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M. Aubry and J.-M. Lemaire proved in [1] that a simply connected 5-manifold M admits a free circle action if and only if $H^*(M)$ is free (or, what amounts to the same (see [2]), that M is a connected sum of S^3-bundles over S^2). We show in Section 1 that $H^*(M)$ being free is as well equivalent to the existence of a *semifree* action (or, even better: to the existence of an action with only fixed points, free and isolated exceptional orbits) on M.

If M_1 and M_2 are simply connected 5-manifolds with S^1-action one can apply a well-known construction introduced by R.Z. Goldstein and L. Lininger (cf. [8]) to build a new S^1-manifold M with $H^3(M) \cong H^3(M_1) \oplus H^3(M_2) \oplus \mathbb{Z}$ (M is diffeomorphic to a connected sum $M_1 \# M_2 \# S$ where S denotes an S^3-bundle over S^2). Therefore, particular interest should be paid to rational cohomology spheres. Examples of such are connected sums $N_{k,l} := X_{-1}^{\#k} \# M_2^{\#l}$ where

$$\begin{aligned} X_{-1} &= SU(3)/SO(3) \\ \text{and} \quad M_2 &= \{(z_1,\ldots,z_4) \in S^7 \mid z_1^2 + z_2^3 + z_3^3 + z_4^3 = 0\} \end{aligned}$$

for which holds

$$\begin{aligned} H^3(X_{-1}) &\cong \mathbb{Z}/(2), & w_2(X_{-1}) &\neq 0, \\ H^3(M_2) &\cong \mathbb{Z}/(2) \oplus \mathbb{Z}/(2), & w_2(M_2) &= 0. \end{aligned}$$

It is proved in [9] that the manifolds $N_{k,l}$ admit non-trivial circle actions with fixed points. In Section 2 some more examples are presented using results on Brieskorn manifolds due to J. Milnor (cf. [11]). It follows that every irreducible simply connected spin-5-manifold can be endowed with a fixed point free circle action.

That M admits a free S^1-action if and only if M is a connected sum of sphere bundles also holds for differentiable, simply connected 6-manifolds with free cohomology. In Section 3 it is shown that, in contrast to dimension 5, for this class of manifolds the existence of a free S^1-action is equivalent to the existence of an action without fixed points.

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1 Semifree actions on 5-manifolds

If A is an abelian group let $\mathrm{Tors}_{\mathbb{Z}}A$ denote the subgroup of elements of finite order. Let M be a simply connected, closed 5-manifold furnished with an effective, topological action of S^1.

1.1 Lemma. *For the fixed point set M^{S^1} one has $\mathrm{Tors}_{\mathbb{Z}}H^{\mathrm{odd}}(M^{S^1}) = 0$.*

Proof. By [12, Thm. 3] a component $F \subset M^{S^1}$ is a cohomology manifold of dimension 1 or 3. In particular, $H^i(F) = 0$ for $i > 3$. Since $H^1(F) \cong \mathrm{Hom}\,(H_1(F), \mathbb{Z})$ is free and $H^*(F;\mathbb{Q})$ is a Poincaré algebra it remains to consider the case $H^3(F;\mathbb{Q}) \cong \mathbb{Q}$. If $\mathrm{Tors}_{\mathbb{Z}}H^3(F) \neq 0$ there is a prime p and $k > 1$ such that $H^3(F;\mathbb{Z}/(p)) \cong (\mathbb{Z}/(p))^k$. Let $F' \subset M^{\mathbb{Z}/(p)}$ denote the component containing F. Then $H^*(F';\mathbb{Z}/(p))$ is a Poincaré algebra over $\mathbb{Z}/(p)$ of formal dimension 3 (cf. [6], [4]). It follows $H^3(F';\mathbb{Z}/(p)) \cong \mathbb{Z}/(p)$ which implies $F' \neq F$. Therefore S^1 acts non-trivially on F' and one has $F \subset (F')^{S^1}$. Since, again by [12], $(F')^{S^1}$ has positive codimension in F' this is a contradiction to the assumption that $\mathrm{Tors}_{\mathbb{Z}}H^3(F) \neq 0$. □

If X is an S^1-space one has the bundle $X \to X_{S^1} := ES^1 \times_{S^1} X \xrightarrow{\pi} BS^1$ associated to the universal S^1-bundle $S^1 \to ES^1 \to BS^1$ and the equivariant cohomology $H^*_{S^1}(X) := H^*(X_{S^1})$ gets an algebra over $H^*(BS^1) \cong \mathbb{Z}[t]$, $\deg(t) = 2$, via π^*. We define $H^*_{S^1}(X)_1 := H^*_{S^1}(X) \otimes_{\mathbb{Z}[t]} \mathbb{Z}_1$ where $\mathbb{Z}_1$ denotes $\mathbb{Z}$ with the $\mathbb{Z}[t]$ module structure given by evaluating at $t = 1$.

1.2 Lemma. *There is an isomorphism of graded $\mathbb{Z}[t]$ modules*

$$\mathrm{Tors}_{\mathbb{Z}}H^{\mathrm{odd}}_{S^1}(M) \cong \mathrm{Tors}_{\mathbb{Z}}H^3(M) \otimes \mathbb{Z}[t].$$

Proof. Since M is simply connected $H^i(M)$ is free for $i \neq 3$. We set $T := \mathrm{Tors}_{\mathbb{Z}}H^3(M)$, $r := \mathrm{rk}\,H^2(M) = \mathrm{rk}\,H^3(M)$ and consider the E_2-term of the spectral sequence belonging to the fibration $M \to M_{S^1} \to BS^1$:

$$\begin{array}{lllll}
\mathbb{Z} & 0 & \mathbb{Z} & 0 & \mathbb{Z} \\
0 & 0 & 0 & 0 & 0 \\
\mathbb{Z}^r \oplus T & 0 & \mathbb{Z}^r \oplus T & 0 & \mathbb{Z}^r \oplus T \\
\mathbb{Z}^r & 0 & \mathbb{Z}^r & 0 & \mathbb{Z}^r \\
0 & 0 & 0 & 0 & 0 \\
\mathbb{Z} & 0 & \mathbb{Z} & 0 & \mathbb{Z}
\end{array}$$

The summands T survive and no further torsion is generated in odd degrees. Therefore one obtains for $i \geq 1$:

$$\mathrm{Tors}_{\mathbb{Z}}H^{2i+1}_{S^1}(M) = \mathrm{Tors}_{\mathbb{Z}}E_\infty^{2(i-1),3} = \mathrm{Tors}_{\mathbb{Z}}E_7^{2(i-1),3} \cong T.$$

Furthermore, one reads from the Gysin sequence

$$\begin{array}{ccccccc} 0 & \to & H^{2i+1}_{S^1}(M) & \stackrel{\cdot t}{\to} & H^{2i+3}_{S^1}(M) & \to & H^{2i+3}(M) \\ & & \cup & & \cup & & \\ & & T & \stackrel{\cong}{\to} & T & & \end{array}$$

of the fibration $S^1 \to M \simeq ES^1 \times M \to M_{S^1}$ that $\mathrm{Tors}_{\mathbb{Z}} H^{2i+1}_{S^1}(M)$ is mapped isomorphically onto $\mathrm{Tors}_{\mathbb{Z}} H^{2i+3}_{S^1}(M)$ for $i \geq 1$. Thus $\mathrm{Tors}_{\mathbb{Z}} H^{\mathrm{odd}}_{S^1}(M)$ contains no t-torsion and so

$$\mathrm{Tors}_{\mathbb{Z}} H^{\mathrm{odd}}_{S^1}(M) \cong \mathrm{Tors}_{\mathbb{Z}} H^3(M) \otimes \mathbb{Z}[t]. \qquad \square$$

1.3 Corollary. *If the action of S^1 is semifree (i.e. there are only free orbits and fixed points) then $H^*(M)$ is free.*

Proof. Lemma 1.1, the Localization Theorem (cf., e.g. [7]) and Lemma 1.2 yield isomorphisms

$$\begin{aligned} 0 &= \mathrm{Tors}_{\mathbb{Z}} H^{\mathrm{odd}}(M^{S^1}) \otimes \mathbb{Z}[t, t^{-1}] \\ &\cong \mathrm{Tors}_{\mathbb{Z}} \left(H^{\mathrm{odd}}_{S^1}(M) \otimes_{\mathbb{Z}[t]} \mathbb{Z}[t, t^{-1}] \right) \\ &\cong \mathrm{Tors}_{\mathbb{Z}} H^3(M) \otimes \mathbb{Z}[t, t^{-1}] \end{aligned}$$

and so $\mathrm{Tors}_{\mathbb{Z}} H^*(M) = \mathrm{Tors}_{\mathbb{Z}} H^3(M) = 0$. $\square$

Let η_3 denote the non-trivial S^3-bundle over S^2. If $H^3(M) \cong \mathbb{Z}^r$ and if $w_2(M) \in H^2(M; \mathbb{Z}/(2))$ is the second Stiefel-Whitney class of M then according to the classification of Barden (cf. [2]) there is a diffeomorphism

$$M \cong \begin{cases} X_r := (S^2 \times S^3)^{\# r} & : w_2(M) = 0, \\ Y_r := (S^2 \times S^3)^{\# r-1} \# \eta_3 & : w_2(M) \neq 0. \end{cases}$$

M. Aubry and J.-M. Lemaire proved in [1] that the manifolds X_r, $r \geq 0$, and Y_s, $s \geq 1$, are precisely all simply connected, closed 5-manifolds admitting *free* circle actions. Therefore, together with Corollary 1.3 one obtains:

1.4 Proposition. *For a closed, simply connected 5-manifold M are equivalent:*

(i) M admits a free S^1-action.

(ii) M admits an effective, semifree S^1-action.

(iii) $H^*(M)$ is free. $\square$

The following observation shows that Proposition 1.4 can be sharpened by substituting

(ii') M admits an effective S^1-action with only fixed points, free and isolated exceptional orbits.

for assertion (ii). Let M^Σ denote the singular set which is the union of all non-free orbits.

1.5 Remark. *If M^Σ consists only of fixed points and isolated exceptional orbits, i.e.*

$$M^\Sigma = M^{S^1} \cup S^1/(\mathbb{Z}/(m_1)) \cup \ldots \cup S^1/(\mathbb{Z}/(m_r)), \qquad m_i > 1 \quad \textit{for} \quad i = 1, \ldots, r,$$

then $H^*(M)$ is free.
Proof. For $m \neq 0$ it follows from

$$\begin{aligned} H^*_{S^1}\Big(S^1/(\mathbb{Z}/(m))\Big) &= H^*\Big(ES^1 \times_{S^1} S^1/(\mathbb{Z}/(m))\Big) \\ &\cong H^*\left(B\,\mathbb{Z}/(m)\right) \\ &\cong \mathbb{Z}[t]/(mt), \qquad\qquad \deg\,(t) = 2, \end{aligned}$$

that $H^{\text{odd}}_{S^1}(M^\Sigma) \cong H^{\text{odd}}_{S^1}(M^{S^1}) \cong H^{\text{odd}}(M^{S^1}) \otimes \mathbb{Z}[t]$, hence $\text{Tors}_{\mathbb{Z}} H^*(M) = 0$. □

1.6 Remark. *If p is a prime such that $\text{Tors}_{\mathbb{Z}} H^3(M) \otimes \mathbb{Z}/(p) \neq 0$ then $M^{\mathbb{Z}/(p)}$ contains a threedimensional component.*
Proof. Let $E \subset M^\Sigma$ denote the union of the isolated exceptional orbits. If $\{p_1, \ldots, p_s\}$ is the set of primes p with the property that $M^{\mathbb{Z}/(p)} \not\subset M^{S^1} \cup E$ Lemma 1.1, the Localization Theorem and Lemma 1.2 can be applied to obtain

$$\begin{aligned} 0 &= \text{Tors}_{\mathbb{Z}} H^{\text{odd}}(M^{S^1}) \otimes \mathbb{Z}[t, t^{-1}, p_1^{-1}, \ldots, p_s^{-1}] \\ &\cong \text{Tors}_{\mathbb{Z}}\Big(H^{\text{odd}}_{S^1}(M^{S^1} \cup E) \otimes_{\mathbb{Z}[t]} \mathbb{Z}[t, t^{-1}, p_1^{-1}, \ldots, p_s^{-1}]\Big) \\ &\cong \text{Tors}_{\mathbb{Z}}\Big(H^{\text{odd}}_{S^1}(M) \otimes_{\mathbb{Z}[t]} \mathbb{Z}[t, t^{-1}, p_1^{-1}, \ldots, p_s^{-1}]\Big) \\ &\cong \text{Tors}_{\mathbb{Z}} H^3(M) \otimes \mathbb{Z}[t, t^{-1}, p_1^{-1}, \ldots, p_s^{-1}], \end{aligned}$$

Thus $\text{Tors}_{\mathbb{Z}} H^3(M)$ is the sum of its p_i-primary subgroups, $i = 1, \ldots, s$. It follows from $\text{Tors}_{\mathbb{Z}} H^3(M) \otimes \mathbb{Z}/(p) \neq 0$ that $p \in \{p_1, \ldots, p_s\}$ as asserted. □

2 Examples of actions without fixed points on rational cohomology-5-spheres

For integers $a, b \geq 1$ let $\gcd(a, b)$, $\text{lcm}(a, b) \geq 1$ denote their greatest common divisor and their least common multiple.

2.1 Lemma. *Given $k \geq 2$, $g \geq 0$ the unique (up to diffeomorphism, cf. [2]) closed, simply connected 5-manifold M with $H^3(M) \cong (\mathbb{Z}/(k))^{2g}$ and $w_2(M) = 0$ admits a fixed point free S^1-action if there are integers $a_2, a_3, a_4 \geq 2$ relatively prime to k such that*

$$(*) \quad \mathrm{lcm}(a_2, a_3) = \mathrm{lcm}(a_2, a_4) = \mathrm{lcm}(a_3, a_4) =: m$$

$$\text{and} \qquad g = 1 - \tfrac{a_2 a_3 + a_2 a_4 + a_3 a_4 - a_2 a_3 a_4}{2m}.$$

Proof. Let (a_2, a_3, a_4) be a triple as in the assertion. Then

$$M := \{(z_1, \ldots, z_4) \in S^7 \mid z_1^k + z_2^{a_2} + z_3^{a_3} + z_4^{a_4} = 0\}$$

is a simply connected 5-manifold (cf. [5]) on which S^1 acts by

$$S^1 \times M \to M,\ (\lambda, (z_1, \ldots, z_4)) \mapsto (\lambda^m z_1,\ \lambda^{k \frac{m}{a_2}} z_2, \lambda^{k \frac{m}{a_3}} z_3, \lambda^{k \frac{m}{a_4}} z_4).$$

The action is effective because $(*)$ is equivalent to $\frac{m}{a_2}, \frac{m}{a_3}, \frac{m}{a_4}$ being pairwise relatively prime. Hence, if $d > 1$ is a divisor of m there is at most one exponent $k \frac{m}{a_i}$, $2 \leq i \leq 4$, having d as a divisor too. Assume $d = \gcd(m, k \frac{m}{a_2}) > 1$. Then

$$M^{\mathbb{Z}/(d)} = \{(z_1, z_2) \in S^3 \mid z_1^k + z_2^{a_2} = 0\}$$

is the union of exceptional orbits and so $H^{\mathrm{odd}}_{S^1}(M^{\mathbb{Z}/(d)}) = 0$. The only component of M^Σ not being an exceptional orbit is

$$F := M^{\mathbb{Z}/(k)} = \{(z_2, z_3, z_4) \in S^5 \mid z_2^{a_2} + z_3^{a_3} + z_4^{a_4} = 0\}.$$

The induced action of $G := S^1/(\mathbb{Z}/(k))$ on F is free with orbit space F/G the orientable surface of genus $g = 1 - \frac{a_2 a_3 + a_2 a_4 + a_3 a_4 - a_2 a_3 a_4}{2m}$ (cf. [11, Lemma 7.2]). It follows that

$$\begin{aligned} H^*_G(F) &\cong H^*(F/G) \\ &\cong \mathbb{Z}[x_1, y_1, \ldots, x_g, y_g, t]/(x_i x_j, y_i y_j, x_i y_i - x_j y_j; x_i y_j \text{ for } i \neq j; t), \end{aligned}$$

with $\deg(x_i) = \deg(y_i) = 1$ for $1 \leq i \leq g$, $\deg(t) = 2$, and hence

$$H^*_{S^1}(F) \cong \mathbb{Z}[x_1, y_1, \ldots, x_g, y_g, t]/(x_i x_j, y_i y_j, x_i y_i - x_j y_j; x_i y_j \text{ for } i \neq j; k\,t).$$

Evaluating at $t = 1$ yields

$$(\mathbb{Z}/(k))^{2g} \cong H^{\mathrm{odd}}_{S^1}(F)_1 \cong H^{\mathrm{odd}}_{S^1}(M^\Sigma)_1 \cong H^{\mathrm{odd}}_{S^1}(M)_1$$

and thus $H^3(M) \cong (\mathbb{Z}/(k))^{2g}$ by Lemma 1.2. Finally, if M^* denotes the orbit space of the action of the subgroup $\mathbb{Z}/(k)$ and if $p : M \to M^*$ is the corresponding projection then $M^* \cong S^5$ (cf. [3, Thm. V. 9.4.]) implies $w_2(M) = p^* w_2(M^*) = 0$. □

2.2 Examples. *a) $g = 1$ if and only if $\frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4} = 1$. In this case the triple (a_2, a_3, a_4) is $(2,3,6)$ or $(2,4,4)$ or $(3,3,3)$.*
b) If $k \geq 2$ is not a multiple of 6 Lemma 2.1 asserts the existence of a fixed point free circle action on the simply connected 5-manifold M_k with $H^3(M_k) \cong \mathbb{Z}/(k) \oplus \mathbb{Z}/(k)$ and $w_2(M_k) = 0$. It follows that every irreducible (i.e. not splittable into a non-trivial connected sum) simply connected spin-5-manifold admits an S^1-action without fixed points.
c) For $k = 6$ the smallest g supplied by Lemma 2.1 results choosing $a_2 = a_3 = a_4 = 5$. In this case one obtains $g = 6$, i.e. a non-trivial S^1-action on $M_6^{\#6}$.

3 Fixed point free actions on 6-manifolds

Let η_4 denote the non-trivial S^4-bundle over S^2. A result due to R.Z. Goldstein and L. Lininger (cf. [8]) asserts that the manifolds

$$(S^3 \times S^3)^{\#r} \# (S^2 \times S^4)^{\#r-1} \qquad , r \geq 1,$$
$$\text{and} \quad (S^3 \times S^3)^{\#r} \# \eta_4 \# (S^2 \times S^4)^{\#r-2} \quad , r \geq 2,$$

are, up to difeomorphism, precisely all differentiable, closed, simply connected 6-manifolds with free cohomology admitting free S^1-actions. Using the classification theorems obtained by C.T.C. Wall, P.E. Jupp and A. Žubr (cf. [14], [10], [15]) this can be formulated as follows:

3.1 Theorem (Goldstein-Lininger). *Let M be a closed, differentiable, simply connected 6-manifold with free cohomology. M admits a free circle action if and only if all the Euler characteristic $\chi(M)$, the trilinear cup form $\mu(M)$ (cf. [14, Section 3]) and the first Pontryagin class $p_1(M) \in H^4(M)$ vanish.*

These conditions also characterize the manifolds which can be endowed with a *fixed point free* action:

3.2 Lemma. *If M is a closed, differentiable, simply connected 6-manifold admitting a topological, fixed point free circle action then*

$$\chi(M) = 0, \quad \mu(M) = 0, \quad p_1(M) \in \mathrm{Tors}_{\mathbb{Z}} H^4(M).$$

Proof. The first equation holds because the action has no fixed points. Let $X = M/S^1$ be the orbit space and $p : M \to X$ be the corresponding projection. We set

$$r := \frac{\mathrm{rk}\, H^3(M)}{2}, \qquad T := \mathrm{Tors}_{\mathbb{Z}} H^3(M) = \mathrm{Tors}_{\mathbb{Z}} H^4(M)$$

and consider the E_2-term

$$\begin{array}{lllll}
\mathbb{Z} & 0 & \mathbb{Z} & 0 & \mathbb{Z} \\
0 & 0 & 0 & 0 & 0 \\
\mathbb{Z}^{r-1} \oplus T & 0 & \mathbb{Z}^{r-1} \oplus T & 0 & \mathbb{Z}^{r-1} \oplus T \\
\mathbb{Z}^{2r} \oplus T & 0 & \mathbb{Z}^{2r} \oplus T & 0 & \mathbb{Z}^{2r} \oplus T \\
\mathbb{Z}^{r-1} & 0 & \mathbb{Z}^{r-1} & 0 & \mathbb{Z}^{r-1} \\
0 & 0 & 0 & 0 & 0 \\
\mathbb{Z} & 0 & \mathbb{Z} & 0 & \mathbb{Z}
\end{array}$$

of the spectral sequence corresponding to the fibration $M \to M_{S^1} \to BS^1$. One has

$$H^1(M_{S^1}) = 0 \quad \text{and} \quad H^2(M_{S^1}) \cong \mathbb{Z}^r.$$

Since M_{S^1} is a rational cohomology 5-manifold with $H^*(M_{S^1}; \mathbb{Q}) \cong H^*(X; \mathbb{Q})$ (see, e.g., [13, Proposition 3.1]) it follows that $H^4(M_{S^1}; \mathbb{Q}) = 0$ and so

$$\ker (d_2 : E_2^{0,4} \to E_2^{2,3}) \subset T.$$

If $a, b, c \in H^2(M) = E_2^{0,2}$ then

$$d_2(a \cup b) = d_2 a \cdot b + a \cdot d_2 b = 0$$

implies $a \cup b \in T$ and thus $a \cup b \cup c \in \text{Tors}_{\mathbb{Z}} H^6(M) = 0$. Hence, $\mu(M) = 0$. Finally, $H^4(X; \mathbb{Q}) = 0$ implies $p_1(M) = \pi^* p_1(X) \in \text{Tors}_{\mathbb{Z}} H^4(M)$. □

Theorem 3.1 and Lemma 3.2 prove:

3.3 Proposition. *For a differentiable, closed, simply connected 6-manifold with free cohomology the following are equivalent:*

(i) M *admits a free* S^1*-action.*

(ii) M *admits a fixed point free* S^1*-action.*

(iii) $\chi(M) = 0, \mu(M) = 0, p_1(M) = 0.$ □

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