Relative Alpha*

Jens Carsten Jackwerth†

Anna Slavutskaya‡

ABSTRACT

The alpha within a factor model of fund performance could measure current outperformance over risk-adjusted returns; it could be used to identify funds which generate high future performance and thus determine fund flow; and it could be used to find persistence in alpha. We advocate a new measure to evaluate hedge funds - relative alpha. It links each hedge fund to a group of its peers in a straightforward, semi-parametric way. We do not require knowledge of the true factor structure. We show that relative alpha outperforms traditional, absolute alpha (e.g. based on Fung and Hsieh (2001)) along all three dimensions.

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†Jens Jackwerth is from the University of Konstanz, PO Box 134, 78457 Konstanz, Germany, Tel.: +49-(0)7531-88-2196, Fax: +49-(0)7531-88-3120, jens.jackwerth@uni-konstanz.de

‡Anna Slavutskaya is from the University of Konstanz, PO Box 134, 78457 Konstanz, Germany, Tel.: +49-(0)7531-88-3767, Fax: +49-(0)7531-88-3120, anna.slavutskaya@uni-konstanz.de

Investment analysis makes much use of alpha, the intercept in a factor model of returns. First, alpha could measure the current outperformance of a fund over its risk-adjusted return. Here, it is important to account for all relevant factors in order to avoid missing factor bias. Second, alpha could predict future fund performance and thus assets should flow to high alpha hedge funds. Third, alpha could be persistent so that high performance funds today remain high performance funds in the future. While it is not clear that one and the same alpha serves all three goals, in the analysis of hedge funds, the seven factor model of Fung and Hsieh (2001) has become the de facto standard and is used for all three purposes. Our goal is to show that our new measure, relative alpha, performs better than the traditional, absolute alpha measures (e.g. Fung and Hsieh (2001) and others) along all three dimensions.

To argue our case, we first define absolute and relative alpha. Hedge funds are often evaluated in terms of alpha within a factor model:

$$r_t - r_{f,t} = \alpha + \sum_{l=1}^{L} \beta_l X_{l,t} + \varepsilon_t, \tag{1}$$

where $(r_t - r_{f,t})$ is the excess hedge fund return at time t, $X_{l,t}$ is the factor l at time t, β_l is the risk exposure to factor l, and ε_t is a mean zero error term. Given the relatively short life span of the average hedge fund (typically only 36 to 48 monthly observations), popular sets of factors number just seven or eight (e.g. Fung and Hsieh (2001) or Agarwal and Naik (2004)) so that they can be estimated with some precision.

As an alternative, we propose a new measure ("relative alpha"). Relative alpha links each hedge fund to a group of its peers and averages over the expected differences between returns of the hedge fund and returns of each of its peers. Peers are characterized by the low variance of these differences in returns and receive the more weight in our average the lower their variance of differences in returns. Thus, relative alpha measures the outperformance of a hedge fund over its closest peers without resorting to a particular factor model. The kernel-based estimation technique is simple to implement and uses solely hedge fund returns.

We will now argue that relative alpha performs better than absolute alpha along all three dimensions of interest: measuring current outperformance over risk-adjusted returns; identifying funds which generate high future performance and thus determining fund flow; and finding persistence in alpha.

First, in terms of measuring current outperformance, absolute alpha has the intuitive interpretation as the excess skill added to the hedge fund return by the manager beyond the risky investments into the factors. However, hedge funds are sophisticated investment vehicles and the universe of their strategies might lie beyond the seven or eight factors typically employed in factor models. If there are other relevant, omitted factors, then absolute alpha will erroneously include the returns associated with the omitted factors. The Fung and Hsieh (2001) model only explains with its seven factors 41% of the return variation (adjusted R-squared) in our sample, leaving us worried that some of the missing explanatory power is due to omitted factors. Interpreting the seven closest peers as factors, we achieve an adjusted R-squared of 81%. This high value is not surprising, but it shows that we achieve our goal of finding peers that are close to our hedge funds. Also, the average standard error around the alpha estimates is only 0.57 for relative alpha vs. 0.86 for Fung and Hsieh (2001).

Second, we find that relative alpha is better than absolute alpha in identifying funds which generate high future performance and thus determining fund flow. In particular, we use rolling windows of 36 months to estimate both relative and absolute alphas. We then invest in equally weighted portfolios based on hedge funds in the highest deciles, respectively, and measure the out-of-sample monthly returns. On a monthly basis, the relative alpha strategy has higher mean return (1.12%) at lower volatility (2.11%), yielding a Sharpe ratio of 0.49, compared with the absolute alpha strategy with a lower mean return (1.02%) at higher volatility (3.25%), yielding a Sharpe ratio of 0.27 which is about half the Sharpe ratio of the relative alpha strategy. The monthly Sharpe ratio of a top-minus-bottom decile strategy is a surprising 11 times higher for relative over absolute alpha. We wondered if any risk averse investor would prefer the sorts based on relative alpha over the ones based on absolute alpha. Thus, we employ a test if investing into the top relative alpha portfolio stochastically non-dominates (in the second order) investing into the top absolute alpha portfolio. The test of Davidson and Duclos (2013) rejects the null at the 1% level. We conclude that any risk averse investor would rather use a sort based on relative alpha. We obtain the same result when we investigate top-minus-bottom deciles. Relative alpha also dominates the use of the appraisal ratio of Treynor and Black (1973), the manipulation-proof performance measure (MPPM) of Goetzmann, Ingersoll, Spiegel, and Welch (2007), and the strategy distinctiveness index (SDI) of Sun, Wang, and Zheng (2012). Thus, portfolios based on sorting by relative alpha perform significantly better than sorts based on absolute alpha in statistical and economic terms.

Moreover, we find a strong positive relation between fund flows and past relative alpha, i.e. the larger the past relative alpha, the larger the flows into assets under management. We do not find the same pattern for the fund flows and past absolute alpha relation.

Third, alpha is often interpreted as the skill a hedge fund manager possesses. If the managers skill is persistent, then alpha should also be persistent[1]. Tests of alpha persistence show that relative alpha exhibits significant positive coefficients while absolute alpha exhibits insignificant coefficients or even negative coefficients. Our findings for absolute alpha are in line with Capocci and Hübner (2004) who also find very little or no persistence in absolute hedge fund alpha.

Furthermore, we design a simulation study to analyze under which circumstances relative alpha works best. Relative alpha works the better the more omitted variable bias exists. Also, a larger cross-section of hedge funds contributes to the superior performance of relative alpha. Such large cross-section increases the probability for each hedge fund to find a relevant group of peers which spans the investment opportunity set of the hedge fund.

We also show that our results are not qualitatively affected by minor changes in methodology and sample.

Based on a literature review in Section I, the paper develops the hypotheses and introduces the econometric methodology for testing our performance measure in Section II. All data are presented in Section III. Results follow in Section IV while robustness checks are presented in Section V. Simulation study is in Section VI. Section VII concludes.

[1]For mutual funds, Berk and Green (2004) argue that competition should eliminate such skill related alpha. In Berk and van Binsbergen (2013) they thus argue that mutual funds should not exhibit persistence in alpha. However, for hedge funds Glode and Green (2011) show theoretically that potential information spillovers (associated with innovative trading strategy or emerging sector) could lead to persistent alpha after all.

I. Literature

There are several related branches of the literature. First, a number of papers directly try to improve the omitted factor bias by adding additional factors: the factor models of Agarwal and Naik (2004), Fung and Hsieh (2001), or the hedge fund index model of Jagannathan, Malakhov, and Novikov (2010). These models have around 7-10 factors and are thus much richer than a single market factor model or the Fama-French three factor model. In particular, Fung and Hsieh (2001) is nowadays the de facto standard factor model for hedge fund research, but even Fung and Hsieh (2001) only accounts in our sample for 41% of the return variation of hedge funds with their seven factors. Thus, there is concern that the remaining 59% of return variation might contain omitted factors.

Hunter, Kandel, Kandel, and Wermers (2014) augment a conventional factor model with an active peer benchmark. They determine peer-groups of mutual funds based on their investment objectives. While this is a simple approach that significantly improves the selection of successful funds, there are several issues that hinder us from applying this methodology directly to hedge funds. First, hedge fund strategies are not as well specified as the strategies of mutual funds. Not merely is the description of hedge fund strategies vague, but hedge funds may change their strategies according to market conditions, available financial resources, and current management objectives. Second, a four-factor model of Carhart (1997) used for mutual funds is able to explain 60-70% of the variation in returns, which is almost twice as much as the amount of explained variation by typical hedge fund linear factor models.

A paper by Wilkens, Yao, Jeyasreedharan, and Oehler (2013) adopts the endogenous benchmark approach of Hunter et al. (2014) to hedge funds. Wilkens et al. (2013) construct weighted endogenous benchmarks to relate individual funds to their style peers in order to reduce the problem of missing variables. Still, Wilkens et al. (2013) require a linear factor model with the associated problem that about half of the hedge fund return variation is left unexplained. We add to this literature by defining the peer group of a hedge fund to be only hedge funds which are very similar in terms of the variance of the return differences. That allows us to reduce the omitted variable bias greatly and to implicitly offset much of the unexplained variation in hedge fund returns. As a result, we can estimate relative alphas of one hedge fund with respect to its peer group more precisely.

Hoberg, Kumar, and Prabhala (2014) use competition between mutual funds to develop a new measure of skill, customized peer alpha (CPA). They show that this new measure predicts alpha for at least four quarters. Competitors are defined based on the data on mutual fund holdings. Funds are placed into a 3-dimensional (also 2- or 4-) space of characteristics (including size, value-growth orientation, momentum, dividend yield) based on the dollar-weighted average of their stock holdings. Two funds are considered competitors if the spatial distance between them is smaller than a given value. CPA is then measured as a fund's outperformance over its spatial peers. A fund is considered skillful if it is able to beat funds with similar strategies. In our relative alpha approach, we also compare funds to their peers. However, we derive relative alpha directly as a way to reduce the omitted variable bias and do not use the competition concept. The approach of Hoberg et al. (2014) does not work for hedge funds as hedge fund holdings are unknown but for the 13F reports which only apply to US equity holding of large funds ($>$ \$100 million).

We also connect to a second literature on fund flow and fund performance (see e.g. Fung, Hsieh, Naik, and Ramadorai (2008) and Ding, Getmansky, Liang, and Wermers (2009). The results are somewhat contradictive which is due to the different samples and different methodologies. Sirri and Tufano (1998) find that mutual fund flows chase good performance.

As a performance measure they use fractional rank quantiles which represent fund performance relative to other funds in the same period. Getmansky (2012) adopts similar methodology and records a positive relation between flows and past performance for the middle and bottom terciles of hedge funds. At the same time, top performing funds do not grow proportionally as much as the average fund in the industry. Goetzmann, Ingersoll, and Ross (2003) on the contrary reveal that hedge funds demonstrate a decrease in investments, conditional on the past returns.

A third literature concerns the predictability of alpha. The existing literature (Ammann, Huber, and Schmid (2010), Capocci and Hübner (2004)) shows mixed evidence concerning the predictability of absolute alpha. That is, historically measured alpha has little predictive value for future alpha. But that means that allocating investments to past high alpha funds will not lead to high alpha in the future. In contrast, we document strong persistence of relative alpha during our sample.

II. Hypotheses and methodology

We assume that there exists a complete factor model which explains hedge funds without omitted factors and with uncorrelated error terms. Without loss of generality, we assume these factors to have been orthogonalized. In particular, we do not limit ourselves to the seven or eight factors of Fung and Hsieh (2001) or Agarwal and Naik (2004). Thus, our assumed factor model would have perfect explanatory power but for the error term, i.e. an R-squared of close to 1. Obviously, we might not be able to enumerate all these factors, but we do not need to; we argue below that we can still assess our performance measure relative alpha without the explicit knowledge of the full factor model by essentially netting out much of the unknown factor structure. The complete factor models for hedge funds i and j are then as follows:

$$r_{it} - r_{f,t} = \alpha_i + \sum_{k=1}^{K} \beta_{ik} X_{k,t} + \varepsilon_{it}, \tag{2}$$

$$r_{jt} - r_{f,t} = \alpha_j + \sum_{k=1}^{K} \beta_{jk} X_{k,t} + \varepsilon_{jt}, \tag{3}$$

where $(r_{i,t} - r_{f,t})$ is the excess hedge fund return of hedge fund i at time t, $X_{k,t}$ is the factor k at time t, β_{ik} is the risk exposure of hedge fund i to factor k, and ε_{it} is a mean zero error term for hedge fund i. Definitions for hedge fund j are similar.

We next take expectations of the differences in returns. If hedge funds i and j implement identical strategies (i.e. they load on the same risk factors $X_{k,t}$ and have $\beta_{ik} = \beta_{jk}$), then their factor loadings cancel, leaving only differences in alphas:

$$E[r_{it} - r_{jt}] = \alpha_i - \alpha_j + \sum_{k=1}^{K} (\beta_{ik} - \beta_{jk}) E[X_{k,t}] + E[\varepsilon_{it} - \varepsilon_{jt}] = \alpha_i - \alpha_j \tag{4}$$

We can thus obtain relative alpha, the difference in hedge fund i's alpha from hedge fund j's alpha. Now clearly, hedge funds typically do not have perfectly identical betas. Instead, we allow for small discrepancies in betas. Such discrepancies in beta would not even affect the expectation in Equation (4) as long as beta differences are random and uncorrelated from one hedge fund to the next. Also, we allow for more than one peer hedge fund with respect to which fund i's relative alpha is being calculated and suggest taking weighted averages.

For these two additional steps (discrepancies in betas and a larger peer group), we first need a distance measure in order to establish the size of beta discrepancies and second an averaging technique in order to work out the relative alpha with respect to a group of peers.

As a distance measure, we calculate the variance of the return differences:

$$Var[r_{it} - r_{jt}] = (\beta_i - \beta_j)' Cov(X_{k,t})(\beta_i - \beta_j) + \sigma_i^2 + \sigma_j^2, \tag{5}$$

where β_i (β_j) is the $(K \times 1)$ vector of risk exposures of hedge fund i (j), $Cov(X_{k,t})$ is the $(K \times K)$ variance-covariance matrix of the factors (assumed to be bounded from above), and $\sigma_i^2(\sigma_j^2)$ are the variances of the error terms ε_{jt} and ε_{jt}. We assume that the sum of the variances of the error terms (σ^2) is similar in magnitude for all hedge fund pairs[2]. Note that there are no covariances for the error terms as we assumed that all common components are reflected in the factor structure. Using our assumption that the factors are orthogonal to each other, the expression for the variance of return differences becomes:

$$Var[r_{it} - r_{jt}] = \sum_k (\beta_{ik} - \beta_{jk})^2 Var\left(X_{k,t}\right) + \sigma_i^2 + \sigma_j^2. \tag{6}$$

It follows that the variance of return differences is smaller if the funds' betas are closer to each other (i.e. the differences in betas decrease for at least one factor and do not increase for the other factors). We provide more details on this mechanism in our simulation study in Section VI. We will thus use the variance of return differences as a measure of how similar

[2]We use a simulation study in Section VI in order to argue that this assumption is quite reasonable. Namely, the systematic variance component $(\beta_i - \beta_j)' Cov(X_{k,t})(\beta_i - \beta_j)$ increases strongly in to total variance $Var[r_{it} - r_{jt}]$ but $\sigma_i^2 + \sigma_j^2$ stays fairly flat in comparison.

two hedge funds are to each other.

As an averaging technique, we compute relative alpha as the sum of kernel weighted expected return differences. The kernel gives more importance to peer funds which are closer to the hedge fund in terms of the variance of return differences as in Equation (5). Therefore, the relative alpha of hedge fund i is being determined with respect to all other hedge funds by first calculating, one at a time, the expected differences in returns according to Equation (4). Those pairwise expected differences in returns are then aggregated into one relative alpha via kernel weights which add up to one. Relative alpha is calculated as:

$$\Delta_{i,T} = \frac{\sum_{i \neq j} K\left(Var[r_{it} - r_{jt}]/h\right) E[r_{it} - r_{jt}]}{\sum_{i \neq j} K\left(Var[r_{it} - r_{jt}]/h\right)}, \tag{7}$$

where $K(\cdot)$ is a Gaussian Kernel, h is the bandwidth according to Silverman (1986) rule of thumb, $Var[\cdot]$ and $E[\cdot]$ are the variance and expectation of the return differences between hedge funds i and j for t=T-$35,\ldots,T$.

Our results do not change much if we use bandwidths from a fifth to five-times of the value of Silverman (1986) rule of thumb. Kernel estimates are biased on the boundary of the data and we suffer from this problem as we evaluate the kernel estimate at a point where the variance is zero based on variances which are all positive. Thus, we show in our robustness section that results do not change when we use the local regression technique proposed by Hastie and Loader (1993) which better accommodates the boundary bias. Relative alpha is distinct from absolute alpha, but the two alpha measures are related to each other. The correlation between relative alpha and absolute Fung and Hsieh (2001) alpha is on average 0.62 ranging between 0.37 and 0.86, depending on the sample.

A number of advantages emerge which argue in favor of using relative alpha over absolute alpha. First, we do not require knowledge of the true factor model. Thus, we are much less prone to omitted variable bias: if two hedge funds are similar, then they presumably have a rather similar (but possibly partially unknown) factor structure which will cancel out in the relative alpha calculation. Second, implementation is straightforward. Third, relative alpha performs considerably better than absolute alpha along our three dimensions of interest: explaining high current performance, predicting high future performance, and predicting high future alpha. We now detail the methodology for establishing the superior performance of relative alpha in each dimension in turn.

To assess how well alpha measures the current outperformance over risk-adjusted returns, we investigate the adjusted R-squared of the Fung and Hsieh (2001) model for absolute alpha in terms of average and standard deviation. For relative alpha, we use the returns of the seven closest peer funds in lieu of the seven Fung and Hsieh (2001) factors. We can then regress hedge fund returns on those peer fund returns and again obtain values of adjusted R-squared.

To show that relative alpha predicts future high performance, we compare the out-of-sample performance of portfolios based on sorts on relative and absolute alphas. We use 36-month rolling windows to estimate relative and absolute alpha, sort hedge funds into top and bottom deciles, and form equally weighted portfolios. We record returns of top, bottom, and top-minus-bottom portfolios in the 37^{th} month and repeat the procedure by moving one month out. We take care of look-ahead bias by recording a zero return instead of a missing return in case a hedge fund is delisted. Since hedge funds may have a lock-up period, as a robustness check we increase portfolio holding period to 12 months. We report means, standard deviations, and Sharpe ratios for these strategies. Also, we use the test of

Davidson and Duclos (2013) for second order stochastic non-dominance. A rejection of this test has the powerful implication that any risk-averse investor would prefer investing into hedge funds sorted by relative alpha as opposed to sorts based on absolute alpha. A related point is that high alpha should then generate high fund flow as investors allocate investments according to past alpha. We simply regress fund flows for fund i (measured from time *t-1* to t) on its relative (absolute) alpha (measured from time *t-1,...,t-36*).

To analyze persistence of alpha, we adopt a methodology commonly used in the hedge fund literature. We consider consecutive 72 (48, 24)-month periods, starting with the 1^{st}, 37^{th}(25^{th}, 13^{th}), 73^{rd}(49^{th}, 25^{th}), ...observations. Each of these periods is divided into two 36 (24, 12)-month sub-periods: a formation period (1-36^{th} (1-24^{th}, 1-12^{th}) months) and an evaluation period (37-72^{nd} (25^{th}-48^{th}, 13^{th}-24^{th}) months). For each hedge fund which survives the whole 72 (48, 24)-month period, we compute relative alpha in the formation ($\triangle_{1i}$) and the evaluation ($\triangle_{2i}$) periods and estimate the following regression:

$$\triangle_{2i} = a_{\Delta} + b_{\Delta}\triangle_{1i} + \omega_i, \tag{8}$$

where a_{Δ}, b_{Δ} are the parameters to be estimated, and ω_i is an error term. We stack all observations for the different non-overlapping periods and then run the joint regression. Persistence in relative alpha is determined by a significantly positive coefficient b_{Δ}.

The persistence study is repeated for absolute alphas from the seven factor Fung and Hsieh (2001) model:

$$\alpha_{2i} = a_\alpha + b_\alpha \alpha_{1i} + v_i, \tag{9}$$

where $\alpha_{1i}(\alpha_{2i})$ are the alpha estimates in the formation (evaluation) period for fund i; $a_\alpha,\ b_\alpha$ are the parameters to be estimated, and v_i is an error term.

III. Data

For hedge fund information we use the MOAD database described in Hodder, Jackwerth, and Kolokolova (2013). MOAD is a merged database of six commercially available databases (CISDM, Barclays, TASS, HFR, Altvest, and Eurekahedge). We use only USD-denominated, net-of-fees returns with at least 36 month historical returns which leaves us with 11,597 hedge funds. Our sample runs from February 1994 until June 2011. The descriptive statistics of our sample are presented in Table I. We document excess kurtosis and left-skewness in hedge fund returns, suggesting that returns are often not normally distributed.

[Table I about here]

Hedge funds differ from other asset classes in many respects. One of them is the absence of strict regulation. This leads to database biases as reporting is voluntary. We address those biases as follows. First, our joint database is free of survivorship bias because it contains both live and dead funds. Second, to control for the instant history bias, we delete the first 12 months of each hedge fund's returns. We compute our main results based on the reported returns as we find them in the database.

For funds of funds, we extract the USD-denominated, net-of-fees returns in a similar fashion to the hedge fund returns from our database and are left with 9,314 funds of funds. The descriptive statistics of our sample are presented in Table II. Compared to single hedge funds, fund of funds have smaller net-of-fee average return which is explained by the double layer of fees. As a portfolio of funds, they also demonstrate smaller variance.

[Table II about here]

We further use the seven factors of the Fung and Hsieh (2001) model which are available at David A.Hsieh's Hedge Fund Data Library[3].

For the alternative factor models of Capocci and Hübner (2004), Edelman, Fung, Hsieh, and Naik (2012), Agarwal and Naik (2004) we downloaded the risk factors from Thomson Reuters Datastream. Option-based factors from Agarwal and Naik (2004) were graciously provided by the authors.

IV. Results

We now present our results where we first investigate the ability of relative and absolute alpha to assess current hedge fund performance. Next, we analyze the ability to predict future performance and the associated fund flow which should result from such ability. We continue with an investigation of the persistence of relative alpha versus absolute alpha.

[3]https://faculty.fuqua.duke.edu/ dah7/HFData.htm

Last, we document drivers of relative alpha.

A. *Current hedge fund performance*

When focusing on alpha as a measure of current performance, we are much concerned that the returns associated with omitted factors might show up in the alpha of a misspecified factor model. For absolute alpha, we use the adjusted R-squared of the Fung and Hsieh (2001) model in order to assess the explanatory power of the factor model. With an average adjusted R-squared of 41%, the Fung and Hsieh (2001) model leaves quite some variation in returns unexplained. Also, the average standard error around the alpha estimates is fairly large at 0.86.

For our relative alpha measure, we cannot directly obtain comparable quantities. We thus use the returns of the seven closest peer funds as pseudo factors. Then, we are able to repeat the above exercise of computing average adjusted R-squared which at 81% explains significantly more return variation than the Fung and Hsieh (2001) model at all significance levels. Also, the average standard error around the alpha estimates is much tighter at 0.57 which is significantly different from 0.86 at the 10% level.

B. *Predicting future performance*

In assessing the predictive ability of alpha, we turn to out-of-sample returns on decile portfolios of hedge funds, sorted by 36-month in-sample alpha. We roll the sample one month forward and repeat the exercise.

Table III provides monthly results on top decile, bottom decile, and top-minus-bottom decile portfolios. The top relative alpha portfolio delivers a slightly higher mean return (1.12%) at substantially lower standard deviation (2.11%) in comparison to the top absolute alpha portfolio (mean of 1.02% and standard deviation of 3.25% based on Fung and Hsieh (2001)), which is reflected in an almost doubled Sharpe ratio (0.49 for the top relative alpha portfolio vs. 0.27 for the top absolute alpha portfolio). We formally test if investing into the top relative alpha portfolio stochastically non-dominates (in the second order) investing into the top absolute alpha portfolio. The test of Davidson and Duclos (2013) rejects the null at the 1% level.[4] That means that any risk averse investor prefers the top relative alpha portfolio to the top absolute alpha portfolio. The same result obtains when we turn to the top-minus-bottom portfolios.

[Table III about here]

Another important result is that the top-minus-bottom portfolio constructed by sorting on relative alpha has a significantly positive mean return, while the mean return of the top-minus-bottom portfolio constructed by sorting on absolute alpha is insignificant. The Sharpe ratio of top-minus-bottom portfolio is 11 times higher for relative alpha when compared to absolute alpha. We conclude that relative alpha works better in distinguishing between future winners and losers than absolute alpha.

To strengthen our argument, we repeat the analysis by sorting hedge funds into decile portfolios based on alternative performance measures. Again, all results are based on 36-month rolling windows and on recording the subsequent out-of-sample returns. We use

[4]We apply the Davidson and Duclos (2013) test based on the t statistic version. We simulate the distribution of the test statistics in order to account for small samples.

the appraisal ratio of Treynor and Black (1973), the manipulation-proof performance measure (MPPM) of Goetzmann et al. (2007), and the strategy distinctiveness Index (SDI) of Sun et al. (2012). The relevant formulas are collected in Appendix. All three performance measures are second-order stochastically dominated by relative alpha, although the manipulation-proof performance measure only with a p-value of 0.07, see Table IV.

[Table IV about here]

There are several alternatives to the Fung and Hsieh (2001) factor model. As alternatives, we evaluate absolute alpha based on the models proposed in Agarwal and Naik (2004), Edelman et al. (2012), and Capocci and Hübner (2004). Please consult Appendix for details on all the factors used. The results are presented in Table V.

[Table V about here]

Our previous findings are still valid for the factor models of Edelman et al. (2012), and Capocci and Hübner (2004). In Agarwal and Naik (2004), absolute alpha works well in picking the top performers. However, it fails to pick bottom performers properly, i.e. the difference between top and bottom portfolios is statistically insignificant (as opposed to relative alpha). All Davidson and Duclos (2013) tests are significant at the 10% level except for the top portfolios of Agarwal and Naik (2004). The ratios of monthly Sharpe ratios reflect these findings and range from 1.4 to 7.2, reflecting the superiority of relative alpha but for the top portfolios based on Agarwal and Naik (2004) which have the same Sharpe ratio as achieved under relative alpha.

We try to explain the qualitative differences between top and bottom portfolios. In order to do so, we first regress portfolios based on relative alpha on the seven factors of Fung and Hsieh (2001). Table VI summarizes average risk exposures across funds and time. Generally, the estimates from Fung and Hsieh (2001) are quite similar for both portfolios, except for three factors: the size spread factor, the credit spread factor, and the currency trend-following factor. The top portfolio loads positively on the size spread factor (estimated average risk exposure is 0.03), while the bottom portfolio has a negative exposure to this factor (estimated average risk exposure is -0.02). Exposures to the credit spread factor as well as the currency trend-following factor are also significantly higher for the top portfolio (-0.45 and 0.02 respectively) than for bottom portfolio (-0.53 and 0.01). However, there are several issues one should be aware of: a) the exposures are aggregated across two dimensions; b) the estimates are still exposed to the omitted variable bias.

[Table VI about here]

A second way to explain the portfolio construction is to repeat the study within investment styles. Table VII shows the characteristics of top, bottom, and top-minus-bottom portfolios for the largest 6 investment strategies: equity long/short, fixed income, global macro, CTA/managed futures, event driven, and relative value. Naturally, the styles differ in their characteristics: some styles are considered more risky and yield higher returns (CTA/managed futures), while others are safer but less profitable (Equity long/short). Each portfolio based on relative alpha yields higher mean return and equal or lower standard deviation in comparison to absolute alpha. The only exception is CTA/managed futures where the top portfolio based on relative alpha has a higher standard deviation (8.12 for relative alpha vs. 7.89 for absolute alpha top portfolio). On the contrary, the bottom portfolio based on relative alpha has generally a lower return.

[Table VII about here]

Our main results on single hedge funds are also confirmed for funds of funds, see Table VIII: relative alpha works better in differentiating between future winners and losers than absolute alpha.

[Table VIII about here]

C. Fund flow and alpha

To analyze the relation between fund flow and alpha, we simply regress fund flows (from t-1 to *t*) and past relative (absolute) alpha (measured from *t-36* to *t-1*). The results of these regressions are presented in Table IX.

[Table IX about here]

Table 9 documents a significantly positive relation between past relative alpha and current fund flows. Even though our relative alpha approach is novel and has not been applied in the literature before, the underlying idea of comparing hedge funds to their peers seems well-understood according to the data: funds with higher past relative alpha attract more investments. We do not find the same pattern for the absolute alpha: the slope coefficient in the regression is insignificantly different from zero.

D. Persistence of alpha

We know from the hedge fund literature that there is rather mixed evidence on hedge fund absolute alpha persistence. We are therefore interested if relative alpha is more persistent than absolute alpha and present results in Table X. Relative alpha is more persistent than absolute alpha and it is robust to different sample sizes: estimates of the b-coefficients for relative alpha (Equation (8)) are consistently positive and significant for different sample sizes (12, 24, and 36 months) as opposed to the b-coefficients for absolute alpha (Equation (9)). It is worth noting that the b-coefficients for relative alpha are decreasing as we increase the sample size which is consistent with time-decay in persistence.

[Table X about here]

We further analyze the sensitivity of top portfolios to the business cycle by regressing top portfolio returns on a crisis-dummy during the recent financial crisis. We find that both top portfolios (constructed by relative and absolute alphas) are losing, but the magnitude of the crisis-dummy for the relative alpha portfolio is smaller than for the absolute alpha portfolio.

Table 11 demonstrates persistence results for funds of funds. Slope coefficients for relative alpha stay positive for all sample sizes and are significant for the 24 and 12-month samples. Moreover, the significance of the slope coefficients is decreasing in the out-of-sample size. The sign of slope coefficient for absolute alpha is switching for different sample sizes.

[Table XI about here]

E. Results on drivers of relative alpha

In our attempt to see what drives relative alpha, we run panel data regressions. On the right hand side of our regressions we include the seven factors from Fung and Hsieh (2001), a dummy variable indicating whether the fund is open to new investments, the logarithm of assets under management, a dummy variable indicating a high water mark, a dummy variable indicating whether the fund is using leverage, the management fee, and the performance fee. We divide the sample into non-overlapping 12-month periods and compute relative alpha for each fund within these periods. Time-varying explanatory variables (the seven factors of Fung and Hsieh (2001) and assets under managements) are averaged over the same periods. Table XII summarizes several panel data regressions: robust OLS, fixed effect, and random effect. The significance of the coefficients is robust to different methodologies. Apart from some Fung and Hsieh (2001) factors, the following variables have an effect on relative alpha: openness to new investment and performance fee. If a fund is opened to new investment, it has on average lower relative alpha, consistent with the intuition that superior hedge funds (with high relative alpha) are more likely to be closed to new investment. Also, the higher the performance fee, the higher the relative alpha of a fund which is consistent with the role of fees in signaling better performance (see Habib and Johnsen (2012) on mutual funds).

[Table XII about here]

V. Robustness

As a part of our robustness checks, we split the sample into several subsamples: February 1994 - February 2000 (dotcom bubble), March 2000 - July 2007 (intermediate period), and August 2007 - June 2011 (financial crisis). Table XIII summarizes performance of top portfolios based on relative alpha and absolute alpha during the subsamples. During the first subsample (the dotcom bubble from February 1994 - February 2000) both measures are performing similarly and yield a Sharpe ratio of 0.61-0.63. Our main results stay largely unaffected during the last two subsamples, the intermediate period and the financial crisis, i.e. relative alpha outperforms absolute alpha in terms of Sharpe ratios and the Davidson and Duclos (2013) test confirms that any risk averse investor would prefer top relative alpha portfolios. During the recent financial crisis the returns overall decreased dramatically which is also reflected in the portfolios (mean return is 0.71% instead of 1.12% during the whole period for relative alpha and 0.53% instead of 1.02% for absolute alpha).

[Table XIII about here]

Furthermore, our results are robust to a range of minor methodological and sample changes (see Internet Appendix):

- eliminating the hedge funds which are closed to new investment (Table IA.1)
- eliminating small funds with assets under management under $20 million as proposed in Kosowski, Naik, and Teo (2007) (Table IA.2)
- changing the rolling window size to 24 months instead of 36 months (Table IA.3)
- using portfolios of only 20 hedge funds with the largest alphas instead of larger deciles (Table IA.4)
- comparing relative alpha to a random hedge fund portfolio (Table IA.5)
- holding top and bottom portfolios for 12 months instead of 1 month (Table IA.6)
- correcting for boundary bias of the kernel estimates by using locally weighted regressions as proposed in Hastie and Loader (1993) (Table IA.7)

VI. Simulation

We use a simulation study to investigate why relative alpha outperforms absolute alpha. We are concerned about three aspects, namely the poor estimation of absolute alpha due to short time-series, multi-collinear factors, and omitted factors.

We first simulate hedge fund returns by assuming that the true model is the seven factor model of Fung and Hsieh (2001) with independently normally distributed residuals. In order to preserve the empirical characteristics of our hedge fund returns, we set the assumed true parameters equal to the estimated parameters as observed in the data. That gives us empirical residuals from which we compute the variances of the residuals. We then assume that the true residuals are independently normally distributed with variances equal to the empirical variances and zero mean. Given this structure, we can simulate hedge fund returns where the initial cross-sectional dimension is 2000, the time series length is 36 months, and the number of simulation runs is 100.

We next estimate relative alpha based on the simulated returns using our usual methodology as in Equation (7). We report the average value of 0.15 in Table XIV for the true model in the first row (for estimation method Relative alpha) and column two (labeled relative alpha) as we use Equation (7). An absolute alpha value is not available in this case (n/a in column three, labeled absolute alpha). Next we report the true average alpha of 1.20 in the column absolute alpha. Also, we would like to report a relative alpha version which we base on Equation (4). Namely, if two hedge funds are identical in terms of their beta exposures, then

$$E[r_{it} - r_{jt}] = \alpha_i - \alpha_j + \sum_{k=1}^{K} (\beta_{ik} - \beta_{jk}) E[X_{k,t}] + E[\varepsilon_{it} - \varepsilon_{jt}] = \alpha_i - \alpha_j. \tag{10}$$

Thus, we replace in the relative alpha formula of Equation (7) the expectation $E[r_{it} - r_{jt}]$ with the difference in true alpha:

$$\triangle^{\alpha}_{(i,T)} = \frac{\sum_{i \neq j} K\left(Var[r_{it} - r_{jt}]/h\right) \left(\alpha_i - \alpha_j\right)}{\sum_{i \neq j} K\left(Var[r_{it} - r_{jt}]/h\right)}. \tag{11}$$

We compute the modified relative alpha $\triangle^{\alpha}_{(i,T)}$ from Equation (11) and obtain an average value of 0.17, insignificantly different from 0.15 for average relative alpha itself. Next, we estimate the Fung and Hsieh (2001) model for each hedge fund and obtain for the true model exactly the same absolute alpha (estimation method FH7) - the variation introduced by simulating the residuals does not affect the average absolute alpha value. Using the differences of those estimated alphas instead of the expectation $E[r_{it} - r_{jt}]$, we modify Equation (7) yet again:

$$\triangle^{\widehat{\alpha}}_{(i,T)} = \frac{\sum_{i \neq j} K\left(Var[r_{it} - r_{jt}]/h\right) \left(\widehat{\alpha}_i - \widehat{\alpha}_j\right)}{\sum_{i \neq j} K\left(Var[r_{it} - r_{jt}]/h\right)}. \tag{12}$$

Calculating $\triangle^{\widehat{\alpha}}_{(i,T)}$ from Equation (12) yields 0.17 as the average relative alpha version for the estimation method FH7. Finally, we introduce omitted variable bias in the estimation method Market model with just the market and an intercept. The Market model ignores the remaining six factors of the true model. The average absolute alpha under the Market model is 0.95 while the relative alpha version thereof (calculated according to Equation (12)) is 0.05. These estimates based on the estimation method with omitted factors (Market model) are significantly different from the values using the true alphas. A big advantage is thus that our Relative alpha estimation method finds a value for relative alpha (0.15) close to the true value of 0.17, whereas the estimation method Market model with omitted factors is far off at 0.05.

[Table XIV about here]

Next, we vary the length of the time series and show the influence of small sample bias on absolute alpha in comparison to relative alpha. From Table XIV, columns four and five, we see that the small sample (of only 6 instead of 36 monthly returns) affects the estimation methods Relative alpha and Market model but not the estimation method FH7. Using the correct model (FH7) gives - even on very short samples - the correct average alpha (1.20, 0.17 when expressed as relative alpha) which is insignificantly different from the true alpha (1.27, 0.17 when expressed as relative alpha). Relative alpha performs poorly (0.03) on the short samples and the Market model, too (0.05, 0.02 when expressed as relative alpha). The simulation suggests that a minimal sample length is needed for the relative alpha method to work.

Finally, we reduce the cross section from which to pick the peer group from 2000 to 50. Results are in Table XIV, columns six and seven. Using the correct model (FH7) gives identical alphas (0.44, 0.10 when expressed as relative alpha) to the true alphas. The Market model has a hard time yet again due to the omitted factor bias with estimated alphas of -0.01 when expressed as relative alpha. Relative alpha is also performing poorly at 0.06, significantly different from 0.10 for the same value based on the true alphas. It shows that for relative alpha to perform well, a minimal cross sectional dimension is required.

We conclude from our simulation that the good performance of relative alpha is due to its capability of dealing with omitted factor bias. In order to achieve that feat, the method needs reasonably long samples for the estimation (36 months) and a large enough cross section of peer funds (2000) so that the investment opportunity set is being spanned by its peers.

In Section II we argued that the variance of return differences $V[r_i - r_j]$ is a good measure of funds proximity. It consists of a systematic ($\triangle\beta' Cov(X)\triangle\beta$) and an idiosyncratic ($\sigma_i^2 + \sigma_j^2$) component. A concern is our assumption that the idiosyncratic component is fairly constant across hedge fund pairs while the systematic component measures the similarity of the hedge fund pair. Based on our simulation study and for one sample hedge fund, we know both components and plot them together in Figure 1 (sorted by increasing total variance, $V[r_i - r_j]$). We find that the idiosyncratic component ($\sigma_i^2 + \sigma_j^2$) is quite similar for different funds, while the systematic component ($\triangle\beta' Cov(X)\triangle\beta$) increases strongly in total variance, $V[r_i - r_j]$. We are thus comfortable with our argument that the variance of return differentials is a good measure for the similarity in the hedge fund risk exposures.

[Figure 1 about here]

According to Equation (4), for hedge funds with identical strategies, the expectation of return differences is equal to the differences in true alpha. If two hedge funds differ in their strategies (i.e. have different betas), then Equation (4) will not hold perfectly anymore and $E[r_i - r_j] - (\alpha_i - \alpha_j) = \triangle\beta' E[X] + E[\varepsilon_i - \varepsilon_j]$. We argued above that this discrepancy should be small for similar hedge funds where we interpret similarity as small variances in return differences, $V[r_i - r_j]$. Using our simulation, we can depict that relation in Figure 2 where we plot (for one sample hedge fund) the discrepancies $E[r_i - r_j] - (\alpha_i - \alpha_j)$ across variances in return differences, $V[r_i - r_j]$. Figure 2 shows that the smaller the variance, the closer is the expectation of differences, $E[r_i - r_j]$, to the true alpha differences $(\alpha_i - \alpha_j)$. We are thus comfortable with our above assumption that Equation (4) holds reasonably well for similar hedge funds.

[Figure 2 about here]

VII. Conclusion

We propose a novel performance measure, relative alpha, which assesses the out-performance of a hedge fund with respect to a group of peers. It exhibits the intriguing property that omitted factor bias cancels, as the peer group is selected by exhibiting the least variance of return differentials. A nice side effect is that the investor does not even need to know the exact factor structure, nor the omitted factors - simply the similarity of hedge funds according to our distance measure leads to the reduction of omitted factor bias.

Different results obtain for different uses of alpha, but relative alpha tends to beat absolute alpha in all three dimensions. Concerning alpha as a measure of current outperformance of the risk-adjusted return, we find that relative alpha explains more return variation (81%) than the Fung and Hsieh (2001) model (41%). When using alpha in order to predict high future performance, then relative alpha can be used to construct portfolios of hedge funds and the out-of-sample performance of the top decile portfolio second order stochastically dominates sorts based on absolute Fung and Hsieh (2001) alpha, the appraisal ratio, the manipulation-proof performance measure, and the strategic distinctiveness index. Sharpe ratios of relative alpha sorted top portfolios are about twice those of the competitors. Related, we find fund flow is more closely related to relative alpha than it is to absolute alpha.

Finally, using past alpha to predict future alpha, relative alpha is strongly persistent in our sample, as opposed to absolute alpha.

Our results stay robust to various methodological changes and sample manipulations. In our simulation study we show that relative alpha works better than absolute alpha when there are omitted variables, if there is a large number of hedge funds in the cross-section, and if there is a reasonable sample length.

Work lies ahead in several directions. Concerning absolute alpha, there is a lively discussion about skill versus luck as the driving force and we would like to study this question for relative alpha, too. Relative alpha might also predict voluntary delisting of hedge funds from the databases.

Appendix

Performance Measures

Appraisal Ratio

$$AR = \frac{\widehat{\alpha}}{\widehat{\sigma}_{\varepsilon}}, \quad (1)$$

where $\widehat{\alpha}$ is the alpha estimate of a hedge fund, $\widehat{\sigma}_{\varepsilon}$ is the residual standard deviation.

Manipulation-proof Performance Measure of Goetzmann et al. (2007)

$$\widehat{\Theta} = \frac{1}{(1-\rho)\triangle t} ln \left(\frac{1}{T} \sum_{t=1}^{T} [\frac{1+r_t}{1+r_{ft}}]^{1-\rho} \right), \quad (2)$$

where T is the total number of observations, $\triangle t$ is the length of time between observations, r_t is the return of a hedge fund in t, r_{ft} is the risk-free rate, ρ is the relative risk-aversion coefficient.

Strategy Distinctiveness Index of Sun et al. (2012)

$$SDI = 1 - corr(r_t, \mu), \quad (3)$$

where r_t is the return of a hedge fund in t, and μ is the average return of all funds belonging to the same style.

Factor Models

Fung and Hsieh (2001)

1. Bond Trend-Following Factor, lookback straddles
2. Currency Trend-Following Factor, lookback straddles
3. Commodity Trend-Following Factor, lookback straddles
4. Excess return on the S&P 500 index over the risk-free rate
5. Difference in the returns on the Wilshire Small Cap 1750 index and Wilshire Large Cap 750 index
6. The monthly change in the 10-year treasury constant maturity yield
7. The monthly change in the spread between Moody's Baa yield and 10-year treasury constant maturity yield

Agarwal and Naik (2004)

1. Returns on Russel 3000 Index
2. Returns on Morgan Stanely Capital International world excluding US Index
3. MSCI emerging market index
4. Salomon Brothers government and corporate bond index
5. Salomon Brothers world government bond
6. Lehman high yield index
7. Federal Reserve Bank competitiveness-weighted dollar index
8. Goldman Sachs commodity index
9. Factor-mimicking portfolio for size
10. Factor-mimicking portfolio for book-to-market equity
11. Factor-mimicking portfolio for the momentum effect
12. The monthly change in the spread between Moody's Baa yield and 10-year treasury constant maturity yield

13. At-the-money European call on the S&P 500 composite index
14. At-the-money European put on the S&P 500 composite index
15. Out-of-the-money European call on the S&P 500 composite index
16. Out-of-the-money European put on the S&P 500 composite index

Edelman et al. (2012)

1. Bond Trend-Following Factor, lookback straddles
2. Currency Trend-Following Factor, lookback straddles
3. Commodity Trend-Following Factor, lookback straddles
4. Excess return on the S&P 500 index over the risk-free rate
5. Difference in the returns on the Wilshire Small Cap 1750 index and Wilshire Large Cap 750 index
6. The monthly change in the 10-year treasury constant maturity yield
7. The monthly change in the spread between Moody's Baa yield and 10-year treasury constant maturity yield
8. Excess return on the IFC Emerging Markets Index

Capocci and Hübner (2004)

1. Excess return on the Russel 3000 Index
2. Factor-mimicking portfolio for size
3. Factor-mimicking portfolio for book-to-market equity
4. Factor-mimicking portfolio for the momentum effect
5. Excess return of the MSCI World Index excluding US
6. Excess return on the Lehman Aggregate US Bond Index
7. Excess return on the Salomon World Government Bond Index
8. Excess Return of the JP Morgan Emerging Bond Index
9. Excess Return of the Lehman BAA Corporate Bond Index

10. Excess Return of the Goldman Sachs Commodity Index

Tables

Table I Descriptive statistics: Hedge Funds

The summary statistics are the equally weighted cross-sectional averages, standard deviations, minimum, and maximum of the: mean monthly return, μ; the standard deviation of monthly returns, σ; the skewness, Skewness; the excess kurtosis, Kurtosis. The sample is February 1994 to June 2011.

	Mean	Std.dev	Minimum	Maximum
μ	0.87	0.79	-6.68	8.56
σ	4.06	3.07	0.00	38.89
Skewness	-0.18	1.31	-10.33	9.93
Kurtosis	6.53	6.84	1.56	111.81

Table II Descriptive statistics: Funds of Funds

The summary statistics are the equally weighted cross-sectional averages, standard deviations, minimum, and maximum of the: mean monthly return, μ; the standard deviation of monthly returns, σ; the skewness, Skewness; the excess kurtosis, Kurtosis. The sample is February 1994 to June 2011.

	Mean	Std.dev	Minimum	Maximum
μ	0.53	0.35	-2.63	5.26
σ	2.17	1.31	0.12	19.50
Skewness	-0.90	1.22	-12.10	8.66
Kurtosis	7.34	6.78	1.63	153.76

Table III Predicting future portfolio performance: relative alpha vs. absolute alpha
The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test.

HF deciles	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.12	2.11	0.49	1.02	3.25	0.27	<0.01
Bottom	0.40	2.30	0.16	0.85	3.37	0.23	n/a
Top-Bottom	0.72	1.24	0.52	0.17	2.80	0.05	<0.01

Table IV Predicting future portfolio performance: alternative hedge fund performance measures

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on Appraisal ratio of Treynor and Black (1973), Manipulation-Proof Performance Measure (MPPM) of Goetzmann et al. (2007), and Strategy Distinctiveness Index (SDI) of Sun et al. (2012). The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test of each measure against relative alpha portfolios.

HF deciles	Appraisal ratio			DD (2013)	MPPM			DD (2013)	SDI			DD (2013)
	mean	std.dev	Sharpe Ratio	test	mean	std.dev	Sharpe Ratio	test	mean	std.dev	Sharpe Ratio	test
Top	0.75	2.71	0.23	0.01	0.89	3.24	0.23	0.07	0.77	3.70	0.17	0.01
Bottom	0.72	3.80	0.15	n/a	0.80	3.21	0.20	n/a	0.55	1.54	0.29	n/a
Top-Bottom	0.03	3.55	0.00	0.01	0.09	2.96	0.00	0.02	0.22	4.53	0.01	0.00

Table V Predicting future portfolio performance: alternative hedge fund factor models

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on absolute alphas of the Agarwal and Naik (2004), Edelman et al. (2012), and Capocci and Hübner (2004) factor models. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test of each measure against relative alpha portfolios.

HF deciles	Agarwal and Naik (2004)			DD (2013)	Edelmann et al. (2012)			DD (2013)	Capocci and Hübner (2004)			DD (2013)
	mean	std.dev	Sharpe ratio	test	mean	std.dev	Sharpe ratio	test	mean	std.dev	Sharpe ratio	test
Top	0.94	1.96	0.43	0.30	1.18	3.16	0.33	0.07	1.07	2.91	0.32	0.08
Bottom	0.63	2.82	0.17	n/a	0.45	3.47	0.10	n/a	0.61	3.83	0.12	n/a
Top-Bottom	0.31	1.67	0.14	0.00	0.73	3.10	0.19	0.01	0.46	3.05	0.12	0.06

Table VI Risk exposures within top and bottom portfolio deciles

The table demonstrates risk exposure to the factors form Fung and Hsieh (2001) of top and bottom decile portfolios constructed by sorting based on relative alphas. The risk exposures are averaged across time. The last column provides p-values of the mean differences between top and bottom portfolios risk exposures.

Factor	Top	Bottom	Difference	p-value
Alpha	2.60	-0.19	2.79	0.00
Equity Market Factor	0.08	0.07	0.01	0.33
The Size Spread Factor	0.03	-0.02	0.05	0.05
The Bond Market Factor	-0.27	-0.28	0.01	0.50
The Credit Spread Factor	-0.45	-0.53	0.07	0.06
Bond Trend-Following Factor	-0.02	-0.02	0.00	0.22
Currency Trend-Following Factor	0.02	0.01	0.01	0.00
Commodity Trend-Following Factor	0.01	0.01	0.00	0.98

Table VII Predicting future portfolio performance within self-reported styles: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha across the largest self-reported styles: equity long/short, fixed income, global macro, CTA/managed futures, event driven, and relative value. The characteristics include monthly mean and standard deviation.

	Relative alpha			Absolute alpha		
	Top	Bottom	Top-Bottom	Top	Bottom	Top-Bottom
Equity long/short						
mean	1.15	0.71	0.43	1.08	0.91	0.16
std.dev	3.30	6.02	5.19	4.75	6.10	6.34
Fixed income						
mean	0.91	0.00	0.91	0.77	-0.30	1.07
std.dev	1.91	2.71	2.85	1.89	3.30	3.38
Global macro						
mean	0.97	0.24	0.72	0.89	0.44	0.45
std.dev	2.16	1.93	1.63	2.87	2.27	2.75
CTA/managed futures						
mean	1.75	0.66	1.09	1.64	1.00	0.64
std.dev	8.12	5.22	9.50	7.89	6.04	9.69
Event driven						
mean	1.20	0.43	0.77	1.16	0.57	0.59
std.dev	2.73	2.96	1.97	3.02	2.92	2.59
Relative value						
mean	1.16	0.49	0.67	0.94	0.43	0.50
std.dev	1.81	2.00	1.87	2.30	2.01	2.27

Table VIII Predicting future portfolio performance for funds of funds: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test.

HF deciles	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	0.84	2.23	0.35	0.82	2.71	0.26	0.14
Bottom	0.05	1.95	0.01	0.06	2.20	0.01	n/a
Top-Bottom	0.79	1.74	0.42	0.76	2.27	0.30	0.04

Table IX Fund flow vs past alpha

We regress fund flows (measured from *t-1* to *t*) on the estimates of relative (absolute) alpha (measured from t-1,...,*t-36*) and a constant. The table summarizes the OLS estimates, t-statistics, and p-values.

	Relative Alpha			Absolute Alpha		
	coeff.	t-stat	p-val	coeff.	t-stat	p-val
constant	0.01	10.21	0.00	0.01	9.84	0.00
alpha	0.00	3.47	0.00	0.00	0.85	0.40

Table X Persistence of alpha: relative alpha vs. absolute alpha

The table presents estimated slope coefficients (b) from stacked, non-overlapping linear regressions: $\triangle_{2i} = a_\Delta + b_\Delta \triangle_{1i} + \omega_i$ (for relative alpha) and $\alpha_{2i} = a_\alpha + b_\alpha \alpha_{1i} + v_i$ (for absolute alpha). It also provides t-statistics and p-values on the significance of the b estimates. By stacking the regressions, we assume that the slope coefficients are constant across periods.

Sample size	Relative alpha			Absolute alpha		
Formation/Evaluation	b	t-stat	p-val	b	t-stat	p-val
12/12	0.17	11.49	0.00	-0.08	-4.17	0.00
24/24	0.14	5.10	0.00	-0.01	-0.74	0.46
36/36	0.07	2.90	0.00	0.04	1.90	0.06

Table XI Persistence of alpha for funds of funds: relative alpha vs. absolute alpha

The table presents estimated slope coefficients (b) from stacked, non-overlapping linear regressions: $\triangle_{2i} = a_\Delta + b_\Delta \triangle_{1i} + \omega_i$ (for relative alpha) and $\alpha_{2i} = a_\alpha + b_\alpha \alpha_{1i} + v_i$ (for absolute alpha). It also provides t-statistics and p-values on the significance of the b estimates. By stacking the regressions, we assume that the slope coefficients are constant across periods.

Sample size	Relative alpha			Absolute alpha		
Formation/Evaluation	b	t-stat	p-val	b	t-stat	p-val
12/12	0.14	7.49	0.00	0.31	17.21	0.00
24/24	0.17	6.56	0.00	-0.09	-3.43	0.00
36/36	0.04	1.00	0.32	-0.05	-1.04	0.30

Table XII Explanatory panel regressions for relative alpha

The dependent variable is relative alpha. The independent variables are the seven Fung and Hsieh (2001) factors, the logarithm of assets under management, a dummy variable indicating whether the fund is using leverage, a dummy variable indicating whether the fund is using a high water, a dummy variable indicating whether the fund is open to new investments, the management fee, the performance fee, and a constant. All variables are averages over non-overlapping one-year periods. Below the coefficients we report p-values.

Variable	OLS robust	Fixed effect	Fixed effect robust	Random effect	Random effect robust
equity market factor	0.09	0.12	0.12	0.10	0.10
	0.00	0.00	0.02	0.00	0.00
size spread factor	0.09	0.09	0.09	0.08	0.08
	0.02	0.02	0.07	0.02	0.03
bond market factor	0.03	0.09	0.09	0.05	0.05
	0.23	0.03	0.05	0.19	0.07
credit spread factor	0.00	0.08	0.08	0.02	0.02
	0.91	0.10	0.08	0.59	0.48
bond trend-following factor	0.01	0.00	0.00	0.01	0.01
	0.22	0.73	0.71	0.15	0.14
currency trend-following factor	0.02	0.02	0.02	0.02	0.02
	0.13	0.06	0.13	0.00	0.10
commodity trend-following factor	0.00	0.00	0.00	0.00	0.00
	0.97	0.70	0.64	0.93	0.89
log assets under management	0.01	-0.23	-0.23	0.01	0.01
	0.27	0.00	0.00	0.02	0.37
leverage {0,1}	-0.03	-	-	-0.05	-0.05
	0.45	-	-	0.40	0.31
high water mark {0,1}	0.02	-	-	-0.01	-0.01
	0.81	-	-	0.85	0.90
open to new investment {0,1}	-0.23	-	-	-0.26	-0.26
	0.01	-	-	0.00	0.03
management fee	0.17	-	-	0.22	0.22
	0.29	-	-	0.00	0.31
performance fee	0.01	-	-	0.01	0.01
	0.00	-	-	0.02	0.00
constant	-0.36	3.27	3.27	-0.37	-0.37
	0.14	0.00	0.00	0.00	0.23

Table XIII Predicting future portfolio performance for different samples: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test.

Period	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Feb. 1994-Feb. 2000	1.70	2.77	0.61	2.36	3.77	0.63	0.56
Mar. 2000-Jul. 2007	1.10	1.87	0.59	0.75	2.76	0.27	0.01
Aug. 2007-Jun. 2011	0.71	1.90	0.38	0.53	2.66	0.20	0.10

Table XIV Simulation study

The table demonstrates results of the simulation study as described in Section VI. We simulate hedge fund returns by using a factor model that we assume to be true, using 100 simulation runs. In order to preserve the empirical characteristics of our hedge fund returns, we use the seven factors of Fung and Hsieh (2001) and set assumed true parameters equal to the estimated parameters as observed in the data. For the true model (columns two and three), we report two results according to four estimation methods. In column three "Absolute alpha", we report average absolute alphas. In column two "Relative alpha", we report relative alpha based on Equation (7) where we substitute $E[r_{it} - r_{jt}]$ with the expected differences in alpha. In the first row of results, we use our usual estimation method "Relative alpha" as in Equation (7). In the second row, we report "True alpha". In the third row, we use the estimation method "FH7" based on the Fung and Hsieh (2001) model. In the fourth row, we use the estimation method "Market model" where we repeat the calculations using only a single market factor model. In columns four and five, marked "only 6 returns", we repeat the study using only 6 months of observations instead of 36 months. In columns six and seven, marked "Only cross section 50", we repeat the study using only 50 peer funds instead of 2000.

Estimation	True model, 36 returns, cross section 2000		Only 6 returns		Only cross section 50	
method used	Relative alpha	Absolute alpha	Relative alpha	Absolute alpha	Relative alpha	Absolute alpha
Relative alpha	0.15	n/a	0.03***	n/a	0.06***	n/a
True alpha	0.17	1.20	0.17	1.27	0.10	0.44
FH7	0.17	1.20	0.17	1.20	0.10	0.44
Market model	0.05**	0.95***	0.02***	0.05***	-0.01***	0.39

Figures

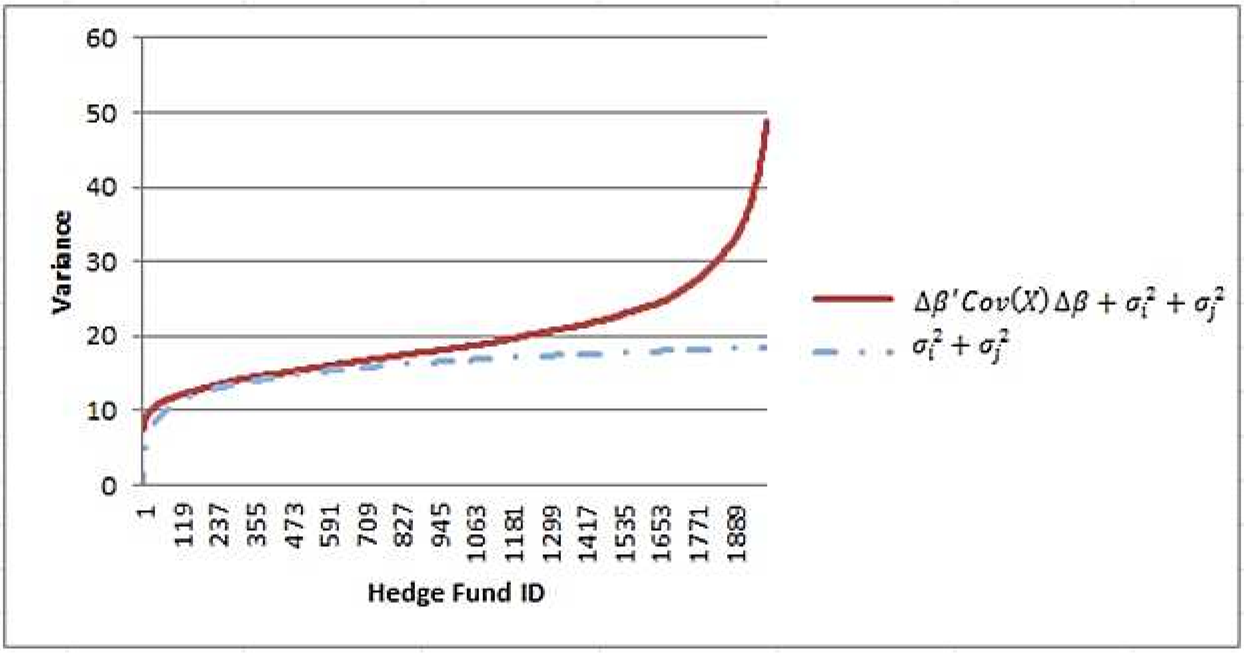

Figure 1. Simulation: variance decomposition

The figure shows the proportion between the idiosyncratic component $(\sigma_i^2 + \sigma_j^2)$ of the variance of return differences $V[r_i - r_j]$ and the systematic component $(\triangle\beta' Cov(X)\triangle\beta)$ for the simulation study based on one randomly selected hedge fund and the differences with all other hedge funds. We sorted the hedge fund peers by increasing total variances on the x-axis and numbered them by integers, using a variable called ID. The idiosyncratic components vary somewhat as a function of $V[r_i - r_j]$ and we thus fit a regression function to the data according to logarithmic function to the data of the model $(\sigma_i^2 + \sigma_j^2) = \text{ a} + \text{b} \ln(ID) + \varepsilon$. All values are in percent per month.

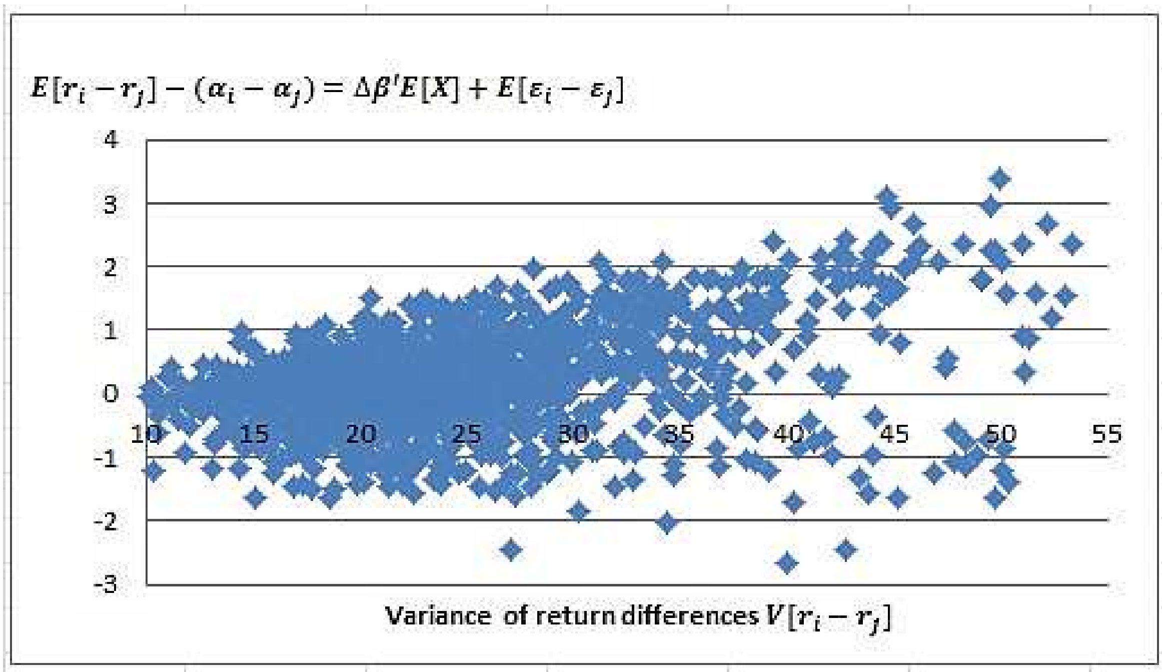

Figure 2. Simulation: Discrepancy of expectation of return differences and differences in alpha vs. variances of return differences

The figure shows the relation between the variance of the return differences $V\,[r_i - r_j]$ and the expression: $E\,[r_i - r_j] - (\alpha_i - \alpha_j) = \sum_{k=1}^{K} (\beta_{ik} - \beta_{jk}) E[X_{k,t}] + E[\varepsilon_{it} - \varepsilon_{jt}]$ for the simulation study based on one randomly selected hedge fund and the differences with all other hedge funds. All values are in percent per month.

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Internet Appendix for "Relative Alpha"

Jens Carsten Jackwerth

Anna Slavutskaya

Table IA.1 Predicting future portfolio performance for funds open to new investment: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test. Here the sample of the hedge funds is restricted to funds which are opened to new investments.

HF deciles	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.10	2.19	0.47	1.05	2.97	0.31	0.01
Bottom	0.30	2.02	0.14	0.51	2.17	0.20	n/a
Top-Bottom	0.80	1.35	0.54	0.54	2.47	0.21	0.00

Table IA.2 Predicting future portfolio performance for large funds: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test. Here the sample of the hedge funds is restricted to the funds with assets under management larger than $20 million.

HF deciles	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.38	2.86	0.44	1.32	3.56	0.34	0.02
Bottom	0.55	2.44	0.20	0.78	2.62	0.27	n/a
Top-Bottom	0.83	2.43	0.31	0.53	3.45	0.13	0.00

Table IA.3 Predicting future portfolio performance for shorter rolling window: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test. Here we use 24-month rolling windows instead of 36-month used in the main runs.

HF deciles	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.26	2.07	0.59	1.07	3.10	0.31	0.01
Bottom	0.56	2.51	0.20	0.60	2.20	0.25	n/a
Top-Bottom	0.70	1.86	0.35	0.46	3.02	0.13	0.00

Table IA.4 Predicting future portfolio performance for smaller portfolios: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom 20 fund portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test. Here we use 20 hedge funds for our portfolios instead of deciles as used in the main runs.

HF 20-fund portfolios	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.13	2.11	0.49	1.03	3.02	0.31	0.00
Bottom	0.50	2.85	0.14	0.71	2.87	0.22	n/a
Top-Bottom	0.63	2.08	0.27	0.33	3.09	0.09	0.00

Table IA.5 Predicting future portfolio performance: relative alpha vs. random portfolio
The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and randomly selected portfolios. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test.

HF deciles	Relative alpha			Random portfolio			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.12	2.11	0.49	0.60	1.77	0.31	0.02
Bottom	0.40	2.30	0.16	n/a	n/a	n/a	n/a
Top-Bottom	0.72	1.24	0.55	n/a	n/a	n/a	n/a

Table IA.6 Predicting future portfolio performance for longer holding period: relative alpha vs. absolute alpha

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order-stochastic non-dominance test. Portfolios are held for a period of 12 months instead of one month as used in the main runs.

HF deciles	Relative alpha			Random portfolio			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.04	2.32	0.34	1.03	3.50	0.20	0.00
Bottom	0.56	3.23	0.12	0.70	3.36	0.15	n/a
Top-Bottom	0.48	2.59	0.14	0.33	3.63	0.04	0.02

Table IA.7 Correcting for the boundary bias of the kernel estimates

The table demonstrates out-of-sample performance characteristics of top, bottom, and top-bottom decile portfolios constructed by sorting based on relative alpha and absolute alpha. The characteristics include monthly mean, standard deviation, and Sharpe ratio. Last column provides p-values of the Davidson and Duclos (DD 2013) second-order stochastic non-dominance test. We correct for the boundary bias of the kernel estimates from the Equation (7) by using the locally weighted regression method proposed by Hastie and Loader (1993).

HF deciles	Relative alpha			Absolute alpha			DD (2013)
	mean	std.dev	Sharpe ratio	mean	std.dev	Sharpe ratio	test
Top	1.11	2.08	0.49	1.02	3.25	0.27	<0.01
Bottom	0.41	2.26	0.16	0.85	3.37	0.23	n/a
Top-Bottom	0.70	1.27	0.51	0.17	2.80	0.05	<0.01