3. Topological pumps and quasicrystals

Oded Zilberberg
Institute for Theoretical Physics, ETH Zurich, 8093 Zürich, Switzerland

Status
A pump consumes energy to perform work that moves a fluid. In topological pumps, an adiabatic and cyclic variation of a Hamiltonian potential moves particles a quantized and robust distance per pump cycle. Such a quantization is ideal for industrial applications, such as variable rate feeders, low-current electronics, slow-light, and spintronics, as well as a metrology standard [40, 84, 85]. Furthermore, topological pumps exhibit topological boundary effects that are potentially-useful as spectrally-protected multiplexors for optical classical and quantum gates [75], cf contribution on quantum optical effects in topological photonics.

Topology is a global property of a system, corresponding to an integral over local geometrical properties. In topological pumps, a spatial competition between length scales generates multiple spectral bands. These bands exhibit a non-vanishing curvature in response to the time-dependent modulation of the model, where the latter can be thought of as a synthetic dimension (see contrib. on synthetic dimensions in photonics). Averaging this time-local geometric property over the whole pump cycle leads to quantized transport; see figure 5 for an example.

Due to their potential applications, pumps were widely-explored in electronic mesoscopic systems, see e.g. [84, 85], and citations within. However, as these systems are inherently open to dissipation channels and diabatic corrections, quantization of the transport is challenging to obtain. As a result, only a few years ago, quantized topological pumping was observed for the first time in more controllable cold atoms experiments [88]. Similarly, only recent developments in photonic metamaterials allow for direct access to the topological boundary modes of the pump using input–output experiments [40, 87].
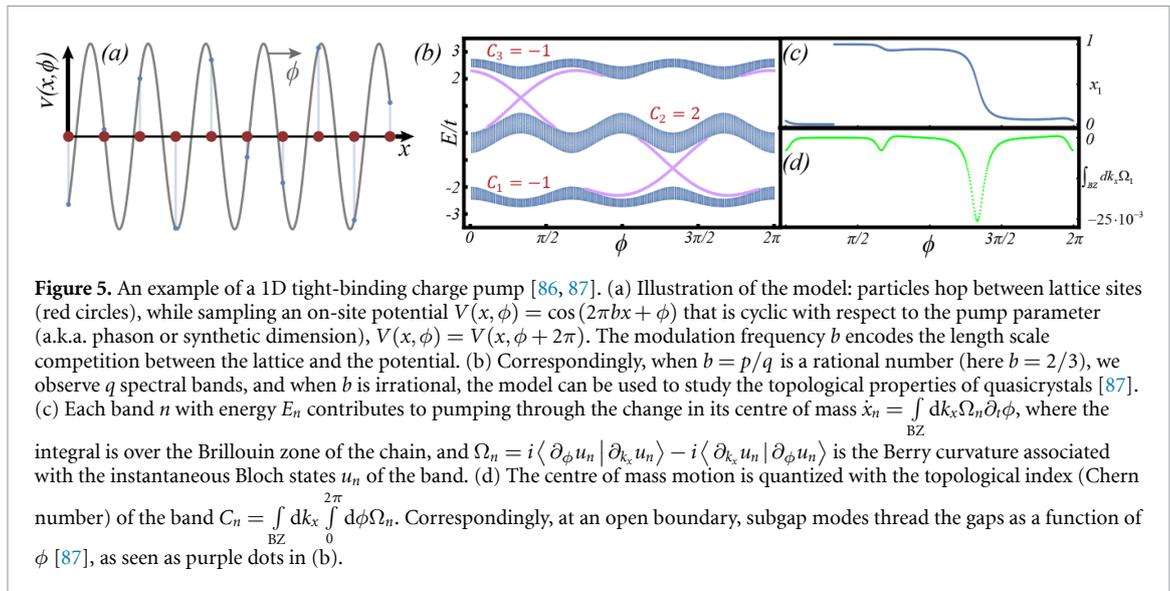
Current and future challenges
Similar to many other topological effects, topological pumps are best understood within a linear, adiabatic, and dissipationless limit. Contemporary challenges are four-fold and involve (a) extending the zoo of pump models and associated pumped quantities within the linear domain, e.g. by exploring topological pumping and their boundary phenomena in higher-dimensions [40, 89], or in new geometries that transfer quantities other than charge, such as spin or even parity [40]; (b) including realistic corrections such as non-Hermitian and diabatic effects to the pump [40, 90]; (c) moving to many-body topological pumps, by considering weak and strong nonlinearities [40]; and (d) harnessing topological pumps for true industrial applications.

We leave detailed discussion of the plethora of effects in (a) to longer reviews [40]. Similarly, the promise of new applications in (d) is omnipresent within the contemporary study of topological effects. Yet, a clear technological advantage is still missing. In the context of (b), which is explicitly relevant to topological photonics, diabatic corrections to topological pumping tend to destroy the topological character alongside the quantization of the pumped charge [84]. Furthermore, loss of particles due to dissipation leads to reduced transmitted power and fidelity of the pump for communication applications. Recently, in an experiment using coupled plasmonic waveguides, quantized pumping in a diabatic system was obtained using a time-dependent modulation of the dissipation. The latter is realized by changing the widths of the waveguides along the propagation axis of the plasmons. This experiment exemplifies the challenges and opportunities in (b), where non-Hermitian effects are harnessed to obtain quantized pumping.

Moving to many-body pumps (c), a wide variety of directions opens up in both fermionic and bosonic realizations. Considering more traditional electronic realizations of pumps, weakly-interacting systems are well understood as long as a non-degenerate ground state evolves along the pump-cycle without closing the many-body gap. At the same time, as topological pumps are intimately related to quantum Hall systems, where interesting fractional topological pumps are predicted to appear in strongly-interacting systems [91]. Moving to topological photonics and driven nonlinear photonic systems, the slow modulation of bifurcated steady-states can lead to novel pumping paradigms, where a new topological classification will be required.

Advances in science and technology to meet challenges
Technology is racing forward to couple linear and nonlinear elements in controllable quantum engineered systems. The controllability of such devices will allow us to realize the various unexplored scenarios described above. With tunable interactions in cold-atom setups, various strongly-correlated scenarios are within reach, where new topological pumping can occur. At the same time, the lifetime of excitations in such experiments is relatively short, such that quasi-adiabatic topological pumping will be challenging. The combined effort of inventing quantized diabatic pumps that can function at short times alongside constant development of longer-lived coherent platforms will push towards realization of more exotic pumps.

Figure 5. An example of a 1D tight-binding charge pump [86, 87]. (a) Illustration of the model: particles hop between lattice sites (red circles), while sampling an on-site potential $V(x,\phi) = \cos(2\pi bx + \phi)$ that is cyclic with respect to the pump parameter (a.k.a. phason or synthetic dimension), $V(x,\phi) = V(x, \phi + 2\pi)$. The modulation frequency b encodes the length scale competition between the lattice and the potential. (b) Correspondingly, when $b = p/q$ is a rational number (here $b = 2/3$), we observe q spectral bands, and when b is irrational, the model can be used to study the topological properties of quasicrystals [87]. (c) Each band n with energy E_n contributes to pumping through the change in its centre of mass $\dot{x}_n = \int_{BZ} dk_x \Omega_n \partial_t \phi$, where the integral is over the Brillouin zone of the chain, and $\Omega_n = i\langle \partial_\phi u_n | \partial_{k_x} u_n \rangle - i\langle \partial_{k_x} u_n | \partial_\phi u_n \rangle$ is the Berry curvature associated with the instantaneous Bloch states u_n of the band. (d) The centre of mass motion is quantized with the topological index (Chern number) of the band $C_n = \int_{BZ} dk_x \int_0^{2\pi} d\phi \, \Omega_n$. Correspondingly, at an open boundary, subgap modes thread the gaps as a function of ϕ [87], as seen as purple dots in (b).

Similarly, nonlinear optics devices with additional slow pumping can lead to new methods to guide light. Here, much remains to be understood concerning the required coherence for such protocols, their topological characterization, as well as, their potential for applications.

Concluding remarks

Topological pumps have only recently been experimentally realized, thus opening up a very active research topic, where many-body effects, time-dependent drives, and dissipation interplay to realize new topological effects in the bulk and at the surface of electronics and photonics systems.

Acknowledgments