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The Multiverse View and Set-Theoretic Practice

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Abstract: Hamkins' multiverse view is a prominent position on the nature of set theory. It is posited against the universe view and proposed as a philosophical theory explaining current set-theoretic practice. This paper confronts the multiverse view with the results of an interview study investigating current set-theoretic practice. The study reveals a heterogeneity of set-theoretic research practices. The multiverse view is found to align well with pluralist research practices but not with absolutist practices. The generalisation claim of the multiverse view fails because of this heterogeneity; only a reduced interpretation of the multiverse view might hold generally. Furthermore, Hamkins' prediction that the community as a whole will not adopt axioms deciding the continuum hypothesis is found probably true, but for different reasons: although part of the community has already accepted axioms beyond ZFC, another part of the community will withdraw from doing so. In conclusion, I argue that Hamkins' multiverse view is best interpreted as a valuable perspective on pluralist set-theoretic practices.

Keywords: multiverse view; set-theoretic practice; qualitative research; pluralism in mathematics

1 Introduction

Hamkins' multiverse view (Hamkins 2012) is a prominent position on the nature of set theory. It is posited against the universe view, which states that set theory aims to describe a unique set-theoretic universe in which every set-theoretic sentence is either true or false. The multiverse view relies on a close connection with set-theoretic practice. The term 'set-theoretic practices' encompasses both set-theoretic research activities and the beliefs of set-theoretic practitioners, which give meaning to their work. This papers confronts the multiverse view with the results of a study investigating current set-theoretic practice.

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The debate between the universe view and the multiverse view, along with numerous other specific positions, is one of the most important and controversial topics in contemporary philosophy of set theory. A central question is whether sentences independent of ZFC have a determinate truth-value. The continuum hypothesis is the most prominent example of such a sentence, but many others exist, such as Suslin's hypothesis and projective determinacy. In this debate, arguments related to set-theoretic practice are becoming increasingly significant. Most notably, Penelope Maddy introduced a detailed meta-philosophy – Second Philosophy – which emphasises the analysis of set-theoretic practice as the foundation for a justified philosophical theory, making set-theoretic practices crucial to her argumentation. Maddy argues in favour of the universe view, maintaining that principles governing set-theoretic practice – such as extrinsic justification of axioms – support a universe view (Maddy 1997). Although some reasons in favour of the multiverse view have emerged (e.g., the study of set-theoretic models is more naturally interpreted within a multiverse framework), Maddy concludes that this evidence does not outweigh the arguments for the universe view, though she acknowledges the heuristic role the multiverse view can play (Maddy 2017).

Another perspective is defended by Carolin Antos (2022). While she also recognizes the significance of set-theoretic practice in this debate, she disagrees with both Maddy and Hamkins. Based on the observation that the study of set-theoretic models is integral to contemporary set-theoretic practice, she argues that set-theoretic models should be regarded as fundamental entities of set theory, on par with sets themselves.

On the other hand, there are several positions that do not rely on set-theoretic practice. Gödel's Platonism is one such example (Gödel 1947), as is Balaguer's full-blooded Platonism (Balaguer 1998). According to Gödel, independent sentences are truth-apt, and mathematicians can determine the truth or falsity of mathematical conjectures by accessing the mathematical realm of sets through their mathematical intuition. In contrast, Balaguer argues that independent sentences are not truth-apt. In his full-blooded Platonism, all possible mathematical objects exist. For instance, this includes bijections between $\mathfrak{c}$ (the size of the continuum) and various cardinal numbers such as $\aleph_1$, $\aleph_2$, or $\aleph_{17}$. This illustrates why, on Balaguer's account, sentences like the continuum hypothesis (CH) do not have a determinate truth-value.

I understand the universe view as the conviction that there is a preferred universe of sets – an objective, real universe in which sentences independent of ZFC have definitive answers. Some set theorists, such as Woodin and Steel, seemingly endorse a universe view. However, even proponents of the universe view work with multiverse structures. Let me clarify. Woodin (2001a, 2001b) argues that through extensive set-theoretic work, set theorists have accumulated substantial evidence supporting the truth of projective determinacy. He suggests that similar research

strategies might ultimately resolve the question of whether the continuum hypothesis is true. Later, Woodin (2017) introduces a set-theoretic research programme centred on the concept of a canonical inner model containing a supercompact cardinal, known as Ultimate-*L*, which is intended to solve the continuum problem. While both Woodin and Steel have engaged with multiverse structures (Woodin 2011; Steel 2014), neither defends a multiverse view as representing the correct domain of sets. Rather, Woodin (2011) aimed to gather evidence that the continuum hypothesis is false, relying on the proof of what Woodin termed the Ω-conjecture. However, as this line of research reached a dead end, Woodin abandoned it in favour of a radically different approach, a shift in perspective analysed by Rittberg (2015). Regarding Steel's work on the axiomatisation of a multiverse structure (as presented in Steel [2014] and further examined by Maddy and Meadows [2020]), Steel should not be interpreted as a multiversist. His multiverse structure includes the assumption that the universes it encompasses satisfy large cardinal axioms, demonstrating his acceptance of axioms beyond ZFC. Additionally, Steel emphasises that his multiverse theory posits the existence of a "core universe," which he considers to be the preferred or real universe of sets. This idea is further analysed by Bagaria and Ternullo (2023). In this sense, both Woodin and Steel hold views that oppose Hamkins' multiverse perspective.

This paper is not the place to provide a full review of the debate between the universe view and the multiverse view. However, it is important to acknowledge that this debate is ongoing, with various types of arguments being presented. Among these, some arguments specifically refer to set-theoretic practice. In this paper, I contribute another perspective of this kind.

Section 2 reviews the details of Hamkins' multiverse view. Section 3 presents my study on current set-theoretic practices. It first motivates two research questions: (1) Does the multiverse view fit with set-theoretic practice? and (2) How probable is it that Hamkins' prediction that the community will not adopt axioms deciding the continuum hypothesis will hold? Then, it describes the sample of 28 set-theoretic practitioners that I interviewed and summarises relevant results of the study. Section 4 discusses the results. Because of a strong heterogeneity of set-theoretic practitioners' views on the nature of set theory, the multiverse view is found to fit well with pluralist practices but not with absolutist practices. The generalisation claim of the multiverse view fails because of this heterogeneity; only a reduced interpretation of the multiverse view might hold generally. Further, Hamkins' prediction is found probably true, but essentially because of the failure of generalisation, which also holds for absolutist viewpoints. In the concluding Section 5, I argue that Hamkins' multiverse view is best understood as a valuable perspective on pluralist set-theoretic practices and highlight two questions worth further exploration in light of the reflections in this paper.

2 Hamkins' Multiverse View

In this section, I describe the most important characteristics of Hamkins' multiverse view. His view is explicitly posited against the universe view defended by other set-theoretic practitioners like Woodin or Steel. Hamkins proposes that set-theoretic reality is not a unique set-theoretic universe, but rather a very broad and liberal multiverse that contains all kinds of set-theoretic universes (1.1). According to Hamkins, his account should be favoured over the universe view, because it is a coherent explanation of the experience of set-theoretic practitioners, who work with various set-theoretic universes (1.2). Based on his multiverse view, Hamkins predicts that a new axiom candidate, say A, solving the continuum hypothesis (CH) will not be accepted in the set-theoretic community, because it would be opposed to the set-theoretic practitioners' experience with the set-theoretic universes satisfying $\neg A$ (1.3).

2.1 Opposition to the Universe View

Hamkins posits his multiverse view against the universe view:

> I shall argue for a contrary position [to the universe view], the *multiverse view*, which holds that there are diverse distinct concepts of set, each instantiated in a corresponding set-theoretic universe, which exhibit diverse set-theoretic truths. Each such universe exists independently in the same Platonic sense that proponents of the universe view regard their universe to exist. (Hamkins 2012, p. 416f)

Instead of a unique, set-theoretic universe, in which every set-theoretic statement is either true or false, the multiverse is rather a collection of all set-theoretic universes. A given statement is true in some of them and false in others.

Since Hamkins claims that each set-theoretic universe in the multiverse exists in a Platonist sense, his view is sometimes related to Balaguer's "fullblooded platonism – which asserts that any mathematical object which possibly *could* exist actually *does* exist" (Balaguer 1995, p. 303, emphases original).[1] In Balaguer's full-blooded Platonism, consistency already yields existence. As a consequence, all set-theoretic models exist. Hamkins and Balaguer share this liberal conception, but their views should not be conflated, as their arguments in support of their positions differ significantly.[2] Before presenting his multiverse axioms, Hamkins specifies:

1 Antos et al. (2015) and Jonas (2020), for example, draw this connection.

2 Balaguer's main argument in favor of full-blooded Platonism is that it resolves the problem of epistemic access while preserving Platonism. Since every consistent mathematical structure exists, mathematicians discover mathematical reality by studying these structures; they do not need to rely on a special cognitive ability to access a Platonic realm. Thus, Balaguer's position is driven by the goal of addressing a theoretical issue in the philosophy of mathematics rather than by ensuring coherence with mathematical practice.

> The background idea of the multiverse, of course, is that there should be a large collection of universes, each a model of (some kind of) set theory. There seems to be no reason to restrict inclusion only to ZFC models, as we can include models of weaker theories ZF, ZF^{-}, KP, and so on, perhaps even down to second-order number theory, as this is set-theoretic in a sense. (Hamkins 2012, p. 436)

This idea is subsequently spelled out in terms of explicit multiverse axioms. One of his most controversial ones is the axiom called 'Well-foundedness Mirage'. It states that every set-theoretic universe in the multiverse is ill-founded from the perspective of another set-theoretic universe (Hamkins 2012, p. 439).

It is important to note that non-Platonist positions are compatible with both a universe view and a multiverse view. A realist position in the philosophy of set theory typically includes two key components: an existence claim and an objectivity claim. However, these are not always both considered necessary for realism. For instance, Jonas (2020) reduces set-theoretic realism to an existence claim alone. More commonly, though, a realist position is identified with the objectivity claim: that set-theoretic sentences are objectively either true or false. These distinctions are important in philosophical analyses of realism.

However, when examining mathematical practice, a methodological challenge arises. In most cases, it is not reliable to attribute a specific philosophical stance to a working mathematician unless there is clear evidence. This is because it presumes that the mathematician is familiar with the nuances of various philosophical positions, which may not always be the case given their different areas of focus. Of course, there are exceptions, such as mathematicians with significant philosophical expertise or those who explicitly argue for particular philosophical positions in their research. My approach is to take seriously the published claims of mathematicians, but to ascribe only broad philosophical convictions to them in general. This is intended to avoid the risk of attributing detailed philosophical positions without sufficient evidence.

These reflections highlight a crucial distinction in perspectives for this paper. I regard Hamkins' multiverse view as a *philosophical position* on the nature of sets. Since he has published his views with detailed argumentation, I take his claims at face value, engage with them seriously, and respond with my own arguments. However, another important focus of this paper will be the *philosophical convictions* of working mathematicians. Many mathematicians hold beliefs about the nature of sets and the meaning of their research practices. Since most of them do not publish philosophical papers, I do not interpret these convictions as formal philosophical positions intended as contributions to philosophical debate. Instead, I view them as personal convictions that reflect individual standpoints regarding their own research activities. My approach here is observational.

Returning to the question of Platonism, I can ascribe a Platonist view to Hamkins, as he has explicitly stated this. However, I cannot ascribe either Platonist or non-Platonist views to most working mathematicians due to a lack of sufficient evidence that would allow for a thorough consideration of the subtleties involved in these philosophical positions. That said, I would like to highlight in this section that both the universe view and the multiverse view posit the existence of a Platonist reality. But according to Hamkins, this reality is a very broad and liberal multiverse of set-theoretic universes. What reasons does he give to justify this position?

2.2 Explanation of the Set-Theoretic Practitioners' Experience

Hamkins' main argument in favour of the multiverse view is its ability to explain the every-day research experience of set-theoretic practitioners. According to him, set-theoretic practitioners have extensive experience with a wide variety of set-theoretic worlds, which they consider all to be "perfectly fine set-theoretically" (Hamkins 2015, p. 137). He describes his observation in detail:

> We know what it is like to live in the universe obtained by adding $\aleph_2$ many Cohen reals over L and what it is like in the $L(\mathbb{R})$ of that extension. We know what it is like in the universe obtained by forcing MA + ¬CH over L, or by forcing with Sacks forcing, or by iteratively adding a dominating real. The resulting universes are places we have been. We are deeply familiar with the universe obtained by forcing with the Laver preparation of a supercompact cardinal and with the model of PFA obtained by Baumgartner's similar construction. This is not to say that we know everything that there is to know about these universes – there will always be more to learn about them.[3] (Hamkins 2015, p. 139)

There are two points worth noting about his argument. First, Hamkins considers the experience of all set-theoretic practitioners to be similar and, therefore, generalises to the whole set-theoretic community. Second, he emphasises that his view is about what set-theoretic practitioners actually *do* (in his words, "their mathematical behavior") and not about what they claim to do: "regardless of what set-theorists might assert, their mathematical behavior demonstrates that they accept the existence of diverse set-theoretic worlds" (Hamkins 2015, p. 136). In other words, his

3 Every standard set theory book, for example (Jech 2006), explains the specific set-theoretic details mentioned in this quotation. Here are the specific references. See Chapter 14 on forcing in general, including the proof of the independence of the continuum hypothesis, and pp. 225–226 for more on Cohen reals. Chapter 16 treats iterated forcing and Martin's axiom. Sacks forcing is introduced in Chapter 15 on pp. 244–245, and the dominating number in Chapter 26 on pp. 532–533. For Laver preparation of a supercompact, see Theorem 31.21 on pp. 607–608. Baumgartner's construction is included in Baumgartner (1984).

multiverse thesis states that all set-theoretic practitioners, independently of what they claim to believe, actually behave as if a set-theoretic multiverse exists, because their every-day research mainly consists in working with different set-theoretic models. Hence, in particular, he claims that supporters of the universe view, despite their official goal of finding out the truth about the set-theoretic universe, are actually finding out truthes about the set-theoretic multiverse.

Owing to this argument, I think that Hamkins' multiverse view should be considered as a serious proposal for a theoretical framework that is meant to explain the every-day research activities of set-theoretic practitioners. The emphasis here is on the latter aspect. In the literature, scholars typically engage with Hamkins' ontological claims. While I briefly address these claims, my primary focus is on his arguments related to set-theoretic practice, which have yet to be thoroughly evaluated. Within this proposal intended to explain set-theoretic research activities, he makes a tangible prediction about the behavior of the set-theoretic community.

2.3 Hamkins' Prediction: General Rejection of New Axioms Deciding CH

Based on the close connection of the multiverse view to the activities of set-theoretic practitioners, Hamkins makes the following prediction. Given the possibility that some priniciple deciding CH is put forward, he predicts

> that set-theorists will object to the claim that the principle is a set-theoretic truth, and furthermore, their objections will arise from a wellspring of deep mathematical experience with the contrary hypothesis. (Hamkins 2015, p. 140)

According to Hamkins, no new axiom candidate deciding CH will ever be accepted by the set-theoretic community.

A curious observation is that the prediction is restricted to axiom candidates that decide CH. As a consequence, the prediction does not apply to new axioms such as the well-known large cardinal axioms and determinacy principles like projective determinacy. It is not immediately clear why Hamkins focuses on axiom candidates deciding CH, because his argument would generalise to all independent sentences. Set theorists have extensive experience working with large cardinal axioms, just as they do with forcing axioms. Set-theoretic practitioners have just as much experience with models of CH and its negation as with models of other independent sentences and their negations, e.g. combinatorial principles such as

$\diamondsuit$ (*Diamond*), the assumption $V = L$, Suslin's hypothesis, "All sets are Lebesgue-measurable" etc.[4]

A relevant distinction between different types of independent sentences of ZFC should be noted in this context. On one hand, there are new axioms proposed as candidates to extend ZFC. These include large cardinal axioms, forcing axioms, determinacy principles, and other philosophically motivated principles (e.g., reflection principles such as those discussed by Roberts (2017) or Welch (2017)). These statements are considered fundamental expressions about sets or as foundational starting points for further deductions. The continuum hypothesis (CH), however, is of a different nature. Set theorists do not view CH or its negation as potential axioms themselves, but rather as sentences that might be resolved through the addition of new axioms. The goal is not to assume CH or its negation, but to deduce its truth or falsity from more fundamental assumptions, if possible.

However, recognising this distinction does not fully explain Hamkins' focus on new axioms that could decide CH. Large cardinal axioms do not determine whether CH is true or false, but forcing axioms like PFA and MM do, as does an axiom expressing V = Ultimate-L. Why, then, does Hamkins differentiate between these types of axioms?

There might be an easy explanation of Hamkins' emphasis of new axioms deciding CH. The continuum hypothesis is the oldest and most famous independent sentence and it is also relatively well-known outside set theory. Therefore, an emphasis of CH is common in the literature in the philosophy of set theory. Discussions on the justification of new axiom candidates often unfold from the question of CH. Indicative contributions are Gödel's influential paper (Gödel 1947) and Woodin's expositions in (Woodin 2001a, 2001b). A similar focus on CH probably guarantees an easy embedding of Hamkins' contribution in the existing literature. In other words, he might proceed that way because everyone does.

This philosophical focus on CH is not mirrored in set-theoretic practice. There are numerous independent statements like Suslin's hypothesis or projective determinacy, many of which are at least as interesting to set-theoretic practitioners as CH. The next section provides informative data illustrating set-theoretic practice.

3 Set-Theoretic Practitioners: An Explorative Study

In a recent study of the set-theoretic community, I analysed the practice of set-theoretic practitioners from different research areas. Between 2017 and 2019, I

4 Find more details on these principles here: the $\diamondsuit$ (Diamond) principle (Jech 2006, p. 191), $V = L$ (Jech 2006, Ch. 13), Suslin's hypothesis (Jech 2006, pp. 274ff), and Solovay's model in which all subsets of the reals are Lebesgue-measurable (Jech 2006, p. 519ff).

conducted an exploratory interview study with 28 set-theoretic practitioners about their research and their views on the set-theoretic independence phenomenon. This sample size allows for an analysis of the set-theoretic community that goes beyond single case studies. A second advantage of this method is the retrieval of information on set-theoretic practices that is difficult to access. For instance, set-theoretic practitioners usually exclude any philosophical views from their research publications, but in anonymised interviews, most of them spoke openly.

The study explored various aspects of the set-theoretic independence problem, with a particular focus on the naturalness judgements of set-theoretic practitioners. The specific methodology, motivation, results, and discussion of the study are detailed in my PhD thesis. For this article, I present and discuss the parts of the study's results that are relevant to the claims about set-theoretic practice found in Hamkins' multiverse view.

Since Hamkins' multiverse view, from here on abbreviated as 'MV', is so closely related to set-theoretic practice, I shall confront his statements about set-theoretic practice with the results of my study. To be precise, by evaluating the claims of MV regarding set-theoretic practice, I am focusing on only a part of Hamkins' broader and more complex position. In the philosophical debate about the nature of sets, including views such as the universe view or pluralism, this aspect of Hamkins' position has received little attention. The main reason is the lack of rigorous evidence needed to evaluate claims about set-theoretic practice. Until now, the literature has offered only limited analyses that shed light on these practices (notable exceptions include Maddy 1988a, 1988b, 1990; Rittberg 2015, 2016). My study is the first to provide data from a larger portion of the set-theoretic community, enabling such an evaluation.

The study provides informative data with regard to some of Hamkins' claims. The goal of the paper is to evaluate these claims in the light of my findings. I address the following two main questions:
Q-Fit: Does MV fit with set-theoretic practice?
Q-Prediction: How probable is it that Hamkins' prediction will hold?

With regard to **Q-Fit**, it is important to note that Hamkins' main focus is what set-theoretic practitioners do and not what they claim to do. I take this into account when I discuss the results of the study with regard to the first research question by differentiating between the activities of set-theoretic practitioners on the one hand and their beliefs on the other. One might assume that it is obvious MV does not accurately describe the practice of set theorists who endorse the universe view (such as Woodin, for instance). However, I believe this interpretation oversimplifies the issue. Hamkins, being a member of the set-theoretic community, is well aware that some members endorse the universe view. Yet, he maintains the multiverse view,

claiming it is supported by the actual practices of set theorists, including those who hold the universe view. As I explained earlier, Hamkins argues that while proponents of the universe view aim to discover the truth about the set-theoretic universe, they are, in fact, uncovering truths about the set-theoretic multiverse. In other words, **Q-Fit** questions whether Hamkins' assertion that "regardless of what set-theorists might assert, their mathematical behavior demonstrates that they accept the existence of diverse set-theoretic worlds" (Hamkins 2015, p. 136) is an accurate description of set-theoretic practice and whether this claim provides a compelling argument for MV.

Another important question in this context is whether an absolutist or pluralist view is more prevalent within the set-theoretic community. For instance, if the universe view is supported by only a few set theorists while the majority endorse a pluralist view, this would lend support to Hamkins' multiverse view.

Hamkins' prediction addressed by **Q-Prediction** is about what set-theoretic practitioners will accept; about their future attitudes. Since the interview study provides a snapshot of current attitudes of set-theoretic practitioners, it allows to determine which outcome is more probable in light of the present findings.

The most crucial interview question in relation to these research questions posed to the interviewees was: "Some set theorists are looking for new axioms to extend ZFC, others think that such aims are pointless. Do you have an opinion on that?" I asked this question at the end of an interview after detailed conversation about the specific research practices of the participant. It usually triggered extensive responses.

I shall start with characterising the sample of 28 interviewees in Section 3.1. Section 3.2 presents relevant results.

3.1 Sample Set

This subsection gives some quantitative evaluations of the sample of 28 professional set theorists to show that the sample possibly represents more than 8 % of the community, and that the sample is diverse in terms of research area, age, gender, location of home institution, and view on the possibility of extending ZFC.[5] First, regarding the size of the current set-theoretic community, there are no conclusive data on the total number of professional set theorists. But a preliminary hint at the

5 Paragraphs describing the methodology and sample of my study are similar across several of my publications, because the amount of data collected exceeds exposition in a single publication. This currently applies to the chapter "The hidden use of new axioms" (forthcoming in: Antos, C.; Barton, N. ; Venturi, G. (eds.) *The Palgrave Companion to The Philosophy of Set Theory*, Palgrave Macmillan) and to my dissertation (Kant 2025).

Table 1: Distribution of research areas.

Combinatorics	13
Descriptive set theory	11
Ergodic theory	4
Inner model theory	8
Forcing axioms	8
Large cardinals and forcing	8
Forcing	8
Set-theoretic and general topology	5
Cardinal characteristics	4
Determinacy and large cardinals	3
Recursion theory	3
Class forcing	2
Set theory of the reals (forcing)	2
Small research areas (very specific)	4

community's size is given by the 'list of homepages of set theorists' managed by the set theorist Jean A. Larson, which lists 323 set theorists. I spoke to 8.6 % of them (by adding one person to the list), and I invited 45 (13.6 %) of them to participate in the study (by adding two people to the list).[6] If one extrapolates that per 43 set theorists, two are not listed, one obtains a total number of 338 set theorists, and 28 out of 338 is still 8.3 %. As said, this is a preliminary evaluation owing to the lack of conclusive data.

Second, regarding the specific research areas of the interviewees, all main research areas in set theory are represented (see Table 1). Each interviewee indicated between 1 and 5 research areas, including some additional smaller ones.

The interviewees are all professional set theorists with a (past) permanent position as a professor of mathematics with a research focus on set theory. One shortcoming of the study in this respect is that the views of the younger generation are not represented. However, the study still represents different generations. Table 2 indicates the years in which the interview participants obtained their PhDs. These years were taken from the *Mathematics Genealogy Project*.[7]

Table 2: Years of obtaining the PhD.

before 1980	1980–1989	1990–1999	after 1999
6	4	9	9

6 See the web page https://people.clas.ufl.edu/jal/set-theory-homepages/\#I, accessed 12 August, 2020.

7 See https://mathgenealogy.org, accessed 11 November, 2020.

With regard to gender, 4 of the interviewees (14 %) are women according to my evaluation (the interviewees were not explicitly asked about their gender). The sample does not seem to be biased in this respect, because the majority of set theorists are male.

Considering the location of their home institution, 15 of the interviewees have a permanent position at a European university, 11 at a university in the USA, and 2 at a university outside Europe and the USA.[8] There are groups outside Europe and the USA that were not included in the interviews, but there is no obvious corresponding bias, because most set-theoretic research seems to be concentrated in these Western regions.

Regarding their views on independence, 11 of the interviewees have an absolutist view and another 11 a pluralist view. Hence, the sample is not biased in this respect. The remaining 6 interviews do not contain a coding in one of these two categories.

These data show that the sample is broad, especially according to the range of different research areas, but also that it is limited in certain respects, for example, it captures mainly the European and US-American research context. As such, it is not biased in some obviously misleading way. All in all, these data provide evidence that a broad cross-section of views are represented in the study.

3.2 Results

The relevant results of the study with regard to **Q-Fit** and **Q-Prediction** are the views of set-theoretic practitioners with an absolutist view (11 out of 28 in the sample, Section 3.2.1) and the views of set-theoretic practitioners with a pluralist view (11 out of 28 in the sample, Section 3.2.2). In each section, a substantial part of the sample has its say.

To guarantee the anonymity of the participants of the study, the source interviews for interview quotations are not indicated. This measure is necessary, because some readers of this article might identify interviewees even with a small amount of information. If two quotes in one paragraph are from the same interview, I shall point to this fact. Otherwise, the quotes are from different interviews.

3.2.1 Absolutist Practitioners

A substantial part of the sample (11 out of 28) demonstrated various kinds of absolutist convictions. This section presents their views. To summarise, many of them believe that there is a universe of sets. Their overall research goal is to find correct

8 Specific locations are not indicated to maintain anonymity.

new axioms, because ZFC is, according to them, not sufficient to settle important problems. Historically, some of them observe that a development towards agreement on the axioms takes place, and many already accept more axioms than ZFC. They explain how their philosophical view is compatible with research practices including forcing and different set-theoretic models. With regard to Hamkins' multiverse view, many of these interviewees explicitly reject it.

In more detail, many interviewees with absolutist views expressed a belief in the universe of sets: "Personally, I imagine it like this: there's V, and there's nothing outside." Other interviewees said things like "we are in this universe of V", or "the real universe satisfies choice". Often, the interviewees relate the idea of the existence of a unique set-theoretic universe V to other ideas such as the goals of set theory.

Absolutist practitioners share the same goal: They want to find correct new axioms. In the following quotation, an interviewee embeds this goal in their wider world view:

> The objective of set theory is to understand the set-theoretic universe, …because this is the universe where all mathematics takes place. And we need to understand what this is because we want to answer mathematical questions. And we know that many of them cannot be answered from the axioms you wrote down. So, it means the axioms you wrote down are not sufficient. You have to write down more axioms that will tell us more. But we have to agree on what features this universe has.

Such a position sometimes correlates with a negative attitude regarding consistency results in terms of their value for set-theoretic research, as the following quotation by another interviewee demonstrates: "For what concern[s] consistency results, I find it mostly a game now because the most interesting consistency results have been proved." Two interviewees use such a game metaphor with a negative connotation. They find contemporary consistency results, especially separation results,[9] insignificant, because these results do not help to achieve their goal.

With regard to the historical development, several interviewees refer to a strategy to achieve their goal, also described by Woodin (2001a), in which set theorists first set the axioms for the theory of $H(\omega_1)$, then for the theory of $H(\omega_2)$, and so on.[10]

They consider themselves partially successful in achieving their goal, because for the theory of $H(\omega_1)$, they are convinced that they found the right axioms:

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9 In a separation result, two close set-theoretic notions are separated, such as two cardinal characteristics or two large cardinal axioms. The difficulty consists in the observation that in most models the two notions are equal, hence, the forcing construction needed for the separations is usually combinatorically very complicated.

10 For a cardinal κ, the set $H(\kappa)$ contains all sets hereditarily of cardinality less than κ. We write: $H(\kappa) = \{ x \mid \mathrm{trcl}(x) < \kappa \}$. For example, $H(\omega_1)$ is the set of all hereditarily countable sets.

> [A]mong those that are, let's say, Platonist, or have a point of view which is not too dissimilar to Platonism, I think, there [are] not [many] questions about the truth of large cardinal axiom[s] or about the fact that we have a good theory of [$H(\omega_1)$] which is given by AD and whatever.[11] Then, we may have diverging opinion on what happens afterwards.

For the theory of $H(\omega_2)$, there is disagreement among set-theoretic practitioners with absolutist views on whether a V = Ultimate-L axiom or a forcing axiom like MM^{++} is more promising, resp. true.

Regarding an essential observation in MV, absolutist practitioners also work with different set-theoretic models and forcing, as emphasised in this quote:

> Forcing has found its way into all parts of set theory. Even inner model theorists when they're building models. Forcing comes into this as well. I'm not saying you can insulate inner model theory from forcing. These things now fit in together, influence each other. As it does with these forcing axioms. Everybody needs forcing. Everybody needs to use forcing in some way.

The interviewee directly added: "But the question whether the forcing models themselves tell us something about V, that's another philosophical question altogether." Some of them explain why working with set-theoretic models does not counteract their belief in the existence of a unique set-theoretic universe V:

> My point of view is that there is a universe of sets, and you can either talk about the theory of this universe of sets or else, inside this universe, you have many other objects which are either sets or classes…. Some classes are models of set theory, some sets are models of set theory and then, you study this structure.

With regard to MV, a majority of interviewees with an absolutist view (7 out of 11) clearly distance themselves from it: "I don't like [the] multiverse view. I don't think it describes really what we do." They reveal as reasons against it that, according to them, the background theory of MV is not clear, it has no formal language, it is incoherent, and the study of models is not the purpose of set theory.[12] The presence of these nuanced objections against MV shows that set-theoretic practitioners discuss MV and that especially absolutist practitioners engage with it, even though finally discarding it.

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11 The interviewee referred to the axiom $\mathrm{AD}^{L(\mathbb{R})}\mathrm{AD}^{L(\mathbb{R})}$. Maddy (2011) gives an overview of the reasons in favour of this axiom (see II.2.(iv)).

12 It is worth noting that there have been attempts to formalise Hamkins' multiverse structure. Gitman and Hamkins (2010) present a model demonstrating the coherence of the axioms suggested by Hamkins, while Gorbow and Leigh (2022) provide a formal background theory compatible with his multiverse framework. I am grateful to one of the reviewers for pointing me to these important references.

3.2.2 Pluralist Practitioners

In contrast to interviewees with an absolutist view, those with a pluralist view (11 out of 28) did only mention the multiverse rarely. Interviewees did not explicitly endorse MV. They used other ways to describe their world views: Interviewees with a pluralist view believe that the set-theoretic community will not adopt further axioms beyond ZFC and they often conclusively accept ZFC with its inherent independence phenomenon. Two of them offer an algebraic view on their work with set-theoretic models. Furthermore, two other interviewees describe how they enjoy working with different set-theoretic models.

Interviewees with a pluralist view reject the idea that ZFC should be extended by further new axioms, which would mean to discard other axioms which are incompatible with the chosen ones: "If I had a guess, I don't really expect any axioms beyond ZFC to be adopted and have this status in the mathematical community that the axioms of ZFC do," said one interviewee. Another one noted:

> It's very, very difficult to make a point that you discovered something that is somehow eternally true, and that everybody must accept. I think that would be a complete failure because the people are simply not going to accept it.

They believe that some of the new axioms may be more popular, but not one is better than all others, because there are always models, in which the candidate axiom is false.

They conclusively accept the theory ZFC and its inherent independence phenomenon: "I'm perfectly happy with ZFC and a pluralism of universes." One interviewee also said that independence is a fact of life.

Interviewees with a pluralist view clearly like to work with all different kinds of set-theoretic axioms and models: "I don't really feel the need to choose between axiom systems either. They're are all out there and you can study all of them, and I think that's all worthy of study." Two interviewees mention the algebraic view on set-theoretic models: According to them, set theorists can study models of set theory like algebraists study various groups or fields. Like no algebraist would be surprised that something is true in one field and something else is true in another field, set theorists shouldn't be surprised either that something is true in one model and false in another. In their world view, there are all these set-theoretic models and one can go from one to the other using forcing. They also think that a new adopted axiom would restrict set theory to just some of these models. It is not clear to them what kind of mathematical benefit this would bring.

Moreover, in two interviews, it becomes very clear how much the interviewees enjoy the work with different models and how they play on the flexibility that is given

by the presence of different set-theoretic models. According to one interviewee, adopting a new axiom would take away a lot of what is enjoyable for them about doing set theory:

> That's something that I find fascinating in set theory, that your world is not fixed, that you can have this manipulation of things, and not everything is determined if it's true or false. You can play a little bit with further truths, or you can have some room to manoeuvre, then it becomes less boring.

Note that in this quotation the game metaphor is used with a positive connotation.

4 Discussion

Based on the results of the study, Sections 4.1 and 4.2 address the research questions **Q-Fit** and **Q-Prediction**, and propose some conclusions.

4.1 Does the Multiverse View Fit with Set-Theoretic Practice?

This section discusses the first research question **Q-Fit**. The confrontation of MV with data on the set-theoretic community results in the following hypotheses: MV can only explain absolutist research practices by supposing a very strong conflict between activities and beliefs (3.1.1). At the same time, it can explain pluralist research practices without supposing such conflict (3.1.2). Since the data demonstrate a heterogeneity of the set-theoretic community with regard to research practices, any generalised claim on such research practices is false. The evidence rather suggests: A part of the community searches for new axioms beyond ZFC and another part conclusively accepts ZFC and rejects a search for new axioms (3.1.3).

4.1.1 Absolutist Practices: MV Supposes a Strong Conflict Between Activities and Beliefs

Hamkins' paraphrase of the universe view (Hamkins 2012, p. 416) corresponds well to the absolutist convictions of some of the interviewees. Since his multiverse view is posited explicitly against such views, how then is it possible that MV might fit with absolutist research practices? Recall that the term 'practices' includes activities *and* beliefs. Hamkins claims that the practitioners' *activities* demonstrates MV, regardless of the practitioners' *beliefs*. Hence, a conflict between activities and beliefs does not counteract his view. My aim is to take his claims seriously, interpret them charitably, and explore how they might hold true. I review the points for and against

a fit, ultimately demonstrating that if MV is assumed to be true, the conflict must be strong.

On the side in favour of a fit between MV and absolutist practices is the observation that set-theoretic practitioners with absolutist views indeed work with different set-theoretic models and with forcing. This is a fundamental fact about contemporary set-theoretic practice in general, also emphasised by Antos (2022), who argues that both sets and models are fundamental entities in current set-theoretic practice.

However, the study suggests that this is the only way in which MV can be understood as being generally correct: Stating that set-theoretic practitioners work with different models and with forcing. Further four points rather support incoherence between MV and absolutist practices.

On MV, the construction of set-theoretic models by forcing is a fundamental research practice. However, some absolutist practitioners consider the mere construction of such models to be insignificant. They do not value certain consistency results such as separation results, because such mathematical results do not support achieving their goal of finding new correct axioms. Recall the negatively connotated game metaphor.

Second, a typical absolutist view is the belief in a unique set-theoretic universe V. Such a universe does not exist on MV. Hence, this belief is false on MV. Closely related to this belief is the goal of searching for new correct axioms, which neither makes sense on MV. Hence, assuming that MV is true, this research goal cannot be achieved.

As a further point of incoherence, the data suggest that absolutist practitioners accept large cardinal axioms and projective determinacy. Some interviewees also provide a strategy and an outlook how they will adopt further new axioms. However, on MV, one cannot accept axioms beyond ZFC and reject other incompatible axioms.

A fourth important point is that absolutist practitioners do not seem to directly discard MV but only after some engagement with its ideas. The data show that many of them engaged with MV, apparently much more than set-theoretic practitioners with pluralist views. Hence, the rejection of MV is a conscious one.

All this does not refute MV, because MV can hold for practitioners who explicitly do not believe in it. However, MV supposes a strong conflict between activities and beliefs for an important part of the set-theoretic community – the absolutist practitioners. Assuming that MV is true implies that absolutist practitioners have false beliefs, a non-sensical and non-achievable research goal, that some of their acceptance attitudes are empty, and that their conscious rejection of MV is misled. In other words, MV implies that absolutist practitioners mostly fail in having true beliefs about their research activities.

If only a small minority of set theorists endorsed absolutist views, this might not pose a problem for MV. However, although the study is not fully representative, a

significant portion of the sample does endorse absolutist views. This suggests that absolutist practitioners are a non-negligible part of the set-theoretic community. Since MV is proposed as a view intended to capture the activities of *all* set theorists, this finding provides evidence against MV.

There is a subtle but crucial distinction in my argument: my conclusion is not merely that absolutist practitioners disagree with MV. Rather, I have identified specific aspects in which the research practices of absolutist practitioners do not align with the practices that would be encouraged by MV. This is not just a propositional objection; it is an observation that several research practices of absolutist practitioners contradict the claim that the true nature of sets is a large multiverse.

This strong conflict is problematic for MV, because the main argument in favour of MV is its potential to explain the research activities of set-theoretic practitioners. But it can only do so by being revisionary with regard to many fundamental beliefs of absolutist practitioners.

4.1.2 Pluralist Practices: Good Alignment

Although pluralist practitioners do not explicitly endorse MV (with Hamkins himself being a notable exception), it aligns well with the pluralist research practices. The data include a minor point of incoherence and many points in favour of a fit between MV and pluralist practices. Since most of these observations are rather uncontroversial, I keep them brief.

A point of incoherence between MV and pluralist practices is that, according to the data, pluralist practitioners are less liberal towards fragments of ZFC than MV is. Although interviewees in rare occasions mentioned such theories, they clearly prefer ZF and ZFC models over models of, for instance, Kripke-Platek set theory or other weak fragments. They would not necessarily adopt principles like the Well-foudnedness Mirage axiom.

All other observations point out a coherence between MV and pluralist practices. Pluralist practitioners believe that the set-theoretic community will not adopt any axioms beyond ZFC, just as MV states. Some of them emphasised that all models are interesting to look at, that models in set theory can be treated like groups or fields in algebra, and that they deeply enjoy the flexibility that is given by the presence of numerous different models.

MV aligns well with these pluralist research practices. Assuming that MV is true, pluralist practitioners have true beliefs on their research activities; in other words, there is a coherence between their activities and their beliefs. The observation that they do not explicitly identify as multiversists does not counteract this argument. I consider MV to be a plausible position on the nature of sets that could underlie

pluralist research practices; however, Hamkins' very liberal conception is not fully reflected in these practices.

4.1.3 Generalisation Fails

MV is proposed as a general thesis for the whole set-theoretic community. It claims to capture the activities of *all* set-theoretic practitioners. If one reduces MV to the claim that set-theoretic practitioners work with different set-theoretic models and with forcing, then such a general claim coincides with the evidence provided by the study.

But the essence of MV is much more elaborate: It assumes the existence of a liberal multiverse encompassing all kinds of set-theoretic models. In the previous section, I showed that MV aligns well with pluralist practices but not with absolutist practices. Regarding the latter, it is revisionary with regard to many important beliefs of absolutist practitioners. This is problematic, because MV claims to be based on the research activities of set-theoretic practitioners, to be very close to the practice.

The study reveals a heterogeneity of research practices within the set-theoretic community. The set-theoretic community does not exclusively pursue either absolutist or pluralist research goals. Instead, there is a broader variety, with some practitioners leaning more towards the absolutist side, others towards the pluralist side, and still others pursuing more individual research goals. With respect to axiom adoption, there are pluralist practitioners who will probably stay with ZFC and not adopt any axioms beyond, and there are absolutist practitioners who will do the contrary and pursue their strategies to adopt further axioms. For example, the different connotations of the game metaphor (playing with models, constructing forcing models) illustrate a heterogeneity of attitudes towards specific mathematical results. Absolutist practitioners use the game metaphor with a negative connotation while pluralist practitioners use it with a positive connotation. MV wipes off such differences of attitudes and gives priority to one of them. In this sense, it is not an appropriate general characterisation of set-theoretic research practices.

In conclusion, the claimed generality fails, because it neglects the heterogeneity in the set-theoretic community with regard to research practices. If the goal is to characterise general set-theoretic research practices, then more modest and non-revisionary accounts would be more appropriate, such as the one of Antos (2022).

This said, further results of the study suggest that the highlighted heterogeneity only applies to the more philosophical beliefs of set-theoretic practitioners but not to value judgements on mathematics or even genuine mathematical beliefs [see Chapter 8 in my dissertation (Kant 2025)].

Let me present a curious observation in this context while remaining within the scope of this article. Hamkins takes a clear stance on value judgements regarding

different set-theoretic models, stating that "we may prefer some of the universes in the multiverse to others, and there is no obligation to consider them all as somehow equal" (Hamkins 2012, p. 417). According to him, set theorists can legitimately find certain set-theoretic universes more natural, interesting, deep, exciting, illuminating, or beautiful than others. These are all instances of value judgements, which play an essential role in absolutist reasoning for new axioms. Maddy (2011) explains that new axioms are justified by extrinsic reasons – namely, by presenting compelling desirability judgements in their favour. My findings indeed suggest that absolutist practitioners rely on extrinsic reasoning when justifying new axioms.

This leads to the following situation: MV allows set-theoretic practitioners to make desirability judgements about different set-theoretic universes, thereby providing a foundational basis for the extrinsic justification of new axioms. But is this sufficient to claim that MV establishes a framework for absolutist reasoning? Some scholars take this approach. Jonas (2020), for instance, examines disagreements among absolutist practitioners – some advocating for forcing axioms, while others support V = Ultimate-L axioms. She concludes that Hamkins' multiverse framework is well-suited to accommodate these disagreements.[13] However, MV cannot accommodate the absolutist belief that one of these many universes is, in fact, the uniquely correct one.

Therefore, in my view, MV is not sufficient to establish a framework for absolutist reasoning. While pluralist, absolutist, and other practitioners can all make desirability judgements, only absolutist practitioners take the further step – after gathering sufficient extrinsic reasons – of concluding that a particular axiom is ultimately acceptable.

To be suffiently precise, my findings also suggest, in alignment with Hamkins' claims, that absolutist and pluralist practitioners may not be as different as they seem when it comes to their value judgements about different set-theoretic models. Indeed, I argue that the role of value judgements in the discourse of the set-theoretic community is crucial for mathematical progress. This broader issue, however, will be explored in greater detail elsewhere.[14]

4.2 Does Hamkins' Prediction Hold?

Hamkins predicts that the set-theoretic community will not adopt axioms deciding CH. To address the second research question **Q-Prediction**, I shall discuss two issues.

13 She uses this observation to argue that, in mathematics, one can maintain a realist position while still accommodating mathematical disagreement.

14 For a start, see (Kant 2025, Chapters 8 and 9).

First, I find that, according to the data, the prediction probably holds, but not for the reasons that MV indicates. Second, the focus on axiom candidates deciding CH is not necessary and is perhaps only included, because these axioms are still an open issue for absolutist practitioners.

In light of the findings, the following is suggested: It is unlikely that the set-theoretic community will adopt axiom candidates deciding CH, such as forcing axioms or a V = Ultimate-L axiom, because part of the community does not adopt *any* axioms beyond ZFC. Pluralist practitioners are not only opposed to adopting new axioms, they also provide reasons for their stance. Therefore, I expect their attitudes to be based on deep convictions that will persist.

Hamkins claims that set theorists will *not* adopt any axioms beyond ZFC that can decide CH, because of their deep mathematical experience with models in which the contrary holds. However, my findings reveal a more nuanced picture: some set theorists have adopted large cardinal axioms and projective determinacy (PD) and are likely to adopt further axioms, including some of those that can decide CH. Therefore, Hamkins' claim that set theorists will not adopt additional axioms deciding CH appears to be incorrect according to my findings. Nevertheless, since pluralist practitioners explicitly state they will refrain from adopting new axioms, Hamkins' broader claim that the set-theoretic community *as a whole* will not adopt further axioms is likely accurate.

Considering this reasoning in detail, the question is raised (again) why Hamkins restricts his prediction to axiom candidates deciding CH and does not generalise to all axiom candidates beyond ZFC. After all, set-theoretic practitioners with pluralist views do not make any difference between them when it comes to the question of their acceptability. According to the results of the study, they will not accept large cardinal axioms and neither will they accept forcing axioms.

Based on the research conducted, I conjecture that the difference has to do with the attitudes not of pluralist practitioners but of absolutist practitioners. For absolutist practitioners, there is indeed a crucial difference between, for instance, large cardinal axioms which do not decide CH and further new axioms that do decide CH. Absolutist practitioners already accept large cardinal axioms, but they are mostly not settled on any further axioms. Some of them favour forcing axioms, others favour an axiom stating that V = Ultimate-L. In particular, absolutist practitioners consider the continuum problem to be an open issue. From the perspective of absolutist practitioners, the claim that new axioms deciding CH will not be adopted cannot be directly refuted, because, although they might have good candidates, the issue is not settled. If, however, the claim was that any axiom beyond ZFC will not be adopted, absolutist practitioners would simply consider this claim to be false, because they themselves already accept large cardinal axioms as extending ZFC.

Although Hamkins excludes new axioms that do not decide CH, such as large cardinal axioms, from his prediction – potentially for the reason just mentioned – he includes new axioms that decide CH, such as forcing axioms. However, my reasoning applies to all new axioms beyond ZFC: the set-theoretic community *as a whole* will not adopt axiom candidates beyond ZFC simply because part of the community – namely, the pluralist practitioners – is unlikely to do so.

5 Conclusions

In this paper, I analysed Hamkins' practice-based argument in support of his multiverse view. Hamkins contends that the research practices of set theorists reveal the existence of multiple set-theoretic universes, regardless of what set theorists explicitly claim. For readers already convinced that the multiverse view does not accurately reflect set-theoretic practices, this paper may not offer much new insight. However, since the literature has not thoroughly examined this aspect of Hamkins' view, my aim was to take his claims seriously, interpret them charitably, and explore how they might hold true. To do so, I conducted a detailed investigation of contemporary research practices, drawing on an extensive study, and compared these findings with Hamkins' assertions about set-theoretic practices.

Reflecting on the initial question of whether MV fits with set-theoretic practice, the findings in this paper indicate that it is worth first taking a step back. There is only a small possibility to maintain MV in general, as it comes with a revisionary account of absolutist research practices. It supposes a strong conflict between activities and beliefs for absolutist practitioners. In this paper, I discussed the implications of this insight. If one wishes to weaken this conflict, then one must reduce MV to the following claim: Set-theoretic practitioners work with different set-theoretic models and they use forcing. However, this interpretation does not do justice to MV; it does not imply that set-theoretic universes have to exist in a Platonist sense, nor that models of various fragments of ZFC have the same status than models of ZFC or that there is no unique set-theoretic universe V. A more modest conception of set-theoretic practices like the one presented by Antos (2022) would be sufficient and can, based on the here presented findings, indeed claim to hold generally for the set-theoretic community *as a whole*.

With regard to Hamkins' prediction that set-theoretic practitioners will not adopt new axioms deciding CH, the study supports that his prediction probably holds, but not for the reasons he indicates. Rather because part of the community will reject such axioms, while another part of the community will probably adopt axioms deciding CH at some point.

After having confronted MV with the results of my study of current set-theoretic practices, I conclude that MV does not provide a description of set-theoretic practices in general, because it does not suitably characterise the practices of absolutist practitioners. Instead, I consider MV to be a plausible philosophical position that might underlie pluralist practices. More generally, since the study revealed a heterogeneity of the set-theorists' research practices, neither a pluralist nor an absolutist viewpoint, such as the one advocated by Maddy, is correct for the whole set-theoretic community. Since this special issue is devoted to the multiverse view, I focused on this position. However, a similar argument could likely be made against Maddy's practice-based universe view, though her relationship to set-theoretic practice differs from that of Hamkins. Therefore, my argument against MV would not translate directly one-to-one.

Let me address an important objection: that the study does not contribute anything to the argument. My response is that it does. While a few set-theoretic practitioners in the research literature defend philosophical views, a key result of the study relevant to my argument is that both absolutist and pluralist research practices are pursued by significant parts of the set-theoretic community. Therefore, from this thorough investigation of current set-theoretic practice, one cannot infer a clear priority for either side. The study also uncovered details about set-theoretic research practices that are crucial for evaluating the potential fit between MV and set-theoretic practice, such as the opposing connotations of the game metaphor: some practitioners enjoy playing with models, while others find this pointless. Also, if we only consult the research literature, the pluralist position might appear closely aligned with Hamkins' multiverse view. However, the study adds much more nuance to the pluralist convictions of practitioners and highlights how they differ from Hamkins' multiverse view (MV endorses a very liberal conception encompassing all types of models, which is not entirely reflected in pluralist attitudes toward set-theoretic models). These insights into set-theoretic practice were necessary for a rigorous analysis of Hamkins' practice-based claims.

Another concern regarding my conclusions might be that they avoid addressing which position is correct. It is true that my conclusion is a negative one, demonstrating that if a philosophical account of set theory seeks to closely relate to set-theoretic practice, it must acknowledge the presence of absolutist, pluralist, and other practitioners within the set-theoretic community. Thus, a general account that prioritises one of these groups does not accurately reflect actual set-theoretic practice. Specifically, regarding Hamkins' multiverse view, I conclude that it is not as well grounded in set-theoretic practices as it claims to be, which is a negative result.

What do we learn from this? Highlighting the heterogeneity of set-theoretic research practices underscores the complexity of the problem. It shows that experts on set-theoretic independence phenomena have varying interpretations. Therefore,

in addressing the debate between the universe and multiverse views, we learn, first, that multiple positions are plausible. Second, if we aim to choose among these positions, we must refine our analytical methods to assess whether a given position is justified. This brings us to further open questions.

– What kind of evidence would be effective in favour or against pluralist or absolutist viewpoints?
– What is the exact relationship between set-theoretic practitioners' research activities and their philosophical beliefs on the nature of set theory?

The first question is methodological. This paper focused in detail on practice-based arguments. However, the debate encompasses a much wider variety of argument types. I suspect that, in some cases, arguments of different kinds do not engage directly with one another. For example, whether arguments based on set-theoretic practice are decisive in the debate between the universe view and the multiverse view seems to depend on the individual. This carries the risk of presenting arguments that address issues unrelated to those raised by an opponent. Maddy provides a proposal for a philosophical methodology addressing such questions, but although set-theoretic practice is a key ingredient in her methodology, her conclusion gives preference to absolutist viewpoints. Pluralist practices are not reflected in her account. Still, her proposal shows what is to be expected from an answer to the methodological question: a proposed method on how to answer questions on set-theoretic nature and a justification why this method is better than others.

Regarding the second question, a few attempts were made. There seems, however, not to be a simple answer. On the one hand, research activities are crucially informed by philosophical beliefs. Woodin's and Hamkins' research programmes are exemplary for such a relationship, as Rittberg (2020) explains in detail. But, regarding the inverse direction, mathematical theorems do not seem to clearly speak in favour of one or the other world view. Although mathematicians sometimes use mathematical theorems to support their world views, such arguments do not seem to be convincing for all set-theoretic practitioners (this also relates to the first question). A very subtle and complex connection seems to exist between these philosophical views and mathematical research activities that would be worth investigating further.

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