

The 8th International Conference on Information Technology and Quantitative Management
(ITQM 2020 & 2021)

Efficiency linkages between cryptocurrencies, equities and commodities at different time frames

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Abstract

This paper examines pricing efficiency of cryptocurrencies and some traditional assets measuring the level of market efficiency with Adjusted Market Inefficiency Measure. The patterns of several cryptocurrencies' price dynamics over the last 4 years are compared with those of traditional assets. Correlation and mutual information matrices for AMIM are obtained using different estimation intervals. The results across different time scales are tested for noise using permutation entropy technique, empirical estimations are represented in statistical complexity plane to show the structure of efficiency links. Usage of AMIM in short window estimation is justified. Efficiency levels seem to be closely connected if judged from the standpoint of information theory at all time frames. Efficiency linkages become more linear at larger analysis periods. Cryptocurrencies seem to be more closely connected to equities, especially S&P500. Bursts of inefficiency on cryptocurrencies markets spread to equity markets and are possibly mediated in bank system. Commodities seem to be more independently priced.

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Peer-review under responsibility of the scientific committee of the The 8th International Conference on Information Technology and Quantitative Management (ITQM 2020 & 2021)

Keywords: Bitcoin; Cryptocurrencies; Pricing Efficiency; Adaptive Markets Hypothesis; Adjusted Market Inefficiency Measure; Permutation Entropy; Statistical Complexity Plane; Kullback-Leibler divergence; Efficiency Structure, Mutual Information

1. Introduction

The concept of market efficiency dates back to the works of Eugene Fama [1]. According to the EMH, the market can be inefficient, show weak, semi-strong and strong forms of efficiency. The EMH framework does not allow to develop instruments to track changes in efficiency level, as the EMH relies heavily on statistical tests as a mathematical tool. Statistical tests assess the state of the market during the estimated period by accepting or rejecting a Null Hypothesis. Thus, the market is assumed to be in a static state in any period under study, which does not allow to capture the evolving nature of pricing within the estimated time frame.

According to the EMH, weak-form efficiency means that the price cannot be predicted from past observations. Thus, price behavior can be described as a stochastic process. Empirical studies showed that even weak-form

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efficiency does not always hold [2],[3]. The overall validity of the EMH is still considered to be a hot topic.[4]

An alternative to the EMH was proposed by Andrew Lo [5],[6]. While deriving the Adaptive Markets Hypothesis, he considers the financial markets to exhibit adaptability to a constantly changing environment. Such an approach allowed Lo to narrow the bridge between economics and biology, and ecology in particular.

Potential of the AMH framework in applied field is particularly interesting in the context of crypto markets. Pricing process on crypto markets is less influenced by institutional factors and more by investors' preferences and their behavior. Therefore, there exists a stronger impact of behavioral patterns.

The behavior of cryptocurrencies has been studied thoroughly lately from different perspectives. First of all, [7] show that crypto market is independent of traditional assets and that there is no major factor that could explain Bitcoin behavior. Although other studies show that Bitcoin is somewhat similar to gold and other commodities, as it has negative correlations with more traditional financial assets [8],[9]. Other studies show that crypto-assets behave autonomously [10],[11].

As for the AMH studies and cryptocurrencies, Bitcoin pricing itself shows evolving efficiency [12]. The same authors [13] show that long memory in the volatility of intra-day bitcoin returns is adaptive. What is more, there is an adaptive pattern of long memory in the volatility of intra-day bitcoin returns [13]. This result leads to the conclusion that Bitcoin market behavior is consistent with the AMH. [14] applied martingale difference hypothesis to measure Ethereum and Bitcoin Efficiency. Their results are consistent with the AMH, showing varying and evolving nature of market efficiency. They also argue that intraday values may mask the shifts in efficiency. In an extensive study for Bitcoin and Ethereum markets using GLS time-varying AR model it was shown that AMH holds for those markets as well [15]. I can't help but mention studies done by [15] and [16]. Noda finds using GLS TV-AR model that efficiency evolves over time both for Bitcoin and Ethereum. The results are also compliant with [17] findings that markets are more efficient after the year 2013. The particular feature of Noda's method is that the results do not depend on sample size and inference is possible even with a small width of window. This study also uses the metric created by [18], they also later applied it to analyze cryptocurrencies [16]. They focus solely on cryptocurrencies and show that before 2017 five largest cryptocurrencies are mostly inefficient, with Ripple being the most efficient and Litecoin being the least efficient between five cryptocurrencies covered. As they use longer time frame (compared to similar previous studies) and more robust measure of inefficiency – they find it that approximate efficiency was not reached in 2013.

Under the AMH, the pricing patterns of cryptocurrencies should have impact on the efficiency of more conventional assets and vice versa. To the best of my knowledge, there were few attempts to check how efficiency levels of different assets affect each other.

What is more, shorter time frames are often neglected for better finite sample approximations to asymptotics. Time frame issue is even of greater interest in the light of the study by [19], where they find out that Bitcoin markets are efficient at 1-minute time frame but are inefficient at 5 minutes time frame. There is a need to examine the time structure of efficiency levels of different assets, as our current understanding remains vague.

This study mainly focuses on efficiency relations between traditional assets and cryptocurrencies at different time scales. I seek to capture the mutual dynamics of cryptocurrencies' and more traditional assets' pricing patterns by assessing their efficiency levels. This study contributes to the existing literature in several ways.

First, as the time structure of efficiency levels of various assets and their connections to each remains unobvious, several time intervals are used for comparison. That helps provide deeper insight into the time structure of mutual efficiency swings. What is more, several studies show that effectiveness of diversification using crypto-assets depends largely on the time-interval of having the cryptocurrency in the portfolio [20],[21],[22]. Very short-term estimates are not guaranteed to be free of noise – thus, there is a need to address the issue of noise in estimates.

Second, I adhere to the AMH framework that enables to capture evolutionary nature of pricing process. The AMH is capable of capturing behavioral patterns in pricing process. Also, it enables to see the shifts in efficiency levels. And I am most interested in seeing the relations between these shifts. Information processing instruments from information theory are applied to strengthen the results of quantitative analysis and to address the issue of noise in the estimates. These tools such as Permutation Entropy, Statistical Complexity Plane, and Mutual Information have already proved themselves worthy in studies of various types of signals starting from signals in physical

processes to signals in complex biological environments and financial markets.

Third, [16] were the first ones to introduce Adjusted Market Inefficiency Measure (AMIM) to cryptocurrencies (this metric seems to be free of flaws inherent in prior metrics, to be discussed later) but they focused mainly on crypto market. Compared to their work, in this study a broader dataset is used, which also captures more traditional assets. Thus, the aim is to study the structure of dependence between efficiency levels, and hopefully, find some patterns even at different time.

Fourth, AMIM is a promising tool, an attempt is made to study its behavior in small sample using heterogeneous empirical data. To be exact, I try to understand how informative this measure is when the data is scarce due to shorter estimation window. I am answering this question using above mentioned instruments from Information Theory.

Last, the dataset contains ten different assets of heterogeneous nature. The dataset also captures the period of rallying prices for the cryptocurrency market, which I hope, will help us shed more light on efficiency relations between assets even in extreme environment. Combined with already mentioned different time frames this will aid in achieving broader understanding of the pricing process and its efficiency between different assets.

2. Methodology

The method used in the study is based largely on the paper by Leirvik and Le Tran [18]. Adjusted Market Inefficiency Measure is a metric that is both easy to construct and implement to study the dynamics of different assets. By the end of the day, I have rolling windows of 16, 31, 61, 91, 121, 181 trade days applied to the dataset. Therefore, I have 1014, 999, 969, 939, 909, 849 estimations of AMIM for each time interval length

There are some problems inherent in estimation with very small window width. First one is the convergence of AMIM to normal distribution – it is not guaranteed in a small sample. As a result, variance-covariance matrix may be less stable. This will result in a greater impact of noise in the resulting estimates. What is more, as AIC is not a consistent model selection criterion, it will probably choose a richer model even if the sample is relatively large. This, in turn, implies more irrelevant variables included in the underlying AR(q) process, which means higher q. Therefore, there is a need in a very robust method of noise processing.

I examine the resulting estimates using Permutation Entropy and Mutual Information tools. Having the Entropy estimates, I can also build a representation of my time series in a Statistical Complexity Plane. Last, but not least, I refer to correlation matrices and mutual information to study the linkages between efficiency levels of different assets. Mutual information is measured in units of bits. Since $I(X;X)$ is the maximum information in bits the series contains, I can build a relative measure – $I(X;Y)/I(X;X)$ which tells how much % of information about X one knows by observing Y.

3. Data

My dataset contains 1031 observations of daily prices for Bitcoin, Ethereum, Litecoin, Ripple, Standard&Poor's 500, Morgan Stanley Capital International World Index, 1-month futures on Brent oil traded on ICE, 1-month futures for electrical energy traded on NYMEX, and also 1-month futures for gold and silver traded on COMEX. The range of data is from the Thirty first of July 2015 till the ninetieth of July 2019. Data includes observations of closing prices during trade days both for crypto and traditional assets. This coincidence of observations enhances comparability of the data. The AR model is then applied to series of daily price return p , where $p = \ln(r_{t+1}/r_t)$. It means that there are 1030 observations of p for each asset.

4. Results and Discussion

Behavior of AMIM depends mainly on the length of rolling window. It can be seen, that even at 181 trade days window there exist leaps that denote periods when the pricing is not efficient. In other words, leaps on the plot denote price trends caught by the underlying AR model.

Estimation with larger window is more reliable, alas, it does not capture the short-term price trends that may

exist. To capture the short-term price trends, I use shorter windows. Estimations via shorter windows are more exposed to noise, now I am trying to address this issue via the methods of information theory. The noisier the resulting estimates – the less information can be retrieved. Let us first examine mutual information, $I(X;X)$ for 10 different assets:

Table 1

Comparison of mutual information and permutation entropy at different time frames

	Bitcoin	Ethereum	Litecoin	Ripple	SP500	MSCI	Brent	Energy	Gold	Silver
$I(X;X)$ at 16 days, bits	3.918	3.918	4.002	4.137	4.085	3.772	4.054	3.991	3.989	3.970
$I(X;X)$ at 181 days, bits	4.220	4.168	4.335	4.335	4.418	4.137	4.137	4.137	4.252	4.220
Δ in bits	0.302	0.250	0.333	0.198	0.333	0.365	0.083	0.146	0.262	0.250
% change	7.70%	6.38%	8.33%	4.79%	8.16%	9.67%	2.06%	3.66%	6.57%	6.30%
PE at 16 days	0.991	0.991	0.987	0.991	0.987	0.994	0.993	0.992	0.984	0.988
PE at 181 days	0.933	0.951	0.916	0.927	0.926	0.956	0.934	0.939	0.949	0.936

The table shows that on average I am winning 0.252 bits when going from 16 to 181 trade days window. This denotes ~6.36% information gain on average from the estimates at 16 days window. Noise indeed is present in the estimates and it for sure lowers the amount of information one can extract, but information loss is not that severe in terms of bits. Permutation Entropy calculations confirm these findings. Estimates in narrow window are exposed to noise and are very random (values close to 1), but they tend to be lower for longer windows, still, absolute difference is not that notable, though the “long” estimates are more deterministic.

Table 2

Values for Statistical Complexity Measure at different time frames

	Bitcoin	Ethereum	Litecoin	Ripple	SP500	MSCI	Brent	Energy	Gold	Silver
SCM at 16 days	0.041	0.039	0.058	0.041	0.059	0.027	0.031	0.035	0.070	0.056
SCM at 181 days	0.288	0.213	0.352	0.309	0.312	0.193	0.284	0.263	0.224	0.274

It is interesting to see, that even at very short windows resulting estimates are not purely random, they show some degree of structure, what is more, all confidence intervals do not reach zero bound, meaning that all the estimates are significantly different from zero even at 16 days trade window.

Once I switch from 16 days window estimation to 181 days – the signal from AMIM gets more deterministic, less exposed to noise. Mutual information calculations show that it starts to carry more information. It is interesting to note, that MSCI efficiency remains to be the most random at 16 and 181 trade days window. Probably, that is due to its broader scope. Randomness of estimates measured by PE and SCM grows approximately linearly with the window length increase. At least, that is what happens at the studied time frame.

Calculations for AMIM result in 6 time series for each of ten assets at different time scales. Correlation matrices for efficiency measure can be applied to the data. AMIM distribution approaches that of normal distribution due to the standardization procedure in larger window estimations. I hope that calculations above may have convinced you that analysis of correlation matrices for 16 days frame may be as meaningful as for 181 days, at least, information loss is not that severe.

At 16 days rolling window one can see that there is no significant correlation between assets. At the same time, as the Ethereum example shows, the shorter the window length the more short-term trends there are.

At 181 trade days window assets show increased correlation between their pricing efficiencies. One can note that commodities efficiency is negatively correlated with that of both traditional and crypto-assets. One exception is the behavior of Ripple. Its inefficiency is relatively strongly correlated with that of gold futures. It is also negatively correlated with MSCI efficiency and Bitcoin. Other crypto assets behavior coincides with the price movements of SP500 and MSCI except for Ethereum. It is negatively correlated with MSCI. All in all, one can see the cryptocurrencies' efficiency is stronger correlated with equity indices, than with commodity futures. This finding supports the point of view expressed by [9] that Bitcoin does not reflect distinctive properties of gold and cannot act

as a hedge against downward market trends.

Table 3

Correlation matrix for AMIM at 16 days rolling window

	Bitcoin	Ethereum	Litecoin	Ripple	SP500	MSCI	Brent	Energy	Gold	Silver
Bitcoin	1	-0,005	0,030	-0,007	0,027	-0,041	0,040	0,007	0,029	-0,004
Ethereum		1	0,093	0,087	0,073	0,052	-0,006	0,004	-0,019	-0,031
Litecoin			1	0,068	-0,003	-0,045	0,015	-0,041	0,002	0,002
Ripple				1	0,081	-0,021	-0,049	-0,059	0,091	-0,011
SP500					1	0,00	0,028	-0,025	0,007	-0,027
MSCI						1	0,013	0,037	0,065	-0,016
Brent							1	0,009	-0,018	0,007
Energy								1	-0,002	0,050
Gold									1	0,067
Silver										1

Table 4

Correlation matrix for AMIM at 181 days rolling window

	Bitcoin	Ethereum	Litecoin	Ripple	SP500	MSCI	Brent	Energy	Gold	Silver
Bitcoin	1	0,128	0,217	-0,122	0,159	0,222	-0,034	0,008	-0,087	0,099
Ethereum		1	0,224	0,121	-0,354	0,206	-0,186	-0,351	-0,057	-0,163
Litecoin			1	-0,025	0,150	0,327	-0,243	-0,307	-0,366	-0,336
Ripple				1	0,152	-0,154	-0,311	-0,321	0,359	-0,172
SP500					1	0,324	-0,300	-0,116	-0,093	-0,082
MSCI						1	-0,180	-0,284	-0,446	-0,212
Brent							1	0,408	-0,097	0,288
Energy								1	-0,024	0,391
Gold									1	0,093
Silver										1

At the same time, I can study the mutual information matrices for the same assets (matrices will also be symmetric due to symmetry property of mutual information). One can see that behavior of cryptocurrencies is more driven by the S&P500 movements, rather than by MSCI price changes. Efficiency levels of Brent and Energy are least useful in the analysis of cryptocurrencies.

Table 5

Relative Mutual Information for AMIM at 16 days rolling window, %

	Bitcoin	Ethereum	Litecoin	Ripple	SP500	MSCI	Brent	Energy	Gold	Silver
Bitcoin		82.99%	87.24%	88.56%	87.24%	79.25%	86.44%	88.56%	91.49%	84.31%
Ethereum			87.24%	88.56%	89.37%	79.25%	88.56%	88.56%	91.49%	84.31%
Litecoin				88.80%	87.51%	81.77%	86.72%	88.80%	91.67%	84.64%
Ripple					87.91%	82.36%	89.17%	89.17%	91.94%	85.14%
SP500						80.10%	86.99%	89.03%	91.84%	84.95%
MSCI							85.91%	88.12%	91.16%	83.70%
Brent								88.94%	91.78%	86.89%
Energy									95.30%	88.25%
Gold										91.16%
Silver										

Table 6

Relative Mutual Information for AMIM at 181 days rolling window, %

	Bitcoin	Ethereum	Litecoin	Ripple	SP500	MSCI	Brent	Energy	Gold	Silver
Bitcoin		90.13%	94.08%	94.08%	96.05%	91.36%	89.38%	89.38%	94.08%	91.36%
Ethereum			96.00%	94.00%	96.00%	89.25%	89.25%	89.25%	96.00%	93.25%
Litecoin				94.23%	96.16%	91.59%	89.66%	89.66%	94.23%	91.59%
Ripple					96.16%	89.66%	89.66%	89.66%	92.31%	91.59%
SP500						89.86%	89.86%	89.86%	92.46%	91.74%
MSCI							91.18%	89.17%	91.94%	91.18%
Brent								91.18%	91.94%	93.20%
Energy									91.94%	91.18%
Gold										93.38%
Silver										

Stock indices are more closely connected to cryptocurrencies rather than to commodities, at least from the standpoint of mutual information. The picture is contrary if one studies correlation matrices. S&P500 efficiency is correlated with Brent efficiency level, while MSCI efficiency is correlated with efficiency of gold. Possible explanation is that correlation is just a measure of linear dependence, maybe, there is a more intricate non-linear connection between cryptocurrencies and stock indices at shorter time span.

Mutual information and correlation matrices point at closer connection of cryptocurrencies with other stock assets, rather than between different cryptocurrencies. It is observable that efficiency pattern of cryptocurrencies is similar between crypto-assets. Equity indices are effectively priced at considerable time periods. Commodities do not reflect major movements of other assets. I see that at the beginning of 2018 equity indices and cryptocurrencies are not adequately priced at the same period. Plots illustrate the evolution of pricing efficiency

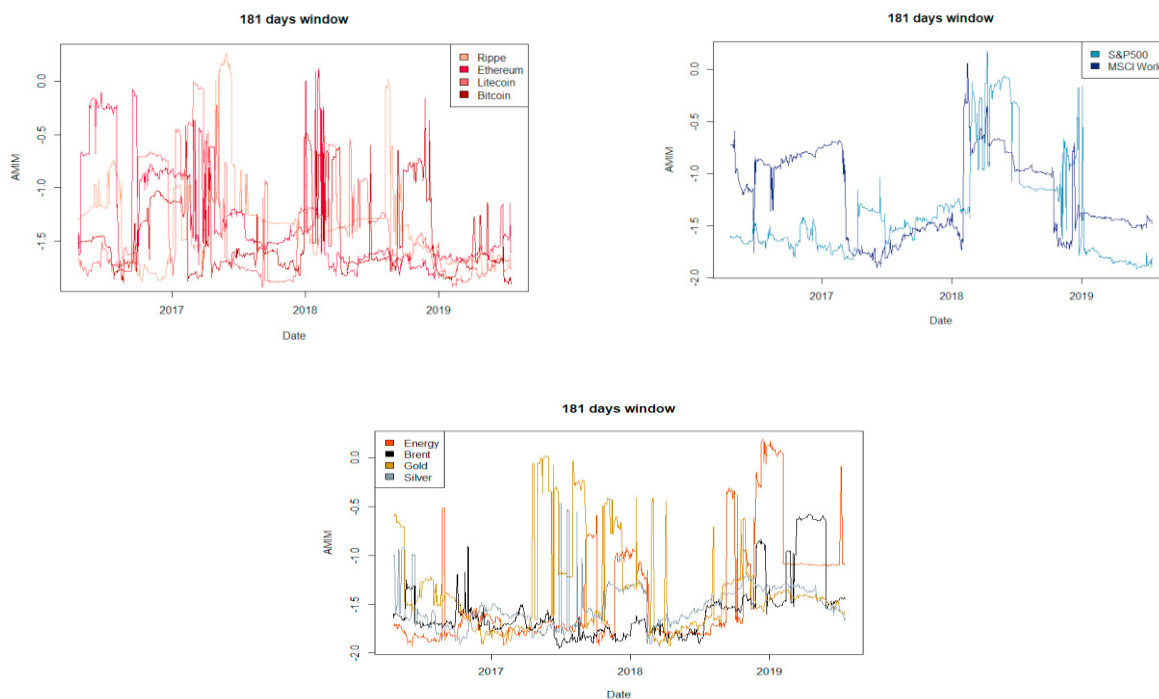


Fig. 1. AMIM for 10 assets at 181 days rolling window

The leaps in efficiency level of commodity assets seem to be disconnected from crypto and stock market fluctuations. At the beginning of 2018 one can observe a burst in inefficiency on crypto-market – which later spread

to stock market. Then prices for cryptocurrencies tend to be more efficient by the end of 2018, which is compliant with the findings by [16]. Leirvik and Le Tran use Moving Average approach and 1-year rolling window, but their estimates also capture inefficiency of Ripple in second quarter of 2018. After all, Ripple is used in banking transactions, possibly, inefficient pricing of Q1 2018 on crypto and stock markets was later somehow mediated by the banking system.

This inefficiency leap at the beginning of 2018 captures the period of rallying prices for crypto assets. Possible explanation for this pattern between markets is that higher returns on crypto market made investors withdraw their money from stock market in search of higher profits, causing inadequate price fluctuations on stock market. This is a nice feature of AMIM – we do not know the drivers of inadequate pricing, the reasons behind it, but we can see how this inefficiency is transmitted between different markets.

Analysis of mutual information and correlation matrices reveals a piece of puzzle about structure of efficiency swings. At shorter time frames assets are less correlated, they are also less entangled from the standpoint of mutual information, but the difference is way less dramatic.

A hypothesis may be stated, based on my analysis of mutual information and correlation matrices, that the structure of connections between efficiency of assets is less linear at shorter time frames and becomes more linear at longer time spans, whereas the level of connection estimated by mutual information does not differ much. This fact should be accounted for while actively managing a portfolio.

5. Conclusion

This study contributes to the existing literature on crypto assets' market efficiency in several ways. First of all, one of the most advanced metrics developed under the Adaptive Markets Hypothesis was applied to ten assets of different nature. The metric used offers a way to smoothly compare efficiency levels. Secondly, scope of the analysis went beyond crypto market, assessing efficiency links between crypto, stock and commodity markets. Thirdly, the analysis performed captures different time spans. Before describing the results of the analysis – the issue of noise in shorter window estimation is addressed. Using non-parametric and robust tools of information theory and statistical mechanics (that have already been used successfully in financial market studies) I show that even in small samples Adjusted Market Inefficiency Measure still captures meaningful phenomena, at least judged by the analysis of information in bits between estimates and by the analysis in Statistical Complexity Plane. These properties of AMIM have not yet been thoroughly studied. An interesting result is shown, that the degree of non-randomness shown by Statistical Complexity Measure for AMIM estimates is linearly dependent on the length of the rolling window, at least this is what happens in the studied time frame. What is more, I try to shed more light on overall structure of efficiency swings between different assets. To understand this structure matrices of relative mutual information and correlation matrices are built.

Evidence opens a way to express a hypothesis: efficiency levels are connected less linearly at shorter time frames, but still somehow entangled as shown by mutual information calculations, efficiency level connections become more linear at longer time spans. This seems to be compliant with Adaptive Markets Hypothesis. At shorter time periods, different groups of investors struggle for exploiting arbitrage opportunities, making market more efficient in the long run. This claim is supported by numerical estimations and thorough quantitative analysis.

Calculations show that crypto-assets are more connected with stock assets, rather than between each other. What is more, price swings between crypto and stock assets tend to be more intertwined. Ripple is an exception and seems to be closely connected to gold, this is probably due to its usage in banking transactions. Commodity assets appear to stay more isolated from stock and crypto-assets. This can be seen from the analysis of mutual information, correlation matrices and the different pattern of time series behavior.

Acknowledgements

The author would like to thank Mikhail Stolbov (MGIMO University) for offering his valuable advice. Author also would like to thank participants of Quantitative Finance Workshop 2020 and reviewers for the valuable feedback.

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