Human Capital and Optimal Policy under Risk and Credit Constraints

Dissertation

zur Erlangung des akademischen Grades
des Doktors der Wirtschaftswissenschaften (Dr. rer. pol.)
an der Universität Konstanz
am Fachbereich Wirtschaftswissenschaften

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Tag der mündlichen Prüfung: 14.02.2011

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Contents

Introduction and Summary

This dissertation consists of five self-contained research papers on optimal tax and education policy with endogenous human capital investment, which have been written during my doctoral studies at the University of Konstanz. Becker (1964) first used the concept of human capital investment to describe all activities to increase skills and resources in people. The internal rate of return on an additional year of schooling is empirically estimated to be somewhere between 5 and 15% (see e.g. Hartog and Oosterbeek, 2007). At the same time we observe large scaled governmental involvement in education in most countries, such as public schooling and subsidies to educational costs. Therefore, we would wonder why governments subsidize education to such an large extent, notwithstanding the high private return to schooling. This thesis deals with two of the arguments which have been brought forward, namely the missing (or imperfect) private insurance against risks in human capital investment and borrowing constraints for investment in human capital. Both market imperfections distort educational decisions, whereby governmental interventions are called for. Chapters 1, 2 and 3 are devoted to analyze how tax and education policy should be optimally designed when private insurance markets for idiosyncratic income risks are absent. Chapters 4 and 5 discuss the optimal tax policy in order to address the problem of borrowing constraint.

Each individual faces a substantial uncertainty in future labor income. There is uncertainty for example on successful graduation from schools and universities, on success in labor markets or on health conditions during education or working. How the level of educational investment affects the extent of income risks, is empirically not clear. On one hand, more education can provide an insurance, because better educated individuals are less likely to become unemployed. On the other hand, within-group wage differentials are higher for more educated workers partly because of higher specification of human capital. Consequently, income risks can affect educational investment in two different ways, as long as no perfect insurance against income risks is available. If education exaggerates income risks, individuals invest less in education in order to reduce their exposure to risks. If education provides an insurance against income risks, private educational investment is from an aggregate point of view too high.

The effect of binding credit constraints on educational investment is, however, unambiguous. Since constrained individuals cannot borrow the sufficient amount of money to finance education, their educational investments are suboptimally low. With binding credit constraints family income would be an important determinant of educational investment. Evidence for causal effect of family income on children's education is found in empirical works. Acemoglu and Pischke (2001) show that an increase of 10% in family income is associated with a 1.4 % increase in the possibility of attending a 4-year college. After comparing the causal effect of family income found in the NLSY97 data with that in the NLSY79 data, Belley and Lochner (2007) conclude that the causal effect of family income has become almost twice as large as 20 years ago. They argue that the increasing income inequality and the increasing intuition fees are responsible for the increasing importance of family income.

The absence of private insurance against income risks and the existence of borrowing constraints can be explained by human capital characteristics: i) the return to human capital investment, namely the labor income, is determined by educational and labor effort of individuals as well as by their innate ability, which are however not observable to private insurers or to creditors. This information asymmetry gives rise to moral hazard and adverse selection problems, which force insurers or creditors to limit their supplies. ii) Human capital is inalienable and thus a bad collateral for loans for educational investment. iii) A perfect insurance contract should be signed in very early stages of life, or before the veil of ignorance is lifted. However, this is not possible since legal arrangements prevent children from entering contracts. (see also Sinn, 1996). These problems cause market imperfections, which justify governmental interventions.

A social insurance against income risks can be provided by distributive labor taxation, because it redistributes between different states of nature and reduces income variance. A social insurance through redistributive labor taxation can be thought as, for instance, an unemployment benefit or health insurance. Furthermore, from a life-cycle perspective, a redistributive labor tax shifts income from periods with high income to ones with low income. By doing so, labor taxation helps individuals to transfer income between periods over life-cycle and works as a substitute to private credit market. However, redistributive labor taxation distorts private educational and labor supply decisions and causes efficiency costs. We have therefore a trade-off in optimal taxation between welfare gain from insurance or alleviating credit constraints and efficiency costs.

The effects of income risks on educational investment are first analyzed by Levhari and Weiss (1974). They show that uninsurable income risks lead to a too low (high) level of private investment in education, if education exaggerates (mitigates) income risks. The reason is that risk-averse individuals want to reduce income risks and adjust their educational investment accordingly. By analyzing optimal policy in a model with risky educational investment, Eaton and Rosen (1980b) argue that a social insurance can be provided by redistributive labor taxation. As a result, the optimal labor tax rate is positive. Anderberg and Andersson (2003) show in a model where the government can centralize educational investment that the effects of education on income risks should be exploited for insurance purpose. If more education hedges against income risks, education should be overprovided. In case that educational investment is not observable and thus cannot be controlled, it is shown by Hamilton (1987) and Grochulski and Piskorski (2010) that capital taxation can be used instead as an indirect education subsidy to encourage educational investment. Moreover, since capital taxation discourages saving and reduces capital income, it encourages labor supply by increasing the marginal utility of income of individuals (see Jacobs and Schindler, 2009 and Kocherlakota, 2005).

Chapter 1 is joint work with Bas Jacobs (Erasmus University of Rotterdam) and Dirk Schindler (University of Konstanz). It has been accepted for publication under the title "Optimal Taxation of Risky Human Capital" in Scandinavian Journal of Economics. In this chapter we analyze the optimal combination of tax policy and education subsidies. In order to achieve an optimal insurance, the marginal welfare gain from insurance must be traded off against the marginal efficiency costs of tax distortions in labor supply and educational investment. Depending on how education affects the extent of income risks, individuals invest either too much or too little in education so as to insure themselves against income risks. Such a self-insurance gives rise to a fiscal externality of educational investment, when labor tax rate is positive. In case of underinvestment in education the marginal return to education is higher than its marginal costs. Consequently, the additional tax revenue raised from the marginal return to education is higher than the losses in tax revenue from tax-deductibility of educational costs. As a result, tax revenue is increased when educational investment increases. In case of overinvestment in education, the opposite happens and an increase of educational investment lowers tax revenue.

Since education subsidies do not depend on future labor income and cannot affect income variance, they have no direct insurance effect. Nevertheless, education subsidies are used in an optimal policy. There are two reasons for the use of education subsidies. Firstly, education subsidies are used to reduce tax distortions in labor supply. Education subsidies encourage educational investment and raise individuals' future wage rate. This has a positive effect on labor supply. Secondly, education subsidies are used to internalize the fiscal externality. If private educational investment is distorted downwards by income risks, education should be more subsidized, since a higher level of educational investment increases tax revenue and has a positive fiscal externality. However, if educational investment is too high, education should be less subsidized (or taxed) to reduce the excessive private educational investment.

Chapter 2 is joint work with Dirk Schindler (University of Konstanz) and is available as Working Paper 2010-05, Department of Economics, University of Konstanz. This chapter extends the analysis in the former chapter by allowing government to additionally tax capital income. Like education subsidies, capital taxation does not provide direct insurance against income risks and is only used as a catalyst to improve social insurance. Specifically, capital taxation allows for a better insurance by reducing tax distortions in labor supply. There are two ways by which capital taxation encourages labor supply. Firstly, Capital taxation is an indirect education subsidy and increases labor supply by encouraging educational investment. Secondly, capital taxation decreases capital income of individuals and has a positive effect on labor supply by increasing the opportunity costs of leisure.

We show that education subsidies and capital taxation are both used as catalysts for social insurance. The optimal subsidy rate as well as the optimal capital tax rate increase with their effectiveness in boosting labor supply, but decrease with their net distortions. Furthermore, we identify "indirect complementarity effects", which reflect the interactions of the two catalysts. In particular, the use of one instrument makes the other instrument more distortive by further distorting the relevant decision margin. Consequently, the more education subsidies (capital taxation) worsen the distortions of capital taxation (education subsidies), the less subsidies (or capital tax) should be employed. Since education subsidies and capital taxation both improve social insurance by reducing labor supply distortions, they are strategic substitutes. If capital tax rate is driven down by for example more intense tax competition, education should then be more subsidized in order to achieve an optimal social insurance.

Chapter 3 results from the joint work with Dirk Schindler (University of Konstanz) and is available as Discussion Paper 09/01, Forschergruppe Heterogene Arbeit, Universität Konstanz. In this paper we pursue the question whether a Norwegian-type two-bracket labor tax system in combination with education subsidies can enhance the efficiency-insurance trade-off and increase social welfare compared to a standard Eaton and Rosen (1980b) world. With a two-bracket labor tax system the skill premium can be taxed more heavily than unskilled labor income by a surtax. Since human capital investment risks are embedded in skill premium, a two-bracket tax system enables a better insurance by reducing the variance of risky labor income more heavily without causing additional tax distortions in unskilled labor supply. To show the efficiency effect of a two-bracket taxation, we analyze the welfare effects of an income-neutral tax reform. Starting from a proportional labor tax, the introduction of a surtax on skill premium and redemption of additional tax revenue as education subsidies increase welfare, if the (potential) efficiency costs of the surtax are more than compensated by its welfare gain from a better insurance. The efficiency costs of a surtax by distorting labor supply and educational investment can be reduced by subsidizing education. In case that the efficiency costs are totally eliminated by education subsidies, such a tax reform is unambiguously welfare-improving.

Chapters 4 and 5 address the problem of credit constraints for educational investment. By simulating a life-cycle model where borrowing is not allowed, Hubbard and Judd (1986) show that progressive taxation increases welfare. The intuition is that progressive taxation alleviates credit constraints by shifting individual tax burden from credit constrained periods to credit unconstrained ones. It is also shown by Hoff and Lyon (1995) that progressive taxation increases welfare by alleviating credit constraints. In their model a lump-sum transfer financed by a proportional labor tax is paid when individuals invest in education. Thus, individuals have to pay a larger share of educational costs by themselves and use less credit to finance education. Since individuals now bear more of the human capital investment risks, the moral-hazard problem associated with credit constraints is mitigated. Andolfatto and Gervais (2006) argue that redistribution from students and retirees to workers is optimal, since it relaxes credit constraints by improving individuals' incentive to repay loans. Lochner and Monge-Naranjo (2002) find that education subsidies have substantially greater effects on educational investment in a model with endogenous credit constraints than in one with exogenous credit constraints. The reason is that education subsidizes increase borrowers' future labor income as a collateral for credits and increase consequently credit limit in private loan markets. In Krüger and Perri (1999), however, progressive taxation tightens credit constraints and a higher tax progressivity decreases welfare. In their model individuals are not allowed to borrow anymore, once they have defaulted on loans. Progressive taxation shifts income from high-income periods to low-income ones, which makes the punishment of no-borrowing less severe. As a result, credit constraints are strengthened.

Chapter 4 is joint work with Bas Jacobs (Erasmus University of Rotterdam). It analyzes optimal tax policy in a two-period life-cycle model where borrowing constraint in private loan markets hampers educational investment of poor individuals. We show that a redistributive income tax is efficient, because it relaxes borrowing constraint by shifting income from high-income period to low-income and thus credit constrained one. Hence, consumption is better smoothed and investment in human capital increases. Because of this efficiency effect the optimal labor tax rate is positive – even in the absence of distributional concerns. Therefore, we provide a case for distortionary income tax on grounds of efficiency only. With heterogenous agents, the equity-efficiency trade-off is less severe because progressive income taxes alleviate capital market imperfections. The optimal labor tax rates with binding credit constraints are higher compared to models without credit constraints. Moreover, there are interactions between redistribution and alleviating credit constraints. If initial wealth is equally distributed, individuals with higher ability are more credit constrained since they invest more in education. Consequently, government can mitigate redistribution problem by alleviating credit constraints to a less extent. Instead, for an equal distribution of ability, credit constraints should be alleviated to a larger extent, because poorer individuals are more credit constrained.

Chapter 5 extends the analysis in chapter 4 by explicitly modeling credit constraints. It is available as Working Paper 2010-04, Department of Economics, University of Konstanz. Credit constraints arise from limited commitment of borrowers to repay loans. An individual decides to default, if he is better off by defaulting than by repaying. We assume that banks can seize a given part of the defaulting individuals' labor income. To avoid defaults, banks limit the maximal credit amount to reduce the incentive for defaulting. Under the assumption of perfect competition in loan markets the equilibrium credit limit is determined by the indifference of borrowers between defaulting and repaying.

The optimal taxation trades the marginal welfare gain from shifting income across different life-cycle periods and from affecting the endogenous credit limit against the marginal efficiency costs of distorting educational investment and labor supply. Compared to models with exogenous credit constraint, the optimal labor tax rate is higher (lower) if taxation raises (lowers) the credit limit in private loan markets. Numerical examples show that redistributive taxation lowers the equilibrium credit limit. The intuition is that redistributive taxation reduces the incentive to invest in education and to work, thus exaggerating the moral hazard problems associated with credit constraint. As a result, credit constraint is tightened.

This dissertation shows that redistributive labor taxation increases welfare due to its insurance effect when private insurance markets against income risks are missing and due to its efficiency effect from alleviating credit constraints. If a social insurance is implemented by redistributive income taxation, education subsidies and capital taxation should be used as complementary instruments to alleviate tax distortions in labor supply and to internalize a fiscal externality. The availability of these two instruments thus allows for a better social insurance. When credit constraints are binding, distributional labor taxation relaxes credit constraints by shifting income from high-income periods to credit constrained ones. The trade-off between equity and efficiency is therefore less severe. Moreover, tax policy affects borrowers' incentive to repay loans and thus the extent of credit constraints. The effects of tax policy on credit constraints should be taken into account when we design optimal policy.

Einleitung und Zusammenfassung

Die vorliegende Dissertation besteht aus fünf unabhängigen Forschungspapieren über optimale Steuer- und Bildungspolitik, welche ich während meines Promotionsstudiums an der Universität Konstanz erstellt habe. Becker (1964) benutzte zuerst den Begriff Humankapitalinvestition zur Bezeichnung aller Aktivitäten zur Weiterentwicklung der individuellen Fähigkeiten und des Wissens. Die Rendite eines zusätzlichen Schuljahrs in Form eines höheren zukünftigen Arbeitseinkommens liegt nach empirischen Untersuchungen zwischen 5 und 15% (siehe z.B. Hartog und Oosterbeek, 2007). Gleichzeitig beobachten wir in vielen Ländern große staatliche Ausgaben für Bildung, z.B. in Form der Bereitstellung eines öffentlichen Schulsystems und der Subventionierung von Bildungsprogrammen. Es stellt sich folglich die Frage, warum der Staat trotz der hohen privaten Renditen Bildung in so großem Umfang subventioniert. Diese Dissertation beschäftigt sich mit zwei Begründungen für den Eingriff des Staates, nämlich fehlenden (oder imperfekten) privaten Versicherungsmärkten gegen Einkommensrisiko und Restriktionen für Bildungskredite. Beide verzerren private Investitionen in Humankapital, was einen staatlichen Eingriff rechtfertigen kann. Kapitel 1, 2 und 3 analysieren, wie Steuer- und Bildungspolitik optimal gestaltet werden sollen, wenn Versicherungsmärkte für ein idiosynkratisches Einkommensrisiko fehlen. Kapitel 4 und 5 beantworten die Frage, wie das Problem der Kreditrestriktionen durch Steuerpolitik optimal gelöst werden kann.

Für jedes Individuum ist die Höhe des zukünftigen Arbeitseinkommens mit großer Unsicherheit verbunden. Es kann Unsicherheit über den erfolgreichen Abschluss des Studiums, über den Erfolg im Arbeitsmarkt oder über die Gesundheit geben. Wie die Bildung das Einkommensrisiko beeinflusst, ist aber empirisch nicht eindeutig. Einerseits kann eine höhere Bildung eine Versicherung gegen das Einkommensrisiko bieten, weil besser gebildete Individuen seltener arbeitslos werden. Andererseits kann eine höhere Bildung aufgrund einer stärkeren Spezialisierung des Humankapitals das Einkommensrisiko erhöhen. Folglich kann das Risiko zwei unterschiedliche Wirkungen auf die Bildung haben, solange keine perfekten Versicherungsmärkte existieren. Wenn Bildung das Einkommensrisiko verstärkt, investieren Individuen weniger in Bildung, um das Risiko zu reduzieren. Wenn Bildung einen Versicherungseffekt hat, investieren Individuen aus einer aggregierten Perspektive zu viel in Bildung.

Im Gegensatz zum Risiko ist der Effekt von Kreditrestriktionen auf die Bildung eindeutig. Kreditrestriktion schränken die Bildungsmöglichkeiten von armen Studenten ein und führen zu einer zu niedrigen Bildungsinvestition. Bei Existenz von Kreditrestriktionen bestimmt das Familieneinkommen die Bildungsinvestition der Kinder. Empirisch kann eine Kausalwirkung des Familieneinkommens auf die Bildung nachgewiesen werden. Nach Acemoglu und Pischke (2001) führt eine 10% Zunahme des Einkommens zu einer 1.4% Zunahme in der Wahrscheinlichkeit, ein vierjähriges College zu besuchen. Belley und Lochner (2007) zeigen, dass sich der Effekt des Familieneinkommens in den letzten 20 Jahren fast verdoppelt hat. Dies ist auf die zunehmende Einkommensungleichheit und auf die steigenden Studiengebühren zurückzuführen.

Das Fehlen von privaten Versicherungsmärkten bzw. die Existenz von Kreditrestriktionen sind mit den Besonderheiten des Humankapitals zu erklären: i) Die Erträge des Humankapitals, nämlich das Arbeitseinkommen, hängen sowohl von Bildungs- und Arbeitsanstrengungen als auch von angeborenen Fähigkeiten ab, die aber nicht beobachtbar sind. Es gibt deshalb ein moral-hazard Problem und ein adverse-selection Problem für die privaten Versicherungsanbieter bzw. die Kreditgeber, die ihr Angebot deshalb einschränken oder ganz einstellen würden. ii) Humankapital ist nicht verpfändbar und wird von Kreditgebern nur eingeschränkt als Sicherheit für Kredite akzeptiert. iii) Eine perfekte Versicherung gegen Einkommensrisiko sollte in sehr früheren Lebensperioden abgeschlossen werden, nämlich bevor die Schleier des Nichtwissens gehoben wird. Das ist aber nicht möglich, weil Kinder nicht geschäftsfähig sind (siehe dazu auch Sinn, 1996). Aus diesen Gründen entsteht Marktversagen, was dann staatliche Interventionen rechtfertigen kann.

Eine redistributive Besteuerung kann eine Versicherung gegen das Einkommensrisiko bieten, da sie Einkommen zwischen unterschiedlichen Zuständen der Zukunft umverteilt und dadurch die Einkommensvarianz reduziert. Eine Versicherung durch eine redistributive Lohnsteuer wird im Folgenden als Sozialversicherung bezeichnet. Unter diesem Begriff ist eine Versicherung im weiten Sinn zu verstehen, die z.B. eine Renten- oder Krankenversicherung beinhalten kann. Außerdem verschiebt eine redistributive Lohnsteuer aus Lebenszyklusperspektive Einkommen von Lebensperioden mit hohem Einkommen nach Perioden mit niedrigem Einkommen. Dadurch hilft eine Lohnsteuer den Individuen, ihr Einkommen zwischen Perioden zu transferieren, und sie stellt deshalb ein Substitut zu privaten Kreditmärkten dar. Gleichzeitig verursacht eine redistributive Lohnsteuer Verzerrungen von Bildungs- und Arbeitsentscheidungen der Individuen. Folglich haben wir einen Trade-off zwischen Wohlfahrtsgewinnen durch Versicherung oder durch Lockerung der Kreditrestriktionen und Effizienzkosten durch Verzerrungswirkungen.

Die Analyse der Effekte des Risikos auf die Bildung geht auf das Papier von Levhari und Weiss (1974) zurück. Sie zeigen, dass das Einkommensrisiko zu einer zu niedrigen (hohen) privaten Bildungsinvestition führt, wenn Bildung das Risiko verstärkt (mindert). Durch eine niedrigere (höhere) Investition in Bildung versuchen Individuen, das Einkommensrisiko zu reduzieren. Eaton und Rosen (1980b) analysieren als erste eine optimale Politik, wenn Bildungsinvestitionen mit Unsicherheit verbunden sind. Sie argumentieren, dass eine Sozialversicherung durch eine redistributive Lohnsteuer bereitgestellt werden kann. Der optimale Steuersatz ist deshalb positiv. Anderberg und Andersson (2003) greifen die Effekte von Bildung auf das Einkommensrisiko auf und argumentieren für den Einsatz von Bildungspolitik als ein Versicherungsinstrument. Wenn Bildung das Einkommensrisiko reduziert, sollte die Bildung gefördert werden, um den Versicherungseffekt der Bildung auszunutzen. Im Fall dass Bildungsinvestitionen nicht beobachtbar und daher nicht kontrollierbar sind, zeigen Hamilton (1987) und Grochulski und Piskorski (2010), dass eine Zinssteuer als eine indirekte Bildungssubvention eingesetzt werden kann, da die Zinssteuer die Opportunitätskosten der Bildung reduziert. Jacobs und Schindler (2009) und Kocherlakota (2005) zeigen, dass eine Zinssteuer das Arbeitsangebot fördern kann, weil sie das Kapitaleinkommen reduziert und damit den Grenznutzen des Einkommens steigert.

Kapitel 1 ist eine gemeinsame Arbeit mit Bas Jacobs (Erasmus University of Rotterdam) und Dirk Schindler (Universität Konstanz). Sie wurde unter dem Titel *Optimal Taxation of Risky Human Capital* im Scandinavian Journal of Economics zur Veröffentlichung angenommen. Dieses Kapitel analysiert die optimale Kombination von Steuerpolitik und Bildungssubventionen, wenn Bildungserträge unsicher sind. Eine Sozialversicherung durch eine redistributive Besteuerung is dann optimal, wenn der marginale Wohlfahrtsgewinn durch Versicherung gerade die marginalen Effizienzkosten der Steuerverzerrung ausgleicht. Abhängig davon, wie Bildung das Einkommensrisiko beeinflusst, investieren die Individuen zu wenig oder zu viel in Bildung, um sich selber gegen das Einkommensrisiko zu versichern. Durch diese Selbstversicherung geht von der Lohnsteuer eine Fiskalexternalität aus. Im Fall einer Unterinvestition in Bildung ist der Grenzertrag der Bildung höher als die Grenzkosten. Bei einer marginalen Erhöhung der Bildung sind die Steuereinnahmen aus dem zusätzlichen Bildungsertrag höher als der Verlust in Steuereinnahmen durch die Steuerabzugsfähigkeit der Bildungskosten. Dadurch steigt das gesamte Steueraufkommen mit einer Erhöhung der Bildung. Im Fall der Überinvestition in Bildung ist es gerade umgekehrt und eine Erhöhung der Bildung senkt das Steueraufkommen.

Eine Bildungssubvention hat im Gegensatz zur Lohnsteuer keinen direkten Versicherungseffekt, da die Höhe der Bildungssubvention nicht von Realisierung des Risikos abhängt. Sie wird trotzdem eingesetzt und zwar aus zwei Gründen: i) Im Wohlfahrtsoptimum soll Bildung subventioniert werden, um die Verzerrung des Arbeitsangebots durch die Lohnsteuer zu reduzieren. Bildungssubventionen steigern Bildungsinvestition und den effektiven Lohnsatz. Folglich steigt das Arbeitsangebot und die Verzerrung im Arbeitsangebot nimmt ab. ii) Bildungssubventionen sollen im Optimum die Fiskalexternalitäten internalisieren. Bei Unterinvestition in Bildung verursachen Bildungssubventionen eine positive Externalität, weil eine Bildungssubvention die Unterinvestition in Bildung mindert. Entsprechend soll der optimale Subventionssatz höher sein. Bei Überinvestition in Bildung verursachen Subventionen dagegen eine negative Externalität, weil sie die Überinvestition in Bildung verstärkt. Demzufolge soll Bildung weniger stark subventioniert werden.

Kapitel 2 ist eine gemeinsame Arbeit mit Dirk Schindler (Universität Konstanz), welche als Working Paper 2010-05, Department of Economics, University of Konstanz, verfügbar ist. Dieses Kapitel erweitert die Analyse aus Kapitel 1, indem die Regierung zusätzlich auch Kapitaleinkommen besteuern kann. Eine Zinsbesteuerung ermöglicht wie eine Bildungssubvention einen verstärkten Einsatz der Lohnsteuer als Sozialversicherung, d.h. Zinsbesteuerung bewirkt ebenfalls keinen Versicherungseffekt, aber sie wirkt wie ein Katalysator zur Verbesserung der Sozialversicherung. Die Rolle der Zinsbesteuerung besteht darin, die Steuerverzerrungen im Arbeitsangebot zu reduzieren. Einerseits ist eine Zinssteuer eine indirekte Bildungssubvention und fördert das Arbeitsangebot durch Förderung der Bildung. Andererseits reduziert eine Zinssteuer das Einkommen aus Ersparnissen und führt zu einem höheren Arbeitsangebot, da die Individuen nun einen höheren Grenznutzen des Einkommens aufweisen.

Wir zeigen, dass sowohl eine Bildungssubvention als auch eine Kapitalbesteuerung als Katalysator für die Sozialversicherung wirken. Je effektiver die beiden Instrumente das Arbeitsangebot stabilisieren und je geringer die Verzerrung der Bildungsentscheidung bzw. der Ersparnisbildung ausfällt, desto höher sind die optimalen Sätze und damit die Versicherungswirkung. Zusätzlich spielen indirekte komplementäre Effekte eine Rolle, welche durch die Interaktion der beiden Katalysatoren ausgelöst werden. Je stärker ein Instrument die Effektivität des anderen Instruments mindert, desto zurückhaltender sollte es verwendet werden. Weil Bildungssubvention und Zinssteuer ähnliche Wirkungen auf die Verbesserung der Sozialversicherung aufweisen, sind die beiden Instrumente strategische Substitute. Wenn die Zinssteuer, z.B. durch einen stärkeren Steuerwettbewerb, sinkt, sollte Bildung stärker subventioniert werden.

Kapitel 3 ist eine gemeinsame Arbeit mit Dirk Schindler (Universität Konstanz), die auch als Discussion Paper 09/01, Forschergruppe "Heterogene Arbeit", Universität Konstanz, verfügbar ist. Wir betrachten in diesem Kapitel die Fragestellung, ob im Vergleich zum herkömmlichen Modell von Eaton und Rosen (1980b) eine progressive Lohnsteuer in Form einer norwegischen Zweistufensteuer in Kombination mit Bildungssubventionen eine bessere Versicherungswirkung hervorrufen kann. Die zusätzlichen Wohlfahrtseffekte einer zweistufigen Lohnsteuer bestehen darin, dass eine Zusatzsteuer auf die Bildungserträge eine bessere Versicherung, nämlich eine stärkere Reduzierung der Varianz der Bildungserträge, ermöglicht, ohne dabei die unqualifizierte Arbeit zusätzlich zu verzerren. Dazu analysieren wir die Wohlfahrtseffekte einer einkommensneutralen Steuerreform. Ausgehend von einer proportionalen Lohnsteuer, sind die Einführung einer Zusatzsteuer und die Rückzahlung der zusätzlichen Steuereinnahmen in Form von Bildungssubventionen wohlfahrtssteigernd, solange die (potenziellen) Effizienzkosten den Wohlfahrtsgewinn nicht übersteigen. Der Wohlfahrtsgewinn resultiert aus einer besseren Versicherung und die Effizienzkosten der Zusatzsteuer können durch die Gewährung der Bildungssubventionen reduziert werden. Unter Umständen können die Effizienzkosten durch Subventionen komplett eliminiert werden. In diesem Fall ist eine solche Steuerreform eindeutig wohlfahrtssteigernd.

In den Kapiteln 4 und 5 widmen wir uns dem Problem der Kreditrestriktionen. Hubbard und Judd (1986) simulieren ein Lebenszyklusmodell ohne Verschuldungsmöglichkeit und zeigen, dass eine progressive Besteuerung die Wohlfahrt steigert. Der wohlfahrtssteigernde Effekt der Besteuerung resultiert aus Lockerung der Kreditrestriktionen, da eine progressive Steuer die individuelle Steuerlast von kreditbeschränkten Perioden hin zu nicht kreditbeschränkten Perioden verschiebt. Der wohlfahrtssteigernde Effekt einer umverteilenden Besteuerung bei Kreditrestriktionen wird ebenfalls von Hoff und Lyon (1995) gezeigt. In ihrem Papier erhöht die Besteuerung den Pauschaltransfer in Perioden, in denen Bildungsinvestitionen getätigt werden. Das senkt die Investitionskosten, die durch Kreditaufnahme gedeckt werden müssen. Folglich tragen die Studenten einen höheren Teil des Risikos der Bildungsinvestitionen selbst und das Moral-hazard Problem wird dadurch gemindert. In einem Drei-Perioden-Lebenszyklusmodell zeigen Andolfatto und Gervais (2006), dass eine Rückwärtsumverteilung, nämlich eine Umverteilung von Studenten und Rentnern zu Erwerbstätigen, die Anreizkompatibilitätsbedingung der Kreditrestriktionen lockert und dadurch das Kreditlimit erhöht. Da eine Bildungssubvention ebenfalls das Kreditlimit durch Lockerung der Anreizkompatibilitätsbedingung steigert, hat die Bildungssubvention einen stärkeren Effekt auf Bildungsinvestitionen als in einem Modell mit fixer Kreditrestriktion (Lochner und Monge-Naranjo, 2002). Durch Simulation in einem Modell mit exogenem Arbeitsangebot und exogener Bildungsinvestition zeigen Krüger und Perri (1999), dass eine Erhöhung der Steuerprogressivität aufgrund des negativen Effekts auf die Kreditrestriktionen die Wohlfahrt reduziert. Weil die Strafe für die Nichtbedienung des Kredits in Form eines Ausschlusses aus dem Kreditmarkt durch eine umverteilende Besteuerung weniger streng wird, verstärkt die Besteuerung die Kreditrestriktionen.

Kapitel 4 ist eine gemeinsame Arbeit mit Bas Jacobs (Erasmus University of Rotterdam). Dieses Kapitel analysiert eine optimale Steuerpolitik bei Kreditrestriktionen für Bildungsinvestitionen. Wir nehmen in diesem Kapitel an, dass die Individuen keinen Studienkredit aufnehmen können. Unter dieser Annahme zeigen wir, dass eine redistributive Lohnsteuer die Verzerrungen durch Kreditrestriktionen mindert, weil die Lohnsteuer das Einkommen von Perioden mit hohem Einkommen hin zu kreditbeschränkten Perioden umverteilt. Dadurch werden die Kreditrestriktionen teilweise aufgehoben, was zu einer höheren Bildungsinvestition und einem höheren Konsum während der Bildungszeit führt. Aufgrund dieser Wohlfahrtseffekte ist bei homogenen Individuen ein positiver Lohnsteuersatz optimal. Damit zeigen wir, dass eine verzerrende Besteuerung auch ohne distributive Zielsetzung aus Effizienzgründen optimal sein kann. Bei heterogenen Individuen gibt es einen abgemilderten Trade-off zwischen Umverteilung und Effizienz, weil Umverteilung einen zusätzlichen Effizienzgewinn durch Lockerung der Kreditrestriktionen bewirkt. Wir ziehen daraus die Schlussfolgerung, dass bei Existenz von

Kreditrestriktionen der optimale Lohnsteuersatz höher sein sollte.

Außerdem sind Interaktionen zwischen Umverteilung und Kreditrestriktionen von Interesse, wenn sich die Anfangsausstattungen der Individuen bezüglich Vermögen und Fähigkeiten unterscheiden. Sind z.B. die Anfangsvermögen identisch, beschränken die Kreditrestriktionen die Bildungsinvestition der klügeren Individuen stärker, weil sie mehr in Bildung investieren. Das hat eine kleinere Einkommensungleichheit zur Folge. Deshalb kann der Staat das Umverteilungsproblem mindern, indem er die Kreditrestriktionen weniger stark lockert. Bei einer gleichen Verteilung der Fähigkeiten sollte der Staat die Kreditrestriktionen stärker lockern, da ärmere Individuen stärker davon betroffen sind.

Kapitel 5 endogenisiert als Erweiterung der Analyse in Kapitel 4 die Kreditrestriktionen. Es ist als Working Paper 2010-04, Department of Economics, University of Konstanz, verfügbar. Kreditrestriktionen resultieren aus beschränkten Möglichkeiten für Kreditnehmer, sich zur Kreditrückzahlung zu verpflichten. Die Kreditnehmer haben dann einen Anreiz, den Kredit nicht zu bedienen, wenn sie sich durch Nichtrückzahlung des Kredits besser stellen können als durch Rückzahlung. Wir nehmen an, dass die Banken im Fall der Nichtrückzahlung einen Teil des Arbeitseinkommens der Kreditnehmer bekommen. Durch Beschränkung der Kredithöhe können die Banken den Anreiz zur Nichtrückzahlung reduzieren. Unter der Annahme perfekter Konkurrenz auf dem Kreditmarkt werden die gleichgewichtigen Kreditrestriktionen durch eine Anreizkompatibilitätsbedingung bestimmt: der Kreditnehmer muss indifferent zwischen Bedienung und Nichtbedienung des Kredits sein.

Bei endogenen Kreditrestriktionen soll die optimale Lohnsteuer die Summe aus dem marginalen Wohlfahrtsgewinn durch Umverteilung zwischen den Lebensperioden und dem marginalen Wohlfahrtseffekt der Veränderung in Kreditrestriktionen durch die Steuerpolitik gegen die marginalen Steuerverzerrungen ausgleichen. Im Vergleich zum Modell mit exogenen Kreditrestriktionen ist der optimale Lohnsteuersatz höher (niedriger), falls die Besteuerung die Kreditrestriktionen mindert (verstärkt). Numerische Beispiele zeigen, dass sich eine redistributive Lohnsteuer negativ auf die Kreditrestriktionen auswirkt. Eine redistributive Lohnsteuer senkt die Anreize zur Bildungsinvestition und zur Arbeit, wodurch sich das moral-hazard Problem im Zusammenhang mit den Bildungskrediten verschärft. Folglich verschärfen sich auch die Kreditrestriktionen.

Zusammenfassend zeige ich in meiner Dissertation, dass eine umverteilende Besteuerung aufgrund des Versicherungseffekts bei fehlenden Versicherungsmärkten gegen Einkommensrisiko und aufgrund des Effizienzeffekts bei Verzerrungen durch Kreditrestriktionen optimal ist. Zur Verbesserung der Versicherungswirkung einer redistributiven Lohnsteuer sollen Bildungssubventionen und Zinssteuer als komplementäre Instrumente verwendet werden, um die Steuerverzerrungen im Arbeitsangebot zu mindern und fiskalische Externalitäten zu internalisieren. Bei Existenz von Kreditrestriktionen wird der Trade-off zwischen Effizienz und Gerechtigkeit verbessert, weil die umverteilende Besteuerung einen Effizienzgewinn durch Lockerung der Kreditrestriktionen auslöst. Zusätzlich beeinflusst die Besteuerung die Anreize der Kreditnehmer, den Kredit zu bedienen, und verändert damit das Ausmaß der Kreditrestriktionen. Folglich soll der Effekt der Besteuerung auf die endogen bestimmten Kreditrestriktionen beim Design der optimalen Politik berücksichtigt werden.

1 Optimal Taxation of Risky Human Capital

1.1 Introduction

Individuals face large labor market risks during their working lives. They may become unemployed, sick, disabled or they may experience loss of skill due to old age, health problems, technological changes, and globalization. In principle, private insurance should be feasible, since individual idiosyncratic income risks can be pooled in the aggregate. However, in the case of human capital, private insurance markets tend to suffer from market failure and private insurance is not (or only to a very limited extent) available due to moral hazard, adverse selection, and various legal limitations in trading claims on human capital (Sinn, 1996).

Although all social insurance policies suffer from moral hazard problems, the government may overcome adverse selection and legal problems by providing mandatory social insurance against human capital risk. Therefore, insuring human capital risks is one of the key roles of modern welfare states. Indeed, virtually all social benefits (for example welfare, unemployment, sickness, disability, health, or old age benefits) provide insurance against the loss of skill. Moreover, if individuals fail to acquire sufficient skills when young, they are liable to become dependent on social insurance benefits later on in life. Thus, human capital policies could be desirable to avoid dependency on the welfare state.

Despite its obvious policy relevance, it is rather surprising that only a limited number of papers have addressed the question of how social insurance should be organized when human capital is subject to non-insurable risks. Moreover, it is not clear whether education policy should be employed as a complementary policy to social insurance. Some earlier papers have analyzed the implications of human capital risks for the design of optimal insurance and/or education policy. See, for example, Eaton and Rosen (1980a, 1980b), Hamilton (1987), Anderberg and Andersson (2003), Grochulski and Piskorski (2010), da Costa and Maestri (2007), and Anderberg (2009). However, these papers do not explicitly derive answers to the following three questions.

Firstly, *is the optimal amount of social insurance higher or lower when education increases or reduces earnings risk?* Anderberg and Andersson (2003, p. 1523) argue that: "If human capital reduces earnings risk, encouraging education would seem to mitigate the insurance / redistribution problem." Hence, if education hedges against labor market risk (increases labor market risk), the government needs to rely less (more) on social insurance. However, this argument is not formally proven.

Secondly, *should education policy correct under- or overinvestment in human capital, or not?* Human capital investment is typically inefficient, because risk-averse individuals reduce their exposure to income risk in the presence of missing insurance markets. Da Costa and Maestri (2007, p. 696) suggest that education policy is optimal if education is a risk-increasing activity: "Optimal policies derived under these assumptions will then prescribe ... educational subsidies to ameliorate the problem of under-investment in human capital." Anderberg and Andersson (2003, p. 1523) argue in contrast that education policy is also needed, but now when education hedges against income risk: "The insight is thus that if education moderates wage uncertainty, a second-best policy should, rather unambiguously, encourage the formation of human capital (relative to the first-best), while if education exacerbates wage uncertainty the overall conclusion is ambiguous."

Thirdly, *does the availability of education policy optimally increase the amount of social insurance or not?* Again, one would expect this to be true. For example, Bovenberg and Jacobs (2005) demonstrate in deterministic settings that optimal education policy typically lowers the cost of redistribution and thus raises optimal tax rates.

We demonstrate that the answers to these three questions are not trivial and can be completely counterintuitive. Indeed, we show that all the suggested answers to the questions raised above are either partially or completely incorrect. We do so by developing a model, which integrates the previously studied approaches in order to characterize optimal linear tax and education policies in risky economies.

We study a two-period life-cycle model of human capital investment, labor supply, and saving. Ex-ante homogenous households differ ex-post due to the realization of idiosyncratic risk in their second-period income. Markets to insure earnings or human capital risks are missing. Therefore, social insurance is welfare enhancing, since we assume that there is no aggregate risk. Social insurance takes the form of a linear income tax. Full insurance is impossible due to the endogeneity of labor supply, which causes a moral hazard problem. We extend the previous literature by employing a completely general earnings function that depends on human capital investment, labor supply and a random variable reflecting the uncertain state of nature. This general earnings function allows both for the possibility that education is a risky activity that increases exposure to labor market risk and the possibility that education reduces exposure, i.e., hedges, against labor market risk. Our paper contributes in five major ways to the existing literature.

Firstly, we provide the answer to the first question raised above. We show that if educational investment increases (reduces) exposure to non-insurable income risks, the risk-premium acts as a (pre-existing) implicit tax (subsidy) on human capital investment. Missing insurance markets therefore result in non-tax distortions, which generate fiscal externalities that need to be taken into account when designing tax and education policies. Income taxes exacerbate (mitigate) underinvestment (overinvestment) in human capital when education increases (decreases) earnings risk. Income taxes should optimally be lower (higher) as a result. Therefore, if education hedges against (increases) labor market risk, it is incorrect to argue that optimal income taxes should be lower (higher) because individuals self-insure by overinvesting (underinvesting).

Secondly, we answer the second question by demonstrating that education subsidies are not used for insurance. Indeed, when there is no social insurance, governments cannot improve upon the laissez-faire outcome by subsidizing education. Intuitively, education subsidies are state-independent and cannot insure income risks. Subsidizing education thus upsets the optimal private response to market risk by distorting investment in human capital. As long as insurance markets are missing and social insurance is unavailable, it is therefore incorrect to argue that governments should correct under- or overinvestment in human capital with education subsidies.

Thirdly, we show that education subsidies are optimally employed in an income tax cum education subsidy policy. Subsidies on education are optimal only in combination with social insurance in order to mitigate the tax and non-tax distortions associated with social insurance. The first role of education subsidies is to reduce the tax distortions on labor supply if education is complementary to labor. Education subsidies then boost labor supply, and thereby indirectly off-set the labor tax distortion on work effort (see also Jacobs and Bovenberg, 2008). The second role of education subsidies is to internalize the fiscal externality caused by under- or overinvestment in education. Da Costa and Maestri (2007) are indeed correct to argue that education should be subsidized if it is a risky activity, but only to the extent that subsidies are needed to internalize the fiscal externality, and not to directly tackle over- or underinvestment (see the previous point).

Fourthly, we answer the third question. When both tax and education policies are optimized, we demonstrate that the design of social insurance becomes *independent* from the question whether education is a risky investment or not. Therefore, ambiguities arise as to whether more or less social insurance is provided compared to the optimal tax policy without education subsidies. This crucially depends on the risk-properties of human capital and the complementarity of education and work. Consequently, optimal education policy does not automatically allow for more social insurance.

Fifthly, our paper is the complement to Anderberg (2009) who analyzes non-linear tax and education policies in comparable settings. We bolster his findings by showing that the risk-properties of human capital are critical in shaping human capital policies also under much weaker informational assumptions. In particular, only aggregate labor incomes and educational investments need to be verifiable to the government for linear policy instruments to be employed. Moreover, our findings suggest that great care should be taken when drawing inferences from the wedges that are now commonly analyzed in the new dynamic public finance literature. We show that a tax wedge on education does not prove that education is optimally taxed at the optimal second-best allocation, which is decentralized through income taxes and education subsidies.

The remainder of this paper is structured as follows. Section 1.2 provides a survey of the literature. Then the model is introduced in section 1.3. Section 1.4 discusses optimal tax and education policies. The paper ends with conclusions and discussion in section 1.5.

1.2 Earlier Literature

Levhari and Weiss (1974) are the first to examine the effect of idiosyncratic risks on human capital formation. Human capital investment can both increase and decrease exposure to income risk depending on the risk-properties of the earnings function. Individuals will self-insure by underinvesting in human capital if this increases the exposure to labor market risk, but will overinvest in human capital if this hedges against labor market risk. Empirically, both possibilities appear to be relevant.[1]

The formal analysis of social insurance with endogenous human capital investment began with the seminal paper by Eaton and Rosen (1980b). They assume a multiplicative earnings function, where labor earnings are a linear product of labor supply, human capital, and a stochastic risk factor. Thus, investments in education raise the exposure to labor market risk. Consequently, private investment in education is driven below the socially desirable level. A distortionary income tax is shown to be welfare-enhancing, since it redistributes income from favorable to unfavorable states of nature. The linear income tax is a partial substitute for missing insurance markers. Consequently, human capital risks are partially insured, and human capital investment increases.

Hamilton (1987) adopts the model of Eaton and Rosen (1980b) to analyze taxes on savings, besides the income tax. Hamilton points out that there remains socially inefficient underinvestment in human capital, since the labor tax cannot eliminate all income risk due to moral hazard in labor supply. Hamilton (1987) shows that taxing savings reduces the opportunity costs of human capital accumulation and is optimal under the (very) strong assumptions of (i) inelastic labor supply and (ii) either zero equilibrium savings or constant absolute risk aversion.

Grochulski and Piskorski (2010) generalize the findings of Hamilton (1987) to non-linear policy instruments without imposing the strong restrictions on the model, while maintaining an earnings function with multiplicative risk. At the same time, they do not allow for education policy, as education is assumed to be non-verifiable to the government. They show that labor supply carries a wedge (i.e., is distorted) for insurance purposes. This is analogous to the optimality of a distortionary labor tax. Moreover, there is an intertemporal wedge in consumption

[1] Hartog (2005) reviews a whole literature that empirically establishes risk-compensation in wages. Palacios-Huerta (2004, 2006) shows that human capital is risky on average. However, he also finds that the human capital premium decreases as workers become better educated, suggesting that human capital investments hedge against labor market risk on the margin.

choices, indicating a role for capital income taxation, for two reasons. Firstly, the intertemporal wedge stimulates labor supply, and indirectly reduces the tax-distortions on labor supply (see also Diamond and Mirrlees, 1978, 1986; Golosov et al., 2003).[2] Secondly, by lowering the opportunity costs, intertemporal wedges give incentives to invest in human capital. This is optimal because the labor tax discourages human capital investments. See also Jacobs and Bovenberg (2010) who obtain the same result even in the absence of risk.

Anderberg and Andersson (2003) are the first to simultaneously optimize linear tax and education policies. They use a stripped-down version of the Eaton and Rosen (1980b) model, while allowing for a more general earnings function, as in Levhari and Weiss (1974). Anderberg and Andersson (2003) assume that the government directly controls educational investment. Also these authors obtain a trade-off between social insurance and distortions in labor supply. In addition, they find that the use of education policy generates a 'revenue creation effect' because labor supply and education are complementary activities, so that education policy can mitigate the tax distortions on labor supply. Moreover, education policy entails an 'insurance effect' depending on the risk-properties of human capital. Their main message is that education should be overprovided relative to first-best rules if it is risk-decreasing and it should be underprovided if it is risk-increasing.

Da Costa and Maestri (2007) and Anderberg (2009) also build on the Eaton-Rosen-Hamilton model, but now assume that human capital investments can be verified by the government so that it can employ education policies, besides capital taxation and non-linear income taxation. In addition to deriving the desirability of wedges on labor supply and saving, da Costa and Maestri (2007) argue that education policy should ensure social efficiency in human capital investment. Anderberg (2009) concludes this result to be erroneous. Aggregate human capital investment should be optimally distorted in a way that depends on the shape of the earnings function similar to Anderberg and Andersson (2003).

[2] Cremer and Gavhari (1995a, 1995b) also demonstrate that the optimal (linear) commodity tax is non-uniform under both linear and non-linear income taxation, when one of the commodities is consumed before the realization of risk and the other thereafter. If we interpret these commodities as consumption today and consumption tomorrow, the result is immediate. Hence, intertemporal wedges or capital income taxes are optimal.

1.3 The Model

We follow Levhari and Weiss (1974) by analyzing a two-period life-cycle model of human capital investment, labor supply, and saving. There is a continuum of ex-ante identical individuals, who differ ex-post due to an idiosyncratic shock θ, which is drawn from a probability distribution $f(\theta)$. We assume $\theta \in \Theta \equiv [\underline{\theta}, \infty)$, where Θ denotes the set of values for θ and $\underline{\theta}$ denotes the lower bound on θ.

Households derive utility from consumption c_1 in period one and consumption c_2 in period two. Moreover, they derive disutility from labor supply l in the second period. There is no labor-leisure choice in the first period. Households maximize a von Neumann-Morgenstern expected utility function, which is assumed to be separable between the sub-utility function of consumption in both periods and the disutility of work:

$$\mathcal{E}[u(c_1, c_2)] - v(l), \quad u_1, u_2, v_l > 0, \quad u_{11}, u_{22}, -v_{ll} < 0. \tag{1.1}$$

$\mathcal{E}$ denotes the expectation operator, i.e., $\mathcal{E}[X] \equiv \int_{\Theta} X df(\theta)$, and subscripts refer to the argument of differentiation. The sub-utility function of consumption is increasing and concave, whereas the disutility function of labor supply is increasing and convex. Furthermore, we impose the Inada-conditions on both sub-utility functions to avoid corner solutions.

In the first period, individuals have a unit time endowment, which is spent on investment in education (e), and work ($1-e$). Consequently, individuals forego labor earnings while learning.[3] The wage per unit of time worked in the first period is normalized to one. In addition, individuals have an exogenous income endowment ω. Apart from investing in education, individuals can borrow and lend in perfect capital markets at a constant real interest rate r. Total savings are denoted by a.[4]

Gross labor income in the second period is represented by a general earnings function, which

[3] Without any loss of generality we could also allow for direct costs of education as long as all educational investments are verifiable and can be subsidized (cf. Bovenberg and Jacobs, 2005).

[4] We assume that the lower bound $\underline{\theta}$ is sufficiently large such that second-period income is always high enough to prevent individuals defaulting on their loans. See Jacobs and Yang (2010) for the analysis of optimal taxation of human capital with imperfect capital markets.

depends on labor supply l and education e:

$$\Phi(\theta, l, e), \quad \Phi_e, \Phi_l > 0, \quad \Phi_{ee} < 0, \quad \Phi_{ll} \leq 0. \tag{1.2}$$

Therefore, both income and the returns to education are risky. We assume that, for any given value of θ, the marginal returns to education are positive and decreasing. Similarly, the marginal returns to labor effort are positive and non-increasing. Furthermore, the random variable θ is assumed to exert a positive effect on income: $\Phi_\theta > 0$. In the remainder of the analysis, we focus on the two cases identified in the literature: (i) educational investment amplifies income risks ($\Phi_{\theta e} > 0$), and (ii) educational investment hedges against income risks ($\Phi_{\theta e} < 0$).

Social insurance takes place through a linear tax system with a positive marginal tax rate t on labor earnings in both periods and a lump-sum transfer T, which can be seen as a negative income tax or a basic income. Without loss of generality the transfer is only given in the second period.[5] Since forgone labor earnings are the only cost of education, all educational investments are tax-deductible. We introduce a flat rate subsidy s on net forgone earnings (i.e., opportunity costs of education). This can be viewed as a subsidy per unit of time enrolled in education.[6] The informational assumptions for employing linear instruments are that only aggregate incomes and education choices need to be verifiable to the government.

Consequently, we can write the first-period and second-period budget constraints as

$$c_1 = (1-t)\,(1-(1-s)\,e) - a + \omega, \tag{1.3}$$

$$c_2 = (1-t)\,\Phi(\theta, l, e) + Ra + T, \tag{1.4}$$

where $R \equiv 1 + r$ is the interest factor.

The timing structure of the model is as follows. The government sets the proportional tax rate t, the subsidy rate s, and the lump-sum transfer T before the choices of households and the revelation of the risk θ. Moreover, educational investment e, savings a, and labor supply l are simultaneously chosen before risk realizes.[7] This implies that first-period consumption is

[5]Since we assume perfect capital markets, individuals can always borrow against the transfer to finance first-period consumption.

[6]We abstract from taxes on saving and refer to Hamilton (1987) and Schindler and Yang (2009) for the analysis of optimal capital taxes in a similar model.

[7]It can be shown that a timing sequence in which labor supply is chosen after uncertainty has been resolved does

pinned down by these choices. After the shock realizes, incomes are earned and second-period consumption takes place.

The household's unconstrained maximization problem can be obtained upon substitution of the household budget constraints into the utility function:

$$\max_{\{e,l,a\}} U(e,l,a) \equiv \mathcal{E}[u((1-t)(1-(1-s)e)-a+\omega,(1-t)\Phi(\theta,l,e)+(1+r)a+T)]-v(l), \tag{1.5}$$

The first-order conditions for this maximization problem are given by[8]

$$\mathcal{E}[u_2(.)\Phi_e(.)] = \mathcal{E}[u_1(.)](1-s), \tag{1.6}$$

$$(1-t)\mathcal{E}[u_2(.)\Phi_l(.)] = v_l(l), \tag{1.7}$$

$$R\mathcal{E}[u_2(.)] = \mathcal{E}[u_1(.)]. \tag{1.8}$$

The first-order conditions for education (1.6) and labor supply (1.7) can be rewritten by employing the risk premia in education and labor supply:

$$\pi_i \equiv -\frac{cov[u_2(.),\Phi_i(.)]}{\mathcal{E}[u_2(.)]\mathcal{E}[\Phi_i(.)]}, \quad i=e,l. \tag{1.9}$$

π_e is the negative of the normalized covariance between marginal utility of consumption and marginal return of human capital. A positive risk premium implies that education increases income risk, since $\pi_e > 0$ corresponds to $\Phi_{\theta e} > 0$. A negative risk premium $\pi_e < 0$, instead, mirrors a risk-reducing effect of education, due to $\Phi_{\theta e} < 0$. Similarly, π_l is the negative of the normalized covariance between marginal utility of consumption and marginal return to labor, representing the risk premium in labor supply. Its interpretation is analogous to the risk premium in educational investment. Note that if individuals would be risk-neutral, both risk premia would be zero. Similarly, risk premia are zero if the marginal returns to education or labor are not state-dependent, i.e., when there is no risk. Both risk premia are also zero when the risk-factor θ enters the earnings function in an additively separable fashion ($\Phi_{\theta e} = \Phi_{\theta l} = 0$),

not change any of the results qualitatively, cf. Cremer and Gavhari (1995a), and Anderberg and Andersson (2003).

[8] In general, the second-order conditions are not automatically satisfied due to the interaction between learning and working, which generates non-linear budget sets. We assume that second-order conditions are always satisfied. This requires that the complementarity between education and labor is sufficiently weak (low Φ_{el}) and absolute risk-aversion is sufficiently large, see also Jacobs et al. (2009).

since education and labor supply do not affect income risk in that case.

Using the definition of π_e, the first-order condition (1.6) can be written as

$$(1-\pi_e)\mathcal{E}\left[\Phi_e\left(\theta,l,e\right)\right] = R\left(1-s\right). \tag{1.10}$$

The risk-adjusted expected marginal return to education is equal to the marginal cost of education. Note that the tax system does not directly affect investment in education, since all costs of education are tax deductible. However, taxation generally affects investment in education indirectly via labor supply. More labor supply raises the returns to human capital investments as long as $\Phi_{el} > 0$. This is the case for all earnings functions discussed in the literature (cf. Jacobs and Bovenberg, 2008). Education subsidies naturally boost educational investments, since they reduce the marginal cost of human capital investment.

If income is risky, the expected marginal return of education can be either higher or lower than marginal costs, depending on the sign of the risk premium π_e. If education increases exposure to labor market risk, $\pi_e > 0$, individuals command a positive risk-premium on their educational investment. Hence, from a social point of view, risk-averse individuals invest too little in education. Missing insurance markets for human capital related income risk thus create an implicit tax on human capital investment. If income risk decreases with education, individuals command a negative risk-premium on their educational investment, $\pi_e < 0$. In that case, risk-averse individuals invest too much in education in order to reduce their exposure to labor market risk. Thus, missing insurance markets create an implicit subsidy on human capital investments. If there is no income risk, condition (1.10) reduces to $\Phi_e\left(.\right) = 1-s$, which is the optimality condition for investment in human capital under certainty.

The first-order condition for labor supply (1.7) can be rewritten using π_l:

$$\frac{v_l(l)}{\mathcal{E}\left[u_2(.)\right]} = (1-t)(1-\pi_l)\mathcal{E}\left[\Phi_l\left(\theta,l,e\right)\right]. \tag{1.11}$$

The marginal rate of substitution between consumption and labor must be equal to the risk adjusted net wage. A higher tax rate reduces the incentives to supply labor. Note that if education raises the wage rate, incentives to supply labor are stronger when individuals are better educated. Thus, education and labor are complementary as long as $\Phi_{el} > 0$. If an increase in labor supply increases risk, $\pi_l > 0$, individuals supply less labor than is socially efficient. If labor supply decreases the exposure to risk ($\pi_l < 0$), the risk premium turns negative, leading to socially inefficient precautionary labor supply. Again, the risk premium acts as an implicit tax (subsidy) on labor if labor supply increases (reduces) exposure to labor market risk, i.e., if $\pi_l > 0$ ($\pi_l < 0$).

Equation (1.8) is the stochastic Euler-equation for consumption. The larger the interest rate, the stronger are the incentives to save, and the more individuals allocate resources to the second period of the life-cycle.

1.4 Optimal Tax and Education Policies

We assume a benevolent government with full commitment, which maximizes social welfare by optimally choosing linear tax and education policies. The intertemporal government budget constraint is given by

$$t\mathcal{E}\left[\Phi\left(\theta,l,e\right)\right]+tR(1-e)=R\left(1-t\right)se+T+G, \tag{1.12}$$

where G is an exogenous revenue requirement. Since income risk is idiosyncratic, tax revenue is deterministic according to the law of large numbers and tax revenue equals its expected value. We abstract from any systematic risk.

Social welfare is (ex ante) expected indirect utility $V\left(.\right)$ of the representative household:

$$V(T,t,s)\equiv\mathcal{E}[u((1-t)\left(1-(1-s)\,\hat{e}\right)-\hat{a}+\omega,(1-t)\,\Phi(\theta,\hat{l},\hat{e})+(1+r)\hat{a}+T)]-v(\hat{l}), \tag{1.13}$$

where hats denote the optimized values for l, e, and a. For later reference, we apply Roy's lemma to find the derivatives of the indirect utility function: $\frac{\partial V(.)}{\partial T}=\mathcal{E}\left[u_2\left(.\right)\right]$, $\frac{\partial V(.)}{\partial t}=-\mathcal{E}[u_2\left(.\right)\left(\Phi\left(.\right)+R\left(1-(1-s)\hat{e}\right)\right)]$, and $\frac{\partial V(.)}{\partial s}=\mathcal{E}[u_2\left(.\right)]R(1-t)\hat{e}$.

The Lagrangian for maximization of social welfare is given by

$$\max_{\{T,t,s\}}\mathcal{L}\equiv\mathcal{E}\left[V(T,t,s)+\eta\left(t\Phi\left(\theta,l,e\right)+tR(1-e)-R\left(1-t\right)se-T-G\right)\right], \tag{1.14}$$

where η denotes the Lagrange-multiplier of the government budget constraint (1.12).

In order to characterize the optimal solutions for the optimal tax and subsidy rates, we introduce the following tax wedges on labor and education:

$$\Delta_l \equiv t\Phi_l(.), \tag{1.15}$$

$$\Delta_e \equiv t(\Phi_e(.) - R) - R(1-t)s. \tag{1.16}$$

Δ_l (Δ_e) measures the increase in tax revenue (measured in monetary units) if labor supply (education) is raised with one unit.

The first-order conditions for this maximization problem are given by:

$$\frac{\partial \mathcal{L}}{\partial T} = \mathcal{E}\left[\frac{\partial V(.)}{\partial T} - \eta + \eta\Delta_l \frac{\partial l}{\partial T} + \eta\Delta_e \frac{\partial e}{\partial T}\right] = 0, \tag{1.17}$$

$$\frac{\partial \mathcal{L}}{\partial t} = \mathcal{E}\left[\frac{\partial V(.)}{\partial t} + \eta(\Phi(.) + R(1-e) + Rse) + \eta\Delta_l \frac{\partial l}{\partial t} + \eta\Delta_e \frac{\partial e}{\partial t}\right] = 0, \tag{1.18}$$

$$\frac{\partial \mathcal{L}}{\partial s} = \mathcal{E}\left[\frac{\partial V(.)}{\partial s} - \eta R(1-t)e + \eta\Delta_l \frac{\partial l}{\partial s} + \eta\Delta_e \frac{\partial e}{\partial s}\right] = 0. \tag{1.19}$$

In the remainder of this section, we first analyze optimal tax and education policies separately. Then, we derive the optimal structure of both tax and education policies simultaneously.

1.4.1 Optimal Lump-sum Transfer

Using Roy's lemma we obtain from (1.17):

$$\mathcal{E}\left[\frac{u_2}{\eta} + \Delta_l \frac{\partial l}{\partial T} + \Delta_e \frac{\partial e}{\partial T}\right] = 1. \tag{1.20}$$

Hence, the expected social marginal value of a unit increase in lump-sum income, including the income effects on the tax base, should be equal to its resource costs, which equal unity.

1.4.2 Optimal Taxation

This section derives the optimal level of social insurance in the absence of education policy ($\bar{s} = 0$). We define the 'insurance characteristic' ξ as the negative of the normalized covariance between gross income Φ and the private marginal value of income u_2:

$$\xi \equiv -\frac{cov\left[\Phi, u_2\right]}{\mathcal{E}\left[\Phi\right]\mathcal{E}\left[u_2\right]} > 0. \tag{1.21}$$

The insurance characteristic ξ measures the (marginal) gain in social welfare of larger income insurance. It is positive, because higher labor income is associated with lower marginal utility of consumption, i.e., $cov\left[\Phi, u_2\right] < 0$. Reducing the variance in earnings by means of redistributive income taxes thus raises social welfare. Indeed, $\xi = 0$ if the government is not concerned about income insurance and all individuals have the same marginal utility of income u_2, or if household income Φ is deterministic, and there is no risk.

Using Roy's lemma and the risk-adjusted Slutsky-equations, we can find the optimal tax rate at the optimal T from (1.18):

$$\frac{t}{1-t} = \frac{\xi}{\varepsilon_{lt} + \pi_e \varepsilon_{et}}, \tag{1.22}$$

where $\varepsilon_{lt} = -\frac{\mathcal{E}[\Phi_l]l}{\mathcal{E}[\Phi]}\frac{\partial l^*}{\partial t}\frac{1-t}{l} > 0$ and $\varepsilon_{et} = -\frac{\mathcal{E}[\Phi_e]e}{\mathcal{E}[\Phi]}\frac{\partial e^*}{\partial t}\frac{1-t}{e}$ are the expected utility compensated elasticities of labor supply and education and where an asterisk $(^*)$ denotes a compensated demand or supply function (see Appendix 1.A.3). These elasticities are weighted by the expected earnings shares of labor and education in total earnings. The expression in (1.22) shows the trade-off between insurance and efficiency. The optimal tax on labor equates the marginal benefits of income insurance (ξ) with the marginal costs of providing it. The optimal tax rate increases when the government attaches a larger social value to income insurance as measured by a higher ξ.

The marginal costs consist of two terms: (i) tax-induced distortions on labor supply $\frac{t}{1-t}\,\varepsilon_{lt}$, and (ii) a fiscal externality, $\frac{t}{1-t}\,\pi_e\varepsilon_{et}$, stemming from the missing insurance markets.[9] The optimal tax decreases if the distortions in labor supply become more severe as indicated by a higher elasticity ε_{lt}. Indeed, if labor supply (and educational investments) would be completely inelastic ($\varepsilon_{lt} = \varepsilon_{et} = 0$), the optimal tax rate would be one hundred percent ($t = 1$).

The optimal tax rate is also determined by the tax elasticity of investments in education, as can be seen from the presence of the term $\pi_e\varepsilon_{et}$. In particular, the income tax may exacerbate or mitigate the non-tax distortions arising from the missing insurance markets. If education

[9]Following Heller and Starrett (1976), we interpret the (fiscal) impact of allocative distortions resulting from a missing market as an externality.

increases the exposure to labor market risk, the risk premium acts as if there is a pre-existing implicit tax on educational investment ($\pi_e > 0$). If educational investments hedge against labor market risk, the risk premium acts as if there is a pre-existing implicit subsidy on educational investment ($\pi_e < 0$). Provided that investment in human capital falls with a higher tax rate ($\varepsilon_{et} > 0$),[10] a higher income tax thus exacerbates (mitigates) underinvestment (overinvestment) in human capital if $\pi_e > 0$ ($\pi_e < 0$). The implicit tax (or subsidy) on education due to non-insurable income risks, thereby, creates a fiscal externality in the presence of positive income taxes.

This can be seen most clearly from the (expected) tax wedge on education $\mathcal{E}[\Delta_e]$, which measures the gain in tax revenue available for redistribution if human capital investment increases by one unit. By applying the first-order equation for optimal human capital investment (1.10), we can rewrite the expected net tax wedge on education as

$$\mathcal{E}[\Delta_e]|_{s=0} = \frac{\pi_e}{1-\pi_e}t. \tag{1.23}$$

$\frac{\pi_e}{1-\pi_e}$ represents the risk wedge on human capital investment. If there is underinvestment ($\pi_e > 0$) the social marginal benefits of an additional unit invested in education are larger than its social marginal costs. Consequently, the cost of the tax deduction on the marginal costs of the investment is smaller than the tax revenue from the marginal benefits of the investment in education, i.e., $\mathcal{E}[\Delta_e]|_{s=0} > 0$. Income taxation will exacerbate socially undesirable underinvestment by further reducing educational investments below first-best levels (if $\varepsilon_{et} > 0$), which decreases tax-revenues. Consequently, optimal tax rates are set lower (ceteris paribus). In the case of overinvestment ($\pi_e < 0$), the opposite holds true. In particular, the public cost of the tax deduction on the marginal costs of the investment is larger than the marginal revenue generated by taxing the returns to education, i.e., $\mathcal{E}[\Delta_e]|_{s=0} < 0$. Social insurance thus reduces socially undesirable overinvestment in human capital and increases tax revenue (if $\varepsilon_{et} > 0$). Optimal tax rates are set higher as a result (ceteris paribus). If education has no effect on the exposure to risk, there is no risk-premium on human capital investment ($\pi_e = 0$). Thus, the implicit tax on education is zero, because all costs are deductible against the rate at which returns are taxed. Hence, the fiscal externality vanishes. In that case, the optimal tax is only determined by the

[10] Although the tax system does not affect human capital investments directly, it does so indirectly by lowering labor supply as long as labor and education are complementary in generating gross income (i.e., $\Phi_{el} > 0$).

labor supply elasticity.[11]

We find the results of Eaton and Rosen (1980b) if we assume that the earnings function exhibits multiplicative risk. The latter implies that education will always increase the exposure to risk and there will be underinvestment, i.e., $\pi_e > 0$. However, Eaton and Rosen (1980b, pp. 712-714) do not derive an explicit expression for the optimal tax rate. We show that the optimal income tax is downward biased due to the negative fiscal externality ($\pi_e > 0$), which is a novel finding. If we assume that human capital investment is exogenous ($\varepsilon_{et} = \pi_e = 0$), we obtain the outcome of Eaton and Rosen (1980a): $\frac{t}{1-t} = \frac{\xi}{\varepsilon_{lt}}$. This formula captures the trade-off between insurance and labor supply distortions. The next proposition summarizes our findings from this subsection.

Proposition 1.1. *The optimal income tax trades off social insurance against the incentives to work, and the internalization of the fiscal externality stemming from missing insurance markets. If education increases (reduces) exposure to labor market risk, the income tax exacerbates (mitigates) the distortions of missing insurance markets on human capital investment.*

1.4.3 Optimal Education Policy

This section derives the optimal education policy for a given level of taxation $\bar{t}$. In doing so, we gain the intuition for the optimal structure of taxes and subsidies when both policy instruments are simultaneously optimized. By using Roy's lemma and the Slutsky-equations, we can rearrange the first-order condition for education subsidies (1.19) to find the optimal subsidy rate for a given $\bar{t}$ at optimal T (see Appendix 1.A.4):

$$\frac{s}{1-s} = \left(\frac{\frac{\varepsilon_{ls}}{\varepsilon_{es}}}{1-\pi_e} + \frac{\pi_e}{1-\pi_e} \right) \bar{t}. \tag{1.24}$$

$\varepsilon_{ls} = \frac{\mathcal{E}[\Phi_l(.)]l}{\mathcal{E}[\Phi(.)]} \frac{\partial l^*}{\partial s} \frac{1-s}{l}$ and $\varepsilon_{es} = \frac{\mathcal{E}[\Phi_e(.)]e}{\mathcal{E}[\Phi(.)]} \frac{\partial e^*}{\partial s} \frac{1-s}{e} > 0$ denote the expected utility compensated elasticities of labor and education with respect to the education subsidy. These elasticities are again weighted by the expected shares of labor and education in total earnings.

The insurance characteristic is absent in the expression for optimal education subsidies. In contrast to the optimal income tax – see previous section – there is no gain in using education

[11] The tax distortions in labor supply are still typically higher than in standard models without endogenous human capital investment, as long as education and labor are complementary in earnings, see, e.g., Jacobs (2005).

subsidies for insurance. Education subsidies are not used at all ($s = 0$) when the income tax is zero ($\bar{t} = 0$). In that case, the only way households can reduce their exposure to risk is to self-insure, i.e., to over- or underinvest in education. This self-insurance is chosen optimally. Education subsidies do not yield additional welfare gains in absence of income taxation ($\bar{t} = 0$), because education subsidies are state-independent. Education subsidies therefore do not directly reduce the exposure to income risk. Consequently, in the absence of tax-provided social insurance, the government cannot improve market outcomes by subsidizing education as this policy would only upset the optimal private responses of individuals to income risk by distorting human capital investment.

There is a role for education policy, however, when the government organizes social insurance through an income tax system ($\bar{t} > 0$). At an exogenously given tax rate $\bar{t} > 0$, education subsidies correct the tax-distortions of the income tax. There are two reasons why education subsidies are optimally employed. First, labor taxation distorts labor supply. If $\varepsilon_{ls} > 0$ education and labor supply are complementary in generating income. Thus, by subsidizing education, the government can indirectly boost labor supply and thereby reduce the tax distortions on labor supply. The higher is the tax rate t, the larger are the distortions on labor supply, and the larger is the need to fight these labor-tax distortions with education subsidies. Similarly, if education and labor are substitutes, $\varepsilon_{ls} < 0$, education should be taxed so as to increase labor supply and to off-set the tax wedge on labor. See also Jacobs and Bovenberg (2008). In both cases the government trades off fewer tax-induced distortions on labor supply against larger subsidy-induced distortions in educational investment. The more education responds to subsidies (larger ε_{es}), the larger is the social cost of undesirable overinvestment, and the lower is the optimal education subsidy.

Second, the education subsidy internalizes the fiscal externality, which is represented by the second term, $\frac{\pi_e}{1-\pi_e}\bar{t}$. Note that this term equals the implicit tax wedge on education in equation (1.23), $\mathcal{E}[\Delta_e]|_{s=0}$, where education subsidies are absent. Consequently, the education subsidy fully internalizes the fiscal externality arising from under- or overinvestment in human capital in the presence of income taxes. The higher is the exogenously given labor tax rate $\bar{t} > 0$, the larger is the fiscal externality $\frac{\pi_e}{1-\pi_e}$ due to the implicit tax on human capital. If education is risk-increasing, $\pi_e > 0$, education should be subsidized more in order to internalize the fiscal externality. If education has a risk-mitigating effect, $\pi_e < 0$, there is an implicit subsidy on human capital which is ceteris paribus off-set by an explicit tax on education.

By combining these two arguments, it becomes apparent that optimal education subsidies are unambiguously positive when education and labor supply are complementary ($\varepsilon_{ls} > 0$) *and* there is underinvestment in education ($\pi_e > 0$). In that case, education subsidies help to reduce both tax distortions in labor supply and to internalize the fiscal externality of underinvestment in education. If, on the other hand, education hedges against labor market risks ($\pi_e < 0$) the two arguments pull in opposite directions as long as education and labor remain complementary ($\varepsilon_{ls} > 0$). Therefore, the sign of the education subsidy cannot be unambiguously determined. In particular, education should be taxed if $\frac{\varepsilon_{ls}}{\varepsilon_{es}} < -\pi_e$. In that case, socially undesirable over-investment in education is relatively large compared to the complementarity of education with labor supply.

These findings are related to Bovenberg and Jacobs (2005) and Jacobs and Bovenberg (2008) who analyze optimal redistribution and education policy with ex ante differing individuals and no income risk. On the one hand, these authors also demonstrate that education subsidies boost labor supply and thereby help to off-set tax-distortions from social insurance. On the other hand, education subsidies generate inequality, due to the ability bias in education. The latter effect is absent in our model, since everyone is identical ex ante.[12] We summarize the findings of this section in the following proposition.

Proposition 1.2. *Education subsidies do not provide income insurance and are only used for efficiency reasons. First, education subsidies boost labor supply when education and work effort are complementary. Education subsidies are higher if labor and education are more complementary. Second, education subsidies (taxes) are used to internalize the fiscal externality. Optimal education subsidies are higher (lower) if there is more under- (over-)investment in human capital.*

1.4.4 Optimal Tax Cum Education Policy

By combining the expressions for the optimal tax and education policies (1.50) and (1.53) from Appendices 1.A.3 and 1.A.4, we obtain the optimal tax rate $\hat{t}$ and education subsidies $\hat{s}$ if the

[12] This is also the reason why education subsidies are not used for insurance.

government simultaneously optimizes income taxes and education subsidies:

$$\frac{\hat{t}}{1-\hat{t}} = \frac{\xi}{\varepsilon_{lt} - \frac{\varepsilon_{ls}}{\varepsilon_{es}}\varepsilon_{et}}, \tag{1.25}$$

$$\frac{\hat{s}}{1-\hat{s}} = \left(\frac{\frac{\varepsilon_{ls}}{\varepsilon_{es}}}{1-\pi_e} + \frac{\pi_e}{1-\pi_e}\right)\hat{t}. \tag{1.26}$$

Note that all statements in section 1.4.3 about the optimal education subsidy for given tax policy carry over to the case in which tax and education policies are simultaneously optimized. For this reason we do not discuss the expression in equation (1.26) further and refer to the previous section.

Our results bolster Anderberg's (2009) findings that the risk-properties of human capital are crucial for the design of optimal human capital policies. While Anderberg (2009) considers a general set of information-rich non-linear policy instruments, our analysis shows that the risk-properties of human capital are also key for optimal human capital policies under linear policy instruments, which are less informationally demanding. Optimal education policies will not ensure aggregate efficiency in human capital investment, since not all income risk will be fully diversified. Moreover, our analysis points out that the fiscal externalities associated with missing insurance markets are crucial for the design of educational policy.

The optimal tax rate $\hat{t}$ is no longer directly affected by the risk-wedge, since, compared to equation (1.22), the risk premium in education π_e ceases to enter the optimal tax formula. Hence, the income tax no longer exacerbates underinvestment if $\pi_e > 0$ and no longer mitigates overinvestment if $\pi_e < 0$. The expression for the optimal income tax confirms that the education subsidy perfectly internalizes the fiscal externality arising from under- or overinvestment in human capital. Education subsidies are a more efficient instrument to internalize the fiscal externality than income taxes, since the latter also distort labor supply. This finding mirrors the results on optimal taxation in the presence of externalities by Sandmo (1975, pp. 92, 95). He shows that externalities should optimally be internalized by only correcting the price of the commodity, which causes the externality, in an additive way ('additive property'). In our case this 'commodity' is education. We also find that the correction term enters additively in expression (1.26) for optimal education subsidies.

The interpretation of the optimal tax rate therefore changes slightly, since it is now exclusively used for insurance purposes. Naturally, the optimal tax rate still increases in the marginal benefits of insurance (ξ) and decreases in higher tax-induced distortions in labor supply (ε_{lt}). The new optimal tax expression (1.25), however, reveals that the optimal income tax increases if education and labor supply become more complementary as indicated by $\frac{\varepsilon_{ls}}{\varepsilon_{es}}$. See also the expression for the optimal education subsidy. Education subsidies boost labor supply if $\varepsilon_{ls} > 0$ and thereby help to off-set the tax distortions on labor effort. Consequently, income taxes increase (ceteris paribus). If education responds very elastically to education subsidies, ε_{es} is large, optimal tax rates are lower, since subsidies are more distortionary and exacerbate overinvestment in education.

The optimal use of education policy does not unambiguously increase optimal income tax rates, for given demand for redistribution ξ, and assuming that the elasticities remain the same. This can be inferred from comparing the optimal tax policy cum education subsidies, in equation (1.25), with the optimal tax rate in equation (1.22), where education policy is absent ($s = 0$). Intuitively, one would expect the optimal tax rate to be higher if the government has more instruments. This conclusion is not necessarily valid in the current second-best setting with multiple distortions. The intuition is only confirmed for the case where $\pi_e > 0$. Without education subsidies, income taxation exacerbates the non-tax distortions from missing insurance markets, which causes a negative fiscal externality. A lower tax rate is thus optimal. With optimal education policy internalizing the fiscal externality, the optimal income tax is unambiguously higher (even if $\varepsilon_{ls} = 0$). However, in the case of overinvestment in human capital due to missing insurance markets ($\pi_e < 0$), the income tax features a positive fiscal externality by mitigating non-tax distortions in human capital investment. When education subsidies, or even education taxes (e.g., if $\varepsilon_{ls} = 0$), are available, there is, however, no longer a role for the income tax to correct for overinvestment in human capital. As a result optimal income taxes may well be lower. The following proposition summarizes our findings.

Proposition 1.3. *If labor and education are more complementary, both the optimal tax rate and optimal education subsidies increase. If the risk premium on education rises, there is a rise in optimal education subsidies as well. If education increases earnings risk, education policy allows for more social insurance compared to tax policy alone. If education hedges against labor market risk, optimal tax rates with education policy could be lower than without education policy if the complementarity between education and labor is sufficiently weak.*

Our findings are importantly related to Hamilton (1987), Anderberg and Andersson (2003) and Anderberg (2009). Hamilton (1987) extends Eaton and Rosen (1980b) and analyzes capital taxes as an indirect education subsidy. Hamilton (1987) is right in pointing out that there remains underinvestment in education when income taxes are optimally set. Consequently, a capital tax could be welfare enhancing, because a capital tax is an indirect education subsidy. Since we assume that education is verifiable, we can allow for direct education subsidies. We have shown that the role for education policy is to internalize the fiscal externality associated with underinvestment. Therefore, we are able to show that the use of education subsidies is always welfare enhancing. Hamilton (1987) needs strong assumptions (constant absolute risk aversion and inelastic labor supply) to show that his education policy is desirable, since in contrast to education subsidies capital taxes also distort savings.

Anderberg and Andersson's (2003) analysis is closest to ours. The major difference is that these authors assume that the government can impose a mandatory level of education centrally. Consequently, there is no fiscal externality in human capital investment – which explains the absence of the risk premium in their optimal tax formula (in equation 11). Education policy then has an insurance effect, because it replaces self-insurance of households in a decentralized setting. Anderberg and Andersson (2003, p. 1523) state that *"The insight is thus that if education moderates wage uncertainty, a second-best policy should, rather unambiguously, encourage the formation of human capital (relative to the first-best), while if education exacerbates wage uncertainty the overall conclusion is ambiguous."* Though this statement is correct, it would be misleading to conclude that education subsidies (or taxes) would be an optimal policy when education decisions are made at the decentralized level. Indeed, if households choose educational investment themselves, there is no insurance effect of educational policy. More importantly, the novel finding of our paper is that in the presence of income taxation, there will be socially 'excessive' underinvestment (overinvestment) by households, compared to the constrained second-best optimal amount of underinvestment (overinvestment). This, ceteris paribus, calls for a policy that *discourages* (*encourages*) educational investment. Even under linear policy instruments it can be misleading to obtain policy recommendations from looking at the optimal wedges on individual choices and comparing these with the first-best choice-rules. The policy implementation in our setting is the polar opposite of what the wedges on education seem to suggest. As has also been stressed by Golosov et al. (2003, 2006), there is generally no clear-cut correspondence between tax wedges and tax rates that would implement optimal second-best allocations. We believe that this could also be an important issue in the recent papers in the new dynamic public finance tradition (see, e.g., da Costa and Maestri, 2007, and Anderberg, 2009). In light of this discussion, we rephrase our results in the following corollary.

Corollary 1.1. *From a positive (negative) tax wedge on education compared to the first-best rule cannot be concluded that education should be subsidized (taxed) if human capital investment is made at the decentralized level and the government only has indirect control over individual choices via subsidies and taxes.*

1.5 Conclusions

This paper analyzed optimal social insurance and education policy. The optimal income tax strikes a balance between benefits of social insurance and the distortions in labor supply. The optimal income tax is higher if education and work are more complementary, since the government can indirectly off-set labor-tax distortions by subsidizing education. Optimal education subsidies unambiguously increase if education and labor are more complementary. In that case, subsidies on education are a more attractive instrument to fight tax distortions on labor supply. The optimal income tax is not determined by the risk-properties of human capital, but optimal education policies are. Education subsidies are not used to off-set under- or overinvestment in human capital in the absence of taxation, because this would upset the optimal private response to market risks. However, the non-insurable risk due to missing insurance markets gives rise to a fiscal externality from income taxation. Boosting education yields more (less) tax revenues if there is underinvestment (overinvestment) in human capital. Subsidizing (taxing) education is optimal in order to internalize the fiscal externality originating from the missing insurance markets, ceteris paribus. Hence, if education is a risky activity there is a strong role for subsidizing education on a net basis to off-set the distortions of social insurance on human capital investments and labor supply. Social insurance will then increase. However, if education hedges against labor market risk, the case for education subsidies is weakened, and social insurance may even be reduced compared to the outcome in the absence of education subsidies. Whether education subsidies or education taxes should be employed is an empirical question that can only be answered by knowing the risk-properties of human capital.

Appendix 1.A

1.A.1 Deriving Slutsky Equations under Risk

In order to derive the expected utility-compensated substitution effects, we calculate how much lump-sum income T an individual should receive (pay) in order to keep its expected utility constant when either the tax rate t or the subsidy rate s changes. This is equivalent to deriving the expenditure function and applying Shephard's lemma.

The utility function is given by

$$\mathcal{E}[u(c_1, c_2)] - v(l), \quad u_1, u_2, v_l > 0, \quad u_{11}, u_{22}, -v_{ll} < 0. \tag{1.27}$$

and the period specific-budget constraints read as follows:

$$c_1 = (1-t)(1-(1-s)e) - a + \omega, \tag{1.28}$$

$$c_2 = (1-t)\Phi(\theta, l, e) + Ra + T, \tag{1.29}$$

where $R \equiv 1 + r$ is the interest factor, a denotes savings, and the general earnings function is given by

$$\Phi(\theta, l, e), \quad \Phi_e, \Phi_l > 0, \quad \Phi_{ee} < 0, \quad \Phi_{ll} \leq 0. \tag{1.30}$$

First, totally differentiate the expected utility function at constant utility to obtain

$$d\mathcal{E}[U] = \mathcal{E}[u_1 dc_1] + \mathcal{E}[u_2 dc_2] - v_l dl = 0. \tag{1.31}$$

Totally differentiating the budget constraints gives

$$dc_1 = (1-t)(1-s)de - da + d\omega - [1-(1-s)e]dt + (1-t)eds \tag{1.32}$$

$$dc_2 = (1-t)\Phi_l dl + (1-t)\Phi_e de + Rda - \Phi dt + dT. \tag{1.33}$$

Substituting the expressions in equations (1.32) and (1.33) into the differentiated utility function (1.31), and applying the first-order conditions for utility maximization, yields

$$\mathcal{E}[u_2\left(R(1-s)e-(\Phi+R)\right)]dt+\mathcal{E}[u_2]R(1-t)eds+\mathcal{E}[u_2]dT+Rd\omega=0. \tag{1.34}$$

After applying Steiner's Rule, setting the changes in the exogenous endowment to zero, $d\omega = 0$, and using the definition of the insurance characteristic $\xi \equiv -\frac{cov[\Phi,u_2]}{\mathcal{E}[\Phi]\mathcal{E}[u_2]} > 0$, the following compensations for tax and subsidy changes are obtained

$$\left.\frac{dT}{dt}\right|_{ds=0}=\frac{\partial T}{\partial t}=(1-\xi)\mathcal{E}[\Phi]+R(1-(1-s)\,e), \tag{1.35}$$

$$\left.\frac{dT}{ds}\right|_{dt=0}=\frac{\partial T}{\partial s}=-R(1-t)e. \tag{1.36}$$

Therefore, the Slutsky-equations are given by

$$\frac{\partial e}{\partial t}=\frac{\partial e^*}{\partial t}-\frac{\partial T}{\partial t}\frac{\partial e}{\partial T}=\frac{\partial e^*}{\partial t}-\left((1-\xi)\mathcal{E}[\Phi]+R(1-(1-s)\,e)\right)\frac{\partial e}{\partial T}, \tag{1.37}$$

$$\frac{\partial l}{\partial t}=\frac{\partial l^*}{\partial t}-\frac{\partial T}{\partial t}\frac{\partial l}{\partial T}=\frac{\partial l^*}{\partial t}-\left((1-\xi)\mathcal{E}[\Phi]+R(1-(1-s)\,e)\right)\frac{\partial l}{\partial T}, \tag{1.38}$$

$$\frac{\partial e}{\partial s}=\frac{\partial e^*}{\partial s}-\frac{\partial T}{\partial s}\frac{\partial e}{\partial T}=\frac{\partial e^*}{\partial s}+R(1-t)e\frac{\partial e}{\partial T}, \tag{1.39}$$

$$\frac{\partial l}{\partial s}=\frac{\partial l^*}{\partial s}-\frac{\partial T}{\partial s}\frac{\partial l}{\partial T}=\frac{\partial l^*}{\partial s}+R(1-t)e\frac{\partial l}{\partial T}. \tag{1.40}$$

where an asterisk $(^*)$ denote a compensated demand or supply function.

1.A.2 Optimal Policy Rules

The Lagrangian for maximization of social welfare is given by

$$\max_{\{T,t,s\}}\mathcal{L}\equiv\mathcal{E}\left[V(T,t,s)+\eta\left(t\Phi\left(\theta,l,e\right)+tR(1-e)-R\left(1-t\right)se-T-G\right)\right], \tag{1.41}$$

where η denotes the Lagrange-multiplier of the government budget constraint. The first-order conditions for this maximization problem are given by:

$$\frac{\partial \mathcal{L}}{\partial T} = \mathcal{E}\left[\frac{\partial V(.)}{\partial T} - \eta + \eta\Delta_l\frac{\partial l}{\partial T} + \eta\Delta_e\frac{\partial e}{\partial T}\right] = 0, \tag{1.42}$$

$$\frac{\partial \mathcal{L}}{\partial t} = \mathcal{E}\left[\frac{\partial V(.)}{\partial t} + \eta\left(\Phi\left(.\right) + R\left(1-e\right) + Rse\right) + \eta\Delta_l\frac{\partial l}{\partial t} + \eta\Delta_e\frac{\partial e}{\partial t}\right] = 0, \tag{1.43}$$

$$\frac{\partial \mathcal{L}}{\partial s} = \mathcal{E}\left[\frac{\partial V(.)}{\partial s} - \eta R(1-t)e + \eta\Delta_l\frac{\partial l}{\partial s} + \eta\Delta_e\frac{\partial e}{\partial s}\right] = 0, \tag{1.44}$$

where we introduced the following tax wedges on labor and education:

$$\Delta_l \equiv t\Phi_l\left(.\right), \tag{1.45}$$

$$\Delta_e \equiv t\left(\Phi_e\left(.\right) - R\right) - R\left(1-t\right)s. \tag{1.46}$$

Using Roy's lemma we obtain from reordering equation (1.17) the expected social marginal value of a unit increase in lump-sum income, including the income effects on the tax base:

$$\mathcal{E}\left[\frac{u_2}{\eta} + \Delta_l\frac{\partial l}{\partial T} + \Delta_e\frac{\partial e}{\partial T}\right] = 1. \tag{1.47}$$

1.A.3 Optimal Taxation

For deriving the optimal labor tax rate (at given education subsidies), we simplify the first-order condition (1.18) by substituting Roy's lemma and the Slutsky-equations (1.37) and (1.38). Reordering terms gives:

$$\begin{aligned} &-\frac{\mathcal{E}[u_2\left(\Phi(\theta,l,e) + R\left(1-\left(1-s\right)e\right)\right)]}{\eta} + \mathcal{E}[\Phi(\theta,l,e)] + R\left(1-\left(1-s\right)e\right) \\ &+\mathcal{E}[\Delta_e]\left(\frac{\partial e^*}{\partial t} - \left((1-\xi)\mathcal{E}[\Phi] + R(1-\left(1-s\right)e)\right)\frac{\partial e}{\partial T}\right) \\ &+\mathcal{E}[\Delta_l]\left(\frac{\partial l^*}{\partial t} - \left((1-\xi)\mathcal{E}[\Phi] + R(1-\left(1-s\right)e)\right)\frac{\partial l}{\partial T}\right) = 0. \end{aligned} \tag{1.48}$$

After using Steiner's Rule, the definition of ξ and the rearranged first-order condition for T

in equation (1.20), we receive

$$\xi\mathcal{E}[\Phi\,(.)] = -\mathcal{E}[\Delta_e]\frac{\partial e^*}{\partial t} - \mathcal{E}[\Delta_l]\frac{\partial l^*}{\partial t}. \tag{1.49}$$

Substituting Δ_e and Δ_l from equations (1.15) and (1.16), as well as $\mathcal{E}[\Phi_e] = \frac{R(1-s)}{1-\pi_e}$ from the household's first-order condition for optimal educational investment (i.e., equation (1.10) in the paper), and rearranging lead to

$$\xi = \frac{t}{1-t}\varepsilon_{lt} + \left(\frac{\pi_e(s+t(1-s))-s}{(1-s)(1-t)}\right)\varepsilon_{et}, \tag{1.50}$$

where $\varepsilon_{et} \equiv -\frac{\mathcal{E}[\Phi_e(.)]e}{\mathcal{E}[\Phi(.)]}\frac{\partial e^*}{\partial t}\frac{1-t}{e}$, and $\varepsilon_{lt} \equiv -\frac{\mathcal{E}[\Phi_l(.)]l}{\mathcal{E}[\Phi(.)]}\frac{\partial l^*}{\partial t}\frac{1-t}{l} > 0$ are the (negative) income weighted *expected utility* compensated elasticities of education and labor with respect to the tax rate. The elasticities are weighted by the expected shares of education and labor in total earnings. Finally, applying $\bar{s} = 0$, rearranging and collecting terms result in expression (1.22) in the paper.

1.A.4 Optimal Education Policy

For deriving the optimal education subsidies at a given labor tax rate $\bar{t} > 0$, we simplify the first-order condition for education subsidies (1.19) by substituting Roy's lemma and the Slutsky-equations (1.39) and (1.40). Reordering terms gives:

$$\mathcal{E}\left[R(1-t)e\left(\frac{u_2}{\eta} + \Delta_l\frac{\partial l}{\partial T} + \Delta_e\frac{\partial e}{\partial T} - 1\right)\right] + \mathcal{E}[\Delta_e]\frac{\partial e^*}{\partial s} + \mathcal{E}[\Delta_l]\frac{\partial l^*}{\partial s} = 0. \tag{1.51}$$

Since educational investment e is deterministic, we can utilize

$$R(1-t)e\,\mathcal{E}\left[\frac{u_2}{\eta} + \Delta_l\frac{\partial l}{\partial T} + \Delta_e\frac{\partial e}{\partial T} - 1\right] = 0, \tag{1.52}$$

which follows from equation (1.20). Hence,

$$\frac{\mathcal{E}[\Delta_e]e}{1-s}\frac{\partial e^*}{\partial s}\frac{1-s}{e} + \frac{\mathcal{E}[\Delta_l]l}{1-s}\frac{\partial l^*}{\partial s}\frac{1-s}{l} = 0. \tag{1.53}$$

Substituting Δ_e and Δ_l from equations (1.16) and (1.15) as well as applying the household's first-order condition for educational investment, $\mathcal{E}[\Phi_e] = \frac{R(1-s)}{1-\pi_e}$, from equation (1.10) in the paper, results in

$$\bar{t}\varepsilon_{ls} + \frac{\pi_e(s + \bar{t}(1-s)) - s}{1-s}\varepsilon_{es} = 0, \tag{1.54}$$

where we defined the subsidy elasticities as the income weighted *expected utility* compensated elasticities, $\varepsilon_{es} \equiv \frac{\mathcal{E}[\Phi_e(.)]e}{\mathcal{E}[\Phi(.)]}\frac{\partial e^*}{\partial s}\frac{1-s}{e} > 0$ and $\varepsilon_{ls} \equiv \frac{\mathcal{E}[\Phi_l(.)]l}{\mathcal{E}[\Phi(.)]}\frac{\partial l^*}{\partial s}\frac{1-s}{l}$. Rewriting yields expression (1.24) in the paper.

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2 catalysts for Social Insurance: Education Subsidies vs. Capital Taxation

2.1 Introduction

In all developed countries, the labor income tax plays an important role as revenue generator for financing public sector expenditures and as a means of income redistribution. As private insurance markets are incomplete (Sinn, 1996), labor taxation is furthermore crucial in providing social insurance against income risks due to uncertain outcomes in education and (productivity) shocks in labor markets (Eaton and Rosen, 1980b). Since globalization has amplified income risks in the last years, the latter role has significantly gained importance. Moreover, we observe since the mid 1980's that globalization has driven down both corporate and personal capital taxes by intensifying tax competition and increasing the elasticity of real savings (see, e.g., Winner, 2005). Furthermore, there is (anecdotal) evidence at least in Europe that education subsidies have decreased as well (i.e., that tuition fees increased). However, both (real) capital taxation and education subsidies are still additional policies for social insurance.

Now, this paper focuses on the insurance role of taxation and on the net efficiency costs, which are unavoidably created in the process. We derive the optimal social insurance package as combination of labor and capital taxation as well as educational policy. The new aspect will be on optimally combining the additional policies in order to foster insurance provision via boosting labor supply. Analogously to catalysts in chemical reactions, capital taxation and education subsidies facilitate providing social insurance by reducing efficiency costs, but they do not provide insurance themselves. Our findings are highly policy relevant, since we show that capital taxation and education subsidies are strategic substitutes. This calls the development described above and in particular the recent restrictive education policies in light of ongoing tax competition in question.

A big chunk of risk in labor income is due to uncertainty associated with education. Educational investment can both mitigate or aggravate the exposure to income risk.[1] Thus, it is natural to bring educational choices into the picture. That leads to a dynamic set-up, where savings in real capital must enter. Since both decision margins are close substitutes (e.g., Nielsen and Sørensen, 1997), any policy which fosters (hampers) human capital investment will obviously harm (promote) real savings and vice versa. Hence, besides providing social insurance, designing an education policy and incorporating the treatment of savings in real capital are some of the most important tasks of the modern welfare state.

The idea of perceiving supplementary instruments as catalysts of social insurance via the labor income tax is – especially with respect to real capital taxation – only just developed. Important forerunners in absence of human capital investment are Jacobs and Schindler (2009) as well as Kocherlakota (2005). The former focus on linear tax instruments and point out that capital taxation mitigates labor tax distortions by intertemporal wealth and substitution effects, which increase opportunity costs of leisure. Following the 'new dynamic public finance' approach (see also Golosov et al., 2006), Kocherlakota shows in a non-linear taxation setting that there is a role for capital taxation, since capital taxes relax the incentive constraints for non-mimicking. Incorporating (unobservable) human capital formation, Hamilton (1987) states that positive capital taxation should overcome underinvestment in education, which is due to uninsurable risk and self-insurance, by decreasing intertemporal opportunity costs of educational investment, if (i) labor supply is exogenous and (ii) either savings are zero or absolute risk aversion is constant. His results are backed by Grochulski and Piskorski (2010), who apply non-linear tax instruments.

Turning to observable educational investment, Anderberg and Andersson (2003) state that education should be overprovided (underprovided) if it is a risk-decreasing (risk-increasing)

[1] Empirical evidence that human capital investment cuts both ways with respect to exposure to risk is provided, e.g., by Palacios-Huerta (2004), Belzil and Hansen (2004), and Hartog (2005). The theoretical analysis dates back to Levhari and Weiss (1974).

activity. In doing so, educational policy exploits the insurance effect of human capital and it complements social insurance by income taxation. In their approach the government directly controls educational investment. Jacobs et al. (2010) make clear that the results from the former paper cannot be transferred to a decentralized setting. Individuals already exploit the insurance effect of education by self-insurance. Still, education subsidies are used for mitigating labor supply distortions by increasing the effective wage rate, but they do not provide insurance. Furthermore, the optimal level of education subsidies crucially depends on internalizing a fiscal externality, which stems from the interplay of labor taxation and over- or underinvestment into education.

To the best of our knowledge, only da Costa and Maestri (2007) and Anderberg (2009) analyze the simultaneous use of education subsidies and capital taxation. They apply a non-linear taxation setting and find a positive intertemporal wedge, indicating a role for capital taxation. However, their results differ as to whether the aggregate level of educational investment should be socially efficient. Moreover, it is generally difficult or even impossible to implement these optimal wedges through tax instruments.

Compared to these contributions, we offer a comprehensive model allowing to derive optimal capital taxation and optimal education subsidies simultaneously and to account for interactions between these two catalysts. To that end, we apply a two-period life-cycle model, where ex-ante homogenous households invest in education, decide on savings and choose labor supply. In the second-period, income realizes according to a general earnings function, which depends on educational investment, labor supply and an idiosyncratic shock. The exposure to risk can be increasing or decreasing in human capital. In any case, second-period consumption is stochastic, and households are heterogenous ex-post. In line with the literature, we assume that insurance markets are missing. Nevertheless, the government can provide social insurance through redistributive income taxation. The policy package consists of a linear income tax accompanied by a lump-sum transfer, a proportional capital tax and linear education subsidies.

Firstly, our analysis leads to the new insights that education subsidies and capital taxes differ both in the way they boost labor supply and in the distortions they induce. The popular argument that only capital taxes should be used, because they are more effective in boosting labor supply, is shown to be wrong. Similarly, the statement that only education subsidies should be used, since they are less distortive, does not hold either. We derive an explicit formula for the optimal education subsidy rate and optimal capital tax rate mix, showing the trade-offs between their net complementarity effects on labor supply and their net distortions of educational investment and real savings respectively. Moreover, we identify "indirect complementarity effects", which reflect offsetting interactions between the two catalysts. In particular, the more education subsidies (capital taxation) counteract the efficiency effect of the other instrument, the less education subsidies (capital taxation) should be employed.

Secondly, by extending the model used by Hamilton (1987), we show that there is no longer a role for capital taxation in internalizing the (fiscal) effect of self-insurance by under- or over-investing into education. Consequently, the additive property (Sandmo, 1975) holds and solely education subsidies are used for correcting inefficient educational investment, because they are the more efficient instrument to control for the education level. Only in the special case where education subsidies are not available, the Hamilton-intuition carries over. By incorporating endogenous labor supply and a general earnings function into our model, we point out that the Hamilton-result also holds under weaker conditions. Furthermore, we show that the optimal capital tax rate can become negative, if there is severe overinvestment in education. In this case, discouraging excessive educational investment by a subsidy on capital income overcompensates its negative effects on labor supply.

Thirdly, we derive a Ramsey-type rule for optimal education subsidies and capital taxes, showing that capital taxes and education subsidies are (strategic) substitutes. Thus, if in a third best scenario capital taxes are constrained due to tax competition and globalization, optimal policy requires an an increase in education subsidies. This result is highly policy-relevant.

Fourthly, we complement results from the 'new dynamic public finance' literature (e.g., Anderberg, 2009) by showing that the main results and the basic intuition in this strand of literature are also valid under linear tax instruments which requires much less information. The advantage of our setting is that the tax structure can be directly implemented and that the driving forces behind the optimal instruments as well as their interaction can be characterized explicitly.

The remainder of the paper is as follows. Section 2.2 introduces the model and sets up the optimal tax problem. Section 2.3 derives the optimal social insurance package. As a benchmark case, we discuss optimal labor taxation, if no other instruments are available. Then, we describe the optimal use of education subsidies and capital taxes as catalysts and finally we analyze the optimal mix of labor tax, education subsidies and capital taxes. Section 2.5 concludes.

2.2 The Model

2.2.1 Technologies and Preferences

We analyze a two-period model, where a continuum of ex-ante identical households decide on their educational investment, consumption and second-period labor supply. We assume that individuals are endowed with an initial wealth ω and with one unit of time in each period. We assume further that education e is pure time investment and that there is no labor-leisure decision in the first period. Educational investment is observable and verifiable. Hence, education costs, i.e., the forgone earnings, reduce the income tax base and they can be taxed or subsidized by educational policy.[2] Apart from investing in education, households can save or borrow in a perfect capital market. Savings are denoted by a. The first-period budget constraint (before taxation and education subsidies) reads

$$a = \omega + (1 - e) - c_1, \tag{2.1}$$

where c_1 is consumption in the first period, and where we have normalized the first-period wage rate as well as the price of consumption to 1.

In the second period, households supply labor l and consume c_2 financed from savings and second period labor income. Gross labor income is represented by a general earnings function, depending on hours worked l and education e:

$$\Phi(\theta, l, e), \quad \Phi_e,\ \Phi_l > 0, \quad \Phi_{ee} < 0, \quad \Phi_{ll} \leq 0. \tag{2.2}$$

θ is an idiosyncratic shock drawn from a probability distribution $f(\theta)$. Therefore, both income and the return to education are risky. We assume that, for any given value of θ, the marginal return to education Φ_e is positive and decreasing. Similarly, the marginal return to labor Φ_l is positive and non-increasing. Furthermore, the random variable θ is assumed to have a positive effect on income: $\Phi_\theta > 0$. In the remainder of the analysis, we focus on the two cases identified in the literature (cf. Levhari and Weiss, 1974): (i) educational investment amplifies income risks ($\Phi_{\theta e} > 0$), and (ii) educational investment hedges against income risks ($\Phi_{\theta e} < 0$). The

[2] Without any loss of generality we could also allow for direct resource costs of education. As long as all inputs are verifiable, this does not change the results (see also Bovenberg and Jacobs, 2003, 2005).

consumer budget constraint in the second-period without taxes is

$$c_2 = \Phi(\theta, l, e) + (1 + r) \cdot a, \tag{2.3}$$

where c_2 is consumption in the second period, and r is the real interest rate.

Households derive utility from consumption and disutility from labor. They maximize a von Neumann-Morgenstern expected utility function. Following common practice in the optimal tax literature under risk we assume the utility function to be additively separable in consumption and leisure (see, e.g., Cremer and Gahvari, 1995a, 1995b; Golosov et al., 2006; Diamond, 2006):[3]

$$EU = \mathcal{E}[U(c_1, c_2, l)] = \mathcal{E}[u(c_1, c_2)] - v(l), \quad u_1, u_2, -v_l > 0, \quad u_{11}, u_{22}, -v_{ll} \leq 0, \tag{2.4}$$

where $\mathcal{E}$ denotes the expectation operator, i.e., $\mathcal{E}[X] \equiv \int_\Theta X df(\theta)$, and Θ is the set of values for θ. The sub-utility function of consumption u is increasing and concave, whereas the disutility function of labor v is increasing and convex. All functions are at least twice differentiable, and we assume the Inada conditions to hold.

Insurance markets to insure (idiosyncratic) income risks are missing[4] due to moral hazard, adverse selection, and, as Sinn (1996, p. 261ff) points out, timing and contract problems. Sinn argues that perfect insurance contracts would have to be signed before the veil of ignorance has lifted, which is hardly possible with respect to, e.g., human capital risks or innate abilities, since parents would have to sign the contracts for their children or even for unborn children. Since a child would have to fulfill these obligations incontestably for all its life, this system would come close to bondage. Thus, Sinn (1996, p. 278) concludes that such insurance *"cannot be provided privately unless the fundamentals of western civil law are called into question."*

Instead, government can provide social insurance by redistributive taxation. We assume a linear income tax system with a marginal tax rate t and a lump-sum transfer T, which can be seen as a negative income tax or basic income. Furthermore, educational investment is subsidized at a flat rate s and education costs are tax deductible. Last but not least, the return

[3]This assumption fulfills the requirements for the Atkinson-Stiglitz theorem to hold. Hence, any non-zero capital tax rate will be due to incomplete insurance markets and providing social insurance.

[4]Note that this assumption simplifies the analysis, but does only affect the level of taxation, not the optimal tax structure.

to savings is taxed at a flat rate τ. Interest expenses on borrowing are subsidized at the same rate, i.e., there is full loss off-set. Taken together, our model is similar to the set-up in Hamilton (1987). However, in our model education can be directly subsidized or taxed, and we allow for a more general risk process, where education can either intensify income risk or hedge against it.

The timing structure of the model is as follows: In period 1 the government sets the proportional labor tax rate t, the subsidy rate s, the capital tax rate τ, and the lump-sum transfer T. After the policies are announced, households choose educational investment e, first-period consumption c_1, and labor supply l simultaneously, before risk realizes.[5] In period 2, (income) risk realizes, net incomes are earned and second-period consumption takes place. Thus, second-period consumption is stochastic, while first-period consumption, working time and education are deterministic.

2.2.2 Households

The representative household faces a stochastic intertemporal budget constraint

$$c_2 = (1-t) \cdot \Phi(e, l, \theta) + R \cdot [\omega + (1-t)(1-(1-s)e) - c_1] + T, \tag{2.5}$$

where $R = 1+r\cdot(1-\tau)$ represents the net interest factor on household saving. Subject to budget constraint (2.5) a household maximizes its expected utility function $EU = \mathcal{E}[u(c_1, c_2)] - v(l)$ by choosing optimal intertemporal consumption, educational investment and second-period labor supply. Consequently, the maximization problem turns into

$$\max_{c_1, l, e} \mathcal{E}[u(c_1, (1-t) \cdot \Phi(e, l, \theta) + R \cdot [\omega + (1-t)(1-(1-s)e) - c_1] + T)] - v(l), \tag{2.6}$$

[5] It can be shown that a timing sequence, in which labor supply is chosen after uncertainty has been resolved, does not change any of the results qualitatively, cf. Cremer and Gavhari (1995a), and Anderberg and Andersson (2003).

and the appropriate first order conditions are

$$\mathcal{E}[u_1] - R \cdot \mathcal{E}[u_2] = 0, \tag{2.7}$$

$$\mathcal{E}[u_2 \cdot \{(1-t)\Phi_e(e,l,\theta) - R \cdot (1-t)(1-s)\}] = 0. \tag{2.8}$$

$$\mathcal{E}[u_2 \cdot (1-t)\Phi_l(e,l,\theta)] - v_l = 0, \tag{2.9}$$

Equation (2.7) implies that for optimal intertemporal allocation of consumption the expected marginal rate of substitution is equal to the net interest factor,

$$\frac{\mathcal{E}[u_1]}{\mathcal{E}[u_2]} = R, \tag{2.10}$$

i.e. the standard Euler equation holds.

From the first order condition for optimal educational investment (2.8) it follows by Steiner's Rule that the risk-adjusted marginal return to educational investment is equal to the present value of marginal investment costs (after subsidization),

$$(1-\pi_e) \cdot \mathcal{E}[\Phi_e] = R \cdot (1-s), \tag{2.11}$$

where $\pi_e = -\frac{cov(u_2,\Phi_e)}{\mathcal{E}[u_2]\mathcal{E}[\Phi_e]} \in (-1,1)$ represents the risk premium in educational investment, measuring disutility from increased exposure to risk. π_e is positive, if education is risk-increasing, i.e., if $\Phi_{\theta e} > 0$, which holds for sector-specific human capital investment. π_e is negative, instead, if education serves as a hedging device and provides insurance against income risks (e.g., general upper secondary education). This is the case, if $\Phi_{\theta e} < 0$.

As we assume educational investment to be observable and tax deductible, the tax system does not directly affect investment in education. However, taxation generally affects investment in education indirectly via labor supply: a tax-induced decrease in labor supply lowers the return to human capital investment as long as $\Phi_{el} > 0$. This is the case for all earnings functions discussed in the literature (cf. Jacobs and Bovenberg, 2008). As a result, taxation reduces the incentives for investing into education. Instead, education subsidies reduce marginal costs and boost educational investment.

Missing insurance markets, however, drive a wedge between expected marginal return to education and the net investment costs, implying

$$\mathcal{E}[\Phi_e] - R \cdot (1-s) = \pi_e \cdot \mathcal{E}[\Phi_e] = \frac{\pi_e}{1-\pi_e} \cdot R \cdot (1-s) \gtrless 0 \quad \text{if} \quad \pi_e \gtrless 0. \tag{2.12}$$

Facing uninsurable income risk, households use educational investment as self-insurance device to reduce their exposure to risk. If education is risk-increasing (risk-decreasing), households invest too little (much) in education from a social point of view, viz., expected marginal returns are higher (lower) than marginal costs. This socially inefficient – but individually rational – level of investment will be the worse, the more risk-averse households are, i.e., the higher the risk premium is in absolute terms.

Accordingly, the first order condition for labor supply (2.9) can be rearranged as

$$(1-\pi_l)(1-t)\mathcal{E}[\Phi_l] = \frac{v_l}{\mathcal{E}[u_2]}, \tag{2.13}$$

where $\pi_l = -\frac{cov(u_2,\Phi_l)}{\mathcal{E}[u_2]\mathcal{E}[\Phi_l]}$ mirrors the risk premium in labor supply. Hence, for optimal labor supply the risk-adjusted net wage rate equals the marginal rate of substitution between consumption and labor. The presence of risk acts as an additional tax on labor, if labor supply is a risk-increasing activity ($\pi_l > 0$). But it acts as a wage subsidy, if higher labor supply reduces the exposure to income risk ($\pi_l < 0$).

Substituting the optimal consumption, educational investment and labor supply in the expected utility function, we receive the expected indirect utility function

$$V(T,t,s,R) = \mathcal{E}[u(\hat{c}_1,\hat{c}_2)] - v(\hat{l}), \tag{2.14}$$

where hats indicate the optimal values.

For later reference, we apply the envelope theorem (Roy's lemma) to find the derivatives of the indirect utility function as $\frac{\partial V}{\partial T} = \mathcal{E}[u_2]$, $\frac{\partial V}{\partial t} = -\mathcal{E}[u_2 \cdot \{\Phi(e,l,\theta) + R \cdot (1-(1-s)e)\}]$, $\frac{\partial V}{\partial s} = \mathcal{E}[u_2] \cdot R \cdot (1-t) \cdot e$ and $\frac{\partial V}{\partial R} = \mathcal{E}[u_2] \cdot [\omega + (1-t)(1-(1-s) \cdot e) - c_1]$.

2.2.3 Government

We assume a benevolent government with full (and credible) commitment. Hence, a time-inconsistency motive does not appear. Without loss of generality, we abstract from an exogenous government revenue requirement. The government chooses policy instruments T, t, s and R to maximize the expected indirect utility $V\left(T,t,s,R\right)$. The informational requirements for employing linear instruments are that only aggregate income, aggregate savings and aggregate education choices need to be verifiable to the government, i.e., taxes can be collected and subsidies can be paid in a withholding fashion (at firm level) and individual incomes need not to be observed.

By the law of large numbers, individual idiosyncratic risks cancel in the aggregate and we find that the government budget constraint is given by

$$t\cdot[\mathcal{E}[\Phi(e,l,\theta)]+(1+r)\cdot(1-(1-s)\cdot e)]+(1+r-R)\cdot[\omega+(1-t)(1-(1-s)\cdot e)-c_1]-(1+r)\cdot s\cdot e=T \tag{2.15}$$

All tax revenue is deterministic at the aggregate level and it is used to finance the lump-sum transfer and education subsidies. We abstract from any systematic risk.[6]

Taken together, the optimization problem can be formulated as:

$$\max_{T,t,s,R} V(T,t,s,R) \quad \text{s.t.} \quad (2.15) \tag{2.16}$$

[6]In case of additional systematic (aggregate) income risks, government's tax revenue would turn risky, as well. This would require an additional insurance device in the form of public consumption for smoothing aggregate shocks over private and public consumption (see, e.g., Kaplow, 1994), but it should not affect our main findings on insuring the idiosyncratic part of risk.

Denoting the Lagrange multiplier as η, the first order conditions are

$$\frac{\partial V}{\partial T} + \eta \cdot \left\{ \Delta_e \cdot \frac{\partial e}{\partial T} + \Delta_l \cdot \frac{\partial l}{\partial T} + \Delta_{c_1} \cdot \frac{\partial c_1}{\partial T} - 1 \right\} = 0, \tag{2.17}$$

$$\frac{\partial V}{\partial t} + \eta \cdot \left\{ \Delta_e \cdot \frac{\partial e}{\partial t} + \Delta_l \cdot \frac{\partial l}{\partial t} + \Delta_{c_1} \cdot \frac{\partial c_1}{\partial t} + \mathcal{E}[\Phi(.)] + R \cdot (1 - (1-s) \cdot e) \right\} = 0, \tag{2.18}$$

$$\frac{\partial V}{\partial s} + \eta \cdot \left\{ \Delta_e \cdot \frac{\partial e}{\partial s} + \Delta_l \cdot \frac{\partial l}{\partial s} + \Delta_{c_1} \cdot \frac{\partial c_1}{\partial s} - R \cdot (1-t) \cdot e \right\} = 0, \tag{2.19}$$

$$\frac{\partial V}{\partial R} + \eta \cdot \left\{ \Delta_e \cdot \frac{\partial e}{\partial R} + \Delta_l \cdot \frac{\partial l}{\partial R} + \Delta_{c_1} \cdot \frac{\partial c_1}{\partial R} - [\omega + (1-t)(1-(1-s) \cdot e) - c_1] \right\} = 0. \tag{2.20}$$

To simplify notations we introduce the (expected) tax wedges

$$\begin{aligned} \Delta_e &= t \cdot \{\mathcal{E}[\Phi_e] - R \cdot (1-s)\} - R \cdot s - \tau r \\ &= t \cdot \frac{\pi_e}{1-\pi_e} \cdot R \cdot (1-s) - R \cdot s - \tau r, \end{aligned} \tag{2.21}$$

$$\Delta_l = t \cdot \mathcal{E}[\Phi_l], \tag{2.22}$$

$$\Delta_{c_1} = -\tau r. \tag{2.23}$$

The tax wedges indicate the (expected) change in total tax revenue, based on behavioral responses of households, due to a marginal change in one of the tax instruments. Thereby, the second equality in equation (2.21) stems from applying the households' first order condition (2.8) twice.

2.2.4 Decision Margins and Distortions

The task of the government is to provide social insurance (i.e., to redistribute between "winners" and "losers"). Income risk can be reduced by implementing a wage tax and granting a deterministic lump-sum transfer. However, this comes at the cost of distorting labor supply and creating a fiscal externality. The latter stems from the fact that the marginal return and the marginal costs of educational investment are not equalized due to self-insurance of households by under- or overinvesting in education. Consequently, a marginal increase in educational investment creates a positive (negative) tax-revenue effect in case of underinvestment (overinvestment) in education (see Jacobs et al. 2010).

In order to alleviate these efficiency costs, both education subsidies and capital taxation can be applied as 'catalysts' for social insurance via labor taxation. Education subsidies increase human capital investment and, thus, the effective wage rate. As a result, education subsidies alleviate labor tax distortions by increasing labor supply. However, this comes at the cost of distorting educational investment.

Capital income taxation fosters labor supply via two channels: First, it works as indirect education subsidy by reducing the opportunity costs of human capital investment. Consequently, it encourages labor supply in the second period, but it distorts educational investment. Second, capital taxation reduces second-period consumption and boosts labor supply by increasing the marginal utility of income, viz., the opportunity costs of second-period leisure.[7] However, the latter effect distorts intertemporal consumption.

In the following, we analyze how these three instruments can be combined optimally in order to balance the net distortions on all margins and to provide an optimal social insurance package. The main question to be answered is, how education subsidies and capital taxation can serve as catalysts to facilitate income insurance.

2.3 The Social Insurance Package

2.3.1 Optimal Transfer Income

Following Diamond (1975), we define the expected net social marginal value of income, including the income effects on the tax base, as $b \equiv \frac{\mathcal{E}[u_2]}{\eta} + \Delta_e \cdot \frac{\partial e}{\partial T} + \Delta_l \cdot \frac{\partial l}{\partial T} + \Delta_{c_1} \cdot \frac{\partial c_1}{\partial T}$. From the first order condition (2.17) we get

$$b \equiv \frac{\mathcal{E}[u_2]}{\eta} + \Delta_e \cdot \frac{\partial e}{\partial T} + \Delta_l \cdot \frac{\partial l}{\partial T} + \Delta_{c_1} \cdot \frac{\partial c_1}{\partial T} = 1. \tag{2.24}$$

Hence, the optimal lump-sum transfer T balances the net marginal value of income (from the society's perspective) against its marginal revenue costs, which are equal to 1.

[7] See Cremer and Gahvari (1995a) or Jacobs and Schindler (2009) for a detailed analysis of this effect.

2.3.2 Labor Taxation without catalysts

First, we derive as a benchmark case the optimal labor tax rate t without catalysts, i.e., when the government can neither use education subsidies nor capital taxation. This case corresponds to the set of instruments in Eaton and Rosen (1980b) when neither educational investment nor savings are verifiable to the government.

Analogously to the distributional characteristic by Atkinson und Stiglitz (1980), we define the insurance characteristic

$$\xi \equiv -\frac{cov(u_2, \Phi(.))}{\mathcal{E}[u_2] \cdot \mathcal{E}[\Phi(.)]} > 0 \tag{2.25}$$

as the negative normalized covariance between the marginal utility of income and income. The insurance characteristic ξ captures the marginal welfare loss of income risk and measures the government's concern for insurance.

Moreover, we define the expected-utility compensated elasticities with respect to the labor tax rate as $\varepsilon_{et} = \frac{\partial e^c}{\partial t}\frac{1-t}{e}$ and $\varepsilon_{lt} = \frac{\partial l^c}{\partial t}\frac{1-t}{l}$. By applying the Slutsky equations and equation (2.24) to eliminate the income effects and by inserting $s = \tau = 0$, the optimal labor tax rate can be derived from equation (2.18) as (see Appendices 2.A.1 and 2.A.2)

$$\frac{t}{1-t} = \frac{\xi}{\omega_l \cdot (-\varepsilon_{lt}) - \pi_e \cdot \omega_e \cdot \varepsilon_{et}}. \tag{2.26}$$

$\omega_l = \frac{\mathcal{E}[\Phi_l l]}{\mathcal{E}[\Phi]}$ and $\omega_e = \frac{\mathcal{E}[\Phi_e e]}{\mathcal{E}[\Phi]}$ are the expected earnings shares of labor and education in total earnings, respectively.

Equation (2.26) represents the standard trade-off between the welfare gain of providing insurance and the efficiency costs of doing so. The higher the benefits from social insurance, as measured by a higher $\xi > 0$, the higher is the optimal tax rate t. However, social insurance comes with an excess burden, in particular from distorting labor supply. We assume that all compensated elasticities maintain the signs they have in a world without uncertainty. Hence $\varepsilon_{lt} < 0$.[8] And the larger $(-\varepsilon_{lt}) > 0$ is ceteris paribus, the higher is the distortion of labor supply and the lower should the optimal tax rate be.

Furthermore, the optimal labor tax rate depends on a fiscal externality $(\pi_e \cdot \omega_e \cdot \varepsilon_{et})$, which can

[8] Though in principle the signs of some of these elasticities are ambiguous due to offsetting insurance effects, this assumption should hold under mild restrictions; see Jacobs and Schindler (2009) for a comprehensive discussion in a related setting as well as Jacobs and Bovenberg (2010) for signing elasticities in a deterministic model.

be of any sign.[9] Note that the labor tax elasticity of educational investment is negative, $\varepsilon_{et} < 0$, because an increase in labor taxation decreases (compensated) labor supply and therefore the utilization of human capital. In case of $\pi_e > 0$ ($\pi_e < 0$), where there is under- (over-)investment in education, a marginal decrease of educational investment decreases (increases) tax revenue. Therefore, increasing the labor tax rate causes a negative (positive) fiscal externality, calling for a lower (higher) labor tax rate.

2.3.3 Education Subsidies and Capital Taxation as catalysts

In case both educational investment and savings in real capital are observable and verifiable, the government can use both instruments as catalysts for social insurance policy.

We define the expected-utility compensated elasticities with respect to the subsidy rate as $\varepsilon_{es} = \frac{\partial e^c}{\partial s}\frac{1-s}{e}$, $\varepsilon_{ls} = \frac{\partial l^c}{\partial s}\frac{1-s}{l}$, and $\varepsilon_{c_1 s} = \frac{\partial c_1^c}{\partial s}\frac{1-s}{c_1}$. Furthermore, the corresponding elasticities with respect to the after tax interest rate R are denoted as $\varepsilon_{eR} = \frac{\partial e^c}{\partial R}\frac{R}{e}$, $\varepsilon_{lR} = \frac{\partial l^c}{\partial R}\frac{R}{l}$, and $\varepsilon_{c_1 R} = \frac{\partial c_1^c}{\partial R}\frac{R}{c_1}$.

The optimal education subsidies follow from combining the first order conditions (2.19) and (2.20) and applying the optimal lump-sum transfer (2.24) (see Appendix 2.A.3) as

$$\frac{s}{1-s} = \left[\frac{\varepsilon_{ls} - \varepsilon_{as}\cdot\frac{\varepsilon_{lR}}{\varepsilon_{aR}}}{\varepsilon_{es} - \varepsilon_{as}\cdot\frac{\varepsilon_{eR}}{\varepsilon_{aR}}}\right]\frac{\omega_l}{\omega_e}\cdot\frac{\hat{t}}{1-\pi_e} + \frac{\pi_e}{1-\pi_e}\cdot\hat{t}. \tag{2.27}$$

By inserting expression (2.27) in equation (2.51) in Appendix 2.A.3, the optimal capital tax rate follows after some rearrangements as

$$\frac{\tau r}{R} = \left[\frac{(-\varepsilon_{lR}) + \varepsilon_{eR}\cdot\frac{\varepsilon_{ls}}{\varepsilon_{es}}}{\varepsilon_{aR} - \varepsilon_{eR}\cdot\frac{\varepsilon_{as}}{\varepsilon_{es}}}\right]\omega_l\cdot\hat{t}. \tag{2.28}$$

We define $\gamma_e = \frac{R\cdot e}{\mathcal{E}[\Phi]}$ and $\gamma_{c_1} = \frac{R\cdot c_1}{\mathcal{E}[\Phi]}$ as shares of expenditure on education and first-period consumption in total earnings, respectively. Moreover, we define the savings elasticity with respect to education subsidies as $\varepsilon_{as} = -\left(\gamma_e\cdot\varepsilon_{es} + \gamma_{c_1}\cdot\varepsilon_{c_1 s}\right) < 0$, which comprises the expenditure-share weighted effects of education subsidies on educational investment and on first-period consumption. We assume that education subsidies increase first-period consumption ($\varepsilon_{c_1 s} > 0$). The reasoning is as follows: education subsidies increase total income by encouraging education and increasing labor supply. The resulting higher labor income increases consumption in

[9] See Jacobs et al. (2010) for a detailed discussion of the fiscal externality.

both periods from consumption-smoothing. The savings elasticity with respect to the net interest rate is defined as $\varepsilon_{aR} = -\left(\gamma_e \cdot \varepsilon_{eR} + \gamma_{c_1} \cdot \varepsilon_{c_1R}\right) > 0$. It is unambiguously positive, because a higher net interest rate R renders both educational investment and first-period consumption less attractive.

The insurance characteristic ξ does not enter either of the two optimal tax rules and both expressions hold for the optimal labor tax rate $\hat{t}$ as well as for an arbitrarily given tax rate $t > 0$. Accordingly, two straightforward results apply both to the optimal education subsidies and to the optimal capital taxation: First, neither catalysts directly provides social insurance, since both capital tax payments and education subsidies received do not affect the variance of income, i.e., they do not vary across the states of nature. Moreover, all households are homogenous ex ante; consequently, there is no ability bias at work either (see Maldonado, 2008, and Jacobs and Bovenberg, 2008, for ability bias in a deterministic world with heterogenous households). Second, neither instrument is used if there is no social insurance. If the labor tax rate is zero, $t = 0$, the only effective insurance device is over- or underinvestment in education, which is optimally chosen by households. This insurance effect cannot be improved by subsidizing education or taxing capital income. Furthermore, in case of $t = 0$ there is no fiscal externality to be corrected for.

From equation (2.27) we find that, *firstly*, optimal education subsidies decrease in distortions caused, which are represented by the denominator in the first term on the right hand side. The more elastic educational investment is with respect to subsidies, $\varepsilon_{es} > 0$, the higher will the excess burden of this instrument be. However, the availability of capital taxation allows for a mitigating complementarity effect: reducing distortions in educational investment can be traded off against distorting real savings, $\frac{\varepsilon_{eR}}{\varepsilon_{aR}} < 0$, and this effect becomes the stronger the more the savings tax base responds to education subsidies, $\varepsilon_{as} < 0$. *Secondly*, education subsidies increase in the marginal efficiency gains from boosting labor supply, as indicated by $\varepsilon_{ls} > 0$ in the numerator of the first term on the right hand side. Due to the complementarity between labor supply and education, education subsidies foster labor supply and counteract the negative incentive effects of labor taxation (see Bovenberg and Jacobs, 2005, Jacobs and Bovenberg, 2008). *Thirdly*, education subsidies interfere with the complementarity effect of capital taxation on labor supply. Capital taxation also alleviates distortions in labor supply, both via fostering education (Jacobs and Bovenberg, 2010) and via intertemporal consumption effects (Jacobs and

Schindler, 2009), but this efficiency gain has to be traded-off against (downwards) distortions in savings, as indicated by $\frac{\varepsilon_{lR}}{\varepsilon_{aR}} < 0$. Since education subsidies distort real savings downwards as well, they worsen the aforementioned trade-off, i.e., applying capital taxes becomes more costly. Hence, education subsidies make the capital tax a less effective instrument to boost labor supply. The stronger this interference ($\varepsilon_{as} \cdot \frac{\varepsilon_{lR}}{\varepsilon_{aR}} > 0$) is, the lower should education subsidies be. They might ceteris paribus even turn negative in order to boost the capital-tax effect. In the following, we will call this interference effect "indirect complementarity effect".

Fourthly, we see from equations (2.27) and (2.28) that the additive property of internalizing externalities in an optimal-tax setting (Sandmo, 1975) holds, if sufficient instruments are available. Contrary to mitigating labor supply distortions, the externality is corrected by relying on education subsidies only and in an additive manner. This is represented by $\frac{\pi_e}{1-\pi_e}$, the second term on the right hand side of (2.27). Depending on the sign and the magnitude of the externality, education subsidies can also turn negative. The risk premium π_e does not explicitly enter the formula for the optimal capital tax rate. Accordingly, when education subsidies are optimally chosen, inefficient educational investment does not affect capital taxation directly. The reason is that targeting the price of the "commodity", which causes the externality, is more efficient (see Sandmo, 1975, pp. 92, 95). In our case, this commodity is education and its relevant price is directly linked with education subsidies.

Turning to the optimal capital taxation as given by equation (2.28), we find that, *firstly*, the capital tax rate decreases in distortions caused in compensated savings, $\varepsilon_{aR} > 0$. The more elastic savings are with respect to the interest rate, the higher are the efficiency losses from capital taxation. However, education subsidies can moderate distortions in savings, traded off against distorting educational investment ($\frac{\varepsilon_{as}}{\varepsilon_{es}} < 0$). This trade-off is the more important, the more a higher interest rate decreases educational investment ($\varepsilon_{eR} < 0$). Hence, this complementarity effect works in favor of higher capital taxes. *Secondly*, capital taxation improves efficiency by fostering labor supply via two channels: (i) By reducing second-period consumption, capital taxation increases the marginal utility of income and thus the opportunity costs of leisure. Consequently, capital taxation ceteris paribus boosts labor supply in the second period (cf. Jacobs and Schindler, 2009). (ii) Capital taxation encourages human capital investment. Therefore, it fosters labor supply by increasing the opportunity costs of leisure on this account, as well. Consequently, capital taxation mitigates labor supply distortions. This is represented by the first term in the numerator, $-\varepsilon_{lR} > 0$. *Thirdly,* there is an "indirect complementarity effect" at work. Education subsidies boost labor supply, but distort educational investment, see the discussion of equation (2.27). This trade-off is the more beneficial, the higher $\frac{\varepsilon_{ls}}{\varepsilon_{es}} > 0$ is. The more a higher interest rate decreases educational investment ($\varepsilon_{eR} < 0$), the more the aforementioned trade-off is improved and the lower should the capital tax be, ceteris paribus.

Taken together the second and the third aspect, the optimal capital tax can also be negative, contrary to models without endogenous educational investment (Cremer and Gahvari, 1995a,b; Jacobs and Schindler, 2009). Capital taxation will be zero in the special case, where its complementarity effect on labor supply ε_{lR} exactly offsets its effect of deteriorating complementarity effect of education subsidies on labor supply, implying $\frac{\varepsilon_{lR}}{\varepsilon_{eR}} = \frac{\varepsilon_{ls}}{\varepsilon_{es}}$. If so, both instruments are, per "unit" of distortion in educational investment, equally effective in boosting labor supply, and capital taxation becomes redundant, since it additionally distorts intertemporal consumption. Generally, capital taxation becomes the less important the more labor supply distortions are mitigated via education subsidies.

We summarize

Proposition 2.1. *If both savings and educational investment are verifiable, both capital taxation and education subsidies are used for mitigating labor supply distortions, but they do not provide any direct insurance. Both instruments increase in their complementarity effect on labor and decrease in induced net distortions and in harming the complementarity effect of the other instrument. The additive property for externalities holds and only education subsidies are used to internalize the external effect of missing insurance markets.*

Compared to models relying only on education subsidies (cf. Jacobs et al., 2010), the availability of capital taxation has significant effects. The intuition can be briefly summarized as follows: first, capital taxation is another way to mitigate distortions in labor supply by indirectly subsidizing educational investment. Second, there is a stand-alone effect of capital taxation on labor supply via intertemporal wealth effects (cf. Jacobs and Schindler, 2009). Therefore, capital taxation has an additional complementarity effect working independently of education.[10] Still, both education subsidies and capital taxation are used. Education subsidies are less distortive in the sense that they distort only educational investment, but they affect labor supply

[10]Without having shown this explicitly, this intertemporal mechanism is also relevant in extensions of models with centrally decided educational investment (e.g., Anderberg and Andersson, 2003).

only by complementarity between education and labor, and they are costly in the sense that the government has to collect tax revenue to finance subsidies. On the other hand, capital taxes not only distort educational investment, but also intertemporal consumption.

Consequently, by extending and generalizing the model by Hamilton (1987) we justify the use of capital taxation, but its role fundamentally changes. In particular, capital taxation is no longer required for internalizing the fiscal externality, but only for alleviating tax distortions in labor supply.

From rearranging and dividing equation (2.27) by equation (2.28), we obtain a Ramsey-type rule for the simultaneous use of education subsidies and capital taxation

$$\frac{\frac{s}{1-s}}{\frac{\tau r}{R}} = \frac{\varepsilon_{aR} - \varepsilon_{lR} \cdot \frac{\varepsilon_{as}}{\varepsilon_{ls}}}{\left(\varepsilon_{es} - \varepsilon_{ls} \cdot \frac{\varepsilon_{eR}}{\varepsilon_{lR}}\right) \cdot \omega_e \cdot (1-\pi_e)} \cdot \frac{\varepsilon_{ls}}{(-\varepsilon_{lR})} + \frac{\varepsilon_{aR} - \varepsilon_{eR} \cdot \frac{\varepsilon_{as}}{\varepsilon_{es}}}{\omega_l \cdot \left[(-\varepsilon_{lR}) + \varepsilon_{eR} \cdot \frac{\varepsilon_{ls}}{\varepsilon_{es}}\right]} \cdot \frac{\pi_e}{1-\pi_e}. \quad (2.29)$$

The second term on the right hand side of equation (2.29) mirrors the effect of the fiscal externality. As implied by the additive property, the relative reliance on education subsidies ceteris paribus increases (decreases) in the magnitude of the fiscal externality π_e in case of underinvestment $\pi_e > 0$ (overinvestment $\pi_e < 0$). The externality matters ceteris paribus the more, the higher the net distortions of capital taxation are ($\varepsilon_{aR} - \varepsilon_{eR} \cdot \frac{\varepsilon_{as}}{\varepsilon_{es}} > 0$), i.e., the more costly capital taxation is. These net distortions are positive from the second order conditions of the governmental optimization problem. The externality matters ceteris paribus the less, the more relevant labor supply is (viz., the larger is the share ω_l) and the better capital taxation can alleviate labor supply distortions (i.e., the higher is $(-\varepsilon_{lR}) + \varepsilon_{eR} \cdot \frac{\varepsilon_{ls}}{\varepsilon_{es}}$).

The first term on the right hand side encompasses two effects: on the one hand there is the standard distortion effect. The more net distortions capital taxation causes in savings relative to risk-adjusted, income-weighted net distortions in educational investment by education subsidies, i.e., the higher is $\frac{\varepsilon_{aR} - \varepsilon_{lR} \cdot \frac{\varepsilon_{as}}{\varepsilon_{ls}}}{\left(\varepsilon_{es} - \varepsilon_{ls} \cdot \frac{\varepsilon_{eR}}{\varepsilon_{lR}}\right) \cdot \omega_e \cdot (1-\pi_e)}$, the more expensive capital taxation is in terms of welfare costs. Hence, the more education subsidies will ceteris paribus be used compared to capital taxation. Note that the "indirect complementarity effects", discussed in equations (2.27) and (2.28), cancel, but that there are alleviating complementarity effects working via labor supply ($\varepsilon_{lR} \cdot \frac{\varepsilon_{as}}{\varepsilon_{ls}} > 0$ and $\varepsilon_{ls} \cdot \frac{\varepsilon_{eR}}{\varepsilon_{lR}} > 0$, respectively) at play, now. On the other hand, contrary to a standard Ramsey rule, the instruments differ in their beneficial effects. Thus, education subsidies are ceteris paribus also the more preferable to capital taxation, the better the former boost labor supply than the latter does, i.e., the higher is $\frac{\varepsilon_{ls}}{(-\varepsilon_{lR})} > 0$.

Taken together, equation (2.29) indicates that education subsidies and capital taxes are (strategic) substitutes (i.e., if one instrument increases, the other one should optimally decrease). This substitutability establishes a policy-relevant linkage between educational policy and competition in personal tax rates on real capital. Winner (2005) provides strong evidence that there is tax competition going on since the mid-eighties by showing a shift from taxing capital to taxing labor. This shift in tax burdens is not only due to corporate tax competition, but also due to a decrease in personal capital income taxes, as can be observed in all OECD countries. If fiercer 'tax competition' is interpreted as globalization, which ceteris paribus raises the elasticity of savings due to a higer mobility of capital, i.e., as an increase in ε_{aR}, we find from equation (2.28) that the optimal capital tax decreases, because it becomes more costly, now. As can be seen from the Ramsey-equation (2.29), education subsidies should be increased relative to capital taxation, - at least as long as there is underinvestment $\pi_e > 0$.

The effect on the absolute level of optimal education subsidies implied by equation (2.27) is, however, ambiguous. On the one hand, education subsidies are less necessary to reduce capital-tax induced distortions in education by decreasing savings ($\varepsilon_{as} \cdot \frac{\varepsilon_{eR}}{\varepsilon_{aR}} > 0$). This ceteris paribus decreases subsidies. On the other hand, it becomes less important that education subsidies hamper the complementarity effect of capital taxation ($\varepsilon_{as} \cdot \frac{\varepsilon_{lR}}{\varepsilon_{aR}} > 0$), since the latter is less effective. This ceteris paribus increases education subsidies. As long as mitigating labor supply distortions has more weight than mitigating distortions in educational investment, education subsidies increase as well. Consequently, under these conditions, capital tax competition should be accompanied by increasing direct subsidies on education. We conclude

Corollary 2.1. *Capital taxation and education subsidies tend to be (strategic) substitutes as long as reducing labor supply distortions matters. Then, a lower capital tax rate induced by capital-tax competition should be accompanied by higher (direct) education subsidies.*

2.3.4 Non-observable Educational Investment

Two relevant special cases can be analyzed. For the first one, where capital taxation is not available, we refer to Jacobs et al. (2010). In this section, we analyze the other polar case, where the government cannot observe educational investment. Hence, education subsidies are not available. This setting allows to study the results in Hamilton (1987). The optimal capital tax rate in absence of education subsidies is derived by setting $s = 0$ in equation (2.51) in Appendix 2.A.3 as

$$\frac{\tau r}{R} = -\left(\omega_l \cdot \frac{\varepsilon_{lR}}{\varepsilon_{aR}} + \pi_e \cdot \omega_e \cdot \frac{\varepsilon_{eR}}{\varepsilon_{aR}}\right) \cdot \hat{t}, \tag{2.30}$$

Again, equation (2.30) balances the marginal efficiency gains against the marginal excess burden of capital taxation. However, without education subsidies, capital taxation has to correct the fiscal externality as well. Hamilton (1987) assumes multiplicative risk, unambiguously implying underinvestment into education. He argues that capital taxation should be used to correct this inefficient educational investment, and he shows that the optimal capital tax rate is positive, if (i) labor supply is inelastic and (ii) either equilibrium savings are zero or there is constant absolute risk aversion. Our approach shows that these very strong assumptions can be relaxed, and it extends the Hamilton-analysis by deriving a closed-form solution for the optimal capital tax.[11]

Equation (2.30) confirms that capital taxation is increasing in the magnitude of the fiscal externality in case of underinvestment (i.e., $\pi_e > 0$). In other words, the more education is distorted downwards as a consequence of uninsurable income risk, the stronger is the need for capital taxation in order to encourage education. Furthermore, the more effective capital taxation is in fostering education ($\varepsilon_{eR} < 0$), the higher should the tax rate be. Contrary to Hamilton (1987), however, we show that the optimal capital tax rate can also turn negative, in case educational investment is a risk-reducing activity (i.e., if there is overinvestment and $\pi_e < 0$) and if the fiscal externality effect dominates the complementarity effect in labor supply, which is described in the following paragraph. In this case, interest income should be subsidized to discourage excessive overinvestment into education.

As we allow for endogenous labor supply, there is a second effect at play. Capital taxation boosts labor supply and moderates distortions from social insurance by fostering educational investment and decreasing second-period consumption, as shown in the previous subsection 2.3.3. Hence, the optimal capital tax rate also increases in the complementarity between capital taxation and labor supply ($\varepsilon_{lR} < 0$).

All these beneficial effects are traded off against distortions in real savings ($\varepsilon_{aR} > 0$). A

[11]Note that the capital tax rate τ also enters the elasticities on the right hand side. As usual in Public Finance, it still highlights in detail the trade-offs determining the optimal tax rate.

higher net interest rate increases the (intertemporal) opportunity costs of human capital investment ($\varepsilon_{eR} < 0$) and it increases the price of first-period consumption ($\varepsilon_{c_1 R} < 0$). Consequently, savings are increased by a higher net interest rate. These distortions decrease the optimal capital tax rate.

Proposition 2.2. *If education subsidies are not available, capital taxation is used for boosting endogenous labor supply and for internalizing the fiscal effect from under- or overinvestment into education. Depending on the risk properties of education ($\pi_e \gtrless 0$) and the magnitude of the fiscal externality, the optimal capital tax rate can be negative as well.*

Grochulski and Piskorski (2010) show that the unobservability of educational investment makes incentive constraints more severe and leads to a larger wedge on real capital investment. The latter is implemented by a higher volatility of marginal capital tax rates. In our linear-taxation model, the optimal capital tax rate tends also to be higher in the absence of education subsidies, but only in case of underinvestment. It is because capital taxation is the only instrument to alleviate labor supply distortions and to internalize the fiscal externality. However, if education is risk-decreasing, capital taxation will be decreased ceteris paribus to fight against the effect of overinvestment in human capital.

2.3.5 Optimal Labor Tax Cum catalysts

Substituting equations (2.27) and (2.28) into equation (2.45) in the appendix finally leads to the optimal labor tax formula, when both education subsidies and capital taxation are optimally chosen:

$$\xi = \frac{\hat{t}}{1-\hat{t}} \cdot \omega_l \left((-\varepsilon_{lt}) + \underbrace{\varepsilon_{et} \left[\frac{\varepsilon_{ls} - \varepsilon_{as} \cdot \frac{\varepsilon_{lR}}{\varepsilon_{aR}}}{\varepsilon_{es} - \varepsilon_{as} \cdot \frac{\varepsilon_{eR}}{\varepsilon_{aR}}} \right]}_{s-effect} - \underbrace{\varepsilon_{at} \left[\frac{(-\varepsilon_{lR}) + \varepsilon_{eR} \cdot \frac{\varepsilon_{ls}}{\varepsilon_{es}}}{\varepsilon_{aR} - \varepsilon_{eR} \cdot \frac{\varepsilon_{as}}{\varepsilon_{es}}} \right]}_{\tau-effect} \right). \tag{2.31}$$

The optimal labor tax rate increases in the welfare gain ξ from reducing income risk, but it decreases in the tax elasticity of labor supply (ε_{lt}). The labor supply distortions are, however, the less severe, the more labor taxation boosts the net complementarity effect of education subsidies by decreasing subsidy-induced distortions in education (i.e., the larger $\varepsilon_{et} < 0$ is in absolute value). This is the "s-effect" in equation (2.31). The same holds true for fostering the net complementarity effect of capital taxation by reducing capital-tax-induced distortions in savings (viz., by having a larger $\varepsilon_{at} > 0$). The latter is the "τ-effect" in equation (2.31). These complementarity effects ceteris paribus increase the labor tax rate, allowing for a better social insurance.

As known from Jacobs et al. (2010), the fiscal externality does not enter the optimal labor tax formula. Thus, with optimal education subsidies, inefficient educational investment does not directly affect the optimal labor taxation any more. Compared to Jacobs et al. (2010), the availability of capital taxation increases the likelihood of better social insurance in equation (2.31), relative to the case without catalysts in (2.26).

Our analysis complements the analysis of optimal non-linear taxation in the 'new dynamic public finance' literature (see Golosov et al., 2006, Diamond, 2006 for a survey). If only real savings are observable, Kocherlakota (2005) and Grochulski and Piskorski (2010) point out that capital should bear a positive wedge for relaxing incentive constraints. For the case of verifiable educational investment, Anderberg (2009) and da Costa and Maestri (2007) show that education should bear a wedge as well, i.e., that both education subsidies and capital taxation are optimally used in order to provide social insurance efficiently.

Our approach confirms their results for the informationally less demanding case of linear tax instruments. The downside of linear taxation is that the tax structure is less flexible, the upside is, however, that the government only has to verify aggregate labor income, aggregate savings and aggregate investment in education. Our analysis sheds light on the driving forces and the main intuition behind optimal intertemporal and educational wedges for relaxing incentive constraints (namely increasing opportunity costs of leisure). In addition, we point out that, under linear tax instruments, the optimal capital tax rate can become negative, if it severely interferes with boosting labor supply by education subsidies. To the best of our knowledge, this result is new to the existing literature.

Another advantage of linear instruments is that they are directly implementable. The reason is that successful (i.e., high-ability) agents cannot profit from mimicking unsuccessful (i.e., low-ability) agents. We derive explicit formulas for the optimal education subsidies and the optimal capital tax rate. Instead, for non-linear taxation in vein of 'new dynamic public finance', implementing the optimal intertemporal wedges is difficult and requires e.g. special assumptions about the distribution of shocks and record keeping (cf. Golosov and Tsyvinski, 2006).

Implementing optimal educational wedges is, except for very special cases, even impossible (see Anderberg, 2009).

2.4 Conclusions

This paper examines the optimal social insurance package in an intertemporal model with endogenous labor supply and educational investment. Whilst income risk is only insured by labor taxation, both education subsidies and capital taxation, if available, serve as catalysts for social insurance by mitigating labor supply distortions. Optimal education subsidies increase in their complementarity effect on labor supply via enhancing education, but they decrease in induced net distortions in educational investment. The optimal capital tax also increases in its complementarity effect, which boosts labor supply both by fostering education and by intertemporal wealth effects. It decreases in its distortions in real savings. Both instruments decrease in interfering with the complementarity effect of the other instrument. Since education subsidies and capital taxation differ both in their benefits and in their distortions caused, both instruments are, generally, used and their net marginal dead weight losses are balanced against each other.

Our results show that capital taxation should be used under less restrictive assumptions in comparison to Hamilton (1987). In case educational investment is not observable, capital taxation is used both for mitigating labor supply distortions and for internalizing a fiscal externality, which results from self-insurance of households by over- or underinvesting into education. If educational investment is verifiable, it follows from our analysis that capital taxation is not used anymore for internalization of the fiscal externality and that the additive property holds (see Sandmo, 1975). This is because education subsidies are the better targeted instrument. Nevertheless, capital taxation still plays a role in a generalized Hamilton-model: it is used for boosting labor supply. Furthermore, our analysis of linear taxation complements the 'new dynamic public finance' literature. We derive closed form solutions for optimal tax rates, which are directly implementable.

Our results have a clear policy implication: if tax competition drives personal capital tax rates down, education subsidies should rather increase. In Europe, (personal) capital taxes are indeed decreasing, but education subsidies have decreased as well. Based on our analysis, this policy should be questioned, if the policy objective is to foster labor supply and to overcome labor market distortions from providing social insurance.

Appendix 2.A

2.A.1 Risk-Adjusted Slutsky Equations

For deriving the risk-adjusted Slutsky equations (see also Cremer and Gahvari, 1995a), we define the expenditure function $X(t,s,R,V)$ as the minimum level of non-labor income T required to attain the expected indirect utility V. $X(.)$ can be obtained from setting $X(t,s,R,V) \equiv T$ for the optimal level of indirect utility V as given in equation (2.14). Consequently, the compensated demand functions are defined as

$$c_i^c(t,s,R,V) \equiv c_i(t,s,R,X(t,s,R,V)), \tag{2.32}$$

where the superscript c denotes a compensated change. By totally differentiating the compensated demand functions for given V and using Shephard's lemma we obtain the following risk-adjusted Slutsky equations with respect to the tax rate t

$$\frac{\partial e}{\partial t} = \frac{\partial e^c}{\partial t} - ((1-\xi)\mathcal{E}[\Phi] + R\cdot(1-(1-s)e))\frac{\partial e}{\partial T} \tag{2.33}$$

$$\frac{\partial l}{\partial t} = \frac{\partial l^c}{\partial t} - ((1-\xi)\mathcal{E}[\Phi] + R\cdot(1-(1-s)e))\frac{\partial l}{\partial T} \tag{2.34}$$

$$\frac{\partial c_1}{\partial t} = \frac{\partial c_1^c}{\partial t} - ((1-\xi)\mathcal{E}[\Phi] + R\cdot(1-(1-s)e))\frac{\partial c_1}{\partial T}. \tag{2.35}$$

The Slutsky equations with respect to changes in the subsidy rate s are

$$\frac{\partial e}{\partial s} = \frac{\partial e^c}{\partial s} + R(1-t)e\frac{\partial e}{\partial T} \tag{2.36}$$

$$\frac{\partial l}{\partial s} = \frac{\partial l^c}{\partial s} + R(1-t)e\frac{\partial l}{\partial T} \tag{2.37}$$

$$\frac{\partial a}{\partial s} = \frac{\partial a^c}{\partial s} + R(1-t)e\frac{\partial a}{\partial T}, \tag{2.38}$$

and the ones with respect to variations in the net (after tax) interest rate R are

$$\frac{\partial e}{\partial R} = \frac{\partial e^c}{\partial R} + (\omega + (1-t)(1-(1-s)\cdot e) - c_1)\,\frac{\partial e}{\partial T} \tag{2.39}$$

$$\frac{\partial l}{\partial R} = \frac{\partial l^c}{\partial R} + (\omega + (1-t)(1-(1-s)\cdot e) - c_1)\,\frac{\partial l}{\partial T} \tag{2.40}$$

$$\frac{\partial c_1}{\partial R} = \frac{\partial c_1^c}{\partial R} + (\omega + (1-t)(1-(1-s)\cdot e) - c_1)\,\frac{\partial c_1}{\partial T}. \tag{2.41}$$

2.A.2 Optimal Income Taxation

From Roy's lemma, equation (2.18) and the Slutsky equations (see Appendix 2.A.1), we find

$$-\left[\mathcal{E}[\Phi(.)](1-\xi) + R\cdot(1-(1-s)\cdot e)\right]\cdot\left\{\frac{\mathcal{E}[u_2]}{\eta} + \Delta_e\cdot\frac{\partial e}{\partial T} + \Delta_l\cdot\frac{\partial l}{\partial T} + \Delta_{c_1}\cdot\frac{\partial c_1}{\partial T}\right\} \tag{2.42}$$

$$+\mathcal{E}[\Phi(.)] + R\cdot(1-(1-s)\cdot e) + \Delta_e\cdot\frac{\partial e^c}{\partial t} + \Delta_l\cdot\frac{\partial l^c}{\partial t} + \Delta_{c_1}\cdot\frac{\partial c_1^c}{\partial t} = 0,$$

where we defined the insurance characteristic

$$\xi = -\frac{cov(u_2, \Phi(.))}{\mathcal{E}[u_2]\cdot\mathcal{E}[\Phi(.)]} > 0. \tag{2.43}$$

Using $b = 1$ from equation (2.24) and rearranging (2.42) result in

$$\xi = -\frac{\Delta_e}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial e^c}{\partial t} - \frac{\Delta_l}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial l^c}{\partial t} - \frac{\Delta_{c_1}}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial c_1^c}{\partial t}. \tag{2.44}$$

Defining the expected-utility compensated elasticities with respect to the tax rate as $\varepsilon_{et} = \frac{\partial e^c}{\partial t}\frac{1-t}{e}$, $\varepsilon_{lt} = \frac{\partial l^c}{\partial t}\frac{1-t}{l}$ and $\varepsilon_{c_1t} = \frac{\partial c_1^c}{\partial t}\frac{1-t}{c_1}$, inserting the definitions of the tax wedges (2.21) to (2.23) and collecting terms, we end up with

$$\xi = -\frac{t}{1-t}\cdot\left[\omega_l\cdot\varepsilon_{lt} + \pi_e\cdot\omega_e\cdot\varepsilon_{et}\right] + \frac{s}{1-s}\cdot\frac{1-\pi_e}{1-t}\cdot\omega_e\cdot\varepsilon_{et} - \frac{\tau r/R}{1-t}\cdot\varepsilon_{at}. \tag{2.45}$$

Thereby $\omega_l = \frac{\mathcal{E}[\Phi_l l]}{\mathcal{E}[\Phi]}$ and $\omega_e = \frac{\mathcal{E}[\Phi_e e]}{\mathcal{E}[\Phi]}$ are the expected shares of labor and education in total earnings respectively. Defining $\gamma_e = \frac{R\cdot e}{\mathcal{E}[\Phi]}$ and $\gamma_{c_1} = \frac{R\cdot c_1}{\mathcal{E}[\Phi]}$ as shares of expenditure on education and first-period consumption in total earnings respectively allows us to define $\varepsilon_{at} = -(\gamma_e\cdot\varepsilon_{et} + \gamma_{c_1}\cdot\varepsilon_{c_1t})$ as the compensated elasticity of savings with respect to the labor tax rate

t. Setting $s = \tau = 0$ leads to equation (2.26) in the text.

2.A.3 Optimal Education Subsidies and Optimal Capital Taxation

Rearranging (2.19), substituting Roy's lemma and separating income and substitution effects deliver

$$R\cdot(1-t)\cdot e\cdot\left\{\frac{\mathcal{E}[u_2]}{\eta}+\Delta_e\cdot\frac{\partial e}{\partial T}+\Delta_l\cdot\frac{\partial l}{\partial T}+\Delta_{c_1}\cdot\frac{\partial c_1}{\partial T}-1\right\}+\Delta_e\cdot\frac{\partial e^c}{\partial s}+\Delta_l\cdot\frac{\partial l^c}{\partial s}+\Delta_{c_1}\cdot\frac{\partial c_1^c}{\partial s}=0. \tag{2.46}$$

Dividing equation (2.46) by $\mathcal{E}[\Phi(.)]$ and applying equation (2.24) lead to

$$\frac{\Delta_e}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial e^c}{\partial s}+\frac{\Delta_l}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial l^c}{\partial s}+\frac{\Delta_{c_1}}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial c_1^c}{\partial s}=0. \tag{2.47}$$

Defining the expected-utility compensated elasticities with respect to the subsidy rate s as $\varepsilon_{es} = \frac{\partial e^c}{\partial s}\frac{1-s}{e}$, $\varepsilon_{ls} = \frac{\partial l^c}{\partial s}\frac{1-s}{l}$ and $\varepsilon_{c_1 s} = \frac{\partial c_1^c}{\partial s}\frac{1-s}{c_1}$, we receive after inserting the tax wedges (2.21) to (2.23) and collecting terms

$$\frac{s}{1-s}\cdot(1-\pi_e)\cdot\omega_e\cdot\varepsilon_{es}=t\cdot(\omega_l\cdot\varepsilon_{ls}+\pi_e\cdot\omega_e\cdot\varepsilon_{es})+\frac{\tau r}{R}\cdot\varepsilon_{as}. \tag{2.48}$$

Thereby, the savings elasticity $\varepsilon_{as} = -(\gamma_e\cdot\varepsilon_{es}+\gamma_{c_1}\cdot\varepsilon_{c_1 s}) < 0$ comprises the expenditure-share weighted effects of education subsidies on educational investment and on first-period consumption. Using the same techniques for optimal capital taxation, the first order condition (2.20) can be reformulated as

$$\begin{aligned}&[\omega+(1-t)(1-(1-s)\cdot e)-c_1]\cdot\left\{1-\left(\frac{\mathcal{E}[u_2]}{\eta}+\Delta_e\cdot\frac{\partial e}{\partial T}+\Delta_l\cdot\frac{\partial l}{\partial T}+\Delta_{c_1}\cdot\frac{\partial c_1}{\partial T}\right)\right\}\\ = \;&\Delta_e\cdot\frac{\partial e^c}{\partial R}+\Delta_l\cdot\frac{\partial l^c}{\partial R}+\Delta_{c_1}\cdot\frac{\partial c_1^c}{\partial R}.\end{aligned} \tag{2.49}$$

Dividing both sides by $\mathcal{E}[\Phi(.)]$ and utilizing the optimal lump-sum transfer (2.24), we receive

$$\frac{\Delta_e}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial e^c}{\partial R}+\frac{\Delta_l}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial l^c}{\partial R}+\frac{\Delta_{c_1}}{\mathcal{E}[\Phi(.)]}\cdot\frac{\partial c_1^c}{\partial R}=0. \tag{2.50}$$

By defining the corresponding elasticities with respect to a change in the after tax interest rate R as $\varepsilon_{eR} = \frac{\partial e^c}{\partial R}\frac{R}{e}$, $\varepsilon_{lR} = \frac{\partial l^c}{\partial R}\frac{R}{l}$ and $\varepsilon_{c_1R} = \frac{\partial c_1^c}{\partial R}\frac{R}{c_1}$ as well as taking the tax wedges (2.21) to (2.23) into account, we end up with

$$\frac{\tau r}{R} \cdot \varepsilon_{aR} = -t \cdot (\omega_l \cdot \varepsilon_{lR} + \pi_e \cdot \omega_e \cdot \varepsilon_{eR}) + \frac{s}{1-s} \cdot (1-\pi_e) \cdot \omega_e \cdot \varepsilon_{eR}, \tag{2.51}$$

The savings elasticity is again defined as $\varepsilon_{aR} = -\left(\gamma_e \cdot \varepsilon_{eR} + \gamma_{c_1} \cdot \varepsilon_{c_1R}\right) > 0$. It is unambiguously positive, because a higher net interest rate R renders both educational investment and first-period consumption less attractive. Inserting equation (2.51) for $\frac{\tau r}{R}$ in equation (2.48) and collecting terms, we arrive at equation (2.27) in the text.

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3 Education, Wage Uncertainty and Flat-Tax Debate

3.1 Introduction

Like savings in real capital, education and accumulation of human capital is another way to transfer resources into the future. However, as real capital investment, investment into human capital is a decision under uncertainty. The risk effects of human capital investment are manifold: On the one hand, investment in human capital reduces the probability of being unemployed (see, e.g. Chapman, 1993) and can act as a kind of unemployment insurance. On the other hand, education increases the risky components in wage income (see, i.e., Mincer, 1974).[1]

Following the Mincer tradition, educational risk then has a negative feedback effect on investment in education, if risk is increasing with educational investment: uninsurable income risks make risk-averse households demand a risk-premium, thereby causing underinvestment in education and an inefficient resource allocation (see Levhari and Weiss, 1974).

What can be done in order to reduce the effects of educational and wage risk and in mitigating "precautionary underinvestment" in education? In case of educational and wage risks, it is often argued that private insurance contracts cannot be signed due to, e.g., adverse selection. However, the government can provide insurance via labor taxation and has to trade-off this welfare gain against induced distortions.[2]

This paper shows that, even in case of elastic supply of skilled labor, this trade-off can be

[1] The results by Mincer (1974) have gained renewed support by, e.g., Carneiro et al (2003).

[2] See the following section for a brief overview of the literature on socially insuring educational risk and our paper's relationship to it.

optimally implemented, if the government can use a progressive labor tax, targeting the source of risk directly, which is in the case of risky human capital formation the skill premium. In order to alleviate the created distortions in educational investment and in labor choice, it is necessary to grant education subsidies. Concerning the latest reforms towards a flat tax system (see section 3.8), our results raise additional and new doubts against the desirability of such a flat tax.

In short, optimal tax and education policy in case of risky human capital formation requires a combination of progressive taxation and education subsidies. This ‘Siamese Twins’ principle is well-known from Bovenberg and Jacobs (2005), who argue that optimal redistribution via progressive income taxation requires education subsidies in order to mitigate distortions. We show in the present paper that this mechanism can also be applied in case of risky skilled labor income in order to insure against ex-post income inequality *and* in order to mitigate ex-ante underinvestment in education, caused by uncertainty and incomplete insurance markets.[3] Thus, insurance motives are another argument for progressive (labor) taxation besides redistributional concerns or efficient human capital investment à la Nielsen and Sørensen (1997).

A progressive wage tax allows to target the risky component in labor income directly, whereas tuition fees (or education subsidies) provide an instrument in order to influence education decisions directly. Although each instrument ceteris paribus enforces distortions in educational and labor choice, the combination of both increases the degrees of freedom for the government.

Do educational and wage risk, however, really matter? One can observe (still) increasing wage inequality among workers of different skill grades. This is driven by increasing educational wage differentials, but also by wage differentials between industries and is – predominantly – driven by skill-biased technological change (Katz and Autor, 1999). Jacobs (2004) supposes that the wage differential will increase even more in the future, due to the growth rate of skilled labor supply falls short of the increase in the demand for skilled workers. In reverse, the increase of and the variance in skilled wages turn into educational risk, because at the time of educational investments there is uncertainty of future wages and of successful graduation. Carneiro et al (2003) show for high-school and college graduates that a large part of variance

[3]This is also the major difference to a setting with ex-ante heterogenous households in a deterministic world, as in, i.e., Bovenberg and Jacobs (2005). Both approaches are very similar from an ex-post point of view, but uncertainty causes inefficient investment incentives and differs therefore ex ante. Moreover, taxation is ceteris paribus less distortive due to an insurance effect.

in returns to education cannot be predicted by students at the time of investment. Moreover, a significant share of about 10% of college graduates suffers from negative returns on educational investment.

In a nutshell, education creates several types of risk: students may fail at university, their acquired skills may deteriorate due to idiosyncratic (technological) shocks, and prospective incomes of skilled workers face a higher variance, because after graduation salaries differ across sectors and professions.[4]

Our approach provides conditions for the 'Siamese Twins'-mechanism as an optimal social insurance device in such a risky economy. Following Nielsen and Sørensen (1997), we apply an OLG-model with a Norwegian-type two-bracket labor tax and extend the model by introducing risky human capital formation in the educational sector. Hereby, ex-ante homogenous households become heterogenous ex-post due to different risk realization.

The remainder of the paper is organized as follows. In section 3.2 we provide a brief overview of related literature, then present the model in section 3.3 and examine household choices in section 3.4. In section 3.5 the optimization problem is set up and welfare-maximizing conditions for optimal tax and education policies are derived. In section 3.6, we analyze the special case of exogenous labor supply first and then generalize the results for endogenous skilled labor supply in section 3.7. Section 3.8 links our results to the current flat tax debate. Section 3.9 concludes.

3.2 Relation to the Literature

The analysis of the effects of income risks on human capital decisions dates back to Levhari and Weiss (1974). However, they neglect the effects of tax instruments on this decision margin. Eaton und Rosen (1980a,b) are the first to show in a seminal paper series that there is a trade-off between distortions in human capital investment and labor supply on the one hand and an insurance effect, provided by a proportional income taxation, on the other hand. The main intuition of their results is that the government can diversify private risk at no costs and grant a deterministic lump-sum transfer. This reduces income risk. Moreover, they show that in case of risk a proportional income tax can increase human capital investment under some assumptions. Their model is extended to incorporate capital income taxation as an indirect subsidy on

[4]See Anderberg and Andersson (2003) for another discussion of the various risk aspects.

education by Hamilton (1987).[5]

Wigger and von Weizsäcker (2001) analyze optimal public insurance against educational risk and show that, due to moral hazard, full insurance is not possible in case of unobservable learning effort and heterogeneity of individuals.

The papers, being closest to the set of governmental instruments used in our approach, are, however, García-Peñalosa and Wälde (2000) and Jacobs and van Wijnbergen (2007). The former paper uses an education subsidy and contingent lump-sum (graduate) taxes, which have to be paid, if human capital investment is successful. The latter paper focuses on capital market failure and adverse selection problems in credit financing. Both papers apply a binary model and exclude endogenous labor supply and its distortions, created by graduate taxes. Therefore, the result of optimal income insurance with governmental full equity stakes in human capital returns in Jacobs and van Wijnbergen can be seen as analogue to our result in the (special) case of exogenous leisure demand.

3.3 The Model

We augment the two-period OLG-model in Nielsen and Sørensen (1997) by stochastic shocks in the individual human capital formation technology and by an explicitly modeled educational sector including tuition fees.

The economy is populated by a continuum of ex-ante homogenous individuals, whose mass is normalized to unity. The representative individual lives for two periods and is endowed with one unit of time in each period. In period 1, the individual invests a fraction e of his time endowment in education and works for the rest of time. Moreover, the household chooses its first period consumption and saves the rest of current income. In period 2, the individual supplies skilled labor L and consumes leisure $l = 1 - L$. The assumption that there is no leisure in the first period simplifies the analysis without loss of generality.[6]

In period 1, the individual has an initial human capital stock of 1 and supplies unskilled labor

[5] Varian (1980) moreover includes a section on non-linear taxation, but focus on risky returns in real capital and uses a model, where savings are the only choice variable. Whilst this model is not able to analyze our questions, the basic intuition of progressive taxation as superior social insurance is similar to our approach.

[6] Leisure in the second period can also be interpreted as retirement. Accordingly, labor time L would then represent utilization of human capital.

only. After investing in education, it acquires a human capital stock $\tilde{H}(e, \theta)$ in period 2,[7] which is a function of time investment e and a random variable θ.

The human capital production technology is supposed to be concave in time effort. Thus, we have a positive, but decreasing marginal productivity, $\tilde{H}_e = \frac{\partial \tilde{H}(e,\theta)}{\partial e} > 0$, $\tilde{H}_{ee} < 0 \quad \forall \theta$. We assume that the marginal productivity of human capital production is large enough at $e = 0$ and is small enough around $e = 1$ to ensure an inner solution. Moreover, we assume that, in the second period, the household can supply unskilled labor without incurring any risk, if it does not invest in education, $e = 0$. This implies $H(0, \theta) = 1$, independently of the realization of θ.

The random variable θ captures the risk in human capital formation. θ can be for instance interpreted as the individual ability to learn, which is unknown ex-ante, or as a sector-specific or technological shock affecting the utilization of specific human capital, or as individual fortune in final exams.[8] Following the Mincer-tradition, we focus throughout the paper on the case that θ has a positive effect on human capital formation, $\frac{\partial H(e,\theta)}{\partial \theta} > 0$. A higher realization of θ can then be interpreted, e.g., as a higher realized learning ability or as a better grade in the university exam, which is relevant for the effective wage of a skilled worker. Furthermore, we assume that θ affects positively the marginal productivity of education, $H_{e\theta} > 0$, as well – implying, i.e., that individuals with higher ability learn more effectively than less able individuals. In other words, $H_{e\theta} > 0$ accords to risk being increasing in schooling and increases the sensitivity of the realized human capital stock to the shock θ. This results in underinvestment in education due to risk, accompanied by a positive risk premium for education, as will be shown later on.[9] The density function of θ is $f(\theta)$, which is known to the individuals and the government.

The idiosyncratic educational risk θ realizes at the beginning of period 2, and an individual then supplies $\tilde{H}(e, \theta) \cdot L$ units of effective (skilled) labor. Thus, the households differ in their human capital stock and are ex-post heterogenous. However, as we assume θ to be an idiosyncratic risk factor, there is no aggregate risk, and in aggregate all stochastic variables will take their expected values. This implies that total human capital stock is deterministic.

The distribution of θ and the human capital formation function are assumed to guarantee that

[7] Variables indicated with a tilde depend on the realization of θ and are stochastic.

[8] The latter argument rests on the idea that better exam grades imply higher wages. However, the success in final exams can be negatively affected, if the student has a bad hair day due to, i.e., illness.

[9] See Levhari and Weiss (1974, pp. 953) and Anderberg and Andersson (2003, pp. 1527) for a related discussion on $H_{e\theta} = \frac{\partial^2 H}{\partial e \partial \theta} \gtrless 0$. Following Levhari and Weiss, the case of $H_{e\theta} > 0$ and, therefore, underinvestment in education strikes us to be more relevant.

$\tilde{H}(e,\theta) > 1$ for all values of θ, given $e > 0$. Any education investment increases the human capital stock of a household due to $\tilde{H}_e > 0$, but this increase is in part stochastic, which implies that second period income and consumption will be risky.

Following the major line of the literature, we assume that private insurance against education risk is not available. This might be caused by market failure due to adverse selection and moral hazard[10] or by the fact that individuals are too young to write insurance contracts, when they decide on their human capital investment (Sinn, 1996).[11]

Individual utility depends on consumption in the two periods, C_1 and C_2, and on second period leisure l. It takes the form

$$U = U(C_1, C_2, l). \tag{3.1}$$

The utility function is assumed to be twice differentiable in all arguments and marginal utilities are assumed to be positive, but decreasing, consequently $U_{C_1},\ U_{C_2},\ U_l\ >\ 0$, respectively $U_{C_1C_1}, U_{C_2C_2}, U_{ll} <\ 0$. Furthermore, we assume the Inada conditions to hold. Thus, marginal utilities of consumption and leisure are decreasing, guaranteeing risk-averse behavior in both periods and inner solutions.

Following Nielsen and Sørensen (1997), the economy is a small open economy. The aggregate production function of a homogenous good has constant returns to scale, and the production function can be formulated in intensive form as $y = f(k)$. Hereby y is output and k represents the physical (or real) capital stock per unit of effective labor. The price of the good is normalized to unity. The world capital market is perfectly integrated, and the real interest rate $r = f'(k)$ is exogenously given from the perspective of the home economy. This also implies that the wage rate per unit of effective labor is determined by $W = f(k) - rk$ and is exogenous as well.

We assume that higher education is acquired at public universities, which are financed by the government. For sake of simplicity, we assume that universities are a "club good:" Consuming higher education is non-rival, but students can be excluded. The educational sector causes fix costs of $\overline{G}$ and these can either be financed by taxes or by tuition fees.

In our model, we adopt the Norwegian two-bracket labor tax system, which is also used in Nielsen and Sørensen (1997). There is a basic tax rate t_1 for labor income below a threshold

[10] Private insurers are supposed to be unable to distinguish between external shocks and endogenous behavior of the assured. See, e.g., Eaton and Rosen (1980b), pp. 707.

[11] Some discussion of this assumption and an opposing view is to be found in Andersson and Konrad (2003).

value $X \cdot W$. If the household earns more labor income than the threshold, the part of income above the threshold is liable to a marginal tax rate t_2. Accordingly, $t_2 > t_1$ implies that the labor tax structure is progressive.

The labor income of the individual is equal to $W \cdot (1-e)$ in period 1, and equal to $W \cdot \tilde{H}(e, \theta) \cdot L$ in period 2. It is assumed that $W \cdot (1-e) < WX < W \cdot \tilde{H}(e, \theta) \cdot L \, \forall \theta$ such that t_1 is the marginal tax rate for unskilled workers. t_2 is the marginal tax rate for the skilled.

The assumption has also two other implications. First, even in the worst state of nature, the household is, after realization of risk, more productive than an unskilled worker, who never attended university, and (marginal) return to human capital will be liable to the high-bracket rate t_2. Second, we implicitly assume that the marginal productivity of the first units of time investment in human capital production is high enough to ensure an inner solution of e, avoiding any problems around the kink in the household budget constraint.

In order to focus on the risk effects of human capital and the insurance property of the labor tax, we want to keep the model as simple as possible and assume that there is no taxation of real capital. An interest tax acts as a subsidy on human capital investment and also calls for either progressive taxation (see Nielsen and Sørensen, 1997) or reduced pre-loaded education subsidies. As will become clear later on, a tax on real capital would not change the main results of our model, but the analysis gets much more complicated.

However, the government can collect tuition fees P_e per semester spent at university. A household has to pay $P_e \cdot e$ in fees, or receives a subsidy, if $P_e < 0$.

Taken together, we apply the model of Nielsen and Sørensen (1997) and augment it by a risky human capital production and an educational sector, where the government can also use tuition fees. Compared to the standard Eaton and Rosen (1980a,b) world, we extend the instruments of the government by a two-bracket tax system and tuition fees. The motivation behind this is that a progressive labor tax is a superior instrument (compared to a proportional tax system) in order to tax and to insure risky returns to human capital, and tuition fees (or subsidies) provide another, direct, instrument in order to control the education decision of households.

3.4 Individual Optimization

At the beginning of period 1, the individual chooses its educational investment e, its first period consumption C_1 and leisure demand in the second period l, without knowing the outcome of its education investment. The individual pays tuition fees of $P_e \cdot e$, and forgoes $(1 - t_1)W \cdot e$ in labor income. The budget constraint of the individual for the first period is:

$$S = w_1(1 - e) - P_e \cdot e - C_1, \tag{3.2}$$

whereby $w_1 = (1 - t_1)W$ is the after tax wage rate for unskilled labor income. S is savings if positive, and private debt if negative.

In period 2, the individual consumes its entire wealth, which is given by the sum of savings in period 1 plus interest, and its after tax labor income in period 2, $(1 - t_2)W \cdot \tilde{H}(e, \theta) \cdot L$. The consumption in period 2, $\tilde{C}_2$, is then equal to

$$\tilde{C}_2 = (1 + r)S + w_2\tilde{H}(e, \theta) \cdot L + (w_1 - w_2)X, \tag{3.3}$$

with $w_2 = (1 - t_2)W$. Remember that labor income of period 2, $W \cdot \tilde{H}(e, \theta) \cdot L$, is liable to two different tax rates: t_1 is the basic tax rate on labor income $W \cdot X$, and t_2 is levied on at least part of the skill premium, $W \cdot (\tilde{H}(e, \theta) \cdot L - X)$.

The intertemporal budget constraint can then be written as

$$\tilde{C}_2 = (1 + r)[w_1(1 - e) - P_e \cdot e - C_1] + w_2 \cdot \tilde{H}(e, \theta) \cdot L + (w_1 - w_2)X \tag{3.4}$$

Due to the risk in the human capital formation process, second period labor income and, therefore, consumption in period 2 are uncertain. The individual only knows the distribution of possible outcomes, given its investment e, when it decides over its consumption, education investment and leisure demand. Thereby, it maximizes a von Neumann–Morgenstern expected utility function, $\mathcal{E}[U(C_1, \tilde{C}_2, l)]$, where leisure l is given by $l = 1 - L$.

The optimization problem of the individual is:

$$\max_{C_1, e, L} \quad \mathcal{E}[U(C_1, \tilde{C}_2, 1 - L)] \quad \text{s.t.} \quad (3.4). \tag{3.5}$$

The first order conditions for this problem are given by:

$$\mathcal{E}[U_{C_1}] - (1+r)\mathcal{E}[U_{C_2}] = 0 \quad \Rightarrow \quad \frac{\mathcal{E}[U_{C_1}]}{\mathcal{E}[U_{C_2}]} - 1 = r, \tag{3.6}$$

$$\mathcal{E}[U_{C_2}\{w_2\, \tilde{H}_e\, L - (1+r)(w_1 + P_e)\}] = 0, \tag{3.7}$$

$$\mathcal{E}[U_l] - w_2 E[U_{C_2}\, \tilde{H}] = 0. \tag{3.8}$$

Equation (3.6) states the standard condition that the marginal rate of time preferences must be equal to the real interest rate r. Equation (3.7) implies that the effort level e is chosen optimally, if risk-adjusted expected marginal productivity and marginal costs of educational investment are equalized. Thereby, costs of educational investment are given by the indirect costs of forgone earnings plus direct costs of tuition fees. Hence, average and marginal costs of investment into education are, in terms of second period consumption, given by $(1+r)(w_1 + P_e)$.

For optimal choice of leisure in equation (3.8), the expected marginal utility of leisure should be as large as the risk-adjusted expected marginal costs, represented by reduced consumption. Higher human capital in the second period implies higher opportunity costs of leisure and, accordingly, reduces leisure demand and boosts labor supply.

From Jacobs (2005) follows that decreasing marginal utilities do not guarantee the second order conditions to be fulfilled, due to a positive feedback effect of educational investment and human capital on labor supply. Assuming, however, the cross-effects of changes in marginal utilities to be sufficiently small, the second order conditions are fulfilled, and we can still focus on the first order conditions.[12]

Applying Steiner's rule, $\mathcal{E}[\tilde{X} \cdot \tilde{Y}] = \mathcal{E}[\tilde{X}]\mathcal{E}[\tilde{Y}] + \text{Cov}(\tilde{X}, \tilde{Y})$, and defining

$$\pi_e = -\frac{\text{Cov}(U_{C_2}, H_e)}{\mathcal{E}[U_{C_2}]\mathcal{E}[H_e]} \tag{3.9}$$

as the negative normalized covariance between marginal utility of second period consumption and marginal return to education, we get

$$w_2 L \overline{H}_e (1 - \pi_e) = (1+r)(w_1 + P_e) \tag{3.10}$$

[12]See appendix 3.A.1 for a formal derivation.

from equation (3.7).

In a deterministic world, there is no risk premium, implying $\pi_e = 0$, and marginal return in human capital must then be equal to marginal return in real capital in the household optimum. Increased labor supply enhances the utilization of human capital, therefore raising the marginal return in education. Consequently, labor supply has a positive effect on education. In a world, where human capital formation is stochastic and $H_{e\theta} > 0$ holds, a risk averse household, however, demands a positive risk premium $\pi_e > 0$ and will end up with socially undesirable underinvestment in human capital: the expected marginal return to human capital is larger than the marginal productivity of physical capital. The reason for the underinvestment are risk averse households wanting to avoid risk and demanding a risk premium for the risky 'asset' education, which is mirrored in the variable π_e. This risk premium acts like a distortionary tax on skilled labor income.

Next, it can be seen from examining equation (3.10), that, if tuition fees are absent ($P_e = 0$), a proportional labor tax ($w_1 = w_2$) affects investment in education only via risk aversion and the risk premium. For the case of exogenous skilled labor supply, returns and costs of human capital investment are reduced proportionally and there is no distortion in labor supply, as already shown by Eaton and Rosen (1980b).

Optimal consumption and education demand functions can be characterized by

$$C_1^* = C_1^*(w_1, w_2, P_e, r),\ e^* = e^*(w_1, w_2, P_e, r),\ l^* = 1 - L^* = 1 - L^*(w_1, w_2, P_e, r) \quad (3.11)$$

and indirect utility of an individual results as

$$V = U(C_1^*(w_1, w_2, P_e, r), e^*(w_1, w_2, P_e), L^*(w_1, w_2, P_e, r), \theta) = V(w_1, w_2, P_e, r, \theta). \quad (3.12)$$

Using the envelope theorem, the derivatives of the indirect utility function w.r.t the two after tax wage rates and tuition fees are given by

$$\frac{\partial V}{\partial w_1} = \mathcal{E}[U_{C_2}][(1+r)(1-e) + X], \quad (3.13)$$

$$\frac{\partial V}{\partial w_2} = \mathcal{E}[U_{C_2}\{\tilde{H}(e,\theta) \cdot L - X\}], \quad (3.14)$$

$$\frac{\partial V}{\partial P_e} = -(1+r)e\mathcal{E}[U_{C_2}]. \quad (3.15)$$

and will be used in the following sections.

3.5 Optimal Taxation and Educational Policy

We assume that governmental expenditures for universities are fixed and do not depend on education demand of households. This exogenous public spending is financed by labor tax revenue and tuition fees.

However, following Nielsen and Sørensen (1997) the government is supposed to implement a Pareto-improving tax reform: in order to avoid windfall gains and/or losses, the old generation still faces the old tax rules, but the young and all following generations are liable to the new, post-reform tax parameters. Thus, the government chooses tax rates on labor income and tuition fees in order to maximize the welfare of a representative consumer, subject to the government's budget constraint, keeping the utility of the current old generation constant.

This allows to implement the new steady-state tax parameters within one period, but requires a transition scheme in order to fulfill both constraints simultaneously. Such a Pareto-improving mechanism, which does not affect the welfare of the current old, can be achieved by using a one-time debt policy in the transition period.[13]

Defining the budget constraint for the transition period and keeping the tax parameters and the stock of debt constant for all following periods, the consolidated intertemporal budget constraint of the government results after some rearrangements as

$$(W - w_1)[(1 + r)(1 - e) + X] + (W - w_2)(\overline{H} \cdot L - X) + (1 + r)P_e \cdot e = \overline{R}, \qquad (3.16)$$

where $\overline{R}$ now is the exogenous public spending minus the yield on tax revenue collected from the old generation during the transition period. As we have assumed only idiosyncratic risk in human capital formation, all risk vanishes in aggregate and total human capital stock in period 2 equals its expected value:

$$\overline{H} = \mathcal{E}[\tilde{H}(e, \theta)]. \qquad (3.17)$$

Thus, tax revenue is deterministic, because for the government educational risk is perfectly

[13]See Nielsen and Sørensen (1997), pp. 318. The advantage of this approach is that one does not focus on steady-state utility only.

diversified by the law of large numbers.

The government chooses the optimal tax rates t_1, t_2 (and thereby the after tax wages w_1, w_2) and tuition fees P_e in order to maximize social welfare. The optimization problem of the government is given by:

$$\max_{w_1, w_2, P_e} V(w_1, w_2, P_e, r, \theta) \quad \text{s.t.} \quad (3.16) \tag{3.18}$$

The first order conditions for this problem are:

$$\frac{\partial V}{\partial w_1} + \lambda \Big\{ -[(1+r)(1-e) + X] + (W - w_2)\overline{H} \cdot \frac{\partial L}{\partial w_1}$$
$$+[(W - w_2)\overline{H}_e L - (1+r)(W - w_1 - P_e)] \frac{\partial e}{\partial w_1} \Big\} = 0, \tag{3.19}$$
$$\frac{\partial V}{\partial w_2} + \lambda \Big\{ -(\overline{H}L - X) + (W - w_2)\overline{H} \cdot \frac{\partial L}{\partial w_2}$$
$$+[(W - w_2)\overline{H}_e L - (1+r)(W - w_1 - P_e)] \frac{\partial e}{\partial w_2} \Big\} = 0, \tag{3.20}$$
$$\frac{\partial V}{\partial P_e} + \lambda \Big\{ (1+r)e + (W - w_2)\overline{H} \cdot \frac{\partial L}{\partial P_e}$$
$$+[(W - w_2)\overline{H}_e L - (1+r)(W - w_1 - P_e)] \frac{\partial e}{\partial P_e} \Big\} = 0 \tag{3.21}$$

with λ as marginal costs of governmental revenue, and $\overline{H}_e$ as deterministic marginal productivity of education investment. The latter is equal to the expected value of marginal productivity across all households, $\overline{H}_e = \mathcal{E}[\tilde{H}_e]$.

3.6 A Special Case: Exogenous Leisure Demand

Before starting the general analysis of optimal education and taxation policies, we analyze first the special case of exogenous skilled labor supply, i.e., $l = 0$ and $L = 1$. This allows to sharpen the intuition for the insurance effect of tax progression.

If leisure is exogenously given, utility only depends on consumption in both periods. Moreover, the threshold value for the upper tax bracket X can be set at $X = 1$ for convenience. First order conditions for household behavior and optimal governmental policy can be inferred from equations (3.6) – (3.8), respectively (3.19) – (3.21) in the previous sections by substituting $L = 1$ and $X = 1$.

Applying (3.13) to (3.15) and the covariance rule, dividing and rearranging conditions (3.19) and (3.20) results in

$$\frac{\text{Cov}(U_{C_2}(\theta), \tilde{H}(\theta))}{\mathcal{E}[U_{C_2}]} \cdot N + A \cdot \left\{ (\overline{H} - 1)\frac{\partial e}{\partial w_1} - [(1+r)(1-e)+1]\frac{\partial e}{\partial w_2} \right\} = 0. \qquad (3.22)$$

Combining conditions (3.19) and (3.21) in the same manner gives

$$A \cdot \left\{ (1+r)e\frac{\partial e}{\partial w_1} + [(1+r)(1-e)+1]\frac{\partial e}{\partial P_e} \right\} = 0, \qquad (3.23)$$

where $A = (W - w_2)\overline{H}_e - (1+r)(W - w_1 - P_e)$ is the tax wedge of education, which measures the extent to which taxation and subsidization reduce the marginal return to education, and $N = -[(1+r)(1-e)+1] + A \cdot \frac{\partial e}{\partial w_1}$ is the tax revenue effect of increasing the net unskilled wage w_1.

Substituting some comparative-static results, equations (3.50) and (3.51) in appendix 3.A.2, in (3.23) and simplifying, we get

$$A \cdot \frac{(1+r)\alpha\mathcal{E}[U_{C_2}](2+r)}{SOC} = 0, \qquad (3.24)$$

where SOC stands for the determinant of the Hessian matrix.

Assuming an inner solution, SOC must be positive. Moreover, expected marginal utility of second period consumption is positive, and we have that $\alpha = \frac{\partial\left\{\mathcal{E}[U_{C_1}] - (1+r)\mathcal{E}[U_{C_2}]\right\}}{\partial C_1} \neq 0$ by assuming the Hessian matrix to be negative definite. Hence, condition (3.24) can only be fulfilled, if

$$A = (W - w_2)\overline{H}_e - (1+r)(W - w_1 - P_e) = 0. \qquad (3.25)$$

This implies that educational investment should not be distorted in the optimum, i.e. neither taxed nor subsidized on a net basis.

From inserting (3.25) in (3.22) then follows

$$\frac{\text{Cov}(U_{C_2}(\theta), \tilde{H}(\theta))}{\mathcal{E}[U_{C_2}]} \cdot N = 0, \qquad (3.26)$$

where N reduces to $-[(1+r)(1-e)+1] < 0$, because the tax base of the basic labor tax rate must be positive as $e \in [0, 1]$.

Therefore, optimal tax policy is described by

$$\text{Cov}(U_{C_2}(\theta), \tilde{H}(\theta)) = 0, \tag{3.27}$$

which states that the marginal utility of second period consumption should be uncorrelated with risk in human capital formation. This is only the case, if second period consumption does not depend on the human capital stock H, implying $w_2 = 0$ and, consequently, $t_2 = 1$ from equation (3.4).

The intuition for this result is as follows: Given our model and the fact that educational risk is assumed to be idiosyncratic, the government can provide full insurance against income risk by taxing away all (risky) returns to human capital. By the law of large numbers, the risk is entirely diversified in the aggregate budget constraint. However, taxing away the entire skill premium will sweep out any incentive for investing in human capital and, therefore, lead to inefficiency.

This can be avoided by using additionally education subsidies. In our case, the government can fully control the education decision, because it simultaneously provides incentives for educational investment via scholarships, paid out per semester spent at university. This can be seen from substituting $t_2 = 1$ in the first order condition (3.7) of household optimization, which leads to

$$P_e = -w_1. \tag{3.28}$$

From (3.28) and the Inada-conditions on the utility function, $t_1 \geq 1$ would imply $w_1 \leq 0$ and $P_e \geq 0$, which cannot be a social optimum, as household income would be zero or even turn negative and there would be no private consumption. It follows that $t_1 < 1$, and thus direct tax progression $t_2 > t_1$.

Next, by applying $t_2 = 1$ and $P_e = -w_1$, equation (3.25) simplifies to

$$\overline{H}_e = (1 + r). \tag{3.29}$$

Hence, optimal tax and education policy guarantees that the socially optimal level of educational investment is achieved and marginal productivities in educational investment and real capital investment are equalized. With full insurance and no distortion in educational investment individuals would choose the efficient level of education. Moreover, there is no distortion in labor supply, because we have assumed that leisure demand is exogenous. The basic tax rate t_1 is (in combination with education subsidies P_e) therefore a lump-sum tax and can be used for balancing the budget. Accordingly, we can simultaneously reach an efficient allocation of resources and full insurance.

Taken together, we can state our first result:

Proposition 3.1. *If educational risk is idiosyncratic and leisure demand is exogenous, optimal tax and education policy is characterized by full insurance and education subsidies. The government taxes skilled labor income with $t_2 = 1$ and subsidizes education directly with negative tuition fees $P_e = -w_1 < 0$.*

Of course this result is an artefact of the assumption of exogenous leisure demand, which leaves educational investment as the only decision margin distorted by the labor tax. However, the results show that private (educational) risk itself is not the problem, even if it is uninsurable in the private sector. The result can be seen as an analogue to the results in Jacobs and van Wijnbergen (2007, Propositions 6 and 8), who argue that optimal risk diversification should imply that all human capital investment is financed by an 'equity stake' of the government, which also takes all risky returns.

In the standard risk models, based on the seminal work by Eaton and Rosen (1980a,b), the government can only use a proportional labor tax and lump-sum transfers. Compared to this set up, extending the governmental instruments by a progressive labor tax and an education subsidy increases the degrees of freedom.[14]

The risk is not embedded in labor income, but in the skill premium. This premium can be taxed directly using the two-bracket tax system. This avoids any effect on returns to unskilled (raw) labor supply. Furthermore, education subsidies ($P_e < 0$) allow to control the education decision and guarantee efficiency, although full insurance is provided. If, instead, a proportional labor tax at a tax rate $t = 1$ would be implemented in such a model and all revenue would be returned in a lump-sum manner, there would be no incentive for investment in human capital at all.

[14] In addition, the major difference to the non-linear taxation result in Varian (1980) is that tuition fees increase the degrees of freedom even more and enable a better alleviation of distortions. Translated into the Varian-setting, this would imply to use additionally investment subsidies, therein.

Therefore, it is worthwhile to extend governmental instruments compared to standard analysis, because this enables the government to cope with risk without sacrificing (more) efficiency.

Last but not least, Proposition 3.1 shows that there is another linkage between income taxes and educational systems. Bovenberg and Jacobs (2005) state that progressive taxation and educational subsidies are 'Siamese Twins'. The intuition is that endogenous education decisions increase the elasticity of labor supply and, thus, increase the costs of redistribution by income taxation. This effect can be mitigated by introducing education subsidies. A similar effect can emerge in Schindler (2007), who introduces tuition fees and an educational sector in the model by Nielsen and Sørensen (1997), where there is a distortionary real capital taxation, which is to be countered by progressive labor taxation.

Proposition 3.1 now shows that progressive taxation and education subsidies are also 'Siamese Twins' due to risky educational investment. In order to improve the insurance function of income taxation, the government must grant scholarships to students. The argument behind this is again as in Bovenberg and Jacobs (2005): education subsidies are needed to avoid efficiency losses and to control the education decision.

At first sight and from an ex post perspective, our approach seems to double Bovenberg and Jacobs (2005). However, ex ante, there are major differences between a setting with heterogenous households versus educational risk in incomplete insurance markets: in the latter case the lack of sufficient insurance leads to inefficient underinvestment in education and taxation is less distortive due to an insurance effect, which mitigates ceteris paribus underinvestment. Nevertheless, it turns out in our analysis that full subsidization of educational investment is necessary in the special case of exogenous labor supply.

Two criticism force on: First, full insurance is supposed to create moral hazard,[15] second, exogenous leisure demand is – as mentioned – unrealistic.

In our model, investing less time than socially efficient, is not optimal given the per-semester subsidies. Hence, nobody will receive transfers, if it does not invest in education and does not pay its taxes on the skill premium. Thus, there is no moral hazard in sense of 'shirking.' However, if the success in human capital formation does not only depend on the time spent at university, but also on the learning intensity – the way *how* time is spent at university, – the

[15] See i.e., Wigger and von Weizsäcker (2001), who examine public versus private financing of higher education and focus on moral hazard problems in section II.5.

moral hazard problem is re-introduced, and full insurance is most likely not to be optimal.

If leisure choice is not exogenous, the (skilled) wage tax will cause major distortions in labor supply. These cannot be (entirely) avoided by education subsidies. Thus, there should be a trade-off between the insurance effect of taxation on the one hand, and efficiency losses on the other hand. Again, full insurance ($t_2 = 1$) seems not to be optimal in such a setting.

Although full insurance appears to be unlikely then, the intuition for some progressive taxation (at $t_1 < t_2 < 1$) and education subsidies, $P_e < 0$ should, however, survive. This will be examined in the next section.

3.7 Educational Risk and Siamese Twins

Turning to the more general case of endogenous second period leisure, skilled labor supply will reduce to $L < 1$, because of the Inada conditions for the utility function. We are going to show conditions under which the intuition for progressive taxation and education subsidization can then be generalized. Let us assume that the government has also the possibility to use a (state-independent) poll tax τ. The governmental budget constraint turns into

$$t_1 \cdot W \cdot [(1+r)(1-e) + X] + t_2 \cdot W \cdot [\overline{H} \cdot L - X] + (1+r) \cdot P_e \cdot e + \tau = \overline{R}. \tag{3.30}$$

In order to show the optimality of progressive taxation and education subsidies, we focus on a balanced budget policy reform concerning the instruments t_2 and P_e. Totally differentiating (3.30) and rearranging gives

$$\frac{dP_e}{dt_2}\Big|_{d\overline{R}=0} = \tag{3.31}$$
$$-\frac{\Big[t_2 \cdot W \cdot \overline{H}_e \cdot L - (1+r)(t_1 \cdot W - P_e)\Big] \cdot \frac{\partial e}{\partial t_2} + t_2 \cdot W \cdot \overline{H} \cdot \frac{\partial L}{\partial t_2} + W \cdot [\overline{H} \cdot L - X]}{\Big[t_2 \cdot W \cdot \overline{H}_e \cdot L - (1+r)(t_1 \cdot W - P_e)\Big] \cdot \frac{\partial e}{\partial P_e} + t_2 \cdot W \cdot \overline{H} \cdot \frac{\partial L}{\partial P_e} + (1+r) \cdot e}.$$

If we first look at the case, where the entire public spending is financed by the poll tax, $\tau = \overline{R}$

and $t_1 = t_2 = P_e = 0$, the balanced budget condition (3.31) simplifies to

$$\left.\frac{dP_e}{dt_2}\right|_{d\overline{R}=0} = -\frac{W \cdot [\overline{H}(e,\theta) \cdot L - X]}{(1+r) \cdot e}. \tag{3.32}$$

In this case, introducing a positive surtax rate will implement a progressive tax system, which is a pure graduate tax. Returning tax revenue as education subsidies, the effect of such a compensated tax reform on social welfare can be calculated from

$$\frac{dV}{dt_2} = \frac{\partial V}{\partial t_2} + \frac{\partial V}{\partial P_e} \cdot \left.\frac{dP_e}{dt_2}\right|_{d\overline{R}=0}. \tag{3.33}$$

Using (3.14) and (3.15), as well as the simplified balanced-budget effect (3.32), we infer from (3.33) at $t_2 = t_1 = P_e = 0$

$$\begin{aligned}\left.\frac{dV}{dt_2}\right|_{t_2=t_1=P_e=0} &= W \cdot \left\{\mathcal{E}[U_{C_2}] \cdot [\overline{H}(e) \cdot L - X] - \mathcal{E}[U_{C_2} \cdot (\tilde{H}(e,\theta)L - X)]\right\} \\ &= (-W) \cdot \text{Cov}(U_{C_2}(\theta), \tilde{H}(\theta)) \cdot L > 0, \end{aligned} \tag{3.34}$$

because sign$\{\text{Cov}(U_{C_2}, \tilde{H}(\theta))\} = \text{sign}\{\text{Cov}(U_{C_2}, \tilde{C}_2(\theta))\} < 0$.

Hence, we can conclude:

Proposition 3.2. *In case of risky human capital formation, it is not optimal to finance the education system by a pure lump-sum tax. Introducing a graduate tax, accompanied by education subsidies, increases social welfare.*

Beginning in an undistorted allocation, a progressive labor tax with $t_2 > t_1 = 0$, which is in fact a graduate tax, insures against income risk and distorts both investment in education and skilled labor supply. These distortions can be mitigated in part by granting education subsidies. The combination of both distorting instruments increases welfare, because around $t_2 = t_1 = P_e = 0$, the welfare increasing insurance effect is more valuable than the net efficiency losses created by distortions.

Whilst Eaton and Rosen (1980b) show that distortionary labor taxation and lump-sum transfers increase welfare, we show that the combination of two distorting instruments can deliver a welfare increasing insurance effect. The intuition of the inelastic leisure demand case still applies: Progressive labor taxation can tackle the risky income base in a better way, and education subsidies are a superior instrument in order to avoid distortions in education.

Analyzing these effects, instead, in an economy, where a flat tax with positive tax rate t is in place, is more realistic, but also much more complicated. The economy is already distorted and increasing the surtax rate t_2 will then amplify these distortions in a non-negligible way. Nevertheless, we are able to derive conditions, for which tax progression is optimal, and we can draw some conclusions on the favorability of a flat tax regime in the following section.

For the analysis to come, we make the following assumptions:

Assumption 3.1. *(i) The Laffer curve concerning tuition fees has a positive derivative around $P_e = 0$, thus $D = \frac{\partial \bar{R}}{\partial P_e}\Big|_{P_e=0} > 0$.*

(ii) The tax base for the surtax is nonzero, accordingly $W \cdot L \geq W \cdot X$.

(iii) Tuition fees have negative effects on uncompensated labor supply and educational investment, $\frac{\partial e}{\partial P_e} < 0$, $\frac{\partial L}{\partial P_e} < 0$.

(iv) Uncompensated labor supply and educational investment depend positively on net skilled wages, hence $\frac{\partial L}{\partial t_2} < 0$, $\frac{\partial e}{\partial t_2} < 0$.

The negative effects of tuition fees and the positive effects of skilled wages on the demand for education are empirically well tested. In Leslie and Brinkman's review (1987) they conclude that the modal result of about 30 empirical studies is a 1.8% enrollment decline per 100 dollar increase in tuition fees. An overview on the literature analyzing the effects of higher skilled wages on enrollment has been provided by Freeman (1986), showing an elasticity of higher education demand to salaries in a range between 0.5 and 2.0.

The estimates for the uncompensated wage elasticity of labor supply are, instead, mostly very low and sometimes even negative, ranging from 0.14 till -0.29 for US men with a median of -0.10 (see Pencavel, 1986). Nevertheless, we assume the uncompensated labor supply not backward bending, implying $\frac{\partial L}{\partial t_2} < 0$. $\frac{\partial L}{\partial P_e} < 0$ is then implied by the fact that an increase in tuition fees will reduce human capital and consequently reduce earnings, leading to a decrease in labor supply.

The welfare effect of altering the surtax rate t_2 in a flat tax environment $t_2 = t_1 = t$ and in the absence of tuition fees $P_e = 0$ can be derived as[16]

$$\left.\frac{dV}{dt_2}\right|_{t_2=t_1=t,P_e=0} = -\frac{W\,\overline{H}\,L\mathcal{E}\left[U_{C_2}\right]\,(1+r)\,e}{D}\cdot \left\{\frac{t}{1-t}\cdot\Big(\varphi\left[\gamma\,\pi_e\,\eta_{eP_e}+\eta_{LP_e}\right]-\left[\gamma\,\pi_e\,\eta_{et_2}+\eta_{Lt_2}\right]\Big)-\pi_H\right\} \tag{3.35}$$

where $\gamma = \frac{\overline{H}_e e}{\overline{H}}$ is the expected production elasticity of human capital. Determining the insurance characteristics, $\pi_H = -\frac{\text{Cov}(U_{C_2},H)}{\mathcal{E}[U_{C_2}]\overline{H}}$ is the negatively normalized covariance between marginal utility of second period consumption and the human capital stock, which is always positive. Furthermore, η_{ij} represents uncompensated elasticities in labor supply and educational investment and $\varphi = \frac{\overline{H}L(1-\pi_H)-X}{(1+r)e} > 1$, because of $X < L$ from Assumption 3.1 and the fact that $W\cdot(\overline{H}L(1-\pi_H)-L)$ is the risk-adjusted return to education before taxes, which has to be positive and larger as the inflated costs of educational investment, $(1+r)We$, in order to have positive educational investment.

According to Assumption 3.1, the denominator of equation (3.35) D is positive. Progressive taxation to be welfare-enhancing, implying $\left.\frac{dV}{dt_2}\right|_{t_2=t_1=t,P_e=0} > 0$, requires thereafter

$$\frac{t}{1-t}\cdot\Big(\varphi\cdot\left[\gamma\pi_e\eta_{eP_e}+\eta_{LP_e}\right]-\left[\gamma\,\pi_e\,\eta_{et_2}+\eta_{Lt_2}\right]\Big)-\pi_H < 0. \tag{3.36}$$

Accordingly, a tax reform introducing both progressive wage tax and subsidization of education has three welfare-relevant effects: First, a progressive tax provides better insurance against income risks and increases the utility of risk-averse individuals. This unambiguously welfare-increasing effect is measured by the magnitude of the insurance characteristics $\pi_H > 0$. Second, progressive wage taxation has negative incentive effects on both labor supply and educational investment, causing an excess burden. Third, though being per-se distorting as well, education subsidies have a positive effect for two reasons: (i) They mitigate the negative effects of increasing t_2 by fostering educational investment and stabilizing labor supply as education and labor supply are complements (at least given Assumption 3.1). (ii) Educational subsidies mitigate the underinvestment problem into education as well, as the marginal costs of education decrease.

[16]See Appendix 3.A.3.

Thus, the allocative net effect, the second, bracketed factor of the first term on the left-hand-side of equation (3.36), is ambiguous.

Taken together, for tax progression to be desirable, the (potentially) harmful net effect of induced distortions in labor supply and educational investment has to be compensated by the welfare-enhancing insurance effect π_H.

If the disincentive effects of increasing the surtax rate t_2 are larger than the effects of education subsidies, mitigating underinvestment in education, the allocative net effect of the tax reform affects welfare indeed negatively. Then, there is the classical trade-off between insurance and efficiency and whether such a tax reform can improve welfare depends on which effect dominates. A welfare-enhancing tax reform then requires that the initial flat tax rate t is not too high:

$$\frac{t}{1-t} < \frac{\pi_H}{\varphi \cdot [\gamma \pi_e \eta_{eP_e} + \eta_{LP_e}] - [\gamma\, \pi_e\, \eta_{et_2} + \eta_{Lt_2}]}. \tag{3.37}$$

The higher the initial tax rate t, the larger are the induced distortions by an increase of t_2 and the less likely is a welfare-improvement. The more risk matters instead, measured by an increase in π_H, the more importance is attached to insurance and the likelier is a welfare-improving by the tax reform, allowing then for a higher initial tax rate t.

If the allocative net effect is instead welfare-enhancing itself, there is no trade-off at all and we can state from examining condition (3.36):

Proposition 3.3. *Starting from $t_2 = t_1 = t > 0$ and $P_e = \tau = 0$, a sufficient condition for a welfare-enhancing introduction of tax progression $t_2 > t_1 > 0$ and simultaneous redemption of additional tax revenue as education subsidies per semester $P_e < 0$ is*

$$|\gamma \pi_e \eta_{et_2} + \eta_{Lt_2}| \leq |\varphi \left(\gamma \pi_e \eta_{eP_e} + \eta_{LP_e}\right)|. \tag{3.38}$$

Proof: According to Assumption 3.1 both $\gamma \pi_e \eta_{eP_e} + \eta_{LP_e}$ and $\gamma \pi_e \eta_{et_2} + \eta_{Lt_2}$ are negative. If $|\gamma \pi_e \eta_{et_2} + \eta_{Lt_2}| \leq |\varphi \cdot (\gamma \pi_e \eta_{eP_e} + \eta_{LP_e})$, the inequality (3.36) is fulfilled irrespectively of the magnitude of the insurance effect $\pi_H > 0$. □

Proposition 3.3 characterizes a situation, where the distortive effects of increased wage taxation are more than compensated by the introduction of educational subsidies. In case Proposition 3.3 holds, the tax reform provides efficiency gains instead of an excess burden and should be implemented even on pure efficiency grounds and irrespectively of any insurance effect.

This case is the more likely the more inelastic labor supply and educational investment react onto changes in the net wages, consequently the lower are the elasticities η_{et_2} and η_{Lt_2}, because distortionary effects are very small then. Moreover, the effect on education is weighted by the product of the risk premium in educational investment π_e and the expected production elasticity of human capital γ, indicating whether labor taxation amplifies underinvestment in education substantially.

The likelihood for fulfilling the condition in Proposition 3.3 increases instead in the sensitivity of labor supply and educational investment with regard to education subsidies, η_{eP_e} respectively η_{LP_e}, measuring the compensating allocative effects. Again, the effect on education is weighted by $\pi_e \cdot \gamma$, measuring the relevance of underinvestment again. Additionally the effect of education subsidies is weighted by φ. $\varphi = \frac{W \cdot [\overline{H}L(1-\pi_H)-X]}{(1+r) \cdot W \cdot e}$ can be interpreted as the risk-adjusted, discounted average return in tax revenue from subsidizing educational investment, as the numerator represents the (risk-adjusted) tax base of returns to education and the denominator gives tax revenue forgone by educational investment. Hence, φ mirrors the self-financing effect of subsidization in education.

How realistic is such a situation and can this proposition be backed by some empirical evidence? In the discussion of Assumption 3.1, we have already seen that labor supply is very inelastic. This implies that the sensitivity of labor supply to education subsidies should also be around zero, because the complementary of labor supply and education works via increasing wages. If we assume $\eta_{Lt_2} = \eta_{LP_e} = 0$, the condition in Proposition 3.3 boils down to

$$|\eta_{et_2}| \leq \varphi \cdot |\eta_{eP_e}|, \tag{3.39}$$

where $\varphi \geq 1$.

Chang and Hsing (1996) report that for the US the elasticity of enrollment of private institutions of higher education (IHE), relative to that of public institutions, w.r.t average tuition fees and costs per student at private IHEs, relative to those at public IHEs, is -13.561 for the years $1990 - 1991$. In our case of pure public schools the elasticity of enrollment w.r.t. tuition fees might be expected to be smaller, but should still not be too small and remain negative. Dynarski (1999) finds that each $1,000$ dollar increase in student benefits by Social Security Student Benefit Program increases the share of high school graduates who attended college before 1996 by 3.6 percentage points. Therefore an increase in tuition fees respectively an increase in student aid (education subsidies) seem to have a significant effect on education demand.

The effect of a higher wage rate on student enrollment has also been estimated by some studies. Freeman (1986) provides an overview on a part of this literature, showing an elasticity of higher education demand to salaries in a range of 0.5 – 2.0. Kodde (1985) reports a smaller elasticity of enrollment to future monthly income of 0.14, using data from Dutch high school graduates in 1982.[17] In a more recent study by Fredriksson (1997) the elasticity of the enrollment rate of higher school leavers w.r.t. the university graduate wage rate is estimated to equal 2.8 for Sweden.

Psacharopoulos (1973), instead, estimates the elasticity of freshman enrollment at public institutions for higher education in Hawaii for the years $1956 - 1968$ to be 0.45 w.r.t relative earnings of college graduates to high school graduates, but to be -1.12 w.r.t tuition fees.

Taken together, the requirements for Proposition 3.3 to be applicable may or may not be fulfilled. Therefore, it is in any case worthwhile to have a closer look at the determinants of the insurance characteristics π_H. In order to be able to derive some clear-cut results here, we make some additional assumptions:

Assumption 3.2. *(i) The subutility function in the second period is separable in consumption and labor supply,* $U_{c_2 l} = 0$.

(ii) There is multiplicative wage risk: $\tilde{H}(e, \theta) = \theta \cdot h(e)$.

(iii) The shock is normally distributed with mean $\mathcal{E}[\theta] = 1$ *and variance* σ_θ^2.

Modeling wage risk in a multiplicative way is in line with Eaton and Rosen (1980a,b) and Hamilton (1987), whereas assuming that it is normally distributed might be a bit cumbersome at first glance. However, this allows to apply a Rubinstein-theorem, which should be a reasonable approximation for other distributions as well. We then conclude:

Proposition 3.4. *Given Assumption 3.2, the insurance effect and therefore the preferability of progression in the wage tax and of introducing direct education subsidies* $P_e < 0$ *are increasing in*

[17] However, he also finds very small responses of enrollment to tuition fees.

(i) expected net labor earnings $(1-t) \cdot W \cdot h(e) \cdot L$,

(ii) global risk aversion of consumption $ARA(C)$,

(iii) and the variance of the shock σ_θ^2.

Proof: See Appendix 3.A.4. □

The more risk is in the economy and the more this risk affects well-being, the more valuable becomes social insurance ceteris paribus – calling then for tax progression. This intuition is mirrored in Proposition 3.4, where the expected net wage income measures a household's exposure to risk and where risk aversion determines, how the household is affected by this exposure. The variance of the shock is finally a measure for the magnitude of the risk in the economy.

Altogether, if the insurance effect of progressive taxation is more important than its net distortionary effects, a progressive income tax in combination with an education subsidy is superior to a proportional (flat) income tax.

If so, progression in the labor tax guarantees superior insurance effects compared to proportional taxation, and education subsidies avoid that the efficiency losses get too strong. Thus, the intuition of the result in case of entirely exogenous leisure demand can also be applied in case of elastic skilled labor supply. However, full insurance seems not to be optimal, because the induced efficiency losses would now be too high, if $t_2 = 1$.

However, we have to note that a non-progressive taxation can nevertheless be optimal under certain conditions. A necessary condition for this to be true can be easily derived from Proposition 3.3.

Corollary 3.1. *A necessary condition for having non-progressive taxation in a Second-best optimum is that*

$$|\gamma \pi_e \eta_{et_2} + \eta_{Lt_2}| > |\varphi \left(\gamma \pi_e \eta_{eP_e} + \eta_{LP_e}\right)|. \tag{3.40}$$

This condition implies that the negative welfare effects of increasing t_2 have to dominate the effects of fostering educational investment by subsidization $P_e < 0$.

In fact, a regressive tax structure might be optimal, then. If the initial flat tax rate t is too high, it matters more to decrease distortions in labor supply directly by reducing the marginal tax rate on skilled labor than to provide enhanced insurance via a progressive tax system and to foster education via subsidies. In a nutshell, tax regression is optimal, in case the net distortions dominate the insurance effect. This can be the case, if and only if Corollary 3.1 is fulfilled, but following our discussion of Proposition 3.3, we think that it is unlikely having the net distortions dominating the insurance effect.

3.8 Implications to the Flat-Tax Debate

How are our results related to the debate whether a flat tax should be implemented and which new insights can be derived?

The main idea of a flat tax dates back to a very influential proposal by Hall and Rabushka (1983). In fact, they proposed a tax system with a tax base, containing all labor income, but tax-exempting capital income on the personal level, and only one, constant tax rate, accompanied by a tax-credit.[18]

This tax system contradicts standard results in optimal taxation models, where non-linear taxation turns out to be beneficial in case society values redistribution.[19] However, flat tax systems are easier to run and some simulations show that the optimal non-linear structure can be well-approximated by a flat tax (Myles, 1995, section 5.4). More important, the standard models on non-linear taxation neglect (labor market) participation distortions, endogenous human capital formation and labor market imperfections. All these tend to decrease marginal tax rates.[20]

These might be the reasons, why flat tax systems got more and more popular and are recently introduced in many Eastern European countries (e.g. Slovakia and Estonia), but also discussed in Western Europe.[21] Nevertheless, a flat tax remains doubtful even if standard optimal taxation results are neglected.

Nielsen and Sørensen (1997) show that a flat tax in the broader sense, incorporating a constant tax rate on capital income, cannot be optimal even in a model without redistribution, if there is endogenous human capital formation. The capital tax acts as subsidy on educational investment and *calls* for progressive taxation in order to mitigate distortions.

Our results now raise doubts whether a flat tax can be optimal in a world with human capital formation, if there are risky returns to education and wage risk in general. These doubts are

[18] See, i.e, Atkinson (1995) for a detailed description and analysis of the flat tax.

[19] See, e.g., Myles (1995, chapter 5) for a survey and Saez (2001) for a more recent contribution.

[20] See, e.g., Jacobs et al. (2007, section 2) or Keen et al. (2008, section 3.1) for a brief overview and discussion.

[21] See, i.e., Keen et al. (2008) and Paulus and Peichl (2008). In Germany, e.g., the conservative party CDU supported a flat-tax like proposal by Paul Kirchhof in its election campaign 2005.

derived in a model with ex-ante homogenous households and without capital taxation, which then should be the most favorable setting for a flat tax. However, in our model a constant flat tax rate $t > 0$ is only optimal if simultaneously

$$\frac{t}{1-t} = \frac{\pi_H}{\varphi\left(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}\right) - \left(\gamma\pi_e\eta_{et_2} + \eta_{Lt_2}\right)} \tag{3.41}$$

and Corollary 3.1, $|\gamma\pi_e\eta_{et_2} + \eta_{Lt_2}| > |\varphi\left(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}\right)|$, are fulfilled.

Thus, there have to be net distortions from introducing tax progression and these distortions have to cancel exactly against the insurance effect at the proportional tax rate t.

Notwithstanding the insurance effect, it is already doubtful, whether Corollary 3.1 can be sustained by empirical observations. Taking into account the insurance effect and Proposition 3.4, it appears then very unlikely that a flat tax rate $t > 0$ guarantees sufficient insurance in a second-best optimum. Insurance and distortionary effects will cancel each other only by pure incidence.

3.9 Conclusions

We have shown that in a two-period model with endogenous, but risky human capital formation, the optimal labor tax structure is most likely to be progressive in order to insure against income risk, and (direct) education subsidies should be used in order to alleviate induced distortions, if the educational risk is idiosyncratic and the government can diversify the risk at no costs.

If leisure demand is entirely inelastic, the government is able to provide full insurance. In the more general case of elastic skilled labor supply, progression survives under some assumptions, when the starting point is a positive proportional labor tax.

In a nutshell, extending the instruments of the government in a standard Eaton-Rosen world leads to the Bovenberg-Jacobs effect of ‘Siamese Twins’, where education subsidies are needed in order to alleviate efficiency losses. Thus, the mechanism identified in Bovenberg and Jacobs (2005) should also carry over in case of risk and helps to avoid inefficient underinvestment in education in a laissez-faire economy.

Our results also apply to the flat tax debate. Here, we raise severe doubts, whether a flat tax can be optimal, even in a model with ex-ante homogenous households and without capital taxation. The reason is that a flat tax is unlikely to guarantee sufficient insurance against risky returns to educational investment and wage risk.

A critical point in the model is the assumption of idiosyncratic risk. The government can diversify the risk at no costs, whereas it is in general assumed that private insurance is not possible. Thus, the critical question, which appears in all such models is: Why can the government do better than private insurers? This problem is easily solved, if the risk is assumed to be aggregate risk. In this case the government can provide diversification of risk on private and public consumption (see e.g., Kaplow, 1994). However, tax revenue then turns risky, which will have major impact on the results. This aspect is left for further research.

Appendix 3.A

3.A.1 Second-Order-Conditions of Household Optimization

The Hessian Matrix in the households' optimization problem is

$$H = \begin{pmatrix} \frac{\partial\mathcal{E}[U]}{\partial C_1 C_1} & \frac{\partial\mathcal{E}[U]}{\partial C_1 e} & \frac{\partial\mathcal{E}[U]}{\partial C_1 l} \\ \\ \frac{\partial\mathcal{E}[U]}{\partial e C_1} & \frac{\partial\mathcal{E}[U]}{\partial ee} & \frac{\partial\mathcal{E}[U]}{\partial el} \\ \\ \frac{\partial\mathcal{E}[U]}{\partial l C_1} & \frac{\partial\mathcal{E}[U]}{\partial le} & \frac{\partial\mathcal{E}[U]}{\partial ll} \end{pmatrix}$$

The second order conditions for maximizing utility require $\mathcal{E}[U_{C_1C_1}] < 0$ and the second leading minor $\mathcal{E}[U_{C_1C_1}] \cdot \mathcal{E}[U_{ee}] - \mathcal{E}[U_{C_1e}]^2$ to be positive. The third leading minor must be negative, implying

$$\begin{aligned} \mathcal{E}[U_{C_1C_1}] \cdot \mathcal{E}[U_{ll}] \cdot \mathcal{E}[U_{ee}] \quad &+ \quad 2\mathcal{E}[U_{el}] \cdot \mathcal{E}[U_{C_1e}] \cdot \mathcal{E}[U_{C_1l}] < \mathcal{E}[U_{C_1C_1}] \cdot \mathcal{E}[U_{el}]^2 \\ &+ \quad \mathcal{E}[U_{ll}] \cdot \mathcal{E}[U_{C_1e}]^2 + \mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{C_1l}]^2. \end{aligned} \tag{3.42}$$

Rearranging leads to

$$\begin{aligned} 2(\mathcal{E}[U_{C_1e}] \cdot \mathcal{E}[U_{el}] \cdot \mathcal{E}[U_{C_1l}] - \mathcal{E}[U_{C_1C_1}] \cdot \mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{ll}]) < \\ \mathcal{E}[U_{C_1C_1}](\mathcal{E}[U_{el}]^2 - \mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{ll}]) + \mathcal{E}[U_{ll}](\mathcal{E}[U_{eC_1}]^2 - \mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{C_1C_1}]) \\ +\mathcal{E}[U_{ee}](\mathcal{E}[U_{C_1l}]^2 - \mathcal{E}[U_{C_1C_1}] \cdot \mathcal{E}[U_{ll}])] \end{aligned} \tag{3.43}$$

If we change the order of Hessian Matrix, it follows that the second leading minors of the respective Hessian Matrix $\mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{ll}] - \mathcal{E}[U_{el}]^2$ and $\mathcal{E}[U_{C_1C_1}] \cdot \mathcal{E}[U_{ll}] - \mathcal{E}[U_{C_1l}]^2$ are both positive. Therefore we have:

$$\mathcal{E}[U_{el}]^2 < \mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{ll}] \tag{3.44}$$

$$\mathcal{E}[U_{C_1l}]^2 < \mathcal{E}[U_{C_1C_1}] \cdot \mathcal{E}[U_{ll}] \tag{3.45}$$

$$\mathcal{E}[U_{C_1e}]^2 < \mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{C_1C_1}] \tag{3.46}$$

Inequations (3.44), (3.45) and (3.46) imply that

$$(\mathcal{E}[U_{el}] \cdot \mathcal{E}[U_{eC_1}] \cdot \mathcal{E}[U_{C_1 l})]^2 < (\mathcal{E}[U_{ee}] \cdot \mathcal{E}[U_{ll}] \cdot \mathcal{E}[U_{C_1C_1})]^2 \tag{3.47}$$

(3.47) states that the absolute value of the three cross-effects in household utility multiplied together should be smaller than the second-derivatives of e, l and C_1.

Using (3.47) in equation (3.43), its left-hand-side is positive and the right-hand-side of (3.43) turns out to be positive as well – due to (3.44) – (3.46).

Therefore, all restrictions for guaranteeing the Hessian matrix to be negative definite, (3.43) – (3.47), are fulfilled if the cross-effects are sufficiently small in absolute values (or even tend to zero).

This is the standard second-order condition in case of positive or negative feedback effects between different household decisions, which we will assume to hold throughout the paper: the cross effects are small enough in comparison to diminishing marginal utilities in order to avoid corner solutions.

3.A.2 Comparative Statics of Household Choice

Assuming exogenous labor supply in the second period and totally differentiating the two first order conditions (3.6) and (3.7) of the individual maximization problem results in

$$\begin{aligned}
&\alpha dC_1 + \beta de = \\
&-\{\mathcal{E}[U_{C_1C_2}][(1+r)(1-e)+1] - (1+r)\mathcal{E}[U_{C_2C_2}][(1+r)(1-e)+1]\}dw_1 \\
&-\{\mathcal{E}[U_{C_1C_2}(\tilde{H}-1)] - (1+r)\mathcal{E}[U_{C_2C_2}(\tilde{H}-1)]\}dw_2 \\
&-\{-(1+r)e\mathcal{E}[U_{C_1C_2}] + (1+r)^2 e\mathcal{E}[U_{C_2C_2}]\}dP_e,
\end{aligned} \tag{3.48}$$

$$\begin{aligned}
&\gamma dC_1 + \delta de = \\
&-\{\mathcal{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}[(1+r)(1-e)+1]] - (1+r)\mathcal{E}[U_{C_2}]\}dw_1 \\
&-\{\mathcal{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}(\tilde{H}-1)] + E[U_{C_2}\tilde{H}_e]\}dw_2 \\
&-\{\mathcal{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}(1+r)(-e)] - (1+r)\mathcal{E}[U_{C_2}]\}dP_e,
\end{aligned} \tag{3.49}$$

whereby

$$\begin{aligned}\alpha &= \frac{\partial \mathcal{E}[U_{C_1}] - (1+r)\mathcal{E}[U_{C_2}]}{\partial C_1},\\ \beta &= \frac{\partial \mathcal{E}[U_{C_1}] - (1+r)\mathcal{E}[U_{C_2}]}{\partial e},\\ \gamma &= \frac{\partial \mathcal{E}[U_{C_2}\{w_2 H_e - (1+r)(w_1+P_e)\}]}{\partial C_1},\\ \delta &= \frac{\partial \mathcal{E}[U_{C_2}\{w_2 H_e - (1+r)(w_1+P_e)\}]}{\partial e}.\end{aligned}$$

Using Cramer's rule the derivatives of optimal education decision e w.r.t. w_1 and P_e can be derived as:

$$\frac{\partial e}{\partial w_1} = \frac{-\alpha\{\eta[(1+r)(1-e)+1] - (1+r)\mathcal{E}[U_{C_2}]\} + \gamma\{\epsilon[(1+r)(1-e)+1]\}}{SOC}, \tag{3.50}$$

$$\frac{\partial e}{\partial P_e} = \frac{\alpha\{\eta(1+r)e + (1+r)\mathcal{E}[U_{C_2}]\} - \gamma\epsilon(1+r)e}{SOC}, \tag{3.51}$$

where

$$\eta = \mathcal{E}[U_{C_2C_2}\{w_2\tilde{H}_e - (1+r)(w_1+P_e)\}],\ \epsilon = \mathcal{E}[U_{C_1C_2}] - (1+r)\mathcal{E}[U_{C_2C_2}],$$

and SOC stands for the determinant of the Hessian matrix.

3.A.3 Proof of Equation (3.35)

Applying $t_2 = t_1 = t$ and $P_e = 0$ in the balanced-budget condition (3.31), leads to

$$\frac{dP_e}{dt_2} = -\frac{tW[\overline{H}_e L - (1+r)]\frac{de}{dt_2} + tW\overline{H}\frac{dL}{dt_2} + W[\overline{H}L - X]}{tW[\overline{H}_e L - (1+r)]\frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e} \tag{3.52}$$

Moreover, from household choice and equation (3.10) follows

$$\overline{H}_e \cdot L - (1+r) = \overline{H}_e \cdot L \cdot \pi_e, \tag{3.53}$$

as $P_e = 0$ and $w_1 = w_2$ due to $t_1 = t_2$.

This allows to rewrite equation (3.52) as

$$\frac{dP_e}{dt_2} = -\frac{tW\,\overline{H}_e\,L\,\pi_e \cdot \frac{de}{dt_2} + tW\,\overline{H}\frac{dL}{dt_2} + W[\overline{H}L - X]}{tW\,\overline{H}_e \cdot L \cdot \pi_e \cdot \frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e} \tag{3.54}$$

Assuming, according to Assumption 3.1, that the Laffer curve is increasing around $P_e = 0$, $\frac{\partial \bar{R}}{\partial P_e}\Big|_{dP_e=0} > 0$, implies that the denominator of (3.54) is positive:

$$D = \frac{\partial \bar{R}}{\partial P_e} = tW\,\overline{H}_e \cdot L \cdot \pi_e \cdot \frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e > 0. \tag{3.55}$$

Substituting equation (3.54) as well as the Envelope effects in (3.14) and (3.15) into equation (3.33) results in

$$\frac{dV}{dt_2}\Big|_{t_2=t_1=t,P_e=0} = -W\mathcal{E}[U_{C_2}\,(HL-X)] + (1+r)\,e \cdot \mathcal{E}[U_{C_2}] \cdot \frac{t\overline{H}_e LW\pi_e\frac{de}{dt_2} + tW\overline{H}\frac{dL}{dt_2} + W[\overline{H}L - X]}{t\overline{H}_e LW\pi_e\frac{de}{dP_e} + tW\overline{H}\frac{dL}{dP_e} + (1+r)e} \tag{3.56}$$

and by factoring out the denominator of the second summand on the right-hand-side

$$\begin{aligned}\frac{dV}{dt_2}\Big|_{t_2=t_1=t,P_e=0} &= \\ -\frac{W}{D} \cdot \Big\{&\mathcal{E}[U_{C_2} \cdot (\tilde{H} \cdot L - X)] \cdot \left[tW\overline{H}_e\,L\,\pi_e \cdot \frac{\partial e}{\partial P_e} + tW\overline{H} \cdot \frac{\partial L}{\partial P_e}\right] \\ &-\mathcal{E}[U_{C_2}](1+r) \cdot e \cdot \left[t\overline{H}_e\,L\,\pi_e \cdot \frac{\partial e}{\partial t_2} + t\overline{H} \cdot \frac{\partial L}{\partial t_2}\right] \\ &+ \Big(\mathcal{E}[U_{C_2} \cdot (\tilde{H} \cdot L - X)] - \mathcal{E}[U_{C_2}][\overline{H} \cdot L - X]\Big) \cdot (1+r) \cdot e\Big\}\end{aligned} \tag{3.57}$$

Applying Steiner's Rule, the first bracket in the last line of (3.57) reduces to

$$\mathcal{E}[U_{C_2} \cdot (\tilde{H} \cdot L - X)] - \mathcal{E}[U_{C_2}][\overline{H} \cdot L - X] = -\mathcal{E}[U_{C_2}]\,\overline{H}\,L\,\pi_H < 0, \tag{3.58}$$

where we have defined the insurance characteristics (according to Feldstein's distributional characteristic) as

$$\pi_H = -\frac{\text{Cov}(U_{C_2}, H)}{\mathcal{E}[U_{C_2}] \cdot \overline{H}} > 0. \tag{3.59}$$

Substituting equation (3.58) into (3.57), factoring out $\overline{H}\ L\ \mathcal{E}[U_{C_2}]\ (1+r)\ e$ now, and relying in the second line on Steiner's Rule again, leaves us with

$$\begin{aligned}\frac{dV}{dt_2}\bigg|_{t_2=t_1=t, P_e=0} &= -\frac{W\overline{H}\ L\ \mathcal{E}[U_{C_2}](1+r)e}{D} \cdot \left\{\frac{\overline{H}\ L \cdot (1-\pi_H) - X}{(1+r)\cdot e} \cdot\right. \\ &\left[\frac{t}{1-t}\frac{\overline{H}_e \cdot e}{\overline{H}} \cdot \pi_e \cdot \frac{(1-t)W}{e}\frac{\partial e}{\partial P_e} + \frac{t}{1-t}\frac{(1-t)W}{L} \cdot \frac{\partial L}{\partial P_e}\right] \\ &- \left.\left[\frac{t}{1-t}\frac{\overline{H}_e \cdot e}{\overline{H}} \cdot \pi_e \cdot \frac{(1-t)}{e}\frac{\partial e}{\partial t_2} + \frac{t}{1-t}\frac{(1-t)}{L} \cdot \frac{\partial L}{\partial t_2}\right] - \pi_H\right\}\end{aligned} \tag{3.60}$$

Evaluating at $t_2 = t_1 = t$ and $P_e = 0$, we define the uncompensated elasticities of labor supply L and educational investment e with respect to the tax rate t_2 and overall education costs $w_1 + P_e = (1-t)W$ as

$$\begin{aligned}\eta_{Lt_2} &= \frac{1-t_2}{L} \cdot \frac{\partial L}{\partial w_2} = \frac{1-t}{L} \cdot \frac{\partial L}{\partial w_2} \\ \eta_{et_2} &= \frac{1-t_2}{e} \cdot \frac{\partial e}{\partial w_2} = \frac{1-t}{e} \cdot \frac{\partial e}{\partial w_2} \\ \eta_{LP_e} &= \frac{w_1 + P_e}{L} \cdot \frac{\partial L}{\partial P_e} = \frac{(1-t)W}{L} \cdot \frac{\partial L}{\partial P_e} \\ \eta_{eP_e} &= \frac{w_1 + P_e}{e} \cdot \frac{\partial e}{\partial P_e} = \frac{(1-t)W}{e} \cdot \frac{\partial e}{\partial P_e}\end{aligned}$$

Moreover, we define $\gamma = \frac{e}{\overline{H}} \cdot \overline{H}_e$ as the (expected) production elasticity of educational investment. Applying these definitions in equation (3.60), we finally end up with equation (3.35) in the text:

$$\begin{aligned}\frac{dV}{dt_2} &= -\frac{W\ \overline{H}\ L\mathcal{E}[U_{C_2}]\ (1+r)\ e}{D} \cdot \\ &\left(\frac{t}{1-t}\left[\frac{\left(\overline{H}L\,(1-\pi_H) - X\right)}{(1+r)e}\,(\gamma\pi_e\eta_{eP_e} + \eta_{LP_e}) - (\gamma\pi_e\eta_{et_2} + \eta_{Lt_2})\right] - \pi_H\right).\end{aligned}$$

3.A.4 Proof of Proposition 3.4

For proving Proposition 3.4, we have to show the decomposition of the insurance effect π_H into the factors mentioned in the proposition. If the shock θ is normally distributed, second period consumption $\tilde{C}_2(\theta)$ is normal as well, and we can apply a Rubinstein-theorem (see Rubinstein, 1976, pp. 421) in order to get

$$\text{Cov}(U_{c_2}, \theta) = \mathcal{E}[U_{c_2 c_2}] \cdot \text{Cov}(C_2, \theta). \tag{3.61}$$

For $t_2 = t_1 = t$ and $P_e = 0$, the household's budget constraint reads

$$\tilde{C}_2 = (1 + r)\ S_1 + (1 - t)\ W\ \theta\ h(e)\ L, \tag{3.62}$$

where $S_1 = (1 - t)\ W\ L - C_1$. Inserting in the right-hand-side of (3.61) results in

$$\begin{aligned}
\mathcal{E}[U_{c_2 c_2}] \cdot \text{Cov}(C_2, \theta) &= \mathcal{E}[U_{c_2 c_2}] \cdot \text{Cov}((1 + r)\ S_1 + (1 - t)\ W\ \theta\ h(e)\ L, \theta) \\
&= \mathcal{E}[U_{c_2 c_2}] \cdot \text{Cov}(\theta, \theta) \cdot (1 - t)\ W\ h(e)\ L
\end{aligned} \tag{3.63}$$

by applying some covariance rules.

Collecting terms and recognizing that $\text{Cov}(\theta, \theta) = \sigma_\theta^2$ is the variance of the shock parameter θ, π_H can be rearranged to

$$\begin{aligned}
\pi_H &= -\frac{\text{Cov}(U_{C_2}, H)}{\mathcal{E}[U_{C_2}] \cdot \overline{H}} = -\frac{\mathcal{E}[U_{C_2 C_2}]}{\mathcal{E}[U_{C_2}] \cdot \overline{H}} \cdot \text{Cov}(C_2, \theta) \cdot h(e) \\
&= -\frac{\mathcal{E}[U_{C_2 C_2}]}{\mathcal{E}[U_{C_2}]} \cdot (1 - t) W\ h(e)\ L \cdot \sigma_\theta^2 = (1 - t) W\ h(e)\ L \cdot ARA(C) \cdot \sigma_\theta^2
\end{aligned} \tag{3.64}$$

whereby $\overline{H} = h(e)$, and $(1 - t)\ W\ h(e)\ L$ is second-period expected wage income of a skilled worker as $\mathcal{E}[\theta] = 1$. Moreover, we have defined global absolute risk aversion in consumption as $ARA(C) = -\frac{\mathcal{E}[U_{c_2 c_2}]}{\mathcal{E}[U_{c_2}]} > 0$ according to Varian (1992, p. 380).

Obviously, the insurance effect π_H is increasing in the three economic variables, mentioned in Proposition 3.4.

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4 Second-Best Income Taxation with Endogenous Human Capital and Borrowing Constraints

4.1 Introduction

This paper examines optimal income taxation and human capital formation in an economy where individuals face credit constraints. Empirical evidence for credit constraints is presented in two strands of the literature. Firstly, (poor) individuals can experience difficulties financing their higher education as shown in Kane (1996), Keane and Wolpin (2001), Plug and Vijverberg (2005), Belley and Lochner (2007), Stinebrickner and Stinebrickner (2008) and Lochner and Monge-Naranjo (2008).[1] Secondly, ample empirical evidence for binding borrowing constraints is found when empirically testing the life-cycle hypothesis in consumption. See Attanasio and Weber (2010) for an excellent overview of this literature. Binding credit constraints preventing individuals to invest optimally in human capital could contribute to persistence in income mobility, result in larger inequality, strengthen segregation of neighborhoods, and decrease economic growth (Loury, 1981; Galor and Zeira, 1993; Durlauf, 1996, and Benabou, 1996a,b; De Gregorio, 1996; Mookherjee and Ray, 2003; Galor and Moav, 2004).

The purpose of this paper is to analyze optimal redistributive tax policies when individuals

[1] Consistent with the presence of credit constraints Kane (1995) and van der Klaauw (2002) identify large impacts of financial aid on college enrollment. Acemoglu and Pischke (2001) show that family income plays an important role in determining educational attainment. Caneiro and Heckman (2002) point out that credit constraints are relevant for about 8% of the youth in the US.

cannot borrow the funds to smooth consumption and to finance human capital investments. To that end, we develop a two-period life-cycle model, where individuals make educational investments in first period and they work in the second period. In doing so, we make four important assumptions: 1) exogenous constraints restrict the amount of borrowing that can be made by individuals in period 1; 2) educational investments are not verifiable to the government; 3) individualized lump-sum taxes are not feasible; 4) age-specific tax instruments are ruled out. The first assumption originates from the fact that human capital is an insufficient collateral for borrowing. Poor individuals (or their parents) cannot borrow for investments in education (of their children). Moreover, legal restrictions against slavery prevent individuals from engaging in a contract that employs future income as a collateral (see also Stiglitz, 1994; Palacios, 2002; Jacobs and van Wijnbergen, 2007). The second assumption assumes that all costs are non-verifiable. This implies that the government cannot directly off-set underinvestment in human capital with education subsidies. Our main results would be insensitive to including some verifiable educational investments, which can be subsidized, as long as some non-verifiable costs remain.[2] The third assumption implies that individualized lump-sum taxes are not possible due to informational constraints that prevent the government from verifying individual abilities and/or individual initial wealth. The last assumption ensures that the tax system cannot discriminate between individuals by their age.

We demonstrate that the optimal income tax is progressive even in representative agent settings where distributional concerns are absent. That is, we provide a case for distortionary income taxation on grounds of efficiency only. The intuition is that, as long as incomes are increasing over the life-cycle, a progressive tax system redistributes resources from later stages to earlier stages in the life-cycle. By taxing later income at higher average rates than current income, while redistributing the revenue through age-invariant lump-sum transfers, the progressive income tax alleviates credit constraints. Hence, not only consumption is smoothed better, but also investment in human capital increases. The labor tax trades off the welfare gains of alleviating credit constraints against the tax distortions in labor supply and human capital for-

[2]It can be argued that a substantial part of educational costs is verifiable, e.g. opportunity costs and institutional, direct costs of education, and therefore subsidizable. Nevertheless, some part of the time and study effort invested in education is non-verifiable and, therefore, difficult to disentangle from working or leisure time. Similarly, it can be difficult for the government to distinguish direct costs of education (books, computers, etc) from pure consumption. Hence, subsidies on direct costs could be difficult to target efficiently and some underinvestment due to capital market imperfections would remain.

mation. The extent to which individuals are credit constrained, and the tax elasticities of labour supply and educational investment determine the optimal tax rate. In an extension of the model with heterogenous agents, we demonstrate that all the results derived under homogenous agents carry over to the more general case with heterogeneous agents. We show that with credit constraints the trade-off between equity and efficiency is less severe, since redistribution generates not only equity gains, but also efficiency gains. Hence, when distributional concerns are allowed for, the case for progressive income taxation is strengthened further.

Our paper relates to the existing literature in a number of ways. The paper conceptually closest to ours is Hubbard and Judd (1986). They simulate a life-cycle model to demonstrate that progressive income taxation is welfare-improving compared to proportional income taxation when credit constraints are binding. The intuition is the same as ours: the progressive income tax redistributes resources over the life-cycle and allows for better consumption smoothing. Our paper, however, provides a formal proof for their finding as a special case of our model in which educational investment is kept exogenous. Hoff and Lyon (1995) also show that redistributive income taxation improves welfare by mitigating adverse-selection in the capital market. Taxing labor income progressively and rebating the tax revenue through lump-sum transfers increases collateralizable wealth. Progressive taxes thereby moderate inefficient overinvestment in education. Our model in contrast emphasizes underinvestment in human capital, which results from binding credit constraints.

Earlier work by Loury (1981), Glomm and Ravikumar (1992), Benabou (1996a, 1996b), and Fernandez and Rogerson (1996; 1998) demonstrates that when credit constraints are binding public provision of education or equalizing expenditure on education among communities can increase income equality, reduce segregation, promote income mobility, and boost economic growth. Tobin (1980) also points out that government policy should help credit constrained individuals to move resources from future to present periods. However, this literature has not yet conducted an analysis of an *optimal* redistributive policy when individuals face binding credit constraints.

Our paper also contributes to an extensive literature, which emphasizes the potentially efficiency-enhancing effects of distortionary taxes in second-best settings. See Van der Ploeg (2006a) for an overview. We show that the introduction of a distortionary tax instrument, can reduce a pre-existing non-tax distortion in the economy, i.e., the credit constraint. For example, Akerlof

(1976) shows that the introduction of a distortionary income tax helps to tame the ‘rate race’ and reduces the individuals’ excessive incentives to work. Related is Layard (1980, 2005) who argues that progressive taxation is welfare-improving because individuals are involved in status races (‘keeping up with the Joneses’) and exhibit habit persistence, both generating excessive incentives to work. Labor-market imperfections arising from trade unions, efficiency wages and search frictions also provide second-best arguments for progressive taxes (see Koskela and Vilmunen, 1996; Pissarides, 1998; and Sørensen, 1999; Boone and Bovenberg, 2002; Van der Ploeg, 2006b; and Bovenberg 2006). Unions set wages above market clearing levels when unemployment benefits improve the outside options of workers. Also firms pay too high efficiency wages in order to recruit, to retain and to motivate workers when they face attractive outside options. Progressive taxes punish both unions and firms to bid up wages, so that wages are moderated, and unemployment decreases.[3] Progressive taxation could also correct search frictions in labor market. Progressive taxation lowers the wage demand by workers, which increases vacancies and expands employment. This is optimal if workers have too much bargaining power compared to firms, i.e., when the Hosios (1990) condition is not met. In the presence of missing insurance markets, progressive taxation redistributes income across different states of natures and improves upon efficiency by partially replacing the missing insurance market (Eaton and Rosen, 1980; Varian, 1980; Jacobs et al., 2010).

The remainder of this paper is organized as follows. Section 4.2 presents our life-cycle model with imperfect credit markets and human capital investment. Optimal tax policies are analyzed in section 4.3 in an economy with representative individuals, which focuses on optimal efficient taxation to relax borrowing constraints. In section 4.4 we extend the model to a heterogeneous agent setting and show how optimal redistribution and alleviating credit constraints determine optimal income taxes. The last section 4.5 concludes.

4.2 Model

The economy is populated by a continuum of individuals living for two periods. The mass of all individuals is normalized to 1. Individuals differ in their ability n and initial wealth ω.

[3] Van Ewijk and Tang (2007) show that education subsidies are optimal in order to off-set the disincentives on human capital investments when the government uses progressive taxes to lower union’s wage demands.

Ability and wealth have a cumulative joint distribution $F(n, \omega)$, which has supports $[\underline{n}, \infty)$ and $[\underline{\omega}, \infty)$. We use subscripts to denote the type of individual by its ability and initial wealth, and superscripts to label the period in the life-cycle.

We consider a two-period life-cycle model with educational investment, labor supply, saving and borrowing constraints. In the first period the individual does not work, but invests in education and consumes. In the second period, the individual supplies labor and consumes all its wealth.

The resource costs of human capital investment $e_{n\omega}$ are non-verifiable. We normalize the unit cost of education and consumption goods to one. Besides educational investment, the individual decides on its consumption in first period $c^1_{n\omega}$ and saving $a_{n\omega}$. Consequently, the first-period budget constraint is

$$a_{n\omega} = -e_{n\omega} + \omega + g - c^1_{n\omega}, \tag{4.1}$$

where g is the time-invariant lump-sum transfer.

Individuals are only allowed to borrow a maximum amount of a_o in the capital market, implying the following borrowing constraint:

$$a_{n\omega} + a_o \geq 0. \tag{4.2}$$

This assumption reflects the fact that individuals have limited access to loans to finance consumption and educational investments. The (exogenous) interest rate r is the same for saving and borrowing.

In the second period, the individual chooses labor supply $l_{n\omega}$. Gross labor income $z_{n\omega}$ depends on educational investment $e_{n\omega}$, labor supply $l_{n\omega}$ and ability n:

$$z_{n\omega} \equiv n l_{n\omega} \phi(e_{n\omega}), \quad \phi' > 0, \quad \phi'' < 0, \tag{4.3}$$

where $\phi\left(e_{n\omega}\right)$ is the production function for human capital with positive but diminishing marginal returns to human capital investment.

Second-period consumption equals after-tax labor income, saving plus interest income and the lump-sum transfer:

$$c_{n\omega}^2 = (1-t)nl_{n\omega}\phi(e_{n\omega}) + (1+r)a_{n\omega} + g, \tag{4.4}$$

where t denotes the labor tax rate. We rule out taxes on saving, since we will focus mainly on credit-constrained individuals. Taxes on saving would not yield any revenues when savings are zero.[4]

The individual characteristics (n and ω) and individual decisions ($e_{n\omega}$, $a_{n\omega}$ and $l_{n\omega}$) are assumed to be private information. In line with Mirrlees (1971) only total labor income is verifiable to the government. Consequently, government has to rely on distortionary labor taxes to redistribute income. With a flat tax rate and positive non-individualized lump-sum transfers the income tax is progressive. Age-specific lump-sum transfers would be available if the transfers could be conditioned on age. However, we rule out age-specific transfers, since most legal systems do not allow for age-discrimination. Therefore, the transfers must be identical in both periods. The non-verifiability of ω implies that the government can neither levy taxes on initial wealth nor condition transfers upon initial wealth of each individual. The informational requirement for levying a flat tax is that the government only needs to verify aggregate labour income.

Individuals derive utility from consumption in both periods and disutility from labor. The utility function is assumed to be separable in consumption and labor:

$$U = u(c_{n\omega}^1, c_{n\omega}^2) - v(l_{n\omega}), \quad u_1, u_2, v' > 0, \quad u_{11}, u_{22}, -v'' < 0, \quad u_{12} \geq 0. \tag{4.5}$$

Without loss of generality we assume that the subutility function u is homogenous of degree one and concave in both arguments. The subscripts refer to the derivatives with respect to the first and the second argument of the utility function, respectively. The disutility of labor $v(.)$ is increasing and convex in $l_{n\omega}$. The individual chooses educational investment $e_{n\omega}$, saving $a_{n\omega}$ and labor supply $l_{n\omega}$ to maximize utility (4.5) subject to the budget constraints (4.1), (4.4), and the credit constraint (4.2).

[4]Hubbhard and Judd (1986) and Aiyagari (1995) show that capital taxation is welfare-improving with binding credit constraints because capital taxation results in redistribution from non-credit constrained individuals (who do save) to credit constrained individuals (who do not save). Consequently, credit constraints are alleviated, but this comes at a price of distorting the saving decisions of the non-constrained individuals.

After substituting budget constraints for $c^1_{n\omega}$ and $c^2_{n\omega}$ we can formulate the following Lagrangian $\mathcal{L}$ for the individual's maximization problem

$$\max_{\{a_{n\omega},e_{n\omega},l_{n\omega}\}} \mathcal{L} \equiv u(-e_{n\omega}+\omega+g-a_{n\omega};(1-t)nl_{n\omega}\phi(e_{n\omega})+(1+r)a_{n\omega}+g) \qquad (4.6)$$
$$-v(l_{n\omega})+\mu_{n\omega}\left(a_{n\omega}+a_o\right),$$

$\mu_{n\omega}$ is the Kuhn-Tucker multiplier on the credit constraint (4.2). The multiplier $\mu_{n\omega}$ is the shadow price for borrowing more than the borrowing limit, i.e., it measures the marginal increase in individual utility if the individuals' borrowing limit a_o increases with one unit. We should note that the shadow price $\mu_{n\omega}$ is different for individuals with different n and ω. The first-order conditions for utility maximization are given by

$$\frac{\partial\mathcal{L}}{\partial a_{n\omega}} = -u_1\left(c^1_{n\omega},c^2_{n\omega}\right)+(1+r)u_2\left(c^1_{n\omega},c^2_{n\omega}\right)+\mu_{n\omega}=0, \qquad (4.7)$$

$$\mu_{n\omega}\geq 0, \quad \mu_{n\omega}=0 \text{ if } a_{n\omega}+a_o>0, \qquad (4.8)$$

$$\frac{\partial\mathcal{L}}{\partial e_{n\omega}} = -u_1\left(c^1_{n\omega},c^2_{n\omega}\right)+u_2\left(c^1_{n\omega},c^2_{n\omega}\right)(1-t)nl_{n\omega}\phi'\left(e_{n\omega}\right)=0, \qquad (4.9)$$

$$\frac{\partial\mathcal{L}}{\partial l_{n\omega}} = u_2\left(c^1_{n\omega},c^2_{n\omega}\right)(1-t)n\phi\left(e_{n\omega}\right)-v'\left(l_{n\omega}\right)=0. \qquad (4.10)$$

If individuals are not credit constrained ($\mu_{n\omega}=0$), the consumption and educational choices of the household can be summarized as

$$\frac{u_1\left(c^1_{n\omega},c^2_{n\omega}\right)}{u_2\left(c^1_{n\omega},c^2_{n\omega}\right)} = (1-t)nl_{n\omega}\phi'(e_{n\omega}) = 1+r. \qquad (4.11)$$

Intertemporal consumption choices are not distorted since the marginal rate of intertemporal substitution in consumption equals one plus the interest rate, which is the marginal rate of intertemporal transformation. The optimality condition for investment in education equates the marginal costs of investing one unit of resources in education $(1+r)$ with the marginal benefits of one unit of resources invested in education $((1-t)nl_{n\omega}\phi'(e_{n\omega}))$. Note that the marginal benefits of education increase if individuals supply more labor. Hence, labor and education are complements in generating gross income. As long as the marginal income tax rate is positive, the tax system distorts educational investments, since the marginal benefits are taxed, whereas the marginal costs are not.

For credit constrained individuals ($\mu_{n\omega} > 0$) we have $a_{n\omega} = -a_o$, and we obtain

$$\frac{u_1\left(c^1_{n\omega}, c^2_{n\omega}\right)}{u_2\left(c^1_{n\omega}, c^2_{n\omega}\right)} = (1-t)nl_{n\omega}\phi'(e_{n\omega}) > 1+r. \tag{4.12}$$

The credit constraint creates a wedge in intertemporal consumption choices, i.e., a difference between marginal rate of intertemporal transformation $(1+r)$ and marginal rate of intertemporal substitution $(\frac{u_1(.)}{u_2(.)})$, implying that individuals would like to transfer more consumption from the second period to the first if they could. Thus, a binding credit constraint makes income in the first period relatively more valuable to the agent than in the second period. Investment in education of credit-constrained individuals is distorted by the borrowing constraint, since the marginal returns to investment in human capital $((1-t)nl_{n\omega}\phi'(e_{n\omega}))$ are larger than the marginal returns to financial saving $(1+r)$.

We can define the implicit tax $\pi_{n\omega}$ on human capital investment arising from the credit constraint as:

$$\pi_{n\omega} \equiv 1 - (1+r)\frac{u_2\,(.)}{u_1\,(.)}. \tag{4.13}$$

$\pi_{n\omega}$ measures to which extent the intertemporal consumption choices are distorted. An intertemporal consumption wedge implies that $\pi_{n\omega} > 0$, and $\frac{u_1(.)}{u_2(.)} > 1+r$. If the credit constraint is slack, there is no distortion caused by imperfect capital markets: $\pi_{n\omega} = 0$, and the standard Euler-equation applies.

Using the definition of $\pi_{n\omega}$, the first-order condition for educational investment can be rewritten as

$$(1-\pi_{n\omega})(1-t)nl_{n\omega}\phi'(e_{n\omega}) = 1+r. \tag{4.14}$$

From this equation we can see that human capital investment is reduced, because the binding credit constraint acts as an implicit tax on the return from human capital investment. Nevertheless, the value of $\pi_{n\omega}$ is different for individuals differing in both n and ω. In particular, it decreases with increasing initial wealth until it becomes zero when individuals are not credit constrained. It increases with ability n – for given levels of initial wealth ω –, because more able agents have a higher marginal return to education ($nl_{n\omega}\phi'\,(.)$) and, consequently, would like to borrow more in order to finance larger investment in education.

First-order conditions are necessary, but not sufficient due to the positive feedback between learning and labor supply. The second-order condition requires that $\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < 0$, where $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega})e_{n\omega}}{\phi(e_{n\omega})}$, $\alpha_{n\omega} \equiv \frac{\phi''(e_{n\omega})e_{n\omega}}{\phi'(e_{n\omega})}$ and $\varepsilon_{n\omega} \equiv \left(\frac{v''(l_{n\omega})l_{n\omega}}{v'(l_{n\omega})}\right)^{-1}$ denote the elasticity of the human capital production function, the elasticity of the marginal return in human capital production function, and the elasticity of labor supply, respectively (see Appendix 4.A.1). A sufficiently low elasticity of labor supply $\varepsilon_{n\omega}$, a sufficiently low elasticity of the human capital production function $\beta_{n\omega}$, and a sufficiently high elasticity of the marginal return to human capital investment (in absolute value) ensure that the feedback between labor supply and education dampens out and interior solutions are obtained. We assume in the remainder that the second-order conditions are always fulfilled.

The first-order conditions and the household budget constraints jointly determine optimal investment in education, labor supply, and consumption choices as functions of the policy parameters, of ability n and of initial wealth ω. By indicating the optimized values with an asterisk, we can write the indirect utility function as

$$V\left(g, t; n, \omega\right) \equiv u(c^{1*}_{n\omega}, c^{2*}_{n\omega}) - v(l^{*}_{n\omega}), \tag{4.15}$$

Applying Roy's lemma yields the following derivatives with respect to the policy instruments: $\frac{\partial V_{n\omega}}{\partial g} = u_1\left(.\right) + u_2\left(.\right)$, and $\frac{\partial V_{n\omega}}{\partial t} = -u_2\left(.\right) n l_{n\omega}\phi\left(e_{n\omega}\right)$.

For later reference, we also derive the Slutsky-equations for education and labor supply (see Appendix 4.A.2). With capital market failures, deriving the compensated demand and supply functions is not trivial, because the exact timing of the compensation to keep utility fixed matters. If the credit constraint is slack, one unit of compensation given in first period is the same as the discounted value of one unit of compensation given in second period. However, if the credit constraint is binding, the value of one unit of compensation given in first period is higher than the discounted value of one unit of compensation in the second-period**.** We derive the Slutsky-equations where a uniform income compensation is given in both periods, e.g. by a higher lump-sum transfer:

$$\frac{\partial e_{n\omega}}{\partial t} = \frac{\partial e^c_{n\omega}}{\partial t} - \frac{u_2(.)}{u_1(.)+u_2(.)} n l_{n\omega} \phi(e_{n\omega}) \frac{\partial e_{n\omega}}{\partial g}, \tag{4.16}$$

$$\frac{\partial l_{n\omega}}{\partial t} = \frac{\partial l^c_{n\omega}}{\partial t} - \frac{u_2(.)}{u_1(.)+u_2(.)} n l_{n\omega} \phi(e_{n\omega}) \frac{\partial l_{n\omega}}{\partial g}, \tag{4.17}$$

where $e^c_{n\omega}$ denotes the compensated demand for education, and $l^c_{n\omega}$ denotes the compensated supply of labor.

4.3 Optimal Taxation without Redistribution

This and the next sections derive optimal tax policies with and without redistributional concerns. We assume that the government is benevolent and has full commitment. That is, the government announces the tax schedule before individuals make their decisions and fully commits to it.[5] In this section we discuss optimal taxation when individuals are all identical and there are, consequently, no redistributional concerns. We therefore suppress the subscripts n and ω. Moreover, we assume that the initial wealth of the representative individual is not sufficient to finance the optimal level of human capital investment. Consequently, the credit constraint is binding and educational investment is inefficiently low. The case with a slack credit constraint is straightforward. In particular, first-best can be obtained, since all individual choices are efficient and the government has access to a lump-sum tax.

We focus on optimal tax policy when age-specific lump-sum transfers are not available to the government. If age-specific lump-sum transfers were available in a setting with a representative agent, it would follow trivially that the credit constraint could be perfectly overcome without any efficiency costs. In particular, a policy with age-specific transfers can be viewed as a government loan where the government provides an amount of lump-sum income to each young individual and requires them to pay it back, including interest costs, by a lump-sum tax when they are old. Consequently, government can act perfectly as a lender to replace the missing capital market without using distortionary taxes on labor income. In heterogeneous agent settings, which we will analyze in the next section, a first-best optimum would require both age-

[5]However, in view of the sunk character of the educational investment, the optimal policy is generally not time-consistent. Therefore, a benevolent government may want to renege on its announcements and re-optimize taxes after investments have been made, see also Pereira (2009).

specific *and* individualized lump-sum transfers, which are not feasible due to the informational constraints we have imposed on n and ω.

The tax system thus consists of a flat tax on labor income and uniform lump-sum transfers in both periods. Without loss of generality we assume that there are no exogenous government expenditures.[6] Tax revenue from labor taxation is used only to finance lump-sum transfers g in both periods. The government budget constraint is therefore given by

$$tnl\phi(e) - g = (1 + r)\, g. \tag{4.18}$$

The net tax payment in the second period should be equal to the transfer in the first period plus interest. Note that we express the government budget constraint in terms of second-period income. We assume that government is not credit-constrained, as opposed to individual households. Intuitively, private markets will make government borrowing available, since the government can effectively collateralize human capital through the tax system. By the government's ability to tax income, the government can secure claims on the future returns from human capital (Jacobs and Van Wijnbergen, 2007). Hence, the government implicitly acts as a lender, but the individual pays back its 'loan' through a labor tax. Consequently, alleviating the credit constraint is costly because labor supply is distorted, and the first-best allocation cannot be obtained any more.

The government chooses g and t in order to maximize indirect utility of the representative individual. The Lagrangian $\mathcal{W}$ for maximizing social welfare is given by

$$\max_{\{g,t\}} \mathcal{W} \equiv V(g,t) + \eta(tnl\phi(e) - g(2+r)), \tag{4.19}$$

where η is the shadow price of public resources. The government optimally chooses the lump-sum transfer g in both periods and the labor tax rate t to maximize the indirect utility of the representative agent.

The optimal uniform lump-sum transfer g satisfies (see Appendix 4.A.3):

$$\frac{u_1(.) + u_2(.)}{\eta} + tnl\phi'(e)\frac{\partial e}{\partial g} + tn\phi(e)\frac{\partial l}{\partial g} = 2 + r, \tag{4.20}$$

[6] An exogenous revenue requirement would not change our main result that progressive income taxes are optimal.

where we used Roy's lemma: $\frac{\partial V}{\partial g} = u_1(.) + u_2(.)$. Equation (4.20) states that the marginal social benefit of providing one unit of income in both periods (including the indirect income effects on the tax bases) should be equal to the marginal resource cost of providing one unit of income in both periods (see also Atkinson and Stiglitz, 1980).

The first-order condition for optimal income tax rate can be reformulated as (see Appendix 4.A.3):

$$\frac{t}{1-t} = \frac{\pi(1-\rho)}{\varepsilon_{lt} + \beta\varepsilon_{et}}. \tag{4.21}$$

where $\varepsilon_{lt} \equiv -\frac{\partial l^c}{\partial t}\frac{1-t}{l}$ and $\varepsilon_{et} \equiv -\frac{\partial e^c}{\partial t}\frac{1-t}{e}$ denote the compensated tax elasticities of labor supply and educational investment, respectively, $\beta \equiv \frac{\phi'(e)e}{\phi(e)}$ is the elasticity of human capital production, and $1-\rho \equiv \frac{1+r}{2+r-\pi} = \left(1+\frac{u_2(.)}{u_1(.)}\right)^{-1}$. The optimal tax rate t trades off the welfare gains of alleviating the credit constraint (numerator) against the efficiency costs of doing so (denominator). At the optimum, the marginal welfare gains of alleviating the credit constraint should be equal to its marginal efficiency costs. The more individuals are credit constrained, as measured by a higher value of π, the larger is the welfare gain of a higher tax rate. The compensated tax elasticities ε_{lt} and $\beta\varepsilon_{et}$ measure the tax distortions on labor supply and educational investment. The more elastic labor supply or educational investments respond to the tax rate t, the larger are tax distortions, and the lower should be the optimal tax rate.

$\rho \equiv \frac{1-\pi}{2+r-\pi}$ measures the inefficiency of an age-independent tax system with uniform lump-sum transfers in both periods compared to a tax system where the transfer is provided only in the first period. In the latter case we would obtain $\rho = 0$. Intuitively, for a given tax rate (and, therefore, for a given level of efficiency costs) the resources available to be transferred to the first period are lower when the same amount has to be transferred to the second period as well. Hence, for one unit of revenue raised in second period by the labor tax only $\frac{1+r}{2+r}$ can be transferred to the first period. Due to the 'leak' of the transfers to the second period, the credit constraint is alleviated to a lesser extent, and the optimal tax rate is lower as a result. The relative share of tax revenue that can be transferred to the first period increases if a higher interest rate (r higher) or more severe capital market failures (π higher) make intertemporal transfers less costly to the government than to households. The reason is that government faces a lower relative price for first-period consumption, i.e. $(1+r)$, than households, i.e. $\frac{1+r}{1-\pi}$.

Note that our efficiency case for progressive income taxation does not rely exclusively on the endogeneity of human capital investments. Indeed, the optimal income tax would be progressive even when human capital would be exogenous ($\beta = 0$). Therefore, we formally prove the numerical findings by Hubbard and Judd (1985) that optimal income taxes are progressive when individuals are borrowing constrained.

Even in the absence of redistributional concerns, the optimal labor tax rate is positive. The distortionary income tax helps to reduce a pre-existing non-tax distortion in capital markets. We thus provide second-best argument for employing distortionary income taxation for efficiency reasons. See also the introduction 4.1 for references to the literature on efficient income taxation in models with rat-races and habit persistence, distorted labor markets, and missing insurance markets.

If investment in education and labor supply would both become perfectly inelastic, the labor tax would become completely non-distortionary and the first-best allocation could be obtained. The labor tax then has become a second-period lump-sum tax, which differs from the first-period lump-sum tax g. The results of this section are summarized in the following proposition.

Proposition 4.1. *The optimal labor tax is positive for efficiency reasons when agents are credit constrained. Taxing labor income relaxes credit constraints because it transfers income from non-constrained towards constrained phases in the life-cycle. The optimal tax rate strikes a balance between the welfare gains from alleviating the credit constraints and the efficiency losses of distortionary taxation on labor supply and human capital investment.*

4.4 Optimal Taxation with Redistribution

Until now we have shown the optimality of positive labor tax assuming identical agents. In this section we allow the individuals to differ in their initial wealth ω and their innate ability n. By doing so, we introduce redistributional concerns in the optimal tax problem. Since we assume that neither ω nor n are observable to the government, individualized lump-sum transfers that are conditioned on either ability or initial wealth are excluded. Consequently, the government has to rely on distortionary labor income taxation to redistribute income between individuals.

Like before, revenues from the labor tax are used to finance the lump-sum transfers in both periods. With heterogenous agents age-specific lump-sum transfers that are conditioned both on initial wealth and on learning ability would be required to achieve the first-best, which is not feasible due to the presumed non-observability of ω and n.

We can write the government budget constraint as

$$t\int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} nl_{n\omega}\phi(e_{n\omega})dF(n,\omega) = (2+r)\,g. \tag{4.22}$$

The government maximizes a social welfare function, which is a concave sum of individual indirect utilities $V\,(g,t;n,\omega)$

$$\int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} \Psi(V\,(g,t;n,\omega))dF(n,\omega), \quad \Psi' > 0, \quad \Psi'' \leq 0. \tag{4.23}$$

The social welfare function is utilitarian if $\Psi' = 1$, and it is Rawlsian if $\Psi' = 0$ for all agents, except for the individual with the lowest utility.

We follow Diamond (1975) in defining the net social marginal valuation of one unit of income, measured in monetary units, for individuals with ability n and initial wealth ω as:

$$b_{n\omega} = \Psi'(V_{n\omega})\frac{u_1(.)+u_2(.)}{\eta} + tnl_{n\omega}\phi'(e_{n\omega})\frac{\partial e_{n\omega}}{\partial g} + tn\phi\,(e_{n\omega})\,\frac{\partial l_{n\omega}}{\partial g}. \tag{4.24}$$

By using Roy's lemma the optimal lump-sum transfer g is such that (see Appendix 4.A.4)

$$\overline{b_{n\omega}} = 2+r, \tag{4.25}$$

where $\overline{b_{n\omega}} = \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} b_{n\omega}dF(n,\omega)$ denotes the average social value of income. The lump-sum transfer g should be set such that the average social marginal valuation of one unit income in both periods equals its resource costs.

In order to characterize the optimal income tax, we define the distributional characteristic of labor income as (see Atkinson and Stiglitz, 1980)

$$\xi = 1 - \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} \frac{b_{n\omega}z_{n\omega}}{\overline{b_{n\omega}}\,\overline{z_{n\omega}}}dF(n,\omega) > 0. \tag{4.26}$$

ξ measures the marginal increase in social welfare expressed in monetary units, as a fraction of taxed labor income, of a marginal redistribution through the tax system. ξ is the negative normalized covariance between $b_{n\omega}$, the welfare weight of agent $n\omega$, and gross labor income $z_{n\omega}$. We (realistically) assume that i) the correlation between endowments and abilities is non-negative, and ii) income effects are sufficiently small, so as to ensure that the distributional characteristic is always positive. That is, a positive value of ξ implies that agents with a higher gross labor income $z_{n\omega}$ have a lower social welfare weight $b_{n\omega}$. ξ is therefore a measure for the strength of redistributional concerns implied by the social welfare function (4.23) given the amount of before-tax inequality. ξ is zero if the social welfare weights $b_{n\omega}$ for all individuals are equal, so that the government does not want to redistribute any income, or if there is no inequality in labor earnings $z_{n\omega}$, so that taxing labor earnings does not redistribute (income differences can only arise from differences in initial wealth).[7]

By using the definition of ξ, equations (4.14) and (4.25) the optimal labor tax t can be expressed as (see Appendix 4.A.4)

$$\frac{t}{1-t} = \frac{\xi + \overline{\pi_{n\omega}\left(1-\rho_{n\omega}\right)}}{\overline{\varepsilon_{lt,n\omega}} + \overline{\beta_{n\omega}\varepsilon_{et,n\omega}}}. \tag{4.27}$$

where $\overline{\pi_{n\omega}\left(1-\rho_{n\omega}\right)} \equiv \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} \frac{b_{n\omega}z_{n\omega}}{\overline{b_{n\omega}z_{n\omega}}}\pi_{n\omega}(1-\rho_{n\omega})dF(n,\omega)$ is a weighted average for the marginal welfare gain of relaxing the credit constraint where $b_{n\omega}z_{n\omega}$ is used as the weight. Similarly, $\overline{\varepsilon_{lt,n\omega}} \equiv \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} \frac{z_{n\omega}\varepsilon_{lt,n\omega}}{\overline{z_{n\omega}}}dF(n,\omega)$ and $\overline{\beta_{n\omega}\varepsilon_{et,n\omega}} \equiv \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} \frac{z_{n\omega}\beta_{n\omega}\varepsilon_{et,n\omega}}{\overline{z_{n\omega}}}dF(n,\omega)$ denote the income-weighted averages of the compensated labor supply and education elasticities, respectively.

Individuals with a different ability n or initial wealth ω are differently affected by credit constraints. Therefore, the individual utility gains from relaxing the credit constraints differ across individuals. For agents that are not constrained at all, the welfare gain of alleviating the credit constraint is zero ($\pi_{n\omega} = 0$). $\overline{\pi_{n\omega}\left(1-\rho_{n\omega}\right)}$ gives the average welfare gain – weighted by welfare weights and gross income – from alleviating all potentially relevant credit constraints. The weighted average tax elasticities in labor and education measure the average efficiency costs of taxing labor income. Therefore, the optimal tax rate shows the trade-off between, on the one hand, the welfare gain from redistribution and alleviating the credit constraints and, on

[7] Because of the non-observability of initial wealth, the government cannot tax initial wealth to redistribute income. For individuals with same labor income, but different initial wealth levels, the distributional characteristic of labor income is zero although they feature different welfare weights $b_{n\omega}$. In this case, the government will not tax labor income because the labor tax cannot help to reduce inequality.

the other hand, the efficiency costs of taxing labor income.

In the presence of credit constraints, taxing labor income improves efficiency by encouraging educational investments and reducing the intertemporal distortions in consumption. As a result, the conflict between redistribution and efficiency is less severe, which can be seen by rewriting equation (4.27) as

$$\frac{t}{1-t}\left(\overline{\varepsilon_{lt,n\omega}}+\overline{\beta_{n\omega}\varepsilon_{et,n\omega}}\right)-\overline{\pi_{n\omega}\left(1-\rho_{n\omega}\right)}=\xi. \tag{4.28}$$

The welfare gains from relaxing credit constraints reduce the efficiency costs of redistributive taxation and increases the optimal tax rate – for a given desire to redistribute income ξ. Even without any desire for income redistribution ($\xi = 0$), the labor tax rate would remain positive, implying a case for distortionary taxation only on grounds of efficiency, as we have demonstrated above for homogenous agents. When credit constraints are irrelevant, ($\overline{\pi_{n\omega}\left(1-\rho_{n\omega}\right)} = 0$), equation (4.27) reduces to the standard expression for the optimal linear income tax with endogenous human capital formation (see, for example, Jacobs, 2005; Bovenberg and Jacobs, 2005).

Although the welfare gains from redistribution ξ and those from alleviating credit constraints enter equation (4.27) additively, they cannot be separated when determining the optimal tax rate t. This can be seen by noting that $b_{n\omega}$ enters the term $\overline{\pi_{n\omega}(1-\rho_{n\omega})}$ through the weights for the individual utility gains from a less tight credit constraint. To see this, suppose that initial wealth is equally distributed. Then, higher-ability individuals feature lower social welfare weights $b_{n\omega}$, and they would be more credit constrained, because they invest more in education. The weighted average welfare gain $\overline{\pi_{n\omega}\left(1-\rho_{n\omega}\right)}$ is lower than that without any redistributional concerns (where $b_{n\omega}$ is constant). The reason is that high-ability individuals have a lower welfare weight $b_{n\omega}$ and therefore the welfare gains for relaxing the credit constraints at high-ability individuals are discounted by lower social welfare weights. Consequently, the optimal tax rate for alleviating the credit constraint is lower than without redistribution. Alternatively, if high-ability individuals remain more credit constrained, they invest less in education, which reduces income inequality and, therefore, mitigates the redistribution problem.

Similarly, if individuals have identical abilities, but differ in initial wealth, individuals with lower initial wealth are more credit constrained and also have a larger welfare weight. As long as initial wealth is not observable, the government can achieve some redistribution by alleviating the credit constraint to a larger extent, since alleviating credit constraints helps the individuals with less initial wealth more than the ones with more initial wealth. The optimal tax rate is therefore higher when redistributional concerns are present, compared to the case where they are absent.

However, the effect of credit constraints on the optimal tax rate tends to be smaller lower if initial wealth and ability are more (positively) correlated. On the one hand, credit constraints are less severe, since high-ability individuals also have more initial wealth to finance their higher investment in education, i.e. $\pi_{n\omega}$ is lower. On the other hand, credit constraints for high-ability individuals are weighted less in view of the redistributional concerns of the government.

We also note that, even without any desire to redistribute income, the optimal tax rate is not the same as that in (4.21). If the government has no redistributional concerns, i.e. $b_{n\omega}$ is constant, we have $\xi = 0$. The optimal tax rate with heterogenous agents is different than that with homogenous agents, since the welfare gains and tax distortions are weighted by gross income. Credit constraints facing high-income agents are more important than credit constraints facing low-income agents, because alleviating credit constraints for them has larger efficiency effects. Similarly, the tax distortions of high-income earners are relatively more important than the tax distortions of low-income earners, since the elasticities are weighted with income. We summarize the results of this section in Proposition 4.2.

Proposition 4.2. *The optimal tax rate trades-off the welfare gains from alleviating credit constraints and equity against the efficiency costs of labor tax. The efficiency-enhancing effects of redistribution improve the trade-off between redistribution and efficiency. For given levels of initial wealth, the optimal income tax is lower if high-ability individuals are relatively more credit constrained and mitigating credit constraints results in larger inequality. For given levels of ability, redistributional concerns raise the income tax, since credit constraints of poorer individuals are weighted more heavily.*

4.5 Conclusion

This paper uses a two-period life-cycle model of saving, labor supply, and human capital investment to analyze optimal income taxes when individuals differ in their ability and their initial wealth. Binding borrowing constraints can prevent individuals from optimally smoothing consumption over their life-cycle and from optimally investing in human capital. We have demonstrated that the optimal linear income tax is always positive – even in the absence of any redistributional concerns. A distortionary income tax is optimal because it relaxes borrowing constraints by redistributing resources from the unconstrained to the borrowing constrained stages of the life-cycle. Hence, redistribution allows for better consumption smoothing and larger investments in human capital. The progressive income tax is thus a second-best instrument to correct the non-tax distortion in the capital market. The equity-efficiency trade-off is therefore less severe when progressive income taxes mitigate capital market imperfections.

In future research we would like to analyze the optimal setting of capital income taxes in our model of saving and learning with heterogeneous agents and borrowing constraints. We expect that capital taxes are positive. Aiyagari (1995) has shown that, with binding credit constraints, capital taxes are optimally positive in infinite horizon Ramsey models. Hubbard and Judd (1986) showed that this is also true in life-cycle models. Jacobs and Bovenberg (2010) demonstrated that capital taxes are optimally positive even in the absence of binding credit constraints, since the capital tax reduces the disincentives of the labor tax on human capital investments.

Appendix 4.A

4.A.1 Second-Order-Conditions

We will first derive the second-order conditions of the household's maximization problem when credit constraints are slack and then when credit constraints are binding.

Slack Credit Constraint

We employ a two-step budgeting procedure to derive the second-order conditions. We assume homothetic sub-utility in consumption u over c^1 and c^2 and we define the real price-index for consumption p_c such that $p_c \equiv \frac{c^2_{n\omega}+(1+r)c^1_{n\omega}}{u(c^1_{n\omega},c^2_{n\omega})}$. Due to homotheticity the consumption price p_c is independent of n and ω. Using the budget constraint

$$(1+r)c^1_{n\omega} + c^2_{n\omega} = (1-t)n\phi\left(e_{n\omega}\right)l_{n\omega} + (1+r)\left(-e_{n\omega}+\omega+g\right)+g, \tag{4.29}$$

we can rewrite the individual maximization problem as an unconstrained maximization problem:

$$\max_{\{e_{n\omega},l_{n\omega}\}} \frac{(1-t)n\phi\left(e_{n\omega}\right)l_{n\omega}}{p_c} + \frac{(1+r)\left(-e_{n\omega}+\omega+g\right)+g}{p_c} - v(l_{n\omega}). \tag{4.30}$$

The first-order conditions are given by

$$(1-t)n\phi'\left(e_{n\omega}\right)l_{n\omega} - (1+r) = 0, \tag{4.31}$$

$$(1-t)n\phi\left(e_{n\omega}\right) - v'(l_{n\omega})p_c = 0. \tag{4.32}$$

Hence, the Hessian matrix with second-order derivatives is

$$H \equiv \begin{bmatrix} (1-t)n\phi''l_{n\omega} & (1-t)n\phi' \\ (1-t)n\phi' & -v''p_c \end{bmatrix}. \tag{4.33}$$

The first principal minor, $(1-t)n\phi''l_{n\omega}$ is negative, because by assumption $\phi''\left(e_{n\omega}\right) < 0$. Therefore, for the Hessian to be negative semi-definite, the second principal minor should be positive:

$$-(1-t)n\phi''l_{n\omega}v''p_c - \left((1-t)n\phi'\right)^2 > 0. \tag{4.34}$$

By defining the wage elasticity of labor supply as $\varepsilon_{n\omega} \equiv \left(\frac{v''(l_{n\omega})l_{n\omega}}{v'(l_{n\omega})}\right)^{-1}$, the elasticity of human capital production function as $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega})e_{n\omega}}{\phi(e_{n\omega})}$, the elasticity of the marginal return to education as $\alpha_{n\omega} \equiv \frac{\phi''(e_{n\omega})e_{n\omega}}{\phi'(e_{n\omega})}$, and using the first-order condition for labor supply, we can rewrite the above inequality as

$$\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < 0. \tag{4.35}$$

Since human capital production function is concave ($\phi''(e_{n\omega}) < 0$), $\alpha_{n\omega}$ is negative. The second-order condition thus requires that the elasticity of labor supply and the elasticity of human capital production function should be sufficiently small and the elasticity of the marginal return to education is sufficiently large (in absolute terms) so as to avoid corner solutions. In the second stage of the budgeting procedure, individuals maximize $u(c^1_{n\omega}, c^2_{n\omega})$ subject to the constraint $p_c u(c^1_{n\omega}, c^2_{n\omega}) = (1+r)c^1_{n\omega} + c^2_{n\omega}$. The associated second-order condition $u_{11}(.)u_{22}(.) - u^2_{12}(.) > 0$ is always satisfied since $u(.)$ is assumed to be strictly concave.

Binding Credit Constraint

With a binding credit constraint, savings are zero ($a = 0$). Hence, we can again obtain an unconstrained maximization problem upon substitution of budget constraints in the utility function:

$$\max_{\{e_{n\omega}, l_{n\omega}\}} u(-e_{n\omega} + \omega + g, (1-t)n\phi(e_{n\omega})l_{n\omega} + g) - v(l_{n\omega}). \tag{4.36}$$

The first-order conditions are given by

$$-u_1(.) + u_2(.)(1-t)n\phi'(e_{n\omega})l_{n\omega} = 0, \tag{4.37}$$

$$u_2(.)(1-t)n\phi(e_{n\omega}) - v'(l_{n\omega}) = 0. \tag{4.38}$$

The Hessian matrix H with second-order partial derivatives is given by

$$H \equiv \begin{bmatrix} \begin{array}{c} u_{11} - 2u_{12}(1-t)n\phi' l_{n\omega} \\ +u_{22}\left((1-t)n\phi' l_{n\omega}\right)^2 + u_2(1-t)n\phi'' l_{n\omega} \end{array} & \begin{array}{c} -u_{12}(1-t)n\phi + u_{22}(1-t)^2 n^2 \phi\phi' l_{n\omega} \\ +u_2(1-t)n\phi' \end{array} \\ \begin{array}{c} -u_{12}(1-t)n\phi + u_{22}(1-t)^2 n^2 \phi\phi' l_{n\omega} \\ +u_2(1-t)n\phi' \end{array} & u_{22}\left((1-t)n\phi\right)^2 - v''(l_{n\omega}) \end{bmatrix}. \tag{4.39}$$

For the Hessian matrix to be negative semi-definite, the principal minors should switch signs. The first principal minor

$$u_{11} - 2u_{12}(1-t)n\phi' l_{n\omega} + u_{22}\left((1-t)n\phi' l_{n\omega}\right)^2 + u_2(1-t)n\phi'' l_{n\omega} < 0, \tag{4.40}$$

is negative since all terms of (4.40) are negative under the assumptions that the consumption utility function is concave in both arguments ($u_{11} < 0$, $u_{22} < 0$), the human capital production function is concave ($\phi'' < 0$), and consumption in two periods are complementary ($u_{12} \geq 0$). The second principal minor should therefore be positive:

$$\begin{aligned}\Big(u_{11} - 2u_{12}(1-t)n\phi' l_{n\omega} + u_{22}\left((1-t)n\phi' l_{n\omega}\right)^2 + u_2(1-t)n\phi'' l_{n\omega}\Big) \times \Big(u_{22}\left((1-t)n\phi\right)^2 - v''\Big) \\ - \Big(-u_{12}(1-t)n\phi + u_{22}(1-t)^2 n^2 \phi\phi' l_{n\omega} + u_2(1-t)n\phi'\Big)^2 > 0.\end{aligned} \tag{4.41}$$

Use the first-order conditions (4.37) and (4.38) and the definitions $\varepsilon_{n\omega} \equiv \left(\frac{v''(l_{n\omega})l_{n\omega}}{v'(l_{n\omega})}\right)^{-1}$, $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega})e_{n\omega}}{\phi(e_{n\omega})}$ and $\alpha_{n\omega} \equiv \frac{\phi''(e_{n\omega})e_{n\omega}}{\phi'(e_{n\omega})}$ to reformulate the above inequality as

$$\begin{aligned}&u_{11}u_{22} + \frac{\alpha_{n\omega}}{e_{n\omega}}u_{22}u_1 - \frac{u_2}{\varepsilon_{n\omega}}\frac{\beta_{n\omega}}{e_{n\omega}}\left(u_{11}\frac{u_2}{u_1} - 2u_{12} + u_{22}\frac{u_1}{u_2} + u_2\frac{\alpha_{n\omega}}{e_{n\omega}}\right) \\ &> u_{12}^2 + \left(u_2\frac{\beta_{n\omega}}{e_{n\omega}}\right)^2 - 2u_{12}u_2\frac{\beta_{n\omega}}{e_{n\omega}} + 2u_{22}u_1\frac{\beta_{n\omega}}{e_{n\omega}}.\end{aligned} \tag{4.42}$$

We assume that the utility function is linear homogenous and we use the properties $u_{11}(.)c_1 = -u_{12}(.)c_2$ and $u_{12}(.)c_1 = -u_{22}(.)c_2$ to derive

$$u_{11}(.)u_{22}(.) - u_{12}^2(.) = 0. \tag{4.43}$$

Using (4.43) we can rewrite (4.42) as

$$\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < -\frac{u_{22}u_1 e_{n\omega}}{u_2^2}\left(1 - \frac{\alpha_{n\omega}\varepsilon_{n\omega}}{\beta_{n\omega}} + 2\varepsilon_{n\omega}\right) - \frac{u_{11}e_{n\omega}}{u_1} + 2\frac{u_{12}e_{n\omega}}{u_2}\left(\varepsilon_{n\omega}+1\right). \tag{4.44}$$

Because $u_{22} < 0$, $u_{11} < 0$ and $\alpha_{n\omega} < 0$, the right-hand-side of equation (4.44) is always positive. Consequently,

$$\alpha_{n\omega} + \beta_{n\omega}\varepsilon_{n\omega} < 0 \tag{4.45}$$

is sufficient for (4.44) to be fulfilled. Therefore, $\alpha_{n\omega}+\beta_{n\omega}\varepsilon_{n\omega}<0$ is the sufficient second-order condition for the households' maximization problem for both the cases of slack and binding credit constraints.

4.A.2 Slutsky Equations

In order to derive the expected utility-compensated substitution effects, we calculate how much lump-sum income g an individual should receive (pay) in order to keep its utility constant ($dU_{n\omega}=0$) when the tax rate t changes. This is equivalent to deriving the expenditure function and applying Shephard's lemma. Totally differentiating the utility function (4.5) and the budget constraints (4.1) and (4.4) yields

$$dU_{n\omega}=u_1(.)dc^1_{n\omega}+u_2(.)dc^2_{n\omega}-v'\left(l_{n\omega}\right)dl_{n\omega}=0, \tag{4.46}$$

$$dc^1_{n\omega}=-de_{n\omega}+d\omega+dg-da_{n\omega}, \tag{4.47}$$

$$dc^2_{n\omega}=(1-t)nl_{n\omega}\phi'\left(e_{n\omega}\right)de_{n\omega}+(1-t)n\phi\left(e_{n\omega}\right)dl_{n\omega}-nl_{n\omega}\phi\left(e_{n\omega}\right)dt \\ +(1+r)da_{n\omega}+dg. \tag{4.48}$$

Substitute $dc^1_{n\omega}$ and $dc^2_{n\omega}$ in $dU_{n\omega}$ to find

$$dU_{n\omega}=\left(u_2(.)(1-t)nl_{n\omega}\phi'\left(e_{n\omega}\right)-u_1(.)\right)de_{n\omega}+u_1d\omega \\ +\left(u_1(.)+u_2(.)\right)dg+\left(u_2(.)(1+r)-u_1(.)\right)da_{n\omega} \\ +\left(u_2(.)(1-t)n\phi\left(e_{n\omega}\right)-v'\left(l_{n\omega}\right)\right)dl_{n\omega}-u_2(.)nl_{n\omega}\phi\left(e_{n\omega}\right)dt=0 \tag{4.49}$$

$\left(u_2(.)(1-t)nl_{n\omega}\phi'\left(e_{n\omega}\right)-u_1(.)\right)de_{n\omega}$ and $\left(u_2(.)(1-t)n\phi\left(e_{n\omega}\right)-v'\left(l_{n\omega}\right)\right)dl_{n\omega}$ are both equal to zero from the first-order conditions. The term $\left(u_2(.)(1+r)-u_1(.)\right)da_{n\omega}$ is equal to zero as well both in case of a binding credit constraint ($da_{n\omega}=0$) and a slack credit constraint ($u_2(.)(1+r)-u_1(.)=0$). Thus, we have

$$dU_{n\omega}=-u_2(.)nl_{n\omega}\phi\left(e_{n\omega}\right)dt+u_1(.)d\omega+\left(u_1(.)+u_2(.)\right)dg=0. \tag{4.50}$$

For utility compensation through a transfer given in both periods, the Slutsky-equations are thus given by

$$\frac{\partial e_{n\omega}}{\partial t} = \frac{\partial e^c_{n\omega}}{\partial t} - \frac{u_2(.)nl_{n\omega}\phi\left(e_{n\omega}\right)}{u_1(.)+u_2(.)}\frac{\partial e_{n\omega}}{\partial g}, \tag{4.51}$$

$$\frac{\partial l_{n\omega}}{\partial t} = \frac{\partial l^c_{n\omega}}{\partial t} - \frac{u_2(.)nl_{n\omega}\phi\left(e_{n\omega}\right)}{u_1(.)+u_2(.)}\frac{\partial l_{n\omega}}{\partial g}. \tag{4.52}$$

4.A.3 Optimal Taxation without Redistribution

This appendix derives the optimal income tax when all agents are identical and age-specific transfers are not available. The Lagrangian for maximization of social welfare is

$$\max_{\{g,t\}} \mathcal{W} \equiv V(g,t) + \eta(tnl\phi\left(e\right) - (2+r)g). \tag{4.53}$$

The first-order conditions for the government's maximization problem are given by

$$\frac{\partial \mathcal{W}}{\partial g} = u_1(.) + u_2(.) - \eta\left(2+r\right) + \eta\left(tnl\phi'\left(e\right)\frac{\partial e}{\partial g} + tn\phi\left(e\right)\frac{\partial l}{\partial g}\right) = 0, \tag{4.54}$$

$$\frac{\partial \mathcal{W}}{\partial t} = -u_2(.)nl\phi\left(e\right) + \eta nl\phi\left(e\right) + \eta\left(tnl\phi'\left(e\right)\frac{\partial e}{\partial t} + tn\phi\left(e\right)\frac{\partial l}{\partial t}\right) = 0. \tag{4.55}$$

Using the Slutsky-equations (4.51) and (4.52) we can derive from (4.54) that

$$\frac{u_1(.)+u_2(.)}{\eta} + tnl\phi'\left(e\right)\frac{\partial e}{\partial g} + tn\phi\left(e\right)\frac{\partial l}{\partial g} = 2+r. \tag{4.56}$$

Using the definitions of the compensated elasticities $\varepsilon_{lt} \equiv -\frac{\partial l^c}{\partial t}\frac{1-t}{l}$ and $\varepsilon_{et} \equiv -\frac{\partial e^c}{\partial t}\frac{1-t}{e}$, (4.55) can be rewritten as

$$-\frac{u_2(.)}{u_1(.)+u_2(.)}\left(\frac{u_1(.)+u_2(.)}{\eta} + tnl\phi'\left(e\right)\frac{\partial e}{\partial g} + tn\phi\left(e\right)\frac{\partial l}{\partial g}\right) + 1 - \frac{t}{1-t}\frac{\phi'\left(e\right)e}{\phi\left(e\right)}\varepsilon_{et} - \frac{t}{1-t}\varepsilon_{lt} = 0. \tag{4.57}$$

Using (4.56) and $\frac{u_1}{u_2} = \frac{1+r}{1-\pi}$ we find

$$\frac{t}{1-t} = \frac{\frac{\pi}{1+\frac{1-\pi}{1+r}}}{\varepsilon_{lt} + \beta\varepsilon_{et}} = \frac{\pi(1-\rho)}{\varepsilon_{lt} + \beta\varepsilon_{et}}. \tag{4.58}$$

where $\beta \equiv \frac{\phi'(e)e}{\phi(e)}$ is the elasticity of human capital production function and $\rho \equiv \frac{1-\pi}{2+r-\pi}$.

4.A.4 Optimal Taxation with Redistribution

This appendix derives the optimal taxation when agents differ in initial wealth and abilities and when the government does not have access to age-specific transfers. The Lagrangian for the government's maximization problem is:

$$\max_{\{g,t\}} \mathcal{W} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left(\Psi\left(V(g,t;n,\omega)\right) + \eta(tnl_{n\omega}\phi\left(e_{n\omega}\right) - (2+r)g)\right) dF(n,\omega). \tag{4.59}$$

The first-order conditions are

$$\frac{\partial \mathcal{W}}{\partial g} = \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left[\Psi'(V_{n\omega})\frac{\partial V_{n\omega}}{\partial g} + \eta t \left(nl_{n\omega}\phi'\left(e_{n\omega}\right)\frac{\partial e_{n\omega}}{\partial g} + n\phi\left(e_{n\omega}\right)\frac{\partial l_{n\omega}}{\partial g}\right) - \eta\left(2+r\right) \right.$$
$$\left. -\eta\left(2+r\right)\right] dF(n,\omega) = 0, \tag{4.60}$$

$$\frac{\partial \mathcal{W}}{\partial t} = \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left[\Psi'(V_{n\omega})\frac{\partial V_{n\omega}}{\partial t} + \eta t \left(nl_{n\omega}\phi'\left(e_{n\omega}\right)\frac{\partial e_{n\omega}}{\partial t} + n\phi\left(e_{n\omega}\right)\frac{\partial l_{n\omega}}{\partial t}\right) \right.$$
$$\left. + \eta nl_{n\omega}\phi\left(e_{n\omega}\right)\right] dF(n,\omega) = 0. \tag{4.61}$$

Using Roy's lemma, (4.60) can be written as

$$\int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} \left(\Psi'(V_{n\omega})\frac{u_1(.)+u_2(.)}{\eta} + tnl_{n\omega}\phi'\left(e_{n\omega}\right)\frac{\partial e_{n\omega}}{\partial g} + tn\phi\left(e_{n\omega}\right)\frac{\partial l_{n\omega}}{\partial g}\right) dF(n,\omega) = 2+r. \tag{4.62}$$

Define $b_{n\omega} \equiv \Psi'(V_{n\omega})\frac{u_1(.)+u_2(.)}{\eta} + tnl_{n\omega}\phi'\left(e_{n\omega}\right)\frac{\partial e_{n\omega}}{\partial g} + tn\phi\left(e_{n\omega}\right)\frac{\partial l_{n\omega}}{\partial g}$ as the social valuation of one unit income for individuals with ability n and initial wealth ω. We derive

$$\overline{b_{n\omega}} = 2+r, \tag{4.63}$$

where $\overline{b_{n\omega}} \equiv \int_{\underline{n}}^{\infty} \int_{\underline{\omega}}^{\infty} b_{n\omega} dF(n,\omega)$. Using Roy's lemma $\frac{\partial V_{n\omega}}{\partial t} = -u_2(.)nl_{n\omega}\phi\left(e_{n\omega}\right)$, the Slutsky-equations (4.51) and (4.52), $\overline{b_{n\omega}} = 2+r$ and $\frac{u_1(.)}{u_2(.)} = \frac{1+r}{1-\pi_{n\omega}}$ we can rewrite equation (4.61) as

$$\int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty}\left(-\frac{b_{n\omega}nl_{n\omega}\phi\left(e_{n\omega}\right)}{\overline{b_{n\omega}}}\left(1-\frac{\pi_{n\omega}}{1+\frac{1-\pi_{n\omega}}{1+r}}\right)-\frac{t}{1-t}nl_{n\omega}\phi\left(e_{n\omega}\right)\left(\beta_{n\omega}\varepsilon_{et,n\omega}+\varepsilon_{lt,n\omega}\right)\right)dF(n,\omega)$$
$$+\overline{z_{n\omega}}=0, \tag{4.64}$$

where $\overline{z_{n\omega}} \equiv \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty} nl_{n\omega}\phi\left(e_{n\omega}\right)dF(n,\omega)$ is average labor income in the economy and $\beta_{n\omega} \equiv \frac{\phi'(e_{n\omega})e_{n\omega}}{\phi(e_{n\omega})}$ denotes the elasticity of the human capital production function for individuals with ability n and initial wealth ω. We define the distributional characteristic as $\xi \equiv 1 - \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty}\frac{b_{n\omega}z_{n\omega}}{\overline{b_{n\omega}}\overline{z_{n\omega}}}dF(n,\omega) = -\frac{cov(b_{nw},z_{n\omega})}{\overline{b_{n\omega}}\overline{z_{n\omega}}}$, which measures the social concern for redistribution. The first-order condition (4.61) can be further simplified to

$$\xi+\overline{\pi_{n\omega}(1-\rho_{n\omega})}=\frac{t}{1-t}\overline{\beta_{n\omega}\varepsilon_{et,n\omega}+\varepsilon_{lt,n\omega}}, \tag{4.65}$$

where $1-\rho_{n\omega} \equiv \frac{1+r}{2+r-\pi_{n\omega}}$ and we define $\overline{\pi_{n\omega}(1-\rho_{n\omega})} \equiv \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty}\frac{b_{n\omega}z_{n\omega}}{\overline{b_{n\omega}}\overline{z_{n\omega}}}\pi_{n\omega}(1-\rho_{n\omega})dF(n,\omega)$ as the weighted average of the welfare gain from alleviating the credit constraint and $\overline{\beta_{n\omega}\varepsilon_{et,n\omega}} \equiv \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty}\frac{z_{n\omega}}{\overline{z_{n\omega}}}(\beta_{n\omega}\varepsilon_{et,n\omega}dF(n,\omega)$ and $\overline{\varepsilon_{lt,n\omega}} \equiv \int_{\underline{n}}^{\infty}\int_{\underline{\omega}}^{\infty}\frac{z_{n\omega}}{\overline{z_{n\omega}}}(\varepsilon_{lt,n\omega})dF(n,\omega)$ denote the weighted-average elasticities of human capital and labor supply. The optimal tax rate can be therefore characterized by

$$\frac{t}{1-t}=\frac{\xi+\overline{\pi_{n\omega}(1-\rho_{n\omega})}}{\overline{\beta_{n\omega}\varepsilon_{et,n\omega}}+\overline{\varepsilon_{lt,n\omega}}}. \tag{4.66}$$

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5 Endogenous Credit Constraints, Human Capital and Optimal Policy

5.1 Introduction

Redistributive policy and education subsidies are often justified by the existence of credit constraints that individuals face when they invest in education. Credit constraints arise from the limited commitment of debitors to repay loans. In addition, moral hazard problems due to the non-observability of effort in education as well as in labor supply make human capital bad collateral and thus make credits for educational investment more difficult to access. In the presence of credit constraints, family income plays an important role in determining educational attainment. Using the changes in income distribution in the US as instrument, Acemoglu and Pischke (2001) find that family income is the main explanation for the different college enrollment rates of children from different income groups. Belley and Lochner (2007) conclude that the importance of income has substantially increased from the early 1980s to the early 2000s, after comparing the causal effects of income on educational attainment found in the NLSY97 date with those found in the NLSY79 data. Lochner and Monge-Naranjo (2008) show further that the increase in the role of family income can only be explained by more severe credit constraints in response to increasing income inequality and rising tuition costs. Evidence for credit constraints is found as well in other empirical studies. Kane (1995) and Van der Klaaum (2002) identify large impacts of financial aid and tuition costs on college enrollment, which indicates the presence of credit constraints. Stinebrickner and Stinebrickner (2008) find an important causal role of credit constraints in explaining the drop-out decision of students from low-income families.

Credit constraints are also shown by Kean and Wolpin (2001) to significantly affect students' consumption and working choices. Kane (1996) argues that borrowing constraints can explain the higher tendency of delayed entrance in college observed in high tuition states.

To mitigate the negative effects of credit constraints, optimal policy should feature public provision of education and redistribution from rich to poor individuals (see e.g. Glomm and Ravikumar, 1992; Fernandez and Rogerson, 1996, 1998 and Benabou, 1996a). However, these studies take credit constraints as given and ignore the fact that governmental policy can change credit constraints by affecting the incentive to repay loans. Therefore, optimal policy derived under the assumption of exogenous credit constraints could be misleading. Krueger and Perri (1999) show that redistributive taxation can exaggerate credit constraints and lead to a lower welfare. Andolfatto and Gervais (2006) argue that education subsides and a pension program financed by taxes on working population may actually lead to a lower level of human capital investment when the effects of taxation on credit constraints are considered. By simulating an educational investment model with endogenously determined credit constraints, Lochner and Monge-Naranjo (2002) find that education subsidies have substantially greater effects than in a similar model with exogenous credit constraints.

This paper aims to analyze optimal tax policy when individuals face credit constraints in educational investment, while taking the effects of taxation on credit constraints into account. To that end, we employ a two-period life-cycle model with identical agents, who invest in education in the first period and work in the second period. There are private banks providing loans for the individuals. We assume that in case of default banks can force the debitor to pay back the loans if the repayment is covered by the collateral, which is a given fraction of the debitor's earnings. Otherwise, banks can only seize that part of the earnings. Moreover, the debitor has to pay a fixed cost associated with the enforcement of repaying. We make the further assumption that educational investment is not verifiable. Educational investment in this paper is mainly secondary and higher education. The costs of education include tuition fees, expenditure for computer and books etc, additional costs for accommodation and forgone earnings. These costs are hardly verifiable except for tuition fees, which are only a small share of the total costs.[1] Labor supply in the second period is not observable either. The non-observability of education

[1] Becker (1964) and Boskin (1975) show that the costs of goods investment and time investment (which is the forgone earnings) in the total costs of education is one-quarter and three-quarters. The share of tuition costs in total costs are thus less than one-quarter.

and labor supply leads to moral hazard problems concerning the collateral, i.e. the debitor may reduce his education or work effort once the credit is granted. To avoid defaults, banks limit the amount of credit the individuals can take. Under the assumption of a perfectly competitive loan market the equilibrium borrowing limit is determined by the condition that the individuals are indifferent between repaying and defaulting.

We show that, when credit constraints are binding, both educational investment and the first period consumption are too low compared to those without credit constraints. When government has access to age-specific transfers, the optimal policy would be to transfer income from the second to the first period via lump-sum transfers, which is in fact governmental loans. When age-specific transfers are not available, government has to use distortionary labor taxes to alleviate credit constraints. We derive the optimal linear labor tax rate, which trades off the welfare effects of redistributing income across the life-cycle and of changing the borrowing limit against the efficiency costs of distorting education and labor supply. Compared to the case where credit constraints are given, the optimal labor tax rate is lower (higher) if redistributive taxation tightens (relaxes) the borrowing limit. Numerical examples show that for reasonable model parameters the borrowing limit decreases with an increase of the labor tax rate. Ignoring the effects of tax policy on credit constraints overestimates the welfare effects of taxation. The intuition is that individuals planning to default invest less in education and work less in order to reduce default costs. Redistributive taxation reduces the incentive to invest in education and to work, thus exaggerating the moral hazard problems associated with credit constraints.

This paper is closely related to Jacobs and Yang (2010), who analyze optimal taxation under exogenous credit constraints. They show that redistributive taxation mitigates credit constraints by redistributing from high-income to low-income (and constrained) periods over the life-cycle. In this paper we endogenize credit constraints and show how this affects the optimal tax policy. Close to our analysis is also Andolfatto and Gervais (2006), who analyze optimal tax policy under endogenous credit constraints. However, they only analyze the optimal age-specific lump-sum transfers and argue that it is optimal to redistribute from young and old to working individuals. In our model the optimal age-specific lump-sum transfers are to redistribute from working to young individuals. The completely opposite results of our paper arise from the different assumptions about default punishment. Andolfatto and Gervais assume that defaulting debitors are punished by being excluded from the capital market. Consequently, redistributing income from old to working individuals relaxes credit constraints by increasing default costs, namely the costs of no-saving for old age. Differently, creditors in our model can seize part of the debitors' earnings. Redistributing from working to young individuals via lump-sum transfers increases the costs of losing part of the earnings and relaxes credit constraints.

Lochner and Monge-Naranjo (2002) show that education subsidies relax credit constraints and have a greater impact on human capital formation than in a similar model with exogenous credit constraints. Intuitively, education subsidies increase educational investment and thus future earnings. Default becomes less attractive since default costs, i.e., losing part of the earnings, increase. Their analysis assumes exogenous labor supply and verifiable educational investment. As a result, the amount that the creditors can seize in case of default is known when the credit is granted. In our paper, however, the non-observability of educational investment and labor supply leads to moral hazard problems, which are the main reason why human capital is badly collateralizable. Krueger and Perri (1999) analyze the effects of the tax system on welfare with binding credit constraints. They show that, under plausible conditions, the increase of tax progressivity reduces welfare by tightening credit constraints. Thus, our results seem to be consistent with their findings. However, they did not consider endogenous educational investment and labor supply decisions.

The remainder of the paper is organized as follows. In section 5.2 we lay out the model environment and derive the borrowing constraints under non-default condition. Section 5.3 analyzes optimal tax policy for both the case of age-specific lump-sum transfers and the case of uniform transfers. Numerical examples are prensented in section 5.4. The last section concludes.

5.2 The Model

The economy is populated by a continuum of identical individuals whose mass is normalized to 1. Individuals live for two periods. In the first period the representative individual has an initial wealth, which he spends for education and consumption. Since we are interested in credit constraints, we assume that the individual does not have sufficient initial wealth to finance its optimal educational investment and consumption in the first period. Without loss of generality, we set the initial wealth to zero. After acquiring human capital, the agent supplies labor in the second period and consumes all its wealth. There are private banks providing loans for individuals on a perfectly competitive capital market. The economy is a small open economy, which implies that banks can raise funds at a constant interest rate r.

5.2.1 Preferences and Human Capital Technology

The utility of the individuals is given by

$$U = u\left(c^1\right) + \beta u\left(c^2\right) - \beta v\left(l\right), \tag{5.1}$$

with c^1 and c^2 denoting the first and the second period consumption respectively and l the second period labor supply. Separability is assumed for simplicity and does not affect the main results of the paper. β is the rate of time preference. The subutility function $u\left(.\right)$ is increasing and concave, whilst the disutility function $v\left(.\right)$ is increasing and convex. Furthermore, we assume that the Inada-conditions are fulfilled to avoid corner solutions.

When the agent is young, it decides on consumption c^1, investment in human capital e and the required borrowing a. The interest rate the agent pays is also r. The costs of human capital investment e are assumed to be only monetary and may include tuition fees, forgone earnings and additional costs for computer, accommodation and books, etc. We assume that these costs are private information, since additional costs for computer, accommodation and books etc. and forgone earnings are badly verifiable. Although tuition fees are observable, the share of tuition fees in the total costs of education is almost zero for public secondary school and small for higher education. In the second period the agent becomes skilled and supplies labor according to the wage rate $w\left(e\right)$ with $w'\left(e\right) > 0$ and $w''\left(e\right) < 0$. Inada-conditions for $w\left(e\right)$ are fulfilled as well.

The tax system consists of a labor tax at a flat rate t and lump-sum transfers, g^1 and g^2, where the superscripts denote the periods. The agent's budget constraints are consequently

$$c^1 = -e + a + g^1, \tag{5.2}$$

and

$$c^2 = \left(1 - t\right) w\left(e\right) l - (1 + r)a + g^2. \tag{5.3}$$

Following the standard literature in optimal taxation, only gross income is observable. Since educational investment is private information, neither wage rate $w(e)$ nor labor supply l can be verified.

5.2.2 First-Best Allocation

For comparison we first describe the optimal allocation with a perfect capital market. The representative agent maximizes utility (5.1) subject to (5.2) and (5.3). As a result, consumption is smoothed according to the Euler's equation $\frac{u_1}{\beta u_2} = 1 + r$, where u_1 and u_2 denote the marginal utility of consumption in the first and the second period respectively. Moreover, the representative agent balances marginal costs of education against its marginal return, $(1-t)\, w'(e)\, l = 1 + r$, and the optimal labor supply is given by $(1-t)\, w(e) = v'(l)$.

Labor taxation distorts educational investment as well as labor supply. Since we have identical agents and thus no distributional concern, there is no need for governmental intervention. First-best allocation could be achieved by setting labor tax rate to zero and using lump-sum taxes to finance exogenous governmental expenditure.

5.2.3 Endogenous Credit Constraint

Credit constraints arise from the agent's limited ability to commit itself to the repayment. We assume that a given fraction γ of the agent's future earnings is taken as collateral. If the agent defaults, banks can get the repayment $(1+r)\, a$ back if it is covered by the collateral. Otherwise, banks can only get the collateral. Furthermore, the defaulting agent has to pay a fixed cost of F, which covers the banks' costs in processing defaults, e.g. costs for courts and garnishees. The default costs C can be summarized as follows:

$$C = \begin{cases} (1+r)a + F & \text{if } (1+r)a \leq \gamma(1-t)w(e)l, \\ \\ \gamma(1-t)w(e)l + F & \text{if } (1+r)a > \gamma(1-t)w(e)l. \end{cases}$$

We assume that banks cannot seize lump-sum transfers the agent receives in the second period, which can be thought of as public goods and social insurance that cannot be seized. Further punishment, as e.g. an exclusion from credit markets as studied in Kehoe and Levine (1993, 2000) and Andolfatto and Gervais (2006), is not considered in our two-period model.

An agent decides to default if the utility of defaulting is higher than that of repaying. Unlike in Lochner and Monge-Naranjo (2002, 2008), educational investment and labor supply in our model are private information that cannot be observed by banks. As a result, the amount of collateral, i.e. the fraction γ of the agent's future earnings, is unknown when credit is extended to the agent in the first period. The agent who plans to default can reduce default costs by reducing educational investment and labor supply. The non-default condition is thus that the agent cannot be better off by defaulting and by adjusting its choices correspondingly. Because the credit market is per assumption perfectly competitive, the equilibrium borrowing limit would be such that the agent is indifferent between repaying and defaulting.

Now we first derive the indirect utility of the agent when it plans to repay. After inserting budget constraints (5.2) and (5.3) in the utility function (5.1), the Lagrangian function for the agent's maximization problem when credit limit is $\overline{a}$ and when the agent repays is

$$\max_{e,a,l} \mathcal{L} = u(-e+g^1+a) + \beta u\left((1-t)w\left(e\right)l - (1+r)a + g^2\right) - \beta v\left(l\right) + \mu\left(\overline{a}-a\right), \tag{5.4}$$

where μ is the Kuhn-Tucker multiplier for the credit constraint $a \leq \overline{a}$. μ gives the shadow price of relaxing credit limit by one euro. We assume that credit constraint is binding, since the case of slack credit constraint is not interesting for us. This assumption implies that

$$a = \overline{a}; \mu = u_1 - \beta(1+r)u_2 > 0, \tag{5.5}$$

$$\frac{u_1}{\beta u_2} = (1-t)w'\left(e\right)l > 1+r, \tag{5.6}$$

$$\frac{v'\left(l\right)}{u_2} = (1-t)\,w\left(e\right). \tag{5.7}$$

A credit constrained agent cannot borrow the optimal amount of credit to finance its consumption and educational investment. As a result, both first period consumption and educational investment are distorted downwards compared to the first-best allocation: $\frac{u_1}{\beta u_2} > 1+r$ and $(1-t)w'\left(e\right)l > 1+r$. The agent would like to consume more and invest more in education if it could borrow more than $\overline{a}$.

Binding credit constraints act like an implicit tax on borrowing and educational investment. We define this implicit tax as

$$\pi = 1 - (1+r)\frac{\beta u_2}{u_1}. \tag{5.8}$$

Accordingly, we can rewrite the first-order-condition for educational investment as

$$(1-\pi)(1-t)\, w'(e)\, l = 1+r. \tag{5.9}$$

Therefore, π measures the extent to which the inter-temporal consumption and educational investment are distorted by credit constraints. The lower the credit limit $\overline{a}$ is, the higher is the implicit tax π and the more severe is the credit constraint. Substituting the optimal consumer decisions given by the first-order-conditions into the utility function (5.1), we get the indirect utility of the repaying agent as a function of tax policy parameters, interest rate and credit limit, $V(t, g^1, g^2, r, \overline{a})$.

The Lagrangian function for the agent's maximization problem if it plans to default is

$$\max_{e,a,l} \mathcal{L} = u\left(-e + g^1 + a\right) + \beta u\left((1-t)w(e)\, l - \min[(1+r)a, \gamma(1-t)\, w(e)\, l] - F + g^2\right) - \beta v(l) + \mu(\overline{a} - a). \tag{5.10}$$

It follows immediately that defaulting always leads to a lower utility if $(1+r)\, a \leq \gamma(1-t)\, w(e)\, l$, since in this case defaulting only causes the additional cost of F and brings no benefit. Consequently, credit constraints can only arise where $(1+r)\, a > \gamma(1-t)\, w(e)\, l$. Therefore, we derive the first-order-conditions only for the case $(1+r)\, a > \gamma(1-t)\, w(e)\, l$.

Since the agent does not repay the loans in the second period, it would borrow as much as possible, i.e. $a_d = \overline{a}$. We use the subscript d to denote the variables in case of default. The first-order-conditions for the defaulting agent are

$$a_d = \overline{a};\ \mu_d = u_{1d} > 0, \tag{5.11}$$

$$\frac{u_{1d}}{\beta u_{2d}} = (1-t)(1-\gamma)\, w'(e_d)\, l_d, \tag{5.12}$$

$$\frac{v'(l_d)}{u_{2d}} = (1-t)(1-\gamma)\, w(e_d)\,. \tag{5.13}$$

We can see that the defaulting agent chooses education and labor supply levels which differ from those of the agent who repays. Again, we get the indirect utility of the defaulting agent as a function of the tax policy parameters, interest rate, credit limit and the punishment parameters, $V_d(t, \gamma, F, g^1, g^2, r, \overline{a})$.

For both maximization problems (5.4) and (5.10) the second-order-conditions require that the marginal utility of consumption should decrease sufficiently fast, the productivity of education in wage rate is not too high and the marginal disutility of labor should increase fast enough (see Appendix 5.A.1). These conditions ensure that the positive feedback between education and labor supply is not too strong such that interior solutions are obtained. We assume that the second-order-conditions are always fulfilled.

We denote the optimal borrowing of the repaying agent in a perfect capital market as a^*, for given tax policy and interest rate. We make the assumption that

$$V\left(t, g^1, g^2, r, a^*\right) < V_d\left(t, \gamma, F, g^1, g^2, r, a^*\right), \tag{5.14}$$

which implies that the agent would default if it can borrow a^*. Consequently, no bank would lend the amount a^*, since they know for sure that the agent would default. This assumption ensures the existence of credit constraints.

The indirect utility V_d is increasing and concave in borrowing limit $\overline{a}$, since $\frac{\partial V_d}{\partial \overline{a}} = \mu_d = u_{1d}$ applies and the first period consumption always increases with borrowing limit. The indirect utility V is increasing and concave in $\overline{a}$ as well, as long as $\overline{a} < a^*$. This is because $\frac{\partial V}{\partial \overline{a}} = \mu = u_1 - (1+r)\beta u_2$ is positive and decreasing in $\overline{a}$ for $\overline{a} < a^*$ and equal to zero for $\overline{a} \geq a^*$. From the concavity of both V and V_d in $\overline{a}$ for $\overline{a} < a^*$, the fact that $V(t, g^1, g^2, r, \overline{a}) > V_d(t, \gamma, F, g^1, g^2, r, \overline{a})$ for very small $\overline{a}$ and the assumption (5.14), we can conclude that in a $(\overline{a}, V)$ diagram V_d would cut V only once from below in the interval $[0, a^*]$. Note that for small loans where $(1+r)\,a \leq \gamma(1-t)\,w(e)\,l$, the first-order-conditions for the optimal choices of the defaulting agent are the same as for the repaying agent and V_d is always lower than V.

We denote the equilibrium borrowing limit as A, and it is determined by the equation

$$V\left(t, g^1, g^2, r, A\right) = V_d\left(t, \gamma, F, g^1, g^2, r, A\right). \tag{5.15}$$

To avoid losses from credit default, a bank will tighten the borrowing limit until the incentive for defaulting vanishes, i.e. when (5.15) holds. Solving the equation (5.15) for A, we get the equilibrium credit limit as the function $A(t, g^1, g^2, r, \gamma, F)$. By construction we have $A < a^*$. In equilibrium, banks lend up to the amount of A. The borrowers take the highest possible loan

A and pay it back in the second period. If the agent borrows more than A, it would default.

Since both the marginal return to education and the net wage rate are lower for the defaulting agent, it is optimal to invest less in education and to work less if the agent plans to default. The following lemma compares the optimal choices in equilibrium by the agent if it plans to default and if it plans to repay.

Lemma 5.1 *In equilibrium the agent who plans to default invests less in education and works less than the agent who plans to repay ($e > e_d$ and $l > l_d$). It follows straightforwardly from the first period budget constraint (5.2) and the condition for equilibrium $V = V_d$ that $c^1 < c_d^1$ and $c^2 > c_d^2$.*

Proof see Appendix 5.A.2.

The comparative statics of the credit limit A depend on how the change in one parameter affects the indirect utility of the repaying agents compared to that of the defaulting ones. By totally differentiating the equation (5.15) and by using the Roy's lemma we can derive:

$$\frac{\partial A}{\partial t} = \beta\frac{u_2 w\left(e\right)l - u_{2d}\left(1-\gamma\right)w\left(e_d\right)l_d}{u_1-\left(1+r\right)\beta u_2 - u_{1d}} \tag{5.16}$$

$$\frac{\partial A}{\partial g^1} = \frac{u_{1d}-u_1}{u_1-\left(1+r\right)\beta u_2 - u_{1d}} > 0 \tag{5.17}$$

$$\frac{\partial A}{\partial g^2} = \beta\frac{u_{2d}-u_2}{u_1-\left(1+r\right)\beta u_2 - u_{1d}} < 0 \tag{5.18}$$

$$\frac{\partial A}{\partial \gamma} = -\frac{\beta u_{2d}\left(1-t\right)w\left(e_d\right)l_d}{u_1-\left(1+r\right)\beta u_2 - u_{1d}} > 0 \tag{5.19}$$

$$\frac{\partial A}{\partial F} = -\frac{\beta u_{2d}}{u_1-\left(1+r\right)\beta u_2 - u_{1d}} > 0 \tag{5.20}$$

$$\frac{\partial A}{\partial r} = \frac{\beta u_2 A}{u_1-\left(1+r\right)\beta u_2 - u_{1d}} < 0 \tag{5.21}$$

Using Lemma 5.1 all comparative statics can be signed except for the effect of the labor tax rate. From $c^1 < c_d^1$ we have $u_1 > u_{1d}$. Moreover, we have $u_1 - (1+r)\beta u_2 - u_{1d} = \mu - \mu_d < 0$, since the shadow price for a marginal increase of borrowing limit in equilibrium is higher for the defaulting agent than for the repaying one.[2] Therefore, $\frac{\partial A}{\partial g^1} > 0$ and the borrowing limit increases with g^1, ceteris paribus. Intuitively, since the repaying agent consumes less in the first period, increasing first period consumption benefits the repaying agent more than the defaulting

[2] In equilibrium the agent is indifferent between repaying and defaulting, $V = V_d$. If the agent can borrow one unit more than the equilibrium borrowing limit, it would default, $V < V_d$. Consequently, we have $\mu < \mu_d$.

one. On the other hand, increasing the second period transfer g^2 tightens the incentive constraint of repaying and lowers the borrowing limit. This is because lump-sum transfer is not seizable and it makes the punishment of losing part of the earnings less severe. A higher interest rate tightens the incentive constraint as well. The higher the interest rate is, the higher is the cost of repaying and the more attractive is defaulting. Making the default punishment more severe, either by increasing the fixed cost F or the fraction γ of income that can be seized, increases the default costs and therefore also the borrowing limit.

However, the effect of increasing t is ambiguous. On the one hand, a higher tax rate harms the defaulting agent more by reducing second period income, since the defaulting agent consumes less in the second period and has a higher marginal utility of consumption. On the other hand, a higher tax rate reduces the after tax seizable income and makes the punishment less severe. The total impact of a higher tax rate depends therefore on which effect dominates.

5.3 Optimal Tax Policy

In this section we first formulate the governmental problem and then derive the optimal tax policy. The government is benevolent and can fully commit to announced tax policy. The tax system consists of a flat labor tax and lump-sum transfers in both periods. The time structure of the model is as follows: the government first announces the labor tax rate and the lump-sum transfers. Then private banks determine the borrowing limit under the non-default condition (5.15). Given the tax policy and borrowing constraints, individuals decide on educational investment, borrowing and labor supply.

We assume without loss of generality that there is no exogenous governmental expenditure[3]. The governmental budget constraint is thus given by

$$tw\,(e)\,l = (1+r)\,g^1 + g^2. \tag{5.22}$$

The government chooses g^1, g^2 and t to maximize the indirect utility of a representative agent, taking the responses of banks in determining credit constraints into account. The Lagrangian

[3] Any exogenous governmental expenditure can be financed by lump-sum taxes in such a way that credit constraint is not affected.

function for the governmental problem is

$$\mathcal{W} = V + \eta \left(tw\left(e\right) l - \left(1 + r\right) g^{1} - g^{2} \right), \tag{5.23}$$

where the lagrangian multiplier η measures the shadow price of governmental revenue.

5.3.1 Optimal Age-Specific Transfers

We first derive the optimal tax policy when the government can use age-specific transfers. With exogenous credit constraints, Jacobs and Yang (2010) argue that the availability of age-specific transfers enables the government to overcome credit constraints perfectly. This is the case if the government provides the amount of money that individuals can not borrow in the private loan market and requires them to pay this transfer back plus interest in forms of lump-sum taxes. In fact, government implements age-specific lump-sum transfers to replace the missing (or imperfect) private credit market. As a result, agents are not credit constrained anymore and the optimal labor tax rate is zero.

In our model with endogenous credit constraints, however, such age-specific transfers would change the individuals' incentive to repay and thus affect the borrowing limit. As shown before, a higher first period transfer increases the borrowing limit whilst a higher second period transfer reduces it. Consequently, age-specific transfers that redistribute income from the second to the first period ($g^{1} = -\left(1+r\right) g^{2} > 0$) increase borrowing limit. The lowest level of lump-sum transfers needed to remove binding credit constraints perfectly is characterized by

$$A\left(g^{1*}\right) + g^{1*} = a^{LP}, \tag{5.24}$$

where a^{LP} is the optimal borrowing in a laissez-faire economy with a perfect credit market. The government should provide the amount of credit to young agents such that the credit constraints are not binding any more. If government transfers more resources than g^{1*}, banks would like to lend more than agents want to borrow. Public lending crowds out private lending. For $g^{1} = a^{LP}$ there would be no private lending any more. Compared to the case of exogenous credit constraints, a lower level of lump-sum transfers is needed to remove credit constraints perfectly.

Andolfatto and Gervais (2006) also analyze the optimal age-specific lump-sum transfers with endogenous credit constraints. However, they argue that the optimal policy should be transferring income from young and old to working agents. The difference of our results to theirs arises from the different assumptions about default punishment. In their model default is punished by being excluded from the capital market, which means that agents cannot save any more for old age if they default. Transferring income from old and young to middle-aged agents makes the costs of no-saving higher and relaxes the incentive constraint of repaying thereafter. Our two-period model, however, does not encompass the punishment through exclusion from capital market. The defaulting agent is punished by losing part of its labor income. Transferring income from adults to young agents not only reduces the credit demand but also relaxes the incentive constraint, since the defaulting agent benefits less from the (mandatory) public lending on which it cannot default.

The first-best result with age-specific transfers arises from the assumption that the government has higher enforcement power than private banks and the agent cannot default on governmental loans. This assumption is not very harmful since the government does have higher enforcement power through the tax system. In addition, government faces lower costs of collecting repayment and a less severe tracking problem than private banks.

5.3.2 Optimal Tax Policy with Uniform Lump-sum Transfers

Age-specific lump-sum transfers (or taxes) are difficult to implement because of age-discrimination, which is against legal norms in some countries. Therefore, in this section, we analyze optimal tax policy with uniform lump-sum transfers, i.e. $g^1 = g^2 \equiv g$. The Lagrangian function for governmental optimization then becomes

$$\mathcal{W} = V + \eta \left(tw\left(e\right) l - \left(2 + r\right) g \right). \tag{5.25}$$

If credit constraints are exogenous, redistributive taxation, i.e. a positive tax rate on labor income and positive lump-sum transfers, alleviates credit constraints by shifting income from high-income to low-income (and constrained) periods. In fact, government still acts like a lender through redistributive taxation to supplement the imperfect capital market (see Jacobs and Yang, 2010). With endogenous credit constraints, there is an additional effect of tax policy, namely its effect on the borrowing limit A, that has to be taken into account when designing the optimal policy.

Analogously to the definition of the net social marginal valuation of income by Diamond (1975) we define the net social marginal valuation of a higher borrowing limit as

$$\psi \equiv \frac{u_1 - (1+r)\,\beta u_2}{\eta} + tw'(e)l\frac{\partial e}{\partial A} + tw(e)\frac{\partial l}{\partial A}. \tag{5.26}$$

ψ reflects the welfare effects, measured in terms of governmental revenue, of a marginal increase in A. The increase of the borrowing limit by one euro increases the utility of the agents by $u_1 - (1+r)\,\beta u_2$, which is positive for constrained agents. The two other terms on the right-hand-side of (5.26) are the effects on tax revenue of a higher borrowing limit due to the induced changes in the agents' choices. A higher A enables the agents to invest more in education and leads to a higher labor supply due to the positive feedback between education and labor[4]. The income effects are thus positive as well.

Using definition (5.26) we can characterize the optimal lump-sum transfers as (see Appendix 5.A.4)

$$\frac{u_1 + \beta u_2}{\eta} + tw'(e)\,l\frac{\partial e}{\partial g} + tw(e)\frac{\partial l}{\partial g} + \psi\frac{\partial A}{\partial g} = 2 + r. \tag{5.27}$$

The left-hand-side of equation (5.27) gives the net social marginal valuation of one euro of transfer given in both periods, including the income effects on tax revenue. The welfare effects of income by affecting the borrowing limit A is given by $\psi\frac{\partial A}{\partial g}$. If a higher income increases the borrowing limit, one unit of income is more valuable than that in case of exogenous credit constraints; and vice versa. In optimum, lump-sum transfers equal the net social marginal valuation of income to its resource costs, $2+r$, both measured in terms of the second period income.

The first-order-condition for the optimal tax rate t can be reformulated as (see Appendix 5.A.4)

$$(1-\rho)\,\pi + \left(\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z}\right)\psi = \frac{t}{1-t}\left(\theta\varepsilon_e + \varepsilon_l\right), \tag{5.28}$$

where $\varepsilon_e \equiv -\frac{\partial e^c}{\partial t}\frac{1-t}{e}$ and $\varepsilon_l \equiv -\frac{\partial l^c}{\partial t}\frac{1-t}{l}$ are the compensated tax elasticities of education expenditure and labor supply, $z \equiv w(e)\,l$ denotes gross labor income, and $\rho \equiv \frac{1-\pi}{2+r-\pi}$. The optimal

[4] Labor supply increases with education as long as the substitution effect dominates the income effect.

income tax balances the welfare gain of alleviating credit constraints against the efficiency costs of doing so. The latter, as given by the right-hand-side of equation (5.28), arises from tax distortions in education and labor supply, measured by the compensated tax elasticities. $\theta \equiv \frac{w'(e)e}{w(e)}$ is the elasticity of gross wage rate w.r.t education. The more important education is, the higher is θ and the higher are the efficiency costs of tax distortion.

The welfare gain of taxing labor income is given by the left-hand-side of equation (5.28). The first term is the welfare effects for fixed borrowing constraints. Taxing labor income and reimbursing tax revenue in forms of lump-sum transfers shifts income from the second to the first period and thus reduces the credit demand. Since credit constrained agents value the first period income more than the second period income ($u_1 - \beta(1+r)u_2 > 0$), such income shifts increase welfare. The more agents are credit constrained, i.e. the higher is the value of π, the higher is the welfare gain of transferring one unit of income from the second to the first period.

However, since the same amount of transfer has to be given in the second period as well, only part of the tax revenue can be transferred to the first period. The parameter $\rho \equiv \frac{1-\pi}{2+r-\pi} = \frac{1}{\frac{1+r}{1-\pi}+1} < 1$ reflects the increase in the uniform transfers if tax revenue is increased by one unit, while taking into account that the relative shadow price of the first period income compared to the second period is $\frac{1+r}{1-\pi}$. Note that for one unit of tax revenue we have $\frac{1+r}{1-\pi}\rho + \rho = 1$, i.e., the values of the first and the second period transfer should sum up to 1. Consequently, $1 - \rho = \frac{1+r}{1-\pi}\rho$ gives the value of the first period transfer. The higher the shadow price $\frac{1+r}{1-\pi}$ is, the higher is the value of the first period transfer and the welfare gain of taxation. A higher interest rate and tighter credit constraint increase the value of the first period transfer and thus the welfare gain of taxation.

The second term on the left-hand-side of (5.28) is the welfare effect of taxation by affecting the borrowing constraints. As defined by (5.26), ψ gives the welfare effects of relaxing borrowing constraints by one euro. The term $\left(\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z}\right)$ gives the total change in borrowing limit for one unit increase in tax revenue. ρ is by definition the increase in g, while taking the relative price of the first period income into account. Therefore, $\rho\frac{\partial A}{\partial g}$ gives the change in A due to higher lump-sum transfers when tax revenue is increased by one unit. Similarly, $\frac{\partial A}{\partial t}\frac{1}{z}$ is the change in A due to a higher labor tax rate, whilst $\frac{1}{z}$ is the required increase in tax rate to increase the tax revenue by one unit, ceteris paribus.

To sum up, the welfare effects of taxing labor income are the sum of the welfare gain of reducing borrowing demand while keeping the borrowing limit as given and the welfare effect of changing the borrowing limit. The aforementioned comparative statics (equation (5.16) to (5.18)) show that both labor tax t and uniform lump-sum transfer g have ambiguous effect on borrowing limit[5]. Therefore, the second welfare effect can be either positive or negative.

Rewriting condition (5.28), we can characterize the optimal tax rate as

$$\frac{t}{1-t} = \frac{(1-\rho)\,\pi + \left(\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z}\right)\psi}{\theta\varepsilon_e + \varepsilon_l} \tag{5.29}$$

The optimal tax rate depends on the total welfare gain of alleviating the credit constraint and its efficiency costs. The higher the total welfare gain and the lower the tax distortions are, the higher is the optimal tax rate.

If the credit constraint is exogenous, equation (5.29) reduces to

$$\frac{t}{1-t} = \frac{(1-\rho)\,\pi}{\theta\varepsilon_e + \varepsilon_l}, \tag{5.30}$$

since $\frac{\partial A}{\partial g} = \frac{\partial A}{\partial t} = 0$ for exogenous credit constraints. This result replicates that in Jacobs and Yang (2010) for identical agents and exogenous credit constraints. Compared to equation (5.30), the optimal tax rate with endogenous credit constraints is additionally determined by the term $\left(\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z}\right)\psi$, which is the welfare effect of taxation by changing the borrowing limit A.

If $\rho\frac{\partial A}{\partial g}+\frac{\partial A}{\partial t}\frac{1}{z} < 0$, a more redistributive taxation tightens the credit constraints. We know from Lemma 5.1 that the agent who plans to default invests less in education and works less in order to reduce default costs. Since redistributive taxation reduces the incentive to invest in education and to work, it exaggerates the moral hazard problems associated with credit constraints. As a result, the borrowing limit decreases when taxation becomes more redistributive. Therefore, redistributive taxation has two opposite effects on welfare. It increases welfare by mimicking governmental loans, but reduces welfare by tightening the credit constraint. Compared to the case of exogenous credit constraints, the optimal tax rate is lower. Moreover, it can even turn negative, if the negative effect of redistributive taxation by reducing borrowing limit dominates its positive effect. In this case, optimal taxation consists of lump-sum taxes and labor income

[5] The comparative static for the uniform transfer is $\frac{\partial A}{\partial g} = \frac{u_{1d}-u_1+\beta u_{2d}-\beta u_2}{u_1-(1+r)\beta u_2-u_{1d}}$.

subsidies, which shifts income from the first to the second period. Such a tax policy is welfare-improving, because the tax-induced increase in the borrowing limit overcompensates the tax-induced increase in borrowing demand.

If $\rho\frac{\partial A}{\partial g} + \frac{\partial A}{\partial t}\frac{1}{z} > 0$, a more redistributive taxation does not only reduce credit demand but also relaxes the borrowing limit. Redistributive taxation is therefore more efficient in mitigating credit constraints compared to the case of exogenous credit constraints. Consequently, a higher labor tax rate is optimal.

Lochner and Monge-Naranjo (2002) show that subsidizing education reduces the incentive to default and thus has a larger welfare effect than in a similar model with exogenous credit constraints. In this paper, we rule out education subsidies by the assumption of unobservable educational investment. Since education subsidies mimic age-specific transfers, the availability of education subsidies would reduce the desirability of labor taxation. However, as long as the share of observable educational costs in total costs is not too high, the effects of education subsidies are limited. Moreover, subsidizing verifiable costs would distort the efficient composition of the verifiable and the non-verifiable investment (see Bovenberg and Jacobs, 2005).

We summarize our results in the following proposition:

Proposition 5.1. *The optimal labor tax rate balances the welfare gain of shifting income across periods and of changing the borrowing limit against the efficiency costs of distorting educational investment and labor supply. If a higher tax rate and higher lump-sum transfers tighten (relax) borrowing constraints, the optimal income taxation is less (more) redistributive compared to the case of exogenous credit constraints.*

5.4 Numerical Examples

This section uses numerical examples to illustrate how the borrowing limit responds to changing tax policy and changing parameters. Moreover, we compare the optimal tax rates with those under exogenous credit constraints. These numerical examples are only for the purpose of illustration. Political interpretation of the results should be taken with caution.

Following Saez (2001) we assume the utility function to be logarithmic:

$$U = \ln c^1 + \beta \ln c^2 - \beta \ln \left(1 + \frac{l^{1+\epsilon^{-1}}}{1+\epsilon^{-1}}\right), \tag{5.31}$$

where ε is a parameter governing the (un)compensated elasticity of labor. Furthermore, we assume the wage rate function to be Cobb-Douglas:

$$w(e) = ne^{\theta}. \tag{5.32}$$

θ is the elasticity of the wage rate w.r.t education and n reflects the individual innate ability in generating income.

For the parameterization of the benchmark case we set the elasticity of the wage rate θ to 0.5. Trostel (1993) uses 0.45 for the elasticity of human capital in time investment and 0.15 for the elasticity in goods investment. Jacobs (2005) uses the values of 0.3 and 0.1 respectively. Therefore the total share of education in the wage rate of 0.5 lies between the value of 0.4 by Jacobs (2005) and 0.6 by Trostel (1993). For the parameter ϵ we take the value of 0.2 as benchmark case. Saez (2001) uses the value of 0.25 and 0.5 to match the empirical estimates of compensated tax elasticity of earnings. With endogenous education in our model, the tax elasticity of earnings is higher than the value of ϵ. Moreover, we set the interest rate r to 0.63, which equals an annual interest rate of 5% for a period of 10 years. The time preference β is assumed to be 0.62 such that $\beta(1+r) \approx 1$. For the punishment parament γ we use the value of 0.2, which is a bit higher than the calibrated value of 13% by Lochner and Monge-Naranjo (2002) for the US economy. The fixed cost F, ability n and initial wealth ω are calibrated to ensure the existence of credit constraints and to avoid corner solutions. We use for the benchmark case $F = 1$, $n = 6$ and $\omega = 5$.

The procedure to find the borrowing limit is as follows: we first calculate the optimal individual borrowing when capital market is perfect. Then we check if the agent can be better off by defaulting when the credit limit is at the level of the optimal borrowing. If this is the case, we reduce the credit limit by one unit and check again if the agent can be better off by defaulting. We keep doing this till the agent cannot be better off by defaulting. The corresponding credit limit is then the endogenous credit limit. To find the optimal tax rate we follow the method used by Jacobs (2005). For each given tax rate we find the uniform lump-sum transfer which maximizes the utility of the representative agent under the governmental budget constraint, whereby the credit constraints are endogenously determined by the respective tax policy. We then search for the tax rate that leads to the highest utility of the agent. For comparison we also calculate the optimal tax rate when the credit limit is held constant. The exogenous credit limit is set to equal the credit limit in laissez-faire. To reduce computation time, we searched for the optimal tax rate in the range between 0 and 0.5 for the case of exogenous credit constraints and searched in the range between -0.2 and 0.4 for the case of endogenous credit constraints[6].

We first depict in Figure 1 the borrowing limit for each given tax rate in the benchmark case, where lump-sum transfer g is chosen optimally. The credit limit falls almost monotonically with increasing tax rate, implying that for our benchmark case labor taxation tightens the credit constraints.

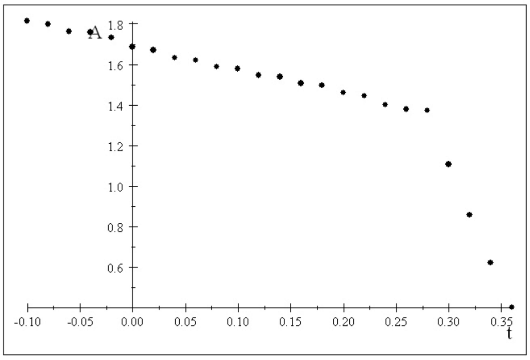

Figure 1: Endogenous borrowing limit

For the benchmark case, the optimal tax rates and the corresponding allocations under endogenous and exogenous credit constraints and in laissez-faire are shown in Table 1. The credit limit is decreased by optimal taxation compared to laissez-faire. However, the distortion caused by credit constraints, which is measured by π, is reduced. This is because labor taxation redistributes income to constrained life-cycle period and thus mitigates the distorting effects of credit constraints. Compared to the case of exogenous credit constraints, the optimal tax rate is lower due to the negative effect of labor taxation on the credit constraints. Moreover, the equilibrium

[6] For the case of exogenous credit constraints the governmental maximization problem is well-behaved. We found that for our simulations the optimal tax rates always lie between 0 and 0.5. With endogenous credit constraints, the indirect utility is almost concave in tax rate and there are only very small local fluctuations. We also found that the global maximum always lies between -0.2 and 0.4. The Gauss-programmes for simulation are available upon request.

education, labor supply and welfare are lower when credit constraints are endogenized[7]. This implies that assuming exogenous credit constraints overestimates the welfare effects of taxation.

	t	g	e	l	A	π	v
Endogenous	0.22	0.778	2.047	1.084	1.442	0.08	2.5727
Exogenous	0.25	0.930	2.223	1.093	1.686	0.01	2.5886
Lassez-faire	0	0	2.032	1.107	1.686	0.30	2.5530

Table 1: Benchmark case simulation, n=6, ε=0.2, θ=0.5, γ=0.2, F=1

Table 2 shows the comparative statics of the borrowing limit. Starting from laissez-faire and from the benchmark case values, the row dA/dx gives the relative change in credit limit for given changes in tax policy or in parameters. Ceteris paribus, increasing the labor tax rate by 1 unit lowers the credit limit by 1.78 unit. However, increasing the lump-sum transfer by 1 unit relaxes the credit limit by 3.78 unit. Consistent to the comparative statics derived in (5.19) to (5.21), a higher γ and a higher F increase the credit limit, whereby increasing γ has a much larger effect than increasing F. A higher interest rate r tightens the credit constraints.

	$dt = 0.01$	$dg = 0.01$	$d\gamma = 0.01$	$dF = 0.1$	$dr = 0.01$
dA/dx	-1.78	3.74	10	1	-1.38

Table 2: Comparative statics

We further simulated the optimal tax rates under exogenous and endogenous credit constraints for different values of the parameters ϵ, γ and θ. The results are reported in Tables 3, 4 and 5 respectively, where the second row gives the optimal tax rates and the third row gives the corresponding utility. The figures in the brackets are values for the case of exogenous credit constraints. For all parameter values the endogeneity of credit constraints reduces the optimal tax rate and the maximized welfare. This suggests that redistributive taxation has a negative effect on the borrowing limit. However, for lower parameter values the optimal tax rates are reduced only to a small degree. When the parameter values are higher, the effect of the endogeneity of credit constraints tends to increase.

We first look at Table 3. Since ϵ measures the wage rate elasticity of labor supply, it reflects the magnitude of the positive feedback effect between labor and education. A higher ϵ implies that the agent would like to invest more in education and to work more. Consequently, credit

[7] As Saez (2001) pointed out, l does not necessarily represent the working time.

constraints are more severe. This can be seen by the fact that the optimal tax rate under exogenous credit constraints increases with ϵ, since a higher tax rate is then needed to alleviate the more severe credit constraints. However, a higher ϵ implies that the agent responds more elastically to the changing credit limit. The negative welfare effects of taxation by reducing credit limit are consequently larger. As a result, the endogeneity of credit constraints lowers the optimal tax rate to a larger degree.

	$\epsilon = 0.1$	$\epsilon = 0.15$	$\epsilon = 0.2$	$\epsilon = 0.25$
t	0.2 (0.22)	0.23 (0.24)	0.22 (0.25)	0.22 (0.27)
v	2.5963 (2.6061)	2.5829 (2.5962)	2.5727 (2.5886)	2.5671 (2.5871)

Table 3: optimal tax rates for different values of ε

Increasing the value of γ reduces the optimal tax rates both in case of endogenous and exogenous credit constraints. A higher value of γ implies higher borrowing limit and less severe credit constraints, since defaulting is less attractive due to higher default costs. Consequently, a lower tax rate is required to mitigate credit constraints. At the same time, the borrowing limit is more responsive to labor taxation, because the after tax income is the more important the higher is the value of γ. As a result, the optimal tax rate decreases much faster than those under exogenous credit constraints.

	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
t	0.29 (0.30)	0.22 (0.25)	0.18 (0.21)	0.06 (0.16)
v	2.5437 (2.5527)	2.5727(2.5886)	2.6036 (2.6233)	2.6360 (2.6518)

Table 3: optimal tax rates for different values of γ

Increasing the value of θ has similar effects on the optimal tax rates as increasing the value of ϵ. The higher is θ, the more important is education and the more the agent would like to invest in education. Therefore, when θ is higher, the credit constraints are more severe and the optimal tax rate is correspondingly higher. As for ε, the difference between the optimal tax rates under exogenous and endogenous credit constraints increases with θ. A higher value of θ implies more distortionary effects of credit constraints and thus a larger negative effect of taxation by reducing credit limit.

	$\theta = 0.4$	$\theta = 0.45$	$\theta = 0.5$	$\theta = 0.55$
t	0.11 (0.12)	0.18 (0.19)	0.22 (0.25)	0.26 (0.31)
v	2.4959 (2.4988)	2.5309 (2.5381)	2.5727 (2.5886)	2.6234 (2.6531)

Table 3: optimal tax rates for different values of θ

Our simulation results suggest that the optimal tax rate should be set lower when the effect of tax policy on credit constraints is considered. When labor is elastic and education is important and also when banks can seize a large share of the earnings, we find a significant effect on the optimal tax policy, when credit constraints are endogenized. Moreover, our simulation shows that, notwithstanding the fact that redistributive taxation reduces the borrowing limit, taxation with reverse redistribution is not very probably to be optimal.

5.5 Conclusion

This paper derives the optimal tax policy when credit constraints arise from the limited commitment of individuals to repay loans for educational investment and consumption smoothing. The optimal redistributive taxation balances the total welfare gain of reducing borrowing demand and of changing the credit limit against the efficiency costs of distorting labor supply and education. If a more redistributive taxation tightens (relaxes) credit constraints, the optimal tax rate is lower (higher) compared to the case of exogenous credit constraints. Thus, assuming exogenous credit constraints leads to a too high (low) tax rate and over- (under) estimates the welfare effects of taxation. Numerical examples show that for reasonable model parameters the optimal tax rate is lower than that under exogenous credit constraints.

In future research it would be interesting to analyze optimal tax policy with endogenous credit constraints when individuals differ in ability and in initial wealth. In this case, the level of borrowing limit at which the individual is indifferent between defaulting and repaying would differ individually and is private information due to the unobservable ability. Consequently, conditions for equilibrium credit constraints would change, which would also have different political implications.

Appendix 5.A

5.A.1 Second-Order-Conditions for the Households' Problem

We first derive the second-order-conditions for the agent's maximization problem when it repays the loans. With binding credit constraints, savings are equal to the borrowing limit $\overline{a}$. Hence, we can obtain an unconstrained maximization problem upon substitution of the two budget constraints (5.2) and (5.3) in the utility function (5.1):

$$\max_{\{e,l\}} u(-e+g^1+\overline{a})+\beta u\Big((1-t)w\left(e\right)l-(1+r)\overline{a}+g^2\Big)-\beta v\left(l\right) \tag{5.33}$$

The first-order-conditions are given by

$$-u_1+\beta u_2(1-t)w'(e)l=0, \tag{5.34}$$

$$\beta u_2(1-t)w(e)-\beta v'(l)=0. \tag{5.35}$$

The Hessian matrix H with the second-order partial derivatives is given by

$$H\equiv\left[\begin{array}{cc}\left(\begin{array}{c}u_{11}+u_{22}\beta(1-t)^2\left(w'(e)\right)^2l^2\\+\beta u_2(1-t)w''(e)l\end{array}\right) & \left(\begin{array}{c}\beta u_{22}(1-t)^2w\left(e\right)w'(e)l\\+\beta u_2(1-t)w'(e)\end{array}\right)\\ \left(\begin{array}{c}\beta u_{22}(1-t)^2w(e)w'(e)l\\+\beta u_2(1-t)w'(e)\end{array}\right) & \left(\begin{array}{c}\beta u_{22}(1-t)^2\left(w(e)\right)^2\\-\beta v''(l)\end{array}\right)\end{array}\right] \tag{5.36}$$

where u_{11} and u_{22} are the second derivative of the first and the second period utility of consumption respectively. For the Hessian matrix to be negative semi-definite, the principal minors should switch signs. The first principal minor

$$u_{11}+u_{22}\beta(1-t)^2\left(w'(e)\right)^2l^2+\beta u_2(1-t)w''(e)l \tag{5.37}$$

is negative if the utility function of consumption and the wage rate function are concave, i.e. $u_{11}<0$, $u_{22}<0$ and $w''\left(e\right)<0$, which are assumed to be fulfilled. The second principal minor should be positive:

$$\Big(u_{11} + u_{22}\beta(1-t)^2 \left(w'(e)\right)^2 l^2 + \beta u_2(1-t)w''(e)l\Big) \times \Big(\beta u_{22}(1-t)^2 \left(w(e)\right)^2 - \beta v''(l)\Big)\Big) \tag{5.38}$$
$$- \Big(\beta u_{22}(1-t)^2 w(e)w'(e)l + \beta u_2(1-t)w'(e)\Big)^2 > 0.$$

Next, define $\delta \equiv \left(\frac{lv''(l)}{v'(l)}\right)^{-1}$ as the elasticity of the marginal disutility $v'(l)$, $\theta = \frac{w'(e)e}{w(e)}$ as the elasticity of wage rate and $\alpha = \frac{w''(e)e}{w'(e)}$ as the elasticity of the marginal return to education. Furthermore, define $(1-t)w(e)l = Z$ as the net labor income. Using the first-order-conditions we can reformulate the above inequation as

$$\left(\frac{u_{11}}{\beta z}\frac{e^2}{\theta^2} + u_{22}Z + u_2\frac{\alpha}{\theta}\right) \times \left(u_{22}Z - \frac{u_2}{\delta}\right) > \left(u_{22}Z + u_2\right)^2 \tag{5.39}$$

Define $-\frac{u_{22}}{u_2}c_2 = \sigma$ as the elasticity of the marginal utility of consumption u_2. σ measures how fast the marginal utility of the second period consumption declines. Further reformulations of equation (5.39) lead to

$$\delta^{-1} > \frac{1 + \sigma\frac{Z}{c_2}\left(\frac{u_{11}}{u_2\beta Z}\frac{e^2}{\theta^2} + \frac{\alpha}{\theta} - 2\right)}{\sigma\frac{Z}{c_2} - \frac{u_{11}}{u_2\beta Z}\frac{e^2}{\theta^2} - \frac{\alpha}{\theta}} \tag{5.40}$$

From the concavity of the consumption utility function and of the wage rate function we have $u_{11} < 0$, $\alpha < 0$ and $\sigma > 0$. Therefore, the right-hand-side of (5.40) is the smaller, the larger $\sigma\frac{Z}{c_2}$, $\frac{u_{11}}{u_2\beta Z}\frac{e^2}{\theta^2}$ and $\frac{\alpha}{\theta}$ are in absolute value. Consequently, the second-order-condition (5.40) requires that the elasticity δ is not too high, the marginal utility of consumption decreases sufficiently fast (σ and u_{11} are not too small in absolute value) and the productivity of education is not too high (the elasticity θ is sufficiently small and the marginal return to education decreases fast enough). These conditions ensure that the positive feedback between education and labor supply dampens and that interior solutions are obtained.

The unconstrained maximization problem for the defaulting agent is

$$\max_{\{e,l\}} u(-e + g^1 + \overline{a}) + \beta u\Big((1-\gamma)(1-t)w(e)\,l - F + g^2\Big) - \beta v(l) \tag{5.41}$$

for the case that $(1+r)\,a > \gamma(1-t)\,w(e)\,l$. Thus, the second-order-conditions are the same as for the repaying agent except that the after-tax income becomes $Z = (1-\gamma)(1-t)\,w(e)\,l$, which does not change the results qualitatively.

5.A.2 Proof of Lemma 5.1

We prove in this appendix that in equilibrium the agent who plans to default invests less in education and work less than the agent who plans to repay. First we establish that the equilibrium credit limit A is characterized by the following inequations:

$$\gamma (1-t) w (e^*) l^* + F > (1+r) A, \tag{5.42}$$

$$\gamma (1-t) w (e_d^*) l_d^* + F < (1+r) A, \tag{5.43}$$

where (e^*, l^*) are the optimal choices of the agent who plans to repay and (e_d^*, l_d^*) are the optimal choices of the agent who plans to default. We denote the indirect utilities for the repaying agent and for the defaulting one as V and V_d respectively. In equilibrium we have $V = V_d$. The first inequation (5.42) can be shown as follows: Suppose that $\gamma (1-t) w (e^*) l^* + F < (1+r) A$, the repaying agent could then achieve a higher utility than $V = V_d$ by defaulting, since its second period consumption would increase while the first period consumption and labor supply remain unchanged. This is not possible since V_d is the highest utility the agent can achieve by defaulting. Let's now look at the case of $\gamma (1-t) w (e^*) l^* + F = (1+r) A$. If the agent changes its plan and decides to default, its utility would remain the same as long as it does not change its education and labor supply choices. However, the choices (e^*, l^*) are not optimal any more for defaulting, since $\frac{u_{1d}}{\beta u_{2d}} > (1-t)(1-\gamma) w' (e^*) l^*$ and $\frac{v'(l^*)}{u_{2d}} > (1-t)(1-\gamma) w (e^*)$. Consequently, the agent can achieve a higher utility than $V = V_d$ by defaulting and optimizing its choices accordingly. This contradicts again the fact that $V = V_d$ is the highest utility the defaulting agent can obtain. Therefore we can conclude that the inequation (5.42) must be fulfilled. The second inequation (5.43) can be shown analogously: if $\gamma (1-t) w (e_d^*) l_d^* + F \geq (1+r) A$, the defaulting agent can achieve a higher utility than $V = V_d$ by repaying its loan and optimizing its choices accordingly. This is a contradiction to the fact that V is the highest utility the repaying agent can obtain. From the two inequations (5.42) and (5.43) it follows that in equilibrium

$$w (e^*) l^* > w (e_d^*) l_d^*.$$

Thus the defaulting agent must have a lower gross labor income than the repaying agent. There are however 3 cases for $w(e^*)\,l^* > w(e_d^*)\,l_d^*$: 1) $e^* > e_d^*$ and $l^* > l_d^*$ 2) $e^* \leq e_d^*$ and $l^* > l_d^*$ 3)$e^* > e_d^*$ and $l^* \leq l_d^*$. Now we show that only the first case is possible. Dividing the first-order-condition for education by the first-order-condition for labor supply we find for the repaying agent that

$$\frac{u_1}{\beta v'(l^*)} = \frac{w'(e^*)\,l}{w(e^*)}, \tag{5.44}$$

and for the defaulting agent that

$$\frac{u_{1d}}{\beta v'(l_d^*)} = \frac{w'(e_d^*)\,l_d^*}{w(e_d^*)}. \tag{5.45}$$

When $e^* \leq e_d^*$ and $l^* > l_d^*$, it follows that $\frac{u_1}{\beta v'(l^*)} < \frac{u_{1d}}{\beta v'\left(l_d^*\right)}$ and $\frac{w'(e^*)l}{w(e^*)} > \frac{w'\left(e_d^*\right)l_d^*}{w\left(e_d^*\right)}$. Therefore, (5.44) and (5.45) cannot be fulfilled simultaneously. Analogously, if $e^* > e_d^*$ and $l^* \leq l_d^*$, we have $\frac{u_1}{\beta v'(l^*)} > \frac{u_{1d}}{\beta v'\left(l_d^*\right)}$ and $\frac{w'(e^*)l}{w(e^*)} < \frac{w'\left(e_d^*\right)l_d^*}{w\left(e_d^*\right)}$. Again, for this case (5.44) and (5.45) cannot be fulfilled simultaneously. Therefore, we can conclude that in equilibrium $e^* > e_d^*$ and $l^* > l_d^*$. It follows straightforward that the defaulting agent consumes more in the first period $c^1 < c_d^1$. Since $V = V_d$ in equilibrium, the repaying agent consumes more in the second period, $c^2 > c_d^2$.

5.A.3 Slutsky Equations

To derive the Slutsky equations we calculate how much lump-sum income g given in both periods an individual should receive (pay) in order to keep its utility constant when the tax rate t changes. This is equivalent to deriving the expenditure function and applying Shephard's lemma. Totally differentiating the utility function (5.1) and the budget constraints of the households (5.2) and (5.3) gives:

$$dU = u_1 dc^1 + \beta u_2 dc^2 - \beta v'(l)\,dl, \tag{5.46}$$

$$dc^1 = -de + dg + da, \tag{5.47}$$

$$dc^2 = (1-t)w'(e)\,l de + (1-t)w(e)\,dl - w(e)\,l dt - (1+r)da + dg. \tag{5.48}$$

Substitute dc^1 and dc^2 in dU to get

$$\begin{aligned} dU &= \left(\beta u_2(1-t)w'(e)\,l - u_1\right) de + u_1 dg + \left(u_1 - \beta u_2(1+r)\right) da \\ &+ \beta\left(u_2(1-t)w(e) - v'(l)\right) dl - \beta u_2 w(e)\, l dt + \beta u_2 dg \\ &= 0. \end{aligned} \tag{5.49}$$

$\left(\beta u_2(1-t)w'(e)\,l - u_1\right) de$ and $\beta\left(u_2(1-t)w(e) - v'(l)\right) dl$ are both equal to zero from the first-order-conditions (5.6) and (5.7). The term $\left(u_1 - \beta u_2(1+r)\right) da$ is equal to zero as well since with binding credit constraints $da = 0$. Thus, we have

$$dU = -\beta u_2 w(e)\, l dt + \left(u_1 + \beta u_2\right) dg = 0. \tag{5.50}$$

The following compensation in g for changes in t is obtained

$$\frac{dg}{dt} = \frac{\beta u_2 w(e)\, l}{u_1 + \beta u_2}.$$

The Slutsky equations are therefore given by

$$\frac{\partial e}{\partial t} = \frac{\partial e^c}{\partial t} - \frac{\beta u_2 w(e)\, l}{u_1 + \beta u_2}\frac{\partial e}{\partial g}, \tag{5.51}$$

$$\frac{\partial l}{\partial t} = \frac{\partial l^c}{\partial t} - \frac{\beta u_2 w(e)\, l}{u_1 + \beta u_2}\frac{\partial l}{\partial g}. \tag{5.52}$$

where e^c and l^c denote the compensated demand function for education and the compensated labor supply function respectively.

5.A.4 Optimal Tax Policy with Uniform Lump-sum Transfers

In this appendix we derive the optimal tax policy when government has no access to age-specific lump-sum transfers, which implies $g^1 = g^2 \equiv g$. The lagrangian function for governmental maximization problem becomes

$$\mathcal{L} = V + \eta\left(tw(e)\, l - (2+r)\, g\right), \tag{5.53}$$

where η is the Lagrangian multiplier for governmental budget constraint. The first-order-conditions are respectively

$$\frac{\partial \mathcal{L}}{\partial t} = \frac{\partial V}{\partial t} + \eta \left(w(e)\, l + tw'(e)\, l \left(\frac{\partial e}{\partial t} + \frac{\partial e}{\partial A}\frac{\partial A}{\partial t} \right) + tw(e) \left(\frac{\partial l}{\partial t} + \frac{\partial l}{\partial A}\frac{\partial A}{\partial t} \right) \right) = 0, \quad (5.54)$$

$$\frac{\partial \mathcal{L}}{\partial g} = \frac{\partial V}{\partial g} + \eta \left(-(2+r) + tw'(e)\, l \left(\frac{\partial e}{\partial g} + \frac{\partial e}{\partial A}\frac{\partial A}{\partial g} \right) + tw(e) \left(\frac{\partial l}{\partial g} + \frac{\partial l}{\partial A}\frac{\partial A}{\partial g} \right) \right) = 0. \quad (5.55)$$

Note that for the constrained agent $a = A$ and $e = e(t, g, A)$. Assuming that credit constraints remain binding in the neighborhood of policy parameters, we use general envelope theorem to get

$$\frac{\partial V}{\partial g} = u_1 + \beta u_2 + \mu \frac{\partial A}{\partial g} = u_1 + \beta u_2 + (u_1 - (1+r)\beta u_2)\frac{\partial A}{\partial g}, \quad (5.56)$$

$$\frac{\partial V}{\partial t} = -\beta u_2 w(e)\, l + \mu \frac{\partial A}{\partial t} = -\beta u_2 w(e)\, l + (u_1 - (1+r)\beta u_2)\frac{\partial A}{\partial t}, \quad (5.57)$$

where μ is the marginal utility of an increase in borrowing limit by one unit and is equal to $u_1 - (1+r)\beta u_2$. We define

$$\psi \equiv \frac{u_1 - (1+r)\beta u_2}{\eta} + tw'(e)\, l \frac{\partial e}{\partial A} + tw(e)\frac{\partial l}{\partial A} \quad (5.58)$$

as the net social marginal valuation of one unit increase in credit limit measured in terms of tax revenue, including the income effect. Using (5.56) and (5.58), we can rewrite the first-order-condition for g as

$$\frac{u_1 + \beta u_2}{\eta} + \psi \frac{\partial A}{\partial g} + tw'(e)\, l \frac{\partial e}{\partial g} + tw(e)\frac{\partial l}{\partial g} = 2 + r. \quad (5.59)$$

The optimal lump-sum transfer requires that the net social marginal valuation of income should be equal to its resource costs $2+r$, whereby the effect of income on the borrowing limit is taken into account. Using (5.57), the Slutsky equations (5.51) and (5.52), the definition of ψ (5.58) and the equation (5.59), the first-order-condition for tax rate t can be reformulated as

$$w\left(e\right)l-\frac{\beta u_2 w\left(e\right)l}{u_1+\beta u_2}\left(2+r\right)+\frac{\beta u_2 w\left(e\right)l}{u_1+\beta u_2}\psi\frac{\partial A}{\partial g}+\psi\frac{\partial A}{\partial t}=\frac{t}{1-t}\theta\varepsilon_e w\left(e\right)l+\frac{t}{1-t}\varepsilon_l w\left(e\right)l \tag{5.60}$$

where we define the elasticity of wage rate in education as $\theta\equiv\frac{w'(e)e}{w(e)}$ and the tax elasticities of education and labor supply as $\varepsilon_e\equiv-\frac{\partial e^c}{\partial t}\frac{1-t}{e}$ and $\varepsilon_l\equiv-\frac{\partial l^c}{\partial t}\frac{1-t}{l}$ respectively. Using the definitions $\pi\equiv1-\left(1+r\right)\frac{\beta u_2}{u_1}$ and $\rho\equiv\frac{1-\pi}{2+r-\pi}$ we derive the following equation for the optimal tax rate

$$\left(1-\rho\right)\pi+\left(\rho\frac{\partial A}{\partial g}+\frac{\partial A}{\partial t}\frac{1}{z}\right)\psi=\frac{t}{1-t}\left(\theta\varepsilon_e+\varepsilon_l\right), \tag{5.61}$$

where $z\equiv w\left(e\right)l$ is the gross labor income. The optimal tax rate balances the welfare gain of alleviating credit constraints, including the welfare effect of induced changes in the borrowing limit, against the efficiency costs of distorting educational investment and labor supply.

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Erklärung

Ich versichere hiermit, dass ich die vorliegende Arbeit mit dem Thema

Human Capital and Optimal Policy under Risk and Credit Constraints

ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertig habe. Die aus anderen Quellen direkt oder indirekt übernommenen Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Weitere Personen, insbesondere Promotionsberater, waren an der inhaltlich materiellen Erstellung dieser Arbeit nicht beteiligt.[8] Die Arbeit wurde bisher weder im In- noch im Ausland in gleicher oder ähnlicher Form einer anderen Prüfungsbehörde vorgelegt.

Konstanz, den 12. 10. 2010

(Hongyan Yang)

[8] Siehe hierzu die Abgrenzung auf der folgenden Seite.

Abgrenzung

Ich versichere hiermit, dass ich Kapitel 5 der vorliegenden Arbeit ohne Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertig habe.

Kapitel 1 entstammt einer gemeinsamen Arbeit mit Herrn Bas Jacobs (Erasmus University of Rotterdam) und Herrn Dirk Schindler (University of Konstanz). Meine individuelle Leistung bei der Erstellung dieser Arbeit ist 30%.

Kapitel 2 entstammt einer gemeinsamen Arbeit mit Herrn Dirk Schindler (University of Konstanz). Meine individuelle Leistung bei der Erstellung dieser Arbeit ist 50%.

Kapitel 3 entstammt einer gemeinsamen Arbeit mit Herrn Dirk Schindler (University of Konstanz). Meine individuelle Leistung bei der Erstellung dieser Arbeit ist 50%.

Kapitel 4 entstammt einer gemeinsamen Arbeit mit Bas Jacobs (Erasmus University of Rotterdam). Meine individuelle Leistung bei der Erstellung dieser Arbeit ist 60%.

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