italy has been stable at historically low levels for longer periods of time. Consequently, the investigation of the behaviour of the Phillips curve in periods of low inflation has become possible as the low-inflation data series have sufficient length to detect possible changes in the relationship. In chapter five Phillips curves are estimated using a nonlinear estimation technique. The technique allows, at least to a certain degree, to identify empirically which of the theoretical models is the underlying model for the possible nonlinearity of the Phillips curve. Most of the previous studies did not consider several theoretical models that make the relationship nonlinear and rather assumed one of them, thereby neglecting other possible important dynamics. The empirical part of this paper studies the type of nonlinearity that is supported by the data using an approach that allows the data to determine whether the relationship between real activity and inflation is better described by a nonlinear specification, rather than by a linear one. Furthermore it will determine what theoretical foundation of nonlinearity is preferred by the data. Hence, this study will take theory as a guide but let the data decide, if there is nonlinearity in the Phillips curve, and if so, what type of nonlinearity. The results of the empirical studies provide evidence for a convex Phillips curve, whereas the source of the convexity appears to be different in Germany, compared to France and Italy. The policy implications for the identified underlying models are given in chapter six. Chapter seven concludes and gives an outlook for future research.

## **2. Theoretical Foundations of Nonlinearities in the Phillips Curve**

There exist several theoretical approaches to model the relationship between inflation and real activity. As the aim of this study is to investigate the possibility of nonlinearities in the Phillips curve, this chapter surveys four theoretical models that

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<sup>5</sup> In terms of real GDP.

imply a nonlinear Phillips curve. These models have in common, that they predict that the slope of the Phillips curve is a function of macroeconomic conditions, and is therefore nonlinear. The models either differ in the source of nonlinearity or predict different shapes of the curve. This chapter therefore gives special emphasis to the determinants of the nonlinearity and the shape of the Phillips curve. Formal models will not be presented, as these can be found in the referenced literature. The purpose of this chapter is to give a theoretical background for the empirical work to be conducted. Furthermore, it is essential to compare these models and give the intuition behind them, especially focussing upon the mechanisms determining the shape of the Phillips Curve. Sections 2.1 to 2.4 each present a model verbally, explaining the theoretical framework, main assumptions and implications for the shape of the Phillips curve. The empirical evidence based on micro data supporting the models' assumptions will also be presented. Section 2.5 compares the models and summarizes the relevant implications.

## 2.1 The Capacity Constraint Model

The capacity constraint model relies upon the assumption that firms find it difficult to increase their production capacity in the short run. Due to bottlenecks in the production process, inflation accelerates more during periods of high aggregate demand than during periods of low demand. The analytical derivation of the model presented here can be found in Evans (1985).

### 2.1.1 The Theoretical Framework

The model presented by Evans (1985) works in a simple theoretical framework. The economy's output consists of  $n$  different goods that are produced in  $n$  different heterogeneous sectors. Demand for good  $i$  ( $i=1, \dots, n$ ) is determined by the relative price of the good, aggregate output, and an exogenous taste parameter. The prices are flexible and market clearing. The firms use labour as the only input factor and each sector is faced with a specific labour market<sup>6</sup>. For each sector exists specialized workers, hence

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<sup>6</sup> This model has been used as an example to show how the shortage of the input factor labour affects the shape of the Phillips curve. Translating this model in a more realistic framework, firms may be faced with bottlenecks in all input factor markets. Examples are the shortage of labour, material inputs, capital, and of technological progress. See Macklem (1997) for an overview of possible sources for capacity constraints.

each sector is faced with a different labour supply. The labour market is characterized by the assumption that workers in sector  $i$  only supply a fixed amount of labour, which cannot be exceeded. Furthermore, there exists a unique minimum wage floor in each sector. Thus, there exist two different states in the labour market for a particular sector. One state is that the labour market for sector  $i$  is in excess supply. There exists unemployment due to the assumption of a minimum wage and employment is determined by demand, whereas the wage is fixed at its minimum level. The other possible state is when labour market is in excess demand. The wage is determined by demand while supply is equal to its maximum. This second situation can also be referred to as a bottleneck, as firms wish to produce more but are constrained by the shortage of an input factor. Hence, the short side of the labour market determines the production of goods in each sector<sup>7</sup>. Dynamic is included into the model by allowing the adjustment of the minimum wage in each sector to be a function of past excess demand in the sector. Workers can move between the sectors over time. This assumption can be justified by the assumption that workers that switch jobs to a different type of job have to get time consuming training before they can work in another sector. Net inflow in a sector depends on the expected shadow relative wage<sup>8</sup>. In the short run, the distribution of relative wages over the  $n$  sectors determines the degree of imbalance in the economy. This imbalance is reflected in the proportion of sectors facing shortages in their specific labour supply, which is denoted by  $b$  ( $0 \leq b \leq 1$ ) henceforth. Thus, the share of the firms reacting to a demand shock by increasing output is  $1-b$  and the share  $b$  reacts by changing the price of their good. When aggregating over the economy this implies that an increase in aggregate demand yields to more and more firms being constrained by capacity. Hence,  $b$  increases and  $1-b$  decreases. This also implies that more firms react by changing their price and less by changing their output. With a higher  $b$ , the share of firms faced with bottlenecks in their production<sup>9</sup>, inflation is more reactive to an increase in aggregate demand. Considering the whole economy,  $b$  rises with aggregate demand, and therefore inflation is more sensitive to an increase in aggregate demand

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<sup>7</sup> This condition is referred to as the 'min' condition (Barro and Grossman, 1976).

<sup>8</sup> The difference between the market clearing wage in the sector and the average wage in the economy is defined as the shadow relative wage in the sector.

<sup>9</sup> The model also has a long run equilibrium that is different to the short run implications, as there is dynamic assumed in the economy by a shifting taste parameter and labour that shifts flexible over the sectors in the long run. These long run implications, however, are relevant for the discussion of an asymmetric short run Phillips curve, hence it will not be presented in this study. The cases when  $b=0$  and  $b=1$  can be interpreted as the Keynesian and the classical case respectively.

when current aggregate demand is already high. This yields an upward sloping, nonlinear aggregate supply curve<sup>10</sup>.

### 2.1.2 The Implied Phillips Curve Relationship

The Evans model described above implies a convex short run aggregate supply curve. The main assumptions for this result are the sector specific constrained labour supply and the minimum wages. The heterogeneity of the sectors allows the aggregate supply curve to be a curve<sup>11</sup>. Furthermore, the labour supply is restricted to one sector in the short run, but not in the long run. As a result of the constraint, a sector that is faced with strong demand, cannot exceed production above a maximum, which is the amount that can be produced with the sector specific labour supply. By aggregation over all the sectors, the slope of the aggregate supply curve increases with the share  $b$  of firms that produce at their maximum capacity and cannot supply enough to meet demand for their good. For the cyclical behaviour of the economy this means, that as a response to an increase in aggregate demand, there comes a point in the cycle, when demand begins to outstrip supply and inflation will begin to rise. Hence, the trade-off between inflation and output is higher at states of economy with higher excess demand than at moderate levels of excess demand. This implies that the slope of the Phillips curve depends on the state of the business cycle and hence a general formulation according to equation (4) of the Phillips curve relationship implied by this model is

$$(\pi - \pi^e) = f(y - \bar{y}) \quad (5)$$

with  $f'(y - \bar{y}) > 0$  and  $f''(y - \bar{y}) > 0$  where  $(y - \bar{y})$  is the output gap and  $\pi$  is inflation.

### 2.1.3 Empirical Evidence from Micro Data

The capacity constraint model outlined in the previous subsection gives a theoretical framework for a nonlinear Phillips curve. As the purpose of this study is to detect whether nonlinearities can be found in the data, the first step is to see whether there exists empirical evidence from micro data, which justifies the assumptions of the model. If the source of the output inflation trade-off were capacity constraints, then these constraints should be detectable in the data on the firm level for the countries considered

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<sup>10</sup> See Evans (1985) for a proof.

<sup>11</sup> Assuming that all sectors are equal would yield an L-shaped aggregate supply curve.

in this study. The ECB (2002) has published a study that investigates the European labour market. The study finds that the Euro Area labour market is characterized by “the existence of high unemployment and, at the same time firms in euro area countries report difficulties in recruiting workers” (ECB (2002), p.4). This coexistence of unsatisfied labour supply and labour demand suggests that there is a difficulty in the matching process. Firms have difficulties increasing their labour input in the short run. The labour supply itself might not be the cause of the problem, but rather the difficulty in finding workers and the initial training of newly hired employees. This empirical finding is consistent with the assumption that firms have difficulties increasing production capacity in the short run. The result shows that the capacity constraint model as presented by Evans (1985) might well rely upon assumptions that exist in the countries considered in this study.

## 2.2 The Costly Adjustment Model

The costly adjustment model implies an asymmetric output inflation relationship, where the slope of the Phillips curve changes with the average level of inflation. The main assumption of this model is that of menu costs, which creates a trade-off between output and inflation. Ball *et al.* (1988) and Ball and Mankiw (1994) present a theoretical model where menu costs make it costly for firms to adjust their prices and hence, at low levels of inflation, the loss from suboptimal prices can be less than the costs that the firm incurs for changing prices. Therefore, menu costs cause asymmetric price adjustments, which create an asymmetric relationship between output and inflation.

### 2.2.1 The Theoretical Framework

As the name already suggests, the main assumption of this model is the presence of menu costs, which are fixed and predetermined and have to be paid whenever changing the nominal price of a good<sup>12</sup>. The assumption of monopolistic competition allows for price setting. Furthermore, it is assumed that the price setting is staggered, which means that only a share of the firms set their price each period. Assume that an economy with  $n$  heterogeneous sectors is faced with a sector specific demand shock in sector  $i$

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<sup>12</sup> The formal model is presented in Ball *et al.* (1988).

( $i=1, \dots, n$ ), which causes the profit maximising relative price in sector  $i$ ,  $p_i^*$ , to change. Under the assumption, that it is costly to change the price, a firm in this sector is faced with a choice: it either pays the menu costs and adjusts the price to  $p_i^*$ , or it takes the loss from its current relative price  $p_i$  being different from the profit maximising price  $p_i^*$ . This profit maximising relative price changes each period, as the overall price level changes every period (caused by other firms changing their prices), whereas the nominal price of firm  $i$  does not. As the menu costs are fixed and predetermined, each firm  $i$  at time  $t$  ( $t=1, \dots, T$ ) computes an optimal value  $\lambda_i$  for the time period for that the firm will keep its price fixed. This period  $\lambda_i$  denotes the time between two price changes, which are changed by paying the menu costs. The price set by the firm is simply the average of the expected loss-minimizing prices during the period  $[t, t+\lambda_i]$ . During periods of low inflation, the loss from not adjusting the price  $p_i$  to its optimal value  $p_i^*$  is low, hence the frequency of price adjustment is lower (i.e.  $\lambda_i$  is higher). Thus, the frequency of price adjustments  $\lambda_i$  is implicitly determined by the price level. And the other way round, the price level is determined by  $\lambda_i$  for all  $i=1, \dots, n$ . Ball *et al.* (1988) show, that there exists a unique symmetric Nash equilibrium for the choice of all firms, where all firms set their frequency to be the same  $\lambda$  for a given average rate of inflation. The assumption of staggered price setting implies that there are  $\lambda$  groups of firms, which change their prices simultaneously, and each group changes their optimal price at another point in time during the interval  $[t, t+\lambda]$ . The frequency  $\lambda$  is an increasing function of the average rate of inflation. Hence, shocks to demand influence inflation slowly in periods of low inflation, since firms change their prices infrequently. During periods of high average inflation, the aggregate price level reacts more quickly to a given shock in aggregate demand. This means that the slope of the short run Phillips curve is steeper in periods of high inflation, as inertia is lower, and positive demand shocks push up prices more rapidly. This mechanism can be assumed for the wage setting process as well. The assumption of costly adjustment in the wage setting process seems plausible, as negotiations are costly for both firms and workers, implying that it is not optimal to have instantaneous contracts. Furthermore, the level of inflation affects the optimal duration of contracts, as in a world of low inflation both firms and workers are better off with longer contracts. In addition, in a decentralised capitalist economy labour contracts are typically staggered, which means that aggregate wages always exhibit some level of stickiness, no matter what the frequency and size of shocks are, since it takes time for all wage contracts to be revised. The mechanism is the same as

for the costly adjustment of prices. Hence, in periods of low inflation, longer wage contracts are more frequent in the economy.

A second model that yields an asymmetric output-inflation relationship is a model from Ball and Mankiw (1994). The model works in a similar framework, with the assumption of the presence of positive trend inflation. Furthermore the model is modified by the assumption that firms have to pay menu costs only when they wish to adjust their prices between a fixed period  $[t, t+\lambda]$ . Price changes at  $t$  and  $t+\lambda$  are costless. In the model described above, inflation determines the length of period  $[t, t+\lambda]$  endogenously. In this model, trend inflation causes firms relative prices to decline automatically in the period between price adjustments. In this environment, positive shocks to firms' desired prices trigger greater adjustment than do negative shocks of the same size. Therefore, positive and negative shocks to demand have different effects on price adjustments and thereby to inflation. Prices are more flexible upwards than downwards, given a positive and negative demand shock of the same absolute size. This implies a Phillips curve that is steeper at periods of higher trend inflation<sup>13</sup>.

### 2.2.2 The Implied Phillips Curve Relationship

Both models imply that the level of inflation determines the shape of the Phillips curve, which is convex in both cases. The assumption of menu costs causes a trade-off between inflation and output<sup>14</sup>. The main difference between the two models is that in the first model presented, the frequency of price changes is determined by the average rate of inflation. The frequency is higher in periods of high average inflation. The

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<sup>13</sup> Burstein (2002) further develops this model by allowing the firms to make price plans for longer periods, where also the size of the period  $\lambda$  can be changed by paying a fixed cost. The findings from this model is quite similar to the finding from the model presented in this paper: The Phillips curve becomes steeper in periods of high inflation and flatter with moderate Inflation. For the purpose of this paper, the main point is that the short-run trade-off between real activity and inflation in the costly adjustment model implies a convex relationship, with the slope being determined by the level of inflation.

<sup>14</sup> One could argue that small menu costs cannot explain the size and duration of macroeconomic fluctuations. This would imply that the costly adjustment models are not realistic and hence not relevant for the explanation of a possible convexity of the short run Phillips curve. This argument is not necessarily true. The effect of small menu costs on the price setting behaviour of firms can cause large effects on the macroeconomic level as described in Mankiw (1985). He shows that the costs from a non-adjustment are second order for a firm, even when taking into account the macroeconomic disturbances. The social costs of these non-adjustments, however, are first order. Hence, a rationally acting firm has incentives not to adjust its price. If many firms in the economy have these incentives, this can cause large business cycles in the economy. In a similar argumentation, Akerlof and Yellen (1985) also show that, under the assumption of near-rationality, non-maximising behaviour of firms can only cause "insignificant" costs to the single firms but can have large effects to the business cycle.

second model assumes that the frequency is predetermined and the menu costs have to be paid only if firms wish to make price adjustments between the fixed periods. Therefore, the assumption of trend inflation makes the sensitivity of the price level to a positive demand shock larger than to a negative shock, as a positive demand shock is partially offset automatically by inflation. In both models, menu costs are the reason why firms do not adjust their prices instantly and both imply a convex Phillips curve, where the slope of the curve is determined by inflation. Therefore, the Phillips curve derived from the costly adjustment model can be expressed as

$$(\pi - \pi^e) = \beta(\bar{\pi}) \cdot (y - \bar{y}) \quad (6)$$

with  $\beta(\bar{\pi}_1) < \beta(\bar{\pi}_2)$  and  $\bar{\pi}_1 < \bar{\pi}_2$ , where  $\bar{\pi}$  is the average inflation rate and  $\pi^e$  the expected inflation rate<sup>15</sup>.

### 2.2.3 Empirical Evidence from Micro Data

Cecchetti (1986) collected data on U.S. magazine prices and found that prices are changed, on average, every seven years during the 1950s but every three years during the high inflation of the 1970s. Buckle and Carlson (1995) also found that the average duration between price changes is shorter in periods of higher inflation. These empirical findings give support to the predictions from the costly adjustment model: The adjustment occurs more frequently during high inflation periods and hence nominal rigidities are decreased, which makes inflation less reactive to changes in nominal demand. The argument that menu costs are very small and trivial might have to be given a further thought. Levy *et al.* (1997) find that the costs of changing prices in U.S. supermarkets amount to between 0.5 percent and one percent of revenues. Hoffmann and Kurz-Kim (2004), Baudry *et al.* (2004), and Veronese *et al.* (2004) study the behaviour of price setting in Germany, France and Italy respectively using micro data from the CPI components<sup>16</sup>. The studies found that the average period between price changes in the components of the harmonized index of consumer prices (HICP) is about 15 to 20 months, excluding rents. These studies confirm that, on the micro level, pricing frequency depends upon the inflation rate. Therefore, a convex Phillips curve, where the slope of the curve is determined by the average level of inflation, has a theoretical

<sup>15</sup> All other notations remain as already specified above.

<sup>16</sup> All these papers use the components of the prices collected by the respective national statistical offices to compute the HICP. These prices are highly disaggregated and therefore give a good picture of the pricing behaviour on the firm level.

foundation and empirical support from micro data for the countries considered in this thesis.

## **2.3 The Downward Nominal Wage Rigidity Model**

The third theoretical model that implies asymmetries in the Phillips curve is the downward nominal wage rigidity model. This model is based on the idea that workers are more reluctant to accept a decrease in their nominal wages than in their real wages, due to money illusion, institutional or behavioural factors. Yates (1998) and Stiglitz (1986) give excellent overviews of the theories that explain the observed rigidities in wages. This section gives an overview of a general framework for the theories of downward nominal wage rigidity.

### **2.3.1 The Theoretical Framework**

The concepts of money illusion and wage fairness are used in this model to derive the asymmetric Phillips curve relationship. If employees suffer from money illusion they will accept real wage decreases more readily if nominal wages do not decrease. This assumption implies that wages are not flexible enough to be at a market clearing efficient level, which in this model is the reason for the classical dichotomy to break down. This implies that the real effects of negative shocks to demand could be diminished by allowing steady positive inflation to erode real wages. In periods of low inflation, the effect of a negative demand shock would have higher effects on unemployment than a positive shock of the same size. This is due to the fact, that the shock cannot be offset by a decrease in real wages. Therefore, at low levels of inflation, excess supply would have less impact over inflation and more impact over real activity than excess demand shocks. This means that there is a clear asymmetry at these low inflation levels. Akerlof *et al.* (1996) show in a model under the assumption of money illusion of workers, that positive inflation rates lead to more efficiency and that low inflation can have long-run effects on the real economy.

The assumption of efficiency wages yields to downward nominal wage rigidity<sup>17</sup>, too. The assumption of efficiency wages is that workers are more productive when above market clearing wages are paid. According to this hypothesis employees become more productive when wages are adjusted upwards. This implies that firms find it in their best interest to raise wages quickly in response to positive shocks in order to keep productivity high. When faced with negative shocks, firms are more hesitant to adjust real wages as they wish to increase productivity. As a result, positive demand shocks have less impact over the real economic activity and more over the inflation rate. The analogous argument can also be made for prices<sup>18</sup>. This requires the assumption that customers are not able to perfectly observe the quality of a good before buying it. Whenever a customer buys a specific good  $i$ , he assumes that firm's prices are set according to marginal costs and hence a fall in the nominal price of the good is interpreted as a fall in quality. This argument also relies on the assumption that customers suffer from money illusion, as they cannot distinguish between falls in the aggregate price level and a fall in the relative price of a good. Hence, firms might be inhibited from lowering prices, as they fear that customers might interpret this as a fall in quality.

### 2.3.2 The Implied Phillips Curve Relationship

Put together, the downwards nominal wage rigidity model implies an asymmetric relation between inflation and unemployment in the short-run. The determinant of the slope of the Phillips curve is inflation, like in the costly adjustment model. The slope of the Phillips curve should be lower at low inflation rates and steeper when inflation is high; therefore, the curve is also convex and the slope is determined by the level of inflation. Therefore, the models implications for the Phillips curve are the same as for the costly adjustment model, which are expressed in equation (6) in the previous section.

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<sup>17</sup> Stiglitz (1986) presents the formal model. A general explanation is given for instance in Blanchard and Fisher (1989, ch 9).

<sup>18</sup> See Stiglitz (1984).

### 2.3.3 Empirical Evidence from Micro Data

The question of whether downward nominal wage rigidity is present in the wage data was investigated by several studies. Kahneman *et al.* (1986) found in US survey data, that most workers find nominal wage cuts to be unfair. Holden and Wulfsberg (2004) have recently investigated the existence of downward nominal wage rigidity in Europe. Using industry data, they found that the fraction of wage cuts prevented due to downward nominal wage rigidity has fallen over time, from 70 percent in the 1970s to 20 percent in the 1990s, but the number of industries affected by downward nominal wage rigidity has increased. Dessy (2002) explored the distribution of wage changes in the European countries and found that the changes in nominal wage cuts show clear evidence for downward nominal rigidity. These studies of micro data show that there is empirical evidence for the existence of downward nominal wage rigidity. The main part of this paper is to check empirically, whether this phenomenon has effects on the shape of the Phillips curve, as shown by the model in this paragraph.

## 2.4 The Strategic Pricing Model

This section gives an example for a theoretical model that implies a concave shape of the Phillips curve. What is denoted as the strategic pricing model henceforth was originally presented in Stiglitz (1984). The intuition behind the model is that in monopolistically competitive markets, firms are less threatened by the entry of potential competitors in a recession and hence charge higher prices than they do during boom phases. In expansions, the firms in the market keep prices down to avoid entrance of competitors. Therefore, the price level is less sensitive to a positive shock to demand than to a negative shock.

### 2.4.1 The Theoretical Framework

Assume that an economy consists of two markets, where different goods are produced. One is a competitive and the other is an oligopolistic market. In the oligopolistic market, potential entrants do not know what the marginal costs of the firms in that market are. What they know, is that the marginal costs are either  $c_1$  or  $c_2$  with  $c_1 < c_2$ . Only if the

costs are equal to  $c_2$  will the market entrance be profitable for the potential entrants. For simplification, let the firms set prices for two periods with discount factor  $r$ . Potential entrants observe the prices set in period one and consequently decide about entering the market in period two with probability  $\omega$ . The firms currently in the oligopolistic market know that the potential rivals do not know the marginal cost function. If their costs are equal to  $c_2$  the firm in the oligopolistic market charges the lowest possible price, which is  $p_2=c_2$ . If their costs are  $c_1$  firms charge a price  $p_1$  that is sufficiently low to keep potential entrants away from entering the market. The potential entrants observe the price set in period one. If the price is equal to  $p_2$ , they enter the market with probability  $\omega$ . This set up results in a standard screening self-selection problem. A firm with costs  $c_1$  has to decide upon its price such that the profit is maximised, taking into account the signalling effect of its price. Let the profit function of a firm in the market be  $R(p_i, c_i)$  with  $i=1,2$ . In a two period setting, the low-cost firm charges the price that satisfies the condition that the price  $p_1$  is set sufficiently low such that the high cost firm can not imitate it, which can be expressed in the condition

$$R(p_2, c_2) \left( 1 + \frac{(1-\omega)}{1+r} \right) = R(p_1, c_2) \left( 1 + \frac{1}{1+r} \right). \quad (7)$$

This equation defines the price  $p_1$  set by a low cost firm, as all other variables are fixed. The left hand side of equation (7) is the sum over the two periods of the profits for a high cost firm, weighted by the probability of entrance  $\omega$ . The second period is discounted by the discount factor  $r$ . If other firms enter, the profit of a firm with costs  $c_2$  in period two is assumed to be zero. If  $\omega$  is equal to zero, the low-cost firm charges its highest possible price  $p_1=p_2$ . With an increasing probability of entering the market, the price  $p_1$  decreases. Assume that outsider firms have more potential to enter a market during expansions than during recessions and hence

$$\frac{\partial \omega(y - \bar{y})}{\partial (y - \bar{y})} > 0 \quad (8)$$

where  $y - \bar{y}$  is the state of the business cycle with a positive value reflects a boom and a negative a recession. Given that  $p_2$  is fixed at  $p_2=c_2$ , solely  $p_1$  determines the aggregate price level. By equation (7)  $p_1$  is implicitly determined by  $\omega$ , and  $R(p_1, c_1)$  is an increasing function in  $p_1$ . Hence, from equation (7) follows that  $p_1$  is decreasing in  $\omega$  which implies by (8) that  $p_1$  is a positive, concave function of the output gap. Intuitively, this means that the higher the probability of entering the market, the lower is the price charged by the low cost firms, which allows them to signal that entering the market would be unattractive.

### 2.4.2 The Implied Phillips Curve Relationship

The Stiglitz model implies a concave shape of the Phillips curve. The probability of market entrance increases in expansions and hence the mark-up over marginal costs is tightened. During expansions, firms are reluctant to raise their prices as they wish to keep out new competitors. Hence, if the price level outside the oligopolistic market increases as a response to an increase in aggregate demand, the monopolistic firms slow down the increase in the aggregate price level as they increase the price by less relative to the firms in the competitive market. This makes inflation less sensitive to an increase in demand in an expansion than in a recession. Rewriting equation (4) according to the strategic pricing model yields

$$(\pi - \pi^e) = f(y - \bar{y}) \quad (9)$$

with  $f'(y - \bar{y}) > 0$  and  $f''(y - \bar{y}) < 0$ . The other three models have been supported by empirical evidence from micro data. Given the strategic pricing and that the market form is assumed, evidence from micro data should be hard to find and does not yet exist, as far as this study could find. Hence, the empirical support for this model has to be found in the macroeconomic data.

## 2.5 Conclusions from Theory

The models presented above all give a theoretical underpinning for a nonlinear Phillips curve. The main assumptions and the implications for the determination of the shape of the short run aggregate supply curve are summarised in Table 2-1. The capacity constraint model differs from the other convex models as the source of the curvature lies in the shortage of input factors, while the other convex models derive the Phillips curve relationship from price setting behaviour. The increase in inflation as a response to a positive shock in aggregate demand in the costly adjustment and downwards nominal wage rigidity model is higher at lower rates of inflation. These models are very similar from the point of view that high inflation forces prices and wages to be set at more efficient levels. During low inflation periods, inertia is higher in both models. Therefore, the determinant of the slope of the Phillips curve is the average level of inflation. The capacity constraint model assumes that prices are fully flexible, but the input factor labour is restricted in the short run and there exists minimum wage floors. The Stiglitz model differs from the others by the implied concave shape, resulting from strategic

behaviour of oligopolistic firms. Here, prices do not respond symmetrically to a change in aggregate demand for strategic reasons and the determinant for the slope of the Phillips curve is the state of the business cycle. For the question of the nonlinearity of the Phillips curve, it is important not just to determine the shape of the curve, but also to evaluate which determinant causes the relationship to be nonlinear. The studies from micro data show that all the imperfections the convex models rely upon seem to exist in reality. The coexistence of more than one model might be the case in reality, as they do not contradict each other. However, for successful monetary policy, the dominant model has to be identified, as the determinant of the curvature has to be known by the policy makers.

**Table 2-1 Theoretical Models: Overview**

	<b>Main Assumption</b>	<b>Determinant of the shape of the Phillips curve</b>	<b>Shape of the Phillips curve</b>
<b>Capacity Constraint Model</b>	Short run restriction of input factors	Excess Demand	Convex
<b>Costly Adjustment Model</b>	Staggered and costly price setting	Level of Inflation	Convex
<b>Downward Nominal Wage Rigidity Model</b>	Money illusion, efficiency wages	Level of Inflation	Convex
<b>Strategic Pricing Model</b>	Strategic pricing to avoid market entrance of competitors	Excess Demand	Concave

Table 2-1 compares the models that imply a nonlinear Phillips curve. These underlying theoretical models in the Phillips curve can be identified by a combination of the shape of the curve and the determinant of the curvature. Only the costly adjustment and the nominal wage rigidity model cannot be distinguished from each other.

### **3. An Overview of the Empirical Literature**

This section gives an overview of the literature on empirical studies of Phillips curve nonlinearities. Before the 1990s, Phillips curves were estimated, for the most part, by using linear estimation techniques. The nonlinearity issue became popular in the mid 1990s due to more sophisticated econometric techniques and software available. As empirical studies for the European countries are rare, the different approaches, independent of the region studied, are described to give an idea of the empirical

literature and the methodologies used. Dependent on the methodology, the results for Canada and the U.S., the countries that have been the most analysed, give mixed findings. These approaches and results are surveyed in this chapter. In addition, the evidence on European countries is reviewed. Furthermore, a review table of the empirical literature on nonlinearities in the Phillips curve is presented, which can be found in Appendix B.

### 3.1 Empirical Evidence in favour of a Convex Phillips Curve

The largest share of the empirical studies found a convex Phillips curve. These studies can be divided in two groups: one that assumes the excess demand to be the determinant of the slope of the Phillips curve, and the other group assumes the inflation rate to be the determinant of a change in the relationship between inflation and real activity. These assumptions correspond with assuming the capacity constraint model and the downward nominal wage rigidity or costly adjustment model as underlying theoretical models respectively. The methodologies and the outcomes of these studies are discussed in this paragraph.

#### 3.1.1 Convexity Determined by the Real Activity Measure

The capacity constraint model predicts that the state of the economy's real sector determines the slope of the Phillips curve. Some researchers assume a kinked functional form to estimate a Phillips curve, assuming capacity constraints during high economic activity. This means that regressors have asymmetric effects, depending on whether they are positive or negative. Equation (10) shows a general formulation of the estimation equation

$$\pi_t = \alpha_0 + \sum_i \alpha_i \pi_{t-i} + \beta^+ gap_t^+ + \beta gap_t + \gamma z_t + \varepsilon_t \quad (10)$$

where  $i$  is the maximum number of lagged inflation rates,  $gap^+$  is the positive value of the gap and  $z$  is a control variable for supply shocks. Hence, if the gap is positive, the coefficient describing the Phillips curve relationship is  $(\beta^+ + \beta)$ , whereas the coefficient

is  $\beta$  for states of the economy when the gap is negative<sup>19</sup>. This specification assumes the kink to be at the point where output equals potential output to approximate the curved relationship.

Turner (1995) uses this methodology and he estimates Phillips curve equation (10) for the G7 countries individually and finds significant asymmetric effects from the output gap with inflationary effects of positive gaps being larger than the deflationary effects of negative gaps for Canada, Japan, and the U.S.. Notably, he could not reject the linear model for the European countries France and Germany. Clark *et al.* (1996) also estimate a kinked line for the US Phillips curve. They found evidence for convexity for US quarterly data from 1964 to 1990 using lagged inflation, inflation expectations, the gap indicator and a separate term for the output gap when the gap is positive. These studies however fail to control for supply shocks (see next section for a discussion of the importance of including another variable to control for supply shocks into the estimation). Gordon (1997), including supply shock control variables, rejects the hypothesis of nonlinearity for the US data over the period 1955 to 1996.

Another possibility to detect nonlinearity is to include quadratic (k=2) and cubic (k=3) terms into the estimation equation

$$\pi_t = \alpha_0 + \sum_i \alpha_i \pi_{t-1} + \sum_k \beta_k \text{gap}_t^k + \gamma z_t + \varepsilon_t \quad (11)$$

If  $\beta_2$  is significantly positive, this indicates a convex relationship, while a negative coefficient  $\beta_2$  indicates a concave relationship.  $\beta_3$  is included to account for a possible cubic shape, but is not found to be significant in any of the studies choosing this specification.

Laxton *et al.* (1995) and Clark *et al.* (1996, 2001) estimate equation (11) and find support for the model including the quadratic term, which outperforms the linear model. Laxton *et al.* (1995) use pooled data for the G7 countries for 1967 to 1991 and they find that the coefficient of the quadratic gap term is high and statistically significant which points to convexity. Clark *et al.* (1996, 2001) also find evidence for convexity using US data. They also did not control for supply shocks. The problem with pooling the data is another crucial issue, which is analysed in section 3.4.1. Bean (2000) estimates equation

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<sup>19</sup> This specification could also detect concavity, when the coefficient  $\beta^+$  is negative. However, this is a possibility that has not been found in any of the studies reviewed.

(11) using squared values ( $k=2$ ) of the gap measure in a panel regression, finds for a number of OECD countries<sup>20</sup> mild evidence for a convex trade-off. The coefficient of the squared output gap is positive and has a t-statistic between one and two, neither of which are high enough to reject the null hypothesis of  $\beta_2=0$ .

Pyyhtiä (1999) estimates both equation (10) and (11) (with  $k=2$ ) for Austria, Germany, Finland, France, Italy, the Netherlands, and Spain. He found that, when estimating equation (11) using pooled data, the quadratic term is insignificant when using ordinary least squares (OLS) and seemingly unrelated regression (SUR). On the other hand, when using the generalized method of moments (GMM) with the lagged output gap as an instrument, Pyyhtiä (1999) finds that the coefficient of the squared gap measure  $\beta_2$  is significant. However, he notes that the model is very sensitive to the choice of instruments<sup>21</sup>. When estimating equation (11) for each country separately, the coefficient of the quadratic term  $\beta_2$  is significant for Germany and Italy, though the coefficient on the level of the output gap  $\beta_1$  for Germany is insignificant. Furthermore, Pyyhtiä (1999) estimates a kinked function as specified in equation (10). This equation is slightly different as he modifies the variable *gap* in equation (10) to be equal to the output gap, when it is negative, and zero otherwise. He found that the coefficients on the positive output gaps  $\beta^+$  are positive and significant for Italy, Germany, the Netherlands, and Spain. However, he concludes that the negative gap is "only weakly" (Pyyhtiä (1999), p 20) significant for Germany, and insignificant for the other countries. Mayes and Virèn (2004) also estimate the Phillips curves for the EU countries for the period 1985 to 2001 using the kinked specification. They find that only coefficients on the positive output gaps  $\beta^+$  are significant in all countries except Spain and Finland. This finding is similar to the reported results from Pyyhtiä (1999), although the result for Spain is different.

Debelle and Vickery (1997) and Debelle and Laxton (1997) study the Phillips curves in Australia and the U.S., and Canada and the UK respectively. They use a Kalman filter technique<sup>22</sup> to estimate the Phillips curve simultaneously with the nonaccelerating inflation rate of unemployment (NAIRU) by treating the NAIRU as an unobserved component. They use a specification, that uses a gap measure that assumes a capacity

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<sup>20</sup> Excluding Mexico, Iceland, and Turkey.

<sup>21</sup> He does not specify clearly what other instruments he used and what the outcome of these estimations was.

<sup>22</sup> See Hamilton (1994, Ch 13) for a description.

constraint, i.e. the estimation equation (10) with  $\beta^+ = 0$  and  $gap = (NAIRU - u)/NAIRU$ , where  $u$  is the unemployment rate, and  $NAIRU$  is the estimated simultaneously with the Phillips curve. The advantage of this technique is that the  $NAIRU$  is model-consistent and can be interpreted as the unemployment rate, which is consistent with no change in the inflation rate. Aguiar and Martins (2002) use a similar approach using aggregated Euro Area data and conclude that they cannot reject the linearity hypothesis. Other studies estimating kinked linear curves and cubic or quadratic functional forms are listed in the overview table in the appendix.

### 3.1.2 Convexity Determined by the Level of Inflation

Yates (1998), on the other hand, considers the possibility that the change in the relationship might be determined by the level of inflation. This is in contrast to the other studies, which assume the state of the business cycle to change the slope of the Phillips curve. Yates (1998) estimates a system of seemingly unrelated regressions for six OECD countries to find evidence of whether downward nominal price rigidities can be found in the Phillips curve. He uses two lags of inflation and two lags of the output gap as regressors. He multiplies the regression coefficients of the gap measures with dummy variables that take a value of one when inflation is rising from the previous period and zero otherwise. Furthermore, he includes the same set of gap measures into the regression and multiplies the coefficients with dummy variables that take the value of one when inflation is falling and zero otherwise. He tests whether the coefficients of the negative and positive change in inflation are the same by using an F-Test, which does not reject the hypothesis of equal coefficients in periods of falling and periods of rising prices<sup>23</sup>. This is not in line with the costly adjustment and downward nominal wage rigidity models. These models predict that the change is determined by the level of inflation, not the size of the change from one period to the other. The specification of Yates (1998) tests if inflation reacts differently to an increase in the variable  $gap$ , when inflation increases from the previous period than when it decreases. Thereby, the

<sup>23</sup> Yates (1998) estimates a SURE of the form

$$\pi_t = \alpha + \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \delta_0 D^+ gap_t + \delta_1 D^+ gap_{t-1} + \delta_2 D^+ gap_{t-2} + \varphi_0 D^- gap_t + \varphi_1 D^- gap_{t-1} + \varphi_2 D^- gap_{t-2}$$

where  $D^+$  and  $D^-$  denote the dummies for increasing and decreasing inflation periods respectively. If the first difference of inflation of  $t$  and  $t-1$  is positive,  $D^+$  is one and  $D^-$  is zero, and the other way around for a negative first difference of inflation. Then he tests whether the hypothesis  $\delta_0 + \delta_1 + \delta_2 = \varphi_0 + \varphi_1 + \varphi_2$  can be rejected.

absolute size of the increase or decrease is not relevant in this specification. According to the models, the specification would be better, if it would test whether the slope of the Phillips curve changes when the change in inflation is very high. Since the models predict that the relationship changes when inflation is much different from what was expected<sup>24</sup>, this could yield more reliable results. This could be the reason for the result, that the hypothesis of equal coefficients cannot be rejected.

Dupasquier and Ricketts (1997, 1998) are the first in this strand of literature, that test different determinants of the non-linearity for Canada and the US without imposing the source of nonlinearity a priori. They use a state space framework and estimate a Phillips curve with treating the coefficient of the gap  $\beta^{uc}$  as a time-varying unobserved component. The measurement equation they used is

$$\pi_t = \alpha\pi_t^e + (1 - \alpha)\pi_{t-1} + \beta_t^{uc} gap_t + \varepsilon_t \quad (12)$$

where  $\pi_t^e$  is the expected inflation rate and  $\varepsilon_t$  is an iid error term<sup>25</sup>. The unobserved coefficient is then defined in the transition equation, which defines the unobserved coefficient  $\beta_t^{uc}$  as a function of its own lag, a further explanatory variable  $X$ , and an iid error term  $v_t$ .

$$\beta_t^{uc} = \alpha_0 + \rho\beta_{t-1}^{uc} + \gamma X_{t-1} + v_t \quad (13)$$

They estimate different equations, which include different variables for the explanatory variable  $X_{t-1}$ . They run one regression inserting the output gap for  $X_{t-1}$ . If the state of the business cycle would be the determinant of the slope of the Phillips curve,  $X_{t-1} = gap_{t-1}$  would be significant in the transition equation. To examine whether the level of inflation is the determinant of the slope, the ex post probabilities from a three state Markov switching model (MSM)<sup>26</sup> for the moderate and high inflation states (see footnote) are inserted for  $X_{t-1}$  in the transition equation. If one of these variables are significant, the change in the output-inflation relationship is determined by the inflation

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<sup>24</sup> The costly adjustment and downward nominal wage rigidity models predict that the price responsiveness increases only when the price level is much different from what was expected by economic agents.

<sup>25</sup> The model uses the restriction, that the sum of the coefficients of lagged and expected inflation rate is one. This assumption is made to restrict the trade-off between output and inflation to be valid only in the short-run.

<sup>26</sup> The three state Markov Switching Model assumes three states of inflation, one state of low, one of moderate, and one of high inflation. The probabilities of the state faced with in the next period given the state in the current period are being estimated and, using these probabilities, the inflation is being forecasted. This implies, that the expectations are allowed to be different in the three states. The methodology applied in their paper is described in Ricketts and Rose (1995).

rate<sup>27</sup>. This would point towards the two models that imply the average level of inflation to be the determinant of a change in the Phillips curve relationship. Dupasquier and Ricketts (1997, 1998) could only find weak support for the significance of the output gap in both countries and conclude that there is significant time-variation in the trade-off between output and inflation, but the source of the nonlinearity is unclear. The mixed results might be due to the fact that supply shocks have not been included into the regression. Nevertheless, this approach is one of only a few that consider more than one determinant of the slope of the Phillips curve.

### 3.2 Empirical Evidence in favour of a Concave-Convex Phillips Curve

Filardo (1998) uses a piecewise linear Phillips curve similar to the estimation equation (10). He modifies equation (10) by estimating three different coefficients describing the inflation-output trade-off, one for a weak, one for a balanced and one for a overheated economy. He finds that during times, when the economy is weak and overheated, the sensitivity of inflation to output is higher than during balanced times. Moreover, he found that the trade-off is lower during recessions than during boom phases. Filardo (1998) also tests for the equality of the two coefficients during high and low economic activity, and rejects the null hypothesis of equality. This evidence can be seen as a possible explanation for the apparently contradictory results on the shape of the US Phillips curve.

Barnes and Olivei (2003) estimate a threshold model for the US with two states, one for the unemployment close to the NAIRU, and one when the unemployment rate deviates above a specific threshold from the NAIRU. This methodology allows the authors to estimate two different coefficients for the slope of Phillips curve. One slope for states when the economy is around its potential and the other describes the relationship during periods of very high and very low cyclical unemployment. They find that the latter coefficient is significantly higher than the coefficient that describes the trade-off during moderate states of the economy. This finding contradicts Filardo's (1998) finding, in which the coefficients of high and low unemployment states are not equal. Barnes and Olivei (2003) control for supply shocks, what Filardo does not. But the restriction that a

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<sup>27</sup> Both the high and the moderate inflation states are included to allow the relationship between output and inflation to change at both, moderate and high inflation levels.

large gap has the same effect on inflation, no matter if it is positive or negative, does not have theoretical support. Furthermore, the lower and higher bounds are estimated at an unemployment rate of about 4 and 7.5 percent respectively. The authors graph the unemployment rate against time used for their estimations in their paper. When drawing a horizontal line at 4 and 7.5 percent of unemployment, there remain only a few data points that lie outside these two lines. This seems doubtful, as it might well be that the high and low state coefficient are masking outliers rather than giving an indication of the true relationship.

### 3.3 Empirical Evidence in favour of a Concave Phillips Curve

As noted before, most of the empirical studies on nonlinearities conclude either that the Phillips curve is linear or convex. However, concavity in the Phillips curve has also been found in the U.S. data. Stiglitz (1997), citing unpublished research from Council of Economic Advisers, concluded that US data provides some evidence of a concave Phillips curve<sup>28</sup>. He referred to the oligopolistic model explained in chapter three.

Eisner (1997) estimates a traditional Phillips curve similar to the estimation equation (10) of the form

$$\pi_t = \alpha_0^h + \alpha_0^l + \sum_i \alpha_i \pi_{t-i} + \sum_j \beta_j^h u_{t-j}^h + \sum_j \beta_j^l u_{t-j}^l + \gamma z_t + \varepsilon_t \quad (14)$$

for the U.S. where inflation  $\pi_t$  is a function of lagged inflation, current and lagged unemployment ( $u$ ) and control variables  $z_t$  for supply shocks and dummies for periods of the Nixon price controls.  $i=1, \dots, 20$  and  $j=1, \dots, 5$  indicate the number of lags, that are initially included into the estimation. This specification corresponds to estimation equation (...), with the difference that also lagged unemployment variables are included into the model.  $u^h = u$  if unemployment is higher or equal to the NAIRU and zero otherwise. Analogous  $u^l = u$  if unemployment is below the NAIRU and zero otherwise. The constants  $\alpha^h$  and  $\alpha^l$  correspond to  $u^h$  and  $u^l$  respectively. In his estimation results, the sum of the coefficients  $\beta^h$  are significant and negative, while the sum of the coefficients  $\beta^l$  were negative, and hence wrongly signed, and not statistically significant. The absolute value of the sum of the  $\beta^h$  is considerably lower than the corresponding value of the low-unemployment coefficients. He concluded that high unemployment

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<sup>28</sup> Unfortunately, Stiglitz names neither the methodology nor the data range of the study he refers to.

appears to reduce inflation. In estimating separate regressions for low and high unemployment, Eisner finds that "an increase of one percentage point of unemployment above the NAIRU would (...) lower inflation by 2.2 percentage points. But each reduction of unemployment by one percentage point below the NAIRU would raise inflation by only 0.65 percentage points" (Eisner (1997), p. 210-11). This would indicate a concave Phillips curve, according to Eisner.

### 3.4 Conclusion and Shortcomings

The fact that researchers using different approaches come to different conclusions reflects the inherent uncertainty associated with the empirical measurement of the shape of the short-run Phillips curve. There are several possibilities that could explain the dissimilarity in the empirical findings and the different conclusions drawn from them. Appendix B on page 70 gives an overview of the empirical studies on nonlinearities in the Phillips curve and their different conclusions. In this section, two fields that might explain the discrepancies are addressed: One is the measurement of data and the econometric specification and the other the econometric and theoretical methodology.

#### 3.4.1 Measurement and Data Problems

One problem in the data is the uncertainty associated with the way the output gap and unemployment gap are calculated (see discussion in the next chapter). This paper gives an overview of different approaches to measure real activity in paragraph 4.1.2 and compares the performance of these measures in the linear model. Another problem might be that a large number of the studies neglects the impact of supply shocks. This, however, is important when thinking about straightforward aggregate supply and demand theory, as both can cause inflation, a shift in supply and demand<sup>29</sup>. A third criticism is that some of these studies use pooled datasets. Pooling the data assumes that all countries in the data set have the same Phillips curve shape. The authors (e.g. Laxton *et al.* (1995)) assume that the source for the nonlinearity are capacity constraints. Pooling the data assumes that the capacity constraints are the same in all the studied

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<sup>29</sup> The book with the collected essays of Gordon (2004) gives a good overview of the importance of supply shocks and that they account to a large part for the finding of a disappearance of the Phillips curve during some periods.

countries, which is an assumption that might cause estimation problems. To make this point clearer, assume as an example that there are two countries in the dataset and the capacity of an input factor in only one country is limited in the short run<sup>30</sup>. Thus, the Phillips curve relationship is convex in this country but linear in the other. The estimation of a kinked or quadratic functional form yields evidence for a convex curve caused by the convex relationship in one country, although in the other country the curve might be linear or have another microfoundation that implies another shape of the curve. Treating these countries as homogenous in the estimation might lead to results that will fail to find different relationships in the different countries. Therefore, each country will be estimated separately. This is especially interesting for the Euro Area countries as, if the countries' Phillips curves have different shapes, there will be considerable implications for monetary policy. Using too highly aggregated data (e.g. Aguiar and Martins (2002)) in the estimations might yield a similar problem<sup>31</sup>. Appendix B shows which studies use supply shock control variables and which measures of inflation expectations and real activity are used in the respective papers.

### 3.4.2 The Theoretical Foundation

As described in the chapter on the theoretical foundations of nonlinearities in the Phillips curve, its slope can be determined by different factors. The last two columns of the summary table in Appendix B shows that most of the researchers did not test for the source of the change in the Phillips curve relationship and just assumed the excess demand to be the determinant of the slope, thereby neglecting the costly adjustment and downward nominal wage rigidity models. Using the results of this estimated model for policy recommendations might cause significantly wrong predictions. For instance, the capacity constraint model would imply that when excess demand approaches an estimated threshold, the model predicts that inflation accelerates no matter what the level of inflation currently is. Using this model as a policy maker would imply that monetary policy should react to avoid an increase in inflation, which is costly to reverse.

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<sup>30</sup> One country has restrictive labour laws like a legal constraint on the hours worked per week and the laws that make it difficult to fire workers once they are hired are very restrictive, and other countries, where these laws are liberal.

<sup>31</sup> Note that this argument is relative: data on the country level are regional aggregates, too. However, using aggregate Euro Area data might cause problems due to different institutional surroundings, at least before the start of the European Monetary System (EMS).

However, if the true source of the nonlinearity is the costly adjustment model and inflation is currently at a level that does not force the firms to increase their prices more frequently, then the models' predictions are wrong. This argument can also be made the other way around, as well as for the other theoretical microfoundations. Hence, neglecting the theoretical models that imply different determinants of a change in the Phillips curve relationship might also be a reason for the mixed empirical results. In conclusion, despite some exceptions<sup>32</sup>, the empirical studies just assume a determinant for the nonlinearity, without any prior testing. Another issue related to the econometric techniques might add to the contradictory results. As described in the review of the empirical literature in the Appendix, the type of nonlinear relationship is approximated by a linear specification with kinks or quadratic terms included. There might be a problem in using this methodology since the functional form and the location, where the slope of the line changes, are just assumed by the researchers. This issue is also discussed in paragraph 5.2 when describing the methodology used. Only a few studies use a nonlinear estimation technique rather than piecewise linear specifications to estimate the Phillips curve (see appendix B). The approach used in this study allows for both, concavity and convexity, and tests if a linear model is appropriate. It also tests for the different determinants drawn from theory that may cause the nonlinear relationship.

#### **4. Estimation and Evaluation of the Linear Phillips Curve Specification**

As described in the previous chapter, one of the most challenging tasks in estimating the Phillips curve is choosing the model and measuring the variables. The choice of the data and the problems in measuring the variables and the specification of the econometric estimation equation used in this study are presented in this section.

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<sup>32</sup> Dupasquier and Ricketts (1998), Eliasson (2001), and Fauvel et al (2004). See Appendix B for more details.

## 4.1 Data

This section gives an overview of the data used in the study and discusses measurement and stationarity issues. The last part of this section gives a graphical representation of the data to give an initial idea regarding the relationship.

### 4.1.1 Data Description and Sources

Data for inflation and demand pressure are constructed to measure the empirical stability of the Phillips curve. The data used for this study are all obtained from the OECD Economic Outlook Database and are in quarterly frequency from 1969Q1 to 2004Q3 (if not stated otherwise). The gross domestic product (GDP) and unemployment measures are seasonally adjusted numbers. The concept of potential output and the natural rate of unemployment needs a careful estimation in order to get reasonable and reliable estimates in empirical work. The main task is to decompose the observed output series into a non-stationary trend, a stationary cycle, and a random component. As shown by Canova (1998), output gaps differ dramatically depending on the detrending method. Given this reason this studies uses gaps measured with different econometric and statistical approaches to test firstly the quality of the gap measure before using the best performing measure for the nonlinear estimation. In this paragraph, the data description for the variables used in the quantitative analysis is presented and the derivation of the excess demand indicators is discussed. This section defines the data sources and the different variables' names, the different methods used to calculate the demand pressure indicators, the measurement of inflation expectations, and further statistical issues are discussed in section 4.1.2.

*Inflation.* This study uses the consumer price index (CPI) excluding food and energy to measure the price level. The CPI data used excludes the prices of the items food and energy in order to exclude components that push up volatility in the series and that are mostly supply side driven. The difference between the log of the consumer price index and the value of the same quarter in the previous year is used as inflation data.

*Potential output gap.* This data is available in the OECD Database. The OECD measures potential output by using a production function approach as described in

Giorno *et al.* (1995). The gap is measured as the percentage deviation of real GDP from its potential. This variable is denoted as "OECD gap".

*Hodrick-Prescott filter.* Another measure for the output gap is the Hodrick-Prescott filtered trend of GDP. The Hodrick-Prescott (HP) filtered detrended series is calculated using a smoothing parameter of 1600 as suggested for quarterly data frequency.

*Capacity utilisation.* The capacity utilisation rate for the industrial sector is used as another indicator of demand pressure. As a measure of the business cycle, the deviation from the long-run average will be used. The resulting series is named "excess capacity" henceforth<sup>33</sup>.

*Unemployment gap.* The variable named "Unemployment gap" in the quantitative part of this study is computed by using the OECD measure of the NAWRU (non-accelerating wage inflation rate of unemployment). The gap is calculated as described in the previous bullets.

*Import Price Index.* The first difference of the logarithm of the import price index is used to control for supply shocks.

#### 4.1.2 Discussion on Measurement

This study uses the first difference of the log of the CPI excluding food and energy as a measure for the inflation rate. The ECB's second pillar, however, focuses on the harmonized consumer price index (HICP). The problem is that there does not exist enough history for that time series. The ECB sometimes also refers to a rate of "inflation excluding the more volatile items of energy and unprocessed food" (e.g. ECB (2004), p.5). That makes this series to be the closest approximation available with adequate history to the inflation measure that the ECB is focussing upon for their

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<sup>33</sup> The name "excess capacity" is not completely correct, as the long term average of the capacity utilisation is at about eighty percent of the capacity utilisation rate. Excess capacity is, from definition, every percentage below 100 percent. The measure obtained from survey data, and therefore it would be more appropriate to name the variable "more than half of the firms produce close to their maximum capacity". Furthermore, the sign has been inverted, to have a positive value of the variable indicating excess demand in the economy and a negative excess supply.

monetary policy decisions. Especially during periods of high oil prices, the measure excluding energy seems to be more appropriate.

In the business cycle literature, there is a line of research concerned with the accuracy of the output gap measure<sup>34</sup>. Conventional estimates of the Phillips curve use measures of either the output gap or unemployment gap as an indicator of demand pressure. This study reviews different measures that are from different theoretical or statistical backgrounds. For comparability purposes, the indicators have been re-scaled by dividing each series by its standard deviation and the sign of the unemployment gap is inverted. Correlation matrices for the gap measures are reported in Appendix A. The following section gives a short description of the different methodologies in estimating output gaps that are used in the following econometric section. The estimation of the real activity measures used in this study are can be broadly divided in three groups: non-structural methods, structural methods, and direct measures. To capture these different methodologies in this study, each of these three methods is applied to obtain different measures of real activity.

*Univariate, non-structural methods.* Broadly speaking, this phrase includes all methods that are based on some statistical procedure that extracts a trend from a series without using any other data and that are not based on economic theory. These are described and compared in Cogley (1997)<sup>35</sup>. As an example for a non-structural measure of the *gap* the most frequently used method, the HP filter, is being used. The interest in non-structural methods is partly motivated by the fact that they require less information than theory-based methods. Nevertheless, non-structural univariate methods also have several shortcomings. The HP filter for example needs the choice of a smoothing parameter, that is chosen rather ad hoc. Furthermore, the economic theory and hence the nature of the business cycle is being neglected. In this study, HP filter has been used as described previously, which yields the variable that is named "HP Gap" in this study.

*Structural Measures.* Structural measures have an advantage over non-structural methods by using economic theory to determine the trend. Potential output is estimated

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<sup>34</sup> Chagny and Döpke (2001) give an overview over the different measures of the output gap in the euro zone.

<sup>35</sup> It is hard to distinguish between structural and non-structural approaches. Some of the structural, theory based methods, such as the production function approach based on only one production factor, often turn out to be more or less a trend extraction method. Moreover, some of these theory-based methods use trends or filters as inputs for estimation. Hence the term "structural" should be treated carefully.

by using a production function. Depending on the quantity of input factors available in the economy at the point in time considered, the output that can be generated from them can be estimated. Approaches based on production functions try to unearth the nature of constraints that limit output (for example labour, capital, or global factor productivity). Giorno *et al.* (1995) give a detailed description of the OECDs procedure and production function approach to measure the "OECD gap" series, which is used as an example for a structural measure in this study.

A further structural measure in this study is the "unemployment gap". The OECD applies the Kalman filter technique to calculate the measure of the NAWRU. This is a structural measure in the sense that the Phillips curve relationship is used to compute the NAWRU. This means that the NAWRU is estimated as an unobserved component in the and together with the wage Phillips curve estimation equation. However, some of the studies reviewed (e.g. Mourougane and Ibaragi (2004)) use the *gap* calculated as a deviation from the non-accelerating inflation rate of unemployment (NAIRU) data series as a right hand side variable in the Phillips curve estimation equation, without estimating the NAIRU and the Phillips curve simultaneously. Thus, the gap series that is computed by this measure is the unemployment that is "accelerating inflation rate of unemployment" by definition. Recall that the NAIRU is calculated using data from the Phillips curve relationship. Regressing this measure on inflation would more or less mean that one has influenced the values used creating the desired output. Thus the NAIRU would cause an endogeneity problem in the regression, as the Phillips curve relationship was already used to compute the NAIRU. Therefore, it can be suspected that when using this measure the estimation results show a clear relationship by definition. To use the NAWRU rather than the NAIRU would at least not cause this kind of a masking effect, although it is clear that wage inflation and inflation move quite simultaneously in the medium term.

*Direct measures.* Survey measures are an ideal addition to the gap measures from GDP and Unemployment rates. Firms are surveyed monthly as to how much of their maximum capacity is currently being made use of. One of the advantages of this method is that survey measures are not due to revisions. Hence in the most recent years, the survey measure is more accurate than the other measures as the other ones are still under revision. In this study simply the long run mean of the series has been used as the rate that is defined as the point, where the economy is neither in excess supply nor in

excess demand. The time series is sufficiently long and there is no reason to believe that there is a long-term trend in the capacity utilisation rate<sup>36</sup>.

*Inflation expectations.* Unfortunately there does not exist survey data for the Euro Area countries for inflation expectations that dates back to 1970. Paloviita (2004) has used real-time estimates from the OECD forecasts as empirical proxies for economic agents' inflation expectations in her study. This data would be a good proxy for expectations, but this data set is only available in annual frequency. Another good proxy for inflation expectations were break-even inflation rates from inflation linked bonds, but these are not issued in all of the three countries considered. Therefore, this study estimates inflation expectations using an assumption about the nature of inflation expectation formation and model inflation expectations using econometric techniques. The formation of inflation expectations, however, is another crucial issue that economists still do not agree upon. There are two main approaches that explain the nature of the formation of expectations. On the one hand there is the assumption of rational expectations from the new classical economics, which has come under attack for assuming too much information on the part of agents<sup>37</sup>. On the other hand, the assumption of adaptive expectations is an unsatisfactory concept because it assumes that agents do not react to the systematic mistakes they make. Both criticisms of the assumptions on the formation of inflation expectations are realistic. An intermediate assumption would be that the economic agents learn about the inflation process, but they do not have complete information and include their errors made in one period when forming their future expectations. Therefore, this study uses the intermediate approach, a learning process in building expectations. It is then assumed that expectations are built using only a limited set of information<sup>38</sup>. It is assumed that agents use a rule of thumb to forecast inflation. They use the currently observed inflation rate and the unemployment gap<sup>39</sup> to form their expectations about the inflation in the next period. The assumption

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<sup>36</sup> A measure of the Business Survey also yields good output gap measures but the series for the European countries, except Germany, start only in 1985. An excellent description of this method is given in Flaig and Ploetscher (2000).

<sup>37</sup> See for example Fuhrer (1997) and Roberts (2001).

<sup>38</sup> See Sargent (1993).

<sup>39</sup> The unemployment gap is measured by using the percentage deviation of unemployment from its HP filtered trend. This is to be consistent with the idea that agents use an easy rule of thumb. The main intention of this procedure is to model the expectations that the agents have formed in the past using the information available at that time. There are two criticisms that should be mentioned. One is that the HP filter is applied up to the most recent data and the gap measured by this method in the past might have yielded significantly different results, as the HP filter has an end point adjustment problem. The other criticism concerns the data. Using real-time data would make these estimates more appropriate...

of learning allows a time variation of the parameters in the expectation model. A state space model is used to model this expectation process, assuming the learning starts at the point in time where the OECD data series start, which is in 1964. Therefore, the data used later in the Phillips curve estimations starts in 1969, to allow some time for the learning procedure. A formal description and a more detailed discussion of the method can be found in Appendix C.

*German Reunification.* In the German data, there exists the reunification problem. Therefore, this study uses reindexed data for GDP and unemployment by using the Western German growth rates for the period leading up to 1991. From that point on, the GDP data for reunited Germany have been used and the level of 1992 for reunited Germany is the basis for the pre-1992 series, which allows to lift the data up to the level of 1992 by using the growth rates for Western Germany<sup>40</sup>, thereby avoiding a level-break in the data series.

### 4.1.3 Graphical Representation of the Data

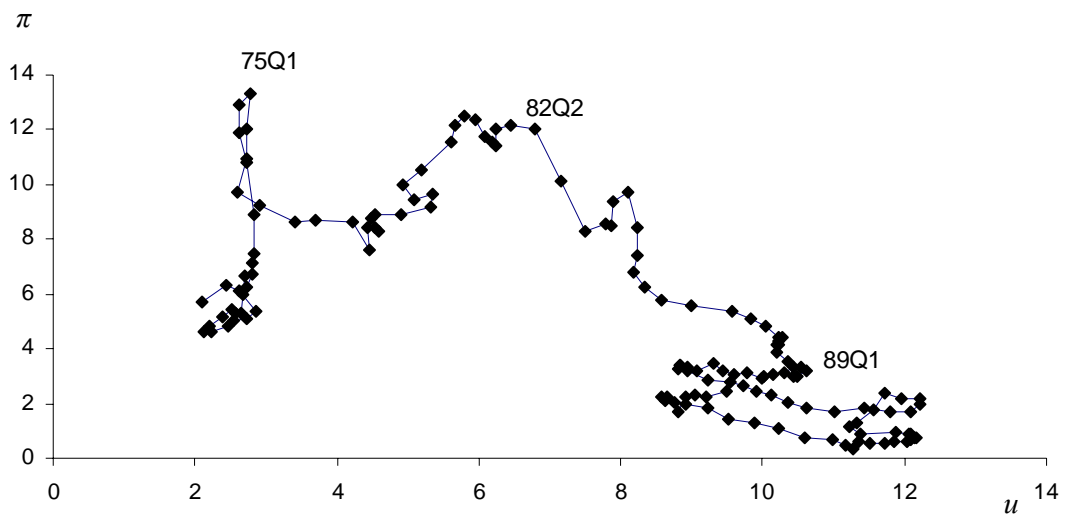
To get an idea of the data and the relationship between unemployment and inflation a simple graph will be presented. The scatter plots of the European countries are illustrated by Figure 4-1 to Figure 4-3. The combinations of the unemployment rate and Inflation show some periods in which the points fall along a fairly stable, downward-sloping curve, though several do not follow that relationship. According to these scatter plots, there is no evidence for a long run trade-off, which is in line with the expectation-augmented Phillips curve specification. These graphs, however, are not useful to detect the relationship according to equation (4).

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Unfortunately there is no real-time data available for these countries, the frequency and the length of the time series. Survey measures could circumvent this problem as they are not revised. Survey measures are not available with a longer history, as the learning process is already assumed to start in 1964. Therefore the unemployment gap is chosen as a proxy for the real-time real activity, as it is the only data series available for each country from the OECD database that dates back to 1964.

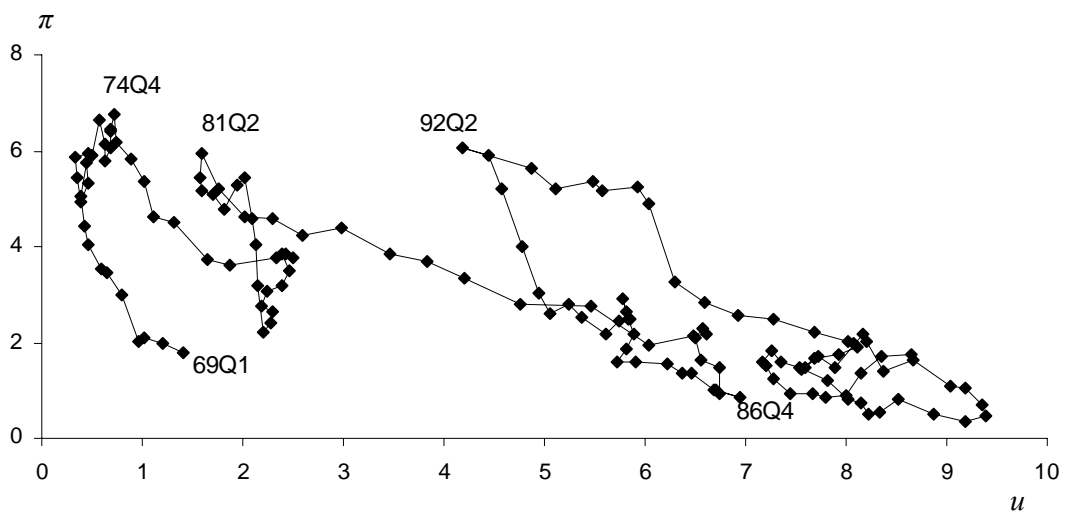
<sup>40</sup> Clearly there is the problem that this procedure just lifts up the level to the reunited German series but this is not just a level effect but a structural break with a change in the trend and the Phillips curve relationship is likely to change substantially for several years. I am bearing this caveat in mind for the interpretation of the estimation results.

**Figure 4-1 Unemployment and Inflation in France 1969Q1- 2004Q3<sup>41</sup>**



The scatter plot of the French inflation and unemployment data shows no relationship in the first half of the 1970, when inflation rose from around five up to almost fourteen percent at a stable rate of unemployment. During the 1980s, the downward sloping relationship was quite obvious and since then, there has been rising and falling unemployment at quite stable rates of inflation, thus there was no obvious trade-off between inflation and unemployment.

**Figure 4-2 Unemployment and Inflation in Germany 1969Q1- 2004Q3<sup>42</sup>**

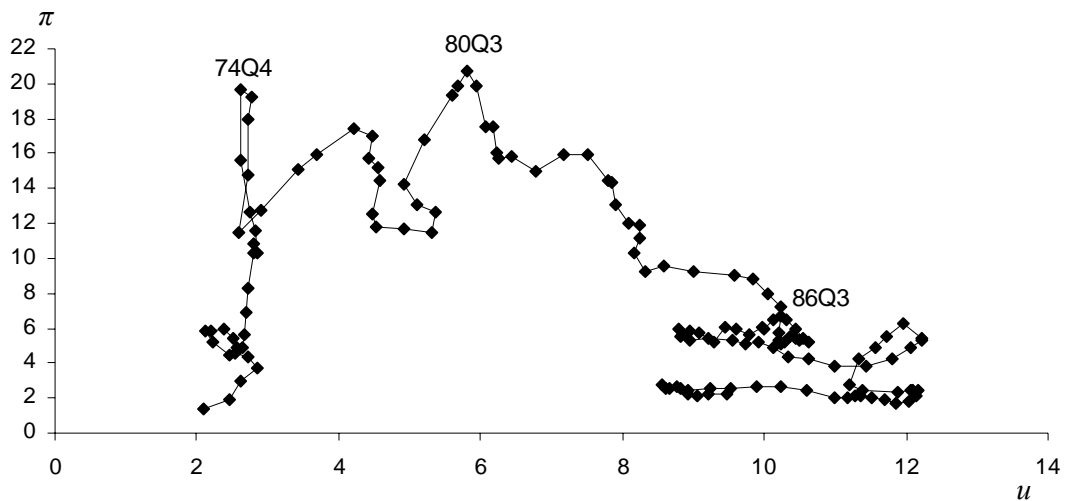


<sup>41</sup> Source: OECD Economic Outlook Database (2004).  $\pi$  denotes the annual inflation rate,  $u$  the unemployment rate.

<sup>42</sup> Source: OECD Economic Outlook Database (2004).  $\pi$  denotes the annual inflation rate,  $u$  the unemployment rate.

The German series shows a downward sloping relationship in the periods from 1969 to 1974, in the first half of the 1980s and again from 1992 on until 1999. This relationship does not seem to exist between the previously mentioned periods and since the beginning of phase two of the European Monetary Union.

**Figure 4-3 Unemployment and Inflation in Italy 1969Q1- 2004Q3<sup>43</sup>**



The Italian graph is quite similar to the French in that it has a similar shape over the given periods. What is striking in the Italian series is the much higher average inflation rates compared to the two other countries.

These graphs are plotted to give an idea of the empirical relationship, one can clearly see that there seems to exist a relationship for some periods. For other periods, however, the relationship does not seem to exist. This might be because there is only a short-run relationship. The points that do not follow the downward sloping relationship might be due to the fact that inflation might be influenced by one of these control variables, such as supply shocks or changes in inflation expectations. Hence, it is not yet possible to draw any conclusions from these graphs but they are useful to give an initial idea of the data.

<sup>43</sup> Source: OECD Economic Outlook Database (2004).  $\pi$  denotes the annual inflation rate,  $u$  the unemployment rate.

#### 4.1.4 Stationarity

All demand indicator variables as well as the import inflation series are stationary, Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests have been performed but are not reported here, as all tests reject the unit root at least at the 10% significance level. A more crucial issue is the stationarity of the inflation data. Pyyhtiä (1999) estimated Phillips curves for the Euro area using inflation levels in the regression, despite that he wrote that "there is a certain kind of unit root problem". However, he does not report any unit root tests. In Appendix D, the unit root tests for the inflation data are described and the results are reported. The results are similar to the tests for the CPI data from the International Monetary Fund (IMF) for the period 1960 to 2001 used in the study of Hoogenveen (2004). The ADF and PP test for some countries do not reject the null of a unit root, while the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test does not reject the hypothesis of stationarity at a 10% significance level. Explanations for these mixed results are low power of the tests, relatively short data ranges and structural breaks in the series. Oil price shocks<sup>44</sup>, the German reunification, new inflation targets of the respective European central banks, or the change of the central bank could cause the structural change in the data. The last two columns in Table D-4 show a unit root test that accounts for a level shift. Choosing the break date as described in the appendix, the series for Germany appears to have a structural break after the reunification, while the other countries are more affected by the oil price shocks in the 1970s. The German series is stationary on the five percent level when accounting for the level shift, while the other series still have mixed results. Note, that the ADF, the PP and the test including a structural break have statistics close to rejecting the null of a unit root. Furthermore, Li *et al.* (2004) test monthly inflation series from 1970 to 2003 for the countries considered in this paper and report ADF and PP tests. They reject the null hypothesis of a unit root at the five percent significance level for both tests used for all the countries. Therefore, inflation will be used in levels and not in first differences for the Phillips curves estimations assuming the price levels are  $I(1)$ . This is in line with the other studies reviewed in the previous chapter. Also, from a theoretical point of view, it is realistic to assume inflation to be stationary. The same reasoning applies for inflation expectations, which have almost the same test statistics (which is due to the high correlation between lagged inflation and the expectation).

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<sup>44</sup> Though the oil price shocks lasted only for a short period, the level shift in inflation can be for a longer time span because of second round effects.

## 4.2 A Linear Benchmark Model

Before examining whether the relationship between indicators of excess demand and inflation is nonlinear, a necessary first step is to estimate a linear model to have a benchmark to compare the results of a linear and a nonlinear model to. Already in the previous chapter the different specifications of the estimation equations of the papers reviewed are striking. This shows the challenging task of choosing an appropriate estimation specification. One first being the inclusion of inflation expectations. If only inflation expectations are included on the right hand side of the estimation equation, the model is purely forward looking. A hybrid specification includes a forward looking and a backward looking component of inflation. Dupuis (2004) assesses the empirical performance of competing specifications of the Phillips curve for the US. He assesses both the question of whether a backward looking, a forward looking, or a hybrid specification yields the best empirical results and which measure of the gap does best from an empirical point of view. He finds that the observed persistence of inflation suggests that lags as well as expectations of inflation would be required in an appropriate empirical specification. The empirical limitations of the purely forward looking Phillips curve have led the largest number of researchers choosing a hybrid version of the Phillips curve. That is also the formulation that will be undertaken in this study's estimations. A general specification of the Phillips curve estimation equation is the model that Gordon (1997) calls the "triangle model" of inflation. From that specification, the inflation process is driven by three basic determinants: inertia, demand, and supply. A general equation capturing these determinants as right hand side variables is

$$\pi_t = \alpha_0 + \sum_j \alpha_j \pi_{t-j} + \beta^D D_t + \beta^S z_t + \varepsilon_t \quad (15)$$

where inertia is captured by the lagged (annual) inflation terms  $\pi_{t-j}$ , and  $j$  is the maximum number of lags included in the regression,  $D_t$  is a measure for excess demand, and  $z_t$  is a vector of supply shocks,  $\alpha_j$ ,  $\beta^D$  and  $\beta^S$  are coefficients,  $\alpha_0$  is the intercept, and the independent identically distributed (iid) error term is denoted as  $\varepsilon_t$ . Note that this model is purely backward looking assuming that expectations are adaptive. This assumption is too strong and hence inflation expectations are included in the model. For the hybrid specification, the triangle model is extended by a measure of inflation

expectations. Hence the estimation equation is

$$\pi_t = \alpha_0 + \alpha_e \pi_t^e + \sum_j \alpha_j \pi_{t-j} + \beta^D D_{t-i} + \beta^S z_t + \varepsilon_t \quad (16)$$

with  $\pi_t^e$  denoting the expected inflation rate. The different indicators of demand pressure are used in the equation (16). The lag  $i=1$  of the demand pressure indicator, which yields the highest estimated coefficient, is used in the models<sup>45</sup>. To control for supply shocks, the first difference of the logarithm of the import price index will be included. The lagged import inflation is included in equation for the variable  $z$ <sup>46</sup>. Regarding the dynamics it is not possible to impose a certain lag structure of past inflation variables from theory. The first estimated equation includes eight lags of inflation and the t-values are used as a guide to decrease the number of lags to achieve a model without error autocorrelation. The most parsimonious equations are reported in Table 4-1 to Table 4-3. These tables show the estimated coefficients and diagnostic tests, as well as tests that are useful to detect possible nonlinearities. These tests are reported in the last three rows of the tables, the description and interpretation of the tests are given in section 4.3, whereas the following subsections of 4.2. focus upon interpreting the coefficients.

### 4.2.1 France

The resulting regression coefficients for France are reported in Table 4-1 on the next page. The coefficients of the inflation expectation series are significant at the one percent level only in the model using the capacity utilisation rate and are insignificant in the other models. This finding is likely to be caused by the measurement of the inflation expectations. As the lagged inflation rate is used as a right hand side variable to compute the one-step-ahead forecasts of inflation<sup>47</sup>. Similar results for inflation expectations are reported in the other countries' estimations. This problem would probably not arise if real-time data for the gap measure would be used in the inflation

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<sup>45</sup> Several regressions of equation (16) have been conducted previously that showed that the specification  $i=1$  yields the highest coefficient in the majority of the regressions.

<sup>46</sup> For larger countries, a change in import inflation might not just cause the domestic inflation to change but also vice versa. Domestic inflation might influence the import inflation. Including import inflation with a lag accounts for the possible reverse causality problem.

<sup>47</sup> The correlations of lagged and expected inflation are very high. This might produce a problem as the two variables are close to being collinear, which is not longer problematic, just the standard errors might be too high.

expectations model<sup>48</sup>. Coefficients of lagged inflation rates and the inflation expectations sum up close to unity, despite not imposing the constraint that the sum must equal one. The coefficients of the gap measures are significant and signed as expected, except the excess capacity, which is not significantly different from zero. The results seem robust with respect to the choice of the gap indicator. The linearity tests are described and interpreted in paragraph 4.3.

**Table 4-1 Linear Regression France**

<b>Dependent Variable: <math>\pi_t</math></b>				
	<b>Unemployment Gap</b>	<b>OECD Gap</b>	<b>HP Gap</b>	<b>Capacity Utilisation</b>
<i>const</i>	0.13**	0.17***	0.07	0.02
$\pi_t^c$	0.09	0.13	0.08	1.06***
$\pi_{t-1}$	1.21***	1.17*	1.28***	0.27
$\pi_{t-2}$	-0.37***	-0.37***	-0.40***	-0.41***
$\pi_{t-4}$	-0.25**	-0.24**	-0.17*	-0.20*
$\pi_{t-5}$	0.29***	0.28***	0.19**	0.25**
gap <sub>t-1</sub>	0.18***	0.18***	0.12***	0.06
$\pi_{t-1}^{imp}$	0.02**	0.01**	0.01*	0.00
$R^2$	0.99	0.99	0.99	0.99
Adj $R^2$	0.99	0.99	0.99	0.99
S.E.	0.40	0.39	0.41	0.37
$\Sigma \pi$	0.98	0.96	0.97	0.98
<b>Linearity Tests</b>				
RESET	0.31	0.12	0.99	2.97**
ARCH-LM	2.54**	2.27**	5.32***	5.50***
BDS	0.05***	0.03*	0.03	0.07***

\*\*\*, \*\*, \* denote 1%, 5% and 10% significance level.  $\pi^{imp}$  is the annual growth rate of the import price index, gap is the gap measure of the respective columns.  $\Sigma \pi$  denotes the sum of coefficients of lagged inflation rates. All data series end in 2004Q3, but the gap series are available only from different points in time. Model (1) and (2) from 1971Q1, model (3) from 1969Q1 and (4) from 1976Q1. The linearity tests are described in the following paragraph. RESET and ARCH-LM report the F-statistic, BDS the BDS test statistic where the significance values are calculated with bootstrapped p-values.

<sup>48</sup> As noted before, data sets for the real-time output gap are not yet available. It would be a very useful task to collect a dataset (for example from OECD publications) and include that data into the estimations. The preferred specification would be the model of inflation expectations as presented here and using business survey data or real-time GDP forecasts instead of the HP unemployment gap. This, however, is being left open for future research.

## 4.2.2 Germany

The resulting regression coefficients for Germany are reported in Table 4-2. Inflation expectations are not significant. This is analogous to the results for France and indicates a data problem in measuring inflation expectations. Coefficients of lagged inflation rates sum up close to unity, but are lower when compared to France. All gap measures are significant and signed as expected. Changing the gap measure changes neither the size of the gap coefficient nor all other coefficients considerably, which indicates the robustness of the results to the choice of the excess demand indicator.

**Table 4-2 Linear Regression Germany**

Dependent Variable: $\pi_t$				
	Unemployment Gap	OECD Gap	HP Gap	Capacity Utilisation
<i>const</i>	0.20 <sup>***</sup>	0.17 <sup>***</sup>	0.14 <sup>**</sup>	0.01
$\pi_t^c$	-0.19	0.16	-0.16	-0.54
$\pi_{t-1}$	1.40 <sup>***</sup>	1.10 <sup>***</sup>	1.44 <sup>***</sup>	1.81 <sup>***</sup>
$\pi_{t-2}$	-0.27 <sup>**</sup>	-0.30 <sup>***</sup>	-0.31 <sup>***</sup>	-0.28 <sup>**</sup>
$\pi_{t-4}$	-0.30 <sup>***</sup>	-0.30 <sup>***</sup>	-0.28 <sup>***</sup>	-0.27 <sup>**</sup>
$\pi_{t-5}$	0.53 <sup>***</sup>	0.57 <sup>***</sup>	0.59 <sup>***</sup>	0.55 <sup>***</sup>
$\pi_{t-6}$	-0.26 <sup>***</sup>	-0.29 <sup>***</sup>	-0.33 <sup>***</sup>	-0.31 <sup>***</sup>
gap <sub>t-1</sub>	0.18 <sup>***</sup>	0.13 <sup>***</sup>	0.14 <sup>***</sup>	0.16 <sup>***</sup>
$\pi_{t-1}^{imp}$	0.01 <sup>*</sup>	0.01	0.01	0.01 <sup>*</sup>
$R^2$	0.97	0.97	0.97	0.97
Adj $R^2$	0.97	0.96	0.97	0.97
S.E.	0.34	0.35	0.35	0.35
$\Sigma \pi$	0.90	0.93	0.94	0.97
<b>Linearity Tests</b>				
RESET	3.96 <sup>***</sup>	2.21 <sup>*</sup>	2.35 <sup>*</sup>	2.21 <sup>*</sup>
ARCH-LM	6.23 <sup>***</sup>	3.60 <sup>***</sup>	2.40 <sup>**</sup>	1.99 <sup>**</sup>
BDS	0.03 <sup>**</sup>	0.03 <sup>**</sup>	0.02 <sup>*</sup>	0.02 <sup>*</sup>

\*\*\*, \*\*, \* denote 1%, 5% and 10% significance level.  $\pi^{imp}$  is the annual growth rate of the import price index, gap is the gap measure of the respective columns.  $\Sigma \pi$  denotes the sum of coefficients of lagged inflation rates. All models include data from 1969Q1 to 2004Q3. The linearity tests are described in the following paragraph. RESET and ARCH-LM report the F-statistic, BDS the BDS test statistic where the significance values are calculated with bootstrapped p-values. There is a large break in the data series for only one quarter in 1992Q4, which has been interpolated (see also stationarity tests with level shift).

### 4.2.3 Italy

The results for Italy reported in Table 4-3 differ from the results for Germany and France in the fact that inflation expectations are wrongly signed, this is likely to be due to the data problem mentioned previously. However, the inflation expectations are not significantly different from zero. The sum of the inflation coefficients does not indicate a misspecification as they are close to unity. The Phillips curve relationship, which is captured in the coefficient of the gap measure, differs more between the different gap measures. The production function approach measure, the OECD gap, is significant at the one percent level and higher than in the other countries. A striking result is also the

**Table 4-3 Linear Regression Italy**

<b>Dependent Variable: <math>\pi_t</math></b>				
	<b>Unemployment Gap</b>	<b>OECD Gap</b>	<b>HP Gap</b>	<b>Capacity Utilisation</b>
<i>const</i>	0.03	0.16	0.03	-0.11
$\pi_t^c$	-0.98	-0.62	-0.85	-0.85
$\pi_{t-1}$	2.47***	2.02***	2.32***	2.32***
$\pi_{t-2}$	-0.49***	-0.44***	-0.49***	-0.48***
$\pi_{t-4}$	-0.55***	-0.53***	-0.54***	-0.52***
$\pi_{t-5}$	0.86***	0.82***	0.88***	0.87***
$\pi_{t-7}$	-0.34***	-0.29***	-0.35***	-0.35***
gap <sub>t-1</sub>	0.12*	0.26***	0.18***	0.17**
$\pi_{t-1}^{imp}$	0.03***	0.02***	0.02***	0.03***
$R^2$	0.99	0.99	0.99	0.99
Adj $R^2$	0.99	0.99	0.99	0.99
S.E.	0.63	0.60	0.62	0.63
$\Sigma \pi$	0.97	0.96	0.98	0.99
<b>Linearity Tests</b>				
RESET	3.97***	4.70***	3.82***	2.91***
ARCH-LM	7.81***	5.79***	8.31***	7.58***
BDS	0.11***	0.13***	0.13***	0.11***

\*\*\*, \*\*, \* denote 1%, 5% and 10% significance level.  $\pi^{imp}$  is the annual growth rate of the import price index, gap is the gap measure of the respective columns.  $\Sigma \pi$  denotes the sum of coefficients of lagged inflation rates. All models include data from 1969Q1 to 2004Q3. The linearity tests are described in the following paragraph. RESET and ARCH-LM report the F-statistic, BDS the BDS test statistic where the significance values are calculated with bootstrapped p-values.

standard error of the regressions, which is higher than that of the other countries. These differences could also be caused by nonlinearity.

The result showing expected inflation is not significant seems to be at odds with the new Phillips curve. However, this is a finding that several studies agree with (e.g. Fuhrer (1997) for the US data) and is likely to be caused by the data problem mentioned above. The other coefficients are all signed as expected and the values are reliable.

### 4.3 Testing Linearity against an Unspecified Alternative

For detecting nonlinearity, an important step in the modelling cycle is to check the residuals of the linear specification using a range of diagnostic tests. When nonlinearity is present in the model but neglected by estimating with an ordinary least squares (OLS) estimator, there should be some kind of systematic pattern in the residuals. In this subsection, Portmanteau<sup>49</sup> residual tests are reported. These tests do not have a specific alternative hypothesis, and as a result they are useful in detecting nonlinearity, but not in determining the nature of the nonlinearity. These tests are standard tests for nonlinearity, as suggested by Enders (2004) for example. A further step is to test the model against the specified alternative, i.e. the nonlinear estimation. This approach is given in the next chapter. The tests explained in the following three subsections are reported with the results in last three rows in Table 4-1 to Table 4-3.

#### 4.3.1 The RESET

The Regression Error Specification Test (RESET) is one of the most popular tests against misspecification of functional form. The intuition behind the test is that the residuals of a truly linear model estimated by OLS should not be correlated with the regressors. This can be tested by estimating the linear model and then estimate a second OLS regression by regressing these residuals on the left and right hand side variables of the original model. In this analysis this means regressing the residuals of regression (16) on the right hand side variables of the regression and the inflation rates. The formal econometric test is performed as follows. The regression equation is rewritten for

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<sup>49</sup> Portmanteau tests are defined as residual-based tests, that do not have a specific alternative hypothesis.

simplification. Let  $\beta = (a_0, \alpha^e, \dots, a_j, \beta^D, \beta^z)'$  be a vector of the coefficients from the regression (16),  $x_{m \times 1}$  be the vector of regressors  $x_t = (1, \pi_t^e, \pi_{t-1}, \dots, \pi_{t-j}, D_t, z_t)'$ , with  $u_t$  denoting the error term. Thus equation (16) can be rewritten as

$$\pi_t = \beta'x_t + u_t \tag{17}$$

with  $\hat{u}_t = \pi_t - \hat{\pi}_t$  as regression residuals, where  $\hat{\pi}_t = \hat{\beta}'x_t$  and  $t=1, \dots, T$ . By estimating the parameters of

$$\hat{u}_t = \delta'x_t + \sum_{j=2}^h \xi_j \hat{\pi}_t^j + v_t \tag{18}$$

by OLS and computing the sum of squared residuals (SSR) of the models (17) and (18) denoted by  $SSR_0 = \sum_t \hat{u}_t^2$  and  $SSR = \sum_t \hat{v}_t^2$  respectively, the RESET test can be performed. An F-statistic is computed by

$$F = \frac{(SSR_0 - SSR)/(h-1)}{SSR/(T-m-h)} \tag{19}$$

where m equals the number of elements of x. If  $E[x_t u_{t-s}] = 0$  for all s, then (19) is exactly F(h-1, T-m-h) distributed under  $H_0: \xi_j = 0$  for all  $j=2, \dots, h$ . If one of the coefficients  $\xi$  is statistically significant, this may be an indication of nonlinearity or any other kind of misspecification. Lütkepohl and Krätzig (2004) suggest, that a value for h=2 or h=3 is sufficient to give an impression of whether or not the relevant model defects are present in this model, which can be detected by this test. Granger and Teräsvirta (1993, Ch 6) find this test to be very useful to detect nonlinearity if the number of variables under the linearity hypothesis is large while the time series is short. Hence, the power of the test is appropriate in this linear model. The RESET tests of the residuals of the linear models in Table 4-1 to Table 4-3 show that the null hypothesis cannot be rejected for the French models, except for the model using the excess capacity as gap indicator. The null can be rejected for all other regressions. This indicates that there is an  $\xi_j$ , that is significantly different from zero, which might be evidence of nonlinearity in these models.

### 4.3.2 The McLeod-Li Method

The McLeod-Li method is also a diagnostic test of the residuals of the linear regression<sup>50</sup>. In principle, this test is the same as fitting an autoregressive conditional heteroskedasticity (ARCH) model to the estimation residuals. The test statistic is

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<sup>50</sup> See McLeod and Li (1983).

computed by regressing the residuals of (17) on the lagged squared values of the residuals of (17).

$$\hat{u}_t = \alpha_0 + \sum_{s=1}^q \alpha_s \hat{u}_{t-s}^2 + \eta_t \tag{20}$$

with  $SSR_0 = \sum_t \hat{u}_t$  and  $SSR = \sum_t \hat{\eta}_t$ . The test statistic can be computed as

$$LM = \frac{(SSR_0 - SSR) / q}{SSR / (T - m - q - 1)} \tag{21}$$

which has an approximate  $F(q, T-m-q-1)$  distribution under  $H_0: \alpha_s=0$  for all  $s=1, \dots, q$ . It turns out that the McLeod-Li test is the exact Lagrange multiplier (LM) test for ARCH errors. The results of these tests are all highly significant for the regressions presented in Table 4-1 to Table 4-3 This is an indication of ARCH in the errors which might be caused by nonlinearity in the models.

### 4.3.3 The BDS Test

The BDS test is a Portmanteau test for time-based dependence in a series. It can be used for testing against a variety of possible deviations from independence including linear dependence, non-linear dependence, or chaos. The test checks whether the regression residuals are independent and identically distributed (iid). If the regression is estimated by OLS though the true model is nonlinear, this test is useful to detect the dependence of the residuals. The BDS testing results for independence, as described in Brock *et al.* (1996) are reported in the last row in Table 4-1 to Table 4-3 This paragraph gives an intuition behind the test and describes the procedure. To perform the test, a distance is chosen and denoted by  $\kappa$ <sup>51</sup>. Then, a pair of observations of the residual series is chosen. If the residuals are truly iid, then the probability  $p_1(\kappa)$  of the distance between these points being less than or equal to  $\kappa$  is asymptotically constant

$$P(|u_t - u_s| \leq \kappa) = p_1(\kappa). \tag{22}$$

To test whether all points have a constant probability of having a distance of being less than or equal to  $\kappa$ , one is choosing sets of  $m$  pairs and moves through the consecutive observations of the sample in order. That is, given an observation  $s$  and an observation  $t$  of the residual series  $u_t$ , a set of pairs is constructed  $\{(u_s, u_t), (u_{s+1}, u_{t+1}), \dots, (u_{s+m-1}, u_{t+m-1})\}$  where  $m$  is the so-called embedding dimension. Let the joint probability of every

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<sup>51</sup> The tests in this study use a value of  $\kappa=0.3$ .

pair of points satisfying the condition be

$$P(|u_t - u_s| \leq \kappa, |u_{t-1} - u_{s-1}| \leq \kappa, \dots, |u_{t+m-1} - u_{s+m-1}| \leq \kappa) = p_m(\kappa). \quad (23)$$

Now, if the residuals are independent, by definition, the joint probability equals the product of the single probabilities, and hence

$$p_1^m(\kappa) = p_m(\kappa). \quad (24)$$

These probabilities have to be estimated and therefore the condition does not hold exactly. The ratio of sets that satisfy this condition is

$$p_{m,n}(\kappa) = \frac{2}{(n-m+1)(n-m)} \sum_{s=1}^{n-m+1} \sum_{t=s+1}^{n-m+1} \prod_{j=0}^{m-1} I_{\kappa}(X_{s+j}, X_{t+j}) \quad (25)$$

where  $n$  equals the number of observations and  $I_{\kappa}$  is the indicator function defined as

$$I_{\kappa}(x, y) = \begin{cases} 1 & \text{if } |x - y| \leq \kappa \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

These ratios can be used to construct a test statistic for independence

$$b_{m,n}(\kappa) = c_{m,n}(\kappa) - c_{1,n-m+1}(\kappa)^m \quad (27)$$

which is under the assumption of independence

$$\sqrt{n-m+1} \frac{b_{m,n}(\kappa)}{\sigma_{m,n}(\kappa)} \rightarrow N(0,1) \quad (28)$$

$$\sigma_{m,n}^2(\kappa) = 4(k^m + 2 \sum_{j=1}^{m-1} k^{m-j} c_1^{2j} + (m-1)^2 c_1^{2m} - m^2 k c_1^{2m-2}) \quad (29)$$

where  $c = c_{1,n}(\kappa)$  and

$$k = k_n(\kappa) = \frac{6}{n(n-1)(n-2)} \sum_{t=1}^n \sum_{s=t+1}^n \sum_{r=s+1}^n h_{\kappa}(X_t, X_s, X_r) \quad (30)$$

$$h_{\kappa}(i, j, k) = \frac{1}{3} [I_{\kappa}(i, j)I_{\kappa}(j, k) + I_{\kappa}(i, k)I_{\kappa}(k, j) + I_{\kappa}(j, i)I_{\kappa}(i, k)] \quad (31)$$

where  $c_{1,n}(\kappa)$  and  $k_n(\kappa)$  are consistent and the most efficient estimators of  $c$  and  $k$  respectively<sup>52</sup>. The problematic in performing the BDS test is that applications with fewer than 500 observations are not reliable, since the distribution of the BDS test statistic can be quite different from the asymptotic normal distribution. To compensate for this, this study has calculated bootstrapped p-values for the test statistic. The test values are reported in Tables 4-1 to 4-3 with the embedded dimension  $m=5$ . The tests show that the errors are not normally distributed in almost all of the regressions. The

<sup>52</sup> See Brock *et al.* (1996).

interpretations of the tests described in this section show that there is evidence for possible nonlinearity. As already mentioned, these tests are also useful to detect other misspecifications, hence a more explicit conclusion can be drawn only when the nonlinear estimation results indicate that these problems have been removed.

## 5. Nonlinear Estimation

### 5.1 Aims of the Econometric Specification

One criticism drawn from the empirical literature review is that the theoretical foundation for nonlinearities in the Phillips curve have been neglected in the studies, with only a few exceptions. This is an issue this study addresses. To find out which underlying model determines the shape of the Phillips curve, the empirical study uses an exclusion strategy to identify the dominant underlying model as close as possible. This is done by identifying two characteristics of the estimated Phillips curve. First, the curvature has to be investigated, i.e. whether the curve is convex or concave. If it is concave, the oligopolistic strategic pricing model is the favoured model. There is no need for a further step. If it is convex, the oligopolistic strategic pricing model can be excluded and there is a second characteristic that has to be identified, which is whether the slope of the curve is determined by the size of the gap (capacity constraint model) or by the average rate of inflation (downward nominal rigidity or costly adjustment). This strategy is summarized in Table 5-1.

**Table 5-1 Determination of the Underlying Model**

		Determinant of the slope of the Phillips curve	
		Inflation	Gap
<b>Shape of the Phillips curve</b>	Convex	Costly adjustment/ DNWR <sup>53</sup>	Capacity Constraint
	Concave	-	Strategic Pricing

<sup>53</sup> Downwards nominal wage rigidity.

## 5.2 Methodology

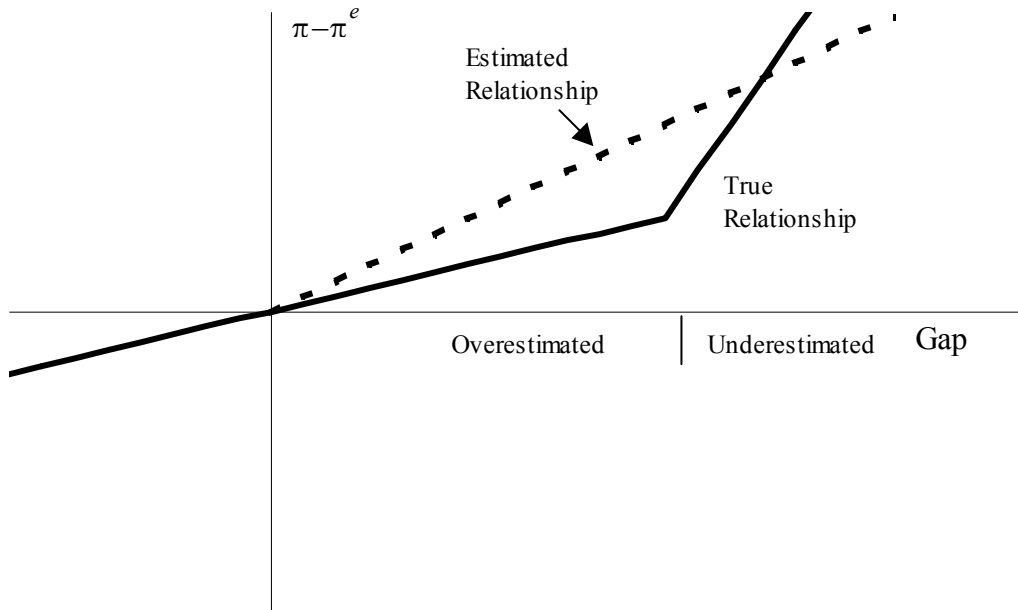
The literature review shows that there are some shortcomings in the specification and estimation of nonlinear Phillips curves. The most important point is that the theoretical foundation is for the most part neglected. Most of the studies assume one theory without considering the other. Another point is that most studies did not apply a nonlinear estimation technique, which should yield more exact results. Rather, they estimate two linear coefficients, one for a positive and one for a negative gap. The capacity constraint model, however, predicts that the relationship changes when the excess demand (supply) is very high. During periods of moderate economic activity, the highest share of firms should not already be faced with bottlenecks. Therefore, there does not seem to be much logic behind the assumption that the relationship must change at a point of zero. The same argument can be made when assuming the costly adjustment model to be the underlying model. The theoretical model predicts that firms change their prices when inflation is much higher than expected. It is more realistic that the price policy does not change if inflation is at rates the agents are accustomed to, and that changes occur at inflation rates that are extraordinarily high. Figure 5-1 on the next page shows a graph of the relevant Phillips curve relationship that makes this point clear. Estimating the model, assuming the kink at the gap equal to zero, would still yield a convex shape but would overestimate the slope on the left side of the true kink and underestimate it on the right side. This means that using the estimation results for policymaking would imply that the costs of fighting inflation would be underestimated at levels of high economic activity (on the right side of the kink in the graph<sup>54</sup>). This implies that monetary policy makers might overestimate the costs of fighting inflation and therefore react too aggressively to an increase in excess demand. The analogous argumentation applies for an overestimation of the slope.

A further point of criticism is that it is not realistic that the relationship "jumps" from one coefficient to another. It seems logical to assume that the so-called regime switching between a high and a low regime, i.e. high and low slope of the Phillips curve relationship, occurs via a smooth transition between one regime and the other. To estimate a nonlinear model with two regimes without making any a priori assumptions

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<sup>54</sup> Note that the terms underestimated and overestimated in the graph are related to the slope of the relationship. An underestimation of the slope implies an overestimation of the costs of decreasing inflation.

**Figure 5-1 Possible Estimation Error Using a Kinked Functional Form**



about the shape of the relationship and the "switching point" a nonlinear estimator is required. All these shortcomings can be circumvented by using a smooth transition regression (STR) model. In the STR model, multiple regimes along with endogenous switching between them define the economy. This methodology is presented in paragraph 5.2.1. The advantages of the STR model over the specifications chosen in the majority of the previous studies are presented in section 5.2.2.

### 5.2.1 The Smooth Transition Regression Model

A general formulation of the standard STR model is

$$\pi_t = x_t' \phi + x_t' \theta G(\gamma, c, s_t) + \varepsilon_t \tag{32}$$

with  $t=1, \dots, T$  and  $\varepsilon_t \sim iid(0, \sigma^2)$ . The vector  $x_t' = (1, \pi_{t-1}, \dots, \pi_{t-p}; w_{1t}, \dots, w_{kt})'$  with  $p+k=m$  and  $w_t$  with  $w_t' = (\pi_{t-1}^{imp}, gap_{t-1})'$  is a vector of exogenous variables. In this model  $k=2$  and  $p=7$  denotes the maximum lag chosen in the linear specification, which is different for each country. Furthermore,  $\phi = (\phi_0, \phi_1, \dots, \phi_m)'$  and  $\theta = (\theta_0, \theta_1, \dots, \theta_m)'$  are parameter vectors of the linear and nonlinear part respectively and  $G(\gamma, c, s_t)$  is a continuous function, which is further specified together with  $\gamma$ ,  $c$  and  $s_t$  below. This model can be interpreted by rearranging (32) to

$$\pi_t = [\phi + \theta G(\gamma, c, s_t)] x_t + \varepsilon_t \tag{32a}$$

which is a model with varying parameters, which vary between two values  $\phi$  to  $\phi + \theta$ , depending on the so-called transition function  $G(\gamma, c, s_t)$ . The transition function is denoted by  $G(\gamma, c, s_t)$ , which is bounded between zero and unity and continuous in the transition variable  $s_t$ . A transition variable can be an element of  $x_t$  or a time trend  $t^{55}$ . For the study of the Phillips curve relationship, it is convenient to assume a logistic transition function, which is

$$G(\gamma, c, s_t) = \left( 1 + \exp \left\{ -\gamma \prod_{k=1}^K (s_t - c_k) \right\} \right)^{-1} \quad (33)$$

where  $\gamma > 0$  is the slope parameter<sup>56</sup>. A logistic smooth transition regression (LSTR) model where  $K=1$  yields the LSTR(1) and with  $K=2$  the LSTR(2) model. The vector  $c = (c_1, \dots, c_K)'$  denotes location parameters with  $c_1 \leq \dots \leq c_K$ . The slope parameter  $\gamma > 0$  determines the smoothness of the transition and the vector  $c$  the location of the transition. For a LSTR(1) model, the parameters  $\phi + \theta G(\gamma, c, s_t)$  change monotonically from  $\phi$  to  $\phi + \theta$  as a function of  $s_t$ .  $c$  is the value of  $s_t$ , where the transition function is equal to 0.5. That follows from (32a) and (33). The model has two regimes that are different and the transition between the regimes is smooth<sup>57</sup>. To let the data decide whether the Phillips curve is convex or concave, and which theory described is dominant in the model, the two parameter vectors  $\phi$  and  $\theta$  are being estimated, restricting the elements of  $\theta$  to be zero except the parameter describing the Phillips curve relationship  $\theta_{gap}$ . If the parameter  $\phi_{gap}$  is positive, which can be assumed from theory, then a positive value of the coefficient of the gap measure as an element in  $\theta_{gap}$  indicates convexity and a negative value a concave curve. For a LSTR(2) model, the curve is s-shaped and the parameters of the model change symmetrically around the estimated midpoint  $\frac{1}{2}(c_1 + c_2)$ . The transition function has the smallest value at this midpoint. This model is appropriate if inflation has the same dynamic behaviour at large and small values of the transition variable. Hence, the two extremes for both very small and very large values of the transition variable  $s_t$  are the same, but the regime between these extremes is different. This has no clear theoretical foundation for the Phillips curve, none of the theoretical models suggest such behaviour. However, recall the empirical study by Filardo (1998), which finds that the US curve is convex for a

<sup>55</sup> In a special case  $t$  can be chosen as the transition variable, then the model can be interpreted as a linear model with time-varying parameters.

<sup>56</sup> See Teräsvirta(2004).

<sup>57</sup> If the slope parameter  $\gamma$  is very large the model converges to a two-state regime switching model.

positive output gap and concave for a negative output gap. The modelling cycle is performed as suggested in Teräsvirta (1998) and reported in the remainder of this chapter. The first step is the specification. The procedure described in the following section 5.2 tests the linear model in chapter four against the LSTR specification and if the linear model is rejected, the type of LSTR (i.e. LSTR(1) or LSTR(2)) model is then chosen. The second step is the estimation of the parameters, which is performed in section 5.3. Finally, the LSTR model, if chosen in step one, is evaluated by testing the models residual properties and for coefficient constancy.

### 5.2.2 Advantages of the Nonlinear Specification

The advantage over simply adding basic nonlinear terms like in equation (11) is that the STR model allows for the possibility that the asymmetry will not be precisely the same for each regime. In other words, a convex or concave Phillips curve is not likely to be invariant over time any more than a linear Phillips curve. The advantage of the STR model over other types of threshold or regime dependent models is that it allows for a smooth transition between regimes rather than by a "jump". The next advantage is that the location parameter of the transition is estimated and therefore not assumed a priori, as in the studies assuming a kinked functional form. Furthermore, for the policy insight that can be drawn from the Phillips curve relationship, STR models have a further advantage: a nonlinear relationship can as well be modelled via a nonparametric approach. This approach, however, is a too general way of specifying the curve for this purpose<sup>58</sup>. As the aim is to put an economic structure on the estimation technique, the STR model allows to sufficiently restrict the set of possible nonlinear models to the models presented in the theoretical part. This technique allows one to narrow down the choice to the models referred to above. This also includes restricting the family of STR models to a more specific class, which will be presented in the following section. Furthermore, a STR model is locally linear and thus allows easy interpretation. This is especially important to allow statements about the costs of a disinflation in terms of output, i.e. the sacrifice ratio. Eliasson (2001) is the only study that uses the STR

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<sup>58</sup> A further problem is the number of regressors as nonparametric approaches face the "curse of dimensionality", which means that a nonparametric approach is inappropriate in this model. A semiparametric method, however, would work in this specification. Using a semiparametric method would be an issue for future research, e.g. for testing the robustness of these results.

methodology to estimate Phillips curves for Australia, Sweden, and the US. However, she does not restrict the models to the class of models that have theoretical microfoundations, but also allows for STR specifications, which do not have theoretical foundations. Therefore, this study is innovative in two points:

firstly in studying the shape of the Phillips curve for these Euro Area countries by investigating possible nonlinearities in the Phillips curve by the determinants of the nonlinearity for the Euro Area countries. Secondly in applying the STR modelling technique to determine the underlying model for a nonlinear Phillips curve, which is a more structural approach compared to the approach in Eliasson (2001).

### 5.3 Nonlinear Model Specification

The Portmanteau tests performed in chapter four are, as noted before, not helpful in testing the linear model against a certain specification of a nonlinear model. In this section, the linear model is tested against the LSTR specification. The testing procedure has two steps. First, it is tested whether the linear model is adequate allowing for different transition variables, that can be a possible transition variable from theory. If the linear model is rejected, a LSTR(1) or LSTR(2) model has to be chosen. The testing procedure is described in the following two subsections. As the measures for excess demand have all performed well in the linear part, and the results did not change substantially with the measure of the gap, the OECD gap measure is used as proxy for demand pressure for the nonlinear equations. This measure is computed from a production function approach and hence is a structural measure and does not already contain Phillips curve information, as the NAWRU does. The theoretical background of this measure makes it to be the most accurate one.

#### 5.3.1 Testing Linearity Against the Nonlinear Specification

The logistic transition function (33) can as well be constant, if  $\gamma = 0$  then  $G(0, c, s_t) = 1/2$  for all values of  $c$  and  $s_t$ . Hence the STR model nests the linear model and the linearity test can be performed by testing the null of  $\gamma = 0$ . Linearity versus STR can be tested

by an LM test but this is problematic<sup>59</sup>. An F-Test is computed instead, as recommended by Granger and Teräsvirta (1993) and Teräsvirta (2004), which is derived in the following three steps. First, the residual sum of squares of the regression of  $\pi_t$  on  $x_t$  is computed  $SSR_0 = \frac{1}{T} \sum_{t=1}^T \hat{\varepsilon}_t^2$ . Second, from the regression

$$\hat{\varepsilon}_t = \alpha'_0 x_t + \sum_{s=1}^3 \alpha'_s \tilde{x}_t s_t^s + v_t \quad (35)$$

the residual sum of squares is computed  $SSR_1 = \frac{1}{T} \sum_{t=1}^T \hat{v}_t^2$ . Third, the F-test statistic

$$F = \frac{(SSR_0 - SSR_1) / 3m}{SSR_1 / (T - 4m - 1)} \quad (36)$$

is computed. Under

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$$

the test-statistic corresponding to (35) has an approximate F-distribution with  $3m$  and  $T - 4m - 1$  degrees of freedom. Theory suggests two variables that could perform as the transition variable, these are lagged inflation, as a proxy for average inflation, and the measure for real activity. If the null is rejected for both transition variables, one chooses the transition variable, which was most strongly rejected as measured by its p-value.

### 5.3.2 Choosing the Type of the Model

The next step in the modelling cycle is to choose the type of STR model. Note, that if linearity was not rejected in the first step, the modelling cycle ends and the linear model

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<sup>59</sup> Like many nonlinear models, the STR model is only identified under the alternative, not the null hypothesis due to the nuisance parameters  $\theta$  and  $c$ . Therefore, the transition function in the STR model (33) is substituted by a third-order Taylor approximation around the null. The model (32) for LSTR(1) is assumed, which allows testing both for LSTR(1) and LSTR(2) (this has been suggested by Teräsvirta (1998, 2004). The test is based on the transformed equation

$$\pi_t = x'_t \beta_0 + \sum_{j=1}^3 (\tilde{x}_t s_t^j)' \beta_j + \varepsilon_t^*, \quad t=1, \dots, T \quad (34)$$

where  $x_t = (1, \tilde{x}_t)'$  and  $\varepsilon_t^* = \varepsilon_t + \tilde{x}_t' \theta R(\gamma, c, s_t)$  where  $R(\gamma, c, s_t)$  is the remainder.  $\varepsilon_t^* = \varepsilon_t$  under the null hypothesis within this equation is

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$$

because each  $\beta_j = \gamma \tilde{\beta}_j$  for  $j=1,2,3$ , where  $\tilde{\beta}_j \neq 0$  is a function of the parameters in the original STR specification. Because  $\varepsilon_t^* = \varepsilon_t$  under  $H_0$ , the asymptotic distribution theory is not affected if an LM-type test is used. The asymptotic distribution theory of the resulting  $\chi^2$ -test requires the existence of  $E[s_t \tilde{x}_t \tilde{x}_t']$ . The test statistic has an asymptotic  $\chi^2$ -distribution when the null is valid. However, the statistic can be distorted in a small sample, and hence an F-statistic is computed instead, as the F-statistic has better small sample properties (the empirical size of the test remains close to the nominal size while power is good), as shown by Granger and Teräsvirta (1993, Ch 7) and Teräsvirta (1998, 2004).

is chosen. Then, when linearity is rejected, tests have to be performed to decide between the LSTR(1) and LSTR(2) model. The test makes use of the auxiliary regression (35). The following sequence of null hypotheses is defined

$$H_{03} : \beta_3 = 0$$

$$H_{02} : \beta_2 = 0 | \beta_3 = 0$$

$$H_{01} : \beta_1 = 0 | \beta_2 = \beta_3 = 0.$$

Granger and Teräsvirta (1993, ch 7) show, that in this test sequence, if the rejection of  $H_{03}$ , measured by the p-value, is the strongest of the three tests, then a LSTR(2) model is chosen. Otherwise a LSTR(1) model is selected. All three hypotheses can be rejected simultaneously, hence the strongest rejection indicates the best choice of the model. For a detailed derivation of the test see Granger and Teräsvirta (1993) and Teräsvirta (1994). For the Phillips curve models, two variables are considered as transition variables, i.e. the lag of inflation, as a proxy for average inflation, and the gap measure. Table E-1 in the appendix shows the p-values of the F-tests.

### 5.3.3 Estimation of Parameters

The parameters of the STR model are calculated using conditional maximum likelihood. The log-likelihood is maximised numerically, which requires good starting values for the algorithm to work. Therefore, a gridsearch is performed to find values of  $\gamma$  and  $c$ . Thereby the remaining parameters  $\phi$  and  $\theta$  are estimated conditionally on  $\gamma$  and  $c$ . For each value of  $\gamma$  and  $c$  the residual sum of squares is computed. The values that correspond to the minimum of that sum are taken as starting values. Initially, the full model is estimated with the lag structure from the linear model as a starting point. Then, the model's size is reduced by using the t-values as a guide to exclude redundant variables, thereby the inflation expectations have been excluded from the model to increase the degrees of freedom. In the transition part, all variables but the gap measures are excluded. This allows the coefficient of the gap to vary either with the level of inflation or with the size of the gap. The latter would provide evidence for the capacity constraint model if  $\theta_{gap} > 0$  and for the oligopolistic strategic pricing model otherwise. If the coefficient varies with the level of inflation, this would give evidence for the costly adjustment model and the downward nominal rigidity model if  $\theta_{gap} > 0$  and has no theoretical model for interpretation otherwise. The determination of the underlying

theoretical model using the STR estimation results is illustrated in Table 5-2, which corresponds to Table 5-1.

**Table 5-2 Determination of the Underlying Model Using STR**

		Determinant of the slope of the Phillips curve (=transition variable)	
		$s_t = \pi_{t-1}$	$s_t = gap$
Shape of the Phillips curve	Convex ( $\theta_{gap} > 0$ )	Costly adjustment/ DNWR	Capacity Constraint
	Concave ( $\theta_{gap} < 0$ )	-	Strategic Pricing

### 5.4 Evaluation of the Nonlinear Model

This section gives a description of several tests for the nonlinear model. The usual residual tests as the RESET are not powerful in detecting misspecification of the STR model<sup>60</sup>. The tests performed are suggested by Granger and Teräsvirta (1993) and Teräsvirta (2004).

#### 5.4.1 Test of No Additive Nonlinearity

After the STR has been estimated, it has to be checked whether there is remaining nonlinearity in the model. The test assumes that the type of the remaining nonlinearity is again of the STR type. An alternative is defined as

$$\pi_t = x_t' \phi + x_t' \theta G(\gamma_1, c_1, s_{1t}) + x_t' \psi H(\gamma_2, c_2, s_{2t}) + u_t \tag{37}$$

where  $H(\gamma_2, c_2, s_{2t})$  is another transition function and  $u_t$  is the *iid* error term. Assuming for simplicity that  $H(0, c_2, s_{2t}) = 0$ , the null of no additive nonlinearity is defined as  $\gamma_2 = 0$ . As for the test of paragraph 5.3.1, the model is not identified under the null and hence the transition function  $H(\gamma_2, c_2, s_{2t})$  is approximated by its Taylor expansion around  $\gamma_2 = 0$ . The auxiliary model is

$$\pi_t = x_t' \beta_0 + x_t' \theta G(\gamma_1, c_1, s_{1t}) + \sum_j (\tilde{x}_t s_{2t}^j)' \beta_j + u_t^* \quad j=1,2,3. \tag{38}$$

<sup>60</sup> See Teräsvirta (2004).

and is used to test this alternative, where  $u_t^* = u_t + z_t' \psi R_3(\gamma_2, c_2, s_{2t})$ , with  $R_3$  being the remainder from the polynomial approximation. The test is performed by regressing the residuals of the smooth transition regression on  $(\tilde{x}_t' s_{2t}, \tilde{x}_t' s_{2t}^2, \tilde{x}_t' s_{2t}^3)'$  and the partial derivatives of the log-likelihood function with respect to the parameters of the model. The null is defined as

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$$

and the transition variables  $s_{2t}$  are again the lag of inflation as a proxy for average inflation and the output gap. Furthermore, all variables but the output gap are excluded from the second nonlinear part by restricting the corresponding parameter to zero. The resulting F-statistics are given in the same way as in the test for linearity. The p-values of the results are reported in Table 5-3.

#### 5.4.2 Test of Parameter Constancy

A next step in the evaluation of the model is to test for misspecification. The STR model has to be tested for parameter constancy. The null hypothesis of the test performed is parameter constancy. Consider the STR model from equation (32)

$$\pi_t = x_t' \phi + x_t' \theta G(\gamma, c, s_t) + u_t \quad (39)$$

with the modification  $\phi(t) = \phi + \lambda_\phi L_\phi(\gamma_\phi, c_\phi, t^*)$  and  $\theta(t) = \theta + \lambda_\theta L_\theta(\gamma_\theta, c_\theta, t^*)$ , where  $t^* = t/T$ . Under the alternative, the parameter  $\theta$  of the STR model is assumed to be time varying and vary smoothly between  $\theta$  and  $\theta + \lambda_\theta$  with the transition function  $L_\theta$ , which is defined as a logistic function

$$L_\phi(\gamma_\phi, c_\phi, t^*) = \left( 1 + \exp \left\{ -\gamma_\phi \prod_{k=1}^K (t^* - c_{k\phi}) \right\} \right)^{-1} \quad \gamma > 0, \quad K=1,2. \quad (40)$$

This smooth transition between the parameters is defined as analogous for the parameter  $\phi$ <sup>61</sup>. The null hypothesis of parameter constancy is

$$H_0 : \gamma_\theta = \gamma_\phi = 0$$

and the alternative is that either one or both of the parameters are positive. For this test, only the coefficient of the output gap variable is being tested for constancy<sup>62</sup>. Again, the

<sup>61</sup> Note that the alternative hypothesis defines a time varying smooth transition regression model (TV-STR). A univariate TV-STAR model is developed by Lundbergh *et al.* (2003).

<sup>62</sup> The parameters  $\gamma$  and  $c$  are assumed to be constant.

model is only identified under the alternative<sup>63</sup>. Hence the following nonlinear auxiliary regression is used for the test

$$\pi_t = x_t' \beta_0 + \sum_{j=1}^3 (x_t(t^*)^j)' \beta_j + \sum_{j=1}^3 \{x_t(t^*)^j G(\gamma, c, s_t)\}' \beta_{j+3} + u_t^* \quad (41)$$

The null of parameter constancy thus becomes

$$H_0 : \beta_1 = \beta_2 = \dots = \beta_6 = 0$$

and all but the parameters of the output gap are excluded from the parameter constancy test, in order to detect the possible nonconstancy in the relevant coefficient. The LM test for testing the null is performed by regressing the residual of the regression (32),  $\hat{u}_t$ , on the right hand side variables of (40). Analogous to the test described in the initial model selection step, an F-version of the test is preferred to the asymptotic theory. The results of these tests when only testing the constancy of the parameter of the output gap are presented in Table 5-3.

## 5.5 Estimation Results of the Nonlinear Models

Recall the estimation equation, which has the following form

$$\pi_t = x_t' \phi + x_t' \theta G(\gamma, c, s_t) + \varepsilon_t \quad (32b)$$

with  $t=1, \dots, T$  and  $\varepsilon_t \sim iid(0, \sigma^2)$  where  $x_t' = (1, \pi_{t-1}, \dots, \pi_{t-p}; w_{1t}, \dots, w_{kt})'$  with  $p+k=m$  and  $w' = (\pi_{t-1}^{imp}, gap)'$  and hence  $k=2$ . Furthermore,  $\theta = (0, \dots, 0, \theta_{gap})'$  allows only the coefficient of the gap measure to change in the specification. The F-test described previously tests whether the variable  $gap$  or  $\pi_{t-1}$  is favoured as a transition variable by the data and the results are reported in Table 1 in Appendix E. These tests suggest LSTR(1) models for Italy for both possible transition variables, where the rejection of the linear model is strongest for the inflation rate as the transition variable. For Germany, the model is linear for the  $gap$  as a transition variable and a LSTR(1) model is appropriate for the lagged inflation rate. The model for France, with the gap as the

<sup>63</sup> This familiar complication is dealt with in the same way as before. The parameters  $\phi(t)$  and  $\theta(t)$  as defined in equation (39), is expanded into a Taylor series around the null. A first order Taylor expansion around  $\gamma_{\psi} = 0$  has the form (after reparametrization)

$$T(\gamma_{\psi}, c_{\psi}, t^*) = \frac{1}{2} \left( 1 + \gamma_{\psi} \left\{ \delta_0^{(\psi)} + \delta_1^{(\psi)} t^* + \delta_2^{(\psi)} (t^*)^2 + \delta_3^{(\psi)} (t^*)^3 \right\} \right) + R_1(\gamma_{\psi}, c_{\psi}, t^*),$$

$\psi = \phi, \theta$ , where  $R_1$  denotes the remainder. Substituting  $T-R_1$  into equation (39) yields the auxiliary equation (41).

transition variable suggests a LSTR(2) model, and a LSTR(1) with the lagged inflation rate as a transition variable. When running a LSTR(2), the transition occurs due to only four outlying observations, which can be observed from the graph when plotting the estimated transition function  $G(\hat{\gamma}, \hat{c}, s_t)$  against its argument  $s_t = gap$ . Hence a LSTR(1) model is estimated. Firstly, the models are estimated with the suggested transition variable. The results are reported in Table 5-3 on the next page. As for France and Italy, the tests of no remaining nonlinearity are rejected for the respective other transition variable, the models are estimated, each with the other transition variable. The chosen transition variable is reported in row  $s_t$ . The inflation expectations have been included in the models but were not significant in any of them. Therefore, expectations are excluded from the nonlinear models<sup>64</sup>.

### 5.5.1 France

The results from the F-tests for France indicate that the *gap* should be chosen as the transition variable, which points towards the capacity constraint model. The estimated coefficients with the *gap* as the transition variable are reported in the first column of Table 5-3 on the next page. The estimation output shows that the coefficients of the lagged inflation variables sum up to 0.96, which is very close to the result from the linear equation. The Phillips curve relationship can now be divided in two extreme regimes of the business cycle: one low-activity regime, where the coefficient describing the slope of the Phillips curve is 0.17, which is close to the linear model. The other is a high-activity regime, where the coefficient is 0.76, which shows that the model is convex. In between, the transition is smooth. Hence, the sensitivity of inflation to an increase in the output *gap* is higher at high excess demand, which is a result that is in line with the capacity constraint model. The changing point between low and high inflation regimes is estimated by the location parameter denoted by  $c$ , which has a value of 0.22. The location parameter is the point, where the transition from one regime to the other is at its midpoint, i.e. where the transition function is 0.5. This shows that the regime changes around a *gap* with a size of 0.22, at a *gap* with approximately -0.2 the low regime starts

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<sup>64</sup> As noted before, the problems with the high correlation with the lagged inflation rate and the insignificance might be due to the high collinearity. The estimator would be still consistent in that case, but the additional information in that measure appears to be marginal, and is therefore excluded.

**Table 5-3 STR Coefficient Table**

<b>Dependent Variable: <math>\pi_t</math></b>					
$s_t$	<b>France</b>	<b>France</b>	<b>Germany</b>	<b>Italy</b>	<b>Italy</b>
	<b>gap</b>	<b><math>\pi</math></b>	<b><math>\pi</math></b>	<b><math>\pi</math></b>	<b>gap</b>
<b>Low regime</b>					
<i>const</i>	0.18	0.08	0.15**	0.44*	0.31**
$\pi_{t-1}$	1.26***	1.21***	1.27***	1.25***	1.30***
$\pi_{t-2}$	-0.34***	-0.31***	-0.34***	-0.33***	-0.37***
$\pi_{t-4}$	-0.24***	-0.20**	-0.31***	-0.53***	-0.53***
$\pi_{t-5}$	0.28***	0.27***	0.62***	0.65***	0.67***
$\pi_{t-6}$			-0.30***		
$\pi_{t-7}$				-0.14***	-0.13***
<i>Gap<sub>t-1</sub></i>	0.17**	0.08	0.08*	0.17**	0.27*
$\pi_{t-1}^{imp}$	0.01**	0.02***	0.00	0.01***	0.02**
$\Sigma \pi$	0.96	0.97	0.93	0.90	0.94
<b>High regime</b>					
<i>Gap<sub>t-1</sub></i>	0.59***	0.32***	0.12*	0.33***	0.53**
$c_l$	0.22	5.75***	3.96***	10.03***	0.21
$\gamma$	11.61	12.76	11.85	5.02	17.75
$\Sigma gap$	0.76	0.41	0.20	0.50	0.80
$R^2$	0.99	0.99	0.97	0.99	0.99
<i>Adj R<sup>2</sup></i>	0.99	0.99	0.97	0.99	0.99
<i>S.D.</i>	0.38	0.37	0.33	0.60	0.58
<i>ARCH-LM</i>	1.16	1.90*	1.13	4.26***	0.00***
<b>Test of no remaining nonlinearity</b>					
$\pi$	0.02	0.67	0.63	0.57	0.10
<i>gap</i>	0.94	0.19	0.10	0.04	0.75
<b>Test of parameter constancy</b>					
H	0.00	0.11	0.27	0.00	0.11

\*\*\*, \*\*, \* denote 1%, 5% and 10% significance level.  $\pi^{imp}$  is the annual growth rate of the import price index, gap is the gap measure of the respective columns.  $\Sigma \pi$  denotes the sum of coefficients of lagged inflation rates.  $\Sigma gap$  is the sum of the gap coefficients of the linear and the nonlinear part.  $\gamma$  denotes the pace of the transition and  $s_t$  and  $c_l$  are the transition variable and the location parameter respectively. A constant is included in the regressions but not reported in the table. The tests of no remaining nonlinearity and parameter constancy are discussed in section 5.4. H denotes the null of the parameter constancy test, the values are the p-values of the test.

transmitting into a high regime with an increasing excess demand<sup>65</sup>. At a gap size of approximately 0.5, the high regime coefficient determines the output-inflation relationship<sup>66</sup>. To get a better idea of the transition dynamics, the transition function  $G(\hat{\gamma}, \hat{c}, gap)$  is plotted against the output gap series in Figure F-1 in the appendix. The average of the rescaled output gap is -0.49, which indicates that the change in the relationship starts to change at an above average excess demand. Assuming that there is neither excess demand nor supply in the economy at this point of -0.49, a gap of -0.2 indicates weak excess demand. This result is consistent with the theory where the share of firms facing capacity constraints increases during an upturn of the economy. Unlike in the linear regression, the test of no ARCH in the errors cannot be rejected. The null of parameter constancy of the Phillips curve parameter, however, is clearly rejected. Furthermore, there is evidence for remaining nonlinearity when including the lagged inflation rate as the transition variable in the test for no remaining nonlinearity as described in 5.4.1.

Therefore, the model is estimated again using the lagged inflation rate as a transition variable, which is reported in the second column. The results reported in the second column show that the coefficients describing the Phillips curve relationship are both lower than in the first estimation. The coefficient determining the slope of the Phillips curve in a low inflation regime is 0.08 with a p-value of 0.13 and 0.41 in a high inflation regime, which implies a convex Phillips curve, too. The estimate of the changing point between low and high inflation regimes is at 5.75 percent inflation. The transition function is plotted in Figure F-2 in the appendix. The relevant coefficient describing the slope of the Phillips curve is higher during periods of high inflation. During periods with inflation rates below approximately three percent, the estimated coefficient is 0.08, whereas it increases with an increase in average inflation converging towards a coefficient value of 0.41. This would be evidence for the costly adjustment and downward nominal wage rigidity model. When evaluating the model, there is evidence for ARCH in the error terms, which could indicate a misspecification. The test of parameter constancy cannot be rejected, which indicates that this specification yields a more stable coefficient than the previous one. Moreover, there is no evidence for remaining nonlinearity from the gap or inflation.

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<sup>65</sup> The estimate of the location parameter is not significant at the 10 percent level, but it has a p-value of 0.12.

<sup>66</sup> The parameter  $c_t$  has a p-value of 0.11, hence the null of  $c_t=0$  can be rejected with a probability of 89 percent. For the second regression for Italy with the gap as the transition variable, the p-value is 0.16.

Thus both models seem to exist in the data for France. However, when plotting the transition function with the inflation rate as the transition variable against time (not reported here), the coefficient of 0.41 is valid for the period 1970 to the early 1980s, then, the coefficient decreases to the value of about 0.08. This is due to the inflation process in France, which was very high during the 1970s and decreased during the 1980s to a lower average inflation rate. It could be possible that the downwards nominal wage rigidity and costly adjustment models are the dominant models during that period, later after the mid 1980s the capacity constraint model would be the dominant model. When putting this result into a historical perspective, the time of the change in the mid 1980s is striking. The change in the regime in the early 1980s coincides with the time of a significant change in the French labour laws, specifically with the introduction of minimum wages and a maximum number of weekly working hours enacted between 1981 and 1983. During that period, prices and wages continued to increase and the French Franc was devaluated in June 1982 and March 1983. Furthermore, unemployment increased during the period of dirigism. After the breakdown of this regime, the period of a transition towards the "Franc Fort" began. Therefore, it could also be the case that the relationship between inflation and output has changed during the early 1980s, not due to a change in inflation but due to a change to another monetary policy regime. This change may not be noticeable as there is no data available to control for the regime shift in monetary policy. Nevertheless, the change in monetary policy influences both the inflation rate and the expectations of economic agents and therewith the Phillips curve relationship. Therefore, the change in the relationship determined by the level of inflation might be a result of the changing institutional environment.

In conclusion, there is evidence for nonlinearity in the Phillips curve for France. First, the errors of the linear model indicate that there might be nonlinearity in the data. Second, when performing the model selection test, the nonlinear model is preferred by the data, which confirms the linearity test from the OLS regression residuals. The underlying microfoundation of the model, however, is less clear. There is evidence that both inflation and the gap change the Phillips curve relationship. The downward nominal wage rigidity and costly adjustment models and the capacity constraint model do not contradict each other and they could co-exist. It could also be the case that the capacity constraint model is the underlying model and there was an institutional change in the relationship in the mid 1980s. This seems fair to assume since the change from one coefficient to the other is a single change during a period that also affected by a

change in the monetary policy strategy. Therefore, an accompanied change in expectations might be the reason why there is evidence that the coefficients are non-constant when estimating the capacity constraints.

### 5.5.2 Germany

In contrast to the model for France, the model selection test prefers the LSTR(1) only for inflation as transition variable, while the linear model is preferred when choosing the gap as the determinant of the nonlinearity. The sum of the coefficients of the STR model with the suggested transition variable of the lagged inflation rates is only slightly lower compared to the OLS estimates. This is similar to the finding for the model for France. The coefficient describing the slope of the Phillips curve has a value of 0.08 at an inflation rate of below 3.5 percent. At an inflation rate above that value, the pressure of an increase of the output gap on the inflation rate increases. At the high inflation regime, which is above an inflation rate of approximately 4.5 percent, the estimated coefficient is has a value of 0.2, which is almost double when compared to the OLS estimate of 0.13. This indicates that when the inflation rate is above 3.5 percent in Germany firms increase the frequency of price changes, which makes the price level more reactive towards an increase in the output gap. The transition function is plotted against the lagged inflation rate in Figure F-3. When plotting the transition function against time, the relevant coefficient is changing for several times during the observation period. This result differs from the Italian and French models, where the coefficient only changes once during the observation period. Furthermore, the capacity constraint model does not seem to play a role and rather the costly adjustment and downward nominal rigidity models are dominant. As the changing point is already at low inflation rates, which can be observed from Figure F-3 the output costs for bringing inflation down become higher sooner, at inflation rates of about 2.5 percent. Given the outstanding position of the Bundesbank in fighting inflation compared to the other European central banks, the acceleration of inflation could well be starting at comparably low inflation rates<sup>67</sup>. In the case of costly adjustment of prices, economic agents are used to low inflation and therefore start changing their prices more frequently already at these rather moderate inflation rates. The test of no ARCH in the errors

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<sup>67</sup> When running a LSTR(2) with the output gap as transition variable as suggested by the F-Test, this yields a model that is a LSTR(2) only due to a few observations on both sides. These observations are captured in the residuals when running the model formulated in this section.

cannot be rejected which is an improvement compared to the linear model. Furthermore, the tests of parameter constancy and no remaining nonlinearity do not show evidence for a wrong or incomplete specification. As the ECB was designed with the Bundesbank as a model, the result of parameter constancy and no additive nonlinearity are pleasing. This might be evidence for the conjecture that the constancy problems in the Italian and French estimations might be caused by the change of the monetary policy strategy and the change of the central bank. There are some caveats that should be noted. The result that the *gap* is not a determinant of the slope of the Phillips curve might be caused by the reunification data problem. The SSR of the nonlinear model might be high as the potential output might be due to missmeasurement during the reunification period and also the inflation has behaved differently during that period.

### 5.5.3 Italy

The model selection tests for Italy also prefer the inflation rate as the transition variable, as the rejection of the linear model is stronger. Compared to the German model, the level of the inflation rate where the relationship changes is at approximately 4.5 percent. Below that value, the relevant coefficient is almost half of the value of the linear regression. The high inflation regime is at an inflation rate above 14 percent and the Phillips curve coefficient is at 0.5. Between the high and low inflation regimes, the transition of the coefficient from 0.17 to 0.5 occurs smoother than in the other models, as can be observed from Figure F-4. Given that the inflation rate in Italy has decreased since the start of the second stage of the European Monetary Union (EMU), this could also be evidence for the increased credibility of the central bank, similarly to the French model. The transition starts around 1981, where the Banca d'Italia became independent from the Italian Treasury. The transition function has been below 0.1 since 1987, which is about the time when the Banca d'Italia achieved a reduction to only five percent of inflation, compared with more than 20 percent in 1980. Therefore, the same argumentation as for the French model applies.

Thus, as the suggested model for the *gap* as a transition variable also points towards a LSTR(1) model, hence another regression with the *gap* as transition variable was run. This regression is reported in the last column of Table 5-3. The results are also realistic with the coefficient at a negative output *gap* being 0.27 and changing at a state of the

economy that is neither in excess demand nor supply. In a state with a gap at a value of 0.5, the coefficient is 0.8, i.e. much higher. This model is favoured since, when plotting the transition function against time, the higher coefficient applies also to the time of the boom in the 1990s, which indicates that this transition is not being influenced by the change in monetary policy and central bank. The plot of the transition function of the equation with the inflation level as the transition variable indicates a state of a high output inflation trade-off only until the mid 1980s, and it has been low since 1987.

Though still the costly adjustment model cannot be excluded, and the underlying determinant of the output inflation relationship probably is a "bit of both". According to the test of parameter constancy, the constancy of parameters can be rejected for the costly adjustment and downward nominal rigidity model, but not for the capacity constraint model. The reason might be that the change in the monetary policy strategy also considerably changed the relationship, independent of the current level of inflation. The argumentation is the same as in the French model. Furthermore, there is a clear indication of ARCH in the errors. The system might be misspecified when using the gap as transition variable during the high inflation period in the 1970s and 1980s and correctly specified during the most recent years, and the other way around with regard to the inflation level as the transition variable. Given that the inflation rate in Italy has been much higher than in the other countries and that a high level of inflation has been found to be correlated with a high variability of inflation in cross section studies of industrialised countries<sup>68</sup>. Thus, this may as well be the cause for the ARCH in the errors.

## 5.6 Summary

The regression results of the STR models for Germany, France, and Italy provide some evidence for the costly adjustment and downward nominal wage rigidity models to be a theoretical explanation for the empirical finding of nonlinearity in the Phillips curve. For France and Germany, the diagnostic tests indicate parameter constancy and no remaining nonlinearity, whereas the ARCH in the errors are removed only in the model for Germany. The results for Germany also differ from the other countries as there is no evidence for the *gap* to be the source for the nonlinearity and thereby there is no evidence for the capacity constraint model or the strategic pricing model. In contrast,

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<sup>68</sup> See Lucas (1973) for a seminal study.

the models for Italy and France indicate that the relationship changes with the state of the business cycle, too. However, the evaluation of these models are different for the two countries. The capacity constraint model for Italy shows evidence for parameter constancy and no remaining nonlinearity. On the other hand, there is still evidence for ARCH in the errors. For the specification of the capacity constraint model for France, there is evidence for remaining nonlinearity with the gap measure as the transition variable and the test of parameter constancy is significantly rejected. This however is not the case when specifying the model for France with the inflation measure as the transition variable. When interpreting this result, one should keep in mind that the relationship in the inflation determined nonlinearity for France and Italy only changes once during the sample period.

Thus, the dominant models appear to be the costly adjustment and downward nominal wage rigidity models for Germany. For Italy and France conversely, the capacity constraint model is the preferred model by the data, whereas the change in the relationship determined by the inflation rate appears to be driven by a single change in the institutional surrounding rather than the microfoundation models. What appears to be obvious is that the Phillips curve is convex and therefore there is no evidence for the strategic pricing model.

## **6. Policy Implications**

For monetary policy makers the existence of the Phillips curve relationship implies that that a contractionary monetary shock raises unemployment and decreases output temporarily and leads to a delayed gradual fall in inflation. Assuming that the short-run Phillips curve is linear means that a positive shock in aggregate demand will have the same short-run impact on output and inflation, regardless of the state of the economy in the business cycle or the current level of inflation. In response to that, a contractionary monetary shock can be created by monetary policy for bringing inflation back to its initial level. The costs in terms of output are the same in this case of a linear Phillips curve as the output gains from the positive shock to demand. The main finding of this empirical study is that there is evidence for the existence of a convex Phillips curve in France, Germany, and Italy. A convex Phillips curve has, depending on the underlying model, several implications for monetary policy, which will be shortly outlined below.

The results for Germany indicate that the source of the nonlinearity is the average rate of inflation. The estimation results indicate that the monetary authorities may find it much more difficult to achieve lower inflation rates in a low inflation period, compared to a period when inflation is relatively high. The benefits of lower inflation have to be larger in order to justify a disinflation. On the other hand, during low inflation rates, the sensitivity of inflation to an increase or decrease of excess demand is lower. Thus, the monetary authorities have more time to react to a change in excess demand. This is an advantage, as the evaluation of the current state of the economy is hard to find in real time, due to revisions and trend extraction problems. Furthermore, monetary policy feeds through to the real economy with a lag. Thus, during low inflation periods, the policy makers have time to observe the current state of the economy and react when they have more certainty about the situation of the business cycle. This provides a rationale for targeting low inflation rates, since policy makers are thus given time to react and therefore allowing them to avoid rushed policy decisions. A disadvantage of the low rates of inflation is that prices and wages are set at levels that might be inefficient and inflation cannot erode these imbalances (see Akerlof *et al.* (1996) for a discussion).

For the other countries, the data shows that both excess demand and the level of inflation are the source of the convexity of the Phillips curve. As described in the previous chapter, the capacity constraint model is preferred for France and Italy as the historical change in the institutional surrounding might cause structural changes, which make the models nonlinear determined by the inflation rate. Therefore, there is evidence that firms, that are faced with bottlenecks in the production process in boom phases, cause the nonlinearity and that the slope of the curve changes with the state of the business cycle. This implies that, when the economy is initially at a low state in the cycle, easing monetary conditions will have larger effects on output and lower effects on inflation. On the other hand, if the economy is initially strong, a monetary expansion will mainly affect prices. These are direct effects of the capacity constraint model underlying these findings. There are also indirect effects of the capacity constraint model that are described in Laxton *et al.* (1995) for instance. The greater the variance of output, the lower the average level of output will be. To see this, imagine two situations, one hypothetical situation, where there are no fluctuations in output denoted by  $\bar{y}$  and a more realistic situation with fluctuations in output. In order for there to be no trend in inflation in an economy with a convex Phillips curve and fluctuations, the sum over

time of all negative output gaps relative to  $\bar{y}$  will have to be larger than the sum of the positive output gaps. If this was not the case, inflation would have an upwards trend, as it would rise with each cycle because positive output gaps increase inflation more than negative gaps decrease it. Therefore, the smaller the deviations of output from its potential, the higher the average level of output. This makes a clear case for stabilisation policies.

Schaling (2004) develops a theoretical model that develops alternative monetary policy rules assuming a convex capacity constraint type Phillips curve. The usual monetary policy rule is the well-known Taylor rule, which is linear. Schaling (2004), for instance, shows that in the optimal monetary policy rule the interest rate is a nonlinear function of the deviation of inflation from its target and of output from its potential. Laxton *et al.* (1995) also simulate the influence of the convex capacity constraint type Phillips curve to the inflation and interest rate and compare the results to the linear case when assuming delayed response of monetary policy by one year. From the results, the influence of the positive output shock to inflation and interest rate is twice as strong in the convex Phillips curve case.

Therefore, the policy implications from the capacity constraint models, like found for France and Italy, make the case for avoiding too expansive monetary policy as the sacrifice costs of disinflationary monetary policy required to compensate for earlier expansions can be very high. A pre-emptive tightening in response to inflationary pressures helps to prevent the economy from moving too far up the Phillips curve where inflation begins to rise more rapidly, thereby avoiding the need for a larger negative output gap in the future to reverse this sizeable increase in inflation. The model for Germany indicates a change in the relationship when inflation is above 3.5 percent. Keeping inflation lower would be beneficial for monetary policy as the time to react to changes in real activity is longer. On the other hand, monetary authorities may find it more costly to achieve lower inflation when current inflation is low than when it is relatively high. This means that the benefits of lower inflation have to be larger in order to justify a disinflation when inflation is already low. However, each statement or recommendation about monetary policy should be taken with "a pinch of salt". The Lucas critique<sup>69</sup> says that effects of changes in monetary policy might be unpredictable

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<sup>69</sup> See Lucas (1976).

as the change in the policy also implies a change in the aggregate relationships. Thus, if policymakers attempt to take advantage of statistical relationships, effects operating through expectations may cause the relationships to break down.

## 7. Conclusion

The empirical study of this thesis shows that there is evidence for a convex Phillips curve in France, Germany, and Italy. While for France and Italy, there is evidence for the source of the convexity being the state of the business cycle, the data for Germany suggests that the slope of the Phillips curve changes with the average level of inflation. Albeit the estimations for Italy and France show more unbalanced results, the historical and institutional surroundings give explanations for the failure to reject the tests of parameter constancy. The policy implications from these models are interesting, as they give a rationale for both stabilising and forward looking monetary policy, and the target of low inflation rates. As the results differ from one country to the other, the challenge for the ECB in carrying out monetary policy is to combine the different Phillips curve relationships in the European countries in order to obtain an optimal monetary policy strategy. An aim for future research should also be to quantify the Phillips curve relationship for the other Euro Area countries to allow for clearer statements regarding monetary policy implications for the ECB. Furthermore, the inflation expectations should be included using more promising data in the expectations model described in Appendix C. One example would be to use the learning approach described in this paper but use either real time or survey data for the estimation. To test the robustness of the results, it would be interesting to see whether a semiparametric approach would yield similar shapes of the curve, without imposing any restrictions from theory. Another nonparametric method to detect the shape of the curve would be the test from Abrevaya and Jiang (2005). This test allows the testing of curvature in a multivariate nonparametric regression model. It would also be desirable to properly account for the possible changes in the relationship due to changes in institutional surroundings. These changes would be captured in the expectations, which are modelled in this study with the learning approach, but the data series seem to be the problem in the estimation of expectations. These are shortcomings in this study that could be addressed in future research.

An advantage in this study is that it uses economic theory to put structure on the empirical study. This study's main criticism on most of the empirical studies on the nonlinearities in the Phillips curve reviewed in Appendix B is that important theoretical foundations have been neglected when estimating the relationship. When estimating the slope of the Phillips curves without considering the theories that make the Phillips curve nonlinear, researchers might more or less satisfactorily answer the question of whether the Phillips curve *is* nonlinear. But for monetary policy it is even more important to answer the question of *why* it is nonlinear. Both of these questions are answered in this thesis for the countries studied here. Therefore, considering the theoretical foundations is of high importance for empirically studying the nonlinearity in the Phillips curve. This is an important point that already Kant (1770) makes by saying<sup>70</sup>

*Theory without empirics is empty. Empirics without theory is blind.*

*I. Kant (1724-1804)*

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<sup>70</sup> Translated from German.

## Appendix

### A. Correlations

**Table A-1 Correlation Matrix France**

	<b>HP Gap</b>	<b>Capacity Utilisation</b>	<b>OECD Gap</b>	<b>Unemployment Gap</b>
<b>HP Gap</b>	1.00			
<b>Capacity Utilisation</b>	0.61	1.00		
<b>OECD Gap</b>	0.81	0.61	1.00	
<b>Unemployment Gap</b>	0.43	0.49	0.80	1.00

**Table A-2 Correlation Matrix Germany**

	<b>HP Gap</b>	<b>Capacity Utilisation</b>	<b>OECD Gap</b>	<b>Unemployment Gap</b>
<b>HP Gap</b>	1.00			
<b>Capacity Utilisation</b>	0.51	1.00		
<b>OECD Gap</b>	0.48	0.56	1.00	
<b>Unemployment Gap</b>	0.43	0.37	0.75	1.00

**Table A-3 Correlation Matrix Italy**

	<b>HP Gap</b>	<b>Capacity Utilisation</b>	<b>OECD Gap</b>	<b>Unemployment Gap</b>
<b>HP Gap</b>	1.00			
<b>Capacity Utilisation</b>	0.49	1.00		
<b>OECD Gap</b>	0.20	0.81	1.00	
<b>Unemployment Gap</b>	0.29	0.50	0.59	1.00

## B. Overview: Evidence on Nonlinearities in the Phillips Curve

Table B-1. Literature Review: Empirical Evidence (page 1)

Authors	Country	Time Period	Data Frequency	Method	Gap Measure	Inflation expectations measure	Control Variables	Outcome	Tested for source of nonlinearity?	Determinant of the trade-off
Laxton <i>et al.</i> (1995)	pooled G7	1967-1991	Annual	Linear, convex and s-shaped function	Two sided 5 year moving average of GDP	Inflation expectations modelled by auxiliary regression	Included in inflation expectations proxy but not in relevant regression	Convex	No	Gap
Turner (1995)	G7	1960 -1994	Annual	A separate term in the regression when the gap is positive	HP filtered GDP	Not included	Import price inflation rate	Convex model in US, Japan, Canada; Linear in UK and Italy; Close to reject linear model in Germany and France	No	Gap
Clark <i>et al.</i> (1996)	US	1964-1990	Quarterly	A separate term in the regression when the gap is positive	Output gap as a 25 quarter centred moving average estimate of trend GDP	Michigan inflation expectation survey data	No control for supply shocks	Convex	No	Gap
Gordon (1997)	US	1955-1996	Quarterly	Triangle model, time varying NAIRU	Time varying NAIRU (Kalman Filter)	No expectations included	Control for supply shocks	Linear	No	Gap
Debelle and Laxton (1997)	US, Canada, UK	1971-1995	Quarterly	Capacity constraint model, Kalman Filter	Unobserved component in a capacity constraint model	No expectations included	No control for supply shocks	Convex	No	Gap

Table B-1. Literature Review: Empirical Evidence (page 2)

Authors	Country	Time Period	Data Frequency	Method	Gap Measure	Inflation expectations measure	Control Variables	Outcome	Tested for source of nonlinearity?	Determinant of the trade-off
Debelle and Vickery (1997)	Australia	1959-1997	Quarterly	Capacity constraint model, Kalman Filter	Unobserved component in a capacity constraint model	Australia: Derived from bond yields	No control for supply shocks	Convex	No	Gap
Yates (1998)	US, UK, Sweden, France, Italy, Denmark	1800-1938	Annual	A separate term in the regression for positive or negative lagged inflation rates in a SURE (assumes kinked form)	HP filtered GDP	No expectations included	No control for supply shocks	Linear	No	Inflation
Dupasquier and Ricketts (1998)	US, Canada	1963-1995	Quarterly	State space approach with unobserved coefficient of output gap	SVAR Approach and HP filter	US: Michigan Survey, Canada: Proxy by a three state Markov switching model	No control for supply shocks	Canada: Linear; US: Convex	Yes	Gap and Inflation
Filardo (1998)	US	1959-1997	Quarterly	Piecewise linear form with three possible states, OLS	HP filtered GDP	Michigan inflation expectation survey data	No	Convex-concave	No	Gap
Pyyhtiä (1999)	Seven Euro Area countries (pooled)	1976-1997	Annual	(1) Quadratic and (2) kinked linear specification: OLS, SURE, and GMM	HP filtered GDP	OECD real time forecasts	No	(1) Quadratic term significant when GMM used, not otherwise (2) Convex Ger, Ita.	No	Gap

Table B-1. Literature Review: Empirical Evidence (page 3)

Authors	Country	Time Period	Data Frequency	Method	Gap Measure	Inflation expectations measure	Control Variables	Outcome	Tested for source of nonlinearity?	Determinant of the trade-off
Bean (2000)	OECD Countries (Pooled)	1951-1992	Annual	Quadratic specification, two estimations: (1) Panel with country specific constants (2) Pooled Least Squares	Capacity utilisation	No	No	"Mild" convexity	No	Gap
Eliasson (2001)	US, Sweden and Australia	1977-1979	Quarterly	STR	No gap included (unemployment rate as regressors, assuming that NAIRU is captured in the constant)	Australia: Derived from bond yields. US: Michigan Survey. Sweden: NIER Survey	Only for Sweden, German Inflation as proxy for import inflation	Sweden: Convex-Concave, Australia: Convex US: Linear	Yes	Inflation (Australia), Unemployment (rate) (Sweden)
Clark <i>et al.</i> (2001)	US	1964-1995	Quarterly	A separate term in the regression when the gap is positive	Output gap as a 25 quarter centred moving average estimate of trend GDP	Michigan inflation expectation survey data	No	Convex	No	Gap
Aguiar and Martins (2002)	Euro Area Aggregate	1970-2001	Quarterly	Quadratic, hyperbole, and exponential specification	Unobserved component simultaneously with the Phillips curve	Measure computed from lagged inflation rates	No	Linear	No	Gap
Barnes and Olivei (2003)	US	1961-2002	Quarterly	Piecewise linear form with two possible states, where the state when unemployment is close to its natural rate is different	NAIRU estimated simultaneously with the Phillips curve	No	Control for supply shocks	Higher trade-off during high deviations of unemployment from NAIRU	No	Gap

Table B-1. Literature Review: Empirical Evidence (page 4)

Authors	Country	Time Period	Data Frequency	Method	Gap Measure	Inflation expectations measure	Control Variables	Outcome	Tested for source of nonlinearity?	Determinant of the trade-off
Fauvel <i>et al.</i> (2004)	US	1959-2001	Quarterly	Hamiltons (2001) Test for Nonlinearity	Model in first differences with level of unemployment	No	Control for supply shocks	Lagged inflation changes the relationship	Yes	Inflation, Gap
Mayer and Viren (2004)	EU 15 countries Panel	1985-2001	Quarterly	A separate term in the regression when the gap is positive, SURE	HP filtered GDP	No	Import price inflation rate	Convex relationship for all countries except Spain and Finland	No	Gap
Mourougane and Ibaragi (2004)	Japan	1960-2002	Quarterly	OLS, Wald tests for breaks at low inflation rates	NAIRU series, output gap, capacity utilisation, surveys	No	Import price inflation rate	Break in the relationship at inflation rate of 0.5	No	Inflation
This thesis	France, Germany, Italy	1969-2004	Quarterly	LSTR(1)	OECD output gap from production function approach	Learning approach	Import price inflation rate	Germany: Inflation determines the slope; Italy and France: both, real activity and inflation determine the slope	Yes	Gap, Inflation

### C. The Inflation Expectations Model

This section is related to the measurement of the inflation expectations. It gives a description of the state space model used for the computation of the expectations, for a detailed description and proofs see Harvey (1989) and Basdevant (2003). A general formulation of a state space model consists of at least two equations. The so-called measurement equation can be written as

$$Y_t = Z_t A_t + \varepsilon_t \quad \varepsilon_t \sim N(0, H_t) \quad (\text{A1})$$

where  $Y_t$  is a  $m \times 1$  vector of measured variables,  $Z_t$  is a matrix of parameters of dimension  $m \times p$ ,  $A_t$  is a  $p \times 1$  vector of unobserved variables and  $\varepsilon_t$  the error term with mean zero and variance  $H_t$ . The unobserved variables are determined by the so-called state equation, which is

$$A_t = T_t A_{t-1} + \eta_t \quad \eta_t \sim N(0, Q_t) \quad (\text{A2})$$

with  $T_t$  being a matrix of parameters and  $\eta_t$  the error term with mean zero and variance  $Q_t$ . Furthermore the initial vector  $A_0$  has a mean  $a_0$  and a covariance matrix  $P_0$  and  $E(\varepsilon_t \eta_s') = 0$  and  $E(\varepsilon_t A_0') = 0 \quad \forall (t, s)$ . Define the optimal estimator of  $A_t$  as  $a_t$ . The following equations define the so-called Kalman equations. This estimator is based upon  $Y_t$ , its covariance matrix  $P_t$ , and the estimator based on the information up to  $t-1$   $a_{t|t-1}$  and the covariance  $P_{t|t-1}$ . From equation (A2), the predicted estimate of  $A_t$  is

$$a_{t|t-1} = T_t a_{t-1} \quad \text{with} \quad P_{t|t-1} = T_t P_{t-1} T_t' + Q_t \quad (\text{A3})$$

When  $Y_t$  is known,  $a_t$  is updated from  $a_{t|t-1}$ , which yields the filtered estimate of  $A_t$  which is

$$a_t = a_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} (Y_t - Z_t a_{t|t-1}) \quad (\text{A4})$$

with  $F_t = Z_t P_{t|t-1} Z_t' + H_t$  and  $P_t = P_{t|t-1} - P_{t|t-1} Z_t' F_t^{-1} Z_t P_{t|t-1}$ . These equations define the Kalman equations. Next, these equations are used to model the inflation expectations. As explained in the main part, the aim of the expectation measure is to model inflation expectations, that are not adaptive, as agents do not make systematic errors, and not perfectly rational, as agents do not have perfect information. Basically the learning process is assumed to follow the following rule

$$\pi_{t+1} = \alpha_{1,t} + \alpha_{2,t} \pi_{t-1} + \alpha_{3,t} \text{gap}_{t-1} + \varepsilon_t \quad (\text{A5})$$

where agents form their expectations about the inflation rate in the next period by using the previous inflation rate and their estimate of the unemployment gap. It is assumed

that this is the only information that they find to be relevant for their inflation forecast. Defining  $X_t \equiv (1, \pi_{t-1}, gap_{t-1})$  and  $A_t \equiv (\alpha_{1,t}, \alpha_{2,t}, \alpha_{3,t})'$ , with

$$A_t = A_{t-1} + \frac{a_t}{t} R_t^{-1} X_t' (\pi_{t+1} - X_t A_{t-1}) \quad (A6)$$

where  $R_t = \frac{1}{t} \sum_{\tau=1}^t a_\tau X_\tau' X_\tau$  and  $a_t$  is a sequence of positive numbers. This formula is being used to model a process of "updating" the coefficients that are used to weight the previous inflation rate and the gap. To give the intuition behind that, assume that  $a_t=1$ . That would correspond to a recursive least squares. In recursive least squares the equation is estimated repeatedly, using ever larger subsets of the sample data. The first three observations are used to form the first estimate of  $\pi_4$ . The next observation of the gap is then added to the data set and observations are used to compute the second estimate of  $\pi_5$ . This process is repeated until all the sample points have been used, yielding estimates of the vector  $A_t$ . The computation of recursive least squares assumes that the coefficients are stable, the updating of the coefficients uses the new information and assumes "new" coefficients, that have been stable all through the process. As I have assumed a learning process, the coefficients are assumed to be time-varying, which allows the assumption of learning. The equations defining the formation of expectations by updating the weights put on previous inflation rates and the unemployment gap can be cast into the Kalman filter equations which yields

$$A_t = A_{t-1} + P_{t-1} X_t' f_t^{-1} (\pi_{t+1} - X_t A_{t-1}) \quad (A7)$$

$$P_t = P_{t-1} - P_{t-1} X_t' X_t P_{t-1} f_t^{-1} + Q_t \quad (A8)$$

where  $P_t = \frac{1}{t} R_t^{-1}$  and  $f_t = X_t P_{t-1} X_t' + \frac{1}{a_t}$ . This corresponds to the following state space model I have used for computation. Recall equation (A5)

$$\pi_{t+1} = \alpha_{1,t} + \alpha_{2,t} \pi_{t-1} + \alpha_{2,t} gap_{t-1} + \varepsilon_t \quad (A5)$$

with  $Var(\varepsilon_t) = \frac{1}{a_t}$  and

$$\alpha_{i,t} = \alpha_{i,t-1} + \eta_{i,t} \quad (A9)$$

for  $i=1,2,3$  with  $Var(\eta_t) = 0$ . This allows the coefficients to be updated from the previous estimate. To restrict the variance of the state equation to zero  $Q_t$  is set to zero. The inflation expectations are formed in the following manner. First, the agents form the expectations for the inflation of the next period. Afterwards, they observe the

current inflation rate and update their belief. Thus the predicted estimate of the inflation rate one period ahead is

$$\hat{\pi}_{t+1} = \alpha_{1,t|t-1} + \alpha_{2,t|t-1}\pi_{t-1} + \alpha_{3,t|t-1}gap_{t-1}. \quad (\text{A10})$$

## D. Stationarity Tests

**Table D-1 Unit Root Test for  $\Delta\text{CPI ex Food and Energy}$**

	lag length	ADF	PP	KPSS	DF-GLS	LSL	Break date
<b>France</b>	8	-1.08(c)	-0.95(-)	0.65(c)	-1.25(c)	-1.20(c)	1977Q1
<b>Germany</b>	5	-2.22(c)*	-2.02(c)	0.38(c)	-2.03(c)**	-5.15(c)**	1992Q4
<b>Italy</b>	8	-0.87(-)	-0.77(-)	0.31(c)	-1.08(c)	-1.31(c)	1975Q3

Tested variable: Annual inflation rate (Difference of Logarithm of CPI excluding food and energy)

*Augmented Dickey Fuller Test(ADF)*. The ADF tests the H0 of a unit root. The test is performed with the lags reported in the first column. Including a constant is denoted by (c) and no intercept and trend by (-). The lags are selected by AIC. \*, \*\*, \*\*\* denote the 10%, 5%, and 1% significance level respectively. The significance values follow from the MacKinnon (1996) one-sided p-values.

*Phillips-Perron(PP)*. Phillips and Perron (1988) propose an alternative nonparametric method of controlling for serial correlation when testing for a unit root. The H0 is that the series has a unit root. The estimated equation has less parameters, so the Phillips-Perron test has more degrees of freedom. \*, \*\*, \*\*\* denote the 10%, 5%, and 1% significance level respectively. The significance values follow from the MacKinnon p-values.

*Kwiatkowski-Phillips-Schmidt-Shin(KPSS)*. The KPSS test differs from the other unit root tests described here in that the series is assumed to be stationary under the null. The test is a LM statistic with a limited  $\chi^2$  distribution, the critical values are from Kwiatkowski et al (1992). All series tested with the KPSS test do not reject the null of stationarity.

*Lanne-Saikkonen-Lütkepohl(LSL)*<sup>71</sup>. This tests is based on the assumption that the series contains a level shift. The other tests may have a very low power if a shift in the level in the series is simply ignored<sup>72</sup>. The model is assumed to be follow the process

$$y_t = \mu_0 + \mu_1 t + f_t(\psi)' \gamma + u_t \quad (\text{A11})$$

where  $\psi$  and  $\gamma$  are unknown parameters or parameter vectors with  $\psi > 0$ . The error terms are generated by an AR(p) process. The time series is assumed to include a exponential shift function  $f$

$$f_t(\psi) = \begin{cases} 1 - \exp\{-\psi(t - T_B + 1)\}, & t < T_B \\ 0 & , t \geq T_B \end{cases} \quad (\text{A12})$$

The shift function is based on the exponential distribution function which allows for a nonlinear gradual shift to a new level starting at time  $T_B$ , both  $\psi$  and  $\gamma$  are scalars. Saikkonen and Lütkepohl (2002) and Lanne *et al.* (2001) propose unit root tests for this model based on estimating the deterministic term first by a GLS procedure under the null of a unit root and then subtracting it from the original series. To this adjusted series an ADF type test is performed including terms to correct for estimation errors in the parameters of the deterministic part. The break date is chosen at the point which minimizes the GLS objective function used to estimate the parameters of the deterministic part. The chosen date is reported in the table. Critical values are from Lanne *et al.* (2002).

## E. Linearity Tests

### E.1 Test Linear Model vs. STR

**Table E-1 P-Values of the Model Selection F-Tests**

		$H_0$	$H_{03}$	$H_{02}$	$H_{01}$	Suggested Model
<b>France</b>	$\pi_{t-1}$	0.00	0.51	0.23	0.00	LSTR(1)
	gap	0.04	0.56	0.03	0.09	LSTR(2)
<b>Germany</b>	$\pi_{t-1}$	0.00	0.74	0.04	0.00	LSTR(1)
	gap	0.33	0.23	0.26	0.41	Linear
<b>Italy</b>	$\pi_{t-1}$	0.01	0.01	0.07	0.20	LSTR(1)
	gap	0.00	0.97	0.01	0.00	LSTR(1)

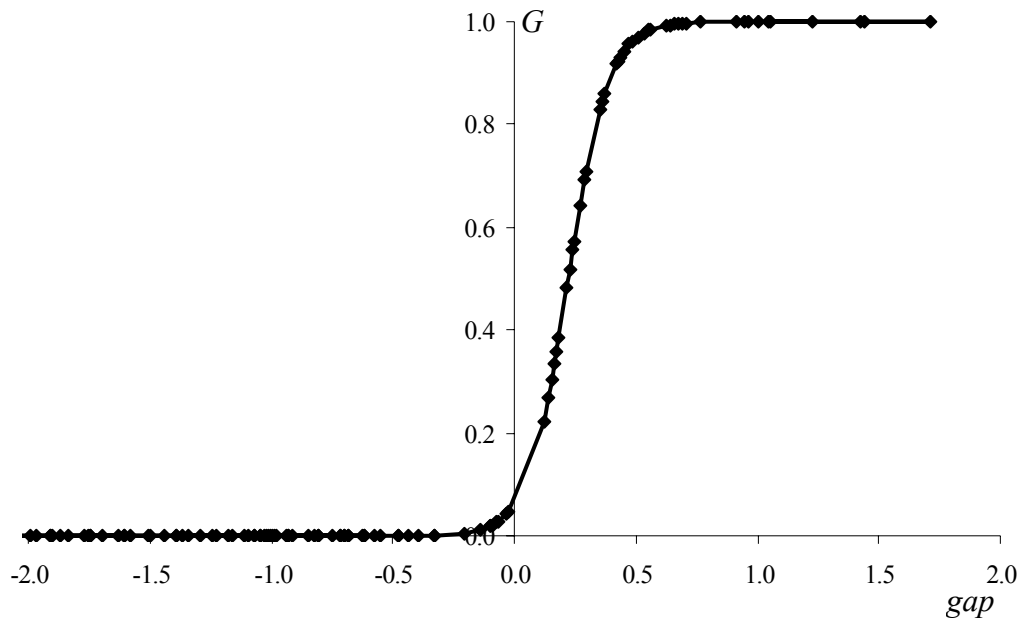
<sup>71</sup> The test is referred to as "unit root test with level shift" (in Lütkepohl and Krätzig (2004) for example). I have used the publishers' names for the indication, analogous to the other tests.

<sup>72</sup> See Perron (1989).

## F. Graphs Transition Functions

The following Graphs show plots of the transition function (ordinate), denoted by  $G$ , and the respective transition variable (abscissa), denoted by  $\pi$  and  $gap$  respectively.

**Figure F-1 Transition Function France (Gap)**



**Figure F-2 Transition Function STR France (Inflation)**

